Neutrino mixing and the solar neutrino problem

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The theory of neutrino mixing and oscillations is considered. Besides the old theory of neutrino oscillations based on K^0, $\bar{K}^0$ oscillations, a modified theory of neutrino mixing and oscillations is presented. A theory of resonant neutrino oscillation (MSW effect) in matter is also given. A critical analysis is made of Wolfenstein's equation, from which this effect is obtained. We consider a theory of the enhancement of the oscillations of neutrinos of different masses in matter that arises through the weak interaction of a virtually oscillating neutrino with matter of very great thickness (the sun). We analyze possible ways of solving the problem of the solar neutrino deficit. We estimate the vacuum neutrino mixing angle under the assumption that the solar neutrino deficit is due to neutrino oscillations. The following conclusion is drawn: To demonstrate the existence of oscillations of the solar neutrinos, an experiment that gives a model-independent test of it is needed.
[S1063-7796(96)00201-9]

1. INTRODUCTION

This paper is devoted to a discussion of the problem of neutrino mixing (oscillations) and the solar neutrino deficit.

The history of neutrino physics began with the letter of Pauli to the Tübingen Physical Society,[1] in which he postulated the existence of a new particle, the neutrino, in order to explain the observed continuous spectrum of electrons from nuclear beta decay. Having learnt about this hypothesis of Pauli, Fermi proposed his famous theory of beta decay.[2] An interaction induced by a neutrino was observed for the first time in the experiment of Reines and Cowan.[3]

Besides the electron neutrino emitted in nuclear beta decay, two other neutrinos have been discovered: the muon neutrino[4] and the tau neutrino.[5] The further development of experiment and theory have led to the creation of the modern theory of the electroweak interaction,[6] which in the extended form including all lepton and quark generations has become known as the standard model of the electroweak interaction.

The suggestion that, by analogy with K^0, $\bar{K}^0$ oscillations, there could be neutrino oscillations (i.e., that there could be neutrino–antineutrino oscillations $\nu \leftrightarrow \bar{\nu}$) was considered by Pontecorvo[7] in 1957. It was subsequently conjectured by Maki *et al.*[8] (1962) and Pontecorvo[9] (1967) that there could be mixing (and oscillations) of neutrinos of different species.

Despite the great successes of the standard model, it still needs extension and generalization, since in its present form it does not explain the existence of families of quarks and leptons and does not describe their masses and mixing. Study of neutrino properties is one of the approaches that can lead to new physics, and this makes the study of neutrinos very interesting. One of the interesting questions is that of the existence of a neutrino mass, for which there are experimental indications.[10] Besides this, the fact that neutrinos participate symmetrically together with charged leptons in the charged current of the weak interaction indicates that the neutrino may be a massive particle.[11]

Another aspect of neutrino physics is the problem of lepton families and the associated conservation laws for the lepton number. The conservation laws for the lepton number can be tested to high accuracy by means of neutrinos.

The problem of solar neutrinos arose after the first experiment performed to measure the flux of neutrinos from the sun by the $^{37}Cl-^{37}Ar$ method.[12] The flux was found to be several times smaller than expected from calculations made in accordance with the standard solar model (SSM).[13] It was suggested in Ref. 14 that the solar neutrino deficit could be explained by neutrino oscillations.

Bahcall and others made a more accurate determination of the SSM parameters and calculated the luminosity and flux of neutrinos from the sun[15] that can result from the nuclear processes[16] that take place within it. Subsequently, when the results of the experiment at Kamiokande[17] confirmed the existence of the deficit relative to the SSM calculations, one of the attractive approaches to the explanation of the solar neutrino deficit became resonant enhancement of neutrino oscillations in matter.[18] Resonant enhancement of neutrino oscillations in matter was obtained from Wolfenstein's equation for neutrinos in matter.[19] It was noted in Ref. 20 that Wolfenstein's equation for neutrinos in matter is an equation for neutrinos in matter in which they interact with the matter not through the weak interaction but through a hypothetical interaction that is left–right symmetric (for a more detailed discussion of this question, see Sec. 3). This then raised the problem of establishing the mechanism of resonant enhancement of neutrinos in matter.

Experimentalists later there were then obtained the first results of the Gran Sasso $^{71}Ga-^{71}Ge$ experiment,[21] that within a 3σ limit did not disagree with the SSM calculations. The new data from the SAGE experiment[22] are fairly close to the Gran Sasso results.

In Ref. 23, the present author proposed a new mechanism of enhancement of neutrino oscillations in matter that is realized through the weak interaction of oscillating neutrinos with matter if the thickness of this matter is sufficiently great (see Sec. 3). Data appeared[24] that could be interpreted as a manifestation of oscillations of atmospheric neutrinos observed through the thickness of the earth. Besides the facilities mentioned above that detect solar neutrinos, new facilities are currently being prepared (Super-Kamiokande[25] and SNO[26]) for detection of neutrinos from the sun, and the data obtained from them (SNO) can be used for a model-independent analysis of the neutrino oscillations.

Section 2 is devoted to discussion of the theory of neutrino mixing and oscillations: lepton numbers, elements of the standard theory of the electroweak interaction, the neutrino mixing scheme, and neutrino oscillations. In Sec. 3, we consider aspects of neutrino oscillations in matter: resonant neutrino oscillations in matter, and the enhancement of the oscillations of neutrinos of different masses in matter. In Sec. 4, we discuss possible ways of solving the problem of the solar neutrino deficit. We describe briefly the experimental facilities that measure the flux of neutrinos from the sun.

2. THEORY OF NEUTRINO MIXING AND OSCILLATIONS

Before we discuss mixing and oscillations, we formulate the conservation laws for the lepton numbers and give the elements of the electroweak theory.

1. Lepton numbers

Up to now, it has been established that there exist three families of leptons and quarks (we give the quark sector, since in what follows we shall use the analogy of the lepton and quark sectors):

$$u\nu_e; \quad c\nu_\mu; \quad t\nu_\tau; \qquad (2.1)$$

$$de; \quad s\mu; \quad b\tau.$$

Following the existing experimental data, we shall assume that each family of leptons has its corresponding conserved lepton number (l_e, l_μ, l_τ):[27]

$$\sum_i l_e^i = \text{const},$$

$$\sum_i l_\mu^i = \text{const}, \qquad (2.2)$$

$$\sum_i l_\tau^i = \text{const}.$$

Accordingly, all the lepton numbers—electron, muon, tau—are conserved separately in processes. It is here supposed that the conservation of these lepton numbers must be associated with definite types of interaction. For example, the lepton numbers are (separately) conserved in the strong, electromagnetic, and weak (we have in mind exchange of W and Z^0 bosons) processes. However, in the extended theory of the electroweak interaction with inclusion of a matrix of Kobayashi–Maskawa type these lepton numbers are not conserved, since this matrix contains nondiagonal elements that mix the lepton numbers. A matrix of Kobayashi–Maskawa type parametrizes processes with allowance for nonconservation of the lepton numbers, and one naturally assumes that such nonconservation must be based on dynamical theories.[20]

In Table I, we give upper limits for the branching ratios

TABLE I. Upper limits for the branching ratios of processes forbidden by the conservation law for lepton charges.[27]

Process	R
$\mu \to e\gamma$	$<4.9\cdot 10^{-11}$
$\mu \to ee\bar{e}$	$<1.0\cdot 10^{-12}$
$\mu \to e2\gamma$	$<7.2\cdot 10^{-11}$
$K_L \to e^{\pm}+\mu^{\mp}$	$<8\cdot 10^{-6}$
$\mu^- + \text{I} \to e^+ + \text{Sb}$	$<3\cdot 10^{-10}$

of processes forbidden by the lepton-number conservation law (2.2). These can be used to obtain corresponding bounds on the parameters of matrices of Kobayashi–Maskawa type or the corresponding theories with nonconservation of the lepton numbers.

It can be seen from the expression (2.1) that there is an analogy between the quark and lepton sectors. Then, proceeding from the experimental fact of the mixing of the d, s, and b quarks, one can suppose that the neutrinos ν_e, ν_μ, and ν_τ may also be mixed, i.e., there will be nonconservation of the lepton numbers that will lead to neutrino oscillations. In what follows, we shall need to consider the electroweak theory, and we therefore give the elements of this theory.

2. Elements of the standard theory of the electroweak interaction

At the present time, all available experimental data agree well with the standard theory of the electroweak interaction[6] proposed by Glashow, Weinberg, and Salam. We give the main elements of this theory. The Lagrangian of the theory contains left and right doublets of leptons and quarks,

$$\Psi_{lL} = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L, \quad \Psi_{lR}, \quad l = e, \mu, \tau,$$

$$\Psi_{iL} = \begin{pmatrix} u \\ d \end{pmatrix}_L^{i=1}, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L^{i=2}, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L^{i=3}, \qquad (2.3)$$

$$\Psi_{iR} = u_R, d_R; \quad c_R, s_R; \quad t_R, b_R,$$

and right singlets of charged leptons and quarks.

The theory is based on the local group $SU(2)_L \times U(1)$ and contains two coupling constants g, g'. The covariant derivatives have the form

$$\partial_\alpha \Psi_{lL} \to \left[\partial_\alpha - ig\,\frac{\tau^i}{2}A_\alpha^i - ig' Y_L^{\text{lep}} B_\alpha\right]\Psi_{lL},$$

$$\partial_\alpha \Psi_{iL} \to \left[\partial_\alpha - ig\,\frac{\tau^i}{2}A_\alpha^i - ig' Y_L^{\text{quark}} B_\alpha\right]\Psi_{iL},$$

$$\partial_\alpha \Psi_{lR} \to \left[\partial_\alpha - i\,\frac{g'}{2}\,Y_R^{\text{lep}} B_\alpha\right]\Psi_{lR},$$

$$\partial_\alpha \Psi_{iR} \to \left[\partial_\alpha - i\,\frac{g'}{2}\,Y_R^{\text{quark}} B_\alpha\right]\Psi_{iR}, \qquad (2.4)$$

where A_α^i, B_α are the gauge fields associated with the groups $SU(2)_L$ and $U(1)$; Y is the hypercharge of the leptons and quarks.

The analog of the Gell-Mann–Nishijima relation in the considered case is

$$Q=T_3^W+\frac{Y}{2}, \tag{2.5}$$

where Q is the electric charge, and T_3^W is the third projection of the isospin.

For the lepton and quark hypercharges, we obtain from the expression (2.5)

$$Y_L^{\text{lep}}=-1, \quad Y_L^{\text{quark}}=\frac{1}{3}, \tag{2.6}$$

$$Y_R^{\text{lep}}=-2, \quad Y_R^{\text{quark}}=2e_q,$$

where e_q is the electric charge of the corresponding quarks.

Using the standard scheme, we can pass from (2.4) and (2.6) to the following expression for the interaction Lagrangian:

$$\mathscr{L}_I=igj^{K,\alpha}A_\alpha^K+ig'\frac{1}{2}j^{Y,\alpha}B_\alpha, \tag{2.7}$$

where

$$j^{K,\alpha}=\sum_{i=1}^{3}\bar{\Psi}_{i,L}\gamma^\alpha\frac{\tau^K}{2}\Psi_{i,L}+\sum_{l=e,\mu,\tau}\bar{\Psi}_{1,L}\gamma^\alpha\frac{\tau^K}{2}\Psi_{l,L}, \tag{2.8}$$

and

$$\frac{1}{2}j^{Y,\alpha}=j^{em,\alpha}-j^{3,\alpha} \tag{2.9}$$

($j^{em,\alpha}$ is the electromagnetic current of the quarks and leptons).

On the transition from the fields A_α^3, B_α to the fields Z_α, A_α:

$$Z_\alpha=A_\alpha^3\cos\theta_W-B_\alpha\sin\theta_W, \qquad A_\alpha=A_\alpha^3\sin\theta_W+B_\alpha\cos\theta_W \tag{2.10}$$

the interaction Lagrangian for the fields Z_α, A_α acquires the form

$$\mathscr{L}_I^0=i\frac{g}{2\cos\theta_W}j^{0,\alpha}Z_\alpha+iej^{em,\alpha}A_\alpha, \tag{2.11}$$

where $j^{0,\alpha}=2j^{3,\alpha}-2\sin^2\theta_W j^{em,\alpha}$ is the neutral current of the standard model.

