Relativistic theory of the evolution of nucleon structure in nuclei

V. V. Burov, A. V. Molochkov, and G. I. Smirnov
Joint Institute for Nuclear Research, Dubna

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The experimental and theoretical studies of lepton deep-inelastic scattering on nuclear targets are reviewed. The main problems which must be solved in analyzing the x and A dependences of the deep-inelastic cross sections are outlined. Several of the models proposed for studying the contribution of nuclear effects to the structure functions $F_2(x)$ are analyzed. A relativistic field-theoretical approach to studying lepton deep-inelastic scattering on very light nuclei is developed. The modifications of the nucleon structure in D, ^{3}H, ^{3}He, and ^{4}He are studied for the first time within a unified approach. It is found that the modifications evolve in a manner completely different from that observed earlier for heavy nuclei. In particular, it is found that the pattern of $F_2(x)$ modifications represented in terms of the ratios $F_2^A/F_2^{N(D)}$ is determined by the values $(1-x_3)=0.32$ (D/N), 0.16 (^{3}He/D), and 0.08 (^{4}He/D). These results allow the definition of the class of modifications of the bound-nucleon structure and the introduction of the unit of nucleon-structure modification. Theoretical justification of the concept of two-stage evolution of the nucleon structure as a function of A, the first stage occurring for $A \leq 4$ and the second for $A > 4$, is also obtained. The long-standing problem of the nature of the EMC effect is explained as a modification of the nucleon structure in the field of the nuclear forces in a three-nucleon bound system. © *1999 American Institute of Physics.* [S1063-7796(99)00106-0]

1. INTRODUCTION

The proton and neutron differ most significantly from all the other representatives of the large family of hadrons in that their lifetimes in the free state are very long: ~890 sec for the neutron and practically infinity for the proton. They are therefore our principal source of experimental information about the structure of strongly interacting particles. The generic name for these two particles is the nucleon, and they are viewed as the constituents of the smallest representative of the macroscopic world, the atomic nucleus. Nucleons are stable inside the nucleus, which explains the extensive use of nuclear targets in high-energy physics experiments. The deep-inelastic scattering (DES) of leptons on free protons and on the nucleons of a nucleus A has led to the discovery of the nucleon constituents, quarks, and has become a fundamental experimental tool for studying hadron structure.

In the scattering process

$$l+A \rightarrow l'+X, \tag{1}$$

a lepton l with momentum k is scattered on a nucleus A with initial four-momentum P and large four-momentum transfer $q=k-k'$. Inclusive experiments on DES record only the final lepton with momentum k', and X is used to denote the unobserved final hadron state of the reaction. In the lowest order in the electromagnetic coupling constant $\alpha=e^2/4\pi$ this process can be depicted schematically by a one-photon exchange graph, as in Fig. 1.

In this approximation, the cross section for the reaction (1) can be written as a contraction of hadron and lepton tensors:[1]

$$d\bar{\sigma} \propto \frac{\alpha^2}{q^4} L^{\mu\nu}(k,k') W_{\mu\nu}(P,q). \tag{2}$$

The lepton tensor $L^{\mu\nu}$ describes lepton emission by a hard photon. Since the lepton is assumed to be a point particle, the expression for $L^{\mu\nu}$ takes the simple form

$$L_{\mu\nu}(k,k') = \frac{1}{2} \sum_{s'} \bar{u}^{s'}(k') \gamma_\mu u^s(k) \bar{u}^s(k) \gamma_\mu u^{s'}(k'). \tag{3}$$

All the information about the target and its nuclear properties is contained in the hadron tensor, which has the form

$$W_{\mu\nu}(P,q) = \frac{1}{2} \sum_n \langle P|J_\mu^+|n\rangle \langle n|J_\nu|P\rangle (2\pi)^4 \times \delta^4(P+q-p_n). \tag{4}$$

This definition allows the hadron tensor of the nucleus to be related to the amplitude for forward Compton scattering by means of the unitarity relation:

$$W_{\mu\nu}(P,q) = \frac{1}{2\pi} \operatorname{Im} T_{\mu\nu}(P,q). \tag{5}$$

In the case of electron (muon) scattering on an unpolarized target, the tensor $W_{\mu\nu}$ can be written most generally as

$$W^{\mu\nu}(P,q) = W_1(\nu,q^2) g^{\mu\nu} + \frac{W_2(\nu,q^2)}{M^2} P^\mu P^\nu + \frac{W_4(\nu,q^2)}{M^2} q^\mu q^\nu + \frac{W_5(\nu,q^2)}{M^2} (P^\mu q^\nu + q^\mu P^\nu), \tag{6}$$

where $\nu=q_0$ is the photon energy and the W_i are the target structure functions. Owing to the condition for gauge invariance

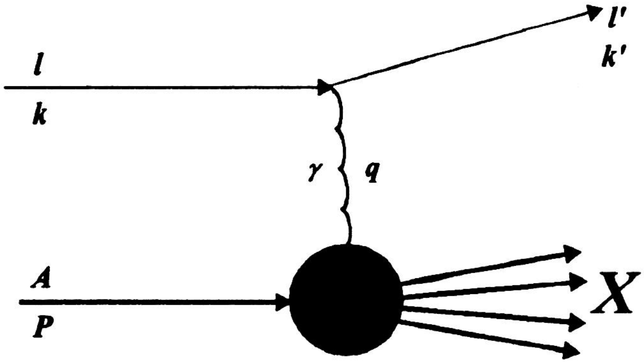

FIG. 1. Graph schematically depicting deep-inelastic scattering in the one-photon approximation.

$$q_\mu W^{\mu\nu}(P,q)=0, \tag{7}$$

the hadron tensor depends only on two structure functions:

$$W_{\mu\nu}(P,q)=W_1(\nu,q^2)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{W_2(\nu,q^2)}{M^2}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\times\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right).$$

In the Bjorken limit ($-q^2=Q^2\to\infty$, $\nu\to\infty$) the condition for scale invariance is realized, and so it is possible to change over to structure functions independent of q^2:

$$MW_1(\nu,q^2)\to F_1(x),$$
$$\nu W_2(\nu,q^2)\to F_2(x), \tag{8}$$

where F_1 and F_2 are scale-invariant structure functions (SFs), and $x=-q^2/(P\cdot q)$ is a new scale-invariant variable, sometimes called the Bjorken x variable. Using (8), $W_{\mu\nu}$ can be written as

$$W_{\mu\nu}(P,q)=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)F_1(x)+\frac{1}{P\cdot q}\times\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right)F_2(x). \tag{9}$$

The pioneering experiments in the late 1960s at SLAC, in which the scale invariance of the nucleon structure functions was discovered, led to the development of quark–parton models of hadrons. Ten years later a new generation of experiments confirmed the predicted logarithmic violation of scale invariance, which proved fundamental for the development of quantum chromodynamics (QCD) as the theory of strong interactions. Further studies of the isospin asymmetry in DES and experiments using polarized targets and incident particles showed that the ideas about the spin and isospin structure of the nucleon current at that time did not agree with the experimental results.[2] A possible interpretation involved the assumption that the isospin symmetry of the quarks in the nucleon is violated. It became obvious that the real picture of the internal nucleon structure is much more complicated than that assumed in the quark–parton model. This stimulated interest in precision tests of the validity of the Gottfried,[3] Ellis–Jaffe,[4] and Bjorken[5] sum rules (SRs).

The binding of the nucleon constituents (quarks and gluons) possesses the unique feature that its structure is preserved at any of the energy transfers accessible at present. It is not excluded that the nonemission of quarks from inside the nucleon is a fundamental property of the nucleon which is preserved at higher energies. This phenomenon, which is known as *confinement*, makes it impossible to study nucleon structure directly, independently of any model representations. The details of the structure hidden from the observer can be compared to the contents of a *black box*, the standard methods of studying which consist of studying the dependence of the response of the object to various perturbations. By making various assumptions about the internal structure of the nucleon, it becomes possible to arrive at a model which is consistent with the dependence obtained experimentally. However, under the conditions of an actual experiment it is impossible to test the initial (model) assumptions in the entire range of kinematical variables. This statement is also true of the most accessible nucleon target, a proton target, and, to an even greater degree, a neutron target, for which deuterium targets are used. In the latter case it is simply impossible to obtain information about the neutron structure from the experimental data without model assumptions about the effects of the proton and neutron binding in the deuteron. In other words, it turns out that the experimental study of the nucleon structure requires as accurate as possible *a priori* information about this structure.

These problems were manifested most acutely just when it became possible to make a precision comparison of the deuteron structure and the structure of a nucleon bound in a heavy nucleus. It was traditionally assumed that not only protons but also nuclei can be used as nucleon targets in DES, as long as the latter are treated as a set of quasifree nucleons. The neglect of nuclear effects was based on the assumption that the binding energy in the nucleus can be neglected in comparison with the energy transfer from the lepton to the nucleus, so that the nucleon and the nucleus can be viewed as systems isomorphic to each other. The incorrectness of this idea was demonstrated by the EMC (European Muon Collaboration) experiment, which measured the ratio of the SF of iron $F_2^{\mathrm{Fe}}(x)$ and that of the deuteron $F_2^{\mathrm{D}}(x)$ (Ref. 6). The EMC experiment showed that the structure of nucleons bound in the nucleus is changed in a nontrivial manner: at intermediate values of x, the SF of the free nucleon (a deuteron in the EMC experiment) proved to be considerably larger than $F_2^{\mathrm{Fe}}(x)$. First of all, this modification of the SF contradicted traditional nuclear-structure models. Second, the nature of the variation of $F_2^{\mathrm{Fe}}(x)$ as a function of x was only in qualitative agreement with the ideas about nuclear binding effects. It proved impossible to obtain quantitative agreement with the data, no matter what model was used to describe the effect.[7–9] The observed modification of the SF of a free nucleon inside the nucleus was called the *EMC effect*.

A large number of models and multiple variations of them have been devised in order to reconstruct the pattern of modifications of the structure of a free nucleon by a nuclear medium. They are discussed in several reviews (see, for example, Refs. 8 and 9). These models fall into three basic groups:

(a) explanation of the EMC effect by taking into account the nucleon separation energy, the relativistic Fermi motion, and nucleon–nucleon correlations;[10,11]

(b) the hypothesis that the EMC effect is related to nonnucleon degrees of freedom;[12–17]

(c) the suggestion that the quark confinement radius changes.[18–20]

In spite of the qualitative differences between these approaches to explaining the nature of the effect, many of them reproduced the general pattern of the effect, and some even satisfactorily described the experimental data in a limited kinematical region (as a rule, for $0.3<x<0.7$). Agreement was obtained at the price of introducing fitted parameters. Nevertheless, attempts to describe the experimental data simultaneously in a wider range of x and for various nuclei proved unsuccessful. Naturally, given this situation, it was impossible to reject categorically many of the models and decide upon a unique mechanism for the EMC effect.

Improvement of the model description of the effect continues to this day. Of the recent publications, it is worth mentioning an approach which proposes an additional QCD evolution,[21] a model of phenomenological double Q^2 rescaling,[22] and a model which deals with the EMC effect in the range $10^{-3}<x<0.7$ by parametrizing the nuclear binding and swelling of the nucleon.[23] A parton model for describing the distribution of sea quarks and gluons in nuclei was proposed in Ref. 24. In addition to studying the EMC effect in heavy nuclei, that study considered the parton distributions in the deuteron, and a relation between the deuteron and neutron SFs was obtained.

A method which is largely an alternative to all the other approaches was developed in Refs. 25 and 26. In order to avoid the problems of theoretically describing the nuclear structure for a finite value of the atomic number A, the authors of Ref. 26 studied the modification of the nucleon structure in infinite nuclear matter. The cross section for the reaction (1) in nuclear matter can be estimated by extrapolating the cross sections measured for a number of finite values of A to the region $A\to\infty$, assuming an $A^{-1/3}$ dependence.[25]

We think that these new studies can be viewed as a good argument for the importance of solving the problem of finding a model-free description of the modification of the structure of the bound nucleon in relation to that of the free nucleon.

The wealth of experimental data which has by now been obtained by the various nuclear collaborations SLAC, BCDMS, NMC, and E665 allows the explanation of a number of fundamental regularities in the x and A dependences of the ratios of the structure functions of heavy nuclei and the deuteron. The most important result from our point of view is the discovery of a universal x dependence (the same for all nuclei with mass $A\geqslant 4$) of the ratio $r^A(x)=F_2^A(x)/F_2^D(x)$, first in a limited range of the variable x,[27] and later in the entire range accessible to current experiments, $10^{-3}<x<0.95$ (Refs. 28–30). The main feature of the discovered regularities is the fact that the three points x_i ($i=1,2,3$) at which $F_2^A(x_i)=F_2^D(x_i)$ are independent of A. However, this result was used only as a technical method of taking into account the A dependence, and not as the key to understanding the nature of the EMC effect. It is therefore natural that not only did the proposed models not reproduce this regularity, but they even failed to describe the current experimental data with a reasonable degree of reliability. Meanwhile, the experimental information already accumulated on lepton DES on nuclei for $A\geqslant 4$ convincingly shows that the binding of the nucleons in the nucleus, and also the saturation of nuclear binding effects for $A\to 4$, are the main mechanisms modifying the structure of the free nucleon.[29,30] It should be noted that numerous theoretical studies emphasize the importance of taking into account the nucleon binding mechanism in the nucleus.[11,31,32] However, they do not attach enough importance to the effect of saturation of the binding forces in few-nucleon systems.

It seems to us that the problem of describing the EMC effect is most often reduced to a search for the necessary conditions ensuring that the ratio $F_2^A(x)/F_2^D(x)$ is different from unity. Here there is extensive use of both little-studied subnucleon degrees of freedom[13,15,18–20] and exotic nuclear constituents,[14,33–40] and of the nucleon–nucleon interaction model (Refs. 11, 12, 31, 32, and 41).[1)] As a result, the success in describing the data and, consequently, understanding the evolution of $F_2(x)$ in a nuclear medium have depended to a large degree on the accuracy of describing the nucleon structure and the nucleon–nucleon interaction.

In order to break this closed cycle, it is necessary to reconstruct the pattern of relative changes in the structure of the bound nucleon on the basis of the general properties of the nucleon and the nucleon–nucleon interaction, which can be obtained from the properties of symmetry and analyticity. On the other hand, the fact that a nucleon located in a nucleus is changed leads to additional possibilities for the theoretical study of nucleon structure and the determination of some of the nucleon properties on the basis of the most general principles.

It is our conviction that the key problem is the systematic relativistic description of nuclear binding effects. The most suitable object for studying the basic effects is the simplest nuclear system: the deuteron. As will be shown below for the example of calculating $F_2^D(x)$, the relativistically covariant approach[42] allows the nuclear effects observed in DES to be explained within a unified physical picture admitting a relatively simple interpretation. The relativistic expressions for the n-nucleon Green function and for the vertex function describing the nuclear state in terms of nucleon degrees of freedom determine a relation between the dynamical structure of the nucleon and the time interval Δt separating the observed nucleon and the residual nucleus (a nucleon in the case of a deuterium target)—the nonsimultaneity of the nucleons in the nucleus. The explanation amounts to attributing the modification of the structure of a nucleon bound in the deuterium nucleus $[F_2^D(x)/F_2^N(x)\neq 1]$ to the nonsimultaneity of the nucleons in the nucleus. This theoretical result makes it easy to demonstrate the relation to the nonrelativistic approach, in particular, how the rescaling of the Bjorken variable x follows from the dependence on the relative time contained in the Compton amplitude of the off-shell nucleon.

Generalization of the result to light nuclei ($A \leqslant 4$) allows study of the evolution of $F_2(x)$ with varying A and establishment of the *sufficient* condition for modifications of the effect of off-shell nucleon deformation, which for $A>4$ is the EMC effect, to be manifested. Thanks to the calculations performed for $A \leqslant 4$ (Refs. 43 and 44), the principal differences in the evolution of the nucleon structure in the lightest nuclei and in a nuclear medium with $A>4$ have been established. The concept of two types of evolution allows the results to be easily generalized to heavy nuclei and makes it possible to describe practically all the existing experimental data on DES on nuclei.[30]

Moreover, the studies which have been carried out make it possible to obtain a number of general consequences for the QCD sum rules which can be checked by using nuclear data. Since most of the current and planned experiments at SLAC, CERN, and HERA[45-47] to check the QCD sum rules and study the neutron SF are based on the data on DES on light nuclei, such studies are very important from the practical point of view.

The present review is devoted to the problem of developing a model-independent approach to studying the evolution of nucleon structure in nuclei. In Sec. 2 we study the fundamental approximations used in analyzing DES on nuclei. Section 3 is devoted to the principal field-theoretical methods of studying the properties of bound states for the example of an n-nucleon system. In Sec. 4 these methods are used to analyze DES on the deuteron, and the main effects of nuclear binding are studied for the example of the deuteron. In Sec. 5 this method is extended to light nuclei and used to study the A dependence of the structure-function ratios F_2^A/F_2^N and F_2^A/F_2^D. In the final section we summarize the main conclusions and results of our study.

2. THE FUNDAMENTAL APPROXIMATIONS

Let us consider the standard assumptions used to analyze DES on nuclei in field models:

- the one-boson approximation in the bound-state equation;
- treatment of the DES amplitude as an incoherent sum of amplitudes on individual constituents;
- representation of the hadron tensor of the bound nucleon in terms of scalar structure functions in the same form as for the free nucleon.

The first assumption allows us to solve, for example, the Bethe–Salpeter equation[48] or the quasipotential bound-state equation[49] in the meson–nucleon theory. In this case the interaction between nucleons can be represented as an infinite ladder of one-meson exchanges. It has been shown[48,50-53] that in this approximation the Bethe–Salpeter equation gives a good description of such fundamental properties of the deuteron as its mass, binding energy, and magnetic and quadrupole moments. It has also been somewhat successful at describing the deuteron elastic form factors.[48,54]

The second assumption makes it possible to treat the squared amplitude $W^A_{\mu\nu}$ of DES on the nucleus as the sum of the squared amplitudes for scattering on the individual constituents. As will be shown in Sec. 4.1, the justification for this is the suppression of the interference terms in $W^A_{\mu\nu}$ proportional to $1/(Q^2)^l$ with $l \geqslant 2$ (Ref. 42). Therefore, the contribution of interference terms is important at small Q^2, where they can significantly affect the Q^2 dependence of $W^A_{\mu\nu}$. The first and second approximations are not independent because the interference terms are determined by the nuclear wave function at large relative momenta. The nature of the Q^2 dependence of $W^A_{\mu\nu}$ at small Q^2 can prove to be important for analyzing DES on nuclei at small x. Therefore, both the first and the second assumptions may prove to be unjustified in this kinematical region.

The available experimental data for DES on nuclei pertain mainly to the region $x>10^{-3}$ and $Q^2>1$ GeV^2. Since experiments show[8,55] that the ratio F_2^A/F_2^D is independent of Q^2, in the calculations we shall restrict ouselves to the Bjorken limit, where the first and second approximations are well justified.

The third assumption allows the hadron tensor of a virtual nucleon to be represented in the form (9). This representation is valid when the nontrivial differences between scattering on free and bound nucleons are small. There are three such differences, which we shall refer to as off-shell effects:

- the impossibility of using the condition of gauge invariance for the bound nucleon in the form (7);
- the contribution of antinucleon degrees of freedom;
- the nonsimultaneity of bound nucleons.

Since it is impossible to use the condition (7), which has been formulated systematically only for physical particles, the expression for $W^N_{\mu\nu}$ for a bound nucleon turns out to be more complicated than (9). In general, as analysis in the quark–parton model has shown,[56] the amplitude for DES on a bound nucleon can be constructed by means of 14 structure functions, of which only three are important for describing the SF of the bound nucleon in the Bjorken limit. The choice of the actual number of SFs entering into the expansion of the hadron tensor depends strongly on the model assumptions.

