Theory of the Fermi liquid taking into account fragmentation and retardation effects

S. G. Kadmenskiĭ
Voronezh State University, Voronezh

Fiz. Élem. Chastits At. Yadra **28**, 391–448 (March–April 1997)

In this review it is shown that the Fermi liquid theory of Landau and Migdal is inapplicable to the phonon parts of the excitation spectra of liquid ^{3}He, electrons in solids, and atomic nuclei. A sum rule for the one-particle strength function of a Fermi system is constructed. It is concluded that the self-consistent field coincides with the generalized Hartree–Fock potential of the Fermi system. A relation between the self-consistent potential and the shell and optical potentials of fermions is found. A nonlinear scheme is developed for calculating the static and dynamical characteristics of Fermi systems with the systematic inclusion of fragmentation and retardation effects. It is shown that the interaction of nucleons with collective surface oscillations is crucial in describing the static and dynamical characteristics of nuclei, including superfluid effects. The characteristics of the real parts of the nucleon optical potentials are calculated using realistic nucleon–nucleon forces. The calculational scheme of optical folding potentials is generalized to the case of composite particles. [S1063-7796(97)00302-1]

INTRODUCTION

It is claimed that the theory of the Fermi liquid constructed by Landau[1,2] for continuous, uniform Fermi systems, and then developed further by Migdal[3] for finite Fermi systems, describes the properties of such apparently different systems as liquid He, many-electron atoms, solids, nuclei, and neutron stars at sufficiently low excitation energies of these systems (low temperatures). The contribution of this theory to the understanding of the physical properties of Fermi systems is unarguable.

However, by now a considerable amount of evidence has been gathered which suggests that the Fermi liquid theory is inapplicable even for the relatively low-energy part of the excitation spectrum of some Fermi systems.

The main concept of this theory is the fermion quasiparticle, which is viewed as the quantum of an elementary excitation of the Fermi system.[1] The concept of quasiparticle is applicable under the condition that the energy E_λ of a quasiparticle with quantum numbers λ, measured from the chemical potential of the system μ, is much larger than the quasiparticle decay width Γ_λ: $|E_\lambda-\mu|\gg\Gamma_\lambda$. For continuous Fermi systems at zero temperature $T=0$, this decay is wholly related to the possibility of decay of the quasiparticle into several quasiparticles, and at $T\neq 0$ it is related also to the probability of change of the quasiparticle state owing to collisions with other quasiparticles. For finite Fermi systems, whose excitation spectrum near the Fermi surface is discrete, there is no quasiparticle decay at energies lower than the threshold for particle emission from the system because the law of energy conservation governing the decay of a quasiparticle into several quasiparticles is not satisfied. In this case quasiparticle fragmentation occurs. This is associated with the mixing of the one-quasiparticle state with more complicated states of the system containing multiquasiparticle components (three, five, and so on). Then Γ_λ, which in infinite Fermi systems is treated as the quasiparticle decay width, can be viewed as the spreading width characterizing the degree to which the one-quasiparticle strength is divided among real states of the finite Fermi system.

The relation $|E_\lambda-\mu|\gg\Gamma_\lambda$ turns out to be valid in some Fermi liquids for the low-energy part of the excitation spectrum and, accordingly, at low temperatures T. However, several systems do not follow this rule, mainly owing to the existence in these Fermi systems of collective branches of excitations of the phonon type. Examples of such systems are electrons in solids, liquid ^{3}He, and nuclei.

In studies of the decay of electron quasiparticles in metals associated with electron interactions with vibrations of the crystal lattice (the quanta of which are called phonons), it has been shown[4] that the concept of quasiparticle can be used in the regions $|E_\lambda-\mu|\ll\omega_D$ and $|E_\lambda-\mu|\gg\omega_D$, where ω_D is the photon Debye frequency. For $|E_\lambda-\mu|\approx\omega_D$, where the quasiparticle excitation energy is comparable to the phonon frequencies, the width Γ_λ becomes close to the excitation energy, and the concept of conduction quasiparticle–electrons loses meaning.

A similar situation is also found in liquid ^{3}He for $|E_\lambda-\mu|\approx\omega_0$, where ω_0 is the characteristic frequency of zero-sound phonons, when the width Γ_λ associated with quasiparticle decay into a phonon and a quasiparticle becomes comparable to $|E_\lambda-\mu|$. This is reflected in the behavior of the specific heat $C_V(T)$ of liquid ^{3}He, where for $T>0.1$ K the linear dependence predicted by the Fermi-liquid theory[1] changes to a highly nonlinear dependence.[5]

An even more dramatic situation is observed in nuclei. From the viewpoint of quantum-field methods,[2,3] the Fermi liquid theory is valid if in the exact one-particle Green function $G_\lambda(\varepsilon)$ near the Fermi surface $\varepsilon\approx\mu$ it is possible to isolate a pole term of the form $a_\lambda/(\varepsilon-E_\lambda)$, where a_λ (called the renormalization constant) depends weakly on the label λ and coincides with the one-particle spectroscopic factor of the Fermi system in the state with quantum numbers λ and energy E_λ. The experimental one-nucleon spectroscopic

 1063-7796/97/020159-21$10.00

factors a_λ for nuclei of the type "magic ± 1 nucleon" take values[6] $a_{\lambda_F} \geq 0.7$ for the ground states of these nuclei $E_{\lambda_F} = \mu$ and are decreased by several orders of magnitude owing to fragmentation effects in transitions to excited states with $|E_\lambda - \mu| \approx \mu \ll E_F$. Noticeable differences of a_λ from unity begin to appear practically from the nuclear ground state, because the frequencies ω_0 of collective surface excitations only slightly exceed the distances between adjacent one-quasiparticle levels with different values of λ.

The Fermi liquid theory also encounters serious problems when dealing with continuum states of fermions, which play the dominant role in analyzing elastic scattering and decays involving Fermi particles for finite Fermi systems.

Successful attempts to go beyond the Fermi-liquid theory have been made repeatedly (see, for example, the review of Ref. 5 on the analysis of the properties of various continuous and finite Fermi systems, the review of Ref. 7 on the theory of nuclear matter, and especially the reviews of Refs. 8 and 9 on the quasiparticle–phonon model of the nucleus, developed to deal with fairly high excitation energies). However, the conditions limiting the applicability of the Fermi liquid theory and the generalization of this theory to systematically include fragmentation and retardation effects are studied in Refs. 10–22, and the present review is based on main results of those studies.

1. PARTICLE FRAGMENTATION IN A "CLOSED" FERMI SYSTEM

To study the problem of quasiparticle fragmentation in strongly interacting Fermi systems, let us consider the properties of the exact one-particle fermionic Green function $G(\mathbf{r},\mathbf{r}',\varepsilon)$. It satisfies the Dyson equation[3]

$$(\varepsilon - T(\mathbf{r}))G(\mathbf{r},\mathbf{r}',\varepsilon) - \int \sum (\mathbf{r},\mathbf{r}'',\varepsilon)\cdot G(\mathbf{r}'',\mathbf{r}',\varepsilon)d\mathbf{r}'' = \delta(\mathbf{r}-\mathbf{r}'), \quad (1)$$

where $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$ is the mass operator, $T(\mathbf{r})$ is the kinetic energy operator, $\mathbf{r}$ is the set of spatial, spin, and also isospin (in the case of nucleons) coordinates of the fermion, and $\delta(\mathbf{r}-\mathbf{r}')$ is a δ function in the spatial variables and a Kronecker delta in the spin (isospin) variables. For continuous Fermi systems the Lehmann expansion for $G(\mathbf{r},\mathbf{r}',\varepsilon)$ takes the well known form[3]

$$G(\mathbf{r},\mathbf{r}',\varepsilon) = \sum_p \frac{\langle N_0|\Psi(\mathbf{r})|p\rangle\langle p|\Psi^+(\mathbf{r}')|N_0\rangle}{\varepsilon - \varepsilon_p + i\delta} + \sum_{p'} \frac{\langle N_0|\Psi^+(\mathbf{r}')|p'\rangle\langle p'|\Psi(\mathbf{r})|N_0\rangle}{\varepsilon - \varepsilon_{p'} - i\delta}, \quad (2)$$

where $\Psi(\mathbf{r})$ $[\Psi^+(\mathbf{r})]$ is the particle annihilation (creation) operator; $\varepsilon_p = E_p - E_{N_0}$ and $\varepsilon_{p'} = E_{N_0} - E_{p'}$, where E_{N_0} and $|N_0\rangle$ are the energy and wave function of the ground state of the system of N interacting particles; and E_p, $E_{p'}$ and $|p\rangle$, $|p'\rangle$ are the energies and wave functions of the states of the system with $(N+1)$ and $(N-1)$ particles, respectively.

Equation (2) requires considerable rearrangement for the case of finite Fermi systems, because such systems are open. In fact, at energies $\varepsilon_p \geq 0$, $\varepsilon_{p'} \leq 2\mu$, where μ is the chemical potential of the system (for simplicity we shall not take into account the variation of the chemical potential μ as a function of the particle number N), channels of fermion emission to the continuum are opened, so that the states $|p\rangle$ and $|p'\rangle$ become quasistationary, and the energies E_p and $E_{p'}$ must respectively be replaced by the complex energies $\tilde{E}_p - i\Gamma_p/2$ and $\tilde{E}_{p'} - i\Gamma_{p'}/2$.

In this case, the situation can be simplified by carrying out the investigation in two stages. First we consider the case of a "closed" finite Fermi system and analyze the particle fragmentation in this limiting case. Then we "open" the Fermi system and study the consequences. To close a finite Fermi system we use the R-matrix technique,[23] which effects the construction of a complete orthonormal basis of internal multiparticle wave functions of the system. In this case the radial wave functions $\varphi_{\alpha p}(r)$ and $\varphi_{\alpha p'}(r)$ obtained by projecting the functions $|p\rangle$ and $|p'\rangle$ on the internal functions of all the open decay channels α are required to be extremal at $r = R_\alpha$, where R_α is the radius of the Wigner–Eisenbud sphere for the channel α:

$$\left.\frac{d\varphi_{\alpha p}(\mathbf{r})}{d\mathbf{r}}\right|_{\mathbf{r}=R_\alpha} = \left.\frac{d\varphi_{\alpha p'}(2)}{d\mathbf{r}}\right|_{\mathbf{r}=R_\alpha} = 0.$$

Then the states $|p\rangle$ and $|p'\rangle$ become stationary, their energies are real, and the Lehmann expansion in the form (2) turns out to be valid also for finite Fermi systems.

Let us now introduce the energy distribution of the residues of the one-particle Green function, $P(\mathbf{r},\mathbf{r}',\varepsilon)$, which coincides with the exact one-particle strength function of the Fermi system:

$$P(\mathbf{r},\mathbf{r}',\varepsilon) = \frac{1}{\pi}|\mathrm{Im}G(\mathbf{r},\mathbf{r}',\varepsilon) = \sum_p \langle N_0|\Psi(\mathbf{r})|p\rangle \times\langle p|\Psi^+(\mathbf{r}')|N_0\rangle\times\delta(\varepsilon-\varepsilon_p) + \sum_{p'} \langle N_0|\Psi^+(\mathbf{r}')|p'\rangle\langle p'|\Psi(2)|N_0\rangle\cdot\delta(\varepsilon - \varepsilon_{p'}). \quad (3)$$

Now we can define the kth moment of the distribution $P(\mathbf{r},\mathbf{r}',\varepsilon)$ as

$$I^{(k)}(\mathbf{r},\mathbf{r}') \equiv \int \varepsilon^k\cdot P(\mathbf{r},\mathbf{r}',\varepsilon)d\varepsilon = \sum_p \langle N_0|\Psi(\mathbf{r})|p\rangle \cdot\langle p|\Psi^+(\mathbf{r}')|N_0\rangle\times\varepsilon_p^k + \sum_{p'} \langle N_0|\Psi^+(\mathbf{r}')|p'\rangle\langle p'|\Psi(\mathbf{r})|N_0\rangle\cdot\varepsilon_{p'}^k.$$

Using the commutation relations for the fermion creation and annihilation operators,

$$\Psi(\mathbf{r})\Psi^+(\mathbf{r}') + \Psi^+(\mathbf{r}')\Psi(\mathbf{r}) = \delta(\mathbf{r}-\mathbf{r}')$$

and the properties of completeness and orthonormality of the states $|p\rangle$ and $|p'\rangle$ for the zeroth moment $I^{(0)}(\mathbf{r},\mathbf{r}')$, we obtain

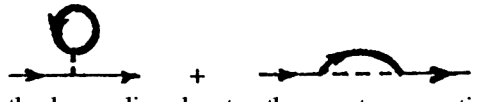
$$I^{(0)}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \quad (4)$$

This expression (4), is the well known sum rule for the residues at the poles of the one-particle Green function.[3]

Using the method of variational derivatives of the S matrix, the mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$ can be written as[11]

$$\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)=\Sigma_0(\mathbf{r},\mathbf{r}')+\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon). \tag{5}$$

The quantity $\Sigma_0(\mathbf{r},\mathbf{r}')$ is the nonretarded part (independent of ε) of the mass operator. In systems with instantaneous two-particle fermion interaction potentials $V(\mathbf{r},\mathbf{r}')$ it is given by the skeleton graphs of first order in $V(\mathbf{r},\mathbf{r}')$:

 (6)

where the heavy line denotes the exact one-particle Green function $G(\mathbf{r},\mathbf{r}',\varepsilon)$ and the dashed line denotes the potential $V(\mathbf{r},\mathbf{r}')$, and it coincides with the nonlocal Hartree–Fock potential:

$$\hat{\Sigma}_0(\mathbf{r},\mathbf{r}')\cdot\varphi(\mathbf{r})\equiv\int V(\mathbf{r},\mathbf{r}')\cdot\rho(\mathbf{r})d\mathbf{r}'\cdot\varphi(\mathbf{r})$$
$$-\int V(\mathbf{r},\mathbf{r}')\rho(\mathbf{r},\mathbf{r}')\varphi(\mathbf{r}')d\mathbf{r}', \tag{7}$$

where $\rho(\mathbf{r},\mathbf{r}')$ is the one-particle density matrix and $\rho(\mathbf{r})\equiv\rho(\mathbf{r},\mathbf{r})$ is the fermion density.

The skeleton graphs of the mass operator in second and higher orders in $V(\mathbf{r},\mathbf{r}')$ contain energy denominators which depend on ε and therefore enter into $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$, the retarded part of the mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$.

The quantity $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$ has the Lehmann expansion[11]

$$\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)=\sum_t\frac{\langle 0|b(\mathbf{r})|t\rangle\langle t|b^+(\mathbf{r}')|0\rangle}{\varepsilon-\varepsilon_t+i\delta}$$
$$+\sum_{t'}\frac{\langle 0|b^+(\mathbf{r}')|t'\rangle\langle t'|b(\mathbf{r})|0\rangle}{\varepsilon-\varepsilon'_t-i\delta}, \tag{5'}$$

where

$$b(\mathbf{r})=\frac{\delta f}{\delta\Psi^+(\mathbf{r})};\quad \varepsilon_t=E_t-E_0;\quad \varepsilon_{t'}=E_0-E_{t'},$$

E_0 and $|0\rangle$ being the energy and wave function of the ground state of the system of N noninteracting fermions; $E_t,E_{t'}$ and $|t\rangle,|t'\rangle$ the energies and wave functions of the systems of $(N+1)$ and $(N-1)$ noninteracting fermions. The sums over t (t') do not include states containing only one particle (one hole) owing to the irreducibility of the mass operator.

Substituting the Lehmann expansion (2) into (1) and letting ε go to ε_p, we obtain the equation for the amplitude of the residue at the pole ε_p of the exact one-particle Green function:

$$(\varepsilon_p-\hat{H}_0(\mathbf{r}))\langle N_0|\Psi(\mathbf{r})|p\rangle-\int\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon_p)$$
$$\cdot\langle N_0|\Psi(\mathbf{r}')|p\rangle d\mathbf{r}'=0, \tag{8}$$

where $\hat{H}_0(\mathbf{r})$ is an integral operator written symbolically as $\hat{H}_0=T+\hat{\Sigma}_0$. In the limit $\varepsilon\to\varepsilon_{p'}$, we obtain an equation analogous to (8) for the amplitude $\langle N_0|\Psi^+(\mathbf{r})|p'\rangle$. Using Eq. (8), the first moment can be written as

$$I^{(1)}(\mathbf{r},\mathbf{r}')=\hat{H}_0(\mathbf{r})\cdot\delta(\mathbf{r}-\mathbf{r}')$$
$$+\int\tilde{\Sigma}(\mathbf{r},\mathbf{r}'',\varepsilon)p(\mathbf{r}'',\mathbf{r}',\varepsilon)d\varepsilon, \tag{9}$$

where the second term on the right-hand side of (9) is the weighted average value $\tilde{\Sigma}_{ave}(\mathbf{r},\mathbf{r}')$ of the retarded part of the mass operator $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$ over the distribution $P(\mathbf{r},\mathbf{r}',\varepsilon)$.

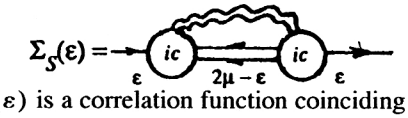
Substituting the Lehmann expansion (5) into (9), using (3) for $P(\mathbf{r},\mathbf{r}',\varepsilon)$, and regrouping the terms in the sums over p,p' and t,t', we obtain

$$\tilde{\Sigma}_{ave}(\mathbf{r},\mathbf{r}')=-\sum_t\int\langle 0|b(\mathbf{r})|t\rangle$$
$$\times\Big\langle t|b^+(\mathbf{r}'')|0\rangle G(\mathbf{r}'',\mathbf{r}',\varepsilon_t)d\mathbf{r}''$$
$$-\sum_{t'}\int\langle 0|b^+(\mathbf{r}'')|t'\rangle$$
$$\times\langle t'|b(\mathbf{r})|0\rangle G(\mathbf{r}'',\mathbf{r}',\varepsilon_{t'})d\mathbf{r}''.$$

Because, as follows from (1), $G(\mathbf{r},\mathbf{r}',\varepsilon)$ vanishes at the poles of the mass operator $\varepsilon=\varepsilon_t$ ($\varepsilon_{t'}$), we have $\tilde{\Sigma}_{ave}(\mathbf{r},\mathbf{r}')=0$. Then the first moment of the distribution $I^{(1)}\times(\mathbf{r},\mathbf{r}')$ is entirely determined by the Hamiltonian $\hat{H}_0$, in which the potential is the nonretarded Hartree–Fock potential $\hat{\Sigma}_0(\mathbf{r},\mathbf{r}')$ (7):

$$I^{(1)}(\mathbf{r},\mathbf{r}')=\hat{H}_0(\mathbf{r})\cdot\delta(\mathbf{r}-\mathbf{r}'). \tag{10}$$

Equations (4) and (10) are valid for finite and infinite Fermi systems of both the "normal" and the superfluid type. In the latter case, the retarded part of the mass operator $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$ contains, in addition to normal terms, the pole superfluid term $\Sigma_s(\mathbf{r},\mathbf{r}',\varepsilon)$, written graphically as[3]

 (11)

where $C(\varepsilon)$ is a correlation function coinciding with the amplitude of the transition of a particle into a hole and a Cooper pair (shown by the double wavy line), and the symbol corresponds to the one-particle Green function $\bar{G}(\mathbf{r},\mathbf{r}',\varepsilon)$, which does not contain graphs of the type (11).