Note that the Lagrangians (2.7) and (2.11) are obtained for Dirac (particles) leptons and quarks with charges g, g' or e, g, using the principle of local gauge invariance. If $SU(2)_L\times U(1)$ gauge invariance is required, the masses of all the particles must be equal to zero (i.e., in this theory particles cannot have masses[28,29]). To obtain masses of the particles, the standard theory of the electroweak interaction based on the assumption of $SU(2)_L\times U(1)$ gauge invariance is broken spontaneously [down to $U(1)$] through the Higgs mechanism.[30] We briefly consider this mechanism.

A doublet of scalar Higgs fields

$$\Phi=\begin{pmatrix}\Phi^{(+)}\\ \Phi^0\end{pmatrix}$$

with hypercharge equal to unity (2.5) is introduced. It is assumed that this doublet interacts with the vector and fermion fields in such a way that local gauge invariance is not broken. To the Lagrangian of the electroweak theory there is added the Higgs potential $V(\Phi^+,\Phi)$,

$$V(\Phi^+,\Phi)=k(\Phi^+,\Phi)^2-\mu^2(\Phi^+,\Phi), \tag{2.12}$$

(k and μ^2 are positive constants), which leads to vacuum degeneracy and to a nonvanishing vacuum expectation value $\langle\Phi^0\rangle$ of the field Φ^0:

$$\langle\Phi^0\rangle=\sqrt{\frac{\mu^2}{2k}}. \tag{2.13}$$

This means that (fixing the vacuum state) we can generate a mass term of the fields of the intermediate bosons, fermions, and Higgs boson. As an example, we consider the scheme for obtaining quark masses. For this purpose, we use a Lagrangian of Yukawa type that is an invariant:

$$\mathscr{L}_1=-\frac{\sqrt{2}}{\lambda}\sum_{\substack{i\\ q=d,s,b}}\bar{\Psi}_{iL}M_{iq}^{(1)}q_R\bar{\Psi}+\text{H.c.} \tag{2.14}$$

$$\mathscr{L}_2=-\frac{\sqrt{2}}{\lambda}\sum_{\substack{i\\ q=u,c,t}}\bar{\Psi}_{iL}M_{iq}^{(2)}q_R\bar{\Psi}+\text{H.c.},$$

where $M^{(1)}$, $M^{(2)}$ are complex 3×3 matrices, and

$$\bar{\Phi}=i\tau_2\Phi^*=\begin{pmatrix}\Phi^{0*}\\ -\Phi^{+*}\end{pmatrix} \tag{2.15}$$

is a doublet of Higgs fields with hypercharge −1.

Taking into account (2.13) and using the gauge invariance of the Lagrangian (2.12), (2.14), we can choose (in the unitary gauge)

$$\Phi(x)=\begin{pmatrix}0\\ \frac{\lambda+H^0(x)}{\sqrt{2}}\end{pmatrix}, \quad \bar{\Phi}=\begin{pmatrix}\frac{\lambda+H^0(x)}{\sqrt{2}}\\ 0\end{pmatrix}, \tag{2.16}$$

where $H^0(x)$ is the neutral scalar Higgs field.

Substituting (2.16) in (2.14), we obtain for the quark masses the expressions

$$\mathscr{L}_1=-\bar{p}_LM_1'p_R+\text{H.c.}, \qquad \mathscr{L}_2=-\bar{n}_LM_2'n_R+\text{H.c.}, \tag{2.17}$$

where

$$p_{L,R}=\begin{pmatrix}u_{L,R}\\ c_{L,R}\\ t_{L,R}\end{pmatrix}, \quad n_{L,R}=\begin{pmatrix}d_{L,R}\\ s_{L,R}\\ b_{L,R}\end{pmatrix}.$$

Thus, the elements M_1', M_2' of the quark mass matrix are equal to the constants of the quark–Higgs-boson Yukawa coupling up to the factor $\sqrt{2}/\lambda$.

The matrices M_1' and M_2' in (2.17) can be diagonalized in the standard manner:

$$M_1'=U_Lm_1U_R^+, \quad M_2'=V_Lm_2V_R^+, \tag{2.18}$$

where $U_{L,R}$ and $V_{L,R}$ are unitary matrices, and m_1 and m_2 are diagonal matrices with positive elements. Then the expression for the mass Lagrangian (2.17) acquires the form

$$\mathscr{L}=\mathscr{L}_1+\mathscr{L}_2=-\bar{p}'m_1p'-\bar{n}'m_2n'$$

$$=\sum_{q=u,\dots} m_q\bar{q}'q; \quad p'=p'_L+p'_R, \quad n'=n'_L+n'_R, \tag{2.19}$$

where

$$p'_{L,R}=U^+_{L,R}p_{L,R'} \quad n'_{L,R}=V^+_{L,R}n_{L,R} \tag{2.20}$$

and $q'(x)\to d'(x),s'(x),\dots,t'(x)$ are quark fields with definite masses.

If (2.20) is taken into account, the expression for the charged and neutral quark currents takes the form

$$j^{(+)\alpha}=2\bar{p}_L\gamma^\alpha n_\alpha=2\bar{p}'_L\gamma^\alpha U_{\rm KM}n'_L,$$

$$j^{0,\alpha}=\bar{p}'_L\gamma^\alpha p'_L-\bar{n}'_L\gamma^\alpha n'_L-2\sin^2\theta_W\left[\frac{2}{3}\bar{p}'\gamma^\alpha p'+\left(-\frac{1}{3}\right)\bar{n}'\gamma^\alpha n'\right], \tag{2.21}$$

where $U_{\rm KM}=U^+_L V_L$. The subscript KM of the matrix U is in recognition of Kobayashi and Maskawa, who first obtained the form of this matrix in parametrized form.[31]

In principle, the entire procedure (2.14)–(2.21) with the quark masses can be repeated for the leptons if it is assumed that the neutrinos are massive and if right components are introduced for them. Before we proceed to the next subsection, we note that the entire above procedure for the quark masses is purely formal, since in a more rigorous approach it is necessary to formulate a common group that unites the quarks and bosons, and it is in the framework of this group that the nonconservation of the flavor numbers must occur and, in principle, the masses of the quarks and leptons must be calculated (or relations between the masses must arise). Much work has been done in this direction,[32] but the problem with the quark and lepton masses is still not yet solved. For this reason, it is evidently more correct to regard matrices like $U_{\rm KM}$ as phenomenological matrices that parametrize the quark mixing (as we have already noted, the same phenomenological matrix can be introduced for the leptons). As an example of the construction of a dynamical theory with quark mixing, we mention Ref. 33, in which this problem was solved by the introduction of three doublets $B^\pm,C^\pm,D^\pm$ of heavy vector bosons.

3. Scheme of neutrino mixing

Following the established tradition, we describe neutrino mixing by means of the introduction of a mass Lagrangian. In addition, for the reason given in Ref. 34, we shall not consider Majorana neutrinos. (From the complete set of available experimental results, it appears that all the existing fermions are Dirac particles.) The interaction between the fermions in the electroweak theory is introduced by means of the principle of local gauge invariance (2.4), i.e., through this principle the fermions acquire a charge. In the case of a Majorana particle its neutrality (i.e., the fact that this particle has no charge) means that it cannot be given an electroweak interaction.

We first consider the case of mixing of two neutrino species [the mixing is constructed by analogy with K^0, $\bar{K}^0$ mesons (Pontecorvo)], and we then consider the more general case.

The mass Lagrangian of the two neutrino species (ν_e,ν_μ) can be written in the form

$$\mathscr{L}'_M=-\left[m_{\nu_e\nu_e}\bar{\nu}_e\nu_e+m_{\nu_\mu\nu_\mu}\bar{\nu}_\mu\nu_\mu+m_{\nu_e\nu_\mu}(\bar{\nu}_e\nu_\mu+\bar{\nu}_\mu\nu_e]\right.$$

$$\equiv-(\bar{\nu}_e,\bar{\nu}_\mu)\begin{pmatrix} m_{\nu_e\nu_e} m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} m_{\nu_\mu\nu_\mu}\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}. \tag{2.22}$$

In the expression (2.22), the third term is responsible for the neutrino mixing and does not conserve the lepton numbers l_e and l_μ. It is obvious that the introduction of such a term is equivalent to the introduction of an interaction that does not conserve lepton numbers. By means of the substitution

$$\nu_e=\cos\theta\nu_1+\sin\theta\nu_2,$$

$$\nu_\mu=-\sin\theta\nu_1+\cos\theta\nu_2 \tag{2.23}$$

we can diagonalize the mass Lagrangian $\mathscr{L}'_M$, and then

$$\mathscr{L}'_M=-[m_1\bar{\nu}_1\nu_1+m_2\bar{\nu}_2\nu_2], \tag{2.24}$$

where

$$m_{1,2}=\frac{1}{2}[(m_{\nu_e\nu_e}+m_{\nu_\mu\nu_\mu})\pm[(m_{\nu_e\nu_e}-m_{\nu_\mu\nu_\mu})^2+4m^2_{\nu_\mu\nu_e}]^{1/2}]$$

and the angle θ is determined by

$$\tan 2\theta=2m_{\nu_e\nu_\mu}/(m_{\nu_\mu\nu_\mu}-m_{\nu_e\nu_e}). \tag{2.25}$$

It can be seen from the expression (2.25) that if $m_{\nu_e\nu_e}=m_{\nu_\mu\nu_\mu}$, then the mixing angle will be $\pi/4$, irrespective of the value of $m_{\nu_e\nu_\mu}$.

We now turn to consideration of the general case. In it, the mass Lagrangian will have the form

$$\mathscr{L}_M=-\bar{\nu}_R M\nu_L+\text{H.c.}\equiv\sum_{l,l'=e,\mu,\tau}\nu_{l'R}M_{l'l}\nu_{lL}+\text{H.c.}, \tag{2.26}$$

where M is a complex 3×3 matrix. It should be noted that ν_R is absent in the Lagrangian of the electroweak interaction (2.3), (2.4). By means of the transformation

$$M=VmU^+, \tag{2.27}$$

where V and U are unitary matrices, we reduce $\mathscr{L}_M$ to diagonal form:

$$\mathscr{L}_M=-\bar{\nu}_R m\nu_L+\text{H.c.}\equiv\sum_{k=1}^{3} m_k\bar{\nu}_k\nu_k+\text{H.c.}, \tag{2.28}$$

where

$$m_{ik}=m_k\delta_{ik},$$

$$\nu'_L=U^+\nu_L,\quad \nu'_R=V^+\nu_R,\quad \nu'=\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}. \tag{2.29}$$

We note that the Lagrangian (2.26) is invariant with respect to the global gauge transformation

$$\nu_k(x) \to e^{i\Lambda}\nu_k(x)$$

or

$$l(x) \to e^{i\Lambda} l(x), \quad l=e,\mu,\tau,$$

i.e., the lepton numbers are separately not conserved (i.e., the neutrinos are mixed), but there does appear the lepton number l, which is associated with the gauge transformation $e^{\pm i\Lambda}$, and this will be conserved.

4. Neutrino oscillations

The theory of neutrino oscillations is constructed by analogy with $K^0 \leftrightarrow \bar{K}^0$ oscillations (a more detailed study of $K^0 \leftrightarrow \bar{K}^0$ oscillations will be given below). We assume that the state vector of the neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ participating in the weak interactions is a superposition of state vectors with different masses. We then ask: How will a neutrino beam behave? It is clear that at a certain distance from the source the neutrino state vector acquires additional phases due to the fact that the different neutrinos have different masses. Then the probability of finding a neutrino of a definite species will be a periodic function of the distance between the source and the detector. This effect is called neutrino oscillation.[7]

We first consider oscillations of two neutrino species, ν_e and ν_μ, and we then give the expressions for the oscillations in the case of three neutrinos. The particles ν_1, ν_2 [see (2.23) and (2.24)] with masses m_1 and m_2 evolve in time in accordance with the law

$$\nu_1(t)=e^{-iE_1t}\nu_1(0); \quad \nu_2(t)=e^{-iE_2t}\nu_2(0) \tag{2.30}$$

where $E_k^2=(p^2+m_k^2)$, $k=1,2$.

