Pionic double charge exchange in the quasiparticle random-phase approximation

Wiesław A. Kamiński
Institute of Physics, Maria Curie-Skłodowska University, 20-031 Lublin, Poland

Fiz. Élem. Chastits At. Yadra **26**, 362–383 (March–April 1995)

A description of the pionic double charge-exchange reaction is given within the proton–neutron quasiparticle random-phase approximation. The approach is tested in the case of iron ^{56}Fe, and fairly good agreement of the calculated quantities with recent data is found. The observed resonance-like behavior of the energy dependence of the cross section is explained semiquantitatively in terms of the two-nucleon process without invoking exotic mechanisms like dibayrons or multiple quark clusters. © *1995 American Institute of Physics.*

1. INTRODUCTION

Investigations concerning double charge exchange (DCX), both experimental and theoretical ones, have attracted much interest during the last decade. Studies of double charge exchange of pions on nuclei are attractive for many reasons. Because the charge conservation law ensures that at least two nucleons must be involved in pionic DCX on a nucleus, the reaction can be regarded as a promising source of specific information about the short-range correlations between bound nucleons. Some authors also hope that the reaction will make it possible to study the expected difference between the neutron and proton densities in nuclei. Depending on the choice of the target nucleus, the DCX process may populate neutron- or proton-rich nuclei far from the stability region.[1–4] It is also possible to obtain information from the reaction about double isobaric analog states,[5,6] double isovector dipole resonances,[7,8] or some exotic states of nuclear matter such as states of three or four neutrons.[9,10] (For a recent competent review of the pionic charge-exchange reactions, see Ref. 11.)

In most theoretical investigations of the DCX reaction in the low-energy region one assumes that the process is sequential: If two nucleons are correlated in space, then one can expect that a neutral pion π^0 emitted from the first charge-exchange reaction on one nucleon has a good chance of initiating a second charge-exchange process on the correlated partner. Further, one expects that the pion interacts during the two-step exchange only with valence nucleons, and thus the core plays only a passive role. This picture is quite natural for transitions to the final double analog state, since during such a transition all the quantum numbers of the nucleons remain unchanged except the third component of the isospin (change of a neutron into a proton). In a nonanalog transition like the transition to the ground state of $T\neq 1$ nuclei, the core can play an active role due to the antisymmetrization of the total wave function, which allows the core nucleons to participate actively in the reaction.[12]

Most of the theoretical approaches in the domain of low pion energy are theories involving the plane-wave impulse approximation—PWIA. Some of them account for distortion effects (distorted-wave impulse approximation—DWIA; coupled-channel techniques). The conclusions are not unique, and we can find statements about either the importance[15–17] or unimportance of the distortion.[13,14] Moreover, all calculations of DCX differential cross sections and angular distributions taking into account even simple correlated nuclear wave functions show fairly satisfactory agreement with the data, while theories without such correlations disagree with the experiments by an order of magnitude or more. Some authors argue[13,14,18–20] that correlation effects are so important that it is impossible to see other effects such as those of reaction dynamics or the pion distortion unless nucleon–nucleon short-range correlations are taken into account properly. It is also not clear what roles are played by the initial- and final-state interactions in this context.

From this point of view the nuclear structure involved in the DCX models is a very sensitive aspect of theoretical interpretations of the process. Here one can find pronounced differences ranging from very simple shell-model approaches[13,14] to the generalized seniority scheme,[21] and to very advanced realistic treatments.[22,23] Of course, the problem of nuclear structure will display its complexity as one deals with heavier nuclei. This is the reason why most existing DCX theoretical treatments concentrate on light nuclei. Recently, Vergados[24] proposed a treatment of the DCX reactions in the context of any shell model in which one separates reaction amplitudes into two parts, one of which depends on the nuclear wave function, while the other is connected with characteristics of the charge-exchange process between a pion and a nucleon. It can help in some respects, but even here one has the problem of explicit construction of the excited states of the intermediate nucleus and the Green functions.

In a series of papers[22,23,25] a new approach to the DCX process in the framework of the proton–neutron quasiparticle random-phase approximation (pn–QRPA) was developed. The model utilizes wave functions in large configuration spaces for both protons and neutrons in the initial, intermediate, and final nuclei, and there is no need for a closure approximation.

The study of the DCX process is especially interesting for the medium-heavy nuclei, for which numerous data exist, and therefore this case provides one more subtle test of the theory. Another point is the observation of the resonance-like behavior around the pion energy T_π=50 MeV. In contrast to the experimental observations, microscopic treatments did not predict a rise of the cross sections in this energy domain.

 1063-7796/95/020148-10$10.00

Only recently, Schepkin proposed a non-nucleon (dibaryon) mechanism[26,27] as a partial explanation of such a behavior of the low-energy cross sections. I am going to show that there is a chance to shed some light on this intriguing behavior in the framework of an ordinary two-nucleon mechanism.

2. DOUBLE CHARGE-EXCHANGE PROCESS

2.1. Brief description of the chosen charge-exchange operators

The formalism of the charge-exchange process for low-energy pions was applied to the DCX reactions on calcium,[22] germanium,[25] and tellurium[23] targets. I shall briefly recapitulate the main features of the theory, but in a more general form than in previous papers.

For the simplest local πNN interaction Lagrangian known as the pseudoscalar coupling, one can construct in the nonrelativistic approximation the effective pion–nucleon Hamiltonian[23,29]

$$h_p(\mathbf{q}) = -\sqrt{2}i\,\frac{f}{m_\pi}\,\Psi_N^\dagger \boldsymbol{\sigma}\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{x}}\tau_+\Psi_N, \tag{1}$$

which generates the p-wave pion–nucleon interaction only. In Eq. (1), $\boldsymbol{\sigma}$ and τ_+ are the Pauli and isospin raising operators, respectively; $\Psi_N^\dagger$ (Ψ_N) are the nucleon creation (annihilation) field operators, and the momentum transfer is taken to be $\mathbf{q}$.

A possible s-wave contribution to the πNN interaction is obtained from phenomenological considerations, taking into account a composite meson-exchange mechanism.[23,29,30] It has the form

$$h_s(\mathbf{q}) = 4\pi\,\frac{\lambda_1}{m_\pi}\,\sqrt{2}\Psi_N^\dagger\omega_q\tau_+\Psi_N. \tag{2}$$

In both Hamiltonians the plane-wave approximation for the pion wave functions was used. The constants f and λ_1 are determined to reproduce the experimental data for nucleon–nucleon and nucleon–pion elastic scattering.[29,30] Of course, in the case of bound nucleons they can, in general, be modified and fitted separately in each DCX reaction of interest. For the purpose of preserving the important features of the model, and because of the approximations used in the construction of the charge-exchange operators, we will apply these constants with experimentally determined values $f/4\pi = 0.08$ and $\lambda_1 = 0.046$ (Ref. 30 and references cited there).

The second-quantization procedure applied to the nucleon field allows one to express the interaction Hamiltonians in terms of creation and annihilation operators for the protons $(c_p^\dagger, c_p)$ and neutrons $(c_n^\dagger, c_n)$ as

$$h_p(\mathbf{q}) = -\sqrt{2}i\,\frac{f}{m_\pi}\sum_{pn}\left[\int d^3x\,\psi_p^*(\mathbf{x})\boldsymbol{\sigma}\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{x}}\psi(\mathbf{x})\right]c_p^\dagger c_n \tag{3}$$

and

$$h_s(\mathbf{q}) = 4\pi\,\frac{\lambda_1}{m_\pi}\,\sqrt{2}\omega_q\sum_{pn}\left[\int d^3x\,\psi_p(\mathbf{x})^*\psi_n(\mathbf{x})\right]c_p^\dagger c_n. \tag{4}$$

Here $\psi_a(\mathbf{x})$ is the solution of the Schrödinger equation for any average nuclear potential, e.g., the harmonic-oscillator or Woods–Saxon potential with $a = p$ or n for protons and neutrons, respectively.

