Antisymmetric tensor fields

M. V. Chizhov*

Cosmic Research and Technology Center, Physics Department, Sofia University, Sofia, Bulgaria

Fiz. Élem. Chastits At. Yadra **26**, 1322–1372 (September–October 1995)

The existence of two different types of particle with unit spin is discussed. They are described by two inequivalent representations of the Lorentz group, the vector potential and the rank-2 antisymmetric tensor field. The properties of antisymmetric tensor fields, their role in theory, and their phenomenological consequences are reviewed. The equations of motion are given for massless particles of arbitrary spin. The existence of new particles of higher spin follows from these equations. © *1995 American Institute of Physics.*

1. INTRODUCTION

A key concept in all of fundamental physics is that of the field. The introduction of new fields into a theory unavoidably leads to new physical consequences and must be consistent with the available experimental data. In this review we shall study the properties of the fundamental rank-2 antisymmetric tensor field. Off the mass shell, it transmits an interaction with unit spin and is the complement of the photon. The logical consistency of such a theory and analysis of recent experimental data lead to the conclusion that such particles can exist in nature.

The enormous diversity of elementary particles can be described in quantum field theory by a restricted set of fields. The scalar, spinor, vector, and symmetric tensor fields are sufficient for describing all four types of interaction and the matter particles involved in them which occur in nature. This is because the parameters of the corresponding fields vary in very wide ranges. For example, particles can be massless and have infinite interaction range, or the opposite: they can be very heavy with mass of order 100 GeV and lead to a nearly pointlike interaction with a range of 10^{-16} cm. Gauge theories serve as the basis for constructing all four types of interaction: the theories describing the strong, the electromagnetic, the weak, and the gravitational forces are all gauge theories.[1] The particles carrying these interactions are referred to as gauge bosons.

Gauge symmetry requires the absence of a bare mass term in the Lagrangian for gauge bosons. Therefore, difficulties arise in quantum field theory when we attempt to make gauge particles massive "by hand." This leads to a nonrenormalizable theory. The Higgs mechanism is currently used to preserve the renormalizability of the theory and at the same time give mass to the gauge bosons.[2] This requires the addition to the theory of scalar fields with nonzero vacuum expectation value $\langle\phi\rangle_0=v\neq 0$; so far, these have not been discovered experimentally. Their interaction with gauge bosons leads to dynamical generation of the boson mass. Since a nonzero vacuum expectation value of the scalar fields can also give rise to a mass for matter particles, in the construction of such a theory it is conventionally assumed that all particles are massless. This is also good because only particles with identical masses can be members of the same multiplet. Therefore, in this case no difficulties arise in the construction of unified theories. All the necessary masses appear as a result of spontaneous symmetry breaking, when the scalar fields acquire nonzero vacuum expectation values. Scalar fields give masses to all particles with which they interact.

In addition to mass, the other invariant of the Poincaré group is spin, while for massless particles it is helicity. The fundamental particles known at present have spin 1/2, 1, or 2. The Weyl spinor ψ_α and its conjugate $\psi^*_\alpha\equiv\psi_{\dot\alpha}$ are the fundamental spinor representations (1/2, 0) and (0, 1/2), respectively, of the Lorentz group. These rank-1 spinors describe particles with spin 1/2. The dotted and undotted spinor indices $\dot\alpha$ and α take the values 1 and 2. Weyl spinors with different types of indices are related by the P transformation corresponding to spatial reflection and separately are not invariants under this transformation. Dirac spinors are constructed as the direct sum of a Weyl spinor and its conjugate and transform as the reducible representation (1/2,0) $\oplus$ (0,1/2). They are therefore invariant under the parity transformation. Since P invariance is violated in nature, it is natural to choose from the start Weyl spinors rather than Dirac spinors as the fundamental particles of matter.

Spinors of higher rank can be constructed using the fundamental Weyl spinors.[3] An arbitrary combination of Weyl spinors is reducible, because the spinor algebra contains the invariant antisymmetric spinors $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot\alpha\dot\beta}$ with undotted and dotted indices. An irreducible spinor of higher rank can be formed from a symmetric combination of Weyl spinors with undotted indices and from a symmetric combination of conjugate Weyl spinors with dotted indices by multiplication. All possible irreducible combinations of n undotted and m dotted fundamental Weyl spinors transform as $(n/2,m/2)$ representations of the Lorentz group and describe particles with spin $j=(n+m)/2$.

The real representation $(j/2,j/2)$ is the best studied and most commonly used one in the case of integer spin. The Pauli matrices $(\sigma^m)_{\alpha\dot\beta}=(\sigma_0,\boldsymbol{\sigma})_{\alpha\dot\beta}$, where σ_0 is the unit matrix, can be used to transform from spinor indices to Lorentz indices. Here and below we shall use Greek letters for spinor indices and Latin letters for Lorentz indices. The traceless tensor $g^{m_1m_2}\phi_{m_1m_2\ldots m_j}=0$, symmetric in the Lorentz indices of $\phi_{m_1\ldots m_j}$, describes Bose particles with spin j (Ref. 4). In fact, it is easily verified that the dimension of the $(j/2,j/2)$ representation is equal to $(j+1)^2$ and coincides with the number of components of the symmetric traceless tensor $\phi_{m_1 \dots m_j}$: $C^j_{j+3} - C^{j-2}_{j+1} = (j+1)^2$, which is equal to the number of degrees of freedom off the mass shell. However, to obtain the physical number of components for particles with spin j: $2j+1$, it is necessary to impose on j^2 additional constraints like the Lorentz conditions $\partial_{m_1}\phi^{m_1 \dots m_j} = 0$. In the massless case, in addition, gauge transformations isolate the physical components with maximum helicity $\lambda = \pm j$. An analogous procedure for constructing fields with higher spin is also used for Fermi particles, which are described by the $(j/2+1/4, j/2-1/4)$ and $(j/2-1/4, j/2+1/4)$ representations or the equivalent traceless spin tensors $\psi^{\alpha}_{m_1 \dots m_{j-1/2}}$ and $\psi^{\dot\alpha}_{m_1 \dots m_{j-1/2}}$, which are symmetric in the Lorentz indices.

In this description it is taken for granted that for a massless particle with spin j the physical components are those with maximum helicity $\lambda = \pm j$. In fact, all the known particles are described by only these representations. The real photon is transverse, which corresponds to the maximum value of the helicity $\lambda = \pm 1$, and the graviton has physical components with helicity $\lambda = \pm 2$. It is natural here to ask whether or not it is possible for a particle to exist which has spin j but physical components corresponding to other, nonmaximum values of the helicity $\lambda = \pm(j-1), \pm(j-2), \dots$. These would correspond to particles which are, so to speak, complementary to the existing, say, photon and graviton, i.e., their partners. Such particles can be described by other representations of the Lorentz group which are not equivalent to traceless symmetric tensors $\phi_{m_1 \dots m_j}$ (spin tensors $\psi^{\alpha}_{m_1 \dots m_{j-1/2}}$).

In this review we concentrate on representations of unit spin (1,0) and (0,1), which correspond to rank-2 spinors symmetric in the spinor indices: $\psi_{\alpha\beta}$ and $\psi_{\dot\alpha\dot\beta}$, respectively. We can go from spinor indices to Lorentz ones using the decomposition of the product of Pauli matrices into symmetric and antisymmetric parts:

$$(\sigma^m \hat\sigma^n C)_{\alpha\beta} = g^{mn} C_{\alpha\beta} - \frac{i}{2} \varepsilon^{mnab} (\sigma_a \hat\sigma_b C)_{\alpha\beta},$$

$$(C \hat\sigma^m \sigma^n)_{\dot\alpha\dot\beta} = g^{mn} C_{\dot\alpha\dot\beta} + \frac{i}{2} \varepsilon^{mnab} (C \hat\sigma_a \sigma_b)_{\dot\alpha\dot\beta}, \qquad (1)$$

where $(\hat\sigma_m)^{\dot\alpha\beta} = (C^{-1}\sigma^T_m C)^{\dot\alpha\beta}$, $C_{\alpha\beta} \equiv \varepsilon_{\alpha\beta}$ is the charge-conjugation matrix, and ε^{nmab} is the completely antisymmetric tensor, with $\varepsilon^{0123} = +1$. We see from (1) that the three complex components of the symmetric spinor $\psi_{\alpha\beta}$ can be associated with the three complex components of the antisymmetric anti-self-dual tensor $T^-_{mn} = (T_{mn} - \tilde T_{mn})/\sqrt{2}$ (Ref. 5):

$$\psi_{11} = (iT^-_{02} - T^-_{01})/\sqrt{2},$$

$$\psi_{22} = (iT^-_{02} + T^-_{01})/\sqrt{2},$$

$$\psi_{12} = \psi_{21} = T^-_{03}/\sqrt{2}, \qquad (2)$$

where $\tilde T_{mn} = i/2 \varepsilon_{mnab} T^{ab}$ is the tensor which is the dual of the real antisymmetric tensor T^{ab}. The complex-conjugate components of the symmetric spinor $\psi_{\dot\alpha\dot\beta}$ correspond to the components of the antisymmetric self-dual tensor $T^+_{mn} = (T_{mn} + \tilde T_{mn})/\sqrt{2} = (T^-_{mn})^*$. The antisymmetric tensor field as a fundamental field rather than as the Maxwell stress tensor was first introduced by Kemmer.[6] The properties of this field and its interactions with other fields were studied further by Ogievetskiĭ and Polubarinov.[7] In their study a rank-2 antisymmetric tensor field A_{mn} was introduced as a gauge field with the gauge transformations $A_{mn} \to A_{mn} + \partial_m \lambda_n - \partial_n \lambda_m$. The authors introduced the very suitable name "notof"[1)] for this field, because its properties turned out to be complementary to those of the photon. Antisymmetric tensor gauge fields will be studied in Sec. 2 of this review.

Antisymmetric tensor gauge fields became particularly interesting after the appearance of Ref. 8, in which it was shown that such fields arise naturally in the dual models of string theory. At the present time they are an integral component of models of extended supergravity[9] and ensure anomaly cancellation in these theories.[10] The efforts to quantize antisymmetric tensor gauge fields led to the discovery of a new construction: "ghosts for ghosts" or pyramids of ghosts.[11] Unfortunately, models of extended supergravity are rather far from phenomenology, and their study is of purely theoretical interest. Therefore, antisymmetric tensor gauge fields have not yet found any physical applications.

There is another important symmetry in elementary-particle physics in addition to gauge symmetry: conformal invariance.[12] In four-dimensional space-time the free Maxwell action for the electromagnetic field is simultaneously gauge and conformally invariant. However, this property is not preserved for the action of an antisymmetric tensor gauge field

$$\mathscr{A}_{\text{gauge}} = \frac{1}{2\cdot 3!} \int d^4x F_{mnk} F^{mnk}$$

$$= \int d^4x \left[\frac{1}{4} (\partial_k A_{mn}) \partial^k A^{mn} - \frac{1}{2} (\partial_m A^{mk}) \partial^n A_{nk} \right], \qquad (3)$$

where $F_{mnk} = \partial_m A_{nk} + \partial_k A_{mn} + \partial_n A_{km}$ is a gauge-invariant, completely antisymmetric strength tensor of rank three. The conformally invariant action for the antisymmetric tensor field T_{mn},

$$\mathscr{A}_{\text{conformal}} = \int d^4x \left[\frac{1}{4} (\partial_k T_{mn}) \partial^k T^{mn} - (\partial_m T^{mk}) \partial^n T_{nk} \right], \qquad (4)$$

differs from (3) and leads to different equations of motion. We shall refer to these fields as antisymmetric tensor matter fields. They are studied in Sec. 3 of this review.

The Green functions for antisymmetric tensor matter fields can be constructed uniquely using conformal field theory.[13] They also arise naturally in models of extended conformal supergravity (Refs. 14–16) and lead to cancellations of axial and conformal anomalies.[17] Models constructed using such fields are renormalizable and possess interesting properties.[18] For example, the coupling constant of a pseudovector gauge field and an antisymmetric tensor matter field possesses asymptotically free behavior, even in the Abelian case.

In contrast to antisymmetric tensor gauge fields, antisymmetric tensor matter fields can interact with the known matter particles of spin 1/2 via an ordinary Yukawa coupling $t\bar{\psi}\sigma^{mn}\psi T_{mn}$, avoiding the problem of self-consistency. If the antisymmetric tensor particles were massless, this would lead to a new interaction with infinite range. For such an interaction to be consistent with the experimental data it is necessary to assume that the Yukawa constant t is unnaturally small. However, there is another more natural solution to this problem, in which antisymmetric tensor matter fields acquire mass dynamically as a result of spontaneous symmetry breaking.

A massive antisymmetric tensor field leads to a nearly pointlike effective tensor interaction which can interfere with the standard weak $V-A$ interaction. There are already experimental data favoring this possibility.[19,20] However, only extremely precise experiments to measure, in particular, the electron energy spectrum in muon decay and direct production of tensor particles at future accelerators will be able to answer this question definitively.

2. ANTISYMMETRIC TENSOR GAUGE FIELDS

The study of various representations of the Lorentz group and, in particular, representations associated with antisymmetric tensor fields has a long history in field theory.[6] However, insufficient attention has been paid to this subject, owing to the absence of experimental data supporting the existence of such particles. The basic monographs on quantum field theory[21] do not discuss fields which transform according to nonstandard representations of the Lorentz group. Our task will be to study the simplest nonstandard representations of the Lorentz group which arise even in the description of particles with unit spin. Since particles with unit spin can transform as a rank-2 symmetric spinor $\chi_{\alpha\beta}$ and its conjugate $\chi_{\dot{\alpha}\dot{\beta}}$ or as a mixed spinor $\psi^{\dot{\alpha}}_{\beta}$, one can speak of two types of particle with unit spin. It is well known that the mixed spinor $\psi^{\dot{\alpha}}_{\beta}$ corresponds to a 4-vector and describes the photon in the massless limit. Then what particle is described by a rank-2 symmetric spinor $\chi_{\alpha\beta}$ and its conjugate $\chi_{\dot{\alpha}\dot{\beta}}$? To answer this question it is necessary to write down the equations of motion for these fields. We shall try to follow the historical sequence in constructing the equations of motion.

2.1. Massive particles with unit spin

The equations of motion for spinor fields of higher rank were first obtained by Dirac.[22] If the theory contains a mass parameter m_0, it is possible to write down a system of first-order equations relating the various spinors:

$$\left.\begin{aligned} \hat{p}^{\dot{\alpha}\beta}\chi_{\beta\lambda}&=m_0\psi^{\dot{\alpha}}_{\lambda}, \\ \hat{p}_{\dot{\alpha}\beta}\psi^{\dot{\alpha}}_{\lambda}&=m_0\chi_{\beta\lambda}, \end{aligned}\right\} \tag{5}$$

where $\hat{p}^{\dot{\alpha}\beta}=p^n(\hat{\sigma}_n)^{\dot{\alpha}\beta}$. Kemmer rewrote these equations in more usual tensor notation, transforming from spinor to Lorentz indices.[6] He found that these equations are equivalent to the equations

$$\begin{aligned} &\partial_a f^{ab}=m_0A^b, \\ &\partial_aA_b-\partial_bA_a=m_0f_{ab}. \end{aligned} \tag{6}$$

Here the rank-2 antisymmetric tensor f_{ab} corresponds to the symmetric spinor $\chi_{\alpha\beta}$, and the vector A_a corresponds to the mixed spinor $\psi^{\dot{\alpha}}_{\beta}$. If the vector A_a is interpreted as the vector potential of a particle with unit spin and the second equation in (6) is viewed as the definition of the field strength $F_{ab}=m_0f_{ab}$ for this particle, these equations can be rewritten as

$$\begin{aligned} &\partial_aF^{ab}=m_0^2A^b, \\ &F_{ab}=\partial_aA_b-\partial_bA_a. \end{aligned} \tag{7}$$

These are none other than the Proca equations.[23] Owing to the antisymmetry of the field strength F_{ab}, the first equation in (7) automatically gives the Lorentz condition $\partial_aA^a=0$. This condition removes the extra scalar degree of freedom. Therefore, Eqs. (7) describe a vector particle with unit spin and mass m_0. It is well known that if the limit $m_0\to0$ is taken, these equations together with the Lorentz condition will describe a massless photon.