If these particles propagate without interaction, then

$$\nu_e(t)=\cos\theta e^{-iE_1t}\nu_1(0)+\sin\theta e^{-iE_2t}\nu_2(0), \tag{2.31}$$

$$\nu_\mu(t)=-\sin\theta e^{-iE_1t}\nu_1(0)+\cos\theta e^{-iE_2t}\nu_2(0).$$

Using the expressions for ν_1 and ν_2 in (2.23) and substituting them in (2.31), we obtain

$$\nu_e(t)=[e^{-iE_1t}\cos^2\theta+e^{-iE_2t}\sin^2\theta]\nu_e(0)+[e^{-iE_1t} - e^{-iE_2t}]\sin\theta\cos\theta\nu_\mu(0), \tag{2.32}$$

$$\nu_\mu(t)=[e^{-iE_1t}\sin^2\theta+e^{-iE_2t}\cos^2\theta]\nu_\mu(0)+[e^{-iE_1t} - e^{-iE_2t}]\sin\theta\cos\theta\nu_e(0).$$

The probability that a neutrino produced as ν_e at the time $t=0$ will be in the state ν_μ at time t is given by the square of the absolute magnitude of the amplitude $\nu_\mu(0)$ in (2.32), i.e.,

$$P(\nu_e \to \nu_\mu)=|(\nu_\mu(0)/\nu_e(t))|^2 = \frac{1}{2}\sin^2 2\theta[1-\cos((m_2^2-m_1^2)/2p)t], \tag{2.33}$$

where it is assumed that $p \gg m_1, m_2$; $E_k \simeq p+m_k^2/2p$.

The expression (2.33) describes oscillations of the neutrino flavor. The mixing angle θ characterizes the degree of mixing. The probability $P(\nu_e \to \nu_\mu)$ changes in accordance with a periodic law with the distance, with periodicity determined by the expression $L_0=2\pi(2p/|m_2^2-m_1^2|)$. These oscillations are a typical interference effect due to the nondiagonal form of the mass matrix (2.22).

The probability $P(\nu_e \to \nu_e)$ that the neutrino ν_e produced at the time $t=0$ is still ν_e at time t is given by the square of the absolute magnitude of the amplitude $\nu_e(0)$ in (2.32). Since the states (2.32) are normalized,

$$P(\nu_e \to \nu_e)+P(\nu_e \to \nu_\mu)=1. \tag{2.34}$$

Thus, we see that the flavor oscillations due to the nondiagonality of the neutrino mass matrix lead to nonconservation of the lepton numbers l_e and l_μ. However, the total lepton number $l=l_e+l_\mu$ is conserved, as can be seen from (2.34).

We now consider oscillations of three neutrino species. To this end, we shall use an expression of the type (2.20) and write it in components:

$$\nu_e=\sum_{k=1}^{3} U_{lk}^*\nu_k, \quad l=e,\mu,\tau, \tag{2.35}$$

where ν_k is the state vector of a neutrino with momentum p and mass m_k. The expression (2.35) is based on the fact that the mass differences of the neutrinos ν_k are so small that a coherent superposition of state vectors of neutrinos with different masses can be formed in the weak interactions:

$$\nu_k(t)=e^{-iE_kt}\nu_k(0). \tag{2.36}$$

Then

$$\nu_l(t)=\sum_{k=1}^{3} e^{-iE_kt}U_{lk}^*\nu_k(0). \tag{2.37}$$

Using the unitarity of the matrix U, we can rewrite (2.37) in the form

$$\nu_l(t)=\sum_{l'=e,\mu,\tau}\nu_{l'}(0)\sum_{k=1}^{3} U_{l'k}e^{-iE_kt}U_{lk}^* \tag{2.38}$$

and, introducing the notation

$$a_{\nu_{l'}\nu_l}(t)=\sum_{k=1}^{3} U_{l'k}e^{-iE_kt}U_{lk}^*, \tag{2.39}$$

we obtain

$$\nu_l(t)=\sum_{l'=e,\mu,\tau} a_{\nu_{l},\nu_{l}}(t)\nu_l'(0), \tag{2.40}$$

where $a_{\nu_{l'}\nu_l}(t)$ is the probability amplitude of the transition $\nu_l \to \nu_{l'}$. The corresponding probability of the transition $\nu_l \to \nu_{l'}$ is

$$P_{\nu_{l'}\nu_l}(t)=\left|\sum_{k=1}^{3} U_{l'k}e^{-iE_kt}U_{lk}^*\right|^2 \tag{2.41}$$

It is obvious that

$$\sum_{l'=e,\mu,\tau} p_{\nu_{l'}\nu_l}(t)=1. \tag{2.42}$$

Using $E_k \simeq p+m_k^2/2p$, we can rewrite the expression (2.41) in the form

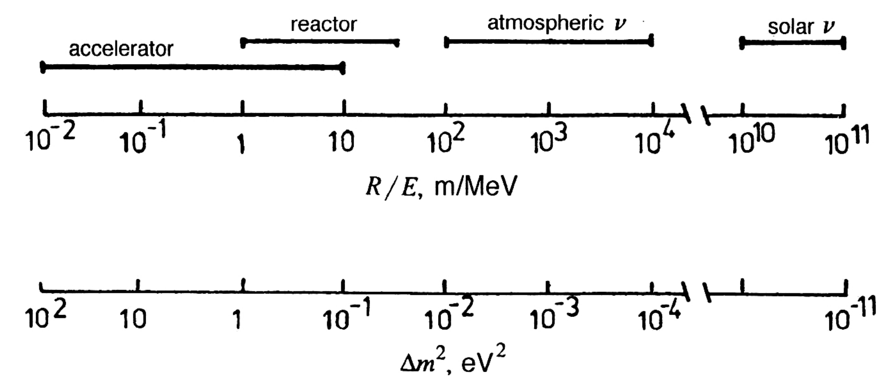

FIG. 1. Ranges of values of R/E and Δm^2 (eV^2) accessible in different experiments.

$$p_{\nu_{l'}\nu_l}(R,p)=\sum_{k=1}^{3}|U_{lk}|^2|U_{l'k}|^2+2\sum_{k>j}|U_{l'k}U_{lk}^*U_{l'j}^*U_{lj}|\times\cos\left[\frac{m_k^2-m_j^2}{2p}R-\Phi_{kj,l'l}\right], \tag{2.43}$$

where R is the distance from the source to the detector, and

$$\Phi_{kj,l'l}=\arg(U_{l'k}U_{lk}^*U_{l'j}^*U_{lj}).$$

The expression (2.43) can be rewritten in the form

$$P_{\nu_{l'}\nu_l}(R,p)=\delta_{ll}+2\sum_{k>j}|U_{l'j}U_{lj}^*U_{l'k}^*U_{lk}|\times\left[\cos\left(\frac{m_j^2-m_k^2}{2p}R-\Phi_{jk,l'l}\right)-\cos\Phi_{jk,l'l}\right]. \tag{2.44}$$

It can be seen from the expressions (2.33) and (2.44) that the neutrino oscillation length is determined by

$$L_0(\mathrm{m})=\frac{2.48E\ (\mathrm{MeV})}{|m_k^2-m_j^2|\ (\mathrm{eV}^2)};\quad k\neq j;\quad k,j=1-3, \tag{2.45}$$

where $E=pc$ is the energy.

It can be seen from (2.33), (2.43), and (2.45) that the neutrino oscillations depend on the ratio of the distance R to the energy E. For clarity, Fig. 1 shows the sensitivity ranges of the different oscillation experiments with axes (R/E) and Δm^2 (the square of the mass difference).

So far, we have considered the standard theory of neutrino oscillations. However, it must be noted that this theory describes neutrino oscillations purely kinematically, i.e., all the parameters of the mass matrix M in (2.26) are constants and do not depend on the time. However, as we noted earlier, such a nondiagonal mass matrix must correspond to a dynamical theory in which the lepton numbers are not conserved. If this is the case, then the constant parameters of the nondiagonal mass matrix are averaged quantities obtained from a dynamical theory. Then the picture of the neutrino oscillations that we have given above is a rough averaged description of the actual picture of neutrino oscillations. In a more detailed dynamical description of the neutrino oscillations, new features of the oscillations are manifested. We now turn to a qualitative dynamical analysis of the oscillations of $K^0,\bar K^0$ mesons, since this process has been studied fairly well, and we then turn to the application of our results to the case of neutrino oscillations.[28,29,23]

$K^0,\bar K^0$ oscillations. We shall assume that K^0, $\bar K^0$ are produced in strong interactions (we shall discuss the case when K^0, $\bar K^0$ are produced in weak processes below); then the mass matrix of the $\bar K^0$, K^0 mesons will have a diagonal form (i.e., strangeness is conserved in the strong interactions). The production time of the K^0, $\bar K^0$ mesons is the characteristic time $\Delta t_{\mathrm{str.}}\simeq 10^{-23}$ s of the strong interaction:

$$\begin{vmatrix} m_{K^0K^0} & 0 \\ 0 & m_{\bar K^0\bar K^0}\end{vmatrix},\quad m_{K^0K^0}=m_{\bar K^0\bar K^0}\equiv m_{K^0}. \tag{2.46}$$

However, besides the strong interaction there is also the weak interaction of quarks, which leads to nonconservation of strangeness, and then the mass matrix (2.46) becomes nondiagonal:

$$\begin{vmatrix} m_{K^0K^0} & m_{K^0\bar K^0} \\ m_{\bar K^0K^0} & m_{\bar K^0\bar K^0}\end{vmatrix} \tag{2.47}$$

[estimates of the nondiagonal term of the mass matrix (2.47) and corresponding references can be found in Ref. 34]. We now see that since the mass term $m_{K^0\bar K^0}$ arises dynamically, a certain time is needed for it to occur, and this time is the characteristic time $\Delta t_{\mathrm{weak}}\simeq 10^{-8}$ s of the weak interaction. As a result, the more detailed consideration of the process of evolution of the K^0, $\bar K^0$ mesons shows that the process of K^0, $\bar K^0$ production and the process of nonconservation of strangeness, leading to oscillations of the K^0, $\bar K^0$ mesons, take place with their own separate characteristic times, and it is obvious that these processes must be considered separately (as we have already noted, in the standard approach[7,9,35] only an averaged kinematic description of the K^0, $\bar K^0$ oscillations is given).