2.2. Transformation to quasiparticles

Because of the quasiparticle character of the RPA, which we will use to describe the structure of the nuclei involved in the charge-exchange process, one needs to transform the expressions (3) and (4) to the Bogolyubov–Valatin quasiprotons $(a_p^\dagger, a_p)$ and quasineutrons $(b_n^\dagger, b_n)$,

$$a_{pm_p}^\dagger = u_p c_{pm_p}^\dagger + v_p(-1)^{j_p+m_p}c_{p-m_p}, \quad a_{pm_p} = (a_{pm_p}^\dagger)^\dagger, \tag{5}$$

$$b_{nm_n}^\dagger = u_n c_{nm_n}^\dagger + v_p(-1)^{j_n+m_n}c_{n-m_n}, \quad b_{nm_n} = (b_{nm_n}^\dagger)^\dagger, \tag{6}$$

where the coefficients u and v satisfy the well-known relation $u_{p(n)}^2 + v_{p(n)} = 1$. After the transformations (5) and (6) are performed, the p- and s-wave Hamiltonians have the form

$$h_p(\mathbf{q}) = -\sqrt{2}i\,\frac{f}{m_\pi}\sum_{pn,JM}\mathscr{F}_{pn}^{JM}(\mathbf{q})\mathscr{R}_{pn}^{JM}; \tag{7}$$

$$h_s(\mathbf{q}) = -4\pi\,\frac{\lambda_1}{m_\pi}\,\sqrt{2}\omega_q\sum_{pn}(2j_p+1)^{1/2}\delta_{pn}\mathscr{R}_{pn}^{00}. \tag{8}$$

In Eqs. (7) and (8) the transition density operator $\mathscr{R}_{pn}^{JM}$ is given by the formula

$$\mathscr{R}_{pn}^{JM} = u_p v_n C^\dagger(pnJM) + v_p u_n \tilde{C}(pnJM) + u_p u_n D(pnJM) - v_p v_n \tilde{D}^\dagger(pnJM). \tag{9}$$

The proton–neutron pair creation and annihilation operators and the additional one-body operators needed in the construction of the operator (9) are defined in the usual way:

$$C^\dagger(pnJM) = \sum_{m_p,m_n}(j_p m_p j_n m_n|JM)a_{pm_p}^\dagger b_{nm_n}^\dagger; \tag{10}$$

$$C(pnJM) = [C^\dagger(pnJM)]^\dagger, \quad \tilde{C}(pnJM) = (-1)^{J+M}C(pnJ-M); \tag{11}$$

$$D^\dagger(pnJM) = \sum(j_p m_p j_n m_n|JM)a_{pm_p}^\dagger(-1)^{j_n+m_n}b_{nm_n}; \tag{12}$$

$$D(pnJM) = [D^\dagger(pnJM)]^\dagger, \quad \tilde{D}(pnJM) = (-1)^{J+M}D(pnJ-M). \tag{13}$$

In Eq. (7) we follow Ref. 23 for a definition of the function $\mathscr{F}_{pn}^{JM}$:

$$\mathscr{F}_{pn}^{JM}(\mathbf{q}) = \sum_{(m)}(-1)^{j_n-m_n}(j_p m_p j_n - m_n|JM) \times\left[\int d^3x\,\psi_p^*(\mathbf{x})\boldsymbol{\sigma}\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{x}}\psi_n(\mathbf{x})\right]. \tag{14}$$

After some algebra the function $\mathscr{F}_{pn}^{JM}$ can be written in a more compact form

$$\mathcal{F}^{JM}_{pn}(\mathbf{q}) = \sqrt{4\pi}\sqrt{6}Y^*_{JM}(\Omega_q)G^J_{pn}(q) \tag{15}$$

by setting apart the form factor G^J_{pn}:

$$G^J_{pn}(q) = (-1)^{j_p+j_n}\hat{j}_p\hat{l}_p\hat{j}_n\hat{l}_n\sum_{l''=0}(-1)^{l_p+l_n-l''/2}$$
$$\times(l_p0l_n0|l''0)(J010|l''0)$$
$$\times R^{l''}_{pn}(q)\begin{Bmatrix}\frac{1}{2} & l_p & j_p\\ \frac{1}{2} & l_n & j_n\\ 1 & l'' & J\end{Bmatrix}. \tag{16}$$

In this equation the symbol { } is the $9j$ symbol (Fano), and (|) represents a Clebsch–Gordan coefficient. Further, Y_{JM} denotes the spherical harmonic depending on the solid angle Ω_q. The coefficient $\sqrt{6}$ in Eq. (15) comes from the reduced matrix element of the nuclear spin operator $\boldsymbol{\sigma}$. We have also used the abbreviation $\hat{\jmath} = \sqrt{2j+1}$ and the corresponding expression for $\hat{l}$. In Eq. (16) we have introduced the overlap integral $R^{l''}_{pn}$ between the radial nuclear wave functions and the radial part of the plane-wave pion,

$$R^{l''}_{pn}(q) = \int_0^\infty drqr^2 j_{l''}(qr)R_{n_pl_p}(r)R_{n_nl_n}(r). \tag{17}$$

Its explicit form depends on the choice of the radial nucleon wave functions $R_{n_pl_p}$ and $R_{n_nl_n}$. In the case of harmonic-oscillator wave functions one can find a very compact analytical expression for this integral.[23]

3. QRPA MODEL FOR THE DCX REACTION

3.1. DCX amplitude and intermediate excited states

In second-order perturbation theory the DCX transition amplitude is given by[23,25,37]

$$F_{if}(\mathbf{k},\mathbf{k}') = \sum_{mm'}\frac{\langle f,0^+;\pi^-(\mathbf{k}')|\hat{O}|mJM\rangle\langle mJM|\overline{m'JM}\rangle^*\langle\overline{m'JM}|\hat{O}|i(\text{G.S.});\pi^+(\mathbf{k})\rangle}{D(E_i,E^J_m,E^J_{m'},\mathbf{q})}. \tag{18}$$

In this equation $|i(\text{G.S.}),\pi^+(\mathbf{k})\rangle$ denotes the ground state of the initial nucleus (A,Z) and an incoming positive pion with momentum $\mathbf{k}$ and initial energy $(k^2+m_\pi^2)^{1/2}$. By analogy, $|f,\pi^-(\mathbf{k}')\rangle$ stands for an arbitrary state (ground or excited) of the final nucleus $(A,Z+2)$ and an outgoing negative pion with momentum $\mathbf{k}'$. The quantity $\mathbf{q}$ is the momentum transfer. Note that in the expression (18) we have assumed that the charge-exchange operators are the nonrelativistic Hamiltonians (3) and (4). It should be stressed that the denominator in Eq. (18) differs in each case of the interaction (3) or (4). The Hamiltonian (3) represents a contribution of pion absorption on a nuclear pair. From the general rules the Hamiltonian is known to be a small part of the pair absorption at low energies. But in the DCX reaction it can play a more important role.[55] The denominator in this case has the simple form $E_i + \omega_k - (E^J_m + E^J_{m'})/2$. The double scattering of a pion by two nucleons in the sequential mechanism is caused by the interaction (4). Of course, the denominator for this channel has no explicit form, and one can calculate the nuclear matrix elements of the charge-exchange operator including the propagator of the intermediate neutral pion. An integration over the intermediate pion momentum is understood.

