The problem of Okubo–Zweig–Iizuka rule violation in nucleon–antinucleon annihilation at rest

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A review of the problem of whether the violation of the OZI rule in nucleon–antinucleon annihilation at rest can be explained in the framework of conventional mechanisms is given in detail. While the vector-dominance model and the rescattering model qualitatively describe the OZI-rule violation in the reactions $\bar{p}p \to \phi\gamma$ and $\bar{p}p \to \phi\pi^0$ for the annihilation from the S state of the protonium atom, the latter model cannot explain the fact that the annihilation into $\phi\pi^0$ from the P state is not seen and the OZI rule in the reaction $\bar{p}p \to f_2'\pi^0$ is not satisfied. We also discuss what information about the OZI-rule violation can be extracted from the reaction $\bar{p}p \to \phi\pi^+\pi^-$ and decays of the J/Ψ meson.
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1. INTRODUCTION

The Okubo–Zweig–Iizuka (OZI) rule[1] was proposed originally for the explanation of several unusual phenomena, in particular the fact that the width of the decay $\phi \to 2\pi$ is much smaller than the width of the decay $\phi \to 2K$, although the phase space in the first case is much greater and the process $\phi \to 2\pi$ is not forbidden by any conservation law. As argued by Lipkin,[2] a more relevant name for this rule would be AZ (Aleksander–Zweig).

In its present formulation the OZI rule says that processes described by disconnected quark diagrams (i.e., diagrams which can be connected by only gluon lines) are suppressed.

There exist many papers in which the decays of the J/Ψ and Υ mesons are considered in the framework of the three-gluon mechanism, and the agreement between theory and experiment is rather impressive (see, e.g., Ref. 3). The success of these calculations was treated by some physicists as the first proof of asymptotic freedom in QCD. On the other hand, attempts to substantiate the OZI rule in the framework of QCD encounter serious difficulties (see, e.g., Refs. 4–7 and references therein). In particular, the problem of whether the OZI rule applies to baryons is not clear,[8–12] but anyway the usual point of view is that any substantial violation of this rule in some process is a signal that some unusual physics plays an important role in this process.

The recent experimental data on $\bar{p}p$ and $\bar{p}n$ annihilation at rest obtained by the Asterix, Crystal Barrel, and Obelix groups[13–16] at LEAR have shown that the branching ratios of the reactions $\bar{p}p \to \phi\gamma$, $\bar{p}p \to \phi\pi^0$, and $\bar{p}n \to \phi\pi^-$ are much larger than expected from naive OZI-rule estimations. Indeed, let θ be the ϕ–ω mixing angle such that the ω and ϕ states are constructed from the u, d, and s quarks as follows:

$$\omega = \frac{1}{\sqrt{6}}(\sqrt{2}\cos\theta + \sin\theta)(u\bar{u} + d\bar{d}) + \frac{1}{\sqrt{3}}(\cos\theta - \sqrt{2}\sin\theta)s\bar{s},$$

$$\phi = \frac{1}{\sqrt{6}}(\cos\theta - \sqrt{2}\sin\theta)(u\bar{u} + d\bar{d}) - \frac{1}{\sqrt{3}}(\sqrt{2}\cos\theta + \sin\theta)s\bar{s}. \quad (1)$$

Then if θ takes the values (36–39)° (see, for example, Ref. 17), the ϕ/ω production ratio takes the values

$$\left|\frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sin\theta}\right|^2 = (0.2-4.2)\cdot 10^{-3},$$

while in practice[13–16]

$$\mathrm{Br}(\bar{p}p \to \phi\gamma)/\mathrm{Br}(\bar{p}p \to \omega\gamma) = 0.243 \pm 0.086, \quad (2)$$

$$\mathrm{Br}(\bar{p}p \to \phi\pi^0)/\mathrm{Br}(\bar{p}p \to \omega\pi^0) = 0.096 \pm 0.015, \quad (3)$$

$$\mathrm{Br}(\bar{p}n \to \phi\pi^-)/\mathrm{Br}(\bar{p}n \to \omega\pi^-) = 0.083 \pm 0.025. \quad (4)$$

The ratio of the corresponding phase volumes is 0.853 for the reaction (2) and 0.849 for the reactions (3) and (4). Therefore the discrepancy between theory and experiment is very large.

The extent of the violation of the OZI rule in other reactions of nucleon–antinucleon annihilation is given, for example, in Ref. 18.

A rather simple explanation of the OZI-rule violation in the reaction (2) has been proposed by Locher, Lu, and Zou;[19] for completeness we describe this explanation in Sec. 2. However, the main purpose of the present paper is to review the state of the art in explaining the data (3) and (4) in the framework of the so-called rescattering model considered by Locher, Lu, and Zou,[19] Locher and Lu,[20] and Buzatu and Lev.[21,22] The main question here is whether the explanation given in those references is reliable [and then there is no reason to think that something unusual happens in the reactions (3) and (4)] or whether this explanation is clearly insufficient (leaving the problem of the OZI-rule violation open). A discussion of some aspects of this problem is given in Ref. 23.

In the present paper we do not consider explanations of the OZI-rule violation in other models, for example, in models in which the OZI-rule violation is an instanton effect,[24] in the model of hidden strangeness,[25,26] in the Skyrme model,[27] and others (a review of different explanations can be found in Ref. 28). All such models suggest from the beginning that an explanation of the OZI-rule violation in the reactions (3) and (4) can be obtained only in the framework of unconventional mechanisms.

As follows from isotopic invariance, the reactions $\bar{p}p \to \phi\pi^0$ and $\bar{p}n \to \phi\pi^-$ can be easily related to each other (see, for example, Refs. 21 and 29 and Sec. 9).

In Secs. 3 and 4 we show that there exist many options in choosing the form of the amplitude in the rescattering model; in particular, we mention two essentially different choices, called model A and model B. Neither of these models has theoretical advantages in comparison with the other (or perhaps model B is substantiated to a greater extent), but, as shown in Sec. 5, fairly good agreement with the data can be obtained in model A, while, as shown in Sec. 6, model B gives values substantially below the data.

However, the success of model A immediately poses the problem of why the reaction $\bar{p}p \to \phi\pi^0$ is not seen when the proton and the antiproton annihilate from the P state of the hydrogen-like $\bar{p}p$ atom. This problem is considered in Sec. 7.

As shown in Sec. 8, the important process for understanding the OZI-rule violation is $\bar{p}p \to f_2'\pi^0$, since the rescattering contribution to this process is negligible.

The conclusion about the OZI-rule violation in the process (4) follows from the data of the Obelix Collaboration[15,16] on the reaction $\bar{p}d \to p\phi\pi^-$ when the proton can be considered as a spectator, i.e., its momentum $\mathbf{p}$ is such that $|\mathbf{p}| < 200$ MeV/c. However, the same degree of OZI-rule violation has been observed in the case when $|\mathbf{p}| \in (400,800)$ MeV/c. Therefore the problem arises of whether the reason for the OZI-rule violation in this case is the same [i.e., the OZI-rule violation in the reaction (4)] or whether some nuclear effects are important. This problem is considered in Sec. 9.

In Sec. 10 we consider the problem of what can be learned about the rescattering contribution by taking into account the existing data on certain decays of the J/Ψ meson. Finally, as shown in Sec. 11, an analog of model A in the reaction $\bar{p}p \to \phi\pi^+\pi^-$ is inconsistent, since the corresponding amplitude does not satisfy the unitarity relation. Therefore this reaction poses additional problems for understanding the OZI-rule violation.

2. THE REACTION $\bar{p}p \to \phi\gamma$ IN THE VECTOR-DOMINANCE MODEL

We describe in this section the explanation of the OZI-rule violation in the reaction $\bar{p}p \to \phi\gamma$ proposed in Ref. 19.

Consider first the reaction $\bar{p}p \to \phi\rho$. The amplitude of this reaction can be written in the form

$$A_{\bar{p}p \to \phi\rho} = F(k_1^2 = m_\rho^2, \ldots) e_{\mu\nu\rho\sigma} e_1^{*\mu} e_2^{*\nu} k_1^\rho k_2^\sigma, \tag{5}$$

where $\mu,\nu,\rho,\sigma = 0,1,2,3$, $e_{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor ($e_{0123} = -1$), e_1 and k_1 are the polarization vector and the four-momentum of the ρ meson, respectively, e_2 and k_2 are the corresponding quantities for the ϕ meson, a sum over repeated indices is assumed, and m_ρ is the ρ-meson mass. The function F in this expression depends on the polarizations of the proton and antiproton and on the masses of all particles in question, but we assume that the proton, antiproton, and ϕ meson are always on-shell, the proton and antiproton are at rest, and only the dependence of F on k_1 is explicitly indicated.

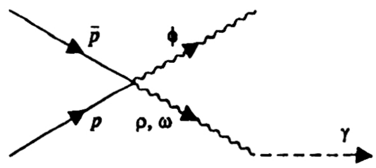


FIG. 1. Vector-dominance model for the reaction $\bar{p}p \to \phi\gamma$.

In the framework of the vector-dominance model the amplitude of the reaction $\bar{p}p \to \phi\gamma$ is described by the diagrams shown in Fig. 1. By analogy with Eq. (5), the amplitude of the reaction corresponding to the ρ meson in the intermediate state can be written in the form

$$A_{\bar{p}p \to \phi\gamma} = F(k_1^2 = 0, \ldots) c_{\rho\gamma} e_{\mu\nu\rho\sigma} e_3^{*\mu} e_2^{*\nu} k_3^\rho k_2^\sigma, \tag{6}$$

where e_3 and k_3 are the polarization vector and the four-momentum of the photon, respectively, and $c_{\rho\gamma}$ is a constant describing the strength of the $\rho \to \gamma$ transition.

Let us introduce the quantity

$$g(k_1^2) = \sum |F(k_1^2, \ldots)|^2, \tag{7}$$

where Σ implies that we take the average value over all the initial polarizations and sum over the final ones. Following Ref. 19, we also express $c_{\rho\gamma}$ in terms of the universal constant f_ρ (Ref. 30): $c_{\rho\gamma} = e m_\rho^2 / f_\rho$. Then it follows from Eqs. (5)–(7) that the ratio of the branching ratios for the reactions $\bar{p}p \to \phi\gamma$ and $\bar{p}p \to \phi\rho$ is given by

$$\frac{\mathrm{Br}(\bar{p}p \to \phi\gamma)}{\mathrm{Br}(\bar{p}p \to \phi\rho)} = \left[\frac{g(0)}{g(m_\rho^2)}\right] \frac{e^2}{f_\rho^2} \left(\frac{k_{\gamma\phi}}{k_{\rho\phi}}\right)^3, \tag{8}$$

where $k_{\gamma\phi}$ is the c.m. momentum in the $\gamma\phi$ system and $k_{\rho\phi}$ is understood analogously.

The authors of Ref. 19 do not take into account the dependence of g on k_1^2, and they assume that g is some constant. Then taking into account the fact that $e^2/4\pi = 1/137$, $f_\rho^2/4\pi = 2.5$, and $\mathrm{Br}(\bar{p}p \to \phi\rho) = (3.4 \pm 1.0) \cdot 10^{-4}$ according to Ref. 14, the result of Ref. 19 is

$$\mathrm{Br}(\bar{p}p \to \phi\gamma) = 1.27 \cdot 10^{-5}, \tag{9}$$

in excellent agreement with the experimental result 1.0×10^{-5} in Ref. 14. The authors of Ref. 19 also discuss the contribution of the ω meson, but this contribution is not very important.

It is interesting to note that in the model described above the unexpectedly large value of $\mathrm{Br}(\bar{p}p \to \phi\gamma)$ is a consequence of the purely kinematical factor $(k_{\gamma\phi}/k_{\rho\phi})^3$, which is equal to 13.1. Although the success of the simple model proposed in Ref. 19 is rather impressive, it is necessary to take into account the fact that the additional assumption used

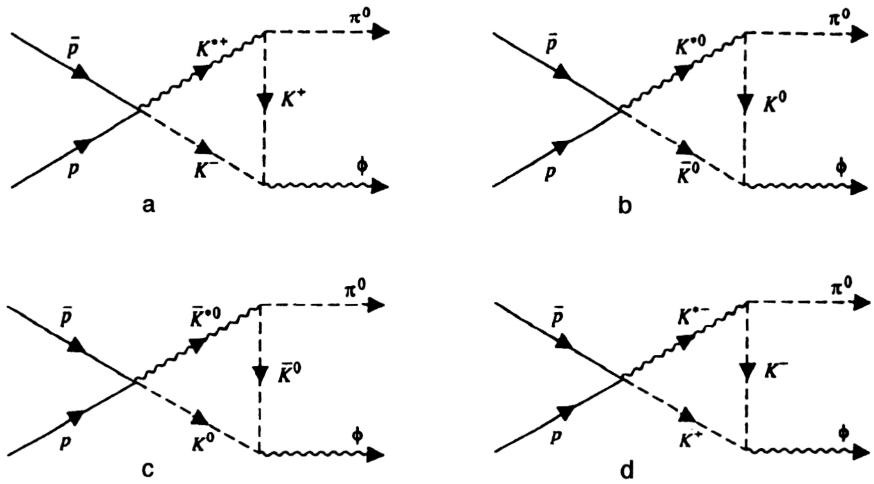


FIG. 2.

in deriving the result is that the dependence of the function g on the off-shell nature of the ρ meson is not important. It is clear that at the present stage of the theory of strong interactions we cannot test the correctness of this assumption.

3. THE PROBLEM OF CALCULATING THE PROCESS $\bar{p}p \to \phi\pi^0$ WITH K^*K INTERMEDIATE STATES

As has been pointed out by several authors (see, e.g., Refs. 31–33), a large amplitude of some OZI-forbidden transitions may be a consequence of the possibility that they can go via two-step processes in which each individual transition is OZI-allowed.

As an example, we first consider the contribution of K^*K intermediate states to the reaction $\bar{p}p \to \phi\pi^0$. There exist four diagrams, shown in Fig. 2, and, as easily follows from isotopic invariance, the contributions of these diagrams in the channel with isospin $I=1$ and spin $S=1$ are equal to each other. To calculate these contributions we have to know the amplitudes of the reactions $\bar{p}p \to K^{*+}K^-$, $K^{*+} \to \pi^0 K^+$, and $K^+K^- \to \phi$ entering into diagram (a) of Fig. 2. We use p_1 and p_2 to denote the four-momenta of the initial proton and antiproton, respectively, k_1 and k_2 to denote the four-momenta of the final π^0 and ϕ mesons, respectively, k_1', k_2', and k_3' to denote the four-momenta of the K^{*+}, K^-, and K^+ mesons, respectively, and e and e' to denote the polarization vectors of the ϕ and K^{*+} mesons, respectively. The initial proton is described by the Dirac spinor $u(p_1)$, and the initial antiproton is described by the Dirac spinor with negative energy $v(p_2)$. We also use m, m_π, m_K, and m_ϕ to denote the proton mass and the masses of the corresponding mesons.

