Semileptonic decays of heavy baryons[1)]

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The current status of the physics of hadrons containing a single heavy quark (b or c) is reviewed. The results of studies of semileptonic decays of heavy baryons with $J^P = \frac{1}{2}^+$ using the relativistic quark model including light-quark confinement are also presented. In particular, the semileptonic flavor-changing decay modes $b \to c$, $b \to u$, $c \to s$, and $c \to d$ are described. The form factors of these decays are calculated in the limit of infinite mass of the heavy quarks, $m_Q \to \infty$ (the Isgur–Wise limit). For transitions $b \to c$ expressions are obtained for the baryonic form factors in the leading order in $1/m_Q$ together with their first-order corrections. It is shown that the Ademollo–Gatto theorem is satisfied when the velocities of the initial and final baryons are equal ($v^\mu = v'^\mu$). The form factors obtained are used to calculate the observables of semileptonic decays of heavy baryons: the decay widths, differential distributions, leptonic spectra, and asymmetry parameters. These results are compared with those obtained using other theoretical approaches. © *1995 American Institute of Physics.*

1. INTRODUCTION: STATUS OF THE PHYSICS OF HADRONS CONTAINING A SINGLE HEAVY QUARK

The last few years have brought rapid development of the physics of hadrons composed of light quarks q (u, d, and s) and heavy quarks Q (c and b). Heavy quarks are those whose masses satisfy the condition $m_Q \gg \Lambda_{\rm QCD}$, where $\Lambda_{\rm QCD} \sim 0.2$ F is the QCD scale parameter. We should first of all state that we shall not deal with the t quark at all in this review, because its mass is too large to give rise to bound hadronic states. Weak decays of heavy hadrons are a unique tool for determining the elements of the Cabibbo–Kobayashi–Maskawa matrix, for studying phenomena lying outside the scope of the standard model, and also for studying the internal structure of hadrons.

The increased interest in such processes is primarily related to the new possibilities in experimental research in intermediate-energy physics. In particular, workers at the ARGUS[1] and CLEO[2] setups have studied semileptonic decays of mesons containing heavy quarks in both the initial and the final states: $B \to De\nu$, $B \to D^*e\nu$. Decays with transition of a heavy meson into a light one, $D \to Ke\nu$ and $D \to K^*e\nu$, have been studied in the E653 (Ref. 3) and E691 (Ref. 4) experiments and in experiments performed by the CLEO group.[5] The decay of the vector meson D^*, $D^* \to K^-\mu^+\nu_\mu$, has been observed by the E687 group.[6] Semileptonic decays of the D_s meson, $D_s \to X + \mu + \nu_\mu$ ($X = \Phi$, K^*, ρ, η, η'), have been studied by the Fermilab collaboration E653 (Ref. 7).

The ARGUS experiment to study the nonleptonic decay $D^0 \to K_s^0\pi^+\pi^-$ should also be noted.[8] The Cabibbo-forbidden two-particle decay modes of the neutral D^0 meson $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ have been studied by the E687 group.[9] The branching ratios of two-particle decays of the vector $D^*(2010)$ meson $D^* \to D\pi$ and $D^* \to D\gamma$ (Ref. 10) and the mass difference of the D^* and D mesons have been measured by the CLEO collaboration.[11] Quite original experiments to determine the lifetime of the $\bar{B}^0$, B^- and B^0, B^+ mesons have been performed by the ALEPH,[12] DELPHI,[13] and OPAL Collaborations.[14] Precision measurements of the lifetimes of the D^0 and D^+ mesons have been made at the E687 setup.[15] The B_s-meson mass was first measured by the ALEPH Collaboration.[16]

For a long time, experimental studies of heavy baryons have focused on the study of processes involving only charmed particles. Semileptonic decays of Λ_c^+ baryons, $\Lambda_c^+ \to \Lambda e^+ X$ and $\Lambda_c^+ \to \Lambda\mu^+ X$, have been observed at the DORIS II setup by the ARGUS group[17] and at the CERS setup by the CLEO group.[18,19] Nonleptonic decays of the Λ_c baryon, $\Lambda_c \to \Lambda\pi$ and $\Lambda_c \to \Sigma\pi$, have been studied by the CLEO[18] and ARGUS groups.[20] Multiparticle nonleptonic decays of the charmed Λ_c baryon, $\Lambda_c \to pK^-\pi^+X$, have been studied at the NA32 setup by the ACCMOR collaboration at CERN.[21] The rare decay $\Lambda_c \to pK^+K^-$ has been observed at the E687 setup.[22] The ARGUS group[23] has discovered a new charmed baryon $\Lambda_c^{*+}(2627)$ in the decay $\Lambda_c^{*+} \to \Lambda_c^+\pi^+\pi^-$.

However, in the last few years there has been considerable progress in the study of processes involving baryons containing the b quark. In particular, the Λ_b baryon was first observed in the decay $\Lambda_b \to J/\Psi\Lambda$ in the proton–antiproton collider at CERN.[24] At LEP the ALEPH and OPAL groups have studied semileptonic decays of Λ_b baryons: $\Lambda_b \to \Lambda_c Xe\nu$ (Refs. 25 and 26). The Λ_b-production process has been studied in Z^0-boson decays and the lifetime has been measured by the DELPHI Collaboration.[13,27] In these experiments it has proved possible to extract rather useful information about the physical characteristics of heavy hadrons. In particular, the masses and lifetimes of heavy hadrons have been measured, together with the branching ratios of weak decays and the polarization characteristics.

From the theoretical point of view, this lively interest in weak decays of heavy hadrons is mainly due to the discovery of a new type of strong-interaction symmetry: a spin–flavor symmetry in the world of heavy quarks (the Isgur–Wise symmetry; Refs. 28 and 29) and to the development of a heavy-quark effective theory (HQET; Refs. 28–43). The latter is a perturbative computational scheme for studying the

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properties of hadrons containing a single heavy quark.

The Isgur–Wise symmetry is based on the following physical picture. Let us consider a system consisting of a heavy quark surrounded by a cloud of light quarks and gluons. This cloud is humorously referred to by Isgur as "brown muck," owing to the fact that the properties of a configuration cannot be described from first principles, i.e., by a perturbative treatment within the standard model of strong interactions. The typical size of this system R is comparable to the strong-interaction radius $R \sim 1/\Lambda_{\rm QCD} \sim 1\,{\rm F}$, and the average momentum transfer between the heavy quark and the light constituents is of order $\Lambda_{\rm QCD}$, i.e., their interaction arises only from "soft" gluons. However, the soft gluons do not "feel" the quantum numbers of the heavy quark, so that the properties of the light degrees of freedom in such hadrons are independent of the flavor (or the mass) of the heavy quark (flavor symmetry) and the orientation of the spin of the heavy quark (spin symmetry). Therefore, hadrons containing a single heavy quark and differing only by the orientation of the spin of this quark or by its flavor in the limit $m_Q \to \infty$ (the Isgur–Wise limit) have the same configuration of the light degrees of freedom.

The appearance of the Isgur–Wise symmetry is easily demonstrated for the example of heavy mesons. Here there is a direct analogy with the properties of hydrogen-like atoms. Let us consider a meson consisting of nonrelativistic heavy and light quarks Q and q, the interaction of which is described by a QCD potential similar to the Coulomb potential of QED. In the limit $m_Q \gg m_q$ the wave functions of mesons containing a c or b quark coincide, and the reduced mass

$$\mu_{\rm red} = \frac{m_Q m_q}{m_Q + m_q} \to m_q \tag{1}$$

is independent of the heavy-quark mass. Therefore, there is $b \leftrightarrow c$ flavor $SU(2)$ symmetry, in spite of the significant difference between the b- and c-quark masses. In atomic physics the analogy with the Isgur–Wise flavor symmetry is manifested in the fact that different isotopes having identical electron shell structure possess identical chemical properties.

Another aspect of the Isgur–Wise symmetry, spin symmetry, is easily demonstrated for this system. Let us consider the part of the total Hamiltonian of the Qq system which corresponds to the spin–spin interaction between the heavy and light quarks, describing the hyperfine splitting

$$\Delta H \propto \frac{l_Q\, l_q}{m_Q m_q} \vec{S}_Q \cdot \vec{S}_q \delta^3(\vec{x}). \tag{2}$$

It is easily seen that in the limit $m_Q \to \infty$ this part of the Hamiltonian will be suppressed relative to the kinetic part. Therefore, the orientation of the heavy-quark spin does not affect the dynamics of the light degrees of freedom. Since the total spin of the system is conserved, the spin of the heavy quark will also be conserved. Therefore, the spin operator of the heavy quark is an $SU(2)$ generator. A spin $SU(2)$ symmetry thus arises. Again here there is an analogy with hydrogen-like atoms, for which the energy levels of the hyperfine splitting are nearly degenerate.

The Isgur–Wise symmetry has formed the basis for a heavy-quark effective theory (HQET; Refs. 28–44). In order to explain the main idea behind this approach, let us consider a hadron of mass M moving with velocity v which contains a heavy quark of mass m_Q possessing velocity v_Q. The dynamics of the heavy quark is described by the standard QCD Lagrangian:

$$\mathscr{L}_Q = \bar{Q}(i\not{D} - m_Q)Q, \tag{3}$$

where D is the covariant derivative. The starting point of the HQET is the rather simple hypothesis that the velocities of the heavy quark and the hadron coincide up to corrections of order $1/m_Q$:

$$v_Q^\mu = v^\mu + k^\mu/m_Q, \tag{4}$$

where k^μ is the so-called residual momentum, which is much smaller than the mass of the heavy quark. In the Isgur–Wise limit $m_Q \to \infty$ we obviously have $v_Q \to v$. Therefore, QCD interactions do not change the velocity of the heavy quark. This is the *Georgi velocity superselection rule.*[30] In other words, in the HQET the velocity becomes a conserved physical quantity, i.e., a quantum number. Expressing the heavy-quark momentum p_Q in terms of the hadron 4-velocity, we obtain

$$p_Q^\mu = m_Q v_Q^\mu = m_Q v^\mu + k^\mu. \tag{5}$$

This expression implies that the heavy quark is nearly on-shell ($p_Q^2 \to m_Q^2$, because $v^2 = 1$). Then the heavy-quark fields are redefined in such a way that in the Isgur–Wise limit the dependence of the kinetic-energy operator on the quark mass vanishes. Additional fields are introduced for this: the annihilation operator h_v^+ of a heavy quark with 4-velocity v and the creation operator h_v^- of a heavy antiquark with 4-velocity v, which are related to the initial quark fields as

$$h_v^+(x) = e^{i m_Q v\cdot x} P_+ Q(x), \quad h_v^-(x) = e^{i m_Q v\cdot x} P_- Q(x), \tag{6}$$

where $P_\pm = (1 \pm \not{v})/2$ are projection operators.

The new fields satisfy the equations of motion

$$\not{v} h_v^\pm(x) = \pm h_v^\pm(x). \tag{7}$$

Obviously, in the rest frame of the heavy quark $v = (1, \vec{0})$ the operator h_v^+ corresponds to the two upper components of the field Q, and h_v^- corresponds to the two lower components. In terms of the new fields the QCD Lagrangian for the heavy quark is written as

$$\mathscr{L}_h = \bar{h}_v^+ iv\cdot D h_v^+ - \bar{h}_v^-(iv\cdot D + 2m_Q)h_v^- + \bar{h}_v^+ i\not{D}_\perp h_v^- + \bar{h}_v^- i\not{D}_\perp h_v^+, \tag{8}$$

where $D_\perp^\mu = D^\mu = v^\mu v\cdot D$ is the covariant derivative orthogonal to the quark velocity v: $v\cdot D_\perp^\mu = 0$. In the Isgur–Wise limit $m_Q \to \infty$, i.e., when the heavy quark is nearly on-shell, the field behaves as $h_v^-(x) \to 0$ and we arrive at the HQET Lagrangian in leading order in the $1/m_Q$ expansion:

$$\mathscr{L}_{\rm HQET}^{(0)} = \bar{h}_v^+ iv\cdot D h_v^+. \tag{9}$$

Then, performing the functional integral

$$Z_h = \int \delta\bar{h}_v^+ \int \delta h_v^+ \int \delta\bar{h}_v^- \int \delta h_v^- \exp\left(i \int dx\, \mathscr{L}_h\right) \tag{10}$$

over the antiquark fields $\bar{h}_v^-$ and h_v^-, we obtain

$$Z_h=\int \delta\bar{h}_v^+ \int \delta h_v^+ \exp\left(i\int dx\ \mathscr{L}_{\rm HQET}\right), \tag{11}$$

where

$$\mathscr{L}_{\rm HQET}=\bar{h}_v^+ i v\cdot D h_v^+ + \lim_{\varepsilon\to+0} \bar{h}_v^+ \not{D}_\perp^\mu \frac{1}{2m_Q iv\cdot D - i\varepsilon} \not{D}_\perp^\mu h_v^+ \tag{12}$$

is the desired Lagrangian of the heavy-quark effective theory,[43] which is conveniently written as an expansion in powers of $1/m_Q$, with the leading-order term and the correction of order $1/m_Q$. For this we use the identity

$$P_+ i\not{D}_\perp i\not{D}_\perp P_+ = P_+\left[(iD_\perp)^2+\frac{g_s}{2}\sigma_{\alpha\beta}G^{\alpha\beta}\right]P_+, \tag{13}$$

where $[iD^\alpha iD^\beta]=ig_sG^{\alpha\beta}$ is the gluon field-strength tensor and g_s is the QCD coupling constant. Then for the HQET Lagrangian we obtain

$$\begin{aligned}
\mathscr{L}_{\rm HQET}&=\mathscr{L}^{(0)}_{\rm HQET}+\mathscr{L}^{(1)}_{\rm HQET}+O(1/m_Q^2),\\
\mathscr{L}^{(0)}_{\rm HQET}&=\bar{h}_v^+ iv\cdot Dh_v^+,\\
\mathscr{L}^{(1)}_{\rm HQET}&=\frac{1}{2m_Q}\bar{h}_v^+(iD_\perp)^2h_v^+ + \frac{g_s}{4m_Q}\bar{h}_v^+\sigma_{\alpha\beta}G^{\alpha\beta}h_v^+ .
\end{aligned} \tag{14}$$

The leading term in the Lagrangian of the effective theory, the Lagrangian $\mathscr{L}^{(0)}_{\rm HQET}$, possesses a new type of symmetry, the Isgur–Wise spin–flavor symmetry (Refs. 28–30), which is not present in the QCD Lagrangian. The spin symmetry is associated with the non-Abelian group $SU(2)$. The spin operators S_i^+ $(i=1,2,3)$ of the fields h_v^+ are introduced as follows:

$$S_i^+ = i\varepsilon_{ijk}[\not{e}_j,\not{e}_k](1+\not{v})/2, \tag{15}$$

where e_k^μ is an orthonormal set of spacelike vectors orthogonal to the velocity of the heavy quark v_μ: $e_{\mu j}e_k^\mu=-\delta_{jk}$, $v_\mu e_k^\mu=0$. Here the operators S_i^+ satisfy the commutation relations

$$[S_i^+,S_j^+]=i\varepsilon_{ijk}S_k^+, \tag{16}$$

i.e., they are $SU(2)$ generators. The flavor symmetry of the Lagrangian $\mathscr{L}^{(0)}_{\rm HQET}$ arises from the fact that the quark mass is absent in it. If we have the situation where N_h heavy quarks move with identical velocities v, the original Lagrangian can be extended to $\mathscr{L}^{h(0)}_{\rm HQET}$:

$$\mathscr{L}^{h(0)}_{\rm HQET}=\sum_{i=1}^{N_h}\bar{h}_v^{+i} iv\cdot Dh_v^{+i}, \tag{17}$$

which is obviously invariant under the group $SU(N_h)$ of rotations in flavor space. Finally, combining the two types of symmetry (spin and flavor), we arrive at the $SU(2N_h)$ Isgur–Wise spin-flavor symmetry in the heavy-quark sector.