The DCX amplitude (18) contains all the terms coming from two different sets of intermediate states $\{|mJM\rangle\}$ and $\{|\overline{m'JM}\rangle\}$ generated within the proton–neutron quasiparticle random-phase approximation (pn–QRPA). In principle, the two sets obtained by the QRPA on the initial and on the final nucleus are identical and describe the excited states of the same intermediate nucleus $(A,Z+1)$. But, since both calculations are not exact solutions of the many-body problem, they lead to slightly different solutions $\{|mJM\rangle\}$ and $\{|\overline{m'JM}\rangle\}$ for the intermediate wave functions:

$$|mJM\rangle = Q^{m\,\dagger}_{JM}|\text{RPA};(\text{A,Z})\rangle \equiv \sum_{(pn)}[X^m_{(pn)J}C^\dagger(pnJM) - Y^m_{(pn)J}\tilde{C}(pnJM)]|i;(\text{G.S.})\rangle, \tag{19}$$

$$|\overline{m'JM}\rangle = \bar{Q}^{m'\,\dagger}_{JM}|\text{RPA};(\text{A,Z}+2)\rangle \equiv \sum_{(pn)}[\bar{X}^{m'}_{(pn)J}C^\dagger(pnJM) - \bar{Y}^{m'}_{(pn)J}\tilde{C}(pnJM)]|f;(\text{G.S.})\rangle. \tag{20}$$

Here $X(\bar{X})$ and $Y(\bar{Y})$ are the forward and backward amplitudes, respectively, and p and n stand for the proton and neutron quasiparticle states [cf. Eqs. (5) and (6)]. As we have pointed out, the states (19) and (20) are not mathematically equivalent. In particular, the intermediate states belonging to different sets are not orthogonal, and this is a reason for including their overlaps $\langle mJ_m|\overline{m'J_{m'}}\rangle$ in the sum (18). Some authors have applied a similar procedure in calculations of double beta decay.[35,36] We have also used this scheme in our previous description of the DCX processes on calcium, germanium, and tellurium isotopes.[22,23,25]

In Eq. (18), E_i is the initial energy of the parent (target) nucleus. Usually, one can also adopt the average QRPA excitation energies $(E^J_m + E^J_{m'})/2$ in the denominator according to the above-mentioned procedure of allowing for the inequivalence of the two sets of intermediate states. It is worth emphasizing that the amplitude (18) does not contain the usual closure approximation in which one takes, instead of a sum over states in the odd–odd nucleus with their indi-

vidual energies, some average energy equivalent for all states. In our calculations we use explicitly the intermediate QRPA states, their structure, and the corresponding excitation energies in the denominator of Eq. (18). In this respect we are able to find individual contributions coming from the different multipolarities J^π and to estimate the importance of the analog and nonanalog routes in the DCX reaction.

3.2. Excited states of the daughter nucleus

In Refs. 22 and 23 one can find expressions for the DCX amplitude in the case of ground- and analog-state transitions. There are no fundamental difficulties in obtaining more general formulas for transitions to any state of the final (A,Z+2) nucleus. By analogy with what was done above [Eqs. (19) and (20)], one can generate such states by the following ansatz:

$$
\begin{aligned}
|\nu\,\mathcal{J}.\,\mathcal{M}\rangle &= \bar{Q}^{\nu\dagger}_{\mathcal{J}.\mathcal{M}}|\mathrm{RPA};\mathrm{A},\mathrm{Z}+2)\rangle \\
&\equiv \Bigg\{ \sum_{(pp')} [\bar{\mathcal{X}}^{\nu}_{(pp')}\mathcal{J}A^\dagger(pp'\,\mathcal{J}.\,\mathcal{M}) \\
&\quad - \bar{\mathcal{Y}}^{\nu}_{(pp')}\mathcal{J}\bar{A}(pp'\,\mathcal{J}.\,\mathcal{M})] \\
&\quad + \sum_{(nn')} [\bar{\mathcal{X}}^{\nu}_{(nn')}\mathcal{J}B^\dagger(nn'\,\mathcal{J}.\,\mathcal{M}) \\
&\quad - \bar{\mathcal{Y}}^{\nu}_{(nn')}\mathcal{J}\bar{B}(nn'\,\mathcal{J}.\,\mathcal{M})] \Bigg\} |f;\mathrm{G.S.}\rangle. \qquad (21)
\end{aligned}
$$

The creation and annihilation pair operators for protons, $A^\dagger$, A, and for neutrons, $B^\dagger$, B, are defined in full analogy with Eq. (10). The symbols $\bar{\mathcal{X}}^{\nu}$ and $\bar{\mathcal{Y}}^{\nu}$ stand for the forward and backward amplitudes. They, as well as the other amplitudes $X,Y,\bar{X},\bar{Y}$, are determined by solving the appropriate QRPA equation of motion for the states in the initial, intermediate, and final nucleus. Details of the structure of the QRPA equations and their solutions can be found in Refs. 22 and 23.

3.3. Matrix elements and the DCX cross section

Using the above expressions for the Hamiltonians (3) and (4), and also the definitions of the intermediate [Eqs. (19) and (20)] and final [Eq. (21)] states, one can find the following formulas for the matrix elements needed to write the amplitude (18) in an explicit form.

The matrix element for the s-wave charge-exchange operator contributing to the transition between the intermediate state $|m'J^\pi M\rangle$ in the nucleus (A,Z+1) and the final state $|\nu\,\mathcal{J}.\,\mathcal{M}\rangle$ of the daughter nucleus (A,Z+2) is

$$
\begin{aligned}
&\langle \nu\,\mathcal{J}.\,\mathcal{M};\pi^-(\mathbf{k}')|h_s|\overline{m'J^\pi M}\rangle \\
&= 4\pi\frac{\lambda_s}{m_\pi}\sqrt{2}\,\omega_{k'}\delta_{J\,\mathcal{J}}\delta_{M.\,\mathcal{M}} \\
&\quad\times \sum_{p\le p',n''} (-1)^{j_p+j_{n''}}[(\bar{\mathcal{X}}^{\nu}_{(pp')}\mathcal{J}\bar{X}^{m'}_{(p'n'')J}\bar{u}_p\bar{u}_{n''} \\
&\quad - \bar{\mathcal{Y}}^{\nu}_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(p'n'')J}\bar{v}_p\bar{v}_{n''})\delta_{pn''} \\
&\quad + (-1)^J(\bar{\mathcal{X}}^{\nu}_{(pp')}\mathcal{J}\bar{X}^{m'}_{(pn'')J}\bar{u}_{p'}\bar{u}_{n''} \\
&\quad - \bar{\mathcal{Y}}^{\nu}_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(pn'')J}\bar{v}_{p'}\bar{v}_{n''})\delta_{p'n''}]. \qquad (22)
\end{aligned}
$$

The matrix element for the p-wave charge-exchange operator contributing to the transition between the intermediate state $|m'J^\pi M\rangle$ and the final state $|\nu\,\mathcal{J}.\,\mathcal{M}\rangle$ is

$$
\begin{aligned}
&\langle \nu\,\mathcal{J}.\,\mathcal{M};\pi^-(\mathbf{k}')|h_p|\overline{m'J^\pi M}\rangle \\
&= i\frac{f}{m_\pi}\sqrt{4\pi}\sqrt{12}\sum_{p\le p',n''J''M''}\hat{J}\hat{J}''(JMJ''M''|\,\mathcal{J}.\,\mathcal{M}) \\
&\quad\times(-1)^{j_p+j_{p''}+J+J''}Y^*_{J''}(\Omega_{k'}) \\
&\quad\times\left[\begin{Bmatrix} j_{n''} & j_{p'} & J \\ \mathcal{J} & J'' & j_p \end{Bmatrix} G^{J''}_{pn''}(k')\right. \\
&\quad\times(\bar{\mathcal{X}}^{\nu}_{(pp')}\mathcal{J}\bar{X}^{m'}_{(p'n'')J}\bar{u}_p\bar{u}_{n''} - \bar{\mathcal{Y}}^{\nu}_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(p'n'')J}\bar{v}_p\bar{v}_{n''}) \\
&\quad + (-1)^{\mathcal{J}}\begin{Bmatrix} j_{n''} & j_p & J \\ \mathcal{J} & J'' & j_{p'} \end{Bmatrix} G^{J''}_{p'n''}(k') \\
&\quad\times\left.\left(\bar{\mathcal{X}}^{\nu}_{(pp')}\mathcal{J}\bar{X}^{m'}_{(pn'')J}\bar{u}_{p'}\bar{u}_{n''}\right.\right. \\
&\quad\left.\left. - \bar{\mathcal{Y}}^{\nu}_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(pn'')J}\bar{v}_{p'}\bar{v}_{n''}\right)\right]. \qquad (23)
\end{aligned}
$$

Equations (22) and (23) are written only for proton–proton quasiparticle excitations in the final nucleus. In the complete expressions one must add analogous terms for neutron–neutron excitations.

