$D=10$ superstring: Lagrangian and Hamiltonian mechanics in twistor-like Lorentz-harmonic formulation[1),2)]

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The Lagrangian and Hamiltonian mechanics of a recently proposed twistor-like Lorentz-harmonic formulation of the $D=10$, $N=$IIB Green–Schwarz superstring are discussed. The equations of motion are derived, and the classical equivalence of this formulation to the standard one is proved. The complete set of covariant and irreducible first-class constraints generating the gauge symmetries of the theory, including κ symmetry, is presented. The algebra of all gauge symmetries and the symplectic structure characterizing the set of second-class constraints are derived. Thus, a basis for the covariant BRST–BFV quantization of the $D=10$ superstring in the twistor-like approach is constructed.

1. INTRODUCTION

Superstrings in $D=10$ (Refs. 1–3) are discussed as a possible basis for constructing a self-consistent quantum theory of gravity and a unified theory of all interactions. However, its covariant quantization is hampered by the problem of κ-symmetry covariant description because this fermionic symmetry[4] is infinitely reducible in the standard superstring formulation.[1,2] Unfortunately, the existing modern schemes[5–7] of covariant quantization have been developed only for systems with a finite level of constraint reducibility. (We recall that such a problem appears already in the superparticle theory.[3,8])

Progress in solving the problem of covariant quantization is necessary for a correct choice of the superstring ground state among the infinite number of solutions for $D=10$ superstring compactification. As a result, infinitely many different effective 4-dimensional theories appear instead of a unique 10-dimensional one.[9]

One way to solve the problem of covariant superstring quantization is to use the fact that the reducibility level of the symmetries is not invariant under possible reformulations of the theory.[5–7] In other words, two classically equivalent theories may have different levels of reducibility of their symmetries. Thus, a formulation of superstring theory which includes auxiliary variables and is classically equivalent to the standard formulation[1–3] may have either a finite level of reducibility of κ symmetry or even irreducible κ symmetry.[3)]

This approach has been opened up in the pioneering works of Nissimov, Pacheva, and Solomon.[14–16] They have extended the phase space of the $D=10$, $N=2$ Green–Schwarz superstring by adding the vector SO(1,9)/[SO(1,1)⊗SO(8)] harmonic variables $(u_m^{[\pm 2]}, u_m^{(i)})$ (see Ref. 13), with the two light-like vectors $u_m^{[\pm 2]}$ replaced by bilinear combinations of the $D=10$ bosonic spinors $v^{\alpha\pm}$: $u_m^{[\pm 2]}=v^{\alpha\pm}\sigma_{m\alpha\beta}v^{\beta\pm}$.

The characteristic feature of the approach[14–17] is the formulation of the action functional in the Hamiltonian formalism using the Lagrange-multiplier method. The "harmonic" variables $(v^{\alpha\pm}, u_m^{(i)})$ and the momentum degrees of freedom canonically conjugate to them are involved in the action principle through constraints chosen in such a way that the additional variables are pure gauge ones. Thus, equivalence of the "harmonic" superstring formulation[14–16] with the standard Green–Schwarz one is achieved.

The use of these variables permitted Nissimov, Pacheva, and Solomon to solve the problem of the covariant decomposition of the Grassmannian constraints into irreducible first- and second-class ones. The second-class fermionic constraints were transformed into first-class constraints using the auxiliary fermionic variables, and covariant quantization of the Brink–Schwarz superparticle and Green–Schwarz superstring theories was carried out[14–16] (see also Ref. 17).

In parallel, the twistor approach[58] to superparticle and superstring theories has been developed (Refs. 24, 40, 42, 43, 45, and 46). It is closely related to the approach of Nissimov, Pacheva, and Solomon; in particular, both approaches use bosonic spinor variables as the auxiliary ones. However, the twistor approach puts forward a new concept explaining the nature of the set of auxiliary bosonic spinor variables necessary for covariant decomposition of the Grassmannian constraints of superparticle or superstring theories. This concept, proposed in Refs. 28–30, interprets the chosen bosonic spinor variables as the "superpartners" of the target superspace Grassmannian coordinate field $\theta^{\alpha I}$ with respect to world-sheet supersymmetry.

Such a treatment of the bosonic spinor variables reduces the arbitrariness in their choice and, in particular, fixes their number as $N(D-2)$, where N is the number of target-space supersymmetries and D is the dimension of the target space-time.

On the basis of the twistor approach the "mysterious" κ symmetry is presented as the nonlinearly realized world-sheet supersymmetry when all auxiliary fields are eliminated using their equations of motion.[28–30]

In the superfield realization of the twistor approach the infinitely reducible κ symmetry with algebra that is closed only on mass shell is replaced by the local world-sheet supersymmetry transformations,[28–30] which are irreducible and have the algebra closed off the mass shell. Thus, the twistor approach seems to be a relevant basis for the covariant superstring quantization, alternative to that developing in Refs. 14–17.

Doubly supersymmetric superfield action functionals

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have been proposed for the superparticle and heterotic superstring in $D=3$, 4, 6, 10 (Refs. 28–32 and 34–40) as well as for the $D=3$, $N=2$ Green–Schwarz superstring.[59] There are some problems in the construction of such superfield action functionals for $N=2$ Green–Schwarz superstrings in $D=4$, 6, 10 (Ref. 59), and, up to now, this problem remains open. Nevertheless, a component twistor formulation[19,23,46] exists for these cases. These formulations are related to the superfield ones rewritten in terms of components, when all the auxiliary variables, except for the bosonic spinor ones, are eliminated from the action using algebraic equations of motion. Therefore, in twistor-like component superstring formulations[19,23,46] the world-sheet supersymmetry is realized nonlinearly, i.e., represented as a κ symmetry, and its algebra is closed only on the mass shell. However, the κ symmetry remains irreducible in this formulation, and the number of auxiliary bosonic spinor variables (twistors) remains the same as in superfield ones. Hence, these formulations[19,23,46] still give possibilities to investigate the machinery of the twistor approach in solving problems related to the task of covariant superstring quantization.

The $D=10$, $N=$IIB superstring formulation,[23,46] being invariant under the (nonlinearly realized) extended local $n=(8,8)$ world-sheet supersymmetry, includes in its configuration space two sets of auxiliary Majorana–Weyl bosonic spinor fields (twistor components) $v^{+}_{\alpha A}(\tau,\sigma)$ and $v^{-}_{\alpha\dot{A}}(\tau,\sigma)$ ($\alpha=1,...,16$; $A=1,...,8$; $\dot{A}=1,...,8$) taking values in 8-dimensional s and c spinor representations of the "transverse" SO(8) group. These twistor components are the superpartners of the Majorana–Weyl Grassmannian spinors $\theta^{\alpha 1}(\tau,\sigma)$ and $\theta^{\alpha 2}(\tau,\sigma)$ under the world-sheet supersymmetry transformations.

Comparing the component twistor-like formulation with the one proposed by Nissimov, Pacheva, and Solomon,[14–17] we conclude that they differ not only in the form of the action functionals, but also in the sets of auxiliary bosonic spinor fields. More precisely, one can say that the additional twistor variables in the set[20,21,23] can be obtained by taking the square root of the transverse vector harmonic variables $u^{(i)}_m$ belonging to the NPS set.[14–17] In other words, the harmonic fields[14–17] are composite objects constructed from the twistor variables.[20,21,23] The importance of the latter difference for the problem of covariant superstring quantization can be shown only by further investigations of the classical and quantum dynamics of superstrings in the twistor approach. Here we note only that the square-root operation leads to nontrivial consequences in many cases.[51]

Note that auxiliary spinor variables similar to the twistor variables discussed above have been previously used by Wiegmann[57] for the description of the $N=1$, $D=10$ heterotic and the Neveu–Schwarz–Ramond fermionic string in the covariant light-cone gauge. The paper of Ref. 57 is closely related to the Lund–Regge geometric approach[60] and, especially, to its gauge interpretation,[61] where the 2-dimensional SO(1,1) and SO($D-2$) gauge fields and the Cartan embedding forms used in Ref. 57 have been introduced. However, Wiegmann does not consider the problem of constructing a covariant Hamiltonian formalism for the original heterotic-string phase space extended by the addition of twistor variables in an arbitrary gauge. Instead, the author of Ref. 57 eliminates the original physical variables of the heterotic string $\theta^{\alpha}(\tau,\sigma)$ i.e., the Grassmannian target-space spinor coordinates, by means of functional integration. On the other hand, the original phase space of the heterotic string reduced in this way is extended by the addition of effective gauge fields[61] generated by the differential forms of embedding. As a result, the Hamiltonian structures of the twistor[22,23,46] and the effective[57] actions differ in principle.

For all the above-mentioned reasons we regard the investigation of the Lagrangian and Hamiltonian structures of the $D=10$, $N=$IIB Green–Schwarz superstring in the component twistor-like formulation[22,23,46] as a problem worthy of attention. This is just the suggested problem in the present paper.

Here we follow Refs. 18–23 and 46 and realize the twistor variables for the $D=10$, $N=$IIB Green–Schwarz superstring as the pure spinor Lorentz harmonics which parametrize the SO(1,9)/[SO(1,1)⊗SO(8)] coset. These harmonics are obtained by taking the square root of the basic vectors of the moving Cartan set attached to the superstring worldsheet. Newman and Penrose were the first to consider this interpretation of the twistor components for $D=4$.[44]

In Refs. 18, 19, 62, 42, and 43 the Newman–Penrose dyads were used for the description of massless superparticles, null superstrings, and null supermembranes. In particular, those papers demonstrated the fundamental role of the component twistor formulation for the actions of null super-p-branes (i.e., massless superparticles for $p=0$, null superstrings for $p=1$, and null super-p-branes for $p=2$) in 4-dimensional space-time for the solution of the problem of covariant constraint splitting and their conversion[7] into Abelian first-class constraints. As a result of the component twistor approach the problem of covariant BRST–BFV quantization of null super-p-branes in $D=4$ was solved.[42,43]

In the case $D=4$ the Newman–Penrose dyads were used to construct vector fields $\vec{u}^{(n)}(\tau,\sigma^M)$ of the Cartan moving set (an isotropic tetrad[44]) attached to the world-hypersheet of the (null) super-p-brane. The twistor-like null super-p-brane action is the first-order form functional constructed using the vector composed from this moving-frame set.

This observation leads to a generalization of the Lorentz-harmonic approach[18,19,42,43] to the description of superstring and other extended supersymmetric objects (for example, supermembranes) in higher dimensions D.[22,23,45,46] The proposed generalization implies the need to consider the D-dimensional spinor harmonics as generalized "dyads." Therefore, if the first-order form action with auxiliary vector variables is known, the problem of the twistor-harmonic description of the superstring (and super-p-branes) imbedded in the D-dimensional space-time is reduced to the problem of constructing a realization of the Cartan set (moving-frame system) $u^{(n)}_m(\tau,\sigma^M)\equiv u^{(n)}_m(\xi^\mu)$ in terms of the spinor $2^{[D/2]}\times 2^{[D/2]}$ harmonic matrix

$$v^a_\alpha \in \mathrm{Spin}(1,D-1), \quad \alpha=1,...,2^\nu; \quad a=1,...,2^\nu \quad (1.1)$$

with $\nu=[D/2]$ or $(D-2)/2$ for Majorana–Weyl spinors in $D=10$ (mod8) (Refs. 18–23, 42, 43, 45, and 46).

But such a task can be solved easily. The orthonormal set

$$u_m^{(n)} u^{m(l)} = \eta^{(n)(l)} = \mathrm{diag}(1,-1,\ldots,-1) \quad (1.2)$$

belongs to the SO(1,$D-1$) group. The double covering of this group is Spin(1,$D-1$). Thus, the connection of the set $u_m^{(n)}$ with the harmonic variable matrix v_α^a is defined by means of a "square-root" type of universal relation

$$u_m^{(n)} \equiv 2^{-\nu} v_\alpha^a (C\Gamma_m)^{\alpha\beta} v_\beta^b (\Gamma^{(n)} C^{-1})_{ab}. \quad (1.3)$$

As a result of Eq. (1.1), the relation (1.3) may be rewritten in the following forms:

$$u_m^{(n)} (\Gamma^m C^{-1})_{\alpha\beta} = v_\alpha^a (\Gamma^{(n)} C^{-1})_{ab} v_\beta^b \quad (1.4a)$$

$$u_m^{(n)} (C\Gamma_{(n)})^{ab} = v_\alpha^a (C\Gamma_m)^{\alpha\beta} v_\beta^b. \quad (1.4b)$$

This is possible because, in the general case, the identities

$$\mathrm{Sp}(v^T C\Gamma_{m_1\ldots m_k} v \Gamma^{(n)} C^{-1}) = 0, \quad (\text{when } k>1), \quad (1.5a)$$

$$\mathrm{Sp}(v^T C\Gamma_m v \Gamma^{(n_1)\ldots(n_k)} C^{-1}) = 0, \quad (\text{when } k>1) \quad (1.5b)$$

are satisfied for the matrix $v_\alpha^a \in \mathrm{Spin}(1,D-1)$ (1.1).

The relations (1.1)–(1.3) are the basis of the twistor-like Lorentz-harmonic approach to super-p-brane theories.

The approach discussed above has been called the harmonic one, because the condition (1.1) is realized not by expressing the matrix v_α^a as an exponential function of the Spin(1,9) Lie-algebra generators, but by the requirement that the matrix v_α^a satisfy a set of so-called harmonicity conditions

$$\Xi_M(v) = 0. \quad (1.6)$$

These conditions ensure fulfillment of all the relations (1.5a), (1.5b), as well as of the relations (1.3), by definition. Moreover, it is more convenient to use them than the straightforward exponential parametrization [this fact was already evident in the case of the compact space SU(2)/U(1) (Ref. 47)].

For the case of the $D=10$ superstring, the matrix v_α^a has one SO(1,9) Majorana–Weyl spinor index $\alpha=1,\ldots,16$ and one 16-dimensional index a of the right product of the SO(1,1) and SO(8) groups. The latter can be decomposed into two SO(1,1)⊗SO(8) invariant subsets of indexes $a=(\overset{+}{A},\overset{-}{\dot{A}})$. Here $A=1,\ldots,8$ and $\dot{A}=1,\ldots,8$ are the indices of (s) and (c) spinor representations of SO(8) and the ± symbols denote the Weyl weight under the transformations from the SO(1,1) group (which is identified with the Lorentz group of the world-sheet in the formulation of Refs. 22, 23, and 46). Accordingly, the 16×16 harmonic matrices v_α^a are decomposed into the two 16×8 blocks[20,21]

$$v_\alpha^a = (v_{\alpha A}^{+}, v_{\alpha\dot{A}}^{-}), \quad (1.7)$$

which transform covariantly under the left SO(1,9) and right SO(1,1)⊗SO(8) transformations.

The corresponding $\mathrm{SO}(1,9)_L \otimes [\mathrm{SO}(1,1)\otimes \mathrm{SO}(8)]_R$ invariant splitting of the composed Cartan set (1.3) has the form

$$u_m^{(n)} \equiv (u_m^{(0)}, u_m^{(1)}, \ldots, u_m^{(9)}) \equiv (u_m^{\{f\}}, u_m^{(i)}) \quad (1.8a)$$

$$u_m^{\{f\}} = (u_m^{(0)}, u_m^{(9)}) = (\tfrac{1}{2}(u_m^{[+2]} + u_m^{[-2]}),$$

$$\tfrac{1}{2}(u_m^{[+2]} - u_m^{[-2]})), \quad (1.8b)$$

$$u_m^{(i)} = (u_m^{(1)}, \ldots, u_m^{(8)}), \quad (1.8c)$$

where the vectors $u_m^{[\pm 2]}$, $u_m^{(i)}$ are defined by the relations[20,21]

$$u_m^{[+2]} = \tfrac{1}{8}(v_A^{+} \tilde{\sigma}_m v_A^{+}) \equiv \tfrac{1}{8} v_{\alpha A}^{+} \tilde{\sigma}_m^{\alpha\beta} v_{\beta A}^{+}, \quad (1.9a)$$

$$u_m^{[-2]} = \tfrac{1}{8}(v_{\dot{A}}^{-} \sigma_m v_{\dot{A}}^{-}), \quad (1.9b)$$

$$u_m^{(i)} = \tfrac{1}{8}(v_A^{+} \tilde{\sigma}_m v_{\dot{A}}^{-}) \gamma_{A\dot{A}}^{i}. \quad (1.9c)$$

The contracted SO(1,9) spinor indices are omitted in (1.9b), (1.9c) and in the following formulas.

The harmonicity conditions (1.6) have the following form in our case:[20–23,46]

$$\Xi_{m_1\ldots m_4} = u^{m(n)} \eta_{(n)(l)} \Xi^{(l)}_{m_1\ldots m_4 m} = 0, \quad (1.10a)$$

$$\Xi_0 \equiv u_m^{[-2]} u^{m[+2]} - 2 = 0, \quad (1.10b)$$

where the expression

$$\Xi^{(n)}_{m_1\ldots m_5} \equiv \mathrm{Sp}(v^T \tilde{\sigma}_{m_1\ldots m_k} v \sigma^{(n)})$$

$$\equiv v_\alpha^a (\tilde{\sigma}_{m_1\ldots m_5})^{\alpha\beta} v_\beta^b (\sigma^{(n)})_{ab} = 0 \quad (1.11)$$

vanishes as a consequence of Eq. (1.10a).[46] The last expression of the type (1.5a) vanishes identically because of the antisymmetry property of the matrix $(\tilde{\sigma}_{m_1\ldots m_3})^{\alpha\beta}$ under spinor-index permutations.

The orthogonality conditions (1.2) are satisfied as a general consequence of the expressions (1.9), the conditions (1.10a), and the famous identity

$$\tilde{\sigma}_m^{\alpha\{\beta_\sigma \gamma\delta\}m} \equiv \tfrac{1}{3}(\tilde{\sigma}_m^{\alpha\beta} \tilde{\sigma}^{\gamma\delta m} + \text{cyclic permutations}(\alpha,\beta,\gamma))$$

$$= 0. \quad (1.12)$$

The normalization conditions for the composed set (1.2), (1.9) are satisfied as a result of the harmonicity conditions (1.10a), (1.10b) and the identity (1.12).

Thus, an orthonormal set in $D=10$ space-time has been constructed in terms of generalized dyads. After the construction of the first-order form superstring action using the auxiliary vector variables $n_m^{[\pm 2]}$, which belong to the moving-frame system, the twistor-like form of the superstring action can be achieved by the simple replacement of the $n_m^{[\pm 2]}$ by the composed vectors $u_m^{[\pm 2]}$ (1.9).

In this way the action for the $D=10$, $N=$IIB superstring was constructed.[22,23]

Here we continue the program outlined in Refs. 22 and 23.

Lagrangian and Hamiltonian mechanics of the twistor-like Lorentz-harmonic formulation of the superstring are constructed. The equations of motion are derived. The decomposition of the constraints into covariant and irreducible first- and second-class ones is carried out. We compute the algebra of the gauge symmetries of the theory in the Hamiltonian formalism and present the symplectic structure characterizing the set of second-class constraints. Thus, we get all necessary information for the conversion of the second-class constraints into Abelian first-class ones (see Ref. 7); the construction of the classical BRST charge and covariant quantization will be the subject of future work.

The paper is organized as follows.

To elucidate the description of the superstring in the twistor-like formulation we consider the bosonic-string formulation with auxiliary vector variables in detail. This is done in Sec. 2, where the derivation of the equation of motion and the construction of the Hamiltonian formalism for systems with harmonic variables are discussed using this simple example. For the convenience of the reader, the description is closed in this section.

In Sec. 3 we describe the twistor-like Lorentz-harmonic superstring formulation[22,23] and discuss its equivalence to the standard one.[1,3] Here we derive all the equations of motion for this superstring formulation.

Section 4 is devoted to the construction of the Hamiltonian formalism.

The primary constraints are derived, and the so-called covariant momentum densities for the harmonic variable are introduced in the Sec. 4.1. It is demonstrated that these momentum variables generate the current algebra of SO(1,9) on the Poisson brackets.

In Sec. 4.2 the Dirac prescription for checking the constraint conservation during the evolution is carried out, and the covariant and irreducible first-class constraints are derived.

In Sec. 5 the first-class constraints are redefined. This redefinition leads to a simplification of the algebra generated by them on the Poisson brackets. Such an algebra is presented in the Sec. 5.1. The symplectic structure of the system of second-class constraints is derived in Sec. 5.3. The relation between the well-known Virasoro constraints and the reparametrization-symmetry generators of the twistor-like formulation[22,23] is discussed in Sec. 5.3.

Our notation for the Majorana–Weyl spinor indices in $D=10$ coincides with that of Refs. 14 and 15 except for a different choice of the metric signature [see Eq. (1.2)].

2. BOSONIC STRING IN THE CARTAN MOVING-FRAME FORMULATION

2.1. Action principle and equations of motion

To elucidate the description of the superstring in the twistor-like Lorentz-harmonic approach we consider the bosonic-string formulation with the following action functional:

$$S\equiv\int d^2\xi L(\xi)=\int d^2\xi e(\xi)(-(\alpha')^{-1/2}e^{\mu}_{\rm f}\partial_\mu x^{\rm m}n^{\{\rm f\}}_{\rm m}+c). \tag{2.1}$$

This formulation uses two D-dimensional vector fields $n^{\{\rm f\}}_{\rm m}=(n^{(0)}_{\rm m},n^{(D-1)}_{\rm m})$ belonging to the Cartan set[4] (moving-frame system) $n^{(\rm l)}_{\rm m}=(n^{\{\rm f\}}_{\rm m},n^{(\rm i)}_{\rm m})$ attached to the string world-sheet and defined by the orthonormality conditions

$$\Xi^{(\rm n)(\rm l)}\equiv n^{(\rm n)}_{\rm m}n^{\rm m(\rm l)}-\eta^{(\rm n)(\rm l)}=0, \tag{2.2}$$

with the Minkowski metric tensor $\eta^{(\rm n)(\rm l)}={\rm diag}(+,-,...,-)$. Another set of auxiliary fields that is used is the world-sheet zweibein $e^{\rm f}_{\mu}(\xi)$ [$\mu=(\tau,\sigma)$; $f=0,1$]:

$$e^{\mu}_{\rm g}e^{\rm f}_{\mu}=\delta^{\rm f}_{\rm g},\quad e^{\rm f}_{\mu}e^{\nu}_{\rm f}=\delta^{\nu}_{\mu},\quad e\equiv\det(e^{\rm f}_{\mu}). \tag{2.3}$$

The two-dimensional Lorentz group SO(1,1) acts on the flat indices f,g of the zweibein $e^{\rm f}_{\mu}(\xi)$ as well as on the 2-valued index {f} labeling the vectors from the set $n^{\{\rm f\}}_{\rm m}(\xi)$.[5] The basis of a two-dimensional vector space can always be chosen to be composed of two light-like vectors with definite and opposite weights under the SO(1,1) group. Thus, it is convenient to work in terms of the light-like zweibein components

$$\begin{aligned}
&e^{\rm f}_{\mu}=(\tfrac{1}{2}(e^{[+2]}_{\mu}+e^{[-2]}_{\mu}),\quad \tfrac{1}{2}(e^{[+2]}_{\mu}-e^{[-2]}_{\mu})),\\
&e^{\mu}_{\rm f}=(\tfrac{1}{2}(e^{\mu[-2]}+e^{\mu[+2]}),\quad \tfrac{1}{2}(e^{\mu[-2]}-e^{\mu[+2]})),\\
&e^{[+2]}_{\mu}e^{\mu[+2]}=0=e^{[-2]}_{\mu}e^{\mu[-2]},\\
&e^{[+2]}_{\mu}e^{\mu[-2]}=2=e^{[-2]}_{\mu}e^{\mu[+2]},\\
&\varepsilon^{\mu\nu}=\tfrac{1}{2}e(e^{\mu[+2]}e^{\nu[-2]}-e^{\mu[-2]}e^{\nu[+2]}),\\
&(\varepsilon^{01}=-\varepsilon_{01}=1),\\
&g^{\mu\nu}=\tfrac{1}{2}(e^{\mu[+2]}e^{\nu[-2]}+e^{\mu[-2]}e^{\nu[+2]}),\\
&\sqrt{-g}\equiv e\\
&\delta^{\nu}_{\mu}=\tfrac{1}{2}(e^{[+2]}_{\mu}e^{\nu[-2]}+e^{[-2]}_{\mu}e^{\nu[+2]}),\\
&\varepsilon_{\mu\nu}e^{\mu[-2]}e^{\nu[+2]}=2/e,
\end{aligned} \tag{2.4}$$

and the light-like vectors

$$\begin{aligned}
&{\rm n}^{\{\rm f\}}_{\rm m}=(\tfrac{1}{2}({\rm n}^{[+2]}_{\rm m}+{\rm n}^{[-2]}_{\rm m}),\quad \tfrac{1}{2}({\rm n}^{[+2]}_{\rm m}-{\rm n}^{[-2]}_{\rm m})),\\
&{\rm n}^{[\pm 2]}_{\rm m}={\rm n}^{\{0\}}_{\rm m}\pm{\rm n}^{\{1\}}_{\rm m}={\rm n}^{(0)}_{\rm m}\pm{\rm n}^{(D-1)}_{\rm m},\\
&{\rm n}^{[+2]}_{\rm m}{\rm n}^{\rm m[+2]}=0={\rm n}^{[-2]}_{\rm m}{\rm n}^{\rm m[-2]},\quad {\rm n}^{[+2]}_{\rm m}{\rm n}^{\rm m[-2]}=2
\end{aligned} \tag{2.5}$$

[cf. (1.8b)].[6]

The variation of the action (2.1) with respect to the inverse zweibeins $e^{\mu}_{\rm f}$ gives the relation

$$e^{\rm f}_{\mu}(\xi)=\partial_\mu x^{\rm m}{\rm n}^{\{\rm f\}}_{\rm m}/c(\alpha')^{1/2}, \tag{2.6}$$

which is a simple expression for the form $e^{\rm f}(d\xi,\xi)=d\xi^{\mu}e^{\rm f}_{\mu}(\xi)$ of the world-sheet, induced by embedding of the world-sheet into the D-dimensional Minkowski space-time. Taking into account Eq. (1.6), we can eliminate the auxiliary zweibein field from the action (2.1):

$$-(\alpha')^{-1/2}e^{\mu}_{\rm f}\partial_\mu x^{\rm m}n^{\{\rm f\}}_{\rm m}=-2ce=-2\ \det(\partial_\mu x^{\rm m}n^{\{\rm f\}}_{\rm m}), \tag{2.7}$$

$$S_{\rm V\text{-}Z}=-(c\alpha')^{-1}\int d^2\xi\ \det(\partial_\mu x^{\rm m}n^{\{\rm f\}}_{\rm m}), \tag{2.8}$$

The resulting action (2.8) coincides with the one from Ref. 41, where the auxiliary vector fields from the Cartan moving set were introduced for the first time for constructing string and superstring actions.