A similar situation also occurs when studying the contribution of antinucleon degrees of freedom, whose role is not yet completely clear. The contribution of these degrees of freedom to various observables is usually parametrized as nuclear effects associated with the quark structure of nucleons or the contributions of various non-nucleon degrees of freedom. For example, the x dependence of the ratio F_2^{Fe}/F_2^D in the region $x=0.2-0.7$ has been explained[40] by studying antiquark degrees of freedom, the presence of which may be related to the existence of antinucleons in the relativistic nucleus. It is also known that antinucleon degrees of freedom play an important role in analyzing the static characteristics of nuclei. It has been shown[50,51] that the problem of the combined description of the deuteron quadrupole and magnetic moments can only be solved after including the contribution of relativistic P waves in the wave-function normalization. The presence of such waves is associated with antinucleon degrees of freedom. It was subsequently shown that the P-wave contribution to various static and dynamical characteristics of the deuteron corresponds to the contribution of paired meson currents in the nonrelativistic approach.[53,57] It is obvious that this off-shell effect cannot be discarded on the basis of only the approximation of small relative momenta of the nucleons in the nucleus, because it affects the low-energy properties of the nucleus.

Thus, to analyze consistently off-shell effects in DES on nuclei it is necessary to understand the role played by a possible change in the gauge-invariance condition and the presence of antinucleon degrees of freedom within the approach used.

As will be shown in Sec. 3.1, the third off-shell effect leads in momentum space to the appearance of a dependence on the zeroth component of the nucleon relative momentum (the relative energy). We note that the distribution function $f(y)$ in the linear convolution

$$F_2^{\rm D}(x)=\int dy F_2^N(x)(x/y)f(y) \tag{10}$$

does not contain any dependence on the relative energy. Moreover, the use of this expression can be justified only in nonrelativistic models, where the nucleon binding is taken into account as additional degrees of freedom[58,59] or is expressed in terms of the nucleon separation energy.[11] The simplest method of introducing a nontrivial dependence on the relative energy is to derive the two-dimensional convolution formula for the nuclear SF:

$$F_2^A(x)=\int dy dp_N^2 \tilde{F}_2^N(x/y,p_N^2)f(y,p_N^2), \tag{11}$$

where the distribution function reflects the dependence on the squared nucleon four-momentum $p_N^2\neq m^2$. In this approach, to calculate $\tilde{F}_2^A(x/y,p_N^2)$ it is necessary to make additional model assumptions not only about the structure of the off-shell nucleon, but also about the properties of the distribution function $f(y,p_N^2)$ in the nucleus.

This approach has been developed in several studies.[49,56,60,61] For example, in Ref. 60, Eq. (11) was derived by using an effective diagrammatic technique in the quark model of the nucleus. In a later study[49] it was shown that the p_N^2 dependence in the nucleon SF reduces to allowance for the change of the boundary conditions for the kinematical characteristics of quarks in the bound nucleon. This makes it possible to reduce the two-dimensional convolution formula (11) to the one-dimensional one (10) with modified argument of the distribution function $f(y)$. The reduction of the two-dimensional convolution formula to the one-dimensional one was later realized in the general case within a field-theoretical formalism independent of the quark model of the nucleon.[62] It was shown that this trivial kinematical inclusion of the dependence on the relative energy in $f(y)$ leads to violation of the baryon sum rule:

$$f(y)=\int \frac{d^4k}{(2\pi)^4}\tilde{f}(k)y\delta\left(y-\frac{M_{\rm D}/2-k_0+k_3}{M_{\rm D}}\right), \tag{12}$$

$$\int dy f(y)\neq 1, \tag{13}$$

where k is the relative 4-momentum of the nucleons bound in the deuteron at rest. An essentially analogous result was obtained by using the quark model of the nucleon in Ref. 61, where it was shown that only the kinematical inclusion of the p_N dependence leads to a change of the number of valence quarks in the nucleon.

Thus, it becomes necessary to introduce a dynamical dependence of the quark distribution in the nucleon on the square of the total nucleon momentum p_N^2. However, as was shown in Ref. 61, the inclusion of the dynamics, while allowing the number of valence quarks in the nucleon to be preserved, also leads to a significant weakening of the p_N^2 dependence of the ratio $F_2^A(x)/F_2^{\rm D}(x)$.

Since these approaches are based on the quark model for the nucleon, they allow the inclusion of a possible off-shell deformation of the nucleon structure in the nucleus on the basis of information about the quark structure of the nucleus. That is, for a consistent microscopic picture of nuclear effects in DES, it is necessary to construct a quark model of the nucleus in which the nucleon degrees of freedom can be isolated. Such a model has been developed in Ref. 63 for the case of infinite nuclear matter. This model is based on the model of a quark bag in which the quarks exchange scalar and vector mesons. It can also be written formally as the Walecka model,[64] but with the additional condition that the average scalar field is changed to the result for the change of the internal nucleon structure.[65] The extension of this model to finite A by means of the local-density approximation led to overestimation of the nuclear binding effects by a factor of two to three.[66]

An alternative approach free from the uncertainties associated with quark models can be developed within the Bethe–Salpeter formalism on the basis of the general properties of the nucleon Green functions. In this approach the nucleon is treated as a four-dimensional *black box* with the structure functions as the input dependences. The problem of analyzing off-shell effects can be reduced to the study of the deformation of the black box.[42] In this approach the nuclear structure function is expressed in terms of the SFs of the physical nucleons and their derivatives on the mass shell.

3. THE FORMALISM

In the present section we study the formalism for describing DES on a system of bound nucleons.

In analyzing processes involving bound states in local field theory, it is necessary to consider particles which are not asymptotically free. A consistent procedure allowing all the information about the physical states contained in the matrix elements to be carried over to a product of field operators is the reduction technique proposed in Ref. 67. This technique is based on the assumption that the physical states in the matrix elements can be treated as being asymptotically free, and the interaction is switched on adiabatically. Therefore, bound states, which do not satisfy this assumption, are excluded from consideration. In order to include bound states, the reduction technique must be supplemented by a procedure allowing expectation values in such states to be expressed in terms of the vacuum expectation values.

This problem is solved in nonrelativistic field theory by introducing an external classical field which allows the bound-state dynamics to be described as particle motion in a potential well. As a result, the calculation of the expectation value in a bound state reduces to the calculation of the expectation value in a one-particle state, and binding effects are taken into account by introducing the momentum distribution of this particle. This simple approach can be used to obtain the nuclear structure function in the form of a convolution.[11] The role played by relativistic corrections and off-shell effects remains unclear. In quasipotential approaches, the solution of the nucleon bound-state problem reduces to deriving an analog of the Schrödinger equation with a covariant three-dimensional potential.[49,68–75] Then the calculation of expectation values in bound states is essentially the same as in the nonrelativistic case. However, in contrast to nonrelativistic approaches, the quasipotential method allows qualitative study of the role of relativistic and off-shell effects in deep-inelastic scattering.[49,56,61]

A method of calculating the expectation values of T products of local operators in bound states was suggested in Ref. 76. It is being actively used and developed at present.[43,48,52,77,78] The essence of the method is that expectation values in bound states are expressed in terms of the vacuum expectation values of a T product of local operators and the matrix elements for the transition between the vacuum and the bound state. We shall consider the application of this method to direct processes involving a bound state of n nucleons.

3.1. The Bethe–Salpeter amplitude

Since the calculation of the DES cross section reduces to analysis of the forward Compton scattering amplitude (5), we shall restrict ourselves to considering the procedure for calculating the matrix element of the T product of local operators $\eta(y_1)\dots\eta(y_k)$ in bound states A:

$$\langle A|T(\eta_1(y_1)\dots\eta_k(y_k))|A\rangle, \tag{14}$$

where $\eta_i(y_i)$ is the set of current operators determining the nucleon interaction with the external fields. In general, some of the operators from the set $\eta_i(y_i)$ may coincide. We shall use the fact that in a definite kinematical region the joint propagation of n interacting nucleons occurs via the formation of a bound state. In this case the first term of the series expansion of the n-nucleon Green function in intermediate states has the form

$$\langle 0|T(\Psi(x_1)\dots\Psi(x_n)\bar\Psi(x_1')\dots\bar\Psi(x_n')|0\rangle$$
$$=\int\frac{d^3P}{(2\pi)^3}\sum_\alpha\langle 0|T(\Psi(x_1)\dots\Psi(x_n))|A(\alpha_1P)\rangle$$
$$\times\langle A(\alpha,P)|T(\bar\Psi(x_1'))|0\rangle\,\theta(\min\{x_{0_i}\}-\max\{x_{0i}'\})+\dots. \tag{15}$$

Here α denotes the set of discrete quantum numbers of the state A, and P denotes the total momentum of the state which, owing to energy conservation, coincides with the total momentum of the system of fields Ψ. The θ function arising in the expansion of the T product in the matrix element (15) ensures that the causality condition is satisfied:

$$\min\{x_{0_i}\}>\max\{x_{0_i}'\}. \tag{16}$$

The maximum and minimum coordinates can be determined by introducing the average coordinate of n fields conjugate to the total momentum P,

$$X=\frac{\Sigma_i^n x_i}{n}, \tag{17}$$

and the nucleon coordinates relative to this point,

$$\tilde x_i=X-x_i. \tag{18}$$

As a result, the maximum and minimum coordinates can be defined as

$$\max\{x_{0_i}\}=X_0+|\max\{\tilde x_{0_i}\}|,$$
$$\min\{x_{0_i}\}=X_0+|\max\{\tilde x_{0_i}\}. \tag{19}$$

Using the integral representation for the θ function,

$$\theta(x_0)=\frac{i}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{ip_0x_0}}{p_0+i\delta}dp_0, \tag{20}$$

we obtain the necessary expression relating the vacuum expectation value of the nucleon operators and the matrix elements for the transition from the vacuum to the bound state:

$$\langle 0|T(\Psi(x_1)\dots\Psi(x_n)\bar\Psi(x_1')\dots\bar\Psi(x_n')|0\rangle$$
$$=\sum_\alpha\int\frac{d^3P}{(2\pi)^3}\frac{dP_0}{2\pi}e^{i(P_0-E)(X_0-X_0')}$$
$$\times e^{-i(P_0-E)(|\max\{\tilde x_{0_i}\}|+|\max\{\tilde x_{0_i}'\}|)}$$
$$\times\frac{\langle 0|T(\Psi(x_1)\dots\Psi(x_n))|A(\alpha,)\rangle\langle T(\bar\Psi(x_1)\dots\bar\Psi(x_n))|0\rangle}{P_0-E+i\delta}. \tag{21}$$

This expression is inconvenient in that it involves two sets of coordinates simultaneously, $\{x_i\}$ and $\{X,\tilde x_i\}$. To eliminate this we shall change over to using the second set of coordinates everywhere in the calculations. Owing to translational invariance, the following relation holds for the fields Ψ:

$$\Psi(x+\tilde x)=e^{-i\hat Px}\Psi(\tilde x)e^{i\hat Px}. \tag{22}$$

Replacing x_i by $X-\tilde x_i$ in the operators Ψ and $\bar\Psi$, and shifting by X, using the transformation (22), we introduce the matrix elements for the transition from the vacuum to the bound state in the space of relative nucleon locations:

$$\langle 0|T(e^{-i\hat PX}\Psi(x_1)e^{i\hat PX}\dots e^{-i\hat PX}\Psi(x_n)e^{i\hat PX})|A\rangle$$
$$=\chi^A_{\alpha,P}(\tilde x_1,\dots\tilde x_n)e^{i(EX_0-\mathbf{P}\cdot\mathbf{X})},$$
$$\langle A|T(e^{i\hat PX}\bar\Psi(x_1')e^{-i\hat PX}\dots e^{i\hat PX}\bar\Psi(x_n')e^{-i\hat PX})|0\rangle$$
$$=\bar\chi^A_{a',P}(\tilde x_1'\dots\tilde x_n')e^{-i(EX_0-\mathbf{P}\cdot\mathbf{X})}. \tag{23}$$

Although the functions χ^A_α formally depend on n variables, only $n-1$ of them are independent, owing to the equation

$$\sum_i^n \bar{x}_i=0.$$

As a result of these transformations, Eq. (21) takes the form

$$\langle 0|T(\Psi(\bar{x}_1)\dots\Psi(\bar{x}_n)\Psi(\bar{x}'_1)\dots\Psi(\bar{x}'_n)|\rangle$$
$$=\sum_\alpha \int \frac{d^4P}{(2\pi)^4} e^{iP(X-X')} \frac{\chi^A_{\alpha,P}(\bar{x}_1\dots\bar{x}_n)\bar{\chi}^A_{\alpha,P}(\bar{x}'_1,\dots\bar{x}'_n)}{P_0-E+i\delta}. \quad (24)$$

Since the integral with respect to P_0 is determined by the behavior of the integrand near the pole at $P_0=E$, we have omitted the exponential factor $\exp\{-i(P_0-E)(|\max\{\bar{x}_{0i}\}|+|\max\{\bar{x}'_{0i}\}|)\}$.

The unknown functions $\chi^A_{\alpha,P}$ and $\bar{\chi}^A_{\alpha,P}$ introduced above and entering into the matrix elements for the transition from the vacuum to the bound state in (23) describe the nuclear state in terms of the degrees of freedom of the virtual nucleons and are called Bethe–Salpeter amplitudes. These amplitudes allow the solution of a fundamental problem, namely, the expression of the expectation values in bound states in terms of the vacuum expectation values.

3.2. Analysis of the matrix elements in the Bethe–Salpeter formalism

In order to explain how the matrix element (14) can be related to the nucleon Green functions and the Bethe–Salpeter amplitudes, we consider the matrix element

$$\langle 0|T(\Psi(x_1)\dots\Psi(x_n)\eta(y_1)\dots\eta(y_k)\Psi(x'_1)\dots\Psi(x'_n)|0\rangle \quad (25)$$

near the singularity of the n-nucleon bound state at $P^2=M^2$.

We expand the time-ordered product in the matrix element (14) in a product of matrix elements of Ψ, η, and Ψ. For this we choose the maximum and minimum zeroth components from the set $\{x_i\}$ in accordance with (19) and write the T product as two terms:

$$T(\Psi(x_1)\dots\Psi(x_n))\eta(y_1)\dots\eta(y_k)\Psi(x'_1)\dots\Psi(x'_n)$$
$$=T(\Psi(x_1)\dots\Psi(x_n))T(\eta(y_1)\dots\eta(y_k))$$
$$\times T(\Psi(x'_1)\dots\Psi(x'_n))\,\theta(X_0-Y_0-|\max\{\bar{x}_{0_i}\}|$$
$$-|\max\{\bar{y}_{0_i}\}|)\,\theta(Y_0-X'_0-|\max\{\bar{x}'_{0i}\}|-\max\{\bar{y}_{0_i}\}|)$$
$$+T(\Psi(x'_1)\dots\Psi(x'_n))T(\eta(y_1))T(\Psi(x_1)\dots\eta(y_k))$$
$$\times T(\Psi(x_1)\dots\Psi(x_n))\,\theta(X'_0-Y_0-|\max\{\bar{x}'_{0_i}\}|$$
$$-|\max\{\bar{y}_{0_i}\}|)\,\theta(Y_0-X_0-|\max\{\bar{x}_{0_i}\}|-|\max\{\bar{y}_{0_i}\}|).$$

Inserting a complete set between T products, we rewrite (14) as a sum over states from the complete set:

$$\langle 0|T(\Psi(x_1)\dots\Psi(x_n)\eta(y_1)\dots\eta(y_k)\Psi(x'_1)\dots\Psi(x'_n)|0\rangle$$
$$=\sum_R \langle 0|T(\Psi(x_1)\dots\Psi(x_n))|R\rangle\langle R|T(\eta(y_1)\dots\eta(y_k))$$
$$+|R\rangle\times\langle R|T(\Psi(x'_1)\dots\Psi(x'_n))|\rangle\,\theta(X_0-Y_0-|\max\{\bar{x}_{0_i}\}|$$
$$-|\max\{\bar{y}_{0_i}\}|)\,\theta(Y_0-X'_0-|\max\{x'_{0_i}\}|$$
$$-|\max\{\bar{y}_{0_i}\}|)+\sum_R \langle 0|T(\Psi(x'_1)\dots\Psi(x'_n))|R\rangle$$
$$\times\langle R|T(\eta(y_1)\dots\eta(y_k))|R\rangle$$
$$\times\langle R|T(\Psi(x_1)\dots\Psi(x_n))|0\rangle\,\theta(X'_0-Y_0-|\max\{\bar{x}'_{0_i}\}|$$
$$-|\max\{\bar{y}_{0i}\}|)\,\theta(Y_0-X_0-|\max\{\bar{x}_{0_i}\}|$$
$$-|\max\{\bar{y}_{0_i}\}|), \quad (26)$$

where Σ_R denotes a summation over discrete quantum numbers and an integration over continuous variables.

Since we are interested in the behavior of the matrix element (14) near the pole at $P^2=M^2$, it is sufficient to study the contribution to (26) from the lowest bound state corresponding to this pole. The bound state in question corresponds to the first term of the series (26) with $R=A$. Using the integral representation (20) for the θ function, we obtain the following expression for the matrix element (25):

$$\langle 0|T(\Psi(x_1)\dots\Psi(x_n)\eta(y_1)\dots\eta(y_k)\Psi(x'_1)\dots\Psi(x'_n)|0\rangle$$
$$=\sum_{\alpha,\alpha}{}' \int \frac{d^3P}{(2\pi)^3}\frac{d^3P'}{(2\pi)^3}\langle 0|T\Psi(x_1)\dots\Psi(x_n)|A,P\rangle\langle A,P|T\eta(y_1)\dots\eta(y_k)|A,P'\rangle\langle A,P'|T\Psi(x'_1)\dots\Psi(x'_n)|0\rangle$$
$$\times\int\frac{dP_0}{(2\pi)}\frac{dP'_0}{(2\pi)}\frac{e^{i(P_0-E)(X_0-Y_0)}e^{i(P'_0-E')(Y_0-X'_0)}e^{i(P_0-E)(\max\{\bar{x}_{0_i}\}-y/2)}e^{i(P'_0-E')(\max\{x'_{0_i}\}-y/2)}}{(P_0-E+i\delta)(P'_0-E'+i\delta)}. \quad (27)$$