Consider the amplitude $\bar{p}p \to K^{*+}K^-$. If all the particles are on-shell, the only amplitude in the channel with $I=S=1$ which survives when the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are small is

$$M^{(11)}_{\bar{p}p\to K^{*+}K^-} = f^{(11)}_{\bar{p}p\to K^{*+}K^-} \times [\bar{v}(p_2)\gamma^\mu u(p_1)] e_{\mu\nu\rho\sigma} e'^{*\nu} k_1'^\rho k_2'^\sigma, \quad (10)$$

where $f_{\bar{p}p\to K^{*+}K^-}$ is some constant and γ^μ is the Dirac γ matrix. The total cross section corresponding to the amplitude (10) can be calculated in a standard way, and the result is

$$\sigma^{(11)}_{\bar{p}p\to K^{*+}K^-} = |f^{(11)}_{\bar{p}p\to K^{*+}K^-}|^2 \frac{(3m^2+2p^2)k'^3}{12\pi p}, \quad (11)$$

where $\mathbf{p}$ is the proton momentum in the c.m. frame of the $\bar{p}p$ system, $p=|\mathbf{p}|$, and k' is the magnitude of the c.m. momentum for the $K^{*+}K^-$ system.

By analogy with Eqs. (10) and (11), the amplitude of the reaction $\bar{p}p \to \phi\pi^0$ has the form

$$M_{\bar{p}p\to\phi\pi^0} = f_{\bar{p}p\to\phi\pi^0} [\bar{v}(p_2)\gamma^\mu u(p_1)] e_{\mu\nu\rho\sigma} e^{*\nu} k_1^\rho k_2^\sigma, \quad (12)$$

where $f_{\bar{p}p\to\phi\pi^0}$ is some constant, and the total cross section corresponding to the amplitude (12) has the form

$$\sigma_{\bar{p}p\to\phi\pi^0} = |f_{\bar{p}p\to\phi\pi^0}|^2 \frac{(3m^2+2p^2)k^3}{12\pi p}, \quad (13)$$

where k is the magnitude of the c.m. momentum for the $\phi\pi^0$ system.

The amplitude of the reaction $K^{*+} \to \pi^0 K^+$ has the form

$$M_{K^{*+}\to\pi^0K^+} = f_{K^{*+}\to\pi^0K^+} (k_1 - k_3')_\mu e'^\mu, \quad (14)$$

and a standard calculation shows that the width of the decay is

$$\Gamma_{K^{*+}\to\pi^0K^+} = \frac{|f_{K^{*+}\to\pi^0K^+}|^2 k_{\pi K}^3}{6\pi m_*^2}, \quad (15)$$

where $k_{\pi K}$ is the magnitude of the c.m. momentum in the πK system. If Γ_* is the total K^{*+} width, it is easy to show that $\Gamma_* = 3\Gamma_{K^{*+}\to\pi^0K^+}$.

By analogy with Eqs. (14) and (15), the amplitude of the reaction $K^+K^- \to \phi$ is given by

$$M_{K^+K^-\to\phi} = f_{K^+K^-\to\phi} (k'_{2\mu} - k'_{3\mu}) e^{\mu *}, \quad (16)$$

and the width of the decay $\phi \to K^+K^-$ is

$$\Gamma_{\phi\to K^+K^-} = \frac{|f_{K^+K^-\to\phi}|^2 k_{K\bar{K}}^3}{6\pi m_\phi^2}, \quad (17)$$

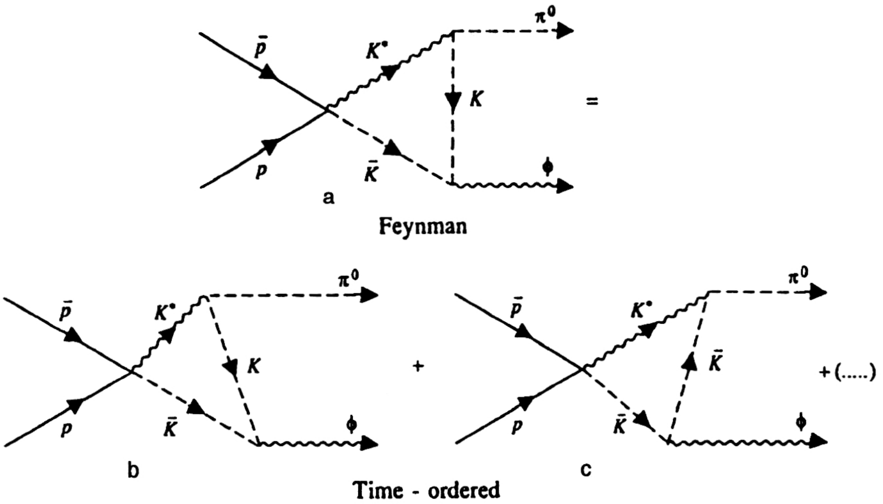


FIG. 3.

where $k_{K\bar{K}}$ is the magnitude of the c.m. momentum in the $K\bar{K}$ system. Since the ϕ decays into $K\bar{K}$ in 87% of the events, it is easy to show that $2\Gamma_{\phi\to K^+K^-}=0.87\Gamma_\phi$, where Γ_ϕ is the total width of the ϕ.

Taking into account Eqs. (10), (14), and (16), and also the fact that all four diagrams in Fig. 2 give equal contributions, we can write, for the amplitude of the reaction $\bar{p}p \to \phi\pi^0$,

$$M_{\bar{p}p\to\phi\pi^0}=8i[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e^{*\lambda}k_1^\nu \times\int f^{(11)}_{\bar{p}p\to K^{*+}K^-}f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi} \times k_1'^\rho k_2'^\sigma(k_2'^\lambda-k_3'^\lambda) \times\frac{\delta^{(4)}(k_1'-k_1-k_3')\delta^{(4)}(k_2-k_2'-k_3')}{(2\pi)^4[k_1'^2-(m_*-i\Gamma_*/2)^2](k_2'^2-m_K^2+i0)} \times\frac{d^4k_1'd^4k_2'd^4k_3'}{k_3'^2-m_K^2+i0}. \tag{18}$$

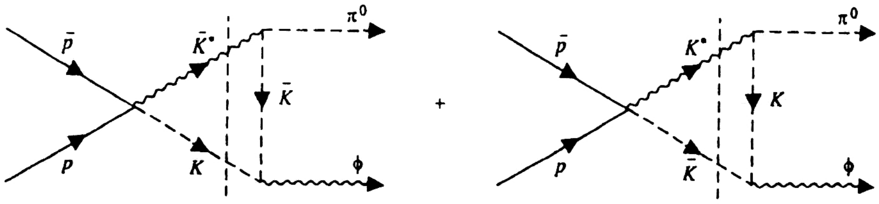
Let us note that the term with $k_1'^\nu k_1'^\beta$ in the propagator $\Pi^{\nu\beta}=(k_1'^\nu k_1'^\beta/m_*^2-g_{\nu\beta})$ of the K^* meson ($g_{\nu\beta}$ is the metric tensor in Minkowski space) does not contribute to the amplitude (18), since $e_{\mu\nu\rho\sigma}k_1'^\nu k_1'^\rho=0$, and for the same reason $k_1^\nu-k_3'^\nu$ can be replaced by $2k_1^\nu$. We have also taken into account the fact that the K^* meson is a Breit–Wigner resonance, and therefore the propagator of the K^* meson depends on the complex mass $(m_*-i\Gamma_*/2)$.

In the general case the quantities $f_{\bar{p}p\to K^{*+}K^-}$, $f_{K^{*+}\to\pi^0K^+}$, and $f_{K^+K^-\to\phi}$ entering into Eq. (18) differ from the corresponding quantities in Eqs. (10), (14), and (16), since the K^{*+}, K^-, and K^+ mesons are off-shell. One might assume that the dependence of these quantities on the off-shell form factors is not strong and neglect this dependence. However, the integral in Eq. (18) strongly diverges in this case. Therefore we should either introduce form factors "by hand" or try to estimate the amplitude (18) with the help of additional assumptions.

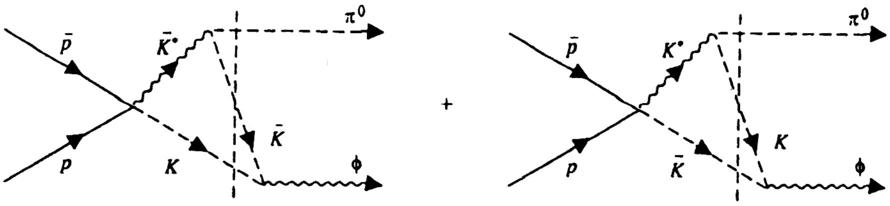
It is important to note that the covariant Feynman approach does not fully agree with our physical intuition that the process $\bar{p}p\to\phi\pi^0$ can be described as $\bar{p}p\to(K^*\bar{K}+\bar{K}^*K)\to K\bar{K}\pi\to\phi\pi^0$. As a rule, one Feynman diagram contains the contribution of a few diagrams of the "old-fashioned" time-ordered perturbation theory. In particular, the three vertices in the Feynman diagram in Fig. 2 are not necessarily time-ordered as we assume. For example, the Feynman diagram in Fig. 3 contains the contributions of diagrams (a) and (b) of the time-ordered perturbation theory. Diagram (a) indeed describes the process $\bar{p}p\to\phi\pi^0$ as $\bar{p}p\to(K^*\bar{K}+\bar{K}^*K)\to K\bar{K}\pi^0\to\phi\pi^0$, while diagram (b) describes the unphysical process $\bar{p}p\to K^*\bar{K}\to K^*\bar{K}\phi\to\phi\pi^0$, since the virtual $\bar{K}$ meson in this diagram decays into $\bar{K}$ and ϕ, and then the interaction between K^* and $\bar{K}$ leads to the production of π^0.

The difficulties with the interpretation of Feynman diagrams and with the divergence in Eq. (18) can be partly overcome if we assume that the main contribution to the integral in Eq. (18) is given by the residues at the poles of the propagators of some intermediate particles. According to our interpretation of the process $\bar{p}p\to\phi\pi^0$, we choose two possibilities, which we call model A and model B. In model A we drop Γ_* in Eq. (18) and replace $[(k_1'^2-m_*^2+i0)\times(k_2'^2-m_K^2+i0)]^{-1}$ by $(-2i\pi)^2\theta(k_1'^0)\theta(k_2'^0)\delta(k_1'^2-m_*^2)\delta(k_2'^2-m_K^2)/2$. Analogously, in model B we replace $[(k_2'^2-m_K^2+i0)(k_3'^2-m_K^2+i0)]^{-1}$ by $(-2i\pi)^2\theta(k_2'^0)\theta(k_3'^0)\delta(k_2'^2-m_K^2)\delta(k_3'^2-m_K^2)/2$. Schematically, model A can be described by Fig. 4a, i.e., K^* and $\bar{K}$ in the diagram of Fig. 4a are on the mass shell. Analogously, model B can be described by Fig. 4b, i.e., $\bar{K}$ and K in the diagram of Fig. 4b are on the mass shell.

One might think that from the theoretical point of view model B seems more substantiated than model A. Indeed, as shown in Refs. 34 and 35, the on-shell approximation is connected with the unitarity relation for the S matrix, but this relation must be formulated only in terms of stable particles. In particular, $K\bar{K}\pi^0$ is an admissible intermediate state, while $K^*\bar{K}$ is not. In addition, the vertices $K^{*+}\to\pi^0K^+$ and $K^+K^-\to\phi$ entering into the amplitude $K^*\bar{K}\to\phi\pi^0$ in model A are not necessarily time-ordered, and therefore this amplitude contains the contribution of not only the process $K^*\bar{K}\to K\bar{K}\pi^0\to\phi\pi^0$ but also the contribution of the un-



a
Model A



b
Model B

FIG. 4.

physical process $K^*\bar{K}\to K^*\bar{K}\phi\to\phi\pi^0$. However, as shown in Refs. 20 and 21, the numerical results in model A are in qualitative agreement with the experimental data. For this reason we investigate below the consequences of both model A and model B.

4. THE PROBLEM OF CALCULATING THE PROCESS $\bar{p}p\to\phi\pi^0$ WITH $\rho^+\rho^-$ INTERMEDIATE STATES

As shown in Refs. 19 and 20, $\rho^+\rho^-$ intermediate states may essentially contribute to the process $\bar{p}p\to\phi\pi^0$. There exist two diagrams describing the process $\bar{p}p\to\phi\pi^0$ via $\rho^+\rho^-$: $\bar{p}p\to\rho^+\rho^-\to\pi^+\pi^0\rho^-\to\phi\pi^0$ and $\bar{p}p\to\rho^+\rho^-\to\rho^+\pi^-\pi^0\to\phi\pi^0$ (see Fig. 5), and the contributions of these diagrams are equal to each other if $I=S=1$. To find these contributions we need the expressions defining the amplitudes $\bar{p}p\to\rho^+\rho^-$, $\rho^+\to\pi^+\pi^0$, and $\rho^-\pi^+\to\phi$.

When $I=S=1$, a possible choice of the amplitude which survives in the limit when $\mathbf{p}_1$ and $\mathbf{p}_2$ are small is

$$M^{(11)}_{\bar{p}p\to\rho^+\rho^-}=f^{(11)}_{\bar{p}p\to\rho^+\rho^-}[\bar{v}(p_2)\gamma^\mu u(p_1)]\times[e'^*_{1\mu}(Pe'^*_2)-e'^*_{2\mu}(Pe'^*_1)], \tag{19}$$

where $e'_i\,(i=1,2)$ are the polarization four-vectors of the ρ^+ and ρ^- mesons, respectively, and $P=p_1+p_2$. We take into account the fact that the C parity of the $\rho^+\rho^-$ system should be equal to -1.

There also exist two other amplitudes which satisfy all the necessary conditions. One of them was used in Refs. 19 and 20, and the corresponding result is small (see the discussion in Ref. 20). The contribution of the other, which is cubic in $\mathbf{k}'_1-\mathbf{k}'_2$, is expected to be small too. Following Ref. 22, we describe here the calculations with the amplitude given by Eq. (19).

A standard calculation shows that the total cross section $\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}$ has the form

$$\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}=|f^{(11)}_{\bar{p}p\to\rho^+\rho^-}|^2\,\frac{(3m^2+2p^2)(E_\rho^2+m_\rho^2)k'^3}{6\pi p m_\rho^4}, \tag{20}$$

where now k' is the magnitude of the c.m. momentum in the $\rho^+\rho^-$ system, m_ρ is the mass of the ρ meson, and $E_\rho=(m_\rho^2+k'^2)^{1/2}$.

