Renormalization of BRS charges

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Normalization of BRS charges and related renormalization-group equations are discussed in connection with color confinement. © *1997 American Institute of Physics.* [S1063-7796(97)00405-1]

1. INTRODUCTION

Quantum chromodynamics (QCD) is characterized, among other things, by BRS invariance and asymptotic freedom, and many of its fundamental properties are governed by the renormalization group (RG). In a series of papers[1–3] and in a review article[4] the problem of color confinement has been discussed, and we refer to these papers for the mathematical details. It has been concluded that color confinement is an inevitable consequence of unbroken non-Abelian gauge symmetry and asymptotic freedom. A key feature of this theory consists in renormalization of BRS charges. In this connection, let us consider the renormalization constants of BRS charges and related RG equations.

2. BRS INVARIANCE

The total Lagrangian density for QCD is given by

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}\cdot F_{\mu\nu}-\bar{\psi}(\gamma_\mu D_\mu+m)\psi+\partial_\mu B\cdot A_\mu+\frac{\alpha}{2}B\cdot B$$
$$+i\partial_\mu\bar{c}\cdot D_\mu c, \tag{2.1}$$

where both the color and flavor indices have been suppressed; B denotes the Nakanishi–Lautrup auxiliary fields, c and $\bar{c}$ are anticommuting Hermitian scalar fields, called Faddeev–Popov ghost fields, and the constant α is the gauge parameter. The covariant derivatives are defined by

$$D_\mu\psi=(\partial_\mu-igT\cdot A_\mu)\psi, \tag{2.2}$$

$$D_\mu c=\partial_\mu c+gA_\mu\times c, \tag{2.3}$$

where the antisymmetric cross product is defined in terms of the structure constant of the Lie algebra.

The BRS transformations of the gauge and quark fields, denoted by δ and $\bar{\delta}$, respectively, are defined by replacing the infinitesimal gauge function by c or $\bar{c}$. For instance,

$$\delta A_\mu=D_\mu c, \quad \bar{\delta}A_\mu=D_\mu\bar{c}, \tag{2.4}$$

$$\delta\psi=ig(c\cdot T)\psi, \quad \bar{\delta}\psi=ig(\bar{c}\cdot T)\psi. \tag{2.5}$$

For the auxiliary fields B and the fields c and $\bar{c}$, local gauge transformations are not defined, and their BRS transformations are defined so as to leave the total Lagrangian (2.1) invariant; specifically,

$$\delta\mathscr{L}=\bar{\delta}\mathscr{L}=0. \tag{2.6}$$

Then the BRS transformations of these auxiliary fields are given by[1,2,4]

$$\delta B=0, \quad \delta\bar{c}=iB, \quad \delta c=-\frac{1}{2}g(c\times c), \tag{2.7}$$

$$\bar{\delta}\bar{B}=0, \quad \bar{\delta}c=i\bar{B}, \quad \bar{\delta}\bar{c}=-\frac{1}{2}g(\bar{c}\times\bar{c}), \tag{2.8}$$

where $\bar{B}$ is defined by

$$B+\bar{B}-ig(c\times\bar{c})=0. \tag{2.9}$$

The conserved BRS charges Q_B and $\bar{Q}_B$ are defined, respectively, as the generators of the corresponding BRS transformations by

$$\delta\phi=i[Q_B,\phi]_\pm, \quad \bar{\delta}\phi=i[\bar{Q}_B,\phi]_\pm, \tag{2.10}$$

where the minus (plus) sign must be chosen when the field ϕ is even (odd) in the ghost fields c and $\bar{c}$. Both charges are Hermitian and nilpotent. For instance, we have

$$Q_B^\dagger=Q_B, \quad Q_B^2=0. \tag{2.11}$$

The equation for the gauge field follows from the Lagrangian density (2.1) and can be expressed as

$$\partial_\mu F_{\mu\nu}+gJ_\nu=i\delta\bar{\delta}A_\nu, \tag{2.12}$$

where J_ν denotes the color current density.

It is then clear that

$$\partial_\lambda(\delta\bar{\delta}A_\lambda^a)=0. \tag{2.13}$$

3. RENORMALIZATION OF THE BRS CHARGE Q_B

The Lagrangian density (2.1) is evidently invariant under the scale transformation

$$c\to e^\lambda c, \quad \bar{c}\to e^{-\lambda}\bar{c}. \tag{3.1}$$

The corresponding Noether current is given by

$$j_\mu=i(\partial_\mu\bar{c}\cdot c-\bar{c}\cdot D_\mu c) \tag{3.2}$$

and satisfies the conservation law

$$\partial_\mu j_\mu=0. \tag{3.3}$$

We define a conserved Hermitian operator Q_c by

$$Q_c=\int d^3x j_0(x). \tag{3.4}$$

This operator satisfies commutation relations of the form

$$i[Q_c,\phi]=N\phi, \tag{3.5}$$

where N denotes the number of c fields minus the number of $\bar{c}$ fields involved in ϕ as factors; N is called the ghost number of the field ϕ and is clearly an integer. The conservation of the ghost number implies that Green's functions or the vacuum expectation values of the chronological products of operators survive only when they involve equal numbers of c and $\bar{c}$ fields in the products.