Taking into account the dynamics, we now give a phenomenological description of the process of K^0, $\bar K^0$ oscillations. We treat the process of the production of K^0, $\bar K^0$ mesons as a quasistationary process with characteristic time $\Delta t_{\mathrm{str.}}\simeq 10^{-23}$ s. During this characteristic time, the weak interaction will give rise to strangeness nonconservation, and as a result the K^0, $\bar K^0$ mass matrix will become nondiagonal. We estimate the probability of this process:

$$W(t=\pi\Delta t_{\mathrm{str.}})=\frac{(1-e^{-t/\Delta t_{\mathrm{weak}}})}{(1-e^{-t/\Delta t_{\mathrm{str.}}})}\simeq\pi\,\frac{\Delta t_{\mathrm{str.}}}{\Delta t_{\mathrm{weak}}}\simeq\pi\cdot 10^{-15}, \quad (2.48)$$

where $1-e^{-t/\Delta t}$ is the probability of decay of the quasistationary state during the characteristic time Δt. Thus, after a time $t\simeq 10^{-23}\pi$ s the K^0, $\bar{K}^0$ will become nondiagonal with probability $\pi\cdot 10^{-15}$. In this case, to determine what will happen to the quasistationary system it is necessary to diagonalize the mass matrix (2.47):

$$\begin{vmatrix} m_{K^0K^0} & m_{K^0\bar{K}^0} \\ m_{\bar{K}^0K^0} & m_{\bar{K}^0\bar{K}^0} \end{vmatrix} \to \begin{vmatrix} m_{K_1^0K_1^0} & 0 \\ 0 & m_{K_2^0K_2^0} \end{vmatrix}, \quad (2.49)$$

$$K_1^0=\frac{K^0+\bar{K}^0}{\sqrt{2}}, \quad K_2^0=\frac{K^0-\bar{K}^0}{\sqrt{2}},$$

$$m_{K_1^0K_1^0}=m_{K^0K^0}+m_{K^0\bar{K}^0},$$

$$m_{K^0K^0}=(m_{K_1^0K_1^0}+m_{K_2^0K_2^0})/2,$$

and the mixing angle is $\theta=\pi/4$, since $m_{K^0K^0}=m_{\bar{K}^0}m_{\bar{K}^0}$. It can be seen from (2.49) that in this case the mass eigenstates will be the states K_1^0, K_2^0. Thus, with probability $\pi\cdot 10^{-15}$ there will be production of K_1^0, K_2^0, while in the remaining cases there will be production of K^0, $\bar{K}^0$ mesons.

Let us now consider what will then happen to the K^0, $\bar{K}^0$ mesons (we shall assume that they move in vacuum) owing to the presence of the strangeness-nonconserving weak interaction.

As we have noted, the K^0, $\bar{K}^0$ mesons produced in the strong interactions are an eigenstate of this interaction, and they are produced on definite mass shells:[36]

$$P_{K^0}^2=m_{K^0}^2, \quad P_{\bar{K}^0}^2=m_{\bar{K}^0}^2. \quad (2.50)$$

Because of the presence of the weak interaction, strangeness will not be conserved, as a result of which the mass matrix becomes diagonal [see (2.49)]. Diagonalization of this mass matrix leads to the occurrence of the states K_1^0 and K_2^0. Further, since the K^0, $\bar{K}^0$ mesons are produced and move in vacuum and are on the corresponding mass shells, the K^0, $\bar{K}^0$ states are transformed into a superposition of K_1^0 and K_2^0, and this process must be dynamical (it is obvious that such a process can be approximately described by characteristics that are kinematic averages, but one will then lose information about the details of the process). Further, the K_1^0 and K_2^0 will decay,[37] and on the background of these decays there will be oscillations of the K^0, $\bar{K}^0$ mesons. The oscillation length will be given by [see (2.33) and (2.45)]

$$L_{\mathrm{osc}}(\mathrm{m})=\frac{2.48p(\mathrm{MeV})}{4m_{K^0\bar{K}^0}m_{K^0}(\mathrm{eV}^2)}. \quad (2.51)$$

It is interesting to note that because the K^0 and $\bar{K}^0$ mesons consist of $\bar{s}$, d and $\bar{d}$, s quarks (the masses of the s and d quarks are different) the masses of the K^0 and $\bar{K}^0$ mesons are equal, and there will be a real and not virtual oscillation of the K^0, $\bar{K}^0$ mesons.

We now briefly discuss the mixing and oscillation of the K^0, $\bar{K}^0$ mesons in the case when they are produced in weak interactions. It should be noted that such production channels of these mesons are difficult to observe because of the competition of other channels. However, in principle the production of K^0, $\bar{K}^0$ mesons in weak interactions is possible. This is due to the fact that the charged current

$$j_F^{\mu}=(\bar{u}\bar{c})_L\gamma^{\mu}V\begin{pmatrix} d \\ s \end{pmatrix}_L, \quad (2.52)$$

where $V=\begin{vmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{vmatrix}$ is the Cabibbo matrix,[38] can be decomposed into two terms: a diagonal term proportional to $\cos\theta_C$ and a nondiagonal term proportional to $\sin\theta_C$. Thus, we have the weak interaction without allowance for quark mixing and with allowance for quark mixing. In the weak interactions without quark mixing (with conservation of strangeness) the mass eigenstates are the K^0, $\bar{K}^0$ mesons, and these are identical to the mass eigenstate of the strong interaction. In contrast, in the weak interactions with quark mixing the mass eigenstates are the K_1^0, K_2^0 states. Decomposing the weak-interaction current in this manner, and knowing that the contributions of these parts are in the ratio $\tan\theta_C$ (θ_C is the Cabibbo angle), we can estimate the probability with which the states K^0, $\bar{K}^0$ and K_1^0, K_2^0 are produced in the weak interactions. The ratio of the amplitudes of these processes will be proportional to

$$\frac{A(K_1^0,K_2^0)}{A(K^0,\bar{K}^0)}\simeq\tan^2\theta_C, \quad (2.53)$$

and the probability will be

$$P=\left|\frac{A(K_1^0,K_2^0)}{A(K^0,\bar{K}^0)}\right|^2\simeq\tan^4\theta_C. \quad (2.54)$$

Thus, in the weak interactions the K_1^0, K_2^0 mesons are produced with probability P, while the K^0, $\bar{K}^0$ mesons are produced with probability $1-P$. The oscillations of the produced K^0, $\bar{K}^0$ mesons will proceed in the same way as the oscillations described above.

The following question arises: How would the oscillations of the K^0, $\bar{K}^0$ mesons in vacuum occur if the masses of the K^0 and $\bar{K}^0$ mesons were different? A detailed discussion of this question with allowance for the decay widths was given in Ref. 36.

In order to apply our results to the case of neutrino oscillations, we here discuss briefly a well-known example from particle physics, the vector-dominance model, in which mixing of particles with different masses occurs.

$\gamma\leftrightarrow\rho^0$ oscillations. Let us now consider the mixing (oscillations) that occur in the vector-dominance model.[39] In this model, one considers mixing of the vector fields of the strong, $V_{\mu}(\rho^0)$, and electromagnetic, A_{μ}, interactions. The original fields $\begin{pmatrix} V_{\mu} \\ A_{\mu} \end{pmatrix}$ are mixed when the strong and electromagnetic interactions are included:[40]

$$V'_{\mu}=\cos\varphi V_{\mu}-\sin\varphi A_{\mu},$$

$$A'_{\mu}=\sin\varphi V_{\mu}+\cos\varphi A_{\mu}, \quad (2.55)$$

where $\cos\varphi=G/\sqrt{G^2+e^2}$, and G and e are the strong and electromagnetic coupling constants.

By virtue of the gauge invariance of the electromagnetic interaction and isospin conservation in strong interactions, the mass matrix of the fields V_μ and A_μ must be diagonal:

$$\begin{vmatrix} m_A^2 & \mu^2 \\ \mu^2 & m_V^2 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & m_\rho^2 \end{vmatrix}. \tag{2.56}$$

How will $\gamma \leftrightarrow \rho^0$ oscillations occur? Since the fields A_μ and V_μ have very different masses, there cannot be real oscillations between them as happens between K^0 and $\bar{K}^0$ ($m_{K^0} = m_{\bar{K}^0}$). There must be virtual oscillations between these fields (particles). This means that the transitions to the fields A'_μ and V'_μ must be virtual (there cannot be a real transition of a photon into a ρ^0 meson with subsequent decay of the meson). To make such a transition ($\gamma \to \rho^0$) real (and not virtual), the photon must participate in an interaction in order to go over to the mass shell of the ρ^0 meson.

Let us consider the system of oscillating particles as a bound system. Phenomenologically, the extent to which the particles are bound will be determined by the nondiagonal terms of the mass matrix [this is the mass matrix (2.49) in the case of K^0, $\bar{K}^0$ mesons, and M in (2.27) in the case of neutrinos] or, more precisely, by the ratio of the nondiagonal to the diagonal terms of the mass matrix:

$$A_{ik} = m_{ik}/m_{ii}, \quad i \neq k, \quad i,k = 1-2(3). \tag{2.57}$$

If $A_{ik} \ll 1$, then the given system can be regarded as a weakly bound system and the physical states ν_e, ν_μ, ν_τ or K^0, $\bar{K}^0$ will be realized as real states (and not virtual states), i.e., it is mainly these states that will be produced. If $A_{ik} < 1$, then it will be mainly the eigenstates of the corresponding diagonalized mass matrix that are produced, i.e., ν_1, ν_2, ν_3 and K_1^0, K_2^0. The asymptotic states (i.e., when ν_e, ν_μ, ν_τ and K^0, $\bar{K}^0$ become superpositions of ν_1, ν_2, ν_3 and K_1^0, K_2^0) appear when $A_{ik} \ll 1$. Whether these asymptotic states and oscillations are real or virtual will be determined by the ratios of the diagonal elements of the mass matrix (for example, in the case of K^0, $\bar{K}^0$, when $m_{K^0} = m_{\bar{K}^0}$, the oscillations will be real).

Knowing that the lepton numbers l_e, l_μ, l_τ are well-conserved in the standard theory of the weak interaction,[6] we can conclude that the neutrino system [see the mass matrix (2.27)] is a weakly bound system (i.e., $m_{ik}/m_{ii} \ll 1$, where $i \neq k,i$, for $k = 1-3$). Then it is the physical states of the neutrinos ν_e, ν_μ, ν_τ that will be mainly produced (the mass matrix is diagonal). The asymptotic states (i.e., when ν_e, ν_μ, ν_τ are superpositions of ν_1, ν_2, ν_3 and there is nonconservation of l_e, l_μ, l_τ) will be realized in the case of motion of the produced neutrinos ν_e, ν_μ, ν_τ in vacuum.

We now turn to the question of the neutrino oscillations, taking as an example $K^0 \leftrightarrow \bar{K}^0$ and $\gamma \leftrightarrow \rho^0$ oscillations, and we classify the possible types of neutrino oscillations.

1) If the ν_e, ν_μ, ν_τ masses are equal, then there must be real oscillations between these neutrinos, in complete analogy with K^0, $\bar{K}^0$ oscillations. In this case, the asymptotic states (i.e., the states ν_1, ν_2, ν_3) will be realized in vacuum as real states (by analogy with K_1^0, K_2^0 mesons).

2) If the ν_e, ν_μ, ν_τ masses are different, then there must be virtual oscillations between them by analogy with the $\gamma \leftrightarrow \rho^0$ oscillations (the asymptotic states will also be realized as virtual states). To observe in this case real oscillations of a neutrino, it must participate in an interaction for its transition onto the corresponding mass shell. For example, if the neutrino ν_e oscillates into the ν_μ, ν_τ neutrinos and these neutrinos have decay modes, then these oscillations and decay modes can be observed if ν_e participates in an interaction and there is a transition to the corresponding ν_μ, ν_τ mass shells.

On this basis it is not difficult to analyze intermediate cases in which the masses of two neutrinos are the same. The above analysis shows that if the masses of the neutrinos ν_e, ν_μ, ν_τ are different, then the oscillations described by the expressions (2.33) and (2.43) will be virtual and the oscillation lengths L_0 [see the expression (2.45)] will be determined by the squares of the differences of the ν_1, ν_2, ν_3 masses. An important difference between this picture of the neutrino oscillations and the old picture of the oscillations is that a neutrino remains on its mass shell if it moves in vacuum.

In the following section, we shall discuss the consequences of this difference.