The operators of order $1/m_Q$ present in $\mathscr{L}_{\rm HQET}$ are easily interpreted in the rest frame of the heavy quark.[43] The operator

$$O_{\rm kin}=\frac{1}{2m_Q}\bar{h}_v^+(iD_\perp)^2h_v^+ \to -\frac{1}{2m_Q}\bar{h}_v^+(i\vec{D})^2h_v^+ \tag{18}$$

is none other than a gauge-invariant addition to the kinetic-energy operator describing the descent of the heavy quark from the mass shell. The operator

$$O_{\rm mag}=\frac{g_s}{4m_Q}\bar{h}_v^+\sigma_{\alpha\beta}G^{\alpha\beta}h_v^+ \to -\frac{g_s}{m_Q}\bar{h}_v^+\vec{S}\cdot\vec{B}_c h_v^+ \tag{19}$$

is the non-Abelian analog of the Pauli term describing the chromomagnetic interaction of the heavy-quark spin with the gluon field, where $\vec{S}$ is the spin operator and $B_c^i=-\frac{1}{2}\varepsilon^{ijk}G^{jk}$ are the components of the chromomagnetic field.

The spin–flavor symmetry leads to numerous relations between the properties of hadrons containing a single heavy quark.[44] One of the clearest manifestations of the Isgur–Wise symmetry is the spectroscopy of heavy hadrons. In particular, owing to this symmetry, hadron states involving c and b quarks can be classified according to the quantum numbers of the light degrees of freedom (the flavor, the spin J_l, the spatial parity P_l, and so on). Therefore, owing to the spin symmetry, each fixed value of the light-quark spin J_l corresponds to a pair of degenerate hadron states with total spin $J=J_l\pm 1/2$. A typical example is the experimental values of the mass difference of vector and pseudoscalar states containing the charmed quark, $m_{D^*}-m_D\simeq 142$ MeV, and the bottom quark, $m_{B^*}-m_B\simeq 46$ MeV. Obviously, the mass splitting of heavy hadrons is rather small and is of order $1/m_Q$. Flavor symmetry predicts that the mass differences of hadrons with different quantum numbers of the light degrees of freedom approximately coincide for c and b hadrons. For example, owing to this symmetry it is possible to expect that the following mass formulas hold: $m_{B_s}-m_B\simeq m_{D_s}-m_D\simeq 100$ MeV, $m_{B_1}-m_B\simeq m_{D_1}-m_D\simeq 555$ MeV, and so on. The first mass formula has been brilliantly confirmed experimentally in 1993. The ALEPH group at LEP[16] was the first to measure the mass of the B_s meson: $m_{B_s}=5.368\pm 0.005$ GeV, which is in good agreement with the Isgur–Wise symmetry: $m_{B_s}^{\rm theor}\simeq 5.379$ GeV.

An important dynamical consequence of the Isgur–Wise symmetry is the group relations between the relativistic form factors of weak decays of heavy hadrons (Refs. 28, 29, 40, and 41). Owing to the spin–flavor symmetry, in the description of the dynamics of the transition $H(Qq)\to H'(Q'q)$ the corresponding matrix element factorizes. That is, the transition amplitude for $H\to H'$ is the product of the transition amplitudes of the heavy quarks $Q\to Q'$ and of the light quarks. The transition from one heavy quark to another is described by the matrix element of the current operator $J=\bar{Q}'\Gamma Q$. Using the $SU_L(2)\times SU_R(2)$ group relations, it has been shown[28] that the form factors of weak decays of heavy mesons of the type $B\to Dl\nu$ are described by a universal function $\xi(\omega)$, where ω is the scalar product of the 4-velocities of the initial and final hadrons. This result has been obtained for the case where the heavy quark acts as a static source of the color field. The generalization of this method to the case of moving heavy quarks has been proposed in Ref. 29. Here use is made of the fact that the amplitude of the light degrees of freedom is independent of the heavy-quark mass m_Q for $m_Q\to\infty$, while the ratio p_Q^μ/m_Q is strictly fixed. However, all these investigations have been carried out for 4-momenta of the light degrees of freedom which are small compared with m_Q.

The authors of Ref. 45 have considered the possibility of violation of the universal function $\xi(\omega)$ via breaking of the heavy-quark symmetry. Their analysis is performed using the parton model for large momentum transfers. In this approach the structure of the meson H_i is described by a pair of valence quarks: a heavy quark Q_i and a light antiquark $\bar{q}$. Here the light degrees of freedom and the heavy quarks are coupled only through the distribution function relating the quark quantum numbers to the hadron quantum numbers.

Two possible symmetry breakings are considered:

(1) The momentum of the light degrees of freedom depends on the heavy-quark mass m_Q, while the other constant quantities are of order $\Lambda_{\rm QCD}$ for $m_Q\to\infty$.

(2) The interaction between the heavy and light quarks induced by the large momentum p_T in the function Φ_i has range of order m_Q^{-1} and vanishes for $m_Q\to\infty$ (the chromomagnetic interaction).

Analysis of these two cases has shown that the relations obtained for the weak form factors of heavy mesons are satisfied in the entire kinematical region of the $b\to c$ transition and also the $b\to u$, $c\to s$, and $c\to d$ transitions. Unfortunately, it is not possible to obtain the specific form of the Isgur–Wise function by using only the spin–flavor symmetry. It is only known that the function $\xi(\omega)$ must be normalized to unity at maximum momentum transfer, i.e., for $v=v'$ and $v\cdot v'=1$ (Ref. 28):

$$\xi(\omega=1)=1. \tag{20}$$

An upper limit (the Bjorken limit) on $\xi(\omega)$ has also been obtained:[31,46]

$$\xi(\omega)\leq\sqrt{\frac{2}{1+\omega}}. \tag{21}$$

Any other predictions about this function are model-dependent.

The explicit form of the Isgur–Wise function has been obtained by the QCD sum-rule method[47] on the basis of calculations of the three-point Green function $T(p,p')$. The masses of the initial meson M and of the final meson M' were written as $M=m_b+\bar{\Lambda}$ and $M'=m_c+\bar{\Lambda}$, where the parameter $\bar{\Lambda}$ has the meaning of the binding energy. Accordingly, the momenta of the initial state P and of the final state P' are expressed in terms of their hadron 4-velocities as

$$P^\mu=(m_Q+\bar{\Lambda})v^\mu,\quad P'^\mu=(m_{Q'}+\bar{\Lambda})v'^\mu. \tag{22}$$

The light-quark propagator contains both perturbative and nonperturbative contributions. The nonperturbative contribution was determined by using a nonlocal quark condensate. Using the sum rule for the vertex function, the following form of the Isgur–Wise function was obtained:

$$\xi(\omega)=\approx\exp(-0.37\sqrt{\omega^2-1}). \tag{23}$$

The main idea of Ref. 48 is that the argument of the universal function $\xi(\omega)$ can vary throughout the entire kinematical region not only with q^2, but also with the mass ratio of the heavy mesons. Therefore, if the hadronic form factor at $q^2=0$ is known for arbitrary masses, it is possible to obtain the explicit form of the Isgur–Wise function. Here the form factors at maximum momentum transfer are determined by the corresponding overlap integrals of the meson wave functions in the infinite-momentum frame:

$$\langle M_2|f(u)|M_1\rangle=\int_0^1 du\,\varphi_{M_2}^*(u)f(u)\varphi_{M_1}(u), \tag{24}$$

where $u=k/p_T$ is the ratio of the (infinite) meson momentum and the momentum transfer p_T. In Ref. 48 the integral (24) was estimated by using the relativistic oscillator model proposed by Bauer, Stech, and Wirbel.[49]

Using the ground state of the relativistic scalar harmonic oscillator

$$\varphi_M(u)=N_M\sqrt{u(u-1)}\exp\left\{-\frac{M^2}{2\omega^2}\left(1-u-\frac{\alpha}{M}\right)^2\right\}, \tag{25}$$

where the constant N_M is determined from the normalization condition for the meson wave function, $\langle M|M\rangle=1$, the parameter ω^2 is the expectation value of the momentum transfer, $\omega^2=\langle p_T^2\rangle$, and the quantity

$$\alpha=M-m_Q^{\rm const}+O\left(\frac{1}{m_Q}\right) \tag{26}$$

determines the difference between the meson mass and the effective constituent mass of the heavy quark, the following expression was obtained[49] for the Isgur–Wise function:

$$\xi(\omega)=\frac{2}{\omega+1}\exp\left[-(2\rho^2-1)\frac{\omega-1}{\omega+1}\right],\quad \rho\simeq1. \tag{27}$$

The Isgur–Wise function has also been obtained in the confined-quark model: the relativistic quark model based on certain assumptions about the hadronization and confinement of light quarks:[50]

$$\xi_{\rm CQM}(\omega)=0.4\cdot\Phi(\omega)+\frac{1.2}{1+\omega}, \tag{28}$$

where

$$\Phi(\omega)=\frac{1}{\sqrt{\omega^2-1}}\ln(\omega+\sqrt{\omega^2-1}). \tag{29}$$

The amplitudes of semileptonic baryon decays have a more complicated spin structure than the amplitudes of the analogous processes involving heavy mesons. Isgur and Wise[40] were the first to obtain model-independent relations between the baryon form factors describing the transitions $\frac{1}{2}^+\to\frac{1}{2}^+$ and $\frac{1}{2}^+\to\frac{3}{2}^+$. It was shown that in the Isgur–Wise limit the form factors of heavy baryons satisfy the group relations and can be expressed in terms of three unknown universal functions: $\zeta(\omega)$, $\eta(\omega)$, and $\iota(\omega)$ (Ref. 38).

In this approach the matrix element of semileptonic decays of heavy hadrons is determined by the currents

$$\mathscr{V}_\nu^{ji}\equiv\bar{\mathscr{Q}}^j\gamma_\nu\mathscr{Q}_i;\quad \mathscr{A}_\nu^{ji}\equiv\bar{\mathscr{Q}}^j\gamma_\nu\gamma_5\mathscr{Q}_i, \tag{30}$$

which are related to the ordinary weak currents

$$V_\nu^{ji}\equiv\bar{Q}^j\gamma_\nu Q^i;\quad A_\nu^{ji}\equiv\bar{Q}^j\gamma_\nu\gamma_5 Q_i \tag{31}$$

as

$$J_\nu^{ij}=C_{ji}\mathscr{J}_\nu^{ji}+\dots . \tag{32}$$

We note that in the leading-log approximation the coefficient functions C_{ji} depend on the scalar product of the 4-velocities of the initial and final hadrons as

$$C_{ji}(\omega)=\left[\frac{\alpha_s(m_i)}{\alpha_s(m_j)}\right]^{a_I}\left[\frac{\alpha_s(m_j)}{\alpha_s(\mu)}\right]^{a_L}. \tag{33}$$

In the specific case of $b\to c$ transitions $a_I=-6/25$ and $a_L=8/27[\omega r(\omega)-1]$, where

$$r(\omega)=\frac{1}{\sqrt{\omega^2-1}}\ln(\omega+\sqrt{\omega^2-1}). \tag{34}$$

For example, the most convenient parametrization of the matrix element of the current J_μ^{ji} for the semileptonic decay $\frac{1}{2}^+\to\frac{1}{2}^+l+\nu_l$ has the form

$$M_{\text{inv},\mu}^{ji}(v,v')=\langle B_{Q_j}(v',s')|J_\mu^{ji}|B_{Q_i}(v,s)\rangle$$
$$=\bar{u}(v',s')\Gamma_{J_\mu}^{ji}u(v,s), \tag{35}$$

where for the vector current

$$\Gamma_{V_\mu}^{ji}=F_1^{ji}(\omega)\gamma_\mu+F_2^{ji}(\omega)v_\mu+F_3^{ji}(\omega)v'_\mu, \tag{36}$$

and for the axial curent

$$\Gamma_{A_\mu}^{ji}=G_1^{ji}(\omega)\gamma_\mu\gamma_5+G_2^{ji}(\omega)v_\mu\gamma_5+G_3^{ji}(\omega)v'_\mu\gamma_5. \tag{37}$$