The sum rule (10) can also be obtained using the commutator technique.[24] For this, using the definition (3'), we write the expression for the first moment $I^{(1)}(\mathbf{r},\mathbf{r}')$ as

$$I^{(1)}(\mathbf{r},\mathbf{r}')=\langle N_0|[[\Psi(\mathbf{r}),H]_-,\Psi^+(\mathbf{r}')]_+|N_0\rangle.$$

Replacing $\hat{H}$ in this expression by the Hamiltonian of the system with instantaneous two-particle interaction $V(\mathbf{r},\mathbf{r}')$ and expanding the commutator (anticommutator) brackets, we obtain

$$I^{(1)}(\mathbf{r},\mathbf{r}')=T(\mathbf{r})\cdot\delta(\mathbf{r}-\mathbf{r}')+\delta(\mathbf{r}-\mathbf{r}')\cdot\int V(\mathbf{r},\mathbf{r}_1)$$
$$\times\langle N_0|\Psi^+_{(\mathbf{r}_1}\Psi(\mathbf{r}_1)|N_0\rangle-V(\mathbf{r},\mathbf{r}')$$
$$<N_0|\Psi^+(\mathbf{r}')\Psi(\mathbf{r})|N_0.$$

The second and third terms on the right-hand side of this equation are the Hartree and Fock (7) potentials, respectively, so that $I^{(1)}(\mathbf{r},\mathbf{r}')$ coincides with the sum rule (10).

Now let us introduce the set of eigenfunctions and eigenvalues of the operator $\hat{H}_0(\mathbf{r})$:

$$\hat{H}_0(\mathbf{r})\cdot\varphi_\lambda(\mathbf{r})=\varepsilon_\lambda\cdot\varphi_\lambda(\mathbf{r}). \tag{12}$$

We transform the Green function $G(\mathbf{r},\mathbf{r}',\varepsilon)$ and the mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$ to the λ representation. Then the distribution function $P(\mathbf{r},\mathbf{r}',\varepsilon)$ becomes the distribution function $P_{\lambda\lambda'}(\varepsilon)$:

$$P_{\lambda\lambda'}(\varepsilon)=\sum_p \langle N_0|a_\lambda|p\rangle\langle p|a^+_{\lambda'}|N_0\rangle\cdot\delta(\varepsilon-\varepsilon_p) + \sum_{p'} \langle N_0|a^+_{\lambda'}|p'\rangle\langle p'|a_\lambda|N_0\rangle\cdot\delta(\varepsilon-\varepsilon_{p'}), \tag{13}$$

where a_λ is the particle annihilation operator in the state λ. Then the sum rules (4) and (11) become

$$\sum_p \langle N_0|a_\lambda|p\rangle\langle p|a^+_{\lambda'}|N_0\rangle+\sum_{p'} \langle N_0|a^+_{\lambda'}|p'\rangle\langle p'|a_\lambda|N_0\rangle = \delta_{\lambda\lambda'}; \tag{14}$$

$$\sum_p \langle N_0|a_\lambda|p\rangle\langle p|a^+_{\lambda'}|N_0\rangle\cdot\varepsilon_p+\sum_{p'} \langle N_0|a^+_{\lambda'}|p'\rangle \times\langle p'|a_\lambda|N_0\rangle\cdot\varepsilon_{p'}=\varepsilon_\lambda\cdot\delta_{\lambda\lambda'}. \tag{15}$$

Equation (15), which physically means that the centroid of the one-particle strength $P_{\lambda\lambda}(\varepsilon)$ is equal to the Hartree–Fock energy ε_λ, was first obtained in Ref. 24, where only the instantaneous two-particle interaction was taken into account. In Ref. 25 the sum rule (15) was generalized to the case where a virtual phonon appears in the retarded part of the mass operator in second-order perturbation theory. In Ref. 26 the sum rule was obtained in the quasiparticle–phonon model (8) taking into account the nucleon–phonon interaction in all orders of perturbation theory. Here the amplitudes for direct transitions more complicated than transitions of a quasiparticle into a quasiparticle plus a phonon are assumed to be to zero.

Equations (14) and (15) demonstrate the fundamentally different roles of the nonretarded $\hat{\Sigma}_0(\mathbf{r},\mathbf{r}')$ and retarded $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$ parts of the mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$. The quantity $\Sigma_0(\mathbf{r},\mathbf{r}')$ actually forms, via the Hamiltonian $\hat{H}_0(\mathbf{r})$ (12), "quasiparticles" (set in quotes) described by the set of quantum numbers λ, normalized wave functions $\varphi_\lambda(\mathbf{r})$, and energies ε_λ. The inclusion of $\tilde{\Sigma}(\mathbf{r},\mathbf{r}',\varepsilon)$ leads to the fragmentation of these quasiparticles over exact multi-quasiparticle states $|p\rangle$ and $|p'\rangle$ of the Fermi system with the same quantum numbers λ. Here the total probability of finding a "quasiparticle" λ in all the exact states p and p' is unity, and the energy centroid of the fragmentation of the "quasiparticles" λ over exact states of the system coincides with the value ε_λ.

A different definition of quasiparticle is used in the Fermi-liquid theory.[1–3] Since the quasiparticle energy E_λ coincides with the exact excitation energy of the Fermi system, among all the exact states $|p\rangle$, $|p'\rangle$ of the system of $(N+1)$ $(N-1)$ particles with quantum numbers λ, the state of lowest energy $|p_0\rangle$ ($|p'_0\rangle$) is distinguished. Its energy is $\varepsilon_{p_0}(\varepsilon_{p'_0})$ and is taken to be equal to the quasiparticle energy E_λ. The quasiparticle wave function $\varphi_\lambda(\mathbf{r})$ is then found by solving Eq. (8) for $\varepsilon_p=\varepsilon_{p_0}=E_\lambda$:

$$(E_\lambda-H_0(\mathbf{r}))\Phi_\lambda(z)-\int\tilde{\Sigma}(\mathbf{r},\mathbf{r}'',E_\lambda)\cdot\Phi_\lambda(\mathbf{r}'')d\mathbf{r}''=0. \tag{16}$$

However, here an essential contradiction arises. Since the residue $\langle N_0|a_\lambda|p_0\rangle$ at the pole $\varepsilon=\varepsilon_{p_0}$ of the exact one-particle Green function $G_{\lambda\lambda}(\varepsilon)$, referred to as the renormalization constant a_λ, is smaller than unity, the wave function $|p_0\rangle$ contains not only the quasiparticle state φ_λ with probability $a_\lambda<1$, but also an admixture of other more complicated multi-quasiparticle components with probability $(1-a_\lambda)$. Then the state $|p_0\rangle$ cannot be viewed as a one-quasiparticle state, and so the energy of this state ε_{p_0} cannot be assumed to coincide with the quasiparticle energy, because the admixtures of multi-quasiparticle components must change the energy ε_{p_0} compared to the energy of the purely one-quasiparticle state.

This contradiction can be resolved only in the single case where $|1-a_\lambda|\ll 1$, where the state $|p_0\rangle$ becomes a purely one-quasiparticle state. This case has been analyzed in Ref. 12.

For all real Fermi liquids, in which a_λ differs significantly from unity, it is natural to reject the quasiparticle representation of Fermi-liquid theory[1–3] and instead use the "quasiparticles" defined by Eq. (12). Then the nonretarded part of the mass operator $\hat{\Sigma}_0(\mathbf{r},\mathbf{r}')$ coinciding with the Hartree–Fock potential should be viewed as the true self-consistent field of the Fermi liquid. In the case of nuclei and nuclear matter, this imposes strong constraints on the choice of parameters of the realistic nucleon–nucleon forces $V(\mathbf{r},\mathbf{r}')$. In particular, potentials $V(\mathbf{r},\mathbf{r}')$ with a hard repulsive core must be discarded as unphysical, because for such $V(\mathbf{r},\mathbf{r}')$ there is no procedure for eliminating the divergences in the calculation of the Hartree–Fock potential $\hat{\Sigma}_0(\mathbf{r},\mathbf{r}')$, in contrast to the Brueckner procedure[7] in the calculation of the exact nucleon mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$.

2. THE "QUASIPARTICLE" WAVE FUNCTIONS AND ENERGIES AND THE STANDARD SHELL MODEL OF THE NUCLEUS

Let us study the properties of the Schrödinger equation (12) and the "quasiparticle" wave functions $\varphi_\lambda(\mathbf{r})$ in more detail using the results of Ref. 21. Applying the shift operator to the fermion spatial coordinate, we transform the nonretarded part of the mass operator $\Sigma_0(\mathbf{r},\mathbf{r}')$ to the mixed representation[3] $\hat{\Sigma}_0(\mathbf{r},\hat{\mathbf{p}})$, where $\hat{\mathbf{p}}$ is the fermion momentum operator. Then, expanding the operator $\hat{\Sigma}_0(\mathbf{r},\hat{\mathbf{p}})$ in a series in powers of $\hat{\mathbf{p}}$ through terms of second order, we obtain

$$\Sigma_0(\mathbf{r},\hat{\mathbf{p}})=\Sigma_0(\mathbf{r},0)+\frac{1}{6}\hat{p}_\alpha\left.\frac{\partial^2\Sigma_0(\mathbf{r},\hat{\mathbf{p}})}{\partial\hat{\mathbf{p}}^2}\right|_{\hat{\mathbf{p}}=0}\hat{p}_\alpha. \tag{16'}$$

Substituting the expansion (16′) into (12), we find

$$\left(\hat{p}_\alpha \frac{1}{2m^*(\mathbf{r})}\hat{p}_\alpha+\Sigma_0(\mathbf{r},0)-\varepsilon_\lambda\right)\varphi_\lambda(\mathbf{r})=0, \tag{17}$$

where the fermion effective mass $m^*(\mathbf{r})$ is given by

$$\frac{1}{m^*(\mathbf{r})}=\frac{1}{m}+\frac{1}{3}\frac{\partial^2\Sigma_0(r,\hat{p})}{\partial\hat{\mathbf{p}}^2}\bigg|_{\hat{\mathbf{p}}=0}. \tag{18}$$

We shall seek the quasiparticle function $\varphi_\lambda(\mathbf{r})$ normalized to unity in the form[21]

$$\varphi_\lambda(\mathbf{r})=f(\mathbf{r})\bar{\varphi}_\lambda(\mathbf{r}), \tag{19}$$

where we find the renormalization factor $f(\mathbf{r})$ from the condition that the gradient terms of the form $\hat{\mathbf{p}}\cdot\bar{\varphi}_\lambda(\mathbf{r})$ vanish in the equation for $\bar{\varphi}_\lambda(\mathbf{r})$:

$$\left(\hat{\mathbf{p}}\frac{1}{2m^*(\mathbf{r})}\right)f(\mathbf{r})=-\frac{1}{m^*(\mathbf{r})}(\hat{\mathbf{p}}f(\mathbf{r})). \tag{20}$$

The solution of this equation has the simple form

$$f(\mathbf{r})=\left(\frac{m^*(\mathbf{r})}{m}\right)^{1/2}. \tag{21}$$

Then for the function $\bar{\varphi}_\lambda(\mathbf{r})$ we obtain the Schrödinger equation:

$$\left(\frac{\hat{\mathbf{p}}^2}{2m}+V(\mathbf{r},\varepsilon_\lambda)-\varepsilon_\lambda\right)\bar{\varphi}_\lambda(\mathbf{r})=0, \tag{22}$$

where the energy-dependent potential $V(\mathbf{r},E)$ can be treated as a one-particle self-consistent fermion potential:

$$V(\mathbf{r},E)=\frac{m^*(\mathbf{r})}{m}\Sigma_0(\mathbf{r},0)+V^{\mathrm{grad}}(\mathbf{r})+\left(1-\frac{m^*(\mathbf{r})}{m}\right)E. \tag{23}$$

The gradient potential $V^{\mathrm{grad}}(\mathbf{r})$, which appears because of the presence of the gradient term in (17), has the form

$$V^{\mathrm{grad}}_{(\mathbf{r})}=\frac{1}{2m}\left\{\frac{(\hat{\mathbf{p}}^2m^*(\mathbf{r}))}{2m^*(\mathbf{r})}-(3/4)\frac{(\hat{\mathbf{p}}m^*(\mathbf{r}))}{(m^*(\mathbf{r}))^2}\right\}. \tag{24}$$

We note that the function $\bar{\varphi}_\lambda(\mathbf{r})$ has normalization of the form

$$\int \varphi^*_{\lambda'}(\mathbf{r})\cdot\varphi_\lambda(\mathbf{r})d\mathbf{r}=\int \bar{\varphi}^*_{\lambda'}(\mathbf{r})\frac{m^*(\mathbf{r})}{m}\bar{\varphi}_\lambda(\mathbf{r})d\mathbf{r}=\delta_{\lambda\lambda'}. \tag{25}$$

It is interesting to compare the self-consistent potential $V(\mathbf{r},E)$ Eq. (23) to the shell potential obtained in the Fermi-liquid theory.[1–3] For this it is necessary to study Eq. (16) for the function $\varphi_\lambda(\mathbf{r})$. Transforming the mass operator $\Sigma(\mathbf{r},\mathbf{r}',\varepsilon)$ to the mixed representation $\Sigma(\mathbf{r},\hat{\mathbf{p}},\varepsilon)$, expanding $\Sigma(\mathbf{r},\hat{\mathbf{p}},\varepsilon)$ in a series in powers of $\hat{\mathbf{p}}$ and $(\varepsilon-\mu)$, and restricting ourselves to terms of lowest order, we obtain

$$\Sigma(\mathbf{r},\hat{\mathbf{p}},\varepsilon)=\Sigma(\mathbf{r},0,\mu)+\frac{\partial\Sigma(\mathbf{r},0,\varepsilon)}{\partial\varepsilon}\bigg|_{\varepsilon=\mu}(\varepsilon-\mu)+\frac{1}{6}\hat{p}\alpha\times\frac{\partial^2\Sigma(\mathbf{r},\hat{\mathbf{p}},\mu)}{\partial\hat{\mathbf{p}}^2}\bigg|_{\hat{\mathbf{p}}=0}\cdot\hat{p}_\alpha. \tag{26}$$

Substituting Eq. (26) into (16), we write this equation in Hermitian form:[28]

$$\left\{a^{1/2}(\mathbf{r})\left[\hat{p}_\alpha\frac{1}{2\bar{m}^*(\mathbf{r})}\hat{p}_\alpha+\Sigma(\mathbf{r},0,\mu)-\frac{\partial\Sigma(\mathbf{r},0,\varepsilon)}{\partial\varepsilon}\bigg|_{\varepsilon=\mu}\cdot\mu\right]\times a^{1/2}(\mathbf{r})-E_\lambda\right\}\Phi^{\mathrm{sh}}_\lambda(\mathbf{r})=0, \tag{27}$$

where $\Phi^{\mathrm{sh}}_\lambda(\mathbf{r})\equiv[a(\mathbf{r})]^{-1/2}\Phi_\lambda(\mathbf{r})$ is the shell wave function $\Phi^{\mathrm{sh}}_\lambda(\mathbf{r})$, normalized to unity, and the renormalization constant $a(\mathbf{r})$ has the form

$$a(\mathbf{r})=\left\{1-\frac{\partial\Sigma(\mathbf{r},0,\varepsilon)}{\partial\varepsilon}\bigg|_{\varepsilon=\mu}\right\}^{-1}. \tag{28}$$

Now we can write the shell function $\Phi^{\mathrm{sh}}_\lambda(r)$ as

$$\Phi^{\mathrm{sh}}_\lambda(\mathbf{r})=\frac{\bar{f}(\mathbf{r})}{[a(\mathbf{r})]^{1/2}}\cdot\bar{\Phi}^{\mathrm{sh}}_\lambda(\mathbf{r}), \tag{29}$$

where the renormalization factor $\bar{f}(\mathbf{r})$ is given by (21) with the effective mass $m^*(\mathbf{r})$ replaced by $\bar{m}^*(\mathbf{r})$, which is given by (18) with $\Sigma_0(\mathbf{r},\hat{\mathbf{p}})$ replaced by $\Sigma(\mathbf{r},\mathbf{p},\mu)$. Then the function $\bar{\varphi}^{\mathrm{sh}}_\lambda(\mathbf{r})$ satisfies the Schrödinger equation

$$\left\{\frac{\hat{\mathbf{p}}^2}{2m}+\bar{V}^{\mathrm{sh}}(\mathbf{r},E_\lambda)-E_\lambda\right\}\bar{\Phi}^{\mathrm{sh}}_\lambda(\mathbf{r})=0, \tag{30}$$

where the shell potential $\bar{V}^{\mathrm{sh}}(\mathbf{r},E)$ is given by

$$\bar{V}^{\mathrm{sh}}_{(\mathbf{r},E)}=\frac{\bar{m}^*(\mathbf{r})}{m}\cdot\Sigma(\mathbf{r},0,\mu)+\bar{V}^{\mathrm{grad}}_{(\mathbf{r})}+\frac{m^*_0(\mathbf{r})}{m}(1-a(\mathbf{r}))\mu+\left(1-\frac{m^*_0(\mathbf{r})}{m}\right)\cdot E, \tag{31}$$

with

$$m^*_0(\mathbf{r})=\frac{\bar{m}^*(\mathbf{r})}{a(\mathbf{r})}, \tag{32}$$

and the gradient potential $\bar{V}^{\mathrm{grad}}(\mathbf{r})$ coincides with the potential of Eq. (24) with $m^*(\mathbf{r})$ replaced by $\bar{m}^*(\mathbf{r})$.

In the case of nuclei, it has been shown[3,5] that $m^*_0(\mathbf{r})\approx m$; that is, $\bar{m}^*(\mathbf{r})\approx a(\mathbf{r})m$. Then the potential $V^{\mathrm{sh}}(\mathbf{r},E)$ becomes independent of the energy E_{sh} and coincides with the standard shell potential $V^{\mathrm{sh}}(\mathbf{r})$ widely used in nuclear structure calculations:[3]

$$V^{\mathrm{sh}}(\mathbf{r})=\frac{\bar{m}^*(\mathbf{r})}{m}\Sigma(\mathbf{r},0,\mu)+\bar{V}^{\mathrm{grad}}_{(\mathbf{r})}+\left(1-\frac{\bar{m}^*(\mathbf{r})}{m}\right)\mu. \tag{33}$$

Here $\bar{f}(\mathbf{r})=[a(\mathbf{r})]^{1/2}$, and $\Phi^{\mathrm{sh}}_\lambda(\mathbf{r})=\bar{\Phi}^{\mathrm{sh}}_\lambda(\mathbf{r})$.

When comparing the self-consistent potential $V(\mathbf{r},E)$, Eq. (23) and the shell potential $V^{\mathrm{sh}}(\mathbf{r})$ Eq. (33), we take into account the fact that the retarded part of the mass operator $\tilde{\Sigma}(\mathbf{r},\hat{\mathbf{p}},\varepsilon)$ is small for $\varepsilon=\mu$. In Ref. 3 in the derivation of the dispersion relation for the mass operator it was in fact assumed that $\tilde{\Sigma}(\mathbf{r},\hat{\mathbf{p}},\mu)=0$. In infinite Fermi systems the quantity $\tilde{\Sigma}(\mathbf{r},\hat{\mathbf{p}},\mu)$ is nonzero only owing to the breaking of the symmetry between "particle" and "hole" states due to the finiteness of the Fermi momentum. In the case of finite Fermi systems the quantity $\tilde{\Sigma}(\mathbf{r},\hat{\mathbf{p}},\mu)$ can turn out to be nonzero

also owing to the asymmetric distribution of one-quasiparticle energies for particle and hole states.