3. NEUTRINO OSCILLATIONS IN MATTER

We consider the theory of resonant neutrino oscillations in matter[18,41] (the MSW effect) obtained from Wolfenstein's equation.[19] We make a critical analysis of Wolfenstein's equation. We then give the theory of the enhancement of the oscillations of neutrinos of different masses in matter.[23]

1. Theory of resonant neutrino oscillations in matter

To describe neutrino oscillations in matter, Wolfenstein[23] proposed the equation

$$i \frac{d\nu_\Phi}{dt} = \left(p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W} \right) \nu_\Phi, \tag{3.1}$$

where p, $\hat{M}^2$, and $\hat{W}$ are the momentum, square of the neutrino mass matrix in vacuum, and the matrix that takes into account the interaction of the neutrino with matter;

$$\nu_\Phi = \begin{pmatrix} \nu_{\nu_e} \\ \nu_{\nu_\mu} \end{pmatrix}; \quad \hat{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}.$$

The mixing in (3.1) is due to the fact that the matrix $\hat{M}^2$ is nondiagonal. If the neutrino energy is not too high, $S \ll G_F^{-1}$, and the thickness of the layer of matter is less than the absorption length, then interactions of neutrinos with matter reduce to elastic forward scattering through angle θ^0. Expressions for W_e and W_μ can be obtained by considering the change in the wave function during the time dt. It is due to the appearance of an additional wave, which is the result of the addition of the waves resulting from scattering by particles in a layer $dR \simeq c\,dt$:

$$d\nu_\alpha = -iW_\alpha \nu_\alpha \cdot dR,$$

$$W_\alpha = \sum f_i^\alpha(0) N_i \cdot p^{-1} \quad (\alpha = e,\mu), \tag{3.2}$$

where $f_i^\alpha(0)$ is the amplitude for ν_α scattering by the ith component of the matter ($i = e,p,n$), and N_i is the concentration of the distribution of the components.

The physical consequences for the oscillations are determined by the difference of the diagonal terms in the matrices M^2 and W, namely

$$W=W_e-W_\mu=\sum \Delta f_i N_i p^{-1}, \tag{3.3}$$

where $\Delta f_i=f_i^e(0)-f_i^\mu(0)$.

It can be seen from (3.3) that if the interaction of ν_e and ν_μ were the same, there would be no effect of matter. If an effect is to exist, nonsymmetric interactions of the neutrinos ν_e and ν_μ are needed. Since at low energies there is no scattering through the charged current for ν_μ, ν_τ, it follows that

$$\Delta f_i(0)=\sqrt{2}G_F\cdot p, \quad W=\sqrt{2}G_F N_e. \tag{3.4}$$

Mikheev and Smirnov[18,42] showed that Eq. (3.1) has a solution of resonance nature (MSW effect), and this leads to large neutrino oscillations. This enhancement of neutrino oscillations in matter stimulated great interest in connection with the possibility of explaining by means of it the discrepancy between the experimentally measured neutrino fluxes from the sun and calculations made in accordance with the SSM.

In vacuum, $\hat{W}=0$, and then (3.1) has the form

$$i\frac{d\nu_\Phi}{dt}=\left(p\hat{I}+\frac{\hat{M}^2}{2p}\right)\nu_\Phi. \tag{3.5}$$

The relationship between ν_Φ and $\nu=(\nu_1,\nu_2)$ (2.23) is established by diagonalizing the mass matrix $\hat{M}^2$:

$$\nu_\Phi=U(\theta)\nu, \quad U(\theta)=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix},$$

$$U(\theta)^+MU(\theta)=m^2=\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}. \tag{3.6}$$

The neutrino oscillation length will be determined by the expression (2.45):

$$L_0=\frac{4\pi p}{|m_1^2-m_2^2|}, \quad E=pc. \tag{3.7}$$

If allowance is made for the contribution of the neutrino interaction with matter (3.3), Eq. (3.1) can be rewritten in the form

$$i\frac{d\nu_\Phi}{dt}=\hat{H}\nu_\Phi, \quad \text{where } \hat{H}=\begin{vmatrix}H_e & \frac{1}{2}\bar{H}\\ \frac{1}{2}\bar{H} & H_\mu\end{vmatrix},$$

$$\bar{H}=-\frac{\Delta m^2\sin 2\theta}{2p};$$

$$H=H_e-H_\mu=\frac{\Delta m^2\cos 2\theta}{2p}+\frac{\Sigma\Delta f_i N_i}{2p}. \tag{3.8}$$

In a medium with variable density, $N_i=N_i(x)$, and Eq. (3.8) is a system of differential equations with variable coefficients.

The intrinsic length (diffraction length) is defined as

$$L^0=2\pi W^{-1}=2\pi\left(\frac{\Sigma\Delta f(0)N_i}{p}\right)^{-1}$$
$$=2\pi m_N(\sqrt{2}G_F\rho Y_e)^{-1}$$
$$=(3\cdot 10^7\ \text{m})(\rho(\text{g/cm}^3)\cdot 2Y_e)^{-1}, \tag{3.9}$$

where Y_e is the number of electrons per nucleon.

Using (3.9), we can write H and $\bar{H}$ in the form

$$H=-2\pi L_0^{-1}\left(\cos 2\theta-\frac{L_0}{L^0}\right),$$

$$\bar{H}=2\pi L_0^{-1}\sin 2\theta. \tag{3.10}$$

We determine the matrix $U_m(\theta_m)$ that diagonalizes the mass matrix for the case of neutrino interaction with matter:

$$\nu_\Phi=\hat{U}_m(\theta_m)\nu_m, \quad \hat{U}_m(\theta_m)=\begin{pmatrix}\cos\theta_m & \sin\theta_m\\ -\sin\theta_m & \cos\theta_m\end{pmatrix},$$

$$\hat{U}_m^+(\theta_m)\hat{H}\hat{U}_m(\theta_m)=\hat{H}_m=\text{diag}(H_1^d,H_2^d), \tag{3.11}$$

where θ_m is the mixing angle in matter.

It follows from (3.8) and (3.11) that

$$\sin 2\theta_m=\bar{H}(H^2+\bar{H}^2)^{-1/2}, \tag{3.12}$$

$$H_{1,2}^d=\frac{1}{2}[H_e+H_\mu\pm(H^2+\bar{H}^2)^{-1/2}],$$

$$M_i^2=2p\cdot H_i^d, \tag{3.13}$$

where ν_{im} and M_i^2 are the analogs of ν_i and m_i^2 in matter.

The dependence of the mixing parameter $\sin^2 2\theta_m$ on the neutrino energy or on the density of the medium has a resonance nature. It follows from (3.12) and from the expression (3.10) for H and $\bar{H}$ that

$$\sin^2 2\theta_m=\sin^2 2\theta\cdot[(\cos 2\theta-L_0(L^0)^{-1})^2+\sin^2 2\theta]^{-1}, \tag{3.14}$$

if (resonance condition!)

$$\cos 2\theta\cong\frac{L_0}{L^0}, \quad \sin^2 2\theta_m\cong 1, \quad \theta_m\cong\frac{\pi}{4}. \tag{3.15}$$

We recall that from (2.45) and (3.9) it follows that $L_0/L^0\sim\rho E$ $(E\simeq pc)$.

The half-width of the resonance at half-height is determined by the mixing angle in vacuum:

$$\Delta(L_0/L^0)=\frac{L_0}{L^0}\tan 2\theta=\sin 2\theta. \tag{3.16}$$

It can be seen from (3.16) that the smaller the vacuum mixing angle, the narrower the resonance peak.

As can be seen from (3.15), the resonance density $(\rho_R=\rho Y_e)$ is

$$\rho_R=-m_N\Delta m^2\cos 2\theta(2\sqrt{2}G_F E)^{-1}. \tag{3.17}$$

We rewrite (3.17) in the form

$$\frac{E\rho_R}{\Delta m^2}=-m_N(2\sqrt{2}G_F)^{-1}. \tag{3.18}$$

It can be seen from (3.18) that if the medium has a variable density (for example, for the sun), the resonance condition will be satisfied for neutrinos in a wide energy range.

The probability of detecting the neutrino ν_e at the distance R from the source is

$$P(E,R,\ldots)=1-\sin^2 2\theta_m \sin\frac{2\pi R}{L_m}, \qquad (3.19)$$

where $L_m=\sin 2\theta_m/\sin 2\theta$.

It can be seen from (3.14) that as $L_0/L^0\rightarrow 0$

$$\sin^2 2\theta_m \simeq \sin^2 2\theta,$$

and as $L_0/L^0\rightarrow\infty$

$$\sin^2 2\theta_m \simeq \sin^2 2\theta\left(\frac{L^0}{L_0}\right)^2.$$

A more complete exposition of the theory of the MSW effect can be found in the review of Ref. 41.

In the studies of Ref. 20, an analysis was made of Wolfenstein's equation (3.1). This is a Schrödinger equation (or, rather, Pauli equation) for the function $\nu_{\Phi}=\left(\begin{smallmatrix}\nu_{\nu_e}\\ \nu_{\nu_\mu}\end{smallmatrix}\right)$, where $\nu_e=\nu_{Le}+\nu_{Re}$, $\nu_\mu=\nu_{L\mu}+\nu_{R\mu}$. Thus, this equation is a left–right symmetric equation for spinor functions. This equation contains the term $\hat{W}$, which arises from the weak interaction (contribution of W bosons) and contains only a left interaction of the spinors (2.8) (neutrinos), and is substituted in the left–right symmetric equation (3.1) without indication of its left origin. We then see that Eq. (3.1) is an equation that includes a term $\hat{W}$ that arises not from the weak interaction but from a hypothetical left–right symmetric interaction. Moreover, the obtained effect of resonant enhancement of neutrino oscillations in matter is due precisely to this hypothetical left–right symmetric interaction. To take into account correctly the left-handed nature of the weak interaction, it is evidently necessary to formulate an equation, not of Schrödinger type, but of Dirac type.

As can be seen from Eqs. (3.1) and (3.13), if enhancement of neutrino oscillations is to occur in matter, the result of the interaction of the neutrinos with matter must reduce to a change in the effective mass of the ν_e neutrino. If the effective mass of the ν_e neutrino increases in matter as a result of the contribution of the hypothetical interaction and $m_{\nu_e}\simeq m_{\nu_\mu}$, then the mixing angle θ will be of order $\theta\simeq\pi/4$. In this connection, it will be interesting to see whether the contribution of the weak interaction of neutrinos in matter leads to a change in the effective ν_e mass.

The standard fermion mass Lagrangian has the form

$$L_{M_0}=-\bar{\Psi}M_0\Psi. \qquad (3.21)$$

In all the existing methods and approaches in nonrelativistic and relativistic quantum theory[43] used to obtain (calculate) masses (or self-energies) of particles, it is assumed that the left and right components of the fermions participate symmetrically in the interactions. Then with allowance for the contribution of the interaction to the fermion mass, M_{ef}, the mass Lagrangian takes the form

$$L_{M'}=-\bar{\Psi}M_0\Psi-\bar{\Psi}M_{ef}\Psi=-\bar{\Psi}(M_0+M_{ef})\Psi=-\bar{\Psi}M'\Psi. \qquad (3.22)$$

We now determine the fermion (neutrino) mass Lagrangian with allowance for the fact that the right component of the fermion (neutrino) does not participate in the (weak) interactions (2.8). Then the fermion (neutrino) mass Lagrangian will have the form

$$L_M=-\bar{\Psi}M_0\Psi-\bar{\Psi}_L M_{ef}\Psi_R-\bar{\Psi}_R M_{ef}\Psi_R. \qquad (3.23)$$

Since Ψ_R, $\bar{\Psi}_R$ do not participate in the (weak) interactions, we obtain

$$L_M=-\bar{\Psi}M_0\Psi+0; \quad M=M_0. \qquad (3.24)$$

Thus, the weak interaction cannot contribute to the mass Lagrangian.

It is then natural to pose this question: What happens to a neutrino as it passes through matter? From the previous discussions, we arrive at the following conclusion. As it passes through matter, the left component of the neutrino, interacting with the matter through W bosons, acquires an additional kinetic energy, which we cannot convert into an effective mass (as, for example, can be done for an electron in a metal, in which the effective electron mass can change), since the right component of the neutrino does not interact with the matter through the W bosons.

A more complete discussion of the issue associated with the neutrino mass can be found in Refs. 28 and 29.