Kemmer also gave another interpretation of Eqs. (6). He assumed that a particle with unit spin can be described by an antisymmetric tensor potential $A_{ab}=i\varepsilon_{abmn}f^{mn}/2$, and that the role of the field strength for this particle will then be played by the vector A^a, or, more precisely, its dual, fully antisymmetric tensor of rank three, $f_{abc}=i\varepsilon_{abcd}A^d$. The equations of motion dual to (6) are

$$\begin{aligned} &\partial_aA_{bc}+\partial_bA_{ca}+\partial_cA_{ab}=m_0f_{abc}, \\ &\partial_af^{abc}=m_0A^{bc}. \end{aligned} \tag{8}$$

Equations (6) and (8) for free massive particles are equivalent, because a dual transform relating them exists. However, this equivalence is lost when an interaction is included.[24]

If the field strength for the antisymmetric tensor field A_{ab} is redefined as $F_{abc}=m_0f_{abc}$, Eqs. (8) take the form

$$\begin{aligned} &F_{abc}=\partial_aA_{bc}+\partial_bA_{ca}+\partial_cA_{ab}, \\ &\partial_aF^{abc}=m_0^2A^{bc}. \end{aligned} \tag{9}$$

Owing to the antisymmetry of the field strength F_{abc}, the second equation in (9) leads to a Lorentz type of condition:

$$\partial_aA^{ab}=0. \tag{10}$$

Because of the antisymmetry of A^{ab}, only three of the four conditions (10) are independent. These conditions allow the removal of three of the six degrees of freedom of the antisymmetric tensor field A^{ab}. Therefore, Eqs. (9) also describe a particle with unit spin and mass m_0.

Owing to the duality of Eqs. (6) and (8), they represent two equivalent ways of describing free massive particles with unit spin. We shall show that for an antisymmetric tensor field there exists yet another equation, equivalent to the others, which also describes a massive particle with unit spin.

It is easiest to show this using the projection-operator formalism.[25] The free action is quadratic in the fields, so that the most general form of the kinetic term for an antisymmetric tensor field is $A_{ab}\Box O^{abcd}A_{cd}$, where the operator O^{abcd} can be expanded in a complete set of projection operators of unit spin:

$$P^{-}_{abcd}=\tfrac{1}{2}[g_{ac}\pi_{bd}-g_{ad}\pi_{bc}-g_{bc}\pi_{ad}+g_{bd}\pi_{ac}],$$

$$P^{+}_{abcd}=\mathbf{1}_{abcd}-P^{-}_{abcd}. \qquad (11)$$

The unit operator in (11) has the form $\mathbf{1}_{abcd}=\frac{1}{2}[g_{ac}g_{bd}-g_{ad}g_{bc}]$, and $\pi_{ab}=\partial_a\Box^{-1}\partial_b$. This is yet another way of showing that an antisymmetric tensor field is described by the (1,0) ⊕ (0,1) representation. The operators $P^{\pm}$ have three properties:

(a) Orthonormality: $P^{\lambda}_{abef}P^{\lambda'}_{efcd}=\delta^{\lambda\lambda'}P^{\lambda}_{abcd}$.

(b) They form the decomposition of unity: $\mathbf{1}_{abcd}=\Sigma_{\lambda}P^{\lambda}_{abcd}$.

(c) Completeness: $O_{abcd}=\Sigma_{\lambda}w_{\lambda}P^{\lambda}_{abcd}$.

Using the projection operators, we write the equation $\Box P^{+}_{abcd}A^{cd}=m_0^2A_{ab}$, which exactly coincides with the equation for the antisymmetric tensor field following from (9). If we act on both sides of it by the operator P^{-}, owing to orthogonality (a), we arrive at the constraint $P^{-}_{abcd}A^{cd}=0$, which is equivalent to (10). In the language of projection operators this implies a "cutting out" of the state with unit spin from the antisymmetric tensor field. For the other physical degrees of freedom, which correspond to a particle with unit spin, because of property (b) we obtain the Klein–Gordon equation: $(\Box-m_0^2)P^{+}_{abcd}A^{cd}=0$.

If we now construct the analogous equations involving P^{-}, we arrive at yet another equation for the antisymmetric field: $\Box P^{-}_{abcd}A^{cd}=m_0^2A_{ab}$ or

$$\partial_a\partial^cA_{cb}-\partial_b\partial^cA_{ca}=m_0^2A_{ab}. \qquad (12)$$

This equation is used in chiral theory for an alternative description of the ρ meson in terms of an antisymmetric tensor field.[26] Using the substitution $\partial^aA_{ab}=m_0A_b$, we immediately see that (12) is equivalent to the Proca equations (7). The property of completeness (c) guarantees that no other equations exist for massive antisymmetric tensor fields.

2.2. Massless particles with unit spin

We shall be interested in the case of massless antisymmetric tensor particles. We therefore take the limit $m_0\to 0$ in (9). This is the case studied by Ogievetskiĭ and Polubarinov.[7] They showed that even the free equations (7) and (9) in the massless limit describe different particles and are inequivalent. To see this, we write down the equations of motion for the vector potential A_m following from (7) (the Maxwell equation) in the massless case,

$$\Box A_m-\partial_m\partial_nA^n=0, \qquad (13)$$

and the equation of motion for the tensor potential A_{mn} following from (9) (the equation for the notof),

$$\Box A_{mn}-\partial_m\partial_kA^{kn}+\partial_n\partial_kA^{km}=0. \qquad (14)$$

We note that the last equation (14) can be obtained from the principle of least action for (3). Equations (13) and (14) are invariant under gauge transformations $\delta A_m=\partial_m\lambda$ of the vector potential and

$$\delta A_{mn}=\partial_m\lambda_n-\partial_n\lambda_m, \qquad (15)$$

of the tensor potential, where $\lambda(x)$ and $\lambda_m(x)$ are arbitrary functions.

We shall show that the free notof has only one state with longitudinal polarization, while the physical photon has two states with transverse polarization. It follows from the auxiliary conditions

$$\partial_mA^m=0,\quad \partial_mA^{mn}=0 \qquad (16)$$

that all the components of the vector potential A_m and of the tensor potential A_{mn} satisfy the d'Alembert equations:

$$\Box A_m=0,\quad \Box A_{mn}=0. \qquad (17)$$

They can therefore be expanded in momentum space in plane waves with positive and negative frequencies:

$$A_m(x)=\int d^3pA_m(\mathbf{p})e^{ipx}+\text{H.c.},$$

$$A_{mn}(x)=\int d^3pA_{mn}(\mathbf{p})e^{ipx}+\text{H.c.},$$

where $p_0=|\mathbf{p}|$. To count the number of states we expand $A_m(\mathbf{p})$ and $A_{mn}(\mathbf{p})$ in a complete basis $e_m^{(1)}$, $e_m^{(2)}$, $p_m=(p_0,-\mathbf{p})$, and $\bar{p}_m=(p_0,\mathbf{p})$ with properties

$$(e^{(i)}e^{(j)})=-\delta_{ij},\quad (e^{(i)}p)=(e^{(i)}\bar{p})=0,\quad p^2=\bar{p}^2=0, \qquad (18)$$

where the vector $\bar{p}_m$ is obtained from p_m by reversing the particle direction of motion. The expansions can be written as

$$A_m(\mathbf{p})=\sum_{i=1}^{2}\alpha_ie_m^{(i)}+\beta p_m+\gamma\bar{p}_m,$$

$$A_{mn}(\mathbf{p})=\delta(e_m^{(1)}e_n^{(2)}-e_n^{(1)}e_m^{(2)})+\sum_{i=1}^{2}\varepsilon_i(e_m^{(i)}p_n-e_n^{(i)}p_m)$$

$$+\sum_{i=1}^{2}\eta_i(e_m^{(i)}\bar{p}_n-e_n^{(i)}\bar{p}_m)+\xi(p_m\bar{p}_n-p_n\bar{p}_m).$$

The auxiliary conditions (16) eliminate all terms containing $\bar{p}_m$ (i.e., $\gamma=\eta_i=\xi=0$), and gauge invariance makes the components containing p_m unimportant. We introduce the unit vectors of right- and left-circular polarization $e_m^{\pm}=(e_m^{(1)}\pm ie_m^{(2)})/\sqrt{2}$ corresponding to the spin projections on the direction of motion (helicities) $\lambda=\pm 1$. Now it can easily be shown that the free notof actually possesses a single polarization state with zero helicity,

$$A_{mn}(\mathbf{p})=i\delta[e_m^{+}(\mathbf{p})e_n^{-}(\mathbf{p})-e_n^{+}(\mathbf{p})e_m^{-}(\mathbf{p})], \qquad (19)$$

while the free photon has two components with helicities $\lambda=\pm 1$:

$$A_m(\mathbf{p})=\sum_{\lambda=\pm 1}\alpha_{\lambda}^{*}e_m^{\lambda}(\mathbf{p}), \qquad (20)$$

where $\alpha_{\pm}=(\alpha_1\pm i\alpha_2)/\sqrt{2}$. Therefore, the action (3) corresponds to the gauge theory of a spinless particle.

2.3. The non-Abelian antisymmetric tensor field

Using the method of dual transformations,[27] let us demonstrate in yet another way that the theory of a free antisymmetric tensor gauge field is equivalent at the classical level to the theory of a one-component scalar field. For this we rewrite the action (3), using the first-order derivative formalism:

$$\mathscr{A}_{\text{gauge}}=\int d^4x\left[\frac{1}{6}A_m\varepsilon^{mnpq}F_{npq}+\frac{1}{2}A_mA^m\right], \tag{21}$$

with the introduction of an auxiliary field A_m. The equations of motion for this field have the form

$$A^m=-\tfrac{1}{6}\varepsilon^{mnpq}F_{npq}=-\tfrac{1}{2}\varepsilon^{mnpq}\partial_nA_{pq}. \tag{22}$$

If this solution is substituted into (21), we again arrive at the action (3) for the antisymmetric tensor gauge field A_{mn}. Up to an integral of a total derivative the action (21) can be rewritten as

$$\mathscr{A}_{\text{gauge}}=\int d^4x\left[\frac{1}{64}A_{mn}\varepsilon^{mnpq}F_{pq}+\frac{1}{2}A_mA^m\right], \tag{23}$$

where $F_{pq}=\partial_pA_q-\partial_qA_p$ is the antisymmetric strength tensor for the auxiliary field A_m. Now varying (23) with respect to the antisymmetric tensor gauge field A_{mn}, we arrive at the following equations of motion:

$$F_{mn}=\partial_mA_n-\partial_nA_m=0, \tag{24}$$

which is essentially a constraint on the auxiliary field A_m. The solution of (24) is the gradient of an arbitrary scalar field: $A_m=\partial_m\phi$. Substituting this solution into (23), we arrive at the usual action for a scalar field,

$$\mathscr{A}_{\text{scalar}}=\frac{1}{2}\int d^4x(\partial_m\phi)\partial^m\phi,$$

which describes a particle with a single degree of freedom and zero helicity.

The action for the antisymmetric tensor gauge field A_{mn} written in the form (23) admits a direct generalization to the non-Abelian case:[28]

$$\mathscr{A}_G=\int d^4x\left[\frac{1}{4}A^a_{mn}\varepsilon^{mnpq}F^a_{pq}+\frac{1}{2}(A^a_m)^2\right], \tag{25}$$

where $F^a_{pq}=\partial_pA^a_q-\partial_qA^a_p+f^a_{bc}A^b_pA^c_q$, and the indices a, b, and c parametrize the adjoint representation of some compact Lie group G with structure constants f^a_{bc}. This action is invariant under the gauge transformations

$$\delta A^a_{mn}=\nabla_m\xi^a_n-\nabla_n\xi^a_m,\quad \delta A^a_m=0, \tag{26}$$

where $\nabla_m\xi^a_n=\partial_m\xi^a_n+f^a_{bc}A^b_m\xi^c_n$ is the covariant derivative.

To transform to the action which is of second order in the derivatives for the antisymmetric tensor gauge field A^a_{mn} we need to eliminate the auxiliary field A^a_m from (25) using the equations of motion

$$A^a_m=-\tfrac{1}{2}K^{-1a}_{mn\ \ b}\varepsilon^{npqr}\partial_pA^b_{qr}, \tag{27}$$

where $K^{-1a}_{mn\ \ b}$ is the inverse of the matrix K^{mna}_b:

$$K^{mna}_b=g^{mn}\delta^a_b+\tfrac{1}{2}\varepsilon^{mnpq}f^a_{bc}A^c_{pq},$$

$$K^{mla}_c K^{-1c}_{ln\ \ b}=\delta^m_n\delta^a_b. \tag{28}$$

The action obtained in this manner is nonpolynomial. On the other hand, variation of (25) with respect to the antisymmetric tensor gauge field A^a_{mn} leads to a condition on the auxiliary field A^a_m:

$$F^a_{mn}=\partial_mA^a_n-\partial_nA^a_m+f^a_{bc}A^b_mA^c_n=0. \tag{29}$$

Its solution, as is well known, can be expressed in terms of the matrix g acting in the adjoint representation of the Lie group G:

$$A_m=g^{-1}\partial_mg, \tag{30}$$

where the matrices A_m belong to the Lie algebra of this group: $A_m=iA^a_mT_a/2$. Here the T_a are linearly independent matrices in the adjoint representation of the Lie algebra, normalized by the condition Tr $T_aT_b=2\delta_{ab}$. Substituting the solution (30) into (25), we arrive at the action for the σ model:

$$\mathscr{A}_\sigma=\int d^4x\ \mathrm{Tr}(\partial_mg^{-1})\partial^mg.$$

We have thus shown that at the classical level the theory of a non-Abelian antisymmetric tensor gauge field is equivalent to the σ model.

2.4. Quantization of the antisymmetric tensor field

Before turning to the quantization of the antisymmetric tensor gauge field, let us construct the Hamiltonian for the action (25) and explain the meaning of the auxiliary fields that have been introduced. For this we eliminate the variable A^a_0 by using the equations of motion

$$A^a_0=\nabla_iB^a_i,$$

and rewrite the action (25) in explicitly Hamiltonian form:

$$\mathscr{A}_G=\int d^4x\left(A^a_i\partial_0B^a_i-\frac{1}{2}(A^a_i)^2-\frac{1}{2}(\nabla_iB^a_i)^2+A^a_{0i}T^a_i\right), \tag{31}$$

where $B^a_i=-\tfrac{1}{2}\varepsilon_{ijk}A^a_{jk}$, $T^a_i=\tfrac{1}{2}\varepsilon_{ijk}F_{jk}$; $i,j,k=1,2,3$. From this we see that A^a_i and B^a_i are canonical variables, $H=\tfrac{1}{2}(A^a_i)^2+\tfrac{1}{2}(\nabla_iB^a_i)^2$ is the Hamiltonian, A^a_{0i} are Lagrange multipliers, and T^a_i are constraints on the canonical variables.

Although the classical theory of the antisymmetric tensor gauge field is equivalent to the theory of the scalar field, the quantization of antisymmetric tensor gauge fields is not a simple problem. The reason is that the actions (23) and (25) describe systems with functionally dependent constraints of the first kind. Owing to the Bianchi identities

$$\nabla_iT_i^{\ a}=\tfrac{1}{2}\varepsilon_{ijk}\nabla_iF^a_{jk}\equiv0,$$

only two of the three constraints T^a_i are independent. By means of these constraints we can eliminate two of the three components of the canonically conjugate momentum A^a_i, leaving only a single independent one. The gauge invariance (26), in turn, also allows us to eliminate two of the three components of the antisymmetric tensor field $A^a_{ij}=-\varepsilon_{ijk}B^a_k$. Thus, as expected, after canonical quantization we are left with only a single pair of independent canonical variables.

The canonical quantization of the free (Abelian) antisymmetric tensor gauge field was performed in Ref. 29. Attempts at covariant quantization of this field[11] led to the discovery of a new phenomenon: "ghosts for ghosts." The functional dependence of the constraints makes it necessary to modify the quantization procedure developed for Yang–Mills fields.[30] Since the action for the Faddeev–Popov ghosts is itself gauge-invariant, it is necessary again to apply to it the procedure for covariant quantization with the introduction of additional ghost fields. The gauge $\nabla^m A^a_{mn}=0$ requires the introduction of anticommuting vector ghost fields $\bar{C}^a_m$ and C^a_m with BRST transformations $\delta C^a_n=(\nabla^m A^a_{mn})\Lambda$, where Λ is an anticommuting Grassmann variable independent of the space-time point. From this we immediately see that the constraints $\nabla^m C^a_m=0$ are imposed on the vector ghost fields. Detailed analysis shows that the solution of this problem requires the introduction of two more commuting scalar ghost fields $\bar{\phi}^a$ and ϕ^a together with a third Nielsen–Kallosh ghost.[31]

The fundamental requirement in gauge field quantization is unitarity of the S matrix.[32] For the examples of the theories of the Yang–Mills field and the gravitational field, Feynman showed that the restoration of unitarity requires the introduction of fictitious particles with anomalous statistics, essentially ghost fields. The ghost fields introduced above satisfy the formal unitarity condition, allowing only a single physical degree of freedom to propagate in loops. The rank-2 antisymmetric tensor field has six degrees of freedom off the mass shell, the vector ghost fields with anomalous statistics each have four degrees of freedom, and the scalar ghost fields have three degrees of freedom, leaving one physical degree of freedom: $6-2\times 4+3=1$. One method of quantizing systems with dependent constraints was proposed in Ref. 33, where a non-Abelian antisymmetric tensor gauge field was canonically quantized in the unitary gauge and the correct transition to covariant gauges was made. Thus, an explicitly unitary S matrix was obtained in Ref. 33.