Thus, we can choose a convention in which c and $\bar{c}$ share the same renormalization constant:

$$c^{(0)}=\tilde{Z}_3^{1/2}c, \quad \bar{c}^{(0)}=\tilde{Z}_3^{1/2}\bar{c}. \tag{3.6}$$

The gauge field is renormalized as

$$A_\mu^{(0)}=Z_3^{1/2}A_\mu . \tag{3.7}$$

In these equations the superscript (0) is attached to "unrenormalized" operators.

On the basis of field equations derived from (2.1) we can obtain the identity

$$\langle A_\mu^a(x),B^b(y)\rangle=-\delta_{ab}\partial_\mu D_F(x-y), \tag{3.8}$$

where D_F denotes the free massless propagator, and a and b are color indices. This identity holds in the renormalized and unrenormalized versions of the operators A_μ and B, and we can derive from Eqs. (3.7) and (3.8) the relation

$$B^0=Z_3^{-1/2}B . \tag{3.9}$$

Now we compare the following two identities in the renormalized and unrenormalized versions that follow from Eqs. (2.7):

$$\{Q_B,\bar{c}\}=B, \tag{3.10}$$

$$\{Q_B^{(0)},\ \bar{c}^{(0)}\}=B^{(0)}. \tag{3.11}$$

Inserting Eqs. (3.6) and (3.9) in the above equations, we find

$$Q_B^{(0)}=(Z_3\tilde{Z}_3)^{-1/2}Q_B . \tag{3.12}$$

Thus, we have obtained the renormalization constant for Q_B. It is necessary, however, to pay some attention to the problem of fixing the renormalization constant for $\bar{Q}_B$.

4. RENORMALIZATION OF THE BRS CHARGE Q_B

So far, we have introduced three conserved charges Q_B, $\bar{Q}_B$, and Q_c which satisfy the commutation relations

$$i[Q_c,Q_B]=Q_B, \quad i[Q_c,\bar{Q}_B]=-\bar{Q}_B, \tag{4.1}$$

$$Q_B^2=\bar{Q}_B^2=Q_B\bar{Q}_B+\bar{Q}_BQ_B=0. \tag{4.2}$$

They form a graded algebra called the BRS algebra.

In the Landau gauge ($\alpha=0$), however, Nakanishi and Ojima[5] have found two more conserved charges, denoted by $Q(c,c)$ and $Q(\bar{c},\bar{c})$, which are obtained by replacing $\bar{c}$ by c and $\bar{c}$ by c in Q_c, respectively.

Here we give their commutation relations:

$$i[Q_c,Q(c,c)]=2Q(c,c), \tag{4.3}$$

$$i[Q_c,Q(\bar{c},\bar{c})]=-2Q(\bar{c},\bar{c}), \tag{4.4}$$

$$[Q(c,c),Q(\bar{c},\bar{c})]=4iQ_c, \tag{4.5}$$

$$[Q(c,c),\bar{Q}_B]=2iQ_B, \tag{4.6}$$

$$[Q(\bar{c},\bar{c}),Q_B]=-2i\bar{Q}_B, \tag{4.7}$$

$$[Q(c,c),Q_B]=[Q(\bar{c},\bar{c}),\bar{Q}_B]=0. \tag{4.8}$$

In other gauges ($\alpha\neq 0$) these two extra charges are no longer conserved, but the above commutation relations are still valid. Thus, the role played by this extended algebra is similar to that of current algebra.

Furthermore, they also satisfy the commutation relations

$$[Q(c,c),\bar{c}]=2ic, \tag{4.9}$$

$$[Q(\bar{c},\bar{c}),c]=-2i\bar{c}. \tag{4.10}$$

With the convention given by Eq. (3.6) we find nonrenormalization of $Q(c,c)$ and $Q(\bar{c},\bar{c})$ on the basis of these commutation relations. Thus, we can use Eqs. (4.6) and (4.7) to conclude that the renormalization constants for Q_B and $\bar{Q}_B$ should be the same, namely, we have

$$\bar{Q}_B^{(0)}=(Z_3\tilde{Z}_3)^{-1/2}\bar{Q}_B, \tag{4.11}$$

in conformity with Eq. (3.12).