It turned out[38] that in the decay $\Lambda_b\to\Lambda_c e\nu$ only the two form factors F_1 and G_1 do not vanish in the limit $m_Q\to\infty$. The two form factors coincide and can be expressed in terms of the function $\zeta(\omega)$:

$$F_1^{ji}(\omega)=G_1^{ji}(\omega)\equiv C^{ji}(\omega)\zeta(\omega), \tag{38}$$

and the corresponding matrix element has the form

$$M_{\text{inv},\mu}^{ji}(v,v')=C^{ji}(\omega)\zeta(\omega)\bar{u}(v')\gamma_\mu(1+\gamma_5)u(v). \tag{39}$$

For the decays $\Omega_b(\Sigma_b)\to\Omega_c(\Sigma_c)e\nu$ and $\Omega_b(\Sigma_b)\to\Omega_c^*(\Sigma_c^*)e\nu$ the relations between the form factors are more complicated. Nevertheless, all the form factors can be expressed in terms of the two functions $\eta(\omega)$ and $\iota(\omega)$. An interpretation of the results of Isgur and Wise using the tensor formalism for describing the symmetry properties of the baryon wave functions has been given by Georgi.[41] The Bjorken sum rules in the Isgur–Wise limit have been obtained in Ref. 51 for semileptonic decays of Ω_b baryons $\Omega_b\to\Omega_c(\Omega_c^*)l\nu_l$; these can be used to obtain model-independent bounds on the behavior of the baryon form factors. In Ref. 35 the HQET was used to study semileptonic decays of heavy baryons and their exclusive creation in e^+e^- annihilation. Using the HQET Lagrangian (14), relations were obtained between the semileptonic form factors of baryons containing the b quark, taking into account the $1/m_Q$ corrections (Refs. 52–54). It turned out that the form factors determining the decay $\Lambda_b\to\Lambda_c e\nu$ are expressed in terms of the axial form factor $G_1(\omega)$ (Refs. 52 and 53). Nonleptonic decays of heavy baryons were studied by using the effective theory in Ref. 38. It was shown that the Isgur–Wise symmetry makes it possible to relate the matrix elements of the transitions $\Lambda_b\to\Lambda_c D_s$ and $\Lambda_b\to\Lambda_c D_s^*$. The explicit form of the form factors of semileptonic decays of baryons containing the b quark have been obtained by using the QCD sum-rule method.[66]

Exclusive semileptonic decays of heavy baryons have been studied in considerable detail by using the spectator quark model (Refs. 55–64). In this approach the baryon wave functions are constructed with allowance for the fact that in the limit of infinite mass of the heavy quark the spins of the heavy and light degrees of freedom "decouple." As a result, there is factorization of the contributions of heavy and light quarks to the baryon wave function $B_{\alpha\beta\gamma}$, which is constructed as the direct product of the free-quark wave functions:

$$B_{\alpha\beta\gamma}=\Psi_\alpha\Psi_\beta\Psi_\gamma \tag{40}$$

and obeys the Dirac equation

$$\left(\frac{\not p}{m_Q}-1\right)_\alpha^{\alpha'}B_{\alpha'\beta\gamma}=0. \tag{41}$$

Assuming that the velocities of the heavy quarks and of the heavy baryons are equal, $p_Q/m_Q=P/M$, it has been shown that the wave function of the heavy baryon also obeys the Bargmann–Wigner equation:

$$(\not P-M)_\alpha^{\alpha'}B_{\alpha'\beta\gamma}=0. \tag{42}$$

The baryon wave functions have been constructed on the basis of symmetry principles and by taking into account the Bargmann–Wigner equation. The matrix elements of the processes $\frac{1}{2}^+\to\frac{1}{2}^+$, $\frac{1}{2}^+\to\frac{3}{2}^+$, and $\frac{3}{2}^+\to\frac{3}{2}^+$ were written in terms of the tensor function $L_{\mu\nu}$, which is determined by the universal functions of the effective theory, $\zeta(\omega)$, $\eta(\omega)$, and $\iota(\omega)$.

According to the results obtained in the heavy-quark effective theory,[40] the matrix element of the decay $\Lambda_b\to\Lambda_c$ in the spectator quark model is described[55] by a single universal function $F_\Lambda(\omega)$, while the transitions $\Sigma_b\to\Sigma_c$ and $\Sigma_b\to\Sigma_c^*$ are determined by the two independent form factors F_L and F_T. However, the spectator quark model does not allow the functions F_Λ, F_L, and F_T to be found explicitly. The momentum dependence of the hadronic form factors was modeled in accordance with the power-counting rules of QCD.[65] That is, the dependence of the baryon form factors on the squared momentum transfer q^2 was chosen to be of the form

$$F(q^2)=F(0)\left(\frac{m_{FF}^2}{m_{FF}^2-q^2}\right)^n, \tag{43}$$

where according to the power-counting rules[65] n varies from 2 to 4, depending on the type of form factor, and m_{FF} is the mass of the corresponding lightest resonance in the vector channel with a given flavor structure. This is either the $B_c^*(6.34\text{ GeV})$ meson, if the weak decay of a b-containing baryon with flavor change $b \to c$ is considered, or the $D_s^*(2.11\text{ GeV})$ meson, if decay with flavor change $c \to s$ is considered, and so on. The spectator quark model has been used to obtain a detailed description of semileptonic decays of charmed[58] and b-containing baryons[59] (the decay widths, differential distributions, and lepton spectra) and of nonleptonic decays of charmed baryons.[61] In connection with future experiments the study of Ref. 62 is particularly noteworthy, as it gives predictions for polarization effects in semileptonic decays of Λ_c and Λ_b baryons.

Semileptonic decays of heavy baryons $\Lambda_b \to \Lambda_c e\nu$ and $\Sigma_b \to \Sigma_c e\nu$ have also been studied by using the nonrelativistic quark model.[67] The decay widths and differential distributions have been calculated.

The decay $\Lambda_b \to \Lambda_c e\nu$ is also the subject of Refs. 68 and 69, in which calculations were performed in the quark model, which is based on the picture of a baryon as a bound state of a quark and a diquark. Here the wave functions in the *infinite-momentum frame* were used as the baryon wave functions. The difference between the approaches is that in Ref. 68 the Drell–Yan formalism[70] was used, while in Ref. 69 the wave functions of the Bauer–Stech–Wirbel model[49] were used in the actual calculations.

In this study we analyze semileptonic decays of heavy baryons in the confined-quark model (CQM; Refs. 71–83), the relativistic quark model taking into account the confinement of light quarks. The CQM was originally proposed as a tool for studying the properties of light hadrons (Refs. 71–78). Several years ago the model was generalized to the physics of hadrons containing a heavy quark (b or c; Refs. 50 and 79–81). Semileptonic decays of charmed and b-containing hadrons (both mesons and baryons) were described.

In this review we focus on baryon physics. We shall calculate the form factors of semileptonic flavor-changing ($b \to c$, $b \to u$, $c \to s$, and $c \to d$) decays of heavy baryons in the limit of infinite mass of the heavy quarks (the Isgur-Wise limit). For the semileptonic decays $\Lambda_b \to \Lambda_c l\,\bar{\nu}_l$ and $\Sigma_b \to \Sigma_c l\bar{\nu}_l$ we calculate the $1/m_Q$ corrections to the leading order of the asymptotic expansion. The results are consistent with the Ademollo–Gatto theorem.[84] The Isgur–Wise function of the decay $\Lambda_b \to \Lambda_c l\bar{\nu}l$ is used for a detailed analysis of the observables of the cascade decay $\Lambda_b \to \Lambda_c[\to \Lambda_s \pi] + W[\to l\nu_l]$: the decay widths, the differential distributions $d\Gamma/d\omega$, the lepton spectra $d\Gamma/dE_l$, and the asymmetry parameters. The results are compared with those of other models.

2. KINEMATICAL AND DYNAMICAL CHARACTERISTICS OF SEMILEPTONIC DECAYS OF HEAVY BARYONS

Here we shall discuss the kinematical and dynamical characteristics of semileptonic decays of heavy baryons $B_i(p) \to B_f(p') + l(k_1) + \bar{\nu}_l(k_2)$ accompanied by a change of flavor $i \to j$. The total momentum of the lepton pair is $q = k_1 + k_2$. We shall neglect the electron mass in our discussion. The masses of the baryons in the initial and final states will be denoted by M_{B_i} and M_{B_j}, respectively. For the heavy-quark mass we introduce the notation m_Q, with $Q = b, c$. The symbol $\bar{\Lambda}$ denotes the binding energy, i.e., the mass difference of the heavy baryon and the heavy quark contained in the baryon: $\bar{\Lambda} = M_{B_Q} - m_Q$. The corresponding invariant matrix element is written as

$$M(B_i \to B_f l\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{if} l^\mu(q) J_\mu(v, v'), \tag{44}$$

where $G_F \simeq 1.166\ \text{GeV}^{-2}$ is the Fermi constant, $l^\mu = \bar{u}_\nu \gamma^\mu (1 - \gamma^5) u_l$ is the leptonic current, $J_\mu(v, v')$ is the weak hadronic current, V_{if} is the corresponding element of the Kobayashi–Maskawa matrix, and v and v' are the 4-velocities of the heavy baryons in the initial and final states, respectively.

We shall start from the hypothesis of a left-handed structure (left-handed chirality) of the weak charged quark current (i.e., the transition $i \to f$) in complete correspondence with the predictions of the standard model of electroweak interactions, i.e., we shall take the weak spin matrix to be of the form

$$O_\mu = \gamma_\mu (1 + \gamma_5), \quad \gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}. \tag{45}$$

This hypothesis of the standard model has been convincingly confirmed in the case of charged quark currents composed of the fields of heavy and light quarks. For the $b \to c$ transition this question remains open. The sensitivity of the polarization characteristics of the semileptonic decay $B \to D^* l\nu$ to the sign of the chirality of the $b \to c$ transition has been analyzed in detail in Ref. 85. For this the symmetry group of the standard model was extended to the group $SU_L(2) \times SU_R(2) \times U_Y(1)$, i.e., a right-handed W boson was introduced into the theory. It was shown that mixing of the left- and right-handed quark currents is not impossible, and the contribution of the right-handed $b \to c$ transition can be very important. Here the experimental study of the semileptonic decay of the B meson $B \to D^* l\nu$ did not provide a solution to this problem. In connection with this there are great hopes for theoretical and experimental studies of the analogous decay in the heavy-baryon sector, $\Lambda_b \to \Lambda_c l\nu$. We are planning to analyze the chirality of the $b \to c$ transition in the semileptonic decay of a b-containing baryon in the near future. In the present study we restrict ourselves to the choice of left-handed chirality of the $b \to c$ transition.

The weak hadronic currents $J_\mu(v, v')$ are linear combinations of the relativistic form factors depending on the kinematical variable ω:

$$\omega = \frac{pp'}{M_i M_f} = \frac{M_i^2 + M_f^2 - q^2}{2 M_i M_f}. \tag{46}$$

For $J_\mu(v, v')$ we shall use the parametrization from the heavy-quark effective theory.[40,41]

• *The transition $B_i \to B_f$*:

$$J_\mu(v,v')=\bar{u}_f(v')\Lambda_\mu(v,v')u_i(v),$$

$$\Lambda_\mu(v,v')=F_1(\omega)\gamma_\mu+F_2(\omega)v_\mu+F_3(\omega)v'_\mu+G_1(\omega)\gamma_\mu\gamma_5+G_2(\omega)v_\mu\gamma_5+G_3(\omega)v'_\mu\gamma_5. \quad (47)$$

• *The transition* $B_i\to B_f^*$:

$$J_\mu(v,v')=\bar{B}_f^{*\alpha}(v')\Lambda_{\mu\alpha}(v,v')B_f(v),$$

$$\Lambda_{\mu\alpha}(v,v')=\gamma_\mu v_\alpha[K_1(\omega)+\gamma_5N_1(\omega)]+v_\mu v_\alpha[K_2(\omega)+\gamma_5N_2(\omega)]+v'_\mu v_\alpha[K_3(\omega)+\gamma_5N_3(\omega)]+g_{\mu\alpha}[K_4(\omega)+\gamma_5N_4(\omega)]. \quad (48)$$

In the Isgur–Wise limit the form factors of semileptonic $b\to c$ decays of heavy baryons satisfy the group relations:[40]

• *The transition* $\Lambda_b\to\Lambda_c$:

$$F_1(\omega)=G_1(\omega)=\zeta(\omega),\quad F_2(\omega)=F_3(\omega)=G_2(\omega)=G_3(\omega)\equiv 0. \quad (49)$$

The function $\zeta(\omega)$ satisfies the normalization condition $\zeta(1)=1$. It is also referred to as the Isgur–Wise baryon function (or the Isgur–Wise function of the decay $\Lambda_b\to\Lambda_c l\bar{\nu}_l$). It is important to know the slope of this function at the point $\omega=1$, which is usually characterized by the rms radius ρ:

$$\zeta(\omega)=1-\rho^2(\omega-1)+\ldots . \quad (50)$$

• *The transition* $\Sigma_b\to\Sigma_c(\Sigma_c^*)$:

$$F_1(\omega)=G_1(\omega)=-\frac{\omega+1}{\sqrt{3}}N_1(\omega)+\frac{1}{2\sqrt{3}}N_4(\omega)=-\frac{\omega-1}{\sqrt{3}}K_1(\omega)+\frac{1}{2\sqrt{3}}K_4(\omega)=-\tfrac{1}{6}[2\omega\eta+(\omega-1)\iota],$$

$$F_2(\omega)=F_3(\omega)=\frac{2}{\sqrt{3}}N_1(\omega)=\tfrac{2}{3}\eta, \quad (51)$$

$$G_2(\omega)=-G_3(\omega)=\frac{2}{\sqrt{3}}K_1(\omega)=\tfrac{2}{3}[\eta+\iota],$$

$$N_2(\omega)=K_2(\omega)\equiv 0,\quad N_i(\omega)=-K_i(\omega),$$

$$i=3,4;\quad F_2(\omega)-G_2(\omega)=-\frac{2}{\sqrt{3}}N_3(\omega).$$

The authors of Ref. 51 obtained model-independent restrictions (Bjorken sum rules) on the choice of form factors of semileptonic decays of the Ω_b baryon, which, as is well known, is equivalent to the Σ_b baryon, because it contains a pair of light quarks with identical quantum numbers. If we introduce a combination of the functions $\eta(\omega)$ and $\iota(\omega)$:

$$\xi_1=\eta-\frac{\omega-1}{2}\iota,\quad \xi_2=-\tfrac{1}{2}\iota, \quad (52)$$

then the sum rules give the following upper bounds for the functions $\xi_1(\omega)$ and $\xi_2(\omega)$:

$$1\geq\frac{2+\omega^2}{3}|\xi_1|^2+\frac{(\omega^2-1)^2}{3}|\xi_2|^2+\frac{\omega-\omega^3}{3}(\xi_1\xi_2^*+\xi_2\xi_1^*). \quad (53)$$