Thus, the action (2.1) is the first-order form representation for the "antisymmetric" action from Ref. 41.

Now let us discuss the relation of our string formulation (2.1) to the standard Dirac–Nambu–Goto and Polyakov ones.

It will be proved below that the variation of the action (2.1) with respect to the auxiliary vector fields $n_m^{\{f\}}(\xi)$ leads to the nontrivial equations

$$\partial_\mu x^m n_m^{(i)}=0, \qquad (2.9)$$

which means that the vectors $n_m^{(i)}$ are orthogonal to the string world-sheet. The completeness of the moving-frame system

$$n_m^{(n)}\eta_{(n)(l)}n_p^{(l)}=\eta_{mp}$$

makes it possible to express $\partial_\mu x^m(\xi)$ in terms of $n_m^{\{f\}}(\xi)$,

$$\partial_\mu x^m(\xi)=c(\alpha')^{1/2}e_\mu^g\eta_{gf}\,n_m^{\{f\}}, \qquad (2.10a)$$

and vice versa,

$$n_m^{\{f\}}=c^{-1}(\alpha')^{-1/2}\eta^{fg}e_g^\mu\partial_\mu x_m. \qquad (2.11)$$

Taking into account Eqs. (2.7) and (2.11), as well as the definitions

$$e_f^\mu\eta^{fg}e_g^\nu\equiv g^{\mu\nu}, \quad e=\sqrt{-g},$$

we can rewrite the action S (2.1) in the form

$$S_P=-(2c\alpha')^{-1}\int d^2\xi\sqrt{-g}g^{\mu\nu}\partial_\mu x^m\partial_\nu x_m,$$

which is the known string action introduced in Ref. 48. On the other hand, Eq. (2.10) leads to the following expression for the induced metric:

$$g_{\mu\nu}=\partial_\mu x^m(\xi)\partial_\nu x_m(\xi)/c^2\alpha', \qquad (2.10b)$$

which results in the relation

$$e=\sqrt{-g}=\det{}^{1/2}(\partial_\mu x^m(\xi)\partial_\nu x_m(\xi))/c^2\alpha. \qquad (2.10c)$$

The substitution of Eqs. (2.10a) and (2.10c) into the functional (2.1) leads to the Dirac–Nambu–Goto action

$$S_{D\text{-}N\text{-}G}=-(c\alpha')^{-1}\int d^2\xi(\det(\partial_\mu x^m(\xi)\partial_\nu x_m(\xi)))^{1/2}.$$

Finally, the variation of the action S (2.1) with respect to $x^m(\xi)$ gives the equation

$$\partial_\mu(ee_f^\mu n_m^{\{f\}})=0, \qquad (2.12)$$

which can be rewritten in the standard form (see Ref. 48)

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu x^m)=0, \qquad (2.13)$$

using Eq. (2.11).

However, the derivation of Eq. (2.9), which is crucial for the conclusion presented above, is not such a simple task. First of all, we note that the variational problem with respect to the $n_m^{\{f\}}$ fields is a problem with a conditional extremum, since it is necessary to take into account the orthonormality conditions (2.2). It can be reformulated as a variational problem with an absolute extremum if we extend the action (2.1) by adding the conditions $\Xi^{(n)(l)}$ (2.2) with corresponding Lagrange multipliers (see Ref. 41). Another way to obtain the right equations of motion is to restrict the class of admissible variations $\delta n_m^{(l)}$ to variations which preserve the orthonormality conditions (2.2). The use of this method does not require the introduction of Lagrange multipliers and seems simpler for the solution of variational problems characterized by complicated structure of the constraints. The same method will be used below to study D=10 superstring dynamics in the twistor-like formulation.[22,23]

2.2. Admissible variations for the variables

Let us consider an arbitrary set of D independent vector variables $n_m^{(l)}$ in D-dimensional space. The condition of independence has the form $\det(n_m^{(l)})\neq 0$. Thus, the set of variables $n_m^{(l)}$ considered as a D×D matrix belongs to the GL(D,R) group. An arbitrary variation with respect to $n_m^{(l)}$,

$$\delta=\delta n_m^{(l)}\partial/\partial n_m^{(l)} \qquad (2.14a)$$

can be rewritten in the form

$$\delta=(n^{-1}\delta n)_{(k)}^{(l)}(n_m^{(k)}\partial/\partial n_m^{(l)}). \qquad (2.14b)$$

In Eq. (2.14b), $(n^{-1}\delta n)_{(k)}^{(l)}\equiv(n^{-1})_{(k)}^m\delta n_m^{(l)}$ is the Cartan differential form, which is invariant under left GL(D,R) transformations. The differential operators $n_m^{(l)}\partial/\partial n_{m(k)}$ in (2.14b) can be considered as covariant derivatives (see Ref. 47) for the GL(D,R) group.

Let us restrict the right GL(D,R) transformations [acting on the numbers (l) of the vectors $n_m^{(l)}$] to be only from the Lorentz group SO(1,D−1). Then the invariant metric tensor

$$\eta^{(n)(l)}=\mathrm{diag}(+,-,\ldots,-)$$

appears, and it becomes possible to lower and raise the indices in the brackets. After this step we can transform Eq. (2.14b) to the form

$$\delta=(n^{-1}\delta n)_{(k)(l)}(n_m^{(k)}\partial/\partial n_{m(l)}) \qquad (2.14c)$$

and decompose the GL(D,R) covariant derivatives $n^{(l)}\partial/\partial n_{(k)}$ into symmetric and antisymmetric parts:

$$(n\partial/\partial n)^{(k)(l)}=\tfrac{1}{2}(\Delta^{(l)(k)}+K^{(l)(k)}),$$

$$\Delta^{(l)(k)}=n_m^{(k)}\partial/\partial n_{m(l)}-n_m^{(l)}\partial/\partial n_{m(k)}, \qquad (2.15a)$$

$$K^{(l)(k)}=n_m^{(l)}\partial/\partial n_{m(k)}+n_m^{(k)}\partial/\partial n_{m(l)}. \qquad (2.15b)$$

The corresponding decomposition of the Cartan differential form is defined by the relations

$$(n^{-1}\delta n)_{(k)(l)}=\tilde{\Omega}_{(k)(1)}(\delta)+S_{(k)(1)}(\delta),$$

$$\tilde{\Omega}_{(k)(l)}(\delta)=(n^{-1}\delta n)_{[(k)(l)]}\equiv\tfrac{1}{2}((n^{-1}\delta n)_{(k)(l)}-(n^{-1}\delta n)_{(l)(k)}), \qquad (2.16a)$$

$$S_{(k)(l)}(\delta)=(n^{-1}\delta n)_{\{(k)(l)\}}\equiv\tfrac{1}{2}((n^{-1}\delta n)_{(k)(l)}-(n^{-1}\delta n)_{(l)(k)}). \qquad (2.16b)$$

Taking into account Eqs. (2.15) and (2.16), the expression (2.14) for an arbitrary variation can be represented in the form

$$\delta=\tfrac{1}{2}\tilde{\Omega}_{(k)(l)}(\delta)\Delta^{(l)(k)}+\tfrac{1}{2}S_{(k)(l)}(\delta)K^{(l)(k)}, \qquad (2.17)$$

It is easy to show that the $\Delta^{(l)(k)}$ and $K^{(l)(k)}$ operators generate the gl(D,R) Lie algebra

$$[\Delta_{(l_1)(l_2)},\Delta_{(k_1)(k_2)}]=-2\eta_{(l_1)[(k_1)}\Delta_{(k_2)](l_2)}+2\eta_{(l_2)[(k_1)}\Delta_{(k_2)](l_1)}, \qquad (2.18a)$$

$$[\Delta_{(l_1)(l_2)}, K_{(k_1)(k_2)}] = 2\eta_{(l_1)\{(k_1)}K_{(k_2)\}(l_2)} - 2\eta_{(l_2)\{(k_1)}K_{(k_2)\}(l_1)}, \tag{2.18b}$$

$$[K_{(l_1)(l_2)}, K_{(k_1)(k_2)}] = 2\eta_{(l_1)\{(k_1)}\Delta_{(k_2)\}(l_2)} + 2\eta_{(l_2)\{(k_1)}\Delta_{(k_2)\}(l_1)}, \tag{2.18c}$$

with the subalgebra (2.18a) of the Lorentz group produced by $D(D-1)/2$ $\Delta_{(l)(k)}$ operators. The operators $K_{(l)(k)}$ are associated with the factor space

$$GL(D,R)/SO(1,D-1)$$

and the number of them is equal to the number of orthonormality conditions $\Xi^{(l)(k)}$.

Now it is evident that the admissible variation can include only $\Delta_{(l)(k)}$ operators. This statement is true because the Lorentz rotations are the only transformations which preserve the orthonormality of the set. However, we can arrive at this statement in a more formal way. This will help us to understand the more complicated case of spinor moving-frame variables (i.e., Lorentz harmonics[20–23]).

The action of the Δ and K operators on the variables $n_m^{(l)}$ can be easily determined [see Eqs. (2.15)]:

$$\Delta_{(l_1)(l_2)} n_{(l)m} = 2\eta_{(l)[(l_1)} n_{(l_2)]m}, \tag{2.19a}$$

$$K_{(l_1)(l_2)} n_{(l)m} = 2\eta_{(l)\{(l_1)} n_{(l_2)\}m}. \tag{2.19b}$$

Thus, we have

$$\Delta_{(l_1)(l_2)}\Xi_{(k_1)(k_2)} = 2\eta_{(k_1)[(l_1)}\Xi_{(l_2)](k_2)} + 2\eta_{(k_2)[(l_1)}\Xi_{(l_2)](k_1)}, \tag{2.20a}$$

$$K_{(l_1)(l_2)}\Xi_{(k_1)(k_2)} = 4\eta_{(k_1)\{(l_1)}\eta_{(l_2)\}(k_2)} + 2\eta_{(k_1)\{(l_1)}\Xi_{(l_2)\}(k_2)} + 2\eta_{(k_2)\{(l_1)}\Xi_{(l_2)\}(k_1)}. \tag{2.20b}$$

Equations (2.20) justify the statement that the $\Delta_{(l)(k)}$ operators preserve the orthonormality conditions (2.2):

$$\Delta_{(l_1)(l_2)}\Xi_{(k_1)(k_2)}\Big|_{\Xi=0} = 0, \tag{2.21a}$$

At the same time,

$$K_{(l_1)(l_2)}\Xi_{(k_1)(k_2)}\Big|_{\Xi=0} = 4\eta_{(k_1)\{(l_1)}\eta_{(l_2)\}(k_2)} \tag{2.21b}$$

Hence the operators $K_{(l)(k)}$ destroy the orthonormality. Moreover, the differential form (2.16b), related to the operator $K_{(l_1)(l_2)}$ [see (2.17)], is reduced to a complete differential of the orthonormality condition $\Xi_{(l_1)(l_2)}$ on the surface (2.2):

$$S_{(l_1)(l_2)}\Big|_{\Xi=0} = \tfrac{1}{2} d\Xi_{(l_1)(l_2)} \tag{2.21c}$$

Thus, the variations

$$\delta\Big|_{\Xi=0} = \tfrac{1}{2}\Omega^{(l)(k)}(\delta)\Delta_{(k)(l)} \tag{2.14d}$$

are admissible [i.e., they preserve the orthonormality conditions (2.2)].

In Eq. (2.14d) the covariant $SO(1,D-1)$ derivative has the form (2.15a), and the expressions for the Cartan forms (2.16a) can be reduced to

$$\Omega^{(k)(l)}(\delta) = \tilde{\Omega}^{(k)(l)}(\delta)\Big|_{\Xi=0} = n_m^{(k)}\delta n^{m(l)} = -n_m^{(l)}\delta n^{m(k)} \tag{2.22}$$

on the surface defined by the orthonormality conditions (2.2).

It is interesting to note that Eq. (2.14a) can be considered as the definition of the covariant derivatives $\Delta_{(l)(k)}$. Thus, $\Delta_{(l)(k)}$ can be understood as derivatives with respect to the Cartan forms $\Omega^{(k)(l)}(\partial)$.

If the variables become the fields living on the world-sheet,

$$n_m^{(l)} = n_m^{(l)}(\xi^\mu),$$

then we must use the variational analogs $\tilde{\Delta}(\xi)$ of the operators Δ,

$$\tilde{\Delta}^{(k)(l)}(\xi) \equiv n_m^{(l)}(\xi)\,\delta/\delta n_{m(k)}(\xi) - n_m^{(k)}(\xi)\,\delta/\delta n_{m(l)}(\xi), \tag{2.23}$$

and for the admissible variation we must use the form

$$\delta\Big|_{\Xi(\xi)=0} = \tfrac{1}{2}\int d^2\xi\,\Omega^{(l)(k)}(\delta)\tilde{\Delta}_{(k)(l)}(\xi) \tag{2.24}$$

instead of one defined by Eq. (2.14d).

Now we are ready to discuss the derivation of Eq. (2.9).

Taking into account Eq. (2.24), it is easy to see that the variation of the action (2.1) with respect to the fields $n_m^{\{f\}}$ is defined by the relation

$$\delta S \equiv \int d^2\xi\,\delta L(\xi) = \int d^2\xi\, e(\xi)(-(\alpha')^{-1/2} e_f^\mu \partial_\mu x^m \delta n_m^{\{f\}})$$
$$= -(\alpha')^{-1/2}\int d^2\xi\, e e_f^\mu \partial_\mu x^m \Omega^{(l)(k)}(\delta)(\Delta_{(k)(l)} n_m^{\{f\}})(\xi). \tag{2.25}$$

We stress that the simple covariant derivative (2.15a) is used in the last part of Eq. (2.25). This is the result of application of the variational derivative (2.23) included in the previous part of this equation.

Hence, we can conclude that:

i) The right equations of motion for the fields have the form of variations of the action (2.1) with respect to the Cartan forms (2.22):

$$\delta S/\delta\Omega^{(l)(k)}(\delta) = 0 \tag{2.26}$$

[these equations automatically take into account the orthonormality conditions (2.2)].

ii) These equations can be represented in terms of the Lagrangian density and ordinary covariant derivatives as follows:

$$\Delta^{(k)(l)} L(\xi) \equiv (n_m^{(l)}(\xi)\partial/\partial n_{m(k)})(\xi)$$
$$- n_m^{(k)}(\xi)\partial/\partial n_{m(l)}(\xi))L(\xi) = 0. \tag{2.27}$$

This statement is true for similar cases when there are no time derivatives of the fields in the action.

iii) The equations of motion are defined by the result of the action of the ordinary covariant derivatives (2.15a) on the fields $n_m^{(l)}$.

Thus, the equations of motion for the $n_m^{(l)}$ fields have the form

$$e e_f^\mu \partial_\mu x^m \Delta_{(k)(l)} n_m^{\{f\}} = 0. \tag{2.28}$$

We can specify them as follows, using Eq. (2.19a):

$$e e_f^\mu \partial_\mu x^m n_{m[(k)}\delta_{(l)]}^{\{f\}} = 0. \tag{2.29}$$

Thus, it is evident that the equations of motion for the fields $n_m^{(l)}$ have nontrivial consequences only for the cases (k)={f} or (l)={f}. The equations (2.29) are satisfied identically when (k)≠{f} and (l)≠{f}. This is a consequence of the SO(8) gauge symmetry of the action (2.1). The operators $\Delta^{(i)(j)}$ generate these transformations.

Equation (2.29) reduces to the relation

$$ee^{\mu}_{[f}\partial_{\mu}x^m n_{m\{g\}]}=0 \tag{2.30}$$

when both indices (k) and (l) belong to the {f} set. Equation (2.30) is satisfied identically if Eq. (2.6) is taken into account. This fact corresponds to the SO(1,1) gauge symmetry of the action (2.1).

Hence the unique nontrivial consequence of Eq. (2.28) corresponds to the variation of the action with respect to the Cartan form $\Omega^{\{f\}(i)}$ describing the variations from the coset SO(1,D−1)/[SO(1,1)×SO(D−2)]. It has the form of the relation

$$ee^{\mu}_{f}\partial_{\mu}x^m n_m^{(i)}=0 \tag{2.31}$$

and is equivalent to Eq. (2.9).

Hence the equations of motion for our bosonic-string formulation (2.1) have been derived using the variational principle based on the concept of admissible variations (2.22), (2.24) of the fields. It is a simple task to derive the same equations of motion using arbitrary variations and the extended action functional completed by products of the orthonormality conditions $\Xi^{(n)(l)}$ (2.2) on the Lagrange multipliers (see Ref. 41 for this approach applied to the second-order form action).

However, for the case of the twistor-like Lorentz-harmonic formulation of the superstring (see Sec. 3) the form of the variational principle described here simplifies the calculation significantly.

2.3. Hamiltonian formalism and covariant momentum densities

We now discuss the Hamiltonian formalism for the bosonic-string formulation (2.1). The first-order form of the action principle results in the following fact. All the expressions for the momentum-density variables

$$P_M(\xi)=-\partial L/\partial(\partial_{\tau}X^M)\equiv(P_m(\xi),P^m_{(l)}(\xi)), \tag{2.32}$$

canonically conjugate to the configuration-space coordinates of the theory,

$$X^M(\xi)\equiv(x^m(\xi),\quad n_m^{(l)}(\xi)), \tag{2.33}$$

result in some constraints. For our formulation of the bosonic string these primary constraints have the form

$$P_m-(\alpha')^{-1/2}ee^{\tau}_{f}n_m^{\{f\}}\approx 0, \tag{2.34a}$$

$$P^m_{(l)}\approx 0, \tag{2.34b}$$

However, the orthonormality conditions (2.2) must be considered as additional primary constraints

$$\Xi^{(n)(l)}\equiv n_m^{(n)}n^{m(l)}-\eta^{(n)(l)}\approx 0, \tag{2.35}$$

if the canonical momentum densities $P^m_{(l)}(\xi)$ for the variables $n_m^{(l)}(\xi)$ are used. Such an extension of the set of constraints makes the Hamiltonian mechanics more complicated in our case (see Ref. 41). But the corresponding complication for the case of the twistor-like formulation of the D=10 superstring[22,23] becomes drastic. Indeed, in this formulation[22,23] the complicated harmonicity conditions (1.10), (1.11) appear instead of the orthonormality conditions (2.2).

For what follows, it is important to develop a method which allows us to eliminate conditions like (2.2) from the set of constraints and to discuss them as strong relations. Such a method was used, in fact, in Refs. 13–17, 20, and 21 and was considered briefly in Ref. 13 for the superparticle case (see also Refs. 18 and 49). Here we justify this method in detail for the case of the bosonic-string formulation (2.1). Such a justification elucidates the discussion for the case of the twistor-like superstring formulation.

Let us return to the primary constraints (2.34). The first of them (2.34a) can be decomposed into two relations, using the orthonormality conditions (2.2):

$$P_0^{\{f\}}\equiv n^{m\{f\}}P_m\approx(\alpha')^{-1/2}ee^{f\tau}\equiv(\alpha')^{-1/2}e\,\eta^{fg}e^{\tau}_{g}, \tag{2.36a}$$

$$P_0^{(i)}\equiv n^{m(i)}P_m\approx 0. \tag{2.36b}$$

Equations (2.36) mean that the variables can be discussed as the matrix of the Lorentz transformations which relate an arbitrary coordinate frame to the fixed one, where the string momentum density $P_0^{(l)}$ has only two nonvanishing components (which coincide with the τ components of the zweibein density ee^{μ}_{f})

$$P_0^{(l)}\equiv(P_0^{\{f\}},P_0^{(i)})=((\alpha')^{-1/2}ee^{f\tau},0)=n^{m(l)}P_m. \tag{2.37}$$

A similar interpretation of the Cartan–Penrose representation, rewritten in terms of the D=4 Lorentz-harmonic matrix was given in Ref. 18.

We shall extend this interpretation to the case of the harmonic sector and form the $SO(1,D-1)_L$-invariant momentum densities

$$P_{(l)(k)}\equiv n_{m(k)}P^m_{(l)}=-n_{m(k)}\partial L/\partial(\partial_{\tau}n_m^{(l)}). \tag{2.38}$$

After the division of $P_{(k)(l)}$ into symmetric,

$$\Sigma_{(l)(k)}\equiv n_{m(k)}P^m_{(l)}+n_{m(l)}P^m_{(k)}, \tag{2.39}$$

and antisymmetric,

$$\Pi_{(l)(k)}\equiv n_{m(k)}P^m_{(l)}-n_{m(l)}P^m_{(k)}, \tag{2.40}$$

parts, we obtain D(D+1)/2 symmetric and D(D−1)/2 antisymmetric constraints equivalent to (2.34b):

$$\Sigma_{(k)(l)}\approx 0, \tag{2.41a}$$

$$\Pi_{(k)(l)}\approx 0. \tag{2.41b}$$

The Poisson brackets are defined by the relations

$$[P_M(\tau,\sigma),X^N(\tau,\sigma')]_P\equiv-[X^N(\tau,\sigma'),\quad P_M(\tau,\sigma)\}_P =\delta^N_M\delta(\sigma-\sigma'), \tag{2.42a}$$

or

$$[(F,G)_P \equiv \int d\sigma(\delta F/\delta P_M(\sigma)\delta G/\delta X^M(\sigma) - \delta F/\delta X^M(\sigma)\delta G/\delta P_M(\sigma))],$$

$$\equiv \int d\sigma(\delta F/\delta P_m(\sigma)\delta G/\delta x^m(\sigma) - \delta F/\delta x^m(\sigma)\delta G/\delta P_m(\sigma)) + \int d\sigma(\delta F/\delta P^m_{(l)}(\sigma)\delta G/\delta n^{(l)}_m(\sigma) - \delta F/\delta n^{(l)}_m(\sigma)\delta G/\delta P^m_{(l)}(\sigma)), \tag{2.42b}$$

where $F \equiv F[X^M(\sigma),P_M(\sigma)]$ and $G \equiv G[X^M(\sigma),P_M(\sigma)]$ are arbitrary functionals defined on the phase space of the system.

