Macroscopic model for magnetic resonances in spherical nuclei

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A quantum–macroscopic nuclear model is described which interprets magnetic resonances in terms of the torsional elastic vibrations of a spherical nucleus. A summary is given of the basic predictions of this model for the energies, the transition current densities, the total excitation probability, the magnetic oscillator strength, and the spread width as functions of the multipole order and the atomic and masss numbers. The cross sections computed in the PWBA and the DWBA are presented for $M\lambda$ resonances excited by means of (e,e') scattering. Strong emphasis is placed on comparison of the theoretical predictions with the experimental data on magnetic resonances with $\lambda \geqslant 2$ excited by electrons inelastically scattered on spherical nuclei.

1. INTRODUCTION

Theoretical studies of collective excitations of nuclei using the methods of the macroscopic physics of continuous media have been directed toward the ultimate goal of constructing an adequate dynamical theory of nuclear matter. The fact that many of the integrated characteristics of the ground state and collective excitations of the nucleus such as the binding energy, the centroids of the resonance energies, the total excitation probabilities, and the spread widths vary smoothly with increasing mass number indicates that the "size effect"—the dependence of these typical parameters on the nuclear radius and shape—plays an extremely important role. Study of the nuclear response using collective models based on specific assumptions about the macroscopic properties of nuclear matter has precisely the goal of determining these "size" laws. By treating the nucleus as a macroscopic particle of condensed matter and comparing the theoretically predicted mass-number dependences of, for example, the energiess and probabilities with experiment, it is possible to judge how well the continuum model reflects the properties of real nuclear matter.

As is well known, the initial study of nuclear dynamics was based on representation of the nucleus as a drop of incompressible, charged, nonviscous liquid. However, the experimental information which is available at the present time allows us to state with certainty that the representation of nuclear matter as a liquid is unsatisfactory for a number of reasons. Here we shall present only a few arguments illustrating this statement, in order to stress the fact that the physical interpretation of the *magnetic* collective response of a nucleus in terms of the theory of continuous media necessarily leads to the conclusion that nuclear matter possesses the features of an elastic continuum, not a liquid.

Let us first briefly discuss the main assumptions on which the liquid-drop model of the nucleus is based. According to the phenomenological description of nuclear properties used in this model, it is assumed that the destructive effect of the Coulomb repulsion is stabilized by attractive nuclear forces modeled by surface-tension forces. The equations governing both the equilibrium and the dynamical response of the nucleus are taken to be the classical equations of an ideal liquid (the continuity equation and the Euler equation). An immediate consequence of the heuristic hypothesis that the behavior of nuclear matter is similar to that of a nonviscous liquid is that the drop model admits only a spherical equilibrium shape of the nucleus. This fact is already an indication of the inadequacy of the liquid hypothesis, since it does not reflect the available data about the equilibrium shape of nuclei. The predictions of the liquid-drop model for the dynamical properties of nuclei disagree just as strongly with experiment.

The main ideas behind the hydrodynamical description of the collective motions of nucleons impose strong constraints on the nature of the collective excitations of the nucleus, as they allow the existence in the nuclear spectrum of only a single collective branch of excitations associated with nuclear surface oscillations. Surface collective modes are identified as excitations of the electric type, since these excited states are characterized by nonzero values of the electric multipole moments. It is known from the classical electrodynamics of continuous media that an external electromagnetic perturbation of a charged, incompressible drop can induce only harmonic distortions of its equilibrium spherical shape: $R(t)=R(1+\alpha_{\lambda\mu}(t)Y_{\lambda\mu}(\hat{\mathbf{r}}))$. These arise from the excitation of oscillations of the electric current density $\mathbf{j}(\mathbf{r},t)=(eZ/A)n_0\delta\mathbf{V}(\mathbf{r},t)$ with irrotational velocity field: $\delta\mathbf{V}(\mathbf{r},t)=\nabla r^{\lambda}Y_{\lambda\mu}(\hat{\mathbf{r}})\dot{\alpha}_{\lambda}(t)$. Here n is the particle number density, $Y_{\lambda\mu}(\hat{\mathbf{r}})$ is the spherical harmonic of multipole order λ, and $\alpha_{\lambda}(t)$ is the amplitude of the collective oscillations. The theoretical estimate of the energy of, for example, the quadrupole nuclear surface mode obtained using the liquid-drop model with the standard set of mass-formula parameters turns out to be about 1.5–2 times higher than the observed energies of the lowest-lying 2^{+} excitations of spherical nuclei strongly influenced by the shell structure, and 3–4 times lower than the locations of the energy centroids of the giant quadrupole resonances (see Fig. 1). This blatant disagreement with experiment indicates the need to revise the liquid-drop representation of nuclear matter. It should also be noted that the quantum nature of the nucleon Fermi distribution in the ground state is not taken into account at all in the liquid-drop model.

Perhaps the most important defect of the liquid-drop rep-

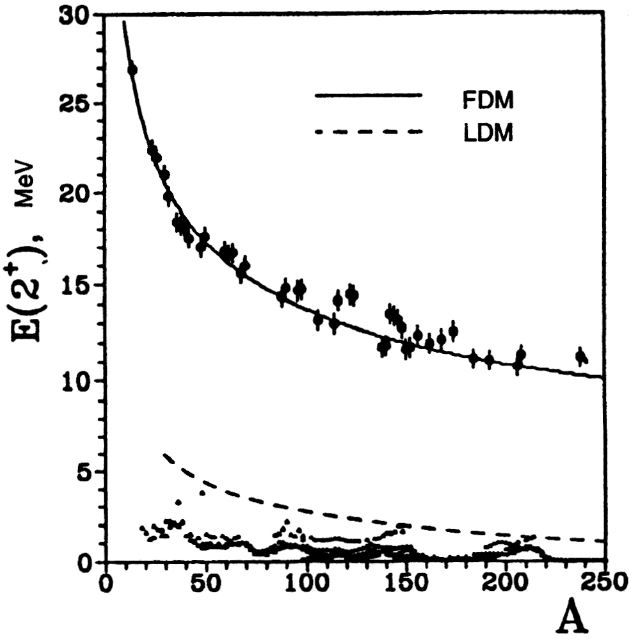

FIG. 1. Experimental data on the energies of $E(2^+)$ quadrupole excitations as a function of the nuclear mass number A: the triangles show the energies of the lowest 2^+ states, and the circles show the locations of the energy centroids of isoscalar electric quadrupole resonances. The solid line was calculated theoretically by using the fluid-dynamical model (FDM), and the dashed line is the prediction of the liquid-drop model (LDM).

resentation of nuclear matter is the impossibility in principle of describing the magnetic response of the nucleus, whereas the existence of magnetic collective modes in the spectra of many nuclei is a firmly established fact.[1–5] In fact, states of the magnetic type lie outside the scope of the hydrodynamical description, because transitions of a drop of charged ideal liquid from the equilibrium state to excited states with nonzero magnetic multipole moment are not realized. This can be verified by substituting the above expression for the current into the definition of the nuclear magnetic multipole moment of order λ (Refs. 6–9):

$$\mathscr{M}(M\lambda,\mu)=\frac{-1}{c(\lambda+1)}\int \mathbf{j}\cdot[\mathbf{r}\times\nabla]r^{\lambda}Y_{\lambda\mu}(\hat{\mathbf{r}})d\tau. \quad (1.1)$$

It is easily checked that this substitution makes (1.1) vanish identically (see also Ref. 6, p. 22). Since oscillations of the potential flux are the only allowed type of eigenvibrations of an incompressible, nonviscous liquid, it immediately follows from these arguments that the standard liquid-drop model in principle does not allow for the possibility of describing magnetic collective modes.

The inadequacy of the liquid-drop approach to the description of the magnetic excitations of a nucleus has been pointed out by Holzwarth and Eckart, though from a somewhat different viewpoint. In their short but important article of Ref. 10 they suggested that the collective magnetic quadrupole response of a spherical nucleus can be viewed as a manifestation of transverse (multipole order $\lambda=2$) oscillations of the nucleon flux. In Ref. 10 the local velocity field in spherical coordinates with fixed polar axis z has the form

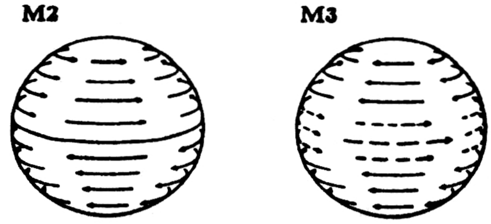

FIG. 2. Geometrical picture of magnetic quadrupole (left) and octupole (right) twist responses of a spherical nucleus.

$\delta V_x=-yz\dot{\alpha}$, $\delta V_y=xz\dot{\alpha}$, $\delta V_z=0$, where α is a geometrically infinitesimal angle of rotation of the collective nucleon flux about the z axis. In a uniformly charged spherical macroscopic particle modeling the nucleus the distinguished direction (in this case the direction of the polar axis) about which vibrations of the excited solenoidal electric current occur can be determined only by the direction of the electromagnetic field penetrating the nucleus, where this field is generated by, for example, a scattered charged particle. It is easily verified that rotational vibrations of this type lead to a nonzero quadrupole magnetic moment of the nucleus in an excited state. The geometrical picture of such oscillations is shown on the left-hand side of Fig. 2. The upper and lower hemispheres undergo axially symmetric shear oscillations of opposite phase, which are expressively referred to by the authors of this model as the "nuclear twist" (Refs. 10–14). This mechanism of "magnetization" of an even–even nucleus (i.e., a transition from the ground to an excited state with nonzero magnetic moment) is generated by bulk solenoidal vibrations of the macroscopic current density. We emphasize the fact that the excitation of torsional vibrations does not lead to fluctuations of the mass density, i.e., they can occur in an incompressible continuum. These vibrations are described in terms of the oscillations of a solenoidal displacement field of a continuous medium.

It is well known from the classical theory of continuous media that the ability to support both longitudinal and transverse undamped vibrations is a property of an ideally elastic medium,[15,16] but not of a nonviscous liquid. In the latter at nonzero equilibrium temperature only essentially longitudinal pressure waves can propagate.[16,17] Transverse shear vibrations in a finite spherical mass of condensed matter, referred to as torsional vibrations, are perhaps the main feature which indicates that the matter is an elastic medium. In a drop of ideal liquid excitation of shear oscillations is impossible, since the hydrodynamical equations do not allow for the appearance of anisotropy in the stress distribution in a perturbation of the equilibrium state. If we accept the idea that the $M2$ resonance is a manifestation of torsional vibrations[10–14] of quadrupole symmetry, its experimental detection can, from the viewpoint of the physics of condensed media, be viewed as direct proof that nuclear matter is elastic.

In Refs. 18–21 these ideas were used as the basis of a description of the magnetic nuclear response, using the elastic-sphere model. The Lamé equation,[1)] the fundamental equation for the vibrations of classical ideally elastic matter,

[16] was used as the equation governing the collective nucleon dynamics. The fact that an adequate continuum description (in the variables of the theory of continuous media: the density, velocity field, displacement field, stress field, and so on) of nucleon collective oscillations in the excitation of nuclear multipole resonances can be obtained not using classical hydrodynamics, but using equations which can reflect the features of the elastic-like behavior of Fermi matter, was first discussed in the studies of Bertsch[24,25] devoted to the analysis of the energy systematics of isoscalar electric giant resonances in nuclei. The arguments given in Refs. 24 and 25 stimulated the development of a quantum–macroscopic theory of the continuous nuclear medium, which is referred to as "nuclear fluid dynamics." At present this theory continues to develop and is viewed as the best model of nuclear Fermi matter.[2)] Using this continuum model of nuclear matter, it is possible to show with mathematical rigor that the existence of magnetic isoscalar resonances of orbital nature is a consequence of the nucleon Fermi motion and of the related dynamical deformation of the Fermi surface determining the quantum nature of the restoring force of transverse nuclear oscillations. In the modern theory of continuous media the equations of the model considered in the present review are known as the equations of the thirteen-moment approximation[26] based on the quantum kinetic equation (see, for example, Refs. 25 and 27–30, where these equations are derived and the microscopic justification of nuclear fluid dynamics is given). The description of collective nuclear motion based on quantum kinetics can be found in Refs. 31–34.