Then, neglecting the contribution of $\tilde{\Sigma}(\mathbf{r},\hat{\mathbf{p}},\mu)$ to the potential $V^{\rm sh}(\mathbf{r})$ (33), we obtain the following relation between the potentials $V(\mathbf{r},E)$ (23) and $V^{\rm sh}(\mathbf{r})$ (33):

$$V(\mathbf{r},E)=V^{\rm sh}(\mathbf{r})+\left(1-\frac{m^*(\mathbf{r})}{m}\right)(E-\mu). \tag{34}$$

If we take into account the volume nature of the distribution of $m^*(\mathbf{r})$ in finite Fermi systems and neglect the off-diagonal matrix elements $(m^*(\mathbf{r}))_{\lambda\lambda'}$ $(\lambda\neq\lambda')$ between functions $\Phi^{\rm sh}_\lambda(\mathbf{r})$ compared to the diagonal elements $(m^*(\mathbf{r}))_{\lambda\lambda}$, the functions $\bar{\Phi}_\lambda(\mathbf{r})$ and $\Phi^{\rm sh}_\lambda(\mathbf{r})$ will be proportional to each other, and the eigenenergies ε_λ and E_λ will turn out to be related simply as

$$(\varepsilon_\lambda-\mu)\frac{m^*_{\lambda\lambda}}{m}=E_\lambda-\mu. \tag{35}$$

To estimate the matrix element $m^*_{\lambda\lambda}$ we can use the expression

$$m^*_{\lambda\lambda}=\int \Phi^{\rm sh*}_\lambda(\mathbf{r})\cdot m^*(\mathbf{r})\cdot\Phi^{\rm sh}_\lambda(\mathbf{r})d\mathbf{r}\gtrsim m^*(0).$$

It then follows from (35) that for a state λ_F lying on the surface of the Fermi system, where $E_{\lambda_F}=\mu$, the energies E_{λ_F} and ε_{λ_F} coincide: $E_{\lambda_F}=\varepsilon_{\lambda_F}=\mu$. The spacing between the energy levels ε_λ with different values of λ grow $m/m^*(0)$ times faster than the corresponding spacings between the analogous shell levels E_λ. As will be shown below, for nuclei the quantity $m/m^*(0)\approx 1.4$, so that the energies of relatively high-lying resonance states of the particle or hole type in nuclei will differ catastrophically from the predictions of the shell model and the Fermi liquid theory.

When the off-diagonal matrix elements $m^{ast}_{\lambda\lambda'}$ are neglected, the (normalized to unity) shell wave function $\Phi^{\rm sh}_\lambda(\mathbf{r})$ and the function $\bar{\varphi}_\lambda(\mathbf{r})$ are proportional to each other, so that the function $\Phi_\lambda(\mathbf{r})$ (19) normalized to unity is given by

$$\Phi_\lambda(\mathbf{r})=\frac{m^*(\mathbf{r})}{m^*_{\lambda\lambda}}\cdot\Phi^{\rm sh}_\lambda(\mathbf{r}),$$

and its modulus significantly exceeds that of the function $\Phi^{\rm sh}_\lambda(\mathbf{r})$ in the surface region of the nucleus. This can lead to a significant renormalization of the matrix elements of operators corresponding to surface physical quantities on the basis of the functions $\Phi_\lambda(\mathbf{r})$ compared to the analogous matrix elements constructed on the basis of the standard shell functions $\Phi^{\rm sh}_\lambda(\mathbf{r})$.

3. THE HIGHER ENERGY MOMENTS OF THE ONE-PARTICLE STRENGTH FUNCTION

Using Eq. (8), the kth-order moment of the distribution $P_{\lambda\lambda'}(\varepsilon)$ (13) can be written as[14]

$$I^{(k)}_{\lambda\lambda'}=\sum_{n=0}^{k}\varepsilon^{k-n}_\lambda\cdot C^n_k\int\tilde{\Sigma}_{\lambda\lambda_1}(\varepsilon)\cdots\tilde{\Sigma}_{\lambda_{n-1}\lambda_n}(\varepsilon)$$

$$\cdot P_{\lambda_n\lambda'}(\varepsilon)d\varepsilon, \tag{36}$$

where C^n_k is the binomial coefficient ($C^0_k=1$). From Eq. (36) we find the recursion relation

$$I^{(k)}_{\lambda\lambda'}=\varepsilon_\lambda I^{(k-1)}_{\lambda\lambda'}+\int\tilde{\Sigma}_{\lambda\lambda_1}(\varepsilon)P_{\lambda_1\lambda'}(\varepsilon)\cdot\varepsilon^{k-1}d\varepsilon.$$

Substituting the Lehmann expansion (5′) for the retarded part of the mass operator into this and representing ε^{k-1} as a binomial $[(\varepsilon-\varepsilon_t)+\varepsilon_t]^{k-1}$, we obtain the recursion relation

$$I^{(k)}_{\lambda\lambda'}=\varepsilon_\lambda I^{(k-1)}_{\lambda\lambda'}+\sum_{n=0}^{k-2}\left\{\sum_t\langle 0|b_\lambda|t\rangle\langle t|b^+_{\lambda_2}|0\rangle\cdot\varepsilon^n_t\right.$$

$$\left.+\sum_{t'}\langle 0|b^+_{\lambda_2}|t'\rangle\langle t'|b_\lambda|0\rangle\cdot\varepsilon^n_{t'}\right\}\cdot I^{(k-2-n)}_{\lambda_1\lambda'}. \tag{36′}$$

Then the second moment becomes

$$I^{(2)}_{\lambda\lambda'}=\varepsilon^2_\lambda\delta_{\lambda\lambda'}+\sum_t\langle 0|b_\lambda|t\rangle\langle t|b^+_{\lambda'}|0\rangle+\sum_{t'}\langle 0|b^+_{\lambda'}|t'\rangle$$

$$\times\langle t'|b_\lambda|0\rangle. \tag{36″}$$

Equation (36″) is the generalization of the analogous expression obtained in Ref. 28 using an effective potential instead of the operator b_λ. We note that the convergence of the sum over $t(t')$ in (36″) is determined by the convergence of the matrix elements $\langle 0|b_\lambda|t\rangle$ $(\langle 0|b^+_{\lambda'}|t'\rangle)$ on the interval $|\varepsilon_t-\varepsilon_\lambda|\lesssim E_F$ for short-range nuclear forces.[29] Therefore, the second moment of the distribution $P_{\lambda\lambda'}(\varepsilon)$ (13) in this case is determined by states $t(t')$ in a large energy range significantly exceeding the characteristic width of the distribution $P_{\lambda\lambda'}(\varepsilon)$ (Ref. 28).

4. THE FERMI LIQUID THEORY TAKING INTO ACCOUNT FRAGMENTATION AND RETARDATION

The question arises of how to extend the Fermi liquid theory[1-3] away from the basic assumption of small excitation energies of the Fermi system.

A similar problem of generalizing the Fermi liquid theory taking into account fragmentation and retardation has been solved in Ref. 14. Let us consider the main points of this new approach. First, instead of the shell basis of functions $\Phi^{\rm sh}_\lambda(\mathbf{r})$ we use the basis of functions $\Phi_\lambda(\mathbf{r})$. Second, to find the Green function $G_{\lambda\lambda'}(\varepsilon)$ we use the Dyson equation (1), written symbolically as

$$G=G_0+G_0\cdot\tilde{\Sigma}\cdot G, \tag{37}$$

where $G^0_{\lambda\lambda'}(\varepsilon)=G^0_\lambda(\varepsilon)\delta_{\lambda\lambda'}$ is the one-particle Green function corresponding to the Schrödinger equation (12). Third, the retarded part of the mass operator $\tilde{\Sigma}_{\lambda\lambda'}(\varepsilon)$ is expressed in terms of a specific skeleton graph determined by the properties of the Fermi system under study. The choice of this graph is one of the central problems of the new approach. In Ref. 15 the operator $\tilde{\Sigma}_{\lambda\lambda'}(\varepsilon)$ for magic nuclei is chosen in a form which includes transitions of the quasiparticle–quasiparticle plus phonon type corresponding to a skeleton graph of the form

which is equivalent to the expression

$$\tilde{\Sigma}=g\cdot G\cdot D\cdot g, \tag{38}$$

where the phonon Green function $D_\alpha(\omega)$ (the wavy line on the graph) for phonons with quantum numbers α is defined as

$$D_\alpha(\omega)=\frac{2\omega_\alpha}{\omega^2-\omega_\alpha^2+i\delta}, \tag{39}$$

and the phonon vertex part $g^\alpha_{\lambda\lambda'}$ is given by a graph of the form

corresponding to the equation

$$g=uGGg, \tag{40}$$

where U is a four-pole block, irreducible in the particle-hole channel and determined from the self-consistency condition.[30]

The system of equations (37)–(40) differs noticeably from the analogous system of linearized equations used in the theory of finite Fermi systems[3] and obtained from the system (37)–(40) by replacing the exact Green function in Eqs. (38) and (40) by its pole part. At the same time, this new system of equations is close in structure to the analogous system of equations of the quasiparticle-phonon model,[8] differing only by Eq. (40), for which in Ref. 8 the random-phase approximation (RPA) is used and the exact Green functions $G_{\lambda\lambda'}(\varepsilon)$ are replaced by the unperturbed functions $G^0_{\lambda\lambda'}(\varepsilon)$.

The strong nonlinearity of the system of equations (37)–(40) reflects the physical nonlinearity inherent in one-particle motion in all real multiparticle Fermi systems. Some of the solutions of this system could in principle turn out to be nontrivial ones corresponding to poorly studied, strongly nonlinear processes such as, for example, soliton-type solutions. It is important that physicists and mathematicians see the importance of studying the possible existence of such processes.

The system of equations (37)–(40) can be solved using the Lehmann expansion (2) for the Green function $G_{\lambda\lambda'}(\varepsilon)$. This expansion includes all the poles, not just the one singled out in the Fermi liquid theory.[2,3] For finding the energies and residues at the poles of the exact Green function, the authors of Ref. 15 proposed a rapidly converging iteration scheme for solving this system of equations. This scheme works as follows.

The Green function $G^0_{\lambda\lambda'}$ is used as the zeroth approximation on the right-hand sides of (38) and (40), and the retarded part of the mass operator $\tilde{\Sigma}_{\lambda\lambda'}$ is found. This is then substituted into the Dyson equation (37) to give the Green function $G_{\lambda\lambda'}(\varepsilon)$. This Green function is substituted into (38) and (40) and the operator $\tilde{\Sigma}_{\lambda\lambda'}$ is found again. The procedure is then repeated.

Two approximations were used to hasten the convergence and decrease the computer time needed for this iteration scheme. First, for each iteration we discarded all the weak poles of the one-particle Green function $G_{\lambda\lambda'}(\varepsilon)$ and the background Green function $D_\alpha(\omega)$, which give small contributions to the sum rule for the residues of the one-particle Green function (14) and to the value of the squared modulus of the dynamical nuclear deformation parameter β^2_α. Second, all the poles of the one-particle Green function and the phonon Green function lying inside a small energy range Δ were replaced by a single pole with the average energy and summed residue.

The formalism described above was used to calculate the one-particle spectroscopic characteristics of nuclei of the "magic +1 nucleon" type, ^{209}Pb, ^{207}Pb, ^{207}Tl, and ^{209}Bi, and the phonon spectrum of the doubly magic nucleus ^{208}Pb. The complete one-particle basis λ included all levels ε_λ up to excitation energy 25 MeV. For states λ corresponding to the continuum we used the harmonic-oscillator wave functions, with the parameters chosen from the condition that the location of one-particle resonance states be described. Calculations showed that further enlargement of the basis of states λ does not lead to any change in the calculated characteristics. The calculational scheme included phonons of multipole order $2\leqslant L\leqslant 8$. The rapid convergence of the iteration procedure is indicated by the fact that only three iterations proved sufficient in actual calculations.

From Table I, where we give the experimental and theoretical values of the one-nucleon spectroscopic factors S_λ for low-lying states of nuclei of the type "magic +1 nucleon" near ^{208}Pb, we see that the values of $S^{\rm exp}_\lambda$ coinciding with the renormalization constant of the Fermi liquid theory[3] a_λ (28) vary from 0.97 to 0.3 in the range of excitation energies up to 3.5 MeV, which indicates that the Fermi liquid theory is inapplicable even to nuclei near the Fermi surface, as discussed above. In turn, the theoretical values of the spectroscopic factors $S^{\rm th}_\lambda=a_{\lambda p_0}/2j+1$, where $a_{\lambda p_0}$ is the residue at the pole of the exact one-particle Green function closest to the Fermi surface for given quantum numbers nlj, decrease in going from the first to the third iteration using phonons constructed on the basis of the random-phase approximation. The inclusion of phonon fragmentation leads to further decrease of the values of $S^{\rm th}_\lambda$. In spite of the spread in the experimental values $S^{\rm exp}_\lambda$, we see from this table that the decrease of the one-nucleon spectroscopic factors with increasing excitation energy is reproduced well enough by the calculations including phonon fragmentation.

To illustrate the nature of the phonon fragmentation in the ^{208}Pb nucleus, in Fig. 1 we show the dynamical deformation parameters $\beta_{L\pi}$ calculated for phonons with $L^\pi=2^+$ and $L^\pi=3^-$ as a function of their frequencies ω_α in the RPA and including their fragmentation.[15] We see that the inclusion of phonon fragmentation significantly changes the energy spectrum.

As an illustration of the results of the calculations of the one-particle strength functions $P_{\lambda\lambda'}(\varepsilon)$ for one-particle resonance states performed in Ref. 15, in Fig. 2 we show the histogram of the distribution $P_{\lambda\lambda}(\varepsilon)$ for the $1i_{11/2}$ state in ^{209}Bi. We see that the inclusion of phonon fragmentation

TABLE I. One-nucleon spectroscopic factors S_λ of low-lying states of nuclei of the type "magic ±1 nucleon" in the vicinity of ^{208}Pb.

State		$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$3i_{13/2}$	$2f_{7/2}$	$1h_{9/2}$
Energy, MeV		0.0	0.57	0.89	1.63	2.34	3.41
^{207}Pb, Theory	A	0.83	0.82	0.80	0.79	0.70	0.76
	B	0.84	0.83	0.81	0.78	0.61	0.63
	C	0.76	0.74	0.70	0.67	0.47	0.43
^{207}Pb, Experiment	Ref. 31	···	(0.97)	···	(0.68)	(0.84)	(0.47)
	Ref. 32	···	0.97	···	0.71	0.89	0.65

State		$2g_{9/2}$	$1i_{11/2}$	$1j_{15/2}$	$3d_{5/2}$	$4s_{1/2}$	$2g_{7/2}$	$3d_{3/2}$
Energy, MeV		0.0	0.80	1.42	1.58	2.03	2.51	2.56
^{209}Pb, Theory	A	0.80	0.86	0.70	0.83	0.90	0.76	0.79
	B	0.80	0.86	0.54	0.80	0.88	0.67	0.71
	C	0.70	0.77	0.46	0.70	0.83	0.54	0.61
^{209}Pb, Experiment	Ref. 33	0.76	0.86	0.49	0.84	0.79	0.86	0.79
	Ref. 34	0.66	0.75	0.71	0.62	0.70	0.81	0.88

State		$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$
Energy, MeV		0.0	0.35	1.34	1.67	3.48
^{207}Tl, Theory	A	0.92	0.90	0.84	0.80	0.76
	B	0.92	0.91	0.84	0.80	0.61
	C	0.86	0.84	0.74	0.69	0.47
^{207}Tl, Experiment	Ref. 35	···	···	(0.74)	(0.65)	(0.41)

State		$1h_{9/2}$	$2f_{7/2}$	$1i_{13/2}$	$2f_{5/2}$	$3p_{3/2}$	$3p_{1/2}$
Energy, MeV		0.0	0.68	1.26	2.42	2.87	3.30
^{209}Bi, Theory	A	0.90	0.89	0.83	0.81	0.87	0.81
	B	0.90	0.87	0.76	0.70	0.80	0.59
	C	0.82	0.79	0.67	0.58	0.71	0.51
^{209}Bi, Experiment	Ref. 36	0.80	0.86	0.90	0.81	0.44	0.30
	Ref. 37	0.54–1.00	0.65–1.12	0.52–0.94	0.66–1.14	0.58–1.03	0.49–0.90

Note: A denotes the calculation in the first iteration, B is the calculation with RPA phonons, and C is the calculation taking into account phonon fragmentation. The values of S_λ^{exp} renormalized to constant coefficient are given in parentheses.

significantly changes the distribution $P_{\lambda\lambda}(\varepsilon)$, making it closer to the experimental one.

5. SUPERFLUID AND PAIR CORRELATIONS IN NUCLEI AND THE NUCLEON–PHONON INTERACTION

The phenomenon of nuclear superfluidity is well understood theoretically[38,39,3] and confirmed experimentally.[28] In many respects it is analogous to superconductivity in metals.[40] In both cases, Cooper pairs of identical fermions are formed in the Fermi system, and a gap appears in the energy spectrum of the one-quasiparticle excitations of the system. In the case of superconductors, the effective interaction between the electrons forming the Cooper pair is almost completely determined by the electron interaction with the phonon excitations of the crystal lattice.[40] However, the nature of the forces leading to the formation of Cooper pairs in nuclei remained unclear for a long time. The problem was studied in detail in Refs. 41–43, 13 and a very curious solution was obtained.