Further, we also need two other matrix elements.

The matrix element for the s-wave charge-exchange operator contributing to the transition between the ground state $|i;\mathrm{G.S.})\rangle$ of the parent nucleus and the intermediate states $|mJ^\pi M\rangle$ is

$$
\begin{aligned}
&\langle mJ^\pi M|h_s|i;(\mathrm{G.S.});\pi^+(\mathbf{k})\rangle \\
&= \sqrt{2}\left(4\pi\frac{\lambda_s}{m_\pi}\right)\omega_k\delta_{J0}\delta_{M0}\left[\sum_{pn}\delta_{pn}(X^m_{(pn)J}u_pv_n\right. \\
&\quad\left. + Y^m_{(pn)J}v_pu_n)\right]. \qquad (24)
\end{aligned}
$$

The matrix element for the p-wave charge-exchange operator contributing to the transition between the initial ground state $|i;\mathrm{G.S.}\rangle$ of the parent nucleus and the intermediate states $|mJ^\pi M\rangle$ is

$$
\begin{aligned}
&\langle mJ^\pi M|h_p|i;(\mathrm{G.S.});\pi^+(\mathbf{k})\rangle \\
&= i\sqrt{12}\frac{f}{m_\pi}\sqrt{4\pi}(-1)^JY^*_{JM}(\Omega_k) \\
&\quad\times\left[\sum_{pn}G^J_{pn}(k)(X^m_{(pn)J}u_pv_n + Y^m_{(pn)J}v_pu_n)\right]. \qquad (25)
\end{aligned}
$$

The expressions and definitions (18) and (22)–(25) al-

low us to write down an explicit formula for the DCX amplitudes in the most general case of the transition to any final state $|\nu \mathcal{J}. \mathcal{M}\rangle$:

$$F^s_J(\mathbf{k},\mathbf{k}') = -\delta_{J\mathcal{J}}\left(4\pi\frac{\lambda_s}{m_\pi}\right)^2 \omega_k\omega_{k'}\sum_{m,m'}\frac{\langle mJ^\pi|\overline{m'J^\pi}\rangle}{D(E_i,E^J_m,E^J_{m'},\mathbf{q})}$$
$$\times\left\{\sqrt{2}\sum_{n'',p\le p'}(-1)^{j_p+j_{n''}}[(\mathcal{J}^\nu_{(pp')}\mathcal{J}\right.$$
$$\times\bar{X}^{m'}_{(p'n'')J}\bar{u}_p\bar{u}_{n''}-\mathcal{V}^\nu_{(pp')}\mathcal{J}\bar{Y}^{m}_{(p'n'')J}\bar{v}_p\bar{v}_{n''})\delta_{pn''}$$
$$+(-1)^J(\mathcal{J}^\nu_{(pp')}\mathcal{J}\bar{X}^{m'}_{(pn'')J}\bar{u}_{p'}\bar{u}_{n''}$$
$$-\mathcal{V}^\nu_{(pp')}\mathcal{J}\bar{Y}^{m}_{(pn'')J}\bar{v}_{p'}\bar{v}_{n''})\delta_{p'n''}]$$
$$\left.+\sum_{p'',n\le n'}\left[\begin{pmatrix}p\to n\\p'\to n'\\n''\to p''\end{pmatrix}\right]\right\}$$
$$\times\left[\sqrt{2}\delta_{J0}\sum_{pn}\hat{j}\delta_{pn}(X^m_{(pn)J}u_pv_n-Y^m_{(pn)J}v_pu_n)\right]. \tag{26}$$

$$F^p_J(\mathbf{k},\mathbf{k}') = -\left(\frac{f}{m_\pi}\right)^2\sum_{m,m'}\frac{\langle mJ^\pi|\overline{m'J^\pi}\rangle}{E_i+\omega_k-\frac{E^J_m+E^J_{m'}}{2}}$$
$$\times\sum_{J'',\mathcal{M}}\hat{J}\hat{J}''(-1)^{\mathcal{J}+\mathcal{M}}\{Y_{J''}(\Omega_{k'})\otimes Y_{J''}(\Omega_k)\}_{\mathcal{J},\mathcal{M}}$$
$$\times\left\{\sqrt{12}\sum_{p\le p',n''}(-1)^{j_p+j_{p''}}\left[\begin{Bmatrix}j_{n''}&j_{p'}&J\\\mathcal{J}&J''&j_p\end{Bmatrix}\right.\right.$$
$$\times G^{J''}_{pn''}(k')(\mathcal{J}^\nu_{(pp')}\mathcal{J}\bar{X}^{m'}_{(p'n'')J}\bar{u}_p\bar{u}_{n''}$$
$$-\mathcal{V}^\nu_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(p'n'')J}\bar{v}_p\bar{v}_{n''})$$
$$+(-1)^{\mathcal{J}}\begin{Bmatrix}j_{n''}&j_p&J\\\mathcal{J}&J''&j_{p'}\end{Bmatrix}G^{J''}_{p'n''}(k')$$
$$\times(\mathcal{J}^\nu_{(pp')}\mathcal{J}\bar{X}^{m'}_{(pn'')J}\bar{u}_{p'}\bar{u}_{n''}$$
$$\left.\times-\mathcal{V}^\nu_{(pp')}\mathcal{J}\bar{Y}^{m'}_{(pn'')J}\bar{v}_{p'}\bar{v}_{n''})\right]$$
$$\left.+\sqrt{12}\sum_{p'',n\le n'}\left[\begin{pmatrix}p\to n\\p'\to n'\\n''\to p''\end{pmatrix}\right]\right\}$$
$$\times\left[\sqrt{12}\sum_{pn}G^J_{(pn)J}(X^{m'}_{(pn)J}u_pv_n-Y^{m'}_{(pn)J}v_pu_n)\right]. \tag{27}$$

The full amplitude $F_{if}(\mathbf{k},\mathbf{k}')$ is taken to be a sum of all multipolarities allowed for each part of the charge-exchange operator. The selection rule stemming from the angular-momentum and parity conservation laws limits the transition due to the s-wave operator (4) to 0^+ intermediate states, whereas the p-wave operator (3) has nonzero contributions for all so-called pion-like intermediate states $(0^-,1^+,2^-,3^+,...)$. The differential cross section is normalized in such a way that

$$\frac{d\sigma}{d\Omega}(\theta,q)=\left|\frac{1}{4\pi}\sum_{J^\pi}[F^s_J(\mathbf{k},\mathbf{k}')+F^p_J(\mathbf{k},\mathbf{k}')]\right|^2, \tag{28}$$

where $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ is the momentum transfer in the DCX process.

As mentioned above, we have used throughout this paper the nonrelativistic approximation for the charge-exchange operator. In general, the relativistic corrections can influence the differential cross section (28). One can expect a negligible role of such terms for the p-wave part of the transition amplitude and a larger contribution to the s part. The plane-wave approximation for pions used in this paper can also be a source of inaccuracies for the cross section. I shall not treat the pion distortions in this paper.