The $\rho^+\to\pi^+\pi^0$ amplitude and the decay width of the ρ meson can be written by analogy with Eqs. (14) and (15):

$$M_{\rho^+\to\pi^+\pi^0}=f_{\rho^+\to\pi^+\pi^0}(k_1-k'_3)_\mu e'^\mu_1,$$

$$\Gamma_{\rho^+\to\pi^+\pi^0}=\frac{|f_{\rho^+\to\pi^+\pi^0}|^2k^3_{\pi\pi}}{6\pi m_\rho^2}, \tag{21}$$

where k_1 and k'_3 are the four-momenta of the π^0 and π^+, respectively, and $k_{\pi\pi}$ is the magnitude of the c.m. momentum in the $\pi\pi$ system.

The $\pi^+\rho^-\to\phi$ amplitude has the form

$$M_{\pi^+\rho^-\to\phi}=f_{\pi^+\rho^-\to\phi}e_{\mu\nu\rho\sigma}e^{\mu*}e'^\nu_2k_2^\rho k'^\sigma_2, \tag{22}$$

where k'_2 is the 4-momentum of the ρ^-. A direct calculation shows that the decay width $\Gamma_{\phi\to\pi^+\rho^-}$ is given by

$$\Gamma_{\phi\to\pi^+\rho^-}=\frac{|f_{\phi\to\pi^+\rho^-}|^2k^3_{\pi\rho}}{12\pi}, \tag{23}$$

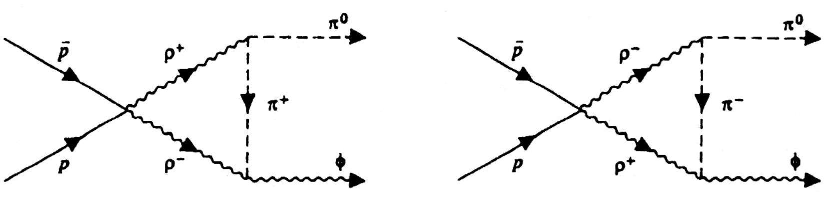


FIG. 5.

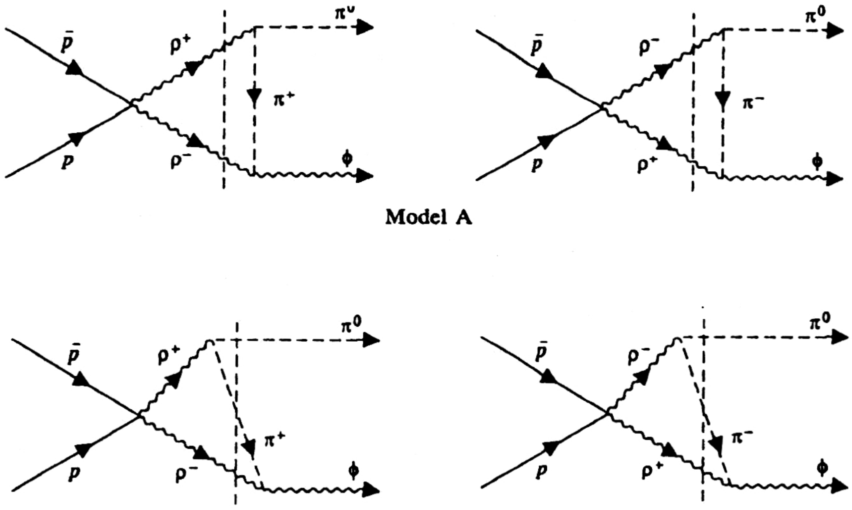


FIG. 6.

where $k_{\pi\rho}$ is the magnitude of the c.m. momentum in the $\pi\rho$ system. Since the ϕ decays into $\pi\rho$ in 12% of the events, it is obvious that $\Gamma_{\phi\to\pi^+\rho^-}=0.12\Gamma_\phi/3$.

As follows from Eqs. (19), (21), and (22), the amplitude of the process $\bar{p}p\to\phi\pi^0$ corresponding to the Feynman diagrams in Fig. 5 can be written in the form

$$M_{\bar{p}p\to\phi\pi^0}=2i[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\alpha\beta\gamma\delta}e^{\alpha *}k_2^\gamma P^\nu \times\int f_{\bar{p}p\to\rho^+\rho^-}f_{\rho^+\to\pi^+\pi^0}f_{\pi^+\rho^-\to\phi}k_2'^\delta(k_1-k_3')^\rho \times\left[\left(\frac{k'_{1\mu}k'_{1\rho}}{m_\rho^2}-g_{\mu\rho}\right)\delta_\nu^\beta-\left(\frac{k'_{1\nu}k'_{1\rho}}{m_\rho^2}-g_{\nu\rho}\right)\delta_\mu^\beta\right] \times\frac{\delta^{(4)}(k_1'-k_1-k_3')\delta^{(4)}(k_2-k_2'-k_3')}{(2\pi)^4[k_1'^2-(m_\rho-i\Gamma_\rho/2)^2][k_2'^2-(m_\rho-i\Gamma_\rho/2)^2]} \times\frac{d^4k_1'd^4k_2'd^4k_3'}{k_3'^2-m_\pi^2+i0}, \tag{24}$$

where δ is the Kronecker symbol.

As in Eq. (18), the integral in Eq. (24) diverges if no form factors are introduced into the vertices $\bar{p}p\to\rho^+\rho^-$, $\rho^+\to\pi^+\pi^0$, and $\rho^-\pi^+\to\phi$. By analogy with Sec. 3 we use the on-shell approximation, where the intermediate states are either $\rho^+\rho^-$ or $\rho\pi\pi$. We again refer to the corresponding models as model A and model B, respectively. These models correspond to the cuts of the Feynman diagrams as shown in Fig. 6.

5. THE CONTRIBUTION OF K^*K AND $\rho^+\rho^-$ INTERMEDIATE STATES IN MODEL A

As follows from the prescription described in Sec. 3, Eq. (18) in model A reads

$$M_{\bar{p}p\to\phi\pi^0}=-8i[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e_\lambda^* k_1^\nu k_2^\rho f^{(11)}_{\bar{p}p\to K^{*+}K^-} \times\int f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi}k_2'^\sigma k_2'^\lambda\theta(k_1'^0)\theta(k_2'^0) \delta(k_1'^2-m_*^2)\times\frac{\delta(k_2'^2-m_K^2)\delta^{(4)}(k_1+k_2-k_1'-k_2')d^4k_1'd^4k_2'}{(2\pi)^2[(k_1'-k_1)^2-m_K^2+i0]}, \tag{25}$$

where we have taken into account the fact that $(k_{2\lambda}e^\lambda)=0$. The quantity $f^{(11)}_{\bar{p}p\to K^{*+}K^-}$ in this expression is the same as in Eq. (10), since the K^* and $\bar{K}$ are on-shell.

It is convenient to consider Eq. (25) in the c.m. frame of the $\bar{p}p$ system, which, at the same time, is the c.m. frame of the $K^*\bar{K}$ and $\phi\pi^0$ systems. The vector P in this frame of reference has the components $P^0=\sqrt{s}$, $\mathbf{P}=0$, and therefore Eq. (25) can be written in the form

$$M_{\bar{p}p\to\phi\pi^0}=\frac{-i}{4\pi^2k}f^{(11)}_{\bar{p}p\to K^{*+}K^-}[\bar{v}(p_2)\gamma^i u(p_1)]e_{ikl}k^k \times\int do' f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi} \times\frac{k'^l(E_{\bar{K}}e^{0*}+k'^m e^{m*})}{a-x}, \tag{26}$$

where $a=(2E_*E_\pi+m_K^2-m_*^2-m_\pi^2)/2kk'$, $E_\pi=(m_\pi^2+k^2)^{1/2}$, $E_*=(m_*^2+k'^2)^{1/2}$, $E_{\bar{K}}=(m_K^2+k'^2)^{1/2}$, $k=|\mathbf{k}|$, $k'=|\mathbf{k}'|$, $\mathbf{k}\equiv\mathbf{k}_1$, $\mathbf{k}'\equiv\mathbf{k}_1'$, $\mathbf{n}=\mathbf{k}/k$, $\mathbf{n}'=\mathbf{k}'/k'$, $x=\mathbf{nn}'$, do' is the element of solid angle corresponding to the unit vector n', and a sum over repeated indices $i,k,l,m=1,2,3$ is assumed.

Let us consider the integrals

$$I^l=\int f(x,s)k'^l do', \quad I^{lm}=\int f(x,s)k'^l k'^m do', \tag{27}$$

where $f(x,s)$ is an arbitrary function of x and s. It is easy to show that

$$I^l=2\pi\frac{k'}{k}k^l\int_{-1}^1 f(x,s)dx, \quad I^{lm}=\pi(k')^2\int_{-1}^1 f(x,s) \times\left[(1-x^2)\delta^{lm}+(3x^2-1)\frac{k^lk^m}{k^2}\right]dx. \tag{28}$$

Then it follows from Eqs. (12) and (26)–(28) that

$$f_{\bar{p}p\to\phi\pi^0}=\frac{i(k')^2}{4\pi k\sqrt{s}}f^{(11)}_{\bar{p}p\to K^{*+}K^-}\times\int_{-1}^{1}f_{K^{*+}\to\pi^0K^+}(k_3'^2)f_{K^+K^-\to\phi}(k_3'^2)\times\frac{1-x^2}{a-x}\,dx. \quad (29)$$

We explicitly note that $f_{K^{*+}\to\pi^0K^+}$ and $f_{K^+K^-\to\phi}$ depend on the off-shell form factor for the K meson with four-momentum k_3'. The importance of taking into account this form factor has been pointed out in Refs. 19 and 20. Following these references, we write

$$f_{K^{*+}\to\pi^0K^+}(k_3'^2)=f_{K^{*+}\to\pi^0K^+}\frac{\Lambda-m_K^2}{\Lambda-k_3'^2},$$

$$f_{K^+K^-\to\phi}(k_3'^2)=f_{K^+K^-\to\phi}\frac{\Lambda-m_K^2}{\Lambda-k_3'^2}, \quad (30)$$

where now the quantities $f_{K^{*+}\to\pi^0K^+}$ and $f_{K^+K^-\to\phi}$ are the same as in Eqs. (14) and (16). Then we get from Eq. (29) the final result

$$f_{\bar{p}p\to\phi\pi^0}=\frac{i(k')^2}{4\pi k\sqrt{s}}f^{(11)}_{\bar{p}p\to K^{*+}K^-}f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi}\times\int_{-1}^{1}\frac{1-x^2}{a-x}\left[\frac{\Lambda-m_K^2}{\Lambda+2E_*E_\pi-m_*^2-m_\pi^2-2kk'x}\right]^2\times dx. \quad (31)$$

As follows from Eqs. (11), (13), (15), (17), and (31),

$$R\equiv\frac{\sigma_{\bar{p}p\to\phi\pi^0}}{\sigma^{(11)}_{\bar{p}p\to K^{*+}K^-}}=0.87\cdot\frac{3}{8}\frac{kk'\Gamma_*\Gamma_\phi m_*^2m_\phi^2}{s(k_{\pi K}k_{K\bar{K}})^3}\left|\int_{-1}^{1}\frac{1-x^2}{a-x}\left[\frac{\Lambda-m_K^2}{\Lambda+2E_*E_\pi-m_*^2-m_\pi^2-2kk'x}\right]^2dx\right|^2. \quad (32)$$

Since for the amplitudes $\bar{p}p\to K^{*+}K^-$ and $\bar{p}p\to\phi\pi^0$ we assume the structure defined by Eqs. (10) and (12), Eq. (32) can be valid only if the value of p is rather small. In Ref. 22 the dependence of R on the laboratory momentum p_{lab} in the range (0–0.4) GeV/c [which corresponds to values of p in the range (0–0.2) GeV/c] has been calculated. Following Refs. 19 and 20, the values 1.2 GeV^2, 2 GeV^2, and ∞ have been chosen for Λ (the last value means the absence of off-shell form factors). The result of Ref. 22 is that R practically does not depend on p_{lab} in the range (0–0.4) GeV/c.

In Refs. 13 and 14 the branching ratio of the reaction $\bar{p}p\to\phi\pi^0$ has been measured not for annihilation in flight but for annihilation at rest from the S state of the hydrogen-like $\bar{p}p$ atom. When $p\to 0$, only the contribution of the S wave survives in Eq. (32). Assuming that the $\bar{p}p$ system in the hydrogen-like atom is unpolarized and taking for the branching ratio $\text{Br}(\bar{p}p\to K^{*+}K^-)^{(11)}$ its experimental value $5.85\cdot 10^{-4}$ (Ref. 36), the result for the branching ratio $\text{Br}(\bar{p}p\to\phi\pi^0)$ is $2.9\cdot 10^{-4}$, $0.99\cdot 10^{-4}$, and $0.4\cdot 10^{-4}$ for $\Lambda=\infty$, $\Lambda=2$ GeV^2, and $\Lambda=1.2$ GeV^2, respectively. According to Ref. 13, $\text{Br}(\bar{p}p\to\phi\pi^0)=(4.0\pm 0.8)\cdot 10^{-4}$, and according to Ref. 14, $\text{Br}(\bar{p}p\to\phi\pi^0)=(5.8\pm 0.4)\cdot 10^{-4}$. We conclude that if the off-shell form factor for the K meson does not strongly depend on k_3', then the contribution of K^*K intermediate states in model A is in fairly good agreement with the experimental data.

The calculation of the contribution of $\rho^+\rho^-$ intermediate states can be carried out by analogy with the above calculation. Using Eqs. (19), (21), (22), (27), and (28), we get

$$f_{\bar{p}p\to\phi\pi^0}=\frac{i(k')^3}{8\pi m_\rho^2\sqrt{s}}f^{(11)}_{\bar{p}p\to\rho^+\rho^-}f_{\rho^+\to\pi^+\pi^0}f_{\pi^+\rho^-\to\phi}F(s), \quad (33)$$

where

$$F(s)=\int_{-1}^{1}\left[(1-x^2)(E_\rho E_\pi-kk'x)+2E_\rho\left(\frac{E_\rho kx}{k'}-E_\pi\right)-2xE_\phi\left(E_\rho x-E_\pi\frac{k'}{k}\right)\right]\times\left[\frac{\Lambda-m_\pi^2}{\Lambda+2E_\rho E_\pi-2kk'x-m_\rho^2-m_\pi^2}\right]^2\times\frac{dx}{2E_\rho E_\pi-2kk'x-m_\rho^2-i0}. \quad (34)$$

In contrast to the K^*K case, now the kinematical conditions are such that all three intermediate particles can be on-shell, in contradiction with the Peierls theorem.[37] In turn, this theorem follows from the fundamental fact that the S matrix can be formulated only in terms of stable particles. However, such a situation is only a formal difficulty, which occurs because we drop Γ_ρ in the propagators of the ρ^+ and ρ^- mesons and treat these mesons as stable particles.

