Charge-exchange reactions involving nucleons and light ions at low and intermediate energies

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The main purpose of this review is to analyze resonance phenomena in charge-exchange processes of the type $a+A \to b+X$ (a, $b=p$, π, n, ^{3}He, t), both on free hadrons and on nuclei; inelastic charge exchange of protons and ^{3}He nuclei is considered in the most detail. There is now a large accumulation of experimental material on these processes, which requires a quantitative interpretation in the framework of a unified "microscopic" approach. The present review contains a brief summary of the main properties of form factors, spectroscopic factors, and sum rules for nucleons and composite particles. A discussion is given of typical methods of analyzing inclusive nuclear reactions in terms of effective numbers of nuclear clusters. An account is given of the formalism needed to describe processes involving pions, nucleons, and Δ isobars; attention is focused on charge-exchange reactions as a very simple and typical example. Consideration is given to the problem of determining the optimal set of phenomenological parameters of the $\pi+\rho+g'$ model, and also to the interference of the various diagrams of the effective theory. The last part of the review is devoted to a discussion of the reaction of charge exchange on nuclei with excitation of Δ isobars in terms of effective numbers; it is demonstrated that an important role is played by a number of many-body processes which are characteristic of reactions on nuclei but absent in the case of the reaction on free nucleons. Mention is also made of a number of general problems in the analysis of resonance reactions in nuclear physics and particle physics, and the prospects for further research are discussed.

INTRODUCTION

The main purpose of this review is to analyze resonance phenomena in charge-exchange processes of the type $a+A \to b+X$ (a, $b=p$, π, n, ^{3}He, t), both on free hadrons and on nuclei; inelastic charge exchange of protons and ^{3}He nuclei is considered in the most detail. There is now a large accumulation of experimental material on these processes (see the review of Ref. 1 and references therein), which requires a quantitative interpretation in the framework of a unified "microscopic" approach.

Inclusive reactions of the type (e,e'), (p,p'), (π,π'), (p,n), $(^3\text{He},t)$, $(^6\text{Li},^6\text{He})$ are a primary source of information about the effective NN and $N\Delta$ interactions, reaction mechanisms, and nuclear structure at intermediate energies. The study of these reactions led to the discovery of Gamow–Teller and other charge-exchange resonances, extended our knowledge of spin forces in nuclei, provided new information about non-nucleon degrees of freedom in the nucleus, and so forth. Among such processes, it is of special interest to consider the charge-exchange reactions (p,Δ^{++}), $(p,n)_\Delta$, $(^3\text{He},t)_\Delta$, $(^6\text{Li},^6\text{He})_\Delta$ involving the Δ isobar, since by analyzing these reactions it is possible to obtain information about the role of the Δ isobar in the spin and isospin components of the nuclear forces. There have been numerous studies of charge-exchange reactions by both theoreticians and experimentalists (see, for example, Refs. 1–3 and references therein). Information about the reaction mechanism on free nucleons is particularly important. The point is that, because of the relatively small ($\approx 5\%$ at $T_p \approx 1$ GeV) renormalization of the interaction in nuclei,[4–7] information about the free processes makes it possible in principle to understand the mechanisms of charge-exchange reactions on nuclei and to investigate the spin-isospin modes of excitation in nuclear matter.

However, at the present time there is no complete picture of the phenomenon. In particular, the existing approaches do not make it possible to distinguish correctly the effects of nuclear structure from the $(p,n)_\Delta$ reaction mechanism at incident-proton energies $T_p \geqslant 1$ GeV.[2] The analysis of the process is considerably complicated by collective excitations of the type (ΔN^{-1}), i.e., a Δ-isobar-nucleon "hole."[5–8] For intermediate energies, the renormalization of the $NN-N\Delta$ interaction in going from the vacuum to the nucleus has not been sufficiently investigated (see Refs. 2 and 4). As always, there remains the debatable technique of taking into account nuclear distortions in the entrance and exit channels in the energy region under consideration.[9–11] Finally, the analysis of inclusive cross sections has its own specific features due to the insensitivity to the details of the structure of the target nucleus.[9,10] Following the terminology of Refs. 9 and 10, we shall refer to this property as the "universality" of inclusive cross sections. In theoretical calculations, universality manifests itself in the fact that the results of the calculations are practically independent of the choice of the basis functions of the shell model. For example, if, instead of Woods–Saxon wave functions, one uses oscillator wave functions, semiclas-

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sical wave functions, or even wave functions of a free Fermi gas, the values of the cross sections are changed by only 5–10%.[9,10] Experimentally, universality manifests itself in the fact that the charge-exchange cross sections containing information on the $NN-N\Delta$ interactions and on the reaction mechanism are smooth functions of their arguments—the mass number A and the energy of the incident particle.