We return to Wolfenstein's equation and consider whether this equation, with the hypothetical (left–right symmetric) interaction contained in it, can lead to the existence of resonant enhancement of oscillations in matter. The answer is evidently clear. In such a form, this equation leads to resonant enhancement of the oscillations in matter. However, a question again arises regarding the equation itself: Is it possible to substitute simultaneously in this equation the nondiagonal mass term $\hat{M}/2p$ responsible for the neutrino mixing and the term $\hat{W}$ associated with the hypothetical left–right symmetric interaction? This question arises because in the region of action of the hypothetical interaction (an action identical to the weak interaction), i.e., at distances of order $1/m_W$, no appreciable nonconservation of lepton numbers is found. It may then be concluded that the nonconservation of the lepton numbers occurs at shorter distances, i.e., the nondiagonal mass term $\hat{M}^2$ must arise at shorter distances. We then have the following question: Can this mass term, arising at shorter distances, be sensitive to a mass that arises at distances of order $1/m_W$? The answer to this question is again evidently clear: The mass term $\hat{M}^2$ can be sensitive only to the masses that arise at distances comparable with the distance at which this mass term itself arises. In other words, if we wish to take into account the contribution of the hypothetical interaction in Eq. (3.1) to the neutrino mass, we must take into account only the part of the mass that arises from this hypothetical interaction at distances of the order of the distances at which the mass term $\hat{M}^2$ arises. Thus, we have arrived at the conclusion that Eq. (3.1) in such a form does not take into account the fact that the terms $\hat{M}^2$ and $\hat{W}$ arise at different distances and that for the correct formulation of the equation they should be the same.

In confirmation of this problem, we give an example associated with quark mixing. In the considered example, the analog of the hypothetical left–right symmetric interaction in Eq. (3.1) is the strong interaction, and the analog of the mass term $\hat{M}^2$ responsible for the neutrino mixing (we can evidently assume that there is an interaction that does not conserve the lepton numbers behind this matrix) is mixing of the s and d quarks or the weak interaction responsible for this mixing. We consider weak Λ^0 decay.[37] The matrix element of this decay is proportional to $\sin\,\theta_C$ (θ_C is the Cabibbo angle[38]). Since the strange quark in the Λ^0 hyperon is a constituent quark (as is well known,[44] constituent quarks have masses of the order of 300 MeV) and $m_s^0 \simeq m_d^0$, the mixing angle θ_C, which can be expressed in terms of the masses of these quarks [$\sin\,\theta_C \sim \sqrt{m_d^0/m_s^0}$ (Ref. 45)], must be large because of the contribution of the strong interaction to the masses of the s and d quarks in the Λ^0 hyperon ($\theta_C \simeq \pi/4$). Indeed, since the weak interaction, which mixes the d and s quarks, acts at short distances and is not sensitive to the masses generated by the strong interaction at the distances characteristic of it, the mixing angle θ_C must be determined by the quark current masses m_d and m_s (Ref. 45), and as a result this mixing angle θ_C is not increased by the contribution of the strong interaction.

In conclusion, we note that the oscillation enhancement effect obtained from Eq. (3.1) is a beautiful effect, but, unfortunately, the two shortcomings of this equation noted above make it necessary to examine further the possible existence of this effect.

We now discuss a different mechanism[23] of neutrino oscillation enhancement in matter.

2. Theory of the enhancement of the oscillations of neutrinos of different masses in matter

As we noted in Sec. 2, if the masses of the different neutrino species are different, then only virtual neutrino oscillations are possible, and if real neutrino oscillations are to occur, the neutrinos must participate in interactions in order to go over to the corresponding mass shells, by analogy with transition of a photon into a ρ^0 meson in the vector-dominance model.

We estimate the probability of transition of a neutrino of one species ν_l into a neutrino of a different species $\nu_{l'}$ ($m_{\nu_l} \neq m_{\nu_{l'}}$) as the neutrino passes through matter. Transitions to the mass shell of the corresponding neutrino will occur in the case of weak interaction of neutrinos in matter (by analogy with the γ–ρ^0 transition). We shall assume that the mass difference of the neutrinos ν_l and $\nu_{l'}$ is fairly small, and, therefore, it can be assumed that the probability of transition to the $\nu_{l'}$ mass shell will be proportional to the total elastic cross section $\sigma^{\rm el}(p)$ of weak interaction of the neutrino (for simplicity, we shall consider oscillations of two neutrinos). Then the elastic interaction length of neutrinos in matter with density, charge, and atomic number ρ, z, A and momentum p will be determined by the expression

$$\Lambda_0 \sim \frac{1}{\sigma^{\rm el}(p)\rho(z/A)}. \qquad (3.25)$$

If the mass difference between the neutrinos is sufficiently large, then this can be taken into account by using the technique of the vector-dominance model.[39] As noted above, we shall assume that this difference is a very small quantity, and therefore we shall use the expression (3.25).

The elastic scattering of neutrinos in matter corresponds to the real part Re $f_i(p,0)$ of the forward scattering amplitude, which is related to the refraction coefficient by

$$p(n_i-1) \simeq \frac{2\pi N_e f_i(p,0)}{p}, \quad i=\nu_e,\nu_\mu,\nu_\tau. \qquad (3.26)$$

Noting that[19]

$$f_i(p,0) \simeq \frac{\sqrt{2}}{2\pi} G_F p \left(\frac{M_i^2}{M_W^2}\right),$$

$$\text{if } i=\nu_e, \quad M_i^2 = M_W^2,$$

$$\text{if } i=\nu_\mu,\nu_\tau, \quad M_i^2 = M_{z^0}^2, \qquad (3.27)$$

we obtain

$$p(n_i-1) \simeq \sqrt{2} G_F N_e \left(\frac{M_i^2}{M_H^2}\right).$$

The phase of the elastic scattering amplitude changes by 2π over the length

$$\Lambda_0 \simeq \frac{2\pi}{\sqrt{2} G_F \rho(z/A)} = 2\pi L^0. \qquad (3.28)$$

(The absorption, or the imaginary part of the forward scattering amplitude, can be ignored for neutrinos that do not have high energies.)

Now, knowing that the elastic interaction length of neutrinos in matter is Λ_0, we must estimate the probability of oscillation of a neutrino when it passes through matter of thickness L. This probability is

$$P(L) = 1 - e^{-2\pi L/\Lambda_0}. \qquad (3.29)$$

Further, using the expressions (3.28) and (3.29), we can determine the neutrino oscillation probability $\rho_{\nu_l\nu_{l'}}(L)$ for different thicknesses L. For this, we average over R the expression for the neutrino oscillation probability:[35]

$$P_{\nu_l\nu_{l'}}(R) = \frac{1}{2}\sin^2 2\theta_{\nu_l\nu_{l'}}\left(1-\cos 2\pi\frac{R}{L_0}\right), \qquad (3.30)$$

where $L_0 = 4\pi p/\Delta m^2$, as a result of which we obtain

$$\bar{P}_{\nu_l\nu_{l'}}(R) = \frac{1}{2}\sin^2 2\theta_{\nu_l\nu_{l'}}.$$

Then the oscillation probability $\rho_{\nu_l\nu_{l'}}(L)$, or the mixing angle β, for the case $\Lambda_0 \geqslant L_0$ will be determined by the following expressions:

a) for thicknesses L comparable with Λ_0,

$$\rho_{\nu_l\nu_{l'}}(L) = \frac{1}{2}\sin^2 2\beta \simeq \bar{P}_{\nu_l\nu_{l'}} = \frac{1}{2}\sin^2 2\theta_{\nu_l\nu_{l'}}, \qquad (3.31)$$

where $\beta \simeq \theta_{\nu_l\nu_{l'}}$;

b) for very large thicknesses L, $L/\Lambda_0 > 1/\sin^2 2\theta_{\nu_l\nu_{l'}} \gg 1$,

$$\rho_{\nu_l\nu_{l'}}(L) = \frac{1}{2}\sin^2 2\beta \simeq \frac{1}{2} \tag{3.32}$$

with $\beta \simeq \pi/4$;

c) for intermediate thicknesses L,

$$\frac{1}{2}\sin^2 2\theta_{\nu_l\nu_{l'}} \leqslant \rho_{\nu_l\nu_{l'}}(L) \leqslant \frac{1}{2} \tag{3.33}$$

or

$$\theta_{\nu_l\nu_{l'}} \leqslant \beta \leqslant \frac{\pi}{4}.$$

In the case $L_0 \geqslant \Lambda_0$, we also have expressions of the type (3.31)–(3.33), but in this case it is necessary to replace Λ_0 by L_0, and then the thickness of the matter will be determined in units of L_0. In addition, since the oscillation length L_0 in this case increases with increasing neutrino momentum [see (3.30)], the number of oscillation lengths $n = L/L_0$ that fit into the given thickness L will decrease with increasing neutrino momentum, and, accordingly, the neutrino oscillation probability $\rho_{\nu_l\nu_{l'}}(L)$ will decrease with increasing momentum.

We consider in more detail the question of the neutrino oscillation probability for intermediate values of the number of interactions n. The probability distribution of n-fold elastic interaction of neutrinos for thicknesses L with mean value $\bar{n} = L/\Lambda_0$ will be determined at not too large $\bar{n}$ by the Poisson distribution

$$f(n,\bar{n}) = \frac{(\bar{n})^n}{n!}e^{-\bar{n}}, \tag{3.34}$$

which at large $\bar{n}$ becomes the Gaussian distribution

$$f(n,\bar{n},\bar{n}) = \frac{1}{\sqrt{2\pi\bar{n}}}e^{-(n-\bar{n})^2/2\pi}. \tag{3.35}$$

The probability of transition of the neutrino ν_l into ν_l and $\nu_{l'}$ as a result of n-fold elastic interaction of the neutrino is determined by the recursion relations ($\theta \equiv \theta_{\nu_l\nu_{l'}}$)

$$\rho^{(1)}_{\nu_l\nu_l} = 1 - (1 - e^{-B})\frac{1}{2}\sin^2 2\theta,$$

$$\rho^{(1)}_{\nu_l\nu_{l'}} = (1 - e^{-B})\frac{1}{2}\sin^2 2\theta,$$

$$\rho^{(n)}_{\nu_l\nu_l} = \rho^{(n-1)}_{\nu_l\nu_l}\left(1 - (1 - e^{-B})\frac{1}{2}\sin^2 2\theta\right) + \rho^{(n-1)}_{\nu_l\nu_{l'}}(1 - e^{-B})\frac{1}{2}\sin^2 2\theta),$$

$$\rho^{(n)}_{\nu_l\nu_{l'}} = \rho^{(n-1)}_{\nu_l\nu_{l'}}\left(1 - (1 - e^{-B})\frac{1}{2}\sin^2 2\theta\right) + \rho^{(n-1)}_{\nu_l\nu_l}(1 - e^{-B})\frac{1}{2}\sin^2 2\theta). \tag{3.36}$$

If we set $B = 2\pi$, then the term e^{-B} in (3.36) can be ignored. For average estimates, we can use the fact that

$$\bar{n} = \int f(n,\bar{n},\bar{n})dn = \bar{n}. \tag{3.37}$$

Then the mean probability of neutrino oscillation will be

$$\rho(\nu_l \to \nu_l) \simeq \rho^{(\bar{n})}_{\nu_l\nu_l},$$

$$\rho(\nu_l \to \nu_{l'}) \simeq \rho^{(\bar{n})}_{\nu_l\nu_{l'}}. \tag{3.38}$$

From (3.36) and (3.38), if $\sin^2 2\theta$ is a sufficiently small quantity ($\sin^2 2\theta \ll 1$), we can obtain the expression [in (3.36), we have retained the terms of first order in $\sin^2 2\theta$]

$$\rho(\nu_e \to \nu_e) = 1 - \bar{n}\frac{1}{2}\sin^2 2\theta,$$

$$\rho(\nu_e \to \nu_\mu) = \bar{n}\frac{1}{2}\sin^2 2\theta. \tag{3.39}$$

In deriving the expressions (3.31)–(3.33), we have used the approximation (which is good at high energies) $L_0 = L^0_e \simeq L^0_\mu \simeq L^0_\tau$. In fact, the interaction lengths of ν_e and ν_μ, ν_τ differ at low energies, since the ν_e scattering occurs through the neutral and charged currents (2.8) and (2.11), whereas the ν_μ, ν_τ scattering occurs through the neutral current. The ratio of $L^0_\mu(L^0_\tau)$ to L^0_e can be found from (2.8) and (2.11) and is

$$L^0_\mu = L^0_\tau; \quad \delta = \frac{L^0_\mu}{L^0_e} \simeq 2.49. \tag{3.40}$$

Then the ratio of the mean numbers of interaction lengths of the ν_e, ν_μ, ν_τ doublets will be

$$\bar{n}_e/\bar{n}_\mu = \bar{n}_e/\bar{n}_\tau \simeq 2.49. \tag{3.41}$$

If we take into account (3.40) (i.e., at low energies), then the picture of the neutrino oscillation enhancement in matter will depend strongly on the vacuum mixing angle θ. If

$$\sin^2 2\theta \geqslant \frac{1}{\delta}, \tag{3.42}$$

then there will be a predominant transition into a ν_e neutrino. This means that when the ν_e, ν_μ, ν_τ neutrinos pass through matter there will be a predominant transition to the ν_e neutrino, while the ν_e neutrino will mainly remain in the same state (this effect of neutrino oscillation enhancement will have a strong effect on the neutrino composition in the case of supernova explosions).

