Elastic scattering of hadrons at high energies

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A review is given of models of elastic scattering of hadrons that describe the experimental data on pp and $\bar{p}p$ scattering in a wide range of energies and momentum transfers. © *1995 American Institute of Physics.*

1. INTRODUCTION

This review is devoted to the physics of diffraction scattering of hadrons, in particular, elastic scattering of protons and antiprotons at high energies.

Diffraction collisions are peripheral, i.e., they take place at small values of the momentum transfer or, equivalently, at large values of the impact parameter. Such collisions are also called "soft," in contrast to "hard" collisions, which take place at small values of the impact parameter.

The interest in diffraction processes is explained by the following facts:

1) Soft collisions make the dominant contribution to the total cross sections.

2) Rich experimental material has been accumulated on elastic collisions, especially on pp and $\bar{p}p$ scattering. In their accuracy, these data are greatly superior to the data on hard processes.

3) The "soft" physics is outside the range of applicability of perturbative chromodynamics, and this stimulates the development of nonperturbative methods in QCD.

In the framework of the theory of Regge poles, diffraction scattering takes place with the exchange of a vacuum trajectory—the Pomeron, the nature of which is largely obscure.

Several fine reviews[1] have been devoted to a discussion of various aspects of elastic and diffraction scattering. A special series of conferences, called Blois Workshops after the location of the first conference of this series, is devoted to this subject. The proceedings of five Blois Workshops have been published in the reviews of Ref. 2.

In our review, we devote our main attention to models that aim at a quantitative description of the experimental data in a wide kinematic range. There are not too many such models. The most critical test of the validity of the various theoretical constructions is the description of the complicated dependence of the differential cross section for elastic scattering on the energy and on the angular variable. It is to this question that we devote the main attention in our review, referring the reader to the cited studies for the many other aspects of the theory and phenomenology of elastic and diffraction scattering. In particular, except in individual cases we shall not consider the spin structure of the scattering amplitude.

2. REVIEW OF THE EXPERIMENTAL DATA

2.1. Total cross sections and the ratios σ_{el}/σ_t and σ_t/B

It is well established that the total cross sections of all scattering processes rise with increasing energy, beginning with the energies of the Serpukhov accelerator. Figure 1 gives data on the total cross sections for pp and $\bar{p}p$ scattering, for which the picture is the most complete. We note that recent measurements using the Tevatron at 1.8 TeV indicate a possible acceleration in the growth of the cross sections.

Figure 2 gives data on the integrated elastic cross section (a) and on the ratios σ_{el}/σ_t and σ_t/B (b and c). Both ratios increase with the energy, violating geometrical scaling.

The cross-section difference $\Delta\sigma=\sigma_t^{\bar{p}p}-\sigma_t^{pp}$ decreases with the energy as a power (Fig. 3):

$$\Delta\sigma=52(s/2m)^{-0.58}\ \text{mb}. \tag{2.1}$$

The behavior (2.1) is usually attributed to the contribution of secondary Reggeons to the scattering amplitude. In the absence of an asymptotic C-odd contribution, the difference (2.1) must tend to zero. There has recently been wide discussion in the literature[4,5] of the possible existence of an asymptotic C-odd exchange, called the odderon.

The dynamical origin of the growth of the total cross section and its conjectured asymptotic behavior are the subject of investigation of many authors. Anticipating a more detailed discussion of various aspects of the cross-section growth, we merely mention here a simple and effective empirical parametrization that was proposed by Donnachie and Landshoff:[6]

$$\sigma_t(\bar{p}p)-\sigma_t(pp)=70s^{-0.56}, \tag{2.2a}$$

$$\tfrac{1}{2}[\sigma_t(\bar{p}p)+\sigma_t(pp)]=150s^{-0.56}+22.7s^{\varepsilon}. \tag{2.2b}$$

The term s^{ε} in (2.2b) corresponds to the contribution of a "supercritical" Pomeron, but Donnachie and Landshoff[6] determined the parameter value $\varepsilon\approx0.08$ from experiment, and found it to be much smaller than estimates based on the results of calculations in the framework of perturbative quantum chromodynamics.

The parametrization (2.2) is attractive above all through its simplicity. The various contributions from the secondary

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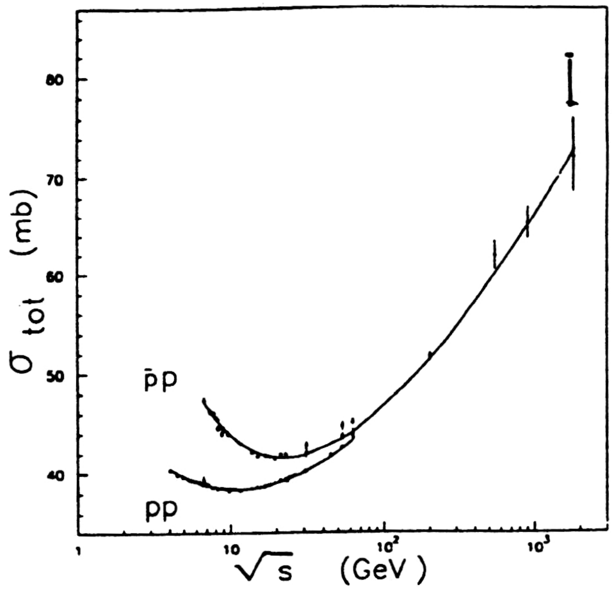

FIG. 1. Total cross sections for pp and $\bar{p}p$ scattering.

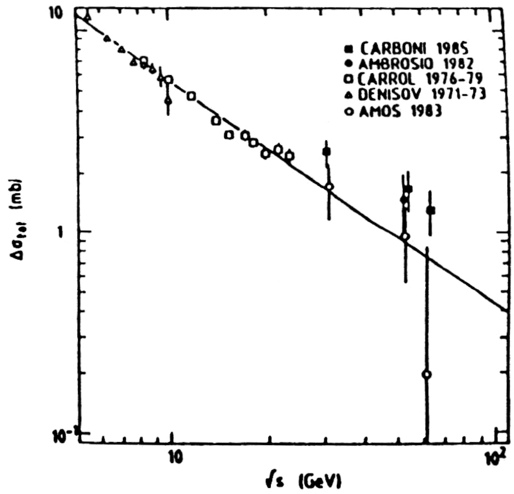

FIG. 3. Difference of the total cross sections of $\bar{p}p$ and pp scattering.

Reggeons and multiple scatterings associated with them are collected together into a single effective power (2.2a). The Pomeron contribution in (2.2b) is, despite the power-law dependence, closer, on account of the smallness of the parameter ε, to models of moderate growth, for example, the dipole Pomeron model. Only future measurements of the total cross sections will make it possible to determine the region of applicability of the parametrization (3.2), and also a possible alternative to it.

2.2. Elastic scattering

Hadron scattering at high energies has a pronounced diffractive nature that is analogous to the phenomenon of diffraction in optics. This analogy was already established in the scattering of nuclei, and its essence is that the nucleus can be regarded as an absorbing sphere, behind which destructive interference takes place between the incident and the scattered wave, as a result of which a "shadow" appears behind the target. This picture is also confirmed by data on the angular distribution of particles in the elastic scattering of hadrons at high energies. The measured differential cross section for elastic pp and $\bar{p}p$ scattering at high energies (Fig. 4) can be divided into three regions:

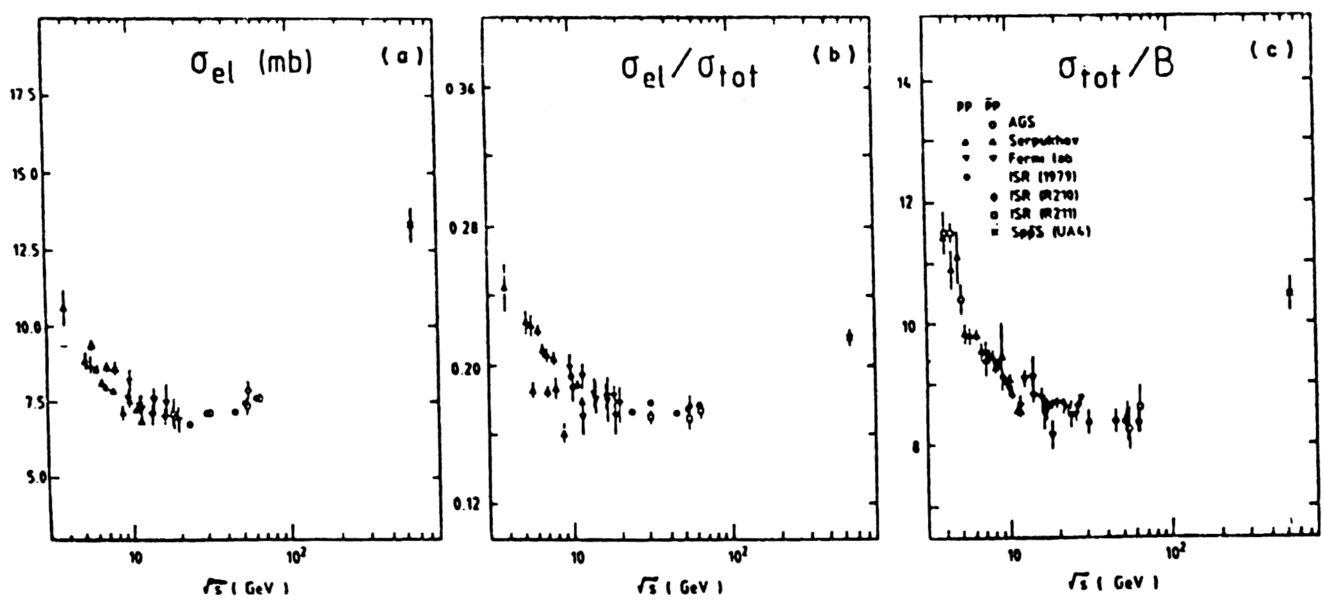

FIG. 2. a) Elastic cross section of pp and $\bar{p}p$ scattering; b) ratio of the elastic to the total cross section: σ_{el}/σ_t; c) ratio of the total cross section to the slope of the peak: σ_t/B.