In addition, we obtain the following bound on the radius ρ_1 of the form factor ξ_1:

$$\rho_1^2\geq\tfrac{1}{3}-\tfrac{2}{2}\xi_2(1). \quad (54)$$

The $1/m_Q$ corrections to the form factors of the semileptonic decay $\Lambda_b\to\Lambda_c l\nu_l$ were found in Ref. 52 by using the HQET. In particular, it was shown that the five form factors $(F_{i=1,2,3},\,G_{i=2,3})$ can be expressed in terms of a single form factor G_1 and the dimensional parameter $\bar{\Lambda}=M_{\Lambda_b}-m_b=M_{\Lambda_c}-m_c$:

$$F_1(\omega)=G_1(\omega)\left[1+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\frac{\bar{\Lambda}}{1+\omega}\right],$$

$$F_2(\omega)=G_2(\omega)=-G_1(\omega)\frac{1}{m_c}\frac{\bar{\Lambda}}{1+\omega}, \quad (55)$$

$$F_3(\omega)=-G_3(\omega)=-G_1(\omega)\frac{1}{m_b}\frac{\bar{\Lambda}}{1+\omega}.$$

The $1/m_c$ corrections to semileptonic decays of the Ω_b baryon were obtained by using the HQET in Ref. 54. Whereas in the decay of the Λ_b baryon $\Lambda_b\to\Lambda_c l\nu_l$ the relations between the form factors are quite universal, for the decays $\Omega_b\to\Omega_c l\nu_l$ and $\Omega_b\to\Omega_c^* l\nu_l$ the picture is quite different. In particular, in addition to the two functions η and ι describing decays of the Ω_b baryon in the Isgur–Wise limit, five additional functions and one dimensional parameter $\bar{\Omega}=M_{\Omega_b}-m_b=M_{\Omega_c}-m_c$ are also needed.

For semileptonic decays of the Λ_Q baryon into light baryons in the Isgur–Wise limit (i.e., when the mass of the heavy quark is $m_Q\to\infty$) the corresponding vertex part $\Lambda_\mu(v,v')$ has the following form:[35,55]

• *The transition* $\Lambda_Q\to\Lambda_q$:

$$\Lambda_\mu(v,v')=L(v,v')\gamma_\mu(1+\gamma_5),$$

$$L(v,v')=\zeta_1(\omega)+\not{v}\zeta_2(\omega). \quad (56)$$

In the limit where the light-quark mass is $m_q\to\infty$ we should obtain the vertex part for $\Lambda_b\to\Lambda_c$ transitions, so that for $m_q\to\infty$ we have the form factor $\zeta_2(\omega)\to 0$ and $\zeta_1(\omega)\to\zeta(\omega)$. This implies that $\zeta_2(\omega)\propto 1/m_q$, and the function $\zeta_1(\omega)$ is related to the Isgur–Wise function $\zeta(\omega)$ as $\zeta_1(\omega)=\zeta(\omega)+O(1/m_q)$.

The observables of semileptonic decays (the decay widths, differential distributions, lepton spectra, and asymmetry parameters) will for the sake of convenience be defined in terms of the so-called *helicity amplitudes* $H^{\Gamma}_{\lambda_f\lambda_W}$ (Refs. 59, 62, 67, and 69), where λ_f is the helicity of the baryon in the final state, and λ_W is the helicity of the weak W boson, which is off-shell. The helicity amplitudes are related to the above-defined form factors of semileptonic decays by linear relations.[59,62] Here we restrict ourselves to the observables in the decay of the Λ_b baryon $\Lambda_b\to\Lambda_c[\to\Lambda_s\pi]+W[\to l\nu_l]$. In this case the helicity amplitudes $H^{\Gamma}_{\lambda_f\lambda_W}$ are expressed in terms of the form factors F_i and G_i, for $i=1,2,3$, as[62,69]

$$H^V_{\pm 1/2\,0}=\sqrt{\frac{\omega-1}{\omega_{\max}-\omega}}\,[(M_{\Lambda_b}+M_{\Lambda_c})F_1(\omega)$$

$$+M_{\Lambda_c}(\omega+1)F_2(\omega)+M_{\Lambda_b}(\omega+1)F_3(\omega)],$$

$$H^A_{\pm 1/2\,0}=\pm\sqrt{\frac{\omega+1}{\omega_{\max}+\omega}}\,[(M_{\Lambda_b}-M_{\Lambda_c})G_1(\omega)$$

$$-M_{\Lambda_c}(\omega-1)G_2(\omega)-M_{\Lambda_b}(\omega-1)G_3(\omega)],$$

$$H^V_{\pm 1/2\,1}=-2\sqrt{M_{\Lambda_b}M_{\Lambda_c}(\omega-1)}F_1(\omega),$$

$$H^A_{\pm 1/2\,1}=\mp 2\sqrt{M_{\Lambda_b}M_{\Lambda_c}(\omega+1)}G_1(\omega),$$

where

$$\omega_{\max}=\frac{M^2_{\Lambda_b}+M^2_{\Lambda_c}}{2M_{\Lambda_b}M_{\Lambda_c}}.$$

The semileptonic decay widths are calculated from the expression

$$\Gamma=\int_1^{\omega_{\max}}d\omega\,\frac{d\Gamma}{d\omega},$$
$$\frac{d\Gamma}{d\omega}=\frac{d\Gamma_{T_+}}{d\omega}+\frac{d\Gamma_{T_-}}{d\omega}+\frac{d\Gamma_{L_+}}{d\omega}+\frac{d\Gamma_{L_-}}{d\omega},\tag{57}$$

where the indices T and L denote the partial contributions of the transverse ($\lambda_W=\pm 1$) and longitudinal ($\lambda_W=0$) components of the hadronic current. The partial differential cross sections are

$$\frac{d\Gamma_{T_\pm}}{d\omega}=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\,\frac{M^3_{\Lambda_c}}{6}\,(\omega_{\max}-\omega)\sqrt{\omega^2-1}|H_{\pm 1/2\pm 1}|^2,$$
$$\frac{d\Gamma_{L_\pm}}{d\omega}=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\,\frac{M^3_{\Lambda_c}}{6}\,(\omega_{\max}-\omega)\sqrt{\omega^2-1}|H_{\pm 1/2\,0}|^2,\tag{58}$$

where $H_{\lambda_f\lambda_W}=H^V_{\lambda_f\lambda_W}-H^A_{\lambda_f\lambda_W}$.

The differential distribution $d\Gamma/dE_l$ (or the leptonic E_l spectrum, where E_l is the charged-lepton energy) is calculated from the expression

$$\frac{d\Gamma}{dE_l}=\frac{d\Gamma_{T_+}}{dE_l}+\frac{d\Gamma_{T_-}}{dE_l}+\frac{d\Gamma_{L_+}}{dE_l}+\frac{d\Gamma_{L_-}}{dE_l}.\tag{59}$$

The expressions for the partial lepton spectra have the form

$$\frac{d\Gamma_{T_\pm}}{dE_l}=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\,\frac{M^2_{\Lambda_c}}{8}\int_{\omega_{\min}(E_l)}^{\omega_{\max}}d\omega(\omega_{\max}-\omega)$$
$$\times(1\pm\cos\Theta)^2|H_{\pm 1/2\pm 1}|^2,\tag{60}$$
$$\frac{d\Gamma_{L_\pm}}{dE_l}=\frac{G_F^2}{(2\pi)^3}|V_{bc}|^2\,\frac{M^2_{\Lambda_c}}{4}\int_{\omega_{\min}(E_l)}^{\omega_{\max}}d\omega(\omega_{\max}-\omega)$$
$$\times(1-\cos^2\Theta)^2|H_{\pm 1/2\,0}|^2.$$

Here Θ is the polar angle between the Λ_c baryon and the charged lepton l in the $(l\nu_l)$ center-of-mass frame. The charged-lepton energy and $\cos\Theta$ are related in the standard way:

$$\cos\Theta=\frac{E_l^{\max}-2E_l+M_{\Lambda_c}(\omega_{\max}-\omega)}{M_{\Lambda_c}\sqrt{\omega^2-1}},$$
$$E_l^{\max}=\frac{M^2_{\Lambda_b}-M^2_{\Lambda_c}}{2M_{\Lambda_b}},\tag{61}$$

$$\omega_{\min}(E_l)=\omega_{\max}-2\,\frac{E_l(E_l^{\max}-E_l)}{M_{\Lambda_c}(M_{\Lambda_b}-2E_l)}.\tag{62}$$

We note that polarization effects in weak decays are due to the nontrivial spin structure of the final states (the decay products). Owing to this, the corresponding density matrices $\rho_{\lambda_f\lambda_W;\lambda'_f\lambda'_W}=H_{\lambda_f\lambda_W}H^*_{\lambda'_f\lambda'_W}$ have a nontrivial form. As is well known, the density matrix plays an important role in the quantum physics of mixed states, because it allows the calculation of the expectation value of any physical quantity characterizing the system (including the polarization state). The angular distributions of the W-boson and final-baryon decay products are good analyzers of the structure of the density matrix $\rho_{\lambda_f\lambda_W;\lambda'_f\lambda'_W}$. The dynamics of W-boson decay $W\to l\nu_l$ is well defined in the standard model of electroweak interactions, and so this process has 100% analyzing power. On the other hand, nonleptonic decays of the charmed Λ_c baryon $\Lambda_c\to\Lambda_s\pi$ have by now been studied quite thoroughly experimentally (Refs. 18, 20, and 86). In particular, the asymmetry parameters in Λ_c decay have been measured. As correctly pointed out in the study of Körner and Krämer,[62] in principle not only decay with a pseudoscalar meson in the final state, but also decay with a vector meson can be used as a "nonleptonic analyzer" of semileptonic decays.

For example, a possible candidate might be the nonleptonic decay $\Lambda_c^+\to\Lambda_s\rho^+$. Here, in studying the polarization characteristics of weak decays of heavy baryons, we restrict ourselves to the nonleptonic decay $\Lambda_c\to\Lambda_s\pi$.

Let us consider the polarization characteristics in the weak two-cascade decay $\Lambda_b\to\Lambda_c[\to\Lambda_s\pi]+W[\to l\nu_l]$. First let us consider the decay of an unpolarized Λ_b baryon. The corresponding 4-dimensional angular distribution has the form[62]

$$\frac{d\Gamma}{d\omega\;d\cos\Theta\;d\chi\;d\cos\Theta_\Lambda}=\mathrm{Br}(\Lambda_c\to\Lambda_s+\pi)$$

$$\times\frac{G_F^2}{2\pi^4}|V_{bc}|^2\,\frac{M^2_{\Lambda_c}}{12}\,(\omega_{\max}-\omega)\sqrt{\omega^2-1}$$

$$\times\Big(\frac{3}{8}(1\pm\cos\Theta)^2|H_{1/2\ 1}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda)$$
$$+\tfrac{3}{8}(1\mp\cos\Theta)^2|H_{-1/2-1}|^2(1-\alpha_\Lambda\cos\Theta_\Lambda)$$
$$+\tfrac{3}{4}\sin^2\Theta[|H_{1/2\ 0}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda)$$
$$+|H_{-1/2\ 0}|^2(1-\alpha_\Lambda\cos\Theta_\Lambda)]$$
$$\mp\frac{3}{2\sqrt{2}}\alpha_\Lambda\cos\chi\sin\Theta\sin\Theta_\Lambda[(1$$
$$\pm\cos\Theta)\mathrm{Re}(H_{-1/2\ 0}H^*_{1/2\ 1})$$
$$+(1\mp\cos\Theta)\mathrm{Re}(H_{1/2\ 0}H^*_{-1/2-1})]\Big). \tag{63}$$

Here Θ_Λ is the polar angle between the baryons Λ_s and Λ_b in the rest frame of the Λ_c baryon, and χ is the angle between the $(l\nu_l)$ and $(\Lambda_s\pi)$ planes in the Λ_b rest frame. The upper and lower signs in Eq. (63) correspond to the final lepton states $l^-\bar{\nu}_l$ and $l^+\nu_l$, respectively. $\mathrm{Br}(\Lambda_c\to\Lambda_s+\pi)$ is the branching ratio of the nonleptonic decay $\Lambda_c\to\Lambda_s+\pi$. The parameter α_Λ is the asymmetry parameter of the nonleptonic decay $\Lambda_c\to\Lambda_s\pi$.