As follows from Eqs. (20), (21), (23), and (33),

$$R_1=\frac{\sigma_{\bar{p}p\to\phi\pi^0}}{\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}}=0.12\frac{3}{4}\left(\frac{kk'}{k_{\pi\rho}k_{\pi\pi}}\right)^3\frac{\Gamma_\rho\Gamma_\phi m_\rho^2}{s(s+4m_\rho^2)}|F(s)|^2. \quad (35)$$

In Refs. 19 and 22 the result for R_1 as a function of p_{lab} has been calculated for the cases $\Lambda=1.2$ GeV^2, $\Lambda=2$ GeV^2, and $\Lambda=\infty$. The dependence of R_1 on p_{lab} also turned out to be weak, but it is not clear what is the upper bound for those p_{lab} for which the result is still valid. If $p_{\text{lab}}=0$, then $R_1=1.13\cdot 10^{-3}$, $R_1=3.2\cdot 10^{-3}$, and $R_1=7.01\cdot 10^{-3}$ for these three cases, respectively. The experimental value of $\text{Br}(\bar{p}p\to\rho^+\rho^-)^{(11)}$ at rest is unknown, but the theoretical model developed in Ref. 38 predicts the value $23.6\cdot 10^{-3}$. Then the contribution of $\rho^+\rho^-$ intermediate states to $\text{Br}(\bar{p}p\to\phi\pi^0)$ at rest is $1.9\cdot 10^{-4}$ if $\Lambda=\infty$. Therefore, as first noted in Ref. 19, model A predicts a rather substantial contribution of $\rho^+\rho^-$ intermediate states to the branching ratio of the reaction $\bar{p}p \to \phi\pi^0$.

As argued by Lipkin, Geiger, Isgur, and others (see, e.g., Refs. 5 and 7), a possible reason for the OZI-rule violation is the interference of amplitudes corresponding to different intermediate states. For example, Lipkin[5] argues that "the contributions from the K^+K^- and $K^{*+}K^{*-}$ intermediate states have the same phase, and this is opposite to the phase of the contribution from the K^+K^{*-} and K^-K^{*+} states." This problem has also been discussed by Sapozhnikov[39] and Zou.[40] It has also been noted by Locher[41] that if in the diagrams in Fig. 2 the K^* mesons are replaced by K mesons, then the corresponding contribution is equal to zero. Indeed, the $KK\pi$ coupling is equal to zero, since three (0^-) particles cannot couple (parity and angular-momentum conservation). It is also not clear which diagrams describing $K^*\bar{K}^*$ intermediate states can compensate the diagrams in Fig. 2. We will see in Sec. 11 that these intermediate states are natural for the reaction $\bar{p}p \to \phi\pi^+\pi^-$ but not for $\bar{p}p \to \phi\pi^0$. On the other hand, it is important to stress that in the theory of strong interactions any conclusion about the dominant role of some finite set of diagrams can be based only on intuition, which often does not work. Thus, any explanation of the OZI-rule violation taking into account only a finite set of diagrams can be at best qualitative.

6. THE CONTRIBUTION OF $K\bar{K}\pi^0$ AND $\rho\pi\pi^0$ INTERMEDIATE STATES IN MODEL B

As follows from the prescription described in Sec. 3, Eq. (18) in model B reads

$$M_{\bar{p}p \leftarrow \phi\pi^0} = 4if^{(11)}_{\bar{p}p \to K^{*+}K^-} f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi} \times [\bar{v}(p_2)\gamma^\mu u(p_1)] e_{\mu\nu\rho\sigma} e^*_\lambda k_1^\nu \times \int \frac{k_3'^\rho k_2'^\sigma (k_2'^\lambda - k_3'^\lambda)\delta^{(4)}(k_2 - k_2' - k_3') d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{16\pi^2 \omega_K(\mathbf{k}_2')\omega_K(\mathbf{k}_3')[(k_1+k_3')^2 - (m_* - i\Gamma_*/2)^2]}, \tag{36}$$

where $\omega_K(\mathbf{k}) = (m_K^2 + \mathbf{k}'^2)^{1/2}$, we take into account the fact that the constants $f_{K^{*+}\to\pi^0K^+}$ and $f_{K^+K^-\to\phi}$ are the same as in Eqs. (14) and (16), and no form factor is introduced into the vertex $\bar{p}p \to K^*\bar{K}$.

It is obvious that

$$e_{\mu\nu\rho\sigma} k_3'^\rho k_2'^\sigma = e_{\mu\nu\rho\sigma}(k_2'^\rho + k_3'^\rho)(k_2'^\sigma - k_3'^\sigma)/2,$$

and therefore Eq. (36) can be written in the form

$$M_{\bar{p}p \to \phi\pi^0} = 2if^{(11)}_{\bar{p}p \to K^{*+}K^-} f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi} \times [\bar{v}(p_2)\gamma^\mu u(p_1)] e_{\mu\nu\rho\sigma} e^{*\lambda} k_2^\rho k_1^\nu I^\sigma_\lambda, \tag{37}$$

where $I^{\sigma\lambda}$ is the relativistic symmetric tensor

$$I^{\sigma\lambda} = \int \frac{(k_2'^\sigma - k_3'^\sigma)(k_2'^\lambda - k_3'^\lambda)\delta^{(4)}(k_2 - k_2' - k_3') d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{16\pi^2 \omega_K(\mathbf{k}_2')\omega_K(\mathbf{k}_3')[(k_1+k_3')^2 - (m_* - i\Gamma_*/2)^2]}. \tag{38}$$

This tensor depends only on k_1 and k_2, and therefore the general form of $I_{\sigma\lambda}$ is

$$I_{\sigma\lambda} = c_1 g_{\sigma\lambda} + c_2 k_{1\sigma}k_{1\lambda} + c_3 k_{2\sigma}k_{2\lambda} + c_4(k_{1\sigma}k_{2\lambda} + k_{2\sigma}k_{1\lambda}). \tag{39}$$

It is obvious that only $c_1 g_{\sigma\lambda}$ contributes to Eq. (37). The simplest way of calculating c_1 is to consider Eq. (38) in the reference frame where the final ϕ meson is at rest. The magnitude of the pion momentum in this reference frame is $q = (\sqrt{s}k)/m_\phi$, and, as follows from Eqs. (38) and (39),

$$\frac{k_{K\bar{K}}}{4\pi^2 m_\phi} \int \{do' k'^i k'^l / [m_\pi^2 + m_K^2 + m_\phi(m_\pi^2 + q^2)^{1/2} + 2qk_{K\bar{K}}x - (m_* - i\Gamma_*/2)^2]\} = -c_1\delta_{il} + c_2 q_i q_l, \tag{40}$$

where $\mathbf{q}$ is the pion momentum, $\mathbf{k}'$ is the momentum of the $\bar{K}$ meson, $x = \mathbf{q}\mathbf{k}'/qk_{K\bar{K}}$, and we integrate over the solid angle corresponding to the unit vector $\mathbf{n} = \mathbf{k}'/k_{K\bar{K}}$. Then the quantity c_1 can be easily calculated by analogy with the calculation of the quantity c_1 in Sec. 5, and the final result for $f_{\bar{p}p \to \phi\pi^0}$ is

$$f_{\bar{p}p \to \phi\pi^0} = -if^{(11)}_{\bar{p}p \to K^{*+}K^-} f_{K^{*+}\to K^+\pi^0} f_{K^+K^-\to\phi} \frac{(k_{K\bar{K}})^2}{4\pi\sqrt{s}k} \times \left[2b + (1-b^2)\ln\left(\frac{b+1}{b-1}\right)\right], \tag{41}$$

where $b = [m_\pi^2 + m_K^2 + m_\phi(m_\pi^2 + q^2) - (m^* - i\Gamma/2)^2]/2qk_{K\bar{K}}$ and we have taken into account that the fact that

$$\int_{-1}^{1} \frac{(1-x^2)dx}{b-x} = 2b + (1-b^2)\ln\left(\frac{b+1}{b-1}\right). \tag{42}$$

By analogy with the derivation of Eq. (32) we now get

$$\frac{\sigma_{\bar{p}p \to \phi\pi^0}}{\sigma^{(11)}_{\bar{p}p \to K^{*+}K^-}} = 0.87 \frac{3}{8} \frac{kk_{K\bar{K}}\Gamma_*\Gamma_\phi m_*^2 m_\phi^2}{sk_{\pi K}^3 k'^3} \times \left|2b + (1-b^2)\ln\left(\frac{b+1}{b-1}\right)\right|^2. \tag{43}$$

A simple numerical calculation shows that if $s = 4m^2$, then $\sigma_{\bar{p}p \to \phi\pi^0} \approx 10^{-4} \cdot \sigma^{(11)}_{\bar{p}p \to K^{*+}K^-}$. Therefore the contribution of $K\bar{K}\pi^0$ intermediate states in model B is negligible.

Let us now consider the contribution of $(\rho^+\pi^- + \rho^-\pi^+)\pi^0$ intermediate states in model B. In this model Eq. (24) reads

$$f_{\bar{p}p \to \phi\pi^0}[v(p_2)\gamma^\mu u(p_1)] e_{\mu\nu\rho\sigma} e^{\nu *} k_1^\rho k_2^\sigma = -if^{(11)}_{\bar{p}p \to \rho^+\rho\_} f_{\rho^+\to\pi^+\pi^0} f_{\pi^+\rho^-\to\phi}[v(p_2)\gamma^\mu u(p_1)] \times e_{\alpha\beta\gamma\delta} e^{\alpha *} k_2^\gamma \times \int \frac{(2\pi)^4 \delta^{(4)}(k_2 - k_2' - k_3') d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{[2(2\pi)^3]^2 \omega_\rho(\mathbf{k}_2')\omega_\pi(\mathbf{k}_3')[(k_1+k_3')^2 - (m_\rho - i\Gamma_\rho/2)^2]} \times k_2'^\delta [(k_1 - k_3')_\mu P_\beta - g_{\mu\beta}(P, k_1 - k_3')], \tag{44}$$

where $\omega_\rho(\mathbf{k}') = (m_\rho^2 + \mathbf{k}'^2)^{1/2}$, $\omega_\pi(\mathbf{k}') = (m_\pi^2 + \mathbf{k}'^2)^{1/2}$.

It is obvious that

$$\int \frac{(2\pi)^4 k_2'^{\delta} k'^{\mu} \delta^{(4)}(k_2-k_2'-k_3')d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{[2(2\pi)^3]^2 \omega_\rho(\mathbf{k}_2')\omega_\pi(\mathbf{k}_3')[(k_1+k_3')^2-(m_\rho-i\Gamma_\rho/2)^2]}$$
$$=c_1 g^{\mu\delta}+c_2 k_1^\mu k_1^\delta+c_3 k_2^\mu k_2^\delta+c_4 k_1^\mu k_2^\delta+c_5 k_2^\mu k_1^\delta, \qquad (45)$$

where the c_i $(i=1,...,5)$ are certain relativistically invariant quantities. As follows from Eq. (44), we have to calculate only c_1, c_2, and c_5. It is convenient to calculate these quantities in the reference frame where the final ϕ meson is at rest and to use Eqs. (28). The final result is [cf. Eq. (35)]

$$\frac{\sigma_{\bar{p}p\to\phi\pi^0}}{\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}}=0.12\frac{3}{16}\frac{kk_{\pi\rho}}{k'^3k^3_{\pi\pi}}\frac{\Gamma_\rho\Gamma_\phi m_\rho^6}{s(E_\rho^2+m_\rho^2)}|F_1(s)|^2, \qquad (46)$$

where, as in Eq. (35), k' is the magnitude of the c.m. momentum in the $\rho^+\rho^-$ system, and

$$F_1(s)=\int_{-1}^{1}\frac{dx}{2m_\pi^2+2\omega_\pi(k_{\pi\rho})+2qk_{\pi\rho}x-(m_\rho-i\Gamma_\rho/2)^2}$$
$$\times\left\{\frac{1}{2}(s-m_\phi^2)\left[x-\frac{k_{\pi\rho}}{2q}(1-3x^2)\right]-\frac{1}{2}(s+m_\phi^2)\right.$$
$$\times\left[\frac{\omega_\pi(k_{\pi\rho})x}{m_\phi}-\frac{\omega_\pi(q)k_{\pi\rho}}{2m_\phi q}(1-3x^2)\right]$$
$$\left.-k_{\pi\rho}q(1-x^2)\right\}. \qquad (47)$$

A simple numerical calculation shows that if $s=4m^2$, then Eq. (46) can be written as

$$\sigma_{\bar{p}p\to\phi\pi^0}=3.13\cdot 10^{-5}\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}. \qquad (48)$$

Therefore, if we again assume that $\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}=23.6\cdot 10^{-3}$ (Ref. 38), then the $(\rho^+\pi^-+\rho^-\pi^+)\pi^0$ intermediate states in model B do not play an important role.

7. THE RELATION BETWEEN THE BRANCHING RATIOS OF THE REACTIONS $\bar{p}p\to\phi\pi^0$ AND $\bar{p}p\to K^*\bar{K}$ IN ANNIHILATION FROM THE P STATE OF THE HYDROGEN-LIKE $\bar{p}p$ ATOM

In contrast to the annihilation $\bar{p}p\to\phi\pi^0$ from the S state of the hydrogen-like $\bar{p}p$ atom, the branching ratio of this annihilation from the P state is small, and the reaction $\bar{p}p\to\phi\pi^0$ from the P state has not yet been observed. The data on the annihilation $\bar{p}p\to K^*\bar{K}$ from the P state are also much more scarce than for the annihilation from the S state, but experiments which are under way are expected to give more detailed information on the $\bar{p}p$ annihilation from the P state. In view of the above discussion it is interesting to investigate the prediction of model A for the ratio of the rates of the reactions $\bar{p}p\to\phi\pi^0$ and $\bar{p}p\to K^*\bar{K}$ in the annihilation from the P state. More precisely, since the annihilation $\bar{p}p\to\phi\pi^0$ from the P state can take place only in the channel with $I=1$, $S=0$, model A makes it possible to give predictions on the quantity $\mathrm{Br}(\bar{p}p\to\phi\pi^0)/\mathrm{Br}(K^{*+}K^-)^{(10)}$. One might think that in model A this quantity should be of the same order as in the case of the annihilation from the S state and hence the explanation of the OZI-rule violation in the framework of the rescattering mechanism is inconsistent. We first describe a calculation in Ref. 42 which shows that there is nevertheless a possibility that model A explains both the large value of $\mathrm{Br}(\bar{p}p\to\phi\pi^0)/\mathrm{Br}(K^{*+}K^-)$ in the annihilation from the S state and a small value of the same quantity in the annihilation from the P state. Then we discuss the criticism of this mechanism in Refs. 40 and 43.