In this review we present a collective model of isoscalar magnetic resonances, following Refs. 35–40. The authors of those articles developed a fluid-dynamical model, generalized to the case of arbitrary multipole order, of magnetic resonances correlated with the predictions for the high-multipole-order dipole mode and quadrupole magnetic resonance made in Refs. 10–14. We also make extensive use of the conclusions of the studies devoted to magnetic excitations of heavy spherical nuclei, using both macroscopic (Refs. 18–21, 29, and 41–46) and microscopic (Refs. 31 and 47–56) methods for theoretically describing the collective magnetic response of spherical nuclei. A fairly complete description of the current status of the experimental physics of magnetic excitations can be found in the review of Raman, Fagg, and Hicks.[5] In the present review we pay special attention to problems of the practical determination of the torsional response of spherical nuclei from the data on the cross sections for electron inelastic scattering with magnetic-resonance excitation.

Our review is organized as follows.

In Sec. 2 we formulate the physical principles forming the basis of the quantum concept of the elasticity of nuclear matter and the variational method of solving the nuclear normal-mode problem, a manifestation of which is isoscalar multipole resonances. In particular, we show that the equations of nuclear fluid dynamics in the long-wavelength limit admit two types of solution. The first (poloidal) solution corresponds to spheroidal vibrations associated with electric isoscalar resonances. The second (toroidal) corresponds to the torsional vibrations responsible for magnetic isoscalar resonances. Thus, it is emphasized that isoscalar $E\lambda$ and $M\lambda$ resonances can be described within a unified approach.

The mechanism for long-wavelength torsional vibrations is described in detail in Sec. 3, where we also give an analytic derivation of the expressions for the multipole energy spectrum and the probability of excitation of twist resonances. Here we compare the model predictions with the data from the Darmstadt linear accelerator on the summed characteristics of the magnetic quadrupole resonance in spherical nuclei.

Section 4 is devoted to the theoretical description of the process of electron inelastic scattering on spherical nuclei. We present expressions obtained analytically for the collective transition current densities and the magnetic form factors for the excitation of twist resonances. The results of numerical calculations of the cross sections for (e,e') scattering in the distorted-wave Born approximation (DWBA) are given. These transition current densities are used to estimate the total reduced probabilities for the excitation of $M\lambda, T=0$ resonances, and their oscillator strengths are calculated. The predictions are compared with the DALINAC data on the cross sections for the excitation of the $M2$ resonance in ^{140}Ce.

The analysis of the high-energy dipole magnetic response of spherical nuclei is the subject of Sec. 5. We give estimates of the location of the energy centroid, the probability, and the cross section for the excitation of the low-lying 1^+ mode as a function of the nuclear atomic number and mass number. The predictions are compared with the linear-electron-accelerator (Bates LINAC) data on the appearance of this mode in the cross section for electron inelastic scattering on ^{208}Pb.

In Sec. 6 we describe the fluid-dynamical model of the damping of local rotational vibrations on the basis of the concept of the viscosity of nuclear matter introduced in the analysis of fission. The analytical dependence of the spread widths of $M\lambda, T=0$ resonances on the mass number and multipole order of the excitation is given.

In the Conclusion we discuss the results in this review and give the main conclusions about the macroscopic treatment of magnetic resonances following from nuclear fluid dynamics.

2. THE VARIATIONAL METHOD OF THE FLUID-DYNAMICAL DESCRIPTION OF NUCLEON COLLECTIVE MOTION

The current understanding of the quantum–macroscopic features of the collective nuclear response has been achieved thanks to extensive research carried out in recent years using the widely recognized and mutually complementary methods of nuclear fluid dynamics. These are described in the reviews of Refs. 25, 29–31, and 57–62, and in the monographs of Refs. 9 and 28, where the physical principles of nuclear fluid dynamics are given in exhaustive detail and the studies published up to the beginning of the current decade are used.

2.1. The equations of nuclear fluid dynamics

The constructive statement of nuclear fluid dynamics is that an adequate macroscopic description of nucleon collective motion can be given in terms of the thirteen local variables of the theory of continuous media: the mass density distribution $\rho(\mathbf{r},t)$, the three components of the average velocity of the excited flux $V_i(\mathbf{r},t)$, and the ten components of the symmetric elastic stress tensor $P_{ij}(\mathbf{r},t)$, the dynamics of which is described by the following closed system of equations:[35]

$$\frac{d\rho}{dt}+\rho\,\frac{\partial V_k}{\partial x_k}=0, \tag{2.1}$$

$$\rho\,\frac{dV_i}{dt}+\frac{\partial P_{ik}}{\partial x_k}+\rho\,\frac{\partial U}{\partial x_i}=0, \tag{2.2}$$

$$\frac{dP_{ij}}{dt}+P_{ik}\,\frac{\partial V_j}{\partial x_k}+P_{jk}\,\frac{\partial V_i}{\partial x_k}+P_{ij}\,\frac{\partial V_k}{\partial x_k}=0, \tag{2.3}$$

where d/dt is the total (substational) derivative and n is the particle number density. Here and below, there is understood to be a summation over repeated indices. The first equation (2.1) is the well known continuity equation. Equation (2.2) describes the motion of the nuclear-matter flux. The collective excitations of the nucleus are classified according to the type of perturbed average velocity field of the nucleon motion. The quantity U is the nuclear internal energy density divided by the nucleon mass. Equation (2.3) controls the dynamics of the internal strains. The nondiagonal structure of the elastic stress tensor provides for the possibility that external perturbations can induce a collective response of the nucleon Fermi system, accompanied by anisotropic distortions in the distribution of internal strains inside the nucleus. This can occur either in an elastic medium or in a viscous liquid. In an ideal (nonviscous) liquid the perturbations propagate without spoiling the equilibrium isotropic state. The introduction of the collective nuclear dynamics of Eq. (2.3) into the description actually implies identification of the behavior of continuous nuclear matter with that of ideally elastic matter, the characteristic dynamical feature of which is the ability to support both longitudinal and transverse undamped vibrations, since, as shown in Ref. 23, Eqs. (2.1)–(2.3) can be exactly reduced to the basic equations of linear elasticity theory (more precisely, the equations governing the vibrations of ideally elastic matter). This is one of the main reasons why in nuclear fluid dynamics the nucleon collective response is interpreted in terms of the vibrations of an elastic continuum. It is interesting to observe that the method described in the present review can be used to analytically obtain the eigenmode spectrum of long-wavelength elastic spheroidal and torsional vibrations of a sphere, whereas the canonical method of elasticity theory based on the Lamé equation does not permit the unique solution of this problem.[23]

As noted in the Introduction, the possibility in principle of exciting transverse shear vibrations in a spherical volume of incompressible nuclear matter is a consequence of the fact that the nucleus is essentially a quantum Fermi system of particles distributed in single-particle orbits of the mean field in accordance with the Pauli principle. In the version of the fluid-dynamical model given below the quantum nature of the nuclear medium is reflected in the fact that the parameters of the ground state of a spherical nucleus of radius $R=r_0A^{1/3}$ F are calculated in the Thomas–Fermi approximation for a Fermi system of nucleons degenerate in spin and isospin (Refs. 14, 18, 24, 27, and 35). In modeling the nucleus as a spherical macroscopic particle of continuous, incompressible Fermi matter, it is usually assumed that the mass and charge are uniformly distributed throughout the nuclear volume, while the stress distribution is isotropic owing to the spherical symmetry and is described by a second-rank tensor: $P_{ik}^{\rm eq}=P_0\delta_{ik}$. It should be noted that the assumption that the elastic properties of the nucleus can be described in terms of a second-rank stress tensor is to a large degree heuristic. In the "moment method" developed by Mikhaĭlov and Bal'butsev and coworkers[29] the description of the nuclear dynamics is constructed by using stress tensors of higher rank (through fifth rank).

2.2. The normal-mode Hamiltonian

Let us consider the linear oscillations of a nucleus, neglecting density fluctuations, i.e., assuming that the nuclear medium is incompressible. The linearized equations of motion describing small deviations of the nucleus from equilibrium have the form

$$\frac{\partial\delta V_k}{\partial x_k}=0, \tag{2.4}$$

$$\rho_0\,\frac{\partial\delta V_i}{\partial t}+\frac{\partial\delta P_{ik}}{\partial x_k}+\rho_0\,\frac{\partial\delta U}{\partial x_i}=0, \tag{2.5}$$

$$\frac{\partial\delta P_{ij}}{\partial t}+P_0\left(\frac{\partial\delta V_i}{\partial x_j}+\frac{\partial\delta V_j}{\partial x_i}\right)+\delta_{ij}\left(\delta V_k\,\frac{\partial P_0}{\partial x_k}\right)=0. \tag{2.6}$$

The eigenfrequencies of the oscillations can be obtained by using the variational principle, which is actually a modern formulation of Rayleigh's "method of normal coordinates."[22,63] The starting point of the method is the energy-balance equation, which is obtained by multiplying Eq. (2.5) by δV_i and then integrating over the volume of the nucleus:

$$\frac{\partial}{\partial t}\int_v \frac{1}{2}\,\rho_0\delta V^2 d\tau-\int_v \delta P_{ij}\,\frac{\partial\delta V_i}{\partial x_j}\,d\tau$$
$$+\oint_s[\rho_0\delta U\delta V_i+\delta P_{ij}\delta V_j]_s d\sigma_i=0. \tag{2.7}$$

This equation controls the energy conservation in the oscillation process. Next we separate the space and time dependences of the multipole fluctuations of the potential and the velocity:

$$\delta U(\mathbf{r},t)=\phi^\lambda(\mathbf{r})\alpha_\lambda(t),\qquad \delta V_i(\mathbf{r},t)=a_i^\lambda(\mathbf{r})\dot\alpha_\lambda(t). \tag{2.8}$$

Here $a_i^\lambda(\mathbf{r})$ are the components of the instantaneous displacement field, and $\alpha_\lambda(t)$ is treated below as an amplitude (a normal coordinate) in accordance with the Bohr and Mottelson treatment of nuclear collective oscillations.[7] Taking into account the expression for the velocity fluctuation (2.8), from Eq. (2.6) we find that the fluctuation of the stress tensor is expressed as a traceless symmetric tensor of the form

$$\delta P_{ij}(\mathbf{r},t) = -\left[P_0\left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i} \right) + \delta_{ij}\left(a_k^\lambda \frac{\partial P_0}{\partial x_k} \right) \right] \alpha_\lambda . \tag{2.9}$$

Substituting (2.8) and (2.9) into (2.7), we can verify that the energy-balance equation (2.7) reduces to the oscillator Hamiltonian (the energy of linear oscillations)

$$H = \frac{B_\lambda \dot{\alpha}_\lambda^2}{2} + \frac{C_\lambda \alpha_\lambda^2}{2}, \tag{2.10}$$

which is an integral of the nuclear collective motion. From this procedure of deriving the collective nuclear Hamiltonian it follows that the mass parameter B_λ and the stiffness parameter C_λ are given by[68]

$$B = \int_v \rho_0 a_i^\lambda a_i^\lambda d\tau, \tag{2.11}$$

$$C = \frac{1}{2} \int_v P_0 \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i} \right)^2 d\tau + \oint_s \left[P_S \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i} \right) a_j^\lambda - \left(\rho_0 \phi^L - a_j^\lambda \frac{\partial P_0}{\partial x_j} \right) a_i^\lambda \right] d\sigma_i . \tag{2.12}$$

Therefore, the problem of finding the eigenfrequencies of nuclear oscillations reduces to the calculation of the velocity fluctuations (or, equivalently, the instantaneous displacement field) and the potential fluctuations. The equilibrium values of the density ρ_0, the bulk pressure P_0, and the surface pressure P_S are treated as the main parameters of the model reflecting the specific features of the nuclear structure. For example, in the generalized Thomas–Fermi method these parameters are calculated in the local-density approximation using Skyrme or Honey forces.

