Wigner functions of essentially nonequilibrium systems

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We discuss the S-matrix interpretation of perturbation theory for the Wigner-function generating functional at finite temperature. For the sake of definiteness the concrete particle-physics problem of high-temperature initial-state dissipation into a cold state is considered from the experimental and theoretical points of view. The temperature is introduced in the theory in a typical microcanonical way. The perturbation theory contains two-temperature (of the initial and final states) Green functions. Two possible boundary conditions are considered. One of them is the usual one in a field-theory vacuum boundary condition. The corresponding generating functional of the Wigner functions can be used in particle physics. Another type of boundary condition assumes that the system under consideration is in the environment of black-body radiation. This leads to the usual Kubo–Martin–Schwinger boundary condition in the equilibrium (one-temperature) limit. The comparisons of the S-matrix approach with Schwinger–Keldysh real-time finite-temperature field theory and with the nonstationary statistical-operator approach of Zubarev are considered. The range of applicability of the finite-temperature description of dissipation processes is shown. © *1999 American Institute of Physics.* [S1063-7796(99)00301-0]

1. INTRODUCTION

At the very beginning of this century P. and T. Ehrenfest proposed a model to visualize Boltzmann's interpretation of irreversibility phenomena in statistics. The model is extremely simple and fruitful.[1] It considers two boxes with $2N$ numbered balls. Choosing the number $l=1,2,...,2N$ *randomly*, one must take the ball with the label l from one box and put it into the other one. Starting from the highly "nonequilibrium" state with all balls in one box, there is a tendency to equalization of the numbers of balls in the boxes. Thus, there is an irreversible[1)] flow toward a preferred (equilibrium) state. One can hope[1] that this model reflects the physical reality of nonequilibrium processes with an initial state very far from equilibrium. A theory of such processes with (nonequilibrium) flow toward a state with maximal entropy should be sufficiently simple to give definite theoretical predictions.

In order to make the discussion less formal, we will be concerned with concrete physical problems. For instance, particle creation processes are a good laboratory for investigation of the general properties of relativistic nonequilibrium processes. Indeed, considering the multiplicity n as the characteristic of the final-state entropy, we can choose asymptotically large $n\gg\bar{n}(s)$, where the mean multiplicity $\bar{n}(s)$ naturally defines the scale of n. Then one can expect, noting the above-mentioned general property of the nonequilibrium flow, that the theory of processes with practically total dissipation of the initial-state kinetic energy into particle masses should be extremely simple. For this reason it is natural to start the discussion with the region $n\gg\bar{n}(s)$. We maintain this condition in constructing the theory.

The theory of dissipative processes has general significance from the thermodynamical point of view, and we concentrate our attention on this important problem. There is also a practical aspect of our problem. At $n\gg\bar{n}(s)$ the cross sections $\sigma_n(s)$ fall off rapidly and are too small ($<nb$). Noting also the problem of triggering such a final state, the experimenters must have sufficient reasons to examine them. The main arguments are as follows: at $n\gg\bar{n}(s)$ we have a unique opportunity (i) to examine *pure* (practically without admixture of hadrons), *cold* (an optimal condition for investigation of collective phenomena in a system), and *dense* (in this case the QCD interaction constant α_s is small) quark plasma (CQGP) and (ii) to realize experimentally the decay of a very hot (at high energies) initial state in the "inflational" regime, with "frozen" nonperturbative degrees of freedom of the hadron system.

It is known from high-energy hadron experiments that the cross sections σ_n have a maximum at $n\sim\bar{n}(s)$, where $1\ll\bar{n}(s)\ll n_{\max}$ and $n_{\max}=\sqrt{s}/m_h$ is the maximum available multiplicity at a given energy $\sqrt{s}$ (m_h is the hadron characteristic mass). This indicates that in hadron processes the nonequilibrium flow is not equal to zero [$\bar{n}(s)\gg1$], but the most probable processes do not lead to the state with maximal entropy [$\bar{n}(s)\ll n_{\max}$]. (The early models were based on the assumption that the final state of inelastic hadron processes has maximal entropy, $\bar{n}(s)\sim n_{\max}$.[3])

The preferred processes at $n\sim\bar{n}$ are due to excitation of hadronic nonperturbative degrees of freedom described by the creation of hadron constituents from the vacuum: the kinetic motion of the partons leads to increasing (because of the confinement phenomenon) polarization of the vacuum and to its instability with respect to quark creation.[4] In other words, there is a long-range correlation among the hadron constituents. In this special correlation the conservation-law constraints are implied. They are important in the dynamics, since each conservation law decreases the number of

dynamical degrees of freedom by at least one unit, i.e., it has a nonperturbative effect [this must explain why $\bar{n}(s) \ll n_{\max}$]. Moreover, in so-called integrable systems each independent integral of the motion (in involution) reduces the number of degrees of freedom by two units. As a result, there is no stochastization in such systems,[5] i.e., the nonequilibrium flow is equal to zero. But it will be argued that at very high multiplicities this effect is negligible. Thus, if

$$\bar{n}(s) \ll n < n_{\max},$$

we will see that the particle creation processes are close to Markovian, in accordance with Boltzmann's idea. The reason for this phenomenon is the faster falloff of the soft channels of hadron creation, compared with the hard channels in the asymptotic region of n.

Discarding nonperturbative effects, the creation of a high-multiplicity final state can be described by the standard methods of QCD. We will demonstrate the dominance of processes with the minimal number of QCD jets in the high-multiplicity region. This means that the high-multiplicity processes are stationary and Markovian.[2)] This result is in agreement with Boltzmann's general concept of nonequilibrium flows.

Thus, the high-multiplicity processes are "unshadowed" by nonperturbative and complicated perturbative effects. This will allow us to investigate not only the new state of the pure colored plasma but also the structure of the fundamental Lagrangian. These conclusions are not evident, and we start our discussion with a brief review of the arguments.

It must be noted that the experimental investigation of high-multiplicity processes in the deep asymptotic region in n seems unrealistic. But in considering moderate $n \gg \bar{n}$ we cannot be sure that the final state is in equilibrium. Investigation of fractal dimensions in multiparticle hadronic processes at high energies reveals the presence of considerable fluctuations.[6] This requires a theory of dissipation processes with a nonequilibrium final state.

There is another aspect of the problem. At present our understanding of hadronic processes is by no means capable of giving any quantitative prediction. The above-mentioned prediction concerning the absence of nonperturbative contributions to hadronic processes "works" only in the deep asymptotic region in n. Thus, at moderate $n \gg \bar{n}$ we cannot be sure that they do not have an important influence. That is why we will concentrate our attention in this paper on the search for an economic (thermodynamical) description of the dissipative processes, trying to find the connections of our S-matrix approach with the other approaches. It is important to note that the proposed formalism allows us to separate the dynamical aspect of the problem from the purely descriptive one (see also the concluding section).

This central problem of the formalism can be solved by noting that our dissipative problem contains an element of dynamics, since it depends crucially on the boundary condition. Therefore, we adopt the S-matrix formalism, which is natural for describing the time evolution of dissipative systems. For this purpose the amplitudes

$$\langle (p)_m | (q)_n \rangle = a_{n,m}(p_1, p_2, \ldots, p_m; q_1, q_2, \ldots, q_n)$$

of the m-into-n-particle transition will be introduced. (The in- and out-states must be composed of mass-shell particles.[7]) Moreover, to incorporate the boundary condition $n \gg m$ we must calculate the probability by integrating over the particle momenta:

$$r(P; n.m) \sim \int |a_{n,m}|^2 = \int \langle (p)_m | (q)_n \rangle \langle (q)_n | (p)_m \rangle,$$

since the amplitude a_{nm} is a function of too many variables $(p_1, p_2, \ldots, p_m; q_1, q_2, \ldots, q_n)$. This standard method of particle physics practically solves our problem.

Nevertheless, it is desirable to use thermodynamical language as the most economic one, i.e., the formalism which uses the minimal number of parameters (temperature, chemical potential, etc.) for the description of the system.

The field-theoretical description of statistical systems at finite temperature is usually based on the formal analogy between imaginary time and inverse temperature β ($\beta = 1/T$).[8] This approach is fruitful[9] for the description of the static properties of a system, but it requires a complicated mathematical formalism for the analytic continuation to real time[10] if we want to clarify the dynamical aspects. The first important quantitative attempt to construct a real-time finite-temperature field theory[11] encountered a problem of pinch singularities. Further investigation of the theory made it possible to demonstrate a mechanism of cancellation of these unphysical singularities.[12] This was achieved by doubling the number of degrees of freedom:[13,14] the Green functions of the theory represent a 2×2 matrix. It certainly makes the theory more complicated, but the operator formalism of the thermo-field dynamics[15] shows the unavoidable character of this complication.

The Schwinger–Keldysh real-time finite-temperature field-theoretical description[13,14] of statistical systems is based on the Kubo–Martin–Schwinger (KMS)[16,17] boundary condition for a field:

$$\Phi(t) = \Phi(t - i\beta).$$

This formal trick introduces into the formalism the temperature $T = 1/\beta$ but unavoidably leads to *equilibrium* fluctuation–dissipation conditions[18] (see also Ref. 19). Besides this, we must have a two-temperature theory describing the kinetic-energy dissipation process (for the initial and final states separately). It is evident that in such a theory with two temperatures it is impossible to use the KMS boundary condition.

In the S-matrix approach a finite-temperature description can be introduced (e.g., Ref. 20 and references cited therein) by taking into account, for instance, the fact that

$$d\Gamma_n = |a_{n,m}| \prod_1^n \frac{d^3 q_i}{(2\pi)^3 2\epsilon(q_i)}, \quad \epsilon(q) = (q^2 + m_h^2)^{1/2},$$

is the differential measure of the final state. Then we can define the temperature as a function of the initial energy through the equation of state, i.e., it is proportional to the mean energy of the created particles. Such an introduction of the temperatures as a Lagrange multiplier is obvious for the microcanonical approach.[16] The initial-state temperature will

be introduced in the same way. Using standard terminology,[21] we will deal with "mechanical" perturbations only,[22] and it will not be necessary to divide the perturbations into "thermal" and "mechanical" ones.[23]

Introducing the temperature as a Lagrange multiplier, we must assume that the temperature fluctuations are small (Gaussian). Otherwise the notion of temperature becomes meaningless.

The "working" idea concerning nonequilibrium processes is based on the assumption that the evolution of the system goes through several phases. In the first, "fast" phase the s-particle distribution functions $\mathbf{D}_s$, $s>1$, depend strongly on the initial conditions. But at the end of this phase the system "forgets" the initial-state information. The second phase is the "kinetic" one. One can expect that the space–time fluctuations of thermodynamical parameters in this phase are large-scale, i.e., there are macroscopic domains in which the subsystems are in equilibrium, with Gaussian fluctuations of the thermodynamical parameters. In the last, "hydrodynamical" phase the whole system is described by macroscopic parameters. We will see that the Schwinger–Keldysh formalism[10,13,14] is applicable for the "hydrodynamical" phase only.