It can be shown that the variables $\Sigma_{(k)(l)}(\xi)$ and $\Pi_{(k)(l)}(\xi)$ realize a vector representation of the gl(D,R) current algebra on the Poisson brackets (2.42):

$$[\Pi_{(l_1)(l_2)}(\sigma), \quad \Pi_{(k_1)(k_2)}(\sigma')]_P = 2(\eta_{(1_1)[(k_1)}\Pi_{(k_2)](1_2)} - \eta_{(1_2)[(k_1)}\Pi_{(k_2)](1_1)}\delta(\sigma-\sigma'), \tag{2.43a}$$

$$[\Pi_{(l_1)(l_2)}(\sigma), \quad \Sigma_{(k_1)(k_2)}(\sigma')]_P = -2(\eta_{(1_1)\{(k_1)}\Sigma_{(k_2)\}(1_2)} - \eta_{(1_2)\{(k_1)}\Sigma_{(k_2)\}(1_1)}) \times \delta(\sigma-\sigma'), \tag{2.43b}$$

$$[\Sigma_{(l_1)(l_2)}(\sigma), \quad \Sigma_{(k_1)(k_2)}(\sigma')]_P = -2(\eta_{(1_1)\{(k_1)}\Pi_{(k_2)\}(1_2)} + \eta_{(1_2)\{(k_1)}\Delta_{(k_2)\}(1_1)}) \times \delta(\sigma-\sigma'). \tag{2.43c}$$

The constraints $\Pi_{(k)(l)}$ (2.40) form a representation of the SO(1,D−1) current algebra and, consequently, do not change the constraints (2.35) in the weak sense:

$$[\Pi_{(l_1)(l_2)}(\sigma), \quad n_{(l)m}(\sigma')]_P = -2\eta_{(l)[(l_1)}n_{(l_2)]m}\delta(\sigma-\sigma'), \tag{2.44}$$

$$[\Pi_{(l_1)(l_2)}(\sigma), \quad \Xi_{(k_1)(k_2)}(\sigma')]_P = -2(\eta_{(k_1)[(1_1)}\Xi_{(1_2)](k_2)} + \eta_{(k_2)[(1_1)}\Xi_{(1_2)](k_1)}) \times \delta(\sigma-\sigma'). \tag{2.45}$$

Thus, it is natural to consider $\Pi_{(k)(l)}$ as the (covariant) momentum variables for the degrees associated with the Lorentz subgroup SO(1,D−1) of the group GL(D,R) (i.e., with the orthonormal set).

Unlike $\Pi_{(k)(l)}$, the symmetric constraints $\Sigma_{(k)(l)}$ do not preserve the orthonormality conditions (2.35). Indeed,

$$[\Sigma_{(l_1)(l_2)}(\sigma), \quad \Xi_{(k_1)(k_2)}(\sigma')]_P = 4\eta_{(k_1)\{(1_1)}\eta_{(1_2)\}(k_2)}\delta(\sigma-\sigma') + 2(\eta_{(k_1)\{(1_1)}\Xi_{(1_2)\}(k_2)} + \eta_{(k_2)\{(1_1)}\Xi_{(1_2)\}(k_1)})\delta(\sigma-\sigma'), \tag{2.46}$$

or, in the weak sense,

$$[\Sigma_{(l_1)(l_2)}(\sigma), \quad \Xi_{(k_1)(k_2)}(\sigma')]_P \approx 4\eta_{(k_1)\{(l_1)}\eta_{(l_2)\}(k_2)}\delta(\sigma-\sigma'). \tag{2.47}$$

Thus, it is natural to consider the combinations of the phase variables $P^m_{(l)}$ and $n^{(l)}_m$ represented by $\Sigma_{(k)(l)}$ and $\Xi^{(k)(l)}$ as new canonically conjugate variables describing D(D+1)/2 degrees of freedom. Owing to their vanishing in the weak sense, the phase variables $\Xi^{(k)(l)}$ and $\Sigma_{(k)(l)}$ can be eliminated from the string dynamics by the transition from Poisson brackets to Dirac ones (see Ref. 50):

$$[F,G]_D \equiv [F,G]_P + \tfrac{1}{4}\int d\sigma[F,\Xi^{(k)(l)}(\sigma)]_P[\Sigma_{(k)(l)}(\sigma),G]_P - \tfrac{1}{4}\int d\sigma[F,\Sigma_{(k)(l)}(\sigma)]_P[\Xi^{(k)(l)}(\sigma),G]_P. \tag{2.48}$$

The momentum variables remaining after the elimination of $\Sigma_{(k)(l)}$ are the covariant momentum densities $\Pi_{(k)(l)}$. Thus, it is important to express the Dirac brackets (2.48) in terms of $\Pi_{(k)(l)}$. With this in mind, let us discuss the change of variables from the momentum densities $P^m_{(l)}$ to $\Pi_{(k)(l)}$ and $\Sigma_{(k)(l)}$. It is based on the obvious relation

$$P^{(l)m} = (n^{-1})^m_{(r)}n^{(r)}_{m'}P^{m'(l)} = (n^{-1})^m_{(r)}P^{(r)(l)} = \tfrac{1}{2}(n^{-1})^m_{(r)}\Pi^{(r)(l)} + \tfrac{1}{2}(n^{-1})^m_{(r)}\Sigma^{(r)(l)}. \tag{2.49}$$

Using (2.49), we find that

$$\delta/\delta\Pi^{(r)(l)}(\sigma) = \int d\sigma'\,\delta P^{(s)m}(\sigma')/\delta\Pi^{(r)(l)}(\sigma)\,\delta/\delta P^{(s)m}(\sigma') = \tfrac{1}{2}(n^{-1})_{m(r)}\delta/\delta P^{(l)}_m - \tfrac{1}{2}(n^{-1})_{m(l)}\delta/\delta P^{(r)}_m \tag{2.50}$$

$$\delta/\delta\Sigma^{(r)(l)}(\sigma) = \int d\sigma'\,\delta P^{(s)}_m(\sigma')/\delta\Sigma^{(r)(l)}(\sigma)\,\delta/\delta P^{(s)}_m(\sigma') = \tfrac{1}{2}(n^{-1})_{m(r)}\delta/\delta P^{(l)}_m + \tfrac{1}{2}(n^{-1})_{m(l)}\delta/\delta P^{(r)}_m \tag{2.51}$$

and, consequently,

$$\delta/\delta P^{(l)m} = n^{(r)}_m(\delta/\delta\Pi^{(r)(l)} + \delta/\delta\Sigma^{(r)(l)}). \tag{2.52}$$

Using the representation (2.52), the change of the momentum densities can be made in the Poisson and Dirac brackets. Then Eq. (2.48) can be represented in the form

$$[F,G]_D = \int d\sigma(\delta F/\delta P_m(\sigma)\delta G/\delta x^m(\sigma) - \delta F/\delta x^m(\sigma)\delta G/\delta P_m(\sigma)) + \int d\sigma(\delta F/\delta\Pi^{(r)(l)}(\sigma)\tilde{\Delta}^{(r)(l)}(\sigma)G - \tilde{\Delta}^{(r)(l)} \times(\sigma)F\ \delta G/\delta\Pi^{(r)(l)}(\sigma)) + \int d\sigma(\delta F/\delta\Sigma^{(r)(l)}(\sigma)\tilde{K}^{(r)(l)}(\sigma)G - \tilde{K}^{(r)(l)}(\sigma)F\ \delta G/\delta\Sigma^{(r)(l)})(\sigma)) + \tfrac{1}{4}\int d\sigma[F,\Xi^{(k)(l)}(\sigma)]_P[\Sigma_{(k)(l)}(\sigma),G]_P - \tfrac{1}{4}\int d\sigma[F,\Sigma_{(k)(l)}(\sigma)]_P[\Xi^{(k)(l)}(\sigma),G]_P, \tag{2.53}$$

where the variational covariant derivatives $\tilde{\Delta}$ are defined by Eq. (2.23).

Let us discuss some functionals $\tilde{F}$ and $\tilde{G}$ which are independent of the variables $\Sigma_{(k)(l)}(\sigma)$:

$$\tilde{F}\equiv\tilde{F}[x^m(\sigma),P_m(\sigma),n_m^{(l)}(\sigma),\Pi_{(k)(l)}(\sigma)], \qquad (2.54)$$

$$\tilde{G}\equiv\tilde{G}[x^m(\sigma),P_m(\sigma),n_m^{(l)}(\sigma),\Pi_{(k)(l)}(\sigma)].$$

The Dirac brackets (2.53) coincide with (the "covariant" version of) the Poisson ones on the class of such functions:

$$[\tilde{F},\tilde{G}]_D$$

$$=\int d\sigma(\delta\tilde{F}/\delta P_m(\sigma)\delta\tilde{G}/\delta x^m(\sigma)-\delta\tilde{F}/\delta x^m(\sigma)\delta\tilde{G}/\delta P_m(\sigma))$$

$$+\int d\sigma(\delta\tilde{F}/\delta\Pi^{(r)(l)}(\sigma)\tilde{\Delta}^{(r)(l)}(\sigma)\tilde{G}-\tilde{\Delta}^{(r)(l)}$$

$$\times(\sigma)\tilde{F}\delta\tilde{G}/\delta\Pi^{(r)(l)}(\sigma))=[\tilde{F},\tilde{G}]_P. \qquad (2.55)$$

Thus, the ordinary Poisson brackets (2.55), together with the strong relations (2.2), can be used for the function with the properties (2.54).

Therefore we need not include the orthonormality conditions (2.2) in the list of Hamiltonian constraints if the phase space includes only the covariant momentum densities $\Pi_{(k)(l)}(\sigma)$ (2.40) for the variables. Such momentum densities are characterized by the property

$$[\Pi_{(k)(l)}(\sigma),\ \Xi^{(k)(l)}(\sigma')]_P|_{\Xi=0}=0. \qquad (2.56)$$

A similar prescription will be used below to investigate the Hamiltonian mechanics for the twistor-like superstring formulation.[22,23] It makes it possible to take into account the complicated harmonicity conditions (1.10) and (1.11) as "strong" relations and to eliminate them from the list of Hamiltonian constraints.

To clarify the nature of the covariant momentum densities $\Pi_{(k)(l)}(\sigma)$ we shall prove that they can be defined as derivatives of the Lagrangian density with respect to the τ components of the Cartan differential form (2.22):

$$\Pi_{(l)(k)}(\sigma)=-\partial L/\partial\Omega_\tau^{(k)(l)}(\sigma). \qquad (2.57)$$

The components $\Omega_\mu^{(k)(l)}=(\Omega_\tau^{(k)(l)},\ \Omega_\sigma^{(k)(l)})\equiv\Omega^{(k)(l)}(\partial_\mu)$ of the Cartan differential form $\Omega^{(k)(l)}(d)$ (2.22) with respect to the holonomic basis $d\xi^\mu=(d\tau,d\sigma)$ are defined by the relation

$$\Omega^{(k)(l)}(d)=n_m^{(k)}\delta n^{m(l)}=d\xi^\mu\Omega_\mu^{(k)(l)}=d\tau\Omega_\tau^{(k)(l)}$$

$$+d\sigma\ \Omega_\sigma^{(k)(l)}. \qquad (2.58)$$

Indeed, using the completeness of the set of differential forms $\tilde{\Omega}^{(k)(l)}(\delta)$ and $S^{(k)(l)}(\delta)$ (2.17), we can decompose the derivative with respect to $\partial_\tau n_m^{(l)}$ as follows:

$$\partial/\partial(\partial_\tau n_m^{(l)})=\tfrac{1}{2}\partial\tilde{\Omega}_\tau^{(k)(l)}/\partial(\partial_\tau n_m^{(l)})\partial/\partial\tilde{\Omega}_\tau^{(k)(l)}$$

$$+\tfrac{1}{2}\partial S_\tau^{(k)(l)}/\partial(\partial_\tau n_m^{(l)})\partial/\partial S_\tau^{(k)(l)}. \qquad (2.59)$$

Multiplying Eq. (2.59) by $n_m^{(s)}$ and taking the antisymmetric part of the resulting expression, we get the relation

$$\partial/\partial\tilde{\Omega}_\tau^{(s)(l)}=n_{m(s)}\partial/\partial(\partial_\tau n_m^{(l)})-n_{m(l)}\partial/\partial(\partial_\tau n_m^{(s)}). \qquad (2.60)$$

Now the expression (2.57) coincides with (2.40) [see also (2.35)], and we conclude that the covariant momentum density characterized by the property (2.56) is defined by the expression

$$\Pi_{(l)(k)}=-\partial L/\partial\Omega_\tau^{(k)(l)}(\sigma)=n_{m(l)}P_{(k)}^m-n_{m(k)}P_{(l)}^m. \qquad (2.61)$$

Finally, we note that the covariant momentum density (2.61) is the "classical analog" for the variational covariant derivative (2.23). This statement means that the Poisson bracket of $\Pi_{(k)(l)}$ with any admissible functional defined on the configuration space (2.33) coincides with the action of the variational covariant derivative (2.23) on the same functional:

$$[\Pi^{(l)(k)}(\sigma),\ \tilde{F}[x^m,n_m^{(r)}]]_P=\tilde{\Delta}^{(l)(k)}(\sigma)\tilde{F}[x^m,n_m^{(r)}]. \qquad (2.62)$$

The properties (2.56), (2.61), and (2.62) of the covariant momentum density $\Pi_{(k)(l)}$ will help us to find the corresponding variable for the twistor-like superstring formulation[22,23] and, thus, to simplify the investigation of its Hamiltonian mechanics (see Sec. 4).

3. D=10 SUPERSTRING IN THE TWISTOR-LIKE LORENTZ-HARMONIC FORMULATION

3.1. Action functional

The twistor-like action functional for the $D=10$, $N=$IIB superstring has the form[22,23]

$$S=S_1+S_{W-Z}, \qquad (3.1)$$

$$S_1=\int d^2\xi e(\xi)(-(\alpha')^{-1/2}e_f^\mu\omega_\mu^m n_m^f+c)=\int d\tau d\sigma e$$

$$\times[-(\alpha')^{-1/2}(e^{\mu[+2]}u_m^{[-2]}+e^{\mu[-2]}u_m^{[+2]})\omega_\mu^m+c]$$

$$\equiv\int d\tau d\sigma e(c+\tfrac{1}{16}(\alpha')^{-1/2}e^{\mu[+2]}\omega_\mu^m(\nu_{\dot{A}}^-\tilde{\sigma}_m\nu_{\dot{A}}^-)$$

$$+\tfrac{1}{16}(\alpha')^{-1/2}e^{\mu[-2]}\omega_\mu^m(\nu_{\dot{A}}^+\tilde{\sigma}_m\nu_{\dot{A}}^+)), \qquad (3.1a)$$

$$S_{W-Z}\equiv-(c\alpha')^{-1}\int d\tau d\sigma\varepsilon^{\mu\nu}[i\omega_\mu^m(\partial_\nu\theta^1\sigma_m\theta^1$$

$$-\partial_\nu\theta^2\sigma_m\theta^2)+\partial_\mu\theta^1\sigma^m\theta^1\partial_\nu\theta^2\sigma_m\theta^2]. \qquad (3.1b)$$

Here

$$\omega_\mu^m=\partial_\mu x^m-i(\partial_\mu\theta^1\sigma^m\theta^1+\partial_\mu\theta^2\sigma^m\theta^2)\equiv\partial_\mu x^m$$

$$-i(\partial_\mu\theta^{\alpha 1}\sigma^m{}_{\alpha\beta}\theta^{\beta 1}+\partial_\mu\theta^{\alpha 2}\sigma^m{}_{\alpha\beta}\theta^{\beta 2}) \qquad (3.2)$$

are the coefficients of the pullback of the D=10, N=2B supersymmetric Cartan form[51] on the world-sheet,

$$\omega^m=dx^m-i(d\theta^1\sigma^m\theta^1+d\theta^2\sigma^m\theta^2)=d\xi^\mu\omega_\mu^m,$$

x^m (m=0,1,...,9) are the ordinary (flat) space-time coordinates, and $\theta^{\alpha l}=(\theta^{\alpha 1},\theta^{\alpha 2})$ (α=1,...,16) are the fermionic (Grassmannian) coordinates of the D=10, N=2B superspace which have the properties of Majorana–Weyl spinors with respect to the SO(1,9) group; $\sigma^m{}_{\alpha\beta}$ are the symmetric 16×16 Pauli matrices for $D=10$ space-time (see Refs. 14 and 15 for the notation). The conventions for the world-sheet zweibeins $e_\mu^{[\pm 2]}$, $e^{\mu[\pm 2]}$ are collected in Eqs. (2.4).

The action (3.1) differs from the trivial supersymmetrization ($\partial_\mu x^m \Rightarrow \omega_\mu^m$) of the (moving-frame) bosonic-string formulation (2.1) by:

i) addition of the Wess–Zumino term (3.1b);

ii) replacement of the fundamental moving-frame vectors $n_m^{(l)}$ (2.5) by the compound ones $u_m^{(l)}$ (1.8), (1.9) formed from the bosonic spinor variables (1.1):

$$n_m^{(l)} \Rightarrow u_m^{(l)} \equiv \tfrac{1}{16}\mathrm{Sp}(v\tilde{\sigma}_m v^T \sigma^{(l)}) \equiv \tfrac{1}{16} v_\alpha^a \tilde{\sigma}_m^{\alpha\beta} v_\beta^b \sigma_{ab}^{(l)}, \quad (3.3)$$

$$v_\alpha^a = (\nu_{\alpha A}^+, \nu_{\alpha \dot{A}}^-) \in \mathrm{Spin}(1,9). \quad (3.4)$$

The orthonormality conditions (1.2) for the composed set (3.3) are straightforward consequences of the relation (3.4). To arrive at the decomposition (1.8), (1.9) of the set of composed moving-frame vectors (3.3) the following σ-matrix representation must be used:

$$\sigma_{ab}^0 = \mathrm{diag}(\delta_{AB}, \delta_{\dot{A}\dot{B}}) = \tilde{\sigma}^{0ab},$$

$$\sigma_{ab}^9 = \mathrm{diag}(\delta_{AB}, -\delta_{\dot{A}\dot{B}}) = -\tilde{\sigma}^{9ab},$$

$$\sigma_{ab}^{(i)} = \begin{bmatrix} 0 & \gamma_{A\dot{B}}^i \\ \tilde{\gamma}_{\dot{A}B}^i & 0 \end{bmatrix} = -\tilde{\sigma}^{(i)ab}, \quad (3.5)$$

$$\sigma_{ab}^{[+2]} \equiv (\sigma^0 + \sigma^9)_{ab} = \mathrm{diag}(2\delta_{AB}, 0) = (\tilde{\sigma}^0 - \tilde{\sigma}^9)^{ab} = \tilde{\sigma}^{[-2]ab},$$

$$\sigma_{ab}^{[-2]} \equiv (\sigma^0 - \sigma^9)_{ab} = \mathrm{diag}(0, 2\delta_{\dot{A}\dot{B}}) = (\tilde{\sigma}^0 + \tilde{\sigma}^9)^{ab} = \tilde{\sigma}^{[+2]ab}.$$

In Eqs. (3.5), $\gamma_{A\dot{B}}^i$ are the σ matrices for the SO(8) group (see Ref. 3), and $\tilde{\gamma}_{\dot{A}B}^i \equiv \gamma_{B\dot{A}}^i$.

The presence of the Wess–Zumino term (3.1b) in the action (3.1) leads to invariance of this action under the κ-symmetry transformations, whose explicit form was presented in Refs. 23 and 46. There are also the obvious reparametrization symmetry and the gauge symmetry under the right product of the SO(8) and SO(1,1) groups.

The SO(8) gauge symmetry transformations result in arbitrary rotations of the eight spacelike composed vectors $u_m^{(i)}$ [see Eq. (1.8)] among themselves, and the SO(1,1) ones result in pseudorotations of the vectors $u_m^{\{f\}}$. To achieve invariance of the action functional (3.1), they must be identified with the world-sheet Lorentz-group transformations acting on the "flat" indices of the zweibeins e_μ^f [see Eq. (2.4)].

The relations (3.4) together with the gauge symmetry under the right product of the SO(1,1) and SO(8) groups permit us to identify the space of harmonic variables $\{(\nu_{\alpha A}^-, \nu_{\alpha\dot{A}}^+)\}$ with the coset space SO(1,9)/[SO(1,1)⊗SO(8)].[22,23,46] We stress that the so-called "boost" symmetry is absent in the superstring formulation (3.1), in contrast to the formulations of the D=10 Green–Schwarz heterotic superstring presented in Refs. 38 and 39. The reason for this difference will be discussed below.

3.2. Harmonic variables, composed moving-frame vectors, and admissible variations

The relation (3.4) is realized by the requirement that the variables $v_\alpha^a = (\nu_{\alpha A}^-, \nu_{\alpha\dot{A}}^+)$ must satisfy the harmonicity conditions (1.10):[20,21,46]

$$\Xi_{m_1 \ldots m_4} = u^{m(n)} \eta_{(n)(l)} \Xi_{m_1 \ldots m_4 m}^{(l)} \equiv u^{m(n)} \eta_{(n)(l)} \mathrm{Sp}(v\tilde{\sigma}_{m_1 \ldots m_4 m} v^T \sigma^{(n)}) = 0, \quad (3.6a)$$

$$\Xi_0 \equiv u_m^{[-2]} u^{m[+2]} - 2 \equiv \tfrac{1}{8}(\nu_{\dot{A}}^- \tilde{\sigma}_m \nu_{\dot{A}}^-) \tfrac{1}{8}(\nu_A^+ \tilde{\sigma}_m \nu_A^+) - 2 = 0. \quad (3.6b)$$

(We stress that the equality[20,21]

$$\Xi_{m_1 \ldots m_5}^{(n)} \equiv \mathrm{Sp}(v\tilde{\sigma}_{m_1 \ldots m_k} v^T \sigma^{(n)}) = 0$$

results from Eqs. (3.6a); see Ref. 46.)

It is easy to see that Eqs. (3.6) kill the 210+1=211 degrees of freedom and reduce the numbers of independent variables included in v_α^a to 45=256−211=dimSO(1,9). The equivalence of the restrictions (3.6) to the relation (3.4) was discussed in detail in Ref. 46.

It is necessary to introduce the inverse harmonic matrix

$$(v^{-1})_a^\alpha \equiv (v^{-1})_a^\alpha \equiv (\nu_A^{-\alpha}, \nu_{\dot{A}}^{+\alpha})^T. \quad (3.7)$$

In contrast to the case of $D=4$ (Refs. 18, 19, 42, and 43), its elements cannot be expressed in terms of the harmonic variables $\nu_{\alpha A}^+$, $\nu_{\alpha\dot{A}}^-$ in a simple and covariant way. This is explained by the impossibility of transforming the subscript $D=10$ Majorana–Weyl spinor index into the superscript one, since they describe representations with different chiralities. Therefore it is convenient to consider the 256 variables $\nu_A^{-\alpha}$, $\nu_{\dot{A}}^{+\alpha}$ as independent harmonics and to supplement the set of harmonicity conditions by the 256 relations for the inverses of the matrices $(v^{-1})_a^\alpha$ and v_α^a:

$$(v^{-1})_a^\alpha v_\alpha^b = \delta_a^b:$$

$$\Xi_{AB}^{[0]} \equiv \nu_A^{-\alpha} \nu_{\alpha B}^+ - \delta_{AB} = 0, \quad \Xi_{A\dot{B}}^{[-2]} \equiv \nu_A^{-\alpha} \nu_{\alpha\dot{B}}^- = 0, \quad (3.8)$$

$$\Xi_{\dot{A}B}^{[+2]} \equiv \nu_{\dot{A}}^{+\alpha} \nu_{\alpha B}^+ = 0, \quad \Xi_{\dot{A}\dot{B}}^{[0]} \equiv \nu_{\dot{A}}^{+\alpha} \nu_{\alpha\dot{B}}^- - \delta_{\dot{A}\dot{B}} = 0$$

(256−256=0, so that additional degrees of freedom are not included in the theory).

Note that the distinction in the SO(1,1) weights ± for the same SO(8) [(s) or (c) spinor] index structure will help us to distinguish the harmonics $\nu_{\alpha A}^-$, $\nu_{\alpha\dot{A}}^+$ in (3.4) from $\nu_A^{-\alpha}$, $\nu_{\dot{A}}^{+\alpha}$ in (3.7) in the expressions where the SO(1,9) spinor indices are contracted and omitted [see, for example, Eqs. (3.11)].