The very existence of superfluid and pair correlations in nuclei implies that the effective interaction $v(\mathbf{r}_1,\mathbf{r}_2)$ between identical nucleons in a singlet state in the nucleus is attractive. At the same time, the analogous interaction potential $v_{in}(\mathbf{r}_1,\mathbf{r}_2)$ in nuclear matter at normal density ρ_0 has a value close to zero,[7] which leads to the absence of superfluidity in nuclear matter. It can be expected that the potential $v(\mathbf{r}_1,\mathbf{r}_2)$ is close to the potential $v_{in}(\mathbf{r}_1,\mathbf{r}_2)$ in the interior of all sufficiently heavy nuclei, where the nucleon density $\rho(\mathbf{r})$ coincides with the nuclear matter density ρ_0. Therefore, the pairing in nuclei is wholly associated with their finite extent. There are two possible sources for the appearance of the effective attraction of the potential $v(\mathbf{r}_1,\mathbf{r}_2)$. The interaction of a pair of nucleons located in the outer region of the nucleus, where the nucleon density is low and the gas approximation is applicable, is accurately determined by the attractive vacuum potential $v_{ex}^0(\mathbf{r}_1,\mathbf{r}_2)$. Since the transition from the potential $v=v_{ex}^0$ in the external region to the value $v=v_{in}$ at the center of the nucleus is continuous, we can introduce an interpolation potential $v^0(\mathbf{r}_1,\mathbf{r}_2)$ of the form[3]

$$v^0(\mathbf{r}_1,\mathbf{r}_2)=\{v_{in}\cdot y^n(\mathbf{R})+v_{ex}(1-y^n(\mathbf{R}))\}f(\mathbf{r}), \qquad (41)$$

where

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2; \quad \mathbf{R}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2}; \quad y(\mathbf{R})=\frac{\rho(\mathbf{R})}{\rho_0}, \qquad (42)$$

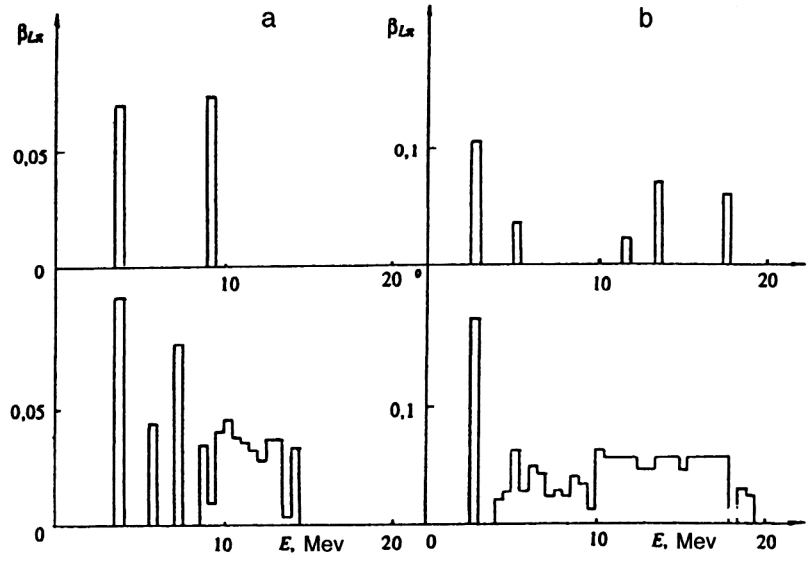


FIG. 1. Dynamical deformation parameters $\beta_{L\pi}$ for phonons with $L^{\pi}=2^{+}$ (a) and $L^{\pi}=3^{-}$ (b) for the ^{208}Pb nucleus. The upper part of the two figures corresponds to the calculations for RPA phonons, and the lower part to calculations including phonon fragmentation.

and n is a parameter set equal to 1 or 2/3. Here it is assumed that, owing to the short-range nature of the nuclear force, the radial form factor of the potential $f(r)$ is weakly renormalized in going from the outside to the inside of the nucleus. Calculations of the pairing effects in nuclei of the type "magic +2 nucleons," where the energy gap in the one-particle excitation spectrum is zero, i.e., where there is two-nucleon pairing but no superfluidity, have been performed in Ref. 44 using Eq. (41) with the form factor $f(\mathbf{r})$ chosen to be a δ function. It turned out that to obtain the experimental values of the pairing energies in such nuclei for $v_{ex}=v_{ex}^{0}$, v_{in} must be repulsive and take values $v_{in}\lesssim\frac{1}{6}|v_{ex}^{0}|$, which is in reasonable agreement with the condition that v_{in} be small ($v_{in}\approx 0$) in nuclear matter. Unfortunately, the use of a δ-function potential makes it necessary to artificially cut off the equations for the Cooper-pair amplitude in momentum space owing to the logarithmic divergence at large momenta. Therefore, the authors of Ref. 41 changed over from zero-range potentials to finite-range ones ($r_0\approx 1.2$ fm), which are equivalent from the viewpoint of the scattering problem, but more physical. In addition, they automatically eliminate the problem of the logarithmic divergence. Owing to the significant decrease of the absolute values of the pair matrix elements, the description of the experimental pairing energies in this case for $v_{ex}=v_{ex}^{0}$ required strong attraction inside the nucleus, $v_{in}\approx v_{ex}^{0}$ (Ref. 41). If we calculate the value of v_{ex} necessary for pairing to be absent in nuclear matter ($v_{in}\approx 0$), it turns out to be significantly larger than the value v_{ex}^{0}: $v_{ex}\approx 3v_{ex}^{0}$ (Ref. 42). This result contradicts the condition obtained above: $v_{in}\approx 0$, $v_{ex}\approx v_{ex}^{0}$. This implies that for some reason, explanation of the experimental values of the paring energies requires that the effective interaction $v(\mathbf{r}_1,\mathbf{r}_2)$ have an additional source of attraction compared to the potential $v^{0}(\mathbf{r}_1,\mathbf{r}_2)$ Eq. (41) with $v_{in}=0$ and $v_{ex}=v_{ex}^{0}$. It is reasonable to assume that this source might be the interaction of two nucleons via the exchange of phonons, the quanta of collective surface oscillations of finite nuclei. This idea was realized in Refs. 43 and 13 for nuclei of the type "magic +2 nucleons" and "magic +3 nucleons," and for spherical superfluid nuclei. Here it was necessary to solve the problem of including retardation for the amplitude of Cooper pair production. We note that a similar problem was solved earlier for superconductors by the method of cancellation of dangerous graphs only in second-order perturbation theory in the retarded electron-phonon interaction.[40]

6. THE ONE-PARTICLE GREEN FUNCTION IN A SUPERFLUID FERMI SYSTEM

Let us continue our study of finite Fermi systems using the methods of Ref. 13. We write the Dyson equation (1) for the one-particle Green function in graphical form taking into

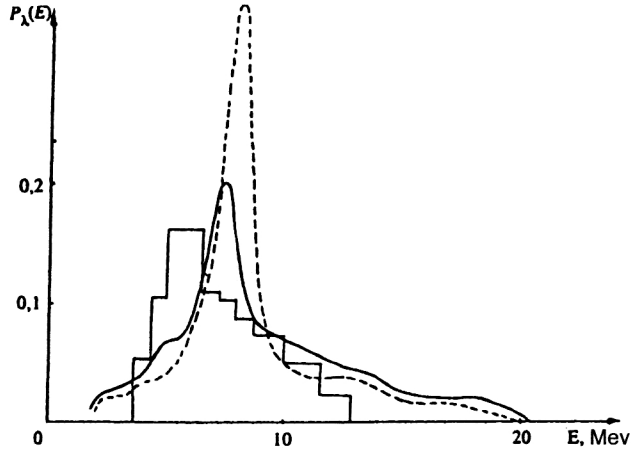
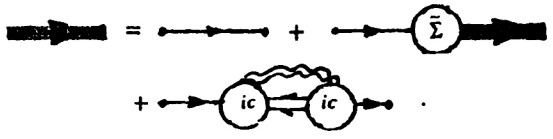


FIG. 2. One-particle strength function of the $1i_{11/2}$ state in ^{209}Bi. The histogram is the experimental data, (Ref. 36) the dashed line is the calculation using RPA phonons, and the solid line is the calculation taking into account fragmentation and phonons. The experimental data are taken from Ref. 35.

account the superfluidity term $\Sigma_s(\varepsilon)$ (11) in the mass operator:

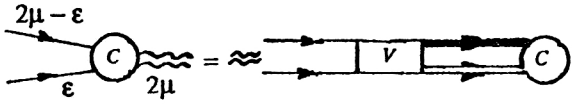


(43)

Using the fact that the multiple index λ takes the form $\lambda = nljm$ in spherical systems, owing to conservation of total angular momentum j, its projection m, and parity, the off-diagonal elements ($\lambda \neq \lambda'$) of the Green function $G_{\lambda\lambda'}(\varepsilon)$, the retarded part of the mass operator $\tilde{\Sigma}_{\lambda\lambda'}(\varepsilon)$, and the correlation function $C_\lambda(\varepsilon)$ are related to the change of the principle quantum number n by ± 2, ± 4, and so on. The effect of such terms is small (see, for example, Refs. 42, 44, and 14). Therefore, in Eq. (43) we can keep only the diagonal terms, writing it in the analytic form[13]

$$G_\lambda(\varepsilon) = G^0_\lambda(\varepsilon) + G^0_\lambda(\varepsilon)\tilde{\Sigma}_\lambda(\varepsilon)G_\lambda(\varepsilon) - G^0_\lambda(\varepsilon)C_\lambda(\varepsilon)\bar{G}_\lambda(2\mu-\varepsilon)\cdot C_\lambda(\varepsilon)G_\lambda(\varepsilon). \quad (44)$$

In turn, the correlation function $C_\lambda(\varepsilon)$ satisfies the equation[3]



(45)

where V is the four-pole block irreducible in the particle–particle channel, which plays the role of the effective potential of the two-nucleon interaction in the pairing problem. Since the four-pole block V is symmetrized, the right- and left-hand sides of Eq. (45) are invariant when ε and $2\mu-\varepsilon$ are interchanged. Then, instead of the function $C_\lambda(\varepsilon)$, we introduce the function $\tilde{C}_\lambda(\varepsilon')$ even in the variable $\varepsilon' \equiv \varepsilon - \mu$. The retarded part of the mass operator $\tilde{\Sigma}_\lambda(\varepsilon)$ is written as

$$\tilde{\Sigma}_\lambda(\varepsilon) = {}_1\tilde{\Sigma}_\lambda(\varepsilon') + {}_2\tilde{\Sigma}_\lambda(\varepsilon'),$$

where ${}_1\tilde{\Sigma}_\lambda(\varepsilon')$ is an even and ${}_2\tilde{\Sigma}_\lambda(\varepsilon')$ is an odd function of the variable ε'. Then for Eq. (44) we obtain

$$G_\lambda(\varepsilon) = \frac{[\varepsilon' - {}_2\tilde{\Sigma}_\lambda(\varepsilon')] + [(\xi_\lambda - \mu) + {}_1\tilde{\Sigma}_\lambda(\varepsilon')]}{\Omega_\lambda(\varepsilon')}, \quad (46)$$

where

$$\Omega_\lambda(\varepsilon') \equiv \varepsilon'^2\left[1 - \frac{2\tilde{\Sigma}_\lambda(\varepsilon')}{\varepsilon'}\right]^2 - [\varepsilon_\lambda - \mu - {}_1\tilde{\Sigma}_\lambda(\varepsilon')]^2 - [\tilde{C}_\lambda(\varepsilon')]^2. \quad (47)$$

Since the function $\Omega_\lambda(\varepsilon')$ (47) is even in the variable ε', each pole $\mu + E_{\lambda\eta}$ of the Green function $G_\lambda(\varepsilon)$ corresponds to a pole $\mu - E_{\lambda\eta}$, where $E_{\lambda\eta}$ is positive and given by

$$E_{\lambda\eta} = [(\varepsilon_{\lambda\eta} - \mu)^2 + \Delta^2_{\lambda\eta}]^{1/2}, \quad (48)$$

where

$$\varepsilon_{\lambda\eta} = [\varepsilon_\lambda - \mu + {}_1\tilde{\Sigma}_\lambda(E_{\lambda\eta})]/(1+q_{\lambda\eta}); \quad (49)$$

$$\Delta_{\lambda\eta} = \tilde{C}_\lambda(E_{\lambda\eta})/(1+q_{\lambda\eta}); \quad (50)$$

$$q_{\lambda\eta} = -{}_2\tilde{\Sigma}_\lambda(E_{\lambda\eta})/E_{\lambda\eta}. \quad (51)$$

Using (46), let us construct a Lehmann expansion of the type (2) for $G_\lambda(\varepsilon)$:

$$G_\lambda(\varepsilon) = \sum_\eta \left[\frac{a_{\lambda\eta}u^2_{\lambda\eta}}{\varepsilon - \mu - E_{\lambda\eta} + i\delta} + \frac{a_{\lambda\eta}v^2_{\lambda\eta}}{\varepsilon - \mu + E_{\lambda\eta} - i\delta}\right], \quad (52)$$

where

$$a_{\lambda\eta}u^2_{\lambda\eta} = \frac{(1+q_{\lambda\eta})[E_{\lambda\eta} + \varepsilon_{\lambda\eta} - \mu]}{\dot{\Omega}_\lambda(E_{\lambda\eta})}; \quad (53)$$

$$a_{\lambda\eta}v^2_{\lambda\eta} = \frac{(1+q_{\lambda\eta})[E_{gl\eta} - \varepsilon_{\lambda\eta} + \mu]}{\dot{\Omega}_\lambda(E_{\lambda\eta})}, \quad (53')$$

where the dot denotes the derivative with respect to ε', and the coefficients $u_{\lambda\eta}$ and $v_{\lambda\eta}$ are defined such that $u^2_{\lambda\eta} + v^2_{\lambda\eta} = 1$. Adding and multiplying (53) and (53′), we find

$$a_{\lambda\eta} = \frac{2E_{\lambda\eta}(1+q_{\lambda\eta})}{\dot{\Omega}_\lambda(E_{\lambda\eta})}; \quad (54)$$

$$u^2_{\lambda\eta} = 1/2\left[1 + \frac{\varepsilon_{\lambda\eta} - \mu}{E_{\lambda\eta}}\right]; \quad v^2_{\lambda\eta} = 1/2\left[1 - \frac{\varepsilon_{\lambda\eta} - \mu}{E_{\lambda\eta}}\right]. \quad (55)$$

If the superfluidity correlations are switched off, i.e., $\tilde{C}_\lambda = 0$, we have the following: $\mu \to \mu^0$, $\varepsilon_{\lambda\eta} \to \varepsilon^0_{\lambda\eta}$, the coefficients $u_{\lambda\eta}$ and $v_{\lambda\eta}$ take the "normal" values $u^0_{\lambda\eta} = 0$ and $v^0_{\lambda\eta} = 1$ for $\varepsilon^0_{\lambda\eta} \leqslant \mu^0$ and $u^0_{\lambda\eta} = 1$ and $v^0_{\lambda\eta} = 0$ for $\varepsilon^0_{\lambda\eta} \leqslant \mu^0$ and the constant $a_{\lambda\eta}$ becomes the residue $a^0_{\lambda\eta}$ for the one-particle "normal" Green function $G^n_\lambda(\varepsilon)$. We note that Eq. (48) for $E_{\lambda\eta}$ and Eq. (53) for the coefficients $u^2_{\lambda\eta}$ and $v^2_{\lambda\eta}$, found for poles of the one-particle Green function of arbitrary complexity,[13] coincides with the standard expressions for E_λ and u^2_λ, v^2_λ (Ref. 3), obtained when only one-quasiparticle poles are included in the Green function. Comparing the Lehmann expansion (49) with the general form of this expansion in (2), we find that

$$a_{\lambda\eta}u^2_{\lambda\eta} = |\langle N_0|a_\lambda|p\rangle|^2; \quad E_p - E_{N_0} = E_{\lambda\eta} + \mu;$$

$$a_{\lambda\eta}v^2_{\lambda\eta} = |\langle N_0|a^+_\lambda|p'\rangle|^2; \quad E_{N_0} - E_{p'} = \mu - E_{\lambda\eta}.$$

Combining $a_{\lambda\eta}u^2_{\lambda\eta}$ and $a_{\lambda\eta}v^2_{\lambda\eta}$ and summing over indices p, p', we obtain the sum rule

$$\sum_\eta a_{\lambda\eta} = 1,$$

which coincides with the sum rule obtained above (Eq. 14).

7. EQUATION FOR THE CORRELATION FUNCTION TAKING INTO ACCOUNT FRAGMENTATION AND RETARDATION EFFECTS

Equation (45) for the correlation function $C_\lambda(\varepsilon)$ is written analytically as

$$C_\lambda(\varepsilon)=\int \frac{d\varepsilon_1}{2\pi i} V_{\lambda\lambda,\lambda_1\lambda_1}(\varepsilon-\varepsilon_1)\cdot \bar{G}_{\lambda_1}(2\mu-\varepsilon_1) \times \bar{G}_{\lambda_1}(\varepsilon_1)C_\lambda(\varepsilon_1). \quad (56)$$

The four-pole block V irreducible in the particle–particle channel is described by an expression of the form[3]

$$V=W+U(GG)_{\rm ph}\Gamma,$$

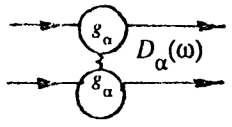
where W is a four-pole block irreducible simultaneously in the particle-particle and particle-hole channels, U is a four-pole block irreducible in the particle-hole channel and introduced earlier in Eq. (40), and $(GG)_{\rm ph}$ is the product of two one-particle Green functions in the particle-hole channel. The scattering amplitude Γ of two particles in a medium in turn satisfies the equation

$$\Gamma=U+U(GG)_{\rm ph}\Gamma.$$

The solution of this equation can be written as[3]

$$\Gamma=\Gamma'+\tilde{\Gamma},$$

where the pole part $\tilde{\Gamma}$ of the total amplitude Γ has been isolated explicitly. It depends on the energy $\omega=\varepsilon-\varepsilon_1$ transferred in the particle–particle channel. This part $\tilde{\Gamma}$ can be written as[3]

$\tilde{\Gamma}=\sum_\alpha$  , (57)

where g_α is the phonon production amplitude, satisfying (40), and the wavy line corresponds to the phonon Green function $D_\alpha(\omega)$ (39).

As noted in Ref. 30, nuclei contain a branch of collective excitations associated with isoscalar, spin-independent, strongly collectivized phonons ($\alpha=\alpha_0$), which correspond to surface oscillations of the nucleus and have an origin analogous to classical oscillations of the capillary type. The surface component of the wave function of such phonons significantly exceeds the volume component associated with excitations of the zero-sound type.

The exchange of such strongly collectivized surface phonons can cause strong attraction in the particle-particle channel.[43] Let us therefore explicitly isolate from $\tilde{\Gamma}$ the terms containing $\alpha=\alpha_0$, which correspond to these phonons. The rest of the amplitude $\tilde{\Gamma}$ associated with the large number of zero-sound phonons and therefore having weaker ω dependence will be included along with the nonpole terms in Γ'. Then we can write the four-pole block V as

$$V=V_0+\sum_{\alpha_0} g_{\alpha_0}D_{\alpha_0}(\omega)g_{\alpha_0},$$

where

$$V_0=W+\Gamma'-U-\sum_{\alpha\neq\alpha_0} g_\alpha D_\alpha(\omega)g_\alpha,$$

with the value V_0 corresponding to the effective retarded interaction in the particle–particle channel. It can be written in the form (41) (Ref. 3). Then Eq. (56) is rewritten as[13]

$$C_\lambda(\varepsilon)=\int \frac{d\varepsilon_1}{2\pi i} G^n_{\lambda_1}(2\mu-\varepsilon_1)G_{\lambda_1}(\varepsilon_1)C_{\lambda_1}(\varepsilon_1) \times\left[V_{0\lambda\lambda,\lambda_1\lambda_1}+\sum_{\alpha_0} g^2_{\alpha_0\lambda\lambda_1}\cdot D_{\alpha_0}(\varepsilon-\varepsilon_1)\right]. \quad (58)$$

The quadratic dependence of $D_\alpha(\varepsilon-\varepsilon_1)$ on $(\varepsilon-\varepsilon_1)$ in Eq. (58) allows construction of the Lehmann expansion for the correlation function $\tilde{C}_\lambda(\varepsilon')$ (Ref. 43):

$$\tilde{C}_\lambda(\varepsilon')=\tilde{C}^0_\lambda+\sum_{\rm p}\frac{\tilde{C}_{\lambda r}}{\varepsilon'^2\varepsilon^2_\lambda(r)+i\delta}. \quad (59)$$

Substituting (59) into (58) and using the Lehmann expansion (49) for $G_{\lambda_1}(\varepsilon_1)$ and the analogous representation with $\tilde{C}_\lambda=0$ for $\bar{G}_{\lambda_1}(2\mu-\varepsilon_1)$, we can obtain[43] a system of coupled nonlinear equations for the coefficients $\tilde{C}^0_\lambda$ and $\tilde{C}_{\lambda r}$ and energies $\varepsilon_\lambda(p)$. This system can be solved iteratively, with the zeroth iteration corresponding to the use of only the homogeneous term for $\tilde{C}_\lambda(\varepsilon')$, the first iteration obtained using this homogeneous term on the right-hand side of (58), and so on.