The S-matrix finite-temperature description outlined above can be realized not only for a uniform temperature distribution (we have taken the first step in this direction, wishing to introduce the initial and final temperatures separately). Thus, by introducing cells of a measuring device (calorimeter) and introducing the energy–momentum shells of each cell separately, we can introduce the individual temperatures in each cell. This can be done, since in the S-matrix theory the measurement performed by free (mass-shell) particles, i.e., the measurement of the energy (and momentum), can be performed in each cell separately. This allows us to capture the "kinetic" phase as well (if the number of calorimeter cells is large enough). In this phase the multiparticle distribution functions $\mathbf{D}_s$, $s>1$, are functionals of the one-particle distribution function $\mathbf{D}_1$ only. This means that we have a "shorter" description of the nonequilibrium medium.[24] We will return to this question in Sec. 4 in considering the range of applicability of the thermal description of dissipative processes.

The microcanonical description assumes that the energy of the system is known with arbitrary accuracy. Introducing the measurement cells and corresponding energy shells, we assume that the energy in each cell can be measured with arbitrary accuracy. This is why we must work in the framework of the Wigner-function formalism.[25]

Wigner proposed the function $W(q,R)$ for the quantum-state phase-space description:[26]

$$W(q,R)=\int dr e^{iqr}\Psi(R+r/2)\Psi^*(R-r/2),$$

where $\Psi(x)$ is the wave function of the state. The existence of other approaches must be mentioned.[27] But as will be seen below, Wigner's description is most natural for us.

In the classical limit $\hbar=0$ the function $W(q,r)$ coincides with the phase-space probability distribution function. It obeys the equation[28]

$$\dot{W}=\{W,H\}+O(\hbar),$$

which coincides with the Liouville equation only in the classical limit $\hbar=0$.

The extension of Wigner's idea to the relativistic case uses the connection between Wigner's approach and the inclusive description of inelastic scattering processes.[25,29] But the Wigner functions are not directly measurable quantities because of the quantum uncertainty principle $\Delta q\Delta r\sim\hbar$. This restriction leads to the impossibility of taking measurement (calorimeter) cells with arbitrarily small 4-dimension Δr and defines the natural domain of applicability of the Wigner-function approach. If we are to use the Wigner-function description of experiments, the corresponding theory must take into account this restriction. A discussion of this question is given in Sec. 4.

Thus, in our terms we can use the thermodynamical formalism if we can apply to the nonstationary medium the "shortened" description. In this case the mean values of the correlation functions over space–time are negligible, and the fluctuations of the thermodynamical parameters are small (Gaussian). Another approach should also be mentioned. According to the proposal[32] that equilibrium in the nonuniform nonstationary medium may be attained in small regions more quickly than in the whole system, the maximum of the entropy in these restricted domains of the system can be used for the construction of a "local equilibrium density matrix" (LDM).[32] But the LDM is applicable for the description of processes in which dissipation may be disregarded.[30] Nevertheless, if the energy–momentum density of the nonstationary flow is considerably smaller than the energy density of the matter, then the former can be taken into account perturbatively, considering the LDM as the initial condition. This modifies the LDM to the "nonstationary density matrix" (NDM) of Zubarev[32] by introducing an infinitesimal interaction with a heat bath to get increasing entropy. We will return to this question in Sec. 5.

The S matrix will be introduced phenomenologically, using the ordinary reduction formalism of quantum field theory. This naturally makes it necessary to introduce boundary conditions for the interacting fields $\Phi(\sigma_\infty)$, where σ_∞ is an infinitely remote hypersurface (e.g., Ref. 31). The value of $\Phi(\sigma_\infty)$ specifies the environment of the system.

We start with the vacuum boundary condition $\Phi(\sigma_\infty)=0$ familiar for a field theory. This theory can be applied in particle physics. The simplest choice $\Phi(\sigma_\infty)\neq 0$ assumes that the system under consideration is surrounded by blackbody radiation. This "boundary condition" restores the Schwinger–Keldysh[10] real-time finite-temperature field theory[12] from the S-matrix formalism in the "hydrodynamical" phase and gives a dynamical interpretation of the KMS periodic boundary condition.

One should recognize also that the last choice of boundary condition is not unique: one can consider another organization of the environment of the system. The S-matrix interpretation is capable of showing how to adapt the formalism to an arbitrary environment.[3)] It should broaden the potentialities of the real-time finite-temperature field-theoretical methods, for instance, for heavy-nucleus high-

energy interactions. It is also of special interest to consider topological effects, but for the reasons considered above, in this paper the discussion will be confined to perturbation theory (see also the concluding section).

The central purpose of this review paper is to describe the connections between the ordinary S-matrix description and the currently popular real-time finite-temperature field theories. We wish to discuss:

- The QCD-jet dominance in deep asymptotics in n (Sec. 2). In that section we show why the *real-time* formalism is needed to describe the dissipative process.
- The S-matrix interpretation of the Schwinger–Keldysh theory (Sec. 3). In that section the uniform-temperature description of the state will be introduced in the spirit of the microcanonical description. It is shown how to make explicit calculations to demonstrate the equivalence of this microcanonical description and the ordinary (Gibbs) canonical formalism.
- The range of applicability of the finite-temperature description (Sec. 4). In that section the necessity and sufficiency of Bogolyubov's "shortened" description is discussed.
- The S-matrix description of media with a nonuniform temperature distribution (Sec. 5). In that section the Wigner-function formalism is introduced. The range of its applicability in describing experiments is demonstrated.
- The comparison of our S-matrix approach with the "nonstationary statistical operator" of Zubarev[32] (Sec. 6). In that section the main distinction between the S-matrix (microcanonical) theories and Zubarev's (canonical) perturbation theories is explained.
- Concluding remarks (Sec. 7). In that section a method of including nonperturbative effects in the formalism is discussed.

2. PHENOMENOLOGY

To construct the phenomenology[33] of high-multiplicity processes let us introduce the classification of the asymptotics in n. For this purpose it is useful to consider the "grand partition function":

$$T(z,s)=\sum_n z^n\sigma_n(s), \quad T(1,s)=\sigma_{\text{tot}}(s).$$

Strictly speaking, a summation over n is performed up to $n_{\max}$. But we can extend the summation up to infinity[4)] if the weight z is sufficiently small, $0<z<z_{\max}$. Thus, $T(z,s)$ can be regarded as a nontrivial function of z with sufficient accuracy. Note that $z_{\max}>1$, since $\sigma_n(s)$ decreases with n.

If we know $T(z,s)$, then $\sigma_n(s)$ is defined by an inverse Mellin transformation. This gives the (usual in thermodynamics) equation (of state):

$$n=z\frac{\partial}{\partial z}\ln T(z,s). \tag{2.1}$$

Solving this equation, we can estimate the asymptotics of σ_n:

$$\sigma_n(s)\sim e^{-n\ln\bar{z}(n,s)}, \tag{2.2}$$

where $1<\bar{z}(n,s)\ll z_{\max}$ is the smallest solution of Eq. (2.1).

It follows from (2.2) that for $n\to\infty$ the solution of (2.1) must tend to a singularity z_s of $T(z,s)$, and the character of the singularity is not important. Thus, we must consider three possibilities:

a) $z_s=z_a=1$; b) $z_s=z_b=\infty$; c) $z_s=z_c$, $1<z_c<\infty$.

Following Lee and Yang,[34] there are no singularities for $0<z<1$.

Let us consider now the physical content of this classification.

a) $z_s=1$.

It is known that the singularity $z_s=1$ reflects a first-order phase transition.[34] To find σ_n for this case we adopt Langer's analyses.[35] Introducing the temperature $1/\beta$ instead of the total energy $\sqrt{s}$, we can use the isomorphism with the Ising model. For this purpose we divide the spatial volume into cells, and if there is a particle in a cell, we write (-1). Otherwise, we write $(+1)$. This is the model of a lattice gas well described by the Ising model. We can regulate the number of downward spins, i.e., the number of created particles, by the external magnetic field $\mathbf{H}$. Therefore, $z=\exp\{-\beta\mathbf{H}\}$, and $\mathbf{H}$ is the chemical potential.

The corresponding partition function in the continuous limit[35] (see also Ref. 36) has the form

$$R(\beta,z)=\int D\mu e^{-\int dx\{(1/2)(\vec{\partial}\mu)^2-\epsilon\mu^2+\alpha\mu^4-\lambda\mu\}}, \tag{2.3}$$

where $\epsilon\sim(1-\beta_c/\beta)$ and $\lambda\sim\mathbf{H}$, with critical temperature $1/\beta_c$.

If $\beta_c>\beta$, there is no phase transition and the potential has one minimum at $\mu=0$. But if $\beta_c<\beta$, there are two degenerate minima at $\mu_\pm=\pm\sqrt{\epsilon/2\alpha}$ if $\lambda=0$. Switching on $\mathbf{H}<0$, the left-hand minimum at $\mu_-\sim-\sqrt{\epsilon/2\alpha}$ becomes absolute, and the system will tunnel into this minimum (see also Ref. 37). This process describes particle creation as a process of spin flips.

Equation (2.1) gives, for $n\to\infty$,

$$\ln\bar{z}\sim n^{-1/3}>0.$$

As a result,

$$\sigma_n\sim e^{-an^{2/3}}>O(e^{-n}), \quad a>0,$$

i.e., we have a decrease slower than e^{-n}. The quasiclassical calculation shows that the functional determinant is singular at $\mathbf{H}=0$. It must be stressed that in the Ising-model description used here the chemical potential deforms the ground state. As a result, the quasiclassical approximation is applicable, since $\ln\bar{z}\ll1$, i.e., since the processes of spin flip are rare in the high-multiplicity region. It is easy to show in this approximation[35] that the functional determinant is singular at $\mathbf{H}=0$, i.e., at $z=1$.

Note that $\bar{z}$ *decreases* to 1 with n. This unusual phenomenon must be explained. The mechanism of particle creation considered above describes the "fate of the false vacuum."[37] In the process of decay of an unstable state, clusters of a new phase of size X are created. If the clusters have dimension $X>X_c$, its size increases, since the volume energy $(\sim X^3)$ of a cluster becomes better than the surface-

tension energy ($\sim X^2$). This condition defines the value of X_c. The wall of a "critical" cluster accelerates, i.e., the work needed to add one particle to a cluster decreases with $X>X_c$. This explains why $\bar{z}$ decreases with n, noting that $\ln \bar{z}$ is proportional to the Gibbs free energy per particle.

The mechanism of particle creation described above assumes that we have prepared the *equilibrium* system in the unstable phase at $\mu_+ \sim +\sqrt{\epsilon/2\alpha}$, and in going to another state at $\mu_- \sim -\sqrt{\epsilon/2\alpha}$ the system creates particles. The initial state may be the QGP, and the final state may be a hadron system. Therefore, we must describe the way in which the quark system was prepared.

Following the Lee–Yang picture of a first-order phase transition[34] (see also Ref. 36), there is no phase transition in a finite system (the partition function cannot be singular for finite $n_{\max}$). This means that the multiplicity (and the energy) must be high enough to see the phenomena described above.

b) $z_s=\infty$.