It is easy to prove that the composed vectors $u_m^{(n)}$ in (3.3) can be expressed in terms of the inverse harmonic matrix (3.7) as well as the ordinary one (3.4) [see Eqs. (1.9)]:

$$u_m^{(l)} \equiv \tfrac{1}{16}\mathrm{Sp}(v\tilde{\sigma}_m v^T \sigma^{(l)}) = \tfrac{1}{16}\mathrm{Sp}(v^{-1}\sigma_m (v^{-1})^T \tilde{\sigma}^{(l)}). \quad (3.9)$$

In terms of the harmonic variables $\nu_{\alpha A}^-$, $\nu_{\alpha\dot{A}}^+$ and $\nu_A^{-\alpha}$, $\nu_{\dot{A}}^{+\alpha}$, Eqs. (3.9) can be specified as follows [see Eqs. (1.8) and (1.9), and the σ-matrix representation (3.5)]:

$$u_{\rm m}^{(\rm l)}=(u_{\rm m}^{\{\rm f\}},u_{\rm m}^{(\rm i)})\equiv(\tfrac{1}{2}(u_{\rm m}^{[+2]}+u_{\rm m}^{[-2]}),\quad u_{\rm m}^{(\rm i)}),$$
$$\tfrac{1}{2}(u_{\rm m}^{[+2]}-u_{\rm m}^{[-2]})),\tag{3.10}$$
$$u_{\rm m}^{[+2]}=\tfrac{1}{8}(\nu_{\rm A}^{+}\tilde{\sigma}_{\rm m}\nu_{\rm A}^{+})=\tfrac{1}{8}(\nu_{\dot{\rm A}}^{+}\sigma_{\rm m}\nu_{\dot{\rm A}}^{+})\equiv\tfrac{1}{8}\nu_{\dot{\rm A}}^{+\alpha}\sigma_{{\rm m}\alpha\beta}\nu_{\dot{\rm A}}^{+\beta},\tag{3.11a}$$
$$u_{\rm m}^{[-2]}=\tfrac{1}{8}(\nu_{\dot{\rm A}}^{-}\tilde{\sigma}_{\rm m}\nu_{\dot{\rm A}}^{-})=\tfrac{1}{8}(\nu_{\rm A}^{-}\sigma_{\rm m}\nu_{\rm A}^{-})\equiv\tfrac{1}{8}\nu_{\rm A}^{-\alpha}\sigma_{{\rm m}\alpha\beta}\nu_{\rm A}^{-\beta},\tag{3.11b}$$
$$u_{\rm m}^{(\rm i)}=\tfrac{1}{8}(\nu_{\rm A}^{+}\tilde{\sigma}_{\rm m}\nu_{\dot{\rm A}}^{-})\gamma_{\rm A\dot{A}}^{\rm j}=-\tfrac{1}{8}(\nu_{\rm A}^{-}\sigma_{\rm m}\nu_{\dot{\rm A}}^{+})\gamma_{\rm A\dot{A}}^{\rm j}.\tag{3.11c}$$

The orthonormality conditions (1.2) can be specified as follows:

$$u_{\rm m}^{(\rm n)}u^{\rm m(k)}=\eta^{(\rm n)(\rm k)}={\rm diag}(1,-1,\ldots,-1):\tag{3.12}$$
$$u_{\rm m}^{[+2]}u^{\rm m[+2]}=0,\quad u_{\rm m}^{[-2]}u^{\rm m[-2]}=0,\tag{3.12a,b}$$
$$u_{\rm m}^{[\pm2]}u^{\rm m(i)}=0,\tag{3.12c}$$
$$u_{\rm m}^{[+2]}u^{\rm m[-2]}=2,\quad u_{\rm m}^{(\rm i)}u^{\rm m(j)}=-\delta^{(\rm i)(\rm j)}.\tag{3.12d,e}$$

To justify them explicitly the identity (1.12) and the consequences (1.4) of the harmonicity conditions (3.6) and (3.8) must be used (see Ref. 46 for details). For the studied D=10 superstring case the relations (1.4) can be specified as follows:

$$u_{\rm m}^{(\rm l)}\sigma_{\alpha\beta}^{\rm m}={\rm v}_{\alpha}^{\rm a}\sigma_{\rm ab}^{(\rm l)}{\rm v}_{\beta}^{\rm b}:\tag{3.13}$$
$$u_{\rm m}^{[+2]}\sigma_{\alpha\beta}^{\rm m}=2\nu_{\alpha\rm A}^{+}\nu_{\beta\rm A}^{+},\tag{3.13a}$$
$$u_{\rm m}^{[-2]}\sigma_{\alpha\beta}^{\rm m}=2\nu_{\alpha\dot{\rm A}}^{-}\nu_{\beta\dot{\rm A}}^{-},\tag{3.13b}$$
$$u_{\rm m}^{(\rm i)}\sigma_{\alpha\beta}^{\rm m}=(\nu_{\alpha\rm A}^{+}\nu_{\beta\dot{\rm A}}^{-}+\nu_{\beta\rm A}^{+}\nu_{\alpha\dot{\rm A}}^{-})\gamma_{\rm A\dot{A}}^{\rm j},\tag{3.13c}$$
$$u_{\rm m}^{(\rm l)}\tilde{\sigma}^{{\rm m}\alpha\beta}=({\rm v}^{-1})_{\rm a}^{\alpha}\tilde{\sigma}^{(\rm l)\rm ab}({\rm v}^{-1})_{\rm b}^{\beta}:\tag{3.14}$$
$$u_{\rm m}^{[+2]}\tilde{\sigma}^{{\rm m}\alpha\beta}=2\nu_{\dot{\rm A}}^{+\alpha}\nu_{\dot{\rm A}}^{+\beta},\tag{3.14a}$$
$$u_{\rm m}^{[-2]}\tilde{\sigma}^{{\rm m}\alpha\beta}=2\nu_{\rm A}^{-\alpha}\nu_{\rm A}^{-\beta},\tag{3.14b}$$
$$u_{\rm m}^{(\rm i)}\tilde{\sigma}^{{\rm m}\alpha\beta}=-(\nu_{\rm A}^{-\alpha}\nu_{\dot{\rm A}}^{+\beta}+\nu_{\rm A}^{-\beta}\nu_{\dot{\rm A}}^{+\alpha})\gamma_{\rm A\dot{A}}^{\rm j},\tag{3.14c}$$
$$u_{\rm m}^{(\rm l)}\tilde{\sigma}_{(\rm l)}^{\rm ab}={\rm v}_{\alpha}^{\rm a}\tilde{\sigma}_{\rm m}^{\alpha\beta}{\rm v}_{\beta}^{\rm b}:\tag{3.15}$$
$$u_{\rm m}^{[+2]}\delta_{\rm AB}=(\nu_{\rm A}^{+}\tilde{\sigma}_{\rm m}\nu_{\rm B}^{+}),\tag{3.15a}$$
$$u_{\rm m}^{[-2]}\delta_{\dot{\rm A}\dot{\rm B}}=(\nu_{\dot{\rm A}}^{-}\tilde{\sigma}_{\rm m}\nu_{\dot{\rm B}}^{-}),\tag{3.15b}$$
$$u_{\rm m}^{(\rm i)}\gamma_{\rm A\dot{B}}^{\rm i}=(\nu_{\rm A}^{+}\tilde{\sigma}_{\rm m}\nu_{\dot{\rm B}}^{-}),\tag{3.15c}$$
$$u_{\rm m}^{(\rm l)}\sigma_{(\rm l)\rm ab}=({\rm v}^{-1})_{\rm a}^{\alpha}\sigma_{{\rm m}\alpha\beta}({\rm v}^{-1})_{\rm b}^{\beta}:\tag{3.16}$$
$$u_{\rm m}^{[-2]}\delta_{\rm AB}=(\nu_{\rm A}^{-}\sigma_{\rm m}\nu_{\rm B}^{-}),\tag{3.16a}$$
$$u_{\rm m}^{[+2]}\delta_{\dot{\rm A}\dot{\rm B}}=(\nu_{\dot{\rm A}}^{+}\sigma_{\rm m}\nu_{\dot{\rm B}}^{+}),\tag{3.16b}$$
$$u_{\rm m}^{(\rm i)}\gamma_{\rm A\dot{B}}^{\rm i}=-(\nu_{\rm A}^{-}\sigma_{\rm m}\nu_{\dot{\rm B}}^{+}).\tag{3.16c}$$

For the derivation of the equations of motion let us consider the concept of an admissible variation for the case of spinor harmonic variables. This is a variation which does not destroy the harmonicity conditions (3.6) and (3.8) [or, equivalently, the relation (3.4)]. Such a variation was discussed in detail for the case of the fundamental variables in Sec. 2 [see Eqs. (2.14a)–(2.14d)]. Thus, we can omit some obvious steps in the discussion of the spinor harmonic case.

An arbitrary variation of the variables ${\rm v}_{\alpha}^{\rm a}$ and $({\rm v}^{-1})_{\rm a}^{\alpha}\equiv{\rm v}_{\rm a}^{\alpha}$ of the type

$$\delta=\delta{\rm v}_{\alpha}^{\rm a}\frac{\partial}{\partial{\rm v}_{\alpha}^{\rm a}}+\delta{\rm v}_{\rm a}^{\alpha}\frac{\partial}{\partial{\rm v}_{\rm a}^{\alpha}},\tag{3.17}$$

can be written in the form

$$\delta=\left({\rm v}^{-1}\delta{\rm v}\right)_{\rm a}^{\rm b}\left({\rm v}_{\alpha}^{\rm a}\frac{\partial}{\partial{\rm v}_{\alpha}^{\rm b}}-{\rm v}_{\rm b}^{\alpha}\frac{\partial}{\partial{\rm v}_{\rm a}^{\alpha}}\right),\tag{3.18}$$

where the conditions (3.8) were used explicitly. To specify Eq. (3.18) we use Eq. (3.11), the consequences (3.13)–(3.16) of the harmonicity conditions, and the known identities (see, for example, Refs. 14 and 15)

$$S_{\alpha\beta}\equiv S_{\{\alpha\beta\}}=\tfrac{1}{16}\sigma_{\alpha\beta}^{\rm m}{\rm Sp}(\tilde{\sigma}_{\rm m}S)+\tfrac{1}{5!16}(\sigma^{\rm m_1\ldots m_5})_{\alpha\beta}{\rm Sp}(\tilde{\sigma}_{\rm m_1\ldots m_5}S),\tag{3.19a}$$

$$A_{\alpha\beta}\equiv A_{[\alpha\beta]}=-\tfrac{1}{3!16}(\sigma^{\rm m_1m_2m_3})_{\alpha\beta}{\rm Sp}(\tilde{\sigma}_{\rm m_1m_2m_3}A),\tag{3.19b}$$

$$F_{\alpha}^{\beta}=\tfrac{1}{16}\delta_{\alpha}^{\beta}{\rm Sp}(F)-\tfrac{1}{32}(\sigma^{\rm m_1m_2})_{\alpha}^{\beta}{\rm Sp}(\sigma_{\rm m_1m_2}F)$$
$$+\tfrac{1}{4!16}(\sigma^{m_1\ldots m_4})_{\alpha\beta}Sp(\sigma_{m_1\ldots m_4}F).\tag{3.19c}$$

Indeed, by varying Eq. (3.15) in the form

$$\delta u_{\rm m}^{(\rm l)}\tilde{\sigma}_{(\rm l)}^{\rm ab}=2(\delta{\rm v}^{\rm a}\tilde{\sigma}_{\rm m}{\rm v}^{\rm b})$$
$$=({\rm v}^{-1}\delta{\rm v})_{\rm c}^{\rm a}\tilde{\sigma}_{(\rm n)}^{\rm cb}u_{\rm m}^{(\rm n)}+({\rm v}^{-1}\delta{\rm v})_{\rm c}^{\rm b}\tilde{\sigma}_{(\rm n)}^{\rm ca}u_{\rm m}^{(\rm n)})\tag{3.20}$$

and contracting the result (3.20) with the 10×16 matrix $(u^{\rm m(k)}\sigma_{(\rm k)\rm bd})$, we obtain

$$u^{\rm m(k)}\delta u_{\rm m}^{(\rm l)}(\sigma_{(\rm k)}\tilde{\sigma}_{(\rm l)})_{\rm d}^{\rm a}=10({\rm v}^{-1}\delta{\rm v})_{\rm d}^{\rm a}+(\tilde{\sigma}_{(\rm k)}{\rm v}^{-1}\delta{\rm v}\sigma^{(\rm k)}).\tag{3.21}$$

It is easy to see that the left-hand side of Eq. (3.21) can be represented in the form $u^{\rm m(k)}\delta u_{\rm m}^{(\rm l)}(\sigma_{(\rm k)(\rm l)})$. This results from the vanishing of the expression $u^{\rm m(k)}\delta u_{\rm m}^{(\rm l)}\eta_{(\rm k)(\rm l)}=1/2\delta\Xi^{(\rm k)(\rm l)}\eta_{(\rm k)(\rm l)}=0$, which is a consequence of the orthonormality conditions (3.12). The right-hand side of Eq. (3.21) can be transformed using the identities (3.19c) for ${\rm v}^{-1}\delta{\rm v}$ and the relations

$$\tilde{\sigma}^{(\rm k)}\sigma_{\rm m_1\ldots m_{2r}}\sigma_{(\rm k}=(10-4{\rm r})\tilde{\sigma}_{\rm m_1\ldots m_{2r}}$$
$$=(-1)^{\rm r}(10-4{\rm r})\sigma_{\rm m_1\ldots m_{2r}}.\tag{3.22}$$

Thus, we derive from Eq. (3.21) the relation

$$u^{\rm m(k)}\delta u_{\rm m}^{(\rm l)}(\sigma_{(\rm k)(\rm l)})$$
$$=10({\rm v}^{-1}\delta{\rm v})+\tfrac{1}{4}(\sigma^{\rm m_1m_2}){\rm Sp}(\sigma_{\rm m_1m_2}{\rm v}^{-1}\delta{\rm v})$$
$$+\tfrac{1}{64}(\sigma^{\rm m_1\ldots m_4}){\rm Sp}(\sigma_{\rm m_1\ldots m_4}{\rm v}^{-1}\delta{\rm v}).\tag{3.23}$$

Contracting Eq. (3.23) with the matrices I, $\sigma_{(\rm k)(\rm l)}$, and $\sigma_{\rm m_1\ldots m_4}$, we obtain the relations

$${\rm Sp}({\rm v}^{-1}\delta{\rm v})=0\Leftrightarrow\nu_{\rm A}^{-}\delta\nu_{\rm A}^{+}=-\nu_{\dot{\rm A}}^{+}\delta\nu_{\dot{\rm A}}^{-},\tag{3.24}$$

$$-\tfrac{1}{8}{\rm Sp}({\rm v}^{-1}\delta{\rm v}\sigma^{(\rm k)(\rm l)})=u^{\rm m(k)}\delta u_{\rm m}^{(\rm l)}=\Omega^{(\rm k)(\rm l)}(\delta),\tag{3.25}$$

$$\mathrm{Sp}(\mathrm{v}^{-1}\delta\mathrm{v}\sigma_{\mathrm{m}_1\ldots\mathrm{m}_4})=0, \tag{3.26}$$

which are straightforward consequences of the harmonicity conditions (3.6) and (3.8). Taking into account Eqs. (3.24)–(3.26), it is easy to derive from the identity (3.19c) the following expression for $\mathrm{v}^{-1}\delta\mathrm{v}$:

$$(\mathrm{v}^{-1}\delta\mathrm{v})=\tfrac{1}{4}\Omega^{(\mathrm{k})(\mathrm{l})}(\delta)(\sigma_{(\mathrm{k})(\mathrm{l})}). \tag{3.27}$$

Thus, the admissible variation which preserves the harmonicity conditions (3.6) and (3.8) has the form

$$\delta=\tfrac{1}{2}\,\Omega^{(\mathrm{k})(\mathrm{l})}(\delta)\Delta_{(\mathrm{l})(\mathrm{k})}, \tag{3.28}$$

which coincides with Eq. (2.14d). However, in Eq. (3.28) the SO(1,9) Cartan forms $\Omega^{(\mathrm{k})(\mathrm{l})}(\delta)$ are defined by Eq. (3.25) in terms of the spinor harmonic variables (3.4) and (3.7), and the covariant derivatives $\Delta_{(\mathrm{k})(\mathrm{l})}$ are defined as follows:

$$\Delta^{(\mathrm{l})(\mathrm{k})}\equiv\tfrac{1}{2}(\sigma^{(\mathrm{k})(\mathrm{l})})_{\mathrm{a}}^{\mathrm{b}}\left(\mathrm{v}_{\alpha}^{\mathrm{a}}\frac{\partial}{\partial\mathrm{v}_{\alpha}^{\mathrm{b}}}-\mathrm{v}_{\mathrm{b}}^{\alpha}\frac{\partial}{\partial\mathrm{v}_{\mathrm{a}}^{\alpha}}\right). \tag{3.29}$$

Taking into account the definition of the composed vectors (3.3) [or (3.9)], we can obtain the action of the covariant derivatives $\Delta_{(\mathrm{l}_1)(\mathrm{l}_2)}$:

$$\Delta_{(\mathrm{l}_1)(\mathrm{l}_2)}u_{(\mathrm{l})\mathrm{m}}=2\eta_{(\mathrm{l})[(\mathrm{l}_1)}u_{(\mathrm{l}_2)]\mathrm{m}}. \tag{3.30}$$

Equation (3.30) coincides with Eq. (2.19a). It can also be shown that the operators $\Delta_{(\mathrm{l}_1)(\mathrm{l}_2)}$ generate the Lorentz-group algebra (2.18a).

For the ensuing discussion it is useful to specify Eqs. (3.25), (3.28), and (3.29) as follows (the contracted spinor indices are omitted):

$$\delta=\tfrac{1}{2}\Omega^{(\mathrm{k})(\mathrm{l})}(\delta)\Delta_{(\mathrm{l})(\mathrm{k})}$$

$$=\Omega^{(0)}(\delta)\Delta^{(0)}+\Omega^{[\mp2]\mathrm{j}}(\delta)\Delta^{[\pm2]\mathrm{j}}-\tfrac{1}{2}\Omega^{(\mathrm{i})(\mathrm{j})}(\delta)\Delta^{(\mathrm{i})(\mathrm{j})}, \tag{3.31}$$

$$\Omega^{(\mathrm{k})(\mathrm{l})}(\delta)=(-2\Omega^{(0)}(\delta),\ \Omega^{[\mp2](\mathrm{j})}(\delta),\ \Omega^{(\mathrm{i})(\mathrm{j})}(\delta)): \tag{3.32}$$

$$\Omega^{(0)}(\delta)\equiv-\tfrac{1}{2}\Omega^{[+2][-2]}(\delta)=\tfrac{1}{2}u^{\mathrm{m}[-2]}\delta u_{\mathrm{m}}^{[+2]}$$

$$=-\tfrac{1}{2}u^{\mathrm{m}[+2]}\delta u_{\mathrm{m}}^{[-2]}=\tfrac{1}{4}(\nu_{\mathrm{A}}^{-}\delta\nu_{\mathrm{A}}^{+}-\nu_{\dot{\mathrm{A}}}^{+}\delta\nu_{\dot{\mathrm{A}}}^{-})=\tfrac{1}{2}\nu_{\mathrm{A}}^{-}\delta\nu_{\mathrm{A}}^{+}, \tag{3.32a}$$

$$\Omega^{[+2](\mathrm{i})}=u_{\mathrm{m}}^{[+2]}\delta u^{\mathrm{m}(\mathrm{i})}=\tfrac{1}{4}\nu_{\dot{\mathrm{A}}}^{+}\tilde{\gamma}_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\delta\nu_{\mathrm{A}}^{+}\equiv\tfrac{1}{4}\nu_{\dot{\mathrm{A}}}^{+\alpha}\tilde{\gamma}_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\delta\nu_{\alpha\mathrm{A}}^{+}, \tag{3.32b}$$

$$\Omega^{[-2](\mathrm{i})}=u_{\mathrm{m}}^{[-2]}\delta u^{\mathrm{m}(\mathrm{i})}=\tfrac{1}{4}\nu_{\mathrm{A}}^{-}\gamma_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\delta\nu_{\dot{\mathrm{A}}}^{-}, \tag{3.32c}$$

$$\Omega^{(\mathrm{i})(\mathrm{j})}=u^{\mathrm{m}(\mathrm{i})}du_{\mathrm{m}}^{(\mathrm{j})}=-\tfrac{1}{8}(\nu_{\mathrm{A}}^{-}\gamma_{\mathrm{AB}}^{\mathrm{ij}}\delta\nu_{\mathrm{B}}^{+}+\nu_{\dot{\mathrm{A}}}^{+}\tilde{\gamma}_{\dot{\mathrm{A}}\dot{\mathrm{B}}}^{\mathrm{ij}}\delta\nu_{\alpha\dot{\mathrm{B}}}^{-}), \tag{3.32d}$$

$$\Delta^{(\mathrm{l})(\mathrm{k})}=(-2\Delta^{(0)},\Delta^{[\mp2](\mathrm{j})},\Delta^{(\mathrm{i})(\mathrm{j})}): \tag{3.33}$$

$$\Delta^{(0)}\equiv-\tfrac{1}{2}\Delta^{[+2][-2]}$$

$$=\nu_{\mathrm{A}}^{+}\partial/\partial\nu_{\mathrm{A}}^{+}-\nu_{\dot{\mathrm{A}}}^{-}\partial/\partial\nu_{\dot{\mathrm{A}}}^{-}-\nu_{\mathrm{A}}^{-}\partial/\partial\nu_{\mathrm{A}}^{-}+\nu_{\dot{\mathrm{A}}}^{+}\partial/\partial\nu_{\dot{\mathrm{A}}}^{+}, \tag{3.34a}$$

$$\Delta^{[+2](\mathrm{i})}=\nu_{\mathrm{A}}^{+}\gamma_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\partial/\partial\nu_{\dot{\mathrm{A}}}^{-}-\nu_{\dot{\mathrm{A}}}^{+}\tilde{\gamma}_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\partial/\partial\nu_{\mathrm{A}}^{-}, \tag{3.34b}$$

$$\Delta^{[-2](\mathrm{i})}=\nu_{\dot{\mathrm{A}}}^{-}\tilde{\gamma}_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\partial/\partial\nu_{\mathrm{A}}^{+}-\nu_{\mathrm{A}}^{-}\gamma_{\mathrm{A}\dot{\mathrm{A}}}^{\mathrm{j}}\partial/\partial\nu_{\dot{\mathrm{A}}}^{+}, \tag{3.34c}$$

$$\Delta^{(\mathrm{i})(\mathrm{j})}=\tfrac{1}{2}(\nu_{\mathrm{A}}^{+}\gamma_{\mathrm{AB}}^{\mathrm{ij}}\partial/\partial\nu_{\mathrm{B}}^{+}+\nu_{\dot{\mathrm{A}}}^{-}\tilde{\gamma}_{\dot{\mathrm{A}}\dot{\mathrm{B}}}^{\mathrm{ij}}\partial/\partial\nu_{\dot{\mathrm{B}}}^{-}+\nu_{\mathrm{A}}^{-}\gamma_{\mathrm{AB}}^{\mathrm{ij}}\partial/\partial\nu_{\mathrm{B}}^{-}$$

$$+\nu_{\dot{\mathrm{A}}}^{+}\tilde{\gamma}_{\dot{\mathrm{A}}\dot{B}}^{ij}\partial/\partial\nu_{\dot{B}}^{+}, \tag{3.34d}$$

$$\Delta_{(\mathrm{l}_1)(\mathrm{l}_2)}u_{(\mathrm{l})\mathrm{m}}=-2\eta_{(\mathrm{l})[(\mathrm{l}_1)}u_{(\mathrm{l}_2)]\mathrm{m}}:$$

$$\Delta^{(0)}u_{\mathrm{m}}^{[\pm2]}=\pm2u_{\mathrm{m}}^{[\pm2]},\quad \Delta^{(0)}u_{\mathrm{m}}^{(\mathrm{j})}=0, \tag{3.35}$$

$$\Delta^{(\mathrm{i})(\mathrm{j})}u_{\mathrm{m}}^{[\pm2]}=0,\quad \Delta^{(\mathrm{i})(\mathrm{j})}u_{\mathrm{m}}^{(\mathrm{i}')}=-2\delta^{\mathrm{i}'[\mathrm{i}}u_{\mathrm{m}}^{(\mathrm{j}])}, \tag{3.36}$$

$$\Delta^{[+2](\mathrm{i})}u_{\mathrm{m}}^{[-2]}=2u_{\mathrm{m}}^{(\mathrm{i})},\quad \Delta^{[+2](\mathrm{i})}u_{\mathrm{m}}^{[+2]}=0,$$

$$\Delta^{[+2](\mathrm{i})}u_{\mathrm{m}}^{(\mathrm{j})}=\delta^{\mathrm{ij}}u_{\mathrm{m}}^{[+2]}, \tag{3.37}$$

$$\Delta^{[-2](\mathrm{i})}u_{\mathrm{m}}^{[-2]}=0,\quad \Delta^{[-2](\mathrm{i})}u_{\mathrm{m}}^{[+2]}=2u_{\mathrm{m}}^{(\mathrm{i})},$$

$$\Delta^{[-2](\mathrm{i})}u_{\mathrm{m}}^{(\mathrm{j})}=\delta^{\mathrm{ij}}u_{\mathrm{m}}^{[-2]}. \tag{3.38}$$

In the notation (3.33) the Lorentz-group algebra (2.18a) generated by the operators $\Delta_{(\mathrm{l})(\mathrm{k})}$ takes the form of the relations

$$[\Delta^{(0)},\Delta^{(\mathrm{i})(\mathrm{j})}]=0, \tag{3.39a}$$

$$[\Delta^{(\mathrm{i})(\mathrm{j})},\Delta^{(\mathrm{i}')(\mathrm{j}')}]=2\delta^{\mathrm{j}[\mathrm{i}'}\Delta^{(\mathrm{j}')]\mathrm{i}}-2\delta^{\mathrm{i}[\mathrm{i}'}\Delta^{(\mathrm{j}')]\mathrm{j}}, \tag{3.39b}$$

$$[\Delta^{(0)},\Delta^{[\mp2](\mathrm{j})}]=\mp2\Delta^{[\mp2](\mathrm{j})}, \tag{3.39c}$$

$$[\Delta^{(\mathrm{i})(\mathrm{j})},\Delta^{[\mp2](\mathrm{i}')}]=-\delta^{\mathrm{ii}'}\Delta^{[\mp2]\mathrm{j}}+\delta^{\mathrm{ji}'}\Delta^{[\mp2]\mathrm{i}}, \tag{3.39d}$$

$$[\Delta^{[+2](\mathrm{i})},\Delta^{[-2](\mathrm{j})}]=\delta^{\mathrm{ij}}\Delta^{(0)}+2\Delta^{(\mathrm{i})(\mathrm{j})}. \tag{3.39e}$$

The nature of the operators $\Delta^{(0)}$, $\Delta^{[\mp2](\mathrm{j})}$, and $\Delta^{(\mathrm{i})(\mathrm{j})}$ becomes evident from Eqs. (3.35)–(3.39). The operators $\Delta^{(\mathrm{i})(\mathrm{j})}$ and $\Delta^{(0)}$ generate the SO(8) and SO(1,1) transformations, respectively. The operators $\Delta^{[\mp2](\mathrm{j})}$ generate the transformations from the coset space SO(1,9)/SO(1,1)⊗SO(8). One of them can be associated with the "boost" symmetry appearing in the Lorentz-harmonic approach to superparticle theory.[20,21] But in the present formulation of $D=10$ superstring theory such symmetry is absent.[22,23]

Now we are ready to discuss the equations of motion for the $D=10$ superstring theory (3.1). The procedure is similar to the one discussed in Sec. 2.