Using (23) to go over to relative variables, we rewrite this expression as

$$\langle 0|T(\Psi(\bar{x}_1)\ldots\Psi(\bar{x}_n)\eta(\bar{y}_1)\ldots\eta(\bar{y}_k)\bar{\Psi}(\bar{x}_1')\ldots\bar{\Psi}(\bar{x}_n')|0\rangle$$

$$=\sum_{\alpha,\alpha'}\int\frac{d^4P}{(2\pi)^4}\frac{d^4P'}{(2\pi)^4}e^{i(P_0-E)(\max\{\bar{x}_{0_i}\}-y/2)}$$

$$\times e^{i(P_0'-E')(\max\{\bar{x}_{0_i}'\}-y/2)}$$

$$\times\frac{\chi^A_{\alpha,P}(\bar{x}_1,\ldots\bar{x}_n)\langle A|T(\eta(\bar{y}_1)\ldots\eta(\bar{y}_k))|A\rangle\bar{\chi}^A_{\alpha',P}(\bar{x}_1',\ldots\bar{x}_n')}{(P_0-E+i\delta)(P_0'-E'+i\delta)}$$

$$\times e^{iPX}e^{-iP'X'}e^{i(P-P')Y}. \quad (28)$$

On the other hand, owing to the unitarity of the n-nucleon Green function, related to the amplitude (15) as

$$G_{2n}(x_1\ldots x_n,z_1,\ldots z_n)$$

$$=i\langle 0|T(\Psi(x_1)\ldots\Psi(x_n)\bar{\Psi}(x_1')\ldots\bar{\Psi}(x_n'))|0\rangle, \quad (29)$$

the matrix element (25) can be rewritten identically as

$$\langle 0|T(\Psi(x_1)\ldots\Psi(x_n)\eta(y_1)\ldots\eta(y_k)\bar{\Psi}(x_1')\ldots\bar{\Psi}(x_n')|0\rangle$$

$$=\int d^4z_1\ldots d^4z_nd^4z_1'\ldots d^4z_n'd^4z_1''\ldots d^4z_n''d^4z_1'''\ldots d^4$$

$$\times z_n'''G_{2n}(x_1\ldots x_n,z_1,\ldots z_n)G_{2n}^{-1}(z_1\ldots z_n,z_k',\ldots z_n')$$

$$\times\langle 0|T(\Psi(z_1)\ldots\Psi(z_n)\eta(y_1)\ldots\eta(y_k)$$

$$\times\bar{\Psi}(z_1'')\ldots\bar{\Psi}(z_n'')|0\rangle\times G_{2n}^{-1}(z_1'',\ldots z_n'',z_1''',\ldots z_n''')$$

$$\times G_{2n}(z_1''',\ldots z_n''',x_k',\ldots x_n'). \quad (30)$$

Taking into account the behavior of the n-nucleon Green function (24) near the pole at $P^2=M^2$, we compare this expression with (28):

$$\sum_{\alpha,\alpha'}\int\frac{d^4P}{(2\pi)^4}\frac{d^4P'}{(2\pi)^4}\int dz_1\ldots dz_ndz_1'\ldots dz_n'e^{iPX}$$

$$\times e^{-iP'X'}\frac{\chi^A_{\alpha,P}(\bar{x}_1,\ldots\bar{x}_n)\bar{\chi}^A_{\alpha,P}(z_1,\ldots z_n)}{P_0-E+i\delta}$$

$$\times\bar{G}_{2n+k}(z_1\ldots z_n,\bar{y}_1\ldots\bar{y}_n,z_1'\ldots z_n')$$

$$\times\frac{\chi^A_{\alpha',P'}(z_1',\ldots z_n')\bar{\chi}^A_{\alpha',P'}(\bar{x}_1',\ldots\bar{x}_n')}{P_0'-E'+i\delta}$$

$$=\sum_{\alpha,\alpha'}\int\frac{d^4P}{(2\pi)^4}\frac{d^4P'}{(2\pi)^4}e^{i(P_0-E)(\max\{\bar{x}_{0_i}\}-y/2)}$$

$$\times e^{i(P_0')(\max\{\bar{x}_{0_i}'\}-y/2)}e^{i(P-P')Y}$$

$$\times\frac{\chi^A_{\alpha,P}(\bar{x}_1,\ldots\bar{x}_n)\langle A|T(\eta(\bar{y}_1)\ldots\eta(\bar{y}_k))|A\rangle\bar{\chi}^A_{\alpha'}(\bar{x}_1',\ldots\bar{x}_n')}{(P_0-E+i\delta)(P_0'-E'+i\delta)}$$

$$\times e^{iPX}e^{-iP'X'}, \quad (31)$$

where $\bar{G}_{2n+k}$ is used to denote the truncated Green function, defined as

$$\bar{G}_{2n+k}(x_1\ldots x_n,y_1\ldots y_k,x_k',\ldots x_n')$$

$$=i\int d^4z_1\ldots d^4z_nd^4z_1'\ldots d^4z_n'G_{2n}^{-1}(x_1\ldots x_n,z_k,\ldots z_n)$$

$$\times\langle 0|T(\Psi(z_1)\ldots\Psi(z_n)\eta(y_1)\ldots\eta(y_k)\bar{\Psi}(z_1')\ldots$$

$$\times\bar{\Psi}(z_n')|0\rangle G_{2n}^{-1}(z_1',\ldots z_n',x_1'\ldots x_n'). \quad (32)$$

Multiplying both sides of the integrand in (31) by $(P_0-E)(P_0'-E')$ and letting P_0 tend to E and P_0' to E', we obtain an expression for calculating the expectation value in the bound state of the T product of a set of local operators:

$$\langle A|T(\eta(\bar{y}_1)\ldots\eta(\bar{y}_k))|a\rangle e^{i(P-P')Y}$$

$$=\int dz_1\ldots dz_ndz_1'\ldots dz_n'\bar{\chi}^A_{\alpha,P}(z_1,\ldots z_n)$$

$$\times\bar{G}_{2n+k}(z_1\ldots z_n,\bar{y}_1\ldots\bar{y}_n,z_1'\ldots z_n')\chi^A_{\alpha',P'}(z_1',\ldots z_n'). \quad (33)$$

This expression relates the amplitude for scattering on a bound state of n nucleons with irreducible Green function $\bar{G}_{2n+k}$, describing scattering on n virtual nucleons, to the unknown functions $\chi^A_{\alpha,P}$ and $\bar{\chi}^A_{\alpha,P}$ describing the nuclear state in terms of the nucleon degrees of freedom, the equation for which must be found.

3.3. The Bethe–Salpeter equation

Equation (24) establishes a relation between the functions $\chi^A_{\alpha,P}$ and the n-nucleon Green function G_{2n}. Thus, having the equation for G_{2n}, we can obtain an equation satisfied by the amplitude $\chi^A_{\alpha,P}$. Here it is insufficient to know G_{2n} only perturbatively, since the analysis of the behavior of the Green function near the bound-state pole requires summation of the entire perturbation series. Let us therefore see what general equations for the Green functions G_{2n} can be obtained without invoking perturbation theory.

The propagation of a free nucleon from the point x_1 to the point x_2 is described by the free Green function $S^0_{(1)}$ satisfying an equation of the form

$$(i\hat{\partial}_{x_1}-m)S^0_{(1)}(x_1,x_2)=\delta(x_1,x_2), \quad (34)$$

where m is the nucleon mass. In the case of a nucleon interacting with its own field, a term taking into account this self-interaction appears on the right-hand side of the equation for the exact Green function $S_{(1)}$:

$$(i\hat{\partial}_{x_1}-m)S_{(1)}(x_1,x_2)=\delta(x_1-x_2)$$

$$-\int dx_1'\bar{G}_2(x_1,x_1')S_{(1)}(x_1',x_2). \quad (35)$$

Comparing (34) and (35), we see that the function $\bar{G}_2(x_1,x_1')$ satisfies the Dyson equation

$$\bar{G}_2(x_1,x_2)=S^{0^{-1}}_{(1)}(x_1,x_2)-S^{-1}_{(1)}(x_1,x_2), \quad (36)$$

i.e., it coincides with the one-nucleon irreducible self-energy part. The Green function $S_{(2)}$ describing the joint propagation of two physical nucleons which do not interact with each other satisfies an equation of the form

$$(i\hat{\partial}_{x_1}-m^*)\otimes(i\hat{\partial}_{y_1}-m^*)S^0_{(2)}(x_1,x_2,y_1,y_2)$$
$$=\delta(x_1-x_2)\,\delta(y_1-y_2), \tag{37}$$

where $m^*=m-\bar{G}_2(x_1,x_2)$. Inclusion of the interaction between the nucleons leads to the appearance of an additional term on the right-hand side:

$$(i\hat{\partial}_{x_1}-m^*)\otimes(i\hat{\partial}_{y_1}-m^*)G_4(x_1,x_2,y_1,y_2)$$
$$=\delta(x_1-x_2)\,\delta(y_1-y_2)$$
$$+\int dx_1' dy_1' \bar{G}_4(x_1,x_1',y_1y_1')G_4(x_1',x_2,y_1',y_2). \tag{38}$$

Comparing (37) and (38), we obtain the two-particle analog of the Dyson equation, namely, the inhomogeneous Bethe–Salpeter equation:[79]

$$\bar{G}_4(x_1,x_2,y_1,y_2)=S^{-1}_{(2)}(x_1,x_2,y_1,y_2)$$
$$-G_4^{-1}(x_1,x_2,y_1,y_2). \tag{39}$$

By analogy with the one-nucleon case, the function describing the interaction between the nucleons coincides with the irreducible self-energy part of the two-nucleon system. Generalizing to the case of n nucleons, we obtain the equation

$$\bar{G}_{2n}(x_1\ldots x_n,x_1'\ldots x_n')=S^{-1}_{(n)}(x_1\ldots x_n,x_1'\ldots x_n')$$
$$-G^{-1}_{2n}(x_1\ldots x_n,x_1'\ldots x_n'), \tag{40}$$

where the function $S_{(n)}$ is a direct product of n-nucleon propagators:

$$S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n')=\langle 0|T(\Psi(x_1)\Psi(x_1))|0\rangle$$
$$\otimes\ldots\langle 0|T(\Psi(x_n)\Psi(x_n))|0\rangle. \tag{41}$$

Using (40) for G_{2n}, we obtain an integral equation with $\bar{G}_{2n}$ as the kernel:

$$G_{2n}(x_1\ldots x_n,x_1'\ldots x_n')$$
$$=S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n')+\int dz_1\ldots dz_n dz_1'\ldots dz_n' S_{(n)}$$
$$\times(x_1\ldots x_n,z_1\ldots z_n)\bar{G}_{2n}(z_1\ldots z_n,z_1'\ldots z_n')$$
$$\times G_{2n}(z_1'\ldots z_n',x_1'\ldots x_n'). \tag{42}$$

Thus, the exact n-nucleon Green function is the solution of the integral equation which relates the two unknown Green functions G_{2n} and $\bar{G}_{2n}$ to each other.

There are several ways of solving this problem. In particular, by analogy with the Dyson equation, the Bethe–Salpeter equation can be studied by using the technique of dispersion relations. For this we introduce a generalization of the spectral representation for the exact one-particle Green function for the case of n particles—the Nakanishi *integral representation of perturbation theory*.[80] This approach has been used successfully for solving the Bethe–Salpeter equation in the case of two scalar particles.[81] On the other hand, (42) offers an excellent possibility for modeling the two-nucleon Green function if $\bar{G}_{2n}$ is introduced explicitly.

The separable form of the kernel of this equation is the most convenient:

$$\bar{G}_{2n}(z_1\ldots z_n,z_1'\ldots z_n')=\sum_{ij}^{\infty} c_{ij}g_i(z_1\ldots z_n)g_j(z_1'\ldots z_n').$$

In this case the problem of solving the integral equation is replaced by the problem of solving a linear system of equations. This approach has been used successfully to describe the two-nucleon system.[82] We note that a certain combination of the approaches based on use of a separable potential and the spectral representation taking into account the analytic properties of the two-nucleon Green function can serve as the foundation for constructing a relativistic separable ansatz for the function $\bar{G}_{2n}$ (Ref. 83).

The second, most commonly encountered form of $\bar{G}_{2n}$ can be obtained by perturbative methods. Let us consider the iterative solution of (42). We take the zeroth iteration to be

$$G^{(0)}_{2n}(x_1\ldots x_n,x_1'\ldots x_n')=S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n').$$

Substituting this expression into (42), we obtain the first iteration:

$$G^{(1)}_{2n}(x_1\ldots x_n,x_1'\ldots x_n')$$
$$=S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n')+\int dz_1\ldots dz_n dz_1'\ldots dz_n' S_{(n)}$$
$$\times(x_1\ldots x_n,z_1\ldots z_n)\bar{G}_{2n}(z_1\ldots z_n,z_1'\ldots z_n')$$
$$\times S_n(z_1'\ldots z_n',x_1'\ldots x_n'). \tag{43}$$

In the course of successive iterations we obtain the expansion of the exact n-nucleon Green function in an infinite series in powers of $\bar{G}_{2n}$:

$$G_{2n}(x_1\ldots x_n,x_1'\ldots x_n')$$
$$=S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n')$$
$$+\sum_m^{\infty}\int dz_1\ldots dz_n dz_1'\ldots dz_n'\ldots dz_1^{(m)}\ldots dz_n^{(m)} S_{(n)}$$
$$\times(x_1\ldots x_n,z_1\ldots z_n)\bar{G}_{2n}(z_1\ldots z_n,z_1'\ldots z_n')S_n$$
$$\times(z_1'\ldots z_n',x_1'\ldots x_n')\ldots$$
$$\times\bar{G}_{2n}(z_1^{(m-1)}\ldots z_n^{(m-1)},z_1^{(m)}\ldots z_n^{(m)})S_{(n)}$$
$$\times(z_1^{(m)}\ldots z_n^{(m)},x_1'\ldots x_n'). \tag{44}$$

On the other hand, the function G_{2n} can be expanded in a perturbation series in a specific nucleon–nucleon interaction model (for example, the meson–nucleon model):

$$G_{2n}(x_1\ldots x_n,x_1'\ldots x_n')=\sum_i G^{(i)}_{2n}(x_1\ldots x_n,x_1'\ldots x_n'),$$

$$G_{2n}^{(0)}(x_1\ldots x_n,x_1'\ldots x_n')=S_{(n)}(x_1\ldots x_n,x_1'\ldots x_n'). \quad (45)$$

Comparing the series (44) and (45) term by term, we obtain an expression relating $\bar{G}_{2n}$ to the terms of the perturbation series for G_{2n}:

$$\bar{G}_{2n}(x_1\ldots x_n,x_1'\ldots x_n')=\sum_m^{\infty}\sum_{m_1+m_2=m}\frac{1}{m+1}\times\int dz_1\ldots dz_n,dz_1'\ldots dz_n' G_{2n}^{(m_1)^{-1}}\times(x_1\ldots x_n,z_1\ldots z_n)\times G_{2n}^{(m+1)}(z_1\ldots z_n,z_1'\ldots z_n')\times G_{2n}^{(m_2)^{-1}}(z_1'\ldots z_n',x_1'\ldots x_n'). \quad (46)$$

In the meson model of the nucleon–nucleon interaction, the first term of this series ($m=1$) corresponds to the one-boson exchange approximation in the kernel of (42) (the ladder approximation). The solution of (42) in the ladder approximation allows a part of the perturbation series for G_{2n} to be summed, and the low-energy properties of nucleon–nucleon scattering to be described quite accurately.[48] Thus, Eq. (40) specifying the nucleon–nucleon forces allows, in principle, the determination of the exact n-nucleon Green function. Assuming the presence of a bound-state pole in G_{2n}, an analogous equation can be obtained for the Bethe–Salpeter amplitude (23).

We substitute (24) into (42), multiply both sides of the resulting expression by (P_0-E), and take $P\to E$. As a result, we obtain

$$\chi_{a,P}^A(x_1,\ldots x_n)=\int dz_1\ldots dz_n dz_1'\ldots dz_n' S_{(n)}\times(x_1\ldots x_n,z_1\ldots z_n)\bar{G}_{2n}\times(z_1\ldots z_n,z_1'\ldots z_n')\chi_{\alpha,P}^A(z_1',\ldots z_n'). \quad (47)$$

Thus, the matrix element for the transition between the vacuum and the n-nucleon bound state satisfies a homogeneous integral equation with kernel $\bar{G}_{2n}$, which is related to the exact n-nucleon and one-nucleon Green functions by Eq. (40).

We shall need Eq. (47) in momentum space for the rest of the calculations. By means of a Fourier transform, the Bethe–Salpeter amplitude χ can be rewritten in momentum space as

$$\chi_\alpha(P,k_1\ldots k_{n-1})=\int d^4x_1\ldots d^4x_n \times e^{-i\Sigma_i^n k_i x_i}\chi_{\alpha,P}(x_1\ldots x_n), \quad (48)$$

where $k_n=P-\Sigma_i^{n-1}k_i$. Then the Bethe–Salpeter equation takes the form

$$\chi_\alpha^A(P,k_1,\ldots k_{n-1})=S_{(n)}(P,k_1,\ldots k_{n-1})\times\int\frac{d^4k_{n-1}}{(2\pi)^4}\cdots\frac{d^4k_{n-1}}{(2\pi)^4}\bar{G}_{2n}\times(P,k_1\ldots k_{n-1},k_1'\ldots k_{n-1}')\chi_\alpha^A\times(k_1',\ldots k_{n-1}'). \quad (49)$$

Since (49) and (47) are homogeneous integral equations, the Bethe–Salpeter amplitude is defined up to some constant. It is therefore necessary to introduce a normalization condition. Since the solutions of these equations do not possess any direct physical meaning and cannot be normalized on the basis of probability arguments, it is necessary to use a normalization in terms of known expectation values. For the latter we can take, for example, the expectation value of the baryon current at zero momentum transfer:

$$\langle A|J_\mu(0)|A\rangle=iP_\mu. \quad (50)$$

Using (33) to express this matrix element in terms of the Bethe–Salpeter amplitude, we obtain the following normalization condition for χ_α^A:

$$\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{n-1}}{(2\pi)^4}\frac{d^4k_1'}{(2\pi)^4}\cdots\frac{d^4k_{n-1}'}{(2\pi)^4}\bar{\chi}_\alpha^A(P,k_1,\ldots k_{n-1})\times\tilde{G}_{2n+1\mu}(q=0,P,K_1\ldots k_{n-1},k_1'\ldots k_{n-1}')\times\chi_\alpha^A(P,k_1',\ldots k_{n-1}')=iP_\mu. \quad (51)$$

Using the fact that at zero momentum transfer the exact truncated photon–n-nucleon vertex $\bar{G}_{2n+1\mu}$ is related to the derivative of the n-nucleon Green function with respect to the total momentum,[84]

$$\bar{G}_{2n+1\mu}(q=0,P,k_1\ldots k_{n-1},k_1'\ldots k_{n-1}')=-\frac{\partial}{\partial P_\mu}G_{2n}^{-1}(P,k_1\ldots k_{n-1},k_1'\ldots k_{n-1}'), \quad (52)$$

and expressing G_{2n}^{-1} using (40), we obtain the normalization condition for the functions χ_α^A:

$$\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_n}{(2\pi)^4}\frac{d^4k_n'}{(2\pi)^4}\cdots\frac{d^4k_n'}{(2\pi)^4}\bar{\chi}_\alpha^A(P,k_1,\ldots k_{n-1})\times\left[S_{(n)}^{-1}(P,k_1,\ldots k_{n-1})\left\{\frac{\partial}{\partial P_\mu}S_{(n)}\times(P,k_1,\ldots k_{n-1})\right\}S_{(n)}^{-1}(P,k_1,\ldots k_{n-1})+\frac{\partial}{\partial P_\mu}\bar{G}_{2n}(P,k_1\ldots k_{n-1},k_1'\ldots k_{n-1}')\right]\times\chi_\alpha^A(P,k_1',\ldots k_{n-1}')=iP_0. \quad (53)$$

As will be shown in Sec. 4, this condition allows us to construct a theory of DES on the nucleus such that the baryon and momentum sum rules can be satisfied simultaneously.

Let us conclude this section by noting several important features of the Bethe–Salpeter amplitude which we shall use below.

First, the Bethe–Salpeter amplitude depends on the zeroth component of the relative coordinate (the relative time) of the nucleons, which, according to (33), is reflected in the dynamical observables of the n-nucleon bound state. In momentum space this dependence leads to a dependence on the zeroth component of the nucleon relative momentum (the relative energy), which, as will be shown below, is the main reason for the observed nuclear effects in DES.

Second, Eq. (24) gives some information about the analytic structure of χ^A_α. The Green function G_{2n} in momentum space contains a series of singularities, some of which, according to (24), are also present in the Bethe–Salpeter amplitude. These are poles associated with the external nucleon propagators, cuts in the relative momenta, and poles associated with the various bound states formed either by some or by all of the nucleons. The latter are isolated explicitly in (24) and therefore do not contribute to χ^A_α. The first type of singularity can also be explicitly isolated by introducing, instead of the Bethe–Salpeter amplitude, a quantity referred to as the Bethe–Salpeter vertex function:

$$S_{(n)}(P,k_1\ldots k_{n-1})\Gamma^A_\alpha(P,k_1\ldots k_{n-1}) = \chi^A_\alpha(P,k'_1,\ldots k'_{n-1}). \tag{54}$$

Thus, we have at our disposal an object which is free of the two strongest singularities: the poles of the n-nucleon bound state, and the poles and cuts present in the one-nucleon propagator. In Sec. 5.1 we shall study a method of isolating the singularities of the vertex function which arise owing to the presence of m-nucleon ($m<n$) bound states in the nucleus, corresponding to the notion of nuclear clustering.

4. DEEP INELASTIC SCATTERING ON THE DEUTERON

The simplest stable bound state of nucleons observed in nature is the deuteron. This state has the lowest binding energy of all the nuclei ($\epsilon=2.224$ MeV). The deuteron binding energy is so small that by very simple arguments based on nonrelativistic quantum mechanics it can be concluded that the nucleons in this system do not interact during a large fraction of the time. Therefore, in most theoretical and experimental studies of the structure of matter, the deuteron is identified as an isoscalar nucleon.[8,55] In particular, the observed differences between the structure functions of heavy nuclei and the deuteron are usually regarded as evidence for distortion of the structure function of the free nucleon. Despite the fact that this approach appears qualitatively reasonable, many of the results based on measurements of the deuteron structure function turn out to be inconsistent. The clearest example of a difficulty which may be encountered in analyzing experiments using deuteron targets occurs when attempting to extract information about the neutron SF from the data on DES on the deuteron and the proton.[85–88] On the one hand, the experimental test of the QCD sum rules[85,87] using F_2^D leads to a quite unexpected result contradicting the traditional ideas about the parton structure of the nucleon: violation of the Gottfried sum rule. On the other hand, in several theoretical studies it has been shown that F_2^n cannot be uniquely extracted from the deuteron data.[88,89] The theoretical uncertainties can be so large that they are quantitatively comparable to the violation of the Gottfried sum rule.