Let us return to the integral (2.3) to investigate the case $\beta_c>\beta$. In this case the potential has one minimum at $\mu=0$. The external field $\mathbf{H}$ creates the mean field $\bar{\mu}=\bar{\mu}(\mathbf{H})$, and the integral (2.3) must be calculated by expanding it near $\mu=\bar{\mu}$. As a result, in the quasiclassical approximation ($\bar{\mu}$ increases with increasing n),

$$\ln R(\beta,z)\sim(\ln z)^{4/3}.$$

This gives $\ln \bar{z}\sim n^3$ and $\ln \sigma_n \sim -n^4$, i.e.,

$$\sigma_n < O(e^{-n}).$$

There is another possible interpretation of case (b). For this case we can put

$$\ln T(z,s)=n_0(s)+\bar{n}(s)(z-1)+O((z-1)^2) \qquad (2.4)$$

for $|z-1|\ll 1$. By definition, $n_0(s)=\ln \sigma_{\text{tot}}$. The experimental distribution of $\ln T(z,s)-n_0(s)$ for various energies shows that the contributions of terms $O((z-1)^2)$ increase with energy.[38] The hadron "standard model" (SM) assumes that

$$\ln T(z,s)=n_0(s)+\bar{n}(s)(z-1)$$

is the Born term in the perturbation series (2.4). There are various interpretations of this series, e.g., the multiperipheral model, the Regge-pole model, the heavy-color-string model, QCD multiperipheral models, etc. In all these models $n_0=a_1+a_2\ln s$, $0\leqslant a_2\ll 1$, and $\bar{n}(s)=b_1+b_2\ln s$, $b_2>0$. The second ingredient of the hadron SM is the assumption that the mean value of the created-particle momentum transfer is $\langle k\rangle=\text{const}$, i.e., is energy- (and multiplicity-) independent. It can be shown that under these assumptions

$$\ln T(z,s)=n_0(s)+\sum_n c_n(s)(z-1)^n, \quad c_1\equiv\bar{n}, \qquad (2.5)$$

is *regular* at finite values of z (Ref. 38) and is able to give predictions that are well confirmed by experiment.

Inserting (2.5) into (2.1), we find that $\bar{z}(n,s)$ is an increasing function of n. Therefore,

$$\sigma_n < O(e^{-n}). \qquad (2.6)$$

But the SM has a finite range of validity: beyond $n\sim\bar{n}^2$ the model must be changed, since it is impossible to preserve the condition $\langle k\rangle=\text{const}$ at higher multiplicities.[39]

We should stress once more that only the two possibilities (a) and (b) can be deduced from the representation (2.3) (see also Ref. 35). But nevertheless there is another possibility:

c) $1<z_s<\infty$.

Let us assume now that

$$T(z,s)\sim\left(1-\frac{z-1}{z_c-1}\right)^{-\gamma}, \quad \gamma>0. \qquad (2.7)$$

Then, using the normalization condition $(\partial T(z,s)/\partial z)|_{z=1}=\bar{n}_j(s)$, we find that $z_c(s)=1+\gamma/\bar{n}_j(s)$. The singular structure (2.7) is impossible in the SM because of the condition $\langle k\rangle=\text{const}$ But if $|z-1|\ll 1$, we have the estimate (2.4). The difference between the SM and (c) is seen only for $1-(z-1)/(z_c-1)\ll 1$, i.e., in the asymptotics in n or in the asymptotics in the energy. The singular structure is familiar for the "logistic" equations of QCD jets (e.g., Ref. 40).

In our case $\bar{z}=z_c+O(\bar{n}_j/n)$, and at high energies $[\bar{n}_j(s)\gg 1]$

$$\sigma_n\sim e^{-\gamma n/\bar{n}_j}=O(e^{-n}). \qquad (2.8)$$

Therefore, *comparing (2.6) and (2.8), we can conclude that at sufficiently high energies, i.e., if $\bar{n}_j\gg\bar{n}$, where $\bar{n}$ is the SM mean multiplicity, the mechanism (c) must dominate in the asymptotics in n.*

This is a general, practically model-independent, prediction. It has the experimentally important consequence that at high energies there is wide range of multiplicities where the SM mechanism of hadron creation is negligible. In other words, the CQGP of high-multiplicity processes is a dynamical consequence of jets and SM mechanisms. In the transition region between "soft" (SM) and "hard" (jet) processes one can expect "semihard" processes of minijet dominance.

The multiplicity distribution in jets has an interesting property, noted many decades ago by Volterra in his mathematical theory of populations.[41] In our terms, if the one-jet partition function has a singularity at $z_c^{(1)}(s)=1+\gamma/\bar{n}_j(s)$, then the two-jet partition function must be singular at

$$z_c^{(2)}(s)=1+\frac{\gamma}{\bar{n}_j(s/4)}>z_c^{(1)}(s),$$

and so on. Therefore, at high energies and for $n>\bar{n}_j(s)$ the jet number must be minimal (with exponential accuracy). This means that for $n\to\infty$ the processes of hadron creation have a tendency to be Markovian (with a sharp increase of the transverse momentum $\langle k\rangle$), and it is only in the last stage that the (first-order) phase transition (colored plasma) $\to$(hadrons) may be seen.

One can say that in the asymptotics in n we consider an "inflational" channel of thermalization which is so fast[5] that the usual confinement forces are "frozen" and do not play an important role in the final colored-plasma creation.

3. S-MATRIX INTERPRETATION OF REAL-TIME FINITE-TEMPERATURE FIELD THEORIES

3.1. Vacuum boundary conditions

The starting point of our calculations is the n-into-m-particle transition amplitude $a_{n,m}$, the derivation of which is a well known procedure in perturbation theory. For this purpose the $(n+m)$-point Green function $G_{n,m}$ is introduced.[42] To calculate the nontrivial elements of the S matrix one must put the external particles on the mass shell. Formally, this procedure means amputation of the external legs of $G^c_{n,m}$ and, further, multiplication by the free-particle wave functions. As a result, the m-into-n-particle transition amplitude $a_{n,m}$ in the momentum representation has the form

$$a_{n,m}((q)_n;(p)_m)=(-i)^{n+m}\prod_{k=1}^{m}\hat{\phi}(q_k)\prod_{k=1}^{n}\hat{\phi}^*(p_k)Z(\phi). \tag{3.1}$$

Here we introduce the "annihilation" operator

$$\hat{\phi}(q)=\int dx e^{-iqx}\hat{\phi}(x),\quad \hat{\phi}(x)=\frac{\delta}{\delta\phi(x)}, \tag{3.2}$$

where $\hat{\phi}^*(p_k)$ is the "creation" operator, and q_k and p_k are the momenta of the incoming and outgoing particles. In (3.1),

$$Z(\phi)=\int D\Phi e^{iS(\Phi)-iV(\Phi+\phi)}$$

is the generating functional. The total action was divided into two parts, where $S(\Phi)$ is the free part and $V(\Phi,\phi)$ describes the interactions. At the very end one must take the auxiliary field $\phi=0$.

To ensure convergence of the integral (3.1) over the scalar field Φ, the action $S(\Phi)$ must contain a positive imaginary part. Usually, for this purpose Feynman's $i\varepsilon$ prescription is used. It is better for us to shift the time contour infinitesimally to the upper half-plane,[10,43] i.e., to the Mills contour

$$C_+ : t\to t+i\varepsilon,\quad \varepsilon>0,$$

and after all the calculations to return the time contour to the real axis, $\varepsilon\to+0$.

In Eq. (3.1) the integration is performed over all field configurations with the standard vacuum boundary condition

$$\int d^4x\partial_\mu(\Phi\partial^\mu\Phi)=\int_{\sigma_\infty}d\sigma_\mu\Phi\partial^\mu\Phi=0,$$

which assumes zero contribution from the surface term.

Supposing that the particle number and momenta are insufficient for us, we introduce the probability

$$r(P)=\sum_{n,m}\frac{1}{n!m!}\int d\omega_n(q)d\omega_m(p)\times\delta^{(4)}\Big(P-\sum_{k=1}^{n}q_k\Big)\delta^{(4)}\Big(P-\sum_{k=1}^{n}p_k\Big)|a_{n,m}|^2, \tag{3.3}$$

where

$$d\omega_n(q)=\prod_{k=1}^{n}d\omega(q_k)=\prod_{k=1}^{n}\frac{d^3q_k}{(2\pi)^3 2\epsilon(q_k)},$$

$$\epsilon=(q^2+m_h^2)^{1/2},$$

is the Lorentz-invariant phase-space element. We assume that the energy–momentum conservation δ function was extracted from the amplitude. It was divided into two parts:

$$\delta^{(4)}\Big(\sum q_k-\sum p_k\Big)=\int d^4P\delta^{(4)}\Big(P-\sum q_k\Big)\times\delta^{(4)}\Big(P-\sum p_k\Big). \tag{3.4}$$

It is not too hard to see that, apart from the phase-space volume,

$$r=\int d^4P r(P)$$

is the imaginary part of the amplitude $\langle\text{vac}|\text{vac}\rangle$. Therefore, by computing $r(P)$ the standard renormalization procedure can be applied, and new divergences will not arise in our formalism.

The Fourier transformation of the δ functions in (3.3) allows us to write $r(P)$ in the form

$$r(P)=\int\frac{d^4\alpha_1}{(2\pi)^4}\frac{d^4\alpha_2}{(2\pi)^4}e^{iP(\alpha_1+\alpha_2)}\rho(\alpha_1,\alpha_2),$$

where

$$\rho(\alpha_1,\alpha_2)=\sum_{n,m}\frac{1}{n!m!}\int\prod_{k=1}^{n}\{d\omega(q_k)e^{-i\alpha_1 q_k}\}\times\prod_{k=1}^{m}\{d\omega(p_k)e^{-i\alpha_2 p_k}\}|a_{n,m}|^2. \tag{3.5}$$

Introduction of the "Fourier-transformed" probability $\rho(\alpha_1,\alpha_2)$ means only that the phase-space volume is not fixed exactly, i.e., it is proposed that the 4-vector P is fixed with some accuracy if the α_i are fixed. The energy and momentum in our approach are still locally conserved quantities, since the amplitude a_{nm} is translationally invariant. Thus, we can perform the transformation

$$\alpha_1\sum q_k=(\alpha_1-\sigma_1)\sum q_k+\sigma_1\sum q_k\to(\alpha_1-\sigma_1)\times\sum q_k+\sigma_1 P,$$

since the 4-momenta are conserved. The choice of σ_1 fixes the reference frame. This degree of freedom of the theory was considered in Refs. 44 and 45.

Inserting (3.1) into (3.5), we find that

$$\rho(\alpha_1,\alpha_2)=\exp\Big\{i\int dxdx'(\hat{\phi}_+(x)D_{+-}(x-x',\alpha_2)\times\hat{\phi}_-(x')-\hat{\phi}_-(x)D_{-+}(x-x',\alpha_1)\hat{\phi}_+(x'))\Big\}\times Z(\phi_+)Z^*(\phi_-), \tag{3.6}$$

where D_{+-} and D_{-+} are the positive- and negative-frequency correlation functions:

$$D_{+-}(x-x',\alpha)=-i\int d\omega(q)e^{iq(x-x'-\alpha)}$$

describes the process of particle creation at the instant of time x_0 and its absorption at x_0', with $x_0>x_0'$, and α is the center-of-mass 4-coordinate. The function

$$D_{-+}(x-x',\alpha)=i\int d\omega(q)e^{-iq(x-x'+\alpha)}$$

describes the opposite process, $x_0<x_0'$. These functions obey the homogeneous equations

$$(\partial^2+m^2)_x G_{+-}=(\partial^2+m^2)_x G_{-+}=0,$$

since the propagation of mass-shell particles is described.