In the case when

$$\sin^2 2\theta \ll 1 \tag{3.42}$$

there will be an enhancement of neutrino oscillations in matter (i.e., there will be a transition of ν_e into ν_μ, ν_τ), but at the same time it must be borne in mind that the mean number of interaction lengths of the ν_μ and ν_τ will be reduced by a factor δ and, accordingly, $\bar{n}$ in the expression (3.39) will be replaced by $\bar{n}_\mu, \bar{n}_\tau$.

In the expression (3.39), we can restore the time dependence over which the averaging in (3.30) was done. Then

$$\rho(\nu_e \to \nu_\mu) = \bar{n} \sin^2 2\theta \sin^2\left(\frac{2\pi R}{L_0}\right). \tag{3.43}$$

In conclusion, we note that the considered mechanisms of neutrino oscillation enhancement in matter lead merely to a change of the neutrino mixing angle, and for their realization there must exist a vacuum neutrino mixing angle.

4. POSSIBLE WAYS OF SOLVING THE PROBLEM OF THE SOLAR NEUTRINO DEFICIT

Before we proceed to a discussion of possible ways of solving the problem of the solar neutrino deficit, we give material needed for this: the elements of the standard solar model (SSM) and results calculated in this model; a brief description of experimental facilities to study the neutrino flux from the sun together with measurements with them of the neutrino fluxes.

1. Elements of the SSM

It is well known[16] that the luminosity of the sun (and stars) derives from nuclear processes that take place within the stars. In the simplest form, the equation of the balance of the gravitational forces and the pressure forces that arise because of the energy release is

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{4.1}$$

where r is the distance from the center, and $P(r)$, $\rho(r)$, $T(r)$, and $M(r)$ are, respectively, the pressure, density, temperature, and mass. The values of $M(r)$ and $\rho(r)$ are related by

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r). \tag{4.2}$$

The flux $L(r)$ of energy (at given r) per unit time through the sphere of radius r is

$$L(r) = -4\pi r^2 \frac{ac}{3k\rho(r)} \cdot \frac{dT^4(r)}{dr},$$

where k is the opacity, which depends on the nuclear composition of the star, a is the Stefan–Boltzmann constant, and c is the speed of light. The total generated energy (without allowance for the entropy term, which can be ignored) is

$$\frac{dL(r)}{dr} = 4\pi\rho(r)\varepsilon. \tag{4.3}$$

Finally, the observed total luminosity is

$$L_\odot = \int_0^{R_\odot} \frac{dL(r)}{dr}\, dr. \tag{4.4}$$

In (4.3) and (4.4), ε is the energy produced per unit time and mass:

$$\varepsilon = \sum_{i,j} (Q - \langle q_\nu \rangle)_{ij} n_i(r) n_j(r) \langle \sigma v \rangle_{ij} \cdot \frac{1}{(1+\delta_{ij})}, \tag{4.5}$$

where Q is the energy from the reaction of nuclei i and j, $\langle q_v \rangle$ is the mean energy carried away by neutrinos, $n_i(r)$ and $n_j(r)$ are the densities of nuclei i and j, σ is the total cross section, v is relative velocity between i and j, δ_{ij} is the Kronecker symbol, and $\langle \sigma v \rangle$ denotes an average over the Maxwell–Boltzmann distribution.

TABLE II. The solar pp cycle.

Reaction	Fraction as a %	Energy ν (MeV)
$p+p \to {}^2H + e^+ \nu_e$	100	$\leqslant 0.420$
$p+e^- + p \to {}^2H + \nu_e$	0.4	1.442
${}^2H + p \to {}^3He + \gamma$	100	
${}^3He + {}^3He \to \alpha + 2p$	85	
${}^3He + {}^3He \to {}^7Be + \gamma$ or	15	
${}^7Be + e^- \to {}^7Li + \nu_e$	15(90%)	0.861
	(10%)	0.383
${}^7Li + p \to 2\alpha$	15	
${}^7Be + p \to {}^8B + \gamma$	0.02	
${}^8B \to {}^8Be^* + e^+ + \nu_e$	0.02	$\leqslant 15$
${}^7Be^* \to 2\alpha$		

The cross section $\sigma(E)$ can be written in the form

$$\sigma(E) = \frac{S(E)}{E} e^{(-2\pi\eta)},$$

$$\eta = Z_i Z_j \left(\frac{e^2}{\hbar v}\right). \tag{4.6}$$

The function $S(E)$ depends weakly on the energy, and therefore

$$\langle \sigma v \rangle = \left[\frac{8}{\pi\mu(KT)^3}\right]^{1/2} f_0 \int_0^\infty dE S(E) \exp\left(-2\pi\eta - \frac{E}{KT}\right), \tag{4.7}$$

where μ is the reduced mass of nuclei i and j, and f_0 is a quantity that takes into account the effect of screening.

When Eq. (4.4) is solved, it is necessary to specify the initial temperature of the sun, which is usually assumed to be of order 10^7 K, or 1 keV.

In Tables II and III, we give lists of the two pp and CNO reaction cycles, which are the main sources of the energy and neutrino fluxes from the sun.

In addition, to solve Eq. (4.4) we need detailed information about the opacity k and the chemical composition of the sun. Then, knowing $L_\odot$, $M_\odot$, and $R_\odot$, we can numerically solve Eq. (4.4) and obtain all the output parameters of the sun. Figure 2 gives the energy spectrum of neutrinos[15] from various sources. Estimates were also made[15] of the temperature dependences of the neutrino fluxes, which were found to be proportional to T^{18}, T^8, and $T^{-1.2}$ for ${}^8B$, ${}^7Be$, and pp-

TABLE III. Main reactions of the CNO cycle in the sun.

Reaction	Energy ν (MeV)
${}^{12}C + p \to {}^{13}N + \gamma$	
${}^{13}N \to {}^{13}C + e^- + \bar{\nu}_e$	$\leqslant 1.199$
${}^{13}C + p \to {}^{14}N + \gamma$	
${}^{14}N + p \to {}^{15}O + \gamma$	
${}^{15}O \to {}^{15}N + e^+ + \nu_e$	<1.732
${}^{15}N + p \to {}^{12}C + \alpha$	

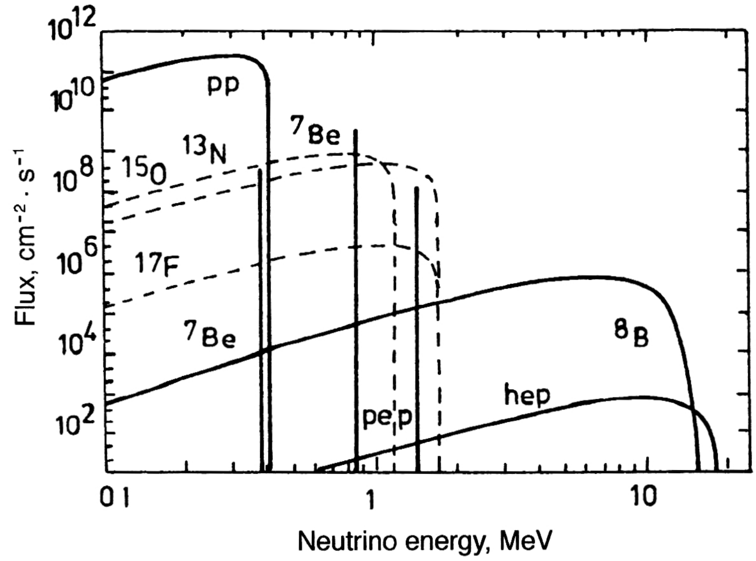

FIG. 2. Spectrum of solar neutrinos calculated in the standard solar model. The ordinate is the flux density of the solar neutrinos at the surface of the earth. The flux densities of monoenergetic neutrinos (*pep*, ^{7}Be) are given in units of cm^{-2}·s^{-1}, and those of neutrinos with a continuous spectrum in units of cm^{-2}·s^{-1}·MeV^{-1}.

neutrinos, respectively. In Table IV, we give the calculated fluxes of solar neutrinos from various sources, and also an estimate of their error (3σ).

2. Brief description of experimental facilities to measure the flux of neutrinos from the sun

We give the neutrino fluxes measured at these facilities and as calculated in accordance with the SSM.

The ^{37}Cl–^{37}Ar experiment (Homestake, USA). Since the seventies, Brookhaven National Laboratory[12,46] has measured the solar neutrino flux using a detector that is a large reservoir filled with $3.8\cdot 10^5$ liters of perchlorethylene (C_2Cl_4) ($2.2\cdot 10^{30}$ atoms) at a depth of 1400 m (4000 m water equivalent). The ^{37}Ar isotope produced by interaction of neutrinos with ^{37}Cl nuclei is periodically extracted to determine the number of its atoms. The half-life of ^{37}Ar is $T_{1/2}\simeq 35$ days. The neutrino detection threshold is $E_{\rm thr}=0.814$ keV. Table V gives the mean number of produced ^{37}Ar atoms in SNU together with SSM calculations (1 SNU represents 1 capture per second in 10^{36} target atoms). It can be seen from Table V that the discrepancy between the calculation and experiment is more than three units.

Kamiokande (Japan). The Kamiokande detector is a cylinder of height 16 m and diameter 19 m filled with 3000 tons of water (H_2O) (Ref. 17) situated at a depth of 2700 m water equivalent (the useful volume of the detector is 680 ton). The detection threshold is 7.5 MeV. Neutrinos are detected through the process of neutrino scattering by electrons: $\nu + e^- \rightarrow e^- + \nu$.

The measured flux of solar neutrinos relative to the calculated data in the SSM and other models is:

Data/SSM$=0.5\pm0.04\pm0.06$, Bahcall (Ref. 15),

Data/SSM$=0.66\pm0.05\pm0.08$, Turch–Chieze (Ref. 47),

Data/SSM$=0.80\pm0.06$, Dar–Shaviv (Ref. 48).

The Gran Sasso ^{71}Ga–^{71}Ge experiment (Italy). The ^{71}Ga–^{71}Ge GALLEX detector[21] in Gran Sasso is a tank filled with 29.5 tons of ^{71}Ga (in a solution of gallium hydrochloride $GaCl_4$). The neutrino detection threshold is $E_{\rm thr}=0.233$ MeV. The ^{71}Ge atoms produced in the tank are periodically extracted (the half-life is $T_{1/2}=11.43$ days) and counted.

The ^{71}Ga–^{71}Ge SAGE experiment (Baksan, Russia). The Baksan ^{71}Ga–^{71}Ge detector[22] contains 55 tons of metallic gallium and consists of seven reactors. As in the GALLEX experiment, the produced ^{71}Ge atoms are periodically extracted from the reactors and their numbers are counted.