In Ref. 27 this variational approach was used to calculate the eigenenergies of electric isoscalar resonances. The Fermi-gas approximation was used to estimate the bulk pressure. Here the surface was assumed to be free of stress: $P_S = 0$. It was also assumed that the internal energy density can be neglected, $\delta U = 0$, owing to the incompressibility and the saturation of nuclear forces by fluctuations. In this approximation Eqs. (2.4)–(2.6) reduce to the standard wave equation for velocity fluctuations δV_i (as for the stress fluctuations δP_{ij}). The latter, in turn, reduces to the Helmholz equation:

$$\Delta \delta \mathbf{V} + k^2 \delta \mathbf{V} = 0, \tag{2.13}$$

if the time dependence of the main dynamical variables of the nucleon collective motion is assumed to be harmonic. The many calculations carried out by using nuclear fluid dynamics show that the observed giant-resonance energies are reproduced well-assuming that long-wavelength ($k \to 0$) oscillations are excited. In the long-wavelength approximation Eq. (2.13) written in terms of the instantaneous displacement field becomes the vector Laplace equation plus (in the case of incompressibility) the solenoidal condition:

$$\Delta \mathbf{a}^\lambda = 0, \quad \mathrm{div}\, \mathbf{a}^\lambda = 0. \tag{2.14}$$

This equation has only two independent solutions which are regular at the origin. In spherical geometry they are orthogonal vector solenoidal (poloidal and toroidal) fields[63,64] with opposite spatial parity. The latter feature allows the electric and magnetic isoscalar responses of the nucleus to be associated with the type of displacement field excited.

2.3. Spheroidal eigenmodes of the nucleus: electric isoscalar resonances

In nuclear fluid dynamics electric resonances are interpreted in terms of spheroidal oscillations of the nucleus. Such oscillations correspond to the poloidal solution of the vector Laplace equation:[63]

$$\mathbf{a}(\mathbf{r}) = \mathrm{curl\ curl\ } \mathbf{r} r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}) = (\lambda+1) \nabla r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}). \tag{2.15}$$

The energy spectrum of isoscalar electric multipole resonances, which was first obtained by Nix and Sierk in Ref. 27 (see also Refs. 58, 62, and 66–68) is given by

$$E(E\lambda, T=0) = \hbar \omega_F \left[\frac{2}{5} (2\lambda+1)(\lambda-1) \right]^{1/2}, \tag{2.16}$$

where ω_F is the fundamental frequency of the collective oscillations of the spherical Fermi system of nucleons,

$$\omega_F = \frac{v_F}{R} = \frac{\hbar}{2mr_0^2} (9\pi)^{1/3} A^{-1/3}, \tag{2.17}$$

and v_F is the Fermi velocity. The solid line in Fig. 1 shows the result of the calculations performed in Ref. 27 using (2.16). As stressed in Ref. 27, Eq. (2.16) reproduces the observed energies of quadrupole electric isoscalar resonances with three-percent accuracy. This surprising agreement with the theoretical prediction for the energy of such a strongly collectivized nuclear response as the giant isoscalar quadrupole resonance indicates that its formation is to a large degree determined by the quantum elasticity of nuclear matter. Keeping this in mind, in the following sections devoted to the magnetic isoscalar response we use the same approximations and physical assumptions.

The physical content of the concept of the elasticity of nuclear matter can be understood from the following arguments. Treating the nuclear ground state as a finite Fermi system of nucleons saturated in spin and isospin, we see that the equilibrium isotropic stress tensor (the pressure) is represented in momentum space by a Fermi sphere with radius fixed by the Fermi velocity v_F, since the equilibrium pressure in the Fermi system is given by $P_0 = \rho_0 v_F^2/5$. Keeping this picture in mind, the stress fluctuations δP_{ij} introduced constructively in the linearization of Eqs. (2.1)–(2.3) by means of the substitution

$$P_{ij} \to P_0 \delta_{ij} + \delta P_{ij} \tag{2.18}$$

(together with the substitutions $\rho \to \rho_0 + \delta\rho$ and $V_i \to V_i^0 + \delta V_i$, where $\delta\rho = 0$ owing to incompressibility and $V_i^0 = 0$, because it is assumed that there are no fluxes in the ground state) are treated as quadrupole distortions of the Fermi sphere. This interpretation is possible because the stress-fluctuation tensor δP_{ij} possesses the same symmetry properties as the quadrupole-moment tensor (in particular, it has zero trace). In the perturbation of the nuclear ground state, which in momentum space corresponds to perturbation of the nodal structure of the orbits of the single-particle Fermi motion filling the Fermi sphere, a reverse coherent reaction of the nucleon orbits arises, which tends to return the distorted Fermi sphere to the equilibrium spherically symmetric state. In the coordinate space of the nuclear volume the distortions of the Fermi sphere are manifested as anisotropic shear stresses whose local distribution is described by the tensor (2.9). Therefore, the restoring force of elastic deformations $F=C_\lambda \alpha_\lambda$ is a force returning the Fermi sphere to the equilibrium spherically symmetric form, and the distribution of the internal strains to the equilibrium isotropic form. These arguments illustrate the quantum origin of nuclear elasticity, which, as stressed in Ref. 28, is not so much related to the details of the nuclear shell structure, but rather is a consequence of more general factors: the nucleon Fermi motion and the dynamical deformation of the Fermi surface.

3. COLLECTIVE MODEL OF THE TORSIONAL MAGNETIC RESPONSE OF THE NUCLEUS

In nuclear fluid dynamics the isoscalar magnetic collective modes of orbital nature are associated with the excitation of a purely rotational displacement field and are described by the second of the two independent solutions of the vector Laplace equation (2.14). The solution of this equation found in Ref. 35 which is regular at the origin is called the toroidal field[63] and has the form

$$\mathbf{a}_\lambda(\mathbf{r})=\operatorname{curl}\, \mathbf{r} r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}). \tag{3.1}$$

The field (3.1) corresponds to the excitation of differential–rotational oscillations of the collective nucleon flux. As shown in Sec. 2, the equations of classical hydrodynamics used as the foundation of the standard liquid-drop model have no solutions corresponding to this type of collective nuclear excitation. In Ref. 23 it was proven with mathematical rigor that the equations of nuclear fluid dynamics (2.4)–(2.6) can be reduced to the equation for an ideally elastic continuous medium. In connection with this, it is natural to assign to nuclear matter the physical properties of an elastic continuum. Magnetic twist resonances are one of the most characteristic manifestations of this fundamental property.

The rotational nature of the collective torsional vibrations can be verified by writing the velocity field $\delta\mathbf{V}$ in the form well known from classical mechanics:

$$\delta\mathbf{V}=[\mathbf{r}\times\mathbf{\Omega}], \tag{3.2}$$

where

$$\mathbf{\Omega}=-\nabla r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}})\dot{\alpha}_\lambda$$

is the angular-velocity field of the rotational motion, which is obviously a local vector function. The collective amplitude α_λ in the geometrical sense is the infinitesimal azimuthal angle of rotation of the collective nucleon flux about the axis whose direction, for example, in nuclear excitation by inelastically scattered electrons, is specified by the direction of the transverse (toroidal) component of the electromagnetic field penetrating the nucleus and induced by the electron flux. We see that the fluid-dynamical model extends the ideas about the rotational collective degrees of freedom of the nucleus. These collective vibrations of the nucleon flux have the nature of a differential (rather than rigid-body) rotation, which, as stressed above, can be caused only by the elastic properties of the nuclear Fermi system.

3.1. The eigenmodes of torsional nuclear vibrations and the energy spectrum of $M\lambda, T=0$ resonances

The mass parameter B_λ and the stiffness parameter C_λ of torsonal oscillations of the nucleus are calculated by using the same expressions (2.11) and (2.13) as the parameters of electric resonances, which in itself demonstrates the generality of the fluid-dynamical method of describing the two types of resonance. Substituting (3.1) into (2.11), we obtain the following expression for the inertial parameter:

$$B_\lambda=M\,\frac{\lambda(\lambda+1)}{2\lambda+1}\,\langle r^{2\lambda}\rangle. \tag{3.3}$$

The torsional stiffness parameter calculated in the Thomas–Fermi approximation from Eq. (2.13) is

$$C_\lambda=M\,\frac{\langle \nu^2\rangle}{3}\,\lambda(\lambda^2-1)\langle r^{2\lambda-2}\rangle, \tag{3.4}$$

where $M=mA$ is the nuclear mass, $\langle v^2\rangle$ is the average velocity of the nucleon Fermi motion (in the Fermi-gas approximation $\langle v^2\rangle=\frac{3}{5}v_F^2$), and $\langle r^\lambda\rangle$ is the radial moment of order λ. In the following calculations we use the Fermi approximation for the particle-distribution density:

$$n_0(r)=n(0)[1+\exp\{(r-R)/a\}]^{-1}. \tag{3.5}$$

The details of the calculations of the integrals determining the mass parameter and the stiffness parameter are given in Appendix 1.

It follows from the multipole dependence of the torsional stiffness parameter (3.3) that an incompressible nucleus does not admit long-wavelength dipole torsional vibrations: the frequency of the dipole mode vanishes. It is easily verified that excitation of the dipole torsional displacement field leads to rigid-body rotation of the nucleus as a whole without any change in its internal state. In fact, in the case $\lambda=1$ the components of the velocity field have the form $\delta V_x=\Omega y$, $\delta V_y=-\Omega x$, $\delta V_z=0$, which corresponds to the velocity-field distribution in rigid-body rotation of the nucleus with angular velocity $\Omega=\dot{\alpha}_\lambda$. Excitation of the dipole field contributes only to the kinetic energy of the collective Hamiltonian (2.10), while the potential energy vanishes with the torsional stiffness parameter. Recalling the basic equation from mechanics for the rotational kinetic energy $T_{\rm rot}=\frac{1}{2}J\Omega^2$, we see that the mass coefficient is simply the nuclear moment of inertia: $B_\lambda=J_\lambda$. It is easily verified that for $\lambda=1$ the calculated mass coefficient exactly coincides with the moment of inertia of a solid sphere: $J_1=\frac{2}{5}MR^2$.

In nuclear fluid dynamics the multipole oscillation energies $E_\lambda=\hbar\omega_\lambda$ are identified with the energy centroids of collective excitations (in this case, isoscalar resonances). The full multipole spectrum of energies $E(M\lambda,T=0)$

TABLE I. Theoretical dependence of the energy centroids and reduced excitation probabilities of isoscalar magnetic twist resonances in spherical nuclei on the mass number and atomic number.