Instead of studying the angular distribution (63), we can also consider the distributions depending on one of the spherical angles defining the geometry of the cascade decay: Θ_Λ, Θ, or χ. For example, the distribution as a function of the polar angle Θ_Λ is obtained by integrating the 4-dimensional distribution (63) over the two angles Θ and χ in the ranges $0\leqslant\Theta\leqslant\pi$ and $0\leqslant\chi\leqslant2\pi$:

$$\frac{d\Gamma}{d\omega\ d\cos\Theta_\Lambda}\propto1+\alpha\alpha_\Lambda\cos\Theta_\Lambda, \tag{64}$$

where the asymmetry parameter α is defined as

$$\alpha=\frac{|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2-|H_{-1/2\ 0}|^2}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2}. \tag{65}$$

In order to obtain the distribution in the polar angle Θ, we integrate the angular distribution (63) over the polar angle Θ_Λ in the range $0\leqslant\Theta_\Lambda\leqslant\pi$ and the azimuthal angle χ in the range $0\leqslant\chi\leqslant2\pi$:

$$\frac{d\Gamma}{d\omega\ d\cos\Theta}\propto1\pm2\alpha'\cos\Theta+\alpha''\cos^2\Theta, \tag{66}$$

where the asymmetry parameters α' and α'' are expressed in terms of the helicity amplitudes as

$$\alpha'=\frac{|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+2(|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2)}, \tag{67}$$

$$\alpha''=\frac{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2-2(|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2)}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+2(|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2)}. \tag{68}$$

The distribution in the azimuthal angle χ is obtained from (63) by integrating over the polar angles in the ranges $0\leqslant\Theta_\Lambda\leqslant\pi$ and $0\leqslant\Theta\leqslant\pi$:

$$\frac{d\Gamma}{d\omega\ d\chi}\propto1\mp\frac{3\pi^2}{32\sqrt{2}}\gamma\alpha_\Lambda\cos\chi, \tag{69}$$

where the azimuthal asymmetry parameter γ is given by the expression

$$\gamma=\frac{2\ \mathrm{Re}(H_{-1/2\ 0}H^*_{1/2\ 1}+H_{1/2\ 0}H^*_{-1/2-1})}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2}. \tag{70}$$

In connection with the experiments planned at CERN on the observation of semileptonic decays of polarized Λ_b and Λ_c baryons, the theoretical description of the polarization characteristics in these decays is of interest. Here the density matrix of the Λ_c baryon consists of the following elements:

$$\rho_{1/2\ 1/2}=|H_{1/2\ 1}|^2(1-P\cos\Theta_P)+|H_{1/2\ 0}|^2(1+P\cos\Theta_P),$$
$$\rho_{1/2-1/2}=\rho_{-1/2\ 1/2}=-P\sin\Theta_P\ \mathrm{Re}(H_{1/2\ 0}H^*_{-1/2\ 0}), \tag{71}$$
$$\rho_{-1/2-1/2}=|H_{-1/2-1}|^2(1+P\cos\Theta_P)+|H_{-1/2\ 0}|^2(1-P\cos\Theta_P),$$

where P is the degree of polarization of the incident baryon Λ_b and Θ_P is the polar angle between the polarization vector of the baryon Λ_b and the baryon momentum Λ_c.

The corresponding 4-dimensional angular distribution is given by the expression[62]

$$\frac{d\Gamma}{d\omega\ d\cos\Theta_P d\chi_P d\cos\Theta_\Lambda}$$
$$=\mathrm{Br}(\Lambda_c\to\Lambda_s+\pi)\times\frac{G_F^2}{2\pi^4}|V_{bc}|^2\frac{M^2_{\Lambda_c}}{24}(\omega_{\max}-\omega)$$
$$\times\sqrt{\omega^2-1}(|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2+|H_{1/2\ 1}|^2$$
$$+|H_{-1/2-1}|^2+\alpha_\Lambda\cos\Theta_\Lambda(|H_{1/2\ 0}|^2$$
$$-|H_{-1/2\ 0}|^2+|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2)$$
$$+P\cos\Theta_P(|H_{1/2\ 0}|^2-|H_{-1/2\ 0}|^2-|H_{1/2\ 1}|^2$$
$$+|H_{-1/2-1}|^2)+P\alpha_\Lambda\cos\Theta_\Lambda\cos\Theta_P$$
$$\times(|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2-|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2)$$
$$-2P\alpha_\Lambda\sin\Theta_\Lambda\sin\Theta_P\cos\chi_P$$
$$\times\mathrm{Re}(H_{1/2\ 0}H^*_{-1/2\ 0})]), \tag{72}$$

where Θ_Λ is the polar angle between the Λ_s and Λ_b baryons in the rest frame of the Λ_c baryon, and $(\pi-\chi_P)$ is the angle between the plane formed by the Λ_b polarization vector and the W-boson momentum and the $(\Lambda_s\pi)$ plane in the Λ_b rest frame.

We define the two-dimensional angular distributions as in the case of decays of unpolarized baryons. In particular, integrating (72) over the angles Θ_Λ and χ_P in the ranges $0\leqslant\Theta_\Lambda\leqslant\pi$ and $0\leqslant\chi_P\leqslant2\pi$, we obtain

$$\frac{d\Gamma}{d\omega\ d\cos\Theta_P}\propto1-\alpha_P P\cos\Theta_P, \tag{73}$$

where the asymmetry parameter α is defined as

$$\alpha_P=\frac{|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2-|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2}. \quad (74)$$

Then, integration of the distribution (72) over the angles Θ_Λ and Θ_P in the ranges $0\leqslant\Theta_\Lambda\leqslant\pi$ and $0\leqslant\Theta_P\leqslant 2\pi$ leads to the angular distribution as a function of the angle χ_P:

$$\frac{d\Gamma}{d\omega\, d\chi_P}\propto 1-\frac{\pi^2}{16}P\alpha_\Lambda\gamma_P\cos\chi, \quad (75)$$

where the azimuthal asymmetry parameter γ_P is given by

$$\gamma_P=\frac{2\ \mathrm{Re}(H_{1/2\ 0}H^*_{-1/2\ 0})}{|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2}. \quad (76)$$

In this study we shall calculate the expectation values of the asymmetry parameters defined as follows. In the expressions for the asymmetry parameters it is necessary to partially integrate the numerator and denominator independently over the kinematical variable ω with weight $(\omega_{\max}-\omega)\sqrt{\omega^2-1}$ in the range $1\leqslant\omega\leqslant\omega_{\max}$. For example, the expectation value of the asymmetry parameter α characterizing the distribution in the angle Θ_Λ in the decay of an unpolarized Λ_b baryon is given by

$$\langle\alpha\rangle=\frac{\int_1^{\omega_{\max}}d\omega(\omega_{\max}-\omega)\sqrt{\omega^2-1}[|H_{1/2\ 1}|^2-|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2-|H_{-1/2\ 0}|^2]}{\int_1^{\omega_{\max}}d\omega(\omega_{\max}-\omega)\sqrt{\omega^2-1}[|H_{1/2\ 1}|^2+|H_{-1/2-1}|^2+|H_{1/2\ 0}|^2+|H_{-1/2\ 0}|^2]}, \quad (77)$$

and so on. The expectation values of the asymmetry parameters will be denoted by angle brackets $\langle\ldots\rangle$.

In conclusion, we note that we are neglecting the effects of CP (or T) violation, so that all the helicity amplitudes in our study are real.

3. BARYON STRUCTURE IN THE CONFINED-QUARK MODEL

The starting point for describing semileptonic decays of heavy baryons in the CQM is the Lagrangian describing the interaction of baryons with quark fields

$$\mathscr{L}_B(x)=g_B\bar{B}_\alpha(x)J_B^\alpha(x)+\mathrm{H.c.}, \quad (78)$$

where α, a, and i are respectively spin, color, and flavor indices, g_B is the coupling constant, J_B^α is the three-quark current with the corresponding quantum numbers of the baryon B (Refs. 71, 74, 75, and 79), and

$$J_B^\alpha(x)=R^{i_1i_2i_3,\alpha}_{\alpha_1\alpha_2\alpha_3}q^{a_1}_{i_1\alpha_1}(x)q^{a_2}_{i_2\alpha_2}(x)q^{a_3}_{i_3\alpha_3}(x)\varepsilon^{a_1a_2a_3}, \quad (79)$$

where $R^{i_1i_2i_3,\alpha}_{\alpha_1\alpha_2\alpha_3}$ is the product of spin and flavor matrices.

Let us write out the explicit form of the quark currents that we shall need. We note that for baryons with quantum numbers $\frac{1}{2}^+$ there are two equally valid versions of the three-quark currents, the so-called *tensor version* and the *vector version* (see Refs. 74, 75, and 87 for more detail). These two currents are equally valid. In the series of studies carried out by our group (see Refs. 71, 74, 75, and 79) it was shown that the tensor version of the three-quark current is preferable, and we shall use it here. In this case the three-quark currents J_B for baryons $B=\Lambda_Q$, Σ_Q, Λ_s, p, and n have the form

$$J_{\Lambda_Q}=\varepsilon^{abc}\cdot[Q^a(u^bC\gamma^5d^c)+\gamma^5Q^a(u^bCd^c)],$$

$$J_{\Sigma_Q}=\varepsilon^{abc}\cdot\sigma^{\mu\nu}\gamma^5Q^a(u^bC\sigma^{\mu\nu}u^c),$$

$$J_{\Lambda_s}=\varepsilon^{abc}\cdot[s^a(u^bC\gamma^5d^c)+\gamma^5s^a(u^bCd^c)], \quad (80)$$

$$J_p=\varepsilon^{abc}\cdot[u^a(u^bC\gamma^5d^c)+\gamma^5u^a(u^bCd^c)],$$

$$J_n=\varepsilon^{abc}\cdot[d^a(u^bC\gamma^5d^c)+\gamma^5d^a(u^bCd^c)],$$

where $C=\gamma^0\gamma^2$ is the charge-conjugation matrix and $Q=b,c$.

In the case of baryons with quantum numbers $\frac{3}{2}^+$ there is only one version of the three-quark current. In this study we restrict ourselves to only heavy pseudovector baryons Σ_Q^*, which correspond to a current of the form[71,74]

$$J^\mu_{\Sigma_Q^*}=-\varepsilon^{abc}\cdot\left[Q^a(u^bC\gamma^\mu d^c)-\frac{i}{2}\gamma^\nu Q^a(u^bC\sigma^{\mu\nu}d^c)\right]. \quad (81)$$

The full Lagrangian needed for describing semileptonic decays of heavy baryons is of the form

$$\mathscr{L}_{\mathrm{full}}=\sum_B\mathscr{L}_B+\mathscr{L}_{\mathrm{weak}}+\mathrm{H.c.}, \quad (82)$$

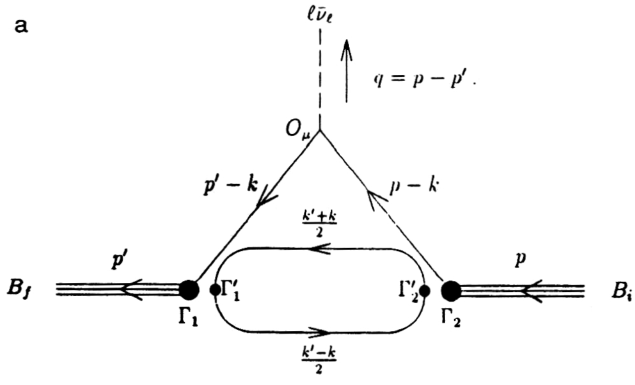
where $\mathscr{L}_{\mathrm{weak}}$ is the standard quark–lepton weak-interaction Lagrangian:

$$\mathscr{L}_{\mathrm{weak}}=\frac{G_F}{\sqrt{2}}l^\mu\bar{q}^a_fV_{if}O_\mu q^a_i. \quad (83)$$

The coupling constant g_B can be calculated by using the so-called *coherence condition*,[71] which implies that the renormalization constant of the baryon wave function is zero:

$$Z_B=1+\frac{3g_B^2}{4\pi^2}\Pi'_B(m_B)=0, \quad (84)$$

where Π'_B is the derivative of the baryon mass operator and m_B is the baryon mass. It should be noted that in our model a



а. Двух-петлевая кварковая диаграмма

b

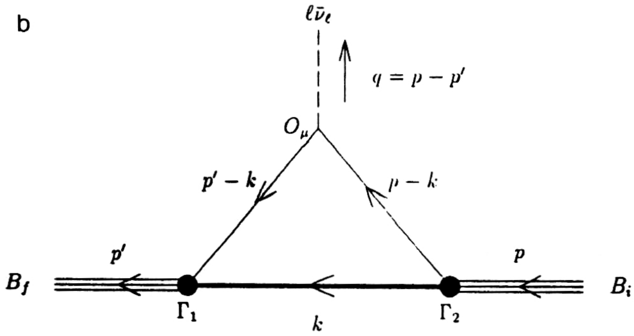


FIG. 1.

the hadron mass spectrum is not calculated from the corresponding equations for bound states (for example, from the Bethe–Salpeter equation), since the latter contain too many free parameters (see Ref. 88, for example). The hadron masses are treated as input parameters of the model and taken from the tables of the Particle Data Group.[86]

Physically, the coherence condition means that the probability of finding a baryon in the "bare" state is zero. In other words, the baryon is a bound state of three quarks. It is important to note that the coherence condition is completely equivalent to the normalization condition for the baryon electromagnetic form factor, $F_{\rm em}(q^2=0)=1$, where the baryon has electric charge ± 1.

In the CQM weak decays of heavy baryons in the leading order in $1/N_c$ are described by the graph shown in Fig. 1a. These graphs do not contain ultraviolet divergences, owing to the *confinement ansatz* for light (u,d,s) quarks. By confinement ansatz we mean the existence of a certain procedure for averaging light quarks over vacuum gluon configurations. In Feynman-diagram language this averaging can be written as an integral of the product of the light-quark propagators of an arbitrary quark diagram with complex measure $d\sigma_z$. For example, in the case where a quark diagram contains only a single light-quark propagator $G_z(p)$, we have

$$G(p)=\int d\sigma_z G_z(p)=\int d\sigma_z \frac{1}{z\Lambda_g-\not{p}}$$

$$=\frac{1}{\Lambda_q}\left[a\left(-\frac{p^2}{\Lambda_q^2}\right)+\frac{\not{p}}{\Lambda_q}b\left(-\frac{p^2}{\Lambda_q^2}\right)\right]. \quad (85)$$

Here the confinement of the propagators describing the propagation of light quarks of different colors is produced independently.

As is well known, in the heavy-quark effective theory in the Isgur–Wise limit[30] the heavy quark is nearly on-shell, i.e., it is in the infrared regime. The infrared asymptote of the one-particle Green function has been studied in the Abelian theory (quantum electrodynamics) in a number of studies (see Refs. 89 and 90, for example). The *infrared propagator* of an electron has the form

$$G(p,\nu)=G(p)s(p^2,\nu), \quad (86)$$

where $G(p)$ denotes the standard propagator of a free fermion field:

$$G(p)=(m-\not{p})^{-1}. \quad (87)$$

The function $s(p^2,\nu)$ is given by the expression

$$s(p^2,\nu)=(1-p^2/m^2)^{-\nu}. \quad (88)$$

Here the parameter ν is related to the gauge-fixing parameter d_l as

$$\nu=(\alpha_{\rm em}/4\pi)(3-d_l), \quad (89)$$

where $\alpha_{\rm em}$ is the fine-structure constant.