Another method of quantization is Becchi–Rouet–Stora–Tyutin (BRST) quantization.[34] The free antisymmetric tensor gauge field was quantized by this method in Ref. 35. A general Hamiltonian method of BRST quantization of systems with dependent constraints of the first kind has been developed by Batalin and Fradkin.[36] However, the proof of unitarity of the S matrix in the physical subspace is a complicated problem which has not been solved in the general case. For example, the Lagrangian method of BRST quantization of a non-Abelian antisymmetric tensor gauge field proposed in Ref. 37 leads to an S matrix which is not unitary in the physical subspace. In Ref. 38 the Hamiltonian method of BRST quantization was applied to the theory of an interacting (non-Abelian) antisymmetric tensor gauge field, and it was shown that the effective Lagrangians obtained by this method and by the method of Ref. 33 are equivalent. As already noted, the latter method leads to a unitary S matrix.

2.5. Antisymmetric tensor fields in theories of gravity

The best known example where we encounter antisymmetric tensor fields is quantum gravity. Localization of the Lorentz group makes it necessary to introduce antisymmetric tensor ghost fields C^{ab} (Ref. 39).

The interaction of antisymmetric tensor gauge particles with the gravitational field was first considered in Ref. 40. The authors studied the renormalizability of this model and the equivalence at the quantum level of the theory of antisymmetric tensor gauge fields interacting with the gravitational field and the theory of a scalar field interacting with gravity. This study was a continuation of earlier ones on the renormalizability of Einstein gravity interacting with scalar fields,[41] photons,[42] Yang–Mills fields,[43] spinors without torsion,[44] spinors with torsion,[45] and quantum electrodynamics.[46] Using the background-field method, at the one-loop level the authors obtained the same counterterms for an external gravitational field interacting with an antisymmetric tensor gauge field as for the case of a scalar field. An essential feature of these calculations was the inclusion of all the ghost fields introduced above in internal loops. However, for the quantum gravitational field the equivalence was shown only by using the equations of motion.

An even more interesting property of field-theoretic models containing fields in nonstandard representations of the Lorentz group is the different contribution of these fields to anomalies.[47] For example, a rank-3 completely antisymmetric tensor field,[48] which has no on-shell degrees of freedom, gives a nonzero contribution to the anomaly of the energy–momentum tensor.[47–49] The anomalous contribution to the trace of the effective energy–momentum tensor in theories of gravity at the one-loop level is proportional to $\varepsilon^{mnab}R_{abcd}\varepsilon^{cdpq}R_{mnpq}$ on the mass shell.[50] The integral of this quantity over all space gives a topological invariant, the Euler characteristic χ:

$$\chi\equiv\frac{1}{32\pi^2}\int d^4x\sqrt{g}(R_{mnab}R^{mnab}-4R_{mn}R^{mn}+R^2).$$

This integral is nonzero in spaces with nontrivial topology. The numerical coefficient A of this anomaly was calculated for fields transforming as standard representations of the Lorentz group after gauge fixing and subtraction of the ghost contribution.[51] A simple expression was obtained for calculating the contribution to this coefficient from a particle with spin transforming as the $(m/2,n/2)$ representation of the Lorentz group:[52]

$$360A=(-1)^{2s}[8-150s^2+90s^4+30t^2(1-t^2+6s^2)], \tag{32}$$

where $s=(m+n)/2$ is the spin of the field and $t=(m-n)/2$. For a scalar particle, $A=1/90$. We note that any particle with spin from zero to two gives a nonzero contribution to A.

To cancel this anomaly it is necessary to choose a field multiplet which would give a net contribution of zero to A. For example, it is possible to choose the field multiplet of extended supergravity with arbitrary N. However, the standard set of fields in these theories leads to an anomaly, except in the case $N=3$. Let us calculate this coefficient for the maximally extended supergravity with $N=8$. Its standard field content is one graviton, eight gravitinos, 28 vector particles, 56 spinors, and 70 scalars. Direct use of (32) gives $A=[848+8\times(-233)+28\times(-52)+7\times56+4\times70]/360=-5$. The solution of this problem appeared quite unexpectedly. Using the elegant technique of dimensional reduction from a higher to a lower number of dimensions,[53] Cremmer, Julia, and Scherk managed to construct $N=8$ supergravity in four dimensions.[54]

It was shown in Ref. 53 that the massless states of the Neveu–Schwarz–Ramond open string[55] give a Yang–Mills supermultiplet in a space of 10 dimensions, while the massless states of the closed string lead to an $N=1$, $d=10$ supergravity multiplet. However, Nahm[56] showed that $N=2$ supergravity can also exist in $d=10$, and the maximum number of dimensions in which a theory with spin 2 can exist is $d=11$. Cremmer, Julia, and Scherk found the lost massless states of the closed string and constructed $N=1$ supergravity in $d=11$. Its superfield content is very simple: a tetrad V_M^A, a Majorana spinor with spin 3/2, ψ_M, and a completely antisymmetric tensor gauge field of rank three, A_{MNP}, where indices written as capital letters take values from 0 to 10. The reduction to four dimensions for the tetrad and the Majorana spinor does not lead to nonstandard fields. However, the reduction of the rank-3 completely antisymmetric tensor gauge field gives, in addition to known fields, seven rank-2 antisymmetric tensor gauge fields A_{mna} (here $a=1,...,7$ are the indices of the compactified space), which have the same number of physical components as seven scalars ϕ_a, and also one rank-3 completely antisymmetric tensor field A_{mnp}, which is auxiliary and has no physical degrees of freedom.

The maximally extended $N=8$ supergravity constructed in this manner contains only 63 true scalars rather than 70; the other seven physical degrees of freedom are replaced by the physical degrees of freedom of the seven rank-2 antisymmetric tensor gauge fields. This set necessarily includes also a new completely antisymmetric tensor field of rank three. In Ref. 47 the contributions to the anomaly of the energy–momentum tensor were calculated for rank-2 antisymmetric tensor gauge fields ($A=1+1/90$) and for the rank-3 completely antisymmetric tensor field ($A=-2$). Now, recalculating the contribution to the anomaly of the energy–momentum tensor from the new supermultiplet, we obtain identically zero.

This property of nonstandard fields has also been used to construct anomaly-free superfield models of gravity with $N\geqslant3$. In Ref. 57 it was shown that such theories of supergravity can be constructed from three basis multiplets with $N=3$, one of which contains an antisymmetric tensor gauge field.

The study of Green and Schwarz[58] has received a great deal of attention. They demonstrated anomaly cancellation in supersymmetric Yang–Mills theory in $d=10$ interacting with $N=1$, $d=10$ supergravity for the particular gauge groups $SO(32)$ and $E_8\times E_8$. Antisymmetric tensor gauge fields play a key role in these cancellations. This study demonstrated the possibility of constructing a consistent quantum theory of superstrings based on these special gauge groups.

Questions related to the use of the low-energy effective string action in cosmology have recently generated considerable interest. This action for the closed string contains three long-range fields: the dilaton ϕ, the antisymmetric gauge field A_{mn}, and the graviton, which are massless excitations of the string. Cosmological solutions with a dilaton and nontrivial field-strength tensor for an antisymmetric tensor gauge field were obtained by Tseytlin.[59] The authors of Ref. 60 gave a general analytic solution for the evolution in the early (but not too early) string era, where only massless bosonic fields dominate in the dynamics. The authors showed that the presence of an antisymmetric tensor gauge field very strongly affects the evolution of the dilaton in four dimensions and can also lead to anisotropic expansion in models with a large number of dimensions. Here one wants expansion of only three-dimensional space.

When discussing the low-energy phenomenology of the antisymmetric tensor gauge field it is necessary to indicate the mechanism by which this field becomes massive. Such a mechanism was actually found in supersymmetric gauge theories of gravity with nonzero topological mass parameter h (Ref. 61). In such theories the antisymmetric tensor gauge field A_{mn} from the gravitational supermultiplet always enters in the combination $A_{mn}+hF_{mn}$ involving the strength tensor F_{mn} for the vector gauge field A_m of the vector supermultiplet. As a result of the gauge transformations (15) for the antisymmetric tensor field with $\lambda_m=-h^{-1}A_m$, this field absorbs the vector field and becomes massive. This mechanism of acquiring mass is in many respects the analog of the Higgs mechanism. A similar phenomenon has also been discovered in other generalized models of gauged supergravity with nonzero mass parameter m (Ref. 62).

3. ANTISYMMETRIC TENSOR MATTER FIELDS

In this section we shall study the antisymmetric tensor field not as a gauge field with the transformations (15), but as a matter field. We shall also give up the constraint (10). This allows us to introduce the action for the Abelian antisymmetric tensor field and the interaction with the antisymmetric tensor current J_{mn} free from the condition $\partial^m J_{mn}=0$. Since there is no gauge symmetry, we can write the conformally invariant action (4) for the antisymmetric tensor field. The Green functions for such fields are uniquely determined in conformally invariant field theory.[13] Such fields were first used as auxiliary fields in $N=2$ conformal supergravity[14,15] for closure of the superconformal transformations for the off-shell field multiplet. Antisymmetric tensor matter fields can be introduced into the standard model of electroweak interactions as physical fields analogous to the Higgs doublet. Recent experiments on semileptonic three-particle decays of mesons[19,20] support this possibility.

3.1. Massless particles of arbitrary spin

The simplest spinor representations of the Lorentz group are the undotted ψ_α and conjugate dotted $\psi_{\dot\alpha}$ fundamental Weyl spinors. The Pauli matrices $(\hat\sigma_m)^{\dot\alpha\beta}$ couple the spinor and Lorentz indices. The simplest action for a free massless spinor particle with spin 1/2 can be found as an invariant bilinear combination of fundamental spinors and the 4-momentum p_m:

$$\mathscr{A}_{1/2}=\int d^4p\,\psi_{\dot\alpha}\hat{p}^{\dot\alpha\beta}\psi_\beta, \quad (33)$$

where $\hat{p}^{\dot\alpha\beta}=p^m(\hat\sigma_m)^{\dot\alpha\beta}$. For the standard representation of the Pauli matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},$$

the equations of motion take the form[63]

$$\hat{p}^{\dot\alpha\beta}\psi_\beta=\begin{pmatrix}p_+&q_-\\q_+&p_-\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}=0. \quad (34)$$

Here we have introduced the longitudinal $p_\pm=p_0\pm p_3$ and transverse $q_\pm=p_1\pm ip_2$ components of momentum relative to the third axis, the spin projection on which has the definite values $\pm 1/2$. The operator $\hat{\mathrm{p}}$ has two eigenvalues $\lambda_+=p_0-|\mathbf{p}|$ and $\lambda_-=p_0+|\mathbf{p}|$, corresponding to solutions with positive $p_0=|\mathbf{p}|$ and negative $p_0=-|\mathbf{p}|$ frequencies, respectively. These two branches of the solutions describe a massless particle with left-handed chirality and an antiparticle with right-handed chirality. Such particles exist in nature and are called neutrinos. The equation of motion for the conjugate spinor has the form

$$p^{\alpha\dot\beta}\psi_{\dot\beta}\equiv(C^{-1})^{\alpha\beta}p_{\beta\dot\alpha}C^{\dot\alpha\dot\beta}\psi_{\dot\beta}=0.$$

The charge-conjugation matrices $C^{\alpha\beta}$ and $C^{\dot\alpha\dot\beta}$ raise the spinor indices of the operator $p_{\alpha\dot\beta}=p_m(\sigma^m)_{\alpha\dot\beta}$.

To generalize the equations obtained above to the case of higher spins, we consider arbitrary representations of the Lorentz group $(m/2,n/2)$ and $(n/2,m/2)$. The spinor $\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}$ and its conjugate $\psi_{\dot\alpha_1\ldots\dot\alpha_m\beta_1\ldots\beta_n}$, which are symmetric in both types of index, describe a particle and an antiparticle with spin $j=(m+n)/2$. By analogy with the preceding discussion, we can construct a very simple invariant action if the operator $\hat{p}^{\dot\alpha\beta}$ $(p^{\alpha\dot\beta})$ is used to associate each undotted (dotted) index of the first spinor with a dotted (undotted) index of its conjugate spinor:

$$\mathscr{A}_j^{(m/2,n/2)}=\int d^4p\,\psi_{\dot\alpha_1\ldots\dot\alpha_m\beta_1\ldots\beta_n}\hat{p}^{\dot\alpha_1\alpha_1}\ldots\hat{p}^{\dot\alpha_m\alpha_m}\times p^{\beta_1\dot\beta_1}\ldots p^{\beta_n\dot\beta_n}\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}. \quad (35)$$

A particle with spin j has $2j+1=m+n+1$ components. However, the spinor $\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}$ has $(m+1)\times(n+1)$ independent components. The number of components is equal to the number of degrees of freedom of a particle with spin j only in the case when it contains indices of a single type (m or n equal to zero). Therefore, in this case no auxiliary constraint like the Lorentz condition on the spinor components is necessary. Using this most economical description of particles with spin j, Weinberg, on the basis of the most general principles of quantum field theory—Lorentz invariance and causality—found the Green functions for the $2j+1$ component functions.[64] If we change from our spinor notation to $2j+1$ component functions, the kinetic term in (35) exactly reproduces the operator structure of the Green functions obtained by Weinberg.

When the spinor contains both types of index, it is necessary to impose the auxiliary condition of transversality:

$$p^{\alpha_1\dot\beta_1}\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}=0, \quad (36)$$

which leads to the necessary reduction of the degrees of freedom: $(m+1)\times(n+1)-m\times n=m+n+1$. Varying (35), we obtain the equations of motion for the spinor $\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}$:

$$\hat{p}^{\dot\alpha_1\alpha_1}\ldots\hat{p}^{\dot\alpha_m\alpha_m}p^{\beta_1\dot\beta_1}\ldots p^{\beta_n\dot\beta_n}\psi_{\alpha_1\ldots\alpha_m\dot\beta_1\ldots\dot\beta_n}=0. \quad (37)$$

For particles with spin greater than unity, the equations of motion contain derivatives of higher than second order and we encounter the problem of unitarity, which is still not solved. We shall therefore restrict ourselves to unit spin and study the equations of motion arising in this case.

3.2. Massless particles of unit spin

Let us first consider the usual (1/2,1/2) representation for a particle with unit spin. We rewrite the action

$$\mathscr{A}_1^{(1/2,1/2)}=\int d^4p\,\psi_{\dot\alpha\beta}\hat{p}^{\dot\alpha\alpha}p^{\beta\dot\beta}\psi_{\alpha\dot\beta}, \quad (38)$$

following from (35), in the form more usual for us, introducing the vector potential A_m: $\psi_{\alpha\dot\beta}=\frac{1}{2}(\sigma^m)_{\alpha\dot\beta}A_m$. Summation over the spinor indices leads to the trace of four σ matrices: $\mathrm{Tr}\ \hat{p}\sigma_n\hat{p}\sigma_m=4p_mp_n-2g_{mn}p^2$. Therefore, (38) takes the form

$$\mathscr{A}_1^{(1/2,1/2)}=-\tfrac{1}{2}\int d^4pA_m^*(p)p^2\bar{g}^{mn}(p)A_n(p), \quad (39)$$

where $\bar{g}_{mn}(p)=g_{mn}-2p_mp_n/p^2$. We note that the operator $\bar{g}_{mn}(p)=P^1_{mn}-P^0_{mn}$ can be represented as the difference of two projection operators of unit spin $P^1_{mn}=g_{mn}-p_mp_n/p^2$ and zero spin $P^0_{mn}=p_mp_n/p^2$. Thus, all the components of the vector potential A_m contribute to the action (39).

Using the Fourier transform

$$A_m(p)=\frac{1}{(2\pi)^2}\int d^4pA_m(x)e^{-ipx},$$

we write the action (39) in x space as

$$\mathscr{A}_1^{(1/2,1/2)}=\int d^4x\left[-\frac{1}{4}F_{mn}F^{mn}+\frac{1}{2}(\partial_mA^m)^2\right]. \quad (40)$$

Here F_{mn} is the gauge-invariant strength tensor of the field A_m. We see that the action (40) is not gauge-invariant and corresponds to the action for the electromagnetic field in a certain gauge. If the Lorentz condition $\partial_mA^m=0$ is imposed, we arrive at the usual description of a massless transverse particle, the photon.