Thus, despite the small binding energy, nuclear effects in the deuteron require detailed study. Since the deuteron is the simplest nuclear system, all the main binding effects can be studied for it in the greatest detail, and all the analytic calculations can be performed consistently and exactly.

We shall use the approach developed in Refs. 42, 76, and 79 and described in Sec. 3 to calculate F_2^D and F_2^D/F_2^N.

4.1. The amplitude of Compton scattering on the deuteron

Owing to the unitarity relation (5), the calculation of the hadronic part of the amplitude for deep-inelastic scattering, $W_{\mu\nu}(P,q)$, reduces to the calculation of the amplitude for forward Compton scattering, $T_{\mu\nu}(P,q)$. According to the definition, the amplitude for Compton scattering on the deuteron, $T^D_{\mu\nu}(P,q)$, can be represented as the expectation value of the T product of nucleon electromagnetic currents J_μ in deuteron states $|D\rangle$:

$$T^D_{\mu\nu}(P,q)=i\int d^4x e^{iqx}\langle D|T(J_\mu(x)J_\nu(0))|D\rangle. \tag{55}$$

Using (33), this definition of $T^D_{\mu\nu}(P,q)$ can be rewritten in terms of the solutions of the Bethe–Salpeter equation for the deuteron, $\Gamma^D(P,k)$, and the two-nucleon Green functions $\bar{G}_{6\mu\nu}$:

$$T^D_{\mu\nu}(P,q)=\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Gamma^S(P,k_1)S_{(2)} \times(P,k_1)\Gamma_{6_{\mu\nu}}(q,P,k_1,k_1,k_2)S_{(2)} \times(P,k_2)\Gamma^S(P,k_2). \tag{56}$$

According to (32), the function $\bar{G}_{6\mu\nu}$ is related to the exact two-nucleon Green function with an insertion describing the Compton scattering of virtual photons on a system of two interacting nucleons:

$$\bar{G}_{6_{\mu\nu}}(q,P,k,k')=\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}G_4^{-1}(P,k,k_1) \times G_{6_{\mu\nu}}(q,P,k_1,k_2)G_4^{-1}(P,k_2,k'), \tag{57}$$

where

$$G_{6_{\mu\nu}}(q,P,k',k)=i\int d^4xd^4yd^4y'd^4Yd^4Y'e^{-iky+ik'y'} \times e^{-iqx}e^{-iP(Y-Y')}\langle 0|T \times\left(\bar{\psi}\left(Y+\frac{y}{2}\right)\bar{\psi}\left(Y-\frac{Y}{2}\right)J_\nu(0)\psi \times\left(Y'+\frac{y'}{2}\right)\psi\left(Y'-\frac{y'}{2}\right)\right)|0\rangle. \tag{58}$$

If a specific form for $\bar{G}_4$ is assumed, the function $\bar{G}_{6\mu\nu}$ can be obtained explicitly. To find the amplitude for Compton scattering on the deuteron, in general it is sufficient to determine the relation between $\bar{G}_{6\mu\nu}$ and the expansion of $G_{6\mu\nu}$ in terms of the functions $\bar{G}_4$.

Expressing the function G_4 using (39), we obtain the following expansion of G_4 in powers of $\bar{G}_4$:

$$G_4(P;k,k')=S_{(2)}(P,k)\Bigg((2\pi)^4\delta(k-k') + \sum_{n\geq 1}\frac{1}{n!}\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_n}{(2\pi)^4} \times\bar{G}_4(P;k,k_1)S_{(2)}(P,k_1)\ldots \times\bar{G}_4(P;k_n,k')S_{(2)}(P,k')\Bigg). \quad (59)$$

Also expanding $G_{6\mu\nu}$ and substituting this expression into (57), we obtain a series whose nth term has the form

$$G^{(n)}_{6\mu\nu}(q,P,k,k')=\sum_{n_1+n_2+n_3=n}\int\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}G_4^{(n_1)} \times(P,k,k_1)\bar{G}^{(n_2)}_{6\mu\nu}(q,P,k_1,k_2)G_4^{(n_3)} \times(P,k_2,k'). \quad (60)$$

Choosing the term of zeroth order in $\bar{G}_4$, we immediately obtain the corresponding contribution to $\bar{G}_{6\mu\nu}$:

$$\bar{G}^{(0)}_{6\mu\nu}(q,P,k,k')=S^{-1}_{(2)}(P,k)[G^{(0)\mathbf{a}}_{6\mu\nu}(q,P,k)(2\pi)^4\delta^4(k-k')+G^{(0)\mathbf{b}}_{6\mu\nu}(q,P,k)(2\pi)^4\delta^4(k-k'-q)]S^{-1}_{(2)}(P,k'). \quad (61)$$

Thus, in the zeroth order in $\bar{G}_4$ the function $\bar{G}_{6\mu\nu}$ contains both the one-nucleon contribution (**a**)

$$G^{(0)\mathbf{a}}_{6\mu\nu}(q,P,k)=G_{4_{\mu\nu}}\left(q,\frac{P}{2}+k\right)\otimes S\left(\frac{P}{2}-k\right) +G_{4_{\mu\nu}}\left(q,\frac{P}{2}-k\right)\otimes S\left(\frac{P}{2}+k\right) \quad (62)$$

and the contribution corresponding to scattering on various nucleons (**b**):

$$G^{(0)\mathbf{b}}_{6\mu\nu}(q,P,k)=G_{3_{\mu\nu}}\left(q,\frac{P}{2}+k\right)\otimes G_{3\nu}\left(q,\frac{P}{2}-k\right) +G_{3_{\mu\nu}}\left(q,\frac{P}{2}-k\right)\otimes G_{3\nu}\left(q,\frac{P}{2}+k\right). \quad (63)$$

The Green functions $G_{4\mu\nu}$ and $G_{3\mu}$ respectively describe Compton scattering on a virtual nucleon and the interaction of a virtual photon with a nucleon.

The first-order contribution to $G_{6\mu\nu}$ depends on $\bar{G}^{(0)}_{6\mu\nu}$ and $\bar{G}^{(1)}_{6\mu\nu}$, and this leads to the expression

$$\bar{G}^{(1)}_{6\mu\nu}(q,P,k,k')=S^{-1}_{(2)}(P,k)G^{(1)}_{6\mu\nu}(q,P,k,k')S^{-1}_{(2)}(P,k') -\int\frac{d^4k''}{(2\pi)^4}\{S^{-1}_{(2)}(P,k)G_4^{(1)} \times(P,k,k'')\bar{G}^{(0)}_{6\mu\nu}(q,P,k'',k') +\bar{G}^{(0)}_{6\mu\nu}(q,P,k,k'')G_4^{(1)} \times(P,k'',k')S^{-1}_{(2)}(P,k')\}, \quad (64)$$

where the function $G^{(1)}_{6\mu\nu}$ is expressed in terms of the Green function $\bar{G}_{5\mu\nu}$ and the zeroth-order term of the function $G_{6\mu\nu}$:

$$G^{(1)}_{6\mu\nu}(q,P,k,k')=\int\frac{d^4k''}{(2\pi)^4}\frac{d^4k'''}{(2\pi)^4}S_{(2)} \times(P,k)\overline{G_{5\mu}}(q,k,k''+q)\bar{G}_4(P,k'' +q,k'''+q)\overline{G_{5\mu}}(q,k,k''+q)\bar{G}_4(P,k'' +q,k'''+q)\overline{G_{5\nu}}(q,k'''+q,k')S_{(2)} \times(P,k')+\int\frac{d^4k''}{(2\pi)^4}\{G_4^{(1)} \times(P,k,k'')\bar{G}^{(0)}_{6\mu\nu}(q,P,k'',k')S_{(2)} \times(P,k')+S_{(2)}(P,k)\bar{G}^{(0)}_{6\mu\nu} \times(q,P,k,k'')G_4^{(1)}(P,k'',k')\}. \quad (65)$$

According to (32), the function $\bar{G}_{5\mu}$ is determined by the Green function $G_{5\mu}$ describing the absorption of a virtual photon by a system of two virtual nucleons.

Following this procedure, we can obtain $\bar{G}_{6\mu\nu}$ in any order in $\bar{G}_4$. However, the general structure of Eq. (56) is such that all the higher contributions reduce to the leading terms already studied. This is easily checked by using (49) to go to higher order in $\bar{G}_4$ in (56).

Substituting the expressions obtained for $\bar{G}_{6\mu\nu}$ into (56) and taking into account the definition (57), we obtain the amplitude for Compton scattering on the deuteron in general form:

$$T^{\mathrm{D}}_{\mu\nu}(P,q)=\int\frac{d^4k}{(2\pi)^4}\Gamma(P,k)G^{(0)}_{6\mu\nu}(q,P,k)\Gamma(P,k) +\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}\frac{d^4k'''}{(2\pi)^4}\Gamma \times(P,k)S^{(2)}(P,k)\bar{G}_{5\mu}(q,P,k,k') \times G_4(P,k',k'')\bar{G}_{5\nu}(q,P,k'',k''') \times S^{(2)}(P,k''')\Gamma(P,k''').$$

In Fig. 2 we show schematically the various contributions to the amplitude of forward Compton scattering on the deuteron. Graphs (a) and (b) correspond to the relativistic impulse approximation. The explicit form of the expressions represented by these graphs is given by the terms (**a**) and (**b**)

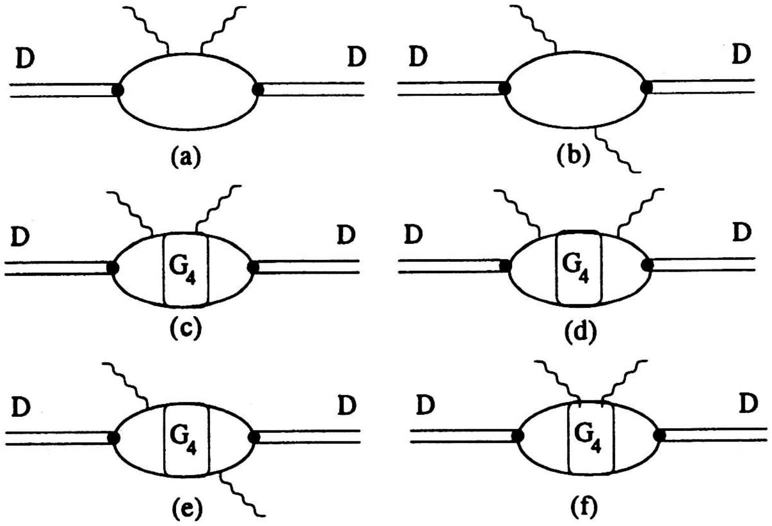

FIG. 2. Graphs schematically depicting the amplitude of forward Compton scattering.

in (61). The graph (b) represents the two-nucleon corrections of zeroth order in $\bar{G}_4$ to the Compton amplitude, the imaginary part of which corresponds to the interference term in the impulse approximation for the hadron tensor. Owing to the large momentum transferred by the photon to the nucleon, the probability that the final state will interfere with the initial state of the residual nucleon is very small. This probability is determined quantitatively by the behavior of the vertex function at large relative momenta, which will lead to suppression of interference terms like $1/(Q^2)^l$ with $l \geqslant 2$. Thus, the contributions to the Compton amplitude represented by graph (b) are sensitive to the deuteron structure at small internucleon separations, which must be manifested in the Q^2 dependence of the ratio of the deuteron and nucleon structure functions. The graphs (c), (d), (e), and (f) represent schematically the various contributions to the Compton amplitude leading to corrections for the final-state interaction in the DES amplitude. The explicit form of these corrections is given by the first term in (65). The contribution of these corrections is significantly suppressed, owing to the nucleon propagators depending on q. On the whole, all these corrections decrease with increasing Q^2 no more slowly than $1/Q^4$.

Thus, in the Bjorken limit it is necessary to include only the one-nucleon contribution from the relativistic impulse approximation. The use of the Bjorken limit is justified, since in this kinematical region the ratio of the nuclear and deuteron SFs is independent of Q^2 (Ref. 55).

4.2. The hadron tensor of the deuteron

Let us consider the hadron tensor of the deuteron in the impulse approximation. Substituting (61) into (56) and discarding the $1/Q^2$ terms, we can write the amplitude for unpolarized Compton scattering on the deuteron as

$$T^{\mathrm{D}}_{\mu}(P,q)=\int\frac{d^4k}{(2\pi)^4}\Gamma^{\mathrm{D}}(P,k)S\left(\frac{P}{2}-k\right)\left(S\left(\frac{P}{2}+k\right)\right.$$
$$\left.\times\bar{G}_{4\mu\nu}\left(q,\frac{P}{2}+k\right)S\left(\frac{P}{2}+k\right)\right)\Gamma^{\mathrm{D}}(P,k). \quad (66)$$

The following representation in terms of Dirac spinors will be valid for the nucleon propagator in general:

$$S^{ss'}(p)=\frac{m}{\tilde{E}}\frac{u^s(\mathbf{p})\bar{u}^{s'}(\mathbf{p})}{(p_0-\tilde{E}+i\delta)}-\frac{m}{\tilde{E}}\frac{v^s(\mathbf{p})\bar{v}^{s'}(\mathbf{p})}{(p_0+\tilde{E}-i\delta)}. \quad (67)$$

Here $\tilde{E}=\sqrt{\mathbf{p}^2+m^2+(\hat{p}+m)\bar{G}_2(p)}$ is the nucleon energy, which becomes the nucleon energy on the mass shell ($E=\sqrt{\mathbf{p}^2+m^2}$) for $p^2=m^2$. In our basic approximation of small relative energy of the nucleons in the deuteron, the contribution of $\bar{G}_2(p)$ can be neglected.

The Green function $\bar{G}_{4\mu\nu}$ is directly related to the amplitude for Compton scattering on the nucleon:

$$T^{\tilde{N}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)=2m\sum_s\bar{u}^s\left(\frac{\mathbf{P}}{2}+\mathbf{k}\right)$$
$$\times\bar{G}_{4\mu\nu}\left(q,\frac{\mathbf{P}}{2}+\mathbf{k}\right)u^s\left(\frac{\mathbf{P}}{2}+\mathbf{k}\right),$$
$$T^{\bar{\tilde{N}}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)=2m\sum_s\bar{u}^s\left(\frac{\mathbf{P}}{2}+\mathbf{k}\right)$$
$$\times\bar{G}_{4\mu\nu}\left(q,\frac{\mathbf{P}}{2}+\mathbf{k}\right)u^s\left(\frac{\mathbf{P}}{2}+\mathbf{k}\right), \quad (68)$$

where $\tilde{N}$ ($\bar{\tilde{N}}$) denotes a bound nucleon (antinucleon). Using the representation (67) and Eq. (68), and taking into account the azimuthal symmetry of the Bethe–Salpeter vertex function, we rewrite the Compton scattering amplitude in terms of the nucleon and antinucleon amplitudes:

$$\frac{1}{6}\sum_{s,s'}T^{\tilde{N}s,s'}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f^{s,s';S}(P,k)$$
$$=T^{\tilde{N}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f(P,k),$$

$$f(P,k)=\frac{1}{3}\sum_{s,S}f^{ss;S}(P,k). \quad (69)$$

This leads to the analog of the convolution formula for the Compton amplitude:

$$T^{\mathrm{D}}_{\mu\nu}(P,q)=\int \frac{d^4k}{(2\pi)^4}\tilde{T}^{\tilde{N}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f^{\tilde{N}}(P,k)+\int \frac{d^4k}{(2\pi)^4}T^{\bar{\bar{N}}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f^{\bar{\bar{N}}}(P,k).$$

The total averaged nucleon Compton scattering amplitude $\bar{G}_{4\mu\nu}(q,P/2+k)$ has singularities associated with the continuum in the intermediate state. Therefore, at large Q^2 the amplitude of the bound nucleon (antinucleon), $T^{\tilde{N}(\bar{\bar{N}})}_{\mu\nu}(P/2+k,q)$, can be related to the hadron tensor of the nucleon in accordance with the unitarity relation (5). As a result, we obtain the convolution formula for the hadron tensor:

$$W^{\mathrm{D}}_{\mu\nu}(P,q)=\int \frac{d^4k}{(2\pi)^4}W^{\tilde{N}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f^{\tilde{N}}(P,k)+\int \frac{d^4k}{(2\pi)^4}W^{\bar{\bar{N}}}_{\mu\nu}\left(\frac{P}{2}+k,q\right)f^{\bar{\bar{N}}}(P,k), \tag{70}$$

where the distribution functions have the form

$$f^{\tilde{N}}(P,k)=\frac{im^2}{2E^3}\frac{1}{\left(\frac{M_{\mathrm{D}}}{2}+2k_0-E+i\delta\right)^2}\left[\frac{\Phi^2_{++}(P,k)}{-k_0-\left(E-\frac{M_{\mathrm{D}}}{2}\right)+i\delta}+\frac{\Phi^2_{+-}(P,k)}{-k_0+\left(E+\frac{M_{\mathrm{D}}}{2}\right)-i\delta}\right],$$

$$f^{\bar{\bar{N}}}(P,k)=\frac{im^2}{2E^3}\frac{1}{\left(\frac{M_{\mathrm{D}}}{2}+k_0+E-i\delta\right)^2}\left[\frac{\Phi^2_{--}(P,k)}{-k_0+\left(E+\frac{M_{\mathrm{D}}}{2}\right)-i\delta}+\frac{\Phi^2_{-+}(P,k)}{-k_0-\left(E-\frac{M_{\mathrm{D}}}{2}\right)+i\delta}\right].$$

The functions Φ are related to the Bethe–Salpeter vertex functions as follows:

$$\Phi^2_{++}(M_{\mathrm{D}},k)=\sum_s \Gamma^S_{\alpha\beta}(M_{\mathrm{D}},k)\sum_s u^s_\alpha(\mathbf{k})\bar{u}^s_\delta(\mathbf{k})\times\sum_s u^s_\beta(-\mathbf{k})\bar{u}^s_\gamma(-\mathbf{k})\Gamma^S_{\delta\gamma}(M_{\mathrm{D}},k),$$

$$\Phi^2_{+-}(M_{\mathrm{D}},k)=-\sum_S \Gamma^S_{\alpha\beta}(M_{\mathrm{D}},k)\sum_s u^s_\alpha(\mathbf{k})\bar{u}^s_\delta(\mathbf{k})\times\sum_s v^s_\beta(\mathbf{k})\bar{v}^s_\gamma(\mathbf{k})\Gamma^S_{\delta\gamma}(M_{\mathrm{D}},k),$$

$$\Phi^2_{-+}(M_{\mathrm{D}},k)=-\sum_S \Gamma^S_{\alpha\beta}(M_{\mathrm{D}},k)\sum_s v^s_\alpha(-\mathbf{k})\bar{v}^s_\delta(-\mathbf{k})\times\sum_s u^s_\beta(-\mathbf{k})\bar{u}^s_\gamma(-\mathbf{k})\Gamma^S_{\delta\gamma}(M_{\mathrm{D}},k),$$

$$\Phi^2_{--}(M_{\mathrm{D}},k)=\sum_S \Gamma^S_{\alpha\beta}(M_{\mathrm{D}},k)\sum_s v^s_\alpha(-\mathbf{k})\bar{v}^s_\delta(-\mathbf{k})\times\sum_s v^s_\beta(\mathbf{k})\bar{v}^s_\gamma(\mathbf{k})\Gamma^S_{\delta\gamma}(M_{\mathrm{D}},k). \tag{71}$$

Thus, we have obtained an expression relating the hadron tensor of the relativistic deuteron to the hadron tensor of the off-shell nucleon and antinucleon in the nucleus. The nucleon and antinucleon contributions are additive.

4.2.1. The deuteron structure function F_2^{D}

In order to calculate the deuteron structure function $F_2^{\mathrm{D}}(x)$, it is necessary to express the corresponding hadron tensors in terms of the scalar structure functions $F_2^N(x)$. This procedure is performed by using the representation (9), which is valid only for free particles. This makes it inapplicable for a nucleon bound in the deuteron. Even if the approximation of small deuteron binding energy is used and possible additional structure functions are neglected, Eq. (9) contains (via the structure functions of the constituents) a dependence on the relative energy of the nucleons in the deuteron, which must be taken into account consistently.