3.4. Ground-state DCX transition

Equations (26) and (27) represent general expressions for the amplitude of the DCX transition into any final state of a daughter nucleus. The DCX transition to the ground state can be simplified to the amplitude[23,37]

$$F^s_{GS}(\mathbf{k},\mathbf{k}')=\left(4\pi\frac{\lambda_s}{m_\pi}\right)^2\omega_k\omega_{k'}\sum_{mm'}\frac{\langle m0^+|\overline{m'0^+}\rangle}{D(E_i,E^0_m,E^0_{m'},\mathbf{q})}$$
$$\times\left\{\left[2\sqrt{2}\sum_{pn}\delta(p,n)\hat{j}_p\left(\bar{X}^{m'}_{(pn)0}\bar{v}_p\bar{u}_n\right.\right.\right.$$
$$\left.-\bar{Y}^{m'}_{(pn)0}\bar{u}_p\bar{v}_n\right)$$
$$\left.\times\left[2\sqrt{2}\sum_{pn}(X^m_{(pn)0}u_pv_n-Y^m_{(pn)0}v_pu_n)\right], \tag{29}$$

$$F^p_{GS}(\mathbf{k},\mathbf{k}')$$
$$=-\left(\frac{f}{m_\pi}\right)^2\sum_{mm',J}\frac{\langle mJM|\overline{m'JM}\rangle}{E_i+\omega_k-\frac{E^J_m+E^J_{m'}}{2}}P_J(\cos\theta_{kk'})$$
$$\times\left\{\left[\sqrt{12}\sum_{pn}(-1)^{j_p+j_n+J}G^J_{pn}(k')(\bar{X}^{m'}_{(pn)J}\bar{v}_p\bar{u}_n\right.\right.$$
$$\left.\left.-\bar{Y}^{m'}_{(pn)J}\bar{u}_p\bar{v}_n)\right]\right.$$
$$\left.\times\left[\sqrt{12}\sum_{pn}G^J_{(pn)J}(k)(X^m_{(pn)J}u_pv_n-Y^m_{(pn)J}v_pu_n)\right]. \tag{30}$$

All symbols in the two last equations have been used already. The only new quantity $P_J(\cos\theta_{kk'})$ is a Legendre polynomial coming from the reduction of the tensor product of spherical harmonics in Eq. (27). We have also used the abbreviation $\delta(p,n) = \delta_{n_p n_n}\delta_{j_p j_n}\delta_{l_p l_n}$.

We would like to note that the s-wave part of the transition operator contains only the route in which the neutron occupying some nuclear single-particle state with definite quantum numbers n,l,j is changed into a proton with exactly the same quantum numbers. These transitions excite the isobaric analog state (IAS) if they are coherently superimposed. However, because of the pairing correlations the situation is more complex and one obtains appreciable transitions through other 0^+ states. In the theory developed here we are able to separate the two types of contributions using a method of identification of the IAS transition which, in general, represents the strongest 0^+ transition.[23] We have already stressed that the p-wave part of the charge-exchange operator gives a contribution to the DCX amplitude only for the pion-like intermediate states and thus causes nonanalog routes which are sensitive to short-range nucleon–nucleon correlations.

4. EXAMPLE OF THE DCX REACTION ON ^{56}Fe

4.1. Details of the calculations

As an example of the application of the theory I shall discuss the ground transition in iron: $^{56}\text{Fe}\rightarrow{}^{56}\text{Ni}$. This reaction was already investigated in the eighties,[38,39] but the data base is still very sparse for nonanalog transitions. For the pion energy 50 MeV three experimental points of the angular distribution have been measured at PSI for the ground-to-ground transition as well as to several individual predominantly 0^+ excited states of ^{56}Ni.[38] Some very preliminary data also exist at the pion energies T_π=35 and 61 MeV. Although the data for the ground-to-ground transition are very limited, one can already conclude the following: The transition exhibits a well pronounced resonance-like behavior. This means that the cross section at T_π=50 MeV is for forward angles an order of magnitude or more larger than at other energies.

The results presented below are achieved with a model space consisting of $0p_{1/2}$, $0p_{3/2}$, $1s_{1/2}$, $0d_{3/2}$, $0d_{5/2}$, $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, and $0f_{7/2}$. The single-particle states used here are calculated with a Coulomb-corrected Woods–Saxon potential. It was assumed that the two types of nucleons—protons and neutrons—occupy the same shells, and calcium ^{40}Ca was taken as the inactive core. The two-body matrix elements needed for the construction of the QRPA matrices were obtained from the *realistic* nuclear-matter G-matrix by solving the Bethe–Goldstone equation (see, e.g., Refs. 40 and 41 for more details)

$$G(\omega)=V+V\frac{Q}{\omega-H_0}G(\omega). \tag{31}$$

In this equation Q is the Pauli projection operator, ω stands for the starting energy, and V is taken to be the nucleon–nucleon realistic one-meson exchange Bonn potential.[42–44] The quantity H_0 is the unperturbed single-particle Hamiltonian. In the present work we have used the harmonic-oscillator Hamiltonian. To take into account the effects of the finite nucleus we solve Eq. (31) with as small an absolute value of the starting energy as −25.0 MeV. This corresponds to an average single-particle energy of −12.5 MeV. The oscillator length used here is b=2.0 fm.

The two-body matrix elements are obtained for nuclear matter. They are not specialized for a given nucleus. Thus, and owing to the finite Hilbert space used, one must renormalize them by multiplying them by factors slightly different from 1.0: g^n_{pair}, g^p_{pair}, g^{pn}_{pp}, and g^{pn}_{ph}. For the ground states of the parent and daughter nuclei one obtains uncorrelated vacuum states by solving the standard BCS equation in the above-mentioned model space. The two renormalization factors g^n_{pair} and g^p_{pair} multiplying the proton and neutron pairing matrix elements $\langle(aa)0|G|(bb)0\rangle$ are fixed by adjusting the empirical pairing gaps Δ^p_a and Δ^n_a to the lowest quasiparticle energy obtained from the gap equation

$$\Delta^{p(n)}_a=\tfrac{1}{2}\,g^{p(n)}_{\text{pair}}\hat{j}_a^{-1}\sum_b \hat{j}_b\Delta_b[(\varepsilon_b-\lambda_{p(n)})^2 + \Delta_b^2]^{-1/2}\langle(aa)0|G|(bb)0\rangle. \tag{32}$$

The empirical pairing gaps are deduced according to the recently published prescription of Moeller and Nix:[45,46]

$$\Delta^{\text{even–even}}_{\text{neutron}}=-\tfrac{1}{8}\,[M(Z,N+2)-4M(Z,N+1)+6M(Z,N)-4M(Z,N-1)+M(Z,N-2)], \tag{33}$$

$$\Delta^{\text{even–even}}_{\text{proton}}=-\tfrac{1}{8}\,[M(Z+2,N)-4M(Z+1,N)+6M(Z,N)-4M(Z-1,N)+M(Z-2,N)]. \tag{34}$$

The expressions (33) and (34) cannot be used for nuclei with a magic number of protons or neutrons. Thus, the pairing gap and the corresponding pairing strengths are estimated by using the adjacent even–even nucleus. Table I contains values of the pairing strengths g^n_{pair} and g^p_{pair} fitted in this way for ^{56}Fe and ^{56}Ni. All the renormalization factors are close to unity. Thus, the bare G-matrix elements of the Bonn potential are already reasonably good.

Solutions of the BCS equations with matrix elements fixed in this manner allow one to evaluate the occupation amplitudes u and v needed for construction of the QRPA equation of motion. To determine the QRPA matrices fully one must also fix two additional renormalization factors, the strength of the particle–particle g^{pn}_{pp} interaction and the strength of the particle–hole g^{pn}_{ph} interaction. For this purpose we used the isobaric analog state (IAS) and the Gamow–Teller state in cobalt ^{56}Co, which are known to be 3.65 and 10.60 MeV, respectively.[47,48] The QRPA energy of these states depends predominantly on the particle–hole strength, and adjustment of them to the experimental energies gives g^{pn}_{ph}=0.8. Details of such a procedure are given in Ref. 23.

TABLE I. Experimental neutron and proton gaps for ^{56}Fe, ^{54}Fe, and ^{58}Ni nuclei obtained from Eqs. (33) and (34). The last two nuclei are used to estimate the strengths in nickel ^{56}Ni because of its double-magic character. (For details, see the text.) (The masses are taken from the mass tables of Ref. 54. The pairing strengths g^n_{pair} and g^p_{pair} were fixed to reproduce the experimental gaps.