3.3. Equations of motion

The equation of motion $\delta S/\delta e_{\mathrm{f}}^{\mu}=0$ gives the expression for the zweibein e_{μ}^{f} in terms of the imbedding functions $\mathrm{x}^{\mathrm{m}}(\xi)$, $\theta^{\alpha\mathrm{I}}(\xi)$ and the composed vectors $u_{\mathrm{m}}^{\{\mathrm{f}\}}$:

$$e_{\mu}^{[\pm2]}=\omega_{\mu}^{\mathrm{m}}u_{\mathrm{m}}^{[\pm2]}/\mathrm{c}(\alpha')^{1/2}. \tag{3.40}$$

This expression is similar to Eq. (2.6), since the zweibein variables are absent in the expression for the additional Wess–Zumino term (3.2).

In straightforward analogy with the bosonic-string formulation [see Sec. 2, Eq. (2.28)], the equations of motion for the harmonic variables $\nu_{\alpha\mathrm{A}}^{+}$, $\nu_{\alpha\dot{\mathrm{A}}}^{-}$ can be represented in the form

$$\sum_{\pm} ee^{\mu[\pm2]}\omega_{\mu}^{\mathrm{m}}\Delta_{(\mathrm{k})(\mathrm{l})}u_{\mathrm{m}}^{[\mp2]}=0 \tag{3.41}$$

using the admissible variations (3.28), which preserve the harmonicity conditions (3.6) and (3.8).

Equations (3.41) are satisfied identically when (k)=(i) and (l)=(j) [see Eq. (3.10)]. This results from the SO(8) gauge symmetry of the action (3.1) generated by the $\Delta^{(i)(j)}$ operators (3.33d). If (k)=[+2], (l)=[−2] (or vice versa), then Eqs. (3.41) reduce to the relation [see Eq. (3.37)]

$$\omega_\mu^{\rm m}(e^{\mu[-2]}u_{\rm m}^{[+2]}-e^{\mu[+2]}u_{\rm m}^{[-2]})=0. \qquad (3.42)$$

Taking into account Eq. (3.40), it can be shown that Eq. (3.42) is satisfied identically. This fact is associated with the SO(1,1) gauge symmetry of the theory. However, the generator of the corresponding symmetry includes terms acting on the zweibein fields in addition to the $\Delta^{(0)}$ operator (3.33a) (see Sec. 4).

Equations (3.41) lead to nontrivial results if and only if (k)=[±2], (l)=(i) (or vice versa). In this case they reduce to the equations [see Eqs. (3.37) and (3.38)]

$$ee^{\mu[\pm 2]}\omega_\mu^{\rm m}u_{\rm m}^{(i)}=0, \qquad (3.43)$$

which can be easily transformed into the form

$$\omega_\mu^{\rm m}u_{\rm m}^{(i)}=0 \qquad (3.44)$$

[cf. Eqs. (2.9) and (2.31)].

Taking into account Eqs. (3.40) and (3.44), and the completeness conditions [which follow from the orthonormality conditions (3.12)]

$$\delta_{\rm m}^{\rm n}=\tfrac{1}{2}u_{\rm m}^{[-2]}u^{{\rm n}[+2]}+\tfrac{1}{2}u_{\rm m}^{[+2]}u^{{\rm n}[-2]}-u_{\rm m}^{(i)}u^{{\rm n}(i)},$$

the coefficients $\omega_\mu^{\rm m}$ (3.2) of the ω-form pullback can be decomposed on the $u_{\rm m}^{[\pm 2]}$ light-like vectors

$$\omega_\mu^{\rm m}=\tfrac{1}{2}{\rm c}(\alpha')^{1/2}(e_\mu^{[-2]}u^{{\rm m}[+2]}+e_\mu^{[+2]}u^{{\rm m}[-2]}) \qquad (3.45)$$

and vice versa,

$$u^{{\rm m}[\pm 2]}=e^{\mu[\pm 2]}\omega_\mu^{\rm m}/{\rm c}(\alpha')^{1/2}. \qquad (3.46)$$

Thus, the vectors $u^{{\rm m}[\pm 2]}$ are tangent to the superstring worldsheet on the shell defined by the equations of motion. On the other hand, the vectors $u^{{\rm m}(i)}$ are orthogonal to the worldsheet on this shell.

Using Eqs. (3.40), (3.45), and (3.46), the classical equivalence of the present $D=10$ superstring formulation with the standard Green–Schwarz one[1] can be readily demonstrated. Substituting Eq. (3.46) into the functional (3.1) and using the definition of the world-sheet metric (2.4), we get the standard action functional[1] [cf. Eqs. (2.9)–(2.13)].

The equation of motion for the $x^{\rm m}(\xi)$ field, $\delta S/\delta x^{\rm m}(\xi)=0$, has the form

$$\partial_\mu\left(e\sum_\pm (e^{\mu[\pm 2]}u_{\rm m}^{[\mp 2]})\right)-\varepsilon^{\mu\nu}(\partial_\mu\theta^1\sigma_{\rm m}\partial_\nu\theta^1 -\partial_\mu\theta^2\sigma_{\rm m}\partial_\nu\theta^2)/{\rm c}(\alpha')^{1/2}=0, \qquad (3.47)$$

which is similar to Eq. (2.12), except for the last term containing Grassmannian degrees of freedom, and it can be easily reduced to the standard form[1]

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\omega_\nu^{\rm m})-\varepsilon^{\mu\nu}(\partial_\mu\theta^1\sigma^{\rm m}\partial_\nu\theta^1-\partial_\mu\theta^2\sigma^{\rm m}\partial_\nu\theta^2)=0, \qquad (3.48)$$

using Eq. (3.48) [see also Eq. (2.1)].

The equations $\delta S/\delta\theta^{\alpha I}(\xi)=0$ have the form

$$(\partial_\mu\theta^I\sigma^{\rm m})_\alpha\left(\sum_\pm e^{\mu[\pm 2]}u^{{\rm m}[\mp 2]}-2(-1)^I\varepsilon^{\mu\nu}\omega_\nu^{\rm m}\right)=0,$$
$$I=1,2, \qquad (3.49)$$

which can be reduced to the standard form[1]

$$(\partial_\mu\theta^I\sigma^{\rm m})_\alpha(\sqrt{-g}g^{\mu\nu}-(-1)^I\varepsilon^{\mu\nu})\omega_\nu^{\rm m}=0, \quad I=1,2 \qquad (3.50)$$

if Eqs. (3.46) are taken into account. However, it is interesting to use Eq. (3.45) and to eliminate the fields $\omega_\nu^{\rm m}$ from Eq. (3.49). We obtain the relations

$$e^{\mu[+2]}\partial_\mu\theta^{\alpha 1}\nu_{\alpha\dot{A}}^-=0, \qquad (3.51a)$$

$$e^{\mu[-2]}\partial_\mu\theta^{\alpha 2}\nu_{\alpha A}^+=0, \qquad (3.51b)$$

when Eqs. (3.13a) and (3.13b) are taken into account.

Therefore the equations of motion for the $D=10$, $N=$IIB superstring in the twistor-like formulation (3.1) have the form of Eqs. (3.40), (3.44)–(3.47), and (3.51). The relations (3.47) and (3.51) are equivalent to the standard equations of motion (3.48) and (3.50) (Ref. 1); however, they have a simpler form. Thus, the twistor-like formulation (3.1) is equivalent to the standard one[1] at the classical level[23] and simplifies the equations of motion substantially.

In the next sections the Hamiltonian formalism for the twistor-like $D=10$ superstring formulation (3.1) is worked out. This formalism is necessary for the covariant superstring quantization using the BFV–BFF scheme.[7]

4. HAMILTONIAN FORMALISM FOR THE D=10 SUPERSTRING IN THE TWISTOR-LIKE FORMULATION

To simplify the Hamiltonian formalism and to clarify the meaning of some constraints, let us reformulate the action principle (3.1) in terms of the zweibein densities

$$\rho_{\rm f}^\mu\equiv(\tfrac{1}{2}(\rho^{\mu[-2]}+\rho^{\mu[+2]}),\tfrac{1}{2}(\rho^{\mu[-2]}-\rho^{\mu[+2]}))$$
$$\equiv ee_{\rm f}^\mu/(\alpha')^{1/2} \qquad (4.1)$$

$$e\equiv\det(e_\mu^{\rm f})=\tfrac{1}{2}\alpha'\varepsilon_{\mu\nu}\rho^{\mu[-2]}\rho^{\nu[+2]},$$
$$(\varepsilon_{01}=-\varepsilon^{01}=-1), \qquad (4.2)$$

instead of the zweibeins $e_\mu^{\rm f}$, $e_{\rm f}^\mu$ themselves:

$$S=S_1+S_{W-Z}, \qquad (4.3)$$

$$S_1=-\tfrac{1}{2}\int d\tau d\sigma[(\rho^{\mu[+2]}u_{\rm m}^{[-2]} +\rho^{\mu[-2]}u_{\rm m}^{[+2]})\omega_\mu^{\rm m}+{\rm c}\ \alpha'\varepsilon_{\mu\nu}\rho^{\mu[+2]}\rho^{\nu[-2]}]$$

$$\equiv-\tfrac{1}{2}\int d\tau d\sigma[(\rho^{\mu[+2]}\tfrac{1}{8}(\nu_{\dot{A}}^-\tilde{\sigma}_{\rm m}\nu_{\dot{A}}^-) +\rho^{\mu[-2]}\tfrac{1}{8}(\nu_A^+\tilde{\sigma}_{\rm m}\nu_A^+))\omega_\mu^{\rm m} +{\rm c}\ \alpha'\varepsilon_{\mu\nu}\ \rho^{\mu[+2]}\rho^{\nu[-2]}], \qquad (4.3a)$$

$$S_{W-Z}\equiv-(c\alpha')^{-1}\int d\tau d\sigma\varepsilon^{\mu\nu}[i\omega_\mu^{m}(\partial_\nu\theta^1\sigma_m\theta^1 - \partial_\nu\theta^2\sigma_m\theta^2)+\partial_\mu\theta^1\sigma^m\theta^1\partial_\nu\theta^2\sigma_m\theta^2]. \quad (4.3b)$$

Here [see Eq. (3.2)]

$$\omega_\mu^{m}=\partial_\mu x^m-i(\partial_\mu\theta^1\sigma^m\theta^1+\partial_\mu\theta^2\sigma^m\theta^2).$$

Of course, the Wess–Zumino term (4.3b) is not modified. However, the twistor-like part of the action (4.3a) includes terms which depend linearly or bilinearly on the densities $\rho^{\mu[\pm2]}$. At the same time, their dependences on the inverse zweibein variables $e^{\mu[\pm2]}$ are more complicated [see Eq. (3.1b)].

4.1. Primary constraints and covariant momentum density

The canonical momentum densities

$$P_M\equiv(P_m,\pi_\alpha^1,\pi_\alpha^2,P_A^{-\alpha},P_{\dot A}^{+\alpha},P_{\alpha A}^{+},P_{\alpha\dot A}^{-},P_{(\rho)\mu}^{[\mp2]}) \equiv-(-1)^M\partial L/\partial(\partial_\tau z^M) \quad (4.4)$$

are conjugate to the configuration-space (target-space) coordinates of the superstring formulation (4.3),

$$z^M\equiv(x^m,\theta^{\alpha1},\theta^{\alpha2},v_{\alpha A}^{+},v_{\alpha\dot A}^{-},v_A^{-\alpha},v_{\dot A}^{\alpha},\rho^{[\pm2]\mu}), \quad (4.5)$$

with respect to the standard Poisson brackets

$$[z^M(\sigma),P_N(\sigma')\}_P=-(-1)^{MN}[P_N(\sigma'),z^M(\sigma)\}_P =-\delta_N^M\delta(\sigma-\sigma'). \quad (4.6)$$

Here the multiplier $(-1)^{MN}$ is equal to -1 if both indices M and N belong to the fermionic variables, and $+1$ otherwise.

The action functional (4.3) is the first-order one in the proper-time derivatives (i.e., in the velocities). Hence all the expressions (4.4) for the canonical momentum densities lead to primary constraints. For the nonharmonic variables such constraints are

$$\Phi_m(\sigma)\equiv P_m-\tfrac{1}{2}\rho^{\tau[+2]}u_m^{[-2]} -\tfrac{1}{2}\rho^{\tau[-2]}u_m^{[+2]}+\frac{i}{c\alpha'}\sum_I(-1)^I\partial_\sigma\theta^I\sigma_m\theta^I\approx0, \quad (4.7a)$$

$$D_\alpha^I(\sigma)\equiv-\pi_\alpha^I+i(\sigma^m\Theta^I)_\alpha\left[P_m-(-1)^I\frac{1}{\alpha'}(\partial_\sigma x_m -i\partial_\mu\theta^I\sigma_m\theta^I)\right]\approx0, \quad (4.7b)$$

$$P_{(\rho)\tau}^{[\pm2]}\approx0, \quad (4.7c)$$

$$P_{(\rho)\sigma}^{[\pm2]}\approx0. \quad (4.7d)$$

For the spinor harmonics

$$(v_{\alpha A}^{+},v_{\alpha\dot A}^{-})=v_\alpha^a, \quad (v_A^{-\alpha},v_{\dot A}^{+\alpha})^T=(v^{-1})_a^\alpha\equiv v_a^\alpha$$

the set of primary constraints consists of the completely trivial relations

$$P_a^\alpha\approx0:\ P_A^{-\alpha}\approx0,\quad P_{\dot A}^{+\alpha}\approx0, \quad (4.8a)$$

$$P_\alpha^a\approx0:\ P_{\alpha A}^{+}\approx0,\quad P_{\alpha\dot A}^{-}\approx0, \quad (4.8b)$$

(which reflect the auxiliary character of the harmonic variables in this formulation) and the harmonicity conditions (3.6) and (3.8), considered as "weak" relations:[50]

$$\Xi_{m_1\ldots m_4}=u^{m(n)}\eta_{(n)(l)}\Xi^{(l)}_{m_1\ldots m_4m} \equiv u^{m(n)}\eta_{(n)(l)}\mathrm{Sp}(v\tilde\sigma_{m_1\ldots m_4m}v\sigma^{(n)})^T\approx0, \quad (4.9a)$$

$$\Xi_0\equiv u_m^{[-2]}u^{m[+2]}-2\equiv\tfrac{1}{8}(v_{\dot A}^{-}\tilde\sigma_m v_{\dot A}^{-})\tfrac{1}{8}(v_A^{+}\tilde\sigma_m v_A^{+})-2\approx0, \quad (4.9b)$$

$$\Xi_{A\dot B}^{[0]}\equiv v_A^{-\alpha}v_{\alpha B}^{+}-\delta_{AB}\approx0,\quad \Xi_{A\dot B}^{[-2]}\equiv v_A^{-\alpha}v_{\alpha\dot B}^{-}\approx0,$$
$$\Xi_{\dot AB}^{[+2]}\equiv v_{\dot A}^{+\alpha}v_{\alpha B}^{+}\approx0,\quad \Xi_{\dot A\dot B}^{[0]}\equiv v_{\dot A}^{+\alpha}v_{\alpha\dot B}^{-}-\delta_{\dot A\dot B}\approx0. \quad (4.9c)$$

The expressions $\Xi^{(l)}_{m_1\ldots m_5}$ (Refs. 20 and 21) included in Eq. (4.9a) vanish as a result of Eq. (4.9a) (Ref. 46) and can be specified as follows:

$$\Xi^{[-2]}_{m_1\ldots m_5}=\tfrac{1}{8}v_{\alpha\dot A}^{-}\tilde\sigma^{\alpha\gamma}_{m_1\ldots m_5}v_{\gamma\dot A}^{-}\approx0, \quad (4.10a)$$

$$\Xi^{[+2]}_{m_1\ldots m_5}=\tfrac{1}{8}v_{\alpha A}^{+}\tilde\sigma^{\alpha\gamma}_{m_1\ldots m_5}v_{\gamma A}^{+}\approx0, \quad (4.10b)$$

$$\Xi^{(i)}_{m_1\ldots m_5}=\tfrac{1}{8}v_{\alpha A}^{+}\gamma^{(i)}_{A\dot A}\tilde\sigma^{\alpha\gamma}_{m_1\ldots m_5}v_{\gamma\dot A}^{-}\approx0. \quad (4.10c)$$

The relations (4.9) and (4.10) are complicated. Thus, it is evident that the computation of the constraint algebra is a hard task if Eqs. (4.9) are understood as a "weak" equality.

Hence, it is important to develop a method which allows us to eliminate the conditions (3.6) and (3.8) from the set of constraints and to discuss them as "strong" relations.[50] Such a method can be devised in a straightforward analogy with the one discussed in Sec. 2 for the case of the bosonic-string formulation.

This means that the concept of covariant momentum density must be used. Now we discuss it for the case of the twistor-like superstring formulation (4.3), using the experience gained in Sec. 2 (and hence omitting some technical details).

Let us recall some properties of the covariant momentum densities which were discussed in Sec. 2. First of all, they must have vanishing [in the weak sense; see Eq. (2.56)] Poisson brackets with the harmonicity conditions (3.6) and (3.8). On the other hand, it is known that the harmonicity conditions (3.6) and (3.8) are the realization of the relation (3.4).[20,21,46] The covariant momentum variables must be canonically conjugate to some parameters of the SO(1,9) group included in the spinor harmonics $(v_{\alpha A}^{+},v_{\alpha\dot A}^{-})=v_\alpha^a$, $(v_A^{-\alpha},v_{\dot A}^{+\alpha})\equiv v_a^\alpha$. Therefore, the covariant momentum densities must be associated with the Lorentz group too, and hence they must generate the SO(1,9)-group algebra on the Poisson brackets.

The other degrees of freedom included in the spinor harmonics $v_{\alpha A}^{+}$, $v_{\alpha\dot A}^{-}$, $v_A^{-\alpha}$, $v_{\dot A}^{+\alpha}$ are killed by the harmonicity conditions (3.6) and (3.8). Henceforth, the harmonic momentum degrees of freedom which cannot be reduced to covariant ones must be conjugate to the harmonicity conditions in the weak sense [see Eqs. (2.46) and (2.47) for the case of the bosonic-string formulation]. We can understand the condition of vanishing of these variables, together with the harmonicity conditions (3.6) and (3.8), as "strong" equalities if the corresponding Dirac brackets are used instead of the Poisson ones (4.6). These Dirac brackets should be analogous to the ones in Eq. (2.48). However, if we discuss a space of functions dependent on only the covariant harmonic momentum densities, these Dirac brackets coincide with the Poisson ones (4.6).

The situation discussed here is similar to the case in which the second-class constraints are solved explicitly (i.e., the superfluous momentum degrees of freedom vanish and the coordinates conjugate to them are expressed in terms of the "physical" ones).[50] The only difference is that the 256 +256 harmonic variables are expressed in terms of the 45 degrees of freedom associated with the SO(1,9) group in an implicit way. This implicit dependence is defined by the harmonicity conditions (3.6) and (3.8) (see Refs. 22, 23, and 46 for details).

Hence, the harmonicity conditions (3.6) and (3.8) can be eliminated from the set of Hamiltonian constraints without changing the Poisson brackets if we define the set of covariant momentum densities with the properties listed above and eliminate all other harmonic momentum variables from the phase space.

The experience of studying the bosonic-string formulation (2.1) gives us a prescription for extracting the covariant momentum densities from the set of canonical ones.

First of all, these densities are the classical analogs of the covariant derivatives (3.31)–(3.34) appearing in the expression (3.31) for the admissible variation, i.e., they can be derived from the expressions (3.31)–(3.34) by the simple replacement of the derivatives $\partial/\partial v$, $\partial/\partial v$ by the canonical momentum densities P_a^α, P_α^a.

On the other hand, they can be derived as the derivatives of the Lagrangian density L of the action (4.3) with respect to $\Omega_\tau^{(k)(l)}$ [where $\Omega_\tau^{(k)(l)}$ are the τ coefficients of the pullbacks of the SO(1,9) Cartan differential forms (3.32) on the worldsheet]. Their form can be derived from Eqs. (3.25) and (3.32) as follows:

$$\Omega_\tau^{(k)(l)}=\Omega^{(k)(l)}(\partial_\tau)=u^{m(k)}\partial_\tau u_m^{(l)}\equiv-\tfrac{1}{8}\mathrm{Sp}(v^{-1}\partial_\tau v\sigma^{(k)(l)} \tag{4.11}$$

or, equivalently, using the representations

$$\Omega^{(k)(l)}(d)=d\tau\Omega_\tau^{(k)(l)}+d\sigma\Omega_\sigma^{(k)(l)} \tag{4.12}$$

for the pullbacks.

Thus, the general expression for the covariant momentum densities in the whole phase space (4.4), (4.5) has the form

$$\Pi^{(l)(k)}\equiv-\partial L/\partial(\Omega_{\tau(k)(l)})=-\tfrac{1}{2}\mathrm{Sp}(v\sigma^{(k)(l)}\partial L/\partial(\partial_\tau v)$$
$$-\partial L/\partial(\partial_\tau v^{-1})\sigma^{(k)(l)}v^{-1}. \tag{4.13}$$

The Poisson brackets of the covariant momentum density with any functional F living in the configuration space of the dynamical system can be represented in the form

$$[\Pi^{(l)(k)}(\sigma),F[v,v^{-1},x,\theta]]_P=\tilde\Delta^{(l)(k)}(\sigma)F[v,v^{-1},x,\theta]. \tag{4.14}$$

Here $\tilde\Delta^{(l)(k)}(\sigma)$ are the variational analogs of the covariant harmonic derivatives (3.29) and (3.33). Thus, these covariant derivatives play the same role for the covariant momentum variables that the ordinary derivatives play for the canonical ones:

$$[P_M(\sigma),F[z^N]]_P=\delta/\delta z^M(\sigma)F[z^N].$$

Moreover, the covariant momentum densities (4.13) generate the current algebra associated with the Lorentz-group algebra (2.18a), (3.39) on the Poisson brackets (4.6),

$$[\Pi^{(l_1)(l_2)}(\sigma),\Pi^{(k_1)(k_2)}(\sigma')]_P$$
$$=[\Delta^{(l_1)(l_2)},\Delta^{(k_1)(k_2)}]|_{\Delta\to\Pi(\sigma)}\delta(\sigma-\sigma'), \tag{4.15}$$

and have vanishing Poisson brackets with the harmonicity conditions (3.6) and (3.8),

$$[\Pi^{(l)(k)}(\sigma),\Xi(\sigma'),]_P|_{\Xi=0}=0, \tag{4.16}$$

when the same harmonicity conditions are taken into account.

Thus, we must leave only the covariant harmonic momentum densities (4.13) in the phase space, which is parametrized now by the following variables:

$$(x^m(\sigma),P_m(\sigma);\quad \theta^{\alpha I}(\sigma),\pi_{\alpha I}(\sigma);$$
$$v_{\alpha A}^{+}(\sigma),v_{\alpha\dot A}^{-}(\sigma),v_A^{-\alpha}(\sigma),v_{\dot A}^{+\alpha}(\sigma),\Pi^{(l)(k)}(\sigma)). \tag{4.17}$$

Only the primary constraints

$$\Pi^{(l)(k)}(\sigma)\approx 0 \tag{4.18}$$

must be taken into account besides the ones presented in Eqs. (4.7). Equations (4.18) replace the whole set (4.8), (4.9) of constraints for the harmonic variables in this approach. The harmonicity conditions (3.6) and (3.8) are understood as a strong equality.

The Poisson brackets are defined by Eqs. (4.14) and (4.15) or by the basic relations

$$[\Pi^{(l)(k)}(\sigma),v_\alpha^a(\sigma'),]_P=\tfrac{1}{2}(v_\alpha\sigma^{(k)(l)})^a\delta(\sigma-\sigma'), \tag{4.19a}$$

$$[\Pi^{(l)(k)}(\sigma),v_a^\alpha(\sigma'),]_P=\tfrac{1}{2}(\sigma^{(k)(l)}v^\alpha)_a\delta(\sigma-\sigma'), \tag{4.19b}$$

which lead to Eqs. (4.15) when the Jacobi identities for the Poisson brackets are taken into account.