The problem can be overcome by integrating (70) with respect to k_0, taking into account the analytic properties of the integrand. This expression is an analytic function containing singularities of the following types: poles and cuts in the nucleon propagators, and poles and cuts in the Bethe–Salpeter vertex function. The singularities of the nucleon propagator (67) are the pole at $p^2=m_N^2$ and the cuts corresponding to the contribution of πN, $2\pi N,\ldots$ states in the physical nucleon. Since the states πN, $2\pi N,\ldots$ lie relatively far in energy from the pole at $p^2=m_N^2$, the cuts in the nucleon propagators can be neglected if the relative energy of the nucleons in the deuteron is assumed to be small.

The singularities of the Bethe–Salpeter vertex function can be fixed by means of the relation between these vertices and the two-nucleon Green function:

$$\Gamma^{\mathrm{D}}_{\alpha\beta}(P,k)\Gamma^{\mathrm{D}}_{\delta\gamma}(P,k')=\lim_{P^2\to M_{\mathrm{D}}^2}(P^2-M_{\mathrm{D}}^2)\times G_{4\alpha\beta\delta\gamma}(P,k,k').$$

Thus, the Bethe–Salpeter vertex function for the deuteron has the same singularities in the relative momentum as the two-nucleon Green function. Since the singularity in this function closest in energy is determined by the cut beginning at $k^2=m_\pi^2$, the singularities in $\Gamma(P,k)$ can be neglected in integrating (70) with respect to k_0, following our original assumptions. This makes it possible to approximate the integral with respect to k_0 in (70) by the residues at the nucleon and antinucleon poles of the corresponding propagators. As a result, we obtain the following expression for the hadron tensor:

$$W^{\rm D}_{\mu\nu}(M_{\rm D},q)=\int\frac{d^3k}{(2\pi)^3}\frac{m^2}{(M_{\rm D}-2E)^2}\Bigg\{\Phi^2_{++}(M_{\rm D},k)\times W^N_{\mu\nu}(\mathbf{k},q)+(M_{\rm D}-2E)\frac{\partial}{\partial k_0}(W^N_{\mu\nu}(k,q)\times\Phi^2_{++}(M_{\rm D},k))_{k_0=k_0^N}+\frac{(M_{\rm D}-2E)^2}{M_{\rm D}^2}\times\Bigg[\Phi^2_{+-}(M_{\rm D},k)W^N_{\mu\nu}(\mathbf{k},q)+\Phi^2_{-+}\times(M_{\rm D},k)\tilde{W}^N_{\mu\nu}(\mathbf{k},q)+M_{\rm D}\frac{\partial}{\partial k_0}(W^N_{\mu\nu}(k,q)\times\Phi^2_{+-}(M_{\rm D},k))_{k_0=k_0^N}+M_{\rm D}\frac{\partial}{\partial k_0}(\tilde{W}^N_{\mu\nu}(k,q)\times\Phi^2_{-+}(M_{\rm D},k))_{k_0=k_0^N}+\frac{M_{\rm D}^2}{(M_{\rm D}+2E)}\cdot\times\frac{\partial}{\partial k_0}(\tilde{W}^N_{\mu\nu}(k,q)\Phi^2_{--}(M_{\rm D},k))_{k_0=k_0^N}+\frac{M_{\rm D}^2}{(M_{\rm D}+2E)^2}\Phi^2_{--}(M_{\rm D},k)\tilde{W}^N_{\mu\nu}(\mathbf{k},q)\Bigg]\Bigg\}. \tag{72}$$

This has the same form as the expansion in powers of the mass defect $(M_{\rm D}-2E)/M_{\rm D}$ of the nucleon in the deuteron. The first result following from this is that all the contributions associated with antinucleons are proportional to the square of the mass defect. Since this quantity is very small for the deuteron $[(M_{\rm D}-2E)/M_{\rm D}\approx 0.01]$, we can conclude that the contribution of antinucleons to the hadron tensor of the deuteron is significantly suppressed.

The main result of these calculations is the expression relating the hadron tensor of the deuteron to the hadron tensors of the on-shell nucleons and their derivatives near the mass shell. Now we can use Eq. (9) and obtain $F_2^{\rm D}$ by means of a projection operator:

$$\tilde{W}^N_j(q,k_i)=P^{\mu\nu}_j\tilde{W}^N_{\mu\nu}(k_i\cdot q,q^2,k_i^2).$$

In the Bjorken limit we can use the metric tensor $g_{\mu\nu}$ as this operator:

$$\lim_{Q^2\to\infty} q^{\mu\nu}W^{N(A)}_{\mu\nu}(P,q)=-\frac{1}{x}F_2^{N(A)}(x).$$

This operator is independent of the relative momentum, and so the derivative of the hadron tensor has the following form:

$$g^{\mu\nu}\frac{d}{dk_{i0}}\tilde{W}^N_{\mu\nu}(k_i,q)=\frac{d}{d(k_i\cdot q)}\tilde{W^N}(k_i\cdot q,q^2,k_i^2)\frac{d(k_i\cdot q)}{dk_{i0}}+2k_{i0}\frac{d}{dk_i^2}\tilde{W^N}(k_i\cdot q,q^2,k_i^2),\times\tilde{W^N}(k_i\cdot q,q^2,k_i^2)=g^{\mu\nu}\tilde{W}^N_{\mu\nu}(k_i,q). \tag{73}$$

The first term in the derivative of the hadron tensor reflects the modification of the structural properties of the bound nucleon determined by the x dependence of its structure functions. The second term reflects the changes in the hadron tensor of the nucleon associated with the change of the nucleon energy. Since the second term is proportional to k_{i0}, whose expectation value is small ($\langle k_0\rangle\simeq\langle M_{\rm D}-2E\rangle$), it can be discarded as a correction of second order in $(M_{\rm D}-2E)/M_{\rm D}$. This allows us to neglect the dependence of $\tilde{W}^N_{\mu\nu}$ on k_i^2:

$$\frac{d}{dk_0}\lim_{Q^2\to\infty}g^{\mu\nu}\tilde{W}^N_{\mu\nu}(P,q)|_{k_0=k_0^N}=\left[\frac{1}{x^2}F_2(x)-\frac{1}{x}\frac{d}{dx}F_2(x)\right]\left(\frac{dx}{dk_0}\right)_{k_0=k_0^N}. \tag{74}$$

Neglecting terms of order $(M_{\rm D}-2E)^2$, we can write the deuteron structure function in the following form:

$$F_2^{\rm D}(x_{\rm D})=\int\frac{d^3k}{(2\pi)^3}\frac{m^2}{4E^3(M_{\rm D}-2E)^2}\Bigg\{F_2^N(x_N)\left(\frac{E-k_3}{M_{\rm D}}+\frac{M_{\rm D}-2E}{2M_{\rm D}}\right)\Phi^2(M_{\rm D},k)-\frac{M_{\rm D}-2E}{M_{\rm D}}x_N\frac{dF_2^N(x_N)}{dx_N}\Phi^2(M_{\rm D},k)+F_2^N(x_N)\frac{F-k_3}{M_{\rm D}}(M_{\rm D}-2E)\frac{\partial}{\partial k_0}\Phi^2(M_{\rm D},k)\Bigg\}_{k_0=E-M_{\rm D}/2}. \tag{75}$$

According to (72), there is no need to neglect the additional structure functions in the representation for $\tilde{W}^N_{\mu\nu}$. The only assumption which must be made in using (9) for the nucleon amplitudes is that the derivatives of the off-shell structure functions are small near the mass shell.

Owing to the normalization condition for the Bethe–Salpeter vertex function, the distribution function in this expression for the deuteron SF satisfies the momentum sum rule

$$\int \frac{d^3k}{(2\pi)^3}\frac{m^2E}{4E^3(M_D-2E)^2}\left\{\left(\frac{E}{M_D}+\frac{M_D-2E}{2M_D}\right)\Phi^2(M_D,k)+\frac{M_D-2E}{M_D}\frac{\partial}{\partial k_0}\Phi^2(M_D,k)\right\}_{k_0=k_0^N}=\frac{M_D}{2} \quad (76)$$

and the baryon sum rule

$$\int \frac{d^3k}{(2\pi)^3}\frac{m^2}{4E^2M_D(M_D-2E)^2}\left\{\Phi^2(M_D,k)+\frac{M_D-2E}{M_D}\frac{\partial}{\partial k_0}\Phi^2(M_D,k)\right\}_{k_0=k_0^N}=1.$$

A physical interpretation for the various contributions in (75) can be proposed. For example, the first term corresponds to the contribution of scattering by the nucleon on the mass shell (the analog of the nonrelativistic impulse approximation). The second term is proportional to the mass defect and, since it can be reduced to one-nucleon contributions (on-shell), it is appropriate to interpret this term as the contribution of nuclear binding effects. The last term cannot be related to one-nucleon contributions, since it depends on the dynamics of the nucleon relative motion. For this reason it should be interpreted as a relativistic two-nucleon effect. A more transparent physical interpretation is possible if we take the nonrelativistic limit. Then we can compare it to the analytic expressions obtained in other nonrelativistic field-theoretical approaches.

4.2.2. The nonrelativistic limit

Let us expand the energy of the bound nucleon in (75) in powers of $\mathbf{p}^2/m^2$ and discard the relativistic two-nucleon corrections. This leads to the following expression for the deuteron structure function:

$$F_2^D(x_D)=\int\frac{d^3k}{(2\pi)^3}\left\{F_2^N(x_N)\left(1-\frac{k_3}{m}\right)\Psi^2(\mathbf{k})-\frac{-T+\epsilon}{wm}x_N\frac{dF_2^N(x_N)}{dx_N}\Psi^2(k)\right\}, \quad (77)$$

where $T=2E-2m$ is the nucleon kinetic energy and $\epsilon=M-2m$ is the binding energy.

In this expression we have introduced the analog of the nonrelativistic wave function $\Psi^2(\mathbf{k})$, related to $\Phi^2(M_D,k)$ as

$$\Psi^2(\mathbf{k})=\frac{m^2}{4E^2M_D(M_D-2E)^2}\{\Phi^2(M_D,k)\}_{k_0=E-M_D/2}.$$

The normalization condition for $\Psi^2(\mathbf{k})$ has the form

$$\int\frac{d^3k}{(2\pi)^3}\Psi^2(\mathbf{k})=1.$$

Let us compare (77) with the results of the calculations in the quasipotential approach:[90]

$$F_2^D(x_D)=\int\frac{d^3k}{(2\pi)^3}F_2^N(x_N)\left(1-\frac{k_3}{m}\right)\Psi^2(\mathbf{k})-\frac{-\langle T\rangle+\epsilon}{2m}x_D\frac{dF_2^N(x_D)}{dx_D}. \quad (78)$$

Here $\Psi(\mathbf{k})$ is the solution of the quasipotential equation with simultaneous nucleons.

Equations (77) and (78) obviously have practically the same structure. In the quasipotential calculation the term containing the derivative of the structure function arose as a result of including the meson corrections associated with the nucleon potential. It is this contribution which ensures that the ratio of the deuteron and nucleon structure functions differs from unity in the range $0.3<x<0.6$.

This result shows that the above interpretation of the first two contributions in (75) is in reasonable agreement with the ideas of the nonrelativistic theory.

In Fig. 3 we compare the results of the calculation using the quasipotential approach [Eq. (78), dashed line] and the approach studied above [Eq. (77), solid line]. The result of including only the first term in these expressions (the nonrelativistic impulse approximation) is shown by the dot–dash line. Comparing the behavior of these curves, we see that the balance between the first and second terms in (77) leads to a small decrease of the ratio F_2^D/F_2^N from unity in the same range of x as that where the EMC effect for heavy nuclei was observed. The difference of the ratio F_2^D/F_2^N from unity is somewhat smaller in the relativistic case, owing to the more complete inclusion of the Fermi motion. The parametrization of the nucleon SFs from Ref. 89 was used for the numerical calculations. The calculation with a more recent parametrization of the proton SF,[86] shown in Fig. 4, gives an additional suppression at small x, which is related to the rapid growth of the nucleon SF in this region.

Thus, the Bethe–Salpeter formalism allows the deuteron structure function to be expressed in terms of the SFs of the constituent proton and neutron. Here the antinucleon contribution is suppressed as the square of the mass defect. The inclusion of the dependence on the relative time in the amplitudes for DES on bound nucleons leads to a modification of the nucleon structure reminiscent of the EMC effect in heavy nuclei. This allows us to conjecture that the nature of the EMC effect can be attributed to the evolution of the off-shell deformation of the bound nucleon from $A=2$ to the values of A at which saturation of binding effects in the nucleon structure sets in.[2)]

Comparison of these expressions and the quasipotential results shows that the binding effects associated with meson corrections in these approaches can be reproduced in the Bethe–Salpeter formalism in the relativistic impulse approximation. Thus, the meson corrections of quasipotential approaches can be viewed as a parametrization of off-shell effects, the most important of which is the nonsimultaneity of the nucleons in the nucleus.

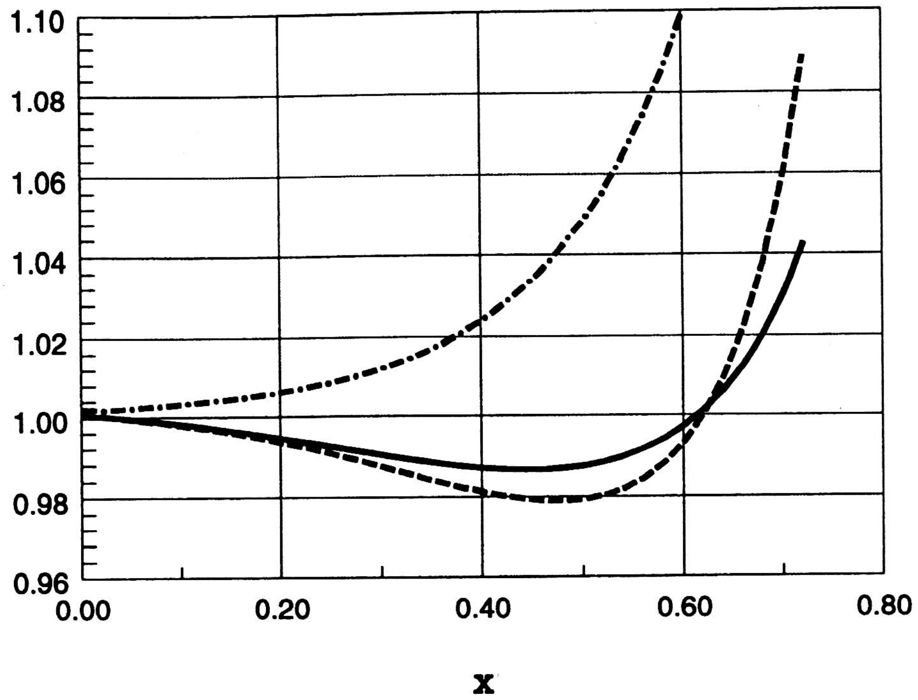

FIG. 3. The ratio of structure functions, $F_2^D/F_2^N(x)$. The parametrization of $F_2^N(x)$ is taken from Ref. 89. The solid line is the relativistic calculation,[90] the dashed line is the calculation in the quasipotential approach,[91] and the dot–dash line is the nonrelativistic impulse approximation [the first term in (77) and (78)].

5. THE STRUCTURE FUNCTIONS OF LIGHT NUCLEI

It is reasonable to use the approach developed above to study the evolution of the nucleon structure as a function of the atomic number A for the lightest nuclei D, ^{3}H, ^{3}He, and ^{4}He.

Strictly speaking, the EMC effect, which is interpreted by most authors as a decrease of the value of the SF of the *free nucleon* in the iron nucleus in the range $0.3<x<0.7$, most likely reflects the differences in the structure of the *deuterium* and helium nuclei. In fact, if we restrict ourselves to the range $10^{-3}<x<0.7$, it is easily checked that the form of the ratio $r(x)=F_2^{4\mathrm{He}}/F_2^{\mathrm{D}}$ is mimicked in heavier nuclei. The universality of the x dependence of the modification of the nucleon structure in nuclei with mass $A\geqslant 4$ was established in Refs. 28 and 29, where the world data on the DES of electrons and muons on nuclei were analyzed. This result obviously indicates that saturation of the modification of the structure function $F_2(x)$ already occurs in the helium nucleus and is manifested as an oscillation of $r^A(x)$ relative to the axis $r^A(x)=1$. The evolution of the modifications from $A=4$ to $A\sim 200$ is manifested as an increase of the oscillation amplitude $a_{\mathrm{EMC}}=1-r^A_{\min}$ by a factor of ~ 3 and is well described as the effect of evolution of the nuclear density.

The stopping of the modification of $F_2(x)$ for $A>4$ is demonstrated most clearly by the unchanging form of $r^A(x)$, fixed by the location of the three points $x_1=0.0615$, x_2

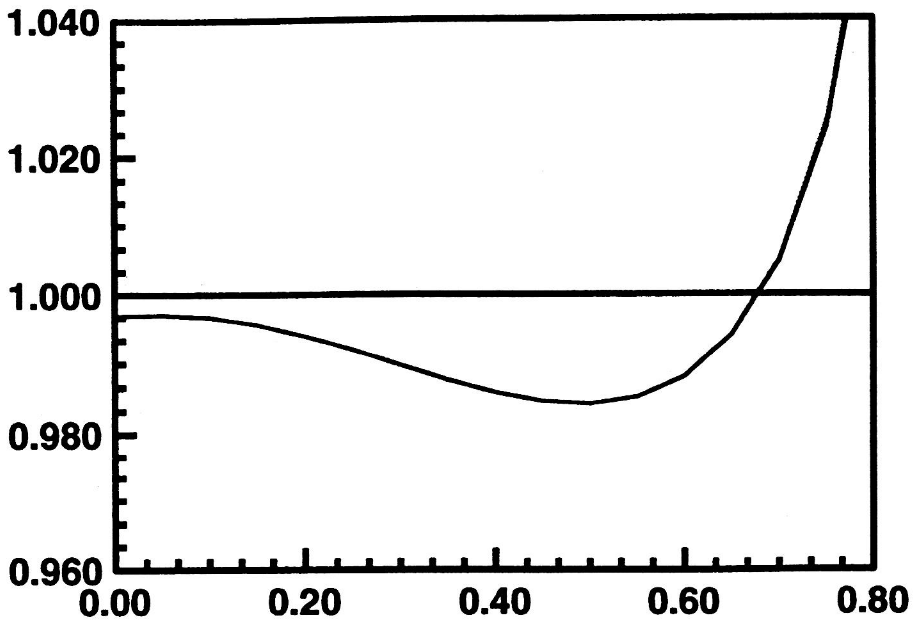

FIG. 4. The ratio of structure functions, $F_2^D/F_2^N(x)$. The parametrization of $F_2^N(x)$ is taken from Ref. 86.

$=0.287$, and $x_3=0.84$ at which $r^A(x)=1$ is independent of A (Ref. 30). The data used in the analysis of Ref. 30 pertain to the last generation of experimental studies to measure $r^A(x)$ and also the effect of modification of the structure functions $F_2^A(x)$ relative to the structure function $F_2^C(x)$ measured on a carbon target.[91] The measurement results are characterized by high statistical accuracy and high reliability, so that much more definite conclusions can be reached regarding the degree to which they disagree with models of the EMC effect. The key to understanding the reasons for this poor description is the presence of two effects instead of one, as has usually been assumed. The first is the rearrangement of the nucleon parton distributions in the field of the nuclear forces due to the two-, three-, or four-nucleon bound system. The second is the preservation of the form of the modification of the parton distributions in heavy nuclei. From our point of view, both effects are of fundamental importance for nucleon structure, and they cannot be understood without theoretical and experimental studies of the chain of modifications $r^D(x)\to r^{A=3}(x)\to r^{A=4}(x)$.

A special feature of very light nuclei is that the accuracy of the measurements for $A=4$ is low at $x>0.7$, while for $A<4$ there are no data at all on the modification of the nucleon structure. However, the preservation of the form of the ratios $r^A(x)$ for $A>4$ leaves us only two possibilities for explaining this phenomenon: the modification of the parton distributions in heavy nuclei is identical to either the modification for $A=3$ or to that for $A=4$. As was shown in Ref. 30, the uncertainty in the location of the point x_3 found from the data for $A>4$ is 0.01, so that it can be used to choose one or the other version of the evolution. There are already grounds for assuming that the parton distributions for $A=4$ have an x dependence considerably different from the dependence characteristic of heavy nuclei. This statement follows from the obvious discrepancy between the coordinates x_3 obtained by approximating the data for ^{4}He and Fe, shown in Fig. 5.

