Fundamental symmetry breaking in nuclear reactions

V. E. Bunakov
*Joint Institute for Nuclear Research, 141980, Dubna**

Fiz. Élem. Chastits At. Yadra **26**, 285–361 (March–April 1995)

A theoretical review is given of P- and (or) T-invariance violating effects in nuclear reactions. It is demonstrated that all of them are characterized by two major enhancement factors—dynamical and resonance ones. The net enhancement effect reaches 5–6 orders of magnitude. Both enhancements are caused by quantum chaoticity (complexity) of compound-nucleus resonances. This complexity, however, demands statistical methods of analysis of observed data in order to extract information on symmetry-breaking interaction strength constants. These methods are also presented and discussed in the review. © *1995 American Institute of Physics.*

It is frequently pointed out that the discovery of America by Columbus nicely illustrates the transformation of scientific hypothesis into discovery. Columbus cherished the idea that the Earth is round and hoped to reach East India by sailing to the West. Notice that:

(a) His idea was by no means original, but he received new information.

(b) He faced enormous difficulties both in search of subsidies and in carrying on with his experiment.

(c) He did not find a new way to India, but discovered a new continent instead.

(d) In spite of all the arguments to the contrary he continued believing that he discovered a new way to the East.

(e) He got neither special respect nor substantial reward in his lifetime.

(f) Since then it was proved without doubt that he was not the first European to reach America.

("Physicists Joking," Mir, Moscow, 1966, p. 66)

1. INTRODUCTION

The phrase "fundamental symmetry" in the title is the frequently used shorthand for P and T invariance. It just reminds us of old times, when both of those symmetries were considered to be most unbreakable. Since then P invariance has lost a good deal of its glamor. Its fundamentality was shaken by the discovery of P violation, first in β decay and then in nucleon–nucleon interactions. For some time it seemed possible that the weak interaction which caused P violation in those two cases might be different for leptonic and baryonic processes. This could have added interest to nuclear P-violation studies. However, the electroweak interaction theory of Weinberg and Salam closed this possibility. The T-invariance fortress remains much more invincible. The only experimental fact now known is CP violation in K-meson decays, discovered 30 years ago. On the basis of the CPT theorem this implies T violation, which can be explained theoretically in an infinite number of ways. All this makes further search for T violation a much more exciting problem of general importance in physics. In other words, T invariance is much more fundamental nowadays than P invariance.

The present wave of interest in P violation in nuclear reactions was boosted by the theoretical predictions[1,2] of a possible huge (6 orders of magnitude) enhancement of these effects in the vicinity of compound-nucleus resonances, which were almost immediately confirmed experimentally.[3] A year later the same huge enhancements were predicted[4] for the effects violating both P *and* T invariance. Similar enhancements of P-conserving T-violating effects were predicted a few years later.[5,6] It seemed to us at that time (and still seems to me now) that those huge enhancements should be primarily used in the experimental search for T-violating effects, since even the establishment of new upper bounds on CP-interaction constants might help a lot in selecting the most promising models of CP violation.

However, this natural line of reasoning does not seem to be well understood and shared. The major part of experimental and theoretical efforts up to now has been concentrated on P-violation effects. Even there a good deal of energy has been wasted on sensation-hunting and theoretical rediscovery of facts known for years.

One of the reasons for this is, according to my experience, a poor understanding of basic and quite general physics, which governs the above enhancements, even by those who work in the field. Many of them naively believe that one can professionally discuss and analyze symmetry breaking in nuclear reactions without any knowledge of nuclear-reaction theory or, at best, with a rudimentary knowledge of the Breit–Wigner formula. This strange belief is partially explained by bad traditions in nuclear physics, where nuclear-structure studies were always considered to be of major im-

 1063-7796/95/020115-33$10.00

portance in spite of the fact that most information for those studies was obtained from nuclear reactions. At a deeper level it comes from the fact that the quantum mechanics of bound states is in all respects much simpler than for the continuum. Therefore it is quite tempting (and often quite misleading) to oversimplify the description of the process by using well-known bound-state analogies.

This unprofessional approach and naive clinging to the bound-state analogies create a fragmentary and wrong impression about the enhancement mechanisms for different observable quantities. For instance, the very popular interpretation of the neutron transmission enhancement in terms of the "structural" enhancement factor $(kR)^{-1}$ does not allow one to understand the significance of P-odd correlation asymmetry measurements in inelastic channels, where the effects might be even larger[7,8] but could not be understood in terms of bound-state parallels. Exactly the same applies to T-violation tests of detailed balance in the isolated-resonance regime, where the net enhancements might be orders of magnitude larger[6,9] than in transmission experiments and are practically restricted only by the experimental energy resolution. All this leads to prejudiced distortion of the "priority scale" for different observables and often makes experimentalists choose rather difficult experiments, which in reality promise no enhancements whatsoever.

Only when the results start to deviate from naive expectations do people start reading papers on nuclear-reaction theory. And the lack of professionalism again shows itself—people start mixing one mechanism with the other, reinvent models which were discarded long ago by professionals, or invent "home-brew" models whose validity was never checked in the description of nonexotic reaction processes. This in turn often leads to aggressively incompetent statements of the type: "Leave me alone with all your fancy reaction models; I have already wasted several days reading some of them and am sure that you are overcomplicating quite simple things."

In view of all this, my present review is primarily addressed to those unprejudiced readers who realize that nuclear-reaction theory is a special branch of nuclear physics developed by generations of professionals. It has only a few features in common with the bound-state spectroscopic theories, and I shall try to emphasize them.

Therefore I shall start with a brief reminder of P-violation theory in cases of bound states (Sec. 2) with special emphasis on its specific enhancements.

Then I shall switch to my main topic of nuclear reactions for isolated resonances, when the average resonance spacing d is much larger than the average resonance width Γ (Sec. 3).

I shall start this section with a short reminder of some results of nuclear-reaction theory for isolated resonances (Sec. 3.1) which will be essential for all the further analysis, namely, the structure of the wave functions for a system of an incident (outgoing) particle and a target (residual) nucleus. In doing this I shall use the best and most physical version of nuclear-reaction theory, namely, the shell model with a continuum, developed by Mahaux and Weidenmüller[10] as the natural realization of Feshbach's unified theory of nuclear reactions. The advantage of this approach over the more popular R-matrix one lies in its much more physical treatment of continuum wave functions, which allows one to describe both direct and compound processes in a unified way.

In a fairly lengthy subsection (Sec. 3.2) we discuss all aspects of P violation in nuclear reactions. Section 3.2.1 contains the analysis of possible P-violating observables. In Sec. 3.2.2 a short historical background is given with special emphasis on how erroneous the bound-state parallels might be. The rather lengthy Sec. 3.2.3 contains an analysis of all possible mechanisms of P violation in nuclear reactions (all 32 terms contributing to the P-violating scattering amplitude). Note that the absolute magnitudes of the corresponding effects are defined by the nucleon–nucleon weak-interaction constants, on which up to now we have only educated guesses and whose extraction from experimental observables should be the ultimate aim of our P-violating investigations. Therefore only estimates of the *relative* contributions from different mechanisms and of their energy behavior are meaningful in the analysis of different competing mechanisms. For this reason I expand in Sec. 3.2.3 on this kind of analysis and emphasize the generality of various enhancement effects specific for each mechanism. It turns out that only two major enhancement factors govern the P-violating amplitudes—the dynamical enhancement $\bar{v}/d \sim \sqrt{N}$ ($\bar{v}$ is the variance of the strong-interaction matrix element between compound states, and $N \sim 10^6$ is the number of basic components which define the complexity of the compound-state wave function) and the resonance enhancement d/Γ. While the former enhancement is well known in the bound-state P-violating theory, the resonance enhancement is a specific feature of continuum spectra which has no bound-state analogs. After analyzing the energy behavior of the P-violating amplitudes we are forced to return to the observables (Sec. 3.2.4) in order to investigate their rather complicated energy dependence caused by the combined influence of the P-violating amplitudes in their numerators and the P-invariant ones in their denominators. This allows us to compare the specific enhancements of all the P-violating observables in different energy regions and to understand in quite general terms the "priority hierarchy" of observables, which is confirmed by experiments. We also show that the "structural" enhancement factor $(kR)^{-1}$ is an artifact of presenting the auxiliary quantities instead of the actually observed ones.

Section 3.3 deals with T violation. In Sec. 3.3.1 we discuss some specific hidden dangers, which make true T-invariance investigations much more subtle than the P-invariance ones, and present the list of "true" T-violating observables. The theory of T violation in nuclear reactions dates back to the late 1950s and is rather dramatic. However, only a few people know about it. Therefore I present a short survey of its development in Sec. 3.3.2. Section 3.3.3 deals with the most important theoretically P-odd T-odd "triple correlation" (TC). An analysis of possible TC enhancements is given there together with an analysis of specific difficulties in its experimental observation. In Sec. 3.3.4 we analyze the P-even T-odd correlation (FC) in neutron transmission, demonstrating both its advantages and drawbacks. In Sec. 3.3.5 we analyze the possibilities of T-violation detailed-balance

tests (TVDB) for two close-lying resonances, when the average spacing d is much larger than the average width Γ. In Sec. 3.3.6 briefly summarize our results on T-violation effects, showing that all of them are governed by the same dynamical and resonance enhancement effects as the P-violating ones. In complete analogy with P violation we present the "priority hierarchy" of T-violating observables and conclude that the most promising results in the near future might be expected from the TVDB tests of Sec. 3.3.5.

In Sec. 4, I discuss the statistical approach to compound-resonance measurements. In Sec. 4.1, I demonstrate that both dynamical and resonance enhancements are quite general consequences of quantum chaos characteristic of the compound nucleus, which was faced and physically understood in "strong" symmetry breaking from the dawn of nuclear physics. Therefore the meaningful analysis of weak symmetry-breaking (WSB) matrix elements extracted from experiments should be done with exactly the same mathematical methods which were successfully applied in studies of "strong" symmetry breaking, namely, with the use of the random-matrix theory and Gaussian ensembles of Wigner and Dyson. Since historically such methods were first applied to the calculation of energy-averaged WSB quantities, I discuss in Sec. 4.2 the practical disadvantages of "unbiased" energy-averaging and come to the idea of "biased" on-resonance ensemble averaging, which is fully described in Sec. 4.3. In Sec. 4.4, I discuss how one should apply the on-resonance theory of Sec. 4.3 to the realistic case of necessarily imperfect experimental measurements (small number of independent on-resonance observations with poor experimental accuracy), concluding that the only appropriate way in this case is given by Bayesian statistics (BS) based on the use of standard conditional probability theory. I also discuss the parallels and differences between the BS results and the empirical maximum-likelihood method (MLM), which has been applied in the experimental analysis of P-violating effects during the past five years. The most obvious disadvantages of the MLM are shown, especially in the typical case when the spins of observed resonances are unknown. In view of this, I only briefly analyze in Sec. 4.5 the sensational "sign-correlation" effect observed in P-violating experiments on a ^{232}Th target, and conclude that its statistical significance was greatly exaggerated and is highly questionable.

In Sec. 5, I present a short summary of the most important and general conclusions and recommendations for the future.

2. *P* VIOLATION IN THE CASE OF NUCLEAR BOUND STATES

We shall start with a brief reminder of the "classical" P-violation experiments in low-energy physics, when only the mixing of bound states was considered. Quite early (see, e.g., Ref. 11), it was realized that experimental observation of interference-type phenomena (i.e., observation of P-odd correlations in the amplitudes of different processes) has an advantage over "brute-force" violation of probabilities (e.g., P-forbidden α decay) because the latter are quadratic in the weak-interaction strength constant F. There are several possible P-odd correlations (see the list in Ref. 11), including the correlation $(\vec{\sigma}_\gamma \cdot \vec{p}_\gamma) = h_\gamma$ between the spin and momentum of the γ quanta emitted by the excited unpolarized nuclei. The value of h is called the helicity and leads to circular polarization of the emitted γ quanta, which can be observed experimentally (see, e.g., Ref. 12). Let us consider this experiment in more detail in order to demonstrate the various enhancement mechanisms which might manifest themselves in it. The wave function Ψ_i of the decaying excited state might be represented as a sum

$$\Psi_i = \psi_1 + \alpha\psi_2 . \tag{1}$$

Here ψ_1 and ψ_2 are the states of opposite parity, while the coefficient α describes the admixture of the state ψ_2 caused by the P-violating weak interaction V_W. Standard first-order perturbation theory gives

$$\alpha = \frac{\langle \psi_1 | V_W | \psi_2 \rangle}{E_1 - E_2} . \tag{2}$$

Circular polarization appears as an interference of the electric E_λ and magnetic M_λ transitions of the same multipolarity λ. Therefore

$$h = 2\alpha \langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle \langle \psi_f | \hat{O}'_\lambda | \psi_2 \rangle , \tag{3}$$

where $\hat{O}_\lambda$ is the "regular" γ-transition operator, which connects the main component ψ_1 of Ψ_i with the final state Ψ_f; $\hat{O}'$ is the "irregular" transition operator, which, owing to the P selection rules, connects only the component ψ_2 with Ψ_f. Then the degree of circular polarization observed will be determined by

$$\delta = \frac{h}{|\langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle|^2 + \alpha^2 |\langle \psi_f | \hat{O}'_\lambda | \psi_2 \rangle|^2} \simeq \alpha \frac{\langle \psi_f | \hat{O}'_\lambda | \psi_2 \rangle}{\langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle} . \tag{4}$$

It is important to note that δ, like *any other observable* which measures the degree of any symmetry breaking, is always normalized by the *total transition probability*. Therefore in general the denominator of (4) contains a sum over all the allowed transitions, $\Sigma_\lambda |\langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle|^2$:

$$\delta = \frac{h}{\Sigma_\lambda |\langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle|^2 + \alpha^2 |\langle \psi_f | \hat{O}'_\lambda | \psi_2 \rangle|^2} . \tag{4a}$$

Only this kind of normalization for observables has real physical meaning—the maximal value of δ is unity, meaning 100% parity violation. This almost trivial rule is taken for granted in all fields of physics, from optics to elementary particles, and all the meaningful enhancements appear only within this normalization. Of course, some odd personalities might introduce a normalization of their own by, say, retaining only the weakest term in the normalization sum. If this term is really small, this would immediately enhance the newly introduced quantity. But such a fictitious "enhancement" would have nothing to do with the physics of the process. I have to mention this triviality only because, as we shall see below, even this standard rule is unprofessionally violated all the time in the majority of experimental (and, alas, even theoretical) publications on P noninvariance in nuclear reactions.

Returning to Eq. (4), we can analyze its structure in order to see the role of different enhancement mechanisms. First of all, we observe that α increases with decreasing level spacing $D=|E_1-E_2|$, as a natural result of perturbation theory. Therefore naively one should expect the average effect to increase linearly with increasing state density $\rho=1/d$ of the system (d here is the average level spacing). However, the increase of ρ is closely connected with increasing complexity of the mixing states ψ_1 and ψ_2. In terms of the basic (so-called "simple configurations") components which make up the compound-state wave function ψ, this means increasing the number N of components and simultaneously the random signs for their admixture coefficients in ψ. Therefore the average value of the matrix element $\langle\psi_2|V_W|\psi_1\rangle\equiv v_P$ will be zero in a proper statistical treatment (see Sec. 4 below), and we can speak only in terms of its variance

$$\tilde{v}_p=\sqrt{(\tilde{v}_p^2)}. \tag{5}$$

Using the simple scaling procedure, we can express v_P in terms of the strong-interaction matrix element v:

$$v_p=F\cdot v, \tag{6}$$

where F is the characteristic ratio of the strengths for the weak and strong interaction, which is given by the phenomenological models as $F\approx 10^{-7}$–10^{-8}.

The average value of $\tilde{v}$ can be estimated from the usual expression for the spreading width Γ_{spr} of the single-particle resonance (which is roughly equal to the imaginary part W of optical-model potential and in the limit of a black nucleus approaches the single-particle level spacing d_0):

$$\Gamma_{\text{spr}}=\frac{2\pi\tilde{v}^2}{d}\approx W\sim d_0. \tag{7}$$

This gives

$$\tilde{v}_p=F\sqrt{\frac{\Gamma_{\text{spr}}d}{2\pi}}\approx F_p\sqrt{d_0 d}. \tag{8}$$

Therefore the variance of α in (2) can be estimated as

$$\tilde{\alpha}=\frac{\tilde{v}_p}{D}=F_p\frac{\tilde{v}}{D}=F_p\sqrt{\frac{\Gamma_{\text{spr}}}{2\pi D}}\approx F_p\sqrt{\frac{d_0}{d}}=F_p\sqrt{N}. \tag{9}$$

Thus, we see that the many-body aspect of the compound nuclear system manifests itself in the systematic enhancement of the P-violating effect by roughly a factor of $\sqrt{N}$, where N is the number of basic components forming the compound states. The number N increases with increasing excitation energy E^* and nuclear mass number, and reaches $\sim 10^6$ for $E^*\approx B_n$ (B_n is the neutron binding energy) in medium and heavy nuclei. This sort of enhancement was considered several times[13–16] and has become known as the *dynamical enhancement*.[15]

One can also see from (4) that increasing the value of the "irregular" amplitude and decreasing the value of the "regular" one leads to an additional enhancement of the effect. This usually happens when for purely structural reasons the "regular" component is strongly forbidden, while the "irregular" one is favored. Therefore this kind of enhancement was called the *structural enhancement*.[15]

These enhancements made possible one of the earliest P-violating observations in the γ channel[17] at the level of $6\cdot 10^{-6}$. However, even earlier[18] similar experiments were performed in the (n,γ) reaction with thermal neutrons, showing the effects up to $\sim 10^{-4}$. As we shall see below, these more impressive results can be easily understood in the framework of nuclear-reaction theory.

As we see, the general trick of any enhancement mechanism is to make the numerator of the observable (4) as large as possible and the denominator small. The same trick will be used in the case of nuclear reactions. Since, however, both the numerators and denominators of the observables in that case are rapidly varying functions of energy, this allows a much larger variety of situations and leads to quite specific enhancements, which we consider in the next sections.

3. NUCLEAR REACTIONS (ISOLATED RESONANCES)

3.1. Elements of nuclear-reaction theory for isolated resonances

We shall introduce the basic results of nuclear-reaction theory which will be extensively used in all our further applications. In doing this we shall follow the approach of Mahaux and Weidenmüller,[10] which is a projection of Feshbach's unified theory of nuclear reactions on the realistic shell-model basis.

The most essential (for our purposes) result of this approach is that in the region of isolated resonances, where $\Gamma\ll d$, the wave function of the system of the incident (outgoing) particle plus the target (residual) nucleus is given by

$$\Psi_{i,f}^{(\pm)}(E)=\sum_k a_{k(i,f)}^{(\pm)}(E)\varphi_k+\sum_c\int b_{c(i,f)}^{(\pm)}(E,E')\chi_c^{(\pm)}\times(E')dE'. \tag{10}$$

Here φ_k is the wave function of the so-called "bound state embedded in the continuum" (BSEC) or, roughly speaking, the wave function of the kth compound state, where all the nucleons of the system occupy only the bound states in the average nuclear potential, but are not allowed to collect all the excitation energy via pairwise collisions on a single particle. The quantity $\chi_c(E)$ is the continuum cth channel wave function which describes the (infinite) motion of a particle in the average field of the target (residual) nucleus. In the particular case of a neutron incident on the ground-state target (elastic channel $c=i$), χ_i is the antisymmetrized product of the target-nucleus wave function in its ground state and the wave function of a neutron moving in the average field of the target. The φ_k and χ_c correspond to Feshbach's projections on closed (Q) and open (P) channels, respectively. Note that unless we switch on the pairwise residual interactions V between the channels (PHQ=QHP in Feshbach's notation) we do not allow the incident neutron to share its energy with the target nucleons. This does not allow this neutron to form a compound resonance. Therefore in the absence of a pairwise interaction V the BSECs would never decay, while the χ's would describe only potential scattering in the mean field. This unphysical situation changes as soon

as we switch on the residual interaction V. Then each resonance k acquires partial decay widths $\Gamma_k^{i,f}$ with amplitudes

$$(\Gamma_k^{i,f})^{1/2}=\gamma_k^{i,f}=(2\pi)^{1/2}\langle\chi_{i,f}(E)|V|\varphi_k\rangle. \tag{11}$$

In other words, the pairwise residual interaction allows the nucleons of the BSEC to collect their total excitation energy on one of the particles and to emit it into the open channel i or f.

For the expansion coefficients a and b in this case the theory gives rather transparent expressions:

$$a_{k(i,f)}^{(\pm)}(E)=\frac{\exp(\pm i\delta_{i,f})}{\sqrt{2\pi}}\frac{\gamma_k^{i,f}}{E-E_k+i\Gamma_k/2}. \tag{12}$$

Here E_k is the energy of a given compound resonance, and $\Gamma_k=\Sigma\Gamma_k^c$ is its total width.

The open-channel wave functions [second part of (10)] are governed by the coefficients

$$b_c^{(\pm)}(E,E')=\delta_{c(i,f)}\delta(E-E')\exp(\pm i\delta_c)+\frac{1}{E^{\pm}-E'}\sum_k a_k^{\pm}(E)\langle\chi_c(E')|V|\varphi_k\rangle. \tag{13}$$

Here $E^{\pm}=E\pm i\varepsilon$ is the usual notation for pole shifts in the complex energy plane.

Let us now simplify the picture, neglecting all the BSECs in the sums of (10) and (13) except the one whose energy E_k is closest to the energy E of the incident neutron (this can always be done when Γ_k is much smaller than the distance between resonances of the same spin and parity). Let us also consider the case when only the neutron elastic-scattering channel is open ($c=i=f$). In this case the continuum term in (10) can be written as

$$\begin{aligned}e^{i\delta_i}\chi_i(E)+a_{k(i)}^{+}(E)\int\frac{dE'}{E+i\epsilon-E'}\langle\chi_i(E')|V|\varphi_k\rangle\\ =e^{i\delta_i}\chi_i(E)+a_{k(i)}^{+}(E)\Big[i\pi\langle\chi_i(E)|V|\varphi_k\rangle\chi_i(E)\\ +\mathscr{P}\int\frac{dE'\chi_i(E)}{E-E'}\langle\chi_i(E')|V|\varphi_k\rangle\Big],\end{aligned} \tag{14}$$

where $\mathscr{P}$ stands for the principal value of the integral.

The first term in (14) describes potential elastic scattering of a neutron. The resonance behavior of $a_k(E)$ [see (12)] shows that the two terms in the square brackets of (14) are the "imprints" of the compound resonance at $E=E_k$ on the elastic continuum. Using Eq. (11), one can express the first of these terms as

$$i\sqrt{\frac{\pi}{2}}\gamma_k^i\chi_i(E). \tag{15}$$

Recalling now that $\chi_i(E)$ describes the single neutron (*valence* particle) in the mean field of a ground-state target, we see that (15) is exactly what was called the single-particle (or *valence*) component u of the compound-resonance wave function in the simplified R-matrix theory. The wave function $\chi_i(E)$ belongs to the continuum, as it should. However, if it has a potential resonance at $E=E_0$ with (single-particle) width Γ_0, one can use the approximate expression[19] valid inside the nuclear potential radius R (for simplicity we suppress the coordinates of the target nucleons):

$$\chi(E)\simeq\sqrt{\frac{\Gamma_0}{2\pi}}\frac{u_0(r)}{E-E_0+i\Gamma_0/2}.$$

Here $u_0(r)$ is the solution of the Schrödinger equation describing particle motion in an average field, which is normalized to unity inside the nuclear volume $r\leqslant R$. Substituting this expression into (15), we get, in the Γ_0 vicinity of $E\approx E_0$,

$$i\sqrt{\frac{\pi}{2}}\gamma_k^i\chi_i(E)\simeq\left(\frac{{}^i\Gamma_k^i}{\Gamma_0}\right)^{1/2}u_0(r), \tag{16}$$

thus restoring the approximate result of the R-matrix approach. The quantity

$$S_k^n=\frac{\Gamma_k^n}{\Gamma_0}\approx\frac{1}{N}$$

is usually called the spectroscopic factor. It determines the probability of finding a single-particle (valence) component in the compound-resonance state, and in the black-nucleus approximation is equal to $1/N\sim(d/\bar{v})^2$, i.e., to the inverse square of the dynamical enhancement factor of (9).