1) the region of the diffraction peak with almost exponential decrease of the cross section with respect to t;

2) the region of the dip and the second maximum;

3) the region of large $|t|$.

This division is rather nominal (especially as regards the "large" $|t|$); in addition, the boundaries between the regions change with the energy, admittedly slowly (logarithmically). The behavior of the observable quantities in these regions is shown in Fig. 4. We consider in detail the behavior of the cross section at very small $|t|$, in the region of the Coulomb–nuclear interference, and the problem of determining the real part of the scattering amplitude, which has recently become the subject of lively discussion.

We recall that in the region of Coulomb–nuclear interference the differential cross section for elastic scattering (without allowance for the spin structure of the amplitude) has the form

$$\frac{d\sigma}{dt}=\pi|T_C+T_h|^2.$$

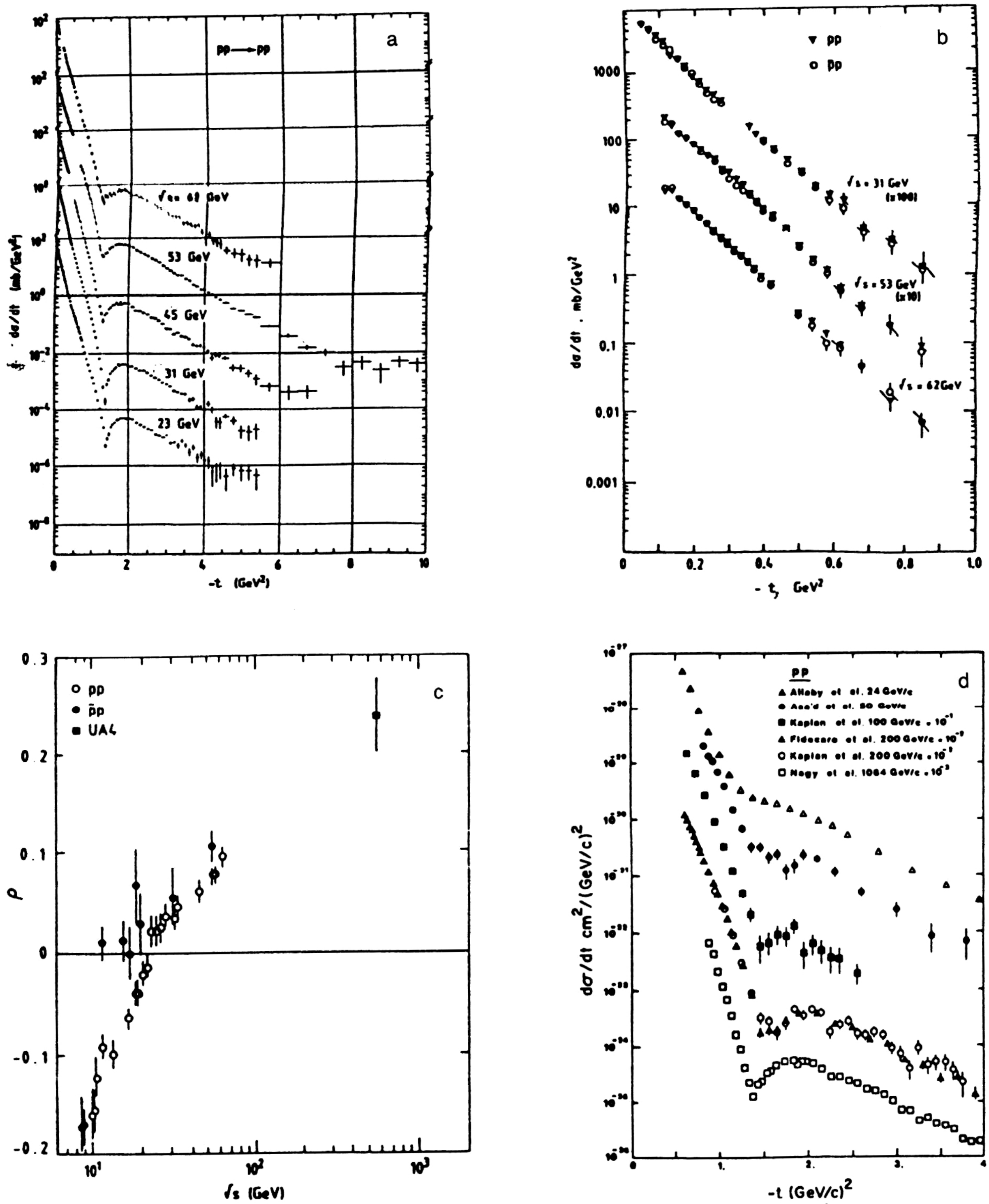

FIG. 4. Differential cross sections of pp and $\bar{p}p$ scattering.

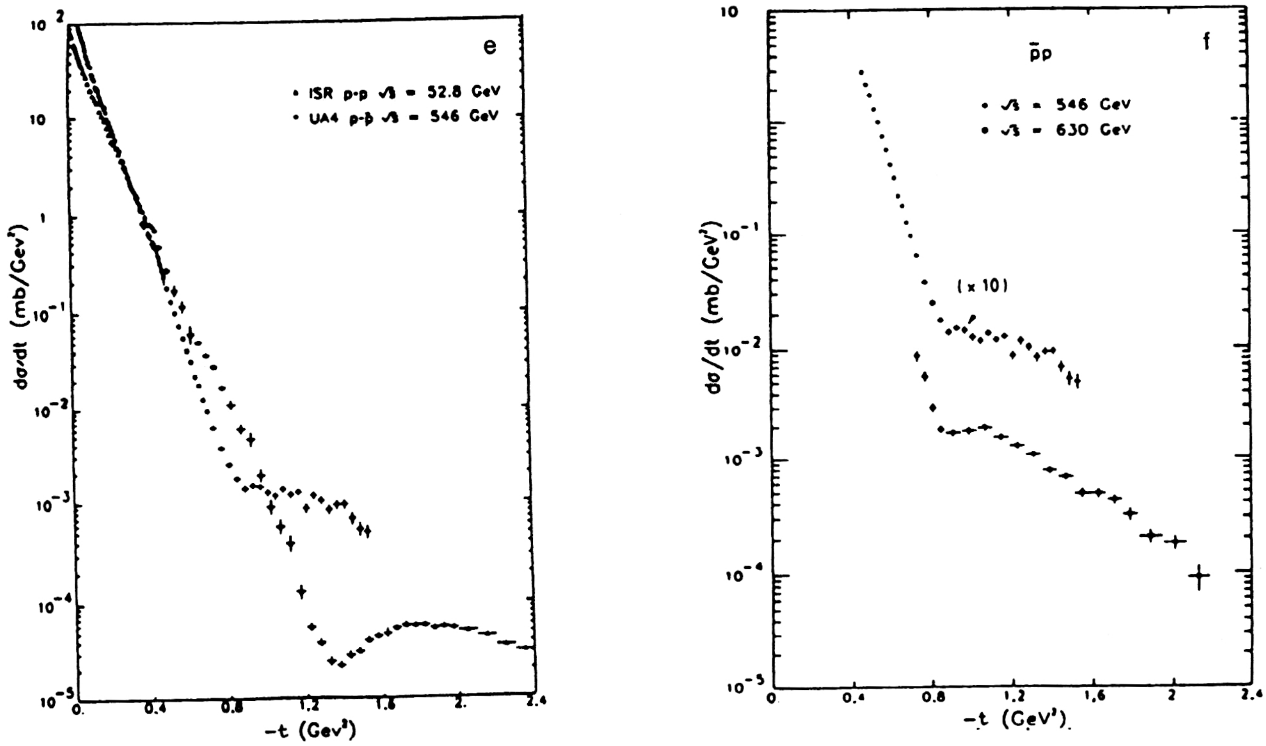

FIG. 4 (*Continued.*)

Here, T_C is the well-known Coulomb amplitude

$$T_C = \pm 2\alpha G^2(t)\exp(\mp i\pi\phi)/|t|,$$

where α is the fine-structure constant, $G(t)$ is the electromagnetic form factor of the proton, and $\phi = \ln(0.08/|t|) - 0.577$. The superscript and subscript correspond to $\bar{p}p$ and pp scattering. The simplest parametrization of the strong-interaction amplitude has the form

$$T_h = (\sigma_t/4\pi)(\rho + i)\exp(-b|t|/2)$$

and contains three adjustable parameters: σ_t, b, and ρ, which is the ratio of the real to the imaginary part of the forward scattering amplitude. For the total cross section, one sometimes uses its known value (determined from a different experiment), and then the problem reduces to variation of the two free parameters b and ρ.

The deviation from an exponential behavior of the diffraction peak (Fig. 5) is usually taken into account by including a quadratic term in the argument of the exponential (the coefficient of which is approximately an order of magnitude smaller than the corresponding coefficient of the linear term). The "break" or "fine structure" of the peak is due, as was shown in Refs. 7 and 8, to the threshold behavior of the amplitude in the unphysical region at $t = 4m_\pi^2$. This effect was studied in detail in the recent studies of Ref. 8. There are some indications[9] that at the Tevatron the peak is "rectified," but this effect cannot yet be regarded as established. Further study of the energy dependence of the "fine structure" of the peak is of great interest for both theory and applications (for example, for the determination of the real part of the amplitude).

The interest in the real part of the amplitude increased in connection with the publication in 1987 of the results of the measurement and analysis of scattering data by the UA4 group,[10] in accordance with which this result does not fit the extrapolations from the region of lower energies, and it therefore generated various theoretical speculations for its explanation (threshold effect, odderon contribution, etc.). At the same time, while the theoreticians were still disputing the possible origin of the anomalously large real part of the amplitude, the experimentalists were able to repeat the UA4 experiment at CERN. According to the results of the UA4/2 group,[11] $\rho = 0.135 \pm 0.015$, corresponding fully to a "normal" value in agreement with the majority of model predictions. The significant discrepancy between the results of analysis of the experimental data in the determination makes an additional and independent analysis of these data a topical problem with the aim of establishing the reasons for the discrepancy and determining more reliably the real part of the forward scattering amplitude. We have already noted that the result of the extrapolation may depend on the employed parametrization of the "fine structure" of the diffraction peak. Despite the fact that the region hidden from "direct measurement" by the Coulomb interaction is small, the curvature of the slope of the peak increases on the approach to $t = 0$ (i.e., on the approach to the branch point at $t = 4m_\pi^2$).