$M\lambda$	$E(M\lambda)=k_\lambda A^{-1/3}$, MeV	$\sum B(M\lambda)\uparrow=\gamma_\lambda Z^2 A^{(2\lambda-4)/3}$, μ^2 F $^{2\lambda-2}$
$M2$	$45\,A^{-1/3}$ MeV	$0{,}7\,Z^2\,\mu^2$ F ^{2}
$M3$	$70\,A^{-1/3}$ MeV	$2{,}3\,Z^2 A^{2/3}\,\mu^2$ F ^{4}
$M4$	$95\,A^{-1/3}$ MeV	$5{,}9\,Z^2 A^{4/3}\,\mu^2$ F ^{6}

$=\hbar\sqrt{C_\lambda/B_\lambda}$ of magnetic resonances treated in terms of the eigenmodes of the long-wavelength torsional vibrations of a spherical nucleus has the form[38,39]

$$E(M\lambda,T=0)=\hbar\left[\frac{\langle v^2\rangle}{3}(2\lambda+1)(\lambda-1)\frac{\langle r^{2\lambda-2}\rangle}{\langle r^{2\lambda}\rangle}\right]^{1/2}. \quad (3.6)$$

In the sharp-boundary approximation this expression becomes[3)]

$$E(M\lambda,T=0)=\hbar\omega_F\left[\frac{1}{5}(2\lambda+3)(\lambda-1)\right]^{1/2}. \quad (3.7)$$

Approximate values of the energy centroids of multipole twist resonances as a function of the mass number are given in Table I. These values were obtained for nuclear-radius parameter $r_0=1.3$ F. The fact that the model predicts that the state lowest in energy is the isoscalar magnetic quadrupole resonance is a consequence of the assumption of the long-wavelength nature of the vibrations excited, which involve the entire mass of the spherical nucleus. As noted above, the quadrupole excitation is associated with torsional vibrations, in which the hemispheres of the spherical nucleus oscillate out of phase, as shown on the left-hand side of Fig. 2. On the right-hand side of this figure we show the displacement distribution characterizing the octupole magnetic response.

The experimental data on magnetic resonances with $\lambda\geqslant 2$ available at present is not as rich as that on electric resonances. Figure 3 shows the energy centroids of $M\lambda, T=0$ resonances as a function of the mass number predicted by nuclear fluid dynamics. Judging from the literature,[5] the most reliable data are those from the Darmstadt linear accelerator (DALINAC), obtained in the study of the cross sections for inelastic scattering at $\theta=165°$ of electrons with energies in the range 20−100 MeV on ^{28}Si, ^{90}Zr, ^{140}Ce, and ^{208}Pb (Refs. 1, 3, and 4). According to Richter, the strength of $M2$ collective excitations of spherical nuclei is localized in the energy range whose centroid is approximated well by the following mass-number dependence:[3,69]

$$E_{\exp}\sim 44A^{-1/3}\ \text{MeV}. \quad (3.8)$$

This dependence, as noted above, indicates the bulk nature of magnetic excitations. As seen from Table II, the experimental systematics of the energies of $M2$ resonances[5] in spherical nuclei agree fairly well with the predictions of nuclear fluid dynamics for the positions of the energy centroids of collective twist excitations. The numerical estimates were obtained for values of the radius parameter $r_0=1.2-1.3$ F and diffuseness parameter $a=0.55-0.6$ F. The spread in the theoretical values of the typical resonance parameters given in Table II is due to the variations of these parameters within the indicated limits.

It should also be noted that the conclusions reached in this model agree with the results obtained by the method of Wigner-function moments when only rank-two deformation tensors are used.[29] One of the important features of the "moment method" is that the determination of the vibration frequencies outside the long-wavelength approximation is taken into account by the self-consistent inclusion of high-multipole-order deformations of the equilibrium Fermi distribution, which are described by higher-rank deformation tensors. The consequences of this effect in describing resonances of higher multipole order $\lambda\geqslant 3$ are discussed in detail in the reviews of Refs. 29 and 61.

An important conclusion of nuclear fluid dynamics is that giant resonances are formed by coherent oscillations of all the nucleons in the full nuclear volume, i.e., they have a bulk rather than a surface nature. In the excitation of surface oscillations the collective motion involves only the peripheral part of the nucleons. It was shown in Ref. 67 that surface excitations are collective to a lower degree than bulk excitations. The bulk nature of giant excitations is also suggested by the $A^{-1/3}$ dependence of the resonance-energy centroid on the mass number.[28] The effect of the diffuseness of the nuclear boundary on the energies of collective excitations has been demonstrated in Refs. 62 and 70. The calculation using a realistic density distribution decreases the values of the energy in relation to the values found when the nuclear boundary is assumed to be sharp.[28]

Study of the temperature dependence of the energy of twist resonances has shown[71] that as the latter increases from

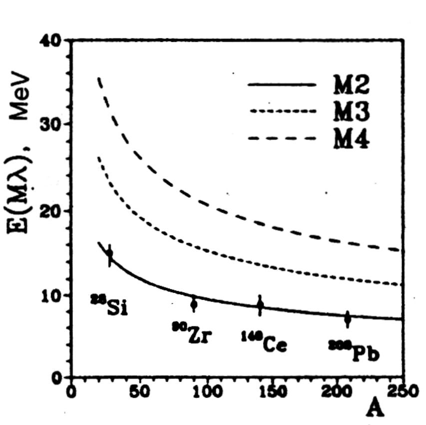

FIG. 3. Theoretical estimates of the location of the energy centroids of isoscalar magnetic quadrupole resonances as a function of mass number (lines). The points are the DALINAC data.[3,4]

TABLE II. Theoretical predictions for the locations of the energy centroids, reduced excitation probabilities, and spread widths of isoscalar magnetic twist resonances in spherical nuclei. The experimental data are the DALINAC data.[3,4]

Element	E (M2), MeV		$\sum B(M2)\uparrow$, $\mu^2 \cdot F^2$		$\Gamma(M2)\uparrow$, MeV
	Theory	Experiment	Theory	Experiment	Theory
^{28}Si	11—13	13—16	230 ± 20	340 ± 20	1,5 ± 0,5
^{90}Zr	8—10	8—10	1300 ± 300	1620	1,0 ± 0,3
^{140}Ce	7,5—9	7,5—10	3100 ± 300	6000 ± 600	0,8 ± 0,3
^{208}Pb	6,5—8	6—8,5	5300 ± 300	8500 ± 750	0,6 ± 0,2

0 to 5 MeV the energy centroids of twist resonances are shifted to higher energies by no more than 1–2%.

When discussing the general trends in the relative location of the energy centroids of magnetic and electric isoscalar resonances in the spectrum of a spherical nucleus we should evaluate the ratio

$$\frac{E(M\lambda,T=0)}{E(E\lambda,T=0)}=\frac{(2\lambda+3)}{2(2\lambda+1)}<1, \quad \lambda\geqslant 2. \tag{3.9}$$

It follows from (3.9) that the energy centroids of magnetic isoscalar resonances of multipole order $\lambda\geqslant 2$ in the energy spectrum are located below the energy centroids of electric isoscalar resonances of the same multipole order. This condition is also satisfied for dipole isoscalar resonances, assuming incompressibility of the nuclear matter. In the calculations of Refs. 18, 29, 39, 72, and 73 carried out by various methods of nuclear fluid dynamics, the isoscalar dipole electric resonance is associated with excitation of poloidal current oscillations of toroidal-like structure in the nuclear volume. The location of the centroid of this resonance is estimated to be $(50-70)A^{-1/3}$ MeV. The experimental systematics of the energies of the dipole magnetic resonance are approximated well by the estimate $41A^{-1/3}$ MeV.

3.2. Total probability for the excitation of $M\lambda, T=0$ twist modes

The macroscopic approach described here allows us to draw quite specific conclusions about the degree to which magnetic resonance excitations are collective, a measure of which is the reduced excitation probability. This characteristic can be defined as the time average of the squared modulus of the magnetic multipole moment:

$$B(M\lambda)=\left(\frac{\hat{J}_f}{\hat{J}_i}\right)^2 \langle|\mathcal{M}(M\lambda,t)|^2\rangle_t. \tag{3.10}$$

Here J_i and J_f are the total nuclear angular momenta in the initial and final states, and $\hat{J}=\sqrt{2\lambda+1}$. The definition of the magnetic moment in terms of electric-current fluctuations is given in the Introduction. Here and below, we present the results of calculations in the frame with fixed polar axis.

We shall consider the nuclear response to a perturbation which does not affect the spin degrees of freedom and does not lift the isospin degeneracy in the nucleon Fermi system. We stress the fact that the rotational fluctuations of the velocity field which we study occur at constant charge $n_e=(eZ/A)n_0$ and mass $\rho_0=mn_0$ density distributions. With these assumptions nucleon collective oscillations are coherently correlated with oscillations of the solenoidal electric current, the spatial distribution of which is characterized by the density[4)]

$$\mathbf{j}_\lambda(\mathbf{r},t)=\frac{eZ}{A} n_0 \text{ curl } \mathbf{r}r^\lambda Y_{\lambda 0}(\hat{\mathbf{r}})\dot{\alpha}_\lambda(t), \tag{3.11}$$

where $\alpha_\lambda(t)=\alpha_\lambda^0 e^{i\omega_\lambda t}$ and α_λ^0 is the amplitude of the zero modes, which according to Bohr and Mottelson is given by[7]

$$\alpha_\lambda^0\equiv\langle|\alpha_\lambda(t)|^2\rangle_t^{1/2}=\left[\frac{\hbar}{2B_\lambda\omega_\lambda}\right]^{1/2}. \tag{3.12}$$

Taking this into account, the reduced probability for excitation of the $M\lambda, T=0$ twist mode is given by

$$B(M\lambda)=\gamma_\lambda Z^2 A^{(2\lambda-4)/3}\mu^2 \ \mathrm{F}^{2\lambda-2}, \tag{3.13}$$

where

$$\gamma_\lambda=\frac{3}{4\pi}\frac{\lambda(2\lambda+1)}{(\lambda+1)}\left[\frac{\lambda-1}{5(2\lambda+3)}\right]^{1/2}(9\pi)^{1/3}r_0^{2\lambda-2}.$$

This expression has been obtained in the approximation of a sharp nuclear boundary, which makes it possible to extract the explicit dependence of $B(M\lambda)$ on the atomic number and mass number.

These arguments show that nuclear transitions to excited states with nonzero magnetic multipole moment can be induced by the excitation of collective rotational oscillations of the nucleon flux. The time-averaged magnetic multipole moment due to such oscillations with frequency ω_λ is given by

$$\mathcal{M}(M\lambda)=\frac{Z}{A}\sqrt{\frac{\lambda}{(\lambda+1)}\frac{2B_\lambda\omega_\lambda}{\hbar^2}}\,\alpha_\lambda^0\left(\frac{e\hbar}{2mc}\right), \quad \lambda\geqslant 2. \tag{3.14}$$

The total probabilities for $M\lambda, T=0$ resonances as a function of atomic number and mass number calculated using Eq. (3.13) are given in Table I. If we restrict ourselves to the sharp-boundary approximation, we see that the analytic expressions for the energy (3.7) and for the excitation probability (3.13) contain a single parameter: the nuclear-radius constant r_0, which gives a slight spread in the numerical estimates for these quantities. We also note the following intramodel correlation between the locations of the energy centroids and the reduced probabilities and the value of the parameter r_0. As r_0 increases the energy of a magnetic twist mode of arbitrary multipole order decreases as r_0^2, and the total reduced probability grows as $r_0^{2\lambda-2}$.

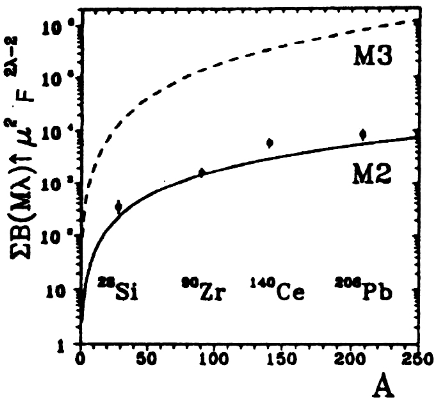

FIG. 4. Total reduced probabilities for isoscalar quadrupole and octupole magnetic resonances as a function of mass number, calculated by using the fluid-dynamical model. The points are the DALINAC data.[3,4]

The question of comparing the reduced excitation probabilities for $M\lambda, T=0$ resonances obtained in this collective model with the experimental data requires special attention. Actual measurements show that the magnetic multipole strength is fragmented among a rather large number of states. For example, according to the data of Ref. 69, in the ^{90}Zr nucleus the strength of $M2$ collective excitations is distributed among 34 states in the energy range 8–10 MeV; the center of localization of the strength is located at ~9 MeV. The summed probability of $M\lambda$ transitions, $\Sigma B_{\text{exp}}(M\lambda)$, is an experimental integral characteristic of the resonance-excitation intensity. According to the point of view usual in macroscopic theories, it is precisely this quantity which should be compared with the theoretical estimates of the probability of exciting the collective mode in question. Such a comparison is made in Table II for spherical nuclei, in whose spectra the magnetic quadrupole resonance was first identified.