Adding this term (16) to the BSEC wave function φ just gives us the R-matrix compound-resonance wave function:

$$\Phi_k=\varphi_k+\sqrt{S_k^i}u_0. \tag{17}$$

Therefore the wave function (10) of the system in the vicinity of an isolated compound resonance E_k can be expressed as

$$\begin{aligned}\Psi_i^{(\pm)}(E)&=a_{k,i}^{\pm}(E)[\varphi_k+\sqrt{S_k^i}u_0]+e^{i\delta_i}\chi_i(E)\\ &\quad+a_k^{\pm}(E)\mathscr{P}\int\frac{dE'\chi_i(E')}{E-E'}\langle\chi_i(E')|V|\varphi_k\rangle\\ &=a_k^{\pm}(E)\Phi_k+e^{i\delta_i}\chi_i(E)\\ &\quad+a_{k,i}^{\pm}(E)\mathscr{P}\int\frac{dE'\chi_i(E')}{E-E'}\langle\chi_i(E')|V|\varphi_k\rangle.\end{aligned} \tag{18}$$

3.2. *P* violation

Let us consider now the case of P violation in neutron-induced reactions, which would demonstrate all the specific features of any symmetry breaking in them.

We shall first discuss the P-violating quantities which might be observed experimentally in these reactions.

3.2.1. *P*-violating observables in neutron-induced reactions

Using a polarized neutron beam, one can observe the P-violating correlation $(\vec{\sigma}_n\cdot\vec{k}_n)$ between the spin $\vec{\sigma}_n$ and the momentum $\vec{k}_n$ of a neutron. There are several possible ways to do it. One can consider the transmission of neutrons with opposite helicities through a target sample and measure the difference of the corresponding total cross sections:

$$\Delta^P_{\text{tot}}=\sigma^+_{\text{tot}}-\sigma^-_{\text{tot}}=\frac{4\pi}{k}\,\text{Im}(f_+-f_-). \tag{19}$$

Here $f_\pm$ determines the forward scattering amplitudes for neutrons with opposite helicities. To obtain the second equality we have used the optical theorem.

The corresponding dimensionless measure of this transmission asymmetry effect is

$$P=\frac{\Delta^P_{\text{tot}}}{\sigma^+_{\text{tot}}+\sigma^-_{\text{tot}}}\simeq\frac{\Delta^P_{\text{tot}}}{2\sigma_{\text{tot}}}. \tag{20}$$

It should be pointed out (see, e.g., Ref. 20) that in reality the experimentalists measure the numbers $N_\pm$ of neutrons with opposite helicities transmitted through the target sample with thickness x and calculate the ratio

$$P_{\text{exp}}=\frac{N_+-N_-}{N_++N_-}. \tag{21}$$

Now for counter efficiency $\epsilon=1$ we have

$$N(x)=N_0e^{-x\sigma_{\text{tot}}\rho}, \tag{22}$$

where N_0 is the intensity of the incident beam and ρ is the density of nuclei in the target sample. Expressing $\sigma^{(\pm)}_{\text{tot}}$ as

$$\sigma^\pm_{\text{tot}}=\sigma^0_{\text{tot}}\pm\frac{\Delta^P_{\text{tot}}}{2}, \tag{23}$$

one can write

$$\begin{aligned}N_+-N_-&=N_0e^{-x\sigma^0_{\text{tot}}\rho}(e^{+x\Lambda^P_{\text{tot}}\rho/2}-e^{-x\Delta^P_{\text{tot}}\rho/2})\\&\simeq N_0e^{-x\sigma^0_{\text{tot}}\rho}\cdot\Delta^P_{\text{tot}}\cdot x\cdot\rho.\end{aligned} \tag{24}$$

Therefore

$$P_{\text{exp}}\simeq\frac{\Delta^P_{\text{tot}}}{2}x\rho. \tag{25}$$

It seems from (25) that since the experimentally observed effect increases with x, one should use very thick targets. However [see (22)], the counting rates $N(x)$ go down exponentially with increasing x. The relative counting statistical error is equal to $1/\sqrt{N}$ and increases exponentially with x:

$$\frac{1}{\sqrt{N}}=\frac{1}{\sqrt{N_0}}\,e^{\sigma_{\text{tot}}\rho x/2}. \tag{26}$$

In order to maximize (25), retaining the minimal possible error (26), one must choose $x\rho\approx(1/\sigma_{\text{tot}})$. Thus, the *actually measured* quantity (25) coincides with the expression (25):

$$P_{\text{exp}}=\frac{\Delta^P_{\text{tot}}}{2\sigma_{\text{tot}}}\equiv P. \tag{27}$$

One can easily see[21] that the same quantity could be obtained with an unpolarized neutron beam. Then P is just a measure of the longitudinal polarization of the initially unpolarized beam arising after passing a distance in the sample equal to the mean free path (hence the symbol P denoting this quantity). Sometimes one measures the difference in the radiative capture cross sections $\sigma^{(\pm)}_{n\gamma}$ and gives the quantity

$$A=\frac{\sigma^+_{n\gamma}-\sigma^-_{n\gamma}}{\sigma^+_{n\gamma}+\sigma^-_{n\gamma}}. \tag{28}$$

The same $(\vec{\sigma}_n\cdot\vec{k}_n)$ correlation in the elastic scattering amplitude also causes the rotation of the neutron polarization around $\vec{k}_n$. The angle of this rotation per unit length of the target sample is determined[22] as follows:

$$\frac{d\Phi}{dz}=\frac{2\pi\rho}{k}\,\text{Re}(f_+-f_-), \tag{29}$$

where ρ is the density of nuclei in the sample. For the same reasons of better statistics the experimentally determined angle Φ is measured for neutrons which traveled a distance z equal to the mean free path $1/\rho\sigma_{\text{tot}}$ in the target:

$$\Phi=\frac{1}{\rho\sigma_{\text{tot}}}\frac{d\Phi}{dz}=\frac{\text{Re}(f_+-f_-)}{\text{Im}(f_++f_-)}. \tag{30}$$

One can also look for the inelastic reaction (n,f) and measure the P-odd correlation $(\vec{\sigma}_n\cdot\vec{k}_f)$ between the initial neutron polarization and the momentum $\vec{k}_f$ of the outgoing particle in channel f. This is done by measuring the asymmetry of the final products with respect to $\vec{\sigma}_n$:

$$\Delta_{nf}=\frac{d\sigma_{nf}}{d\Omega}\uparrow\uparrow-\frac{d\sigma_{nf}}{d\Omega}\uparrow\downarrow. \tag{31}$$

The corresponding dimensionless degree of asymmetry is

$$\alpha_{nf}=\frac{\Delta_{nf}}{\frac{d\sigma_{nf}}{d\Omega}\uparrow\uparrow+\frac{d\sigma_{nf}}{d\Omega}\uparrow\downarrow}. \tag{32}$$

3.2.2. Historical background

The possibility of using low-energy neutron–nucleus interactions and all sorts of neutron coherent scattering processes (neutron optics) in studies of P violation was considered long ago.[21–24] But these theoretical investigations were concerned only with potential-scattering models completely disregarding the presence of compound resonances. Some of these approaches[23,24] made a point of possible enhancement of the effects in the vicinity of a potential (single-particle) p-wave resonance. The first theoretical paper[25] mentioning the possible enhancement of γ-ray circular polarization in the vicinity of a compound resonance remained unnoticed. The first simplified approach to compound-resonance analysis which really encouraged the experimental investigations appeared only in 1980 (Ref. 1; see also Ref. 26). In this approach the p-wave compound resonance was treated in complete analogy with the bound-state case above (see Sec. 2). Indeed, Sushkov and Flambaum took the case of two closely-lying bound states (imitating p and s resonances) with corresponding wave functions ψ_1 and ψ_2. Then in complete analogy with (1) the p-resonance wave function which takes into account the possible parity admixture has the form

$$\Psi(E_p)=\Psi_p+\alpha\Psi_s. \tag{33}$$

Now one might just say that in the case of elastic scattering both states decay by neutron emission and replace the γ-ray transition probabilities in (3) and (4) by the corresponding partial neutron widths Γ_s^n and Γ_p^n. Then one immediately obtains for, say, the P value, in analogy with (4),

$$P \approx \alpha \frac{\sqrt{\Gamma_s^n}}{\sqrt{\Gamma_p^n}}. \tag{34}$$

In the slow-neutron case $\Gamma_p^n \approx (kR)^2 \Gamma_s^n$, where $(kR)^2$ comes from the centrifugal-barrier penetration factor. Thus,

$$P \approx \frac{\alpha}{kR}. \tag{35}$$

The typical value of (kR) for the eV energy region in medium and heavy nuclei is $\sim 10^{-3}$. Thus, in addition to the dynamical enhancement contained in α, they got a particular case of a "structural-enhancement" factor $\sim 10^3$.

This line of argument *sometimes* gives, as we shall see later, the correct order of magnitude of the effect, but it is quite misleading. To start with, the initial equation (33) for the continuum wave function is meaningless, since each continuum wave function with fixed momentum $\vec{k}_n$ is *always* a linear superposition of states with opposite parities (i.e., a superposition of partial waves). Therefore the compound-nucleus wave function $\Psi(E)$ even in the simplest case of slow-neutron elastic scattering *without* any P-violating forces is a sum of p and s compound resonance wave functions Ψ_p and Ψ_s with corresponding "mixture" coefficients [see Eq. (10) or any sound reaction theory]:

$$\frac{e^{i\delta_p}(\Gamma_p^n)^{1/2}}{E-E_p+i\Gamma_p/2} \quad \text{and} \quad \frac{e^{i\delta_s}(\Gamma_s^n)^{1/2}}{E-E_s+i\Gamma_s/2}.$$

Thus, even at the maximum of the p resonance ($E=E_p$) we have

$$\Psi(E_p) \sim \Psi_p + \alpha' \Psi_s,$$

where

$$\alpha' = i \frac{\Gamma_p}{2D} \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right)^{1/2}.$$

Proceeding now with the bound-state arguments which lead us from (33) to (34), we obtain

$$|P| = |\alpha'| \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right)^{1/2} = \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right) \frac{\Gamma_p}{2D} \approx \frac{1}{(kR)^2} \frac{\Gamma_p}{2D} \sim 10^6 \frac{\Gamma}{D}.$$

For the famous La case this would give us $P \approx 10^3$ *without any weak interaction*!

There are also other striking absurdities in (34) and (35): a) Consider their energy behavior. Since all the E dependence enters (34) and (35) essentially through the energy dependence of the partial widths, we see that the effect (35) tends to infinity for very small E (small k). b) We know that the neutron partial widths vary in a rather wide range, obeying the Porter–Thomas law. Equation (34) clearly indicates that the largest P effects would be observed for the smallest Γ_p^n possible—the less observable a p resonance is in the total cross section, the more pronounced it would be in P violation. It is even more tempting to repeat the whole argument for mixing of resonances in higher partial waves (say, $l=3$ ones) with an s-wave resonance. (This is perfectly legitimate if one considers a target with spin $I \geq 2$). Then the "regular" neutron partial widths would be even smaller $[\Gamma_l^n \approx (kR)^{2l} \Gamma_s^n]$, and for $l=3$ we obtain a "structural enhancement" factor $1/(kR)^3 \sim 10^9$ in Eq. (39). This allows the quantity P, which by the definition (20) cannot exceed unity, to reach the value 10^5. Obviously, Sushkov and Flambaum were too good as theorists to be caught in such traps, but I have seen an experimental proposal with clearly stated intentions to hunt for the weakest p resonances in order to obtain maximal P effects. I also know an experimental group which made special efforts to perform transmission experiments with thermal neutrons and was quite disappointed when the effect turned out to be about 10^{-6} instead of the huge increase predicted by Eq. (35). To confuse things even more, the "structural enhancement" of Sushkov and Flambaum is nowadays called in many experimental papers the "kinematic enhancement" (originally this name was given by Shapiro[15] to the typical ratio of electric to magnetic transition amplitudes which might really cause an additional enhancement of the P-violating observables in γ transitions).

In spite of all these inconsistencies, these theoretical results, as I have already pointed out, greatly encouraged the preparation of on-resonance experimental measurements by the Dubna group.[3]

The first proper theoretical treatment of the problem in the framework of nuclear-reaction theory was given by us at the beginning of 1981.[2] We had derived the expressions for Δ_{tot}^P and $\text{Re}(f_- - f_+)$ essential for the description of P violation in neutron transmission. Since the Dubna on-resonance measurements were still in preparation at that time, we had to check our theory[2] by comparing the theoretical relations between the P and Φ values at thermal energies with the existing experimental measurements in Sn performed by the Grenoble group.[27] A few months later, the Dubna group performed the first on-resonance observations in Sn (Ref. 3) and checked our expression for $P(E)$ by comparing their on-resonance results with thermal-energy ones obtained by Lobashov's group in Gatchina.[28] This was the first experimental confirmation of the resonance-enhancement mechanism. Ironically enough, although our expressions for $\Delta_{\text{tot}}^P(E)$ derived in Ref. 2 clearly manifested the resonance enhancement parameter D/Γ, we fully understood its physical meaning and generality only a year later, while finishing a big paper on the general theory of P- and T-violating effects.[7] Some of our expressions obtained in that paper were rederived later in the framework of R-matrix theory.[29,30] The theoretical investigation[31] of the α_{nf}-type correlation in the particular case of $(\vec{p},\alpha)$ reactions is different. The authors obtained fairly large estimates, but did not realize that they encountered a new enhancement mechanism for the α_{nf} correlation. This fact, together with the fundamental possibility of observing P-violating effects of the order of unity, was pointed out in Ref. 8.

3.2.3. P-mixing mechanisms and specific enhancements in neutron-induced reactions

Any proper treatment of P violation in nuclear reactions consists of two steps: 1) expressing the observed quantity in terms of the P-violating part of the T (or S) matrix T_W; 2) calculating T_W in the first-order Born approximation with respect to the weak interaction V_W.

The first part of the task involves the standard theory of reaction kinematics with polarized beams (see, e.g., Refs. 21, 22, and 32). As usual in any angular correlations, this gives rather awkward combinations of vector-coupling coefficients, which are all of the order of unity and do not contribute to the understanding of the essential physics. The general expressions for them can be found in Refs. 7, 29, and 30 (the last two references even give numerical values for some target spins). We shall henceforth omit them in the majority of our expressions. The effects also depend linearly on the incoming-beam polarization p, which will be set equal to unity in all the further expressions. In the case of slow neutrons one can also restrict the neutron angular momenta to the values $l_n=0$ and $l_n=1$.

With these remarks one obtains the following expressions:

$$\Delta_{\text{tot}}^{p}=\frac{\pi}{k^2}\,\text{Im}[\langle p,j=1/2|T_W|s\rangle+\langle s|T_W|p,j=1/2\rangle], \tag{36}$$

$$\frac{d\Phi}{dz}=\frac{2\pi\rho}{k^2}\,\text{Re}[\langle p,j=1/2|T_W|s\rangle+\langle s|T_W|p,j=1/2\rangle], \tag{37}$$

$$\Delta_{nf}=\frac{2\pi}{k^2}\sum_{l_f,l_n}\text{Im}\{\langle l_f,f|T|l_n\rangle\langle(l_f+1),f|T_W|l_n\rangle\}. \tag{38}$$

Here $\langle l_f+1,f|T_W|l_n\rangle$ is the parity-violating element of the T matrix describing the transition from the initial state with $l_n=0,1$ (s and p, respectively) to the final state with angular momentum (l_f+1); f denotes all the additional quantum numbers defining the channel f (j in the case of the elastic p wave); $\langle l_f,f|T|l_n\rangle$ determines the corresponding P-allowed transition.

Now one can use the Born approximation to calculate the P-forbidden transition

$$T_W=\langle\Psi_f^{(-)}|V_W|\Psi_i^{(+)}\rangle. \tag{39}$$

To simplify the problem even more, we shall retain only one s- and one p-wave resonance in the expressions (18) for the initial and final states. In this case the first-order Born amplitude (39) contains nine terms:

$$\begin{aligned}T_W=\langle\Psi_s^{(-)}|V_W|\Psi_p^{(+)}\rangle&=a_p^+(E)\langle\Phi_s|V_W|\Phi_p\rangle a_s^+(E)\\&+e^{i\delta_s}a_p^+(E)\langle\Phi_p|V_W|\chi_s(E)\rangle\\&+a_p^+(E)a_s^+(E)\mathscr{P}\int\frac{dE'}{E-E'}\langle\Phi_p|V_W|\chi_s(E')\rangle\\&\times\langle\chi_s(E')|V|\varphi_s\rangle+e^{i\delta_p}\langle\chi_p(E)|V_W|\Phi_s\rangle a_s^+(E)\\&+e^{i(\delta_s+\delta_p)}\langle\chi_p(E)|V_W|\chi_s(E)\rangle\\&+e^{i\delta_p}a_s^+(E)\mathscr{P}\int\frac{dE'}{E-E'}\langle\chi_p(E)|V_W|\chi_s(E')\rangle\\&\times\langle\chi_s(E')|V|\varphi_s\rangle\\&+a_p^+(E)a_s^+(E)\mathscr{P}\int\frac{dE'}{E-E'}\langle\varphi_p|V|\chi_p(E')\rangle\\&\times\langle\chi_p(E')|V_W|\varphi_s\rangle\\&+a_p^+(E)e^{i\delta_s}\mathscr{P}\int\frac{dE'}{E-E'}\langle\varphi_p|V|\chi_p(E')\rangle\\&\times\langle\chi_p(E')|V_W|\chi_s(E)\rangle\\&+a_p^+(E)a_s^+(E)\mathscr{P}\int\frac{dE'dE''}{(E-E')(E-E'')}\\&\times\langle\varphi_p|V|\chi_p(E')\rangle\langle\chi_p(E')|V_W|\chi_s(E'')\rangle\\&\times\langle\chi_s(E'')|V|\varphi_s\rangle.\end{aligned} \tag{40}$$

In order to understand the physical meaning of each term, it is useful to introduce a graphical technique with the following correspondence rules: a wavy line with indices (p or s) denotes an $l=1$ or $l=0$ neutron in the mean field of the target; a thin solid line denotes the ground-state target; empty circles correspond to strong-interaction amplitudes $\gamma_s^n\exp i\delta_s/\sqrt{2\pi}$ and $\gamma_p^n\exp i\delta_p/\sqrt{2\pi}$; crossed circles correspond to weak-interaction matrix elements; double solid lines with indices correspond to resonance propagators $1/[(E-E_s)+i\Gamma_s/2]$ or $1/[(E-E_p)+i\Gamma_p/2]$ for s or p BSECs φ_s or φ_p; boldface solid lines denote the same propagators, but for the "full" compound-resonance wave functions Φ_s or Φ_p [see Eq. (17)]; a closed loop of neutron and target lines implies a principal-value integration over the neutron energy.

The first term in (40),

$$T_1=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\frac{1}{(E-E_p)+i\Gamma_p/2}\langle\Phi_p|V_W|\Phi_s\rangle\times\frac{1}{(E-E_s)+i\Gamma_s/2}\frac{e^{i\delta_s}\gamma_s^n}{\sqrt{2\pi}}, \tag{41}$$

describes (see diagram 1 of Fig. 1) p-neutron strong absorption into a compound resonance Φ_p, p-resonance propagation, its weak-interaction mixing with a compound resonance Φ_s, propagation of an s resonance, and its strong decay.

The second term,

$$T_2=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\frac{1}{(E-E_p)+i\Gamma_p/2}\langle\Phi_p|V_W|\chi_s(E)\rangle, \tag{42}$$

describes (diagram 2 of Fig. 1) p-neutron strong absorption into a compound resonance Φ_p, p-resonance propagation, and its subsequent "weak" decay into the s-wave continuum state.

The third term,

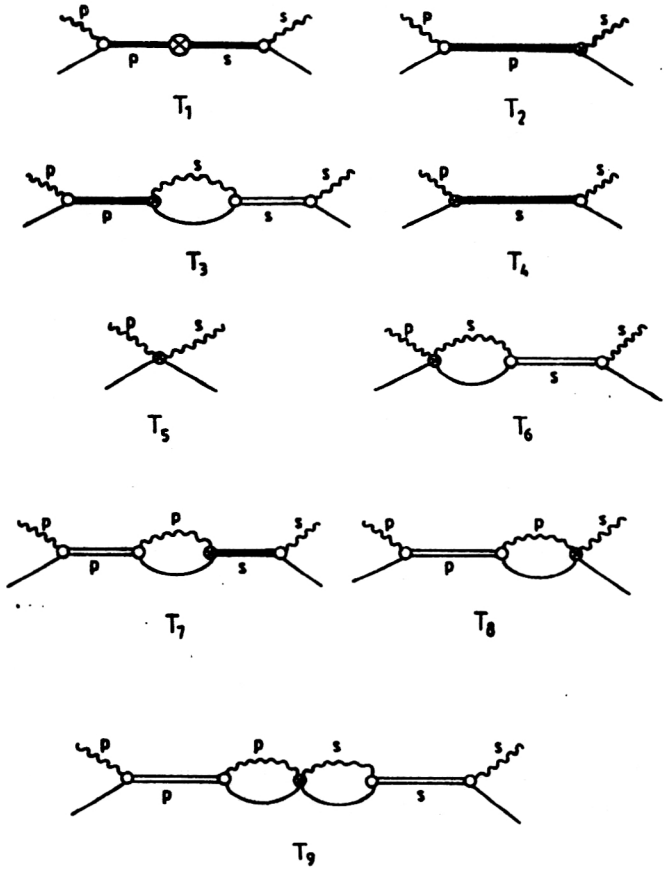

FIG. 1. Diagrams of possible processes contributing to p violation in neutron–nucleus elastic scattering.

$$T_3=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\frac{1}{(E-E_p)+i\Gamma_p/2}\mathscr{P}\times\int\frac{dE'}{E-E'}\langle\Phi_p|V_W|\chi_s(E')\rangle\times\langle\chi_s(E')|V|\Phi_s\rangle\frac{1}{(E-E_s)+i\Gamma_s/2}\frac{e^{i\delta_s}\gamma_s^n}{\sqrt{2\pi}},\qquad(43)$$

describes (diagram 3 of Fig. 1) p-neutron strong absorption, p-resonance propagation of Φ_p, its "weak" decay into the s-neutron continuum with immediate strong reabsorption of the s neutron into the s-resonance state Φ_s. Then there is s-resonance propagation of Φ_s and its strong neutron decay. We see that this is another way of mixing p and s resonances (compare this with T_1) by virtual emission and reabsorption of an s-wave neutron. Processes of this type were first encountered in isospin symmetry breaking, where they played an important role. Historically, they were first analyzed in terms of R-matrix theory and were called "external mixing" processes, in contrast to the "internal mixing" of T_1.

The fourth term T_4 describes p-neutron "weak" absorption forming an s-resonance compound state Φ_s which then decays in a normal "strong" way. It is obvious (see Fig. 1) that this amplitude closely resembles the T_2 amplitude.

The fifth term T_5 describes just the potential scattering of a p-wave neutron in the weak mean field of the target. This type of process was historically first discussed in P violation (see, e.g., Ref. 22) and is important, since all the specific enhancements below should be defined with respect to this simplest amplitude.

The T_6 term describes rescattering of a p-wave neutron in the weak mean field of the target with subsequent "strong" absorption of the created s neutron forming the s-resonance BSEC φ_s. This resonance propagates and then decays in a normal "strong" way governed by the γ_s^n amplitude. This process is topologically close to the process T_8 (see below).

The T_7 term describes strong p-neutron absorption into a p-compound BSEC φ_p, and resonance propagation and strong decay, which is followed by "weak" reabsorption of the neutron into an s-compound resonance state Φ_s. This state then decays in a normal "strong" way. Obviously, T_7 is very similar to the above T_3.

We have already mentioned that T_8 is very similar to T_6. It describes the strong formation of the p-resonance BSEC and its strong decay followed by weak rescattering in the target field. This amplitude was considered recently by Weidenmüller and Lewenkopf.[33,34]

Finally, T_9 describes "strong" p-compound formation of Φ_p and its "strong" decay, which is followed by "weak" rescattering in the target mean field. The s-neutron created in this rescattering is then "strongly" reabsorbed into an s-compound BSEC, which finally decays in a normal "strong" way.

Now that we know all the essential mechanisms contributing to P violation in slow neutron–nucleus elastic scattering, it is appropriate to estimate which of them gives the most important contribution and why. In making such estimates we shall drop all the phase-shift exponentials because in the energy region of interest to us $\delta_s\sim(kR)\sim10^{-3}$, $\delta_p\sim(kR)^2\sim10^{-6}$. Since the total resonance widths in this region are determined essentially by γ emission, we shall assume that $\Gamma_s\approx\Gamma_p=\Gamma$. For simplicity we shall denote $|E_s-E_p|$ by D and assume that $D\sim d$.

We shall tell the reader in advance the result of our analysis carried out back in 1982 (see Ref. 7). The dominant contribution comes from the mechanism of T_1, which is usually called compound–compound $(c-c)$ mixing. Other mechanisms give contributions that are smaller by at least the above factor of dynamical enhancement, $v/D\sim\sqrt{N}\sim10^3$ [see Eq. (9)]. Therefore we shall estimate the ratios of T_1 to all the other amplitudes in (40).