Besides the well-known and observed "break," near which the slope of the peak changes (smoothly) by about 2 GeV^{-2}, there may also be oscillations of the slope as a function of t. Such an effect was found in the case of gradual

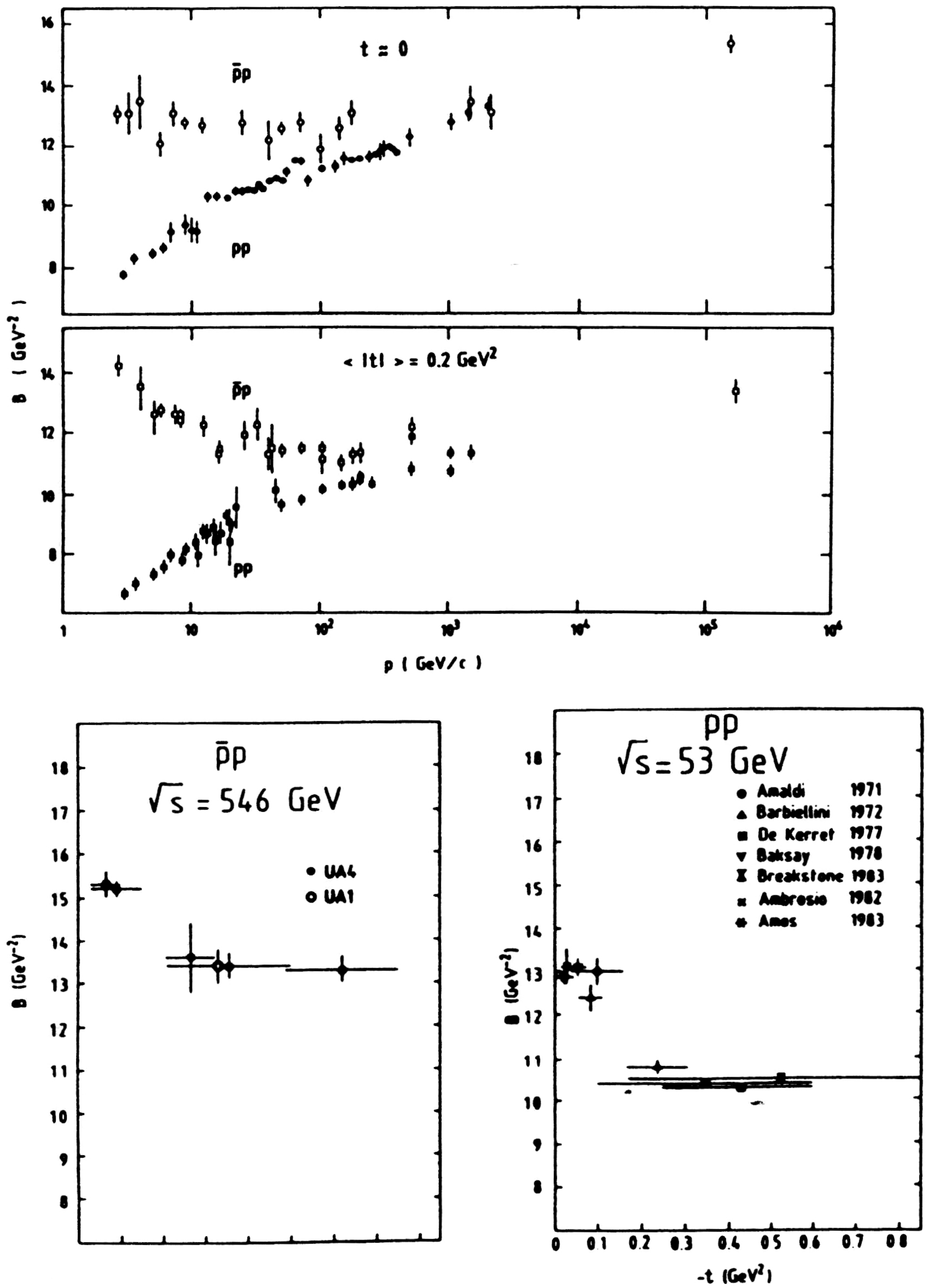

FIG. 5. Slope of the diffraction peak of pp and $p\bar{p}$ scattering as a function of t.

decrease of the interval in which the local slope of the exponential peak was determined.[12]

Interesting results were recently obtained by Selyugin *et al.* from an analysis of UA4/2 data with allowance for the spin-flip amplitude,[13] whose existence at such high energies was predicted in Ref. 14. Using and fixing (as in the UA4/2 analysis) the value $\sigma_t(1+\rho^2)=63.3\pm1.5$ mb determined in the preceding UA4 experiment, practically the same value of ρ as in the UA4/2 analysis was obtained. However, the same analysis of the experimental data, but without the condition of no t dependence of the slope parameter at small momentum transfers, leads to a much larger value of ρ. It was shown that an exponential form of the scattering amplitude with a slope that is practically independent of t can be obtained by taking into account the spin contribution of the scattering amplitude. In this case, ρ reaches the value 0.24

±0.45. However, such an appreciable effect can be regarded as established only after it has been confirmed at lower energies, and for this it is necessary to reconsider the previous data.

3. ELEMENTS OF THE THEORY OF THE ANALYTIC *S* MATRIX AND THE METHOD OF COMPLEX ANGULAR MOMENTA

Our aim is not a systematic exposition of the theory of the analytic S matrix and the method of complex angular momenta, or Regge poles, and therefore we recall only in a fragmentary manner the main propositions and results necessary for further discussion of the models of the high-energy interaction of hadrons. A detailed exposition of the theory can be found in the reviews of Ref. 1 and the book of Ref. 15.

3.1. Analyticity and crossing

The main object of the theory is the S matrix, the operator that transforms the initial state $|i\rangle$ into the final state. The probability amplitude of transition from the initial to the final state is a matrix element of this operator,

$$S_{fi}=\langle f|S|i\rangle,$$

and the transition probability is equal to the square of the modulus of the matrix element:

$$P_{fi}=|S_{fi}|^2.$$

The S matrix is a unitary operator, reflecting the completeness of the set of physical states of free particles.

It is convenient to represent the S matrix in the form of the sum

$$S=I+i\tau, \tag{3.1}$$

where I is the identity operator, and the second term describes the interaction (in the absence of the latter, τ=0). With allowance for the normalization of the states

$$\langle p|p'|\rangle=(2\pi)^3(2p_0)\delta^3(\vec{p}-\vec{p})$$

and the conservation law for the total 4-momentum, Eq. (3.1) for the process $1+2\to3+4$ can be rewritten in the form

$$\langle p_3p_4|S|p_1p_2\rangle=\langle p_3p_4|p_1p_2\rangle+i(2\pi)^4\delta^4 \times(p_1+p_2-p_3-p_4)\langle p_3p_4|T|p_1p_2\rangle,$$

where $T(p_1,p_2,p_3,p_4)$ is a function of the Lorentz invariants formed from the momenta of the particles that participate in the process. It is this function that is usually called the interaction amplitude. Using the widely employed Mandelstam variables

$$s=(p_1+p_2)^2,\quad t=(p_1-p_3)^2,\quad u=(p_1-p_4)^2,$$
$$s+t+u=m_1^2+m_2^2+m_3^3+m_4^2,$$

where $m_1,\dots,m_4$ are the particle masses, and also the fact that the amplitude of the process $1+2\to3+4$ depends on two variables, we represent it in the form

$$T(p_1,p_2,p_3,p_4)=F(s,t).$$

In the case of the interaction of particles of equal masses (for example, elastic nucleon–nucleon scattering) the physical region of the variables

$$s=4(k^2+m^2)\geq4m^2,\quad t=-2k^2(1-\cos\theta),$$
$$u=-2k^2(1+\cos\theta),$$

where k and θ are the initial momentum and scattering angle in the center-of-mass system of particles 1 and 2 (respectively, 3 and 4), is determined by the inequalities

$$\sqrt{s}>4m^2,\quad |\cos\theta|<1.$$

The experimentally observable quantities are the differential cross section

$$\frac{d\sigma}{d\Omega}(1+2\to3+4)=\frac{q}{sk}\frac{1}{64\pi^2}|F(s,t)|^2,$$

$$\frac{d\sigma}{dt}=\frac{1}{64\pi sk^2}|F(s,t)|^2$$

and the total cross section for the interaction of the initial particles, which in accordance with the optical theorem can be expressed in terms of the imaginary part of the amplitude for elastic scattering at zero angle:

$$\sigma_t=\frac{1}{2k\sqrt{s}}\,\mathrm{Im}\,F(s,t=0)=\frac{1}{4ik\sqrt{s}}[F(s+i\varepsilon)-F(s-i\varepsilon)].$$

The analytic properties of the amplitude, as a function of the complex variables, play an extremely important role, both from the point of view of theory and in phenomenological constructions in the analysis of the experimental data. It is assumed that the scattering amplitude is the boundary value of an analytic function of the Mandelstam invariants, regarded as complex variables, with those and only those singularities that are allowed by the unitarity condition. In the general case, the amplitude has poles corresponding to stable (from the point of view of the strong interactions) particles and cuts associated with the thresholds for the production of particles in different channels.

An important consequence of the analyticity and the symmetry properties with respect to space–time reflections and replacement of particles by antiparticles is the relationship between the amplitudes of different scattering processes that has become known as *crossing*. In particular, six processes, which differ in a permutation of the particles or replacement of particles by antiparticles,

$$1+2\to3+4,\quad \bar{3}+\bar{4}\to\bar{1}+\bar{2},$$
$$1+\bar{3}\to\bar{2}+4,\quad 2+\bar{4}\to\bar{1}+3,$$
$$1+\bar{4}\to\bar{2}+3,\quad 2+\bar{3}\to\bar{1}+4$$

will be described by the same amplitude but with different physical regions of the variables s, u, and t. For example, the crossing relation for the amplitudes of the processes $pp\to pp$ and $\bar{p}p\to\bar{p}p$ take the following form (we recall that we are considering a simplified situation, regarding the nucleons as spinless; the complications associated with allowance for the spin are presented in detail in Ref. 15).