The system of nonlinear algebraic equations (38), (40), (49)–(51), (54), and (59), together with the condition of particle number N conservation[3]

$$N=\sum_\lambda\int\left(\frac{d\varepsilon}{2\pi i}G_\lambda(\varepsilon)e^{-i\varepsilon\tau}\right|_{\tau\to 0}=\sum_{\lambda\eta} a_{\lambda\eta}V^2_{\lambda\eta}, \quad (60)$$

form a closed system of equations describing the one-particle and collective spectra of a superfluid nucleus taking into account the retarded nucleon–phonon interaction and quasiparticle fragmentation. The sources of fragmentation are the retarded part of the mass operator $\tilde{\Sigma}(\varepsilon)$ and the presence of retardation in the part of the correlation function $\tilde{C}_\lambda(\varepsilon')$ associated with the retarded nucleon–phonon interaction. This system of equations can be transformed for describing the effects of two-nucleon pairing in nuclei of the type "magic ± 2 nucleons," "magic ± 3 nucleons," in which the energy gap is zero and superfluidity is absent. For this, in all the equations it is necessary to replace the one-particle superfluid Green function $G_\lambda(\varepsilon)$ (44) by the analogous "normal" Green function $G^n_\lambda(\varepsilon)$, with the correlation function $C_\lambda(\varepsilon)=0$. At the same time, the amplitude $\bar{C}_\lambda(\varepsilon)$ for the transition of two nucleons into a Cooper-pair state in such nonsuperfluid nuclei is defined by an equation similar to (58):[3]

$$\bar{C}_\lambda(\varepsilon)=\int \frac{d\varepsilon_1}{2\pi i} G^n_{\lambda_1}(2\mu-\varepsilon_1)G^n_{\lambda_1}(\varepsilon_2)\bar{C}_\lambda(\varepsilon_1) \times\left[V_{0\lambda\lambda,\lambda_1\lambda_1}+\sum_{\alpha_0} g^2_{\alpha_0\lambda\lambda_1}\cdot D^n_{\alpha_0}(\varepsilon-\varepsilon_1)\right], \quad (61)$$

where the bar over all the functions means that the one-particle Green functions $G^n_\lambda(\varepsilon)$ are used to generate them instead of the functions $G_\lambda(\varepsilon)$. This equation is the eigenvalue equation for the energy of the 2μ Cooper pair.

8. THE NUCLEON–PHONON INTERACTION AND PAIR CORRELATIONS IN ATOMIC NUCLEI

To qualitatively estimate the role of the nucleon–phonon interaction in nucleon pairing in nuclei, the systems of equations obtained above were solved using the first iteration. In this case fragmentation effects are completely excluded from all the equations (38), (40), (49)–(51), (54), (59), and (61), i.e., it is assumed that $a_{\lambda\eta}=1$, $q_{\lambda\eta}=0$, $\Delta_{\lambda\eta}=\Delta_\lambda$, and $\varepsilon_{\lambda\eta}=\varepsilon_\lambda$. The equations for $C_\lambda(\varepsilon)$ (58) and $\bar{C}_\lambda(\varepsilon)$ (61) were solved using the first iteration and reduced to the form[13,43]

$$\Delta_\lambda=-\sum_{\lambda_1}\frac{\Delta_{\lambda_1}}{2E_{\lambda_1}}\left\{V_{0\lambda\lambda,\lambda_1\lambda_1}+\sum_{\alpha_0} g^2_{\alpha_0,\lambda\lambda_1}\cdot D_{\alpha_0,\lambda\lambda_1}\right\}; \tag{62}$$

$$2(\mu-\varepsilon_\lambda)\chi_\lambda=(1-2n^0_\lambda)\sum_{\lambda_1}\chi_{\lambda_1}\left\{V_{0\lambda\lambda,\lambda_1\lambda_1}+\sum_{\alpha_0} g^2_{\alpha_0,\lambda\lambda_1}\cdot\bar{D}_{\alpha_0,\lambda\lambda_1}\right\}\chi_{\lambda_1}, \tag{63}$$

where $\Delta_\lambda\equiv\tilde{C}_\lambda(E_\lambda)$ is the energy gap for the superfluid nucleus, χ_λ is the wave function of the Cooper pair of nucleons in the nonsuperfluid nucleus in the λ representation, and $n^0_\lambda=(v^0_\lambda)^2$.

In Eqs. (62)–(63) the phonon Green function $D_{\alpha_0}(\omega)$, Eq. (39) entering into (58) is transformed into the functions $D_{\alpha_0,\lambda\lambda_1}$ and $\bar{D}_{\alpha_0,\lambda\lambda_1}$, where

$$D_{\alpha_0,\lambda\lambda_1}=\frac{E_{\lambda_1}+\omega_{\alpha_0}}{E^2_\lambda-(E_{\lambda_1}+\omega_{\alpha_0})^2},$$

and $\bar{D}$ differs from D in that E_λ is replaced by $|\varepsilon_\lambda-\mu|$.

Equations (62)–(63) were solved by using for $v_0(\mathbf{r}_1,\mathbf{r}_2)$ the interpolation formula (42) with $v_{in}=0$ and $v_{ex}=v^0_{ex}=-950$ MeV·fm^{-1} (Ref. 3) and the form factor in Yukawa form:

$$f(\mathbf{r})=(4\pi r r_0)^{-1}\exp(-r/r_0),$$

where $r_0=1.2$ fm.

For the superfluid spherical nuclei ^{108}Cd, ^{110}Cd, ^{124}Tl, and ^{126}Tl the phonons in Eq. (62) were taken to be the low-lying 2^+, 4^+ phonons and the 2^+ phonon of the giant quadrupole resonance. It has been shown[13] that the main contribution to the nucleon–phonon interaction comes from the low-lying, strongly collectivized 2^+ phonon. The calculated values of Δ_λ were used to obtain the pairing energies of these nuclei. The results of the calculations indicate that the inclusion of nucleon–phonon interactions allows the qualitative description of the experimental pairing energies without the introduction of additional free parameters.

For spherical, nonsuperfluid nuclei of the type "magic ±2 nucleons" and "magic ±3 nucleons" lying near the doubly magic nucleus ^{208}Pb, Eq. (63) was solved[43] taking into account all strongly collectivized isoscalar phonons with $L=0$, $2\leq L\leq16$, including the phonons of the giant resonances. It is only for such a large phonon basis that the pairing energies can be described satisfactorily for all the nuclei studied without the introduction of additional free parameters.

Therefore, in solving the problem of nucleon pairing, only the inclusion of the nucleon–phonon interaction allows the correct derivation of the structure of the nucleon effective interaction in the particle–particle channel. The analogy between superconductivity in metals and nucleon Cooper pairing in nuclei, in which the decisive role is played by fermion–phonon interactions, is thus extended.

9. THE ONE-PARTICLE GREEN FUNCTION AND FERMION ELASTIC SCATTERING

Let us now consider the problem of fermion elastic scattering on a finite Fermi system containing N particles and located in the ground state. The S-matrix element $S_{\mathbf{p}'\mathbf{p}}$ describing elastic scattering can be written as[10]

$$S_{\mathbf{p}'\mathbf{p}}=\lim_{\substack{t\to\infty\\ t'\to-\infty}}\langle N_0|a_{\mathbf{p}'}(t)Sa^+_{\mathbf{p}}(t')|N_0\rangle\cdot e^{-iE_{p'}t/\hbar}\cdot e^{iE_pt'/\hbar}, \tag{64}$$

where $a_{\mathbf{p}}(t)$ is the fermion annihilation operator for momentum $\mathbf{p}$ in the interaction representation and $E_p=p^2/2m$. Then, using the expression for the one-particle Green function in the interaction representation,[45]

$$G_{\mathbf{p}'\mathbf{p}}(t-t')=-i\langle N_0|S^+T\{a_{\mathbf{p}'}(t)a^+_{\mathbf{p}}(t')S\}|N_0\rangle,$$

up to a constant phase factor $e^{i\beta}$ we obtain

$$S_{\mathbf{p}'\mathbf{p}}=\lim_{\substack{t\to\infty,\\ t\to-\infty}}G_{\mathbf{p}'\mathbf{p}}(t-t')\cdot e^{-iE_{p'}t/\hbar}\cdot e^{iE_pt'/\hbar}. \tag{65}$$

We shall use the Dyson equation[45] for the Green function in the form

$$G_{\mathbf{p}'\mathbf{p}}(t-t')=G^0_{\mathbf{p}'\mathbf{p}}(t-t')+\sum_{\mathbf{p}_1\mathbf{p}_2}G^0_{\mathbf{p}'\mathbf{p}_1}(t-t_1)\Omega_{\mathbf{p}_1\mathbf{p}_2}\times(t_1-t_2)G^0_{\mathbf{p}_2\mathbf{p}}(t_2-t')dt_1\cdot dt_2, \tag{66}$$

where $\Omega_{\mathbf{p}_1\mathbf{p}_2}(t_1-t_2)$ is the reducible self-energy operator. The free Green function $G^0_{\mathbf{p}'\mathbf{p}}(t-t')$ for $t>t'$ has the form

$$G^0_{\mathbf{p}'\mathbf{p}}(t-t')=-i\delta_{\mathbf{p}\mathbf{p}'}e^{iE_p(t-t')/\hbar}. \tag{67}$$

Substituting (66) and (67) into (65), we find

$$S_{\mathbf{p}'\mathbf{p}}=\delta_{\mathbf{p}'\mathbf{p}}-2\pi i\cdot\Omega_{\mathbf{p}'\mathbf{p}}(E_p)\cdot\delta(E_p-E_{p'}), \tag{68}$$

where $\Omega_{\mathbf{p}'\mathbf{p}}(\varepsilon)$ is the reducible self-energy operator in the ε representation. Comparing (68) with the expression for the S matrix in terms of the T matrix in the Lippmann–Schwinger theory,[46] we find that the T-matrix element for elastic scattering $T_{\mathbf{p}'\mathbf{p}}$ coincides with the reducible self-energy operator in the ε representation for $\varepsilon=E_p$

$$T_{\mathbf{p}'\mathbf{p}}=\Omega_{\mathbf{p}'\mathbf{p}}(E_p). \tag{69}$$

Expressing the reducible self-energy operator $\Omega_{p'p}(E_p)$ in terms of the mass operator $\Sigma_{p'p}(\varepsilon)$, introduced above in (1), and using the Lippmann–Schwinger equation[46] relating the T matrix to the potential $\hat{V}$ for the interaction of a fermion with a finite Fermi system, we obtain the expression for the potential $\hat{V}$ in coordinate space in terms of the nonlocal and retarded mass operator:

$$\hat{V}=\Sigma(r,r',E_p). \tag{70}$$

Since the poles of the one-particle Green function $G_{p'p}(\varepsilon)$ in the complex ε plane exactly coincide with the poles of the reducible self-energy operator $\Omega_{p'p}(\varepsilon)$, from (68) it follows that the poles of the S-matrix element for elastic scattering $S_{p'p}(\varepsilon)$ coincide with the poles of the one-particle Green function $G_{p'p}(\varepsilon)$.

To go to the optical model,[47,48] it is necessary to average the S-matrix element $S_{p'p}(\varepsilon)$ for elastic scattering over a given energy range Δ containing a sufficiently large number of poles of the matrix element $S_{p'p}(\varepsilon)$ and, therefore, of the Green function $G_{p'p}(\varepsilon)$, but which is sufficiently small compared to the excitation energy of the system containing $(N+1)$ fermions:

$$\bar{S}_{p'p}(\varepsilon)=\int \gamma_\Delta(\varepsilon-\varepsilon')\cdot S_{p'p}(\varepsilon')d\varepsilon',$$

where the averaging function $\gamma_\Delta(x)$ is even in x, a maximum at $x=0$, normalized to unity, and falls off sufficiently rapidly outside the averaging interval Δ. In what follows we shall choose a Breit–Wigner form for the function $\gamma_\Delta(x)$ (Ref. 54):

$$\gamma_\Delta^{BW}(x)=\frac{\Delta}{2\pi}[x^2+\Delta^2/4]^{-1}, \tag{71}$$

which possesses all the above properties and has analytic properties in the complex x plane which are very convenient for calculation.[28] Then the optical potential $\hat{V}^{\rm opt}$ describing the elastic scattering of fermions in the optical model is expressed in terms of the mass operator averaged over energy, which can be written using (5) and (5′) as

$$\hat{V}^{\rm opt}=\bar{\Sigma}(\mathbf{r},\mathbf{r}',E_p)=\Sigma_0(\mathbf{r},\mathbf{r}')+\mathrm{Re}\,\tilde{\bar{\Sigma}}(\mathbf{r},\mathbf{r}',E_p)+i\,\mathrm{Im}\,\tilde{\bar{\Sigma}}(\mathbf{r},\mathbf{r}',E_p), \tag{72}$$

where

$$\mathrm{Re}\,\tilde{\bar{\Sigma}}(\mathbf{r},\mathbf{r}',\varepsilon)=\sum_t \frac{\langle 0|b(\mathbf{r})|t\rangle\langle t|b^+(\mathbf{r}')|0\rangle}{(\varepsilon-\varepsilon_t)^2+\Delta^2/4}\cdot(\varepsilon-\varepsilon_t)+\sum_{t'}\frac{\langle 0|b^+(\mathbf{r}')|t'\rangle\langle t'|b(\mathbf{r})|0\rangle(\varepsilon-\varepsilon_{t'})}{(\varepsilon-\varepsilon_{t'})^2+\Delta^2/4}; \tag{73}$$

$$\mathrm{Im}\,\tilde{\bar{\Sigma}}(\mathbf{r},\mathbf{r}',\varepsilon)=\frac{\Delta}{2}\left\{\sum_t \frac{\langle 0|b(\mathbf{r})|t\rangle\langle t|b^+(\mathbf{r}')|0\rangle}{(\varepsilon-\varepsilon_t)^2+\Delta^2/4}+\sum_{t'}\frac{\langle 0|b^+(\mathbf{r}')|t'\rangle\langle t'|b(\mathbf{r})|0\rangle}{(\varepsilon-\varepsilon_{t'})^2+\Delta^2/4}\right\}. \tag{74}$$

For analyzing the properties of the optical potential Eq. (72) it is useful to analyze the properties of the averaged fermionic strength function.

10. THE AVERAGED ONE-PARTICLE STRENGTH FUNCTION FOR A "CLOSED" FERMI SYSTEM

If the quasiparticle excitation energy $|\varepsilon_\lambda-\mu|$ is sufficiently high, the main contribution to the one-particle strength function $P_{\lambda\lambda'}(\varepsilon)$ (13) comes from highly excited states of the Fermi system p and p' having sufficiently high density. It is then possible to introduce the averaged one-particle strength function $\bar{P}_{\lambda\lambda'}(\varepsilon)$ using the averaging function $\gamma_\Delta(x)$. Owing to the randomness of the phases of the products of amplitudes $\langle N_0|a_\lambda|p\rangle\langle p|a^+_{\lambda'}|N_0\rangle$, the averaged strength function $\bar{P}_{\lambda\lambda'}(\varepsilon)=0$ for $\lambda\neq\lambda'$. Let us therefore consider the diagonal function $\bar{P}_\lambda(\varepsilon)=\bar{P}_{\lambda\lambda}(\varepsilon)$. The kth-order energy moment $\bar{I}^{(k)}_\lambda$ for the function $\bar{P}_\lambda(\varepsilon)$ is given by (3) with $P(\mathbf{r},\mathbf{r}',\varepsilon)$ replaced by $\bar{P}_\lambda(\varepsilon)$. In order for all the moments $\bar{I}^{(k)}_\lambda$ to be finite, it is necessary that the averaging function $\gamma_\Delta(x)$ fall off for large x more quickly than $|x|^{-k-1}$ (for example, the function $\gamma_\Delta(x)$ can be a square function, a Gaussian, and so on). Meanwhile, owing to the parity and normalization of the function $\gamma_\Delta(x)$ the equations

$$\bar{I}^{(0)}_{\lambda\lambda'}=I^{(0)}_{\lambda\lambda'}=\delta_{\lambda\lambda'}; \tag{75}$$

$$\bar{I}^{(1)}_{\lambda\lambda'}=I^{(1)}_{\lambda\lambda'}=\varepsilon_\lambda\delta_{\lambda\lambda'} \tag{76}$$

are satisfied also when a Breit–Wigner form (71) is taken for $\gamma_\Delta(x)$, for which all the higher moments beginning with the second diverge. Let us consider the strength function $\bar{P}_\lambda(\varepsilon)$ averaged over the Breit–Wigner distribution (71). We represent this function as an integral in the complex ε' plane:

$$\bar{P}_\lambda(\varepsilon)=\frac{1}{2\pi^2 i}\oint_C\frac{\Delta}{(\varepsilon-\varepsilon')^2+\Delta^2/4}\cdot G_{\lambda\lambda}(\varepsilon')d\varepsilon', \tag{77}$$

where the closed contour C runs along both sides of the real ε' axis.[28] Substituting into (77) the expression for $G_{\lambda\lambda}(\varepsilon)$ in terms of the mass operator $\Sigma_{\lambda\lambda'}$ obtained from (1), neglecting $\tilde{\bar{\Sigma}}_{\lambda\lambda'}$ for $\lambda\neq\lambda'$ because of the randomness of the signs of the amplitudes $\langle 0|b_\lambda|t\rangle\langle t|b^+_{\lambda'}|0\rangle$, and using the technique of Ref. 54, we obtain

$$\bar{P}_\lambda(\varepsilon)=\frac{1}{2\pi}\frac{2\mathrm{Im}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)+\Delta}{[\varepsilon-\varepsilon_\lambda-\mathrm{Re}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)]^2+\left(\mathrm{Im}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)+\frac{\Delta}{2}\right)^2}, \tag{78}$$

where $\mathrm{Re}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}$ and $\mathrm{Im}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}$ are given by Eqs. (73) and (74).

In analyzing the properties of the distribution (78), it is necessary to take into account the fact that, owing to Eq. (75), it is normalized to unity and, owing to Eq. (76), its first moment is ε_λ. These conditions impose severe constraints on the properties of $\mathrm{Re}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)$ and $\mathrm{Im}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)$. It can be shown rigorously that if $\mathrm{Re}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)$ and $\mathrm{Im}\,\tilde{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)$ are expanded in Taylor series in powers of $(\varepsilon-\mu)$, as is widely

done in studying $\mathrm{Re}\,\bar{\Sigma}(\varepsilon)$ and $\mathrm{Im}\,\bar{\Sigma}(\varepsilon)$ in the Fermi liquid theory,[2,3] theorems (75) and (76) are satisfied in the single case

$$\mathrm{Re}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=0; \quad \mathrm{Im}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=\frac{\Gamma_\lambda}{2}, \tag{79}$$

where Γ_λ is the effective width of the distribution (78), which is independent of the energy ε and coincides with the "spreading" width or the quasiparticle fragmentation width λ in a "closed" finite Fermi system. Then for $\Gamma_\lambda \gg \Delta$ the distribution (78) takes the standard Breit–Wigner form:

$$\bar{P}_\lambda(\varepsilon)=\frac{1}{2\pi}\frac{\Gamma_\lambda}{(\varepsilon-\varepsilon_\lambda)^2+\Gamma_\lambda^2/4}. \tag{80}$$

We note that Eq. (80) for the one-particle averaged strength function was obtained earlier in Ref. 28 using a simpler method. The same result can be arrived at by other methods close in spirit to the one widely used in the R-matrix theory of nuclear reactions and the theory of neutron strength functions.[23,28] At sufficiently high energies ε_t, the summation over t in Eqs. (73) and (74) can be replaced by integration over $d\varepsilon_t$ if we introduce the energy density $\rho(\varepsilon_t)$ of states t and use the matrix element $\overline{|\langle 0|b_\lambda|t\rangle|^2}$ averaged over an interval Δ' which contains sufficiently many levels t and at the same time is sufficiently small compared to the interval Δ ($\Delta' \ll \Delta$).