We shall start with the standard weak potential scattering T_5, which determines the process in the simplest systems of $n-p$ type. In order to estimate T_5 we shall first take the weak-interaction scaling factor F out of $\langle\chi_p(E)|V_W|\chi_s(E)\rangle$. Then we shall proceed by removing the extra barrier penetration factor from the amplitude, thus converting T_5 into the strong-interaction amplitude $\langle\chi_s(E)|V|\chi_s(E)\rangle$, which is roughly equal to the s-wave phase shift $\delta_s\sim(kR)$. Thus,

$$\langle\chi_p(E)|V_W|\chi_s(E)\rangle\approx F\cdot(kR)^2.\qquad(44)$$

The exact calculations of T_5 performed in Ref. 34 show that (44) is correct to within a constant factor of ~ 7. Now we can estimate the ratio

$$\frac{T_1}{T_5} \approx \frac{\gamma_p^n \gamma_s^n \langle \Phi_p | V_W | \Phi_s \rangle}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)} \frac{1}{F(kR)^2}$$

$$\approx \frac{S_n \Theta_0^2 v}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)}. \tag{45}$$

Here we have scaled [see Eq. (6)] the weak-interaction matrix element $v_p = Fv$ and used the standard estimates of the neutron partial widths, factoring out the barrier penetration $(kR)^{2l+1}$, the spectroscopic factor S_n and the "single-particle reduced width" $\Theta_0^2 = 2\hbar^2/mR^2$. The resonance denominators of T_1 give the smallest ratio (45) between E_s and E_p. At this energy point (we consider the case $D \gg \Gamma$),

$$\frac{T_1}{T_5} \approx \frac{S_n \Theta_0^2 v}{D^2} \approx \frac{\Theta_0^2}{d_0} \frac{v}{D} \approx \frac{v}{D} \sim \sqrt{N}. \tag{46}$$

In estimating (46) we assumed that $S_n \sim d/d_0$, where the single-particle level spacing d_0 was taken to be roughly equal to Θ_0^2 (see, e.g., Ref. 12). Thus, we see that under the worst "off-resonance" conditions the $c-c$ mixing mechanism T_1 gives us the dynamical enhancement factor. We also see that this enhancement disappears for the simplest systems with $N \sim 1$. This, however, is not the whole story. We also see from (41) and (45) that in the vicinity of each resonance pole ($E \approx E_s$ or E_p) T_1 presents us with an extra resonance enhancement factor D/Γ, thus providing an overall enhancement of the ratio

$$\left(\frac{T_1}{T_5}\right)_{\text{res}} \approx \frac{v}{D} \frac{D}{\Gamma} = \frac{v}{\Gamma}. \tag{47}$$

This *resonance enhancement* is a specific feature of nuclear reactions which has no analog in the case of bound states. Its meaning is, however, quite transparent—the magnitude of the P-violating effects is proportional to the time τ spent by the incident neutron in the weak-interaction field of the target. The role of the complicated compound resonance is to capture the neutron and keep it inside the compound system for a long time $\tau = \hbar/\Gamma$. This kind of effect was first mentioned by Mahaux and Weidenmüller.[35]

Consider now the ratio of T_1 to T_2:

$$\frac{T_1}{T_2} \approx \frac{\gamma_s^n \langle \Phi_p | V_W | \Phi_s \rangle}{(E-E_s+i\Gamma/2)\langle \Phi_p | V_W | \chi_s \rangle} \approx \frac{v}{(E-E_s+i\Gamma/2)}. \tag{48}$$

In performing the estimate we introduced the scaling $\langle \Phi_p | V_W | \chi_s \rangle \sim F \langle \Phi_s | V | \chi_s \rangle \approx F \cdot \gamma_s^n$. We see again that even in the worst case when $E = E_p$ the quantity T_1 dominates by the dynamical enhancement factor v/D, while at $E = E_s$ the resonance enhancement D/Γ is added.

Since the amplitude T_4 is topologically close to T_2, the ratio T_1/T_4 exhibits exactly the same enhancements with the replacement of E_s by E_p.

All the remaining diagrams in Fig. 1 contain closed loops of principal-value integrals. In estimating those loops we shall follow the arguments of Weidenmüller and Lewenkopf[34,35] for the case of T_8. Crudely, their argument was that the main E' dependence in the integral

$$\mathscr{P} \int \frac{dE'}{E-E'} \langle \varphi_p | V | \chi_p(E') \rangle \langle \chi_p(E') | V_W | \chi_s(E) \rangle \tag{49}$$

comes from the barrier penetration factor $(kR)^{2l+1}$ of $|\chi_p(E')|^2$. This makes it possible to drop the principal-value symbol and remove the integrands at $E' = E$ from the integral. Thus, (49) becomes

$$\langle \varphi_p | V | \chi_p(E) \rangle \langle \chi_p(E) | V_W | \chi_s(E) \rangle \int \frac{dE'}{E'} \frac{\chi_p^2(k'R)}{\chi_p^2(kR)}. \tag{50}$$

In the case of a square-well potential, $\chi_p(kR) \sim j_1(kR)$, and one obtains the analytical result

$$\langle \varphi_p | V | \chi_p(E) \rangle \langle \chi_p(E) | V_W | \chi_s(E) \rangle \cdot \frac{3\pi}{(kR)^3} \approx \gamma_p^n \frac{F}{(kR)}. \tag{51}$$

In order to obtain the final result, we used the above estimate (44) for the weak-interaction amplitude. More exact numerical calculations[34] for the Woods–Saxon potential show that instead of 3π one gets the value $C = 3.1$. This obviously does not affect our order-of-magnitude estimates.

Now we can estimate the ratio of T_1 to

$$T_8 \approx \frac{\Gamma_p^n F}{(E-E_p+i\Gamma_p/2)(kR)}. \tag{52}$$

We see that this ratio is

$$\frac{T_1}{T_8} \approx \frac{\gamma_s^n}{\gamma_p^n} \frac{(kR)}{(E-E_s+i\Gamma_s/2)} \approx \frac{v}{(E-E_s+i\Gamma_s/2)}. \tag{53}$$

Again we observe that even at $E = E_p$ the amplitude T_8 is smaller by the dynamical enhancement factor $v/D \sim \sqrt{N}$.

Since T_6 is topologically close to T_8, the same dynamical enhancement is absent in T_6 even at $E = E_s$, while at $E = E_p$ the resonance enhancement of T_1 is added to the ratio T_1/T_6.

Now we can use the above procedure to estimate the principal-value integral in T_3 [see (43)]:

$$\mathscr{P} \int \frac{dE'}{E-E'} \langle \Phi_p | V_W | \chi_s(E') \rangle \langle \chi_s(E') | V | \varphi_s \rangle$$

$$\approx \langle \Phi_p | V_W | \chi_s(E) \rangle \langle \chi_s(E) | V | \varphi_s \rangle \int \frac{dE'}{E'} \frac{\chi_s^2(k'R)}{\chi_s^2(kR)}$$

$$\approx \frac{F \cdot \gamma_s^n \gamma_s^n}{kR}. \tag{54}$$

Therefore

$$T_3 \approx \frac{\gamma_p^n F (\gamma_s^n)^3}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)(kR)} \tag{55}$$

and

$$\frac{T_1}{T_3} \approx \frac{(kR)v}{\Gamma_s^n} \approx \frac{v}{S_n \Theta_0^2} \approx \frac{d_0}{\Theta_0^2} \frac{v}{d} \approx \frac{v}{d}. \tag{56}$$

In making this estimate we used the same factorization as in (45) and (46) for the neutron width Γ_s^n.

In the same way, we obtain the same estimate (55) for T_7 and (54) for the ratio T_1/T_7.

The only remaining term now is T_9. Each integral in it can be estimated by the method that we have already used several times, giving

$$\mathscr{P}\int \frac{dE'dE''}{(E-E')(E-E'')}\langle\varphi_p|V|\chi_p(E')\rangle \times\langle\chi_p(E')|V_W|\chi_s(E'')\rangle\langle\chi_s(E'')|V|\varphi_s\rangle \approx \gamma_p^n\langle\chi_p(E)|V_W|\chi_s(E)\rangle\gamma_s^n\frac{1}{(kR)^4}\approx\gamma_p^n\gamma_s^n\frac{F}{(kR)^2}. \tag{57}$$

Therefore

$$T_9\approx\frac{\Gamma_p^n F\Gamma_s^n}{(kR)^2(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)} \tag{58}$$

and

$$\frac{T_1}{T_9}\approx\frac{(kR)^2 v}{\gamma_p^n\gamma_s^n}\approx\frac{v}{S_n\Theta_0^2}\approx\frac{d_0}{\Theta_0^2}\frac{v}{d}\approx\frac{v}{d}. \tag{59}$$

To conclude our analysis, we recall that some of the above amplitudes (T_1, T_2, T_3, T_4, and T_7) contained the "full" compound-nucleus functions $\Phi_k=\varphi_k+(S_k^n)^{1/2}u$. Substitution of φ_k instead of Φ_k in the above amplitudes would not affect, as far as we can see, the above estimates of their relative contributions to (40). The additional valence terms $(S^n)^{1/2}u$ would give us seven more amplitudes with the replacement of the corresponding Φ by $(S^n)^{1/2}u$. All of them would contain at least the additional small factor $\sqrt{S^n}\approx 1/\sqrt{N}$ (inverse dynamical enhancement). Therefore in all the amplitudes except T_1 they should be disregarded as small additions to the *already small* amplitudes. In the case of T_1 there will be two "mixed" terms containing the products $(\varphi\cdot\sqrt{S^n}u)$ and one term of the form

$$T_{10}=\frac{\Gamma_p^n}{(\Gamma_p^0)^{1/2}}\frac{\langle u_p|V_W|u_s\rangle}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)}\frac{\Gamma_s^n}{(\Gamma_s^0)^{1/2}}. \tag{60}$$

Since this term contains an extra small factor $S^n\sim 1/N$ in comparison with T_1, it seems that it should be dropped. However, the compound–compound matrix element v_p (more precisely, its variance $\tilde{v}_p$) of (41) goes down with increasing complexity N of the wave function Φ_k as $d_0/\sqrt{N}$ [see Eq. (9)]. Therefore the "single-particle" matrix element $\langle u_p|V_W|u_s\rangle$ should be larger than $\tilde{v}_p$ by roughly a factor $\sqrt{d_0/d}\approx\sqrt{N}$. Thus, the overall ratio T_1/T_{10} is $1/\sqrt{N}$ rather than $1/N$.

An important point is that, in contrast to the partial amplitudes γ^n of (41), whose signs vary randomly from resonance to resonance, all the partial widths in (60) are positive. Similarly, in contrast to the randomly varying sign of v_p in (41), the sign of the single-particle matrix element in (60) determined by u_p and u_s) can vary only over the energy range of the single-particle level spacing d_0. The overall sign of (60) at a given resonance, say, E_p, also seems to be determined by the sign of (E_p-E_s). If, however, we switch to the multiresonance case, we should sum T_{10} at E_p over all the s resonances which can mix with a given p one:

$$\sum_s T_{10}=\frac{\Gamma_p^n}{(\Gamma_p^0)^{1/2}}\frac{\langle u_p|V_W|u_s\rangle}{(E-E_p+i\Gamma_p/2)}\frac{\Gamma_s^0}{(\Gamma_s^0)^{1/2}}\sum_s\frac{S_n(s)}{E-E_s}. \tag{60a}$$

Since the spectroscopic factor saturates to unity over the energy range Γ_{spr} around the position E_{s0} of a single-particle level, we have

$$\sum_s T_{10}\approx\frac{\Gamma_p^n\langle u_p|V_W|u_s\rangle}{(E-E_p+i\Gamma_p/2)}\left(\frac{\Gamma_s^0}{\Gamma_p^0}\right)^{1/2}\frac{1}{E-E_{s0}}. \tag{60b}$$

Therefore the overall sign of the effect due to the valence neutron remains constant over the range d_0. However, the same is true for the mechanisms of T_9 and T_8 [see (52) and (58)]. All these mechanisms, which essentially sprang to life because of the single-particle (valence) contributions $\chi(E)$ to the BSEC wave functions ϕ_k, would give a constant-sign contribution to the effect. But, as we pointed out already in Ref. 7, *all of them lack the factor of dynamical enhancement* $v/D\sim\sqrt{N}$ and for this reason should be dropped. We shall briefly return to this problem in discussing the "sign correlation effect" below (see Sec. 4.5).

Up to now we have considered [see (40) and Fig. 1] only the case when the initial p-wave neutron is transformed by the weak interaction into the final-state s one, i.e., the second term in (36). Repetition of the above analysis for the first term of (36) will add 16 more amplitudes similar to those of Fig. 1. One can easily see that diagrams 1, 3, 5, 7, and 9 are symmetric with respect to the exchange of s- and p-neutron states. Therefore such an exchange will just double the contributions of the corresponding mechanisms to (36). The same exchange in amplitudes 2 and 8 of Fig. 1 would shift their resonance poles to E_s. However, the poles of T_4 and T_6 after this exchange would be shifted to E_p. Therefore the addition of the first term in (36) would just completely restore the symmetry of the whole expression (36) with respect to exchange of the initial and final states, which is expected for *any* elastic-scattering T-invariant amplitude. The relative dominance of the $c-c$ mixture amplitude T_1 remains unaltered.

To summarize, we have seen that proper nuclear-reaction theory allows us to find all the contributions to the weak-interaction elastic scattering amplitude of neutrons on a nuclear target. The leading contribution to this amplitude comes from the compound–compound mixing mechanism T_1 of Eq. (41). This mechanism exhibits two kinds of enhancement factors: a) the dynamical enhancement factor $v/D\sim\sqrt{N}$; b) the resonance enhancement factor D/Γ. The physical reason for both enhancements is the complexity (or quantum chaoticity) of the compound-nucleus resonances. The lack of symmetries characteristic of a quantum chaotic system (see, e.g., Ref. 36) removes all the degeneracies of the independent-particle shell model and thus exponentially decreases the resonance spacing. At the same time it compli-

cates the structure of the compound-resonance wave function, hindering in this way all the decay processes and reducing the total resonance width Γ.

One should also mention that in our analysis we met no traces of the mystical "structural enhancement factor" $1/(kR)$. As mentioned above, theoretically this factor is a false result of inconsistent application of bound-state theory to the continuum nuclear-reaction case.

3.2.4. Back to observables

In the previous subsection we performed an analysis of all possible P-violating amplitudes in the simplest case of elastic neutron–nucleus scattering, understood the physics of their enhancements, and chose our "favorite" $c-c$ mixing amplitude T_1, which exceeds the others by at least a dynamical enhancement factor of $\sqrt{N}\sim 10^3$.

Inserting T_1 into Eqs. (36) and (37), we obtain

$$\Delta_{\text{tot}}^{P}=\frac{2\pi}{k^2}\frac{\gamma_p^n\cdot v_p\cdot\gamma_s^n\cdot e^{i(\delta_s+\delta_p)}}{[(E-E_p)^2+\Gamma_p^2/4][(E-E_s)^2+\Gamma_s^2/4]}\times[(E-E_s)\Gamma_p+(E-E_p)\Gamma_s], \tag{61}$$

$$\frac{d\Phi}{dz}=\frac{4\pi\rho}{k^2}\frac{\gamma_p^n\cdot v_p\cdot\gamma_s^n\cdot e^{i(\delta_s+\delta_p)}}{[(E-E_p)^2+\Gamma_p^2/4][(E-E_s)^2+\Gamma_s^2/4]}\times\left[(E-E_s)(E-E_p)-\frac{\Gamma_s\Gamma_p}{4}\right]. \tag{62}$$

Here v_p stands for the weak-interaction matrix element $i\langle\Phi_p|V_W|\Phi_s\rangle$. The presence of Breit–Wigner denominators shows that both effects exhibit symmetric resonance enhancements in the vicinities of both the s and p resonances. The quantity Δ_{tot}^{P} reaches its maxima at E_s and E_p:

$$\Delta_{\text{tot}}^{P}(E_{\text{res}})\approx\frac{8\pi}{k^2}\frac{\gamma_p^n\gamma_s^n}{\Gamma}\frac{v}{D}. \tag{63}$$

The quantity (62), however, changes sign at the points $E\approx E_{s,p}+\Gamma_s\Gamma-p/4D\approx E_{s,p}$ and reaches its maxima

$$\left(\frac{d\Phi}{dz}\right)_{\text{res}}\approx\pm\frac{8\pi\rho}{k^2}\frac{\gamma_p^n\gamma_s^n}{\Gamma}\frac{v}{D} \tag{64}$$

at the points $E\approx E_{s,p}\pm\Gamma_{s,p}/2$. For the characteristic curves of the energy behavior of (61) and (62), see our paper in Ref. 2.

Provided that the asymmetry in the total cross sections Δ_{tot}^{P} is dominated by the resonance–resonance mixture of T_1, we can write (see, e.g., Ref. 2) for the asymmetry $\Delta_{n,\gamma}^{P}$ in the denominator of Eq. (28)

$$\Delta_{n,\gamma}^{P}=\sigma_{n,\gamma}^{+}-\sigma_{n,\gamma}^{-}\approx\frac{\Gamma_\gamma}{\Gamma}\Delta_{\text{tot}}^{P}\approx\Delta_{\text{tot}}^{P}. \tag{61a}$$

Note that for other mixture mechanisms this might not be true. Even in this case the last equality holds only for low-energy neutrons incident on a nonfissioning target.

We have not yet performed the analysis of the P-violating amplitudes in the inelastic channels, which are essential for the calculation of the quantity $\Delta_{n,f}$ of Eq. (38). This can be easily done in the same way as in the previous subsection. Those who are interested in more details might look through our papers of Refs. 7 and 8. The net result of such an analysis is again the conclusion that $c-c$ mixture amplitudes are dominant. For incident $l_n=0$ and $l_n=1$ those amplitudes are (see Fig. 2)

$$T_{nf}^{(1)}=\langle l_f+1,f|T_W|0\rangle=i\frac{\gamma_s^n\cdot v_p\cdot\gamma_p^f\cdot e^{i(\delta_s^n+\delta_{l_f+1}^f)}}{(E-E_s+i\Gamma_s/2)(E-E_p+i\Gamma_p/2)}, \tag{65}$$

$$T_{nf}^{(2)}=\langle l_f,f|T_w|1\rangle=i\frac{\gamma_p^n\cdot v_p\cdot\gamma_s^f\cdot e^{i(\delta_p^n+\delta_{l_f}^f)}}{(E-E_s+i\Gamma_s/2)(E-E_p+i\Gamma_p/2)}.$$

Both amplitudes exhibit a resonance enhancement of D/Γ at E_s and E_p plus the dynamical enhancement of v/D. However, their ratio is

$$\frac{T^{(2)}}{T^{(1)}}=\frac{\gamma_p^n\gamma_s^f}{\gamma_s^n\gamma_p^f}\sim(kR)\frac{\gamma_s^f}{\gamma_p^f}.$$

Therefore, for low-energy ($E\leq 1$ MeV) neutrons the second diagram contains the *initial channel hindrance* factor (kR) (see Ref. 7), which again has no bound-state analogs, but can be easily understood in terms of nuclear-reaction theory. In contrast to the P-violating elastic amplitudes of the preceding subsection, the inelastic amplitudes are not symmetric with respect to the exchange of s- and p-neutron waves, and it is highly preferable to excite the s-wave compound resonance in the initial neutron channel rather than the p-wave one. This initial channel hindrance leads to even more general and important consequences (see below), making the "inelastic" observables preferable in general to the "transmission" ones. The corresponding "allowed" reaction amplitudes are

$$\langle l_f,f|T|0\rangle=\frac{\gamma_s^n\cdot\gamma_s^f\cdot e^{i(\delta_s^n+\delta_{l_f}^f)}}{(E-E_s+i\Gamma_s/2)}, \tag{66}$$

$$\langle l_f,f|T|1\rangle=\frac{\gamma_p^n\cdot\gamma_p^f\cdot e^{i(\delta_p^n+\delta_{l_f}^f)}}{(E-E_p+i\Gamma_p/2)}. \tag{67}$$

Inserting (65)–(67) into (38) and retaining only the largest terms, we obtain

$$\Delta_{nf}=\frac{2\pi}{k^2}\sum_{l_f}\frac{\gamma_s^f\cdot v_p\cdot\gamma_p^f}{[(E-E_s)^2+\Gamma_s^2/4][(E-E_p)^2+\Gamma_p^2/4]}\times\text{Re}[(E-E_p)\Gamma_s^n\cdot e^{i(\delta_{l_f}^f-\delta_{l_f+1}^f)}]. \tag{68}$$

In this expression we have already neglected the neutron potential phase shifts δ_s and δ_p. When the final channel is a γ-emission one, we get

$$\Delta_{n\gamma_0}=\frac{2\pi}{k^2}\frac{\gamma_s^n\cdot v_p\cdot\gamma_p^f}{[(E-E_s)^2+\Gamma_s^2/4][(E-E_p)^2+\Gamma_p^2/4]}\times(E-E_p)\Gamma_s^n. \tag{69}$$

Note that γ_0 here denotes a *particular* γ transition (say, to the ground state of the initial nucleus). Note also that the effect changes sign near $E=E_p$ in analogy with $d\Phi/dz$ of Eq. (62).

Generally speaking, the most nontrivial part of the P-violation theory ends with the expressions (61), (62), (61a), (68), and (69) for Δ^p_{tot}, $d\Phi/dz$, $\Delta^p_{n,\gamma}$, and $\Delta_{n,f}$. In order to find the dimensionless ratios P, Φ, A, and $\alpha_{n,f}$ observed experimentally, one should just divide those expressions by $2\sigma_{\text{tot}}$, $2\sigma_{n,\gamma}$, or $2d\sigma_{n,f}/d\Omega$, respectively. However, it turns out that even this seemingly simple arithmetical operation is full of intricate tricks, because, as we have already mentioned, the denominators also exhibit rapid and sometimes complicated energy dependences.

Indeed, even in the simplest case of one s- and one p-compound resonance the simplified (i.e., without interference terms) expression for σ_{tot} is

$$\sigma_{\text{tot}}(E)\approx\sigma_s(E)+\sigma_{\text{pot}}(E)+\sigma_p(E)$$
$$\approx\frac{2\pi}{k^2}\left[\frac{\Gamma^n_s\Gamma_s}{(E-E_s)^2+\Gamma^2_s/4}+4(kR)^2\right.$$
$$\left.+\frac{\Gamma^n_p\Gamma_p}{(E-E_p)^2+\Gamma^2_p/4}\right]. \tag{70}$$

Here σ_s, σ_p, and σ_{pot} are the contributions to the total cross section from the s- and p-compound resonances and from potential elastic scattering. As pointed out above, the numerators Δ^p_{tot} and $d\Phi/dz$ display resonance enhancement *both* in s and in p resonances. However, in the energy region of major interest to us (from thermal neutrons to few eV), $\sigma_{\text{tot}}(E)$ and $\sigma_{n,f}(E)$ are dominated by the s-resonance contribution: $\sigma_s(E)/\sigma_{\text{tot}}(E)\sim 1$; $\sigma^s_{n,f}(E)/\sigma_{n,f}(E)\sim 1$. Therefore in this region the resonance enhancement at $E\approx E_s$ is completely canceled in the "observable" ratios:

$$P(E)\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{(E-E_p)^2+\Gamma^2_p/4}\frac{[(E-E_s)\Gamma_p+(E-E_p)\Gamma_s]}{2\Gamma_s}, \tag{71}$$

$$\Phi(E)\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{(E-E_p)^2+\Gamma^2_p/4}\frac{(E-E_s)(E-E_p)-\Gamma_s\Gamma_p/4}{\Gamma_s}, \tag{72}$$

$$\alpha_{n\gamma_0}\approx\frac{\gamma^{\gamma_0}_p}{\gamma^{\gamma_0}_s}\frac{v_p}{(E-E_p)^2+\Gamma^2_p/4}\frac{(E-E_p)}{2}. \tag{73}$$

Since the p-resonance contribution to the cross section is usually *only a small bump* on the large smooth tail of the s resonance (and σ_{pot} is a background of σ_{tot}), all the observables exhibit a characteristic resonance enhancement in the vicinity of E_p, although the last two change sign at $E\approx E_p$ and therefore display more complicated patterns typical of optical dispersion rather than simple Breit–Wigner ones. The largest among them is the "inelastic-channel" observable (73), which at $E\approx E_p\pm\Gamma_p/2$ reaches the value

$$\alpha_{n,\gamma_0}\sim\frac{\gamma^{\gamma_0}_p}{\gamma^{\gamma_0}_s}\frac{v_p}{\Gamma}. \tag{73a}$$

We now encounter a few more specific features of nuclear reactions. Consider the ratio of observables at the maximum, $E\approx E_p\pm\Gamma_p/2$:

$$\left(\frac{P(E)}{\alpha_{n\gamma_0}(E)}\right)_{\max}\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{\gamma^{\gamma_0}_s}{\gamma^{\gamma_0}_p}\frac{D}{\Gamma}\sim(kR)\frac{\gamma^{\gamma_0}_s}{\gamma^{\gamma_0}_p}\frac{D}{\Gamma},$$
$$\left(\frac{\Phi(E)}{\alpha_{n\gamma_0}(E)}\right)_{\max}\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{\gamma^{\gamma_0}_s}{\gamma^{\gamma_0}_p}\frac{D}{\Gamma}\sim(kR)\frac{\gamma^{\gamma_0}_s}{\gamma^{\gamma_0}_p}\frac{D}{\Gamma}. \tag{73b}$$

We see that both observables P and Φ connected with the elastic-channel correlation $(\vec{\sigma}_n\cdot\vec{k}_n)$ contain the already familiar entrance-channel hindrance factor (kR). This demonstrates a very *general law*—if any symmetry-breaking correlation contains a certain power of $\vec{k}_n$, all the corresponding observables will contain hindrance factors (k_nR) of at least the same power. (We shall return to this point in our discussion of T violation below.) This fact *puts all the "transmission" observables of symmetry breaking into an unfavorable position with respect to the inelastic-channel ones* from the very beginning.