In Fig. 4 we show the total reduced probabilities of $M2$ and $M3$ twist excitations as functions of the mass number. The mass-number dependence of the atomic number is parametrized by the well known empirical expression[76]

$$Z=\frac{A}{2+0.015A^{2/3}}, \tag{3.15}$$

which gives a good description of nuclei lying along the β-stability line. The symbols show the DALINAC data given in the reviews of Richter (Refs. 1, 3, and 4). We see that the theoretical predictions for the absolute values of $B(M\lambda)$ derived from the collective fluid-dynamical model under consideration agree fairly well with the experimental data at the qualitative level. It follows from Table II that one reason for the underestimation of the predicted magnetic strength compared with the experimental data is probably oversimplification of the assumptions about the nuclear structure. Actually, in this model the individuality of each nucleus is specified only by its atomic and mass numbers, while features of the shell structure are neglected. This is certainly too strong a simplification. Nevertheless, as can be seen from Fig. 4, the model reproduces the observed mass-number dependence of the $B(M2)$ factor qualitatively correctly.

In this section we have given estimates of the total probabilities for the excitation of magnetic collective modes obtained irregardless of what sort of charged test particle induces the solenoidal oscillations of the electric current inside the nucleus. The calculated probability is a quantitative measure of the intensity of the isoscalar resonance due to coherent in-phase oscillations of the protons and neutrons. Therefore, the probability (3.13) should be treated as the reduced probability for collective response of a nucleus excited by the transverse (toroidal) component of the electromagnetic field. In the next section we consider the theory of the excitation of $M\lambda, T=0$ twist modes in inelastic electron scattering.

4. EXCITATION OF TORSIONAL $M\lambda, T=0$ MODES IN INELASTIC ELECTRON SCATTERING

Experiments on electron scattering on nuclei serve as the source of the most reliable data on equilibrium and dynamical nuclear properties. As is well known, electric giant isoscalar resonances (with $\lambda \geqslant 2$) were discovered precisely in experiments on inelastic electron scattering. Moreover, practically all the information on $E\lambda, T=0$ resonances obtained up to now has been obtained from analysis of the (e,e') cross sections. Since nuclear fluid dynamics interprets magnetic twist resonances on the basis of the same physical principles as electric resonances, it seems to us that the (e,e') reaction must be just as effective a means of exciting that collective branch of the nuclear spectrum. Remarkably, the first consistent theory of (e,e') scattering with the excitation of isoscalar collective states of the electric type developed by Tassie[78] was also based on a macroscopic (liquid-drop) model of nuclear structure. The form of the cross sections for the (e,e') reaction predicted by this model agrees with experiment, which indicates the correctness of the assumptions about the type of collective nucleon motion induced by the incident electrons (i.e., longitudinal convection oscillations of the current with the velocity potential field). Regarding the theory of magnetic (e,e') scattering (with excitation of magnetic collective modes), as far as we know no consistent macroscopic approach based on a collective model of the nucleus has been discussed in the literature.

In this section we present, following Ref. 40, one of the possible versions of the theory of the magnetic multipole nuclear response in inelastic electron scattering based on the collective fluid-dynamical model in which the nucleus is treated as a spherical macroscopic particle of an elastic-like, incompressible Fermi continuum saturated in spin and isospin. By spin degeneracy we mean that the total magnetic moment of the ground state of an even–even spherical nucleus is zero. This mechanism of nuclear "magnetization" by inelastically scattered electrons (i.e., a transition from the ground state to an excited state with nonzero magnetic moment) carries exclusively classical content from the viewpoint of the electrodynamics of continuous media. As we see from the mathematical treatment of magnetic multipole resonances given above, oscillations of the electric current density are described in terms of the velocity field. The classical representation of the current density that is used is not explicitly related to its quantum-mechanical definition or, consequently, to the quantum-mechanical magnetic multipole-moment operator (the latter, as is well known, in the single-particle model is the sum of operators explicitly depending on the orbital and spin angular momenta of the nucleons). In relation to this it should be stressed that while remaining within the macroscopic description of collective nucleon dynamics it would not be appropriate to try to interpret this mechanism in conceptual terms as a microscopic single-particle shell model (i.e., to speak of spin or orbital nature of the transitions). As we have noted in the Introduction, the goal of the macroscopic description of the collective nuclear response is to obtain an adequate dynamical theory of the continuous nuclear medium. The main goal of our study is to reveal the dynamical properties of an elastic continuous medium in the nuclear response and to compare the results with the well known ones obtained by representing the nucleus as a drop of uniformly charged, incompressible liquid.

It must be stressed that at present there is a rather extensive literature on the microscopic analysis of mechanisms for the excitation of magnetic collective states in inelastic electron scattering (see, for example, Refs. 12, 51–53, 70, and 80). In these studies special attention has been paid to dipole and quadrupole collective magnetic excitations. The microscopic features of the excitation in (e,e') scattering of magnetic states of high multipole order in heavy nuclei are discussed in Ref. 54, and the same in light nuclei are discussed in Ref. 80. Judging from the literature, the main object of study of the magnetic response of deformed nuclei is the dipole scissors mode[82–89] (see also references therein). The theoretical study of the distribution of the $M2$ and $M3$ strengths of the response of deformed nuclei is the subject of Refs. 91 and 92, in which the authors used the microscopic quasiparticle–phonon model of nuclear collective excitations.[77]

The main goal of our approach is to supplement the studies mentioned above and reveal the qualitative regularities in the dependence of the integral parameters of magnetic twist resonances extracted experimentally from analysis of the (e,e') reaction on the atomic number, mass number, and multipole order of the excited mode.

4.1. A brief review of the theory of electron scattering

For completeness, let us begin with a brief review of the familiar expressions from the theory of scattering of unpolarized electrons on an unoriented target. The expression for the differential cross section of the (e,e') reaction in the plane-wave Born approximation (PWBA) has the form[92–44]

$$\frac{d\sigma}{d\Omega}=\sigma_M f_{\text{rec}}\left\{\left(\frac{q_\lambda}{q}\right)^2|S^L(q)|^2+\left[\frac{1}{2}\left(\frac{q_\lambda}{q}\right)^2+\tan^2\frac{\theta}{2}\right]|S^T(q)|^2\right\}. \tag{4.1}$$

Here $\sigma_M=[Z\alpha\hbar c\cos(\theta/2)/2E\sin^2(\theta/2)]^2$ is the Mott scattering cross section for unit charge, $f_{\text{rec}}=[1+2E\sin^2(\theta/2)/Mc^2]^{-1}$ is the recoil factor, E is the incident-electron energy, M is the mass of the target nucleus, and θ is the scattering angle. The momentum transfer is $q=\sqrt{q_\lambda^2+\omega^2}$, $q_\lambda=2\sqrt{EE_0}/\hbar c\sin(\theta/2)$, and $\hbar\omega=E-E_0$ is the nuclear excitation energy. The nuclear structure is manifested in the scattering cross section through the longitudinal and transverse form factors. The longitudinal (Coulomb) form factor $|S^L(q)|^2=\Sigma_\lambda|F_\lambda^C(q)|^2$ contains all the information about the spatial distribution of the nuclear charge density. The transverse form factor $S^T(q)$ is related to the transition current density and is the sum of the form factors of electric and magnetic multipole transitions:

$$|S^T(q)|^2=\sum_\lambda\{|F_\lambda^E(q)|^2+|F_\lambda^M(q)|^2\}.$$

Only transverse current oscillations of the nucleons are excited in backward scattering. Therefore, measurement of the cross sections for electron scattering at $\theta=180°$ is the most informative when studying states of the magnetic type.

The magnetic form factor $F_\lambda^M(q)$ is related to the transition current density $J_{\lambda,\lambda}(r)$ by a Fourier–Bessel transform:[95]

$$F_\lambda^M(q)=\frac{\sqrt{4\pi}}{Z}\frac{\hat{J}_f}{\hat{J}_i}\int_0^\infty J_{\lambda,\lambda}(r)j_\lambda(qr)r^2dr, \tag{4.2}$$

where $j_\lambda(qr)$ is the Bessel function of rank λ, J_i and J_f are the total angular momenta of the nucleus in the initial and final states, and $\hat{J}=\sqrt{2\lambda+1}$.

The sole elements of the Born formalism containing information about the nuclear structure are the transition charge and current densities. The characteristics of the excitations calculated in nuclear fluid dynamics carry essentially collective information. Therefore, it is appropriate to compare the predicted transition current densities, form factors, and excitation probabilities of magnetic twist resonances only with the typical integral parameters, appropriately summed over the parameters of the states actually observed.

4.2. The transition current density and form factor of $M\lambda, T=0$ resonances

For analyzing the electron scattering process it is more convenient to use a slightly different, but equivalent to (4.2), representation of the solution of the vector Laplace equation describing long-wavelength multipole torsional oscillations of a spherical nucleus (see, for example, Ref. 96, p. 188):

$$\mathbf{a}_\lambda(\mathbf{r})=-ir^\lambda\mathbf{Y}_{\lambda\lambda1}^0(\hat{\mathbf{r}}),\quad \mathbf{a}_\lambda^*(\mathbf{r})=ir^\lambda[\mathbf{Y}_{\lambda\lambda1}^0(\hat{\mathbf{r}})]^*=\mathbf{a}_\lambda(\mathbf{r}), \tag{4.3}$$

where $\mathbf{Y}_{\lambda\lambda'1}^\mu(\hat{\mathbf{r}})$ are the vector spherical harmonics, possessing the following properties:

$$\int[\mathbf{Y}_{\lambda_1\lambda_2 1}^\mu(\hat{\mathbf{r}})]^*\cdot\mathbf{Y}_{\lambda_1'\lambda_2'1}^{\mu'}(\hat{\mathbf{r}})d\hat{\mathbf{r}}=\delta_{\mu\mu'}\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}, \tag{4.4}$$

$$[\mathbf{Y}_{\lambda_1\lambda_2 1}^\mu(\hat{\mathbf{r}})]^*=(-)^{\lambda_1+\lambda_2+\mu+1}\mathbf{Y}_{\lambda_2\lambda_1 1}^{-\mu}(\hat{\mathbf{r}}). \tag{4.5}$$

The distribution of solenoidal electric current in the nucleus induced by the incident electron in this approach is described by the classical expression:

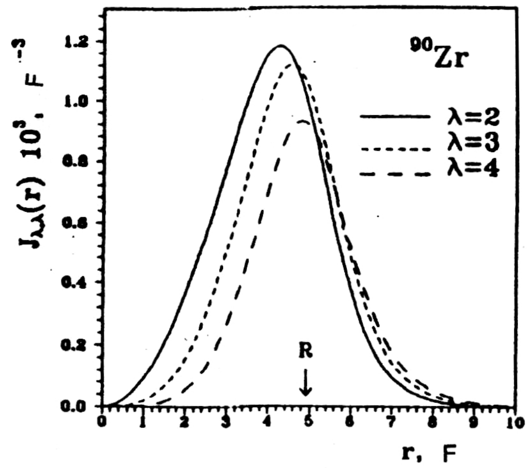

FIG. 5. Collective transition current densities for magnetic quadrupole, octupole, and hexadecapole isoscalar resonances in ^{90}Zr. The calculation was carried out with a realistic particle-number density distribution.