Now let us discuss the form of the canonical Hamiltonian density H_0 which is consistent with the definition of the Poisson brackets and the Hamiltonian equations of motion

$$\partial_\tau f(\sigma)=[f(\sigma),\int d\sigma' H_0(\sigma')]_P. \tag{4.20}$$

The standard expression for the canonical Hamiltonian

$$H_0^{\text{stand}}=-(-1)^M\partial_\tau z^M P_M-L \tag{4.21}$$

has such consistency with Eqs. (4.6) and (4.20). This can be verified by the following formal manipulation. The use of the density (4.21) in Eq. (4.20) written for the simplest function $f=z^M(\tau,\sigma)$ leads to the identity

$$\partial_\tau z^M(\sigma)=\left[z^M(\sigma), \quad \int d\sigma' H_0^{\text{stand}}(\sigma')\right]_P$$

$$=-\int d\sigma' \partial_\tau z^N(\sigma')$$

$$\times[z^M(\sigma), \quad P_N(\sigma')]_P=\partial_\tau z^M(\sigma).$$

To achieve such consistency with Eqs. (4.14), (4.19), and (4.20), we must define the canonical Hamiltonian density in terms of covariant harmonic momentum variables as follows:

$$H_0=-\partial_\tau x^m(\sigma)P_m(\sigma)+\partial_\tau\theta^{\alpha I}(\sigma)\pi_{\alpha I}(\sigma)$$

$$-\tfrac{1}{2}\Omega_\tau^{(k)(l)}(\sigma)\Pi_{(l)(k)}(\sigma)-L(\sigma) \tag{4.22}$$

i.e., instead of the standard combination $\partial_\tau z^M P_M$ (which can be derived by the replacement $\delta\to\partial_\tau$, $\partial/\partial z^M\to P_M$ from the expression for an arbitrary variation $\delta=\delta z^M\partial/\partial z^M$) the expression

$$\tfrac{1}{2}\Omega_\tau^{(k)(l)}(\sigma)\Pi_{(l)(k)}(\sigma)$$

[which can be derived by the replacement $\Omega^{(k)(l)}(\delta)\to\Omega_\tau^{(k)(l)}(\sigma)$, $\Delta_{(l)(k)}\to\Pi_{(l)(k)}(\sigma)$ from the expression for the admissible variation (3.28)] appears in the canonical Hamiltonian.

For the ensuing discussion some specification of the relations (4.13), (4.18), and (4.22) is necessary.

Let us introduce the covariant momentum densities $\Pi^{(0)}$, $\Pi^{[\mp 2](j)}$, and $\Pi^{(i)(j)}$, which are the classical analogs of the covariant derivatives (3.33). In terms of the canonical momentum densities $P_v\equiv(P_A^{-\alpha},P_{\dot A}^{+\alpha})$ and $P_{(v^{-1})}\equiv(P_{\alpha A}^{+},P_{\alpha\dot A}^{-})$ they are defined by the relations

$$\Pi^{(l)(k)}=(-2\Pi^{(0)},\Pi^{[\mp2](j)},\Pi^{(i)(j)})$$

$$=\tfrac{1}{2}\mathrm{Sp}(v\sigma^{(k)(l)}P_v-P_{(v^{-1})}\sigma^{(k)(l)}v^{-1}), \tag{4.23}$$

$$\Pi^{(0)}\equiv-\tfrac{1}{2}\Pi^{[+2][-2]}=\nu_A^+P_A^- -\nu_{\dot A}^-P_{\dot A}^+ -\nu_A^-P_A^+ +\nu_{\dot A}^+P_{\dot A}^-, \tag{4.23a}$$

$$\Pi^{[+2](i)}=\nu_A^+\gamma^i_{A\dot A}P_{\dot A}^+ -\nu_{\dot A}^+\tilde\gamma^i_{\dot A A}P_A^+, \tag{4.23b}$$

$$\Pi^{[-2](i)}=\nu_{\dot A}^-\tilde\gamma^i_{\dot A A}P_A^- -\nu_A^-\gamma^i_{A\dot A}P_{\dot A}^-, \tag{4.23c}$$

$$\Pi^{(i)(j)}=\tfrac{1}{2}(\nu_A^+\gamma^{ij}_{AB}P_B^- +\nu_{\dot A}^+\tilde\gamma^{ij}_{\dot A\dot B}P_{\dot B}^- +\nu_A^-\gamma^{ij}_{AB}P_B^+ +\nu_{\dot A}^+\tilde\gamma^{ij}_{\dot A\dot B}P_{\dot B}^-). \tag{4.23d}$$

It is evident [see Eqs. (4.15) and (3.39)] that the densities $\Pi^{(0)}$ and $\Pi^{(i)(j)}$ generate (on the Poisson brackets) Kac–Moody-like extensions of the SO(1,1) and SO(8) group algebras, respectively. The densities $\Pi^{[\mp2](j)}$ are associated with the coset SO(1,9)/[SO(1,1)×SO(8)].

The Poisson brackets (4.6) can be written in the form

$$[\tilde F,\tilde G]_P=\int d\sigma(\delta\tilde F/\delta P_m(\sigma)\delta\tilde G/\delta x^m(\sigma)-\delta\tilde F/\delta x^m(\sigma)\delta\tilde G/\delta P_m(\sigma)$$

$$-\int d\sigma\delta\tilde F/\delta\theta^{\alpha I}(\sigma)\delta\tilde G/\delta\pi_{\alpha I}(\sigma)$$

$$+\delta\tilde F/\delta\pi_{\alpha I}(\sigma)\delta\tilde G/\delta\theta^{\alpha I}(\sigma)$$

$$+\int d\sigma(\delta\tilde F/\delta\Pi^{(r)(l)}(\sigma)\tilde\Delta^{(r)(l)}(\sigma)\tilde G-\tilde\Delta^{(r)(l)}$$

$$\times(\sigma)\tilde F\delta\tilde G/\delta\Pi^{(r)(l)}(\sigma)) \tag{4.24}$$

for the functionals $\tilde F$ and $\tilde G$ living in the phase space (4.17) [cf. Eq. (2.55)]. All the expressions (4.23) vanish in the weak sense (4.18). Hence the primary harmonic constraints have the following form in this approach:

$$\Pi^{(0)}\equiv-\partial L/\partial\Omega_\tau^{(0)}\approx0, \tag{4.25a}$$

$$\Pi^{[\mp2](j)}\equiv-\partial L/\partial\Omega_\tau^{[\pm2](j)}\approx0, \tag{4.25b}$$

$$\Pi^{(i)(j)}\equiv+\partial L/\partial\Omega_\tau^{(i)(j)}\approx0. \tag{4.25c}$$

Finally, the expression (4.22) for the canonical Hamiltonian can be specified as follows:

$$H_0=-\partial_\tau x^m(\sigma)P_m(\sigma)+\partial_\tau\theta^{\alpha I}(\sigma)\pi_{\alpha I}(\sigma)-\Omega_\tau^{(0)}\Pi^{(0)}$$

$$-\Omega_\tau^{[\pm2](j)}\Pi^{[\mp2](j)}+\tfrac{1}{2}\Omega_\tau^{(i)(j)}\Pi^{(i)(j)}-L, \tag{4.26}$$

where L denotes the Lagrangian density for the action (4.3).

Let us summarize the results of this subsection which define the starting point for the next one.

The phase space of the system is parametrized by the variables (4.17). The Poisson brackets are defined by the standard relations (4.6) for the ordinary variables and by the relations (4.14), (4.15), and (4.19) for the harmonic ones.[7] The canonical Hamiltonian is defined by the relation (4.26). The set of primary constraints includes the relations (4.7a)–(4.7d) and (4.25).

4.2. Irreducible first-class constraints for the $D=10$, $N=2B$ Green–Schwarz superstring

The first-class constraints can be extracted by means of the well-known Dirac procedure of checking the constraint conservation during the evolution.[50]

The evolution of the dynamical variables of a system with constraints is defined by a generalized Hamiltonian, which is the sum of the canonical Hamiltonian and the products of the primary constraints and the corresponding Lagrange multipliers. For the dynamical system with the primary constraints (4.7a)–(4.7d) and (4.25) the generalized Hamiltonian has the form

$$H'=\int d\sigma\ H'(\tau,\sigma),$$

$$H'(\tau,\sigma)=H_0(\tau,\sigma)+\xi_A^{+I}\nu_A^{-\alpha}D_\alpha^I(\sigma)+\xi_{\dot A}^{-I}\nu_{\dot A}^{+\alpha}D_\alpha^I(\sigma) \tag{4.27}$$

$$+[a^{[+2]}u^{m[-2]}+a^{[-2]}u^{m[+2]}$$

$$+a^{(i)}u^{m(i)}]\Phi_m+i\ \alpha^{(0)}\Pi^{(0)}+\tfrac{1}{2}\alpha^{ij}\Pi^{ij}+\alpha^{[\mp2]i}\Pi^{[\pm2]i}$$

$$+\beta^{\mu[\mp2]}P^{[\pm2]}_{(\rho)\mu}.$$

Here the canonical Hamiltonian H_0 is defined by the general expression (4.26) for any dynamical system living on the phase space (4.17). For the superstring formulation (4.3) it has the form

$$H_0\equiv\int d\sigma\ H_0(\tau,\sigma)$$

$$H_0(\tau,\sigma)\equiv-\partial_\tau x^mP_m+\partial_\tau\theta^{\alpha I}\pi_{\alpha I}-\tfrac{1}{2}\Omega_\tau^{(k)(l)}\Pi_{(l)(k)}-L$$

$$= \tfrac{1}{2}(\rho^{[+2]\sigma}u_m^{[-2]} + \rho^{[-2]\sigma}u_m^{[+2]})\omega_\sigma^m + \frac{c\alpha'}{2}\varepsilon_{\mu\nu}\rho^{\mu[+2]}\rho^{\nu[-2]}, \quad (4.28)$$

$$\varepsilon_{01} = -\varepsilon_{10} = -1.$$

The conditions of constraint conservation

$$\frac{d}{d\tau}(\text{constraint}) \approx [(\text{constraint}), H']_P \approx 0 \quad (4.29)$$

must lead either to restrictions on the Lagrange multipliers or to the appearance of "secondary" constraints.[50]

Thus, the requirement of conservation of the constraints $P^{[\pm2]}_{(\rho)\sigma} \approx 0$ (4.7d) leads to the secondary constraints

$$\omega_\sigma^m u_m^{[\mp2]} \pm c\alpha' \rho^{\tau[\mp2]} \approx 0. \quad (4.30a,b)$$

At the same time, conservation of the constraints $P^{[\pm2]}_{(\rho)\tau} \approx 0$ (4.7c) permits us to express the Lagrange multipliers $a^{[\mp2]}$ in terms of the σ components of the world-sheet vector densities $\rho^{\mu[\mp2]}$ (or, more precisely, vice versa):

$$a^{[\mp2]} = \mp\frac{c\alpha'}{2}\rho^{\sigma[\mp2]}. \quad (4.31a,b)$$

Owing to SO(1,1)⊗SO(8) gauge symmetry of the superstring action (3.1), (4.3), the requirement of conservation of the harmonic constraints $\Pi^{(0)} \approx 0$ and $\Pi^{ij} \approx 0$ (4.25a), (4.25d) has no nontrivial consequences. [We recall that $\Pi^{(0)}$ and Π^{ij} generate SO(1,1) and SO(8) transformations on the Poisson brackets.] However, conservation of the other 16 harmonic constraints $\Pi^{[\pm2]i} \approx 0$ (4.25) has nontrivial consequences for the Lagrange multipliers:

$$\rho^{\sigma[\mp2]}\omega_\sigma^m u_m^{(i)} - a^i \rho^{\tau[\mp2]} \approx 0. \quad (4.32a,b)$$

This means that there is no gauge symmetry under the transformations from the coset space SO(1,9)/[SO(1,1)⊗SO(8)] in this formulation. This fact was discussed in detail in Refs. 22 and 23.

The consistency condition for Eqs. (4.32a) and (4.32b) leads to the relation

$$a^i e \equiv a^i(\rho^{\tau[+2]}\rho^{\sigma[-2]} - \rho^{\tau[-2]}\rho^{\sigma[+2]}) = 0,$$

which results in the vanishing of the SO(8) vector Lagrange multiplier,

$$a^i = 0, \quad (4.33)$$

in the case of a nondegenerate world-sheet metric (or, more precisely, nondegenerate world-sheet moving frame). Using Eq. (4.33), we can see that Eqs. (4.32a) and (4.32b) produce the secondary constraint

$$\omega_\sigma^{(i)} \equiv \omega_\sigma^m u_m^{(i)} \approx 0, \quad (4.34)$$

which is the σ component of Eq. (3.44).[8)]

Together with Eqs. (4.30a) and (4.30b), Eq. (4.34) makes it possible to decompose the component of the Cartan-form pullback ω_σ^m onto the basis of two vectors $u_m^{[\pm2]}$ of the target-space moving frame, which, therefore, are tangent to the world-sheet on the mass shell (i.e., on the shell defined by the equations of motion in the target space).

The requirement of preservation of the Grassmannian spinor constraints $D_\alpha^I(\sigma) \approx 0$ (4.7b) gives an expression for the Grassmannian Lagrange multipliers ξ_A^{+2} and $\xi_{\dot A}^{-1}$ in terms of the dynamical variables:

$$\xi_A^{+2} = \frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\partial_\sigma\theta^{\alpha2}\nu_{\alpha A}^+, \quad (4.35a)$$

$$\xi_{\dot A}^{-1} = \frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\partial_\sigma\theta^{\alpha1}\nu_{\alpha\dot A}^-. \quad (4.35b)$$

The other Grassmannian Lagrange multipliers ξ_A^{+1} and $\xi_{\dot A}^{-2}$ remain independent and play the role of parameters of the fermionic κ symmetry in the Hamiltonian formalism. We stress that this symmetry is present in the theory only for a definite choice of the numerical coefficient a' in the Wess–Zumino term (4.3b) of the superstring action (4.3). If this coefficient is different from $\pm1/c\alpha'$, the conservation conditions for the Grassmannian constraints (4.7b),

$$D_\alpha^I(\sigma) \equiv -\pi_\alpha^I + i(\sigma_m\theta^I)_\alpha(P_m - (-1)^I a'(\partial_\sigma x_m - i\partial_\sigma\theta^I\sigma_m\theta^I)) \approx 0,$$

result in the relations

$$\xi_A^{+1}\nu_{\alpha A}^+ u^{m[-2]}(P_m - (-1)^I a'(\partial_\sigma x_m - 2i\partial_\sigma\theta^I\sigma_m\theta^I)) + \xi_{\dot A}^{-1}\nu_{\alpha\dot A}^- u^{m[+2]}(P_m + (-1)^{I+1}a'(\partial_\sigma x_m - 2i\partial_\sigma\theta^I\sigma_m\theta^I)) = \rho^{\sigma[-2]}\partial_\sigma\theta^{\beta2}\nu_{\beta A}^+\nu_{\alpha A}^+(1-(-1)^{I+1}) + \rho^{\sigma[+2]}\partial_\sigma\theta^{\gamma1}\nu_{\gamma\dot A}^-\nu_{\alpha\dot A}^-(1+(-1)^{I+1}). \quad (4.36)$$

From Eq. (4.36) we can obtain relations similar to Eqs. (4.35a) and (4.35b), not only for ξ_A^{+2} and $\xi_{\dot A}^{-1}$, but also for the remaining Grassmannian Lagrange multipliers ξ_A^{+1} and $\xi_{\dot A}^{-2}$. [The primary and secondary constraints (4.7a) and (4.30) must be used in such computations.] Thus, fermionic κ symmetry is absent for the theory if the numerical coefficient in front of the Wess–Zumino term is $a' \neq \pm1/c\alpha'$.

If we choose this coefficient a' to be equal to $-1/c\alpha'$ instead of $+1/c\alpha'$, then the relation (4.36) gives expressions like (4.35a), and (4.35b), but for the Lagrange multipliers ξ_A^{+1} and $\xi_{\dot A}^{-2}$. The Lagrange multipliers ξ_A^{+2} and $\xi_{\dot A}^{-1}$, which remain undetermined in this case, play the role of parameters of the κ-symmetry transformations.

Conservation of the constraints (4.7a),

$$\Phi_m \equiv P_m - \tfrac{1}{2}\rho^{\tau[+2]}u_m^{[-2]} - \tfrac{1}{2}\rho^{\tau[-2]}u_m^{[+2]} + i\frac{1}{c\alpha'}(-1)^I\partial_\sigma\theta^I\sigma_m\theta^I \approx 0,$$

yields the relation

$$u_m^{[+2]}\Big[\tfrac{1}{2}\beta^{\tau[-2]} - \tfrac{1}{2}\partial_\sigma\rho^{\sigma[-2]} - \Omega_\sigma^{(0)}\rho^{\sigma[-2]} + i\alpha^{(0)}\rho^{\tau[-2]} + 2i\frac{1}{c\alpha'}(-1)^I\xi_{\dot A}^{-1}\partial_\sigma\theta^{\alpha I}\nu_{\alpha\dot A}^-\Big] + u_m^{[-2]}\Big[\tfrac{1}{2}\beta^{\tau[+2]} - \tfrac{1}{2}\partial_\sigma\rho^{\sigma[+2]} + \Omega_\sigma^{(0)}\rho^{\sigma[+2]} - i\alpha^{(0)}\rho^{\tau[+2]} + 2i\frac{1}{c\alpha'}(-1)^I\xi_A^{+1}\partial_\sigma\theta^{\alpha I}\nu_{\alpha A}^+\Big]$$

$$+u_{\rm m}^{({\rm i})}[(\alpha^{[+2]({\rm i})}\rho^{\tau[-2]}+\alpha^{[-2]({\rm i})}\rho^{\tau[+2]})$$

$$-\sum_{\pm}\rho^{\sigma[\mp2]}\Omega_\sigma^{[\pm2]({\rm i})}$$

$$-2i\,\frac{1}{c\alpha'}(-1)^{\rm I}(\xi_{\rm A}^{+{\rm I}}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\partial_\sigma\theta^{\alpha{\rm I}}\nu^-_{\alpha\dot{\rm B}}+\xi_{\dot{\rm A}}^{-{\rm I}}\gamma^{\rm i}_{\dot{\rm A}{\rm B}}\partial_\sigma\theta^{\alpha{\rm I}}\nu^+_{\alpha{\rm B}})]=0. \tag{4.37}$$

Here $\Omega_\sigma^{(0)}$, $\Omega_\sigma^{[\pm2](\rm i)}$, and $\Omega_\sigma^{(\rm i)(\rm j)}$ are the coefficients of the pullback of the SO(1,9) Cartan forms (3.34) on the world sheet. They are related to the differential $d\sigma$, and their form can be derived from Eqs. (3.34) by using the relation

$$\Omega(d)=d\xi^\mu\Omega_\mu(\xi)=d\tau\Omega_\tau(\tau,\sigma)+d\tau\Omega_\sigma(\tau,\sigma).$$

The projections of Eq. (4.37) on the composed vectors $u_{\rm m}^{[-2]}$, $u_{\rm m}^{[+2]}$, and $u_{\rm m}^{(\rm i)}$ (3.11) of the moving-frame system (3.10) give us the following expressions for the Lagrange multipliers:

$$\beta^{\tau[-2]}=\partial_\sigma\rho^{\sigma[-2]}+\Omega_\sigma^{(0)}\rho^{\sigma[-2]}-2i\alpha^{(0)}\rho^{\tau[-2]}$$

$$-4i\,\frac{1}{c\alpha'}\,\xi_{\dot{\rm A}}^{-2}\partial_\sigma\theta^{\alpha2}\nu^-_{\alpha\dot{\rm A}}, \tag{4.38a}$$

$$\beta^{\tau[+2]}=\partial_\sigma\rho^{\sigma[+2]}-\Omega_\sigma^{(0)}\rho^{\sigma[+2]}+2i\alpha^{(0)}\rho^{\tau[+2]}$$

$$+4i\,\frac{1}{c\alpha'}\,\xi_{\rm A}^{+1}\partial_\sigma\theta^{\alpha1}\nu^+_{\alpha{\rm A}}, \tag{4.38b}$$

$$\alpha^{[+2](\rm i)}\rho^{\tau[-2]}+\alpha^{[-2](\rm i)}\rho^{\tau[+2]}=\sum_{\pm}\rho^{\sigma[\mp2]}\Omega_\sigma^{[\pm2](\rm i)}$$

$$-2i\,\frac{1}{c\alpha'}\,\xi_{\rm A}^{+1}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\partial_\sigma\theta^{\alpha1}\nu^-_{\alpha\dot{\rm B}}+2i\,\frac{1}{c\alpha'}\,\xi_{\dot{\rm A}}^{-2}\gamma^{\rm i}_{\dot{\rm A}{\rm B}}\partial_\sigma\theta^{\alpha2}\nu^+_{\alpha{\rm B}} \tag{4.38c}$$

$$+2i\,\frac{1}{c\alpha'}\Big[\frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\,\partial_\sigma\theta^{\alpha1}\partial_\sigma\theta^{\beta1}$$

$$+\frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\,\partial_\sigma\theta^{\alpha2}\partial_\sigma\theta^{\beta2}\Big]\nu^+_{\alpha{\rm A}}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\nu^-_{\beta\dot{\rm B}}.$$

Finally, we must verify the conservation of the secondary constraints (4.30a), (4.30b), and (4.34). They can be represented as the projections of one constraint

$$\omega_\sigma^{\rm m}-\tfrac{1}{2}c\alpha'\rho^{\tau[+2]}u_{\rm m}^{[-2]}+\tfrac{1}{2}c\alpha'\rho^{\tau[-2]}u_{\rm m}^{[+2]}\approx0 \tag{4.39}$$

onto the moving-frame vectors $u_{\rm m}^{[-2]}$, $u_{\rm m}^{[+2]}$, and $u_{\rm m}^{(\rm i)}$. The requirement of conservation of this constraint leads to an equation similar to (4.37). Moreover, the projections of this equation onto the moving-frame vectors $u_{\rm m}^{[-2]}$ and $u_{\rm m}^{[+2]}$ coincide with Eqs. (4.38a) and (4.38b), respectively. However, its projection onto the moving-frame vectors $u_{\rm m}^{(\rm i)}$ differs from Eq. (4.38c) and has the form

$$\alpha^{[+2](\rm i)}\rho^{\tau[-2]}-\alpha^{[-2](\rm i)}\rho^{\tau[+2]}$$

$$=\tfrac{1}{2}[\rho^{\sigma[-2]}\Omega_\sigma^{[+2](\rm i)}-\rho^{\sigma[+2]}\Omega_\sigma^{[-2](\rm i)}]$$

$$+2i\,\frac{1}{c\alpha'}\,\xi_{\rm A}^{+1}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\partial_\sigma\theta^{\alpha1}\nu^-_{\alpha\dot{\rm B}}+2i\,\frac{1}{c\alpha'}\,\xi_{\dot{\rm A}}^{-2}\gamma^{\rm i}_{\dot{\rm A}{\rm B}}\partial_\sigma\theta^{\alpha2}\nu^+_{\alpha{\rm B}}$$

$$-2i\,\frac{1}{c\alpha'}\Big[\frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\,\partial_\sigma\theta^{\alpha1}\partial_\sigma\theta^{\beta1}-\frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\,\partial_\sigma\theta^{\alpha2}\partial_\sigma\theta^{\beta2}\Big]$$

$$\times\nu^+_{\alpha{\rm A}}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\nu^-_{\beta\dot{\rm B}}. \tag{4.40}$$

This relation corresponds to the requirement of conservation of the secondary constraints (4.34) and thus is absent in the case of a null superstring [as well as the constraint (4.34) itself]. Consequently, the corresponding "boost" symmetry[20,21] which characterizes the superparticle[18,20,21] and the null-superstring theory[19,42,43] is absent in the case of the twistor-like superstring formulation. This is the case because the superstring action (3.1), (4.3) contains spinor harmonic variables of both types: the $\nu^+_{\alpha{\rm A}}$ harmonics as well as the $\nu^-_{\alpha\dot{\rm A}}$ ones. The eight "boost" symmetries[20,21] consist in the shifting of one of these harmonics by harmonics of another type:

$$\delta\nu^+_{\alpha{\rm A}}={\rm b}^{{\rm i}[+2]}\gamma^{\rm i}_{{\rm A}\dot{\rm A}}\nu^-_{\alpha\dot{\rm A}}\quad\text{or}\quad\delta\nu^-_{\alpha\dot{\rm A}}={\rm b}^{{\rm i}[-2]}\nu^+_{\alpha{\rm A}}\gamma^{\rm i}_{{\rm A}\dot{\rm A}}.$$