Thus, we have at last encountered the factor (kR) in the observables (71) and (72). But, in contrast to naive expectations of bound-state parallels, it is a *hindrance* factor rather than an enhancement one.

On the other hand, we have a factor γ^f_p/γ^f_s in the inelastic-channel observables, which in general might work both ways, but for some special cases might serve in the role of the only true "structural-enhancement" factor (see, e.g., Ref. 8) *increasing the P-violation effects in the inelastic channels practically to the 100% level.*

We also see that the interference patterns of resonance enhancement are rather complicated, which often results in extra resonance enhancements D/Γ (see, e.g., the Φ and P observables). These extra enhancement factors in the most favorable on-resonance situations (see below) might almost compensate the general smallness (kR) pertinent to transmission experiments.

Since historically P-nonconserving effects were first observed at thermal energies, it is instructive to classify the magnitudes of these effects at $E=E_{\text{th}}$:

$$P(E_{\text{th}})\approx-\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{E^2_p}\frac{E_s\Gamma_p+E_p\Gamma_s}{2\Gamma_s}\approx-\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{E_p}\frac{E_s+E_p}{E_p}, \tag{74}$$

$$\Phi(E_{\text{th}})\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{E^2_p}\frac{E_sE_p-\Gamma_s\Gamma_p/4}{2\Gamma_s}\approx\frac{\gamma^n_p}{\gamma^n_s}\frac{v_p}{E_p}\frac{E_s}{\Gamma_s}, \tag{75}$$

$$\alpha_{n\gamma_0}(E_{\text{th}})\approx-\frac{\gamma^{\gamma_0}_p}{\gamma^{\gamma_0}_s}\frac{v_p}{E^2_p}\frac{1}{2}. \tag{76}$$

We see that both P and Φ contain the above strong hindrance factor $\gamma^n_p/\gamma^n_s\sim(kR)\sim 10^{-4}$, which is not present in the inelastic case of α_{n,γ_0}. Therefore the "inelastic" value $\alpha_{n,f}$ is largest (10^{-3}–10^{-4}) at thermal energies and was observed experimentally for γ's and fission fragments in almost "prehistoric" times.[18,37] It was exactly those unbelievably large (compared with the 10^{-7}–10^{-8} effects in np scattering)

effects observed in neutron-induced fission that initiated the Sushkov–Flambaum and Bunakov–Gudkov theoretical studies, which led both groups to the prediction of p-resonance enhancements. Next in magnitude (typically 10^{-5}) comes the value of Φ, which contains an additional large factor E_s/Γ_s. The P value containing instead the factor $(E_s+E_p)/E_p$ is usually smaller (typically 10^{-6}). Consequently, the first experimental observations for them were made later.[27,28] Comparison with experiment nicely confirms the above "hierarchy" of observables (see, e.g., Refs. 2 and 7).

Let us return to the behavior of P in the vicinity of the p-resonance:

$$P(E)\approx \frac{2\pi}{k^2}\frac{v_p}{D}\frac{\Gamma_p}{(E-E_p)^2+\Gamma_p^2/4}\frac{\gamma_p^n\cdot\gamma_s^n}{\sigma_{\rm tot}(E)}. \tag{77}$$

We note that this expression is valid *only* for $D>\Gamma_s/2$.

Since *even at its maximum the p resonance contributes only a small fraction* to $\sigma_{\rm tot}(E_p)$, the overall behavior of $\sigma_{\rm tot}(E)$ in the vicinity of the p resonance is *quite smooth*. Therefore the resonance enhancement of $P(E)$ represented by the Breit–Wigner denominator of (77) fully reveals itself in experiment. Apparently, everything is clear—we have both this resonance-enhancement mechanism and the familiar factor v_p/D of dynamical enhancement in (77).

Nevertheless, we find here the "mythology" of experimentalists, who prefer to stick to naive bound-state analogies, whose physical inconsistency we have already analyzed at length. Instead of presenting the observable P given by (77), they prefer to introduce the *auxiliary* quantity $\mathscr{P}$ by relating the observed $\Delta_{\rm tot}^P$ to a *small fraction* of the observed $\sigma_{\rm tot}$, namely, to the p-resonance contribution $\sigma_p(E)$ [see the third term in (70)]:

$$\mathscr{P}=\frac{\Delta_{\rm tot}^P}{2\sigma_p(E)}=P\frac{\sigma_{\rm tot}}{\sigma_p}. \tag{78}$$

The *purely technical* reason for such a renormalization can be explained as follows. We have already mentioned that the actually measured quantity (25) is

$$P_{\rm exp}(E)=\Delta_{\rm tot}^p(E)\cdot C,$$

where the constant C depends linearly on the target-sample thickness x. In order to optimize the statistical significance of measurements, this x is chosen in such a way that $C\approx 1/2\sigma_{\rm tot}$. When measuring $P_{\rm exp}(E)$ in the Γ_p vicinity of E_p, the experimentalists do not readjust x for each energy point (again because the *smallness* of the ratio $\sigma_p/\sigma_{\rm tot}$ allows this). Therefore the measured value $P_{\rm exp}(E)$ gives a "full-scale" resonance behavior of $\Delta_{\rm tot}^p$ in the vicinity of the p resonance:

$$\Delta_{\rm tot}^P(E)\approx \frac{2\pi}{k^2}\frac{v_p}{D}\frac{\gamma_s^n\gamma_p^n\Gamma_p}{(E-E_p)^2+\Gamma_p^2/4}. \tag{79}$$

In order to avoid quoting a whole set of numbers $P_{\rm exp}(E)$ at all the energy points E measured on the resonance curve (79), the experimentalists prefer to *cancel* the resonance behavior of the effect by normalizing it to $\sigma_p(E)$. This allows them to present only one value of $\mathscr{P}$ instead of the whole resonance curve.

Of course, this makes some sense, although one might prefer to use the knowledge of C to quote directly the measured matrix elements v_p, which will again be one number and, in fact, the only one that we are seeking in performing our experiments. One should, however, realize that this *artificial normalization* by one of the weakest components of the total cross section gives *only an auxiliary quantity without much physical meaning* [see Eq. (4a) of Sec. 2 and the discussion which follows it]. The unphysical normalization of this quantity produces a fictitious enhancement which has nothing to do with reality—one might as well normalize $\Delta_{\rm tot}^P$ by the neutrino cross section and surprise the world with huge unobservable effects. To confuse things even more, nowadays all the experimental papers use for this auxiliary quantity of Eq. (78) the same notation as for the physical observable P, which as already been defined for 20 years by Eq. (20). The Dubna experimental group long ago at least bothered to introduce different notations for those two quantities, although they never advertised the difference between them and always presented $\mathscr{P}$ as the *observed* result. The main reason for such an "absent-minded" mixing of two physically different quantities becomes quite obvious when one represents (78) in a slightly different form:

$$\mathscr{P}=\frac{v_p}{D}\frac{\gamma_s^n}{\gamma_p^n}\sim\frac{v_p}{D}\frac{1}{(kR)}. \tag{80}$$

This is exactly the result which was so easily obtained [see (34) and (35)] in a simple but inconsistent attempt to apply bound-state perturbation theory to the reaction continuum case. The physically meaningful resonance-enhancement mechanism is absorbed in it by the renormalization of $\mathscr{P}$, while instead of it there appears from nowhere the misleading "structural-enhancement" factor $(kR)^{-1}$. If one recalls that the majority of experimental papers and reviews practically start by quoting the simple bound-state expressions (33) and (34), one realizes how tempting it is to take the small step of replacing the observed P by the auxiliary $\mathscr{P}$—there is no need to study reaction theory with its strange terminology of continuum spectra; all the theory one needs to understand the results boils down to the two simple expressions (33) and (34). It is pertinent here to apply the Russian proverb: "Simplicity is worse than robbery."

To summarize, we have shown that the "structural" (or "kinematic") enhancement factor $(kR)^{-1}$ is an artifact produced by the renormalization (78). It immediately disappears when we return to the observable

$$P(E)\equiv\mathscr{P}\frac{\sigma_p(E)}{\sigma_{\rm tot}(E)}\approx P_{\rm exp}, \tag{81}$$

which always contains a small factor $\sigma_p(E)/\sigma_{\rm tot}(E)\sim(\Gamma_p^n/\Gamma_s^n)\sim(kR)^2$, overcompensating the above "structural enhancement."

As to the resonance-enhancement mechanism sitting in the resonance denominator of (77), one often hears naive statements: "Why, it is quite trivial; everybody knows that compound-resonance effects are of Breit–Wigner shape, and we do not need your fancy theories to prove it." This is again a wrong nonprofessional statement which might lead to er-

roneous conclusions. To begin with, the energy behavior of $\Delta^p_{\rm tot}$ in Eq. (61) is more complicated than a simple Breit–Wigner formula combined with bound-state perturbation theory, as it might seem to be from, say, Eq. (79). When one divides it by the energy-dependent $\sigma_{\rm tot}(E)$, the resulting expression becomes even more complicated (see, e.g., Refs. 7 and 8). For instance, the observable P at the p-resonance maximum is given by

$$P(E_p)\approx 8\,\frac{\gamma_p^n}{\gamma_s^n}\,\frac{v_p}{\Gamma_p}\,\frac{(E_p-E_s)}{\Gamma_s}\left[1+\frac{\sigma_p}{\sigma_s}+\frac{\sigma_{\rm pot}}{\sigma_s}\right]^{-1}. \qquad (82)$$

Suppose now that we face a situation when the s and p resonances almost overlap: $E_s\approx E_p$ (the normal Wigner repulsion does not apply to resonances of opposite parity, so this can easily happen). Then the maximal observed effect *goes down* linearly with decreasing spacing $D=|E_s-E_p|$, in contrast to the naive expectations of bound-state perturbation theory and even to our Eq. (77), which was valid only for $D>\Gamma/2$. All these intricacies become quite essential in attempts to calculate the energy-averaged effects (see below). Another illuminating example is provided by the "capture-transmission" observable A of Eq. (28). In view of Eq. (61a), we can write this observable at $E=E_p$ as

$$A\approx\frac{\sigma^s_{n,\gamma}(E_p)\sigma^p_{n,\gamma}(E_p)}{\sigma^s_{n,\gamma}(E_p)+\sigma^p_{n,\gamma}(E_p)}\,B.$$

One can see that, in analogy with all the interference-type quantities, A would be maximal when the s-resonance contribution σ^s to the (n,γ) cross section at $E=E_p$ is exactly equal to the p-resonance one σ^p. Then, and only then, A would reach its maximal possible value

$$A^{\max}\approx 2\,\frac{v_p}{\Gamma}.$$

The observable $P(E)$ differs from A by a factor $\sigma_{n,\gamma}/\sigma_{\rm tot}$. Therefore $P^{\max}$ would *never* reach $2v_p/\Gamma$. However (see Refs. 7, 8, and 38), the optimal situation for $P(E_p)$ occurs again when $\sigma_s(E_p)=\sigma_p(E_p)$. Then

$$P^{\max}(E_p)\approx 2\,\frac{v_p}{\Gamma}\,\frac{\sigma_{n,\gamma}}{\sigma_{\rm tot}}.$$

The famous La resonance with $\mathscr{P}\approx 10\%$ satisfies this condition: $\sigma_p(E_p)\approx\sigma_s(E_p)\approx\sigma_{\rm pot}(E_p)$, thus giving the observed value $P\approx 3\%$.

Equations (78) and (81) and the above considerations show that for *strong* observable p resonances (present beam intensities force us to select just those for measurements) the enhancement of P would be maximal. Since in those cases $\sigma_p(E_p)/\sigma_{\rm tot}(E_p)$ is about 0.1–0.3, the difference between the quoted $\mathscr{P}$ and the observed P would not be too large, although the $\mathscr{P}$ values already look more impressive than the actually measured P. However, with increasing intensities the experimentalists will start observing effects on weaker p resonances, and this difference might rise to orders of magnitude. The impressive 10% effects recently observed in ^{232}Th, when expressed in terms of the physical observables P, turn out to be more modest 1% effects. So it is high time to stop mixing the two quantities and fooling each other.

To conclude this subsection, we should mention that we have paid special attention to transmission measurements of the quantity $P(E)$, since this type of experiment is most popular nowadays. This feeds a constant stream of publications in which the same physical errors in interpretation are repeated again and again.

A special cases is P violation in neutron-induced fission. I am not going to expand on it for several reasons. First of all, this interesting subject is worth a separate review. I shall only mention that there are two theoretical approaches to it. Sushkov and Flambaum (see, e.g., Ref. 26), as in the case of the P quantities, used intuitive bound-state analogies to construct $\alpha_{n,\rm fis}$. Gudkov and I (see Ref. 7 and especially Ref. 39) tried to apply the general expression (68) to this case. However, the most striking fact connected with the experimental observation of $\alpha_{n,\rm fis}$ dates back to the mid-1950s, when it was discovered[40,41] that, in spite of the fact that all the experimental observables in fission are sums over the enormous number of outgoing reaction channels with apparently random signs of γ_f, this summation does not destroy the interference effects in the (n,f) cross section. This difficulty was bypassed at that time by the fission transition-state hypothesis of Bohr.[42,43] Now the *same* story is repeated in the measurements of $\alpha_{n,\rm fis}$. This quantity is also an interference-type phenomenon [see Eq. (68)], and again the summation over all the outgoing channels γ_f does not destroy the P-violation effects caused by the $c-c$ mixing mechanism in each channel. In view of this common origin of the difficulty, we tried to resolve it by generalizing Bohr's hypothesis of transition states. Sushkov and Flambaum used instead a purely classical model of fission-fragment motion plus a hypothesis of a pear-like shape of the fissioning nuclei *at the saddle point*. Both approaches have their weaknesses. Application of the classical-trajectory notion to the analysis of quantum interference effects seems quite hazardous to us. Moreover, while most people agree that for asymmetric fission the fissioning nucleus has a pear-like shape *near the scission point*, the same assumption *on the top of the fission barrier* seems quite dubious and contradicts some experimental evidence.[7,39] We are, however, not very happy with our (or, rather, with Bohr's) transition-state hypothesis because it seems up to now a rather artificial construction in the framework of quantum reaction theory. I am sure that P violation in fission is just an additional beacon in search of a still nonexistent quantum theory of fission.

3.3. *T* violation

3.3.1. Specific intricacies of T invariance and observables

One of the greatest dangers in the analysis of T violation is to follow too closely the parallels with P violation—these parallels might be quite wrong. To demonstrate this point we shall consider the cases of P-odd and T-odd correlations. Everybody knows the mnemonic rule—if the transition operator $\hat{T}$ changes sign under the space-reflection operation $\mathscr{P}$ (is P-odd), then it has nonzero matrix elements between states of opposite parity. This is perfectly true. Therefore if we observe a nonzero amplitude of P-odd correlation (say, $\vec{\sigma}\cdot\vec{k}$), this means a P violation. However, if we observe a

nonzero amplitude of T-odd correlation, this fact in itself in the majority of case has nothing to do with T violation. In order to understand this, let us see how P violation is mathematically connected with P-odd operators (or correlations) $\hat{T}$. Acting with a unitary transformation operator $\mathscr{P}$ on a state $|A\rangle$, we obtain a number π_A (parity quantum number) equal to $+1$ or -1:

$$\mathscr{P}|A\rangle=\pi_A|A\rangle. \tag{83}$$

For operators the space reflection takes the form

$$\mathscr{P}\hat{T}\mathscr{P}^{-1}=\pi_T\hat{T}. \tag{84}$$

Consider now the transition amplitude

$$\langle B|\hat{T}|A\rangle=\langle B|\mathscr{P}^{-1}\mathscr{P}\hat{T}\mathscr{P}^{-1}\mathscr{P}|A\rangle=\pi_B\pi_T\pi_A\langle B|\hat{T}|A\rangle. \tag{85}$$

The first part of this equation uses the fact that $\mathscr{P}\mathscr{P}^{-1}=1$. The second part makes use of (83), (84), and the unitarity of $\mathscr{P}$. The whole equation gives us a selection rule:

$$\pi_A\pi_B\pi_T=1. \tag{86}$$

Thus, the P-odd operator ($\pi_T=-1$) leads to nonzero amplitudes only in the case when $|A\rangle$ and $|B\rangle$ are of opposite parity.

However, the action of the time-reversal operator $\mathscr{T}$ changes the signs of the momenta and spins, and interchanges the initial and final states. Therefore its action on any state $|A\rangle$ cannot be expressed in terms of eigenvalues, as in (83). If one also adds the fact that $\mathscr{T}$ is not unitary, one sees that it is impossible to construct (85) for $\mathscr{T}$ and obtain the selection rule (86).

Therefore $\mathscr{T}$ invariance leads only to two immediate consequences (see, e.g., Refs. 11 and 12). The first is the detailed-balance principle. For a binary process $A+a\rightarrow B+b$ it takes the form

$$\frac{(2s_a+1)(2s_A+1)k_a^2}{(2s_b+2)(2s_B+1)k_b^2}\frac{d\sigma_{ab}/d\Omega}{d\sigma_{ba}/d\Omega}=1. \tag{87}$$

Here s_i are the corresponding particle spins.

The other consequence is the so-called P–A theorem which connects the polarization P and the asymmetry A. For elastic scattering of spin-$\frac{1}{2}$ particles it states that

$$P=A. \tag{88}$$

However, many authors have seriously discussed T-odd correlations, hoping to measure T violation. Was it completely meaningless? The answer is no, but the arguments are quite subtle and tricky (see, e.g., Refs. 44–46). We start with the unitarity of the S matrix:

$$SS^\dagger=1.$$

Inserting this into the expression

$$S=1+i\hat{T}$$

for the transition matrix, we obtain

$$\hat{T}-\hat{T}^\dagger=i\hat{T}\hat{T}^\dagger. \tag{89}$$

In the case of a transition from the initial state $|i\rangle$ to the final state $|f\rangle$ this takes the form

$$\langle f|\hat{T}|i\rangle-\langle f|\hat{T}^\dagger|i\rangle=i\sum_n\langle f|\hat{T}|n\rangle\langle n|\hat{T}^\dagger|i\rangle, \tag{90}$$

where $|n\rangle$ forms a complete set of all possible intermediate states. Up to this point everything was quite exact. Now we begin the approximation. Suppose that the interaction which determines our transition T contains a small parameter F. Then the left-hand side of (90) is linear in F, while the right-hand side is quadratic. Therefore in the first-order approximation

$$\langle f|\hat{T}|i\rangle\approx\langle i|\hat{T}^*|f\rangle. \tag{91}$$

This means that the matrix $\hat{T}$ is Hermitean. Let us now combine (91) with the condition of T invariance (see the definition of the $\mathscr{T}$ operation above):

$$\langle f|\hat{T}|i\rangle=\langle -i|\hat{T}|-f\rangle. \tag{92}$$

Here the minus signs indicate changes of sign for the momenta and spins.

Equations (91) and (92) give

$$\langle f|\hat{T}|i\rangle=\langle -f|\hat{T}|-i\rangle^* \tag{93}$$

or

$$|\langle f|\hat{T}|i\rangle|^2=|\langle -f|\hat{T}|-i\rangle|^2. \tag{94}$$

The last equation means that in the case of $\mathscr{T}$ invariance (92) the transition probability should be an even function under a change of sign of all the spins and momenta. Now (93) shows that the sign-inversion operation for the initial and final states means just a complex conjugation. Therefore the overall sign of the transition probability under this sign inversion is completely determined by the sign of the transition operator $\hat{T}$. If this operator is T-odd, (94) demands that the transition probability should be zero.

Thus, we have seen that nonzero T-odd correlations *are* related to T violation only when the transition matrix $\hat{T}$ is approximately Hermitean and *within the accuracy of this approximation*. The last point is very delicate. Let us consider it in more detail, taking as an example the T-odd correlation $\vec{\sigma}_n\cdot[\vec{k}_e\times\vec{k}_{\tilde{\nu}}]$, which is measured in neutron β decay. Apparently, this is a weak-interaction process, which is governed by the weak-interaction constant F, and therefore the deviations from Hermiticity are

$$T-T^\dagger\sim F^2.$$

However, we must not forget about the final-state electromagnetic interaction (Coulomb scattering of an electron on a proton). This means that the non-Hermitian right-hand side of (90) should contain terms of the type

$$i\langle pe\tilde{\nu}|\hat{T}|p'e'\tilde{\nu}\rangle\langle p'e'\tilde{\nu}|\hat{T}^\dagger|n\rangle. \tag{95}$$

While the second amplitude in (95) is actually of the order of F, the first one is proportional to the fine-structure constant α. Therefore the deviation of $\hat{T}$ from Hermiticity is of the order of αF, and this would imitate a T violation even when it does not exist (see also Ref. 44). In principle one can calculate this final-state interaction correction and subtract it from the experimentally observed value of the T-odd correlation. However, all the existing experiments of this type

were giving only experimental upper bounds on the effect. While the experimental accuracy is more or less easily defined, the accuracy of theoretical estimates of the final-state interactions is usually much less reliable. This makes the estimates of the upper bounds on the "actual" T violation correspondingly unreliable. Therefore experiments of this type are gradually dying out.

If a strong interaction is present in the process, the final-state interaction corrections become of the same order as the Hermitean part of the amplitude, and the situation becomes completely hopeless.

The only exception, when a T-odd correlation provides *direct* evidence of T violation, is the transmission-type experiment. Then the quantity σ_{tot} which determines the transmission can be expressed via the optical theorem [see, e.g., Eq. (19)] as the imaginary part of the zero-angle elastic scattering amplitude $f(0)$. In this case the initial and final states coincide, $|i\rangle=|f\rangle$, and the T-invariance condition (92) by itself [*without approximation of (91)*] immediately gives (93) and (94).

There exist two types of those "true" T-odd correlations in the elastic forward scattering amplitude which can manifest themselves in neutron transmission experiments with nonzero target spins I.

One of them is the correlation $\vec{\sigma}_n(\vec{k}_n\times\vec{I})(\vec{k}_n\cdot\vec{I})$. One can measure this correlation in polarized-neutron transmission through oriented-nuclei samples. Observing that this correlation depends on the angle ϑ between $\vec{k}_n$ and the target alignment axis as $\sin 2\vartheta$, one immediately sees that the best observation conditions would be for $\vartheta=45°$ and for neutron spins $\vec{\sigma}_n$ directed parallel or antiparallel to the $(\vec{k}_n\times\vec{I})$ axis. The presence of a T-violating interaction would cause a difference in σ_{tot} for those two choices of the neutron polarization:

$$\Delta^T=\sigma_{\rightarrow}-\sigma_{\leftarrow}. \tag{96}$$

Using the optical theorem, one can express this quantity in terms of the T-violating part of the scattering amplitude f_T:

$$\Delta^T=\frac{4\pi}{k}\,\text{Im}\,f_T, \tag{97}$$

in complete analogy with Eqs. (19) and (36). Since experimentalists would always prefer to measure relative quantities rather than absolute ones [see (20)–(27)], the experimentally observed T-violation effect will be

$$\beta=\frac{\Delta^T}{\sigma_{\rightarrow}+\sigma_{\leftarrow}}\simeq\frac{\Delta^T}{2\sigma_{\text{tot}}}. \tag{98}$$

Observe that the above correlation is T-violating but P-conserving. The conventional name for it is "fivefold correlation" (FC).