Another feature of the most popular methods of analyzing strong processes[4–8,12–19] (field-theory approaches,[4–8,12] the Glauber approximation,[14] and so forth) is their unmistakable phenomenological character, reflected in the fact that the theoretical formulas contain a large number of phenomenological parameters. This is partly due to the absence of small parameters like a dimensionless coupling constant.[12] In general, the momentum transfers q that are characteristic of the physics of hadronic resonances are also not small in comparison with the momentum (P) of the incident particles. In the region of intermediate energies, the long-wavelength approximation $kR \ll 1$, where k is the wave vector of the incident particle and R is the characteristic dimension of the target, is not satisfied. It should be pointed out that even the existence of a natural small parameter of the theory in the form $\langle V \rangle / E \ll 1$, i.e., the ratio of the average potential energy of a nucleon belonging to the target nucleus in the initial bound state to the kinetic energy of the incident particle, is by no means always a guarantee that the impulse approximation (DWIA) is valid: in the case of reactions on nuclei, resonance excitation of intranucleon degrees of freedom in the system of strongly interacting particles can lead to a substantial enhancement of the contribution from channels that are forbidden by the selection rules for processes on free nucleons.[20]

What we have said above enables us to understand the reasons for the persistent popularity of purely phenomenological approaches involving optical potentials, but it cannot justify them, since at the present time they no longer satisfy the requirements of experiments: there is an urgent need for a "microscopic" theory of processes involving nuclei at intermediate energies. The main difficulty in constructing such a theory, which one would like to see based on the first principles of quantum chromodynamics (QCD),[21] is associated with the fundamentally nonperturbative character of the problems in QCD that arise in the physics of intermediate energies. This is the reason for the phenomenological character of the theory of strong processes at intermediate energies and for the "hybrid" nature of the currently employed approaches (apart from purely kinematic ones[15–19]). These approaches are based on the use of fundamental principles of quantum theory, as well as on phenomenological parametrizations of certain auxiliary form factors in calculations of various observable quantities.

The clearest example of a theory of this kind is the theory of finite Fermi systems. However, the use of this theory is insufficiently correct for the analysis of the studied class of processes, which occur primarily at the surface of the target nuclei. This creates a need for alternative schemes for analyzing processes in the region of intermediate energies.

The inclusive character of most experimental data obtained at intermediate energies facilitates the theoretical analysis to some extent, since inclusive cross sections are insensitive to the fine details of the nuclear structure, such as effects of superfluid mixing of configurations, small deformations of the target nucleus, and so forth.[9,22] It is for this reason that inclusive reactions can be regarded as a convenient tool for studying the reaction mechanisms and the nuclear interaction in the region of intermediate energies. Results from the analysis of inclusive processes can then be used in the investigation of exclusive and "semiexclusive" reactions.

The special features of inclusive reactions enumerated above lead to perfectly natural requirements on the formalism for describing direct processes in the region of intermediate energies.

First, the formalism must be sufficiently universal, i.e., in the framework of a unified approach it must describe a large class of direct processes (including resonance ones), such as quasielastic knockout (p,pX), (e,eX), and $(\alpha,2\alpha)$, fragmentation, transfer of clusters between heavy ions, absorption of slow π^- mesons (π^-,A) and of high-energy γ rays (γ,A), cumulative nuclear reactions with backward emission of fast hadrons, reactions of charge exchange, spin flip, etc.

Second, the formalism must be sufficiently simple from the computational point of view to use it for effective analysis of a wide range of experimental data, and at the same time sufficiently informative to permit possible identification of the reaction mechanism.

Third, in view of the specific features of the studied range of energies ($T \sim 1$ GeV/nucleon), purely potential approximations like the DWBA become insufficiently correct, and therefore the formalism must include both nuclear-physics and field-theory aspects of the problem. Above all, such a formalism must take into account the coupling between the excited "external" degrees of freedom describing the motion of the constituent nucleons of the nucleus and the "internal" degrees of freedom of these constituents, since the interactions of intermediate-energy particles with free nucleons and with nuclei have an appreciable probability of exciting baryon resonances allowed by the energy-momentum conservation laws and by the selection rules.

Finally, a consistent scheme for analyzing inclusive reactions at intermediate energies must include methods of calculating the elementary processes [i.e., cross sections of the type $d\sigma_{a+X\to X+a}/d\Omega_X$; see Eq. (1) below].