We suppose that $Z(\phi)$ can be computed perturbatively. For this purpose the following transformation will be used:

$$e^{-iV(\phi)}=e^{-i\int dx\hat{j}(x)\hat{\phi}'(x)}e^{i\int dxj(x)\phi(x)}e^{-iV(\phi')}$$
$$=e^{\int dx\phi(x)\hat{\phi}'(x)}e^{-iV(\phi')}=e^{-iV(-i\hat{j})}e^{i\int dxj(x)\phi(x)}, \tag{3.7}$$

where $\hat{\phi}$ was defined in (3.2). At the end of the calculations the auxiliary variables j, ϕ' must be taken equal to zero. Using the first equality in (3.7), we find that

$$Z(\phi)=e^{-i\int dx\hat{j}(x)\hat{\Phi}(x)}e^{-iV(\Phi+\phi)}e^{-(i/2)\int dxdx'j(x)D_{++}(x-x')j(x')}, \tag{3.8}$$

where D_{++} is the causal Green function:

$$(\partial^2+m^2)_x G_{++}(x-y)=\delta(x-y).$$

Inserting (3.8) into (3.6), after simple manipulations with differential operators [see (3.7)], we find the expression

$$\rho(\alpha_1,\alpha_2)=e^{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)}$$
$$\times\exp\left\{\frac{i}{2}\int dxdx'(j_+(x)D_{+-}(x-x',\alpha_1)\right.$$
$$\times j_-(x')-j_-(x)D_{-+}(x-x',\alpha_2)$$
$$\times j_+(x')-j_+(x)D_{++}(x-x')j_+(x')+j_-(x)$$
$$\left.\times D_{--}(x-x')j_-(x'))\right\}, \tag{3.9}$$

where

$$D_{--}=(D_{++})^*$$

is the anticausal Green function.

Considering a system with a large number of particles, we can simplify the calculations by choosing the c.m. frame $P=(P_0=E,\vec{0})$. It is also useful[16,20] to rotate the contours of integration over $\alpha_{0,k}$: $\alpha_{0,k}=-i\beta_k$, $\mathrm{Im}\,\beta_k=0$, $k=1,2$. As a result, omitting the unnecessary constant, we will consider $\rho=\rho(\beta_1,\beta_2)$.

External particles play a double role in the S-matrix approach: their interactions create and annihilate the interacting-field system and, on the other hand, they are probes through which a measurement of the system is performed. Since β_k are the quantities conjugate to the particle energies, we will interpret them as the inverse temperatures in the initial (β_1) and final (β_2) states of the interacting fields. But there is the following question: are the constants β_k really "good" parameters to describe the system?

The integrals over β_k,

$$r(E)=\int\frac{d\beta_1}{2\pi i}\frac{d\beta_2}{2\pi i}e^{(\beta_1+\beta_2)E}e^{-F(\beta_1,\beta_2)}, \tag{3.10}$$

where

$$F(\beta_1,\beta_2)=-\ln\rho(\beta_1,\beta_2),$$

can be computed by the stationary-phase method. This assumes that the total energy E is a fixed quantity. The solutions of the equations (of state),

$$E=\frac{\partial F(\beta_1,\beta_2)}{\partial\beta_k},\quad k=1,2, \tag{3.11}$$

give the most probable values of β_k at a given E. The equations (3.11) always have real solutions and, because of the energy conservation law, the two equations in (3.11) have the same solution, with the property[16]

$$\beta_k=\beta(E),\quad \beta>0.$$

Assuming that β is a "good" parameter, i.e., that the fluctuations of β_k are Gaussian, we can interpret $F(\beta_1,\beta_2)$ as the free energy and $1/\beta_k$ as the temperatures. Such a definition of the thermodynamical parameters is in the spirit of the microcanonical description. We will return to this question in Sec. 4.

The structure of the generating functional (3.9) is the same as that of the generating functional of Niemi and Semenoff.[12] The only difference is in the definition of the Green functions, which follows from the choice of the boundary condition (2.6). The Green functions D_{ij}, $i,j=+,-$, were defined on the time contours $C_\pm$ in the complex time plane ($C_-=C_+^*$). This definition of the time contours coincides with the Keldysh time contour.[14] The expression (3.9) can be written in a compact form if matrix notation is used. Note also the doubling of the number of degrees of freedom. This doubling is unavoidable, since the Green functions D_{ij} are singular on the light cone.

3.2. Closed-path boundary conditions

The generating functional $\rho(\alpha_1,\alpha_2)$ has the important factorized structure [see (3.6)]

$$\rho(\alpha_1,\alpha_2)=e^{\hat{N}(\alpha_1,\alpha_2;\phi)}\rho_0(\phi_\pm),$$

where the operator

$$\hat{N}(\alpha_1,\alpha_2;\phi)=\int dxdx'(\hat{\phi}_+(x)D_{+-}(x-x',\alpha_2)\hat{\phi}_-(x')$$
$$-\hat{\phi}_-(x)D_{-+}(x-x',\alpha_1)\hat{\phi}_+(x'))$$

acts on the generating functional

$$\rho_0(\phi_\pm)=Z(\phi_+)Z^*(\phi_-)=\int D\Phi_+ D\Phi_- \exp\{iS(\Phi_+) - iS(\Phi_-) - iV(\Phi_+ + \phi_+) + iV(\Phi_- + \phi_-)\} \tag{3.12}$$

of measurable quantities. All "thermodynamical" information is contained in the operator $\hat{N}(\alpha_1,\alpha_2;\phi)$, and the interactions are hidden in $\rho_0(\phi_\pm)$. One can say that the action of the operator $\hat{N}$ maps the system of interacting fields onto the measurable states. The latter are "labeled" by α_1 and α_2. This property allows us to say that we are dealing with "mechanical" fluctuations only. To regulate the particle number we can introduce into $\hat{N}$ a dependence on the "activities" z_1 and z_2 for the initial and final states separately.

The independent fields ϕ_+,ϕ_- and Φ_+,Φ_- were defined on the time contours C_+,C_-. By definition, the path integral (3.12) describes closed-path motion in the space of the fields Φ. We want to use this fact and introduce a more general boundary condition which also guarantees cancellation of the surface terms in the perturbation framework. We will introduce the equality

$$\int_{\sigma_\infty} d\sigma_\mu \Phi_+ \partial^\mu \Phi_+ = \int_{\sigma_\infty} d\sigma_\mu \Phi_- \partial^\mu \Phi_- . \tag{3.13}$$

The solution of Eq. (3.13) requires that the fields Φ_+ and Φ_- (and their first derivatives $\partial_\mu \Phi_\pm$) coincide on the boundary hypersurface σ_∞:

$$\Phi_\pm(\sigma_\infty)=\Phi(\sigma_\infty),$$

where, by definition, $\Phi(\sigma_\infty)$ is an arbitrary "turning-point" field.

The existence of the nontrivial field $\Phi(\sigma_\infty)$, in the absence of surface terms, has an influence only on the structure of the Green functions

$$G_{++}=\langle T\Phi_+\Phi_+\rangle, \quad G_{+-}=\langle \Phi_+\Phi_-\rangle,$$
$$G_{-+}=\langle \Phi_-\Phi_+\rangle, \quad G_{--}=\langle \bar{T}\Phi_-\Phi_-\rangle, \tag{3.14}$$

where $\bar{T}$ is the antitemporal time-ordering operator. These Green functions must obey the equations

$$(\partial^2+m^2)_x G_{+-}(x-y)=(\partial^2+m^2)_x G_{-+}(x-y)=0,$$
$$(\partial^2+m^2)_x G_{++}(x-y)=(\partial^2+m^2)_x^* G_{--}(x-y)=\delta(x-y), \tag{3.15}$$

and the general solution of these equations,

$$G_{ii}=D_{ii}+g_{ii},$$
$$G_{ij}=g_{ij}, \quad i\neq j, \tag{3.16}$$

contains the undefined terms g_{ij}, which must obey the homogenous equations

$$(\partial^2+m^2)_x g_{ij}(x-y)=0, \quad i,j=+,-. \tag{3.17}$$

The general solution of these equations (they are distinguished by the choice of the time contours $C_\pm$),

$$g_{ij}(x-x')=\int d\omega(q) e^{iq(x-x')} n_{ij}(q), \tag{3.18}$$

is defined by the functions n_{ij}. The latter are functionals of the "turning-point" field $\Phi(\sigma_\infty)$: if $\Phi(\sigma_\infty)=0$, we must have $n_{ij}=0$, and we revert to the theory of the previous section.

Our aim is to define n_{ij}. We can suppose that $n_{ij} \sim \langle \Phi(\sigma_\infty)\cdots\Phi(\sigma_\infty)\rangle$. The simplest assumption gives

$$n_{ij}\sim\langle \Phi_i\Phi_j\rangle\sim\langle \Phi^2(\sigma_\infty)\rangle. \tag{3.19}$$

We will find the exact definition of n_{ij} by starting from the S-matrix interpretation of the theory.

We must assume that there are only free, mass-shell, particles on the infinitely remote hypersurface σ_∞. Formally, this follows from (3.16)–(3.18) and is natural in the S-matrix framework.[7] In other respects the choice of the boundary condition is arbitrary.

Therefore, our aim is to describe the evolution of the system in a background field of mass-shell particles. We will assume that there are no special correlations among the background particles and will take into account only the energy–momentum conservation-law constraints. Quantitatively, this means that the multiplicity distribution of background particles is Poisson-like, i.e., is determined by the mean multiplicity only. This is in the spirit of the definition of n_{ij} in Eqs. (3.18) and (3.19).

Our derivation is the same as in Ref. 45. Here we restrict our discussion to the main quantitative points.

In the vacuum case of Sec. 3.1 the process of particle creation and subsequent absorption was described. In the presence of the background particles this time-ordered picture is wiped out: particles may be absorbed before their creation.

The particle creation and absorption was described by the product of the operator exponent (3.6). One can derive (see also Ref. 45) a generalization of (3.6): the presence of the background particles will lead to the same structure:

$$\rho_{cp}=e^{i\hat{N}(\phi_i^*\phi_j)}\rho_0(\phi_\pm),$$

where $\rho_0(\phi_\pm)$ is the same generating functional [see (3.12)]. But the operator $\hat{N}(\phi_i^*\phi_j)$, $i,j=+,-$, must be changed to take into account the external particle environment.

The operator $\hat{\phi}_i^*(q)$ was interpreted as the creation operator, and $\hat{\phi}_i(q)$ as the annihilation operator [see the definition (3.1)]. Accordingly, the product $\hat{\phi}_i^*(q)\hat{\phi}_j(q)$ acts as the activity operator. Thus, in the expansion of $\hat{N}(\phi_i^*\phi_j)$ we can retain only first nontrivial term:

$$\hat{N}(\phi_i^*\phi_j)=\int d\omega(q)\hat{\phi}_i^*(q)n_{ij}\hat{\phi}_j(q), \tag{3.20}$$

since no special correlation among the background particles should be expected. If external (nondynamical) correlations are present, then higher powers of $\hat{\phi}_i^*\hat{\phi}_j$ will appear in the expansion (3.20).[29] Following the interpretation of $\hat{\phi}_i^*\hat{\phi}_j$, we conclude that n_{ij} is the mean multiplicity of background particles. In (3.20) the normalization condition $N(0)=0$ was used, and a summation over all i,j was assumed. (In the vacuum case only the combinations $i\neq j$ were present.)

Computing ρ_{cp}, we must preserve the translational invariance of the amplitudes and extract the energy–momentum conservation δ functions. We must add to each vertex of an incoming particle in $a_{n,m}$ the factor $e^{-i\alpha_1 q/2}$, and for each outgoing particle the factor $e^{-i\alpha_2 q/2}$, after Fourier transformation of these δ functions.