There is also another correlation, namely, $\vec{\sigma}_n[\vec{k}_n\times\vec{I}]$, which is both P- and T-violating. This "triple correlation" (TC) must be measured with a polarized neutron beam and polarized target nuclei. Performing transmission experiments with beam polarization parallel or antiparallel to the $[\vec{k}_n\times\vec{I}]$ axis, one might observe the cross-section difference

$$\Delta_{PT}=\sigma_{\uparrow}-\sigma_{\downarrow}=\frac{4\pi}{k}\,\text{Im}(f_{\uparrow}-f_{\downarrow}) \tag{99}$$

and the corresponding P- and T-violation effect:

$$\eta=\frac{\Delta_{PT}}{\sigma_{\uparrow}+\sigma_{\downarrow}}\simeq\frac{\Delta_{PT}}{2\sigma_{\text{tot}}}. \tag{100}$$

In complete analogy with P-violating effects [see (29)], this correlation also causes a precession of the neutron spin around the $[\vec{k}_n\times\vec{I}]$ axis. The corresponding value of the rotation angle χ per unit length in a target sample is

$$\frac{d\chi}{dz}=\frac{4\pi\rho}{k}\,\text{Re}(f_{\uparrow}-f_{\downarrow}). \tag{101}$$

Under the optimal experimental conditions $z=1/N\sigma_{\text{tot}}$ the corresponding angle of rotation is

$$\chi=\frac{\text{Re}(f_{\uparrow}-f_{\downarrow})}{\text{Im}(f_{\uparrow}+f_{\downarrow})}. \tag{102}$$

3.3.2. Historical background

Although the general remark that nuclear reactions of strong dynamical complexity are most likely to be sensitive to T violation was made by Henley and Jackobsohn[47] long ago, this remark apparently passed unnoticed until the experimental discovery[48] of CP violation in K-meson decay. In the framework of the CPT theorem that implied T violation. This discovery brought a new wave of experimental and theoretical studies of T violation in nuclear reactions. First, experimental tests of T violation in detailed balance [TVDB; see (87)] were carried out[49] in 1966. Simultaneously, the first publications on nuclear-reaction theory in the presence of T violation appeared.[35,50] Mahaux and Weidenmüller[35] obtained a theoretical expression for T-violating amplitudes in the case of two near-lying compound resonances and were the first to understand this mechanism of resonance enhancement. However, both experimental and theoretical efforts at that time were concentrated on the energy domain of overlapping resonances, $\Gamma\gg d$ (Ericson regime). Therefore Ericson[50] claimed the enhancement parameter to be $\sqrt{W/\Gamma}\sim 10$, where W was supposed to be of the order of the spreading width of Eq. (7). Mahaux and Weidenmüller[35] pointed out that W should be much smaller, reducing the enhancement factor $\sqrt{W/\Gamma}$ to unity. Thus, the possibilities of an isolated-resonance region with really large resonance enhancements $d/\Gamma\gg 1$ remained unnoticed. Much later, Pearson and Richter[51] considered the TVDB for *one* isolated resonance. This case (for its analogs in P violation, see diagrams T_2 and T_4 of Fig. 1) in principle contains a resonance enhancement but lacks the dynamical enhancement factor $\sqrt{N}$ typical of a two-resonance interaction. Moreover, in the case of the TVDB experimental observable (see below) this resonance enhancement is completely canceled by the resonance enhancement of the T-invariant cross section in the denominator. Therefore this mechanism remained unnoticed, and the main theoretical investigations of the TVDB (Refs. 16 and 52–54) were centered on energy-averaged quantities for strongly overlapping resonances.

The full significance of both dynamical and resonance enhancements in T violation was first realized by Gudkov and the present author[4,7] in 1982, when we started the theoretical analysis of the newly suggested[55,56] P- and T-violating triple correlation in neutron transmission and predicted 5–6 orders of possible enhancement for this effect on p resonances interacting with nearby s ones. Later, we studied[38,57] the resonance enhancement of the P-conserving fivefold correlations (96)–(98). Similar investigations were made independently by Barabanov.[58] Ironically, in both investigations the resonance–resonance T-violating term in the amplitude (analog of the term T_1 in P violation) was unnoticed, and only the analogs of T_2 and T_4 were considered. This mistake was finally corrected in 1988 by the present author,[8] who realized the possibilities of both dynamical and resonance enhancements for this type of correlation. Since, however, the FC contains an extra factor $|\vec{k}|$ in comparison with the TC, this results in an extra hindrance factor [see Eqs. (74)–(76) and the discussion following them] of order kR which reduces the overall enhancement of the FC to more modest 2–4 orders of magnitude.

Detailed-balance tests for a close-lying pair of resonances in the isolated-resonance regime were first considered by Weidenmüller and the present author.[6] Both dynamical and resonance enhancement effects were found in that case, together with a possible "true" structural enhancement. However, the measured quantities in the TVDB show an even more complicated interference energy behavior, and the conditions for observing the maximal possible effect are even more involved. The net enhancement for realistic conditions was found to depend essentially on the experimental energy resolution and was estimated by us as 10^3–10^4. However, recently Mitchell and co-workers[9] generalized our analysis by including the angular dependence of the observable quantities. This led to an even more complicated two-dimensional picture for the effect as a function of the energy and angle. However, this more complicated analysis brought even more optimistic estimates. Analyzing their own high-resolution experimental data on (p,p) and (p,α) reactions, obtained at Duke, the authors proved that it is possible to obtain enhancements up to 10^4–10^5. This fact, together with various difficulties characteristic of other types of T-violating experiments (see below), makes the TVDB tests for interfering resonances perhaps the best possible method of observing T noninvariance in the near future.

3.3.3. T and P violation (triple correlation)

The triple-correlation quantities Δ_{PT} and $d\chi/dz$ of (99) and (100) can be analyzed in complete analogy with the P-odd quantities Δ_P and $d\Phi/dz$ (see Refs. 3 and 7) by substituting the T- and P-violating interaction iV_{PT} instead of the P-violating weak interaction V_W into the Born amplitude (39). Analysis of the resulting analog of Eq. (40) shows again that the $c-c$ mixing amplitude

$$T_1^{PT} \approx \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)} \tag{103}$$

dominates, since it contains both factors of dynamical v/D and resonance $D\Gamma$ enhancement.

Inserting this amplitude into (99), we obtain[3,7]

$$\Delta_{PT}=\frac{2\pi}{k^2} G_J \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{[(E-E_p)^2+\Gamma_p^2/4][(E-E_s)^2+\Gamma_s^2/4]} \times[(E-E_s)\Gamma_p+(E-E_p)\Gamma_s] \tag{104}$$

with

$$G_J=\frac{1}{2}\sqrt{\frac{3}{2(2I+1)}}\left[\sqrt{\frac{2I+1}{2I+3}}\,\delta_{J,I+1/2}\delta_{c,I-1/2} + \sqrt{\frac{I}{I+1}}\,\delta_{J,I-1/2}\delta_{c,I+1/2}\right]. \tag{105}$$

Here J is the total compound-resonance spin, and c is the channel spin.

At p- and s-resonance points we have maxima:

$$(\Delta_{PT})_{\text{res}} \approx \frac{8\pi}{k^2} G_J \frac{v_{PT}}{D} \frac{\gamma_p^n \cdot \gamma_s^n}{\Gamma}. \tag{106}$$

In exactly the same way we obtain

$$\frac{d\chi}{dz} = -\frac{4\pi}{k^2} G_J \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{[(E-E_p)^2+\Gamma_p^2/4][(E-E_s)^2+\Gamma_s^2/4]} \times\left[(E-E_p)(E-E_s)-\frac{\Gamma_s\Gamma_p}{4}\right]. \tag{107}$$

Observe that (107) changes sign at $E \approx E_p$ or E_s and has maxima at $E_{p,s} \pm \Gamma_{p,s}/2$:

$$\left(\frac{d\chi}{dz}\right)_{\text{res}} \approx \frac{8\pi}{k^2} G_J \frac{v_{PT}}{D} \frac{\gamma_p^n \cdot \gamma_s^n}{\Gamma}. \tag{108}$$

Introducing the scaling factor λ between the P- and T-violating interaction $\tilde{v}_{PT}$ and the weak one $\tilde{v}_P$, we see that on the average the P- and T-violating observed effects will be enhanced in the same p resonances as the P-violating ones:

$$\eta \approx \lambda P, \tag{109}$$

$$\chi \approx \lambda \Phi. \tag{110}$$

It is worth mentioning that all the present gauge-invariant theoretical models consider only simultaneous P and T violation, giving a wide range of λ between 10^{-4} and 10^{-15} (see, e.g., Refs. 46 and 59). Since the existing experimental constraints on the theory are given only by the case of K-meson decay and by the upper bound on the neutron electric dipole moment (EDM), there is a special branch of CP violation theory called "model-building"—anybody can construct his own brand of CP violation theory, provided that it does not contradict the above two experimental constraints. Moreover, λ enters those constraints in a model-dependent way. Thus *any* additional constraint on λ obtained from TC measurements could help a lot in narrowing the class of acceptable CP violation models. It also seems that after three decades of constantly improved techniques of EDM measurements the experimentalists had already exhausted their possibilities there. Note also that the EDM does not contain

the above six orders of enhancement, which somewhat resemble the good old Wolfenstein enhancement of CP violation in K mesons. Therefore FC measurements might rank as the highest-priority ones from the point of view of "fundamentality."

There are, however, grave difficulties with the idealized transmission experiments that we were analyzing above to search for FC, and we ourselves were the first to realize their presence.[60] Indeed, the simplest method seems to be to choose the $\vec{I}$ and $\vec{k}$ directions along, say, the x and z axes, while directing $\vec{\sigma}$ parallel or antiparallel to the y axis. However, in order to polarize the target we need an external (and rather strong) magnetic field $\vec{H}$. This field would cause a Larmor precession of $\vec{\sigma}$ around $\vec{H}$ as soon as the neutron enters the target. This precession in its turn produces nonzero helicities (of opposite signs for the initial cases of $\vec{\sigma}\uparrow\uparrow\vec{y}$ and $\vec{\sigma}\uparrow\downarrow\vec{y}$). Those helicities would cause the P-odd difference in transmission coming from the "normal" weak interaction and considered in the previous subsection. Moreover, this weak interaction would start rotating $\vec{\sigma}$ around the $\vec{k}$ direction as well [see (62)]. Therefore the neutron spin starts wobbling in three dimensions in an almost unpredictable way. Since all the CP-violating theories agree that $\lambda \ll 1$, those effects would completely camouflage the T-odd correlation that we are looking for. To make the situation worse, even if we manage to keep the target polarized without a strong external magnetic field, we will still face the so-called "nuclear pseudomagnetism" (see Ref. 61). This is the phenomenon caused by the $(\vec{\sigma}\cdot\vec{I})$-dependent part of the nuclear strong interaction, which imitates an external magnetic field causing $\vec{\sigma}$ precession around $\vec{I}$. Since this effect arises from the nuclear interaction, it is quite strong (usually equivalent to several kG of magnetic field).

The only crude remedy that we could suggest in 1984 (see Ref. 60) was to compensate the nuclear pseudomagnetic field by fine-tuning the external field $\vec{H}$ (this is in principle possible, since the direction of pseudomagnetic precession is not correlated with the neutron magnetic moment). We fully realized that such a solution is rather awkward in practice, since one needs to control this compensation by measuring the neutron spin rotation angle with high precision, $\delta\phi \sim 10^{-(3-5)}$. In principle a much higher precision of 10^{-6} was reached[27] in measurements of the P-violating value Φ of Eq. (75), but in our case one should perform these high-precision measurements simultaneously with the measurements of the FC itself.

On publishing this kind of "experimental" proposal[60] we expected to hear the reaction of professional experimentalists. Our expectations lasted for about a decade. Only in 1993 did we receive the first response. One suggestion was presented by the KEK group of Masuda[62] and actually contains a refined version of our magnetic-field fine-tuning. Another suggestion came from the PNPI group of Serebrov.[63] It involves simultaneous measurements of the polarization and asymmetry in the transmission of initially longitudinally polarized neutrons, and seems to be free of the above camouflaging effects. It remains, however, to check the energy dependence of the much more complicated observable suggested in this experiment in order to see whether the above enhancement effects survive in it and to estimate the accuracy of this experiment under realistic conditions.

3.3.4. P-conserving T-violating transmission (fivefold) correlation

We shall start with the T-violating part of the scattering amplitude f_T in Eq. (97). It can be expressed[57,58] for low-energy neutrons ($l=0,1,2$) in the form

$$f_T=\frac{A}{k}\left[3\sqrt{2}\begin{Bmatrix} I & 2 & I \\ I-1/2 & J & I+1/2 \end{Bmatrix}[\langle I+1/2,1|T^{TJ}|I-1/2,1\rangle-\langle I-1/2,1|T^{TJ}|I+1/2,1\rangle]\right.$$
$$+(-1)^{2I}\sqrt{\frac{3}{2I}}[\langle I+1/2,2|T^{TJ}|I-1/2,0\rangle-\langle I-1/2,0|T^{TJ}|I+1/2,2\rangle]\delta_{J,I-1/2}$$
$$\left.+(-1)^{2I}\sqrt{\frac{3}{2(I+1)}}[\langle I+1/2,2|T^{TJ}|I-1/1,0\rangle-\langle I-1/2,0|T^{TJ}|I+1/2,2\rangle]\delta_{J,I+1/2}\right], \qquad (111)$$

$$A=(2J+1)\sqrt{\frac{5I(2I-1)}{(2I+1)(2I+3)}}(-1)^{I+3/2-J}.$$

Here $T=1-S$, where S is the scattering matrix. The notation $\langle c',l'|T|c,l\rangle$ is used for the transition matrix elements, where l and l' are the orbital angular momenta of the initial and final channels, c and c' are the corresponding channel spins, and J is the compound-system spin. Upper T indices denote the T-violating part of the T matrix.

One might introduce, following Mahaux and Weidenmüller,[35] the quantity $\delta S_{\lambda\mu}=S_{\lambda\mu}-S_{\mu\lambda}$ arising in the presence of a T-noninvariant part V_T in the Hamiltonian, and consider the matrix $\delta S_{\lambda\mu}=-T^T_{\lambda\mu}$ in the case of two interacting resonances, using the wave-function expressions (18) for the initial and final channels μ and λ. This would give a whole set of amplitudes similar to those of Fig. 1—the role of the parity quantum number is "mimicked" in our case by the channel spins. The analysis of the various contributions to $T^T_{\lambda\mu}$ gives essentially the same results as in P violation. This analysis was schematically mentioned in connection with detailed-balance tests already in Ref. 35 and later in Refs. 5 and 6. As usual, it boils down[5] to the dominant contribution of the $c-c$ mixing term, which exhibits both dynamical and resonance enhancements:

$$T^T_{\lambda\mu}=4\sqrt{2\pi}\cdot i\,\frac{\gamma_{1\mu}\gamma_{2\lambda}(V_T)_{12}+\gamma_{2\mu}\gamma_{1\lambda}(V_T)_{21}}{(E-E_1+i\Gamma_1/2)(E-E_2+i\Gamma_2/2)}. \qquad (112)$$

Here E_i and Γ_i are the energies and total widths of the mixing compound resonances. The T-violating matrix elements between the compound states Φ_1 and Φ_2, $(V_T)_{12}=\langle\Phi_1|V_T|\Phi_2\rangle=-(V_T)_{21}$, are purely imaginary:

$$(V_T)_{12}=iv_T. \qquad (113)$$

Therefore

$$\text{Im } T^T_{\lambda\mu}=\frac{(\gamma_{1\mu}\gamma_{2\lambda}-\gamma_{2\mu}\gamma_{1\lambda})v_T}{[(E-E_1)^2+\Gamma_1^2/4][(E-E_2)^2+\Gamma_2^2/4]}$$
$$\times[(E-E_1)\Gamma_2+(E-E_2)\Gamma_1]. \quad (114)$$

Substituting (114) into (111) and (97), we get the terms of the $p-p$ resonance mixing (we omit the geometrical factors),

$$\Delta_T(p_1,p_2)\approx\frac{4\pi}{k^2}\frac{(\gamma^n_{1(-)}\gamma^n_{2(+)}-\gamma^n_{2(-)}\gamma^n_{1(+)})\cdot v_T}{[(E-E_1)^2+\Gamma_1^2/4][(E-E_2)^2+\Gamma_2^2/4]}$$
$$\times[(E-E_1)\Gamma_2+(E-E_2)\Gamma_1], \quad (115)$$

and the $s-d$ resonance mixing terms,

$$\Delta_T(s,d)\approx\frac{4\pi}{k^2}\frac{(\gamma^n_{s(-)}\gamma^n_{d(+)}-\gamma^n_{s(+)}\gamma^n_{d(-)})\cdot v_T}{[(E-E_s)^2+\Gamma_s^2/4][(E-E_d)^2+\Gamma_d^2/4]}$$
$$\times[(E-E_s)\Gamma_d+(E-E_d)\Gamma_s]. \quad (116)$$

The indices $(+)$ and $(-)$ stand for the channel spins. The order-of-magnitude estimates for the brackets with γ amplitudes in (115) and (116) are $(\gamma^n_p)^2$ and $\gamma^n_S\gamma^n_d$, respectively. Comparing this with the value Δ^p_{tot} of (61), we see that in both cases Δ_T is smaller by roughly a factor kR arising from the extra $|\vec{k}|$ value in the FC. Otherwise both (115) and (116) show dynamical and resonance enhancements. However, the optimal conditions for the maximal observable $\beta(E)$ of Eq. (98) are different from those of $P(E)$. Detailed analysis (see, e.g., Ref. 5) demonstrates that observations for a strong isolated p resonance [when $\sigma_p(E_p)\approx\sigma_{\text{tot}}(E_p)$] are most favorable. In this case the overall $(kR)^2$ smallness is compensated by the resonance enhancement $(D/\Gamma)^2$ to give

$$\beta_{\max}(E_p)\approx\frac{v_T}{D_{12}}\frac{\sigma_p(E_p)}{\sigma_{\text{tot}}(E_p)}. \quad (117)$$

In the vicinity of the s resonance one gets from (116) the result

$$\beta(E_s)\approx\frac{\gamma^n_d}{\gamma^n_s}\frac{v_T}{D_{sp}}\sim(kR)^2\frac{v_T}{D_{sp}}. \quad (118)$$

The additional small kR factor makes those observations impractical. The main trouble with observations of (116) for a d resonance lies in the fact that exceedingly small Γ^n_d values make those resonances unobservable in σ_{tot}. If only we knew E_d in advance, then

$$\beta(E_d)\approx\frac{\gamma^n_d}{\gamma^n_s}\frac{v_T}{\Gamma_d}\frac{D_{sp}}{\Gamma_s}\frac{\sigma_s(E_d)}{\sigma_{\text{tot}}(E_d)}\sim(kR)^2\frac{v_T}{D_{sd}}\left(\frac{D_{sd}}{\Gamma}\right)^2. \quad (119)$$

Observe that for very small D_{sd} the quantity (119) decreases drastically and transforms into (118) for $D_{sd}\approx\Gamma$. If the d resonance lies sufficiently far from the s resonance (say, $D_{sd}\sim10$ eV), then the resonance enhancement factor $(D_{sd}/\Gamma)^2$ in (119) might almost compensate the smallness of $(kR)^2\sim10^{-5}$–10^{-6}. Since, however, the E_d are not known in advance, the d-resonance enhancement seems to be of purely academic interest, as was pointed out in Ref. 5. This point was misunderstood by the authors of Ref. 30, who "rediscovered" the $s-d$ mixing two years later (see also Ref. 64). Unfortunately, the only known target of ^{165}Ho suitable for FC measurements does not show any p-wave resonances.[64] Therefore the above possibilities of resonance enhancement (117) in the FC were not used by the experimentalists up to now.

3.3.5. Detailed-balance tests (TVDB)

The simplest quantity which describes the T violation in detailed balance TVDB is [see Eq. (87)]

$$\Delta_{DB}(E,\vartheta)=2\frac{\sigma_{ab}(E,\vartheta)-\sigma_{da}(E,\vartheta)}{\sigma_{ab}(E,\vartheta)-\sigma_{da}(E,\vartheta)}. \quad (120)$$

Since, as usual, the Δ_{DB} value is a ratio of experimentally measured quantities, we included the kinematic factors $k_a^2(2s_a+1)(2s_A+1)$ in the quantity $\sigma_{ab}(E,\vartheta)=k_a^2(2s_a+1)(2s_A+1)d\sigma_{ab}/d\Omega(E,\vartheta)$. To simplify our analysis we shall also restrict ourselves to only the E dependence of the cross section, omitting the ϑ dependence for the time being. Thus, (120) would read

$$\Delta_{DB}(E)=2\frac{\sigma_{ab}(E)-\sigma_{ba}(E)}{\sigma_{ab}(E)+\sigma_{ba}(E)}. \quad (121)$$

Any statistically meaningful deviation of this quantity from 0 would imply a T violation. However, the experimental accuracy for absolute cross-section measurements is much lower than for relative ones. Therefore it is preferable to perform measurements at least at two different energy points E_{I} and E_{II} and to construct the quantity

$$\tilde{\Delta}_{DB}(E_{\text{I}},E_{\text{II}})=\frac{\sigma_{ab}(E_{\text{I}})\sigma_{ba}(E_{\text{II}})}{\sigma_{ab}(E_{\text{II}})\sigma_{ba}(E_{\text{I}})}-1. \quad (122)$$

Here one of the points, say, E_{I}, is chosen for the normalization of the ratio. This allows us to cancel most of the systematic errors. This can be seen, since to first order in $\Delta(E)$ we have

$$\hat{\Delta}_{DB}(E_{\text{I}},E_{\text{II}})\simeq\Delta_{DB}(E_{\text{I}})-\Delta_{DB}(E_{\text{II}}). \quad (123)$$

Therefore in most cases we shall proceed by working with the simplest form (121). In doing so we shall use the general expression obtained in Ref. 35 for the difference $\delta S_{ab}=S_{ab}-S_{ba}$ between the S-matrix elements connecting channels a and b and caused by the presence of the T-violating part V_T in the Hamiltonian. As usual, we shall consider the situation for only a pair of close-lying compound resonances 1 and 2. As we have already mentioned in the previous subsection, the use of the wave functions (18) would give, to first order in V_T, amplitudes similar to those considered in P violation (Figs. 1 and 2). These amplitudes are characterized by the same dynamical, v/D, and resonance, D/Γ, enhancement factors. Therefore the dominant contribution to δS_{ab} would come from the $c-c$ mixing amplitude of the type T_1:

$$\delta S_{ab}=4\sqrt{2\pi}\frac{(\gamma_{2a}\gamma_{1b}-\gamma_{2b}\gamma_{1a})\cdot v_T}{(E-E_1+i\Gamma_1/2)(E-E_2+i\Gamma_2/2)}. \quad (124)$$

Here $v_T = -i\langle\Phi_1|V_T|\Phi_2\rangle = i\langle\Phi_2|V_T|\Phi_1\rangle$ is the matrix element of the interaction V_T between the compound-resonance wave functions Φ_1 and Φ_2 [see (17)].