Here we shall study the derivation of the relative changes of the SF $F_2^A(x)$ in relation to the SF of the isoscalar nucleon, $F_2^N(x)=\frac{1}{2}[F_2^p(x)+F_2^n(x)]$, where p and n denote the free proton and free neutron. On the other hand, comparison with the experimental data can be made only for ratios of the structure functions of a nucleus A and the deuteron. This is why it is necessary to represent the results of the calculations both as the ratios A/N and as A/D. Since for the studied range of x $(0.3<x<0.9)$ the experiments (see Ref. 55 and the reviews of Refs. 8 and 9) indicate that $r^A(x)$ is independent of the 4-momentum transfer Q^2, the calculations will be performed in the Bjorken limit.

5.1. Generalization of the formalism for light nuclei

Let us consider the generalization of the approach developed in the preceding section for analyzing DES on light nuclei with $A=3,4$.

The amplitude $T^A_{\mu\nu}$ for forward Compton scattering is defined as the expectation value in nuclear states $|A\rangle$ of the T product of electromagnetic currents:

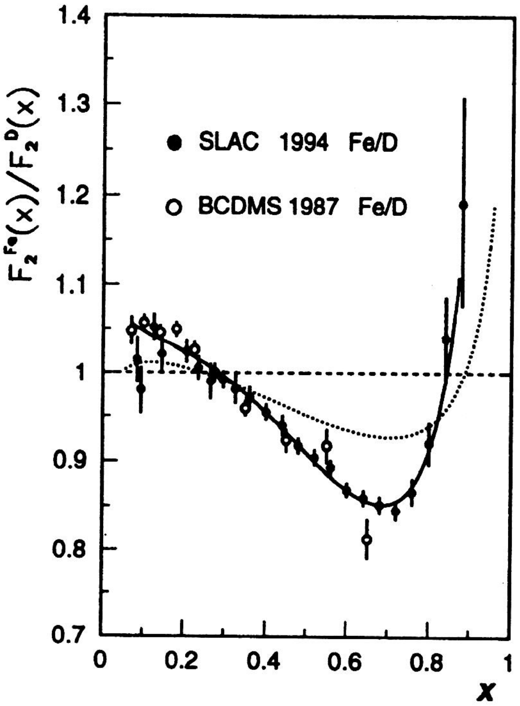

FIG. 5. Ratio of the nuclear structure function $F_2^A(x)$ and the deuteron structure function $F_2^D(x)$. The data for iron are taken from Ref. 55 (black circles) and from Ref. 92 (light circles). The approximation of these data is also shown as the solid line. The measurement results for ^{4}He/D, obtained in Ref. 55, are shown as a parametrization (dotted line).

$$T^A_{\mu\nu}(P,q)=i\int d^4x e^{iqx}\langle A|T(J_\mu(x)J_\nu(0))|A\rangle. \tag{79}$$

According to Eq. (33), $T^A_{\mu\nu}(P,q)$ can be rewritten in terms of the solutions of the Bethe–Salpeter equation for the nucleus, $\Gamma^A(P,k)$, and the n-nucleon Green functions $\bar{G}_{2(n+1)_{\mu\nu}}$:

$$\begin{aligned}T^A_{\mu\nu}(P,q)=\int &\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{n-1}}{(2\pi)^4}\frac{d^4k'_1}{(2\pi)^4}\cdots\frac{d^4k'_{n-1}}{(2\pi)^4}\\ &\times\Gamma^A(P,k_1\ldots k_{n-1})S_{(n)}(P,k_1\ldots k_{n-1})\\ &\times\bar{G}_{2(n+1)_{\mu\nu}}(q;P,k_1\ldots k_{n-1},k'_1\ldots k'_{n-1})S_{(n)}\\ &\times(P,k'_1\ldots k'_{n-1})\Gamma^A(P,k'_1\ldots k'_{n-1}),\end{aligned} \tag{80}$$

where k_i are the relative momenta of the nucleons in the nucleus and P is the total momentum of the nucleus. The function $\bar{G}_{2n}$ is the truncated irreducible n-nucleon Green function, defined by (40).

As in the two-nucleon case, it can be shown that the contribution of all the irreducible corrections to the interaction is suppressed as an additional power $1/(Q^2)^l$ with $l\geq 2$ (Ref. 42). This allows us to neglect all but the term of zeroth order in $\bar{G}_{2(n+1)_{\mu\nu}}$:

$$T^A_{\mu\nu}(P,q)=\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{n-1}}{(2\pi)^4}\Gamma^A(P,k_1\ldots k_{n-1})\times S_{2n}(P,k_1\ldots k_{n-1})\times\overline{G^{(0)}}_{2(n+1)_{\mu\nu}}\times(q;P,k_1\ldots k_{n-1})S_{2n}(P,k_1\ldots k_{n-1})\times\Gamma^A(P,k_1\ldots k_{n-1}),\quad(81)$$

where the Green function $\overline{G^{(0)}}_{2(n+1)_{\mu\nu}}$ is defined in terms of the truncated amplitude for Compton scattering on the nucleon, $\bar{G}_{4\mu\nu}(q,k_i)$:

$$\overline{G^{(0)}}_{2(n+1)_{\mu\nu}}(q,P,k_1\ldots k_{n-1})=\sum_i G_{4\mu\nu}(q,k_i)\otimes S^{-1}_{(n-1)}(k_1,\ldots k_{i-1},k_{i+1},\ldots k_{n-1}).\quad(82)$$

Substituting this expression into (81), we obtain the amplitude for Compton scattering on the nucleus in terms of the nucleon amplitudes:

$$T^A_{\mu\nu}(P,q)=\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{n-1}}{(2\pi)^4}\Gamma^A(P,k_1\ldots k_{n-1})\times\sum_i(S(P,k_i)G_{4\mu\nu}(q;P,k_i)S(P,k_i))\otimes S_{2n-1}(P,k_1,\ldots k_{i-1},k_{i+1},\ldots k_{n-1})\times\Gamma^A(P,k_1\ldots k_{n-1}).\quad(83)$$

In principle, we could carry out the rest of the calculations in complete analogy with the procedure developed for the deuteron, namely, we could isolate the amplitude for Compton scattering on the nucleon and perform an integration over the zeroth component of the relative momentum of the corresponding nucleon. However, in contrast to the two-nucleon case, the vertex function in (83) contains poles corresponding to nucleon–nucleon bound states lying at small relative momenta. If we had the exact solution of (49), we could take these singularities into account and express the Compton amplitude for the nucleus in terms of the nucleon one. However, it is impossible to do this at present, since the solution of (49) for $n\geq 3$ has not been found.

We shall use another method to isolate these singularities, namely, we shall introduce the "bare" Bethe–Salpeter vertex $\mathcal{G}^A$, which is regular in the relative momentum:

$$\Gamma^A(P,k_1,\ldots k_{n-1})=-\int\frac{d^4k'_1}{(2\pi)^4}\cdots\frac{d^4k'_{n-1}}{(2\pi)^4}g_{2n}\times(P,k_1,\ldots k_{n-1};k'_1,\ldots k'_{n-1})\times\mathcal{G}^A(P,k'_1,\ldots k'_{n-1}).$$

All the singularities present in the vertex $\Gamma^A(P,k_1,\ldots,k_{n-1})$ are now determined by the analytic properties of the part of the n-nucleon Green function which is regular in the total momentum at $P^2=M_A^2$:

$$g_{2n}(P,k_1,\ldots k_{n-1};k'_1,\ldots k'_{n-1})=\sum_{m=2}^{n}G_{2m}(P,k_1,\ldots k_{m-1};k'_1,\ldots k'_{m-1})\otimes S_{(n-m)}\times(P,k_1,\ldots k_{m-n-1};k'_1,\ldots k'_{m-n-1}).$$

For example, in the case of ^{3}He we have a pole in G_4 associated with the deuteron and the nucleon–nucleon continuum g_4:

$$G_4\left(\frac{2P}{3}+k,k_1,k'_1\right)=\frac{\Gamma^D\left(\frac{2P}{3}+k,k_1\right)\Gamma^D\left(\frac{2p}{3}+k,k'_1\right)}{\left(\frac{2P}{3}+k\right)^2-M_D^2}+g_4\left(\frac{2P}{3}+k,k_1,k'_1\right).\quad(84)$$

For ^{4}He there are additional poles associated with the ^{3}He and ^{3}H poles. For example, for the neutron–proton–proton Green function there is a ^{3}He pole and a three-nucleon continuum g_6:

$$G_6\left(\frac{3P}{4}+k,k_1,k'_1,k_2,k'_2\right)=\frac{\Gamma^{^3\mathrm{He}}\left(\frac{3P}{4}+k,k_1,k_2\right)\Gamma^{^3\mathrm{He}}\left(\frac{3P}{4}+k,k'_1,k'_2\right)}{\left(\frac{3P}{4}+k\right)^2-M^2_{^3\mathrm{He}}}+g_6\left(\frac{3P}{4}+k,k_1,k'_1,k_2,k'_2\right).$$

Substituting (84) into (83), we find, for example, for ^{3}He,

$$T^{^3\mathrm{He}}_{\mu\nu}(P,q)=\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4K}{(2\pi)^4}\mathcal{G}^{^3\mathrm{He}}(P,K,k)\times\left[\int\frac{d^4k_1}{(2\pi)^4}G_4\left(\frac{2P}{3}+K,k,k_1\right)\times S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\bar{G}_{4\mu\nu}\left(q;\frac{P}{3}+\frac{K}{2}+k_1\right)\times S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\otimes S\left(\frac{P}{3}+\frac{K}{2}-k_1\right)\times G_4\left(\frac{2P}{3}+K,k_1,k'\right)\right]\otimes S\left(\frac{P}{3}-K\right)\times\mathcal{G}^{^3\mathrm{He}}(P,K,k')+\mathcal{G}^{^3\mathrm{He}}(P,K,k)\times\left[\int\frac{d^4k_1}{(2\pi)^4}G_4\left(\frac{2P}{3}+K,k,k_1\right)\times S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\otimes S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\otimes S\left(\frac{P}{3}+\frac{K}{2}-k_1\right)G_4\left(\frac{2P}{3}+K,k_1,k'\right)\right]$$

$$\otimes S\left(\frac{P}{3}-K\right)\overline{G}_{4\mu\nu}\left(q;\frac{P}{3}-K\right)S\left(\frac{P}{3}-K\right)\mathcal{G}^{^3\mathrm{He}}(P,K,k'),$$

and, taking into account the analytic properties of the three-nucleon Green function (84), we obtain

$$T^{^3\mathrm{He}}_{\mu\nu}(P,q)=\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4K}{(2\pi)^4}\bar{\mathcal{G}}^{^3\mathrm{He}}(P,K,k)$$
$$\times\left[\int\frac{d^4k_1}{(2\pi)^4}\frac{\Gamma^{\mathrm{D}}\left(\frac{2P}{3}+K,k\right)\bar{\Gamma}^{\mathrm{D}}\left(\frac{2P}{3}+K,k_1\right)}{\left(\frac{2P}{3}+K\right)^2-M_{\mathrm{D}}^2}\right.$$
$$\times S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\overline{G}_{4\mu\nu}\left(q;\frac{P}{3}+\frac{K}{2}+k_1\right)$$
$$\times S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\otimes S\left(\frac{P}{3}+\frac{K}{2}-k_1\right)$$
$$\left.\times\frac{\Gamma^{\mathrm{D}}\left(\frac{2P}{3}+K,k_1\right)\bar{\Gamma}^{\mathrm{D}}\left(\frac{2P}{3}+K,k'\right)}{\left(\frac{2P}{3}+K\right)^2-M_{\mathrm{D}}^2}\right]$$
$$\otimes S\left(\frac{P}{3}-K\right)\mathcal{G}^{^3\mathrm{He}}(P,K,k')+\bar{\mathcal{G}}^{^3\mathrm{He}}(P,K,k)$$
$$\times\left[\int\frac{d^4k_1}{(2\pi)^4}G_4\left(\frac{2P}{3}+K,k,k_1\right)S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\right.$$
$$\left.\otimes S\left(\frac{P}{3}+\frac{K}{2}-k_1\right)G_4\left(\frac{2P}{3}+K,k_1,k'\right)\right]$$
$$\otimes\frac{u\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right)\bar{u}\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right)}{\left(\frac{P}{3}-K\right)^2-m^2}\overline{G}_{4\mu\nu}\left(q;\frac{P}{3}\right.$$
$$\left.-K\right)\frac{u\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right)\bar{u}\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right)}{\left(\frac{P}{3}-K\right)^2-m^2}\mathcal{G}^{^3\mathrm{He}}(P,K,k'). \quad (85)$$

Returning to the expression (80) for the amplitude of Compton scattering on the deuteron, we note that the structure

$$\int\frac{d^4k_1}{(2\pi)^4}\Gamma^{\mathrm{D}}\left(\frac{2P}{3}+K,k_1\right)S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\overline{G}_{4\mu\nu}\left(q;\frac{P}{3}\right.$$
$$\left.+\frac{K}{2}+k_1\right)S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\otimes S\left(\frac{P}{3}\right.$$
$$\left.+\frac{K}{2}-k_1\right)\Gamma^{\mathrm{D}}\left(\frac{2P}{3}+K,k_1\right), \quad (86)$$

contained in the first term of (85), exactly coincides with this amplitude, but contains an off-shell argument. In the second term of (85) we can isolate the amplitude for Compton scattering on the nucleon:

$$T^{\tilde{N}}_{\mu\nu}\left(\frac{P}{3}-K,q\right)=\bar{u}\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right)\overline{G}_{4\mu\nu}\left(q;\frac{P}{3}-K\right)u\left(\frac{\mathbf{P}}{3}-\mathbf{K}\right). \quad (87)$$

Thus, the amplitude for Compton scattering on ^{3}He can be expressed in terms of the amplitude for scattering on the off-shell deuteron and nucleon. Using the unitarity relation (5), we obtain the corresponding expression for the hadron tensor:

$$W^{^3\mathrm{He}}_{\mu\nu}(P,q)=\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4K}{(2\pi)^4}W^{\mathrm{D}}_{\mu\nu}\left(\frac{2P}{3}+K,q\right)$$
$$\times\bar{\mathcal{G}}^{^3\mathrm{He}}(P,K,k)\frac{\Gamma^{\mathrm{D}}\left(\frac{2P}{3}+K,k\right)\bar{\Gamma}^{\mathrm{D}}\left(\frac{2P}{3}+K,k'\right)}{\left(\left(\frac{2P}{3}+K\right)^2-M_{\mathrm{D}}^2\right)^2}$$
$$\otimes S\left(\frac{P}{3}-K\right)\mathcal{G}^{^3\mathrm{He}}(P,K,k')+\bar{\mathcal{G}}^{^3\mathrm{He}}(P,K,k)$$
$$\times\left[\int\frac{d^4k_1}{(2\pi)^4}G_4\left(\frac{2P}{3}+K,k,k_1\right)S\left(\frac{P}{3}+\frac{K}{2}+k_1\right)\right.$$
$$\left.\otimes S\left(\frac{P}{3}+\frac{K}{2}-k_1\right)G_4\left(\frac{2P}{3}+K,k,k'\right)\right]$$
$$\otimes S\left(\frac{P}{3}-K\right)\mathcal{G}^{^3\mathrm{He}}(P,K,k')\frac{W^N_{\mu\nu}\left(\frac{P}{3}-K\right)}{\left(\frac{P}{3}-K\right)^2-m^2}.$$

Assuming that the relative energy of the fragments is small, we can integrate over the zeroth component of the relative momentum of the fragments and obtain the hadron tensor for ^{3}He expressed in terms of the hadron tensors of these fragments on the mass shell. The amplitudes for scattering on ^{3}H and ^{4}He can be obtained in a similar manner.

5.2. The nuclear structure functions for $A=3,4$

Now let us use the result obtained in the preceding subsection to calculate the SFs of the ^{3}H, ^{3}He, and ^{4}He nuclei.

Acting on the hadron tensor with the projection operator $g_{\mu\nu}$ and introducing the variables

$$x_A=\frac{Q^2}{2P_A\cdot q},$$

$$x_N=\frac{Q^2}{2P_N\cdot q},$$

we isolate the nuclear SF in the Bjorken limit:

$$\lim_{Q^2\to\infty} g^{\mu\nu}W^{N(A)}_{\mu\nu}(P,q)=-\frac{1}{x}F_2^{N(A)}(x).$$

Thus, we obtain F_2^A for ^{3}He and ^{3}H in the form

$$F_2^{^3\mathrm{He}}(x_{^3\mathrm{He}})=\int\frac{d^3k}{(2\pi)^3}\left[\frac{E_p-k_3}{E_p}F_2^p(x_p)+\frac{E_\mathrm{D}-k_3}{E_\mathrm{D}}F_2^\mathrm{D}(x_\mathrm{D})+\frac{\Delta_p^{^3\mathrm{He}}}{E_p}x_p\frac{dF_2^p(x_p)}{dx_p}+\frac{\Delta_p^{^3\mathrm{He}}}{E_\mathrm{D}}x_\mathrm{D}\frac{dF_2^\mathrm{D}(x_\mathrm{D})}{dx_\mathrm{D}}\right]\Phi_{^3\mathrm{He}}{}^2(\mathbf{k}). \quad (88)$$

$$F_2^{^3\mathrm{H}}(x_{^3\mathrm{H}})=\int\frac{d^3k}{(2\pi)^3}\left[\frac{E_n-k_3}{E_n}F_2^n(x_n)+\frac{E_\mathrm{D}-k_3}{E_\mathrm{D}}F_2^\mathrm{D}(x_\mathrm{D})+\frac{\Delta_n^{^3\mathrm{H}}}{E_n}x_n\frac{dF_2^n(x_n)}{dx_n}+\frac{\Delta_n^{^3\mathrm{H}}}{E_\mathrm{D}}x_\mathrm{D}\frac{dF_2^\mathrm{D}(x_\mathrm{D})}{dx_\mathrm{D}}\right]\Phi_{^3\mathrm{H}}{}^2(\mathbf{k}), \quad (89)$$

and for ^{4}He in the form

$$F_2^{^4\mathrm{He}}(x_{^4\mathrm{He}})=\int\frac{d^3k}{(2\pi)^3}\left[\frac{E_p-k_3}{E_p}F_2^p(x_p)+\frac{E_{^3\mathrm{H}}-k_3}{E_{^3\mathrm{H}}}F_2^{^3\mathrm{H}}(x_{^3\mathrm{H}})+\frac{\Delta_p^{^4\mathrm{He}}}{E_p}x_p\frac{dF_2^p(x_p)}{dx_p}+\frac{\Delta_p^{^4\mathrm{He}}}{E_{^3\mathrm{H}}}x_{^3\mathrm{H}}\frac{dF_2^{^3\mathrm{H}}(x_{^3\mathrm{H}})}{dx_{^3\mathrm{H}}}+\frac{E_n-k_3}{E_n}F_2^n(x_n)+\frac{E_{^3\mathrm{He}}-k_3}{E_{^3\mathrm{He}}}F_2^{^3\mathrm{He}}(x_{^3\mathrm{He}})+\frac{\Delta_n^{^4\mathrm{He}}}{E_n}x_n\frac{dF_2^n(x_n)}{dx_n}+\frac{\Delta_n^{^4\mathrm{He}}}{E_{^4\mathrm{He}}}x_{^3\mathrm{He}}\frac{dF_2^{^3\mathrm{He}}(x_{^3\mathrm{He}})}{dx_{^3\mathrm{He}}}\right]\Phi_{^4\mathrm{He}}{}^2(\mathbf{k}), \quad (90)$$

where $\Delta_N^A=-M_A+E_N+E_{A-1}$ is the binding energy of the corresponding nuclear fragment.