Thus, the product $e^{-i\alpha_k q/2}e^{-i\alpha_j q/2}$ can be interpreted as the probability factor of the one-particle (creation+annihilation) process. The probability of the n-particle (creation+annihilation) process is a simple product of these factors if there are no special correlations among the background particles. This interpretation is evident in the c.m. frame $\alpha_k=(-i\beta_k,\vec{0})$.

After these preliminaries it is not hard to show that in the c.m. frame we have

$$n_{++}(q_0)=n_{--}(q_0)=\frac{1}{e^{((\beta_1+\beta_2)/2)|q_0|}-1}\equiv\bar{n}\left(\left|q_0\right|\frac{\beta_1+\beta_2}{2}\right). \tag{3.21}$$

Computing n_{ij} for $i\neq j$, we must take into account the fact that we have one additional particle:

$$n_{+-}(q_0)=\Theta(q_0)(1+\bar{n}(q_0\beta_1))+\Theta(-q_0)\bar{n}(-q_0\beta_1) \tag{3.22}$$

and

$$n_{-+}(q_0)=\Theta(q_0)\bar{n}(q_0\beta_2)+\Theta(-q_0)(1+\bar{n}(-q_0\beta_2)). \tag{3.23}$$

Using (3.21)–(3.23) and the definition (3.16), we find the Green functions (matrix Green functions in real-time finite-temperature field theories were first introduced in Ref. 46):

$$G_{i,j}(x-x',(\beta))=\int\frac{d^4q}{(2\pi)^4}e^{iq(x-x')}\tilde{G}_{ij}(q,(\beta)),$$

where

$$i\tilde{G}_{ij}(q,(\beta))=\begin{pmatrix}\dfrac{i}{q^2-m^2+i\epsilon} & 0\\ 0 & -\dfrac{i}{q^2-m^2-i\epsilon}\end{pmatrix}+2\pi\delta(q^2-m^2)\times\begin{pmatrix}\bar{n}\left(\dfrac{\beta_1+\beta_2}{2}|q_0|\right) & \bar{n}(\beta_2|q_0|)a_+(\beta_2)\\ \bar{n}(\beta_1|q_0|)a_-(\beta_1) & \bar{n}\left(\dfrac{\beta_1+\beta_2}{2}|q_0|\right)\end{pmatrix} \tag{3.24}$$

and

$$a_\pm(\beta)=-e^{(\beta/2)(|q_0|\pm q_0)}.$$

The corresponding generating functional has the standard form

$$\rho_{cp}(j_\pm)=\exp\{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)\}\times\exp\left\{\frac{i}{2}\int dxdx'j_i(x)G_{ij}(x-x',(\beta))j_j(x')\right\}, \tag{3.25}$$

where a summation over repeated indices is assumed.

Inserting (3.25) in the equation of state (3.11), we find that $\beta_1=\beta_2=\beta(E)$. If $\beta(E)$ is a "good" parameter, then the $G_{ij}(x-x';\beta)$ coincide with the Green functions of real-time finite-temperature field theory, and the KMS boundary condition

$$G_{+-}(t-t')=G_{-+}(t-t'-i\beta),$$
$$G_{-+}(t-t')=G_{+-}(t-t'+i\beta) \tag{3.26}$$

is restored. Equation (3.26) can be deduced from (3.24) by direct calculations.

4. APPLICABILITY OF THE FINITE-TEMPERATURE DESCRIPTION

4.1. The Schwinger–Keldysh formalism

There are various approaches to the construction of real-time finite-temperature field theories of Schwinger–Keldysh type (e.g., Ref. 10). All of them use various tricks for analytic continuation of the imaginary-time Matsubara formalism to real time.[47] The basis of these approaches is the introduction of the Matsubara field operator

$$\Phi_M(x,\beta)=e^{\beta H}\Phi_S(x)e^{-\beta H}, \tag{4.1}$$

where $\Phi_S(x)$ is the interaction-picture operator, instead of the Heisenberg operator

$$\Phi(x,t)=e^{itH}\Phi_S(x)e^{-itH}.$$

This introduces an average over a Gibbs ensemble instead of an average over zero-temperature vacuum states.

If the interaction is switched on adiabatically at the instant t_i and switched off at t_f, we have the unitary transformation

$$\Phi(x)=U(t_i,t_f)U(t_i,t)\Phi_S(x)U(t,t_i).$$

Introducing complex Mills time contours[43] to connect t_i to t, t to t_f, and t_f to t_i, we form a "closed-time" contour C (the end-points of the contours C_+ and C_- are joined together). This allows us to write the last equality in the compact form

$$\Phi(x)=T_C\{\Phi(x)e^{i\int_C d^4x'L_{\rm int}(x')}\}_S,$$

where T_C is the time-ordering operator on the contour C.

The corresponding expression for the generating functional $Z(j)$ of the correlation (Green) functions has the form

$$Z(j)=R(0)\langle T_C e^{i\int_C d^4x\{L_{\rm int}(x)+j(x)\Phi(x)\}_S}\rangle,$$

where $\langle\rangle$ denotes an average over the initial state.

If the initial correlations have little effect, we can take an average over a Gibbs ensemble. This is the main assumption of the formalism: the generating functional of the Green functions $Z(j)$ in this case has the form

$$Z(j)=\int D\Phi'\langle\Phi';t_i|e^{-\beta H}T_C e^{i\int_C d^4x j(x)\Phi(x)}|\Phi';t_i\rangle$$

with $\Phi'=\Phi'(x)$. In accordance with (4.1) we have

$$\langle\Phi';t_i|e^{-\beta H}=\langle\Phi';t_i-i\beta|$$

and, as a result,

$$Z(j)=\int D\Phi' e^{i\int_{C_\beta} d^4x\{L(x)+j(x)\Phi(x)\}}, \tag{4.2}$$

where the path integration is performed with the KMS periodic boundary condition:

$$\Phi(t_i)=\Phi(t_i-i\beta).$$

In (4.2) the contour C_β connects t_i to t_f, t_f to t_i, and t_i to $t_i-i\beta$. Therefore it contains the imaginary-time Matsubara part t_i to $t_i-i\beta$. A more symmetric formulation uses the following realization: t_i to t_f, t_f to $t_f-i\beta/2$, $t_f-i\beta/2$ to $t_i-i\beta/2$, and $t_i-i\beta/2$ to $t_i-i\beta$ (e.g., Ref. 12). This case also contains the imaginary-time parts of the time contour. Therefore, Eq. (4.2) represents the analytic continuation of the Matsubara generating functional to real times.

We note that if this analytic continuation is possible in $Z(j)$, then the representation (4.2) gives a good prescription for regularization of the frequency integrals in the Matsubara perturbation theory (e.g., Ref. 10), but nothing new for our problem, since the Matsubara formalism is a formalism for equilibrium states only.

Taking $t_i=-\infty$ and $t_f=+\infty$, and calculating the integral (4.2) perturbatively, we find the coincidence of $Z(j)$ and $\rho(\beta)$ from (3.25) with the Green functions defined in (3.24) if $\beta_1=\beta_2$. This "factorization" of the contributions from the contours C_+ and C_- in the integral (4.2) follows from the Riemann–Lebesgue lemma,[48] which is applicable in the perturbation framework.[12,43] Note the absence of Matsubara parts of the contour, which prevents the factorization, in the derived "S-matrix generating functional" (3.25) by definition (the importance of this circumstance is discussed in Sec. 7).

4.2. Range of the "hydrodynamical" approximation

Let us return now to Eq. (3.5). To find the physical meaning of the quantities $\beta_{1(2)}$ we must show how they can be measured. If there is nonequilibrium flow, it is hard to devise a thermometer (or a thermodynamical calorimeter) which measures locally in space–time the temperatures of these dissipative processes. But another method has been described, namely, to define the temperatures through the equations of state. This is possible in accelerator experiments, in which the total energy E is fixed. Thus, we will define $\beta_{1(2)}$ through the equations of state (3.11), i.e., by regarding $1/\beta_{1(2)}$ as the mean energy of the particles in the initial (final) state. But even if we know the solutions of these equations, we cannot find $\rho(E,z)$ correctly without adding the assumption that the $\beta_{1(2)}$ are "good" quantities, i.e., that the fluctuations near the solutions of (3.11) are small (Gaussian).

This assumption is the main problem in nonequilibrium thermodynamics. The problem in our terms appears as follows: the expansion near $\beta_{1(2)}(E)$ gives asymptotic series in

$$\int \mathbf{D}_s \sim \int \prod \{d\omega(k_i)dr_i\}\langle\varepsilon(k_1)\varepsilon(k_2)\cdots\rangle|_{(r_1,r_2,\ldots)},$$

where $\langle\rangle_{()}$ denotes an average over the fields at fixed points of the phase space $(k,r)_i$. In other words, the fluctuations near $\beta_{1(2)}(E)$ are defined by the values of the inclusive spectra familiar in particle physics. Therefore, the $\beta_{1(2)}(E)$ are "good" quantities if these inclusive spectra are small. But this assumption is too strong. More careful analysis shows that it is sufficient to have the factorization properties[49]

$$\int \prod \{d\omega(k_i)dr_i\}\langle\varepsilon(k_1)\varepsilon(k_2)\cdots\rangle|_{(r_1,r_2,\ldots)}$$

$$-\prod \int d\Omega(k_i)dr_i\langle\varepsilon(k_i)\rangle|_{(r_i)}\sim 0. \tag{4.3}$$

It must be noted that this is the unique solution of the problem, since the expansion near $\beta_{1(2)}(E)$ *unavoidably* leads to asymptotic series with zero radii of convergence.

One may hope to avoid this problem by working always in the energy–momentum representation, i.e., without the introduction of temperatures. Of course, this is possible in particles physics, but if the $\beta_{1(2)}(E)$ are not "good" parameters, this means that all correlations between the created particles are sufficient, i.e., the energy–momentum representation by itself did not solve the problem.

Finally, the factorization property of $\mathbf{D}_s$, $s>1$, is the well known Bogolyubov condition for the "shortened" description of nonequilibrium thermodynamical systems with s-particle distribution functions $\mathbf{D}_s$, $s>1$, expressed in terms of $\mathbf{D}_1$. It is the condition for the applicability of the "hydrodynamical" description, since it assumes that the constant $\beta_1(E)=\beta_2(E)$ is a "good" parameter for the description of the whole system.

Considering a problem with nonzero nonequilibrium flow, it is hard to expect the $\beta_{1(2)}(E)$ to be good parameters, i.e., that the factorization conditions hold. Nevertheless, as was mentioned above, there is a possibility that the *mean values* of the correlation functions are sufficiently small in restricted ranges of the phase space. This is the so-called "kinetic" phase of the process, in which the memory of the initial state has disappeared, an average has been taken over "fast" fluctuations, and we can consider the long-range fluctuations only.