Now, for the numerator of (121) we have

$$\sigma_{ab} - \sigma_{ba} = 2[\mathrm{Re}(\delta S_{ab})\mathrm{Re}(S^0_{ab}) + \mathrm{Im}(\delta S_{ab})\mathrm{Im}(S^0_{ab})]. \tag{125}$$

The corresponding expression for the denominator is

$$\frac{1}{2}(\sigma_{ab}+\sigma_{ba}) = |S^0_{ab}|^2 = \left|\frac{\gamma_{1a}\gamma_{1b}}{E-E_1+i\Gamma_1/2} + \frac{\gamma_{2a}\gamma_{2b}}{E-E_2+i\Gamma_2/2}\right|^2. \tag{126}$$

Then for the case of two weakly overlapping resonances ($\Gamma < |E_1 - E_2| \equiv D$) we obtain (see Ref. 6)

$$\Delta_{DB}(E) = \frac{v_T\cdot(|\gamma_{1b}\gamma_{2a}| - |\gamma_{2b}\gamma_{1a}|)(\Gamma_1|\gamma_{2a}\gamma_{2b}| + \Gamma_2|\gamma_{1a}\gamma_{1b}|)}{[(E-E_2)|\gamma_{1a}\gamma_{1b}| + (E-E_1)|\gamma_{2a}\gamma_{2b}|]^2 + 1/4(\Gamma_2|\gamma_{1a}\gamma_{1b}| + \Gamma_1|\gamma_{2a}\gamma_{2b}|)^2}. \tag{127}$$

The analysis of this expression shows that it reaches its maximal value at the interference minimum of the cross sections $|S^0_{ab}|^2$, i.e., when

$$E = E_0 \equiv \frac{|\gamma_{1a}\gamma_{1b}|E_2 + |\gamma_{2a}\gamma_{2b}|E_1}{|\gamma_{1a}\gamma_{1b}| + |\gamma_{2a}\gamma_{2b}|}. \tag{128}$$

In the case of $\Gamma_1 \approx \Gamma_2 = \Gamma$, this yields for (127) the result

$$\Delta_{DB}(E_0) \simeq 4\,\frac{v_T}{\Gamma}\,\frac{|\gamma_{2b}\gamma_{1a}| - |\gamma_{1b}\gamma_{2a}|}{|\gamma_{1a}\gamma_{1b}| - |\gamma_{2a}\gamma_{2b}|}. \tag{129}$$

Supposing for simplicity that $\Gamma_{1a}\Gamma_{1b} \approx \Gamma_{2a}\Gamma_{2b}$ (i.e., equally strong resonances), we obtain

$$\Delta_{DB}(E_0) = 2\,\frac{v_T}{\Gamma}\left[\left|\frac{\gamma_{2b}}{\gamma_{1b}}\right| - \left|\frac{\gamma_{2a}}{\gamma_{1a}}\right|\right] \equiv \frac{v_T}{\Gamma} f, \tag{130}$$

where $f = 2[|\gamma_{2b}/\gamma_{1b}| - |\gamma_{2a}/\gamma_{1a}|]$, and the position of the interference dip of σ_{ab} is given by

$$E_0 \approx \tfrac{1}{2}(E_1 + E_2).$$

Thus, we observe how the already familiar factors of dynamical and resonance enhancement give in (130) the overall enhancement factor v/Γ. We also see that (130) can sometimes be enhanced even more by the "real" structural enhancement factor f [cf. Eq. (73a) for the "inelastic-channel" observable α_{nf} in P violation]. To be realistic, however, one should take into account the finite experimental energy resolution ΔE, which usually exceeds Γ and smears the whole interference picture, bringing the observed effect down to $(v_T/\Delta E)f$. Making now the conservative assumption $f \approx 1$, we find the overall enhancement factor in $\Delta_{DB}(E_0)$ to be

$$\frac{\tilde{v}}{\Delta E}. \tag{131}$$

Remember that $\tilde{v}$ is the variance of the strong-interaction matrix element [see (9)], which is of the order of 1 MeV. Therefore the net enhancement of the TVDB is about 10^3.

This was essentially the result of our analysis with Weidenmüller.[6] As I have already mentioned, recently the TUNL–Duke group,[9] which is a world authority in fine-resolution experiments, generalized our approach to include the ϑ dependence of Eq. (120) in it. After performing a tedious analysis of their own experimental data on (p,p) and (p,α) reactions, they concluded that there are *real experimental* situations when the enhancement in the TVDB reaches 10^4–10^5. In view of the specific difficulties which mar the TC and FC experiments, this seems to be the most realistic experimental method for T-invariance measurements in the near future.

3.3.6. Brief summary of possible enhancements

Thus, we have seen that the dominant contribution to all the above effects of T violation comes from the c–c mixing amplitude of the type T_1 in Fig. 1. This amplitude contains two basic enhancement mechanisms–dynamical enhancement $\tilde{v}/D$ and resonance enhancement D/Γ. Since, however, the experimental observables are different, those mechanisms manifest themselves differently. Therefore under optimal conditions we might have the following enhancements:

For the TC in the vicinity of E_p, provided that $\sigma_s(E_p) \approx \sigma_p(E_p) \geqslant \sigma_{\mathrm{pot}}$ [see (82a)],

$$\eta_{\max} \sim \frac{\tilde{v}}{\Gamma}, \quad \chi_{\max} \sim \frac{\tilde{v}}{\Gamma}. \tag{132}$$

For the FC in the vicinity of a strong isolated p resonance $[\sigma_p(E_p) \geqslant \sigma_s(E_p) + \sigma_{\mathrm{pot}}(E_p)]$,

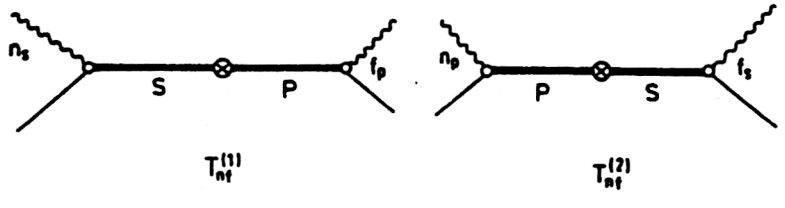

FIG. 2. Two P-violating compound–compound mixing processes in the (n,f) inelastic channel.

$$\beta_{\max} \sim \frac{\tilde{v}}{D}. \tag{133}$$

In this case the extra factor kR with respect to the FC cancels D/Γ.

For the TVDB at the interference minimum of two close-lying resonances,

$$\Delta_{DB}^{\max} \sim \frac{\tilde{v}}{\Gamma} f_1(\Theta), \tag{134}$$

where $f_1(\vartheta)$ might serve as an additional enhancement factor. For realistic conditions, when the experimental energy resolution is $\Delta E > \Gamma$, we have

$$\Delta_{DB}^{\max} \sim \frac{\tilde{v}}{\Delta E} f_1(\Theta). \tag{135}$$

The case of greatest interest to the "high-brow" gauge theories of CP violation is the TC, since it is both T- and P-violating. However, we recall that the estimates (152) were obtained by us[4,7] for an idealized experimental case without Larmor and pseudomagnetic effects.

Among the remaining, purely T-violating effects, the TVDB has obvious advantages over the transmission FC. Its enhancements (135) for already known experimental cases reach 10^4–10^5, while the only available target for the FC shows no p resonances and therefore lacks even the enhancement of (133).

4. STATISTICAL APPROACH TO COMPOUND-RESONANCE MEASUREMENTS

4.1. Nuclear chaos and the need for a statistical approach

The analysis of the previous section shows that in the isolated-resonance regime with $\Gamma \ll d$ (d is the *average* spacing between compound resonances of the same spin) the symmetry-breaking interaction of a pair of close-lying resonances with energy separation D leads to two major enhancements—dynamical enhancement v/D and resonance enhancement D/Γ, which very often combine with other specific factors of nuclear-reaction theory to produce the net enhancement v/Γ, reaching 5–6 orders of magnitude. Both enhancements result from the complexity of nuclear compound resonances and practically disappear in simple nucleon–nucleon scattering [see Eq. (47)]. To be more specific, they manifest quantum chaos, the idea of which is still rejected by the majority of professional "chaotists" but is accepted (at any rate, at an intuitive level) by all nuclear physicists. Establishment of the connection between the fully recognized chaos of classical mechanics and the quantum chaos of nuclear physics is an interesting and promising problem (see, e.g., Ref. 36), whose solution shows that a generic feature of *any* chaoticity (both in quantum and in classical mechanics) is the absence of symmetries in the Hamiltonian of the system. In a compound nucleus this absence of symmetries is caused in the first place by the strong pair-wise "residual" interactions, which remove practically all the degeneracies connected with the mean-field symmetries (let us call this "strong" chaos) and thus lead to an exponential increase of the level density $1/d$. This chaoticity also increases the complexity N of the compound-resonance wave function and randomizes the signs of its basic ("simple") component amplitudes. In plain words this means that the incident nucleon quickly distributes its energy among the target nucleons and gets trapped in the compound system. This, together with small barrier penetration factors, strongly reduces the contribution of particle-emission channels to the resonance total width Γ, leaving only the γ emission. The same complexity of the resonance wave functions considerably reduces the gamma widths. All this results in large compound-resonance lifetimes $\tau \sim 1/\Gamma$ which enter the above enhancement factors. This seems to be a rare occasion when complexity helps us, leading directly to six orders of magnitude of enhancement of the experimentally observed effects.

However, one has to pay for everything. This complexity of the compound-resonance wave functions Φ makes direct calculations of the "weak-chaos" symmetry-breaking (WSB) matrix elements $\langle \Phi_1 | V_{SB} | \Phi_2 \rangle$ completely hopeless. Therefore, even if we observe the WSB effect and manage to extract the corresponding v_{WSB} from it (which might be a problem in itself), we seem to learn nothing about the strength constant of the WSB interaction. Coming back to the origin of strong chaos we see that the above problem with the weak-interaction matrix elements v_{WSB} differs from the same problem with the strong-interaction ones only by the strength constants F [see (6)–(9)] which we are seeking. The problem of strong quantum chaos and strong symmetry-breaking matrix elements was faced and physically understood at the dawn of nuclear physics and led to Niels Bohr's hypothesis of the compound nucleus, which does not "remember" its formation, and to Weisskopf's idea of a black absorbing nucleus. A more refined mathematical technique of nuclear-Hamiltonian random matrices was developed by Wigner, Dyson, Mehta, and other outstanding physicists in the 1950s. In this approach the expansion coefficients c_i of Φ as well as the matrix elements $\langle \Phi_i | V | \Phi_k \rangle$ are considered to be random variables varying from resonance to resonance, while the quantity v for the ensemble of individual resonances obeys the normal-distribution law with zero mean and variance $\tilde{v} = \sqrt{\langle v^2 \rangle}$. This led to various statistical predictions concerning the properties of resonances, which were all brilliantly confirmed experimentally: the Wigner-distribution law for the level spacing, the Porter–Thomas law for the neutron-width distribution, and corresponding laws for the γ-width distributions. Therefore the problem of WSB presented practically nothing new to us. This fact was intuitively recognized in the analysis of WSB for a pair of isolated bound states or resonances from the very beginning (see, e.g., the concept of the dynamical enhancement factor $\sim \sqrt{N}$ in Ref. 15). Therefore all the order-of-magnitude theoretical estimates of the enhancements were made from the very beginning (see, e.g., Refs. 4, 7, and 26) for ensemble-averaged variances. We also predicted the sign randomness of the observed effects in the $c-c$ mixing mechanism and a possible constancy of sign (connected, however, with the loss of the dynamical enhancement factor) for the valence mechanism.[7] However, the intricacies of specific nuclear-reaction en-

hancements for various observables discussed above were so exciting, while the statistics of the experimental observations was so meager, that we were postponing the problem of meaningful analysis for experimentally observed values.

4.2. Energy averaging

Meanwhile, a highly professional and sophisticated statistical theory of symmetry breaking in nuclear reactions was developing. For purely historical reasons it was essentially concentrated on TVDB effects. After the observation was made[47] that in the two-channel case detailed balance follows from unitarity alone (without T invariance), theoretical[50,52–54] and experimental[49,65] interest shifted to the domain of many open channels and strongly overlapping resonances ($\Gamma \gg d$). Even the explicit appearance of Γ in the denominator of the TVDB energy-averaged expression[50] remained unnoticed. Mahaux and Weidenmüller derived the two-resonance expression (112), (124), which two decades later allowed me[5] and Weidenmüller and me[6] to see the large enhancements in the FC and TVDB discussed in the previous section. But they also applied it to the $\Gamma \gg d$ regime of Ericson fluctuations to show that the only enhancement in this regime is the structural factor f [see Eq. (130)]. Later, Moldauer[52] obtained the same results in the R-matrix formalism, again sticking to the $\Gamma \gg d$ region. He also considered the TVDB in direct reactions (see also Ref. 66), proving that the direct-reaction contribution to the T violation is three orders of magnitude smaller than that of the compound-resonance mechanism for $\Gamma \gg d$ (unfortunately, in obtaining this result he formulated it in a somewhat misleading way—the direct-reaction sensitivity to the T violation is three orders of magnitude smaller than that of compound-resonance reactions).

A new wave of statistical approach to the energy-averaged TVDB which involved the full-scale mathematics of random-matrix theory developed in Refs. 67 and 68 started with Refs. 53 and 54. Only in 1989 was the technique developed in those papers first applied by Davis[69] to numerical calculations of the energy-averaged FC and TVDB—numerical, since the method involved the computation of rather complicated multidimensional integrals. An "informed guess" allowed Davis to approximate in semianalytical form the results of exact numerical integration for the energy-averaged values of $\langle \Delta_T^2 \rangle$ [see Eq. (115) for the FC] and $\langle (\sigma_{ab} - \sigma_{ba})^2 \rangle$ [see Eqs. (124) and (125) for the TVDB] in the limit of isolated resonances, $\Gamma < 0.1d$. The corresponding expression for the FC was given (omitting the trivial factor $4\pi/k^2$) as

$$\langle \Delta_T^2 \rangle \approx 4 T_{n(-)} T_{n(+)} \frac{2\pi^2 \langle v_T^2 \rangle}{d^2} \frac{(0.35 - 0.17 \ln t)}{t}, \tag{136}$$

where the neutron transmission coefficients for channel spins $(\pm)$ are given by

$$T_{(\pm)} = \frac{2\pi \Gamma_{n(\pm)}}{d}; \quad t = \frac{2\pi \Gamma}{d}.$$

Since by this time the resonance-enhancement effect for two interacting isolated resonances was already recognized, Ref. 69 was the first recognition of the fact that resonance enhancement survives also in *energy-averaged* effects. Moreover, the need to pay attention to the isolated-resonance regime ($\Gamma \ll d$) was also accepted. However, a statement was made in favor of using *energy averages* of observables for this regime as opposed to the statistics of individual *on-resonance* observations, which I (and the experimentalists) kept in mind all the time. It took me some time and a bit of reasoning to persuade the author of of Ref. 69 that his point of view is more academic than practical. My arguments were as follows: Consider the FC expression (115) in the Γ vicinity of each resonance energy, assuming the typical case $(E_1 - E_2) \approx d$, $\Gamma_1 \approx \Gamma_2 \approx \Gamma$:

$$\Delta_T^{\text{res}} \approx 4 \frac{v_T}{d} \frac{a_{12}}{\Gamma}. \tag{137}$$

Here $a_{12} = \gamma^n_{1(-)} \gamma^n_{2(+)} - \gamma^n_{2(-)} \gamma^n_{1(+)}$, and the factor $4\pi/k^2$ is omitted for the sake of comparison with (136). The same expression for (113) in a typical situation between the resonances, $|E - E_1| \approx |E - E_2| \sim d$, would be

$$\bar{\Delta}_T = 2 \frac{v_T}{d} \frac{a_{12}}{d} \frac{\Gamma}{d}. \tag{138}$$

This expression is a factor of $(d/\Gamma)^2$ smaller than Δ_T^{res} Let us estimate in a simple-minded way the energy-averaged effect (136), starting from (137) and (138) and averaging their squares over an interval d. Then the $(\Delta^{\text{res}})^2$ contribution should be weighted by roughly a factor of Γ/d, while the $(\bar{\Delta})^2$ should have a weighting factor $(d - \Gamma)/d \approx 1$:

$$\langle \Delta_T^2 \rangle \approx (\Delta_T^{\text{res}})^2 \frac{\Gamma}{d} + \bar{\Delta}_T^2 \approx 16 \frac{v_T^2}{d^2} \frac{a_{12}^2}{\Gamma^2} \frac{\Gamma}{d} + 4 \frac{v_T^2}{d^2} \frac{\Gamma^2}{d^2}. \tag{139}$$

Thus, we calculated the contribution to (136) from the interval d around one particular resonance. It remains now to average the parameters v_T^2 and a_{12}^2 over all the possible resonances. Since the γ's in a_{12} are uncorrelated random variables, $\langle a_{12}^2 \rangle = 2\Gamma^n_{(-)} \Gamma^n_{(+)}$. Using the Γ/d smallness, we shall retain in (138) *only the on-resonance contribution*. Thus,

$$\langle \Delta_T^2 \rangle \approx 32 \frac{\Gamma_{n(-)}}{d} \frac{\Gamma_{n(+)}}{d} \frac{\langle v_T^2 \rangle}{d^2} \frac{d}{\Gamma}$$

$$= 4 T_{n(-)} T_{n(+)} \frac{2\pi^2 \langle v_T^2 \rangle}{d^2} \frac{2}{\pi^2 t}. \tag{140}$$

We see that this crude but simple picture almost exactly reproduces the results of (136). The only marked difference is substitution of the factor $2/\pi^2 \approx 0.2$ for $(0.35 - 0.17 \ln t)$. This difference comes essentially because we took from the start a fixed value of d for the interresonance distance $(E_1 - E_2)$. The actual distribution of this distance obeys a more complicated two-level correlation law (see the correlation function R_2 of Ref. 70), which, apart from the Wigner repulsion at very small distances, allows all possible values between 0 and d. If one allows $(E_1 - E_2)$ to vary in this way, the distances smaller than d will contribute more to the resonance effect (137), thus increasing the value of $\langle \Delta_T^2 \rangle$. Indeed,

energy integration of (115) with the correlation function (see Ref. 71) gives an analytical result exactly equal to (136).

The main point of the above simple arithmetic is that *only the small* Γ *vicinity of each resonance contributes* to $\langle\Delta^2\rangle$. It turns out therefore that in order to compare my theoretical value (136) with experiment and to extract $\langle v_T^2\rangle$, I ask the experimentalist to make accurate measurements not only on the resonance curve, but also in the whole "empty" interval $d \gg \Gamma$ between the resonances. I know for sure that all these tedious off-resonance measurements would give null results. I also know that in this way I would decrease the sensitivity of the $\langle v_T^2\rangle$ determination by a factor of d/Γ. In order to compensate this loss, the experimentalist has to increase the accuracy in measuring each null effect by, say, increasing the beam flux by a huge factor $(d/\Gamma)^2$. One should also recall that transmission experiments around the strong s resonances are impossible, or at best suffer from poor statistics. All this makes an energy-averaged calculation *for isolated resonances* almost devoid of any practical meaning. I want to stress this point because of repeated attempts to compare the energy-averaged quantities in this regime with experimentally observed ones. Even in our joint paper,[71] after reading the above critical comments on energy averaging, one meets a vague statement that it is still appropriate for ^{165}Ho, where there are no observable p-wave resonances below 100 eV. This statement is again a purely academic one[1)]—if we observe no p resonances in σ_{tot}, this just means that either the p-resonance spacing d in this energy region is anomalously large or that the Γ_p^n are anomalously small. Both facts should somehow be taken into account in the "unbiased" estimates (136) as additional biasing, and this would lower its value. Even then it would be necessary to remove from the biased estimate (136) the unobservable regions around strong s resonances. One might still hope to get something from averaging the $s-d$ mixture term (116). Apparently, one should get for $\langle\Delta_T^2(s-d)\rangle$ practically the same value as (136). However, here there is another very serious danger of energy averaging. We have already noticed that the $(-\ln t)$ term in brackets of (136) which dominates in our case of small t appears because the unbiased energy-averaging procedure favors the situations when the mixing resonances lie anomalously close to each other $(D_{sp} \ll d)$. But it is exactly in those situations that the denominators of observable β [see (98)] would exhibit strong s-resonance maxima, thus reducing the resonance enhancement of the numerators practically to zero. Therefore the averaged *observed* quantity β would deviate strongly from the calculated value $\langle\langle\Delta^2\rangle\rangle^{1/2}/\langle\sigma_{\text{tot}}\rangle$ because of the strong correlation of the numerator and denominator. In terms of experimentally measured quantities of the type (21)–(25) this means that experimental measurements would be impossible or would give extremely poor statistics in exactly the same energy intervals (near strong s resonances) which contribute most to the calculated values of $\langle\Delta^2(s-d)\rangle$. Together with the loss of sensitivity by a factor of Γ/d and the abundance of strong s-wave resonances in the experimentally observed spectrum of Ho, this just means that Ho measurements might be a waste of time. Conclusions with the same effect concerning the energy-averaged estimates of P-violation observables made by Koonin *et al.*[72] with the aid of optical-model functions were reached by Weidenmüller and Lewenkopf.[34] Thus, the averaged quantity lacks a resonance enhancement and *bears no relation to the experimental observable P of Eq. (20).*

To conclude the discussion of energy averaging, I mention a recent publication[73] on FC experiments with 2-MeV polarized neutrons in ^{165}Ho, where the theoretical analysis is made in the spirit of a very simple model of T violation in direct reactions considered almost 30 years ago by Moldauer.[66] I have already mentioned that Moldauer had actually shown[52,66] that the contribution to the T-violating amplitudes from the compound-resonance mechanisms is about three orders of magnitude larger than from the direct-interaction ones—a fact which is quite obvious in terms of resonance-enhancement physics (the time spent by the particles inside the T-violating nuclear field is much larger for compound processes than for direct ones). Therefore I cannot understand why (apart from its extreme simplicity) the authors of Ref. 73 applied the direct-reaction analysis to their data. This is especially strange, since this experimental energy range might be the most appropriate place to apply the "big guns" of complicated numerical integration developed[69,74] by one of the authors of Ref. 73.

4.3. "On-resonance" ensemble averaging

Having thus discussed the drawbacks of "unbiased" energy averaging in the isolated-resonance regime with $\Gamma \ll d$, we now return to the natural idea discussed in Sec. 4.1; namely, we follow the lines of the well-developed statistical approach to "strong" chaos of neutron resonances and consider the ensemble of weak-interaction v_{pi} values, measured for different p resonances as an ensemble of random variables obeying the normal-distribution law with zero mean and variance $\tilde{v}_p \equiv M = \sqrt{\langle v_p^2\rangle}$:

$$P(v_p) = \frac{1}{\sqrt{2\pi M^2}} \exp\left(-\frac{v_p^2}{2M^2}\right). \tag{141}$$

Thus, any particular value of v_p obtained from one on-resonance measurement is of minor importance, and the main interest is shifted to the variances M. If we are able to extract the value of M from a set of on-resonance observations (and we shall see below how intricate this extraction might be), then we can use the scaling trick [see Eqs. (6)–(8)] and compare this value with its equivalent $\tilde{v}$ for strong interactions, obtained from the well-established spreading width Γ_{spr} of Eq. (7). This comparison would give us the scaling constant

$$F = \frac{M}{\tilde{v}} \tag{142}$$

or, at any rate, an upper bound on it.

Everything seems fine in such a simplified scheme. However, the realistic situation is much more complicated. To begin with, in deriving all the expressions for weak symmetry-breaking quantities of the previous section we retained for simplicity only one resonance in the initial and one

in the final channel. In principle all the observables of Sec. 3 should contain a double sum over all the mixing resonances. Since we consider the on-p-resonance measurements in the isolated-resonance regime, the contributions of all the other p resonances to the observed effect would be smaller by at least a factor of $(d/\Gamma)^2$ and can be discarded. However, the other sum still remains, and the correct expression for, say, $\Delta^P_{\rm tot}$ at E_p would be

$$\Delta^P_{\rm tot}(E_p)=\frac{8\pi}{k^2}\frac{\gamma^n_{pi}}{\Gamma_{pi}}\sum_s\frac{\langle s|V_W|p_i\rangle}{E_p-E_s}\equiv\sum_s A_{is}(v_p)_i,$$

$$A_{is}=\frac{8\pi}{k^2}\frac{\gamma^n_{pi}\cdot\gamma^n_s}{E_p-E_s},\quad (v_p)_i=\langle s|V_W|p_i\rangle. \tag{143}$$

Assuming that all the parameters γ^n_p, γ^n_s, and E_s are known, $\Delta^P_{\rm tot}(E_{pi})$ is a sum of Gaussian-distributed random variables $(v_p)_i$ with fixed coefficients. Since a sum of Gaussian random variables is also a Gaussian random variable, $\Delta^P_{\rm tot}(E_p)$ itself is a Gaussian. However, it is not ergodic in the sense of Ref. 75. Ergodicity here means that the statistical-ensemble average of an observable (whose behavior we know theoretically) is equal to the running average of the same observable taken over a set of experimentally observed resonances. For $\Delta^P_{\rm tot}(E_p)$ to be ergodic, it is necessary that it should be independent of the parameters of the actually investigated resonances. In plain words this means that we should get rid of all the trivial constants known for each particular resonance p_i and consider the new value whose variance is given by $\langle v^2_p\rangle$. Since all the $(v_p)_i$ in (143) have the same variance, such an ergodic variable in the present case would be

$$\Delta^P_i=\frac{\Delta^P_{\rm tot}(E_{pi})}{|\Sigma_s A^2_{is}|^{1/2}}. \tag{144}$$

In the approximation of one s resonance, $\Delta^P_i=(v_p)_i$.