3.2. Asymptotic bounds

The amplitude is a polynomially bounded function of all its variables and satisfies dispersion relations with respect to one and two variables. These properties made it possible to prove rigorously the well-known Froissart bound on the asymptotic behavior of the total interaction cross section:

$$\sigma_t(s) \leq \frac{\pi}{m_\pi^2} \ln^2 s. \tag{3.2}$$

In both theory and phenomenological applications, an important role is played by Pomeranchuk's theorem, which establishes a relationship between the particle and antiparticle interaction cross sections:

$$\frac{\sigma_t^{\bar{A}B}(s)}{\sigma_t^{AB}(s)} \xrightarrow[s\to\infty]{} 1.$$

Under certain additional assumptions (not proved rigorously), an analogous relation holds for the differential cross sections (Cornille–Martin theorem):

$$\frac{d\sigma}{dt}(\bar{A}B) \Big/ \frac{d\sigma}{dt}(AB) \xrightarrow[s\to\infty]{} 1.$$

There is also the Oberson–Kinoshita–Martin theorem. If the total cross section $5\to\infty$ rises in proportion to $\ln^2 s$, then

$$\frac{\text{Im}\, F(s,t)}{\text{Im}\, F(s,0)} \to \phi(\tau), \quad \frac{\text{Re}\, F(s,t)}{\text{Re}\, F(s,0)} \to \frac{d}{d\tau}(\tau,\phi(\tau)),$$

where

$$\tau = -t \ln^2 s.$$

3.3. Regge-pole model

The method of complex angular momenta proceeds from the possibility of expanding the amplitude in a series in partial-wave amplitudes:

$$F(s,t) = 16\pi \sum_{l=0}^{\infty} (2l+1) F_l(t) P_l(z), \tag{3.3}$$

where $P_l(z)$ is a Legendre polynomial, and $z_+ = 1 + 2s/(t-4m^2)$.

We assume that $F_l(t)$ has a pole in the j plane whose position depends on t:

$$F_l(t) \simeq \frac{\beta(t)}{l-\alpha(t)} \quad \text{at } l \approx \alpha(t).$$

This assumption can be justified in the theory of potential scattering, and in strong interactions it is justified above all by experimental data. This pole is called a Regge pole, or Reggeon, and the function $\alpha(t)$ is called its trajectory. Using the unitarity condition, one can show that the residue for the amplitude of the process $1+1\to 3+4$ can be represented in the factorized form

$$\beta(t) \sim \beta_{13}(t) \beta_{34}(t).$$

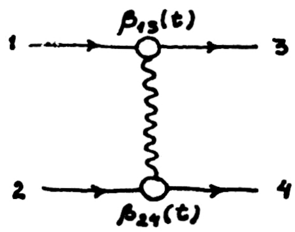

FIG. 6. Diagram with exchange of a Regge pole.

This makes it possible to represent the contribution of the Regge pole to the amplitude $F_R(s,t)$ as $s\to\infty$:

$$F_R(s,t) \sim \beta_{13}(t)\beta_{34}(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{1+\xi e^{-i\pi\alpha(t)}}{-\sin \pi\alpha(t)}, \tag{3.4}$$

where ξ is the signature of the Reggeon, in the form of a diagram (Fig. 6).

Omitting the many interesting and nontrivial properties of Regge poles [analytic and asymptotic properties of the trajectories $\alpha(t)$, their connection with resonances, the systematics of the quantum numbers, etc., all of which is presented in detail in Ref. 15], we consider in detail only the Reggeons that make the main contribution to the processes of elastic nucleon–nucleon scattering at high energies.

In accordance with the optical theorem and the expression (3.4), the Reggeon contribution to the total cross section has a power-law dependence on the energy:

$$\sigma_t(s) \sim s^{\alpha(0)-1}.$$

As s is increased, the Reggeons that have the greatest intercept are the ones that "survive" in the amplitude. It follows from the Froissart bound (3.2) that

$$\alpha(0) \leq 1.$$

Originally, a simple Regge pole with unit intercept $\alpha(0)=1$ was called the Pomeron. Such a Pomeron ensured asymptotic constancy of the total cross sections. However, under the influence of experimental data and the results of perturbative calculations in the framework of QCD the conviction has now grown that the Pomeron is a much more complicated object (we shall return later to a discussion of this question). The Pomeron has vacuum quantum numbers, positive signature $\xi=+1$, and C parity $C=+1$, and its contribution to the amplitude at small $|t|$ is predominantly imaginary. With regard to the trajectory $\alpha_P(t)$, the experimental data on hadron scattering can be described well under the assumption

$$\alpha_P(t) = \alpha_P(0) + \alpha_P' t.$$

At the same time, it follows from the theory that $\alpha_P(t)$ has a branch point at $t = 4m_\pi^2$ and is not, strictly speaking, a linear function of t. In accordance with contemporary ideas, the Pomeron is "composed" of gluons with a possible admixture of quark–antiquark states.

The hypothetical partner of the Pomeron with respect to C parity and signature—the odderon—must have a similar nature. It makes a mainly real contribution to the amplitude, since it has $\xi=-1$.

The f and ω Reggeons come next in importance and magnitude of the contribution to the elastic scattering amplitude. Like the other secondary Reggeons, they are constructed from quark–antiquark states, and they have positive and negative signatures, respectively, and intercepts $\alpha_f(0)\gtrsim 1/2$ and $\alpha_\omega(0)\lesssim 1/2$.

At low energies, the contributions of the ρ and A_2 Reggeons in the hadron interaction cross sections are significant. Allowance for them is important in a combined study of processes involving protons, neutrons, and π and K mesons.

3.4. Impact-parameter representation

The method of impact-parameter amplitudes is very convenient for investigating and constructing high-energy models of the hadron interaction. Expanding the amplitude in a series in the partial-wave amplitudes of the s, and not the t channel [as was done in (3.3)], and using the relationship between the impact parameter b and the angular momentum l,

$$bk=l+1/2,$$

and also the asymptotic property of a Legendre polynomial at large l,

$$P_l(\cos\theta_s)\simeq J_0[(2l+1)\sin\theta/2]=J_0\left[\left(l+\frac{1}{2}\right)\frac{\sqrt{-t}}{k}\right]$$

$$\simeq J_0(b\sqrt{-t}),$$

we readily obtain a new representation for the scattering amplitude that is valid at large s:

$$F(s,t)=8\pi s\int_0^\infty H(b,s)J_0(b\sqrt{-t})bdb$$

$$=4s\int_0^\infty H(s,b)e^{-i\vec{q}\vec{b}}d^2b, \tag{3.5}$$

where $q^2=-t$. The function $H(s,b)$ is called the impact-parameter amplitude. The unitarity condition for this amplitude takes the simple algebraic form

$$\text{Im } H(s,b)=|H(s,b)|^2+G_{\text{in}}(s,b), \tag{3.6}$$

where the function $G_{\text{in}}(b,s)$ describes the contributions of the inelastic processes. The total, elastic, and inelastic cross sections can be expressed in terms of the impact-parameter amplitudes as follows:

$$\sigma_t(s)=8\pi\int_0^\infty \text{Im } H(s,b)bdb, \tag{3.7}$$

$$\sigma_{\text{el}}(s)=8\pi\int_0^\infty |H(s,b)|^2bdb, \tag{3.8}$$

$$\sigma_{\text{in}}(s)=8\pi\int_0^\infty G_{\text{in}}(s,b)bdb. \tag{3.9}$$

3.5. Unitarity

The history of the development of ideas about the Pomeron, which now extends already over more than 30 years, is the history of how, under the influence of ever new experimental data at ever higher energies, both the bare singularity and the scheme for unitarizing it have been modified. It was assumed in the sixties that the total cross sections for hadron interactions tend with increasing energy to constant values. The dominant opinion was that the Pomeron is a simple pole in the plane of the angular momentum. The multi-Pomeron branch points that in accordance with the Reggeon diagrammatic technique must be added to the Pomeron die out with increasing energy. However, this remarkably simple and beautiful picture (the so-called weak-coupling version in Pomeron theory) was found to be in conflict with experimental data. The growth of the total cross sections found at Serpukhov and confirmed later at CERN made it necessary to abandon the idea of the Pomeron as a simple pole with a unit-intercept trajectory.

It was at approximately the same time that quantum chromodynamics was formulated. Being the candidate for the theory of the strong interactions, it must have something to say about the Pomeron. The most important result of a major study of gluon diagrams (initiated by Lipatov and collaborators) is that in QCD the Pomeron is a singularity (or a set of singularities) of the amplitude situated to the right of unity in the j plane. Specifically, the calculations showed that[16]

$$\alpha(0)-1=12\,\frac{\alpha_s}{\pi}\ln 2.$$

However, such a Pomeron, which corresponds to gluon diagrams of ladder type, obviously violates, beginning with certain energies, the unitarity condition and the Froissart bound. Therefore, it must be unitarized in some manner.

On the other hand, a Pomeron with intercept greater than unity was regarded earlier as one of the possibilities for explaining the observed growth of the total cross sections. Under fairly general and obvious assumptions in the framework of the Reggeon diagrammatic technique, it was shown that a summation of multi-Pomeron exchanges restores the correct asymptotic behavior of the amplitude. At the same time, it was found that, depending on the values of the bare intercept and bare three-Pomeron coupling constant and their ratio, there can be different asymptotic regimes of elastic interaction of hadrons with a rising total cross section, namely,

$$\sigma_t(s)\sim(\ln s)^\gamma,\quad 0<\gamma\leqslant 2.$$

The various approaches to the problem of unitarizing the Pomeron can be nominally divided into three groups.

1. We first consider the Pomeron in QCD. The main difficulty is associated with the investigation of the Pomeron contribution in the soft kinematic region. It is necessary to sum the corrections to the Lipatov Pomeron that are due to the interaction of these ladders, i.e., to take into account the multi-Pomeron interaction. At the present time, hopes of success in this undertaking are associated with calculations of the Pomeron contribution to the structure functions $F_2(x,Q^2)$ of deep inelastic scattering of leptons by hadrons. Despite important progress in calculations at large x and Q^2, appreciable difficulties arise precisely on the transition to the soft kinematic region. It is highly probable that the problem of unitarizing the Pomeron is essentially nonperturbative. It

may not only be associated with a summation of exchanges of ladders and their interaction but may also require allowance for diagrams that do not reduce to ladders and also the solution of the problem of quark and gluon confinement within hadrons.