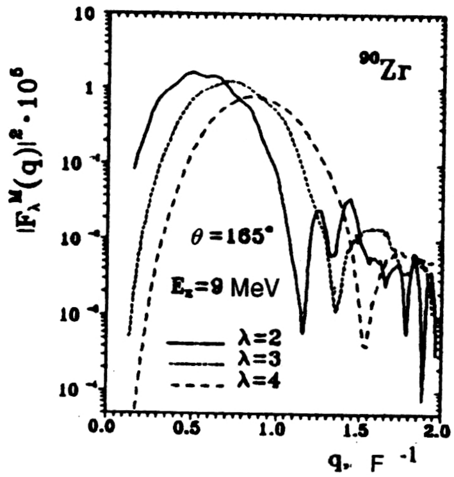

FIG. 6. Collective form factors of magnetic isoscalar resonances excited in electron inelastic scattering at 165° on ^{90}Zr as functions of the momentum transfer q. The calculation was carried out in the distorted-wave approximation.

$$\mathbf{j}_\lambda(\mathbf{r},t)=n_e\delta\mathbf{V}_\lambda(\mathbf{r},t)=n_e\mathbf{a}_\lambda(\mathbf{r})\dot{\alpha}_\lambda(t). \tag{4.6}$$

The collective transition current density of the magnetic torsional response of multipole order λ in a system with fixed polar axis is given by the expression

$$J_{\lambda,\lambda'}(r)=\left\langle\left|\frac{i}{ec}\int \mathbf{j}_\lambda(\mathbf{r},t)\cdot\mathbf{Y}^0_{\lambda\lambda'1}(\hat{\mathbf{r}})d\hat{\mathbf{r}}\right|^2\right\rangle_t^{1/2}$$
$$=N_\lambda n_e(r)r^\lambda,\quad N_\lambda=\frac{\alpha^0_\lambda\omega_\lambda}{ec}=\sqrt{\frac{\hbar\omega_\lambda}{2e^2c^cB_\lambda}}. \tag{4.7}$$

Here, as earlier, $\langle\ldots\rangle_t$ denotes time averaging.

Figure 5 shows the transition current densities (4.7), calculated for ^{90}Zr, corresponding to the excitation of magnetic resonance modes of various multipole orders. It follows, in particular, from this figure that the collective magnetic response of the nucleus has a bulk nature. As the multipole order increases the maximum of the function $J_{\lambda,\lambda}$ moves toward the nuclear surface.

The explicit expression for the collective magnetic multipole form factor of excitations associated with long-wavelength, differentially rotational flux oscillations has the following form in the plane-wave Born approximation:

$$|F^M_\lambda(q)|^2=\frac{4\pi}{Z^2}(2\lambda+1)N^2_\lambda\left|\int_0^\infty n_e r^{\lambda+2}j_\lambda(qr)dr\right|^2. \tag{4.8}$$

In the sharp-boundary approximation this integral can be calculated analytically:[36,38]

$$|F^M_\lambda(q)|^2=\frac{4\pi n_e^2}{Z^2}R^{2(\lambda+3)}(2\lambda+1)N^2_\lambda\left[\frac{(2\lambda+1)j_\lambda(qR)-qRj_{\lambda-1}(qR)}{q^2R^2}\right]^2. \tag{4.9}$$

The transition current densities and form factors calculated here characterize the electron-induced long-wavelength nucleon collective oscillations leading to excited states with nonzero magnetic multipole moments and contain information about the macroscopic distribution of the nucleon flux. As we have already noted, in the fluid-dynamical model of magnetic excitations described above it is not possible, strictly speaking, to reveal quantitatively the role of the spin contribution to the formation of an $M2$ resonance, since the macroscopic electric current density is not split into convection and spin components, as prescribed in the microscopic approach. Moreover, the treatment of the $M\lambda$ nuclear response which we have described does not deal with the question of the distribution of the strength of the magnetic response among isospin channels. Clearly, the problem of the dominance of the spin and convection contributions to the $M\lambda, T=0$ nuclear response competes with the microscopic theory of collective nuclear excitations based on the shell picture of nuclear structure. These problems are discussed in Refs. 12, 48, 50, 53, 56, and 69, in which the 2^- collective response of a spherical nucleus is analyzed from the microscopic point of view. It was noted in Refs. 31 and 89 that the magnetic twist response is analogous, in its macroscopic dynamical nature of the collective nuclear motion, to the isovector 1^+ scissors mode in deformed nuclei.

The results of our calculations for the magnetic form factors obtained in the distorted-wave approximation for ^{90}Zr are shown in Figs. 6–8. The incident-particle energy was varied at fixed scattering angle in such a way that the momentum transfer could change, but a certain level with energy corresponding to the localization center of the magnetic twist resonance would be excited. In Fig. 6 we show the collective magnetic form factors $|F^M_\lambda(q)|^2$ of multipole order $\lambda=2,3,4$ as a function of the momentum transfer, calculated at the scattering angle $\theta=165°$. The region of small momentum transfers ($q<0.5$ F^{-1}) is most suitable for detecting the $M2$ resonance in comparison with compared to

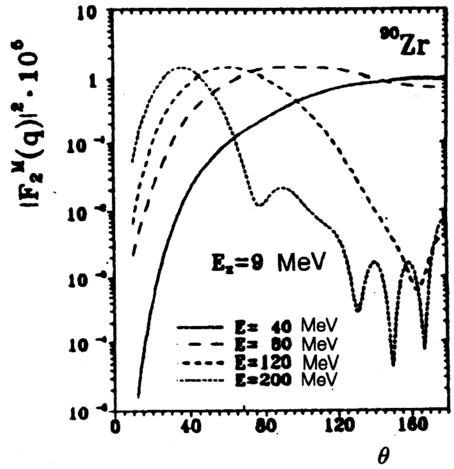

FIG. 7. Dependence of the collective form factor for the quadrupole magnetic resonance on the electron scattering angle θ. The calculation was carried out in the distorted-wave approximation for ^{90}Zr at the indicated values of the incident-electron energy E.

resonance modes of higher multipole order.[36] It is remarkable that for $q\approx 1\ \mathrm{F}^{-1}$, where the hexadecapole form factor has its first maximum, the quadrupole response is at a minimum. A similar picture is observed in the vicinity of the second maximum of the function $|F_4^M(q)|^2$. Figure 7 gives an idea of the form of $|F_2^M(q)|^2$ as a function of the scattering angle θ of electrons with initial energy $E=40$, 80, 120, and 200 MeV; as the incident-electron energy E increases the diffraction minimum is shifted to smaller angles. It follows from our calculations that: (1) magnetic resonances of higher multipole order are excited with noticeably smaller intensity, and (2) the twist effect should be manifested more clearly in heavy nuclei than in light ones.

In Fig. 9 the symbols show the data of the Darmstadt linear electron accelerator on the integrated cross sections of (e,e') scattering on ^{140}Ce for all $M2$ states in the energy range 7.5−10.0 MeV at scattering angle 165° and incident-electron energies in the range from 30 to 55 MeV. This reaction is analyzed from the viewpoint of the microscopic

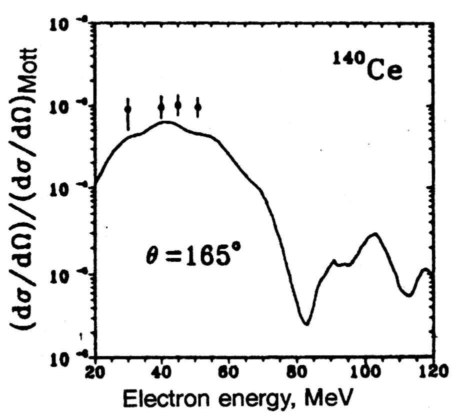

FIG. 8. Cross sections for excitation of the magnetic quadrupole resonance in ^{140}Ce measured at the Darmstadt linear electron accelerator[1] (points). The line is the theoretical calculation using the fluid-dynamical model of the twist response.

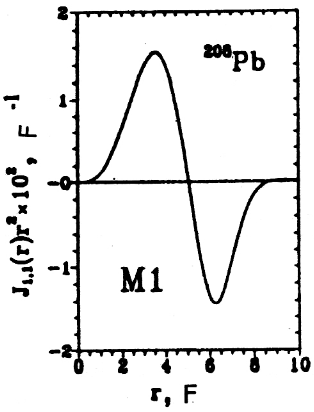

FIG. 9. Transition current density (multiplied by r^2) for the high-energy $M1,T=0$ twist resonance in the ^{208}Pb nucleus predicted by nuclear fluid dynamics.

theory of nuclear structure in Ref. 51. In this figure we also show the theoretical cross sections for excitation of the twist quadrupole resonance, calculated in the PWBA and the DWBA. We see that the experimental and theoretical form factors calculated in this collective model are very similar in shape. According to the logic generally followed in deriving conclusions from collective models, it can be stated that the qualitative agreement between the shapes of the theoretical form factor (which carries information about the spatial distribution of the excited current) and the experimental form factor indicates that the predicted nature of the collective oscillations is correct.

In the theory of electron excitation the reduced probability $B(M\lambda)$ for the transition of a nucleus to a state with magnetic moment of multipole order λ is determined by the integral of the transition current density $J_{\lambda,\lambda}(r)$ as

$$B(M\lambda)=\frac{\lambda}{\lambda+1}\left[\frac{\hat{J}_f}{\hat{J}_i}\int_0^\infty eJ_{\lambda,\lambda}(r)r^{\lambda+2}dr\right]^2$$
$$=\frac{\lambda}{(\lambda+1)}\cdot\frac{2B_\lambda\omega_\lambda}{\hbar}\left(\frac{Z}{A}\right)^2\mu^2,\quad \lambda\geqslant 2. \tag{4.10}$$

It is easily verified that this expression exactly coincides with Eq. (3.13) obtained above. The reduced probability for the excitation of the $M\lambda,T=0$ twist resonance and the experimentally measured total reduced transition probability $\Sigma B(M\lambda)$ are compared in Fig. 4, which we discussed in the preceding section. Here we only add that in the long-wavelength limit the form factor $F_\lambda^M(q)$ is expressed in terms of the reduced excitation probability $B(M\lambda)$:

$$|F_\lambda^M(q)|^2=\frac{4\pi}{e^2Z^2}\frac{q^{2\lambda}}{[(2\lambda+1)!!]^2}\frac{\lambda+1}{\lambda}B(M\lambda). \tag{4.11}$$

From the theory of electron scattering it is known that the long-wavelength approximation ($qR\ll 1$, where R is the nuclear radius) is justified only at relatively low incident-electron energies E. However, as a rule this condition is not realized in experiments. For example, in inelastic electron scattering on ^{90}Zr at $\theta=165°$, we have $qR=1$ for $E=26$ MeV. In this case $|F_\lambda^M(q)|^2$ is no longer proportional

TABLE III. Roots of the dispersion equation determining the excitation frequencies of isoscalar states of the magnetic type.

i	$\lambda=1$	$\lambda=2$	$\lambda=3$	$\lambda=4$	$\lambda=5$
1	5,7635	2,5011	3,8647	5,0946	6,2658
2	9,0950	7,1360	8,4449	9,7125	10,9506
3	12,3229	10,5146	11,8817	13,2187	14,5108

to $B(M\lambda)$, but is determined by the specific features of the transition current density of the excited state. Therefore, it is necessary to use Eq. (4.2) directly in the calculation of the form factors.

The sum rule is an integral measure of the degree to which the nuclear response is collective. The sum rules for magnetic excitations have been studied in Refs. 72 and 96–98. In the macroscopic approach the analog of the sum rules is the oscillator strength of the excitation.[7] According to our calculations, the magnetic oscillator strength of an $M\lambda$ isoscalar torsional response is given by

$$S(M\lambda)=\sum E(M\lambda)B(M\lambda)$$

$$=\beta_\lambda Z^2A^{(2\lambda-5)/3}\ \text{MeV}\cdot\mu^2\text{F}^{2\lambda-2}, \tag{4.12}$$

where $\beta_2=30$, $\beta_3=160$, and $\beta_4=560$.