To describe the relativistically invariant amplitude for the annihilation $\bar{p}p\to\phi\pi^0$ from the P state we have to construct the relativistic wave function describing the $\bar{p}p$ system not in the case when the antiproton and proton have definite momenta, but when they have the definite quantum numbers $L=1$, $S=0$. However, since we need only the ratio of the quantities $\mathrm{Br}(\bar{p}p\to\phi\pi^0)$ and $\mathrm{Br}(\bar{p}p\to K^{*+}K^-)^{(10)}$, the following procedure can be used. We again describe the antiproton and proton by Dirac spinors and write relativistically invariant $\bar{p}p\to\phi\pi^0$ and $\bar{p}p\to K^{*+}K^-$ amplitudes which are of order $|\mathbf{p}|/m$ when $|\mathbf{p}|\to 0$. Therefore, when $|\mathbf{p}|\to 0$, the leading contribution to the corresponding cross sections are given by the P states, and these cross sections are also of order $|\mathbf{p}|/m$. However, the ratio $\sigma_{\bar{p}p\to\phi\pi^0}/\sigma^{(10)}_{\bar{p}p\to K^{*+}K^-}$ when $|\mathbf{p}|\to 0$ becomes just the ratio of the quantities $\mathrm{Br}(\bar{p}p\to\phi\pi^0)$ and $\mathrm{Br}(\bar{p}p\to K^{*+}K^-)$ in the annihilation from the P state of the hydrogen-like $\bar{p}p$ atom if we assume that the $\bar{p}$ and p in this state are unpolarized.

The general form of the $\bar{p}p\to\phi\pi^0$ amplitude with the required properties is

$$M_{\bar{p}p\to\phi\pi^0}=[\bar{v}(p_2)\gamma^5u(p_1)]\left[F_1'(p_1-p_2,e^*)+\frac{F_2'}{m_\phi^2}\right.$$
$$\left.\times(p_1-p_2,k_1-k_2)(k_1-k_2,e^*)\right], \qquad (49)$$

where F_1' and F_2' become constants when $|\mathbf{p}|\to 0$. In contrast to the annihilation from the S state, the amplitude given by Eq. (49) is defined by two unknown constants, since the final $\phi\pi^0$ system has orbital angular momentum either $L=0$ or $L=2$.

It is convenient to consider the amplitude (49) in the c.m. frame. Then we can write

$$M_{\bar{p}p\to\phi\pi^0}=[\bar{v}(p_2)\gamma^5u(p_1)]\left[F_1(\mathbf{pe}^*)+\frac{F_2}{m_\phi^2}(\mathbf{pk})\right.$$
$$\left.\times(\mathbf{ke}^*)\right], \qquad (50)$$

where F_1 and F_2 are linear combinations of F_1' and F_2'. Analogously, we can write

$$M^{(10)}_{\bar{p}p\to K^{*+}K^-}=\left[\bar{v}(p_2)\gamma^5u(p_1)\right]\left[f_1(\mathbf{pe}^*)+\frac{f_2}{m_*^2}(\mathbf{pk}')\right.$$
$$\left.\times(\mathbf{k}'\mathbf{e}'^*)\right], \qquad (51)$$

where f_1 and f_2 are other constants. As easily follows from Eqs. (50) and (51),

$$R_2=\frac{\mathrm{Br}(\bar{p}p\to\phi\pi^0)_{L=1}}{\mathrm{Br}(\bar{p}p\to K^{*+}K^-)^{(10)}_{L=1}}=\left\{k\left[|F_1|^2\left(1+\frac{k^2}{3m_\phi^2}\right)\right.\right.$$
$$+\frac{k^2}{3m_\phi^2}\left(1+\frac{k^2}{m_\phi^2}\right)\left(F_1F_2^*+F_1^*F_2\right.$$
$$\left.\left.\left.+\frac{k^2}{m_\phi^2}|F_2|^2\right]\right)\right\}\Bigg/\left\{k'\left[|f_1|^2\left(1+\frac{k'^2}{3m_*^2}\right)+\frac{k'^2}{3m_*^2}\right.\right.$$
$$\left.\left.\times\left(1+\frac{k'^2}{m_*^2}\right)\left(f_1f_2^*+f_1^*f_2+\frac{k'^2}{m_*^2}|f_2|^2\right]\right)\right\}. \qquad (52)$$

By analogy with the derivation in Sec. 5, we find that in model A

$$M_{\bar{p}p\to\phi\pi^0}=\frac{-ik'\mathbf{p}}{2\pi^2\sqrt{s}}[\bar{v}(p_2)\gamma^5u(p_1)]f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi}$$
$$\times\int\frac{do'(k_{2\lambda}e^{\lambda*})}{(k_1'-k_1)^2-m_K^2}\left[f_1\left(\frac{\mathbf{k}'(k_1k_1')}{m_*^2}-\mathbf{k}\right)\right.$$
$$\left.+\frac{f_2}{m_*^2}\mathbf{k}'\left(\frac{(k')^2(k_1k_1')}{m_*^2}-\mathbf{k}\mathbf{k}'\right)\right]. \qquad (53)$$

Since the relation between the reactions $\bar{p}p\to\phi\pi^0$ and $\bar{p}p\to K^{*+}K^-$ in the annihilation from the S state can be qualitatively explained by assuming that the off-shell form factors in the vertices $K^{*+}\to\pi^0K^+$ and $K^+K^-\to\phi$ do not appreciably suppress the amplitude of $\bar{p}p\to\phi\pi^0$, we do not take into account the contribution of these form factors.

Using Eq. (28), we can derive a relation between the quantities F_i and f_i ($i=1,2$), and the final result is

$$F_i=\frac{ik'}{\pi\sqrt{s}}f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi}\sum_{l=1}^{2}A_{il}f_l, \qquad (54)$$

where

$$A_{11}=\frac{k'}{4km_*^2}\int_{-1}^{1}\frac{(1-x^2)(E_*E_\pi-kk'x)dx}{a-x},$$

$$A_{12}=\frac{k'^2}{4km_*^2}\int_{-1}^{1}\left[\frac{k'(E_*E_\pi-kk'x)}{m_*^2}-kx\right]\frac{(1-x^2)dx}{a-x},$$

$$A_{21}=\frac{m_\phi^2}{2kk'}\int_{-1}^{1}\left\{\frac{(E_*E_\pi-kk'x)}{m_*^2}\left[-\frac{E_Kk'x}{E_\phi k}\right.\right.$$
$$\left.\left.+\frac{k'^2(3x^2-1)}{2k^2}\right]+\frac{E_K}{E_\phi}-\frac{k'x}{k}\right\}\frac{dx}{a-x},$$

$$A_{22}=\frac{m_\phi^2k'}{2m_*^2k^2}\int_{-1}^{1}\left[\frac{k'(E_*E_\pi-kk'x)}{m_*^2}-kx\right]$$
$$\times\left[-\frac{E_Kx}{E_\phi}+\frac{k'(3x^2-1)}{2k}\right]\frac{dx}{a-x}. \qquad (55)$$

It follows from simple numerical calculations and from Eqs. (15), (17), (52), (54), and (55) that

$$R_2=\frac{0.77+0.36yz+0.044y^2}{1.16+0.46yz+0.11y^2}, \qquad (56)$$

where $y=|f_2/f_1|$ and z is the cosine of the relative phase of f_1 and f_2. If $f_2=0$, then $R_2=0.66$, and if $f_1=0$, then $R_2=0.40$. However, in the general case R_2 can take values from $R_{\min}=0.02$ when $y=4.2$, $z=-1$ to $R_{\max}=0.67$ when $y=0.7$, $z=1$. In addition, if we take into account a possible contribution of the off-shell form factors, we can conclude that $\mathrm{Br}(\bar{p}p\to\phi\pi^0)_{L=1}$ and $\mathrm{Br}(\bar{p}p\to K^{*+}K^-)^{(10)}_{L=1}$ are probably of the same order of magnitude. In this case the problem remains of whether the results of the rescattering model for the P-wave annihilation are compatible with the results for the S-wave annihilation. At the same time, one cannot fully exclude the possibility that the first quantity is much smaller than the second one.

As noted by Zou,[40,43] the $L=2$ decay is unlikely to be of similar strength to the $L=0$ decay, because of the strong centrifugal-barrier effect for $L=2$ $K^*\bar{K}$ decay. An experiment which can shed light on the situation is the measurement of the angular distribution in the K^*K system produced in the $\bar{p}p$ annihilation from the P state. If, for example, one of the states with $L=0$ or $L=2$ is dominant, then the destructive interference described above is not possible.

In any case, the value of R of order 10^{-2} which can explain the difference between the situations in the S and P annihilations in the model considered above seems unlikely. However, as argued by Zou,[40,43] the destructive interference is only a minor reason, while there is another more solid and important reason, i.e., the small total decay width of $I=1$ $^1P_1$ protonium.

As noted in Refs. 40 and 43, the fact which is important for understanding the problem under consideration is that, for $\bar{p}p$ annihilation from P states, $K^*\bar{K}$ can come from $^1P_1$, $^3P_1$, and $^3P_2$ states with both isospins 0 and 1, while $\phi\pi$ can come only from the $^1P_1$ state with isospin 1. According to various optical-potential models for protonium annihilation,[44,45] the total decay width for the $I=1$ $^1P_1$ state is only about 1/8 of the summation of the total decay width for all possible P states to $K^*\bar{K}$. The $K^*\bar{K}$ decay width may not be directly proportional to the total decay width for different P states, owing to some dynamic selection rule. It is quite possible that $K^*\bar{K}$ from the $I=1$ $^1P_1$ state is only a very small part of $K^*\bar{K}$ from all the P states. Only this small part can contribute to the rescattering mechanism to the $\phi\pi$ final state. This is in contrast to the case of $\bar{p}p$ annihilation from S states, where the allowed partial wave ($I=1$ $^3S_1$) for $\phi\pi$ is found to be dominant for $K^*\bar{K}$.

Are there another reasons (in addition to optical models) to think that the $K^*\bar{K}$ annihilation from the $I=1$ $^1P_1$ state of protonium is indeed suppressed? As argued by Zou,[40,43] such reasons are the following. First, the Asterix Collaboration found that the branching ratios for $\eta\rho$ and $\eta'\rho$ from P states are much smaller than from S states.[46] The $\eta\rho$ and $\eta'p$ from P states can come only from the $I=1$ $^1P_1$ state. Second, a recent analysis by the Obelix Collaboration[47] shows that $\omega\pi$ is also not seen in $\bar{p}p$ annihilation from the $I=1$ $^1P_1$ state. Thus, the ratio $\phi\pi/\omega\pi$ for P-state annihilation may in fact not be suppressed.

As noted in Refs. 40 and 43, it is desirable to measure the percentage of $K^*\bar{K}$ production from P states that comes from the $I=1\ {}^1P_1$ state. Only after all conventional effects were found to be insufficient to explain the data might we claim any conclusive evidence for new physics, such as strange quarks in the nucleon.[25]

On the other hand, as noted in Ref. 48, although the observations in Ref. 43 are important, the problem is whether they are sufficient to explain the experimental situation, according to which even the upper bound for the ratio of the $\phi\pi$ and $K^*\bar{K}$ channels in the annihilation from the P states is probably of order 10^{-2}. Indeed, according to Ref. 46, the branching ratios of the $\phi\pi$ and $K^{*+}K^-$ channels in the ${}^{33}S_1$ state are $(4.0\pm0.8)\cdot10^{-4}$ and $(5.8\pm0.5)\cdot10^{-4}$, respectively. According to the data in Ref. 47, the branching ratio of the $\phi\pi$ channel in the ${}^{31}P_1$ state is $\leqslant3\cdot10^{-5}$, according to Ref. 49 this quantity is $\leqslant1\cdot10^{-5}$, and the most recent analysis[50] gives the value $\leqslant4.7\cdot10^{-5}$ (with 95% confidence level). At the same time, the data of Refs. 49 and 50 show that when going from liquid to gas targets the yield of $K\bar{K}\pi$ increases.

The data of Ref. 46 show that the branching ratios of the $\eta\rho$ channel are $(0.94\pm0.53)\cdot10^{-3}$ in the P state and $(3.29\pm0.90)\cdot10^{-3}$ in the S state. The ratio of these quantities is about 0.3. The same data for the $\eta'\rho$ channel are $(\sim0.3)\cdot10^{-3}$ and $(1.81\pm0.44)\cdot10^{-3}$, respectively, i.e., the ratio is about 1/6. These values are consistent with the value 1/8 in optical models, but such a degree of suppression of the annihilation of the ${}^{31}P_1$ protonium is one order of magnitude less than what is needed to explain the effect under consideration. In addition, the statistics in the data of the Obelix Collaboration on the angular distribution in the $\omega\pi$ system given in Ref. 47 do not make it possible to clearly distinguish the annihilation from the S and P waves.

We conclude that at the present stage of our understanding of the rescattering mechanism it is not possible to explain the fact that $\phi\pi$ is not seen in the annihilation of the ${}^{31}P_1$ protonium.

8. THE PROBLEM OF OZI-RULE VIOLATION IN THE REACTION $\bar{p}p\to f_2'\pi^0$

In view of the above discussion, it is important to know whether there exist reactions with the property that if the OZI rule in them is violated, then the rescattering model or other conventional mechanisms definitely cannot explain this violation. Following Ref. 51, we show in this section that $\bar{p}p\to f_2'\pi^0$ is a reaction with just such a property.

The situation with regard to f_2-f_2' mixing is analogous to that for $\omega-\phi$ mixing, but the mixing angle is not so close to the ideal one: according to Ref. 17, $\cos\theta=0.78$. Therefore, as follows from the f_2-f_2' analog of Eq. (1), the ratio $\mathrm{Br}(\bar{p}p\to f_2'\pi^0)/\mathrm{Br}(\bar{p}p\to f_2\pi^0)$ should be approximately 0.01. The experimental data on the branching ratio for the annihilation $\bar{p}p\to f_2\pi^0$ at rest are $(3.4\pm0.5)\cdot10^{-2}$, $(2.1\pm0.1)\cdot10^{-1}$, and $(2.0\pm0.6)\cdot10^{-2}$ in the cases of the ${}^1S_0$, ${}^3P_1$, and ${}^3P_2$ states, respectively.[52] Therefore $\mathrm{Br}(\bar{p}p\to f_2'\pi^0)$ is expected to be of order 10^{-4} in the cases of the ${}^1S_0$ and ${}^3P_2$ states, and of order 10^{-3} in the case of the ${}^3P_1$ state. This makes it is necessary to estimate the role of the rescattering contribution in the reaction $\bar{p}p\to f_2'\pi^0$.

The major decay mode of the f_2' meson is $K\bar{K}$, as for the ϕ meson. Therefore, in view of the above discussion, it is reasonable to estimate the role of $(K^*\bar{K}+\bar{K}^*K)$ intermediate states in model A. We shall consider only S-wave annihilation, and we shall see that even the upper bound for the rescattering contribution is much less than the value expected from the OZI rule.