In Refs. 91 and 92 the infrared propagator of the Abelian theory was used as a first approximation in calculations of the Isgur–Wise meson function. We shall proceed in similar fashion. In particular, we shall take the heavy-quark propagator to be

$$S_Q(p,\nu)=\frac{m_Q+\not{p}}{m_Q^2}\left(\frac{1}{1-p^2/m_Q^2}\right)^{1+\nu}, \quad (90)$$

choosing ν to be a free parameter. Therefore, generalization of the CQM to heavy-quark physics leads to the appearance of two additional parameters: the binding energy $\bar{\Lambda}=M_{B_Q}-m_{B_Q}$ and the infrared parameter ν. There are no experimental constraints on the choice of $\bar{\Lambda}$. The binding energy $\bar{\Lambda}$ has been estimated theoretically, using various approaches: potential models, QCD sum rules, and so on. By studying the correlator of two heavy–light quark currents, using the effective theory, it was found that $\bar{\Lambda}=0.50\pm0.07$ GeV (Ref. 93). In our model, by studying leptonic decays of heavy mesons[92] we have obtained a constraint on the choice of the parameters $\bar{\Lambda}$ and ν. In particular, we have shown that the best agreement with the experimental results for the weak constants f_B and f_D is obtained when the parameters $\bar{\Lambda}$ and ν lie in the ranges $0\leqslant\bar{\Lambda}\leqslant0.6$ MeV and $0\leqslant\nu\leqslant1$.

According to the Feynman rules, the vertex part describing the semileptonic decay of a heavy baryon (see Fig. 1a) B_Q into a heavy baryon $B_{Q'}$ and a lepton pair $l\bar{\nu}_l$ is

$$\Lambda_\mu^{QQ'}(\nu,\nu')\propto g_{B_Q}g_{B_{Q'}}\sum_{\Gamma_1\Gamma_2\Gamma_1'\Gamma_2'}K_{\Gamma_1\Gamma_2}\int\frac{d^4k}{\pi^2 i}\Gamma_1 S_{Q'}$$

$$\times(p'-k,\nu)O_\mu S_Q(p-k,\nu)\Gamma_2$$

$$\times \int \frac{d^4k'}{4\pi^2 i} \mathrm{Tr}\left[\Gamma_1' G\left(\frac{k'+k}{2}\right)\Gamma_2' G\left(\frac{k'-k}{2}\right)\right],$$

where $K_{\Gamma_1\Gamma_2}$ are the corresponding group coefficients.

The vertex part describing the semileptonic decay of a heavy baryon B_Q into a light baryon B_q and a lepton pair $l\bar{\nu}_l$ has the form

$$\Lambda_\mu^{Qq}(\nu,\nu') \propto g_{B_Q} g_{B_q} \sum_{\Gamma_1\Gamma_2\Gamma_1'\Gamma_2'} \int \frac{d^4k}{\pi^2 i} \Gamma_1 G(p'-k) O_\mu S_Q$$
$$\times (p-k,\nu)\Gamma_2$$
$$\times \int \frac{d^4k'}{4\pi^2 i} \mathrm{Tr}\left[\Gamma_1' G\left(\frac{k'+k}{2}\right)\Gamma_2' G\left(\frac{k'-k}{2}\right)\right].$$

To avoid the appearance in the matrix elements of ultraviolet divergences and singularities related to free-quark creation, in the CQM it is postulated that the function $G(p)$ is an analytic function in the complex $\not{p}$ plane. The confinement functions $a(u)$ and $b(u)$ are defined as

$$a(u) = \int d\sigma_z \frac{z}{z^2+u}, \quad b(u) = \int d\sigma_z \frac{1}{z^2+u}. \tag{91}$$

The scale parameter Λ_q, which has the dimensions of mass, characterizes the confinement region for light quarks of the corresponding flavor $q=(u,d,s)$. Neglecting the effects of the unitary-symmetry breaking of strong interactions at the heavy-quark mass scale, we shall start from the universal value of the parameter Λ_q, which is the same for u, d, and s quarks: $\Lambda_q = \Lambda_u \equiv \Lambda_d \equiv \Lambda_s$. In Refs. 71–78 it was shown that the physical characteristics of light hadrons depend weakly on the explicit form of the functions $a(u)$ and $b(u)$, and depend only on their integrated characteristics. Here we shall use a very simple set of functions $a(u)$ and $b(u)$, the ones used in Refs. 71–76:

$$a(u) = a_0 \exp(-u^2 - a_1 u),$$
$$b(u) = b_0 \exp(-u^2 + b_1 u). \tag{92}$$

The parameters a_i, b_i, and Λ_q were found from the best description of the experimental data on low-energy hadron physics:[71]

$$a_0 = b_0 = 2, \quad a_1 = 1, \quad b_1 = 0.4, \quad \Lambda_q = 460 \text{ MeV}. \tag{93}$$

Since the calculation of the two-loop graphs (Fig. 1a) involves certain technical difficulties, a *quark–diquark approximation* of the two-loop quark diagrams describing the baryon form factors was suggested in Ref. 75. The basic idea of this approximation is to replace the two-loop quark diagram (Fig. 1a) by a one-loop quark–diquark diagram (Fig. 1b). According to this hypothesis, the vertex parts of semileptonic decays of heavy baryons are written as

$$\Lambda_\mu^{QQ'}(\nu,\nu') \propto g_{B_Q} g_{B_{Q'}} \sum_{\Gamma_1\Gamma_2\Gamma_1'\Gamma_2'} K_{\Gamma_1\Gamma_2} \int \frac{d^4k}{\pi^2 i}$$
$$\times \int d\sigma_z \Gamma_1 S_Q(p'-k,\nu) O_\mu S_Q$$
$$\times (p-k,\nu)\Gamma_2 D_z^{\Gamma_1'\Gamma_2'}(k^2)$$

and

$$\Lambda_\mu^{Qq}(\nu,\nu') \propto g_{B_Q} g_{B_q} \sum_{\Gamma_1\Gamma_2\Gamma_1'\Gamma_2'} K_{\Gamma_1\Gamma_2} \int \frac{d^4k}{\pi^2 i}$$
$$\times \int d\sigma_z \Gamma_1 G_z(p'-k) O_\mu S_Q$$
$$\times (p-k,\nu)\Gamma_2 D_z^{\Gamma_1'\Gamma_2'}(k^2),$$

where $D_z^{\Gamma_1'\Gamma_2'}(k^2)$ is the diquark propagator with the corresponding spin:

$$D_z^{\Gamma_1'\Gamma_2'}(k^2) = d^{\Gamma_1'\Gamma_2'} D_z(k^2), \quad D_z(k^2) = \frac{1}{z^2\Lambda_D^2 - k^2}. \tag{94}$$

Here $d^{\Gamma_1'\Gamma_2'}$ are Lorentz structures dictated by the form of the matrices Γ_1' and Γ_2', and the scale parameter Λ_D, which has the dimensions of mass, characterizes the light-diquark confinement region.

In specific calculations it is convenient to define the parameters $d^{\Gamma_1'\Gamma_2'}$ as

$$d^{PP} = C_{PP}, \; d^{SS} = C_{SS}, \; d^{VV} = C_{VV} g_{\mu\nu}, \; d^{AA} = C_{AA} g_{\mu\nu},$$
$$d^{TT} = C_{TT} g_{\mu\alpha} g_{\nu\beta}, \quad d^{AP} = -d^{PA} = C_{AP}(ik_\mu), \tag{95}$$
$$d^{VT} = -d^{TV} = C_{VT}(ik_\alpha g^{\mu\beta} - ik_\beta \; g^{\nu\alpha}),$$

where $C_{\Gamma_1'\Gamma_2'}$ are numerical coefficients which must ensure preservation of the symmetry properties (for example, gauge invariance) originally possessed by the vertex part of the two-loop diagram. The requirement of gauge invariance imposes the following relations between the parameters $C_{\Gamma_1'\Gamma_2'}$:

$$C_{SS} = C_{PP} = C_{AA} = 3C_{VV} = \tfrac{1}{2} C_{TT}, \quad C_{AP} = C_{PA} = 0. \tag{96}$$

The coefficient C_{VT}, along with the dimensional parameter Λ_D, remains a free parameter in the calculations. The parameter Λ_D was fixed from the condition of obtaining the best description of low-energy nucleon physics:[75] $\Lambda_D = 827.7$ MeV.

It should be noted that the quark–diquark approximation of the two-loop diagrams is closely related to the physics of semileptonic decays, in which a pair of light quarks does not participate in the decay and instead plays the role of a hard core—a diquark. Therefore, the baryons in semileptonic decays are manifested as a bound state of a quark and a diquark.

The technique of calculating the matrix elements in the quark–diquark approximation is demonstrated in the Appendix.

4. FORM FACTORS OF SEMILEPTONIC DECAYS OF HEAVY BARYONS

Let us give the results of the calculations of the form factors of semileptonic decays of heavy baryons $\frac{1}{2}^+ \to \frac{1}{2}^+ l\nu_l$

for various modes (i.e., with change of flavor $b\to c$, $b\to u$, $c\to s$, and $c\to d$). Here in the decays $\Lambda_b\to\Lambda_c l\bar{\nu}_l$ and $\Sigma_b\to\Sigma_c l\bar{\nu}_l$ the form factors are calculated with corrections of order $1/m_Q$.

A. The decay $\Lambda_b\to\Lambda_c l\bar{\nu}_l$:

$$F_1(\omega,\bar{\Lambda},\nu)=G_1(\omega,\bar{\Lambda},\nu)\left[1+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\times\frac{\bar{\Lambda}}{1+\omega}\,\tau(\omega,\bar{\Lambda},\nu)\right],$$

$$F_2(\omega,\bar{\Lambda},\nu)=G_2(\omega,\bar{\Lambda},\nu)=-G_1(\omega,\bar{\Lambda},\nu)\,\frac{1}{m_c}\,\frac{\bar{\Lambda}}{1+\omega}\,\tau(\omega,\bar{\Lambda},\nu),$$

$$F_3(\omega,\bar{\Lambda},\nu)=-G_3(\omega,\bar{\Lambda},\nu)=-G_1(\omega,\bar{\Lambda},\nu)\,\frac{1}{m_b}\,\frac{\bar{\Lambda}}{1+\omega}\,\tau(\omega,\bar{\Lambda},\nu).$$

The function $\tau(\omega,\bar{\Lambda},\nu)$ describes the nontrivial internal structure of heavy baryons. We note that in the HQET $\tau(\omega)\equiv 1$ (Ref. 52). This is explained by the fact that the HQET Lagrangian is a series in powers of $1/m_Q$ consisting of local operators. In our model the form factors are described by a one-loop diagram (Fig. 1b). Here the $1/m_Q$ corrections are obtained in the expansion in powers of $1/m_Q$ of the corresponding loop integral. Therefore, whereas the HQET is a tree approximation to the description of heavy-quark physics, our model makes it possible to take into account nonlocal corrections. It is important to note that the CQM in the Isgur–Wise limit completely reproduces the group relations of the effective theory between the form factors of semileptonic decays.

The form factor $G_1(\omega,\bar{\Lambda},\nu)$ can be expressed in terms of the Isgur–Wise function $\zeta(\omega,\bar{\Lambda},\nu)$ as

$$G_1(\omega,\bar{\Lambda},\nu)=\zeta(\omega,\bar{\Lambda},\nu)\left[1+\bar{\Lambda}\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\rho(\omega,\bar{\Lambda},\nu)\right]. \quad (97)$$

The Isgur–Wise function ζ and also the functions τ and ρ can be expressed in the CQM in terms of the structure integrals Φ_i, with $i=1,2,3$:

$$\zeta(\omega,\bar{\Lambda},\nu)=\frac{\Phi_1(\omega,\bar{\Lambda},\nu)}{\Phi_1(1,\bar{\Lambda},\nu)},\quad \tau(\omega,\bar{\Lambda},\nu)=\frac{\Phi_2(\omega,\bar{\Lambda},\nu)}{\Phi_1(1,\bar{\Lambda},\nu)}, \quad (98)$$

$$\rho(\omega,\bar{\Lambda},\nu)=\frac{\Phi_3(1,\bar{\Lambda},\nu)}{\Phi_1(1,\bar{\Lambda},\nu)}-\frac{\Phi_3(\omega,\bar{\Lambda},\nu)}{\Phi_1(\omega,\bar{\Lambda},\nu)}.$$

The structure integrals $\Phi_i(\omega,\bar{\Lambda},\nu)$ have the following form:

$$\Phi_1(\omega,\bar{\Lambda},\nu)=\int_0^\infty d\alpha\int_0^\infty d\beta\int d\sigma_z\times\frac{(\alpha\beta)^\nu}{[z^2+\alpha^2+\beta^2+2\alpha\beta\omega-2r(\alpha+\beta)]^{1+2\nu}},$$

$$\Phi_2(\omega,\bar{\Lambda},\nu)=(\omega+1)\int_0^\infty d\alpha\int_0^\infty d\beta\int d\sigma_z\times\frac{\alpha(\alpha\beta)^\nu}{[z^2+\alpha^2+\beta^2+2\alpha\beta\omega-2r(\alpha+\beta)]^{1+2\nu}},$$

$$\Phi_3(\omega,\bar{\Lambda},\nu)=\int_0^\infty d\alpha\int_0^\infty d\beta\int d\sigma_z\times\frac{(\alpha\beta)^\nu}{[z^2+\alpha^2+\beta^2+2\alpha\beta\omega-2r(\alpha+\beta)]^{1+2\nu}}\times\left[\frac{1+\alpha(1+\nu)}{2}+(1+2\nu)\alpha\times\frac{(\alpha+\beta-1)^2+\alpha\beta(\omega-3)+\beta}{z^2+\alpha^2+\beta^2+2\alpha\beta\omega-2r(\alpha+\beta)}\right],$$

where $r=\bar{\Lambda}/\Lambda_D$.