TABLE IV. Calculated fluxes of solar neutrinos (the errors correspond to the 3σ level).

Sources	Flux (10^{10} cm^{-2}·s^{-1})
pp	6.0 (1 ± 0.2)
pep	0.014 (1 ± 0.05)
^{7}Be	0.47 (1 ± 0.15)
^{8}B	$5.8\cdot10^{-6}$ (1 ± 0.37)
^{13}N	0.06 (1 ± 0.50)
^{13}O	0.05 (1 ± 0.58)

TABLE V. Reaction rates (in SNU) for a chlorine detector calculated in the SSM and the mean number of atoms (in SNU) detected in the ^{37}Cl–^{37}Ar experiment at Homestake.

Sources	*pep*	^{7}Be	^{8}B	^{15}O	Total
Fluxes (calculation)	0.23	1.7	6.1	0.3	$\simeq7.9\pm2.6$
Fluxes (experiment)	-	-	-	-	2.55($\pm$10%)

TABLE VI. The mean neutrino fluxes measured in the GALLEX and SAGE experiments and the neutrino fluxes (in SNU) calculated in the SSM.

Sources	SSM calculation	GALLEX	SAGE
$pp+pep$	74		
^7Be	36		
^8B	14		
$^{15}N+^{15}O$	8		
Totals	132	$79\pm$ 10 stat. ± 6 syst.	$73\pm$ 18 stat. ± 5 syst.

Table VI gives the data obtained in these experiments on the neutrino flux together with SSM calculations. From this table, it can be deduced that there is a definite solar neutrino deficit.

Super-Kamiokande (Japan). The super-Kamiokande detector[25] will contain 50000 tons of H_2O (useful volume 22000 tons). The detection threshold will be $E_{\text{thr}}=5-7$ MeV. The observations are based on neutrino scattering by electrons. The energy spectrum of the neutrinos will be determined with this detector. Commissioning of the facility is planned for the end of 1996.

SNO (Canada). The SNO detector,[26] which contains 1000 tons of heavy water (D_2O), is placed in a shaft in Sudbury at depth 2073 m water equivalent (Sudbury Neutrino Observatory). The detector is planned to start working in 1996.

Observation of neutrinos is based on the reactions

$$1.\ \nu_x+e^-\rightarrow\nu_x+e^-,\quad E_{\text{thr}}\simeq 6\ \text{MeV},$$

$$2.\ \nu_e+d\rightarrow p+p+e^-,\quad E_{\text{thr}}\simeq 1.45\ \text{MeV},$$

$$3.\ \nu_x+d\rightarrow p+n+\nu_x,\quad E_{\text{thr}}\simeq 2.23\ \text{MeV},$$

$$x=e,\mu\tau.$$

Reaction 1 will proceed through the charged current if $x=e$ and through the neutral current if $x=\mu$, τ, while reaction 2 will proceed only through the neutral current. Comparison of the results obtained in these two reactions (or other pairs) will make it possible to test in a model-independent manner the existence of neutrino oscillations.

Borexino (Gran Sasso, Italy). The new form of this detector[49] will use $\simeq 300$ tons of liquid scintillator in a spherical vessel placed within a cavity filled with ultrapure water (H_2O). The useful volume of the detector will be $\simeq 100$ tons of liquid scintillator. The observation of neutrinos will be based on elastic scattering and excitation of the scintillator nuclei. The energy threshold of neutrino detection will be $E_{\text{thr}}>0.4$ MeV (at purity 10^{-16} g of uranium or thorium impurity for 1 g of the detector). Commissioning of the facility is planned for 1996–97.

3. Possible ways of solving the problem of the solar neutrino deficit

The most traditional ways proposed for solving the solar neutrino problem are:

a) vacuum neutrino oscillations;

b) neutrino oscillations in matter;

c) neutrino spin rotation in the magnetic field of the sun;

d) modification of the SSM;

e) neutrino decay.

We shall briefly discuss these ways of solving the solar neutrino problem.

Vacuum neutrino oscillations. The above experimental results obtained at Homestake, Kamiokande, Gran Sasso (GALLEX), and Baksan (SAGE) show that the relative neutrino fluxes ($P_{\text{exp}}/P_{\text{theor}}$) depend on the neutrino detection thresholds, i.e., on the neutrino energy. It is therefore evidently very difficult to explain these results by vacuum neutrino oscillations.

Neutrino oscillations in matter (in the sun). There now exist numerous publications in which the MSW effect is used to explain the solar neutrino deficit.

The critical analysis of Wolfenstein's equation given in Sec. 3 shows that this effect was not theoretically established. Therefore, we shall regard the expression used to describe the solar neutrino deficit [i.e., the expression (3.19)] as a parametrization formula.

Using this neutrino-oscillation parametrization formula for simultaneous analysis of the $^{37}Cl-^{37}Ar$ results and the Kamiokande and $^{71}Ga-^{71}Ge$ (GALLEX) experiments in a comparison with the neutrino fluxes calculated in the SSM in Ref. 50, the following values were obtained for Δm^2 and $\sin^2 2\theta$:

$$\text{a)}\ \Delta m^2=6\cdot 10^{-6}\ \text{eV}^2,\quad \sin^2 2\theta=7\cdot 10^{-3},$$

$$\text{b)}\ \Delta m^2=8\cdot 10^{-6}\ \text{eV}^2,\quad \sin^2 2\theta=0.6.$$

Using the mechanism of enhancement of the oscillations of neutrinos of different masses in matter (Sec. 3.2), we can estimate the vacuum mixing angle. For this, we determine the mean number of elastic interactions of electron neutrinos produced within the sun in accordance with the expression

$$\bar{n}_{\nu_e}\simeq 40. \tag{4.8}$$

Then $\bar{n}_{\nu_\mu}$ and $\bar{n}_{\nu_\tau}$ can be obtained by using the expression (3.41):

$$\bar{n}_{\nu_\mu}\simeq\bar{n}_{\nu_\tau}\simeq 16. \tag{4.9}$$

For the probability of the $\nu_e\rightarrow\nu_\mu$, ν_τ transition, we obtain from (3.39)

$$P(\nu_e\rightarrow\nu_\mu)\simeq\bar{n}_{\nu_\mu}\cdot\frac{1}{2}\sin^2 2\theta. \tag{4.10}$$

If we assume that the solar neutrino deficit is due to neutrino oscillations, then, using the data of the GALLEX experiment[21] and the SSM predictions[15] for the neutrino flux ($P_{\text{exp}}/P_{\text{theor}}\simeq 0.7$) and the expression (4.10), we obtain

$$\sin^2 2\theta\simeq 3.75\cdot 10^{-2}. \tag{4.11}$$

The expression (4.11) was obtained under the assumption that $\sin^2 2\theta\ll 1$. For the dependence of n (or $\bar{n}$) on the neutrino momentum when $L_0>\Lambda_0$, see Eq. (3.30) and the accompanying text.

Neutrino spin rotation in the magnetic field of the sun. The results obtained in the $^{37}Cl-^{37}Ar$ experiment (Homestake) indicate the existence of an anticorrelation of the neutrino fluxes with the solar activity.[46] A possible explanation of this observation could be the existence of a magnetic moment of the neutrino,[51] which must be fairly large: $\mu_\nu \geqslant 10^{-11}$ μ_B (μ_B is the Bohr magneton). Then as neutrinos pass through the magnetic fields in the upper layers of the sun, the neutrino spin can be rotated, and as a result the neutrinos will become sterile. However, the experimental results at Kamiokande[17] do not confirm the existence of this anticorrelation. On the other hand, it is difficult to justify theoretically the occurrence of such a large magnetic moment of the neutrino.[52] In principle, it is evidently necessary to take into account the possibility of a contribution of such a mechanism to the flux of active neutrinos.

Modification of the SSM. Generally speaking, we cannot rule out the possibility that the SSM[15] requires some corrections, and then the SSM calculations of the neutrino fluxes may be changed. This, in its turn, will entail a change in the interpretation of the experimental results.[48,53]

Neutrino decay. Neutrino decay can exist only in a theory with real vacuum neutrino oscillations (for a criticism of the old theory of neutrino oscillations, see Sec. 2.3). Neutrinos can decay as they pass through the sun in the mechanism of enhancement of the oscillations of neutrinos of different masses, i.e., when ν_e go over into ν_μ, ν_τ. Then if on the decay of ν_μ, ν_τ into ν_e the ν_e have a low energy, so that they pass below the threshold of neutrino detection in the existing experimental facilities, a solar neutrino deficit may be observed. However, because of the fact that neutrinos have a long lifetime,[35,54] it does not appear to be possible to explain by means of such a mechanism the significant reduction in the solar neutrino flux.

5. CONCLUSIONS

1. We have analyzed the old theory of neutrino oscillations constructed by analogy with K^0, $\bar{K}^0$ oscillations. In this theory, the neutrino oscillations are also real when the neutrinos have different masses. This is a consequence of the assumption that the neutrinos are produced with nonconserved lepton numbers (i.e., in a mixed state). It has been shown that such a possibility can be realized with low probability and, moreover, in this case it is not the physical neutrinos (ν_e, ν_μ, ν_τ) but the ν_1, ν_2, ν_3 neutrinos that are produced.

It has been shown that when the neutrinos have different masses there must be virtual neutrino oscillations (see Sec. 2). This conclusion is based on the fact that the neutrinos (ν_e, ν_μ, ν_τ) are produced in weak interactions and are on their mass shells and that if, in addition, there exists (acts) an interaction that does not conserve the lepton numbers, i.e., neutrino oscillations exist, this will occur without departure from the mass shell (i.e., the oscillations will be virtual), by analogy with γ–ρ^0 transitions in the vector-dominance model.

In the framework of this approach, we have constructed a theory of the enhancement of neutrino oscillations in matter. The mechanism of this enhancement is as follows: As a result of the weak interaction of a (virtually) oscillating neutrino with matter there is a transition to the mass shell of the corresponding neutrino (for example, ν_e goes over into ν_μ). If the thickness of matter (the sun) is sufficiently great [see (3.39) and (4.10)], then there will be an accumulation of neutrinos (ν_μ, ν_τ), and this can occur if the thickness of the matter is very great, even going so far as to establish equilibrium between the different neutrino species.

2. We have shown that Wolfenstein's equation, from which the effect of resonant neutrino oscillation enhancement (MSW effect) is obtained, has no relation to the weak interaction, which is a left interaction. This equation contains a contribution of a hypothetical left–right symmetric interaction, i.e., the MSW effect arises from this hypothetical interaction. In addition, the derivation of this equation did not take into account the fact that one can use only part of the effective mass generated by this hypothetical interaction, namely, the part that arises at the distances at which the nonconservation of the lepton numbers occurs (see Sec. 3.1), and not the complete effective mass.

3. Of the total energy released in the sun, only a few percent (2–4%) is carried away by neutrinos (the accuracy with which the flux of thermal energy from the sun is determined is ~1%). If we take into account the very strong temperature dependence (T^{18}) of the fluxes of high-energy solar neutrinos and the uncertainty that may be present in the SSM, it is evidently difficult to draw unambiguous conclusions from comparison of these calculations with the measured fluxes of these high-energy neutrinos.

4. All the existing indications of the possible existence of neutrino oscillations are based on comparison of the measured neutrino fluxes with calculations in the SSM. Therefore it is of fundamental interest to refine and confirm the SSM.

5. To prove the existence of neutrino oscillations, we evidently require a direct experiment for its detection. Therefore the SNO experiment (Canada), in which a direct (model-independent) test of the existence of neutrino oscillations will be possible, is of fundamental interest.

6. As we have already noted, the analogy with quark mixing offers hope of the possible existence of neutrino oscillations. The detection of neutrino oscillations would be a stimulus to the choice of the further path for the development of high-energy physics and the physics of elementary particles beyond the framework of the standard model.

The author is grateful to Professors R. G. Poze and A. A. Tyapkin for constant support and to Professor A. N. Sisakyan for attention and discussions of this work.

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Translated by Julian B. Barbour