5. LOCAL-EQUILIBRIUM HYPOTHESIS

Let us return now to the description of the experimental situation in high-multiplicity experiments. Since at the energies of modern accelerators we have thousands of particles in the final state, it is a hard problem even to count such big numbers. Thus, the number of particles n cannot be considered as a trigger. Moreover, it naturally seems unimportant whether we have a hundred thousand particles or a hundred thousand plus one. To take the first step toward the CQGP it is sufficient to be sure that in experiments the transition of a "hot" initial state into a "cold" final state is examined. For this purpose, ordinary calorimeters can be used.[50]

Thus, we must assume that the energies of the created particles are in the range $\varepsilon_i\leqslant\varepsilon_0$, where ε_0 is fixed by

experiment. Then, using the energy conservation law at a given ε_0, the number of created particles is bounded from below: $n>\sqrt{s}/\varepsilon_0\equiv n_{\min}$. With this constraint the integrated cross section

$$\sigma_{\varepsilon_0}(s)=\sum_{n=n_{\min}}\sigma_n(s)$$

is measured. Choosing $n_{\min}\gg\bar{n}$, i.e., $\varepsilon_0\ll\sqrt{s}/\bar{n}(s)$, we get into the high-multiplicity region. There is also a theoretical possibility of recovering the quantity $\sim\sigma_n$ by calculating the difference $\sigma_{\varepsilon_0}(s)-\sigma_{\varepsilon_0+\delta\varepsilon_0}(s)$.[50]

It is not necessary to measure the energy of each particle to have $n_{\min}\gg\bar{n}$. Indeed, let $\tilde{\varepsilon}_i$ be the energy of the ith group of particles, $\tilde{\varepsilon}_1+\tilde{\varepsilon}_2+\ldots+\tilde{\varepsilon}_k=\sqrt{s}$, and let $\tilde{n}_i$ be the number of particles in the group, $\tilde{n}_1+\tilde{n}_2+\ldots+\tilde{n}_k=n$.[6] Then if $\tilde{\varepsilon}_i<\varepsilon_0$ for $i=1,2,\ldots,k$, we have the inequality $k>n_{\min}$. Therefore, we get into the high-multiplicity domain, since $n\geqslant k$ if $\varepsilon_0\ll\sqrt{s}/\bar{n}(s)$. We can use the calorimeter by requiring that the energy in each cell is $\tilde{\varepsilon}_i<\varepsilon_0$.

The preparation of such an experiment is not a hopeless task, and it may be sufficiently informative. This formulation of the experiment will be the basis of the theory. Theoretically, we should shrink the 4-dimension of the calorimeter cells down to zero, since we do not know *a priori* the cell dimension. Then the cell index i is transformed into the position of a particle r. Thus, we arrive at a contradiction with the quantum uncertainty principle. This forces us to use the Wigner-function formalism, and the first question which must be solved is to find a way to adapt this formalism for the description of our experiment (there are also interesting ideas concerning the applicability of Wigner functions in Ref. 51).

5.1. Vacuum boundary condition

We start our considerations from the assumption that the temperature fluctuations are large-scale. In a cell whose dimension is much smaller than the fluctuation scale of the temperature, we can assume that the temperature is a "good" parameter. (A "good" parameter means that the corresponding fluctuations are Gaussian.)

Let us surround the interaction region, i.e., the system under consideration, by N cells with known space–time positions, and suppose that we can measure the energy and momentum of groups of incoming and outgoing particles in each cell. The 4-dimension of the cells cannot be arbitrarily small in this case because of the quantum uncertainty principle.

To describe this situation we decompose the δ functions in (3.4) into the product of $(N+1)$ δ functions:

$$\delta^{(4)}\Big(P-\sum_{k=1}^{n}q_k\Big)=\int\prod_{\nu=1}^{N}\Big\{dQ_\nu\delta\Big(Q_\nu-\sum_{k=1}^{n_\nu}q_{k,\nu}\Big)\Big\}\times\delta^{(4)}\Big(P-\sum_{\nu=1}^{N}Q_\nu\Big),$$

where $q_{k,\nu}$ is the momentum of the kth incoming particle in the νth cell and Q_ν is the total 4-momenta of the n_ν incoming particles in this cell, $\nu=1,2,\ldots,N$. The same decomposition will be used for the second δ function in (3.4). We must take into account the multinomial character of the particle decomposition into N groups. This gives the coefficient

$$\frac{n!}{n_1!\cdots n_N!}\delta_K\Big(n-\sum_{\nu=1}^{N}n_\nu\Big)\frac{m!}{m_1!\cdots m_N!}\delta_K\Big(m-\sum_{\nu=1}^{N}m_\nu\Big),$$

where δ_K is the Kronecker δ function.

As a result, the quantity

$$r((Q)_N,(P)_N)=\sum_{(n,m)}\int|a_{(n,m)}|^2\times\prod_{\nu=1}^{N}\Big\{\prod_{k=1}^{n_\nu}\frac{d\omega(q_{k,\nu})}{n_\nu!}\delta^{(4)}\Big(Q_\nu-\sum_{k=1}^{n_\nu}q_{k,\nu}\Big)\times\prod_{k=1}^{m_\nu}\frac{d\omega(p_{k,\nu})}{m_\nu!}\delta^{(4)}\Big(P_\nu-\sum_{k=1}^{m_\nu}p_{k,\nu}\Big)\Big\}\tag{5.1}$$

describes the probability of measuring in the νth cell fluxes of incoming particles with total 4-momentum Q_ν and of outgoing particles with total 4-momentum P_ν. The sequence of these two measurements is not fixed.

The Fourier transformation of the δ functions in (5.1) gives

$$r((Q)_N,(P)_N)=\int\prod_{k=1}^{N}\frac{d^4\alpha_{1,\nu}}{(2\pi)^4}\frac{d^4\alpha_{2,\nu}}{(2\pi)^4}\times e^{i\sum_{\nu=1}^{N}(Q_\nu\alpha_{1,\nu}+P_\nu\alpha_{2,\nu})}\rho((\alpha_1)_N,(\alpha_2)_N),$$

where

$$\rho((\alpha_1)_N,(\alpha_2)_N)=\rho(\alpha_{1,1},\alpha_{1,2},\ldots,\alpha_{1,N};\ \alpha_{2,1},\alpha_{2,2},\ldots,\alpha_{2,N})$$

has the form

$$\rho((\alpha_1)_N,(\alpha_2)_N)=\int\prod_{\nu=1}^{N}\Big\{\prod_{k=1}^{n_\nu}\frac{d\omega(q_{k,\nu})}{n_\nu!}e^{-i\alpha_{1,\nu}q_{k,\nu}}\times\prod_{k=1}^{m_\nu}\frac{d\omega(p_{k,\nu})}{m_\nu!}e^{-i\alpha_{2,\nu}p_{k,\nu}}\Big\}|a_{(n,m)}|^2.\tag{5.2}$$

Inserting (3.1) into (5.2), we find

$$\rho((\alpha_1)_N,(\alpha_2)_N)=\exp\Big\{i\sum_{\nu=1}^{N}\int dxdx'[\hat{\phi}_+(x)D_{+-}(x-x';\alpha_{2,\nu})\hat{\phi}_-(x')-\hat{\phi}_-(x)D_{-+}(x-x';\alpha_{1,\nu})\hat{\phi}_+(x')]\Big\}Z(\phi_+)Z^*(\phi_-),\tag{5.3}$$

where ϕ_- is defined on the complex-conjugate contour $C_-:t\to t-i\varepsilon$, and $D_{+-}(x-x';\alpha)$, $D_{-+}(x-x';\alpha)$ are the positive- and negative-frequency correlation functions, respectively.

We must integrate over the sets $(Q)_N$ and $(P)_N$ if the distribution of momentum fluxes over the cells is not fixed. As a result,

$$r(P)=\int D^4\alpha_1(P)D^4\alpha_2(P)\rho((\alpha_1)_N,(\alpha_2)_N), \qquad (5.4)$$

where the differential measure

$$D^4\alpha(P)=\prod_{\nu=1}^{N}\frac{d^4\alpha_\nu}{(2\pi)^4}K(P,(\alpha)_N)$$

takes into account the energy–momentum conservation laws:

$$K(P,(\alpha)_N)=\int\prod_{\nu=1}^{N}d^4Q_\nu e^{i\Sigma_{\nu=1}^{N}\alpha_\nu Q_\nu}\delta^{(4)}\left(P-\sum_{\nu=1}^{N}Q_\nu\right).$$

Explicit integration gives

$$K(P,(\alpha)_N)\sim\prod_{\nu=1}^{N}\delta^{(3)}(\alpha-\alpha_\nu),$$

where $\vec{\alpha}$ is the center-of-mass (c.m.) 3-vector.

To simplify the considerations let us choose the c.m. frame and put $\alpha=(-i\beta,\vec{0})$. As a result,

$$K(E,(\beta)_N)=\int_0^\infty\prod_{\nu=1}^{N}dE_\nu e^{\Sigma_{\nu=1}^{N}\beta_\nu E_\nu}\delta\left(E-\sum_{\nu=1}^{N}E_\nu\right).$$

Correspondingly, in the c.m. frame,

$$r(E)=\int D\beta_1(E)D\beta_2(E)\rho((\beta_1)_N,(\beta_2)_N),$$

where

$$D\beta(E)=\prod_{\nu=1}^{N}\frac{d\beta_\nu}{2\pi i}K(E,(\beta)_N)$$

and $\rho((\beta)_N)$ was defined in (5.3) with $\alpha_{k,\nu}=(-i\beta_{k,\nu},\vec{0})$, $\mathrm{Re}\,\beta_{k,\nu}>0$, $k=1,2$.

We will calculate the integrals over β_k using the stationary-phase method. The equations for the most probable values of β_k,

$$-\frac{1}{K(E,(\beta_k)_N)}\frac{\partial}{\partial\beta_{k,\nu}}K(E,(\beta_k)_N)$$
$$=\frac{1}{\rho((\beta_1)_N)}\frac{\partial}{\partial\beta_{k,\nu}}\rho((\beta)_N),\quad k=1,2, \qquad (5.5)$$

always have unique positive solutions $\tilde{\beta}_{k,\nu}(E)$. We suppose that the fluctuations of β_k near $\tilde{\beta}_k$ are small, i.e., are Gaussian. This is the basis of the local-equilibrium hypothesis.[32] In this case $1/\tilde{\beta}_{1,\nu}$ is the temperature in the initial state in the measurement cell ν, and $1/\tilde{\beta}_{2,\nu}$ is the temperature of the final state in the νth measurement cell.

The last formulation (5.4) implies that the 4-momenta $(Q)_N$ and $(P)_N$ cannot be measured. It is also possible to consider another formulation. For instance, we can suppose that the initial set $(Q)_N$ is fixed (measured) but $(P)_N$ is not. In this case we will have a mixed experiment: $\tilde{\beta}_{1,\nu}$ is defined by the equation

$$E_\nu=-\frac{1}{\rho}\frac{\partial}{\partial\beta_{1,\nu}}\rho,$$

and $\tilde{\beta}_{2,\nu}$ is defined by the second equation in (5.5).

Considering the limit $N\to\infty$, the dimension of the cells tends to zero. In this case we are forced by the quantum uncertainty principle to suppose that the 4-momentum sets (Q) and (P) are not fixed. This formulation becomes purely thermodynamical: we must assume that (β_1) and (β_2) are measurable quantities. For instance, we can fix (β_1) and try to find (β_2) as a function of the total energy E and a functional of (β_1). In this case Eqs. (5.5) become functional equations.

In our microcanonical description the finiteness of the temperature does not affect the quantization mechanism. In fact, one can see from (5.3) that all thermodynamical information is contained in the operator exponent

$$e^{\hat{N}(\phi_i^*\phi_j)}=\prod_\nu\prod_{i\neq j}e^{i\int\hat{\phi}_iD_{ij}\hat{\phi}_j},$$

whose expansion describes the environment, and the "mechanical" perturbations are described by the amplitude $Z(\phi)$. This factorization was achieved by the introduction of the auxiliary field ϕ and is independent of the choice of boundary conditions, i.e., of the choice of the system environment.