The only relativistically invariant amplitude of the process $\bar{p}p\to K^{*+}K^-$ which survives when $p\to0$ and the $K^{*+}K^-$ system is in the state with $I=1$, $S=0$ is

$$M^{(10)}_{\bar{p}p\to K^{*+}K^-}=f^{(10)}_{K^{*+}K^-}[\bar{v}(p_2)\gamma^5u(p_1)](e'^*P), \tag{57}$$

where $f^{(10)}_{K^{*+}K^-}$ is some constant. Then the corresponding cross section is

$$\sigma^{(10)}_{\bar{p}p\to K^{*+}K^-}=\frac{|f^{(10)}_{K^{*+}K^-}|^2sk'^3}{32\pi m_*^2p}. \tag{58}$$

We also need the amplitude of the reaction $K^+K^-\to f_2'$. It has the form

$$M_{K^+K^-\to f'}=f_{K^+K^-\to f'}(k_3'-k_2')_\mu(k_3'-k_2')_\nu e^{*\mu\nu}, \tag{59}$$

where $e^{\mu\nu}$ is the polarization tensor of the final f_2' meson. The corresponding decay width is

$$\Gamma_{f'\to K^+K^-}=\frac{4|f_{K^+K^-\to f'}|^2k^5_{K\bar{K}}}{15\pi m^2_{f'}}, \tag{60}$$

where $k_{K\bar{K}}$ is now the magnitude of the momentum of the K^+ and K^- mesons in the reference frame in which the f_2' meson is at rest. Since the decay of the f_2' meson into $K\bar{K}$ occurs in 72% of the events, the total width of the f_2' meson is $\Gamma_{f_2'}=2\Gamma_{K^+K^-\to f_2'}/0.72$.

As follows from Eqs. (14), (57), and (59), if the form factors are dropped, then the amplitude of the reaction $\bar{p}p\to f_2'\pi^0$ in model A is

$$M_{\bar{p}p\to f_2'\pi^0}=16f^{(10)}_{K^{*+}K^-}f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to f_2'}\times[\bar{v}(p_2)\gamma^5u(p_1)]e^{*\mu\nu}I_{\mu\nu}, \tag{61}$$

where

$$I_{\mu\nu}=\int\frac{(2\pi)^4\delta^{(4)}(k_1+k_2-k_1'-k_2')d^3\mathbf{k}_1'd^3\mathbf{k}_2'}{(2(2\pi)^3)^2\omega_*(\mathbf{k}_1')\omega_K(\mathbf{k}_2')[(k_1'-k_1)^2-m_K^2+i0]}\times\left[\frac{(Pk_1')(k_1k_1')}{m_*^2}-(Pk_1)\right]k_{2\mu}'k_{2\nu}', \tag{62}$$

$\omega_*(\mathbf{k}')=(m_*^2+\mathbf{k}'^2)^{1/2}$, and k_2 is the four-momentum of the final f_2' meson.

The quantity $I_{\mu\nu}$ is the relativistic symmetric tensor which depends only on k_1 and k_2, and, since $P=k_1+k_2$, we can write

$$I_{\mu\nu}=c_1P_\mu P_\nu+c_2g_{\mu\nu}+c_3(P_\mu k_{2\nu}+P_\nu k_{2\mu})+c_4k_{2\mu}k_{2\nu}, \tag{63}$$

where c_i $(i=1,\dots,4)$ are quantities which may depend only on s. Since $e^{\mu\nu}g_{\mu\nu}=e^{\mu\nu}k_{2\mu}=e^{\mu\nu}k_{2\nu}=0$, only the term with c_1 contributes to Eq. (61). Therefore it is sufficient to find only c_1. For this purpose we note that the tensor

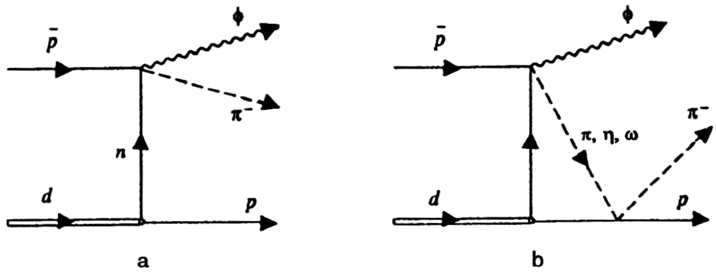


FIG. 7. (a) Pole diagram for the reaction $\bar{p}d\to\phi\pi^-p$ and (b) diagram describing the process $\bar{p}d\to\phi\pi^-p$ proceeding through the rescattering of π, η, and ω mesons produced in the intermediate state.

$$X_{\mu\nu}=\frac{(Pk_2)^2k_{2\mu}k_{2\nu}}{m_{f'}^4}-\frac{(Pk_2)}{m_{f'}^2}(k_{2\mu}P_\nu+k_{2\nu}P_\mu)+P_\mu P_\nu$$
$$-\frac{1}{3}\left(\frac{k_{2\mu}k_{2\nu}}{m_{f'}^2}-g_{\mu\nu}\right)\left[\frac{(Pk_2)^2}{m_{f'}^2}-P^2\right] \quad (64)$$

has the property

$$X^{\mu\nu}g_{\mu\nu}=X^{\mu\nu}k_{2\mu}=X^{\mu\nu}k_{2\nu}=0. \quad (65)$$

Therefore, it follows from Eqs. (63) and (65) that

$$c_1=\frac{I_{\mu\nu}X^{\mu\nu}}{P_\mu P_\nu X^{\mu\nu}}, \quad (66)$$

and it follows from Eq. (61) that

$$M_{\bar{p}p\to f_2'\pi^0}=4f^{(10)}_{K^{*+}K^-}f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to f_2'}$$
$$\times[\bar{v}(p_2)\gamma^5u(p_1)]c_1e^{\mu\nu *}P_\mu P_\nu. \quad (67)$$

An explicit expression for c_1 can be easily obtained in the c.m. frame of the π^0f_2' system (by analogy with Sec. 5). In this frame of reference

$$\frac{(2\pi)^4\delta^{(4)}(k_1+k_2-k_1'-k_2')}{[2(2\pi)^3]^2\omega_*(\mathbf{k}_1')\omega_K(\mathbf{k}_2')}=\frac{k'do'}{16\pi^2\sqrt{s}}, \quad (68)$$

where do' has the same meaning as in Sec. 5.

Taking into account Eqs. (15), (62), (64), and (65)–(67), the final result can be written in the form

$$\frac{\sigma_{\bar{p}p\to f_2'\pi^0}}{\sigma^{(10)}_{\bar{p}p\to K^{*+}K^-}}$$
$$=0.72\frac{45}{2}\frac{kk'\Gamma_{f_2'}\Gamma_*}{sk_{\pi K}^3k_{K\bar{K}}^5m_{f'}^2}$$
$$\times\left|\int_{-1}^{1}\frac{k'E_\pi-E_\pi kx}{m_\pi^2+m_*^2-2E_\pi E_*+2kk'x-m_K^2+i0}\right.$$
$$\times\left\{(E_Kk-E_{f'}k'x)^2-\frac{1}{3}[(E_{f'}E_K-kk'x)^2\right.$$
$$\left.\left.-m_K^2m_{f'}^2]\right\}dx\right|^2. \quad (69)$$

A simple numerical calculation gives for $s=4m^2$ the value $\mathrm{Br}(\bar{p}p\to f_2'\pi^0)=2.66\cdot10^{-2}\,\mathrm{Br}(\bar{p}p\to K^{*+}K^-)^{(10)}$. According to Ref. 36, $\mathrm{Br}(\bar{p}p\to K^{*+}K^-)=(2.1\pm0.4)\cdot10^{-4}$. Therefore even the upper bound for $\mathrm{Br}(\bar{p}p\to f_2'\pi^0)$ is of order 10^{-6}.

It is also possible to calculate the contribution of the $\rho\pi$ channel to the reaction $\bar{p}p\to f_2'\pi^0$. The corresponding amplitude has the same spin structure as the amplitude describing the $(K^*\bar{K}+\bar{K}^*K)$ contribution. A simple numerical calculation gives $\mathrm{Br}(\bar{p}p\to f_2'\pi^0)=(4.08\cdot10^{-4})\cdot\mathrm{Br}(\bar{p}p\to\rho^+\pi^-)^{(10)}$. According to Ref. 52, $\mathrm{Br}(\bar{p}p\to\rho^+\pi^-)^{(10)}=(0.65\pm0.3)\cdot10^{-2}$, and therefore the $\rho\pi$ contribution is also small.

We see that the upper bound for the rescattering contribution to the reaction $\bar{p}p\to f_2'\pi^0$ from the S state is of order 10^{-6}, and by analogy with the calculation in the preceding section we can expect that the upper bound for the rescattering contribution to the reaction $\bar{p}p\to f_2'\pi^0$ from the P states is also of order 10^{-6}. Therefore the role of rescattering in this reaction is negligible, and any violation of the OZI rule in the reaction $\bar{p}p\to f_2'\pi^0$ will be evidence of some unusual phenomena.

According to the preliminary data of the Obelix Collaboration reported in Ref. 39, the ratio of the branching ratios for the $f_2'\pi^0$ and $f_2\pi^0$ annihilations from the P state is in the range $(4-10)\cdot10^{-2}$, and the most recent result for this ratio is $(13\pm2)\cdot10^{-2}$ (Ref. 50). This is one order of magnitude larger than the value predicted by the OZI rule.

9. OZI-RULE VIOLATION IN $\bar{p}d$ ANNIHILATION

As noted in Sec. 1, the data on the reaction $\bar{p}d\to\phi\pi^-p$ are the source of information about the process (4), but this reaction is of interest in its own right. The point is that if the reactions in which the OZI rule is strongly violated involve exotic states (such as hybrids and glueballs), then, as argued by several authors (see, e.g., the review paper of Ref. 53), the masses of these states probably lie in the region 1.4–1.7 GeV/c, i.e., below the threshold of antiproton annihilation on a free nucleon. The above reaction makes it possible to study antiproton annihilation on a bound nucleon at $\sqrt{s}<2m$.

If the process $\bar{p}d\to\phi\pi^-p$ is described by the pole diagram given in Fig. 7a, then it is easy to show that for slow antiprotons the quantity $\sqrt{s}$ for the reaction $\bar{p}n\to\phi\pi^-$ is related to the energy E' of the spectator proton by the relation $s=10m^2-6mE'$. In a recent experiment of the Obelix group[16] the branching ratio of the reaction $\bar{p}d\to\phi\pi^-p$ was measured in the region of proton momenta 0.4–0.8 GeV/c. These values correspond to $\sqrt{s}$ in the range 1.37–1.76 GeV, i.e., in the range of primary interest for our study. We denote the branching ratio of the above reaction by B_2^ϕ, the branching ratio of the reaction $\bar{p}d\to\phi\pi^-p$ at proton momenta in the region 0–0.2 GeV/c by B_1^{ϕ}, and the corresponding branching ratios of the reaction $\bar{p}d \to \omega\pi^- p$ by B_1^{ω} and B_2^{ω}. Then it follows from the data reported in Ref. 16 that

$$B_1^{\phi}=(6.62\pm0.49)\cdot10^{-4}, \quad B_2^{\phi}=(0.93\pm0.22)\cdot10^{-4},$$

$$B_1^{\omega}=(4.97\pm0.89)\cdot10^{-3}, \quad B_2^{\omega}=(8.38\pm1.09)\cdot10^{-4}. \tag{70}$$

Hence we have

$$B_1^{\phi}/B_1^{\omega}=0.13, \quad B_2^{\phi}/B_2^{\omega}=0.11. \tag{71}$$

At the same time, as noted in Sec. 1, the data on the $\phi\omega$ mixing angle[17] and the OZI rule give values of order 10^{-3} for these ratios. Thus, according to the data reported in Ref. 16, the violation of the OZI rule in the reaction $\bar{p}d \to \phi\pi^- p$ at proton momenta in the region 0.4–0.8 GeV/c is as strong as for the reactions (2)–(4).

Following Ref. 54, we investigate in this section whether the above effect is indeed a consequence of the OZI-rule violation in the process $\bar{p}n \to \phi\pi^-$ or whether such a violation is imitated by some nuclear effects in the deuteron.

The amplitude of the reaction $\bar{p}n \to \phi\pi^-$ can be written as

$$A_{\bar{p}n\to\phi\pi^-}=f_{\bar{p}n\to\phi\pi^-}(\bar{u}\gamma^{\mu}v)e_{\mu\nu\rho\sigma}e^{\nu *}p_1^{\rho}p_2^{\sigma}, \tag{72}$$

where $f_{\bar{p}n\to\phi\pi^-}$ is some function of the invariant variables, u is a Dirac spinor describing the initial neutron, v is a Dirac spinor corresponding to negative energy and describing the initial antiproton, e^{ν} is the polarization vector of the ϕ meson, p_1 is the four-momentum of the π^- meson, and p_2 is the four-momentum of the ϕ meson.

At small momenta of the incident antiproton this is the only form of the amplitude that is consistent with the conditions that annihilation proceeds from the state of the $\bar{p}n$ system with spin $S=1$, and that the final $\phi\pi^-$ system be produced in the state with orbital angular momentum $l=1$. It can easily be shown that these conditions follow from the conservation laws for ordinary parity and G parity.

Assuming that $f_{\bar{p}n\to\phi\pi^-}$ is constant and expressing the $d\to pn$ vertex in terms of the nonrelativistic deuteron wave function and Dirac spinors describing the antiproton and neutron in terms of ordinary spinors in the nonrelativistic approximation, we can easily evaluate the contribution of the pole diagram in Fig. 7a to the branching ratio of the reaction $\bar{p}d \to \phi\pi^- p$. The result can be written as

$$B_1^{\phi}=\frac{4m^2 r}{\pi^2 p_0}\,\mathrm{Br}(\bar{p}p\to\phi\pi^0)\int_0^{0.2}(\varphi_0^2(p') + \varphi_2^2(p'))\frac{pp'^2dp'}{2E'\sqrt{s}}, \tag{73}$$

where p_1 is the momentum of the $\phi\pi^-$ system in its c.m. frame, p_0 is the same quantity at $\sqrt{s}=2m$, p' is the final-proton momentum (so that $E'=\sqrt{m^2+p'^2}$), $\varphi_0(p')$ and $\varphi_2(p')$ are the wave functions of the S and D deuteron states in the momentum representation, and r is the ratio of the total cross sections $\sigma_{\bar{p}p}$ and $\sigma_{\bar{p}d}$ near the threshold. We take into account the fact that, owing to isotopic invariance, the amplitude of the reaction $\bar{p}n \to \phi\pi^-$ is greater than the amplitude of the reaction $\bar{p}p \to \phi\pi^0$ by a factor of $\sqrt{2}$. The value of B_2^{ϕ} is determined by the same formula, but the integral with respect to p' is taken from 0.4 to 0.8 GeV/c; B_1^{ω} and B_2^{ω} are given by similar expressions.