2. The second approach is more phenomenological and is based essentially on the general principles of unitarity and analyticity, and also on perturbative QCD results. Experimental data on the properties of various cross sections are here an important and necessary guide. Let us consider this in more detail.

Let the contribution of the bare Pomeron to the hadron elastic scattering amplitude be $f(s,t)$. The unitarized amplitude (not unitary but not violating the unitarity bounds

$$\operatorname{Im} H(s,b)\geqslant 0, \quad |H(s,b)|\leqslant 1 \tag{3.10}$$

for the impact-parameter amplitude) can be constructed as the sum of s-channel multiple scatterings as follows. The unitarized impact-parameter amplitude is represented in the form of the series

$$H(s,b)=\frac{1}{2in}\sum_{n=1}^{\infty}\frac{G(n)}{n!}[2ih(s,b)]^n, \tag{3.11}$$

where $h(s,b)$ is the bare impact-parameter amplitude related to $f(s,t)$ by the transformation (3.5). The function $G(n)$ is determined by the amplitudes for the interaction of two hadrons with n Pomerons. At the same time, the specific choice of the dependence of $G(n)$ on n determines the method of unitarization. Thus, if $G(n)=C^{n-1}$, the amplitude can be unitarized by the quasi-eikonal method. For $C=1$; the series (3.11) is identical to the eikonal representation,

$$H(s,b)=\frac{e^{2ih(s,b)}-1}{2i}, \tag{3.12}$$

where the bare impact-parameter amplitude $h(s,b)$ plays the role of a complex phase. If, however, $G(n)=n!C^{n-1}$, then (3.11) describes a generalized u-matrix unitarization, while for $C=1/2$ it is identical to the original u-matrix model,[1g]

$$H(s,b)=\frac{h(s,b)}{1-ih(s,b)}.$$

3. Many models are constructed in a such a way that from the beginning (by construction) they do not violate the above bounds. Examples here are the model of a maximum Pomeron and maximum odderon (this model will be considered later), and also the dipole Pomeron model with intercept $\alpha_P(0)=1$ (see below).

3.6. Pomeron and odderon at superhigh energies

As will be seen from the following section, the experimental data are often described in models with a supercritical Pomeron and a supercritical odderon. We have already said that they violate unitarity at superhigh energies, and therefore must be unitarized. It is then natural to ask the question: What are the Pomeron and odderon at asymptotic energies?

In discussing this question, we shall not consider quantum chromodynamics, since work on the investigation of the corrections to the Lipatov Pomeron and odderon is not completed. As yet nothing definite can be said about a unitary Pomeron and a unitary odderon in QCD.

Since at superhigh energies the contributions of the secondary Reggeons to the elastic scattering amplitude vanish, the Pomeron (P_0) and the odderon (O_0) must be understood as the crossing-even and crossing-odd components of the amplitude. In iteration models, their properties depend on two circumstances: first, on the form of the bare Pomeron (P_0) and odderon (O_0), and second, on the unitarization procedure, since this procedure is model-dependent. Nevertheless, it is possible to establish some facts and restrictions that do not depend on the details of the unitarization.

Thus, it was shown in Refs. 17 and 18 on the basis of the unitarity condition that in models of a bare supercritical (with intercepts greater than unity) Pomeron and odderon

$$\alpha_-(0)\leqslant\alpha_+(0), \quad \alpha'_-(0)\leqslant\alpha'_+(0)$$

(the subscripts + and − identify, respectively, the Pomeron and the odderon). This conclusion is valid for any of the methods considered above for s-channel unitarization and not only for linear P_0 and O_0 linear trajectories. At the same time,

$$\left|\frac{F_-(s,0)}{F_+(s,0)}\right|\to 0, \quad s\to\infty.$$

Other conclusions depend on whether or not there is degeneracy $[\alpha_+(0)=\alpha_-(0),\ \alpha'_-(0)=\alpha'_+(0)]$ of the P_0 and O_0 trajectories.

For nondegenerate P_0 and O_0, the crossing-odd effects vanish as a power with increasing energy.[18] If the P_0 and O_0 trajectories are degenerate, then it is possible to have a situation in which

$$\Delta\sigma=\sigma_t^{\bar{p}p}-\sigma_t^{pp}\to\text{const}, \quad \rho_{\bar{p}p}\to-\rho_{pp}\to\text{const},$$

and the differential cross sections for pp and $\bar{p}p$ scattering can differ for fixed t and $s\to\infty$.

In the model of a dipole Pomeron and dipole odderon with unit-intercept trajectories unitarized by the quasi-eikonal method,[19] the asymptotic behavior of the amplitudes is determined by the O_0, and not P_0, parameters. Since at the currently achieved energies the P_0 contribution to the amplitudes is dominant, it is to be expected that at certain higher energies a strong interference of the Pomeron and odderon may be observed and, as a consequence, there may be large differences between the pp and $\bar{p}p$ interactions.

Concluding this section, we emphasize that the problem of unitarizing the Pomeron and odderon remains extremely topical and important. It will evidently be even more acute in the study of the structure functions in deep inelastic scattering. At the present time, there are more questions than answers in this problem, and hence there are good possibilities for new investigations.

3.7. Total cross sections and Pomeron models

Before we review models of the elastic scattering amplitudes, we return to the experimental data on the total cross sections and the ratios of the real to the imaginary parts of the amplitudes and give the results of a comparative analysis of some models of the zero-angle scattering amplitudes.[20]

The amplitudes of pp and $\bar{p}p$ scattering were represented in the form

$$A_{pp}^{\bar{p}p}(s,0)=P(s)+f(s)\mp\omega(s), \quad (3.13)$$

where $f(s)$ and $\omega(s)$ are the contributions of the f and ω Reggeons,

$$R(s)=\eta_R(\tilde{s})^{\alpha_R(0)-1}, \quad \tilde{s}=-is/s_0, \quad s_0=1\ \text{GeV}^2,$$

$$R=f,\omega, \quad \eta_f=i, \quad \eta_\omega=1,$$

and for the Pomeron $P(s)$ four well-known methods were considered:

$$P_1(s)=i(p_1+p_2\tilde{s}^\varepsilon), \quad (3.14)$$

$$P_2(s)=i(p_1+p_2\ln\tilde{s}), \quad (3.15)$$

$$P_3(s)=i(p_1+p_2\ln\tilde{s}+p_3\ln^2\tilde{s}), \quad (3.16)$$

$$P_4(s)=i(p_1+p_2\ln^\gamma\tilde{s}). \quad (3.17)$$

The parameters were determined by comparison with data on σ_t and ρ at $\sqrt{s}\geqslant 5$ GeV.

The following conclusions were drawn:

1. The exchange degeneracy of the f and ω trajectories is strongly violated. Fits with $\alpha_f\neq\alpha_\omega$ are significantly better than those in which it is assumed that $\alpha_f=\alpha_\omega$.

2. The use of the expression (3.14) with $p_1\neq 0$ significantly reduces λ^2; at the same time, $\varepsilon=0.0024$, which is much smaller than the value $\varepsilon=0.08$ obtained in the model (2.2) with $p_1=0$ and $\alpha_f=\alpha_\omega$ (Ref. 6).

3. The difference of χ^2 in the models (3.14)–(3.17) is not more than 0.5%, but the values of the parameters are such that for $\sqrt{s}>10^5$ GeV the expressions (3.14)–(3.16) reduce practically to (3.15).

4. The minimum value of χ^2 is achieved in the model (3.17) with $\gamma=0.407$.

The same models were used for the simultaneous description of ten processes of nucleon–nucleon and meson–nucleon interactions. The contributions of the ρ and A_2 Reggeons were taken into account. The conclusions listed above were confirmed.

Thus, at the present time there are no strong grounds for asserting that Froissart or even more rapid growth of the total cross sections is observed, although such growth does not contradict the existing experimental data.

4. MODELS OF ELASTIC SCATTERING

4.1. Rapid-growth model

We introduce combinations of the scattering amplitudes that are even and odd with respect to crossing:

$$F_{pp}=F^{(+)}+F^{(-)},$$

$$F_{\bar{p}p}=F^{(+)}-F^{(-)},$$

each of which we decompose into two terms

$$F^{(\pm)}=F_{AS}^{(\pm)}+F_N^{(\pm)},$$

these being the asymptotic (AS) and nonasymptotic (N) components. Following Ref. 22, we determine these amplitudes at $t=0$, $s\to\infty$:

$$F_{AS}^{(+)}\to isF_1[\ln(se^{-i\pi/2})]^2=sF_1(i\ln^2 s+\pi\ln s),$$

$$F_{AS}^{(-)}\to sO_1[\ln(se^{i\pi/2})]^2=sO_1(-i\pi\ln s+\ln^2 s), \quad (4.1)$$

where F_1 and O_1 are constants. The even amplitude is called the Pomeron, and the odd amplitude is called the odderon. The t dependence is recovered on the basis of the Oberson–Kinoshita–Martin theorem, in accordance with which (see Sec. 3)

$$F^{(+)}(s,t)\to F^{(+)}(s,0)g^{(+)}(\tau),$$
$$F^{(-)}(s,t)\to F^{(-)}(s,0)g^{(-)}(\tau), \quad (4.2)$$

where $g^{(\pm)}$ is an entire function of order 1/2 with respect to τ^2, and τ is the scaling variable

$$\tau=t\ln^2 s.$$

To construct an explicit expression for the scattering amplitude, it is convenient to go over to the complex plane of the angular momentum J. The behavior (4.1) corresponds to the contribution of the triple and double poles to the partial-wave amplitude at $t=0$:

$$f^{(+)}(J,t)=\frac{\beta^{(+)}(J,t)}{[(J-1)^2-tR_+^2]^{3/2}},$$

$$f^{(-)}(J,t)=\frac{\beta^{(-)}(J,t)}{[(J-1)^2-tR_-^2]},$$

where $R_\pm$ are real positive constants.