Let us for the moment forget about this condition and construct the Hamiltonian for the action (40) without using any sort of constraints. We shall need the solutions of the equations of motion following from (39) or (40). For this it is convenient to work in p space, and therefore we shall use the action (39). We introduce a reference frame attached to the vector $\mathbf{p}$ and write $A_m(p)$ as the sum of transverse, longitudinal, and time components:

$$A_m(p)=e_m^1 a_1(k)+e_m^2 a_2(k)+e_m^3 a_3(k)+e_m^0 a_0(k). \quad (41)$$

Here $\mathbf{e}^1$ and $\mathbf{e}^2$ are polarization unit space vectors, orthogonal to each other and to the axis $\mathbf{e}^3$ of the momentum vector:

$$(\mathbf{e}^i\cdot\mathbf{e}^j)=\delta_{ij}, \quad [\mathbf{e}^i\times\mathbf{e}^j]=\varepsilon_{ijk}\mathbf{e}^k \quad (i,j,k=1,2,3),$$

$$\mathbf{e}^3=\frac{\mathbf{p}}{|\mathbf{p}|}, \quad \mathbf{e}^1(-\mathbf{p})=-\mathbf{e}^2(\mathbf{p}), \quad e_0^i=0, \quad (42)$$

and e^0 is the unit time vector: $e_m^0=\delta_{m0}$. Then the action (39) takes the form

$$\begin{aligned}\mathscr{A}_1^{(1/2,1/2)}=\frac{1}{2}\int d^4p\{&a_1^*(p)(p_0^2-\mathbf{p}^2)a_1(p)+a_2^*(p)\\ &\times(p_0^2-\mathbf{p}^2)a_2(p)+a_0^*(p)(p_0^2+\mathbf{p}^2)a_0(p)\\ &+a_3^*(p)(p_0^2+\mathbf{p}^2)a_3(p)\\ &-2p_0|\mathbf{p}|[a_0^*(p)a_3(p)+a_3^*(p)a_0(p)]\}.\end{aligned}$$

This action can be diagonalized by introducing the following linear combinations of the time and longitudinal components: $a_\pm=[a_0(p)\pm a_3(p)]/\sqrt{2}$. In this notation we have

$$\begin{aligned}\mathscr{A}_1^{(1/2,1/2)}=\frac{1}{2}\int d^4p\{&a_1^*(p)(p_0^2-\mathbf{p}^2)a_1(p)+a_2^*(p)\\ &\times(p_0^2-\mathbf{p}^2)a_2(p)+a_+^*(p)(p_0-|\mathbf{p}|)^2a_+(p)\\ &+a_-^*(p)(p_0+|\mathbf{p}|)^2a_-(p)\}. \quad (43)\end{aligned}$$

Now we can easily write down the equations of motion by varying the action (43):

$$\begin{aligned}&(p_0-|\mathbf{p}|)(p_0+|\mathbf{p}|)a_1(p)=0,\\ &(p_0+|\mathbf{p}|)(p_0-|\mathbf{p}|)a_2(p)=0,\\ &(p_0-|\mathbf{p}|)^2a_+(p)=0, \quad (p_0+|\mathbf{p}|)^2a_-(p)=0. \quad (44)\end{aligned}$$

The solutions of (44) for the transverse components, taking into account the reality of the field $A_m(x)$,

$$\begin{aligned}&a_1(p_0,\mathbf{p})=\delta(p_0+|\mathbf{p}|)a_1(\mathbf{p})-\delta(p_0-|\mathbf{p}|)a_2^*(-\mathbf{p}),\\ &a_2(p_0,\mathbf{p})=\delta(p_0+|\mathbf{p}|)a_2(\mathbf{p})-\delta(p_0-|\mathbf{p}|)a_1^*(-\mathbf{p}),\end{aligned}$$

contain both positive-frequency and negative-frequency parts, $a_{1,2}^*(-\mathbf{p})$ and $a_{1,2}(\mathbf{p})$, respectively. The solutions for the linear combinations of time and longitudinal components that we have introduced are

$$\begin{aligned}&a_+(p_0,\mathbf{p})=\delta(p_0-|\mathbf{p}|)a_s(\mathbf{p})+\delta'(p_0-|\mathbf{p}|)a_s'(\mathbf{p}),\\ &a_-(p_0,\mathbf{p})=\delta(p_0+|\mathbf{p}|)a_s^*(-\mathbf{p})-\delta'(p_0+|\mathbf{p}|)a_s'^*(-\mathbf{p}).\end{aligned}$$

These solutions, first of all, contain only single-frequency parts for each of the components $a_\pm$ and, second, contain derivatives of δ functions, which causes the solutions to grow linearly in time. If we restrict ourselves to the class of functions falling off at infinity, we must require that such solutions be absent, i.e.,

$$a_s'(\mathbf{p})=0. \quad (45)$$

This condition exactly coincides with the Lorentz condition. In this case we obtain solutions for $A_\pm(x)$ in the form of single-frequency plane waves.

Now we can write down the expression for the Hamiltonian and substitute the solutions that we have obtained into it. Direct calculations give

$$\begin{aligned}\mathscr{H}=\frac{1}{2}\int d^3x[&(\partial_0\mathbf{A})^2+(\partial_i\mathbf{A})^2+(\partial_0A_0)^2-(\partial_iA_0)^2\\ &+2(\partial_iA_i)^2]=\frac{1}{\pi}\int d^3p\mathbf{p}^2[a_1^*(\mathbf{p})a_1(\mathbf{p})\\ &+a_2^*(\mathbf{p})a_2(\mathbf{p})]. \quad (46)\end{aligned}$$

From this we see that, first, the Hamiltonian is positive-definite and, second, that only the transverse components contribute to the energy. The longitudinal and time components are, so to speak, unphysical in this gauge. If we calculate other dynamical invariants like the total momentum and spin of the system, we again find that the unphysical degrees of freedom cancel. Of course, we have not discovered anything new here; we have only again confirmed the fact that the photon is transverse. However, this example will help us in analyzing the physical degrees of freedom for an antisymmetric tensor matter field, to which we now turn.

The action for fields transforming as the (1,0) and (0,1) representations of the Lorentz group follows from (35):

$$\mathscr{A}_1^{(1,0)}=\int d^4p\,\psi_{\dot\alpha\dot\beta}\hat{p}^{\dot\alpha\alpha}\hat{p}^{\dot\beta\beta}\psi_{\alpha\beta}. \quad (47)$$

As in the case of the electromagnetic field, we transform from spinor to Lorentz indices. The symmetric spinor $\psi_{\alpha\beta}$ in (47) can be expressed in terms of the rank-2 antisymmetric tensor field T_{mn}:

$$\psi_{\alpha\beta}=\frac{i}{8}\,\varepsilon^{abmn}(\sigma_a\hat\sigma_bC)_{\alpha\beta}T_{mn} \quad (48)$$

[see Eq. (2)]. Summing over the spinor indices and taking the trace of the six σ matrices, we arrive at the action for an antisymmetric tensor matter field:

$$\mathscr{A}_1^{(1,0)}=\frac{1}{4}\int d^4pT_{ab}^*(p)p^2\Pi^{abmn}(p)T_{mn}(p), \quad (49)$$

where

$$\Pi_{abmn}(p)=\tfrac{1}{2}[\bar g^{am}(p)\bar g^{bn}(p)-\bar g^{an}(p)\bar g^{bm}(p)]. \quad (50)$$

As in the case of the electromagnetic field, we can write the operator for the kinetic term $\Pi(p)=P^+(p)-P^-(p)$ as the difference of projection operators of unit spin (11). From this representation we see that all six components, vector and pseudovector, of the antisymmetric tensor matter field contribute to the action.

The action (4) for the antisymmetric tensor matter field in x space is conformally invariant. Therefore, just as in the case of gauge symmetry, here the fact that this field is originally massless is important. We shall show that the physical components of the antisymmetric tensor matter field are the longitudinal components of the vector $A_i=T_{0i}$ and the pseudovector $B_i=\frac{1}{2}\varepsilon_{ijk}T_{jk}$ (Ref. 65). For this we calculate the full Hamiltonian of the system, taking into account the equations of motion.

Let us expand the vector $\mathbf{A}(p)=\mathbf{e}^i a_i(p)$ and the pseudovector $\mathbf{B}(p)=\mathbf{e}^i b_i(p)$ in a complete set of orthonormal vectors $\mathbf{e}^i$ (42). Then the action (49) can be written as

$$\mathscr{A}_1^{(1,0)}=\int d^4p\left\{\sum_{\lambda=1}^{2}[a_\lambda^*(p)(p_0^2+\mathbf{p}^2)a_\lambda(p)+b_\lambda^*(p)(p_0^2+\mathbf{p}^2)b_\lambda(p)]+2p_0|\mathbf{p}|[a_1^*(p)b_2(p)+b_2^*(p)a_1(p)-a_2^*(p)b_1(p)-b_1^*(p)a_2(p)]+a_3^*(p)(p_0^2-\mathbf{p}^2)a_3(p)+b_3^*(p)(p_0^2-\mathbf{p}^2)b_3(p)\right\}. \quad (51)$$

The linear substitution

$$a_1(p)=\frac{1}{\sqrt{2}}[c_1(p)+d_2(p)],$$

$$a_2(p)=\frac{1}{\sqrt{2}}[c_2(p)+d_1(p)], \quad a_3(p)=c_3(p);$$

$$b_1(p)=\frac{1}{\sqrt{2}}[d_1(p)-c_2(p)],$$

$$b_2(p)=\frac{1}{\sqrt{2}}[d_2(p)-c_1(p)], \quad b_3(p)=d_3(p)$$

diagonalizes the action (51):

$$\mathscr{A}_1^{(1,0)}=\int d^4p[c_1^*(p)(p_0-|\mathbf{p}|)^2c_1(p)+c_2^*(p)(p_0+|\mathbf{p}|)^2c_2(p)+c_3^*(p)(p_0-|\mathbf{p}|)\times(p_0+|\mathbf{p}|)c_3(p)+(c\to d)]. \quad (52)$$

From the minimum of the action we find the following equations of motion for the c components:

$$(p_0-|\mathbf{p}|)^2c_1(p_0,\mathbf{p})=0, \quad (p_0+|\mathbf{p}|)^2c_2(p_0,\mathbf{p})=0,$$
$$(p_0-|\mathbf{p}|)(p_0+|\mathbf{p}|)c_3(p_0,\mathbf{p})=0. \quad (53)$$

The same equations of motion are valid also for the d components. Taking into acount the reality of the field T_{mn}, the solutions of (53) are found to be

$$c_1(p_0,\mathbf{p})=\delta(p_0-|\mathbf{p}|)c_T(\mathbf{p})+\delta'(p_0-|\mathbf{p}|)c_T'(\mathbf{p}),$$
$$c_2(p_0,\mathbf{p})=\delta(p_0+|\mathbf{p}|)c_T^*(-\mathbf{p})+\delta'(p_0+|\mathbf{p}|)c_T'^*(-\mathbf{p}),$$
$$c_3(p_0,\mathbf{p})=\delta(p_0+|\mathbf{p}|)c_L(\mathbf{p})-\delta(p_0-|\mathbf{p}|)c_L^*(-\mathbf{p}). \quad (54)$$

Here we note that only the longitudinal components contain positive-frequency and negative-frequency parts; the transverse components c_1 and d_1 contain only solutions with positive frequency $p_0=|\mathbf{p}|$, and c_2 and d_2 contain only solutions with negative frequency $p_0=-|\mathbf{p}|$. In addition, the solutions for the transverse components contain derivatives of δ functions, and this leads to solutions which grow linearly in time. Therefore, if we want to restrict ourselves to solutions only in the form of plane waves, it is necessary to require that the components c_T' and d_T' be zero:

$$c_T'(\mathbf{p})=0; \quad d_T'(\mathbf{p})=0. \quad (55)$$

Substituting the solutions (54) into the Hamiltonian

$$\mathscr{H}=\frac{1}{2}\int d^3x[(\partial_0\mathbf{A})^2-(\partial_i\mathbf{A})^2+2(\partial_iA_i)^2+(\mathbf{A}\to\mathbf{B})]$$
$$=\frac{1}{\pi}\int d^3p\,\mathbf{p}^2[a_3^*(\mathbf{p})a_3(\mathbf{p})+b_3^*(\mathbf{p})b_3(\mathbf{p})], \quad (56)$$

we find that it is positive-definite and contains the contribution of only the longitudinal components of the vector and pseudovector fields. Similarly, it can be shown that the contribution of the transverse components also cancels in the calculation of other dynamical invariants and that the total spin is zero. Therefore, the antisymmetric tensor matter field describes scalar and pseudoscalar degrees of freedom.

The antisymmetric tensor matter field describes one degree of freedom more than the antisymmetric tensor gauge field. This follows from the fact that the condition (10) eliminates the two transverse and one longitudinal component of the vector field A_i, whereas the conditions (55) eliminate only the two transverse components. In order to write (55) in covariant form, we must introduce a single auxiliary scalar field φ and generalize the condition (10):

$$\partial^m T_{mn}=\partial_n\varphi. \quad (57)$$

Owing to the arbitrariness of the field φ, the condition (57) leads to the two independent conditions (55). Therefore, in the covariant quantization of the antisymmetric tensor matter field it is necessary to introduce an auxiliary scalar field and the constraints (57). We also note that the conditions (55) select only two of the four transverse components of the vector and pseudovector; fortunately, the other two components cancel in the calculation of dynamical invariants. This is completely analogous to the case of the electromagnetic field, where one Lorentz condition or the condition (45) leads to the elimination of the contribution to the dynamical invariants of one additional scalar degree of freedom.

The antisymmetric tensor matter field, like the electromagnetic field, possesses unphysical degrees of freedom which are not manifested at all on the mass shell. However, we know from experience in working with gauge fields that unphysical degrees of freedom can contribute to closed loops, leading to unitarity violation.[32] Even in the case of the Abelian electromagnetic field interacting with gravity, it is necessary to introduce ghost fields[66] which would cancel this contribution. Therefore, also in our case it is apparently necessary to introduce the corresponding ghost fields, because all the components of the antisymmetric tensor field contribute to the action. Like the antisymmetric tensor gauge field, here again a pyramid of compensating ghost fields arises. Naive counting of the degrees of freedom when the required unitarity condition is satisfied leads to the following set of ghost fields: a pair of anticommuting vector fields $C_m, \bar{C}_m$

and two pairs of commuting scalar fields $D,\bar{D}$ and $E,\bar{E}$ (Ref. 65). However, there is still no systematic quantization of an interacting antisymmetric tensor matter field. The main difficulty is the lack (or lack of knowledge) of a symmetry principle like gauge invariance which would allow a systematic quantization to be done.[67] Nevertheless, this does not prevent us from calculating processes in the lower orders of perturbation theory: tree level plus one-loop quantum corrections with external and nonclosed lines corresponding to antisymmetric tensor matter fields.

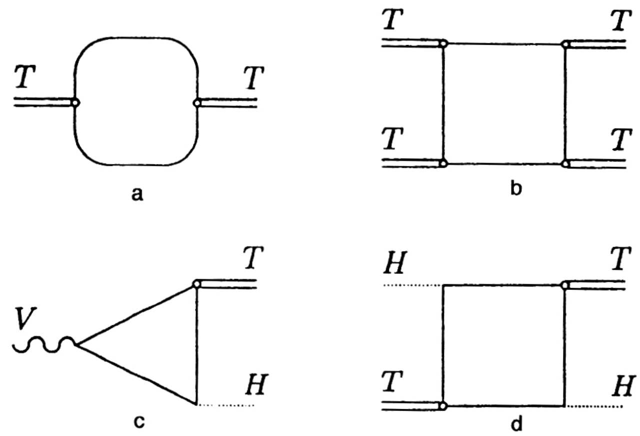

FIG. 1.