5.2. Wigner-function formalism

We will use the Wigner-function formalism in the Carruthers–Zachariasen formulation.[25] For the sake of generality the m-into-n-particle transition will be considered. This will allow us to include heavy ion–ion collisions.

In the previous section the generating functional $\rho((\beta)_N)$ was calculated by dividing the "measuring device" (calorimeter) into N cells. It was assumed that the dimension of the cells of the device tends to zero ($N\to\infty$). Now we will specify the cell coordinates, using Wigner's description.

Let us introduce the distribution function F_n which defines the probability of finding n particles with definite momentum and arbitrary coordinates. These probabilities (cross sections) are usually measured in particle physics. The corresponding Fourier-transformed generating functional can be deduced from (5.3):

$$F(z,(\beta_1)_N,(\beta_2)_N)$$
$$=\prod_{\nu=1}^{N}\prod_{i\neq j}e^{\int d\omega(q)\hat{\phi}_i^*(q)e^{-\beta_{j,\nu}\epsilon(q)}\hat{\phi}_j(q)z_{ij}^\nu(q)}Z(\phi_+)Z^*(\phi_-). \qquad (5.6)$$

The variation of F with respect to $z_{ij}^\nu(q)$ generates the corresponding distribution functions. One can interpret $z_{ij}^\nu(q)$ as the local activity: the logarithm of $z_{ij}^\nu(q)$ is conjugate to the particle number in the cell ν with momentum q for the initial $(ij=21)$ or final $(ij=12)$ states. Note that $z_{ij}^\nu(q)\hat{\phi}_i^*(q)\hat{\phi}_j(q)$ can be considered as the operator of activity.

The Boltzmann factor $e^{-\beta_{i,\nu}\epsilon(q)}$ can be interpreted as the probability of finding a particle with energy $\epsilon(q)$ in the final

state ($i=2$) and in the initial state ($i=1$). The total probability, i.e., the process of creation and subsequent absorption of n particles, is defined by multiplication of these factors.

The generating functional (5.6) is normalized as follows:

$$F(z=1,(\beta))=\rho((\beta)), \tag{5.7}$$

$$F(z=0,(\beta))=|Z(0)|^2=\rho_0(\phi_\pm)|_{\phi_\pm=0}.$$

Here

$$\rho_0(\phi_\pm)=Z(\phi_+)Z^*(\phi_-)$$

is the "probability" of the vacuum-to-vacuum transition in the presence of the auxiliary fields $\phi_\pm$. The one-particle distribution function

$$F_1((\beta_1)_N,(\beta_2)_N;q)=\frac{\delta}{\delta z_{ij}^{\nu}(q)}F|_{z=0}=\{\hat\phi_i^*(q)e^{-\beta_i^{\nu}\epsilon(q)/2}\}\times\{\hat\phi_j(q)e^{-\beta_i^{\nu}\epsilon(q)/2}\}\rho_0(\phi_\pm) \tag{5.8}$$

describes the probability of finding one particle in the vacuum.

We use the definition

$$F_1((\beta_1)_N,(\beta_2)_N;q)=\int dxdx'e^{iq(x-x')}e^{-\beta_{i,\nu}\epsilon(q)}\Big\}\times\hat\phi_i(x)\hat\phi_j(x')\rho_0(\phi_\pm)$$
$$=\int dr\{dye^{iqy}e^{-\beta_{i,\nu}\epsilon(q)}\}\hat\phi_i(r+y/2)\hat\phi_j(r-y/2)\rho_0(\phi_\pm)\} \tag{5.9}$$

and introduce the one-particle Wigner function W_1 (Ref. 25):

$$F_1((\beta_1)_N,(\beta_2)_N;q)=\int drW_1((\beta_1)_N,(\beta_2)_N;r,q).$$

Thus,

$$W_1((\beta_1)_N,(\beta_2)_N;r,q)=\int dye^{iqy}e^{-\beta_{i,\nu}\epsilon(q)}\hat\phi_i(r+y/2)\hat\phi_j(r-y/2)\rho_0(\phi_\pm).$$

This distribution function describes the probability of finding in the vacuum a particle with momentum q at the point r in the cell ν.

Since the choice of coordinates of the device is in our hands, it is natural to adjust the cell coordinate to the coordinate of the measurement r:

$$W_1((\beta_1)_N,(\beta_2)_N;r,q)=\int dye^{iqy}e^{-\beta_i(r)\epsilon(q)}\Big\}\hat\phi_i(r+y/2)\hat\phi_j(r-y/2)\rho_0(\phi_\pm).$$

This choice of the device coordinates lead to the following generating functional:

$$F(z,\beta)=\exp\Big\{i\int dydr[\hat\phi_+(r+y/2)D_{+-}(y;\beta_2(r),z)\times\hat\phi_-(r-y/2)-\hat\phi_-(r+y/2)D_{-+}(y;\beta_1(r),z)\times\hat\phi_+(r-y/2)]\Big\}\rho_0(\phi_\pm), \tag{5.10}$$

where

$$D_{+-}(y;\beta(r),z)=-i\int d\omega(q)z_{+-}(r,q)e^{iqy}e^{-\beta(r)\epsilon(q)},$$

$$D_{-+}(y;\beta(r),z)=i\int d\omega(q)z_{-+}(r,q)e^{-iqy}e^{-\beta(r)\epsilon(q)}$$

are the modified positive and negative correlation functions.

Inclusive partial distribution functions are familiar in particle physics. These functions describe the distributions in the presence of an arbitrary number of other particles. For instance, the one-particle partial distribution function is

$$P_{ij}(r,q;(\beta))=\frac{\delta}{\delta z_{ij}(r,q)}F(z,(\beta))|_{z=1}=\frac{e^{-\beta_i(r)\epsilon(q)}}{(2\pi)^3\epsilon(q)}\int dye^{iqy}\hat\phi_i(r+y/2)\hat\phi_j(r-y/2)\rho(\phi_\pm,(\beta)), \tag{5.11}$$

where Eq. (5.7) has been used.

The mean multiplicity $n_{ij}(r,q)$ of particles in the infinitesimal cell Y with momentum q is

$$n_{ij}(r,q)=\int dq\frac{\delta}{\delta z_{ij}(r,q)}\ln F(z,(\beta))|_{z=1}.$$

If the interactions among the fields are switched off, we find (omitting the indexes)

$$n(r,q_0)=\frac{1}{e^{\beta(r)q_0}-1},\quad q_0=\epsilon(q)>0.$$

This is the mean multiplicity of black-body radiation.

5.3. Closed-path boundary conditions

The formalism developed above allows us to introduce more general "closed-path" boundary conditions. The presence of external black-body radiation flow will reorganize the differential operator $\exp\{\hat N(\phi_i^*\phi_j)\}$ only, and the new generating functional ρ_{cp} has the form

$$\rho_{cp}(\alpha_1,\alpha_2)=e^{\hat N(\phi_i^*\phi_j)}\rho_0(\phi_\pm).$$

The calculation of the operator $\hat N(\phi_i^*\phi_j)$ is strictly analogous to that of Sec. 3. Introducing cells, we find

$$\hat N(\phi_i^*\phi_j)=\int drdy\hat\phi_i(r+y/2)\bar n_{ij}(r,y)\hat\phi_j(r-y/2),$$

where the occupation number $\bar n_{ij}$ carries the cell index r:

$$\bar n_{ij}(r,y)=\int d\omega(q)e^{iqy}n_{ij}(r,q)$$

and $[q_0=\epsilon(q)]$

$$n_{++}(r,q_0)=n_{--}(r,q_0)=\bar{n}(r,(\beta_1+\beta_2)|q_0|/2)$$
$$=\frac{1}{e^{(\beta_1+\beta_2)(r)|q_0|/2}-1},$$
$$n_{+-}(r,q_0)=\Theta(q_0)(1+\bar{n}(r,\beta_2 q_0))$$
$$+\Theta(-q_0)\bar{n}(r,-\beta_1 q_0),$$
$$n_{-+}(r,q_0)=n_{+-}(r,-q_0).$$

For simplicity, the c.m. system was used.

Calculating ρ_0 perturbatively, we find

$$\rho_{cp}(\beta)=\exp\{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)\}\exp\Big\{i\int drdy[\hat{j}_i(r+y/2)G_{ij}(y,(\beta(r))\hat{j}_j(r-y/2)\}, \tag{5.12}$$

where, using matrix notation,

$$iG(q,\beta(r))=\begin{pmatrix}\frac{i}{q^2-m^2+i\varepsilon} & 0\\ 0 & -\frac{i}{q^2-m^2-i\varepsilon}\end{pmatrix}$$
$$+2\pi\delta(q^2-m^2)$$
$$\times\begin{pmatrix}n\left(\frac{(\beta_1+\beta_2)(r)}{2}|q_0|\right) & n(\beta_1(r)|q_0|)a_+(\beta_1)\\ n(\beta_2(r)|q_0|)a_-(\beta_2) & n\left(\frac{(\beta_1+\beta_2)(r)}{2}|q_0|\right)\end{pmatrix} \tag{5.13}$$

and

$$a_\pm(\beta)=-e^{\beta(|q_0|\pm q_0)/2}. \tag{5.14}$$

Formally, these Green functions obey the standard equations in the y space:

$$(\partial^2-m^2)_y G_{ii}=\delta(y),$$
$$(\partial^2-m^2)_y G_{ij}=0,\quad i\neq j,$$

since $\Phi(\sigma_\infty)\neq 0$ reflects the mass-shell particles. But the boundary conditions for these equations are not obvious.

It should be stressed that in our considerations r is the coordinate of the *measurement*, i.e., r is the calorimeter-cell coordinate, and there is no need to divide the interaction region of the QGP into domains (cells). This means that L must be smaller than the typical range of fluctuations of the QGP. But, on other hand, L cannot be arbitrarily small, since this would lead to the assumption of a *local* factorization property of the correlation functions, i.e., to the absence of interactions.

Thus, replacing $\beta\to\beta(r)$, we must assume that $\beta_{1(2)}(r)$ and $z_{+-(-+)}(r,k)$ are constants on the interval L. This prescription adapts the Wigner-function formalism to the case of high multiplicities. It describes temperature fluctuations larger than L and averages fluctuations smaller than L, leading to the absence, on the average, of "non-Gaussian" fluctuations.

This is the typical "calorimetric" measurement, since in a dominant number of calorimeter cells the measured mean values of the energy, with exponential accuracy, are "good" parameters $\sim 1/\beta_2(r,E)$. We will assume that the dimension of the calorimeter cells is $L\ll L_{cr}$, where L_{cr} is the dimension of the characteristic fluctuations at a given n. In the deep asymptotic region in n we must have $L_{cr}\to\infty$. This argument shows that the proposed experiment with a calorimeter as the device for measuring particle energies is sufficiently informative in the high-multiplicity domain.