It is assumed that the functions $\beta^{(\pm)}$ depend weakly on J and exponentially on t:

$$\beta^{(+)}(J,t)=\beta_0(t)+(J-1)\beta_1(t)+(J-1)^2\beta_2(t),$$

$$\beta^{(-)}(J,t)=\beta_0(t)+(J-1)\beta_1(t).$$

By means of an inverse Mellin transformation, we obtain

$$\frac{1}{is}F_{AS}^{(+)}(s,t)=F_1\ln^2\tilde{s}\,\frac{2J_1(R_+\tilde{\tau})}{R_+\tilde{\tau}}\,e^{b_1^{(+)}t}$$
$$+F_2\ln\tilde{s}J_0(R_+\tilde{\tau})e^{b_2^{(+)}t}$$
$$+F_3[J_0(R_+\tilde{\tau})-R_+\tilde{\tau}J_1(R_+\tilde{\tau})e^{b_3^{(+)}t},$$

$$\frac{1}{s}F_{AS}^{(-)}(s,t)=O_1\ln^2\tilde{s}\,\frac{\sin(R_-\tilde{\tau})}{R_-\tilde{\tau}}\,e^{b_1^{(-)}t}$$
$$+O_2\ln\tilde{s}\cos(R_-\tilde{\tau})e^{b_2^{(-)}t}+O_3e^{b_3^{(-)}t},$$

where J_i are Bessel functions, and F_i, O_i, b_i, $b^{(\pm)}$, s_0, and t_0 are constants.

The nonasymptotic Regge contributions (Pomeron, two-Pomeron cut PP, Reggeon R, and cuts RP, and also the odderon and the corresponding cut OP) were chosen in the form

$$F_P^{(+)}(s,t)=C_Pe^{\beta_Pt}\left[i-\cot\left(\frac{\pi\alpha_P(t)}{2}\right)\right]s^{\alpha_P(t)},$$

$$F_{PP}^{(+)}(s,t)=C_{PP}e^{\beta_{PP}t}\left[i\sin\left(\frac{\pi\alpha_{PP}(t)}{2}\right)\right.$$
$$\left.-\cos\left(\frac{\pi\alpha_{PP}(t)}{2}\right)\right]\frac{s^{\alpha_P(t)}}{\ln(se^{-i\pi/2})},$$
$$\alpha_P(t)=1+\alpha_P't,\quad \alpha_{PP}(t)=1+\frac{\alpha_P't}{2},$$
$$F_R^{(\pm)}(s,t)=\pm C_R^{(\pm)}\gamma_R^{(\pm)}e^{\beta_R^{(\pm)}t}$$
$$\times\left[i\mp\begin{matrix}\cot\left(\frac{\pi\alpha_R^{(+)}(t)}{2}\right)\\ \tan\left(\frac{\pi\alpha_R^{(-)}(t)}{2}\right)\end{matrix}\right]s^{\alpha_R^{(\pm)}(t)},$$
$$F_{RP}^{(\pm)}(s,t)=t^2C_{RP}^{(\pm)}e^{\beta_{RP}^{(\pm)}}\binom{i}{1}\left[\sin\left(\frac{\pi\alpha_{RP}^{(\pm)}(t)}{2}\right)\right.$$
$$\left.+i\cos\left(\frac{\pi\alpha_{RP}^{(\pm)}(t)}{2}\right)\right]\frac{s^{\alpha_{RP}^{(\pm)}(t)}}{\ln(se^{-i\pi/2})},$$
$$F_O^{(-)}(s,t)=C_Oe^{\beta_0t}\left[i+\tan\left(\frac{\pi\alpha_O(t)}{2}\right)\right]s^{\alpha_0(t)}(1+\alpha_O(t))$$
$$\times(1-\alpha_O(t)),$$
$$F_{OP}^{(-)}(s,t)=C_{OP}e^{\beta_{OP}}\left[\sin\left(\frac{\pi\alpha_{OP}(t)}{2}\right)\right.$$
$$\left.+i\cos\left(\frac{\pi\alpha_{OP}(t)}{2}\right)\right]\frac{s^{\alpha_{OP}(t)}}{\ln(se^{-i\pi\alpha/2})}.$$

The result of this is a fairly cumbersome model with a large number (in fact, 38) of free parameters. This is the price that must be paid for a sufficiently good description of the experimental data.

4.2. The Donnachie–Landshoff model

This model is based on the following assumptions:

1. The Pomeron is a simple pole with intercept $\alpha(0)>1$. The value $\varepsilon=\alpha(0)-1$ ensures the observed growth of the cross sections, but at the same time the cross section remains below the Froissart bound with a large margin, making it possible to avoid a unitarization procedure up to energies far exceeding the possibilities of future generations of accelerators.

2. Two-Pomeron exchange (rescattering) is introduced to ensure a diffraction structure in the cross section (a dip). The position of this dip is corrected by an additional free parameter, the occurrence of which can be attributed to effective allowance for multiple scattering.

3. The Pomeron is coupled to each quark of the colliding nucleons as an isoscalar photon.

4. In the region of large $|t|$, the contribution of three gluons, which, like the odderon contribution, is C odd, is also important.

5. In the region of low and intermediate energies, the behavior of the scattering amplitude is determined by the standard contribution of secondary Reggeons.

4.3. Multipole Pomeron

Historically, this approach is based on the assumption that a double Pomeranchuk pole (dipole Pomeron,[24–27]) plays a distinguished role. The use of a dipole in addition to a simple pole is an alternative to the model with a simple pole having an intercept greater than unity, in the sense that both models ensure a growth of the cross sections and a positive real part of the forward scattering amplitude, as the experimental data require. However, there is also an important difference—in contrast to a simple "supercritical" Pomeron, a dipole does not violate the Froissart bound (unitarity). A dipole with a linear trajectory has one further attractive property—geometrical scaling (and possibilities of violating it). In the framework of the standard Regge approach, 2 is the maximum multiplicity of a pole that is permitted by unitarity, and in the dipole model, moreover, the cross sections will increase logarithmically. Poles of higher multiplicity raise the power of the logarithm, but the rate at which the peak becomes narrower in Regge theory does not change, remaining logarithmic; in the case of a tripole, this already leads to the inequality $\sigma_t>B$, which contradicts unitarity.

The moderate growth of the cross sections and the approximate geometrical scaling observed for $10\gtrsim\sqrt{s}\lesssim100$ GeV make the dipole Pomeron model[24–27] attractive for the description of diffraction effects in the region of energies of the ISR and SPS accelerators.

It remains an open question whether the dipole Pomeron is merely a simple and convenient parametrization that reflects the properties of diffraction in a restricted range of energies or whether it has some deeper significance.

In the simplest treatment, for $t=0$, the dipole contribution can be identified with the second term in the series expansion in powers of a "supercritical" Pomeron:

$$s^\varepsilon\simeq1+\frac{\varepsilon}{2}\ln s+\dots.$$

If the parameter ε is small, say $\varepsilon\lesssim0.1$, as in the Donnachie–Landshoff model (see Refs. 6 and 23), then in the region of energies of existing accelerators the contribution of the terms higher than the one linear in $\ln s$ can be ignored, and in this sense the dipole Pomeron is close to the Donnachie–Landshoff model (they can be called models of moderate growth, in contrast to models of rapid or Froissart growth). (The values of the parameter ε determined independently in Refs. 6, 23, and 24–27 were found to be similar and much smaller than the popular value $\varepsilon\gtrsim0.3$ obtained from perturbative QCD.)

The multipole model acquires a deeper significance if one includes a second variable, t, or, in Regge language, if the explicit form of the residue is determined. At the same time, it is by no means obvious that the corresponding series is a power-law expansion with respect to a parameter, as occurred in the very simple example given above. In fact, the opposite is more likely, as was found for the model of a dipole Pomeron that reproduces the occurrence and motion of the diffraction minimum in the region of energies of the ISR and SPS accelerators.

Generally speaking, the residues of poles of different multiplicities are arbitrary functions that are not related to each other. If a model is to acquire predictive strength, there must be further considerations that restrict the arbitrariness in the t dependence.

The series of the Glauber–Sitenko multiple-scattering theory is a well-known example in this field, but comparison with experimental data indicates that this series leads only to qualitative agreement (the appearance of many minima in the differential cross section) and requires correction (as is done, for example, in the Donnachie–Landshoff model) for quantitative agreement with the data (position, motion, and filling of the diffraction minima). It is possible that this is a difference between the diffraction of hadrons and nuclei, these being phenomena that generally have much in common.

Such a connection was found fortuitously in work on combination of a simple pole and a dipole. This model was subsequently developed and refined (see Refs. 26 and 27 and the literature cited there), but its essence, associated with a simple and effective mechanism of a diffraction minimum, was preserved and can evidently still serve as a "minimal model" of hadron diffraction.

The dipole Pomeron model is based on assumptions[24] that rely on duality, namely, the assumption that a dependence enters the Regge residue only through the trajectory, and also the assumption that there is a simple (integral) connection between the residues, to the exposition of which we now turn.

It is assumed that the Pomeron is defined by the contribution of an isolated pole of second order in the plane of the angular momentum:

$$a(J,t)=\frac{\beta(J,t)}{[J-\alpha(t)]^2}=\frac{d}{d\alpha}\frac{\beta(J)}{J-\alpha(t)},$$

where the function $\beta(J)$ does not depend on t (in accordance with dual models, this dependence is completely contained in the trajectory). Making a Sommerfeld–Watson transformation, we obtain the following expression for the elastic scattering amplitude:

$$F(s,t)=\frac{d}{d\alpha}\,[e^{-i\pi\alpha/2}G(\alpha)(s/s_0)^\alpha]$$

$$=e^{-i\pi\alpha/2}(s/s_0)^\alpha\left[G'+\left(L-\frac{i\pi}{2}\right)G\right],\quad L\equiv\ln\frac{s}{s_0}. \tag{4.3}$$

The residues G and G' are as yet arbitrary functions. They will be determined below. Noting that the first term in (4.3) (without the logarithm) has the meaning of a simple pole, it is natural to relate the residue to the shape of the diffraction peak, i.e.,

$$G'(\alpha-1)=A\,\exp[b(\alpha-1)]. \tag{4.4}$$

Then G is found by integrating:

$$G(\alpha-1)=A[\exp[b(\alpha-1)]/b-\gamma],$$

where γ is an arbitrary constant. This simple but nontrivial step associated with the determination of the residues was essential for the construction of the model and is of fundamental importance for the mechanism of the diffraction minimum that is built into it. In principle, as a "point of departure" one could specify the function G and then find G' by differentiation; however, in this way we would not obtain, as is readily seen, a simple and effective mechanism of the dip. Indeed, the essence of the model is that besides a growth of the cross sections and the presence of a real part in the forward scattering amplitude (with unit intercept of the trajectory), the model leads to the appearance, in the differential cross section, of a shoulder that evolves with increasing energy into a dip that becomes deeper and moves in the direction of small $|t|$, in agreement with the picture observed in elastic scattering of hadrons up to the energies of the SPS accelerator. We note also that the considered mechanism of the dip in the dipole Pomeron does not depend on the shape of the trajectories.