It is clear that such symmetry is present in theories whose formulation contains only one of the types of harmonics, $\nu^+_{\alpha{\rm A}}$ or $\nu^-_{\alpha\dot{A}}$. This property is satisfied not only in the twistor-like formulation of massless superparticles, null superstrings, and null super-p-branes,[18–21,42,43] but also in the heterotic-string formulations like the ones discussed in Refs. 32, 34, 36, 38, and 40.[9)]

Equations (4.38c) and (4.39) can be solved for the Lagrange multipliers $\alpha^{[+2](\rm i)}$ and $\alpha^{[-2](\rm i)}$:

$$\alpha^{[+2](\rm i)}=\frac{1}{\rho^{\tau[-2]}}\Big[\rho^{\sigma[-2]}\tfrac{1}{2}\Omega_\sigma^{[+2](\rm i)}+2i\,\frac{1}{c\alpha'}\,\xi_{\dot{\rm A}}^{-2}\gamma^{\rm i}_{\dot{\rm A}{\rm B}}\partial_\sigma\theta^{\alpha2}\nu^+_{\alpha{\rm B}}$$

$$+2i\,\frac{1}{c\alpha'}\,\frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\,\partial_\sigma\theta^{\alpha2}\partial_\sigma\theta^{\beta2}\nu^+_{\alpha{\rm A}}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\nu^-_{\beta\dot{\rm B}}, \tag{4.41a}$$

$$\alpha^{[-2](\rm i)}=\frac{1}{\rho^{\tau[+2]}}\Big[\rho^{\sigma[+2]}\tfrac{1}{2}\Omega_\sigma^{[-2](\rm i)}-2i\,\frac{1}{c\alpha'}\,\xi_{\rm A}^{+1}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\partial_\sigma\theta^{\alpha1}\nu^-_{\alpha\dot{\rm B}}$$

$$+2i\,\frac{1}{c\alpha'}\,\frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\,\partial_\sigma\theta^{\alpha1}\partial_\sigma\theta^{\beta1}\nu^+_{\alpha{\rm A}}\gamma^{\rm i}_{{\rm A}\dot{\rm B}}\nu^-_{\beta\dot{\rm B}}. \tag{4.41b}$$

Thus, the verification of the conservation of the constraints under the evolution is complete and, hence, the full set of first-class constraints has been extracted (up to a transition to some linear combinations of them). They can be defined as variations of the generalized Hamiltonian H' in (4.27), (4.28) with respect to the generalized Lagrange multipliers, which may contain the undetermined field parameters of the canonical Hamiltonian playing the role of the Lagrange multipliers for the secondary constraints (besides the original Lagrange multipliers). In the present case we can use, as generalized Lagrange multipliers, the moving-frame density variables $\rho^{\sigma[\mp2]}$, related to the original Lagrange multipliers $a^{[\mp2]}$ by Eqs. (4.31a) and (4.31b).[10]

After substitution of the expressions for the dependent Lagrange multipliers (4.31), (4.33), (4.35), (4.38a), (4.38b), and (4.40) into the expressions (4.27), and taking into account (4.28), the generalized Hamiltonian can be written as follows:

$$H'(\tau,\sigma)=\frac{c\alpha'}{2}\,\rho^{\sigma[+2]}L^{[-2]}(\tau,\sigma)-\frac{c\alpha'}{2}\,\rho^{\sigma[-2]}L^{[+2]}(\tau,\sigma)$$

$$+\xi_{A}^{+1}D_{A}^{-}+\xi_{\dot{A}}^{-2}D_{\dot{A}}^{+}+i\ \alpha^{(0)}D^{(0)}+\tfrac{1}{2}\alpha^{ij}\Pi^{ij}$$

$$+\beta^{\sigma[\mp2]}P^{[\pm2]}_{(\rho)\sigma}. \qquad (4.42)$$

The first-class constraints have the form

$$L^{[-2]}\equiv\frac{2}{c\alpha'}\frac{\partial H'}{\partial\rho^{\sigma[+2]}}$$

$$=u^{[-2]m}L^{1}_{m}+\frac{2}{c\alpha'}\frac{1}{\rho^{[+2]\tau}}\partial_{\sigma}\Theta^{\alpha1}v^{-}_{\alpha\dot{A}}(v^{+\gamma}_{\dot{A}}D^{1}_{\gamma})$$

$$+\frac{1}{c\alpha'}\frac{1}{\rho^{[+2]\tau}}\Omega^{[-2]i}_{\sigma}\Pi^{i[+2]}$$

$$+4i\Big[c\alpha'\rho^{[+2]\tau}\Big]^{-2}\partial_{\sigma}\Theta^{\alpha1}\partial_{\sigma}\Theta^{\gamma1}v^{+}_{\alpha A}\gamma^{i}_{A\dot{B}}v^{-}_{\gamma\dot{B}}\Pi^{i[+2]}$$

$$-\frac{2}{c\alpha'}\partial_{\sigma}P^{[-2]}_{(\rho)\tau}-\frac{2}{c\alpha'}\Omega^{(0)}_{\sigma}P^{[-2]}_{(\rho)\tau}\approx0, \qquad (4.43a)$$

$$\hat{D}^{-}_{A}(\sigma)\equiv\frac{\partial H'}{\partial\xi^{+1}_{A}}=v^{\alpha-}_{A}D^{1}_{\alpha}(\sigma)$$

$$-\frac{2i}{c\alpha'\rho^{[+2]\tau}}\gamma^{i}_{A\dot{A}}v^{-}_{\alpha\dot{A}}\partial_{\sigma}\Theta^{\alpha1}\Pi^{[+2]i}$$

$$+\frac{4i}{c\alpha'}\partial_{\sigma}\Theta^{\gamma1}v^{+}_{\gamma A}P^{[-2]}_{(\rho)\tau}\approx0, \qquad (4.43b)$$

$$L^{[+2]}\equiv-\frac{2}{c\alpha'}\frac{\partial H'}{\partial\rho^{\sigma[-2]}}$$

$$=u^{[+2]m}L^{2}_{m}-\frac{2}{c\alpha'}\frac{1}{\rho^{[-2]\tau}}\partial_{\sigma}\Theta^{\alpha2}v^{+}_{\alpha A}[v^{-\gamma}_{A}D^{2}_{\gamma}]$$

$$-\frac{1}{c\alpha'}\frac{1}{\rho^{[-2]\tau}}\Omega^{[+2]i}_{\sigma}\Pi^{[-2]i}$$

$$-4i[c\alpha'\rho^{[-2]\tau}]^{-2}\partial_{\sigma}\Theta^{\alpha2}\partial_{\sigma}\Theta^{\gamma2}v^{+}_{\alpha A}\gamma^{i}_{A\dot{B}}v^{-}_{\gamma\dot{B}}\Pi^{[-2]i}$$

$$+\frac{2}{c\alpha'}\partial_{\sigma}P^{[+2]}_{(\rho)\tau}-\frac{2}{c\alpha'}\Omega^{(0)}_{\sigma}P^{[+2]}_{(\rho)\tau}\approx0, \qquad (4.43c)$$

$$\hat{D}^{+}_{\dot{A}}(\sigma)\equiv\frac{\partial H'}{\partial\xi^{-2}_{\dot{A}}}=v^{\alpha+}_{\dot{A}}D^{2}_{\alpha}(\sigma)$$

$$+\frac{2i}{c\alpha'\rho^{[-2]\tau}}\tilde{\gamma}^{i}_{\dot{A}A}v^{+}_{\alpha A}\partial_{\sigma}\Theta^{\alpha2}\Pi^{[-2]i}$$

$$-\frac{4i}{c\alpha'}\partial_{\sigma}\Theta^{\alpha2}v^{-}_{\gamma\dot{A}}P^{[+2]}_{(\rho)\tau}\approx0, \qquad (4.43d)$$

$$D^{(0)}\equiv\Pi^{(0)}+2\rho^{[+2]\tau}P^{[-2]}_{(\rho)\tau}-2\rho^{[-2]\tau}P^{[+2]}_{(\rho)\tau}\approx0, \quad (4.43e)$$

$$D^{ij}\equiv\Pi^{ij}\approx0, \qquad (4.43f)$$

$$P^{[\mp2]}_{(\rho)\sigma}\approx0, \qquad (4.43g)$$

where the expressions D^{I}_{α} and L^{I}_{m} (I=1,2) are defined by the relations

$$D^{I}_{\alpha}(\sigma)\equiv-\pi^{I}_{\alpha}+i(\sigma^{m}\theta^{I})_{\alpha}\Big[P_{m}-(-1)^{I}\frac{1}{c\alpha'}(\partial_{\sigma}x_{m}$$

$$-i\partial_{\mu}\theta^{I}\sigma_{m}\theta^{I})\Big], \qquad (4.44)$$

$$L^{I}_{m}\equiv[P_{m}-(-1)^{I}\frac{1}{c\alpha'}\omega^{m}_{\sigma}+\frac{i}{c\alpha'}\sum_{J}(-1)^{J}\partial_{\sigma}\theta^{J}\sigma_{m}\theta^{J}], \qquad (4.45)$$

I,J=1,2.

The first-class constraints (4.43a) and (4.43b) generate reparametrization symmetry with parameters $\rho^{\sigma[\mp2]}$ on the Poisson brackets. The first-class constraints (4.43c)–(4.43f) generate κ-symmetry transformations (with parameters ξ^{+1}_{A} and $\xi^{-2}_{\dot{A}}$), SO(1,1) symmetry (with parameters $\alpha^{(0)}$), SO(8) symmetry (with parameters α^{ij}) and, finally, symmetry under arbitrary shifts of the density components $\rho^{\sigma[\mp2]}$ (with parameters $\beta^{\sigma[\mp2]}$). The last symmetry implies the Lagrange-multiplier nature of the variables $\rho^{\sigma[\mp2]}$.

The connection of the reparametrization-symmetry generators (4.43a) and (4.43c) with the well known Virasoro constraints will be discussed in the next section.

Thus, the complete set of covariant and irreducible first-class constraints for the D=10, N=IIB superstring in the twistor-like Lorentz-harmonic formulation (3.1), (4.3) has been derived.

5. ALGEBRA OF IRREDUCIBLE SYMMETRIES AND SECOND-CLASS-CONSTRAINT SYMPLECTIC STRUCTURE FOR THE D=10, N=IIB GREEN–SCHWARZ SUPERSTRING

5.1. First-class constraints and their algebra

To simplify the algebra of the gauge symmetries generated by the first-class constraints (4.43), let us redefine them, using some linear transformations within the set of first-class constraints. To formulate the results of such a redefinition in a compact form, let us introduce the bosonic and fermionic blocks

$$\tilde{L}^{1}_{m}\equiv L^{1}_{m}-\partial_{\sigma}A^{1}_{m}, \qquad (5.1a)$$

$$\tilde{D}^{1}_{\alpha}(\sigma)\equiv D^{1}_{\alpha}(\sigma)+2i(\sigma^{m}\theta^{1})_{\alpha}A^{1}_{m}, \qquad (5.1b)$$

$$\tilde{L}^{2}_{m}\equiv L^{2}_{m}-\partial_{\sigma}A^{2}_{m}, \qquad (5.2a)$$

$$\tilde{D}^{2}_{\alpha}(\sigma)\equiv D^{2}_{\alpha}(\sigma)+2i(\sigma^{m}\theta^{2})_{\alpha}A^{2}_{m}, \qquad (5.2b)$$

where

$$A^{1}_{m}\equiv u^{[+2]}_{m}P^{[-2]}_{(\rho)\tau}-u^{(i)}_{m}\Pi^{[+2]i}/\rho^{\tau[+2]}\approx0, \qquad (5.3a)$$

$$A^{2}_{m}\equiv u^{[-2]}_{m}P^{[+2]}_{(\rho)\tau}-u^{(i)}_{m}\Pi^{[-2]i}/\rho^{\tau[-2]}\approx0, \qquad (5.3b)$$

and the quantities D^{I}_{α} and L^{I}_{m} (I=1,2) are defined by the relations (4.44), (4.45) or by the expressions

$$L^{1}_{m}\equiv[P_{m}+\frac{1}{c\alpha'}(\partial_{\sigma}x_{m}-2i\partial_{\sigma}\theta^{1}\sigma_{m}\theta^{1}], \qquad (5.4a)$$

Dα1(σ)≡−πα1+i(σmθ1)α[Pm+1cα′ (∂σxm

$$-i\partial_\mu \theta^1 \sigma_m \theta^1)\Bigg], \quad (5.4b)$$

$$L^2_m \equiv [P_m - \frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\partial_\sigma \theta^2 \sigma_m \theta^2], \quad (5.5a)$$

$$D^2_\alpha(\sigma) \equiv -\pi^2_\alpha + i(\sigma^m \theta^2)_\alpha \Bigg[P_m - \frac{1}{c\alpha'}(\partial_\sigma x_m - i\partial_\mu \theta^2 \sigma_m \theta^2)\Bigg]. \quad (5.5b)$$

The algebraic structure associated with the blocks (5.4) and (5.5) is a very simple one:

$$\{D^I_\alpha(\sigma), D^J_\beta(\sigma')\}_P = 2i\delta^{IJ}\sigma^m_{\alpha\beta} L^I_m \delta(\sigma-\sigma'), \quad (5.6a)$$

$$[D^I_\alpha(\sigma), L^J_n(\sigma')]_P = (-1)^I \delta^{IJ} 4i(c\alpha')^{-1}(\partial_\sigma \theta^I \sigma_n)_\alpha \delta(\sigma-\sigma'), \quad (5.6b)$$

$$[L^I_m(\sigma), L^J_n(\sigma')]_P = 2(-1)^I \delta^{IJ}(c\alpha')^{-1}\eta_{mn}\partial_\sigma \delta(\sigma-\sigma'). \quad (5.6c)$$

This set is decomposed naturally onto the two pieces (D^1_α, L^1_m) and (D^2_α, L^2_m), associated with the different light-like directions tangent to the superstring world-sheet. Inside any one such piece the bracket for two fermionic blocks produces the corresponding bosonic one (5.6a), the bracket for a bosonic block with a fermionic one produces the derivative of the corresponding Grassmannian variable (5.6b), and, finally, the bracket for two bosonic blocks is equal to the product of the flat space-time metric and the derivative of a δ function (5.6c). The brackets vanish for any two blocks belonging to different sets (i.e., associated with different light-cone directions).

In contrast to Eqs. (5.6), the algebra of the blocks A^I_m has vanishing brackets for any two blocks from the same set and complicated nonvanishing brackets for the pair A^1_m, A^2_m of blocks associated with the different light-like directions:

$$[A^1_m(\sigma), A^1_n(\sigma')]_P = 0, \quad [A^2_m(\sigma), A^2_n(\sigma')]_P = 0 \quad (5.7a,b)$$

$$[A^1_m(\sigma), A^2_n(\sigma')]_P = (\rho^{\#[+2]}\rho^{\#[-2]})^{-1}[u^{(i)}_m u^{(j)}_n(\delta^{ij}D^{(0)} + 2D^{ij}) + u^{(i)}_m u^{[+2]}_n \Pi^{[-2]i} - u^{[-2]}_m u^{(j)}_n \Pi^{[+2]j}] \approx 0. \quad (5.7c)$$

It is important that the brackets (5.7c) vanish in the weak sense and include on the left-hand side only the harmonic constraints (4.43e) and (4.43f) (which are first-class constraints) and (4.25b) (which are second-class constraints).

Taking into account Eqs. (5.6) and (5.7), we can see that the algebra of the blocks $\tilde{L}^1_m(\sigma)$, $\tilde{D}^1_\alpha(\sigma)$ is defined by the relations

$$\{\tilde{D}^1_\alpha(\sigma), \tilde{D}^1_\beta(\sigma')\}_P = 2i\sigma^m_{\alpha\beta}\tilde{L}^1_m \delta(\sigma-\sigma'), \quad (5.8a)$$

$$[\tilde{D}^1_\alpha(\sigma), \tilde{L}^1_n(\sigma')]_P = -4i(c\alpha')^{-1}(\partial_\sigma \theta^1 \sigma_n)_\alpha \delta(\sigma-\sigma'), \quad (5.8b)$$

$$[\tilde{L}^1_m(\sigma), \tilde{L}^1_n(\sigma')]_P = -2(c\alpha')^{-1}\eta_{mn}\partial_\sigma \delta(\sigma-\sigma') \quad (5.8c)$$

and coincides with the algebra (5.6) written for I=J=1.

The same is true for the algebra of $\tilde{D}^2_\alpha(\sigma)$, $\tilde{L}^2_n(\sigma')$:

$$\{\tilde{D}^2_\alpha(\sigma), \tilde{D}^2_\beta(\sigma')\}_P = 2i\sigma^m_{\alpha\beta}\tilde{L}^2_m \delta(\sigma-\sigma'), \quad (5.9a)$$

$$[\tilde{D}^2_\alpha(\sigma), \tilde{L}^2_n(\sigma')]_P = +4i(c\alpha')^{-1}(\partial_\sigma \theta^2 \sigma_n)_\alpha \delta(\sigma-\sigma'), \quad (5.9b)$$

$$[\tilde{L}^2_m(\sigma), \tilde{L}^2_n(\sigma')]_P = +2(c\alpha')^{-1}\eta_{mn}\partial_\sigma \delta(\sigma-\sigma') \quad (5.9c)$$

[see Eqs. (5.6) with I=J=2].

However, the "crossed" terms have a more complicated form and are completely determined by the brackets (5.7c):

$$\{\tilde{D}^1_\alpha(\sigma), \tilde{D}^2_\beta(\sigma')\}_P = 4i(c\alpha')^{-2}(\partial_\sigma \theta^1 \sigma_n)_\alpha (\partial_\sigma \theta^2 \sigma_n)_\beta [A^1_m(\sigma), A^2_n(\sigma')]_P, \quad (5.10a)$$

$$[\tilde{D}^1_\alpha(\sigma), \tilde{L}^2_n(\sigma')]_P = 2i(c\alpha')^{-2}(\partial_\sigma \theta^1 \sigma_n)_\alpha(\sigma)\partial_{\sigma'}[A^1_n(\sigma), A^2_m(\sigma')]_P, \quad (5.10b)$$

$$[\tilde{L}^1_n(\sigma), \tilde{D}^2_\alpha(\sigma')]_P = 2i(c\alpha')^{-2}(\partial_{\sigma'}\theta^2 \sigma_n)_\alpha(\sigma')\partial_\sigma [A^1_m(\sigma), A^2_n(\sigma')]_P, \quad (5.10c)$$

$$[\tilde{L}^1_m(\sigma), \tilde{L}^2_n(\sigma')]_P = -(c\alpha')^{-2}\partial_\sigma \partial_{\sigma'}[A^1_m(\sigma), A^2_n(\sigma')]_P, \quad (5.10d)$$

as is easy to see from Eqs. (5.6) and (5.7).

The first-class constraints $Y^I_M = (Y^1_M, Y^2_M)$ [see (4.43a)–(4.43d)], which generate κ-symmetry and reparametrization transformations, can be redefined as follows:

$$Y^1_M(\sigma) \equiv (L^1(\sigma), \hat{D}^-_A(\sigma)):$$

$$L^1(\sigma) \equiv 2\rho^{\#[+2]}\tilde{L}^{[-2]} \equiv 2\rho^{\#[+2]}u^{m[-2]}\tilde{L}^1_m(\sigma) + 4(c\alpha')^{-1}\partial_\sigma \theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma), \quad (5.11a)$$

$$\hat{D}^-_A(\sigma) \equiv \nu^{-\alpha}_A(\sigma)\tilde{D}^1_\alpha(\sigma), \quad (5.11b)$$

$$Y^2_M(\sigma) \equiv (L^2(\sigma), \hat{D}^+_{\dot{A}}(\sigma)):$$

$$L^2(\sigma) \equiv 2\rho^{\#[-2]}\tilde{L}^{[+2]} \equiv 2\rho^{\#[-2]}u^{m[+2]}\tilde{L}^2_m(\sigma) - 4(c\alpha')^{-1}\partial_\sigma \theta^{\alpha 2}\tilde{D}^2_\alpha(\sigma), \quad (5.12a)$$

$$\hat{D}^+_{\dot{A}}(\sigma) \equiv \nu^{+\alpha}_{\dot{A}}(\sigma)\tilde{D}^2_\alpha(\sigma). \quad (5.12b)$$

The differences of Eqs. (5.11) and (5.12) from (4.43a)–(4.43d) consist in:

i) addition of the expressions

$$+\frac{2}{c\alpha'}\frac{1}{\rho^{[+2]\#}}\partial_\sigma \Theta^{\alpha 1}\nu^-_{\alpha\dot{A}}(\nu^{-\gamma}_A \tilde{D}^1_\gamma)$$

and

$$-\frac{2}{c\alpha'}\frac{1}{\rho^{[-2]\#}}\partial_\sigma \Theta^{\alpha 2}\nu^+_{\alpha A}[\nu^{+\gamma}_{\dot{A}}\tilde{D}^2_\gamma],$$

which are proportional to the first-class constraints (5.11b) and (5.12b) [or, equivalently, to (4.43b) and (4.43d)], to the constraints (4.43a) and (4.43c), respectively;

ii) multiplication of the resulting expressions by the overall factors $2\rho^{\#[+2]}$ and $2\rho^{\#[-2]}$, respectively.

The algebra of reparametrization and κ-symmetry transformations associated with the same light-like direction tangent to the world-sheet is realized in the form of the bracket relations

$$[Y^1_M, Y^1_N]_P = C^{1K}_{MN} Y^1_N: \quad (5.13)$$

$$[L^1(\sigma), L^1(\sigma')]_P = -4(c\alpha')^{-1}(L^1(\sigma) + L^1(\sigma'))\partial_\sigma \delta(\sigma-\sigma'), \quad (5.13a)$$

$$[L^1(\sigma), \hat{D}^-_A(\sigma')]_P = -4(c\alpha')^{-1}\hat{D}^-_A(\sigma)\partial_\sigma\delta(\sigma-\sigma') + 2(c\alpha')^{-1}\Omega^{(0)}_\sigma(\sigma)\hat{D}^-_A(\sigma)\delta(\sigma-\sigma'), \quad (5.13b)$$

$$\{\hat{D}^-_A(\sigma), \hat{D}^-_B(\sigma')\}_P = i\delta_{AB}(\rho^{\tau[+2]})^{-1}(L^1(\sigma) - 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 1}v^+_{\alpha C}\hat{D}^-_C(\sigma)) \times \delta(\sigma-\sigma'), \quad (5.13c)$$

$$[Y^2_M, Y^2_N]_P = C^{2K}_{MN} Y^2_N: \quad (5.14)$$

$$[L^2(\sigma), L^2(\sigma')]_P = 4(c\alpha')^{-1}(L^2(\sigma) + L^2(\sigma'))\partial_\sigma\delta(\sigma-\sigma'), \quad (5.14a)$$

$$[L^2(\sigma), \hat{D}^+_{\dot{A}}(\sigma')]_P = 4(c\alpha')^{-1}\hat{D}^+_{\dot{A}}(\sigma)\partial_\sigma\delta(\sigma-\sigma') + 2(c\alpha')^{-1}\Omega^{(0)}_\sigma(\sigma)\hat{D}^+_{\dot{A}}(\sigma)\delta(\sigma-\sigma'), \quad (5.14b)$$

$$\{\hat{D}^+_{\dot{A}}(\sigma), \hat{D}^+_{\dot{B}}(\sigma')\}_P = i\delta_{\dot{A}\dot{B}}(\rho^{\tau[-2]})^{-1}(L^2(\sigma) + 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 2}v^-_{\alpha\dot{C}}\hat{D}^+_{\dot{C}}(\sigma)) \times \delta(\sigma-\sigma'), \quad (5.14c)$$

where $\Omega^{(0)}_\sigma(\sigma)$ is the σ component of the SO(1,1) Cartan form (3.32a); they transform as the connection (or gauge-field) component under SO(1,1) gauge transformations.