Nucleus	Δ^n_{exp}, MeV	Δ^p_{exp}, Mev	g^n_{pair}	g^p_{pair}
^{56}Fe	1.360	1.570	0.938	0.993
^{54}Fe	—	1.520	—	0.908
^{58}Ni	1.300	—	.1.030	—

The second factor g^{pn}_{pp} will be treated as a free parameter of the theory, and we will discuss all the reaction observables as functions of it.

4.2. Results and discussion

We calculated the angular distributions and the energy dependence of the cross section for the DCX ground–ground transition on ^{56}Fe. Figure 1 shows the angular distribution for the incident-pion energy T_π=50 MeV. Three curves are presented for three different values of the particle–particle strength g^{pn}_{pp}: 0.8, 1.0, and 1.1. The experimental points were measured at the Paul Scherer Institute by the Tübingen–Karlsruhe group.[38] The angular distribution decreases rapidly as the particle–particle parameter increases. This behavior is observed in a full range of the g^{pn}_{pp} strength up to the value 1.1 for which the QRPA solution tends to a collapse. A similar behavior was also observed in other nuclei[22,23,25] and in other processes, e.g., double beta decay.[35,36] The mechanism for the collapse is connected with an increase of the ground-state correlations by increasing the particle–particle interaction. As a result, the lowest excited QRPA state is pushed down in energy below the ground state. Simultaneously, the cross sections drop by factors of 3–10, depending on the scattering angle. The cross section is reduced, since an increase of g^{pn}_{pp} produces stronger ground-state correlations. This increases the backward amplitudes Y. The terms with Y in Eqs. (29) and (30) become large enough to cancel against the terms with the forward amplitudes X. The magnitude of the DCX cross section decreases rapidly. Comparison of the experimental results and the theoretical predictions (Fig. 1) shows that the physically important domain of g^{pn}_{pp} is the interval 1.0–1.1.

It is interesting to compare the importance of the contributions to the total amplitude coming from the different angular momenta. The contributions for the forward-angle DCX amplitude and for the intermediate states $J^\pi=0^+$, 0^-, 1^+, 2^-, and 3^+ which are most important are presented in Fig. 2. One can notice immediately that a crucial role in pushing the contributions down into agreement with the data is played by the dependence of each partial amplitude on the particle–particle strength g^{pn}_{pp}. The transition through 0^+

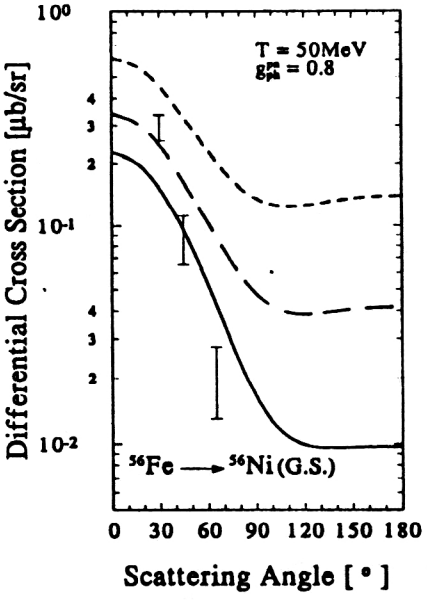

FIG. 1. Angular distribution for the ground-state transition on iron.[56] The particle–hole strength g^{pn}_{ph} is fixed to reproduce the Gamow–Teller and isobaric analog difference in ^{56}Ni. The results for three values of the particle–particle strength are shown: g^{pn}_{pp}=0.8 (short-dashed line), g^{pn}_{pp}=1.0 (long-dashed line), and g^{pn}_{pp}=1.1 (solid line). The experimental data are taken from Refs. 38 and 53.

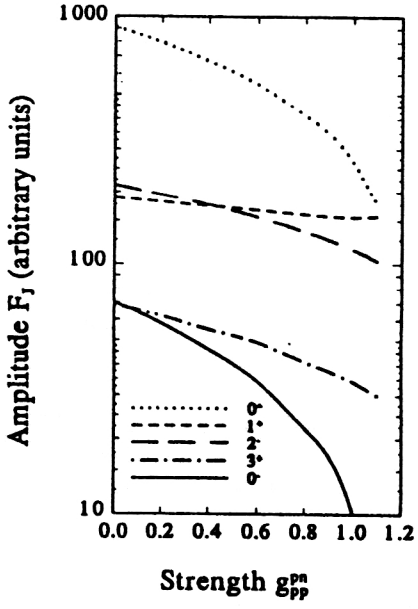

FIG. 2. The most important contributions to the transition amplitude come from the intermediate 0^+ (dotted line), 0^- (solid line), 1^+ (short-dashed line), 2^- (long-dashed line), and 3^+ (dashed-dotted line) states. The results for the pion energy T_π=50 MeV are plotted as a function of the particle–particle strength g^{pn}_{pp}. The particle–hole strength is fixed at 0.8.

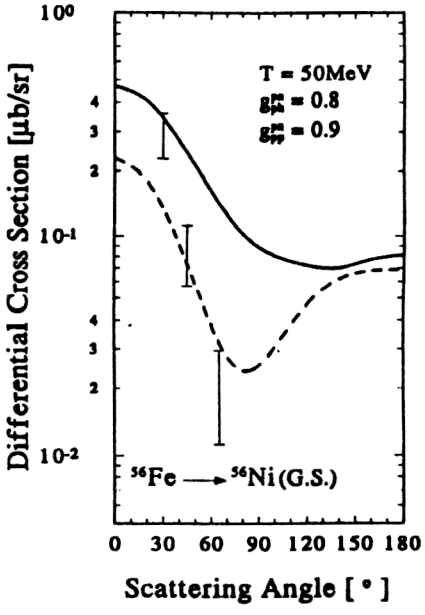

FIG. 3. Angular distributions for two choices of the model space are presented for fixed interaction strengths (g_{ph}^{pn}=0.8, g_{pp}^{pn}=0.9) and for the incident-pion energy T_π=50 MeV. The solid line represents the results of a smaller model space, and the dashed line is for a very large model space. The experimental values are from Refs. 38 and 53. (For details, see the text.)

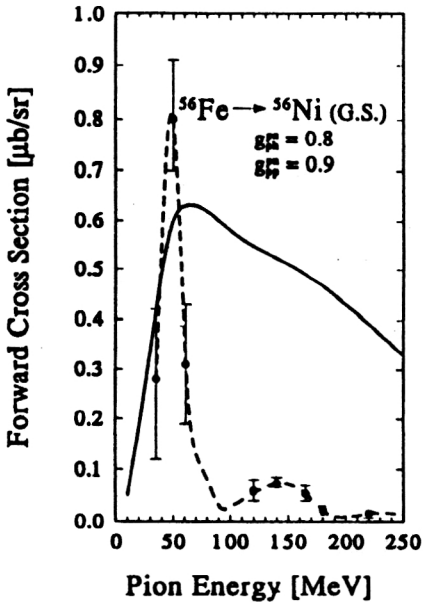

FIG. 4. Energy dependence of the forward-angle (5°) cross section for the ground-state transition on iron ^{56}Fe (solid line). The results are obtained for the small basis. The data indicated by error bars are taken from Refs. 38 and 39 (the short-dashed curve is drawn only to guide the eye).

states with the biggest contribution from the IAS dominates the total amplitude at low g_{pp}^{pn}. But in the physically interesting interval $g_{pp}^{pn} \geq 1.0$ this amplitude is comparable with the 1^+ and 2^- amplitudes. Thus, none of the approaches restricted to the intermediate isobaric analog state give a good description of the cross sections and of the other DCX observables because nonanalog routes play as important a role as transitions through 0^+ states.