5. THE HIGH-ENERGY DIPOLE MAGNETIC RESPONSE OF A SPHERICAL NUCLEUS

As already noted, the dipole twist mode requires special study. The main reason why this mode is absent in the spectrum of magnetic twist resonances (3.6) given above is the assumption that an external perturbation induces only long-wavelength rotational oscillations of the particle flux. In this approximation it is not possible to describe the well established magnetic dipole resonance whose energy centroid is estimated to be at $41A^{1/3}$ MeV (Ref. 5). However, as shown in Ref. 100, without this restriction the equations of nuclear fluid dynamics admit a solution describing shear vibrations which correspond to the collective high-energy $M1,T=0$ mode whose possible existence is discussed in Ref. 2.

Let us return to Eq. (2.13). After substituting (2.8) into it we find that the displacement field obeys the vector Helmholtz equation:

$$\Delta\mathbf{a}(\mathbf{r})+k^2\mathbf{a}(\mathbf{r})=0. \tag{5.1}$$

The solution regular at the origin corresponding to torsional vibrations of the nucleus is given by

$$\mathbf{a}(\mathbf{r},t)=j_1(kr)\mathbf{Y}^0_{111}(\hat{\mathbf{r}}), \tag{5.2}$$

where $k=\omega/c_t$ is the wave vector and $c_t=\sqrt{P_F/\rho_0}=v_F/\sqrt{5}$ is the propagation velocity of transverse axially symmetric torsional vibrations in a spherical nucleus. The frequency ω of these oscillations can be uniquely determined from the boundary condition, which requires the absence of shear on the surface of the spherical nucleus: $n_k\delta P_{ik}=0$, where n_k is the unit vector normal to the nuclear surface. The explicit expression for this condition is

$$\delta P_{r\phi}=0\rightarrow\frac{da_\phi}{dr}-\frac{a_\phi}{r}=0. \tag{5.3}$$

The condition (5.3) leads to the following dispersion relation:

$$z\frac{dj_\lambda(z)}{dz}=j_\lambda(z),\quad z_{\lambda n}=k_{\lambda n}R. \tag{5.4}$$

The numerical values of the roots of this equation for $1\leqslant\lambda\leqslant5$ are given in Table III. The eigenfrequencies of torsional oscillations are given by $\omega_{\lambda i}=(\hbar\omega_F/\sqrt{5})z_{\lambda i}$. The energy of the lowest dipole twist isoscalar response is estimated to be (for $r_0=1.1$ F)

$$E(M1,T=0)=\hbar\omega_{11}=\frac{\hbar^2(9\pi)^{1/3}}{\sqrt{20}mr_0^2}z_{11}A^{-1/3}$$

$$=135A^{-1/3}\ \text{MeV}. \tag{5.5}$$

The assumption of the long-wavelength nature of the oscillations of the flux velocity of intranuclear nucleons is critical only for the dipole response, since it simply excludes it (the energy of the dipole oscillation is zero). As far as $M\lambda$ ($\lambda>2$) resonances are concerned, the energies calculated in the long-wavelength approximation almost exactly coincide with the energies obtained from the Helmholtz equation (5.1) with the boundary condition of no stress at the nuclear surface, which is expressed by the dispersion relation (5.4). The fact that the energy of the high-energy $1^+,T=0$ resonance can be calculated only outside the long-wavelength approximation has been demonstrated in Ref. 46.

At the present time we know of only a single experiment, that of Woodward and Peterson performed at the Bates LINAC[2] using the ^{280}Pb nucleus to find the magnetic dipole strength in the energy range from 19 to 25 MeV, which had been predicted shortly before in Ref. 88 on the basis of microscopic calculations. This experiment measured the cross section for inelastic scattering of 60-MeV electrons at 180° and found a peak at 24 MeV with width of order 1.5 MeV and absolute value of the cross section $d\sigma/d\Omega=50\pm20$ nb/sr.

In order to compare the predictions of nuclear fluid dynamics for the high-energy dipole isoscalar resonance with the data given in Ref. 2, below we present the results of the calculations of the total reduced probability and the trans-

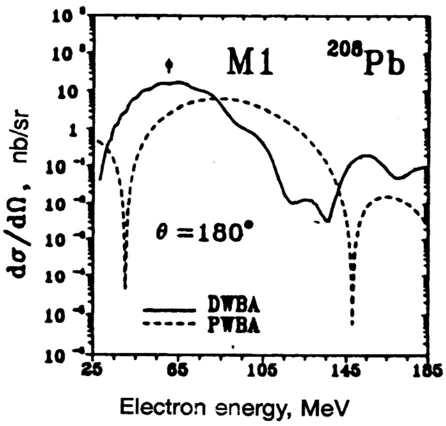

FIG. 10. Cross section for excitation of the high-energy $M1,T=0$ twist resonance in ^{208}Pb predicted by the fluid-dynamical model. The calculations were carried out in both the plane-wave Born approximation (PWBA) and the distorted-wave Born approximation (DWBA). The experimental point is the result of measurement of the cross section for electron inelastic scattering at 180° at an incident-electron energy of 60 MeV. The data are from the Bates LINAC.[2]

verse cross section for the excitation of this mode. In the macroscopic representation the transition current density of the magnetic dipole excitation is given by

$$J_{1,1}(r)=\left\langle\left|\frac{1}{ec}\int \mathbf{j}(\mathbf{r},t)\cdot\mathbf{Y}^{0}_{111}(\hat{\mathbf{r}})d\hat{\mathbf{r}}\right|^2\right\rangle_t^{1/2}$$

$$=N_1 n_e(r) j_1(k_1 r), \tag{5.6}$$

where the constant N_1 depends on the excitation energy as $N_1=[E(M1)/e\hbar c]\alpha_0$. According to Ref. 7, the collective amplitude of the current normal-mode oscillations is determined by the expression $\alpha_0=\langle|\alpha(t)|^2\rangle_t^{1/2}=\hbar/\sqrt{2B_1E(M1)}$. As before, the symbol $\langle...\rangle_t$ denotes time averaging, and B_1 is the mass parameter, given by

$$B_1=\int \rho_0 \mathbf{a}_1^2 d\tau=\frac{3mA}{8\pi}[j_1(k_1r)-j_0(k_1r)j_2(k_1r)]. \tag{5.7}$$

The transition current density shown in Fig. 9 illustrates the bulk nature of these collective transverse oscillations of the nucleon flux.

In the Born approximation the magnetic form factor $F_1^M(q)$ is expressed in terms of the transition current density $J_{1,1}(r)$ as

$$F_1^M(q)=\frac{\sqrt{12\pi}}{Z}\int J_{1,1}(r)j_1(qr)r^2dr$$

$$=\frac{\sqrt{12\pi}}{Z}N_1\int j_1(k_1r)j_1(qr)r^2dr. \tag{5.8}$$

In Fig. 10 we show the differential cross sections for excitation of the $1^+,T=0$ twist mode in ^{208}Pb in the inelastic scattering of electrons of energy $E_e=60$ MeV at 180°, calculated both in the DWBA and in the PWBA.

The total excitation probability $B(M1)$ for the $M1,T=0$ nuclear torsional mode calculated as a function of the atomic number and mass number is given by

$$B(M1)=\frac{3e^2}{2}\left[\int J_{1,1}(r)r^3dr\right]^2\approx 1.0\cdot 10^{-2}Z^2A^{-2/3}\mu^2. \tag{5.9}$$

The results presented in this section can be summarized as follows. Nuclear fluid dynamics predicts a high-energy $M1,T=0$ mode whose strength is expected to be localized in the range 20–25 MeV. In Refs. 10, 18–21, and 42 estimates in the range $E(M1,T=0)=21-27$ MeV are given for the location of the energy centroid of the high-energy $M1,T=0$ resonance. The results of our calculations and the data of the experiment that we have discussed are summarized in Table IV.

6. WIDTHS OF MAGNETIC TWIST MODES

In recent years the relaxation of collective nuclear excitations has been the object of intensive study in nuclear fluid dynamics (Refs. 28, 33, 101, and 102). The microscopic origin of the mechanism studied in this section for the damping of collective nuclear motion is associated with two-particle nucleon collisions, which in the final analysis lead to viscosity of the nuclear matter. Analysis of the damping of collective nuclear motions associated with isoscalar electric resonances with $\lambda\geqslant 2$ has shown that the role of one-body dissipation (Landau damping) is less important than that of two-body dissipation.[102] In the theory of continuous media two-particle dissipation is described macroscopically by the viscosity stress tensor. Shear viscosity has a bulk origin and is characterized by a dynamical viscosity coefficient μ, which in our calculations is treated as a parameter. However, as shown in Ref. 27, this coefficient can be extracted from data on the kinetic energy of fission fragments of heavy nuclei. The coefficient μ is estimated to be[103]

$$\mu=0.03\pm0.01 \text{ TP}, \quad 1 \text{ TP}=0.948\hbar \text{ F}^{-3},$$

where TP stands for terapoise. In the calculations which follow we also use the kinematical viscosity coefficient independent of the density and given by $\nu=\mu/\rho_0$, where ρ_0 is the equilibrium density of nuclear matter. We stress the fact

TABLE IV. Theoretical predictions for the $M1$, $T=0$ twist mode in ^{208}Pb and experimental data[2] on the excitation of the magnetic dipole resonance in the inelastic scattering of 60-MeV electrons at an angle of 180°.

^{208}Pb	Theory	Experiment
$E(M1)$, MeV	22,9	24,0
$B(M1)$, μ^2	2,0	—
$d\sigma/d\Omega$, nb/sr	15,0	50±20

that at fixed value of the dynamical viscosity coefficient our calculations cease to depend on any free parameters. The clearest discussion of the two-body dissipation mechanism can be found in the recent series of studies in Ref. 33, where, in particular, estimates are obtained for the viscosity coefficient on the basis of the kinetic approach developed by those authors (see also Refs. 104 and 105). Microscopic theories of the relaxation of nuclear excitations are described in Refs. 31, 77, 106, and 107. In this section we present the macroscopic calculation of the spread widths of magnetic twist resonances, following Ref. 37.

6.1. The equations of dissipative nuclear fluid dynamics

The macroscopic description of the damping of nucleon collective oscillations is based on the introduction of the dissipative Rayleigh function F, which is defined as the rate of loss of the total energy of collective oscillations and is written as [see, for example, Eq. (3.8) in Ref. 108]:

$$\frac{d}{dt} H(\alpha_\lambda, \dot{\alpha}_\lambda) = F(\dot{\alpha}_\lambda), \tag{6.1}$$

where H is the Hamiltonian of the normal modes (2.10) and the dissipation function is related to the friction coefficient D as $F = \dot{\alpha}_\lambda^2 D_\lambda$. In the Lagrangian treatment the equation of dissipative nuclear fluid dynamics has the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{a}_\lambda} - \frac{\partial L}{\partial a_\lambda} + \frac{\partial F}{\partial \dot{a}_\lambda} = 0, \tag{6.2}$$

where L is the Lagrangian of the torsional normal modes:

$$L = \frac{B_\lambda(\dot{\alpha}^\lambda)^2}{2} - \frac{C_\lambda(\alpha^\lambda)2}{2}. \tag{6.3}$$

The mass parameter B_λ and the torsional stiffness parameter C_λ are defined by Eqs. (3.3) and (3.4) in Sec. 3. Substitution of (6.1) and (6.3) into (6.2) leads to the well known equation for damped linear vibrations:

$$B_\lambda \ddot{\alpha}_\lambda + 2D_\lambda \dot{\alpha}_\lambda + C_\lambda \alpha_\lambda = 0, \tag{6.4}$$

from which it follows that the eigenfrequency ω_λ and the damping coefficient of torsional vibrations γ_λ are

$$\omega_\lambda = (C_\lambda / B_\lambda)^{1/2}, \quad \gamma_\lambda = D_\lambda / B_\lambda. \tag{6.5}$$

The friction coefficient D is given by the integral

$$D_\lambda = \frac{\nu}{2} \int \rho_0 \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i} \right)^2 d\tau, \tag{6.6}$$

which clearly has the same structure as the stiffness parameter. Taking into account the diffuseness of the nuclear surface, the calculated friction coefficient for shear vibrations is

$$D_\lambda = M\nu \langle r^{2\lambda-2} \rangle \lambda(\lambda^2 - 1), \tag{6.7}$$

where μ is the nuclear mass. The result calculated using the sharp-boundary approximation is given below.