In integrating over the vacuum measure $d\sigma_z$ for different values of ν it is necessary to use the identity

$$\int d\sigma_z\,\frac{1}{[z^2+x]^{1+n}}=\frac{\sin(\pi n)}{\pi n}\int_0^\infty\frac{du}{u^n}\int d\sigma_z\times\frac{1}{[z^2+u+x]^2}=-\frac{\sin(\pi n)}{\pi n}\int_0^\infty\frac{du}{u^n}\,b'(u+x).$$

Here $0<n<1$ and $b'(s)=db(s)/ds$.

The function $\rho(\omega,\bar{\Lambda},\nu)$ satisfies the normalization condition $\rho(1,\bar{\Lambda},\nu)=0$. Therefore, in the limit where the velocities of the initial (v) and final (v') baryons are the same, the $1/m_Q$ corrections to the form factor G_1 vanish. In addition, since for $v=v'$ we have

$$\bar{u}_f(v')=\bar{u}_f(v')v_\mu u_i(v)=\bar{u}_f(v')v'_\mu u_i(v),\quad \bar{u}_f(v')\gamma_5 u_i(v)\equiv 0, \quad (99)$$

this also means that in the full matrix element of the decay $\Lambda_b\to\Lambda_c l\bar{\nu}_l$ the corrections of order $1/m_Q$ cancel, i.e., the Ademollo–Gatto theorem is satisfied.

B. The decay $\Sigma_b\to\Sigma_c l\bar{\nu}_l$:

$$F_1(\omega,\bar{\Lambda},\nu)=G_1(\omega,\bar{\Lambda},\nu)\left[1+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\frac{\bar{\Lambda}}{1+\omega}\,\tau(\omega,\bar{\Lambda},\nu)\right],$$

$$F_2(\omega,\bar{\Lambda},\nu)=F_3(\omega,\bar{\Lambda},\nu)=-2G_1(\omega,\bar{\Lambda},\nu)\left[1+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\frac{\bar{\Lambda}}{4(1+\omega)}\,\tau(\omega,\bar{\Lambda},\nu)\right],$$

$$G_2(\omega,\bar{\Lambda},\nu)=-G_3(\omega,\bar{\Lambda},\nu)=2G_1(\omega,\bar{\Lambda},\nu)\Big[1$$

$$+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\frac{3\bar{\Lambda}}{4(1+\omega)}\tau(\omega,\bar{\Lambda},\nu)\Bigg],$$

$$G_1(\omega,\bar{\Lambda},\nu)=\zeta(\omega,\bar{\Lambda},\nu)\left[1+\left(\frac{1}{m_b}+\frac{1}{m_c}\right)\bar{\Lambda}\rho(\omega,\bar{\Lambda},\nu)\right].$$

We note that the form factors of the decays $\Lambda_b\to\Lambda_c l\bar{\nu}_l$ and $\Sigma_b\to\Sigma_c l\bar{\nu}_l$ are expressed in terms of the same functions ζ, τ, and ρ. As in the case of the decay of the Λ_b baryon, the terms proportional to $1/m_Q$ in the matrix element of the decay $\Sigma_b\to\Sigma_c l\bar{\nu}_l$ cancel, in complete agreement with the Ademollo–Gatto theorem.

The results for semileptonic decays of Λ_b and Λ_c baryons into light baryons (Λ_s,p,n) have been obtained in the special case where $\nu=0$.

C. The decay $\Lambda_Q\to\Lambda_s\{p,n\}l\bar{\nu}_l$:

$$F_1(\omega,\bar{\Lambda},\mu_f)=G_1(\omega,\bar{\Lambda},\mu_f)\left[1+\frac{1}{\mu_f}\frac{\bar{\Lambda}}{1+\omega}\tau(\mu,\bar{\Lambda},\mu_f)\right],$$

$$F_2(\omega,\bar{\Lambda},\mu_f)=G_2(\omega,\bar{\Lambda}\mu_f)$$

$$=-G_1(\omega,\bar{\Lambda},\mu_f)\frac{1}{M_f}\frac{\bar{\Lambda}}{1+\omega}\tau_{\rm light}(\omega,\bar{\Lambda},\mu_f),$$

$$F_3(\omega,\bar{\Lambda},\mu_f)=G_3(\omega,\bar{\Lambda},\mu_f)=0,$$

$$G_1(\omega,\bar{\Lambda},\mu_f)=\zeta_{\rm light}(\omega,\bar{\Lambda},\mu_f).$$

Here $\zeta_{\rm light}(\omega,\bar{\Lambda},\mu_f)$ is the Isgur–Wise function for the heavy–light transition, and $\mu_f=M_f/\bar{\Lambda}$.

The functions $\zeta_{\rm light}(\omega,\bar{\Lambda},\mu_f)$ and $\tau_{\rm light}(\omega,\bar{\Lambda},\mu_f)$ can be expressed in the CQM in terms of the structure integrals as

$$\zeta_{\rm light}(\omega,\bar{\Lambda},\mu_f)=\frac{\Phi_1^{\rm light}(\omega,\bar{\Lambda},\mu_f)}{\Phi_1^{\rm light}(1,\bar{\Lambda},\mu_f)},$$

$$\tau(\omega,\bar{\Lambda},\mu_f)=\frac{\Phi_2^{\rm light}(\omega,\bar{\Lambda},\mu_f)}{\Phi_1^{\rm light}(1,\bar{\Lambda},\mu_f)},\qquad(100)$$

where

$$\Phi_1^{\rm light}(\omega,\bar{\Lambda},\mu_f)=\int_0^\infty d\alpha\int_0^{\mu_f}d\beta\times\int d\sigma_z\frac{zs-\alpha-\beta+\mu_f}{z^2\left(s'^2+\frac{\beta}{m_f}(s^2-s'^2)\right)+\alpha^2+\beta^2+2\alpha\beta\omega-2\alpha-\beta\mu_f},$$

$$\Phi_2^{\rm light}(\omega,\bar{\Lambda},\mu_f)=\int_0^\infty d\alpha\int_0^{\mu_f}d\beta\times\int d\sigma_z\frac{2\alpha(\omega+1)}{z^2\left(s'^2+\frac{\beta}{m_f}(s^2-s'^2)\right)+\alpha^2+\beta^2+2\alpha\beta\omega-2\alpha-\beta\mu_f},$$

with $s=\Lambda_q/\bar{\Lambda}$ and $s'=\Lambda_D/\bar{\Lambda}$.

5. THE ISGUR–WISE FUNCTION

In this section we focus on the calculation of the fundamental characteristic of heavy-quark physics, the Isgur–Wise function $\zeta(\omega)$ of the decay $\Lambda_b\to\Lambda_c l\nu_l$. In our model the Isgur–Wise function depends on the parameters $\bar{\Lambda}$ and ν:

$$\zeta(\omega,\bar{\Lambda},\nu)=\frac{\Phi_1(\omega,\bar{\Lambda},\nu)}{\Phi_1(\omega,\bar{\Lambda},\nu)}.\qquad(101)$$

It is important to note that in the diquark approximation the form factors of the decay $\Sigma_b\to\Sigma_c l\nu_l$ are expressed in terms of the Isgur–Wise function $\zeta(\omega,\bar{\Lambda},\nu)$. Taking into account the Bjorken sum rules,[51] this leads to a restriction on the choice of Isgur–Wise function and its radius. Substituting into (52) our results for ζ_1 and ζ_2, namely, $\zeta_1=\omega\zeta$ and $\zeta_2=\zeta$, we obtain an upper bound on the Isgur–Wise function:

$$\zeta^2(\omega,\bar{\Lambda},\nu)\leqslant\frac{3}{1+2\omega^2},\qquad(102)$$

and, accordingly, a lower bound on its radius,

$$\rho^2\geqslant\tfrac{2}{3}.\qquad(103)$$

Let us give the explicit expressions for the function Φ_1 for different values of ν in the range $0\leqslant\nu\leqslant1$.

$\nu=0$:

$$\Phi_1(\omega,\bar{\Lambda},\nu=0)=\int_0^\infty d\alpha\int_0^\infty d\beta\, b(\Delta),$$

where $\Delta=\alpha^2+\beta^2+2\alpha\beta\omega-2r(\alpha+\beta)$ and $r=\bar{\Lambda}/\Lambda_D$.

$0<\nu<1/2$:

Φ1(ω,Λ̄,0<ν<1/2)=− sin(2πν)2πν 0∞ dα0∞ dβ0∞

$$\times \frac{du}{u^{2\nu}} b'(u+\Delta).$$

$\nu=1/2$:

$$\Phi_1(\omega,\bar{\Lambda},\nu=1/2) = -\int_0^\infty d\alpha \int_0^\infty d\beta\, b'(\Delta).$$

$1/2<\nu<1$:

$$\Phi_1(\omega,\bar{\Lambda},1/2<\nu<1) = \frac{\sin(\pi(2\nu-1))}{2\pi\nu(2\nu-1)} \int_0^\infty d\alpha \int_0^\infty d\beta \int_0^\infty \times \frac{du}{u^{2\nu-1}} b''(u+\Delta),$$

where $b''(s)=d^2b(s)/ds^2$.

$\nu=1$:

$$\Phi_1(\omega,\bar{\Lambda},\nu=1) = \frac{1}{2}\int_0^\infty d\alpha \int_0^\infty d\beta\, b''(\Delta).$$

In the case where $\bar{\Lambda}=0$ the Isgur–Wise function has the form

$$\zeta(\omega,\bar{\Lambda},\nu) = \frac{\Gamma(2+2\nu)}{\Gamma^2(1+\nu)} \int_0^\infty d\alpha \frac{\alpha^\nu}{(1+\alpha^2+2\alpha\omega)^{1+\nu}}. \tag{104}$$

In particular, for $\bar{\Lambda}=0$ and $\nu=0,1$ the Isgur–Wise function is

$$\zeta(\omega,0,0) = \frac{\ln(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}},$$
$$\zeta(\omega,0,1) = \frac{3}{\omega^2-1}\left[\frac{\omega \ln(\omega+\sqrt{\omega^2-1})}{\sqrt{\omega^2-1}} - 1\right]. \tag{105}$$

This analysis showed that for values of the infrared parameter ν lying in the range $0 \leq \nu \leq 1/2$ and for any value of the parameter $\bar{\Lambda}$ the radius of the Isgur–Wise function takes values which do not satisfy the Bjorken sum rules. This situation is illustrated in Fig. 2. Here we give the results for the radius of the Isgur–Wise function as a function of the parameters ν and $\bar{\Lambda}$. The numbers label the graphs of the function $\rho^2(\nu)$ for certain values of $\bar{\Lambda}$: *1* ($\bar{\Lambda}=0$ GeV); *2* ($\bar{\Lambda}=0.2$ GeV); *3* ($\bar{\Lambda}=0.4$ GeV); *4* ($\bar{\Lambda}=0.6$ GeV). In turn, the Isgur–Wise function for $0\leq\nu\leq 1/2$ lies above the Bjorken limit (101). Therefore, analysis of the leptonic decays of B and D mesons[92] and the Bjorken sum rules for the Isgur–Wise baryon function[51] impose the following restriction on the choice of the parameter ν: $1/2<\nu\leq 1$. In addition, in fitting the Isgur–Wise function, we tried to obtain the greatest suppression of this function for the maximum value of the kinematical variable ω. This could be done for $\nu=1$. In the graph shown in Fig. 3 we give the results for the lower and upper limit of the Isgur–Wise function with the condition that $\nu=1$ and the parameter $\bar{\Lambda}$ takes reasonable values in the range 0.5 GeV $\leq\bar{\Lambda}\leq$0.6 GeV.

For the best fit of the Isgur–Wise function we take the result obtained for the following choice of the free parameters ν and $\bar{\Lambda}$: $\nu=1$ and $\bar{\Lambda}=0.5$ GeV. It is in this case that we obtain the best agreement with the results of the IMF model,[69] in which the wave functions of baryons in the infinite-momentum frame were used to calculate the Isgur–Wise function.

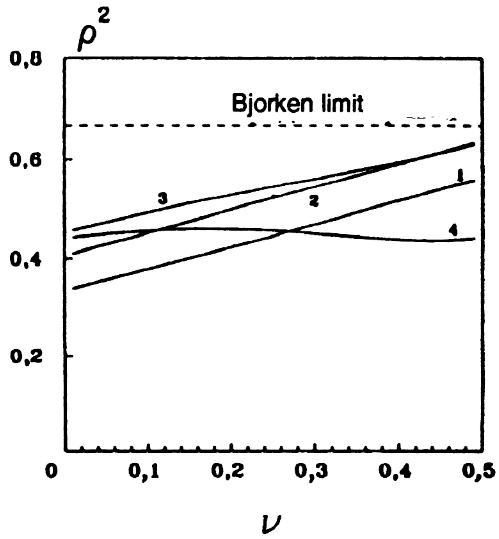


FIG. 2.

In Fig. 4 we compare the results for the Isgur–Wise function obtained in our model ($\nu=1$, $\bar{\Lambda}=0.5$ GeV), using QCD sum rules,[66] and in the IMF model.[69] We see that the Isgur–Wise function obtained by the QCD sum-rule method lies somewhat higher than our curve and the result of Ref. 69. This difference can significantly affect the results for the polarization characteristics of semileptonic decay of the Λ_b baryon.

Let us discuss the results for the radius of the Isgur–Wise function. In the trivial case where $\bar{\Lambda}=0$ the radius depends only on the parameter ν as

$$\rho^2 = \frac{(1+\nu)^2}{3+2\nu}. \tag{106}$$

Obviously, in this case the Bjorken sum rules impose the following lower bound on the parameter ν:

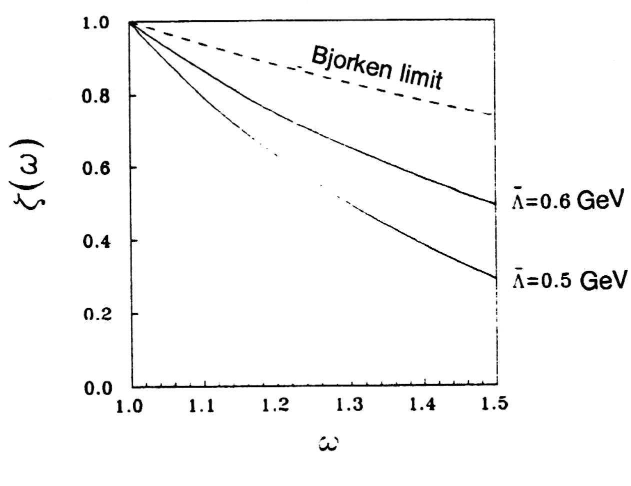


FIG. 3.