Let us now slightly complicate the situation. Before doing this, we shall return to the original expression (61) for $\Delta^P_{\rm tot}$. In order to simplify the derivation, we considered the case of a zero-spin target, $I=0$. In this case the total compound-resonance spin J is completely determined by $\vec{j}=\vec{l}+\vec{s}$ of the neutron, and the partial width of the p resonance admixed to the s one is simply $\Gamma^n_p=\Gamma^n_{p1/2}$, where 1/2 is the neutron j value. If, however, we remove the restriction $I=0$, then the p-resonance partial width would contain two components, $\Gamma^n_p=\Gamma^n_{p1/2}+\Gamma^n_{p3/2}$. In the channel-spin representation [see, e.g., (111)–(116)] this corresponds to two different values ($\pm$) of the channel spin c. Since the T-invariant interaction conserves c, only the widths $\Gamma^n_{p1/2}$ (and the corresponding amplitudes $\gamma^n_{p1/2}$) would enter the expression for $\Delta^P_{\rm tot}$. However, in the majority of cases we know only the total Γ^n_p, while $\gamma^n_{p1/2}$ and $\gamma^n_{p3/2}$ are unknown. Therefore for $I\neq 0$ even the simplest form of on-resonance $\Delta^P_{\rm tot}$ would be

$$\Delta^P_{\rm tot}(E_{pi})=\frac{8\pi}{k^2}\frac{\gamma^n_s}{\Gamma_{pi}(E_s-E_{pi})}\cdot\gamma^n_{p1/2}\cdot v_p\equiv B_{pi}\gamma^n_{pi}v_{pi}, \tag{145}$$

i.e., the product of a known constant B_p and the unknown $\hat{\Delta}_p=\gamma^n_{p1/2}v_p$. Now for different choices of p resonances $\gamma^n_{p1/2}$ behaves as a Gaussian random variable with zero mean and variance $\langle\Gamma^n_{p1/2}\rangle$, which can be easily related to, say, the neutron strength function. The p-wave amplitude $\gamma^n_{p1/2}$ and the matrix element v_p are independent random variables. Thus, we can introduce an on-resonance ensemble of ergodic values:

$$\hat{\Delta}_{pi}=\frac{\Delta^P_{\rm tot}(E_{pi})}{B_i}. \tag{146}$$

These random values are, however, distributed according to the law which governs the distribution of a product of two independent Gaussian random variables:

$$P(\hat{\Delta}_p)=\frac{1}{\pi\omega}K_0(\hat{\Delta}_p/\omega), \tag{147}$$

where K_0 is a Macdonald function and $\omega^2=\langle\Gamma^n_{p1/2}\rangle\langle v^2_p\rangle$ determines the variance of $\hat{\Delta}_p$.

In the many-level generalization we have [see (143)]

$$\Delta^P_{\rm tot}(E_p)=\gamma^n_{p1/2}\sum_s B_{ps}(v_p)_s, \tag{148}$$

and we can introduce the ergodic variable

$$\hat{\Delta}_i=\frac{\Delta^P_{\rm tot}(E_{pi})}{|\Sigma_s B^2_{is}|^{1/2}}, \tag{149}$$

which obeys the same distribution law (147).

Observe how the lack of knowledge of only one additional parameter $\gamma^n_{p1/2}$ of "standard" nuclear spectroscopy complicates the statistical analysis of the WSB and the data interpretation—instead of the well-known analytic Gaussian shape of (142) for Δ^p_i, we get the much more complicated law (148) for $\hat{\Delta}_i$. One can also be sure that the statistical confidence levels of the variances extracted with the help of (148) would be much lower than those obtained using (142).

The case of the T-violating FC [Eq. (116)] when generalized for many levels gives the on-resonance expression:

$$\Delta_T(p)\approx\frac{16\pi}{k^2}\frac{1}{\Gamma_p}\sum_k(\gamma^n_{p1/2}\gamma^n_{pk3/2}-\gamma^n_{p3/2}\gamma^n_{pk1/2})\frac{(v_T)_k(E_p-E_{pk})\Gamma}{(E_p-E_{pk})^2+\Gamma^2/4}. \tag{150}$$

In a rather optimistic case we may know in this expression the parameters of the p resonance where we perform the measurements, namely, E_p, $\Gamma_p\approx\Gamma_{p_k}=\Gamma$, $\gamma^n_{p1/2}$, and $\gamma^n_{p3/2}$. Since our information about p resonances in general is very poor, the rest of the parameters in (150) are very likely to be unknown. The analysis of this situation in Ref. 71 demanded numerical Monte-Carlo simulation for the distribution of distant E_{p_k} and made it possible to construct two approximate analytical expressions for the distribution of the ergodic variable δ_T corresponding to (150). Both of them contain double integrals (see Ref. 71 for details).

This strikingly increasing complexity of distributions with increase of the unknown spectroscopic parameters serves as a good lesson for experimentalists. If they want to extract useful information on WSB interaction constants rather than surprise the world with large P-violation effects,

then they should try to do a good deal of dull work in "standard" spectroscopy in order to determine as many spectroscopic parameters as possible. We shall have to strengthen this statement in the analysis below.

4.4. Analysis of realistic imperfect experimental on-resonance measurements

The distribution analysis of the previous subsection concerned only the statistics for the "theoretical" values of observables arising from the chaotic nature of compound resonances. However, each on-resonance measurement of, say, a $\hat{\Delta}_i$ value can be made with a finite experimental error σ_i, and the experimental results x_i of this measurement in the majority of cases would obey the normal-distribution law

$$P(x_i|\hat{\Delta}_i)=\frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left\{-\frac{(x_i-\hat{\Delta}_i)^2}{2\sigma_i^2}\right\}. \tag{151}$$

Here we have introduced the notation for the conditional probability $P(a|b)$ of a, given b. In our case the measured value $\hat{\Delta}_i$ itself is randomly distributed in accordance with the Gaussian law (141), which we shall denote by $P(\hat{\Delta}_i|M)$. Therefore the connection between the measured value x_i and M will be given by

$$P(x_i|M)=\int d(\hat{\Delta}_i)P(x_i|\hat{\Delta}_i)P(\hat{\Delta}_i|M)$$
$$=\frac{1}{\sqrt{2\pi(\sigma_i^2+M^2)}}\exp\left\{-\frac{x_i^2}{2(\sigma_i^2+M^2)}\right\}. \tag{152}$$

For an infinite set of experimental measurements x_i with small errors σ_i one could plot the curve (152) and extract the M value by applying, say, the least-squares method. Even in this idealized case the problem of finding the confidence intervals ΔM for M is not clearly defined. However, the realistic situation of imperfect measurements is much worse. In practice we might hope to get only a few experimental points x_i with accuracy not exceeding a few σ_i. To dramatize the problem even more, consider a case when after years of hard experimental work we finally get an upper bound $x_1 \leqslant \sigma_1$ in the triple-correlation measurements on an La resonance. What shall we do then in order to connect this upper bound with the corresponding upper bound on M?

Up to now, the fact that we are denoting (151) and (152) as conditional probabilities might seem to be an unnecessary terminological complication of simple things. But when we start considering the above case of imperfect measurements, only the conditional probability theory allows us to solve our problems. Indeed, the exact formulation of the problem is as follows. We have a theoretical expression (152) defining the probability of experimental result x_i (with σ_i) for a given M value. We need to "invert" (152) and find the probability $P(M|x_i)$ of M, given the experimental result $x_i(\sigma_i)$. This problem is in principle easily solved by using the well-known Bayes theorem of standard conditional probability (CPr) theory,

$$P(M|x)\cdot P(x)=P(x|M)\cdot P(M), \tag{153}$$

and putting it in the form

$$P(M|x)=\frac{P(x|M)\cdot P(M)}{P(x)}. \tag{154}$$

According to the same standard CPr theory, the "unconditional" probability is

$$P(x)=\int P(x|M)P(M)dM\equiv N(x). \tag{155}$$

The expressions (153)–(155) are given in any textbook on CPr theory and are accepted by all mathematicians. However, the interpretation of (154) given by Bayes himself forms a special branch of *Bayesian statistics* (BS), which is criticized by the representatives of the more orthodox "frequency" school in mathematical statistics for its "subjectivity" (see an excellent and very brief review of this topic in Ref. 76. Unfortunately, BS is practically unknown to physicists. Unfortunately, since every physicist who ever worked with experimental data or with any version of the above "inverse" problem intuitively felt the need for BS or even tried to apply it without realizing that this is BS. Bayes supposed that *before* we make any kind of measurements x of the physical quantity M, we often have some "*a priori*" knowledge concerning $P(M)$; e.g., *before* measuring a mass we know that it is positive. The Bayes theorem (154) formulates in a mathematically precise way how to combine this "*a priori*" knowledge with the results x of our measurement in order to obtain the "*a posteriori*" probability $P(M|x)$. One might associate this "*a priori*" $P(M)$ with considerations of common sense, which suggests to a physicist that application of orthodox statistical prescriptions to, say, negative experimental x_i obtained in a mass-measurement experiment leads to nonsense—he is sure that the mass is positive, and that only the poor accuracy of his measurement produced the negative x_i. But without BS the physicist does not know how to get out of this trap. The best solution might be to discard this result and use a more precise measuring device. But what to do if this is the best at your disposal (and often the only one in the world)? In the BS approach you just assume the "*a priori*" $P(M)=\Theta(M)$ and proceed through (154) and (155), obtaining a sensible upper limit on the mass as an "*a posteriori*" result of your imperfect measurement (see Ref. 77, where Anderson describes BS as "the correct way to do inductive reasoning from necessarily imperfect data"). In the case when nothing is known "*a priori*" about M the standard assumption is that $P(M)$ is uniform and constant (see Ref. 76).

There is a close, but sometimes misleading, connection between the Bayesian post-probability (BPP) given by (154) and the maximal-likelihood method (MLM) described in numerous manuals on statistics for experimentalists (e.g., Ref. 78). One might characterize the MLM as an attempt to use Bayesian statistics without recognizing it. Indeed, the function $P(x|M)$ of (154) is often called the "likelihood function" $L(M)$, and the MLM says that the best estimate of M is the value $M_{\max}$ which maximizes $P(x|M)$ considered as a

function of M. One easily sees from (154) that in case of complete "*a priori*" ignorance (i.e., $P(M)=\text{const}$) the BPP coincides with the likelihood function to within the normalization (155). Therefore the BPP in this case has exactly the same maximum. As to the definition of the confidence, the MLM usually assigns errors to M by finding the values of M for which $L(M)$ is reduced from its maximal value by a factor of $\exp(-1/2)$. This prescription is obviously based on the assumption that $L(M)$ is a Gaussian centered at $M_{\max}$. For this (and *only for this*) assumption that prescription indeed gives the conventional "one-σ" confidence level of 68%. What is even more important, the ratio of confidence intervals ΔM for 99% and 68% confidence levels in this case is only a factor of 2.6, and this is known to everybody. We shall see, however, that for a small number n of independent experimental measurements $L(M)$ is highly non-Gaussian—in terms of the BPP this means that ΔM for 99% confidence might be larger than ΔM for 68% by a factor of 10^2–10^5 (see, e.g., Ref. 78)! Therefore for small n the MLM confidence prescription becomes senseless. For this reason the most accurate manuals on the MLM warn against the use of this method for small-n cases and vaguely state that the actual accuracy of the $M_{\max}$ definition in the MLM should not exceed the characteristic width (whatever it is) of the $L(M)$ function maximum (see, e.g., Ref. 80).

Thus, we see that the standard MLM coincides with BS only when $P(M)=\text{const}$ and the ensemble of experimental measurements n is large. In all other cases the MLM deviates strongly from BS and should not be applied to data analysis at all.

Since most physicists do not know the ordinary CPr theory (not to mention BS), I shall briefly mention the most dangerous points where CPr theory and BS disagree with our "intuitive" expectations based on a rudimentary knowledge of the ordinary (unconditional) probability in its "frequency" modification.

First of all, the BPP of Eq. (154) (like *any* conditional probability) for two independent measurements *is not* equal to the product of the BPPs for each measurement:

$$P(M|x_1x_2)=\frac{P(x_1x_2|M)P(m)}{N(x_1,x_2)}\neq P(M|x_1)P(M|x_2). \tag{156}$$

Here $N(x_1,x_2)=\int P(x_1,x_2|M)P(M)dM$.

Thus, for n independent on-resonance measurements $x_1,x_2,\ldots,x_n\equiv\{x_i\}^n$ we obtain

$$P(M|\{x_i\}^n)=\frac{\theta(M)}{N(\{x_i\}^n)}\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi(\sigma_i^2+M^2)}}\times\exp\left\{-\frac{x_i^2}{2(\sigma_i^2+M^2)}\right\} \tag{157}$$

with the normalization

$$N(\{x_i\}^n)=\int_0^\infty dM\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi(\sigma_i^2+M^2)}}\times\exp\left\{-\frac{x_i^2}{2(\sigma_i^2+M^2)}\right\}. \tag{158}$$

Expressions of this type were used by us[79] for the analysis of TVDB experiments,[49,65] which were unfortunately performed in the regime of strongly overlapping resonances long before the discovery of resonance enhancement. Our analysis revealed the already mentioned highly non-Gaussian shape of the $P(M|x_{in})$ curves for small n. Therefore in the case of Ref. 49 with its value $n=2$ we obtained at the confidence level 85% an upper bound on the T-violation constant $\xi\leqslant 4\cdot 10^{-3}$, which is comparable to the value $3\cdot 10^{-3}$ quoted in Ref. 49. However, at the confidence level 99% our result was $\xi\leqslant 8.8\cdot 10^{-2}\approx 0.1$. Note the two orders of magnitude difference between the confidence levels 85% (not even 68%!) and 99%. Only after combining the $n=6$ independent observations of Refs. 49 and 65 did the BPP curve start to approach a Gaussian and allow us to obtain $\xi\leqslant 3.5\cdot 10^{-3}$ at the 99% confidence level.

We also observe that for one measurement the $N(x)$ integral of (155) diverges logarithmically at the upper limit M_u *even for infinite accuracy* $\sigma=0$. This means that a single experimental upper bound (however accurate) would *never* allow us to extract an upper bound on the random-variable variance M. This is obvious, since an observed value $x_i\approx 0$ might emerge either in the case of $M\approx 0$ or for large M, as an unlucky fluctuation of the random variable.

All this was true for $p_{1/2}$ resonances. For $p_{3/2}$ ones, which cannot mix with s resonances, we know "*a priori*" that $M=0$. Here the BS results differ drastically from our naive expectations. Since now the "*a priori*" probability is $P(M)=\delta(M)$, from (152)–(155) we obtain

$$P(M|x,3/2)=\frac{P(x|0)}{P(x|0)}\,\delta(M)=\delta(M). \tag{159}$$

The meaning of this purely Bayesian result is also quite simple—if we know *with certainty* the exact value of M before the experiment, this knowledge would not be changed by any further measurements.

The situation becomes much more complicated if we do not distinguish between $p_{1/2}$ and $p_{3/2}$ resonances. Then we can only use their statistical weights and claim that when probing p resonances at random one gets spin 1/2 with probability $p=1/3$ and spin 3/2 with probability $q=2/3$. If only one measurement is made, we can use the ordinary CPr expression

$$P(M|x)=\sum_\beta P(M|x,\beta)P(\beta) \tag{160}$$

in order to combine the BPPs of (157) and (158) (for $n=1$) for spin 1/2 and (159) for spin 3/2 with the aid of the corresponding probabilities $P(\beta)$ (equal to p or q). However, the trivial $\delta(M)$ arising in (160) from the $J=3/2$ term of (159)

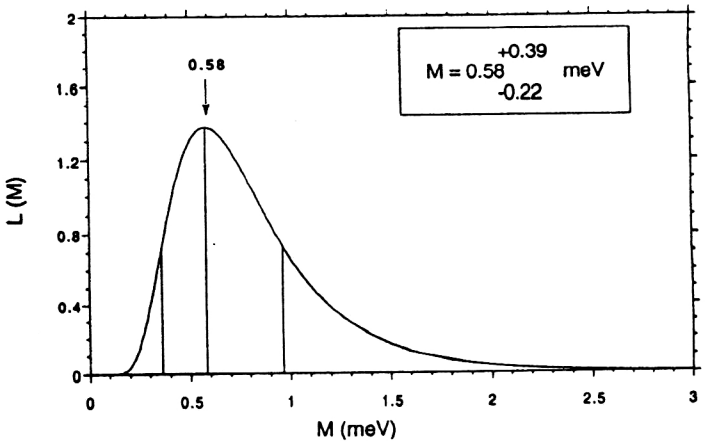

FIG. 3. Likelihood function $L(M)$ for seven $J=1/2$ p resonances in ^{238}U based on experimental data[83] and spin assignment.[84]

does not interest us. Therefore we can easily subtract it from (160) and consider only the $P(M|x1/2)$ term given by (157).

The case of two measurements x_1, x_2 is more subtle. Here we must consider different possibilities. With probability A both resonances have $J=1/2$; with probability B only one of them has $J=1/2$, and it is equally possible that this is either x_1 or x_2. Therefore

$$P(M|x_1x_2)=A\cdot P(M|x_1x_2,1/2)+B[P(M|x_1,1/2) + P(M|x_2,1/2)]. \quad (161)$$

The coefficients A and B are well known in the statistics of Bernoulli trials and are closely related to the binomial-distribution coefficients. The general analysis of Ref. 81 gives, for the case of n measurements,

$$P(M|\{x_i\}^n)=\frac{1}{1-q^n}\sum_{r=1}^{n} p^r q^{(n-r)} \sum_{K_r} P(M|\{x_i\}_r^{K_r},1/2). \quad (162)$$

Here r denotes the number of 1/2 resonances which we might hit in our n trials, and the coefficients in the first sum give the probabilities that this happens. However, for each given r we need to find all possible combinations K_r of r particular x_i values, $\{x_i\}_r^{K_r}$. For each particular combination $\{x_i\}_r^{K_r}$ the BPP $P(M|\{x_i\}_r^{K_r}1/2)$ is given by (157) and (158). The number K_r increases factorially with increasing $r \leqslant n$. Therefore the process of trying all the options in the sums of (163) for, say, $n=30$ would take weeks of fast-speed computer time, while the calculation of each particular $P(M|\{x_i\}_r^{K_r}1/2)$ defining the M distribution for a chosen set of 1/2 resonances would take only seconds. This is another example of how the lack of elementary spectroscopic information on resonance spins enormously complicates the P-violation analysis. Unfortunately, this is not the whole truth. Much worse is the fact that this lack of information on the spins makes the results of all P-violation measurements practically meaningless. We shall show below that *without the spin assignment* all the measurements performed on ^{238}U and ^{232}Th in the last six years allow us to find only the *statistically significant upper bound* $F \leqslant 10^{-6}$ on the strength of the P-violating weak interaction in those nuclei. It is just a lucky chance that the purely phenomenological expression used for the likelihood function in the analysis of these data produced for ^{238}U results close to those obtained with the spin assignment.

Before showing this, let us summarize the difference between the Bayes method and the MLM prescriptions of orthodox statistics in the application to our problem. When resonance spins are known, the only important difference for $J=1/2$ resonances is that the BPPs of (157) and (158) are normalized to unity, while the MLM functions are not. This seemingly insignificant detail leads in the case of small ensembles n to quite different results: in the case of one measurement the BPP cannot be normalized (and we have seen the deep physical reasons behind this), while the standard prescription of the MLM still gives the 68% "confidence level," which is quite meaningless. In the case of $n \geqslant 2$ measurements the BPP curves are normalizable, but the MLM prescription for 68% confidence remains misleading, since: (a) the exp(−1/2) prescription deviates from the actual 68% confidence level; (b) the ΔM intervals for confidence levels of 68% and 99% might differ by several orders of magnitude. With increasing n the difference between BS and the MLM becomes less marked because $L(M)$ gradually approaches a Gaussian. To illustrate this we show in Fig. 3 the $L(M)$ behavior for seven $p_{(1/2)}$ resonances in ^{238}U based on the P-violation measurements of Refs. 82 and 83 and the spin assignment of Ref. 84 together with the "one-σ intervals" of the MLM prescription. Note that even for $n=7$ the function $L(M)$ is still not Gaussian. Therefore the correct definition of 99% confidence would raise the upper limit of M to $M_{up} \approx 1.5$ meV.

In the case of $J=3/2$ the BPP would always give the $\delta(M)$ distribution, while the MLM will not—see $L(M)$ for nine 3/2 resonances in ^{238}U from Ref. 84, shown in Fig. 4. This case shows how misleading the MLM prescriptions might be even for the case of $n=9$. We all understand that for 3/2 resonances the $L(M)$ results should be compatible with $M=0$ and that all the extra maxima of the $L(M)$ curve should be associated only with the poor statistical ensemble (small n). The Bayesian results agree quite well with the above natural expectations—on normalizing the $L(M)$ curve of Fig. 4 and looking for the 99% confidence level we obtain only the upper bound $M \leqslant 1.5$ meV. This upper bound is cer-

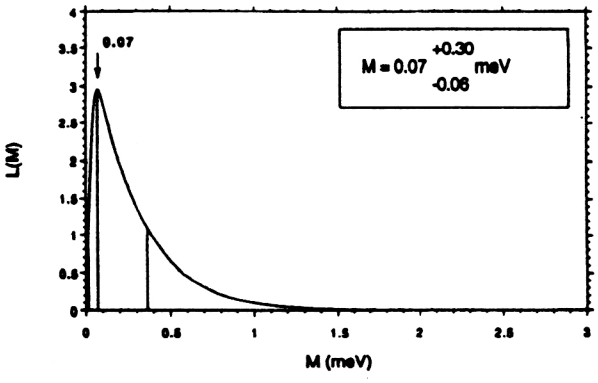

FIG. 4. Likelihood function $L(M)$ for nine $J=3/2$ p resonances in ^{238}U based on experimental data[83] and spin assignment.[84]

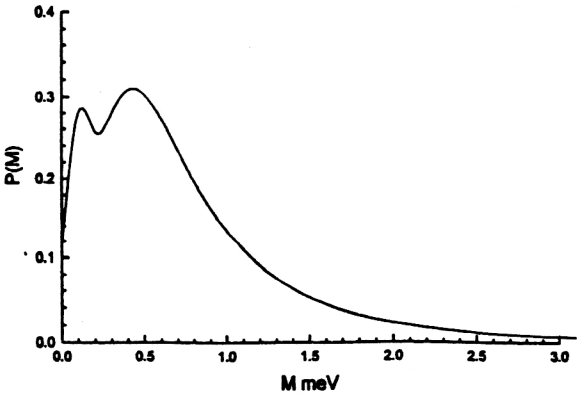

FIG. 6. Bayesian post probability of (162) (normalized to $p=1/3$) for 16 p resonances in ^{238}U without spin assignment.

tainly a rather poor one, but this reflects the basic fact that the $n=9$ ensemble is still a poor one. Bayesian statistics cannot produce miracles and cure this drawback, but it warns us that *the positions of L(M) maxima are much less important than the correct determination of their confidence levels.*

In the case of no spin assignment a *purely empirical* expression was suggested for $L(M)$ in Ref. 82:

$$L(M)=\prod_{i=1}^{n}\left[\frac{p}{\sqrt{2\pi(\sigma_i^2+M^2)}}\exp\left\{-\frac{x_i^2}{2(\sigma_i^2+M^2)}\right\}+\frac{q}{\sqrt{2\pi\sigma_i^2}}\exp\left\{-\frac{x_i^2}{2\sigma_i^2}\right\}\right]. \tag{163}$$

We see that this expression violates both Bayes statistics [since the 3/2 term with coefficient q should be δ-shaped—see (159)] and the rule (159) of conditional probability theory (the conditional probability of n independent measurements differs from a product of n conditional probabilities). The plot of (163) for 16 resonances measured in ^{238}U as given in Ref. 83 is shown in Fig. 5. When the similar plot of Ref. 82 was first demonstrated at the 1989 Alushta School, the response of the experimental audience was: "How do you manage to extract the M values so precisely, when only six of your 17 measured x_i deviate from zero by more than 2σ?" (One should add that now we know that only three of those six nonzero results are *actually* $J=1/2$!) This perfectly sound remark aroused my interest in imperfect-data statistical analysis and finally led to the above Bayesian results. These results, given by (162), are demonstrated for the case of ^{238}U measurements in Fig. 6. Their obvious meaning is (cf. Fig. 4) that without spin assignments the extracted value of M is *compatible with zero and gives only the upper bound* $M\leqslant 3$ *meV at the 95% confidence level,* in complete agreement with sound expectations. The oscillations of the $L(M)$ curve, as in the case of Fig. 4, result only from the poorness of the statistical ensemble.

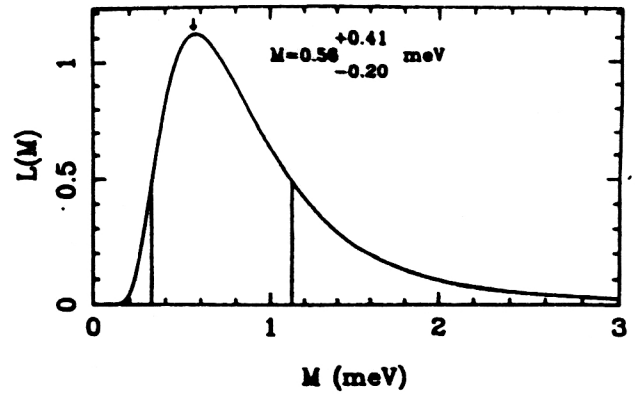

FIG. 5. Likelihood function of (163) for 16 p resonances in ^{238}U without spin assignment (see Ref. 83).