We also examine in this paper the influence of the model space on the final results. Additional calculations of the angular distributions at the pion energy T_π=50 MeV were performed for a "huge" single-particle basis consisting of the states $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$, $1s_{1/2}$, $0d_{3/2}$, $0d_{5/2}$, $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$, $0h_{11/2}$, $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$ for neutrons and $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$, $1s_{1/2}$, $0d_{3/2}$, $0d_{5/2}$, $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, $1f_{7/2}$ for protons. All these single-particle levels are below 5.0 MeV in the Woods–Saxon potential and are either bound or quasibound. In Fig. 3 we present the angular distribution for these two choices of the size of the basis for the particle–particle strength g_{pp}^{pn}=0.9. A change of the shape of the angular distribution by increasing the basis is clearly seen. A minimum around the scattering angle θ=70° appears, which is also in agreement with the Gibbs prediction.[38] Compared with the "small" basis, the angular distribution with the large basis is steeper. The absolute values of the cross section are 2–5 times smaller for the same particle–particle strength g_{pp}^{pn}. This means that one obtains agreement with experiment for smaller values of the particle–particle strength. At this stage of the theory we are not able to separate the two effects which influence the lowering of the cross section, i.e., particle–particle correlations manifested by the magnitude of g_{pp}^{pn} and participation of core nucleons in the DCX process (in the "huge" basis all the nucleons are involved).

Recently measured ground–ground transitions on ^{56}Fe at pion energies T_π=35 and 61 MeV (Ref. 38) together with the earlier data at higher energies from LAMPF (Ref. 39) allow us to systematize the dependence of the DCX cross sections. The experimental observations shown in Fig. 4 exhibit a resonance-like structure near $T_\pi \approx 50$ MeV.

In contrast to this observed behavior, almost none of the theoretical models with a distorted wave or with plane-wave approximations are able to predict even roughly this strong energy dependence for the DCX forward-angle cross section. The microscopic calculations give a rather smooth energy behavior around T_π=50 MeV, except for the predictions of Martemyanov and Shepkin,[26,27] who introduced a dibaryon resonance "by hand" to explain this dependence. These authors proposed a very narrow dibayron resonance formed in the DCX, whose decay into two nucleons is not allowed by the selection rules. The condition for forming this dibaryon is a large overlap of a pair of nucleons (neutron–neutron or proton–proton) in their relative s wave with $J^\pi=0^+$ and T=1. This resonance can appear according to Refs. 26 and 27 at distances less than 1 fm between the nucleons. Taking the estimate of Miller[49] for the 6-quark bag probability to be of the order of a few percent for all nucleon pairs in a nucleus, Martemyanov and Shepkin obtained an energy dependence for the dibaryon mechanism in the double charge-exchange reaction which roughly follows the observed behavior. All such approaches together with the calculations made by Chiang and Zou[50] can be treated as an indication of the importance of quarks in the DCX process.

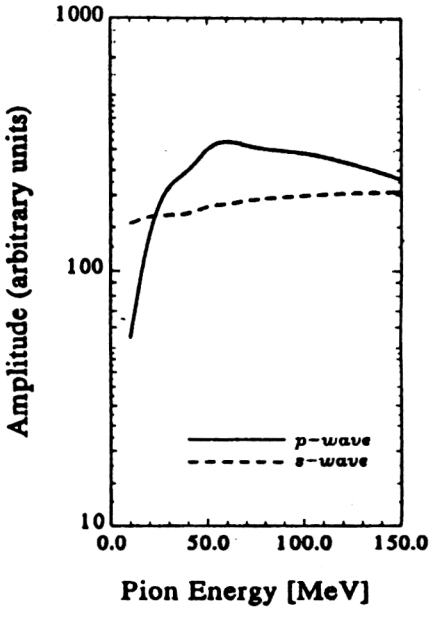

FIG. 5. Energy dependence of the analog (dashed line) and nonanalog (full line) contributions to the ground-state transition amplitude for the scattering angle θ=5°. The interaction strengths g_{ph}^{pn} and g_{pp}^{pn} are taken to be 0.8 and 1.1, respectively.

Our approach, different in spirit, can also give the gross features of the energy dependence of the DCX. The calculated forward (5°) cross section is shown in Fig. 4 as a function of the pion energy up to the resonance region.[1)] One can notice that we are able to explain qualitatively within our mechanism the observed experimental behavior. The curve is not so steep on the high-energy side above 60 MeV, but a peak around T_π=60 MeV is clearly seen. It is rather obvious that we should look for some effects to reduce the DCX amplitude in the higher-energy domain.

A more careful analysis of both the s- and p-wave contributions to the total amplitude at two pion energies, say, 10 and 50 MeV, and at forward angles might supply such a mechanism.[51] In Fig. 5 the dependence of the analog and nonanalog amplitude on the incident-pion energy is shown. The s-wave contribution is almost constant in the full domain of the pion energy. A dramatic increase of the nonanalog amplitude (p-wave contribution) is seen up to the energy T_π=60 MeV. Thus, this component produces the maximum in the cross section. Because the nonanalog route (p-wave component) depends sensitively on g_{pp}^{pn} Refs. 13, 14, and 23, the strong dependence of the amplitude suggests that the particle–particle correlations are of increasing importance as one takes more energetic pions. After the maximum is reached the two components, the s and p wave, remain equally important. One clearly needs some additional mechanism for the reduction if the experimental data are to be reproduced. Such a possibility is offered by improving the s-wave charge-exchange operator by taking its relativistic form.

It is worth noting that Karapiperis and Kobayashi[16] can roughly predict the decrease of the ^{14}C cross section between 50 and 100 MeV. Unfortunately, the cited calculations were not performed for pion energies below 50 MeV. Gibbs and co-workers also proposed a resonance phenomenon in pion scattering, which could be seen even more clearly in DCX around the proper energy T_π=50 MeV.[52] These approaches—including the one presented here—point to a possibility of explaining the observed resonance-like behavior without invoking nonstandard mechanisms. The existing data do not yet discriminate clearly between conventional and more exotic interpretations.

5. FINAL REMARKS

I have investigated the double charge-exchange reaction in the framework of the quasiparticle random-phase approximation (QRPA). The charge-exchange operator was taken in the nonrelativistic form, and the plane-wave approximation was used for the incident, intermediate, and outgoing pions. The approach was applied to the ground-state transition on iron ^{56}Fe. The predicted values underestimate the forward-angle cross section, and thus the calculated angular distribution is flatter than in the experiments.

The amplitudes and cross sections show a smooth dependence on the value of the particle–particle strength g_{pp}^{pn}, which was also observed in earlier calculations of the DCX reaction on calcium,[22] germanium,[25] and tellurium.[23] A comparison with the data allows one to state that the physically important values of the strength g_{pp}^{pn} for iron and nickel nuclei lie in the interval 1.0–1.1. One should stress that the choice of the model space influences the calculated quantities. Because of this effect the g_{pp}^{pn} strength is not unique. The larger the basis, the smaller the particle–particle strength. Moreover, in the larger model space we observed a collapse of the QRPA solution for a g_{pp}^{pn} value as small as 0.9, which may suggest a need for inclusion of higher RPA corrections in the model.

The gross features of the resonance-like shape of the cross section as a function of the pion energy can be reproduced at least semiquantitatively within the conventional $2N$ mechanism. The prediction is not too good, probably because of the approximations that were made. It remains to be seen in the future whether the very speculative idea of a dibaryon resonance in the DCX reaction will prevail. Future development of the approach will make it possible to settle more carefully the questions addressed in this talk.

Last but not least, a sensitivity of the pionic DCX processes to nuclear structure and especially to nucleon–nucleon correlations makes them interesting for double beta decay. In searches for physics beyond the standard model the last reaction has continually received much attention. Grand unified theories predict neutrinoless double beta decay if the neutrino is a Majorana particle with rest mass and/or if weak right-handed currents exist. Combining both phenomena (DCX and double beta decay), we can ensure reliable nuclear matrix elements and thus an accurately defined estimate of the nonstandard physics parameters, such as the average light-neutrino mass, the right-handed weak-current admixtures, and the heavy-neutrino mass.

The author is grateful to all who have contributed to this work. I thank Profs. Amand Faessler and Heinz Clement for fruitful discussions and suggestions. This work was sup-

ported in part by Komitet Badań Naukowych (the State Committee for Scientific Research), Grant No. 2 0347 91.01.

[1)]Generalization of the model for the Δ-isobar degrees of freedom is possible,[37] but we do not intend to discuss this point here.