The three-dimensional momentum distribution $\Phi_A^2(\mathbf{k})$ is defined in terms of the bare Bethe–Salpeter vertex. For example, for ^{3}He we can write

$$\Phi_{^3\mathrm{He}}{}^2(\mathbf{k})=\frac{mM_\mathrm{D}}{4E_pE_\mathrm{D}M_{^3\mathrm{He}}(M_\mathrm{D}-E_p-E_\mathrm{D})^2}\times\left\{\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_1'}{(2\pi)^4}\bar{\mathcal{G}}^{^3\mathrm{He}}(P,k,k_1)S_2\left(\frac{2P}{3}+k,k_1\right)\Gamma^\mathrm{D}\left(\frac{2P}{3}+k,k_1\right)\Gamma^\mathrm{D}\left(\frac{2P}{3}+k,k_1'\right)\times S_2\left(\frac{2P}{3}+k,k_1'\right)\otimes\left(\sum_s u_\alpha^s(\mathbf{k})\bar{u}_\delta^s(\mathbf{k})\right)\times\mathcal{G}^{^3\mathrm{He}}(P,k,k_1')\right\}_{k_0=k_{0_p}}, \quad (91)$$

where $k_{0_p}=M_{^3\mathrm{H}}/3-E_p$.

Since at present there are no realistic solutions of the Bethe–Salpeter equation for a bound system of three or more nucleons, it is necessary to use the phenomenological momentum distribution. It is reasonable to assume that the momentum distributions in (88), (89), and (90) can be related to the distributions extracted from the experimental data. For the numerical calculations we shall use the distributions obtained in Refs. 93 and 94.

The result obtained for the structure functions of the lightest nuclei can be related to the result obtained in the x-rescaling model.[10,11] As an example, let us consider the structure function of ^{3}He. If we regard the integrand in (88) as the first terms of the expansion in the binding energy and take into account the fact that terms above the first order in this quantity are negligible, we can add higher-order terms in such a way that the resulting series can be convolved. As a result, we find the following expression for the ^{3}He structure function:

$$F_2^{^3\mathrm{He}}(x_{^3\mathrm{He}})=\int dy\,d\epsilon\left\{F_2^p\left(\frac{x_{^3\mathrm{He}}}{y-\frac{\epsilon}{M_{^3\mathrm{He}}}}\right)f^{p/^3\mathrm{He}}(y,\epsilon)+F_2^\mathrm{D}\left(\frac{x_{^3\mathrm{He}}}{y-\frac{\epsilon}{M_{^3\mathrm{He}}}}\right)f^{\mathrm{D}/^3\mathrm{He}}(y,\epsilon)\right\}, \quad (92)$$

where $\epsilon=\Delta_p^{^3\mathrm{He}}$ can be interpreted as the separation energy of the corresponding nuclear fragment, and $f^{p(\mathrm{D})/^3\mathrm{He}}(y,\epsilon)$ as the spectral function for the bound proton (deuteron):

$$f^{p(\mathrm{D})/^3\mathrm{He}}(y,\epsilon)=\int\frac{d^3k}{(2\pi)^3}\Phi_{^3\mathrm{He}}{}^2(\mathbf{k})\frac{m}{E_{p(\mathrm{D})}}y\,\delta\left(y-\frac{E_{p(\mathrm{D})}-k_3}{m}\right)\delta(\epsilon-(E_p+E_\mathrm{D}-M_{^3\mathrm{He}})).$$

It should be stressed that both the modification of F_2^N and its evolution from $A=1$ to 4 found in the approach developed in Refs. 42–44 are consequences of the relativistic description of the nuclear structure. In the analytic calculations essential use was made of the fact that the nucleons in the nucleus behave like nonsimultaneous (asynchronous) objects. It is this feature which is the cause of the nuclear-binding effect in $F_2^A(x)$, which appears as a result of the dependence of the hadron tensor of the bound nucleon on τ_i. The approach that we have developed possesses two advantages. First, it allows the results of nonrelativistic models (for example, Ref. 11), in which relativistic effects are introduced by means of parametrizations, to be reproduced in a natural way. Second, the results[44] can be clearly understood by comparison with the results of the x-rescaling model.[11] In fact, by comparing (74) and (88) with (92), it can be shown that rescaling of the Bjorken variable x follows from the dependence of the relative time τ_i contained in the Compton amplitude of the off-shell nucleon. We note that the relation between the nucleon mass and its four-dimensional localization region $r^2\sim 1/m^2$ suggests that the dependence on the relative time τ_i should tend to increase the nucleon localization region. This conclusion is in some sense reminiscent of the model treatment of the increase of the deconfinement radius or nucleon swelling.[8]

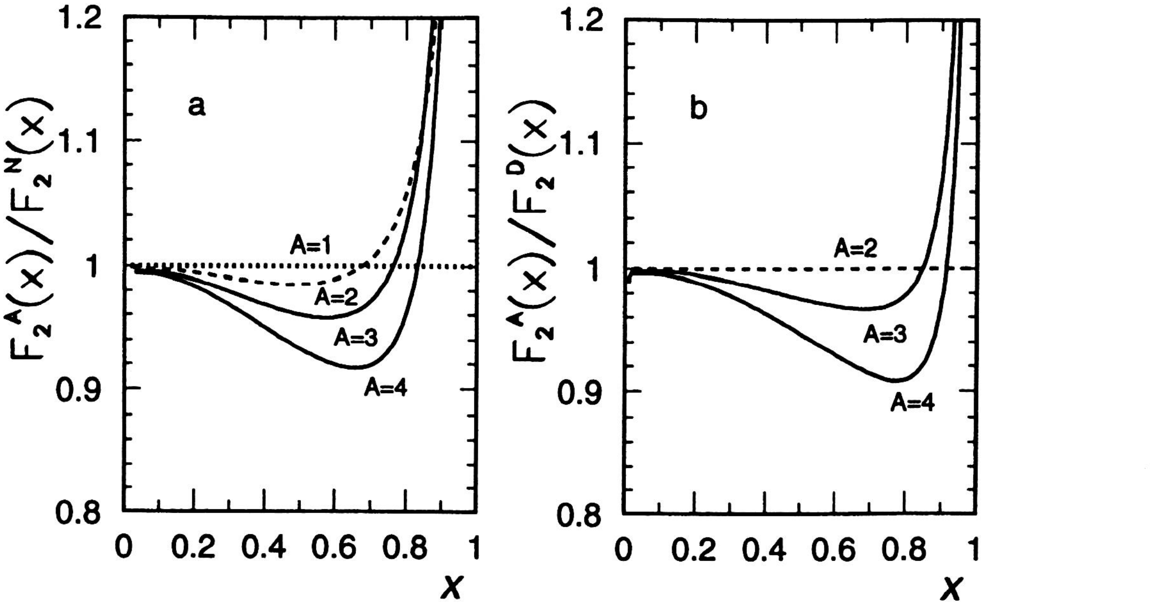

FIG. 6. (a) The ratio of structure functions $F_2^A(x)/F_2^N(x)$. (b) The ratio of the structure functions $F_2^A(x)$ $(A=4)$ and $[F_2^{3\mathrm{H}}(x)+F_2^{3\mathrm{He}}(x)]/2$ to the deuteron structure function $F_2^{\mathrm{D}}(x)$. The dashed line in Fig. 6a shows the result of the calculations for the deuteron, $A=2$. The results for $A=3,4$ are shown by the solid lines.

Nuclear-binding effects are represented in (88) and (90) as the first derivatives of the structure functions of nuclear fragments. Consequently, the structure functions $F_2^{n(p)}(x)$ entering into the expressions determine not only the internal nucleon structure, but also the two-nucleon interaction dynamics. Similarly, $F_2^{\mathrm{D}}(x)$ is responsible for the structure of the two-nucleon bound state and for the dynamics of three-nucleon interactions. According to (75), the derivative of $F_2^{\mathrm{D}}(x)$ can be expressed in terms of the first and second derivatives of F_2^N with the corresponding coefficients. Since the off-shell deformation of the structure of the bound deuteron is determined by the second derivative of F_2^N, it is this term which is responsible for the three-nucleon dynamics. However, the second derivative of F_2^N enters into $F_2^{3\mathrm{He}}$ with a very small coefficient, $\Delta_{\mathrm{D}}^{3\mathrm{He}}\Delta_p^{\mathrm{D}}$, which allows the three-nucleon dynamics to be neglected when studying nuclear-binding effects in DES.

The nucleon structure function needed for the calculations is introduced by parametrizing the data obtained in experiments on lepton DES on protons and deuterons. We have used the parametrization of $F_2^p(x,Q^2)$ obtained recently in Ref. 86. We have found the structure function $F_2^n(x)$ from the measurements of $F_2^p(x)$ and the ratio $F_2^n(x)/F_2^p(x)$ obtained in Ref. 85. Additional calculations in which we used different parametrizations showed that the uncertainties in the measurements of the absolute values of $F_2^{p(n)}(x)$ are suppressed in the calculation of the ratio $r^A(x)$.

The results of the numerical calculations, which show how the SF of the free nucleon, $F_2^N(x)$ $(A=1)$, is related to those of the deuteron $(A=2)$ and helium ($A=3$ and 4), are given in Fig. 6a. The same modifications calculated with respect to $F_2^{\mathrm{D}}(x)$ are shown in Fig. 6b. In contrast to the modifications observed for heavy nuclei with masses $A>4$, the form of the oscillations of the ratio $r^A(x)$ changes for $A\leqslant 4$, causing the coordinate of the intersection point x_3 to move toward larger values of x.

The modifications calculated using the ratio with $F_2^N(x)$ (Fig. 6a) are not only of academic interest. We can use them to show that the distortions of the structure of a nucleon in the deuterium nucleus cannot be viewed as negligible, and the relation $F_2^A(x)/F_2^{\mathrm{D}}(x)\approx F_2^A(x)/F_2^N(x)$ cannot be considered justified. In fact, it follows from the results of the calculations for $A=3$ that the location of x_3 is shifted by 0.08 if $F_2^N(x)$ is replaced by $F_2^{\mathrm{D}}(x)$ (Fig. 6b). This shift is eight times larger than the experimental error in the coordinate x_3 found from a recent analysis of the measurements of $F_2^A(x)/F_2^{\mathrm{D}}(x)$ (Ref. 30). According to Ref. 30, $\overline{x_3}=0.84 \pm 0.01$, independently of A if $A>4$. This accuracy allows the modification of the *deuteron* structure to be reliably distinguished from that of the structure of the *free nucleon.*

It is interesting that the value of $(1-x_3)$, which is ~ 0.32 for $F_2^{\mathrm{D}}(x)/F_2^N(x)$, decreases to ~ 0.16 and ~ 0.08 for the ratios $F_2^{A=3}(x)/F_2^{\mathrm{D}}(x)$ and $F_2^{4\mathrm{He}}(x)/F_2^{\mathrm{D}}(x)$, respectively. Further evolution of the modifications of $F_2^N(x)$ in nuclei heavier than ^{4}He is forbidden by the Pauli principle. It follows from the shape of the curves shown in Fig. 6a and also from the relations between the coordinates of the intersection points x_3 that the modification of the nucleon structure evolves as a saturated process, which is completely consistent with the idea of the rapid saturation of the nuclear binding forces. This phenomenon allows the introduction of a class of x-dependent modifications due to nuclear-binding effects. Within this class there are no mechanisms which could lead to further changes of the form of $r^A(x)$ occurring during the first stage of the evolution, for $A\leqslant 4$. The evolution of the modifications to heavier nuclei, where the EMC effect was discovered, must occur independently of x and should be viewed as the second stage.[29] The concept of the

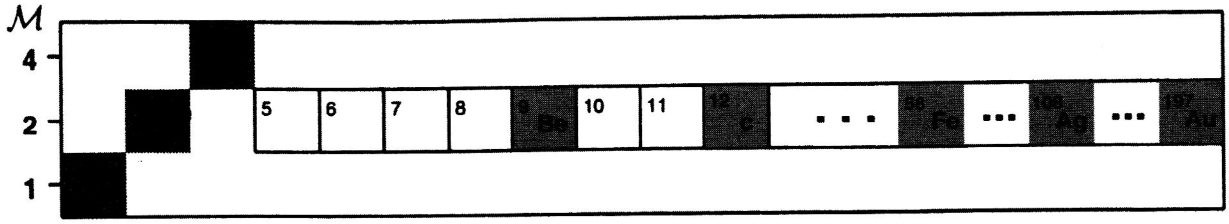

FIG. 7. Degree of distortion of the nucleon structure $\mathcal{M}$ as a function of the mass number A.

evolution of the structure of a nucleon located in a nuclear medium as a two-stage process is crucial for understanding the long-standing problem of the EMC effect.

The results that we have obtained, in particular, the values of $(1-x_3)$, can be used to introduce a quantitative characteristic corresponding to the modification of the nucleon structure as a function of the mass number A. For example, taking as the unit of modification the quantity $\mathcal{M}=1/(1-x_3)$ corresponding to the case of D/N, we find that the modifications of the nucleon structure in nuclei with $A=2$, 3, and 4 occur in the ratio 1:2:4. On the other hand, analysis of the data on nuclei for $A=9-197$ shows that x_3 is independent of A and has average value $\overline{x_3}=0.84\pm0.01$ (Ref. 30). In terms of $\mathcal{M}$, the value of the modification in this case is $\mathcal{M}=2$. It is easily shown that when the nucleon SF is described by the very simple dependence $F_2\sim(1-x)^3$, the characteristic that we have introduced is related to the derivative of F_2 and the actual value of F_2 at the intersection point: $\mathcal{M}\sim F_2'/F_2|_{x=x_3}$. The dependence of $\mathcal{M}$ on the mass number A, shown in Fig. 7, is a convenient illustration of the concept of the two-stage evolution of the nucleon modifications. Nuclei for which $\mathcal{M}$ is reliably determined from the experimental data are located in the region $A\geqslant 9$ and are shown by the shaded cells in Fig. 7.

Since there are no experimental data for nuclei with $A=2,3$, the predictions of Refs. 43 and 44 can be checked only by comparison with the results for the ratio $F_2^{4\text{He}}(x)/F_2^{\text{D}}(x)$, given in Refs. 55 and 95 and shown in Fig. 8. The location of the point where the calculated curve intersects the line $r(x)=1$ corresponds to $x_3=0.913$, which is in good agreement with the data. The small systematic deviation of the curve for $x<0.2$ can be attributed to the parametrization of the nucleon data used to calculate the derivative of F_2^N. Insignificant changes of the parametrization allowing the points x_1 and x_2 to be reproduced lead to a significant improvement of the agreement between theory and experiment for $x<0.2$ (see Fig. 9).

The result of the calculation for $A=3$ is in good agreement with the form of the modification found in experiments on much heavier nuclei, such as iron, silver, and gold nuclei. The agreement is due to the fact that the value $x_3=0.845$ obtained from the theory agrees with experiment within the error. As a result, the modification of the SF ratio in the region $A>4$ occurs simply as an increase of the amplitude of the deviations from the line $r^A(x)=1$ without any change of the x dependence. As was shown in Ref. 30, the results of all experiments on nuclei (except for ^{4}He) are in excellent agreement with the calculation for $A=3$ with the introduction of a scale parameter. The A dependence of the scale parameter is related to the change of the average nuclear density and can be calculated by using the Woods–Saxon potential. In Fig. 10 we give the results of calculating the modification of the nucleon structure in a three-nucleon system (the light shaded region) and show how the nucleon structure evolves in the ^{12}C, ^{56}Fe, and ^{197}Au nuclei (different degrees of shading).

The analysis performed in Refs. 28–30 shows that the second stage $(A>4)$ of the evolution of the nucleon structure in nuclei occurs without distortion of the parton distributions of the nuclear medium. This fact allows the EMC effect to be viewed as a special case of the modifications studied for $A\leqslant 4$. The recurrence of the effect obtained in the calculations for the three-nucleon system in heavy nuclei may indicate that the nuclear forces have the same topology for $A=3$ and $A\geqslant 4$.

5.3. The Gottfried sum rule

Strictly speaking, the experimental verification of the QCD sum rules requires determination of the neutron structure function $F_2^n(x)$ in the entire range of the variable x. It is

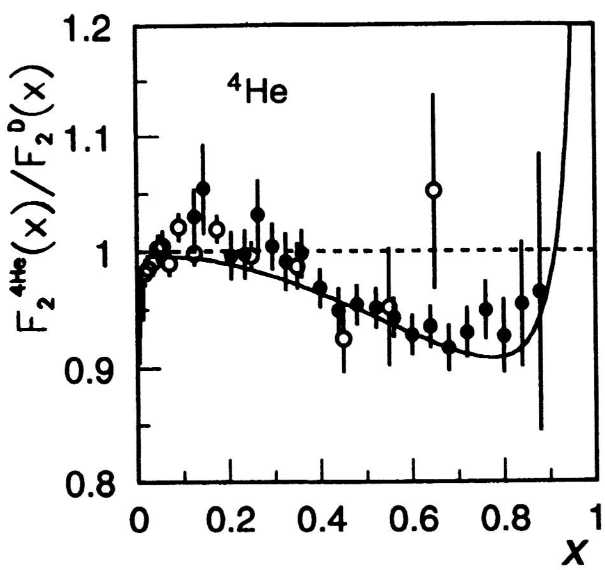

FIG. 8. Results of the calculations of $F_2^{4\text{He}}(x)/F_2^{\text{D}}(x)$ performed in Refs. 43 and 44 (solid line). The experimental results are shown by the dark[55] and light[95] circles.

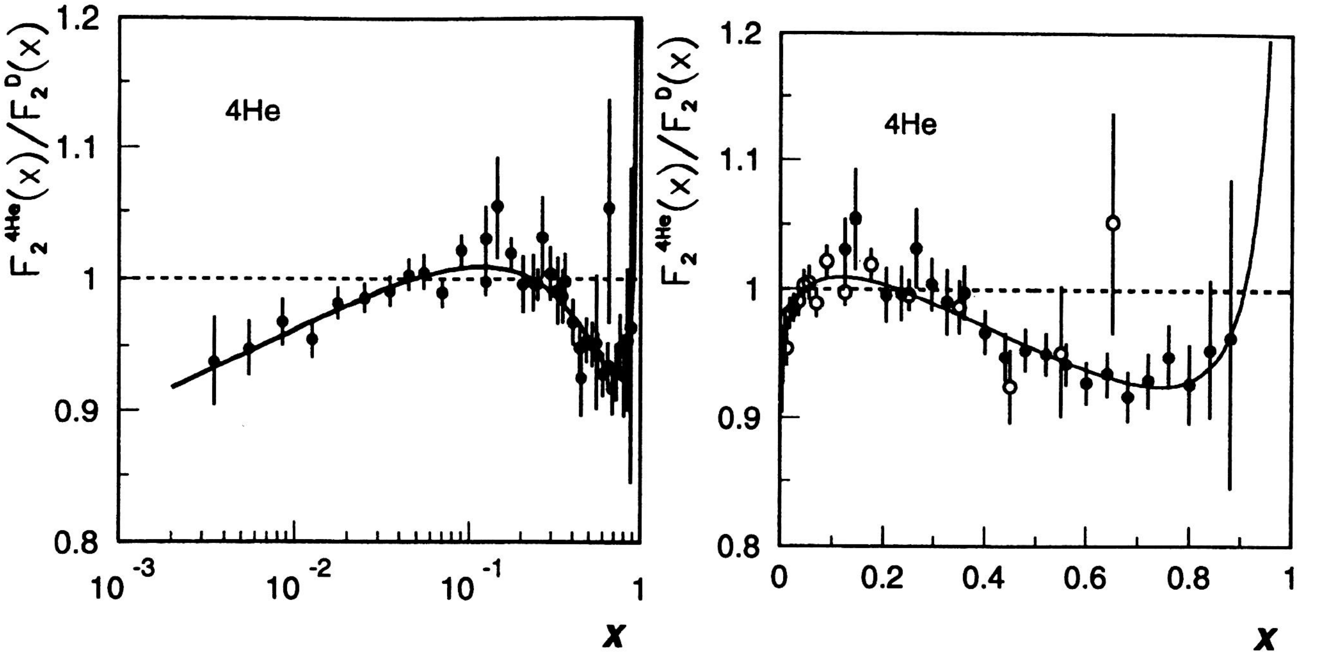

FIG. 9. Results of calculations of $F_2^{4He}(x)/F_2^D(x)$ with a modified parametrization of F_2^N (solid line). The experimental results are shown by the dark[55] and light[95] circles.

therefore natural to expect that the effects studied above will prove important in analyzing the experimental data.