The problem now is to understand the striking similarity between the curves of Fig. 5 obtained by using the erroneous analysis (163) and those of Fig. 3, whose analysis (besides the above remarks on confidence levels) is correct. The simple answer is that this is due to the occurrence of several lucky and unpredictable coincidences.

Let us compare the phenomenology of (163) with the Bayesian expression (162). By removing the constant factors $\exp[-(x_i^2/2\sigma_i^2)]$ from each factor in the product of (163) and representing (163) as an nth power of a binomial we can simplify this comparison. Now we see that the main difference between (162) and (163) lies in the fact that each term of the binomial expansion in (163) contains an extra weight factor $\exp(x_i^2/2\sigma_i^2)$, which exponentially enhances the contribution of each significant deviation $x_i^2/2\sigma_i^2$ of the effect from zero. In the particular ^{238}U case a single 63.5-eV resonance with $x_0\approx 7\sigma_0$ contains an enormous enhancement factor $A_0=10^{10}$. Therefore the maximum of this term, $M^0_{\max}=\sqrt{x_0^2-\sigma_0^2}=0.65$ meV, practically defines the maximum of the whole expression (163). The remaining four statistically significant results, whose maxima lie on both sides of M_0, only slightly shift the overall maximum to $0.58^{+0.5}_{-0.2}$ meV of Ref. 82. In order to estimate the influence of those remaining terms, we observe that the omission of one of the 2σ effects at 57.9 meV in Ref. 83 shifted this maximum to $0.56^{+0.4}_{-0.2}$ meV. Naturally, all the abundant zero effects mentioned above produce no influence whatsoever on the $L(M)$ curve of (163).

Thus, the phenomenological construction (163), which

violates conditional probability theory, has the surprising feature of artificial "exponential" selection of the most statistically significant results, or, to put it differently, exponential suppression of all the null results. Intuitively, one might expect that this lucky feature of (163) (quite unexpected by its creators) is a strange but reasonable way to select 1/2 resonances, since we expect that *on the average* only those resonances would show *marked* deviations from zero. To a certain extent this is true, but only to a certain extent. Let us return to our basic expression (152) to understand how sound this selection might be. First of all, we observe that for *any* value of M (that means 1/2 resonances!) the most probable result is $x_i=0$. The ^{238}U case is not an exception—four out of seven resonances with $J=1/2$ show zero results. However, the width of the distribution curve (152) depends on both M and the σ_i values. Consider first the limiting case of extremely high experimental accuracy, $(M/\sigma_i)^2 \gg 1$. In this case practically all the nonzero results x_i would come from 1/2 resonances with $M\neq 0$. The "contamination" from 3/2 resonances would be negligible because of the small σ values. Therefore in this idealized situation the suppression of null results is perfectly correct, and the expression (163) might be a reasonable approximation of (157).

Consider now the case $(M/\sigma)^2 \simeq 1$. In this case the distributions (152) for $M=0$ and $M\neq 0$ would come closer to each other, and 3/2 resonances would considerably "contaminate" our measurements, contributing many nonzero results x_i. However, if we have a very large ensemble of resonance measurements, $n \gg 1$, and exponentially select the most significant results, we can still be sure that more $x_i \neq 0$ would come from 1/2 resonances. Note that for small-n ensembles this will not work, and the "degree of contamination" coming from 3/2 resonances would vary randomly from ensemble to ensemble.

Finally, consider $(M/\sigma_i)^2 \ll 1$. Then the distributions (152) for $M=0$ and $M\neq 0$ would practically coincide. Only by taking the unrealistic limit of an infinite ensemble, $n\to\infty$, plus some kind of selection of the largest results might one hope to select the 1/2 contributions.

Returning to the ^{238}U results, we know *only after spin assignment* that $M=0.56$ meV. Comparing this with the σ_i of Refs. 82 and 83, we see that the experimental accuracy parameter $(M/\sigma_i)^2$ is more or less evenly distributed between 400 and 0.05. Therefore the chances that nonzero effects come only from 1/2 resonances are roughly "fifty–fifty." Indeed, as we know now, four of the seven nonzero results in U were coming from 3/2 resonances. Therefore the above coincidence of two maximum positions is an unpredictable chance coming essentially from the fact that the main 7σ contribution to (153) *alone* was giving the value $M_0^{\max}$ already within σ_0 of the true M value of Ref. 84. Therefore it would be much simpler and honest to say simply: "We have reasons to hope that the 7σ effect is a 1/2-resonance one, while with the rest we have no guarantees. So we identify the matrix element of this large effect with the variance M." Obviously, the statistical significance of such a statement is quite unpredictable, but so is the statistical significance of results obtained with the much less transparent expression (163).

We can also understand now the results of the Monte-Carlo simulation[85] of the Th case. The authors of Ref. 85 considered an ensemble of $n=7000$ resonances with a nonzero effect in the case when the parameter $(M/\sigma_i)^2$ was distributed in the range from 10^3 to 1. We have already shown above that for such a choice of $(M/\sigma)^2$ an unrealistically large ensemble of 7000 nonzero observations would almost certainly produce the correct results, which was the outcome of Ref. 85. The trouble is that in the actual ^{232}Th of Refs. 86 and 87 we have only seven nonzero effects and do not know in advance the $(M/\sigma_i)^2$ values. Therefore without the spin assignment we cannot even make a clever guess at how many nonzero effects arise from "contaminating" 3/2 resonances and how many null effects come from 1/2 resonances (in U those were four out of seven resonances with $J=1/2$).

Therefore in realistic measurements even the correctness of $M_{\max}$ derived from (163) would always be unpredictable without spin assignment, not to mention the confidence levels given by (163). This unpredictable character of (163) is reflected mathematically in the Bayesian expression (162) and in the results of Fig. 6.

4.5. Sign-correlation effect

Returning to the multilevel expression (145) for Δ^p_{tot}, we see that the signs of the observed effect should vary randomly from resonance to resonance. This comes from the sign-randomness of three quantities in it: the matrix element $(v_p)_i$, the partial amplitudes γ^n_s and γ^n_p, and the energy denominators (E_p-E_s).

However, measurements in ^{232}Th (without spin assignment) demonstrated that seven of the seven statistically significant ($x_i \geq 2\sigma_i$) effects have the same positive sign.[86,87] This poses a question—is it a fluctuation, or does it come from some systematic effect which is not included in (143) and has constant sign? The answer to this question might be best found on the lines of Bayesian statistics (see Ref. 77, where Anderson demonstrates how efficient BS is in "null-hypothesis" tests, discarding the notorious "fifth-force" experiments). However, the authors of Ref. 88 preferred to introduce *ab initio* a constant-sign term in addition to (143). This resulted in a constant-sign addition $B\sqrt{1\ \text{eV}/E}$ (%) to the quoted effect $\mathscr{P}$ [see (78) and (81)]. The magnitude B of this addition was determined in a two-dimensional MLM analysis, using $L(M)$ of (164), modified by the presence of the B term. The result of this analysis was $B=8^{+6.2}_{-6.0}\%$, which is of the same order of magnitude as the effect $\mathscr{P}$ itself (which ranges between 1%–10%).

This highly sensational result was taken at face value as an experimental one by many theorists and experimentalists, and it produced an avalanche of publications with attempts to explain it theoretically. Since it might require a special review to analyze (and even mention) all of them, I will just classify the main trends in those publications. Some of them concentrated on the analysis of a possible sign correlation between the matrix elements $(v_p)_i$, completely ignoring the random signs of the γ's and (E_p-E_s). Others rediscovered our old statement (see, e.g., Ref. 7) that the valence mechanism leads to a sign correlation of the P effects [see (60) for

the valence part T_{10} of the T_1 amplitude), ignoring, however, the fact that we discarded this mechanism in Ref. 7 because it lacks the dynamical enhancement factor $\sqrt{N}\approx 10^3$. The situation is different in Refs. 33 and 34, which concerned the amplitudes of T_8 type. Since all the diagrams containing the wave function χ (see the wavy lines in the loops of T_3 and T_5–T_9 in Fig. 1) describe the motion of valence particle in the target mean field, we called them all the "valence mechanism" in Ref. 7. We estimated them with a crude procedure similar to that of (15) and (16), and on discovering that they lack the dynamical enhancement factor $\sqrt{N}$ we simply stated this fact in Ref. 7. Weidenmüller[33] developed a much more elegant technique of principal-value integral evaluation, which I borrowed from him in the above estimates of Sec. 3.

Practically all the authors concluded that the valence mechanism needs an extra enhancement of 10^2–10^3 in order to explain the above B value. Almost nobody mentioned that this extra factor is exactly the same dynamical enhancement which led to the large observed effects in complex nuclei in the first place.

Nor did anybody question the reliability of the above huge B value obtained from the MLM analysis, whose drawbacks we have just discussed. To those who do not believe in the Bayesian approach, I can suggest a comparison of the above value $B=8^{+6.2}_{-6.0}\%$ with the MLM value $M=0.07^{+0.30}_{-0.06}$ meV in Fig. 4. It is believed by all that the results of Fig. 4 should be compatible with $M=0$. The same must be said about B even by the MLM adepts. Thus, the actual statistical significance of the sign-correlation effect remains an open question, which could be solved, if really necessary, in terms of the Bayesian approach. Since this approach gives only upper bounds even for the M values without spin assignment, the same will be even more true for the B values. Therefore spin assignment in Th is also essential if one really wants to consider the sign-correlation problem seriously.

SUMMARY

Thus, we have seen that the enhancements of *all* the symmetry-breaking effects in nuclear reactions on isolated resonances come essentially from the dynamical factor $\tilde{v}/d\sim\sqrt{N}$ and from the resonance enhancement factor d/Γ, which combine in most optimal situations to the overall factor $\tilde{v}/\Gamma$. For inelastic channels (including the TVDB) there might be an additional structural enhancement factor $f\sim(\gamma_1/\gamma_2)$ [see Eqs. (73a) and (130)]. For "elastic-channel" observables probed in transmission experiments the situation is more complicated. All of them contain, instead of the above f, the barrier-penetration hindrance factors (kR) or $(kR)^2$ (for the FC). However, they also contain an extra resonance enhancement d/Γ [see, e.g., (73b) and (73c)], which might almost completely (in the case of the P-odd correlation) or partially (in the case of the FC) compensate those hindrance factors.

We have also seen that the "structural (kinematic) enhancement factor" $1/(kR)$, so often used to explain the enhancements for the "elastic-channel" P-odd correlations, is merely an artifact of a misleading analogy with bound states in the theory and of quoting the auxiliary value $\mathscr{P}$ instead of the actually observed P in experiments.

Both major enhancements (dynamical and resonance) are quite general results of the quantum chaoticity of compound resonances, which increases the complexity N of the compound-resonance wave functions and reduces their total widths Γ. This reduction is most efficient in the low-energy region of isolated resonances with $\Gamma\ll d$.

The same chaoticity which produces huge enhancements also necessitates the use of statistical methods for the analysis of observables and their proper connection with strength parameters of the symmetry-breaking interactions. This does not lower the reliability of the information obtained as compared with simple nucleon–nucleon interaction processes, provided that one obeys certain general rules and uses the correct statistical methods. In theory this boils down to the use of the methods of the random-matrix approach, whose reliability was established during half a century in neutron-resonance spectroscopy. In processing the experimental data one should use Bayesian statistics (BS), whose significance is gradually being realized by the whole physics community.

Both statistical approaches (in theory and experiment) show an important general feature—lack of information on "standard" spectroscopic parameters immediately complicates the analysis and enormously lowers the statistical significance of the symmetry-breaking observations. The use of BS also strongly favors more independent experimental observations rather than an increase in their individual accuracy.

In this review I had to concentrate essentially on reaction aspects of symmetry breaking and on proper statistical analysis, paying much less attention to the ultimate aim of on-resonance experiments—analysis of new information on symmetry-breaking interaction constants. As mentioned in the Introduction, since the creation of the Weinberg–Salam electroweak interaction theory nobody would be surprised by P violation caused by the weak components of the nucleon–nucleon interaction. Therefore in P violation one should try to study the systematic behavior of F (e.g., its A dependence). Even from this point of view, P violation is only a particular case of symmetry breaking in nuclei. Studies of "strong" symmetry breaking (see Sec. 4.1) are by no means less important—they are only much more advanced. Even in those studies there are still open problems whose solution might be of major importance for P violation—see Weidenmüller's comments on isospin-violating spreading widths and their implications in Ref. 89. Therefore it is even more important to use P violation as "test sites" for future T-violation "on-resonance" experiments by developing the most reliable experimental and theoretical technique.

I have to point out that although on-resonance enhancements were constantly predicted for various T-violating observables since 1982, the experimental situation in this field lacks dynamics. Perhaps too much experimental energy is wasted on P violation, especially in creating and discussing "quasi-sensations." The same applies to FC correlation measurements in the almost hopeless Ho. In view of the facts stated at the end of Sec. 3, the on-resonance TVDB observations seem much more promising than the FC ones in general. I must also repeat that discussion of all the experimentally realistic modifications of TC measurements should be a

highest priority. In planning the experimental strategy of T-violation measurements one should especially follow the rule resulting from BS—more independent on-resonance observations, even with smaller accuracy. Note that in the optimistic case of a nonzero effect one measurement would only produce a sensation, telling us nothing about the interaction constant. In the more probable case of an experimental upper-bound observation one point, however accurately measured, would be quite meaningless.

ACKNOWLEDGMENTS

I officially acknowledge that this work was supported in part by the Russian Fund of Fundamental Investigations. Much less officially but much more warmly, I express my gratitude to all my co-authors in this field. I am especially grateful to Hans Weidenmüller, whose contribution to this work in many capacities (co-authorship, friendly discussions, hospitality, and financial support during my numerous stays in Heidelberg) is simply impossible to measure.

*On leave of absence from Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia.

[1]It is unusual to disagree with one's own publication. The main conception and the analytical part of Ref. 71 were finished before the end of 1989. However, the final text was written only four months later, when I was out of reach in Russia and two other co-authors were in Arizona and Heidelberg, respectively.

[1] O. Sushkov and V. Flambaum, Pis'ma Zh. Éksp. Teor. Fiz. **32**, 377 (1980) [JETP Lett. **32**, 352 (1980)]; in *Proc. of the LNPI 16th Winter School* (Leningrad, 1981), p. 200.
[2] V. Bunakov and V. Gudkov, Z. Phys. A **303**, 285 (1981); in *Proc. of the LNPI 16th Winter School* (Leningrad, 1981), p. 32.
[3] V. Alfimenkov *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **34**, 308 (1981) [JETP Lett. **34**, 295 (1981)].
[4] V. Bunakov and V. Gudkov, JETP Lett. **36**, 328 (1982); Z. Phys. A **308**, 363 (1982).
[5] V. Bunakov, Phys. Rev. Lett. **60**, 2250 (1988).
[6] V. Bunakov and H. A. Weidenmüller, Phys. Rev. C **39**, 70 (1989).
[7] V. Bunakov and V. Gudkov, Nucl. Phys. **A401**, 93 (1983).
[8] V. Bunakov, Phys. Lett. B **207**, 233 (1988).
[9] G. Mitchell *et al.*, in *Time-Reversal Invariance and Parity Violation in Neutron Reactions*, edited by C. Gould, J. D. Bowman, and Yu. P. Popov (World Scientific, Singapore, 1994), p. 167.
[10] C. Mahaux and H. Weidenmüller, *Shell Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969).
[11] R. J. Blin-Stoyle, *Fundamental Interactions and the Nucleus* (North-Holland, Amsterdam, 1973).
[12] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 1 (Benjamin, New York, 1969).
[13] R. Haas *et al.*, Phys. Rev. **116**, 1221 (1959).
[14] R. J. Blin-Stoyle, Phys. Rev. **120**, 181 (1960).
[15] I. S. Shapiro, Sov. Phys. Usp. **11**, 582 (1969).
[16] J. French *et al.*, in *Tests of T-Reversal Invariance in Neutron Physics*, edited by N. Roberson, C. Gould, and J. D. Bowman (World Scientific, Singapore, 1987), p. 80.
[17] V. Lobashov *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **3**, 268 (1966) [JETP Lett. **3**, 173 (1966)].
[18] Yu. Abov *et al.*, Yad. Fiz. **1**, 479 (1965) [Sov. J. Nucl. Phys. **1**, 341 (1965)].
[19] A. I. Baz', Ya. B. Zel'dovich, and A. M. Perelomov, *Scattering, Reactions and Decay in Nonrelativistic Quantum Mechanics* (Israel Program for Scientific Translations, Jerusalem, 1966) [Russ. original, later ed., Nauka, Moscow, 1971].
[20] V. Alfimenkov, Nucl. Phys. **A383**, 93 (1983).
[21] L. Stodolsky, Phys. Lett. B **50**, 352 (1971).
[22] F. Michel, Phys. Rev. **133**, 329 (1964).
[23] G. Karl and D. Tadic, Phys. Rev. C **16**, 1726 (1977).
[24] M. Forte, in *Fundamental Physics with Reactor Neutrons and Neutrinos*, edited by T. von Egidy (Institute of Physics, Bristol, England, 1978), p. 86.
[25] V. Karmanov and G. Lobov, Pis'ma Zh. Éksp. Teor. Fiz. **10**, 332 (1969) [JETP Lett. **10**, 212 (1969)].
[26] O. Sushkov and V. Flambaum, Usp. Fiz. Nauk **136**, 3 (1982) [Sov. Phys. Usp. **25**, 1 (1982)].
[27] M. Forte *et al.*, Phys. Rev. Lett. **45**, 2088 (1981).
[28] V. Kolomensky *et al.*, Phys. Lett. **107**, 308 (1981).
[29] J. Vanhoy *et al.*, Z. Phys. A **333**, 229 (1988).
[30] C. Gould, J. Mod. Phys. A **5**, 2181 (1990).
[31] P. Bizetti and P. Maurenzig, Nuovo Cimento A **56**, 492 (1980).
[32] A. M. Baldin *et al.*, *Kinematics of Nuclear Reactions* (Oxford University Press, London, 1961) [Russ. original, 2nd ed., Atomizdat, Moscow, 1968].
[33] H. A. Weidenmüller, in *Group Theory and Special Symmetries in Nuclear Physics*, edited by J. Drayer and J. Janecke (World Scientific, Singapore, 1992), p. 31.
[34] C. Lewenkopf and H. Weidenmüller, Phys. Rev. C **46**, 2601 (1992).
[35] C. Mahaux and H. Weidenmüller, Phys. Lett. **23**, 100 (1966).
[36] V. Bunakov, in *Proc. of the Second International Seminar on Interactions of Neutrons with Nuclei*, April, 1994 (Dubna, Russia) (in press).
[37] G. Danilyan, Pis'ma Zh. Éksp. Teor. Fiz. **26**, 298 (1977) [JETP Lett. **26**, 422 (1977)]; Usp. Fiz. Nauk **131**, 329 (1980) [Sov. Phys. Usp. **23**, 323 (1980)].
[38] V. Bunakov and V. Gudkov, in *Tests of T-Reversal Invariance in Neutron Physics*, edited by N. Roberson, C. Gould, and J. D. Bowman (World Scientific, Singapore, 1987), p. 175.
[39] V. Bunakov and V. Gudkov, Z. Phys. A **321**, 271 (1985).
[40] V. Sailor, in *Proc. of the International Conference on Peaceful Use of Atomic Energy*, Vol. 4 (United Nations, New York, 1955), p. 199.
[41] J. Lynn and N. Pattenden, *ibid.*, p. 210.
[42] A. Bohr, *ibid.*, Vol. 2, p. 220.
[43] J. Lynn, *The Theory of Neutron Resonance Reactions* (Clarendon Press, Oxford, 1968).
[44] E. Henley and B. A. Jacobsohn, Phys. Rev. Lett. **16**, 708 (1966).
[45] R. Ryndin, in *Proc. of the LNPI 3rd Winter School* (Leningrad, 1968), p. 39.
[46] V. Bunakov and V. Gudkov, in *Proc. of the LNPI 20th Winter School* (Leningrad, 1985), p. 189.
[47] E. Henley and B. A. Jacobsohn, Phys. Rev. **113**, 225 (1959).
[48] J. H. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).
[49] W. von Witsch *et al.*, Phys. Rev. Lett. **22**, 631 (1966); Phys. Rev. Lett. **19**, 524 (1967); Phys. Rev. **169**, 923 (1968).
[50] T. Ericson, Phys. Lett. **23**, 97 (1966).
[51] J. Pearson and A. Richter, Phys. Lett. B **56**, 112 (19
[52] P. Moldauer, Phys. Rev. **165**, 1136 (1968).
[53] D. Boose *et al.*, Phys. Rev. Lett. **56**, 2012 (1986).
[54] D. Boose *et al.*, Z. Phys. A **325**, 363 (1986).
[55] L. Stodolsky, Nucl. Phys. **B197**, 213 (1982).
[56] P. Kabir, Phys. Rev. D **25**, 2013 (1982).
[57] V. Bunakov and V. Gudkov, LNPI Preprint No. 1236 (1986).
[58] A. Barabanov, Yad. Fiz. **44**, 1163 (1986) [Sov. J. Nucl. Phys. **44**, 755 (1986)].
[59] P. Herczeg, in *Tests of T-Reversal Invariance in Neutron Physics*, edited by N. Roberson, C. Gould, and J. D. Bowman (World Scientific, Singapore, 1987), p. 24.
[60] V. Bunakov and V. Gudkov, J. Phys. (Paris) C **3**, Suppl. No. 3, 77 (1984).
[61] V. Baryshevskiĭ and M. Podgoretskiĭ, Zh. Éksp. Teor. Fiz. **47**, 1050 (1964) [Sov. Phys. JETP **20**, 704 (1965)].
[62] Y. Masuda, in *T-Reversal Invariance and P-Violation in Neutron Reactions*, edited by C. Gould, J. D. Bowman, and Yu. Popov (World Scientific, Singapore, 1994), p. 126.
[63] A. Serebrov, *ibid.*, p. 138.
[64] P. Huffman *et al.*, *ibid.*, p. 151.
[65] E. Blanke *et al.*, Phys. Rev. Lett. **51**, 355 (1983).
[66] P. Moldauer, Phys. Lett. B **26**, 713 (1968).
[67] J. Verbaarschot *et al.*, Phys. Rep. **129**, 367 (1985).
[68] J. Verbaarschot *et al.*, Ann. Phys. (N.Y.) **168**, 368 (1984).
[69] E. Davis, Phys. Lett. B **206**, 197 (1989).
[70] M. Mehta, *Random Matrices* (Academic Press, New York, 1967), Chap. 5.

[71] V. Bunakov, E. Davis, and H. Weidenmüller, Phys. Rev. C **42**, 1718 (1990).
[72] S. Koonin *et al.*, Phys. Rev. Lett. **69**, 1163 (1992).
[73] J. Koster *et al.*, in *T-Reversal Invariance and P-Violation in Neutron Reactions*, edited by C. Gould, J. D. Bowman, and Yu. Popov (World Scientific, Singapore, 1994), p. 144.
[74] E. Davis and U. Hartmann, Ann. Phys. (N.Y.) **211**, 334 (1991).
[75] J. French *et al.*, Phys. Lett. B **80**, 17 (1978).
[76] R. J. Barlow, *Statistics* (Wiley, New York, 1989), Chap. 7.
[77] P. W. Anderson, Phys. Today, January 1992, p. 9.
[78] W. T. Eadie *et al.*, *Statistical Methods in Experimental Physics* (North-Holland, Amsterdam, 1971).
[79] V. Bunakov, H.-L. Harney, and A. Richter, Nucl. Phys. **A560**, 71 (1993).
[80] D. Taupin, *Probabilities, Data Reduction, and Error Analysis in the Physical Sciences* (Editions de Physique, Paris, 1989).
[81] V. Bunakov, in *T-Reversal Invariance and P-Violation in Neutron Reactions*, edited by C. Gould, J. D. Bowman, and Yu. Popov (World Scientific, Singapore, 1994), p. 61.
[82] J. D. Bowman *et al.*, Phys. Rev. Lett. **65**, 1721 (1990).
[83] X. Zhu *et al.*, Phys. Rev. C **46**, 768 (1992).
[84] F. Corvi *et al.*, in *T-Reversal Invariance and P-Violation in Neutron Reactions*, edited by C. Gould, J. B. Bowman, and Yu. Popov (World Scientific, Singapore, 1994), p. 79.
[85] J. D. Bowman and E. Sharapov, *ibid.*, p. 69.
[86] C. Frankle *et al.*, Phys. Rev. Lett. **46**, 778 (1991).
[87] C. Frankle *et al.*, Phys. Rev. C **46**, 778 (1992).
[88] J. B. Bowman *et al.*, Phys. Rev. Lett. **68**, 780 (1992).
[89] H. A. Weidenmüller, Nucl. Phys. **A522**, 293c (1991).

This article was published in English in the original Russian journal. It is reproduced here with stylistic changes by the Translation Editor.