A similar procedure can also be followed for the sum over states t'. Then Eqs. (73) and (74) can be rewritten as

$$\mathrm{Re}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=\int_\mu^\infty \frac{d\varepsilon_t\cdot\rho(\varepsilon_t)\overline{|\langle 0|b_\lambda|t\rangle|^2}(\varepsilon-\varepsilon_t)}{(\varepsilon-\varepsilon_t)^2+\Delta^2/4} + \int_{-\infty}^\mu d\varepsilon_{t'}\rho(\varepsilon_{t'})\frac{\overline{|\langle 0|b_\lambda^+|t'\rangle|^2}(\varepsilon-\varepsilon_{t'})}{(\varepsilon-\varepsilon_{t'})^2+\Delta^2/4}; \tag{81}$$

$$\mathrm{Im}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=\frac{\Delta}{2}\int_\mu^\infty d\varepsilon_t\cdot\rho(\varepsilon_t)\frac{\overline{|\langle 0|b_\lambda|t\rangle|^2}}{(\varepsilon-\varepsilon_t)^2+\Delta^2/4} + \frac{\Delta}{2}\int_{-\infty}^\mu d\varepsilon_{t'}\rho(\varepsilon_{t'})\frac{\overline{|\langle 0|b_2^+|t'\rangle|^2}}{(\varepsilon-\varepsilon_{t'})^2+\Delta^2/4}. \tag{82}$$

Now let us represent the product $\overline{|\langle 0|b_\lambda|t\rangle|^2}\rho(\varepsilon_t)$ as a Taylor series in powers $(\varepsilon_t-\mu)^n$, where $n\geq 0$. Then the first integral in (81) diverges for all the terms of this series with $n>0$. In this case the distribution (78) vanishes for all finite ε, which cannot correspond to a real physical situation. This implies that only the term with $n=0$ in the expansion of $\rho(\varepsilon_t)\overline{|\langle 0|b_\lambda|t\rangle|^2}$ in powers $(\varepsilon-\mu)^n$ is nonzero, and we have the expressions

$$\mathrm{Re}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=0; \quad \mathrm{Im}\,\bar{\bar{\Sigma}}_{\lambda\lambda}(\varepsilon)=\pi\cdot[\rho(\varepsilon_t)\overline{|\langle 0|b_\lambda|t\rangle|^2}]_0=\frac{\Gamma_\lambda}{2}, \tag{83}$$

which again lead to the distribution (80). Physically, the result (83) implies that the quantity $\rho(\varepsilon_t)\overline{|\langle 0|b_\lambda|t\rangle|^2}$ is independent of the energy ε_t, i.e., the average value of the squared matrix element $\overline{|\langle 0|b_\lambda|t\rangle|^2}$ is proportional to $\rho(\varepsilon_t)^{-1}=D(\varepsilon_t)$, where $D(\varepsilon_t)$ is the average distance between levels t. The last statement is very natural within the context of the Wigner random-matrix model,[48,49] and is widely used to analyze the structure of highly excited nuclear states.[28,50–52]

It is possible to go outside the distribution (80) while satisfying Eqs. (75) and (76) if the averaged retarded part of the mass operator $\bar{\bar{\Sigma}}_\lambda(\varepsilon)$ is expanded in a Laurent series. Let us demonstrate this for a very simple case. Among the states t in Eqs. (73) and (74) we distinguish the one state t_0 coupled most strongly to the state λ, and we take into account the other states using Eq. (83). Then the distribution $\bar{P}_\lambda(\varepsilon)$ can be written as

$$\bar{P}_\lambda(\varepsilon)=\frac{1}{\pi}\left|\mathrm{Im}\frac{1}{\varepsilon-\varepsilon_\lambda-\bar{\bar{\Sigma}}_\lambda(\varepsilon)}\right|^2, \tag{84}$$

where

$$\bar{\bar{\Sigma}}_\lambda(\varepsilon)=-\frac{i\Gamma_\lambda}{2}+\frac{|\langle 0|b_\lambda|t_0\rangle|^2}{\varepsilon-\tilde{\varepsilon}_{t_0}}. \tag{85}$$

The main feature is the introduction of the complex energy $\tilde{\varepsilon}_{t_0}=\varepsilon_{t_0}-i\Gamma_{t_0}/2$ into the denominator of the second term in (85), where Γ_{t_0} is the fragmentation width of the isolated state t_0. In order for the distribution (84) to satisfy the conditions (75) and (76), Γ_{t_0} must be equal to Γ_λ. Then (84) can be written as

$$\bar{P}_\lambda(\varepsilon)=\frac{\Gamma_\lambda}{2\pi}\left\{\frac{a_1}{(\varepsilon-\varepsilon_1)^2+\Gamma_\lambda^2/4}+\frac{a_2}{(\varepsilon-\varepsilon_2)^2+\Gamma_\lambda^2/4}\right\}, \tag{86}$$

where

$$\varepsilon_{1,2}=\frac{\varepsilon_\lambda+\varepsilon_{t_0}}{2}\pm\left[\frac{(\varepsilon_\lambda-\varepsilon_{t_0})^2}{4}+\left|\langle 0|b_\lambda|t_0\rangle\right|^2\right]^{1/2}; \tag{87}$$

$$a_1=\frac{\varepsilon_1-\varepsilon_{t_0}}{\varepsilon_1-\varepsilon_2}; \quad a_2=\frac{\varepsilon_2-\varepsilon_{t_0}}{\varepsilon_2-\varepsilon_1},$$

with $a_1+a_2=1$.

It is easy to see that the distribution (86) satisfies the conditions (75) and (76). It is physically clear that the state t_0 involved in (86) is a special case of the "doorway" states,[28] which are taken into account, for example, when the coupling of the nucleon elastic scattering channel to the inelastic channels associated with the excitation of low-lying collective nuclear states is taken into account.

We note that the inclusion of the doorway state t_0, while not changing Γ_λ, tends to broaden (84) compared to the distribution (80). Equation (86) can also be generalized to include a large number of doorway states t_0.

Below in our analysis of the properties of the standard optical model for fermions we shall restrict ouselves to the case without doorway states, because the effect of these states is described in the coupled-channel approximation.[28]

11. THE OPTICAL MODEL FOR FERMIONS AND THE AVERAGED RETARDED PART OF THE MASS OPERATOR FOR A FINITE "OPEN" FERMI SYSTEM

The question arises as to how $\tilde{\bar{\Sigma}}_\lambda(\varepsilon)$ changes in going to an "open" Fermi system. This question can be answered using R-matrix theory and the shell-model approach to the description of resonance nuclear reactions.[52] For energies $\varepsilon>0$, at which the channel of fermion emission into the continuum is open, it is necessary to match the interior wave functions $|p\rangle$ and $|p'\rangle$ of a finite Fermi system to the exterior wave functions in the region $r>R_\alpha$ for all open channels α. This matching produces finite widths Γ_p and $\Gamma_{p'}$ of the states $|p\rangle$ and $|p'\rangle$ and a shift of the real parts ε_p and $\varepsilon_{p'}$ of their energies. Then the averaged retarded part of the mass operator, $\tilde{\bar{\Sigma}}(r,r',\varepsilon)$, can be written as

$$\tilde{\bar{\Sigma}}(r,r',\varepsilon)=\tilde{\bar{\Sigma}}_f(r,r')+\Delta\tilde{\bar{\Sigma}}(r,r',\varepsilon). \tag{88}$$

The quantity $\tilde{\bar{\Sigma}}_f(r,r')$ is the fragmentation part of $\tilde{\bar{\Sigma}}(r,r',\varepsilon)$ independent of energy and satisfying relation (79). For energies $\varepsilon_\lambda>0$ the width $\Gamma_\lambda=2\,\mathrm{Im}\,\tilde{\bar{\Sigma}}_f$ determines the probability of a transition per unit time from the one-particle fermionic resonance state ε_λ to the many-quasiparticle states $|p\rangle$ and $|p'\rangle$, which play the role of exact compound states of the system. In other words, the quantity $\mathrm{Im}\,\tilde{\bar{\Sigma}}_f(r,r')$ determines the component of the imaginary part of the fermion optical potential $\hat{V}^{\mathrm{opt}}$ (72) responsible for all processes (elastic and inelastic) occurring via compound-nucleus formation. The quantity $\Delta\tilde{\bar{\Sigma}}(r,r',\varepsilon)$ determines the addition to the retarded part of the mass operator arising from inelastic, multistage, direct, and pre-equilibrium processes unrelated to compound-nucleus formation. The imaginary part of this quantity is nonzero only for fermion energies $\varepsilon\geqslant 0$. The quantity $\Delta\tilde{\bar{\Sigma}}(r,r',\varepsilon)$ can be found using the dispersion relation for the averaged retarded part of the mass operator, which follows from the representation (5′) and the properties of the averaging operator:

$$\mathrm{Re}\,\tilde{\bar{\Sigma}}(r,r',\varepsilon)=\frac{1}{\pi}P\int_\mu^\infty\frac{\mathrm{Im}\,\tilde{\bar{\Sigma}}(r,r',\varepsilon')}{\varepsilon'-\varepsilon}d\varepsilon'-\frac{1}{\pi}p\int_{-\infty}^{\mu}\frac{\mathrm{Im}\,\tilde{\bar{\Sigma}}(r,r',\varepsilon')}{\varepsilon'-\varepsilon}d\varepsilon'. \tag{89}$$

Equation (89) for $\Delta\tilde{\bar{\Sigma}}(r,r',\varepsilon)$ can be written in a different form using the fact that $\Delta\,\mathrm{Im}\,\tilde{\bar{\Sigma}}=0$ for $\varepsilon\leqslant 0$:

$$\mathrm{Re}\{\Delta\tilde{\bar{\Sigma}}(r,r';\varepsilon)-\Delta\tilde{\bar{\Sigma}}(r,r',0)\}=\frac{\varepsilon}{\pi}p\int_0^\infty\frac{\mathrm{Im}\,\Delta\tilde{\bar{\Sigma}}(r,r',\varepsilon')}{\varepsilon'(\varepsilon'-\varepsilon)}d\varepsilon'. \tag{90}$$

A similar relation between the real and imaginary parts of the nucleon optical potential has been used in Ref. 53 to reconstruct the real part of the nucleon optical potential. Going to the mixed representation, Eq. (72) for the nucleon optical potential can be written as

$$\hat{V}^{\mathrm{opt}}=\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)+i\,\mathrm{Im}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E), \tag{91}$$

where

$$\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)=\Sigma_0(\mathbf{r},\hat{\mathbf{p}})+\mathrm{Re}\,\Delta\tilde{\bar{\Sigma}}(\mathbf{r},\hat{\mathbf{p}},E); \tag{92}$$

$$\mathrm{Im}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)=\mathrm{Im}\,\tilde{\bar{\Sigma}}_f(\mathbf{r},\hat{\mathbf{p}})+\mathrm{Im}\,\Delta\tilde{\bar{\Sigma}}(\mathbf{r},\hat{\mathbf{p}},E). \tag{93}$$

Now, expanding all the operators in Eq. (91) in series in powers of the nucleon momentum operator $\hat{\mathbf{p}}$ through terms of second order, we obtain the Schrödinger equation for the optical wave function $\Psi^{\mathrm{opt}}(\mathbf{r})$ of the fermion:

$$\left\{\hat{\mathbf{p}}\frac{1}{2\bar{m}^*(\mathbf{r},E)}\hat{\mathbf{p}}+\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},0,E)+i\,\mathrm{Im}\,\bar{\Sigma}(\mathbf{r},0,E)-E\right\}\Psi^{\mathrm{opt}}_{(2)}=0,94 \tag{94}$$

where $\bar{m}^*$ is the complex effective nucleon mass, defined as

$$\frac{1}{\bar{m}^*(\mathbf{r})}=\frac{1}{m}+\frac{1}{3}\frac{\partial^2\,\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)}{\partial\hat{\mathbf{p}}^2}\bigg|_{\hat{\mathbf{p}}=0}+\frac{1}{3}i\frac{\partial^2\,\mathrm{Im}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)}{\partial\hat{\mathbf{p}}^2}\bigg|_{\hat{\mathbf{p}}=0}. \tag{95}$$

Let us write the function $\Psi^{\mathrm{opt}}(\mathbf{r})$ as

$$\Psi^{\mathrm{opt}}(\mathbf{r})=\tilde{f}(\mathbf{r})\bar{\Psi}^{\mathrm{opt}}(\mathbf{r}) \tag{96}$$

and choose the factor $\tilde{f}(\mathbf{r})$ from the condition that gradient terms of the form $\hat{\mathbf{p}}\bar{\Psi}^{\mathrm{opt}}(\mathbf{r})$ vanish in the equation for $\bar{\Psi}^{\mathrm{opt}}(\mathbf{r})$:

$$\tilde{f}(\mathbf{r})=\left(\frac{\bar{m}^*(r)}{m}\right)^{1/2}. \tag{97}$$

Then the equation for the function $\bar{\Psi}^{\mathrm{opt}}(\mathbf{r})$ (94) becomes

$$\left\{\frac{\hat{\mathbf{p}}^2}{2\bar{m}^*(\mathbf{r},E)}+\bar{\Sigma}(\mathbf{r},0,E)+\tilde{V}^{\mathrm{grad}}_{(\mathbf{r})}-E\right\}\bar{\Psi}^{\mathrm{opt}}(\mathbf{r})=0 \tag{98}$$

or

$$\left(\frac{\hat{\mathbf{p}}^2}{2m}+V^{\mathrm{opt}}(\mathbf{r},E)-E\right)\bar{\Psi}^{\mathrm{opt}}(\mathbf{r})=0, \tag{99}$$

where the energy-dependent complex optical potential $V^{\mathrm{opt}}(\mathbf{r},E)$ has the form

$$V^{\mathrm{opt}}(\mathbf{r},E)=\frac{\bar{m}^*(\mathbf{r},E)}{m}\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},0,E)+V^{\mathrm{grad}}(\mathbf{r})+i\frac{\bar{m}^*(\mathbf{r},E)}{m}\mathrm{Im}\,\bar{\Sigma}(\mathbf{r},0,E)+\left(1-\frac{\bar{m}^*(\mathbf{r},E)}{m}\right)E. \tag{100}$$

If the complex effective mass $\bar{m}^*(r,E)$ is written as

$$\bar{m}^*(r)=\tilde{m}^*(r)+im_1(r) \tag{101}$$

and $[\bar{m}^*(r)]^{-1}$ is expanded in a series in m_1 through first-order terms, then from (95) we obtain

$$\frac{1}{\tilde{m}^*(\mathbf{r})}=\frac{1}{m}+\frac{1}{3}\frac{\partial^2\,\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)}{\partial\hat{\mathbf{p}}^2}\bigg|_{\hat{\mathbf{p}}=0}; \tag{102}$$

$$m_1^*(\mathbf{r}) = -\frac{1}{3}(\tilde{m}^*(\mathbf{r}))^2 \cdot \left.\frac{d^2 \operatorname{Im} \bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)}{d\hat{\mathbf{p}}^2}\right|_{\hat{\mathbf{p}}=0}. \quad (103)$$

Expanding all quantities depending on $\tilde{m}^*(r)$ in series in powers of $m_1^*(r)$ and keeping only the first-order terms, for the optical potential $V^{\text{opt}}(\mathbf{r},E)$ (100) we find

$$\operatorname{Re} V^{\text{opt}}(\mathbf{r},E) = \tilde{V}(\mathbf{r},E), \quad (104)$$

where $\tilde{V}(\mathbf{r},E)$ is a potential differing from the self-consistent potential $V(\mathbf{r},E)$ (23) by the replacement of $\operatorname{Re}\Sigma_0(\mathbf{r},\hat{\mathbf{p}})$ by $\operatorname{Re}\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)$, and

$$\operatorname{Im} V^{\text{opt}}(\mathbf{r},E) = \frac{m_1^*(\mathbf{r},E)}{m}\operatorname{Re}\bar{\Sigma}(\mathbf{r},0,E) + \frac{\tilde{m}^*(\mathbf{r})}{m} \cdot \operatorname{Im}\bar{\Sigma}(\mathbf{r},0,E) + V_1^{\text{grad}}(\mathbf{r}) - \frac{m_1^*(\mathbf{r})}{m}E, \quad (105)$$

where the imaginary part of the gradient potential is defined as

$$V_1^{\text{grad}}(\mathbf{r}) = \frac{1}{2m}\left\{\frac{(\hat{\mathbf{p}}^2 m_1^*(r))}{2\tilde{m}^*(r)} - 3/2\frac{(\hat{\mathbf{p}}m_1^*(r))(\hat{\mathbf{p}}\tilde{m}^*(r))}{(\tilde{m}^*(r))^2} - \frac{(\hat{\mathbf{p}}^2\tilde{m}^*(r))}{2(\tilde{m}^*(r))^2}\cdot m_1(r) + 3/2\frac{(\hat{\mathbf{p}}\cdot\tilde{m}^*(r))^2}{(\tilde{m}^*(r))^3} \times m_1(r)\right\}. \quad (106)$$

Formulas (104) and (105) differ from the expressions for the potential $V(\mathbf{r},E)$ (23) derived earlier by the replacement of $m^*(r)$ by $\tilde{m}^*(r,E)$ and the appearance of terms related to $\Delta\bar{\Sigma}(\mathbf{r},\hat{\mathbf{p}},E)$.

We can prove an important theorem about the phase equivalence of the Schrödinger equations (94) and (99) (Ref. 54). Let us consider the T-matrix element T_{ba} for a transition from the asymptotic state Φ_a to the asymptotic state Φ_b. In the case of elastic scattering the states Φ_a and Φ_b are plane waves with initial and final momenta $\mathbf{p}_a$ and $\mathbf{p}_b$ (Ref. 46):

$$T_{ba} = \langle\Phi_b|T|\Phi_a\rangle = \langle\Phi_b|v|\Psi_a\rangle,$$

where the wave function Ψ_a coincides with the function Ψ^{opt} (94), and the potential V has the form

$$V = \hat{\mathbf{p}}\frac{1}{2\tilde{m}^*(\mathbf{r},E)}\hat{\mathbf{p}} + \bar{\Sigma}(\mathbf{r},0,E) - \frac{\hat{\mathbf{p}}^2}{2m}.$$

Using the transformation (96), it can be rigorously shown that

$$T_{ba} = \langle\Phi_b|V^{\text{opt}}(\mathbf{r},E)|\bar{\Psi}_a\rangle,$$

where $V^{\text{opt}}(\mathbf{r},E)$ is the optical potential Eq. (100), and $\bar{\Psi}_a$ satisfies the Schrödinger equation (99). This implies that the optical potential (100) correctly describes the cross sections and phase shifts of fermion scattering on a finite Fermi system.

12. RELATION BETWEEN THE THEORETICAL AND PHENOMENOLOGICAL NUCLEON OPTICAL POTENTIALS

The parameters of phenomenological nucleon optical potentials possess discrete and continuous indeterminacies. However, if we use Levinson's theorem,[55] according to which the phase shift at zero energy is determined by the number of bound states of the scattering potential, it is possible to find a global set of parameters of the optical potential which gives a sufficiently good description of the total cross sections for compound-nucleus formation and the differential cross sections for the direct part of nucleon elastic scattering for a wide range of target nuclei and incident nucleon energies. Of course, such an optical potential cannot describe the irregularities in the cross sections arising from the strong coupling to doorway states, which are calculated by the coupled-channel method.