[1] K. K. Seth, H. Nann, S. Iversen, M. Kaletka, and J. Hird, Phys. Rev. Lett. **41**, 1589 (1978).
[2] N. Nann, K. Seth, S. G. Iversen, M. D. Kaletka, D. B. Kaletka, D. B. Barlow, and D. Smith, Phys. Lett. **96B**, 261 (1980).
[3] K. K. Seth, S. Iversen, H. Nann, M. D. Kaletka, J. Hird, and H. A. Thiessen, Phys. Rev. Lett. **43**, 1574 (1979); **45**, 147 (1980).
[4] R. Gilman, H. T. Fortune, L. C. Bland, R. R. Kiziah, C. F. Moore, P. A. Seidel, C. L. Morris, and W. B. Cottingame, Phys. Rev. C **30**, 958 (1984).
[5] P. A. Seidel, R. R. Kiziah, M. K. Brown, C. F. Moore, C. L. Morris, H. Baer, S. J. Greene, G. R. Burleson, W. B. Cottingame, C. L. Bland, R. Gilman, and H. T. Fortune, Phys. Rev. Lett. **50**, 1106 (1983).
[6] H. W. Baer, R. L. Burman, M. J. Leitch, C. L. Morris, D. W. Wright, S. H. Rokni, and J. R. Comfort, Phys. Rev. C **37**, 902 (1988).
[7] S. Mordechai, N. Auerbach, M. Burlein, H. T. Fortune, S. J. Greene, C. F. Moore, C. L. Morris, J. M. O'Donnell, M. W. Rawoool, J. D. Silk, D. L. Watson, S. H. Yoo, and J. D. Zumbro, Phys. Rev. Lett. **61**, 531 (1988).
[8] S. Mordechai, H. T. Fortune, J. M. O'Donnell, G. Liu, M. Burlein, A. H. Wuosmaa, S. Greene, C. L. Morris, N. Auerbach, S. H. Yoo, and C. F. More, Phys. Rev. C **41**, 202 (1990).
[9] J. Sperinde, D. Fredrickson, R. Hinkins, V. Perez-Mendez, and B. Smith, Phys. Lett. **32B**, 185 (1970).
[10] J. E. Ungar, R. D. McKeown, D. F. Geesman, R. J. Holt, J. R. Specht, K. E. Stephenson, B. Zeidman, and C. L. Morris, Phys. Lett. **144B**, 333 (1984).
[11] H. Clement, Prog. Part. Nucl. Phys. **29**, 175 (1992).
[12] E. Oset. D. Strottman, and G. E. Brown, Phys. Lett. **73B**, 393 (1978).
[13] E. Bleszynski, M. Bleszynski, and R. J. Glauber, Phys. Rev. Lett. **60**, 1483 (1988).
[14] A. Auerbach, W. R. Gibbs, J. N. Ginocchio, and W. B. Kaufmann, Phys. Rev. C **38**, 1277 (1988).
[15] T. Karapiperis and M. Kobayashi, Phys. Rev. Lett. **54**, 1230 (1985).
[16] T. Karapiperis and M. Kobayashi, Ann. Phys. (N.Y.) **177**, 1 (1987).
[17] K. K. Seth, Nucl. Phys. **A478**, 591c (1988).
[18] M. B. Johnson, E. R. Siciliano, and H. Saraffin, Phys. Lett. **243B**, 18 (1990).
[19] W. R. Gibbs, W. B. Kaufmann, and P. B. Siegel, in *Proc. of the LAMPF Workshop on Pion Double Charge Exchange*, Los Alamos, edited by H. W. Baer and M. J. Leich, Los Alamos National Laboratory Report No. LA-010550-C (1985), p. 90.
[20] S. E. Siciliano, M. B. Johnson, and H. Saraffin, Ann. Phys. (N.Y.) **203**, 1 (1990).
[21] J. N. Ginocchio, Phys. Rev. C **40**, 2168 (1989).
[22] W. A. Kaminski and A. Faessler, Phys. Lett. **244B**, 155 (1990).
[23] W. A. Kaminski and A. Faessler, Nucl. Phys. **A529**, 605 (1991).
[24] J. D. Vergados, Phys. Rev. C **44**, 276 (1991).
[25] W. A. Kaminski and A. Faessler, J. Phys. G **17**, 1665 (1991).
[26] B. V. Martemyanov and M. G. Shepkin, JETP Lett. **53**, 776 (1991).
[27] R. Bilger, B. Broermann, H. Clement, R. Emmrich, K. Fohl, K. Heitlinger, C. Joram, W. Kluge, M. Moll, D. Reule, M. Schepkin, G. J. Wagner, R. Wieser, R. Abela, F. Foroughi, and D. Renker, in *Proc. of the International Nuclear Physics Conference*, Wiesbaden (1992); R. Bilger, H. Clement, K. Fohl, K. Heitlinger, C. Joram, W. Kluge, M. Schepkin, G. J. Wagner, R. Wieser, R. Abela, F. Foroughi, and D. Renker, Z. Phys. A, to be published.
[28] O. Civitarese, A. Faessler, and W. A. Kaminski, J. Phys. G. **17**, 1407 (1991).
[29] T. Erickson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, 1988).
[30] D. S. Koltun and A. Reitan, Phys. Rev. **141**, 141 (1966).
[31] D. Serot, and J. D. Walecka, Adv. Phys. **16**, 1 (1986).
[32] P. G. Blunden, and P. McCorquodale, Phys. Rev. C **38**, 1861 (1988).
[33] Hong Jung, F. Beck, and G. A. Miller, Phys. Rev. Lett. **62**, 2357 (1989).
[34] S. Marcos, M. Lopez-Quelle, and N. van Gizi, Phys. Lett. **257B**, 5 (1991).
[35] O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. **194B**, 11 (1987).
[36] K. Muto and H. V. Klapdor, Phys. Lett. **201B**, 420 (1988).
[37] W. A. Kaminski, *Double Charge-Exchange Reaction with Low-Energy Pions* [in Polish] (Maria Curie-Sklodowska University Press, Lublin, 1991).
[38] R. Bilger, B. M. Barnett, H. Clement, S. Krell, G. J. Wagner, J. Jaki, C. Joram, T. Kirchner, W. Kluge, M. Metzler, R. Wieser, and D. Renker, Phys. Lett. **269B**, 247 (1991).
[39] P. A. Siedl *et al.*, Phys. Rev. C **42**, 1929 (1990) and references cited here.
[40] A. de Shalit and H. Feshbach, *Theoretical Nuclear Physics*, Vol. 1 (Wiley, New York, 1974).
[41] C. Mahaux and H. A. Weidenmüller, Ann. Rev. Nucl. Part. Phys. **29**, 1 (1979).
[42] K. Holinde, Phys. Rep. **68**, 121 (1981).
[43] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[44] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[45] D. G. Madland and J. R. Nix, Nucl. Phys. **A476**, 1 (1988).
[46] P. Moeller and J. R. Nix, Nucl. Phys. **A536**, 20 1992).
[47] F. D. Becchetti, D. Dehnhard, and T. G. Dzubay, Nucl. Phys. **A168**, 151 (1971); T. Caldwell, O. Nathan, O. Hansen, and H. Bork, Nucl. Phys. **A202**, 225 (1973).
[48] Nucl. Data Sheets **48**, 251 (1986) and references cited here.
[49] G. A. Miller, Phys. Rev. **53**, B2008 (1984).
[50] H.-C. Chiang and B.-S. Zou, Nucl. Phys. **A496**, 739 (1989).
[51] W. A. Kaminski and D. Chocyk, to be published.
[52] W. R. Gibbs, W. B. Kauffmann, and J. P. Dedondez, Phys. Lett. **231B**, 6 (1989).
[53] H. Clement, private communication.
[54] A. H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1985).
[55] D. S. Koltun and M. K. Singham, Phys. Rev. C **39**, 704 (1989).

This article was published in English in the original journal. It is reproduced here with the stylistic changes by the Translation Editor.