6. NONSTATIONARY STATISTICAL OPERATOR

One cannot expect an obvious connection between the foregoing S-matrix (microcanonical) approach and Zubarev's[32] approach. The reason is the introduction into Zubarev's formalism of an interaction with a heat bath, external to the system under consideration. This interaction is crucial for the definition of the NSL for the explanation of the trend to a maximal-entropy state, starting the evolution from a local-equilibrium state.[7)]

Therefore, in Zubarev's theory the local-equilibrium state was chosen as the boundary condition. It is assumed that in suitably defined cells of the *system* for a given temperature distribution $T(\vec{x},t)=1/\beta(\vec{x},t)$, where $(\vec{x},t)$ is the index of the cell, the entropy is maximal. The corresponding nonequilibrium statistical operator

$$\rho_z\sim e^{-i\int d^3x\beta(\vec{x},t)T_{00}} \tag{6.1}$$

describes the evolution of a system in the time scale t. Here $T_{\mu\nu}$ is the energy–momentum tensor. It is assumed that the system "follows" the $\beta(\vec{x},t)$ evolution, and the local temperature $T(\vec{x},t)$ is defined as the external parameter which is the regulator of the system dynamics. For this purpose the special $i\varepsilon$ prescription was introduced [it was not shown in (6.1)].[32] It brings the interaction with the heat bath.

The KMS periodic boundary condition cannot be applied for a nonstationary temperature distribution, and for this reason the decomposition

$$\beta(\vec{x},t)=\beta_0+\beta_1(\vec{x},t) \tag{6.2}$$

was proposed in Ref. 30. Here β_0 is a constant, and the inequality

$$\beta_0\gg|\beta_1(\vec{x},t)|$$

is assumed. Then

$$\rho_z\sim e^{-\beta_0(H_0+V+B)}, \tag{6.3}$$

where H_0 is the free part of the Hamiltonian, V describes the interactions, and the term B linear in β_1/β_0 is connected with the deviation of the temperature from the "equilibrium" value $1/\beta_0$. The presence of B perturbations creates "thermal" flows in the system to explain the increasing entropy. Considering V and B as perturbations, one can calculate the observables by averaging over equilibrium states, i.e., adopting the KMS boundary condition. Using standard terminology, one can consider V as a "mechanical" perturbation and B as a "thermal" perturbation.

The quantization problem for the operator (6.3) is connected with the definition of the space–time sequence of mechanical (V) and thermal (B) excitations. This is neces-

sary, since the mechanical excitations influence the thermal ones, and vice versa. It was assumed in Ref. 30 that V and B are commuting operators, i.e., the sequence of V and B perturbations is not sufficient. The corresponding generating functional has the form[30]

$$Z(j)=\exp\left\{-i\int_{C_\beta}d^4x\left(V(-i\hat{j}(x))+\frac{\beta_1(\mathbf{x};\tau)}{\beta_0}T_{00}[-i\hat{j}(x)]\right.\right.$$
$$\left.\left.-\int_{-\infty}^{0}dt_1\frac{\beta_1(\mathbf{x},\tau+t_1)}{\beta_0}T_{00}[-i\hat{j}(\mathbf{x},x+t_1)]\right)\right\}$$
$$\times\mathrm{Tr}(e^{-\beta_0 H_0}T_C e^{i\int_C d^4y j(y)\Phi(y)}),$$

where the time contour C_β was described in Sec. 4.1 and τ is the measurement time.

It is evident that this solution leads to renormalization by the interactions with the external field $\beta(\vec{x},t)$ even without interactions among the fundamental fields Φ. The source of this renormalization is the kinetic term in the energy–momentum tensor T_{00}, i.e., it follows from the "thermal" interactions with the external heat bath. Note the absence of this renormalization in the S-matrix formalism [see, for instance, (3.25)], where the interactions are generated by V perturbations only.

In Ref. 53 the operators V and B are noncommuting, and the B perturbations were switched on after the V perturbations. In this formulation, nondynamical renormalizations are also present, but it is not unlikely that they are canceled at the very end of the calculations.[54]

This formulation with $\beta(\vec{x},t)$ as the external field is reminiscent of the old first-quantized field theory in which matter is quantized but fields are not. It is well known that a consistent quantum field theory requires second quantization. Following this analogy, if we want to take into account consistently the mutual influence of V and B perturbations, the field $\beta(\vec{x},t)$ must be fundamental, i.e., it must be quantized (and the assumption of Ref. 30 becomes true). But this is evidently the wrong idea in the canonical Gibbs formalism. Thus, as in the first-quantized theory, the theory with the operator (6.1) must have a restricted range of validity.[32]

7. CONCLUSION

In our interpretation of the real-time finite-temperature field theory, the statistics and the field quantum dynamics were factorized: the statistics is fixed by the operator $\exp\{\hat{N}(\phi_i^*\phi_j\}$, and a pure field-theoretical dynamics is described by $\rho_0(\phi_\pm)=Z(\phi_+)Z^*(\phi_-)$, where $Z(\phi_\pm)$ is the vacuum-to-vacuum transition amplitude in the presence of the external (auxiliary) fields, $\langle\mathrm{vac}|\mathrm{vac}\rangle_\phi$. We can say that the operator $\exp\{\hat{N}(\phi_i^*\phi_j\}$ maps the system of interacting fields onto a state with definite thermodynamical parameters. We have concentrated our attention in this paper on the structure and origin of the operator $\exp\{\hat{N}(\phi_i^*\phi_j)\}$, and we do not discuss $\rho_0(\phi_\pm)$. But our formalism allows us to use the following "S-matrix" properties, which are new for thermodynamics, to define ρ_0.

First, there is the absence of Matsubara imaginary parts of the time contour in ρ_0, by definition: the approach is purely "real-time." This allows us to construct the formalism without referring to time-asymptotic properties of correlation (Green) functions, and to introduce the temperature description without using the notion of the grand canonical ensemble in constructing the environment of the system, i.e., the measuring device, "by hand."

Moreover, the factorization property discussed here has an important consequence which might allow us to calculate expectation values with high accuracy. Let us consider the theoretical problem of calculating $\rho_0(\phi_\pm)$. To define the functional measure, the orthonormalizability (i.e., unitarity) condition may be used. It leads to the representation[55]

$$\rho_0(\phi)=e^{-i\hat{K}(j,e)}\int DM(\Phi)e^{-U(\Phi,e)}e^{\int dx(v'(\Phi)+j)\phi}, \tag{7.1}$$

where the expansion in the operator

$$\hat{K}(j,e)=2\,\mathrm{Re}\int dx\frac{\delta}{\delta j(x)}\frac{\delta}{\delta e(x)}$$

generates the perturbation series and

$$U(\Phi,e)=V(\Phi+e)-V(\Phi-e)-2\,\mathrm{Re}\int dx e v'(\Phi)$$

weights the quantum fluctuations. The most important term in (7.1) is the measure

$$DM(\Phi)=\prod_x d\Phi(x)\delta(\partial_\mu^2\Phi+m^2\Phi+v'(\Phi)-j),$$

where $v'(\Phi)\equiv\delta V(\Phi)/\delta\Phi(x)$. Thus, solving the equation

$$\partial_\mu^2\Phi+m^2\Phi+v'(\Phi)=j, \tag{7.2}$$

we will find *all* contributions.[8)]

At the very end of the calculations one must put $e=j=0$. Therefore, Eq. (7.2) can be solved by expanding it in j. This shows that (7.1) reproduces at $j=0$ the usual stationary-phase method. Indeed, it can be verified that (7.1) gives the usual perturbation theory.[55]

But Eq. (7.2) gives many more possibilities. Note that the left-hand side of this equation is the sum of known classical forces, and the right-hand side is the quantum force j. Equation (7.2) establishes a local equilibrium between these forces. This solves the old problem of quantization with constraints: it can be done by field transformations in the path integrals, since Eq. (7.2) shows how j must be transformed when the left-hand side is transformed. The presence of derivatives in (7.2) shows that the quantum force must be transformed in the tangent space of fields.[9)]

The right-hand side of Eq. (7.2) may also contain an additional force to describe the external influence on the system of interacting fields. This force was omitted in Eq. (7.2), assuming that the process of particle creation (and absorption) was switched on adiabatically.

As was mentioned above, the action of the operator $e^{-\hat{N}(\beta,z;\phi)}$ on $\rho_0(\phi)$ maps the interacting-field system onto measurable states. Let us consider what this gives. The result of the action has the form

$$\rho(\beta,z)=e^{-i\hat{K}(j,e)}\int DM(\Phi)e^{-U(\Phi,e)}e^{-N(\beta,z;\Phi)},$$

where $N=N_1+N_2$ and

$$N_{1(2)}(\beta,z;\Phi)=\int drd\omega(k)e^{-\beta_{1(2)}(r)\varepsilon(k)}\times z_{+-(-+)}(k,r)|\Gamma(k,\Phi)|^2. \quad (7.3)$$

Here r is regarded as the *index* of the calorimeter cell. This formula needs more careful explanation. Instead of (7.3) we must consider

$$N_{1(2)}(\beta,z;\Phi)=\int drd\omega(k)e^{-\beta_{1(2)}(r)\varepsilon(k)}z_{+-(-+)}(k,r)\times\int dq\,\delta_L(q)\Gamma(k+q,\Phi)\Gamma^*(k-q,\Phi),$$

where L is the scale at which $\beta_{1(2)}(r)$ and $z_{+-(-+)}(k,r)$ can be regarded as constants (L is the dimension of the calorimeter cell). If $L\to\infty$, then $\delta_L(q)$ can be changed to the usual δ function $\delta(q)$, and therefore in this limit we will have (7.3). We have considered this limit, supposing that the measurement is not in contradiction with the quantum uncertainty principle.

Thus, in deriving $N_{1(2)}(\beta,z;\Phi)$ we have used the condition that r is the coordinate of a cell of size L. With this condition,

$$\Gamma(k,\Phi)=\int dxe^{ikx}(\partial_\mu^2+m^2)\Phi \quad (7.4)$$

can be regarded as the order parameter. Indeed, $\Gamma(k,\Phi)$ is the element of the action symmetry group, since it is linear in the field Φ, and the generating functional $R(\beta,z)$ is trivial if $\langle|\Gamma(k,\Phi)|^2\rangle=0$. In this case there is no creation of particles, i.e., there are no measurable asymptotic states (fields).

Indeed, it can be shown[58] that *all* the quantum corrections to the soliton contribution in the $(1+1)$-dimensional sine-Gordon model are equal to zero. This is in accord with the result of Ref. 59 and with the factorizability of the soliton S matrix. Then it is easily seen, by computing the integral in (7.4) by parts, that $\Gamma(k,\Phi_s)=0$, where Φ_s is the soliton solution. This result shows that the hidden symmetry of the sine-Gordon model cannot be broken and that the corresponding (polynomial) integrals of the motion are conserved. The application of this idea to non-Abelian field theory should be fruitful.

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[1)] "What, never (*is the time-reversed flow*)? No, never! What, never? Well, hardly ever." [2]

[2)] The vertex renormalization takes into account the time-reversed fluctuations in the nonequilibrium flow.

[3)] This question was also considered in Ref. 29.

[4)] I.e., if we wish to consider the "thermodynamical limit."

[5)] The parton lifetime with virtuality $|q|$ is $\sim 1/|q|$, and the time needed for the formation of hadrons of mass m_h is $\sim 1/m_h$. Therefore the partons have time to decay before hadron formation if $|q|\gg m$. But this situation is rare, since the thermal motion in the initial stage of the process is high.

[6)] It is assumed that the number of calorimeter cells is $K\geqslant k$.

[7)] This condition is not necessary in the S-matrix formalism, since it is "dynamical" in nature, i.e., it includes the notion of initial and final states as the boundary conditions.

[8)] This means that the unitarity condition is necessary and sufficient for the definition of the path-integral measure for $R_0(\phi_\pm)$.[56]

[9)] This explains why the ordinary transformation of the path integral is impossible, giving wrong results.[57]

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