A detailed comparison of the dipole Pomeron model with data on pp and $\bar{p}p$ scattering has been made in numerous studies.[28–32] A standard set of secondary Reggeons was included, and the main attention was devoted to the role of the Pomeron (and odderon). The presence, in the dipole Pomeron mechanism, of a diffraction dip (at the "Born level," i.e., without calculation of the corrections for multiple scattering) makes this model particularly attractive from the practical point of view (in a minimization procedure), making it unnecessary to calculate a double integral Fourier–Bessel transform. Typical results of fits are shown in Fig. 7.

Overall, these fits are better than other models of elastic diffraction in terms of economy and accuracy. The dipole Pomeron does not violate unitarity explicitly, but nevertheless a unitarization procedure for it is meaningful and has been carried out in the framework of the so-called U-matrix approach, which is an alternative to the eikonal approach. In the U-matrix approach, the unitarized amplitude in the impact-parameter representation is written in the form[1g]

$$T(\rho,s)=\frac{u(\rho,s)}{1-iu(\rho,s)}.$$

The U matrix was chosen in the form

$$u(\rho,s)=\frac{i\sigma_0}{16\pi\alpha' b}\sum_{i=1}^{2}c_i\,\exp(-\rho^2/4R_i^2), \tag{4.5}$$

corresponding to a dipole Pomeron (for simplicity, the contribution of the second Gaussian has been ignored, i.e., we have $c_2=0$). Then for the total, elastic, and inelastic cross sections, and also for the slope of the diffraction peak, the following results were obtained:

$$\sigma_t=\frac{16\pi\alpha'}{\lambda}\ln(1+g)(1+\lambda L),$$

$$\sigma_{\rm el}=\frac{16\pi\alpha'}{\lambda}\left[\ln(1+g)-\frac{g}{1+g}\right](1+\lambda L),$$

$$\sigma_{\rm in}=\frac{16\pi\alpha'}{\lambda}\frac{g}{1+g}(1+\lambda L), \tag{4.6}$$

$$B(s,0)=\frac{1}{8\pi}\frac{\Sigma}{\ln^2(1+g)}\sigma_t,$$

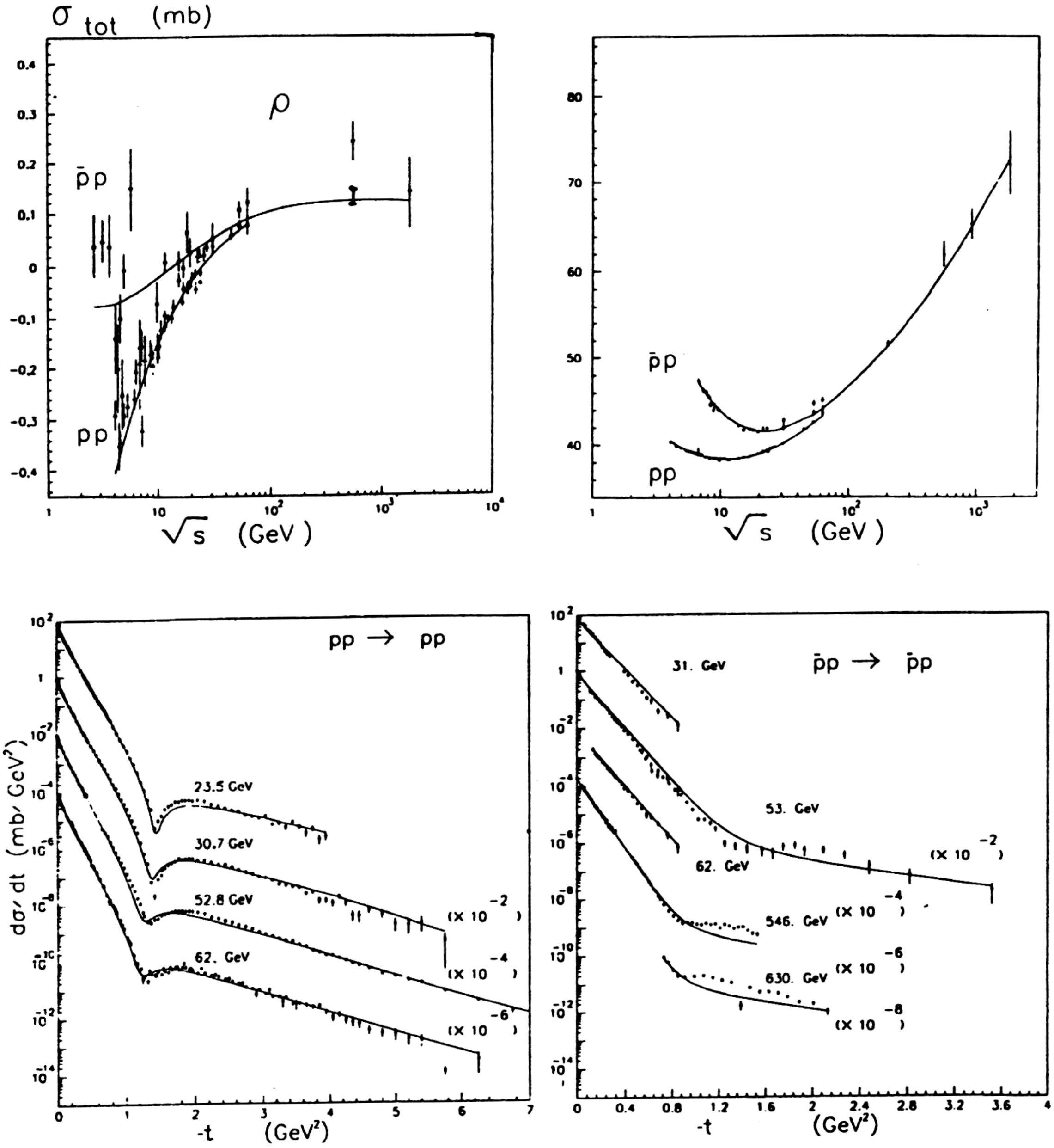

FIG. 7. Results of calculations of cross sections in the dipole Pomeron model.

$$\sum = g\sum_{n=0}^{\infty} (-g)^n(n+1)^{-2}.$$

A remarkable property of the dipole Pomeron is its self-reproducibility with respect to the unitarization procedure—for unit intercept (i.e., when g=const), this procedure leads merely to a renormalization of the constants in (4.6). The ratio of the real to the imaginary part of the scattering amplitude does not change at all as a result of the unitarization:

$$\frac{\text{Re } T(s,0)}{\text{Im } T(s,0)} = \frac{\pi\lambda}{2(1+\lambda L)} = \frac{\text{Re } T_B(s,0)}{\text{Im } T_B(s,0)}.$$

An important dynamical characteristic of the model is the ratio of the elastic to the total cross section, which in the dipole Pomeron model after unitarization takes the form

$$\frac{\sigma_{\text{el}}}{\sigma_t} = 1 - \frac{g}{(1+g)\ln(1+g)},$$

where, we recall,

$$g = s^{\varepsilon}\frac{\sigma_0\lambda}{16\pi\lambda'}.$$

The observed growth of the ratio $\sigma_{\text{el}}/\sigma_t$ uniquely fixes $\varepsilon=0.06$ in this formulation of the model, as a result of which we obtain[27] a "supercritical" dipole Pomeron. Its main difference from the model of a simple pole is that in the case of a dipole Pomeron the growth of the cross sections is redistributed between the exponent [the intercept $\alpha(0)$] and the logarithmic factor from the dipole. Behind this redistribution there is a deep physical significance associated with the "darkening" of the hadrons and the growth of their radius with the energy (more details about this can be found in the literature). From the unitarity condition (from the overlap function) we have

$$G(\rho,s)=\frac{ge^{-x}}{(1+ge^{-x})^2},\quad x=\rho^2/(4\alpha' L),$$

and by means of geometrical arguments the dipole Pomeron model was generalized for inelastic processes of multiparticle production.[27,30] The discussion of these processes goes beyond the scope of the present review. Here, we merely mention that the moderate growth of the cross section for simple diffraction dissociation, measured recently at the Tevatron, supports the dipole Pomeron model, ruling out models of rapid growth.[30]

A promising approach for further development of the dipole Pomeron model is that of a sufficiently rapidly convergent multipole series,[31,32] in accordance with which—in addition to the already discussed simple and moving pole—a correction from a triple Pomeranchuk pole may become significant at the energies of the next generation of accelerators. The essence of this approach is that the contribution of the poles of higher multiplicity is suppressed so much that they will not violate unitarity in the foreseeable future (an analogous "pragmatic" approach to the unitarity condition has already been discussed by us in connection with the Donnachie–Landshoff model in Sec. 2). In the language of Feynman diagrams, this means that there is no need for calculation of the infinite Lipatov gluon ladder (Reggeized two-gluon exchange in QCD),[16] which on account of the large intercept requires unitarization already at current energies. Instead of this, it is sufficient to calculate step by step two-gluon exchange with the minimum number of gluons emitted in the direct channel (one for the dipole, two for the tripole, etc.). Such calculations were recently made in Ref. 31.

This prospect can be confirmed by the following observations: 1) The most recent (in time and energy) points of the total scattering cross section obtained at the Tevatron lie above the logarithmic extrapolation from the region of lower energies; 2) it is difficult to attribute the filling of the diffraction dip at the Tevatron to the odderon alone. It is possible that this is the contribution of the triple pole. The corresponding generalization of the model of the multipole Pomeron is a complicated but interesting problem (see Refs. 31 and 32).