5. RENORMALIZATION GROUP

Having fixed the renormalization constants for the BRS charges, we can write the RG equations for various Green's functions. We shall give some examples related to color confinement.

First of all, we have

$$\delta\bar{\delta}A_\mu^{(0)}=(Z_3\tilde{Z}_3)^{-1}Z_3^{1/2}(\delta\bar{\delta}A_\mu). \tag{5.1}$$

Thus, we have, for instance,

$$\langle(\delta\bar{\delta}A_\mu^a(x))^{(0)},\ A_\nu^b(y)^{(0)}\rangle=\tilde{Z}_3^{-1}\langle\delta\bar{\delta}A_\mu^a(x),\ A_\nu^b(y)\rangle. \tag{5.2}$$

Because of Eq. (2.13), we have a Ward–Takahashi (WT) identity of the form

$$\partial_\mu\langle i\delta\bar{\delta}\,A_\mu^a(x),\ A_\nu^b(y)\rangle=i\delta_{ab}C\partial_\nu\delta^4(x-y). \tag{5.3}$$

The above two-point function and consequently the constant C must satisfy the same RG equation, and we have

$$(\mathscr{D}-2\gamma_{\rm FP})C=0, \tag{5.4}$$

where $\gamma_{\rm FP}$ denotes the anomalous dimension of the Faddeev–Popov ghost fields, $\mathscr{D}$ is given by

$$\mathscr{D}=\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-2\alpha\gamma_v\frac{\partial}{\partial\alpha} \tag{5.5}$$

in the standard notation, and γ_v denotes the anomalous dimension of the gauge field.

In the unrenormalized version of Eq. (5.3) we have $C^{(0)}=1$, but in the renormalized version we have

$$C=Z_3^{-1}+(\text{GIS term}), \tag{5.6}$$

where the "GIS term" denotes the Goto–Imamura–Schwinger term.[6,7] Since the anomalous dimension of Z_3^{-1}, given by $2\gamma_v$, is different from $-2\gamma_{\rm FP}$, in Eq. (5.4) the emergence of the GIS term is unavoidable. In Refs. 2–4 it was shown that C vanishes in gauges in which Z_3^{-1} vanishes, and this is a sufficient condition for confinement, as has been discussed previously in detail.[1–4] It should be stressed here that color confinement is a sort of renormalization effect, since the unrenormalized C can never vanish.

In QED the ghost fields are free, so that $\gamma_{FP}=0$, and $C=1$ also in the renormalized version, indicating the absence of confinement in QED. In QCD the constant C is gauge-dependent and vanishes in certain gauges.[2–4]

Now we turn to three-point functions. In Refs. 1–4 we have given a set of WT identities:

$$\partial_\lambda\langle \delta\bar{\delta}A_\lambda^a(x),\psi^\alpha(y),\bar{\psi}^\beta(z)\rangle = ig_R T_{\alpha\beta}^a[\delta^4(x-y)-\delta^4(x-z)]S_F(y-z), \tag{5.7}$$

$$\partial_\lambda\langle \delta\bar{\delta}A_\lambda^a(x),\ A_\mu^b(y),\ A_\nu^c(z)\rangle = g_R f_{abc}[\delta^4(x-y)-\delta^4(x-z)]D_{F_{\mu\nu}}(y-z), \tag{5.8}$$

where g_R denotes the RG-improved coupling constant. These WT identities enabled us to conclude that color confinement is realized when we can find gauges in which Z_3^{-1} and consequently C vanishes.

In the unrenormalized version the constant g_R agrees with the unrenormalized coupling constant, but in the renormalized version g_R deviates from g, since it satisfies an RG equation

$$(\mathscr{D}-2\gamma_{FP}-\gamma_V)g_R=0. \tag{5.9}$$

We set

$$g_R=g+\Delta, \tag{5.10}$$

where Δ denotes a kind of GIS term, and we find

$$(\mathscr{D}-2\gamma_{FP}-\gamma_V)\Delta=2g\gamma_{FP}+(g\gamma_V-\beta). \tag{5.11}$$

In this case the exact value of g_R is irrelevant in the discussion of color confinement, provided that g_R does not vanish identically, but it is generally different from g.

In QED, however, we have

$$\gamma_{FP}=0,\quad \beta=e\gamma_V. \tag{5.12}$$

Thus, we have

$$e_R=e. \tag{5.13}$$

In conclusion, we would like to emphasize that renormalization plays an essential role in realizing color confinement.

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