3.3. Interactions of the antisymmetric tensor field

There is a one-to-one correspondence (2), (48) between the components of a rank-2 antisymmetric tensor field T_{mn} and rank-2 symmetric spinors $\psi_{\alpha\beta}$ and $\psi_{\dot\alpha\dot\beta}$. Let us write down the simplest Lorentz-invariant Hermitian bare interaction of this field with the fundamental Weyl spinors, using spinor notation:

$$\mathcal{L}_{\rm int}=t[\psi_\alpha\psi^{\alpha\beta}\psi_\beta+\psi_{\dot\alpha}\psi^{\dot\alpha\dot\beta}\psi_{\dot\beta}]. \tag{58}$$

Owing to the requirement of Hermiticity, the interaction (58) contains Weyl spinors ψ_α and their conjugates $\psi_{\dot\alpha}$. Let us define the Dirac bispinor Ψ and the γ matrices in the helicity representation as

$$\Psi=\begin{pmatrix}\psi_\alpha\\ iC^{\dot\alpha\dot\beta}\psi_{\dot\beta}\end{pmatrix}=\begin{pmatrix}\Psi_L\\ \Psi_R\end{pmatrix},\quad \gamma^m=\begin{pmatrix}0 & \sigma^m\\ \hat\sigma^m & 0\end{pmatrix},$$

$$\sigma^{mn}=\frac{i}{2}[\gamma^m,\gamma^n]. \tag{59}$$

Using these definitions and (2) and (48), we rewrite (58) in the more usual form

$$\mathcal{L}_{\rm int}=\frac{t}{4\sqrt{2}}(\bar\Psi_R\sigma^{mn}T^-_{mn}\Psi_L+\bar\Psi_L\sigma^{mn}T^+_{mn}\Psi_R)$$

$$=\frac{t}{4}\bar\Psi\sigma^{mn}\Psi T_{mn}. \tag{60}$$

In obtaining the last equation we have used the identity

$$\frac{i}{2}\varepsilon^{mnab}\sigma_{ab}=\gamma^5\sigma^{mn}. \tag{61}$$

Now we can easily compute the one-loop quantum correction (Fig. 1a) to the action of the antisymmetric tensor matter field (4), (49), assuming that the Dirac particles are massless. The divergent part of the polarization operator,

$$\mathcal{P}_{mnab}(p)=i\left(\frac{t}{4}\right)^2\int\frac{d^4l}{(2\pi)^4}\,{\rm Tr}[\sigma_{mn}(\not p-\not l)^{-1}\sigma_{ab}\not l^{-1}]$$

$$\overset{1/\varepsilon}{=}\frac{1}{12\varepsilon}\left(\frac{t}{4\pi}\right)^2\Pi_{mnab}(p), \tag{62}$$

determines the renormalization of the classical action (4), (49) and exactly reproduces the structure of its kinetic term (50), which is a sign of renormalizability. The ability of quantum corrections to reproduce the structure of the classical action has been used as the foundation of the dynamical theory of composite particles.[68] Thus, it is yet another way of obtaining the classical free action for the antisymmetric tensor field.

The bare interaction (58), (60) at the quantum level also generates the action of the antisymmetric tensor matter field (Fig. 2b):

$$\Delta\mathcal{L}=-\frac{1}{48\varepsilon}\frac{t^4}{16\pi^2}\left[\frac{1}{4}(T_{mn}T^{mn})^2-T_{mn}T^{na}T_{ab}T^{bm}\right]. \tag{63}$$

This action possesses the extremely important property of symmetry under dual transformations:

$$T_{mn}\to T_{mn}\cos(h\lambda)+i\tilde{T}_{mn}\sin(h\lambda);$$

$$\tilde{T}_{mn}\to iT_{mn}\sin(h\lambda)+\tilde{T}_{mn}\cos(h\lambda). \tag{64}$$

Conversely, the requirement of symmetry under the transformations (64) leads uniquely to the action (63). The free action (4) is also invariant under the global transformations

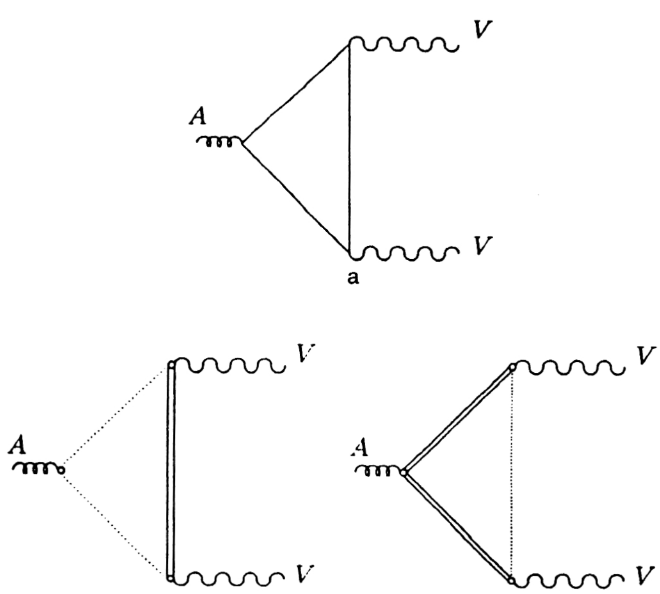

FIG. 2.

(64), when the parameter λ is independent of the spacetime point. This can easily be seen by rewriting the action (4) in terms of the self-dual and anti-self-dual tensors T^+_{mn} and T^-_{mn}:

$$\mathscr{A}_{\text{conformal}} = -\int d^4x(\partial^m T^+_{mk})\partial^n T^-_{nk}, \tag{65}$$

for which the transformations (64) acquire multiplicative form:

$$T^\pm_{mn} \to \exp(\pm ih\lambda)T^\pm_{mn}. \tag{66}$$

Localization of the transformations (66) makes it necessary to introduce the pseudoscalar gauge field A_m and to replace the ordinary derivatives $\partial^m T^\pm_{mn}$ by covariant derivatives $D^m T^\pm_{mn} = (\partial^m \mp ihA^m)T^\pm_{mn}$, which ensure that (65) is invariant under the transformations (66) with arbitrary functions $\lambda(x)$. The gauge field A_m transforms in the usual way: $A_m \to A_m + \partial_m \lambda$. Invariance of the bare interaction (60) under the transformations (64) requires the transformation law for the Dirac spinor $\Psi \to \exp(-ih\lambda/2)\Psi$ and the corresponding covariant derivatives in the kinetic term: $D_m\Psi = (\partial_m + ihA_m/2)\Psi$. We note that the axial charge of the Dirac spinor is half that of the antisymmetric tensor field. A detailed analysis of the renormalizability of the interactions introduced above at the one-loop level has been given in Ref. 18.

For physical applications it is necessary to add to the fields listed above at least one vector field V_m (the photon) and the scalar Higgs field $H^\pm$. As usual, the vector field is introduced through the covariant derivative $(\partial_m - ieV_m)\Psi$. As is well known, the simultaneous introduction of vector and pseudovector fields leads to Adler–Bell–Jackiw axial anomalies,[69] which can spoil the renormalizability of the model. Axial anomalies were discovered in the calculation of the triangle fermion loop with two vector vertices and one axial vertex (Fig. 2a). The presence of such quantum corrections makes it impossible to ensure simultaneously conservation of the vector current, $\partial_m V^m = 0$, and of the axial current,

$$\partial_m A^m = \frac{h}{2}\frac{e^2}{(4\pi)^2}\,\varepsilon^{mnab}F_{mn}F_{ab} \neq 0.$$

Here $F_{mn} = \partial_m V_n - \partial_n V_m$ is the vector field strength.

The simplest way of struggling with these anomalies is to introduce additional fermions with opposite axial charges in order to cancel the contributions from these graphs. Here the story would end if there were no antisymmetric tensor fields. Through their new interaction (Fig. 1c)

$$\mathscr{L}_3 = g[H^+T^-_{mn} + H^-T^+_{mn}]F^{mn} \tag{67}$$

with scalar fields and the vector field strength, they lead to anomalous graphs (Figs. 2b and 2c) of a new type. Omitting the details of the calculations, we give the final nonzero result for the divergence of the pseudovector field:

$$\partial_m A^m = h\frac{e^2}{(4\pi)^2}\,\varepsilon^{mnab}F_{mn}F_{ab} - \frac{h}{2}\frac{e^2}{(4\pi)^2}\,\varepsilon^{mnab}F_{mn}F_{ab}. \tag{68}$$

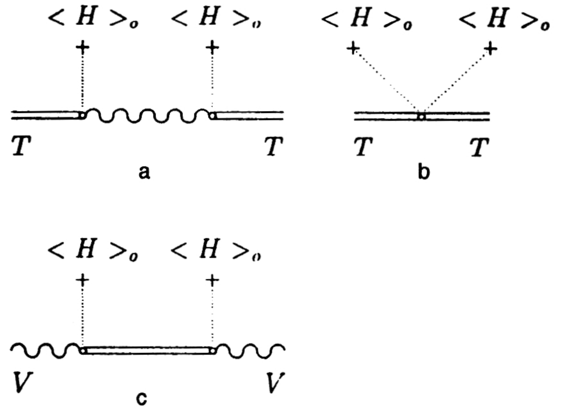

FIG. 3.

The first term on the right-hand side of (68) corresponds to the contributions of the graphs of Fig. 2b, in which the scalar field interacts directly with the pseudovector field, and the second term comes from the graphs of Fig. 2c, where the antisymmetric tensor field forms an axial vertex.

Here it should be stressed that whereas the formal reason for the anomaly in the case of the fermion loop (Fig. 2a) was its linear divergence, in our case the graphs in Figs. 2b and 2c converge. This confirms the view that the appearance of anomalies and violation of the classical conservation laws is not related to the regularization procedure, and that the classical and quantum symmetry properties are, in general, different.[70]

We note that antisymmetric tensor fields are also a source of anomalies, and, in principle, for some special choice of internal symmetry group and particle multiplets it is possible for the contributions of the graphs in Fig. 2 to cancel each other. However, we shall proceed much more simply: we require that the number of Higgs particles and antisymmetric tensor fields be doubled in order to cancel the anomalous contributions in the graphs of Figs. 2b and 2c. We shall use this principle for introducing antisymmetric tensor matter fields into the standard model of electroweak interactions.

Finally, let us discuss one of the most important problems in phenomenology: how to make particles massive. We note that the direct method of giving mass to an antisymmetric tensor matter field, i.e., writing the mass term as $M^2T_{mn}T^{mn} = M^2(\mathbf{B}^2 - \mathbf{A}^2)$, does not work. Because the positivity of the Hamiltonian (56) is spoiled by the indefinite sign of this mass term, tachyon solutions appear and the original consistent formulation of the theory for the massless antisymmetric tensor matter field completely loses its good properties. Therefore, to preserve the symmetry properties we must make the antisymmetric tensor matter field massive dynamically, using the mechanism of spontaneous symmetry breaking. In contrast to the usual Higgs mechanism, here in addition to the scalar field $H^\pm$ with nonzero vacuum expectation value $\langle H^\pm\rangle_0 = M/2g$ we also need the massless vector field V_m. The interaction (67) leads to the polarization operator $\mathscr{P}(p^2) = -(\langle H\rangle_0)^2/p^2 + O(g^0)$ (Fig. 3a) with the necessary pole $1/p^2$ (Ref. 71), which effectively ensures a mass term in the denominator of the propagator of the antisymmetric tensor matter field: $p^2[1+4g^2\mathscr{P}(p^2)] =(p^2-M^2)+O(g^2)$.

However, the inclusion of only the graph in Fig. 3a spoils the structure of the operator $\Pi(p^2)$ in the kinetic term for the antisymmetric tensor matter field. In order to preserve the conformally invariant form of the structure of $\Pi(p^2)$, it is necessary to add the interaction arising from radiative corrections (Fig. 1d),

$$\mathscr{L}_4=-\lambda[(H^+T^-_{mn})^2+(H^-T^+_{mn})^2], \qquad (69)$$

with a fixed constant $\lambda=g^2/2$. Then the effective propagator for the antisymmetric tensor matter field takes the form

$$\langle T(T_{mn}T_{ab})\rangle_0=\frac{2i\Pi_{mnab}(p)}{p^2-M^2}. \qquad (70)$$

Of course, this relation between the interaction constants in (67) and (69) cannot be ensured without some symmetry principle. Actually, exactly the same relation between the constants arises in extended theories of conformal supergravity interacting with the superconformal extended Yang–Mills theory.[72] It should be noted that the vector gauge field in this case also becomes massive,[15] owing to exchange of antisymmetric tensor particles (Fig. 3c). Now we are ready to turn to the extension of the standard model of electroweak interactions by the inclusion of antisymmetric tensor matter fields.

3.4. An extended model of electroweak interactions

The introduction of antisymmetric tensor matter fields into the standard model of electroweak interactions is of both theoretical and purely practical interest. Up to now not a single phenomenological model with unusual matter of this type has been constructed. The interpretation of certain recent experimental data[19,20] also requires the introduction of new tensor interactions.[73] The detailed analysis performed in Ref. 74 makes it possible to eliminate the contradictions and to reconcile all the previous experimental data[75] on the semileptonic radiative decay of the pion, $\pi^-\to e^-\tilde{\nu}\gamma$.

We assume, as usual, the local $SU_L(2)\times U_Y(1)$ symmetry of the weak interactions[76] with the gauge fields $\mathbf{A}_m$ and B_m. The fermionic sector of the matter fields contains several generations of two-component Weyl spinors: left-handed leptonic doublets $L_i=(\nu'_L e'_L)_i$ and right-handed leptonic singlets $(e'_R)_i$, left-handed quark doublets $Q_i=(u'_L d'_L)_i$, and right-handed quark singlets $(u'_R)_i$ and $(d'_R)_i$. Here i is the generation index, and we have omitted the color indices on the quark fields. The prime on a field means that it is a gauge eigenstate.

Let us now consider the bosonic sector of the matter fields. The standard model of electroweak interactions contains a single doublet of scalar Higgs fields. Let us extend this sector by adding antisymmetric tensor matter fields. These fields, like the scalar Higgs fields, interact with left- and right-handed fermions (60). For this Yukawa interaction to be $SU(2)\times U(1)$-invariant, the antisymmetric tensor matter field must be a doublet. The requirement that anomalies be absent in the extended model of electroweak interactions forces us to double the number of fields and to introduce two doublets of Higgs scalars $H_1=(H_1^0H_1^-)$ and $H_2=(H_2^+H_2^0)$ and two doublets of antisymmetric tensor fields $U_{mn}=(U^0_{mn}U^-_{mn})$ and $T_{mn}=(T^+_{mn}T^0_{mn})$ with opposite hypercharges: $Y(H_1)=Y(U)=-1$, $Y(H_2)=Y(T)=+1$. Their minimal interactions with gauge fields are introduced through the covariant derivative $D_m=\partial_m-ig/2\mathbf{T}\cdot\mathbf{A}_m -ig'/2YB_m$, where g and g' are the interaction constants and $\mathbf{T}$ and Y are the generators of the groups $SU(2)$ and $U(1)$, respectively.

Extension of the Higgs sector of the standard model of electroweak interactions by yet another Higgs doublet does not lead to any violation of the relation $\rho=m_W^2/(m_Z^2\cos^2\theta_W)=1$ at tree level, which, as is well known, corresponds to the experimental value $\rho_0=1.0004 \pm 0.0022\pm0.0020$ (Ref. 77). However, there is yet another difficulty associated with extension of the Higgs sector. If we allow arbitrary interactions of Higgs particles with fermions, it is possible for flavor-changing neutral currents to appear when the symmetry is broken. An elegant solution to this problem was found by Glashow and Weinberg.[78] Transitions involving flavor-changing neutral currents at tree level will be absent if the neutral components of one Higgs doublet interact only with quarks of the upper type with charge 2/3, and the neutral components of the other Higgs doublet interact with quarks of the lower type with charge $-1/3$ and with charged leptons. Naturally, this involves an additional symmetry, such as supersymmetry. The most general form of such an $SU(2)\times U(1)$-invariant Yukawa interaction is

$$\mathscr{L}_H=h^u_{ij}\bar{Q}_i(u'_R)_j\,H_1+[h^d_{ij}\bar{Q}_i(d'_R)_j + h^e_{ij}\,\bar{L}_i(e'_R)_j]H_2+\text{H.c.}, \qquad (71)$$

where h^u, h^d, and h^e are, in general, arbitrary nondiagonal matrices.