Let us now consider one of the global sets of parameters of the phenomenological optical potential of the nucleon $V_{ph}^{\text{opt}}(r,E)$, given in Reg. [28] for spherical target nuclei:

$$\operatorname{Re} V_{ph}^{\text{opt}}(r,E) = (V_0(\tau_z) + \beta E)\cdot\left(1+\exp\frac{r-r_0A^{1/3}}{a_0}\right)^{-1} + V_{ls}(\tau_z)\cdot r_{ls}^2\cdot\frac{1}{2}\frac{d}{dr}\left[\left(1+\exp\frac{r-r_{ls}A^{1/3}}{a_{ls}}\right)^{-1}\right]\cdot(\mathbf{l}\cdot\mathbf{s}), \quad (107)$$

where $\tau_Z = +1(-1)$ for the neutron (proton),

$$V_0(\tau_z) = \left(-51+33\tau_z\cdot\frac{N-z}{A}\right)\text{MeV};$$

$$V_{ls}(\tau_z) = \left(22-14\frac{N-Z}{A}\tau_z\right)\text{ MeV};$$

$r_0 = 1.25$ fm; $a_0 = 0.65$ fm; $\beta = 0.3$;

$\gamma_{ls} = 1.1$ fm, $a_{ls} = 0.7$ fm;

$$\operatorname{Im} V_{ph}^{\text{opt}}(r,E) = W_V(\tau_z,E)\left(1+\exp\frac{r-r_wA^{1/3}}{a_w}\right)^{-1} - 4a_w \times W_s(\tau_z,E)\frac{d}{dr}\left[\left(1+\exp\frac{r-r_wA^{1/3}}{aw}\right)^{-1}\right], \quad (108)$$

where $r_w = 1.25$ fm and $a_w = 0.7$ fm, and the ratio of the depth parameters of the volume (w_v) and surface (w_s) absorption potentials has a complicated dependence on the energy and the characteristics A,Z of the target nucleus. Surface absorption prevails at low energies. Volume absorption becomes more important as the energy grows, and dominates at high energies ($E > 100$ MeV). If for all energies we use the variants corresponding to purely volume ($w_s = 0$) or purely surface ($W_v = 0$) absorption, we obtain the following energy dependence of the depth:

$$W_v(E) = -(2+0.006E)\text{ MeV}; \quad W_s(E) = -(2.5+0.3E)\text{ MeV}. \quad (109)$$

Analysis of the properties of phenomenological nucleon optical potentials leads to a number of important conclusions. First, comparison of Eqs. (104) and (107) gives estimates of the real part of the effective nucleon mass at the center of the nucleus $\tilde{m}^*(0)=0.7$ m and the depth of the real potential $\mathrm{Re}\,\bar{\Sigma}(\mathbf{r},0,E)$ at $E=0$ at the center of the nucleus $\mathrm{Re}\,\bar{\Sigma}(0,0,0)\approx -70$ MeV. Second, from (109) for $E=0$ we see the scale of the fragmentation term of the imaginary part of the nucleon optical potential $W_f=W(E=0)\approx -2$ MeV, from which we estimate that the fragmentation width is $\Gamma_\lambda\approx 4.5$ MeV. This value of Γ_λ is considerably lower than the value $100/A^{1/3}$ determined by the energy separation of adjacent one-quasiparticle levels in the nucleus with quantum numbers λ. Physically, this means "moderate" absorption of nucleons in the nucleus, which leads to the appearance of "giant" resonances in the strength function of slow neutrons.[54] Now we can estimate the value of the imaginary part of the effective nucleon mass $m_1(r)$ (103). If for volume absorption we assume that the radius of nonlocality of the potential $\mathrm{Im}\,\bar{\bar{\Sigma}}(r,\hat{p},E)$ is close to the radius of nonlocality of the potential $\mathrm{Re}\,\Sigma_0(r,\hat{p})$, we estimate $m_1(r)$ at the center of the nucleus to be $m_1^*(0)\approx -0.01m$. It can then be expected that the imaginary part of the nucleon optical potential $\mathrm{Im}\,V^{\mathrm{opt}}(\mathbf{r},E)$ acquires an additional term of order $+0.7$ MeV, associated with the first term of Eq. (105), at the center of the nucleus, together with an energy-dependent small term associated with the last term of Eq. (105) and having the form $0.01E$. We note that the sign of the latter is opposite to that of the energy-dependent part of the phenomenological potential $\mathrm{Im}\,V_{\mathrm{ph}}^{\mathrm{opt}}$ (109).

For surface absorption associated with dominance of the surface collective oscillations of the target nucleus, the radius of nonlocality of the potential $\mathrm{Im}\,\bar{\bar{\Sigma}}(\mathbf{r},\hat{\mathbf{p}},E)$ is close to the nuclear radius, so that the estimate of $m_1(r)$ in the surface region must be increased by about an order of magnitude compared to the volume case, giving $m_1(0)\simeq m_1(\mathbf{r})$. Then the first term in (105) can take a large value ≈ 3.5 MeV in the surface region, comparable in modulus to the value of $\mathrm{Im}\,V_{\mathrm{ph}}^{\mathrm{opt}}$ (109). At the same time, the last term in (105) leads to an energy-dependent addition to the imaginary part of the optical potential, which in the surface region takes the value $+0.1E$ and differs in sign from the energy-dependent part of $\mathrm{Im}\,V_{\mathrm{ph}}^{\mathrm{opt}}$ Eq. (109). It follows from these estimates that the phenomenological energy dependence of the imaginary part of the nucleon optical potential $\mathrm{Im}\,V_{\mathrm{ph}}^{\mathrm{opt}}$ Eq. (109) is mainly determined by the energy-dependent term in the retarded part of the mass operator $\Delta\bar{\bar{\Sigma}}$, which is entirely related to inelastic direct and pre-equilibrium processes.

In Refs. 16–21 a detailed study was made of the properties of the real part of the nucleon optical potential for nuclear matter and finite nuclei in the Hartree–Fock approximation, where the addition $\Delta\bar{\bar{\Sigma}}$ related to inelastic processes is not taken into account. The calculations, which used the 14 most common sets of phenomenological vacuum nucleon–nucleon potentials $V(r,r')$, showed that the main features of $\mathrm{Re}\,V_{\mathrm{ph}}^{\mathrm{opt}}$ can be described successfully for several sets. The only problem was with the description of the isospin dependence of $\mathrm{Re}\,V_{\mathrm{ph}}^{\mathrm{opt}}$. The calculated dependence turned out to be weaker than the phenomenological one.

It would be useful to extend the calculations of optical potentials to understand the singularities of their imaginary parts using the generalized Fermi liquid theory taking into account nucleon–nucleon interactions as in Refs. 14,15. Such calculations should be able to describe the surface and volume components of absorption for the fragmentation part of the optical potential. At the same time, it is necessary to estimate the role of the retarded part of the mass operator $\Delta\bar{\bar{\Sigma}}$ associated with inelastic processes, primarily using the dispersion relations (89) and (90).

13. OPTICAL POTENTIALS OF COMPOSITE PARTICLES

Since the dominant contribution to the real part of the nucleon optical potential comes from the Hartree–Fock term, it is natural to construct the real parts of the optical potentials of composite particles in the same approximation.[22]

Let us study the interaction of a composite particle x of mass A_x and a nucleus f of mass A_f. We introduce the coordinates of the center of mass $\mathbf{R}_0$ of the entire system, the center of mass $\mathbf{R}_f$ of the nucleus f, the center of mass $\mathbf{R}_x$ of the particle x, and the relative coordinate $\mathbf{R}$:

$$\mathbf{R}_0=\frac{A_x\mathbf{R}_x+A_f\mathbf{R}_f}{A_x+A_f};\quad \mathbf{R}_f=\frac{\sum_{i=1}^{A_f}\mathbf{r}_i}{A_f};\quad \mathbf{R}_x=\frac{\sum_{k=1}^{A_x}\mathbf{r}_k}{A_x};$$
$$\mathbf{R}=\mathbf{R}_x-\mathbf{R}_f. \tag{110}$$

The coordinate of the ith nucleon includes the radius vector $\mathbf{r}_i$ and the multiple index $\alpha_i=s_i\tau_i$, where s_i and τ_i are the nucleon spin and isospin projections on the z axis. For the nucleus f and particle x we introduce the Jacobi coordinates $\boldsymbol{\xi}_l$ $(1\leqslant l\leqslant A_f-1)$ and $\boldsymbol{\xi}_m$ $(1\leqslant m\leqslant A_x-1)$ using the expressions

$$\boldsymbol{\xi}_l=\sum_{i=1}^{A_f} b_{li}\mathbf{r}_i=\sum_{i=1}^{A_f} b_{li}\mathbf{y}_i; \tag{111}$$

$$\boldsymbol{\xi}_m=\sum_{k=1}^{A_x} b_{mk}\mathbf{r}_k=\sum_{k=1}^{A_x} b_{mk}\mathbf{y}_k, \tag{111'}$$

where

$$\mathbf{y}_i=\mathbf{r}_i-\mathbf{R}_f;\quad \mathbf{y}_k=\mathbf{r}_k-\mathbf{R}_x;\quad \sum_i b_{li}=0;\quad \sum_k b_{mk}=0. \tag{112}$$

For convenience, we choose the standard scheme for determining the coefficients b_{li} (b_{mk}):

$$\boldsymbol{\xi}_1=\mathbf{r}_i-\mathbf{r}_2;\quad \boldsymbol{\xi}_2=\frac{\mathbf{r}_1+\mathbf{r}_2}{2}-\mathbf{r}_3;\ldots;$$
$$\boldsymbol{\xi}_{A_f-1}=\frac{\sum_{i=1}^{A_f-1}\mathbf{r}_i}{A_f-1}-\mathbf{r}_{A_f}, \tag{113}$$

when the Jacobian of the substitution going from the variables $\mathbf{r}_i$ to the variables $\mathbf{R}_f$, $\boldsymbol{\xi}_l$ $(1\leqslant l\leqslant A_f-1)$ is equal to unity, and the coefficient is $b_{l,A_f}=-\delta_{l,A_f-1}$. The reverse variable substitution has the form

$$\mathbf{y}_i=\sum_{l=1}^{A_f-1} a_{il}\boldsymbol{\xi}_l;\quad \sum_{i=1}^{A_f} a_{il}=0;\quad a_{A_f l}=\frac{A_f-1}{A_f}\,\delta_{l,A_f-1}. \tag{114}$$

We then obtain the expressions

$$\mathbf{r}_k=\mathbf{R}_0+\frac{A_f}{A_f+A_x}\mathbf{R}+\sum_{m=1}^{A_x-1}\boldsymbol{\xi}_m a_{mk};$$

$$\mathbf{r}_i=\mathbf{R}_0-\frac{A_x}{A_x+A_f}\mathbf{R}+\sum_{l=1}^{A_f-1} a_{li}\boldsymbol{\xi}_l. \tag{115}$$

Let us consider the Schrödinger equation for the optical wave function Ψ_{xf} describing the elastic scattering of a particle x on a nucleus f:

$$\left\{\sum_{i=1}^{A_f}\frac{\hat{\mathbf{p}}_i^2}{2m}+\sum_{k=1}^{A_x}\frac{\hat{\mathbf{p}}_k^2}{2m}+\sum_{i<j=1}^{A_f}V_{ij}+\sum_{k<s=1}^{A_x}V_{ks}+\sum_{i=1}^{A_f}\sum_{k=1}^{A_x}V_{ik}-E_0\right\}\Psi_{xf}(\mathbf{R}_0,\mathbf{R},\boldsymbol{\xi}_l,\boldsymbol{\xi}_m)=0. \tag{116}$$

This equation can be written as

$$\left\{-\frac{\hbar^2}{2m}\frac{1}{(A_x+A_f)}\frac{\partial^2}{\partial\mathbf{R}_0^2}-\frac{\hbar^2}{2m}\frac{A_x+A_f}{A_xA_f}\frac{\partial^2}{\partial\mathbf{R}^2}-\frac{\hbar^2}{2m}\sum_{i=1}^{A_f}\left(\sum_l b_{li}\frac{\partial}{\partial\boldsymbol{\xi}_l}\right)^2-\frac{\hbar^2}{2m}\sum_{k=1}^{A_x}\left(\sum_m b_{mk}\frac{\partial}{\partial\boldsymbol{\xi}_m}\right)^2+\sum_{i<j=1}^{A_f}V_{ij}+\sum_{k<s=1}^{A_x}V_{ks}+\sum_{i=1}^{A_f}\sum_{k=1}^{A_x}V_{ik}-E_0\right\}\Psi_{xf}(\mathbf{R}_0,\mathbf{R},\boldsymbol{\xi}_l,\boldsymbol{\xi}_m)=0. \tag{117}$$

For convenience, in Eqs. (116) and (117) we have dropped the nucleon spin and isospin indices. We shall seek the wave function of the system in the form

$$\Psi_{xf}=l^{i\mathbf{p}_0\mathbf{R}_0/\hbar}\hat{A}\{\chi_f(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_{A_f-1})\chi_x(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_{A_x-1})\Psi_{xf}^{\mathrm{opt}}(\mathbf{R})\}, \tag{118}$$

where $\hat{A}$ is the antisymmetrization operator:

$$\hat{A}=\left[\frac{(A_l+A_x)!}{A_l!A_x!}\right]^{-\gamma}\sum_{2\ P}(-1)^P P, \tag{119}$$

with the sum over P including only interchanges between nucleons k of the particle x and nucleons i of the nucleus f. The interior wave functions of the nucleus f and particle x are completely antisymmetrized and satisfy a Schrödinger equation of the form

$$\left\{-\frac{\hbar^2}{2m}\sum_{i=1}^{A_l}\left(\sum_{p=1}^{A_l-1}b_{li}\frac{\partial}{\partial\boldsymbol{\xi}_e}\right)^2+\sum_{i<j=1}^{A_l}V_{ij}-E_f\right\}\chi_f=0. \tag{120}$$

Let us multiply (117) on the left by the function $\chi_f^*\chi_x^*$ and integrate over the complete set of internal coordinates of the system. For the terms of the form V_{ik} in the Hamiltonian (117) we include in the sum in (119) the identity permutation P_0 and the permutation of the ith and kth nucleons P_{ik}, while for all the other terms in (117) we limit ourselves to only the permutation P_0, which corresponds to including exchange effects only for interacting nucleons. We write the permutation operator P_{ik} as a product of permutation operators for the nucleon coordinates and quantum numbers α:

$$P_{ik}=P_{ik}(\mathbf{r}_i,\mathbf{r}_k)\cdot P(\alpha_i,\alpha_k). \tag{121}$$

In turn, we write the nucleon coordinate permutation operator in terms of the nucleon coordinate shift operator:

$$P_{ik}(\mathbf{r}_i,\mathbf{r}_k)=\exp\left\{(\mathbf{r}_i-\mathbf{r}_k)\left(\frac{\partial}{\partial\mathbf{r}_k}-\frac{\partial}{\partial\mathbf{r}_i}\right)\right\}, \tag{122}$$

which can be expressed as

$$P_{ik}(\mathbf{r}_i,\mathbf{r}_k)=\exp\left\{(\mathbf{r}_k-\mathbf{r}_i)\cdot\left(\frac{A_x+A_b}{A_xA_l}\frac{\partial}{\partial\mathbf{R}}+\sum_{m=1}^{A_x-1}\frac{\partial}{\partial\boldsymbol{\xi}_m}b_{mk}-\sum_{l=1}^{A_1-1}\frac{\partial}{\partial\boldsymbol{\xi}_l}\cdot b_{li}\right)\right\}. \tag{123}$$

Then Eq. (117) becomes

$$\left\{-\frac{\hbar^2}{2m}\frac{A_x+A_b}{A_xA_l}\frac{\partial^2}{\partial\mathbf{R}^2}+\hat{V}_{xf}(\mathbf{R})-E\right\}\Psi_{xf}^{opt}(\mathbf{R})=0, \tag{124}$$

where $E=E_0-[p_0^2/2m(A_x+A_f)]-\varepsilon_f-\varepsilon_x$ is the energy of the relative motion of the particle and nucleus f. In the case where the particle has equal numbers of protons and neutrons $N_x=Z_x=A_x/2$, the potential $\hat{V}_{xf}$ is written as

$$\hat{V}_{xf}(\mathbf{R})=\int V^H(r)\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r})\rho_x(\mathbf{y}_x)d\mathbf{r}d\mathbf{y}_x-\int V^F(r)\times\rho_f\left(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\quad \mathbf{R}+\mathbf{y}_x-\frac{\mathbf{r}}{A_f}\right)\times\rho_x\left(\mathbf{y}_x,\mathbf{y}_x-\frac{A_x-1}{A_x}\mathbf{r}\right)\exp\left\{-\left(\frac{A_x+A_f}{A_x\cdot A_f}\right)\times\mathbf{r}\frac{\partial}{\partial\mathbf{R}}\right\}\cdot d\mathbf{r}d\mathbf{y}_x, \tag{125}$$

where the potentials $V^H(r)$ and $V^F(r)$ are expressed as a combination of various components of the vacuum nucleon–nucleon potentials and are found in Ref. 16. In Eq. (125) we have introduced the one-particle density matrix $\rho_{f\alpha}(\mathbf{y},\mathbf{y}')$ of the nucleus f (particle x):

$$\rho_{f\alpha}(\mathbf{y},\mathbf{y}')=\sum_{\alpha_1,\ldots,\alpha_{A_f-1}}\int\chi_f^*\left(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_{A_f-2},-\frac{A_f}{A_f-1}\mathbf{y};\boldsymbol{\alpha}_1;\boldsymbol{\alpha}_{A_f-1},\boldsymbol{\alpha}_f\right)\chi_f\left(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_{A_f-2}-\frac{A_f}{A_f-1}\mathbf{y}',\boldsymbol{\alpha}_2,\ldots,\boldsymbol{\alpha}_{A_f-1},\boldsymbol{\alpha}\right)\cdot\left(\frac{A_f}{A_f-1}\right)^3\times\prod_{l=1}^{A_f-2}d\boldsymbol{\xi}_l, \tag{126}$$

which goes into the one-particle density matrix $\rho_{f\alpha}(\mathbf{y})$ of the nucleus f (particle x) in the diagonal case $\rho_{f\alpha}(\mathbf{y})=\rho_{f\alpha}(\mathbf{yy})$.