The brackets of the reparametrization and κ-symmetry generators associated with different light-like world-sheet directions have a more complicated structure. However, they are completely defined by the relations (5.7c):

$$[Y^1_M(\sigma), Y^2_N(\sigma')]_P = C^{1\,2ij}_{MN}\delta(\sigma-\sigma')(\delta^{ij}D^{(0)} + 2D^{ij}):$$

$$[L^1(\sigma), L^2(\sigma')]_P = -16(c\alpha')^{-1}\Omega^{[-2]i}_\sigma\Omega^{[+2]i}_\sigma(\delta^{ij}D^{(0)} + 2D^{ij})\delta(\sigma-\sigma'), \quad (5.15a)$$

$$[L^1(\sigma), \hat{D}^+_{\dot{A}}(\sigma')]_P = 4i((c\alpha')^2\rho^{\tau[-2]})^{-1}\Omega^{[-2]i}_\sigma\partial_\sigma\theta^{\alpha 2}v^+_{\alpha B}\gamma^j_{B\dot{A}}(\delta^{ij}D^{(0)} + 2D^{ij})\delta(\sigma-\sigma'), \quad (5.15b)$$

$$[\hat{D}^-_A(\sigma), L^2(\sigma')]_P = 4i((c\alpha')^2\rho^{\tau[+2]})^{-1}\Omega^{[+2]i}_\sigma\partial_\sigma\theta^{\alpha 1}v^-_{\alpha\dot{B}}\tilde{\gamma}^i_{\dot{B}A}(\delta^{ij}D^{(0)} + 2D^{ij}\delta(\sigma-\sigma'), \quad (5.15c)$$

$$\{\hat{D}^-_A(\sigma), \hat{D}^+_{\dot{A}}(\sigma')\}_P = 4((c\alpha')^2\rho^{\tau[+2]}\rho^{\tau[-2]})^{-1}\partial_\sigma\theta^{\alpha 1}v^-_{\alpha\dot{B}}\gamma^i_{A\dot{B}} \times\partial_\sigma\theta^{\alpha 2}v^+_{\alpha B}\gamma^j_{B\dot{A}}(\delta^{ij}D^{(0)} + 2D^{ij}\delta(\sigma-\sigma'), \quad (5.15d)$$

where $\Omega^{[+2]i}_\sigma$, $\Omega^{[-2]i}_\sigma$ are the components of the pullbacks of the covariant Cartan forms (4.32b) and (4.32c) $[\Omega_\sigma \equiv \Omega(\partial_\sigma)]$ and

$$D^{(0)} \equiv \Pi^{(0)} + 2\rho^{[+2]\tau}P^{[-2]}_{(\rho)\tau} - 2\rho^{[-2]\tau}P^{[+2]}_{(\rho)\tau} \approx 0, \quad (5.16a)$$

$$D^{ij} \equiv \Pi^{ij} \approx 0, \quad (5.16b)$$

are the first-class constraints (4.43e) and (4.43f) generating the SO(1,1) and SO(8) gauge symmetries (on the Poisson brackets).

The fact of closure of the super-reparametrization-symmetry algebra (i.e., the algebra of reparametrization and κ-symmetry transformations) on the SO(1,1) and SO(8) gauge-symmetry transformations is a significant one. It means that SO(1,1)×SO(8) gauge symmetry connects different light-like directions tangent to the world-sheet.

The bracket relations between the SO(1,1) and SO(8) symmetry generators (5.16a), (5.16b) and other first-class constraints are defined by the SO(1,1) weight and the SO(8) index structures of such constraints:

$$[D^{(0)}(\sigma), Y^I_M(\sigma')]_P = w(Y^I_M)Y^I_M(\sigma)\delta(\sigma-\sigma'):$$

$$[D^{(0)}(\sigma), \hat{D}^-_A(\sigma')]_P = -\hat{D}^-_A(\sigma)\delta(\sigma-\sigma'), \quad (5.17a)$$

$$[D^{(0)}(\sigma), \hat{D}^+_{\dot{A}}(\sigma')]_P = +\hat{D}^+_{\dot{A}}(\sigma)\delta(\sigma-\sigma'), \quad (5.17b)$$

$$[D^{(0)}(\sigma), L^{1,2}(\sigma')]_P = 0, \quad (5.17c)$$

$$[D^{(0)}(\sigma), D^{ij}(\sigma')]_P = 0, \quad (5.17d)$$

$$[D^{ij}(\sigma), Y^I_M(\sigma')]_P = (\gamma^{ij})^N_M Y^I_N(\sigma)\delta(\sigma-\sigma'):$$

$$[D^{ij}(\sigma), \hat{D}^-_A(\sigma')]_P = -\gamma^{ij}_{AB}\hat{D}^-_B(\sigma)\delta(\sigma-\sigma'), \quad (5.18a)$$

$$[D^{ij}(\sigma), \hat{D}^+_{\dot{A}}(\sigma')]_P = -\gamma^{ij}_{\dot{A}\dot{B}}\hat{D}^+_{\dot{B}}(\sigma)\delta(\sigma-\sigma'), \quad (5.18b)$$

$$[D^{ij}(\sigma), L^{1,2}(\sigma')]_P = 0, \quad (5.18c)$$

$$[D^{ij}(\sigma), D^{i'j'}(\sigma')]_P = 2\delta^{[i'}D^{j']j]}(\sigma)\delta(\sigma-\sigma'). \quad (5.18d)$$

Thus, the algebra of the first-class constraints

$$Y_\Lambda \equiv (Y^1_M(\sigma), Y^2_N(\sigma), D^{(0)}(\sigma), D^{ij}(\sigma), P^{[\pm 2]}_{(\rho)\sigma}) \equiv (L^1(\sigma), \hat{D}^-_A(\sigma), L^2(\sigma), \hat{D}^+_{\dot{A}}(\sigma'), \times D^{(0)}(\sigma), D^{ij}(\sigma), P^{[\pm 2]}_{(\rho)\sigma})[Y_\Lambda(\sigma), Y_\Sigma(\sigma')]_P = \int d\sigma'' C^\Pi_{\Lambda\Sigma}(\sigma, \sigma'|\sigma'')Y_\Pi(\sigma'') \quad (5.19)$$

is completely specified by Eqs. (5.13)–(5.15), (5.17), and (5.18), except for the bracket relations of them with the remaining two first-class constraints $P^{[\pm 2]}_{(\rho)\sigma} \approx 0$ (4.43g). These brackets all vanish because of the absence of the variables $\rho^{[\mp 2]\sigma}$ in the expressions for the first-class constraints:

$$[P^{[\pm 2]}_{(\rho)\sigma}(\sigma), Y_\Sigma(\sigma')]_P = 0. \quad (5.20)$$

The symmetry generated by the constraints $P^{[\pm 2]}_{(\rho)\sigma} \approx 0$ (4.43g) indicate the Lagrange-multiplier nature of the zweibein densities $\rho^{[\mp 2]\sigma}$ in the present formulation.

5.2. Second-class constraints, their algebra, and symplectic structure

The remaining constraints (4.7) and (4.25) are second-class constraints. They can also be decomposed naturally onto the two sets

$$S_f=(S^1_f,S^2_f)\approx 0 \quad (5.21)$$

associated with different light-like directions tangent to the world-sheet:

$$S^1_f\approx 0: \quad (5.22)$$

$$L^{1(i)}(\sigma)\equiv u^{m(i)}(\sigma)L^1_m(\sigma)=u^{m(i)}[P_m+\frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\partial_\sigma\theta^1\sigma_m\theta^1]\approx 0, \quad (5.22a)$$

$$\nu^{+\alpha}_{\mathring{A}}(\sigma)D^1_\alpha(\sigma)\approx 0, \quad (5.22b)$$

$$\Pi^{[+2](i)}(\sigma)\approx 0, \quad (5.22c)$$

$$\rho^{[+2]\tau}-\tfrac{1}{2}u^{m[+2]}L^1_m(\sigma)\approx 0, \quad (5.22d)$$

$$P^{[-2]}_{(\rho)\tau}(\sigma)\approx 0, \quad (5.22e)$$

$$S^2_f\approx 0: \quad (5.23)$$

$$L^{2(i)}(\sigma)\equiv u^{m(i)}(\sigma)L^2_m(\sigma)=u^{m(i)}[P_m-\frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\partial_\sigma\theta^2\sigma_m\theta^2]\approx 0, \quad (5.23a)$$

$$\nu^{-\alpha}_{A}(\sigma)D^2_\alpha(\sigma)\approx 0, \quad (5.23b)$$

$$\Pi^{[-2](i)}(\sigma)\approx 0, \quad (5.23c)$$

$$\rho^{[-2]\tau}-\tfrac{1}{2}u^{m[-2]}L^2_m(\sigma)\approx 0, \quad (5.23d)$$

$$P^{[+2]}_{(\rho)\tau}(\sigma)\approx 0. \quad (5.23e)$$

The nondegenerate symplectic structure Ω^{IJ}_{fg} with

$$[S^I_f,S^J_g\}_P\approx\Omega^{IJ}_{fg} \quad (5.24)$$

of the set of constraints (5.21) and (5.22) is block-diagonal and is defined by the relations

$$[S^1_f,S^1_g\}_P\approx\Omega^1_{fg}: \quad (5.25)$$

$$\{\nu^{+\alpha}_{\mathring{A}}D^1_\alpha(\sigma),\nu^{+\alpha}_{\mathring{B}}D^1_\alpha(\sigma')\}_P=2i\delta_{\mathring{A}\mathring{B}}u^{m[+2]}L^1_m(\sigma)\delta(\sigma-\sigma') \approx 4i\delta_{\mathring{A}\mathring{B}}\rho^{[+2]\tau}\delta(\sigma-\sigma'), \quad (5.25a)$$

$$[L^{1(i)}(\sigma),\nu^{+\alpha}_{\mathring{B}}D^1_\alpha(\sigma')]_P=-4i(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 1}\nu^{+}_{\alpha A}\gamma^{i}_{A\mathring{B}} \times\delta(\sigma-\sigma'), \quad (5.25b)$$

$$[L^{1(i)}(\sigma),L^{1(j)}(\sigma')]_P=-2(c\alpha')^{-1}(\delta^{ij}\partial_\sigma-\Omega^{ij}_\sigma)\delta(\sigma-\sigma'), \quad (5.25c)$$

$$[\Pi^{[+2](i)}(\sigma),L^{1(j)}(\sigma')]_P=\delta^{ij}u^{m[+2]}L^1_m(\sigma) \times\delta(\sigma-\sigma')\approx 2\delta^{ij}\rho^{[+2]\tau}\delta(\sigma-\sigma'), \quad (5.25d)$$

$$[(\rho^{[+2]\tau}-\tfrac{1}{2}u^{m[+2]}L^1_m)(\sigma),\quad L^{1(j)}(\sigma')]_P =(c\alpha')^{-1}\Omega^{[+2]i}_\sigma\delta(\sigma-\sigma'), \quad (5.25e)$$

$$[P^{[-2]}_{(\rho)\tau}(\sigma),(\rho^{[+2]\tau}-\tfrac{1}{2}u^{m[+2]}L^1_m)(\sigma')]_P=\delta(\sigma-\sigma'), \quad (5.25f)$$

$$[S^2_f,S^2_g\}_P\approx\Omega^2_{fg}: \quad (5.26)$$

$$\{\nu^{-\alpha}_{A}D^2_\alpha(\sigma),\nu^{-\alpha}_{B}D^2_\alpha(\sigma')\}_P=2i\delta_{AB}u^{m[-2]}L^2_m(\sigma)\delta(\sigma-\sigma') \approx 4i\delta_{AB}\rho^{[-2]\tau}\delta(\sigma-\sigma'), \quad (5.26a)$$

$$[L^{2(i)}(\sigma),\nu^{-\alpha}_{B}D^2_\alpha(\sigma')]_P=4i(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 2}\nu^{-}_{\alpha\mathring{A}}\gamma^{i}_{\mathring{A}B}\delta(\sigma-\sigma'), \quad (5.26b)$$

$$[L^{1(i)}(\sigma),L^{1(j)}(\sigma')]_P=2(c\alpha')^{-1}(\delta^{ij}\partial_\sigma-\Omega^{ij}_\sigma)\delta(\sigma-\sigma'), \quad (5.26c)$$

$$[\Pi^{[-2](i)}(\sigma),L^{2(j)}(\sigma')]_P=\delta^{ij}u^{m[-2]}L^2_m(\sigma)\delta(\sigma-\sigma') \approx 2\delta_{\mathring{A}\mathring{B}}\rho^{[-2]\tau}\delta(\sigma-\sigma'), \quad (5.26d)$$

$$[(\rho^{[-2]\tau}-\tfrac{1}{2}u^{m[-2]}L^2_m)(\sigma),\quad L^{2(j)}(\sigma')]_P=(c\alpha')^{-1}\Omega^{[-2]i}_\sigma \times\delta(\sigma-\sigma'), \quad (5.26e)$$

$$[P^{[+2]}_{(\rho)\tau}(\sigma),(\rho^{[-2]\tau}-\tfrac{1}{2}u^{m[-2]}L^2_m)(\sigma')]_P=\delta(\sigma-\sigma'). \quad (5.26f)$$

All the other brackets between pairs of constraints from the same set [either (5.20) or (5.21)] vanish in the strong sense. The brackets between constraints from different sets are all equal to zero in the weak sense:

$$[S^1_f,S^2_g\}_P\approx 0. \quad (5.27)$$

All the bracket relations which are nonvanishing in the strong sense involve the constraints (5.22c) or (5.23c):

$$[S^1_f,S^2_g\}_P\neq 0(\approx 0): \quad (5.28)$$

$$[\Pi^{[+2](i)}(\sigma),\Pi^{[-2](j)}(\sigma')]_P=(\delta^{ij}D^{(0)}+2D^{ij})\delta(\sigma-\sigma')\approx 0, \quad (5.28a)$$

$$[\Pi^{[+2](i)}(\sigma),\nu^{-\alpha}_{B}D^2_\alpha(\sigma')]_P=-\gamma^{j}_{B\mathring{A}}\hat{D}^{+}_{\mathring{A}}(\sigma)\delta(\sigma-\sigma')\approx 0, \quad (5.28b)$$

$$[\nu^{+\alpha}_{\mathring{A}}D^1_\alpha(\sigma),\Pi^{[-2](j)}(\sigma')]_P=\gamma^{j}_{B\mathring{A}}\hat{D}^{-}_{A}(\sigma)\delta(\sigma-\sigma')\approx 0, \quad (5.28c)$$

$$[\Pi^{[+2](i)}(\sigma),L^{2(j)}(\sigma')]_P=\delta^{ij}u^{m[+2]}L^2_m(\sigma)\delta(\sigma-\sigma') \equiv\delta^{ij}(2\rho^{[-2]\tau})^{-1}[L^2(\sigma) + \frac{4}{c\alpha'}\partial_\sigma\Theta^{\alpha 2}\nu^{-}_{\alpha\mathring{A}}\hat{D}^{+}_{\mathring{A}}(\sigma)+\frac{4}{c\alpha'}\partial_\sigma\Theta^{\alpha 2}\nu^{+}_{\alpha A}(\nu^{-\gamma}_{A}D^2_\gamma) - \frac{4}{c\alpha'}\rho^{[-2]\tau}(\partial_\sigma-\Omega^{(0)}_\sigma)P^{[+2]}_{(\rho)\tau} + \frac{4}{c\alpha'}\Omega^{[+2]i'}_\sigma\Pi^{[-2]i'}]\delta(\sigma-\sigma')\approx 0, \quad (5.28d)$$

$$[L^{1(i)}(\sigma),\Pi^{[+2]j}(\sigma')]_P=\delta^{ij}u^{m[-2]}L^1_m(\sigma)\delta(\sigma-\sigma') \equiv-(2\rho^{[+2]\tau})^{-1}[L^1(\sigma)-\frac{4}{c\alpha'}\partial_\sigma\Theta^{\alpha 1}\nu^{+}_{\alpha A}\hat{D}^{-}_{A}(\sigma) - \frac{4}{c\alpha'}\partial_\sigma\Theta^{\alpha 1}\nu^{-}_{\alpha\mathring{A}}(\nu^{+\gamma}_{\mathring{A}}D^1_\gamma)+\frac{4}{c\alpha'}\rho^{[+2]\tau} \times(\partial_\sigma+\Omega^{(0)}_\sigma)P^{[-2]}_{(\rho)\tau}-\frac{4}{c\alpha'}\Omega^{[-2]i'}_\sigma\Pi^{[+2]i'}\delta(\sigma-\sigma')\approx 0, \quad (5.28e)$$

$$[\Pi^{[+2]i}(\sigma),(\rho^{[-2]\tau}-\tfrac{1}{2}u^{m[-2]}L^2_m)(\sigma')]_P$$

$$=-L^{2(i)}(\sigma)\delta(\sigma-\sigma')\approx 0, \qquad (5.28f)$$

$$[(\rho^{[+2]\tau}-\tfrac{1}{2}u^{m[+2]}L^1_m)(\sigma),\Pi^{[-2]i}(\sigma')]_P$$

$$=L^{1(i)}(\sigma)\delta(\sigma-\sigma')\approx 0. \qquad (5.28g)$$

Hence, the symplectic structure (5.24) associated with the irreducible second-class constraints of the Green–Schwarz superstring has been derived in the framework of the twistor-like Lorentz-harmonic formulation.[22,23] Moreover, the second-class constraint algebra

$$[S^I_f,S^J_g\}_P=C^{IJhK}_{fg}S^K_h+C^{IJ\Sigma}_{fg}Y_\Sigma+\Omega^{IJ}_{fg}\approx\Omega^{IJ}_{fg} \qquad (5.29)$$

is described completely by the relations (5.25), (5.26), and (5.28).

5.3. Reparametrization generators and Virasoro conditions

Let us clarify the relation of the first-class constraints (5.11a) and (5.12a) to the well-known Virasoro conditions

$$V^1\equiv[P_m+(c\alpha')^{-1}\partial_\sigma x_m][P^m+(c\alpha')^{-1}\partial_\sigma x^m], \qquad (5.30a)$$

$$V^2\equiv[P_m-(c\alpha')^{-1}\partial_\sigma x_m][P^m-(c\alpha')^{-1}\partial_\sigma x^m], \qquad (5.30b)$$

which generate the reparametrization-symmetry transformation in the standard bosonic-string formulation (see, for example, Ref. 3). The expressions $[P_m+(c\alpha')^{-1}\partial_\sigma x_m]$ and $[P_m-(c\alpha')^{-1}\partial_\sigma x_m]$ are included in the blocks (5.4a), (5.5a) and can be identified with them, or with the blocks (5.1a), (5.2a), up to harmonic and Grassmannian constraints. Thus, we can discuss the forms of the Virasoro-condition generalizations in terms of these blocks.

It is sufficient to discuss only the Virasoro condition (5.30a).

The expression $\tilde{L}^1_m$ is included in the first-class constraint (5.11a), which can be reformulated as

$$u^{m[-2]}\tilde{L}^1_m(\sigma)+2(c\alpha'\rho^{\tau[+2]})^{-1}\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma)\approx 0, \qquad (5.31)$$

and which remains a first-class constraint after multiplication by $u^{n[+2]}\tilde{L}^1_n(\sigma)$:

$$u^{m[+2]}u^{n[-2]}\tilde{L}^1_m(\sigma)\tilde{L}^1_n(\sigma)$$
$$+2(c\alpha'\rho^{\tau[+2]})^{-1}u^{n[+2]}\tilde{L}^1_n(\sigma)\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma)\approx 0. \qquad (5.32)$$

On the other hand, the second-class constraint (5.22a) can be transformed into the constraint

$$\tilde{L}^1_m(\sigma)u^{m(i)}\approx 0, \qquad (5.33)$$

because the block $\tilde{L}^1_m(\sigma)$ (5.1a) differs from $L^1_m(\sigma)$ (5.4a) by the sum of constraints [see Eq. (5.3a)]. The square of the second-class constraint (5.33) is a first-class constraint, by definition. Thus, we have the (dependent) first-class constraint

$$\tilde{L}^1_m(\sigma)u^{m(i)}u^{n(i)}\tilde{L}^1_n(\sigma)\approx 0 \qquad (5.34)$$

in the dynamical system. The linear combination of Eqs. (5.32) and (5.34) in the form

$$u^{m[+2]}u^{n[-2]}\tilde{L}^1_m(\sigma)\tilde{L}^1_n(\sigma)-\tilde{L}^1_m(\sigma)u^{m(i)}u^{n(i)}\tilde{L}^1_n(\sigma)$$
$$+2(c\alpha'\rho^{\tau[+2]})^{-1}u^{n[+2]}\tilde{L}^1_n(\sigma)\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma)\approx 0 \qquad (5.35)$$

can be written as

$$\tilde{L}^1_m(\sigma)\tilde{L}^{m1}(\sigma)+2(c\alpha'\rho^{\tau[+2]})^{-1}u^{n[+2]}\tilde{L}^1_n(\sigma)\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma)\approx 0 \qquad (5.36)$$

(if the completeness conditions

$$\delta^n_m=\tfrac{1}{2}u^{[+2]}_m u^{n[-2]}+\tfrac{1}{2}u^{[-2]}_m u^{n[+2]}-u^{(i)}_m u^{n(i)} \qquad (5.37)$$

for the composed moving-frame variables (1.8), (1.9) is taken into account).

It is easy to see that the first-class constraint (5.36) coincides with the Virasoro condition (5.30a) up to the Grassmannian and harmonic degrees of freedom (which are absent in the standard bosonic-string formulation).

6. CONCLUSION

The classical mechanics of the twistor-like Lorentz-harmonic formulation of the $D=10$, $N=$IIB superstring[22,23] is based on the Lagrangian and Hamiltonian approaches. The equations of motion have been derived [Eqs. (3.40), (3.44), (3.47), and (3.51)] using the concept of the admissible variation (3.31) for the harmonic variables. The complete sets of Lorentz-covariant and irreducible first-class and second-class constraints are presented in Eqs. (4.43a)–(4.43g) and Eqs. (5.28)–(5.33), respectively. The algebra of the gauge symmetries [Eqs. (5.13)–(5.15) and (5.17)–(5.20)] and the symplectic structure associated with the set of second-class constraints [Eqs. (5.25), (5.26), (5.28), and (5.29)] have been calculated.

Thus, we have developed the machinery of the component twistor approach necessary for the next steps towards covariant quantization of the $D=10$ superstring, which consist in providing the conversion[7,54–56] of the second-class constraints into Abelian first-class ones and constructing the classical BRST charge (see Refs. 19, 42, and 43 for the case of null super-p-branes in $D=4$). These steps are under investigation now.

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[3)]Another method consists in attempts to extend the quantization scheme developed by Batalin and Vilkovisky[6] to the case of systems with infinitely reducible symmetries (see Refs. 9–12 and references therein). Such extensions use an infinitely reducible gauge-fixing condition and produce free-type effective actions including an infinite number of fields for superparticles and superstrings. However, the straightforward extension of the BV prescription[6] to systems with infinitely reducible constraints leads to well-known troubles.[10,11] The cohomologies of the superparticle BRST operator calculated in this way differ from the state spectrum of the Brink–Schwarz superparticle obtained from the quantization in the light-cone gauge (see Refs. 10 and 11). To achieve the correct BRST cohomology (i.e., state spectrum) it is necessary to modify not only the BV quantization prescription, but also the initial superparticle or superstring formulation. However, after this step, the second method is reduced to a variant of the first one.

4)The letter l in the round brackets in $n_m^{(l)}$ labels a vector from the moving-frame set $n_m^{(0)}, n_m^{(l)}, \ldots, n_m^{(D-1)}$. It is convenient to separate all the vectors $n_m^{(l)}$ into two sets $n_m^{(l)} = (n_m^{\{f\}}, n_m^{(i)})$, where $i=1,\ldots,D-2$ and $f=0,1$ (so that $n_m^{\{0\}}=n_m^{(0)}$ and $n_m^{\{1\}}=n_m^{(D-1)}$).

5)Thus, the SO(1,1) subgroup of the target-space Lorentz group SO(1,D−1) (acting on the matrix $n_m^{(l)}$ from the right) is identified with the Lorentz group of the string world-sheet in this formulation (cf. Refs. 22 and 23).

6)In this form the coincidence of the variables $n_m^{(l)}$ with the vector harmonics from Ref. 13 is evident. However, these variables were used for the first time for the string and superstring description in Ref. 41.

7)Of course, the simple expressions (4.23) and the initial definition of the Poisson brackets can be used for the calculations, because the Poisson brackets were not changed (see above).

8)It is interesting to note that for the case of a degenerate world-sheet metric, which corresponds to null superstrings (Refs. 19, 42, 43, 52, and 53), Eqs. (4.32a) and (4.32b) become consistent without using Eq. (4.33), and the secondary constraints (4.34) are absent (see Refs. 19, 42, and 43).

9)These formulations can be called half-twistor-like because only one of the Virasoro constraints is "twistorized" (i.e., is solved by using a twistor-like prescription) in them. It is this fact that explains the presence of the (heterotic-superstring) "boost" symmetry in them. Indeed, as is easy to see from the superstring formulation (3.10), (4.3), the inclusion of both types of harmonic variable is necessary precisely for the "twistorization" of both Virasoro constraints. The formulations in which only one of them is "twistorized" can be constructed using only one type of spinor harmonic and, consequently, may have the "boost" symmetry.

10)It is important to note that the choice of the generalized Lagrange multipliers is a very delicate point. If we try to use the components of the world-sheet set $e^{\sigma[\mp 2]}$ as the generalized Lagrange multipliers [instead of the components of the vector densities $\rho^{\sigma[\mp 2]} \equiv (\alpha')^{-1/2} e^{\sigma[\mp 2]}$], then the extraction of the corresponding first-class constraint becomes problematic because of the nonlinear dependence of the resulting expression for the generalized Hamiltonian on $e^{\sigma[\mp 2]}$. In our case this problem can be solved by using the relations (4.31) between the given variables and the original Lagrange multipliers, and requiring that the new generalized Lagrange multiplier must be expressed by a linear relation in terms of the original one.

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