We can also write down a similar interaction with spinor fields for antisymmetric tensor fields:

$$\mathscr{L}_T=t^u_{ij}\bar{Q}_i\sigma^{mn}(u'_R)_jU_{mn}+[t^d_{ij}\bar{Q}_i\sigma^{mn}(d'_R)_j + t^e_{ij}\bar{L}_i\sigma^{mn}(e'_R)_j]T_{mn}+\text{H.c.} \qquad (72)$$

The absence of a symmetry principle leads to a large number of arbitrary parameters. This is a weak spot in the standard model. Let us simplify the model as much as possible, requiring universality of the tensor interaction:

$$t^u_{ij}=t^d_{ij}=t^e_{ij}=\frac{t}{\sqrt{2}}\,\delta_{ij}. \qquad (73)$$

After spontaneous symmetry breaking the neutral components of the Higgs fields acquire nonzero vacuum expectation values: $\langle H_1^0\rangle_0=v_1$ and $\langle H_2^0\rangle_0=v_2$. In supersymmetric generalizations of the standard model of electroweak interactions with two Higgs doublets a parameter β is introduced which characterizes the ratio of their vacuum expectation values: $\tan\beta\equiv v_1/v_2$. The substitution

$$(u'_L)_i=[S_u]_{ij}(u_L)_j,\quad (u'_R)_i=[T_u]_{ij}(u_R)_j,$$

$$(d'_L)_i=[S_d]_{ij}(d_L)_j,\quad (d'_R)_i=[T_d]_{ij}(d_R)_j,$$

$$(e'_L)_i=[S_e]_{ij}(e_L)_j,\quad (e'_R)_i=[T_e]_{ij}(d_R)_j.$$

diagonalizes the quark and charged-lepton mass matrices. In our very simple case (73) such transformations can generate flavor-changing neutral currents in (72) if equality of the unitary matrices $S_{u,d,e}=T_{u,d,e}$ for left- and right-handed quarks is not required. From this it follows directly that the matrices of the Yukawa coupling constants $h^{u,d,e}$ must be Hermitian, so that they can be diagonalized. That is, the requirement (73) that the Yukawa couplings be equal for antisymmetric tensor fields and that flavor-changing neutral currents be absent invokes a higher symmetry of the Yukawa constants for the Higgs particles. Actually, this possibility is now being actively discussed in connection with the search for symmetries for Yukawa coupling constants.[79]

Introducing the Cabibbo–Kobayashi–Maskawa matrix $V_{ij}=[S_u^\dagger S_d]_{ij}$, we can study the mixing for the quarks of the upper or lower type in charged currents. We shall assume that the neutrinos are massless. Therefore, using the degeneracy of the neutrino states, it is possible to factorize $[S_\nu^\dagger S_e]_{ij}$ in the lepton sector from the mixing matrix.

Interactions of the type (67),

$$\mathscr{L}_3=(g_1'\bar{H}_1T^{mn}+g_2'\bar{H}_2U^{mn})F_{mn}+(g_1\bar{H}_1\boldsymbol{\tau}\,T^{mn}+g_2\bar{H}_2\boldsymbol{\tau}U^{mn})\mathbf{G}_{mn}+\text{H.c.}, \tag{74}$$

where $\bar{H}=H^Ti\tau_2$ are transposed doublets and $F_{mn}=\partial_mB_n-\partial_nB_m$ and $\mathbf{G}_{mn}=\partial_m\mathbf{A}_n-\partial_n\mathbf{A}_m+g\mathbf{A}_m\times\mathbf{A}_n$ are the gauge-field strength tensors, give masses to the tensor particles and gauge fields when the symmetry is broken. For the photon to remain massless after the symmetry breaking, it must be required that the equations $g_1'/g_1=-g_2'/g_2=\tan\theta_W$ hold, where θ_W is the Weinberg angle. We note that the interactions (74) lead to mixing of the tensor fields T_{mn} and U_{mn}.

Now we have everything needed to discuss phenomenological consequences. We again note that our model, compared to the standard model of electroweak interactions, contains an additional Higgs doublet and two doublets of antisymmetric tensor matter particles. Their minimal interactions with gauge fields are introduced uniquely in terms of covariant derivatives. We have also written out their fundamental interactions with fermions and their nonminimal interaction with gauge fields which must lead to the physical mass spectrum when the symmetry is spontaneously broken. Here the main criterion was the absence of flavor-changing neutral currents. Of course, there exist many other interactions on which we will not dwell here, since we want to analyze low-energy processes.

3.5. Phenomenological consequences

Interactions of antisymmetric tensor fields with fermions (72) generate new effective interactions of the current-times-current type, in addition to the known interactions arising from electroweak gauge boson exchange. These interactions also contain charged and neutral currents. To avoid contradictions with the experimental data, we must assume that these interactions are relatively weak compared to the standard electroweak interactions. The new tensor interactions turn out to be as though screened. Charged weak currents were discovered long before neutral weak currents, which in the case of charged-particle interactions were screened by electromagnetic interactions. Therefore, the first experimental confirmation of the existence of new tensor interactions should be sought in the background of weak interactions of charged currents.

We shall consider interactions arising only from the charged antisymmetric tensor fields $T^\pm_{mn}$ and $U^\pm_{mn}$. The most general structure of the propagators for these particles after symmetry breaking is

$$\mathscr{P}(q)=\begin{pmatrix}\langle T(T^-T^+)\rangle_0 & \langle T(T^-U^+)\rangle_0\\ \langle T(U^-T^+)\rangle_0 & \langle T(U^-U^+)\rangle_0\end{pmatrix}=\frac{4i}{\Delta_q}\begin{pmatrix}(q^2-m^2)\mathbf{\Pi}^-(q) & \mu^2\mathbf{1}^-\\ \mu^2\mathbf{1}^+ & (q^2-M^2)\mathbf{\Pi}^+(q)\end{pmatrix}, \tag{75}$$

where $\Delta_q=(q^2-m^2)(q^2-M^2)-\mu^4$ and μ, m, and M are arbitrary mass parameters with $M/m=\tan\beta$, because the diagonal mass terms for T_{mn} and U_{mn} arise from the vacuum expectation values v_1 and v_2, respectively. From the form of the interaction (72) and the identities (61) it follows that the fields T^+ and U^- are self-dual and T^- and U^+ are anti-self-dual, so that the operator (75) contains self- and anti-self-dual operators $\mathbf{1}^\pm_{mnab}=\frac{1}{2}(\mathbf{1}_{mnab}\pm i/2\varepsilon_{mnab})$ and $\mathbf{\Pi}^\pm_{mnab}=\mathbf{1}^\pm_{mnkl}\mathbf{\Pi}_{klab}$.

The main difference between tensor interactions and the standard V−A interaction is related to the change of helicity of the fermions involved in the process. Since the standard weak interactions conserve helicity, the decay of the pseudoscalar π meson is strongly suppressed.[80] We can therefore obtain the very first condition on the parameters of the new tensor interaction from the experimental limits derived from these decays.[81] Neglecting the momentum transfer, $q^2\ll\mu^2,m^2,M^2$, we write the effective Lagrangian for semileptonic tensor interactions as

$$\mathscr{L}_{\text{eff}}=-\frac{t^2}{\Delta_0}\bar{u}_i\sigma^{ml}[m^2(1+\gamma^5)+\mu^2(1-\gamma^5)]\times V_{ij}d_j\frac{4q_mq^n}{q^2}(\bar{e}_R\sigma_{nl}\nu_L)+\text{H.c.} \tag{76}$$

The first term in the quark current arises from the exchange of T_{mn} particles, and the second term arises from the mixing of the T_{mn} and U_{mn} fields. To obtain the final form of the interaction (76) we used the useful identities

$$\tfrac{1}{2}(1\pm\gamma^5)\sigma^{mn}\otimes\tfrac{1}{2}(1\pm\gamma^5)\sigma_{mn}=\tfrac{1}{2}(1\pm\gamma^5)\sigma^{ml}\otimes\tfrac{1}{2}(1\pm\gamma^5)\sigma_{nl}\cdot\frac{4q_mq^n}{q^2},$$

$$\tfrac{1}{2}(1\pm\gamma^5)\sigma^{mn}\otimes\tfrac{1}{2}(1\mp\gamma^5)\sigma_{mn}=0. \tag{77}$$

For kinematic reasons, the tensor interaction (76) does not contribute to the semileptonic two-particle decay of the π meson π_{e_2}. However, in Ref. 82 it was shown that, owing to electromagnetic radiative corrections to the tensor interaction (76), the pseudotensor term $\bar{u}\sigma_{mn}\gamma^5d$ leads to generation of an interaction between the leptonic and pseudoscalar quark

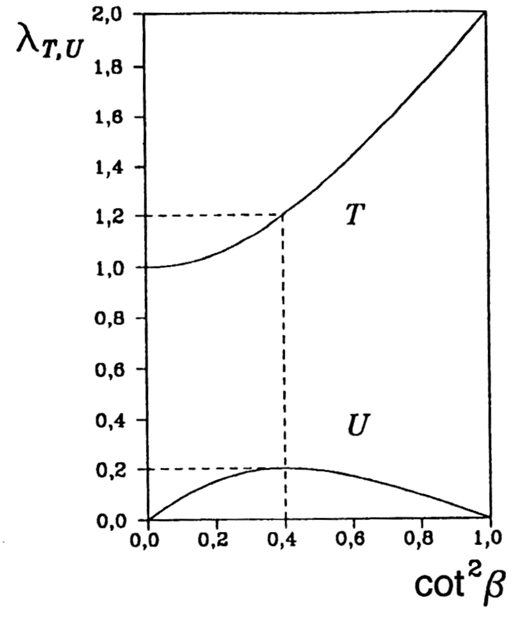

FIG. 4.

curents, to which pion decay is very sensitive.[83] This leads to a strong limit on the tensor interaction constant and practically excludes the possibility of it appearing in any current experiment. This, naturally, makes it impossible to explain the recent experimental data[19,20] in terms of the presence of a new tensor interaction. Our model[84] allows us to avoid this difficulty if we assume that the two mass parameters $\mu^2=m^2$ arising in spontaneous symmetry breaking are equal. Then the pseudotensor quark term $\bar{u}\sigma_{mn}\gamma^5 d$ disappears from (76), and the tensor term $\bar{u}\sigma_{mn}d$ does not contribute to pseudoscalar pion decay because of parity conservation in electromagnetic interactions.

Let us now consider the mass matrix for antisymmetric tensor fields:

$$\mathscr{M}^2=\frac{M^2}{\tan^2\beta}\begin{pmatrix}\tan^2\beta\mathbf{\Pi}^+ & \mathbf{1}^+\\ \mathbf{1}^- & \mathbf{\Pi}^-\end{pmatrix}, \tag{78}$$

which is parametrized by only two parameters: the dimensional mass parameter M and the ratio of the vacuum expectation values $\tan\beta$ of the neutral components of the two Higgs doublets.

The transformation to new fields,

$$T'_{mn}=T_{mn}\cos\varphi+\Pi_{mnab}U^{ab}\sin\varphi,$$

$$U'_{mn}=-\Pi_{mnab}T^{ab}\sin\varphi+U_{mn}\cos\varphi, \tag{79}$$

leads to a diagonal mass matrix $\mathscr{M}^2=M^2\mathrm{diag}(\lambda_T\mathbf{\Pi}^+,\lambda_U\mathbf{\Pi}^-)$ with the eigenvalues

$$\lambda_T=\tfrac{1}{2}[1+\tan^2\beta+\sqrt{(1-\tan^2\beta)^2+4}]/\tan^2\beta,$$

$$\lambda_U=\tfrac{1}{2}[1+\tan^2\beta-\sqrt{(1-\tan^2\beta)^2+4}]/\tan^2\beta,$$

when $\tan\varphi=\tfrac{1}{2}[1-\tan^2\beta+\sqrt{(1-\tan^2\beta)^2+4}]$. The nonnegativity of the eigenvalues of the squared mass matrix leads to the condition $0\leq\cot^2\beta\leq 1$. The curves for the eigenvalues λ_T and λ_U in this mixing-parameter range are shown in Fig. 4.

A remarkable feature of these curves is the presence of a maximum in the mass of the tensor field U'_{mn}. Therefore, the energy of the interaction of spinor particles via U'_{mn} exchange has a minimum at $\cot^2\beta_0=0.4$ in the static limit. Since the current-times-current interaction is always generated by a pair of tensor particles U_{mn} and T_{mn} and the mass of the T_{mn} particles increases monotonically with the mixing parameter $\cot^2\beta$, the energy minimum

$$U\sim\frac{1}{\lambda_U}+\frac{1}{\lambda_T}$$

is reached for a slightly larger value of the mixing parameter $\cot^2\beta=\sqrt{2}-1\approx 0.41$. In other words, we have quite unexpectedly fixed the value of the ratio of vacuum expectation values

$$\tan\beta=\sqrt{1+\sqrt{2}}\approx 1.55 \tag{80}$$

of the neutral components of the two Higgs doublets.

The value of $\tan\beta$ is in excellent agreement with the recent results of analyzing possible values of the parameters of a minimal supersymmetric model with unified coupling constants and masses[85] for t-quark mass in the range $m_t=174\pm 16$ GeV (Ref. 86). It is also interesting to note that this value of $\tan\beta$ corresponds to mixing angle φ of the U_{mn} and T_{mn} fields $\sin^2\varphi=(1-1/\sqrt{3})/2\approx 0.211$, which is surprisingly close to the Weinberg angle θ_W. It is impossible to understand this coincidence of the numerical values of the mixing angles in the gauge and matter sectors while staying within the framework of the standard model. This fact can probably be explained within a unified supersymmetric model containing antisymmetric tensor matter particles.

For phenomenological applications at low energies, let us write down all the effective interactions of leptons and quarks arising from exchange of antisymmetric tensor particles. For example, in the case of muon decay into an electron and an (anti)neutrino, in addition to the ordinary V−A interaction it is necessary to include also the tensor interaction

$$\mathscr{L}_{\mu e}=-\sqrt{2}G_F f_T\bar{\nu}_{\mu L}\sigma_{ml}\mu_R\,\frac{4q^m q_n}{q^2}\,\bar{e}_R\sigma^{nl}\nu_{eL}+\text{H.c.}, \tag{81}$$

where q_m is the 4-momentum transfer between the muon and electron pairs, and the positive dimensionless constant

$$f_T=\frac{\sqrt{2}}{G_F}\frac{t^2}{(1-\cot^2\beta)M^2}>0$$

determines the relative strength of the new tensor interactions in relation to the ordinary weak interactions. The additional interaction for semileptonic decays has the form

$$\mathscr{L}_{qe}=-\sqrt{2}G_F f_T\bar{u}\sigma_{ml}d^\theta\,\frac{4q^m q_n}{q^2}\,\bar{e}_R\sigma^{nl}\nu_L+\text{H.c.}, \tag{82}$$

where $d_i^\theta=V_{ij}d_j$ are mixed states for quarks of the lower type. The richer structure of the tensor interaction arises in the purely quark–quark sector:

$$\mathcal{L}_{ud} = -\sqrt{2} G_F f_T \Big[\bar{u}_L \sigma_{ml} d_R^\theta \cdot \bar{d}_R^\theta \sigma^{nl} u_L + \bar{u}_L \sigma_{ml} d_R^\theta \times \bar{d}_L^\theta \sigma^{nl} u_R + \bar{u}_R \sigma_{ml} d_L^\theta \cdot \bar{d}_R^\theta \sigma^{nl} u_L + \tan^2 \beta \bar{u}_R \sigma_{ml} d_L^\theta \times \bar{d}_L^\theta \sigma^{nl} u_R \frac{4 q^m q_n}{q^2}. \tag{83}$$

All the effective tensor interactions contain only one unknown interaction constant f_T. It is natural to try to fix its value from the experimental data on the semileptonic radiative decay of the pion,[19] the results of which cannot be interpreted within only the standard model[87] and supersymmetric extensions of it.[88] The standard matrix element for this decay can be written as[80]

$$M = M_{IB} + M_{SD}, \tag{84}$$

where

$$M_{IB} = -i \frac{e G_F V_{ud}}{\sqrt{2}} F_\pi m_e \varepsilon_m \bar{e} \left[\left(\frac{k}{kq} - \frac{p}{pq} \right)^m - \frac{i \sigma^{mn} q_n}{2kq} \right] \times (1 - \gamma^5) \nu \tag{85}$$

are the quantum electrodynamical corrections (bremsstrahlung) to pion decay $\pi \to e \nu$, and

$$M_{SD} = -\frac{e G_F V_{ud}}{\sqrt{2} M_\pi} \varepsilon^m \bar{e} \, [F_V e_{mnab} p^a q^b - i F_A (pq \cdot g_{mn} - p_m q_n)] \gamma^n (1 - \gamma^5) \nu \tag{86}$$

are structural-dependent amplitudes parametrized by two form factors F_V and F_A; ε^m is the photon polarization vector; and p, k, and q are the pion, electron, and photon 4-momenta, respectively.