We note that the potential (125) is symmetric in the nucleus f and particle x and goes into the nucleon–nucleus potential $V_{0f}(\mathbf{R})$ for

$A_x=1$, $\rho_x(\mathbf{y}_x)=\delta(\mathbf{y}_x)$:

$$\hat{V}_{0f}(\mathbf{R})=\int V^H(r)\rho_f(\mathbf{R}-\mathbf{r})d\mathbf{r}-\int V^F(r)\cdot\rho_f\left(\mathbf{R}-\mathbf{r},\ \mathbf{R}-\frac{\mathbf{r}}{A_f}\right)\exp\left\{-\frac{A_f+1}{A_f}\,\mathbf{r}\,\frac{\partial}{\partial\mathbf{R}}\right\}\mathbf{r}. \tag{127}$$

For spherical nuclei f and particles x we expand exp{...} in (126) and (127) in a series in powers in $\partial/\partial\mathbf{R}$, keeping terms through second order. Then the potentials (125) and (127) can be written in Hermitian form:[22]

$$\hat{V}_{xf}(\mathbf{R})=V^0_{xf}(\mathbf{R})+\frac{d}{dR_\alpha}\,\tilde{V}_{xf}(\mathbf{R})\,\frac{d}{dR_\alpha}; \tag{128}$$

$$\hat{V}_{0f}(\mathbf{R})=V^0_{0f}(\mathbf{R})+\frac{d}{dR_\alpha}\,\tilde{V}_{0f}(\mathbf{R})\,\frac{d}{dR_\alpha}, \tag{128'}$$

where

$$V^0_{xf}(\mathbf{R})=\int V^H(r)\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r})\rho_x(\mathbf{y}_x)d\mathbf{r}d\mathbf{y}_x-\int V^F(\mathbf{r})\cdot\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\ \mathbf{R}+\mathbf{y}_x-\mathbf{r}/A_f)\times\rho_x\left(\mathbf{y}_x,\mathbf{y}_x-\frac{A_x-1}{A_x}\,\mathbf{r}\right)d\mathbf{r}d\mathbf{y}_x; \tag{129}$$

$$\tilde{V}_{xf}(\mathbf{R})=-\left(\frac{A_x+A_f}{A_xA_f}\right)^2\frac{1}{6}\int V^F(r)r^2\rho_f\left(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\ \mathbf{R}+\mathbf{y}_x-\frac{\mathbf{r}}{A_f}\right)\rho_x\left(\mathbf{y}_x,\ \mathbf{y}_x-\frac{A_x-1}{A_x}\,\mathbf{r}\right)d\mathbf{y}_xd\mathbf{r}; \tag{129'}$$

$$V^0_{0f}(\mathbf{R})=\int V^H(r)\rho_f(\mathbf{R}-\mathbf{r})d\mathbf{r}-\int V^F(r)\rho_f\left(\mathbf{R}-\mathbf{r},\ \mathbf{R}-\frac{\mathbf{r}}{A_f}\right)d\mathbf{r}; \tag{130}$$

$$\tilde{V}_{0f}(\mathbf{R})=-\left(\frac{A_f+1}{A_f}\right)^2\frac{1}{6}\cdot\int V^F(r)\cdot r^2\cdot\rho_f(\mathbf{R}-\mathbf{r},\ \mathbf{R}-\mathbf{r}/A_f)d\mathbf{r}. \tag{130'}$$

Expanding the density matrix of the particle x in a series in powers of $\mathbf{r}$ through terms of second order

$$\rho_x\left(\mathbf{y}_x,\ \mathbf{y}_x-\frac{A_x-1}{A_x}\,\mathbf{r}\right)=\rho_x(\mathbf{y}_x)-1/2\,\frac{\partial\rho_x(\mathbf{y}_x)}{\partial y_{x\alpha}}\,r_\alpha+1/2\,\frac{\partial^2\rho_x(\mathbf{y}_x,\mathbf{y}'_x)}{\partial y'_{x\alpha}\cdot\partial y'_{x\beta}}\bigg|r_\alpha r_\beta\left(\frac{A_x-1}{A_x}\right)^2, \tag{131}$$

substituting this expansion into (129) and (130), and keeping terms of second order in $\mathbf{r}$, we obtain

$$V^0_{xf}(R)=\int V^H(r)\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r})\rho_x(\mathbf{y}_x)d\mathbf{r}d\mathbf{y}_x-\int V^F(r)\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\ \mathbf{R}+\mathbf{y}_x-\mathbf{r}/A_f)\rho_x(\mathbf{y}_x)d\mathbf{r}d\mathbf{y}_x-\frac{1}{6}\left(\frac{A_x-1}{A_x}\right)^2\int V^F(r)r^2P_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\ \mathbf{R}+\mathbf{y}_x-\mathbf{r}/A_f)\cdot\frac{\partial^2\rho_x(\mathbf{y}_x,\mathbf{y}'_x)}{\partial\mathbf{y}'^2_x}\bigg|_{\mathbf{y}'_x=\mathbf{y}_x}d\mathbf{r}d\mathbf{y}_x; \tag{132}$$

$$\tilde{V}_{xf}(\mathbf{R})=-\frac{1}{6}\,\frac{(A_x+A_f)^2}{(A_xA_f)^2}\int V^F(r)r^2\rho_f(\mathbf{R}+\mathbf{y}_x-\mathbf{r},\ \mathbf{R}+\mathbf{y}_x-\mathbf{r}/A_f)\rho_x(\mathbf{y}_x)d\mathbf{y}_xd\mathbf{r}. \tag{133}$$

Using the fact that the effective nucleon mass $m^*_{0f}(\mathbf{R})$ is related to the potential $\tilde{V}_{0f}(\mathbf{R})$ as

$$\frac{1}{m^*_{0f}(\mathbf{R})}=\frac{1}{m_{0f}}+\frac{2\cdot\tilde{V}_{0f}(\mathbf{R})}{\hbar^2}, \tag{134}$$

where $m_{0f}=A_f/A_f+1$ m, Eqs. (132) and (133) can be written as

$$V^0_{xf}(R)=\int V^0_{0f}(\mathbf{R}+\mathbf{y}_x)\rho_x(\mathbf{y}_x)d\mathbf{y}_x+\frac{\hbar^2}{2}\left(\frac{A_x-1}{A_x}\right)^2\left(\frac{A_f}{A_f-1}\right)^2\int\left[\frac{1}{m^*_{0f}(\mathbf{R}+\mathbf{y}_x)}-\frac{1}{m_{0f}}\right]\frac{\partial^2\rho_x(\mathbf{y}_x,\mathbf{y}'_x)}{\partial\mathbf{y}'^2_x}\bigg|_{\mathbf{y}'_x=\mathbf{y}_x}d\mathbf{y}_x; \tag{135}$$

$$\tilde{V}_{xf}(R)=\frac{(A_x+A_f)^2}{(A_x(A_f+1))^2}\int\tilde{V}_{0f}(\mathbf{R}+\mathbf{y}_x)\cdot\rho_x(\mathbf{y}_x)d\mathbf{y}_x=\frac{(A_x+A_f)^2\hbar^2}{2A_x^2(A_f+1)^2}\int\left(\frac{1}{m^*_0(\mathbf{R}+\mathbf{y}_x)}-\frac{1}{m_{0f}}\right)\rho_x(\mathbf{y}_x)d\mathbf{y}_x. \tag{136}$$

Substituting the potential (128) into the Schrödinger equation (124) taking into account Eqs. (135) and (136) and using the technique developed above for analyzing the nucleon optical potential, we obtain

$$\left\{-\frac{\hbar^2}{2m_{xf}}\,\frac{\partial^2}{\partial\mathbf{R}^2}+V^{\text{opt}}_{xf}(\mathbf{R},E)-E\right\}\Psi^{\text{opt}}_{xf}(\mathbf{R})=0; \tag{137}$$

where

$$\text{Re}\,V^{\text{opt}}_{xf}(\mathbf{R},E)=\frac{m^*_{xf}(\mathbf{R})}{m_{xf}}\,V^0_{xf}(\mathbf{R})+V^{\text{grad}}_{xf}(\mathbf{R})+\left(1-\frac{m^*_{xf}(\mathbf{R})}{m_{xf}}\right)E; \tag{138}$$

$$\frac{1}{m^*_{xf}(\mathbf{R})}=\frac{1}{m_{xf}}+\frac{2\tilde{V}_{xf}(\mathbf{R})}{\hbar^2}, \tag{139}$$

and the gradient potential $V^{\text{grad}}_{xf}(\mathbf{R})$ is given by Eq. (24) with $m^*(r)$ replaced by $m^*_{xf}(R)$ and m replaced by $m_{xf}=m\frac{A_xA_f}{A_x+A_f}$. Using (136), equation (139) can be written as

$$\frac{1}{m^*_{xf}(R)}=\frac{1}{m_{xf}}+\frac{(A_x+A_f)^2}{A_x^2\cdot(A_f+1)^2}\int\left[\frac{1}{m^*_{0f}(\mathbf{R}+\mathbf{y}_x)}-\frac{1}{m_{0f}}\right]\rho_x(\mathbf{y}_x)|d\mathbf{y}_x. \tag{140}$$

Let us consider in more detail the properties of the real part of the optical potential for a composite particle (138) that we have obtained. We shall compare it to the analogous potential obtained using the folding procedure with a single folding:[56]

$$\text{Re}\,V^{\text{opt}}_{xf}(\mathbf{R})=\int \text{Re}\,V^{\text{opt}}_{0f}(\mathbf{R}+\mathbf{y}_x)\cdot\rho(\mathbf{y}_x)d\mathbf{y}_x, \tag{141}$$

where $\text{Re}\,V^{\text{opt}}_{0f}$ is the real part of the nucleon optical potential.

We consider the case where the particle x is much smaller than the nucleus $f(A_x\ll A_f)$. Then at the center of the nucleus the effective mass of the composite particle $m^*_{xf}(0)$ is A_x times larger than the effective nucleon mass $m^*(0)$. The folding potential Eq. (141) at the center of the nucleus coincides with the first term of Eq. (138) when the first term of (135) is substituted into it and has a value $\approx A_x\cdot 51$ MeV. At the same time, the effective mass of the composite particle $m^*_{xf}(\mathbf{R})$ (140) smoothly goes into the vacuum value m_{xf} as R grows, but here there is a rather large diffuseness in the nuclear surface region. The gradient term $V^{\text{grad}}_{xf}(\mathbf{R})$ for the particle x has a surface nature and is qualitatively close to the analogous gradient term for the nucleon potential. However, since the depth of the optical potential of the composite particle will be A_x times greater than the depth of the nucleon potential, the gradient term for the composite particle will be A_x times weaker than for the nucleon. The energy dependence for the real part of the optical potential of a composite particle at the center of the nucleus is close to that for the nucleon optical potential, because $m^*_{xf}(0)/m_{xf}=m^*_{0f}(0)/m_{0f}\approx 0.7$. However, owing to the increase of the depth of the x potential by a factor of A_x compared to the nucleon potential, at this energy the energy term in (138) will be A_x times weaker than in the case of the nucleon.

Finally, let us consider the properties of the part of the optical potential of the composite particle determined by the second term in (135). Physically, this term corresponds to the change of the internal kinetic energy E^{kin}_x of the composite particle due to renormalization of the mass of the nucleons of particle x in nucleus f. At the center of the nucleus this term, when substituted into (138), gives an addition of $0.28E^{\text{kin}}_x$ to the depth of the optical potential of the composite particle. If the kinetic energy of a sufficiently heavy particle $x(A_x\gg 1)$ is estimated to be $22A_x$ MeV, the addition to the depth of the potential $\text{Re}\,V^{\text{opt}}_{xf}$ (138) determined by the renormalization of the kinetic energy is $6A_x$ MeV. Then $\text{Re}\,V^{\text{opt}}_{xf}(R,0)$ at the center of the nucleus will have a depth $\approx -45A_x$ MeV and considerably larger diffuseness in the surface region of the nucleus compared to the nucleon optical potential.

As far as the imaginary part of the optical potential of the composite particle is concerned, in addition to the fragmentation term and the terms associated with inelastic channels which do not change the structure of the composite particle, there will also be important terms from the inelastic channels associated with the excitation and decay of the composite particle. Therefore, the depth of the imaginary part of the optical potential of the composite particle will be more than A_x times larger than the analogous depth for the nucleon optical potential. As a rule, this leads to a situation like that of a "black" nucleus in the scattering of practically all composite particles.

We also note that, in general, the Hartree–Fock approximation for the elastic scattering of a composite particle x on a nucleus f allows the reconstruction of the real part of the optical potential $V^{\text{opt}}_{xf}(\mathbf{R},E)$ at values of R large enough that the particle x and nucleus f only weakly overlap. It is therefore possible to neglect unincluded effects such as antisymmetrization, the polarizability of the particle x and the nucleus f, and also the renormalization of the interaction between nucleons due to the other nucleons of the colliding particles.

15. CONCLUSION

Let us briefly summarize the results of this study.

1. It has been shown that the self-consistent one-particle potential of a Fermi system coincides with the Hartree–Fock potential and is close to the real part of the optical potential, while differing significantly from the widely used fermion shell potential.

2. It has been demonstrated the fundamental role of fermion–phonon interactions in the mechanism of quasiparticle fragmentation in Fermi systems. These interactions play a special role in nuclei, where to a large degree they determine the structure of the retarded (normal and superfluid) parts of the mass operator. The theoretical correctness of the quasiparticle–phonon model of the nucleus constructed in Refs. 8 and 9 is thereby confirmed.

3. A nonlinear scheme has been developed for calculating the static and dynamical characteristics of Fermi systems for arbitrary excitation energies, which works well for analyzing the properties of nuclei.

4. A systematic analysis has been carried out to determine the structure of the optical potentials of nucleons and composite particles.

It would be interesting to apply the methods developed here both to study such continuous Fermi liquids as ^{3}He and electrons in solids, and also to generalize and qualitatively improve the results of the theory of finite Fermi systems as applied to many-electron atoms and nuclei.

[1] L. D. Landau, Zh. Éksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys. JETP **3**, 984 (1956)].
[2] L. D. Landau, Zh. Éksp. Teor. Fiz. **32**, 59 (1957) [Sov. Phys. JETP **5**, 101 (1957)].
[3] A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic*

Nuclei, transl. of 1st Russ. ed. (Interscience, New York, 1967) [Russ. original, 2nd ed., Nauka, Moscow, 1983].
[4]A. B. Migdal, Zh. Éksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys. JETP **7**, 996 (1958)].
[5]C. Mahaux *et al.*, Phys. Rep. **120**, 3 (1985).
[6]S. Gales, Ch. Stoyanov, and A. I. Vdovin, Phys. Rep. **3**, 126 (1988).
[7]H. A. Bethe, Ann. Rev. Nucl. Sci. **21**, 93 (1971) [Russ. transl., Mir, Moscow, 1974].
[8]V. G. Solov'ev, Fiz. Élem. Chastits At. Yadra **9**, 810 (1978) [Sov. J. Part. Nucl. **9**, (1978)].
[9]L. A. Malov and V. G. Solov'ev, Fiz. Élem. Chastits At. Yadra **11**, 301 (1980) [Sov. J. Part. Nucl. **11**, 111 (1980)].
[10]S. G. Kadmenskiĭ, Izv. Akad. Nauk SSSR, Ser. Fiz. **26**, 1194 (1962).
[11]S. G. Kadmenskiĭ and L. P. Rapoport, Izv. Vyssh. Uchebn. Zaved. No. 5, 43 (1963).
[12]S. G. Kadmenskiĭ, Izv. Akad. Nauk SSSR, Ser. Fiz. **29**, 331 (1965).
[13]S. G. Kadmenskiĭ and P. L. Luk'yanovich, Yad. Fiz. **49**, 384 (1989) [Sov. J. Nucl. Phys. **49**, 238 (1989)].
[14]S. G. Kadmenskiĭ and P. L. Luk'yanovich, Yad. Fiz. **49**, 1285 (1989) [Sov. J. Nucl. Phys. **49**, 800 (1989)].
[15]S. G. Kadmenskiĭ and P. L. Luk'yanovich, Yad. Fiz. **51**, 650 (1990) [Sov. J. Nucl. Phys. **51**, 411 (1990)].
[16]S. G. Kadmenskiĭ, P. L. Luk'yanovich, and V. V. Lyuboshits, Yad. Fiz. **56**, No. 9, 86 (1993) [Phys. At. Nucl. **56**, 1201 (1993)].
[17]S. G. Kadmenskiĭ, V. V. Lyuboshits, and A. A. Shaĭkina, Izv. Akad. Nauk SSSR, Ser. Fiz. **58**, 15 (1994).
[18]S. G. Kadmenskiĭ, V. V. Lyuboshits, and A. A. Shaĭkina, Yad. Fiz. **58**, 982 (1995) [Phys. At. Nucl. **58**, 910 (1995)].
[19]S. G. Kadmenskiĭ, V. V. Lyuboshits, and A. A. Shaĭkina, Yad. Fiz. **58**, 1222 (1995) [Phys. At. Nucl. **58**, 1143 (1995)].
[20]S. G. Kadmenskiĭ, V. V. Lyuboshits, and A. A. Shaĭkina, Yad. Fiz. **58**, 1606 (1995) [Phys. At. Nucl. **58**, 1514 (1995)].
[21]S. G. Kadmenskiĭ and V. V. Lyuboshits, Yad. Fiz. **59**, 1 (1996) [Phys. At. Nucl. **59**, 5 (1996)].
[22]S. G. Kadmenskiĭ, in *Proc. of Math. LENO. Fifteenth Nucl. Division Conf.*, St. Petersburg, 1995, p. 46.
[23]A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958) [Russ. transl., IL, Moscow, 1960].
[24]M. Baranger, Nucl. Phys. **A149**, 225 (1970).
[25]D. Bes *et al.*, Nucl. Phys. **A293**, 350 (1977).
[26]V. A. Kuz'min, Teor. Mat. Fiz. **70**, 315 (1987).
[27]F. G. Perey, *Direct Interactions and Nuclear Reaction Mechanisms* (Gordon and Breach, New York, 1963), p. 125.
[28]V. A. Khodel, E. E. Saperstein, and M. V. Zverev, Nucl. Phys. **A465**, 397 (1987).
[29]S. G. Kadmenskiĭ *et al.*, Yad. Fiz. **27**, 906 (1978) [Sov. J. Nucl. Phys. **27**, 481 (1978)].
[30]É. E. Sapershteĭn, S. L. Fayans, and V. A. Khodel', Fiz. Élem. Chastits At. Yadra **9**, 221 (1978) [Sov. J. Part. Nucl. **9**, 91 (1978)].
[31]J. Gullot *et al.*, Preprint 79.03, Grenoble.
[32]J. Van de Wiele *et al.*, Nucl. Phys. **A297**, 61 (1978).
[33]G. I. Igo *et al.*, Phys. Rev. **177**, 1381 (1969).
[34]A. F. Jeans *et al.*, Nucl. Phys. **A128**, 229 (1969).
[35]H. Langevin-Joliot, Preprint No. 91. 4006, Orsay (1988).
[36]S. Gales *et al.*, Phys. Rev. C **31**, 94 (1985).
[37]M. I. Martin, Nucl. Data Sheets B **22**, 545 (1977).
[38]V. G. Solov'ev, *Effect of Pair Correlations of the Superconducting Type on the Properties of Nuclei* [in Russian] (Goskomizdat, Moscow, 1963).
[39]V. G. Soloviev, *Theory of Complex Nuclei* (Pergamon Press, Oxford, 1976) [Russ. original, Nauka, Moscow, 1971].
[40]N. N. Bogolyubov, V. V. Tolmachev, and D. V. Shirkov, *A New Method in the Theory of Superconductivity* (Consultants Bureau, New York, 1959) [Russ. original, USSR Academy of Sciences Press, Moscow, 1958].
[41]S. G. Kadmenskiĭ *et al.*, Yad. Fiz. **24**, 400 (1976) [Sov. J. Nucl. Phys. **24**, 208 (1976)].
[42]S. G. Kadmenskiĭ *et al.*, Yad. Fiz. **27**, 906 (1978) [Sov. J. Nucl. Phys. **27**, 481 (1978)].
[43]S. G. Kadmenskiĭ *et al.*, Yad. Fiz. **45**, 942 (1987) [Sov. J. Nucl. Phys. **45**, 585 (1987)].
[44]M. V. Zverev and É. E. Sapershteĭn, Yad. Fiz. **42**, 1082 (1985) [Sov. J. Nucl. Phys. **42**, 683 (1985)].
[45]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N.J., 1963) [Russ. original, Fizmatgiz, Moscow, 1962].
[46]J. Schwinger and B. A. Lippmann, Phys. Rev. **79**, 469 (1950).
[47]H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **90**, 166 (1953); **96**, 448 (1954).
[48]P. E. Hodgson, *The Optical Model of Elastic Scattering* (Clarendon Press, Oxford, 1963) [Russ. transl., Atomizdat, Moscow, 1966].
[49]E. P. Wigner, Ann. Math. **67**, 325 (1958).
[50]S. G. Kadmenskiĭ, V. G. Markushev, and V. I. Furman, Yad. Fiz. **31**, 1175 (1980) [Sov. J. Nucl. Phys. **31**, 607 (1980)].
[51]S. G. Kadmenskiĭ *et al.*, *Strong and Weak Statements in Nuclear Spectroscopy and Nuclear Theory* [in Russian] (Nauka, Leningrad, 1981).
[52]C. Mahaux and H. A. Weidenmuller, *Shell-Model Approach to Nuclear Reactions* (North Holland, Amsterdam, 1969).
[53]G. R. Satchler, Nucl. Phys. **A472**, 591 (1987).
[54]A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vols. 1 and 2 (Benjamin, New York, 1969, 1975).
[55]N. Levinson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **25**, No. 9 (1949).
[56]S. G. Kadmenskiĭ *et al.*, Yad. Fiz. **10**, 730 (1969) [Sov. J. Nucl. Phys. **10**, 422 (1970)].

Translated by Patricia A. Millard