Equation (77) shows that the integral

$$I_D=\int_0^1\frac{dx}{x}\left(2F_2^p(x)-2F_2^D(x)\right),$$

which is usually used to test the Gottfried sum rule experimentally,[87] coincides with the Gottfried integral, apart from a correction proportional to $F_2^N(x=0)$:

$$I_D=\int_0^1\frac{dx}{x}(F_2^p(x)-F_2^n(x))$$

$$-2\frac{\langle M_D-2E_D\rangle_D}{m}F_2^N(x=0).$$

Since the value of the nucleon SF at the origin is unknown and, judging from the recent experimental data,[87] grows rapidly for $x\rightarrow 0$, the value extracted in this way gives a greater

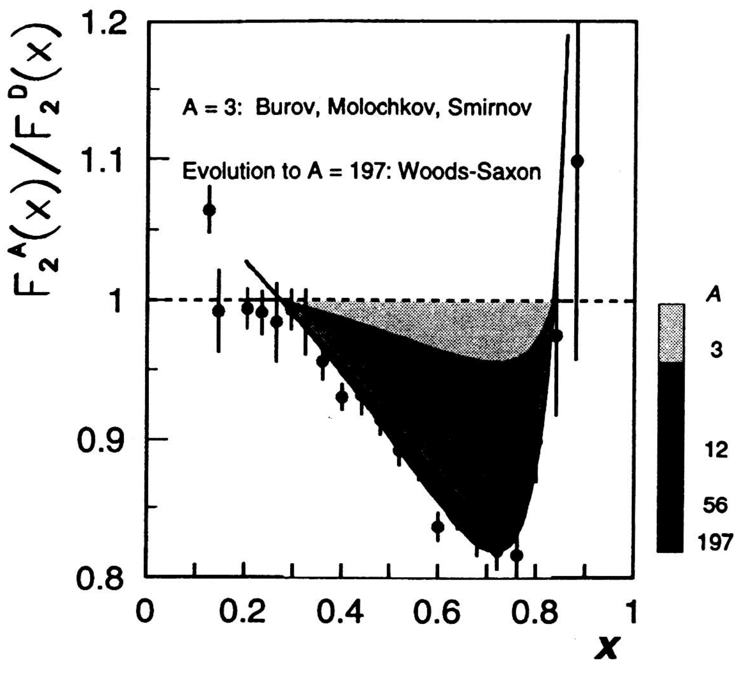

FIG. 10. Evolution of the nucleon structure in nuclei from $A=3$ to $A=197$, obtained by multiplying $F_2^{A=3}(x)/F_2^D(x)$ by a scale parameter. The A dependence of the parameter is determined by the Woods–Saxon potential and is shown by the different degrees of shading. The results of measurements on a gold target were obtained in Ref. 55.

overestimate of the true value of the Gottfried integral, the closer to the origin is the lower limit of integration in an actual experiment.

In recent experiments, the NMC collaboration has found that $I_D(0.004)=0.2281\pm0.0065$ (Ref. 87). It is easily shown that by going to smaller and smaller x in this experiment, at some $x_0=x_1$ it is possible to obtain a value of I satisfying the Gottfried sum rule, $I_D(x_1)=\frac{1}{3}$, and then to overestimate it.

Experiments on DES on the deuteron and proton therefore suggest that the Gottfried sum rule is violated. Meanwhile, there is no reason to assume that the Gottfried integral can be estimated from the available data with any *a priori* specified accuracy.

A quite different situation can arise in analyzing the experimental data on DES on ^{3}He and ^{3}H. Let us calculate the Gottfried integral by replacing the proton and neutron SFs by the SFs of ^{3}He and ^{3}H obtained above [Eqs. (88) and (89)]. As a result, we obtain the expression

$$I_{^3\mathrm{H}}=\int_0^1\frac{dx}{x}(F_2^{^3\mathrm{He}}(x)-F_2^{^3\mathrm{H}}(x))$$
$$=\int_1^0\frac{dx}{x}(F_2^p(x)-F_2^n(x))$$
$$-2\frac{\langle\Delta^p_{^3\mathrm{He}}\rangle^{^3\mathrm{He}}}{m}(F_2^p(x=0)-F_2^n(x=0)).$$

Assuming isospin symmetry of the quark sea for $x\to0$, we can conclude that the last term vanishes and $I_{^3\mathrm{H}}$ coincides with the Gottfried integral. Therefore, an experiment using ^{3}He and ^{3}H targets would allow verification of the Gottfried sum rule independently of model uncertainties and without the need to include nuclear-binding effects.

Let us summarize the results of this section.

The model-independent method developed on the basis of the approach described in Sec. 3 has allowed the SFs of light nuclei to be calculated in terms of the SFs of nuclear fragments and the three-dimensional momentum distribution. As a result, the SF $F_2^A(x)$ was calculated without resorting to numerical solution of Eqs. (49) and (84).

The behavior found for the ratios of SFs of the light nuclei D, ^{3}He, and ^{4}He to the SF of the free nucleon indicates that the modification of the nucleon structure in very light nuclei is a manifestation of the space-like deformation of the parton distribution in the bound nucleon.

The results of the calculations of the ratios of the SFs for ^{3}He and ^{4}He to the SF of the deuteron, which will be checked at TJNAF and HERA, can be regarded as predicting that the EMC effect on heavier nuclei can be understood as a modification of the structure of a nucleon bound in ^{3}He which is enhanced by nuclear-density effects.

These results show that in the region of the EMC effect $(0.3<x<0.9)$ two-nucleon interactions can be viewed as the dominant mechanism in describing nuclear binding forces.

6. CONCLUSION

We have proposed a field-theoretical approach in which nucleon binding effects in the lightest nuclei can be studied relativistically. We have shown that nonsimultaneity of the nucleons in the nucleus is the necessary condition for modification of the nucleon structure.

We have developed a model-independent method allowing the SFs of the ^{3}H, ^{3}He, and ^{4}He nuclei to be calculated in terms of the SFs of the nuclear fragments. For the first time, using a unified approach, we have calculated the modifications of the structure of the free nucleon for the lightest nuclei, $A\leqslant4$. The results of the calculations for $A=2,3$ are the theoretical predictions for the evolution of the nucleon structure in few-nucleon systems.

It has been shown that the structure functions $F_2^{n(p)}(x)$ entering into the expressions for the hadron tensor determine not only the internal structure of the nucleon, but also the dynamics of two-nucleon interactions. The better agreement of the results of the calculations with the available experimental data indicates that nucleon pair interactions give the dominant contribution in the description of nuclear binding forces, at least in the well studied range $0.3<x<0.9$. This also suggests that the inclusion of the nonsimultaneity of the nucleons in the nucleus is the sufficient condition for correctly describing the modification of the nucleon structure.

Both the modification of F_2^N and its evolution from $A=1$ to 4 obtained in our approach are the result of the relativistic treatment of the nuclear structure. Starting from the mutual consistency of the results of theoretical calculations based on understanding of the deuteron structure and the experimental results for $A\geqslant4$, we arrive at the conclusion that the theoretical uncertainties for the ratio $F_2^D(x)/F_2^N(x)$ are small. This, in turn, allows us to reliably distinguish the modification of the deuteron structure from the modification of the structure of the free nucleon on the basis of the available experimental data.

We have found that the most typical, distinguishing feature of the modification of the nucleon structure in nuclei is the location of the coordinate x_3 at which $F_2^A(x)=F_2^D(x)$. It follows from the calculations that the value of $(1-x_3)$, which is ~0.32 for F_2^D/F_2^N, decreases to ~0.16 and ~0.08 for the ratios $F_2^{A=3}/F_2^D$ and $F_2^{^4\mathrm{He}}/F_2^D$, respectively. Choosing the value of $\mathcal{M}=1/(1-x_3)$ as a quantitative measure of the modification and taking it to be unity for the deuteron, we find that the degree of modification increases to $\mathcal{M}=2$ for ^{3}He and ^{3}H, and to $\mathcal{M}=4$ for ^{4}He. On the basis of these results, we have introduced a class of x-dependent modifications due to nuclear-binding effects. Within this class there are no mechanisms which could increase the degree of modification of the nucleon structure.

Our results also allow us to justify theoretically the idea of a two-stage evolution of the structure of a nucleon located in a nuclear medium, where the first stage occurs for $A\leqslant4$ and the second for $A>4$. The second stage of the evolution occurs independently of x and corresponds to degree of modification $\mathcal{M}=2$. The A dependence of the magnitude of the effect is described within the traditional ideas about nuclear structure.

Study of the class of modifications and introduction of the concept of two-stage evolution of the nucleon structure have proved decisive for understanding both the nature of the EMC effect and the reasons for the unsuccessful description of the effect obtained using the various models which have been proposed since the effect was first discovered. The EMC effect is a special case of the modifications studied for $A \leq 4$. The recurrence of the pattern of the effect obtained in the calculations for the three-nucleon system in heavy nuclei may suggest that the nuclear forces for $A=3$ and $A>4$ have the same topology.

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[1)]A more complete list of references on these models is given in the reviews of Refs. 7–9.

[2)]It is reasonable to study the nature of the off-shell deformation by analyzing the spin structure of the bound nucleon.

[1]F. E. Close, *An Introduction to Quarks and Partons* (Academic Press, London, 1979) [Russ. transl., Mir, Moscow, 1982].
[2]J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988).
[3]K. Gottfried, Phys. Rev. Lett. **19**, 1174 (1967).
[4]J. Ellis and R. L. Jaffe, Phys. Rev. D **9**, 1444 (1974); **10**, 1669 (1974).
[5]J. D. Bjorken, Phys. Rev. **168**, 1467 (1966); Phys. Rev. D **1**, 1376 (1970).
[6]J. J. Aubert *et al.*, Phys. Rev. Lett. **123**, 275 (1983).
[7]C. W. Wong, Phys. Rep. **136**, 1 (1986).
[8]M. Arneodo, Phys. Rep. **240**, 301 (1994).
[9]D. F. Geesman, K. Saito, and A. W. Thomas, Ann. Rev. Nucl. Part. Sci. **45**, 337 (1995).
[10]S. V. Akulinichev, S. A. Kulagin, and G. M. Vagradov, Phys. Lett. B **158**, 485 (1985).
[11]S. V. Akulinichev, Phys. Lett. B **357**, 451 (1995).
[12]C. H. Llewellyn Smith, Phys. Lett. B **128**, 107 (1983).
[13]M. Ericson and A. W. Thomas, Phys. Lett. B **128**, 112 (1983).
[14]A. I. Titov, Yad. Fiz. **40**, 76 (1983) [Sov. J. Nucl. Phys. **40**, 50 (1983)].
[15]S. Fredriksson, Phys. Rev. Lett. **52**, 724 (1984).
[16]J. P. Chen *et al.*, Phys. Rev. Lett. **66**, 1283 (1991).
[17]L. P. Kaptar', A. I. Titov, and A. Yu. Umnikov, Fiz. Élem. Chastits At. Yadra **22**, 839 (1991) [Sov. J. Part. Nucl. **22**, 406 (1991)].
[18]F. E. Close, R. G. Roberts, and G. G. Ross, Phys. Lett. B **129**, 346 (1983).
[19]F. E. Close, R. L. Jaffe, and R. G. Roberts, Phys. Lett. B **134**, 449 (1984).
[20]F. E. Close *et al.*, Phys. Rev. D **31**, 1004 (1985).
[21]J.-J. Yang and G.-L. Li, Z. Phys. C **76**, 287 (1997).
[22]Z. He *et al.*, Eur. Phys. J. C **4**, 301 (1998).
[23]D. Indumathi and Wei Zhu, Z. Phys. C **74**, 119 (1997).
[24]S. Kumano and K. Umekawa, Preprint SAGA-HE-130-98, Saga University (1998); hep-ph/9803359.
[25]I. Sick and D. Day, Phys. Lett. B **274**, 16 (1992).
[26]O. Benhar, V. R. Pandharipande, and I. Sick, Preprint JLAB-THY-98-12 (unpublished).
[27]S. Data *et al.*, Phys. Rev. Lett. **52**, 2344 (1984).
[28]G. I. Smirnov, Yad. Fiz. **58**, 1712 (1995) [Phys. At. Nuclei **58**, 1613 (1995)].
[29]G. I. Smirnov, Phys. Lett. B **364**, 87 (1995).
[30]G. I. Smirnov, Preprint E2-99-145, JINR, Dubna (1999); hep-ph/9901422; submitted to Eur. Phys. J. C **31**, (1998).
[31]C. Ciofi degli Atti and S. Liuti, Phys. Lett. B **225**, 215 (1989).
[32]C. Ciofi degli Atti and S. Liuti, Phys. Rev. C **41**, 1100 (1990).
[33]A. Bodek *et al.*, Phys. Rev. Lett. **50**, 1431 (1983).
[34]A. Bodek *et al.*, Phys. Rev. Lett. **51**, 534 (1984).
[35]H. J. Pirner and J. P. Vary, Phys. Rev. Lett. **46**, 1376 (1981).
[36]M. Chemtob and R. Peschanski, J. Phys. G **10**, 599 (1984).
[37]M. Chemtob and R. Peschanski, in *Proceedings of the Nineteenth Rencontre de Moriond*, La Plagne, France, 1984, edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1984), p. 651; Preprint SACLAY-Sph-T-84/49, Centre d'Etudes Nucleaires, Saclay (1984), unpublished.
[38]N. P. Zotov, V. A. Saleev, and V. A. Tsarev, Pis'ma Zh. Éksp. Teor. Fiz. **39**, 81 (1984) [JETP Lett. **39**, 97 (1984)].
[39]L. A. Kondratyuk and M. Zh. Shmatikov, Z. Phys. A **321**, 301 (1985).
[40]A. V. Efremov, Phys. Lett. B **174**, 219 (1986).
[41]L. L. Frankfurt and M. I. Strikman, Phys. Lett. B **183**, 254 (1987).
[42]V. V. Burov and A. V. Molochkov, Nucl. Phys. A **637**, 31 (1998); in *Proceedings of the Seventh Intern. Conf. on Symmetry Methods in Physics*, Dubna, Russia, 1995, edited by A. N. Sissakian and G. S. Pogosyan, E2-96-224, JINR, Dubna (1996), p. 78.
[43]V. V. Burov, A. V. Molochkov, and G. I. Smirnov, JINR Rapid Commun. No. 3[89]-98, 47 (1999).
[44]V. V. Burov, A. V. Molochkov, and G. I. Smirnov, Preprint nucl-th/9904050; submitted to Phys. Lett. B.
[45]K. Abe *et al.*, Phys. Rev. Lett. **79**, 26 (1997).
[46]Compass Proposal, CERN/SPSLC/P297, CERN, Geneva (1996); COMPASS Proposal Addendum 1, CERN/SPSLC 96-30, CERN, Geneva (1996).
[47]K. Ackerstaff *et al.*, Phys. Lett. B **404**, 383 (1997).
[48]M. J. Zuilhof and J. A. Tjon, Phys. Rev. C **22**, 2369 (1980).
[49]F. Gross and S. Luiti, Phys. Lett. B **356**, 157 (1995).
[50]N. Honzawa and Sh. Ishida, Phys. Rev. C **45**, 47 (1992).
[51]V. V. Burov *et al.*, in *Proceedings of the Third Intern. Symp. "Deuteron-95,"* edited by A. M. Baldin, JINR, Dubna, 1996, p. 99.
[52]L. P. Kaptari *et al.*, Phys. Rev. C **54**, 176 (1996).
[53]S. G. Bondarenko *et al.*, Yad. Fiz. **62**, 983 (1999) [Phys. At. Nuclei **62**, 917 (1999)].
[54]V. V. Burov *et al.*, in *Proceedings of the Seventh Intern. Seminar on High Energy Physics Problems,* JINR, Dubna, 1994, p. 90.
[55]J. Gomez *et al.*, Phys. Rev. D **49**, 4348 (1994).
[56]W. Melnitchouk, A. W. Schreiber, and A. W. Thomas, Phys. Rev. D **49**, 1199 (1994).
[57]S. G. Bondarenko *et al.*, Phys. Rev. C **58**, 3143 (1998).
[58]M. Ericson and A. W. Thomas, Phys. Lett. B **148**, 191 (1984).
[59]L. P. Kaptari, K. Yu. Kazakov, and A. Yu. Umnikov, Phys. Lett. B **293**, 219 (1992).
[60]F. Gross and S. Liuti, Phys. Rev. C **45**, 1374 (1992).
[61]S. Kulagin, G. Piller, and W. Weise, Phys. Rev. C **50**, 1154 (1994).
[62]V. V. Burov and A. V. Molochkov, in *Proceedings of the Third Intern. Symp. "Deuteron-95,"* edited by A. M. Baldin, JINR, Dubna, 1996, p. 114; Preprint E2-95-339, JINR, Dubna (1995).
[63]P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).
[64]B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E **6**, 515 (1997).
[65]K. Saito and A. W. Thomas, Phys. Lett. B **327**, 9 (1994).
[66]K. Saito and A. W. Thomas, Nucl. Phys. A **574**, 659 (1994).
[67]F. Low, Phys. Rev. **97**, 1392 (1995); H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **6**, 319 (1957).
[68]A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
[69]R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).
[70]R. H. Thompson, Phys. Rev. D **1**, 110 (1970).
[71]V. G. Kadyshevsky, Nucl. Phys. B **6**, 125 (1968).
[72]K. Holinde, K. Erkelenz, and R. Alzetta, Nucl. Phys. A **194**, 161 (1972).
[73]G. E. Brown and A. D. Jackson, *The Nucleon–Nucleon Interaction* (North-Holland, Amsterdam, 1976) [Russ. transl., Atomizdat, Moscow, 1979].
[74]V. A. Karmanov and I. S. Shapiro, Fiz. Élem. Chastits At. Yadra **9**, 327 (1978) [Sov. J. Part. Nucl. **9**, 134 (1978)].
[75]V. G. Kadyshevskiĭ, R. M. Mir-Kasimov, and N. B. Skachkov, Fiz. Élem. Chastits At. Yadra **2**, 635 (1972) [Sov. J. Part. Nucl. **3**, 69 (1972)].
[76]S. Mandelstam, Proc. R. Soc. London, Ser. A **233**, 248 (1955).
[77]A. Yu. Umnikov and F. C. Khanna, Phys. Rev. C **49**, 2311 (1994).
[78]V. V. Burov *et al.*, Yad. Fiz. **59**, 822 (1996) [Phys. At. Nucl. **59**, 784 (1996)].
[79]E. E. Salpeter and H. A. Bethe, Phys. Rev. **76**, 769 (1949).
[80]N. Nakanishi, *Graph Theory and Feynman Integrals* (Gordon and Breach, New York, 1971).
[81]K. Kusaka and A. G. Williams, Phys. Rev. D **51**, 7026 (1995); K. Kusaka, K. Simpson, and A. G. Williams, Phys. Rev. D **56**, 5071 (1997).

[82] G. Rupp and J. A. Tjon, Phys. Rev. C **41**, 471 (1990).
[83] S. G. Bondarenko, V. V. Burov, and S. M. Dorkin, in *Proceedings of the Intern. Conf. on Mesons and Nuclei at Intermediate Energies*, Dubna, 1994 (World Scientific, Singapore, 1994), p. 539.
[84] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[85] A. C. Benvenuti *et al.*, Phys. Lett. B **237**, 599 (1990).
[86] B. Adeva *et al.*, Phys. Lett. B **412**, 414 (1997).
[87] M. Arneodo *et al.*, Nucl. Phys. B **487**, 3 (1997).
[88] W. Melnitchouk and A. W. Thomas, Phys. Lett. B **377**, 11 (1996).
[89] L. P. Kaptari, A. Yu. Umnikov, and B. Kämpfer, Phys. Rev. D **47**, 3804 (1993).
[90] K. Yu. Kazakov and L. P. Kaptar', Yad. Fiz. **60**, 1593 (1997) [Phys. At. Nucl. **60**, 1448 (1997)].
[91] M. Arneodo *et al.*, Nucl. Phys. B **441**, 12 (1995).
[92] A. C. Benvenuti *et al.*, Phys. Lett. B **189**, 483 (1987).
[93] C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).
[94] R. Schiavilla *et al.*, Nucl. Phys. A **449**, 219 (1986).
[95] P. Amaudruz *et al.*, Nucl. Phys. B **441**, 3 (1995).

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