In order to find the matrix element M_T of the tensor interaction (82) for this decay, it is necessary to calculate the matrix element $\langle \pi | \bar{u} \, \sigma_{mn} \gamma^5 d | \gamma \rangle$ for the tensor quark current. Using the hypothesis of partial conservation of the axial current and the sum rules of quantum chromodynamics, it can be written as[88]

$$\langle \pi | \bar{u} \, \sigma_{mn} \gamma^5 d | \gamma \rangle = \frac{e}{3} \chi \frac{\langle 0 | \bar{q} \, q | 0 \rangle}{F_\pi} \cdot (q_m \varepsilon_n - q_n \varepsilon_m),$$

where $F_\pi = 131$ MeV is the pion decay constant and $\chi = -5.7 \pm 0.6$ GeV^{-2} is the magnetic susceptibility[89] of the quark condensate $\langle 0 | \bar{q} \, q | 0 \rangle = -(0.24 \text{ GeV})^3$. Then the matrix element of the additional tensor interaction has the form

$$M_T = \frac{e G_F V_{ud}}{\sqrt{2}} F_T \left[\varepsilon^m q^n + \frac{(\varepsilon p) q^m - (pq) \varepsilon^m}{(p-q)^2} \times (p-q)^n \right] \bar{e} \, \sigma_{mn} (1 - \gamma^5) \nu, \tag{87}$$

where

$$F_T = \frac{4}{3} \chi \frac{\langle 0 | \bar{q} q | 0 \rangle}{F_\pi} f_T. \tag{88}$$

The first term in (87) coincides with the tensor matrix element proposed in Ref. 73. In our case a characteristic nonlocal matrix element also appears as a result of exchange of antisymmetric tensor matter particles $T_{mn}^\pm$. It should be noted that the two terms in (87) exhaust the possible gauge-invariant structures.

When the electron mass is neglected, the dominant contribution to the pion decay amplitude comes from the squares of the matrix elements M_{IB} and M_{SD}, and also from interference between the matrix elements of the tensor interaction M_T and bremsstrahlung M_{IB}:

$$\frac{d^2 \Gamma}{dx d\lambda} = \frac{\alpha}{2\pi} \Gamma_{\pi \to e\nu} \{ IB(x,\lambda) + a_{SD}^2 [(F_V + F_A)^2 \times SD^+(x,\lambda) + (F_V - F_A)^2 SD^-(x,\lambda)] - a_{SD} F_T I(x,\lambda) \} \tag{89}$$

where $a_{SD} = M_\pi^2 / 2F_\pi m_e$,

$$IB(x,\lambda) = \frac{1-\lambda}{\lambda} \cdot \frac{(1-x)^2 + 1}{x},$$

$$SD^+(x,\lambda) = \lambda^2 (1-x) x^3,$$

$$SD^-(x,\lambda) = (1-\lambda)^2 (1-x) x^3, \quad I(x,\lambda) = (1-\lambda) x^2.$$

In the pion rest frame the variables x and λ are defined as $x = 2E_\gamma / M_\pi$ and $\lambda = 2E_e / M_\pi \sin^2(\theta_{e\gamma}/2)$.

The experiment on the semileptonic radiative decay of the pion[19] was carried out in a large kinematical region: $0.3 < x < 1.0$, $0.2 < \lambda < 1.0$. The measured total decay probability $B^{\exp} = (1.61 \pm 0.23) \times 10^{-7}$ turned out to be smaller than the value expected theoretically: $B^{\text{th}} = (2.41 \pm 0.07) \times 10^{-7}$. Detailed analysis of the experimental data for the differential cross section (89) neglecting the last term gives a negative coefficient in front of the $SD^-(x,\lambda)$ term. Since the functions $I(x,\lambda)$ and $SD^-(x,\lambda)$ have roughly the same form, this result can be attributed to destructive interference between the tensor interaction and bremsstrahlung. We note that our model also predicts the sign of the interference. If we choose $F_T = (1.57 \pm 0.45) \times 10^{-2}$, the differential cross section (89) will adequately describe the experimental data. From (88) we obtain the numerical value of the effective tensor interaction constant:

$$f_T = (1.96 \pm 0.56) \times 10^{-2}. \tag{90}$$

The presence of the tensor matrix element was also discovered experimentally in the semileptonic three-particle decay of the kaon, $K^+ \to \pi^0 e^+ \nu$ (Ref. 20). In Ref. 84 the relativistic quark model was used to analyze the semileptonic radiative decay of the pion, $\pi^- \to e^- \tilde{\nu} \gamma$, and the semileptonic three-particle decay of the kaon, $K^+ \to \pi^0 e^+ \nu$. It was shown that the results of these experiments can be simultaneously described by the single constant f_T of the effective tensor interaction (82).

Naturally, the new tensor interaction will also contribute to nuclear β decay. Careful analysis of the experimental data[73,90] shows that the tensor interaction with constant $f_T \sim 10^{-2}$ is consistent with the available data. It would be interesting to perform new experiments in connection with this. For example, in the DAΦNE laboratory[91] almost half of the decays of ϕ mesons created in e^+e^- annihilation will be into K^+K^- mesons. This offers a good possibility of studying semileptonic three-particle decays of kaons.

The inclusion of a new tensor quark–quark interaction (83) in purely nonleptonic processes is difficult, owing to the presence of nonperturbative quantum chromodynamical effects. However, the success of the standard model in explaining the K_L-K_S mass difference[92] presents a challenge to any extension of it. Although the uncertainty in the theoretical estimates of the K_L-K_S mass difference is rather large (up to 50%; Ref. 93), this difference remains a fundamental source of constraints on the parameters of new interactions. In Ref. 94 it was shown that the contribution to the K_L-K_S mass difference from the new tensor interaction (83) with constant f_T (90) and parameter $\tan\beta$ (80) has the correct sign and amounts to half of its experimental value. We think that the problem of analysis of the $\Delta T=1/2$ rule in the light of new tensor quark–quark interactions (83), which so far has not been studied, is quite interesting.

The most direct method of seeking manifestations of tensor interactions may be analysis of the electron energy spectrum in muon decay.[95] It should be noted that this decay has been bypassed both theoretically and experimentally. As it is free from the difficulties associated with the inclusion of strong interactions, it can be calculated with any accuracy. However, up to now electromagnetic radiative corrections to this decay have been calculated only to one-loop order,[96] and the main decay parameter, the ρ parameter of Michel,[97] has not been measured since 1969 (Ref. 98). Moreover, even the status of the one-loop electromagnetic corrections is not completely clear.[99]

Muon decay is a fundamental process of weak interactions. The Fermi constant G_F is determined from the theoretical equation[100] for the muon lifetime obtained in the standard model and is used as the fundamental parameter in analyzing the radiative corrections of electroweak interactions. However, its accuracy depends on the accuracy of measuring the fundamental parameters of muon decay, which is very often forgotten, and at present it is unjustifiably overestimated.[101]

New tensor interactions (81) lead to new parameters in muon decay which earlier were not taken into account in theoretical analyses and data processing. It has often been assumed that the effective four-fermion interaction of leptons is independent of the momenta. However, since it arises as a result of exchange of an intermediate boson with 4-momentum q_m, nothing prevents it from depending on the momentum transfer. It is interactions of this type which generate antisymmetric tensor particles.

The most general form of the Hamiltonian for muon decay is parametrized by 12 constants $f_i^{\varepsilon\chi}$, which, in general, are complex:

$$\mathscr{H}=\frac{4G_F}{\sqrt{2}}\sum_{\substack{i=S,V,T\\ \varepsilon,\chi=R,L}}\{f_i^{\varepsilon\chi}[\bar{e}_\varepsilon\Gamma^i\nu_n^e][\bar{\nu}_m^\mu\Gamma^i\mu_\chi]+\text{H.c.}\},\quad(91)$$

where the scalar, vector, and tensor interactions are defined as

$$\Gamma^S\otimes\Gamma^S\equiv\mathbf{1}\otimes\mathbf{1};\quad\Gamma^S\otimes\Gamma^S\equiv\gamma_m\otimes\gamma^m;$$

$$\Gamma^T\otimes\Gamma^T\equiv\frac{1}{2}\,\sigma^{ml}\otimes\sigma_{nl}\cdot\frac{4q_mq^n}{q^2}.\quad(92)$$

If we use the old definition $\Gamma^T\otimes\Gamma^T\equiv\frac{1}{2}\sigma^{mn}\otimes\sigma_{mn}$, the normalization of the constants f_T^{RL} and f_T^{LR} does not change, owing to the identities (77), and the two constants f_T^{RR} and f_T^{LL} can be taken to be identically zero. In this sense the new definition (92) is a generalization of the old one and introduces new parameters.

The tensor interaction (81) corresponds to a completely defined choice of constants: $f_T^{RL}=f_T^{LR}=f_T^{LL}=0$, $f_T^{RR}=f_T$. We also assume that in addition to the standard V−A interaction $f_V^{LL}=1$ there are no other interactions and that the corresponding constants are equal to zero. Then the electron energy spectrum in polarized muon decay has the form

$$\frac{d^2\Gamma}{dx\,d\cos\theta}=\frac{G_F^2m_\mu^5}{192\pi^3}\left\{3-2x+rf_T\frac{1}{x}+f_T^2(15-14x)+\frac{\alpha}{2\pi}f(x)+\cos\theta\left[1-2x+rf_T\frac{2-x}{x}+f_T^2(13x-14)+\frac{\alpha}{2\pi}g(x)\right]\right\}x^2,\quad(93)$$

where we have kept terms of first order in the small quantity $r=12m_e/m_\mu\approx5.8\times10^{-2}$, θ is the angle between the momentum of the emitted electron and the muon spin, $x=2E_e/m_\mu$, and $f(x)$ and $g(x)$ are known functions of the one-loop electromagnetic corrections.[96]

The new terms are effectively manifested in the electron energy spectrum by a deviation of the measured old parameters of muon decay from their standard values. The Michel parameter ρ is determined from the isotropic energy spectrum of the electrons, and the experimental accuracy for it, $\rho=0.7518\pm0.0026$ (Ref. 98), is much better than that for other parameters. However, the smallness of the tensor constant f_T still does not permit the contribution of new structures to it to be discerned. It is therefore necessary to carry out new precision experiments which would include processing of the experimental data with allowance for the new parameters.

4. CONCLUSION

We have reviewed the literature devoted to the introduction and use of antisymmetric tensor fields of rank two in quantum field theory. We have not set ourselves the task of giving a complete list of publications on this topic. Our goal was to give a sequential historical outline of the introduction of such fields. We have used the generalizations to obtain new results which supplement and deepen the connections between various approaches. The questions which in our opinion are of key importance have been studied in sufficient detail that complete understanding of them does not require recourse to the original literature. On the other hand, individual facts which help to explain some point are given as references, so that if necessary the details can be found in the original literature or reviews.

The triumph of the standard model of electroweak interactions has temporarily eclipsed the interest in the study, and introduction into the theory, of new fields transforming as nonstandard representations of the Lorentz group. Phenomenological models of elementary particles, including their supersymmetric extensions, use only fields subject to the standard equations of motion. However, outside this area there is still a large class of fields which have not been studied. A real need for the introduction of such fields arises, for example, in the construction of extended theories of supergravity and string theories. The rank-2 antisymmetric tensor field appears naturally in such theories.

Fields transforming as nonstandard representations of the Lorentz group describe particles with nonmaximum values of the helicity. They can therefore play the role of particles with lower spin. For example, the antisymmetric tensor field can replace a scalar particle. On the mass shell they have the same number of purely scalar degrees of freedom and are equivalent. However, off the mass shell the additional degrees of freedom of the antisymmetric tensor field come into play, leading to a richer interaction. As an on-shell scalar, in an interaction this field carries unit spin and is manifested as a vector particle.

The representations of the Lorentz group for unit spin, (1, 0), (0 1), and (1/2, 1/2), describe a scalar and a vector particle on the mass shell. Off the mass shell the former particle leads, in addition to scalar forces, to a vector interaction, exactly as an off-shell photon leads to scalar forces—the Coulomb law. We shall show this by using the argument of Feynman.[32] The Coulomb law arises from the Lorentz-invariant current interaction

$$\frac{J_m^\dagger J^m}{q^2}=\frac{J_0^\dagger J_0}{\mathbf{q}^2}-\frac{\mathbf{J}_T^\dagger \mathbf{J}_T}{q^2}$$

if the current transversality condition $q_m J^m=0$ is used and the longitudinal component $J_L=q_0 J^0/|\mathbf{q}|$ is eliminated. In exactly the same way, the interaction of the antisymmetric tensor fields J_{mn} $(J_{0i}=A_i,\ J_{ij}=\varepsilon_{ijk}B_k)$ with the condition $q_m J^{mn}=0$ $(A_L=0, \mathbf{B}_T=q_0\mathbf{A}_T/|\mathbf{q}|)$,

$$\frac{1}{2}\frac{J_{mn}^\dagger J^{mn}}{q^2}=\frac{\mathbf{A}_T^\dagger \mathbf{A}_T}{\mathbf{q}^2}+\frac{B_L^\dagger B_L}{q^2},$$

leads to the $1/r$ law for the transverse components of the vector current.

Let us now turn to higher spins. The equations of motion for the wave functions are usually sought in the form of first-order differential equations. This is due to the desire to write the wave equations like the Schrödinger equation in the hope of avoiding negative solutions. For example, a particle with spin 3/2 can be described both in the Rarita–Schwinger formalism[102] and in the Bargmann–Wigner formalism[103] by first-order differential equations. The wave functions for spin 3/2 in the Rarita–Schwinger formalism transform as the (1, 1/2) and (1/2, 1) representations, and those in the Bargmann–Wigner formalism transform as the (3/2, 0) and (0, 3/2) representations. In the case of free massive particles these descriptions are equivalent.

However, even using first-order differential equations it is not possible to get rid of negative solutions which, we know, describe antiparticles. Therefore, there is no need for a restriction to first-order differential equations. In our case, particles with spin 3/2 satisfy third-order differential equations [see (37)]. The equations for wave functions transforming as the (1, 1/2), (1/2, 1), and (3/2, 0), (0, 3/2) representations of the Lorentz group are inequivalent and describe different particles. Third-degree equations for wave functions transforming as the (1, 1/2) and (1/2, 1) representations are used in conformal supergravity[17] to describe the gravitino with helicity 3/2 and are equivalent to our equations.

Let us consider the equations of motion for the wave function $\psi_{\alpha\beta\gamma}$ transforming as the (3/2, 0) representation of the Lorentz group. The four components $\psi_1=\psi_{111}$, $\psi_2=\sqrt{3}\,\psi_{\{112\}}$, $\psi_3=\sqrt{3}\,\psi_{\{122\}}$, and $\psi_4=\psi_{222}$ satisfy the matrix equation

$$\begin{pmatrix} p_+^3 & \sqrt{3}p_+^2q_- & \sqrt{3}p_+q_-^2 & q_-^3 \\ \sqrt{3}p_+^2q_+ & p_+(p_+p_-+2q_+q_-) & q_-(2p_+p_-+q_+q_-) & \sqrt{3}p_-q_-^2 \\ \sqrt{3}p_+q_+^2 & q_+(2p_+p_-+q_+q_-) & p_-(p_+p_-+2q_+q_-) & \sqrt{3}p_-^2q_- \\ q_+^3 & \sqrt{3}p_-q_+^2 & \sqrt{3}p_-^2q_+ & p_-^3 \end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}=0, \tag{94}$$

where the symbol $\{\alpha\beta\gamma\}$ denotes complete symmetrization of the indices with a factor of 1/3!. When the momentum vector $\mathbf{p}$ is directed along the third axis $(q_\pm=0)$, the matrix becomes diagonal and the analysis of (94) is simpler. The wave functions ψ_1 and ψ_4 correspond to the maximum values of the helicity $\pm 3/2$ and satisfy the equations of motion with frequency of one sign. These components are analogous to the longitudinal and time components of the electromagnetic field or the transverse components of vector fields transforming as the representations (1, 0) and (0, 1) and are unphysical. Here the physical components are ψ_2 and ψ_3 with helicity $\pm 1/2$, which satisfy the Weyl equations but with an additional factor of p^2.

If we go from spinor to Lorentz indices, a given particle will be described by a spinor $\Lambda_\alpha^{[mn]}$ with two Lorentz indices in which it is antisymmetric and anti-self-dual: $\mathbf{1}^+_{mnab}\Lambda_\alpha^{[ab]}=0$. The spinor $\Lambda_\alpha^{[mn]}$ has six components. The condition

$(\sigma_m \hat{\sigma}_n)_\alpha^\beta \Lambda_\beta^{[mn]}=0$ removes the two extra components which appeared in going to Lorentz indices and leads to the four components of the spinor field introduced above. This new field can be used in extended $N \geqslant 3$ conformal theories of supergravity instead of the spinor Λ_α, in the same way that antisymmetric tensor fields are used instead of scalars.