According to the analysis performed in Ref. 29, $r=0.552$. Then, using the data from Ref. 14, choosing the Reid soft-core model[55] for $\varphi_0(p')$ and $\varphi_2(p')$, and performing a numerical integration, one obtains $B_1^{\phi}=8.7\cdot10^{-4}$ and $B_1^{\omega}=6.4\cdot10^{-3}$, in agreement with the data from Ref. 16, while the values $B_2^{\phi}=0.68\cdot10^{-5}$ and $B_2^{\omega}=1.3\cdot10^{-4}$ obtained in a similar way are significantly smaller than the corresponding results presented in (70). The smallness of B_2^{ϕ} and B_2^{ω} seems natural because the deuteron wave function is small at $p'\in[0.4,0.8]$ GeV/c. By analogy with the Glauber theory and the results obtained in Ref. 56, we can expect that the diagrams in Fig. 7b with π, η, and ω mesons in the intermediate state make an important contribution in this region.

In calculating the contribution of the diagram in Fig. 7b, we will ignore spin effects and the dependence of elementary amplitudes on the Fermi motion of the nucleons inside the deuteron. Calculating the amplitude M corresponding to the diagram in Fig. 7b by the rules of the nonrelativistic diagrammatic technique, we obtain

$$M=-\frac{A_1A_2}{(2\pi)^3\sqrt{m}}\int\frac{\varphi_0(\mathbf{q})d^3\mathbf{q}}{k_X^2-\mu^2+i\mu\Gamma-2\mathbf{k}_X\mathbf{q}}, \tag{74}$$

where k_X is the four-momentum of the intermediate meson X, μ is its mass, Γ is its width, A_1 is the amplitude of the annihilation process $\bar{p}N\to\phi X$ (N is either the proton or the neutron), and A_2 is the amplitude of the process $XN \to \pi^- p$.

Let K be the total laboratory energy of the ϕ meson and $k=\sqrt{K^2-m_{\phi}^2}$ be its momentum. We introduce the function

$$F(K,\mu,\Gamma_{\mu})=\left|-\frac{i}{8\pi k}\int_{q_1}^{\infty}\varphi_0(q)qdq+\int_0^{\infty}\frac{\varphi_0(q)q}{16\pi^2 k}\right. \times\ln\frac{(5m^2-4mK-\mu^2+2kq)^2+\mu^2\Gamma_{\mu}^2}{(5m^2-4mK-\mu^2-2kq)^2+\mu^2\Gamma_u^2} \times\left|dq\right|^2, \tag{75}$$

where $q_1=|5m^2-4mK-\mu^2|/2k$. We denote by p_1 the c.m. momentum in the ϕX system. The square of the invariant energy s for this system depends on E', as above; therefore p_1 is also a function of E'. We denote by $E_{\phi}=\sqrt{m_{\phi}^2+p_1^2}$ the ϕ-meson energy in the c.m. frame of the ϕX system. It is clear that E_{ϕ} is also a function of E'. The process of the X-meson collision with the nucleon is characterized by the invariant quantities $s_1=s_1(K)=9m^2-6mK+m_{\phi}^2$ and $t_1=t_1(E')=2m(m-E')$.

Taking into account the fact that the widths of the π, η, and ω mesons are small, it is possible to calculate the contribution of the amplitude (74) to the branching ratio of the reaction $\bar{p}d\to\phi\pi^- p$, and the results are the following. For the case when the π^0 and π^- mesons are produced in the intermediate state we must take into account the interference of the corresponding diagrams. This is equivalent to extracting from the πN scattering amplitude only the part corresponding to isospin $I=1/2$. Indeed, since the deuteron and the ϕ meson are isoscalar particles, the πN system in the intermediate state can only have isospin 1/2. The contribution of the corresponding diagrams to the branching ratio of the reaction $\bar{p}d\to\phi\pi^-p$ is given by

$$\mathrm{Br}(\bar{p}d\to\phi\pi^-p)=\frac{6r}{\pi p_0}\,\mathrm{Br}(\bar{p}p\to\phi\pi^0)\times\int\int F(K,m_\pi,\Gamma_\pi)[s_1^2-2s_1(m^2+m_\pi^2)+(m^2-m_\pi^2)^2]\left(\frac{d\sigma_{\pi^-p\to\pi^-p}(s_1,t_1)}{dt_1}+\frac{d\sigma_{\pi^-p\to\pi^0n}(s_1,t_1)}{dt_1}-\frac{1}{3}\frac{d\sigma_{\pi^+p\to\pi^+p}(s_1,t_1)}{dt_1}\right)dKdE'. \quad (76)$$

The contribution of the diagram with the η meson in the intermediate state has the form

$$\mathrm{Br}(\bar{p}d\to\phi\pi^-p)=\frac{2r}{\pi p_0}\,\mathrm{Br}(\bar{p}p\to\phi\eta)\times\int\int F(K,m_\eta,\Gamma_\eta)[s_1^2-2s_1(m^2+m_\pi^2)+(m^2-m_\pi^2)^2]\,\frac{d\sigma_{\pi^-p\to\eta n}(s_1,t_1)}{dt_1}\,dKdE'. \quad (77)$$

The contribution of the diagram with the ω meson in the intermediate state is obviously given by Eq. (77), where η is replaced by ω.

In Eqs. (76) and (77) the integration with respect to K at given E' is made over the segment $K\in[K_1,K_2]$, where

$$K_1=\frac{E_\phi(3m-E')-pp'}{\sqrt{s}},\quad K_2=\frac{E_\phi(3m-E^*)+pp'}{\sqrt{s}}. \quad (78)$$

Moreover, the condition

$$K\leqslant\frac{1}{6m}[9m^2+m_\phi^2-(m+m_X)^2]=K_0$$

is imposed because at $s_1\leqslant(m+m_X)^2$ the cross section of the process $XN\to\pi^-p$ must be set equal to zero.

As there are no parametrizations of the differential cross sections for the processes $\pi N\to\pi N$, $\pi^-p\to\eta n$, and $\pi^-p\to\omega n$ as functions of the two variables s_1 and t_1 in the region under consideration, it is reasonable to neglect the dependence of $d\sigma(s_1,t_1)/dt_1$ on t_1, replacing this differential cross section by the expression

$$\frac{d\sigma(s_1t_1)}{dt_1}=\frac{\sigma(s_1)}{s_1}\left\{\left[1-2\frac{(\mu^2+m^2)}{s_1}+\frac{(m^2-\mu^2)^2}{s_1^2}\right]\times\left[1-2\frac{(m_\pi^2+m^2)}{s_1}+\frac{(m^2-m_\pi^2)^2}{s_1^2}\right]\right\}^{-1/2}. \quad (79)$$

Then calculations show that the contributions of rescattering to B_1^ϕ and B_1^ω are much smaller than the contribution of the pole diagram (see above). The contributions to B_2^ϕ of the diagrams with π, η, and ω mesons in the intermediate state are $4.37\cdot10^{-5}$, $1.18\cdot10^{-5}$, and $0.21\cdot10^{-5}$, respectively; the corresponding contributions to B_2^ω are $1.32\cdot10^{-4}$, $0.29\cdot10^{-4}$, and $\leqslant1\cdot10^{-6}$. The contribution of the ω meson is small because only a small part of the spectrum contributes to the integral analogous to (77), in view of the condition $K\leqslant K_0$. If one assumes that the diagrams with π, η, and ω mesons do not interfere, the final results (including the contribution of the pole diagram) are

$$B_2^\phi=7.4\cdot10^{-5},\quad B_2^\omega=2.9\cdot10^{-4}, \quad (80)$$

in qualitative agreement with the experimental data presented in Eq. (70).

For the reaction $\bar{p}d\to\omega\pi^-p$, both the total branching ratios B_1^ω and B_2^ω and the proton spectrum in the momentum range 0.4–0.8 GeV/c were measured in Ref. 16. Equations (76), (77), and (79) enable us to compare the contribution of the diagrams in Figs. 7a and 7b to the proton spectrum with the experimental data of Ref. 16. Figure 8, taken from Ref. 54, shows the experimental data from Ref. 16 and the results of the calculations in Ref. 54 for the individual channels and for the total contribution, found under the assumption that the pole diagram and the diagrams with the π and η mesons do not interfere. Therefore the calculations in Ref. 54 are in qualitative agreement with the data from Ref. 16. As noted in Ref. 54, the results obtained using the Reid soft-core model do not differ significantly from the results of calculations made with the deuteron wave function in the Paris model.[57]

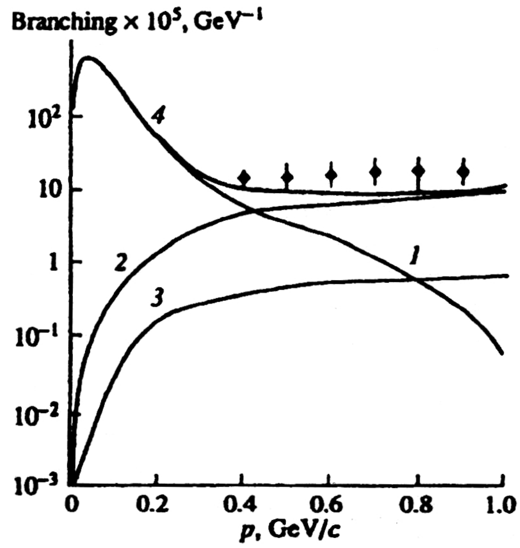


FIG. 8. Calculated relative differential (with respect to the final-proton momentum) branching ratio of the process $\bar{p}d\to\omega\pi^-p$. Contributions of the pole diagram (curve 1) and of the diagrams with π (curve 2) and η (curve 3) mesons in the intermediate state and their total contribution (curve 4) are shown separately (the contribution of the diagram with the ω meson in the intermediate state is negligible).

In Ref. 57 the proton spectrum in the reaction $\bar{p}d \to \phi\pi^- p$ was also calculated, but here experimental data are not yet available.

The qualitative agreement of the above results with the experimental data from Ref. 16 leads to the assumption that the large violation of the OZI rule observed in Ref. 16 may be associated not with exotic nuclear mechanisms in the deuteron, but with OZI-rule violation in the reaction $\bar{p}n \to \phi\pi^-$ (confirmed in the same experiment in the cases when the proton is a spectator) and with rescattering of an intermediate meson; the latter effect is described by the diagrams shown in Fig. 7b. In order to calculate the contribution of these diagrams more reliably, it is necessary to take into account spin effects and the D-wave admixture in the deuteron wave function. However, the main obstacle is that the momentum and spin dependences of the amplitude of the process $XN \to \pi^- p$ are unknown. Locher and Zou,[58] who investigated the reaction $\bar{p}d \to 3\pi N$, calculated diagrams similar to those shown in Fig. 7b under the assumption that the amplitude of the process $XN \to \pi^- p$ can be approximated by several Breit–Wigner amplitudes corresponding to different Δ isobars. Such an approximation is not applicable to our case because (see above) the XN system can only be in a state with isospin $I = 1/2$.

10. J/Ψ DECAYS AS A TEST OF OZI-RULE VIOLATION IN NUCLEON–ANTINUCLEON ANNIHILATION

In this section we consider the problem of whether the investigation of the J/Ψ decays into K^*K and $\phi\pi^0$ can shed light on the OZI-rule violation in the reactions (3) and (4). This problem has been raised in the recent paper of Ref. 59.

As noted in Secs. 3 and 5, one of the main uncertainties in the rescattering mechanism is that the parameter Λ characterizing the vertex $K^* \to K\pi$ is not known, and, as noted in Sec. 5, formally the branching ratio of the reaction $\bar{p}p \to \phi\pi^0$ can be explained assuming that the main contribution is given by the region of integration where K^* is on-shell and $\Lambda = \infty$.

The rescattering contribution to the process $J/\Psi \to \phi\pi^0$ is described by the same four Feynman diagrams as in Fig. 2, but the $\bar{p}p$ pair is replaced by J/Ψ. Therefore the structure of the vertices in these diagrams is known. In particular, the amplitude of the process $J/\Psi \to K^{*+}K^-$ has the form

$$M(J/\Psi \to K^{*+}K^-) = f(K^{*+}K^-) E^\mu e_{\mu\nu\rho\sigma} e'^{*\nu} k_1'^\rho k_2'^\sigma, \tag{81}$$

where $f(K^{*+}K^-)$ is some constant, and E and e' are the polarization vectors of the J/Ψ and K^{*+}, respectively. It is easy to show that the contribution of diagram (a) is equal to that of diagram (d) as a consequence of C invariance, and, analogously, the contribution of diagram (b) is equal to that of diagram (c). The contribution of all four diagrams depends on the quantity $f(K^{*+}K^-) - f(K^{*0}\bar{K}^0)$. If isotopic invariance is not violated, then $f(K^{*+}K^-) = f(K^{*0}\bar{K}^0)$ and the amplitude of the decay $J/\Psi \to \phi\pi^0$ is equal to zero. This is obvious from the fact that the isospin of the J/Ψ is equal to zero, while the isospin of the $\phi\pi^0$ system is equal to one (note that the decay $J/\Psi \to \omega\pi^0$ also is possible only if isotopic invariance is violated). We see that in the rescattering model the decay $J/\Psi \to \phi\pi^0$ can be a consequence of isotopic symmetry breaking in the decays $J/\Psi \to K^*K$.

What is the measure of this breaking? If isotopic invariance is not broken, then the branching ratios $\mathrm{Br}(J/\Psi \to K^{*+}K^-)$ and $\mathrm{Br}(J/\Psi \to K^{*0}\bar{K}^0)$ should be the same, while according to Ref. 60

$$\mathrm{Br}(J/\Psi \to K^{*+}K^- + \mathrm{c.c.}) = (5.26 \pm 0.13 \pm 0.53) \cdot 10^{-3},$$
$$\mathrm{Br}(J/\Psi \to K^{*0}\bar{K}^0 + \mathrm{c.c.}) = (4.33 \pm 0.12 \pm 0.45) \cdot 10^{-3}, \tag{82}$$

and according to Ref. 61

$$\mathrm{Br}(J/\Psi \to K^{*+}K^- + \mathrm{c.c.}) = (4.5 \pm 0.7 \pm 0.8) \cdot 10^{-3},$$
$$\mathrm{Br}(J/\Psi \to K^{*0}\bar{K}^0 + \mathrm{c.c.}) = (4.25 \pm 0.25 \pm 0.65) \cdot 10^{-3}. \tag{83}$$

The values of the corresponding reduced branching ratios given in Ref. 60 are $(1.017 \pm 0.061) \cdot 10^{-3}$ and $(0.836 \pm 0.055) \cdot 10^{-3}$, respectively, while there is practically no difference between the c.m. momenta of the final particles in the $K^{*+}K^-$ and $K^{*0}\bar{K}^0$ systems (these momenta are 1.3713 and 1.3734 GeV/c, respectively). Therefore, although the data do not completely exclude the possibility that isotopic symmetry breaking is negligible, they show that the quantity

$$\epsilon = \frac{\mathrm{Br}(J/\Psi \to K^{*+}K^-) - \mathrm{Br}(K^{*0}\bar{K}^0)}{\mathrm{Br}(J/\Psi \to K^{*+}K^-)} \tag{84}$$

is probably of order 10^{-1}, while, since isotopic symmetry is broken by electromagnetic interactions, this quantity is expected to be of order 10^{-2}.