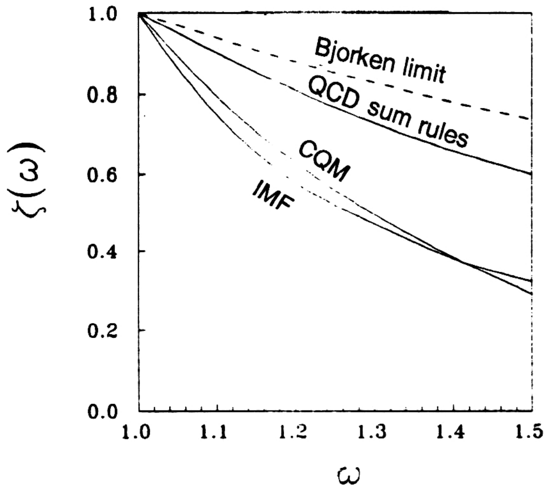


FIG. 4.

$$\nu \geq \frac{\sqrt{10}-1}{3}. \quad (107)$$

The results for ρ^2 obtained in our model for $\nu=1$ and for various values of $\bar{\Lambda}$ are given in Table I. There we also compare our results for the radius of the Isgur–Wise function in the case of the best fit ($\nu=1$, $\bar{\Lambda}=0.5$ GeV) with the results of other approaches: the IMF model[69] and the dipole model (DM; Ref. 69), in which the explicit form of the Isgur–Wise function is modeled as a dipole in accordance with the power-counting rules of QCD[65] for the asymptotes of the hadronic form factors:

$$\rho^2=\begin{cases} 2.25, & \text{CQM}, \\ 3.04, & \text{IMF (Ref. 69)}, \\ 1.78, & \text{DM (Ref. 69)}. \end{cases}$$

6. POLARIZATION CHARACTERISTICS OF THE CASCADE DECAY $\Lambda_b \to \Lambda_c[\to \Lambda_s \pi]+W[\to l\nu_l]$

Let us conclude our discussion by making predictions for the polarization characteristics of the decay $\Lambda_b \to \Lambda_c[\to \Lambda_s \pi]+W[\to l\nu_l]$. Here in the calculations of the corresponding physical quantities we shall use the Isgur–Wise function calculated in the preceding section for the following values of the free parameters ν and $\bar{\Lambda}$: $\nu=1$ and $\bar{\Lambda}=0.5$ GeV. Therefore, we shall work in the Isgur–Wise limit, in which, as is well known, the vertex part of the semileptonic decay $\Lambda_b \to \Lambda_c l \nu_l$ is entirely determined by the Isgur–Wise function. All the needed definitions are given in Sec. 2 above. We shall use the experimental values of the Λ_b and Λ_c baryon masses: $m_{\Lambda_b}=5.64$ GeV and $m_{\Lambda_c}=2.285$ GeV.

TABLE II. Widths of the decay $\Lambda_b \to \Lambda_c l \nu_l$ (in units of 10^{10} sec^{-1}).

Approach	Γ_{total}	Γ_T	Γ_{T_+}	Γ_{T_-}	Γ_L	Γ_{L_+}	Γ_{L_-}
CQM	4.07	1.74	0.47	1.27	2.32	0.11	2.21
IMF(1) (Ref. 69)	4.57	1.88	0.42	1.46	2.69	0.11	2.58
IMF(2) (Ref. 68)	4.89	1.97	0.44	1.53	2.92	0.10	2.82
DM (Ref. 69)	5.51	2.17	0.56	1.61	3.48	0.13	3.35
FQM (Ref. 69)	11.73	3.90	0.94	2.96	11.73	4.09	7.64
SQM (Ref. 59)	4.99	1.93	0.50	1.43	3.06		
NRQM (Ref. 67)	5.9	2.7			3.2		

Let us begin with the calculations of the width of the semileptonic decay $\Lambda_b \to \Lambda_c l \nu_l$. The results of our calculations are given in Table II. For comparison, we give the results of other phenomenological approaches: quark models, in which the baryon wave functions are calculated in the infinite-momentum frame [the IMF(1) in Ref. 68 and the IMF(2) in Ref. 69], the dipole model (DM; Ref. 69), the spectator quark model (SQM; Ref. 59), the free-quark model (FQM; Ref. 69), and the nonrelativistic quark model (NRQM; Ref. 67). We see that our results are in good agreement with those of the approaches of Refs. 68 and 69. It should be noted that our results for the partial widths Γ_{T_+}, Γ_{T_-}, Γ_{L_+}, and Γ_{L_-} admit a quite reasonable explanation.[69] In particular, owing to the left-handed chiral structure of the weak quark current $\bar{c}O_\mu b$, the negative partial widths Γ_{T_-} and Γ_{L_-} dominate over the positive partial widths Γ_{T_+} and Γ_{L_+}, respectively. As in the other approaches, the width Γ_{L_-} dominates over the widths Γ_{T_+} and Γ_{T_-}. Finally, the value of Γ_{L_+} is significantly suppressed in relation to the other partial widths. In Figs. 5 and 6 we give graphs of the differential distributions $d\Gamma/d\omega$ $[d\Gamma_{L_\pm(T_\pm)}/d\omega]$ and the leptonic spectra $d\Gamma/dE_l$ $[d\Gamma_{L_\pm(T_\pm)}/dE_l]$. It should be noted that our results are in good agreement with those of the IMF model.[69] This is the case because in our approaches the behavior of the Isgur–Wise function in the kinematical region $1 \leq \omega \leq \omega_{\max}$ is quite similar. In Table III we give the results of calculations of the asymmetry parameters, which can serve as a prediction for future experiments.

7. CONCLUSION

We have used the CQM to study semileptonic decays of baryons containing a single heavy quark. We have obtained the following results:

- We have obtained expressions for the form factors in the limit of infinitely large masses of the heavy quarks (the Isgur–Wise limit) together with the $1/m_Q$ corrections to the form factors of the decays $\Lambda_b \to \Lambda_c l \nu_l$ and $\Sigma_b \to \Sigma_c l \nu_l$.

TABLE I. Charge radius of the Isgur–Wise function.

$\bar{\Lambda}$, GeV	0.47	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
ρ^2	3.03	2.67	2.43	2.25	2.10	1.99	1.89	1.81	1.73	1.67	1.61	1.56	1.50	1.45

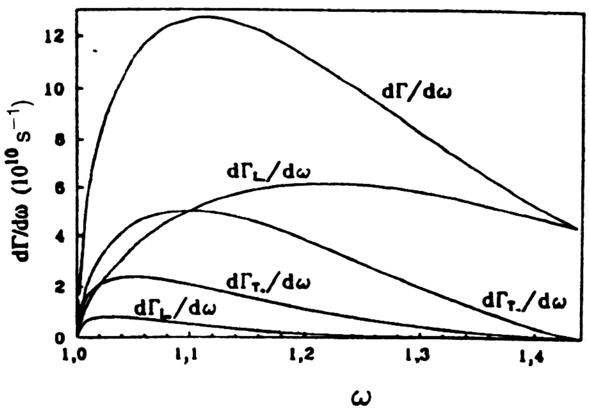


FIG. 5.

- We have shown that in the diquark approximation in the Isgur–Wise limit the form factors of the decays $\Lambda_b \to \Lambda_c l \nu_l$ and $\Sigma_b \to \Sigma_c l \nu_l$ can be expressed in terms of a universal function: the Isgur–Wise function ζ.
- We have calculated the Isgur–Wise baryon function and its rms radius.
- For a particular choice of infrared parameter ν we have obtained suppression of the Isgur–Wise function at the maximum value of the kinematical variable $\omega = v \cdot v'$, where v and v' are the velocities of the initial and final baryon. Here the Isgur–Wise function and its radius satisfy the Bjorken sum rules.[51]
- We have made predictions for future experiments to study the dynamical characteristics of weak decays of the Λ_b baryon. We have calculated the widths, the differential distributions, and the leptonic spectra of the decay $\Lambda_b \to \Lambda_c l \nu_l$. We have also calculated the asymmetry parameters of the cascade decay $\Lambda_b \to \Lambda_c[\to \Lambda_s \pi] + W[\to l \nu_l]$.

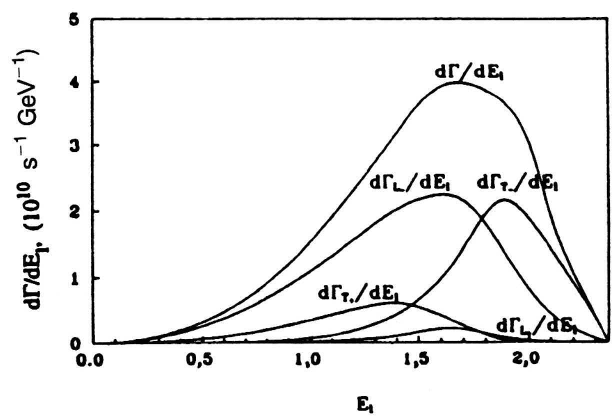


FIG. 6.

Our results are in good agreement with the quark model,[69] in which the baryon wave function is calculated in the infinite-momentum frame. We hope that our work will be useful both to theoreticians interested in heavy-quark physics and to experimentalists involved in experiments on weak decays of heavy baryons.

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TABLE III. Asymmetry parameters.

Approach	$\langle\alpha\rangle$	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$\langle\gamma\rangle$	$\langle\alpha_P\rangle$	$\langle\gamma_P\rangle$
CQM	−0.71	−0.13	−0.46	0.61	0.32	−0.19
IMF(1) (Ref. 69)	−0.71	−0.12	−0.46	0.61	0.33	−0.19
IMF(2) (Ref. 68)	−0.78	−0.14	−0.49	0.53	0.33	−0.15
DM (Ref. 69)	−0.75	−0.12	−0.51	0.57	0.37	−0.17
FQM (Ref. 69)	−0.81	−0.10	−0.60	0.50	0.46	−0.14

8. APPENDIX

Let us demonstrate the technique for calculations in the quark–diquark approximation for the example of the structure integral

$$R(v,v') = \int \frac{d^4k}{\pi^2 i} \int d\sigma_z \frac{1}{[m_{Q'}^2 - (p'-k)^2]^{1+\nu}} \times \frac{1}{[m_Q^2 - (p-k)^2]^{1+\nu}\, z^2\Lambda_D^2 - k^2}.$$

Using the Feynman α parametrization

$$\frac{1}{A_1^{m_1}} \cdots \frac{1}{A_n^{m_n}} = \frac{\Gamma(m_1 + \cdots + m_n)}{\Gamma(m_1)\cdots\Gamma(m_n)} \int_0^1 \times d\alpha_1 \alpha_1^{m_1-1} \cdots \int_0^1 d\alpha_n \alpha_n^{m_n-1} \times \frac{\delta(1-\alpha_1 - \cdots - \alpha_n)}{[A_1\alpha_1 + \cdots + A_n\alpha_n]^{m_1+\cdots+m_n}}$$

and integrating over the virtual momentum k, we obtain

$$R(v,v') = \frac{\Gamma(1+2\nu)}{\Gamma^2(1+\nu)} \int d\sigma_z \int_0^1 d\alpha \alpha^\nu \times \int_0^1 d\beta \beta^\nu \int_0^1 d\gamma \frac{\delta(1-\alpha-\beta-\gamma)}{\Delta^{1+2\nu}},$$

where

$$\Delta = z^2\Lambda_D^2 \gamma + M_i^2\alpha^2 + M_f^2\beta^2 + 2\omega M_i M_f \alpha\beta - 2\bar{\Lambda} \times (M_i\alpha + M_f\beta) + \bar{\Lambda}^2(\alpha^2+\beta^2).$$

Then, making all dimensional quantities dimensionless by means of the parameter $\bar{\Lambda}$, performing the integral over the variable γ, and making the change of variables $\alpha \to \alpha/\mu_i$ and $\beta \to \beta/\mu_f$, where $\mu_i = M_i/\bar{\Lambda}$ and $\mu_f = M_f/\bar{\Lambda}$, we obtain

$$R(\nu,\nu')=\frac{\Gamma(1+2\nu)}{\Gamma^2(1+\nu)}(\mu_i\mu_f)^{1+\nu}\int d\sigma_z\int_0^{\mu_i}d\alpha\int_0^{\mu_f}d\beta \times\left(1-\frac{\alpha}{\mu_i}\right)^{1+\nu}\frac{(\alpha\beta)^\nu}{S^{1+2\nu}},$$

where

$$S=z^2\frac{\Lambda_D^2}{\bar{\Lambda}^2}\left(1-\frac{\alpha}{\mu_i}\right)\left(1-\frac{\beta}{\mu_f}\right)+\alpha^2+\beta^2\left(1-\frac{\alpha}{\mu_i}\right)^2 +2\omega\alpha\beta\left(1-\frac{\alpha}{\mu_i}\right)-2\left(\alpha+\beta\left(1-\frac{\alpha}{\mu_i}\right)\right)+\frac{\alpha^2}{\mu_i} +\beta^2\mu_f\left(1-\frac{\alpha}{\mu_i}\right)^2.$$

In conclusion, let us consider the Isgur–Wise limit ($\mu_i,\mu_f\to\infty$) in the integral $R(\nu,\nu')$:

$$\lim_{\mu_i,\mu_f\to\infty} R(\nu,\nu')=R_{IW}(\nu,\nu')=\frac{\Gamma(1+2\nu)}{\Gamma^2(1+\nu)}(\mu_i\mu_f)^{1+\nu} \times\int d\sigma_z\int_0^{\infty}d\alpha\int_0^{\infty}d\beta\,\frac{(\alpha\beta)^\nu}{(S_{IW})^{1+2\nu}},$$

where

$$S_{IW}=z^2\frac{\Lambda_D^2}{\bar{\Lambda}^2}+\alpha^2+\beta^2+2\omega\alpha\beta-2(\alpha+\beta).$$

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