The spin-2 representations of the Lorentz group contain, in addition to the (1, 1), also the (3/2, 1/2), (1/2, 3/2) and (2,0), (0,2) representations. In our case the wave functions transforming as these representations satisfy fourth-order differential equations. The wave function transforming as the first representation in conformal supersymmetric theories of gravity describes a graviton with helicity ± 2. It can be shown that of the eight components

$$\begin{array}{cccc} & \psi_1=\psi_{111\dot{2}} & \psi_8=\psi_{222\dot{1}} & \\ \psi_2=\psi_{111\dot{1}} & \psi_3=\sqrt{3}\,\psi_{\{112\}\dot{2}} & \psi_6=\sqrt{3}\,\psi_{\{122\}\dot{1}} & \psi_7=\psi_{222\dot{2}} \\ & \psi_4=\sqrt{3}\,\psi_{\{112\}\dot{1}} & \psi_5=\sqrt{3}\,\psi_{\{122\}\dot{2}} & \end{array}$$

of the wave function transforming as the (3/2, 1/2) representation, only the two components ψ_3 and ψ_6 corresponding to the two transverse degrees of freedom of a vector particle with helicity ± 1 are physical on the mass shell, since only these components satisfy the equations $(p^2)^2\psi=0$. In the notation using Lorentz indices, such a particle is described by a real tensor of rank three which is antisymmetric in one pair of indices: $V_{a[mn]}$. The conditions $g^{am}V_{a[mn]}=0$ and $\varepsilon^{abmn}V_{a[mn]}=0$ remove the extra components of the field $V_{a[mn]}$ which unavoidably appear in this description.

Yet another spin-2 field which can be introduced in this manner is the five-component wave function $\psi_1=\psi_{1111}$, $\psi_2=2\psi_{\{1112\}}$, $\psi_3=\sqrt{6}\psi_{\{1122\}}$, $\psi_4=2\psi_{\{1222\}}$, and $\psi_5=\psi_{2222}$, transforming according to the (2, 0) representation. Only the one component ψ_3 with zero helicity is physical on the mass shell. Therefore, in the free case this field is equivalent to the scalar field φ which appears in conformal $N=4$ supergravity and satisfies the equation $(p^2)^2\varphi=0$.

Off the mass shell this field has another four degrees of freedom and carries spin 2. Its wave function with Lorentz indices can be represented as a real rank-4 tensor $\varphi_{[ab][mn]}$ which is antisymmetric in two pairs of indices and symmetric in two pairs. This field has exactly the same symmetry properties as the Riemann tensor R_{abmn}. In order to reduce the number of degrees of freedom to 10, with 5 for a particle with spin 2, $\psi_{\alpha\beta\gamma\delta}$, and 5 for its antiparticle, $\psi_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}$, it is necessary to impose the auxiliary conditions $g^{am}\varphi_{[ab][mn]}=0$ and $\varepsilon^{abmn}\varphi_{[ab][mn]}=0$.

Thus, antisymmetric tensor fields arise naturally in the analysis of spinor representations of the Lorentz group with spin $j \geqslant 1$. Off the mass shell the number of components of their wave function is equal to the dimension of the space of the given spin. On the mass shell the number of degrees of freedom is reduced, and their massless excitations are equivalent to particles of lower spin. This equivalence is lost when an interaction is included. Exchange of antisymmetric tensor fields leads to a richer elementary-particle interaction. The unusual properties of these fields deserve further study.

In conclusion, the author has the pleasant duty of deeply thanking Prof. M. D. Mateev for his interest in this study and useful discussions, and also Prof. V. G. Kadyshevskiĭ for suggesting the writing of this review, which led to a deeper understanding of the topics touched upon here.

*E-mail address: mih@phys.uni-sofia.bg

[1)]The reverse spelling of the Russian word for photon [*Transl. note.*].

[1]N. P. Konopleva and V. N. Popov, *Gauge Fields* (Harwood, Chur, 1981) [Russ. original, 2nd ed., Atomizdat, Moscow, 1980].
[2]P. W. Higgs, Phys. Rev. Lett. **12**, 132 (1964); Phys. Rev. **145**, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964).
[3]E. Cartan, *The Theory of Spinors* (MIT Press, Cambridge, Mass., 1966) [Russ. transl. of earlier ed., IL, Moscow, 1947].
[4]J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970) [Russ. transl., Mir, Moscow, 1973].
[5]B. van der Waerdan, Nachr. Ges. Wiss. Göttingen, 100 (1929); O. Laporte and G. E. Uhlenbeck, Phys. Rev. **37**, 1380 (1931).
[6]N. Kemmer, Proc. R. Soc. London, Ser. A **166**, 127 (1938).
[7]V. I. Ogievetskiĭ and I. V. Polubarinov, Yad. Fiz. **4**, 216 (1966) [Sov. J. Nucl. Phys. **4**, 156 (1967)].
[8]U. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974); E. Cremmer and J. Scherk, Nucl. Phys. **B72**, 117 (1974).
[9]*Supergravities in Diverse Dimensions*, edited by A. Salam and E. Sezgin (North-Holland, Amsterdam and World Scientific, Singapore, 1989).
[10]*School on Supergravity: Supergravity '81*, edited by S. Ferrara and J. G. Taylor (Cambridge University Press, Cambridge, 1982) [Russ. transl., Mir, Moscow, 1985].
[11]M. A. Namazie and D. Storey, Nucl. Phys. **B157**, 170 (1979); P. K. Townsend, Phys. Lett. **88B**, 97 (1979); W. Siegel, Phys. Lett. **93**, 170 (1980).
[12]J. Wess, Nuovo Cimento **18**, 1086 (1960); T. Fulton, F. Rohrlich, and L. Witten, Rev. Mod. Phys. **34**, 442 (1962).
[13]I. T. Todorov, M. C. Mintchev, and V. B. Petkova, in *Conformal Invariance in Quantum Field Theory* (ETS, Pisa, 1978), p. 89.
[14]E. S. Fradkin and M. A. Vasiliev, Lett. Nuovo Cimento **25**, 79 (1979); Phys. Lett. **85B**, 47 (1979).
[15]B. de Wit and J. W. van Holten, Nucl. Phys. **B155**, 530 (1979).
[16]E. Bergshoeff, M. de Roo, and B. de Wit, Nucl. Phys. **B182**, 173 (1981).
[17]E. S. Fradkin and A. A. Tseytlin, Phys. Rep. **119**, 233 (1985).
[18]L. V. Avdeev and M. V. Chizhov, Phys. Lett. **321B**, 212 (1994).
[19]V. N. Bolotov *et al.*, Yad. Fiz. **51**, 717 (1990) [Sov. J. Nucl. Phys. **51**, 455 (1990)]; Phys. Lett. **243B**, 308 (1990).
[20]S. A. Akimenko *et al.*, Phys. Lett. **259B**, 225 (1991); H. Stainer *et al.*, Phys. Lett. **36B**, 521 (1971).
[21]N. N. Bogolyubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, 3rd ed. (Wiley, New York, 1980) [Russ. original, 3th ed., Nauka, Moscow, 1976]; J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965) and *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) [Russ. transl., Nauka, Moscow, 1978]; C. Itzykson and J. B. Zuber, *Introduction to Quantum Field Theory* (McGraw-Hill, New York, 1980) [Russ. transl., Mir, Moscow, 1984].
[22]P. A. M. Dirac, Proc. R. Soc. London, Ser. A **155**, 447 (1936).
[23]A. J. Proca, J. Phys. Radium **7**, 347 (1936).
[24]N. Kemmer, Helv. Phys. Acta **33**, 829 (1960).
[25]P. van Nieuwenhuizen, Nucl. Phys. **B60**, 478 (1973).
[26]J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); G. Ecker *et al.*, Nucl. Phys. **B321**, 311 (1989); D. Kalafatis, Phys. Lett. **313B**, 115 (1993).
[27]E. Cremmer and B. Julia, Nucl. Phys. **B159**, 141 (1979).
[28]D. Z. Freedman, Caltech Report No. 68-624, California Institute of Technology, Pasadena (1977); D. Z. Freedman and P. K. Townsend, Nucl. Phys. **B177**, 282 (1981).
[29]R. K. Kaul, Phys. Rev. D **18**, 1127 (1978).
[30]V. N. Popov and L. D. Faddeev, Preprint 67-36, Institute of Theoretical Physics, Kiev (1967) [in Russian]; L. D. Faddeev and V. N. Popov, Phys. Lett. **25B**, 30 (1967).
[31]N. K. Nielsen, Nucl. Phys. **B140**, 499 (1978); R. E. Kallosh, Nucl. Phys. **B141**, 141 (1978).
[32]R. P. Feynman, Acta Phys. Pol. **24**, 697 (1963).
[33]A. A. Slavnov and S. A. Frolov, Teor. Mat. Fiz. **75**, 201 (1988) [Theor. Math. Phys. (USSR)].

[34] C. Becchi, A. Rouet, and R. Stora, Phys. Lett. **52B**, 344 (1974); I. V. Tyutin, Preprint No. 39, P.N. Lebedev Physics Institute, Moscow (1975) [in Russian].
[35] T. Kimura, Prog. Theor. Phys. **65**, 338 (1981); P. A. Marchetti and M. Tonin, Nuovo Cimento A **63**, 459 (1981).
[36] I. A. Batalin and E. S. Fradkin, Phys. Lett. **122B**, 157 (1983).
[37] L. Baulieu and J. Thierry-Mieg, Nucl. Phys. **B228**, 259 (1983).
[38] S. A. Frolov, Teor. Mat. Fiz. **76**, 314 (1988) [Theor. Math. Phys. (USSR)]; S. A. Frolov and A. A. Slavnov, Phys. Lett. **218B**, 461 (1989).
[39] P. K. Townsend and P. van Nieuwenhuizen, Nucl. Phys. **B120**, 301 (1977).
[40] E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D **22**, 301 (1980).
[41] G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré **1**, 69 (1974).
[42] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 401 (1974).
[43] S. Deser, H.-S. Tsao, and P. van Nieuwenhuizen, Phys. Rev. D **10**, 3337 (1974).
[44] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 411 (1974).
[45] P. van Nieuwenhuizen, in *Relativity, Fields, Strings, and Gravity*, Caracas, 1977.
[46] M. T. Grisaru, P. van Nieuwenhuizen, and C. C. Wu, Phys. Rev. D **12**, 1813 (1975).
[47] M. J. Duff and P. van Nieuwenhuizen, Phys. Lett. **94B**, 179 (1980).
[48] A. Aurilia, D. Christodoulou, and F. Legovini, Phys. Lett. **73B**, 429 (1978).
[49] A. Aurilia, H. Nicolai, and P. K. Townsend, Nucl. Phys. **B176**, 509 (1980).
[50] M. J. Duff, Nucl. Phys. **B125**, 334 (1977).
[51] S. M. Christensen and M. J. Duff, Phys. Lett. **76B**, 571 (1978).
[52] S. M. Christensen and M. J. Duff, Nucl. Phys. **B154**, 301 (1979).
[53] F. Gliozzi, J. Scherk, and D. Olive, Nucl. Phys. **B122**, 253 (1977).
[54] E. Cremmer, B. Julia, and J. Scherk, Phys. Lett. **76B**, 409 (1978).
[55] A. Neveu and J. H. Schwarz, Nucl. Phys. **B31**, 86 (1971); P. Ramond, Phys. Rev. D **3**, 2415 (1971).
[56] W. Nahm, Nucl. Phys. **B135**, 149 (1978).
[57] H. Nicolai and P. K. Townsend, Phys. Lett. **98B**, 257 (1981).
[58] M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984).
[59] A. A. Tseytlin, Int. J. Mod. Phys. D **1**, 223 (1992).
[60] E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D **50**, 4868 (1994).
[61] F. Giani, M. Pernici, and P. van Nieuwenhuizen, Phys. Rev. D **30**, 1680 (1984).
[62] L. J. Romans, Nucl. Phys. **B276**, 71 (1986); Phys. Lett. **169B**, 374 (1986).
[63] H. Weyl, Z. Phys. **56**, 330 (1929).
[64] S. Weinberg, Phys. Rev. **133**, B1318 (1964); **134**, B882 (1964).
[65] L. V. Avdeev and M. V. Chizhov, Preprint E2-94-263 (hep-th/9407067), JINR, Dubna (1994).
[66] G. Sterman, P. K. Townsend, and P. van Nieuwenhuizen, Phys. Rev. D **17**, 1501 (1978).
[67] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton, N.J., 1992).
[68] M. V. Chizhov, Author's Abstract, Candidate's Dissertation R2-82-112, JINR, Dubna (1982) [in Russian]; A. V. Chizhov and M. V. Chizhov, Phys. Lett. **125B**, 190 (1983); M. V. Chizhov, Preprint R2-84-172, JINR, Dubna (1984) [in Russian].
[69] S. L. Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969).
[70] E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. **55B**, 224 (1975); I. A. Batalin and G. A. Vilkovisky, Phys. Lett. **69B**, 309 (1977).
[71] J. Schwinger, Phys. Rev. **125**, 397 (1962).
[72] B. de Wit, J. W. van Holten, and A. Van Proeyen, Nucl. Phys. **B184**, 77 (1981); B. de Wit, P. G. Lauwers, R. Philippe, and A. Van Proeyen, Phys. Lett. **135B**, 295 (1984).
[73] A. A. Poblaguev, Phys. Lett. **238B**, 108 (1990).
[74] A. A. Poblaguev, Phys. Lett. **286B**, 169 (1992).
[75] P. Depommier *et al.*, Phys. Lett. **7**, 285 (1963); A. Stetz *et al.*, Nucl. Phys. **B138**, 285 (1978); A. Bay *et al.*, Phys. Lett. **174B**, 445 (1986); L. E. Piilonen *et al.*, Phys. Rev. Lett. **57**, 1402 (1986); S. Egly *et al.*, Phys. Lett. **175B**, 97 (1986); **222B**, 533 (1989).
[76] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
[77] P. Langacker and J. Erler, Phys. Rev. D **50**, 1304 (1994).
[78] S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[79] P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993); P. F. Harrison and W. G. Scott, Phys. Lett. **333B**, 471 (1994); H. Fritzsch and D. Holtmannspötter, Phys. Lett. **338B**, 290 (1994).
[80] D. A. Bryman, P. Depommier, and C. Leroy, Phys. Rep. **88**, 151 (1982).
[81] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993).
[82] M. B. Voloshin, Phys. Lett. **283B**, 120 (1992).
[83] B. A. Campbell and K. A. Peterson, Phys. Lett. **192B**, 401 (1987); O. Shankar, Nucl. Phys. **B204**, 375 (1982).
[84] M. V. Chizhov, Mod. Phys. Lett. A **8**, 2753 (1993).
[85] C. Kolda *et al.*, Phys. Rev. D **50**, 3498 (1994); W. de Boer, R. Ehret, and D. I. Kazakov, Phys. Lett. **334B**, 220 (1994).
[86] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **50**, 2966 (1994).
[87] I. N. Nikitin, Preprint 90-176, IHEP, Protvino (1990) [in Russian]; Yu. Ya. Komachenko, Yad. Fiz. **55**, 2487 (1992) [Sov. J. Nucl. Phys. **55**, 1384 (1992)]; Yu. Ya. Komachenko and R. N. Rogalyov, Phys. Lett. **334B**, 132 (1994).
[88] V. M. Belyaev and Ia. I. Kogan, Phys. Lett. **280B**, 238 (1992).
[89] B. L. Ioffe and A. B. Smilga, Pis'ma Zh. Éksp. Teor. Fiz. **37**, 250 (1983) [JETP Lett. **37**, 298 (1983)]; Nucl. Phys. **B232**, 109 (1984); V. M. Belyaev and Ya. I. Kogan, Yad. Fiz. **40**, 1035 (1984) [Sov. J. Nucl. Phys. **40**, 659 (1984)].
[90] P. A. Quinn *et al.*, Phys. Rev. D **47**, 1247 (1993); P. Herczeg, Phys. Rev. D **49**, 247 (1994).
[91] *The DAΦHNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Frascati, 1992).
[92] M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[93] M. A. Shifman, Int. J. Mod. Phys. A **3**, 2769 (1988); J. Bijnens, J.-M. Gérard, and G. Klein, Phys. Lett. **257B**, 191 (1991); S. Herrlich and U. Nierste, Nucl. Phys. **B419**, 292 (1994).
[94] M. V. Chizhov, Preprint E2-94-253 (hep-ph/9407237, JINR, Dubna (1994).
[95] M. V. Chizhov, Mod. Phys. Lett. A **9**, 2979 (1994).
[96] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).
[97] L. Michel, Proc. Phys. Soc. London, Sect. A **63**, 514 (1950).
[98] S. E. Derenzo, Phys. Rev. **181**, 1854 (1969).
[99] A. B. Arbuzov, E. A. Kuraev, N. P. Merenkov, and N. V. Makhaldiani, Communication, E4-93-196, JINR, Dubna (1993).
[100] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).
[101] W. Fetscher and H.-J. Gerber, in *Precision Tests of the Standard Electroweak Model*, edited by P. Langacker (World Scientific, Singapore, 1993).
[102] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[103] V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. U.S.A. **34**, 211 (1948).

Translated by Patricia A. Millard