6.2. The spread widths of magnetic resonances

According to the theory of linear nuclear vibrations, the energy $E(M\lambda)$ and the collision widths $\Gamma(M\lambda)$ of nuclear isoscalar resonances in this approach are given by[27]

$$E(M\lambda) = \hbar\omega_\lambda, \quad \Gamma(M\lambda) = \hbar\gamma_\lambda. \tag{6.8}$$

Substituting (6.7) into this, we obtain the following expression for the spread width of an $M\lambda, T=0$ twist resonance:

$$\Gamma(M\lambda) = \hbar\nu(2\lambda+1)(\lambda-1)\frac{\langle r^{2\lambda-2}\rangle}{\langle r^{2\lambda}\rangle}. \tag{6.9}$$

In the sharp-boundary approximation Eq. (6.9) becomes

$$\Gamma(M\lambda) \approx 6.0(2\lambda+3)(\lambda-1)A^{-2/3} \text{ MeV}. \tag{6.10}$$

We therefore find that because of the two-body energy-dissipation mechanism the widths of magnetic resonances fall with increasing mass number as $A^{-2/3}$. As the multipole order of the excitation increases the resonance width grows as $(2\lambda+3)(\lambda-1)$. Numerical estimates of the widths of the quadrupole magnetic isoscalar resonance for some spherical nuclei are given in Table II. These estimates were obtained for the above-quoted values of the parameters of the Fermi distribution for the density.

Noting that the energy and width have a similar multipole dependence, we can write Eq. (6.10) as

$$\Gamma(M\lambda) = \frac{5\nu}{\hbar\nu_F^2}[E(M\lambda)]^2 \text{ MeV}^{-1}. \tag{6.11}$$

It follows from (6.11) that the spread width of a magnetic resonance of given multipole order is proportional to the square of the excitation energy. An analogous dependence of the collision width on mass number also holds for electric isoscalar resonances (Refs. 27, 28, and 33). The analytic dependence of the width of electric isoscalar resonances on the multipole order λ in our calculations has the form

$$\Gamma(E\lambda) = \frac{5\nu}{\hbar\nu_F^2}[E(E\lambda)]^2, \tag{6.12}$$

where $E(E\lambda)$ is given by the Nix–Sierk expression (2.16). Comparing the expressions for the widths (6.11) and (6.12), we find

$$\frac{\Gamma(M\lambda)}{\Gamma(E\lambda)} = \frac{1}{2}\cdot\frac{(2\lambda+3)}{(2\lambda+1)} < 1, \quad \lambda \geq 2, \tag{6.13}$$

i.e., the width of a magnetic isoscalar resonance of given multipole order $\lambda \geq 2$ is always larger than the width of an electric isoscalar resonance of the same multipole order.

These estimates are predictive, since at present there is no experimental information on the widths of magnetic resonances. Our predictions agree rather well with the results of Refs. 28 and 33, where estimates are given for the relaxation parameters of transverse nuclear oscillations. In particular, the $A^{-2/3}$ law for the falloff of the widths of transverse collective modes with increasing mass number predicted in Ref. 28 is reproduced in the case of magnetic resonances with $\lambda \geq 2$ that we have considered.

7. CONCLUSION

Collective magnetic excitations of nuclei are currently the object of active study, stimulated by the experiments performed in Darmstadt (DALINAC),[109] Moscow,[90] Stuttgart,[84] Massachusetts (the Bates LINAC),[110] and other centers. It is therefore timely to review the theoretical investigations which have been carried out using the macroscopic theory of collective nuclear excitations.

One of the goals of the present review was to give a constructive analytic representation of the complete set of measured typical integrated parameters of isosclar magnetic multipole resonances: the locations of the energy centroids, the total excitation probabilities, the spread widths, and the excitation cross sections for electron inelastic scattering. These quantities are represented in the form of power-law functions of the atomic number, mass number, and multipole order of the excitation. This makes it possible to check the predictions of the model experimentally. The fact that the theory described here predicts values of the main typical integrated parameters for the $M2$ collective mode which agree qualitatively with the available experimental data indicates that the macroscopic treatment of the observed quadrupole resonance is adequate.

In the collective model that we have described, magnetic isoscalar resonances are interpreted in terms of torsional oscillations of the nucleus viewed as a spherical macroscopic particle of the nuclear Fermi medium. This emphasizes the fact that nuclear matter possesses the properties of an elastic continuum, and the physical nature of the elasticity of nuclear matter has an essentially quantum origin, since it is a consequence of the nucleon Fermi motion and the associated dynamical deformation of the Fermi surface. In this sense it is appropriate to add that the problem of the elasticity of nuclear matter is the subject of active study in the theory of large-ampliude nuclear motion.[111,112] It can be hoped that the concreteness of the results presented in the present review will prove useful in choosing the direction of future experiments, which will make it possible to determine how reliable are the representations of the elastic-like behavior of nuclear matter.

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APPENDIX 1

It is convenient to calculate the integrals appearing in the text in a system with fixed polar axis. In this case the spherical components of the toroidal instantaneous displacement field a_i corresponding to torsional vibrations of a spherical nucleus have the form

$$a_r=0, \quad a_\theta=0, \quad a_\phi=A_\lambda r^\lambda(1-\mu^2)^{1/2}\frac{dP_\lambda(\mu)}{d\mu}, \tag{A1.1}$$

where $\mu=\cos\theta$ and $P_\lambda(\mu)$ are the Legendre polynomials.

The components of the elastic stress tensor arising in torsional vibrations are

$$\frac{\partial a_1}{\partial x_1}=\frac{\partial a_r}{\partial r}=0,$$

$$\frac{\partial a_2}{\partial x_2}=-\frac{(1-\mu^2)^{1/2}}{r}\frac{\partial a_\theta}{\partial\mu}+\frac{a_r}{r}=0,$$

$$\frac{\partial a_3}{\partial x_3}=\frac{1}{r(1-\mu^2)^{1/2}}\frac{\partial a_\phi}{\partial\phi}+\frac{a_r}{r}+\frac{a_\theta}{r}\frac{\mu}{(1-\mu^2)^{1/2}}=0,$$

$$\frac{\partial a_1}{\partial x_2}=-\frac{(1-\mu^2)^{1/2}}{r}\frac{\partial a_\theta}{\partial\mu}-\frac{a_\theta}{r}=0,$$

$$\frac{\partial a_2}{\partial x_1}=\frac{\partial a_\theta}{\partial r}=0,$$

$$\frac{\partial a_1}{\partial x_3}=\frac{1}{r(1-\mu^2)^{1/2}}\frac{\partial a_r}{\partial\phi}-\frac{a_\phi}{r}$$

$$=-A_\lambda r^{\lambda-1}(1-\mu^2)^{1/2}\frac{dP_\lambda(\mu)}{d\mu},$$

$$\frac{\partial a_3}{\partial x_1}=\frac{\partial a_\phi}{\partial r}=A_\lambda\lambda r^{\lambda-1}(1-\mu^2)^{1/2}\frac{dP_\lambda(\mu)}{d\mu},$$

$$\frac{\partial a_2}{\partial x_3}=-\frac{1}{r}\frac{\partial a_\theta}{\partial\mu}-\frac{a_\phi}{r}\frac{\mu}{(1-\mu^2)^{1/2}}$$

$$=-A_\lambda r^{\lambda-1}\mu\frac{dP_\lambda(\mu)}{d\mu},$$

$$\frac{\partial a_3}{\partial x_2}=-\frac{(1-\mu^2)^{1/2}}{r}\frac{\partial a_\phi}{\partial\mu}$$

$$=-A_\lambda r^{\lambda-1}\left[\mu\frac{dP_\lambda(\mu)}{d\mu}-\lambda(\lambda+1)P_\lambda(\mu)\right].$$

The following basic integrals are used in the calculations:

$$\int_{-1}^{+1}P_\lambda^2(\mu)d\mu=\frac{2}{2\lambda+1},$$

$$\int_{-1}^{+1}(1-\mu^2)\left(\frac{dP_\lambda(\mu)}{d\mu}\right)^2d\mu=\frac{2\lambda(\lambda+1)}{(2\lambda+1)},$$

$$\int_{-1}^{+1}\mu P_\lambda(\mu)\frac{dP_\lambda(\mu)}{d\mu}d\mu=\frac{2\lambda}{(2\lambda+1)},$$

$$\int_{-1}^{+1}\left(\mu\frac{dP_\lambda(\mu)}{d\mu}\right)^2 d\mu=\frac{\lambda(\lambda+1)(2\lambda-1)}{(2\lambda+1)},$$

the last two of which have been obtained by using the recursion relations for the Legendre polynomials.

The expressions given above considerably simplify the awkward calculation of integrals involving the elastic stress tensor. In particular,

$$\frac{1}{2}\int F(r)\left(\frac{\partial a_i^\lambda}{\partial x_j}+\frac{\partial a_j^\lambda}{\partial x_i}\right)^2 d\tau$$

$$=4\pi A_\lambda^2(\lambda-1)(\lambda+1)\int_0^R F(r)r^{2\lambda}dr, \quad \text{(A1.2)}$$

where $F(r)$ is an arbitrary function of r.

[1)]The problem of the eigenvibrations of an elastic sphere has been studied using the Lamé equation by Lamb at the end of the last century.[22] According to Lamb, the eigenmodes of the elastic vibrations of a sphere are characterized by two branches. The first, the branch of spheroidal vibrations, is associated with harmonic deformations of the shape of the sphere. Motions of this type are quite analogous to the oscillations of a liquid drop. The second, the branch of torsional vibrations, is associated with the appearance of bulk shear deformations. A modern study of this problem can be found in Ref. 23. From the viewpoint of this classification, isoscalar electric resonances are described in terms of spheroidal vibrations of the nucleus, and magnetic resonances are associated with the excitation of torsional vibrations.

[2)]The now-common term "nuclear fluid dynamics" is mainly used to stress the difference between the modern, essentially quantum–macroscopic approach to the description of nuclear collective excitations based on the concept of an elastic-like nucleon Fermi continuum, from the earlier "nuclear hydrodynamics," which corresponds to a classical liquid treatment of the continuous nuclear medium.

[3)]In the sharp-boundary approximation the mass parameter (moment of inertia of torsional vibrations) and the torsional stiffness parameter are, respectively, given by[35]

$$R_\lambda=3M\frac{\lambda(\lambda+1)}{(2\lambda+1)(2\lambda+3)}R^{2\lambda} \quad \text{and} \quad C_\lambda=\frac{3}{5}Mv_F^2\frac{\lambda(\lambda^2-1)}{(2\lambda+1)}R^{2\lambda-2}.$$

[4)]Comparing Eq. (3.11) with the expression for the magnetization current well known from classical electrodynamics,[74,75]

$$\mathbf{j}(\mathbf{r},t)=c\ \mathrm{curl}\ \mathbf{M}(\mathbf{r},t),$$

we find

$$\mathbf{M}(\mathbf{r},t)=\frac{eZ}{cA}n_0\mathbf{r}r^\lambda Y_{\lambda0}(\hat{\mathbf{r}})\dot{\alpha}_\lambda(t).$$

Therefore, differentially rotational vibrations of the nucleus accompanied by simultaneous vibrations of the solenoidal current generate the nuclear magnetization field $\mathbf{M}(\mathbf{r},t)$.

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