Concluding this subsection, we recall that the Regge model contains information only about the energy dependence of the scattering amplitude. Duality extended the domain of its predictions, augmenting it with ideas and restrictions on the dependence on another, crossing-symmetric variable.

It was also found to be fruitful to use the geometrical, or s-channel, approach, examples of which we give in the following subsection, where we also discuss a possible synthesis of the two approaches.

4.4. The s-channel approach. The Chou–Yang model, its generalizations, and modifications

In this approach, which was initiated by Chou and Yang,[33] it is assumed that hadrons possess a structure described by a distribution $\rho(x,y,z)$. The scattering of two hadrons A and B along an axis is regarded as the collision of two disks in their center-of-mass system. The interaction is characterized by a transparency or eikonal

$$\Omega(\vec{b})=\int \rho_A(x,y,z)\rho_B(x',y',z')I_{AB}\times(b_x-x'+x,b_y-y'+y)d^3\vec{r}d^3\vec{r}',$$

where I_{AB} determines the interaction and in the simplest case can be chosen in the form of a δ function. Then

$$\Omega(\vec{b})=K_{AB}D_A(\vec{b})\otimes D_B(\vec{b}),$$

where K_{AB} is a constant that does not depend on the impact parameter and is called the absorption parameter.

At high energies and small scattering angles, the z component of the momentum $\mathbf{q}$ is negligibly small, and the expression for the hadron form factor

$$F(q^2)=\int \rho(\vec{r})e^{i\vec{q}\vec{r}}d^3\vec{r}$$

can be written as

$$F(q^2)=\langle D(x,y)\rangle$$

or

$$\langle\Omega(\vec{b})\rangle=K_{AB}F_A(q^2)F_B(q^2).$$

Chou and Yang assumed that the distribution of hadronic matter is analogous to the distribution of electric charge, i.e., that they can both be described by the electromagnetic form factor

$$F_i(q^2)=\left(1+\frac{q^2}{\mu_i^2}\right).$$

Knowledge of the eikonal makes it possible to calculate the scattering amplitude and cross sections.

The Chou–Yang model predicted the appearance of a diffraction structure in the elastic scattering cross sections at high energies, but the prediction is quantitative only because the original model did not contain an energy dependence.

Various authors, including Chou and Yang themselves, attempted a modification of the original model, above all to introduce an energy dependence in it. This procedure is far from unique. Among many possibilities, we mention two that in a certain sense are diametrically opposites: the factorized-eikonal model and the geometric-scaling model. In nature, neither of these two possibilities is evidently realized in pure form, but both are manifested in a certain kinematic region.

Another important shortcoming of the geometrical models—in particular, the Chou–Yang model—is the absence of the complex phase characteristic of Regge models.

In the sense of comparison with experimental data and agreement with them, one of the most developed modifica-

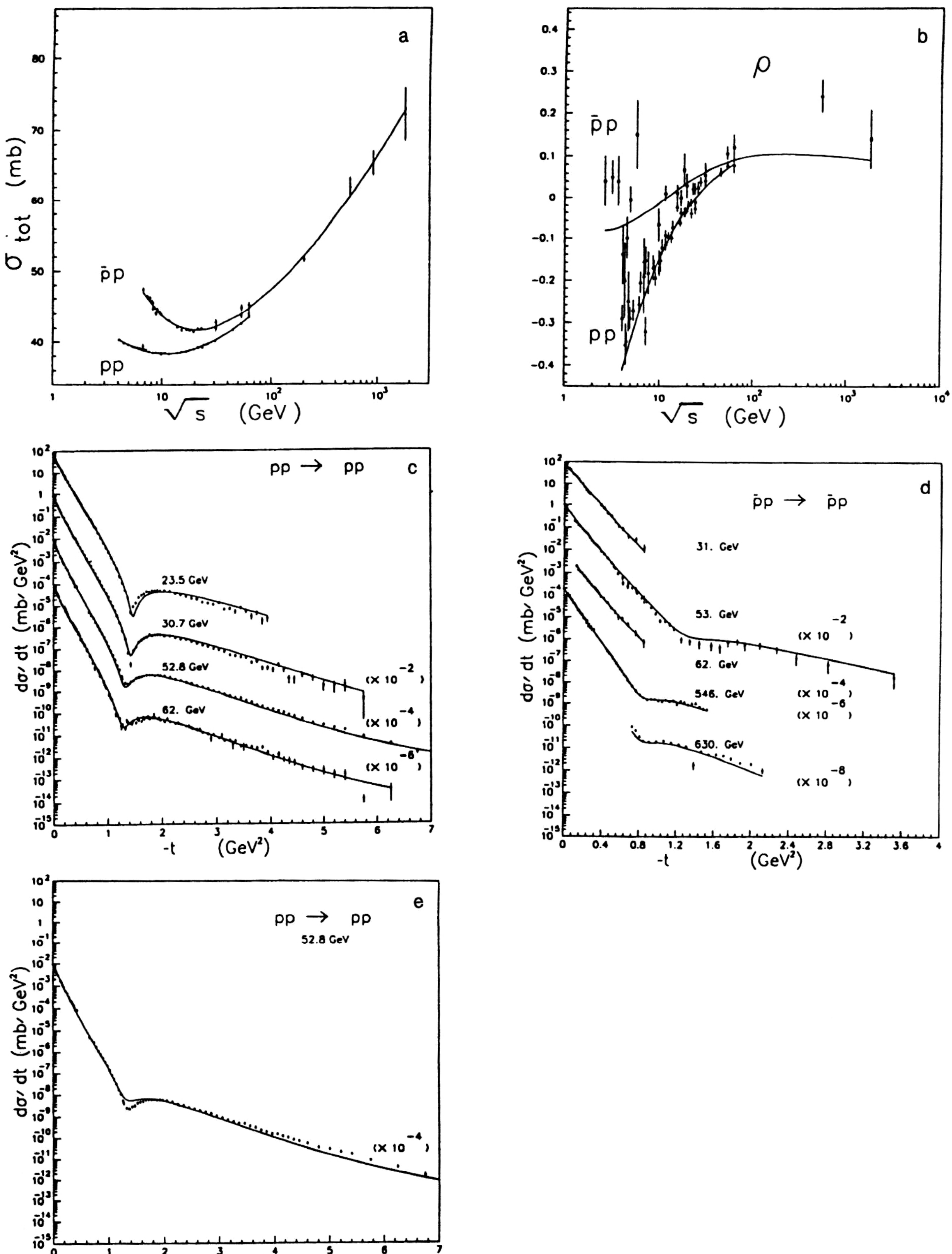

FIG. 8. Results of calculations of cross sections in the model of Ref. 35.

tions of the geometrical models is the model of Bourrely, Soffer, and Wu,[34] in which the Pomeron contribution is defined as

$$A_P^{(\mathrm{BSW})}(s,t)=csS_0(s)\left[\left(1-\frac{t}{m_1^2}\right)\left(1-\frac{t}{m_2^2}\right)\right]^{-2}\frac{a^2+t}{a^2-t},$$

where c is a constant, and $S_0(s)$ is a complex function defined as

$$S_0(s)=\frac{s^c}{(\ln s)^{c'}}+\frac{u^c}{(\ln u)^{c'}}.$$

In this model, in particular, the form factor is not a dipole (as, for example, in the Chou–Yang model) but the product of two simple poles. This is a model in which complexity and an energy dependence are introduced empirically on the basis of a comparison with data.

There is also considerable interest in attempts to combine the s- and t-channel models, i.e., to combine the advantages of the Regge models (signature, energy dependence) and geometrical models (t dependence, physicality).

A possible solution of this problem was found in studies in which the idea is that the Pomeranchuk trajectory has the form

$$\alpha(t)=\alpha_0-\gamma\ \ln(1-\beta t),$$

for which the Regge residue is identical to the dipole form factor. The form of the trajectory $\alpha(t)$ is by no means fortuitous. It is the simplest example of a trajectory that gives a fit between the regions of small and large $|t|$. At small values, it is almost linear, ensuring an exponential shape of the diffraction peak, while at large $|t|$ it behaves logarithmically as the self-similar asymptotic behavior of the dual model requires.[32]

Corresponding calculations and a comparison with data were made in Refs. 35. Here, we merely mention the fundamental difficulty of introducing the Regge signature into the geometrical model. This is due to the calculation of Fourier–Bessel integrals with a complex index.

In the studies, the Pomeron contribution determined in this manner was augmented by the standard contribution of secondary Reggeons with subsequent minimization with respect to the free parameters. Results of fits are given in Fig. 8.

5. CONCLUSIONS

In our review we have tried to draw the attention of the reader to the richness and complexity of the picture of diffraction scattering of hadrons, which requires the construction of an adequate theoretical formalism. The existence of the gulf between calculations in quantum chromodynamics (restricted to perturbation theory) and realistic models that describe the rich set of data hinders progress in this important field of high-energy physics. It is necessary to recognize that however attractive the problem of constructing a microscopic theory of hadron scattering (in the framework of QCD) may be, this aim does not justify an appropriate use of quantum chromodynamics, which remains an attractive theory even if it does not find application in the description of soft processes.

We also wanted to show that there exist many models that successfully describe a restricted group of phenomena in the diffraction scattering of hadrons—for example, the structure of the differential cross section at one given energy—but the number of successful models is rapidly reduced when they are tested in a wider kinematic range. This permits a critical evaluation and selection of viable models and theoretical ideas.

We have deliberately restricted ourselves to a small number of problems—elastic diffraction scattering of protons and antiprotons. We hope that the problems that we have considered and the solutions that we have given for them will stimulate the development of branches of high-energy physics close to elastic diffraction. These are, above all, inelastic hadronic processes (diffraction and nondiffraction, and also deep inelastic scattering of leptons on hadrons, in which diffraction also plays an important role). These fields have become the subject of intensive investigations with the accelerator HERA.

With regard to purely hadronic processes, it is to be expected that a new impetus in their study will arise with the commissioning of the UNK accelerator at Protvino and the LHC at CERN.

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