As noted in Sec. 3, there is no unambiguous way of calculating the diagrams in Fig. 2. If they are calculated in the same way as in Ref. 21, a calculation analogous to that of Ref. 21 gives

$$\frac{\mathrm{Br}(J/\Psi \to \phi\pi^0)}{\mathrm{Br}(J/\Psi \to K^{*+}K^-)} = |\epsilon_1|^2 0.87 \frac{3kk'\Gamma_*\Gamma_\phi m_*^2 m_\phi^2}{128 m_{J/\Psi}^2 (k_{\pi K} k_{K\bar{K}})^3} \times \left| \int_{-1}^{1} \frac{(1-x^2)dx}{a-x} \right|^2 = 0.26 |\epsilon_1|^2, \tag{85}$$

where $m_{J/\Psi}$ is the mass of the J/Ψ meson and $\epsilon_1 = [f(K^{*+}K^-) - f(K^{*0}\bar{K}^0)]/f(K^{*+}K^-)$.

Let us consider the two extreme cases when ϵ_1 is real and when ϵ_1 is imaginary. If ϵ_1 is real, it is obvious that $|\epsilon_1| = |\epsilon|/2$ and therefore

$$\mathrm{Br}(J/\Psi \to \phi\pi^0) = 0.065 |\epsilon|^2 \mathrm{Br}(J/\Psi \to K^{*+}K^-). \tag{86}$$

If ϵ_1 is imaginary, it is obvious that $|\epsilon_1|^2 = |\epsilon|$ and therefore

$$\mathrm{Br}(J/\Psi \to \phi\pi^0) = 0.26 |\epsilon| \mathrm{Br}(J/\Psi \to K^{*+}K^-). \tag{87}$$

We see that if ϵ is of order 10^{-1}, then Eq. (86) is compatible with the upper limit for $\mathrm{Br}(J/\Psi \to \phi\pi^0)$, which is $6.8 \cdot 10^{-6}$ (Ref. 60), while Eq. (87) is not compatible with this limit.

The general conclusion which follows from the above results is that the accuracy of the present data on the branch-

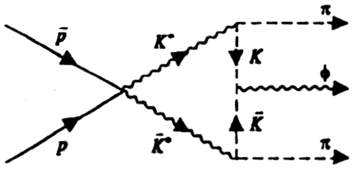


FIG. 9. Diagrams describing the process $\bar{p}p \to K^*\bar{K}^* \to \phi\pi^+\pi^-$.

ing ratios of the decays of J/Ψ into $K^{*+}K^-$, $K^{*0}\bar{K}^0$, and $\phi\pi^0$ does not make it possible to confirm or disprove the rescattering model. This model will be disproved if the right-hand side of Eq. (86) is much larger than the left-hand side.

11. PROBLEM OF THE RESCATTERING CONTRIBUTION TO THE REACTION $\bar{p}p \to \phi\pi^+\pi^-$

The OZI rule in the process $\bar{p}p \to \phi\pi^+\pi^-$ is not strongly violated, since, according to Refs. 13 and 62,

$$\mathrm{Br}(\bar{p}p \to \phi\pi^+\pi^-)/\mathrm{Br}(\bar{p}p \to \omega\pi^+\pi^-)$$

is approximately $7 \cdot 10^{-3}$ for the annihilation from the S state and $9 \cdot 10^{-3}$ for the annihilation from the P state.

Several mechanisms of the reaction $\bar{p}p \to \phi\pi^+\pi^-$ have been considered in Ref. 20, but the results are essentially model-dependent. In view of the small ϕ/ω ratio in the process under consideration, the experimental value of $\mathrm{Br}(\bar{p}p \to \phi\pi^+\pi^-)$ may be simply a consequence of the small deviation of the ϕ–ω mixing angle from the ideal value. Nevertheless, the process $\bar{p}p \to \phi\pi^+\pi^-$ is important for understanding the role of rescattering in the reaction (3). Indeed, a possible, rescattering contribution to this process is given by the diagrams in Fig. 9, where K^* can be either K^{*+} or K^{*0}, and analogously for $\bar{K}^*$. These diagrams contain the same vertices as the diagrams in Fig. 2. Therefore any choice of the vertices compatible with the data on the reaction (3) should also be compatible with the data on the reaction $\bar{p}p \to \phi\pi^+\pi^-$. In particular, the contribution of rescattering diagrams to $\mathrm{Br}(\bar{p}p \to \phi\pi^+\pi^-)$ should not exceed the experimental value.

In calculating the diagrams in Fig. 9 we encounter the same difficulties as in calculating the diagrams in Fig. 2. Since model A has turned out to be successful for describing the reaction (3) for the annihilation from the S state, one might restrict the calculation to the on-shell contribution of the diagrams in Fig. 9. Then the K^* and $\bar{K}^*$ in the $K^*\bar{K}^* \to \phi\pi^+\pi^-$ amplitude (this amplitude is shown in Fig. 10) are both on-shell. We will show in this section that such an amplitude is incompatible with unitarity, and therefore such an analog of model A cannot be used for the analysis of the process $\bar{p}p \to \phi\pi^+\pi^-$.

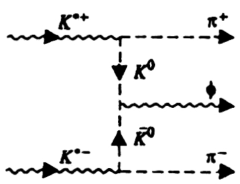


FIG. 10. Feynman diagram for the process $K^{*+}K^{*-} \to \phi\pi^+\pi^-$.

If $M_{K^{*+}K^{*-}}(s,0)$ is the amplitude of elastic $K^{*+}K^{*-}$ scattering at zero angle and $M_{K^{*+}K^{*-}\to n}$ is the amplitude of the $K^{*+}K^{*-}$ transition to some channel n, then, according to the unitarity relation (see, e.g., Ref. 63),

$$\mathrm{Im}\, M_{K^{*+}K^{*-}}(s,0) = \sum_n \int |M_{k^{*+}K^{*-}\to n}|^2 d\Gamma_n, \tag{88}$$

where $d\Gamma_n$ is the volume element of channel n at given s, and Σ implies a sum over the final polarizations. It is obvious that each term in the sum (88) should be finite.

We use $w_{K^{*+}K^{*-}\to\phi\pi^+\pi^-}$ to denote the contribution of the channel $\phi\pi^+\pi^-$ to the sum (88), averaged over the initial polarizations. Let K_1 and K_2 be the four-momenta of the initial K^{*+} and K^{*-} mesons, respectively, k_1 and k_2 be the four-momenta of the final π^+ and π^- mesons, respectively, and k_3 be the four-momentum of the final ϕ meson. Then it follows from Eqs. (14) and (16) that

$$w_{K^{*+}K^{*-}\to\phi\pi^+\pi^-} = \mathrm{const} \int \frac{|f_{K^{*+}\to\pi^0K^+}|^4}{|(K_1-k_1)^2 - m_K^2 + i0|^2} \times \frac{|f_{K^+K^-\to\phi}|^2}{|(K_2-k_2)^2 - m_K^2 + i0|^2} \times \left[\frac{(K_1k_1)^2}{m_*^2} - m_\pi^2\right]\left[\frac{(K_2k_2)^2}{m_*^2} - m_\pi^2\right] \times \left[\frac{(k_3, K_1-K_2-k_1+k_2)^2}{m_\phi^2} - (K_1-K_2-k_1+k_2)^2\right] d\Gamma, \tag{89}$$

where the value of the constant is of no importance for us,

$$d\Gamma = (2\pi)^4 \delta^{(4)}(K_1+K_2-k_1-k_2-k_3) \times \frac{d^3\mathbf{k}_1}{2(2\pi)^3E_+}\frac{d^3\mathbf{k}_2}{2(2\pi)^3E_-}\frac{d^3\mathbf{k}_3}{2(2\pi)^3E_\phi}, \tag{90}$$

and $E_\pm$ are the energies of the corresponding π mesons.

For simplicity we now consider a model in which the total energy of the $K^{*+}K^{*-}$ system is not $2m$ but $2m_*$, i.e., this system is at rest. Let us also neglect m_π. Then a standard calculation gives

$$w_{K^{*+}K^{*-}\to\phi\pi^+\pi^-} = \mathrm{const} \int \frac{|f_{K^{*+}\to\pi^0K^+}|^4 |f_{K^+K^-\to\phi}|^2}{|m_*^2 - m_K^2 - 2E_+m_* + i0|^2} \times \frac{E_+^2E_-^2[4m_*(E_++E_-) - (4m_*^2 - m_\phi^2)]}{|m_*^2 - m_K^2 - 2E_-m_* + i0|^2} dE_+dE_-. \tag{91}$$

For us it is important that if $E_- < m_* - m_\phi/2 \approx 0.38$ GeV, then

$$E_+ \in \left[\frac{4m_*^2 - m_\phi^2}{4m_*} - E_-,\ m_* - \frac{m_\phi^2}{4(m_*-E_-)}\right]. \tag{92}$$

It is obvious from Eq. (91) that the integrand contains singularities at

$$E_{\pm}=\frac{m_*^2-m_K^2}{2m_*}\approx 0.31 \text{ GeV}. \quad (93)$$

Therefore, as follows from Eq. (92), if E_- is given by Eq. (93), then $E_+\in[0.29,0.44]$ GeV. We conclude that the integral in Eq. (91) contains divergences in the integration over both variables E_+ and E_-, and therefore this integral is divergent.

The above model example is useful, since all the calculations can be performed explicitly. However, it is also clear that the integral in Eq. (89) is also divergent when the total energy of the $K^{*+}K^{*-}$ system is equal to $2m$ and the mass of the π meson is not neglected. The point is that $d\Gamma$ is again proportional to dE_+dE_- and there exists an integration region where the denominators of both propagators are equal to zero. The last property is a consequence of the fact that the kinematical conditions allow the reaction

$$K^{*+}K^{*-}\to K^0\bar{K}^0\pi^+\pi^-\to\phi\pi^+\pi^-$$

with both intermediate K mesons on the mass shell. It is also important to note that the choice of the form factors in the vertices $K^*\to K\pi$ and $K\bar{K}\to\phi$ is unimportant, since $f_{K^{*+}\to\pi^0K^+}$ and $f_{K^+K^-\to\phi}$ are constants when all the particles in question are on the mass shell. Therefore the above analog of model A in the reaction $\bar{p}p\to K^*\bar{K}^*\to\phi\pi^+\pi^-$ is incompatible with the unitarity relation.

12. CONCLUSION

Let us briefly summarize the results described in the present paper.

Following Ref. 19, we have shown in Sec. 2 that the OZI-rule violation in the reaction (2) can probably be explained in the framework of the vector-dominance model.

In Secs. 3 and 4 we have discussed two models—model A and model B—describing different on-shell contributions to the reaction $\bar{p}p\to\phi\pi^0$ (see Figs. 4 and 6). We argue that from the theoretical point of view model B is substantiated to a greater extent than model A. Nevertheless, as shown in Sec. 5 and 6, the values of $\text{Br}(\bar{p}p\to\phi\pi^0)$ given by model B are much lower than the experimental data, while model A is in qualitative agreement with the data. At the same time, as shown in Sec. 7, model A is not able to explain the fact that the process $\bar{p}p\to\phi\pi^0$ is not seen when the $\bar{p}p$ system annihilates from the P state of the protonium atom.

The recent data of the Obelix Collaboration on the reaction $\bar{p}p\to f_2'\pi^0$ show that the OZI rule in this reaction is not satisfied, and, as shown in Sec. 8, this fact cannot be explained in the framework of the rescattering model.

Following Ref. 54, we argue in Sec. 9 that the large OZI-rule violation in the reaction $\bar{p}d\to\phi\pi^-p$ for final proton momenta in the range 0.4–0.8 GeV/c is a consequence of the OZI-rule violation in the reaction $\bar{p}n\to\phi\pi^-$.

Following Ref. 59, we argue in Sec. 10 that certain decays of the J/Ψ meson can shed light on the OZI-rule violation in $\bar{p}p$ annihilation at rest, but the accuracy of the existing data is clearly insufficient for drawing any definite conclusions.

Finally, in Sec. 11 it is shown that a certain analog of model A in the reaction $\bar{p}p\to\phi\pi^+\pi^-$ is incompatible with the unitarity relation.

In spite of the partial success of model A, it is important to note that some of the assumptions underlying this model seem questionable. First, it is necessary to check numerically that if the widths of the K^* and ρ mesons are neglected, then the results will not essentially change (this is especially relevant to the question of neglecting Γ_ρ). Second, as argued in Sec. 3, model A does not fully correspond to our assumption that the ϕ meson is created from the K and $\bar{K}$ mesons. Therefore, as pointed out in Refs. 19 and 20, we have to take into account the off-shell form factor for the K meson, but the data agree with model A if this form factor is not very important. The rescattering mechanism also seems questionable from the following simple estimate. Since the K^* meson lives approximately for a time $1/\Gamma_*$ in the frame of reference in which it is at rest, it is easy to see that when the K^* meson decays the distance between the K^* and K mesons in their c.m. frame is $2mk'/\Gamma_*m_*E_K(k')\approx 6$ fm. It seems doubtful that the K^* and K mesons can effectively interact, being separated by such a distance. On the other hand, the analogous distance between the ρ^+ and ρ^- mesons is about 2 fm, but the question arises of whether it is possible to use the concept of a ρ meson in such a process.

To shed light on the problem of the OZI-rule violation in the reaction $\bar{p}p\to\phi\pi^0$, new experimental data and theoretical results are needed. The most important experimental quantities are $\text{Br}(\bar{p}p\to K^{*+}K^-)$ and $\text{Br}(\bar{p}p\to\phi\pi^0)$ when the $\bar{p}p$ system annihilates from the $I=1$ P state of the protonium atom, and $\text{Br}(\bar{p}p\to f_2'\pi^0)$ for the annihilation from the S and P states.

In view of the recent results of the Crystal Barrel Collaboration on $\phi\pi^0$ and $\omega\pi^0$ production in $\bar{p}p$ annihilation in flight,[64] it is also interesting to measure K^*K production and to compare the data with the prediction of the rescattering model.[22]

From the theoretical point of view it is important to carry out calculations not only in the on-shell approximation, but also with the off-shell contribution. The first results in this direction have been obtained in Refs. 59 and 65.

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