Thus, we must seek possible ways of synthesizing the approaches of nuclear physics and of elementary-particle physics. Here we are led naturally to a number of questions which constitute the essence of the problem; many of them go beyond the scope of our review.

• What are the mechanisms of excitation and deexcitation of baryon resonances, especially of the Δ isobar and the Roper resonance $N(1440)$?

• Are these mechanisms different for free nucleons and for nucleons in nuclei?

• What is the reason for the high selectivity of the excitation of baryon resonances in reactions on nuclei?

• What are the optimal conditions for observing particular resonance phenomena?

• What is the sensitivity of the results of calculations to the choice of the model, and how can it be minimized?

• What properties of resonances are determined by the first principles of quantum theory, and what properties can be reproduced on the basis of specific models?

• What kinds of experiments in the physics of intermediate energies are most informative for identifying the reaction mechanism?

Some of the questions in this list were posed in the late 1920s and early 1930s, i.e., during the period when the basic principles of quantum theory were formulated. The questions pertaining directly to the physics of baryon resonances began to appear after 1953, i.e., after Fermi's group discovered the first and most prominent of the nucleon resonances: the $\Delta(1232)$ isobar.

Despite the enormous quantity of impressive results obtained since the discovery of the Δ isobar, an understanding of the true role of this resonance in the physics of intermediate energies began to emerge only by the early 1980s, when it became clear that many effects that had previously been interpreted as a manifestation of short-range nucleon-nucleon correlations are actually due to the contribution of baryon resonances [especially the $\Delta(1232)$ isobar]. There are now strong reasons for supposing that this resonance in the pion nuclear physics of intermediate energies survives as a separate form of baryons even in the strongly interacting nuclear environment and can be regarded as a quasiparticle in exactly the same way as the nucleon.[4–6,23–25]

In attempting to satisfy the enumerated requirements on the formalism, there arises the question of finding a single standard for comparing the cross sections for different direct nuclear processes with emission of nucleons, baryon resonances, and clusters. In our opinion, the concept of "effective numbers" of clusters in the target nucleus makes it possible to find an answer to this question.

Several monographs[9,22,26,27] contain fairly detailed studies and descriptions of direct processes (mainly for the example of quasielastic knockout of nucleons and clusters $X=p$, d, t, ^{3}He, α, whereas we are much more interested in the production of resonances $X=\Delta^{++}$,...), which satisfy, with fairly good accuracy, a relation of the type

$$\frac{d\sigma_{A(a,aX)B}}{d\Omega_X}=\tilde{N}\,\frac{d\sigma_{a+X\to X+a}}{d\Omega_X}, \qquad (1)$$

where $d\sigma_{A(a,aX)B}/d\Omega_X$ is the cross section for knockout of the final cluster X from the nucleus A, $d\sigma_{a+X\to X+a}/d\Omega_X$ is the analogous free cross section, and $\tilde{N}$ is the effective number of clusters X in the target nucleus A.

Factorized relations of the type (1) arise in a natural way in the impulse approximation (see Ref. 28), and also in certain other cases[20] (for more details, see Ref. 27).

In most of the studied cases, the value of $\tilde{N}$ is determined by the properties of the target nucleus A and by the absorption factors for the incident particle and detected fragment in the nuclear medium; it is practically independent of the reaction mechanism. Fulfillment of the relation (1) actually means that the DWIA is applicable. Therefore the discrepancy between experimental data and the results of inclusive DWIA calculations contains valuable information about the reaction mechanism.

The results of studies of the effective numbers $\tilde{N}$ summarized in Refs. 9, 22, 26, 27, and 29 indicate a smooth A and E dependence of the total effective numbers in going from one target nucleus to another, and also for moderate variation of the incident-particle energy in the region $T>0.6$ GeV/nucleon. This behavior of the effective numbers indicates the possibility of semiclassical estimates of the dependence $\tilde{N}=\tilde{N}(E,A)$.

Thus, the study of resonance phenomena in inclusive nuclear reactions leads to an obvious succession of problems: 1) study of the combinatorial properties of the effective numbers; 2) investigation of the phenomenon of nuclear distortions; 3) field-theoretical analysis of free processes; 4) identification of the reaction mechanisms; 5) analysis of the general physical conception of a resonance.

From this standpoint, we consider in our review inelastic charge exchange of protons and ^{3}He nuclei with excitation of the Δ isobar as the most typical example of charge-exchange processes of the type $a+A\to b+X$. The first two sections are devoted to the main results obtained in nuclear physics in the development of the method of effective numbers; $NN\to N\Delta$ processes are discussed in Secs. 3–5; in Sec. 6 we consider the results of analysis of the reactions $A(p,n)_\Delta B$ and $A(^3\mathrm{He},t)_\Delta B$ in the language of effective numbers.

1. SUM RULES FOR NUCLEONS, DEUTERONS, *t*, ^{3}He, AND α PARTICLES, AND THEIR PROPERTIES

Let us consider a real or virtual process of decay of a nucleus A into fragments X and $A-X$:

$$A\to X+(A-X). \qquad (2)$$

We define the form factor of particle X in channel c in accordance with Ref. 30:

$$\Psi_{Xc}(R)=\left\langle \hat{A}\left\{\frac{\sigma(R-R')}{R'}\,\mathfrak{U}_{Xc}(\mathbf{R}')\right\}\middle|\,\Psi_{\sigma_i}^{J_iM_i}\right\rangle, \qquad (3)$$

where $\Psi_{\sigma_i}^{J_iM_i}$ is the wave function of the parent nucleus A with spin J_i, spin projection M_i, and other quantum numbers σ_i; $\mathfrak{U}_{Xc}$ is the channel function of particle X, c is the complete set of channel quantum numbers, including the spins and the spin projections of the fragments, and $\hat{A}$ is the operator of antisymmetrization between the nucleons of the daughter nucleus $A-X$ and of the fragment X.

For the nucleon channel, in the standard notation for the spherical and spin functions and for the Clebsch-Gordan coefficients, the expression for the channel function $\mathfrak{U}_{Nc}(\mathbf{R}')$ has the form[31]

$$\mathfrak{U}_{Nc}(\mathbf{R})=\sum_{\substack{m_S,m_l\\ M_f,m}}(J_f,j,M_f,m|J_c,M_c)\times(l,\tfrac{1}{2}m_f,m_S|j,m)Y_{lm_l}(\mathbf{R})\chi_{1/2,m_S}(\boldsymbol{\sigma})\Psi_{\sigma_f}^{J_fM_f} \qquad (4)$$

where $\Psi_{\sigma_f}^{J_fM_f}$ is the wave function of the daughter nucleus and $\hat{A}$ is the antisymmetrization operator

$$\hat{A}=\frac{1}{\sqrt{N}}\sum_{p}(-1)^{p}\hat{P}\{\ldots\},\tag{5}$$

in which the sum over p includes all distinct permutations between separable neutrons (for definiteness, we assume that it is a neutron that is separated, and we introduce the neutron normalization $1/\sqrt{N}$ in the antisymmetrization operator) and neutrons belonging to the residual nucleus.

To calculate the expression (3) with the channel function (4), we use the shell basis in the jj coupling scheme. We introduce the standard spin-orbit function

$$\Phi_{jlm}(\mathbf{X})=\sum_{m_S,m_l}(1,\tfrac{1}{2},m_l,m_S|j,m)Y_{lm_l}(\mathbf{r})\chi_{1/2,m_S}(\boldsymbol{\sigma})\tag{6}$$

and the radial shell wave function $\mathfrak{U}_{nlj}(r)$. Then the complete basis function $\varphi_{nljm}(\mathbf{X})$ can be written in the form

$$\varphi_{nljm}(\mathbf{X})=\mathfrak{U}_{nlj}(r)\Phi_{jlm}(\mathbf{X}).\tag{7}$$

The subscript n in (7) is the radial quantum number. For states of the discrete spectrum, it takes integer values. In the case of the continuum, we shall use for n the asymptotic momentum $\mathbf{p}$ of the nucleon.

Let us expand the expression $[\delta(R-R')/R']\Phi_{jlm}(\mathbf{X})$ in Eq. (3) in a series in terms of the complete set of single-particle functions $\varphi_{nljm}(\mathbf{X})$ in the form

$$|\chi_{jlm}(\mathbf{R})\rangle\equiv\left|\frac{\delta(R-R')}{R'}\Phi_{jlm}(\mathbf{X})\right\rangle=\sum_{n}\mathfrak{U}_{nlj}(R)R\varphi_{nljm}(\mathbf{R},\boldsymbol{\sigma})\tag{8}$$

and go over to the second-quantization representation. Then, according to Ref. 30, we have

$$|\chi_{jlm}(\mathbf{R})\rangle=\sum_{n}\mathfrak{U}_{nlj}(R)Ra_{\nu}^{+}|0\rangle,\tag{9}$$

where a_{ν}^{+} is the operator of creation of a nucleon in the shell state with quantum numbers $\nu\equiv n,l,j,m$.

Substituting (9) into (3), we obtain the final expression for $\Psi_{Nc}(R)$:

$$\Psi_{Nc}(R)=\sum_{n}G_{n}^{1}(J_i,\sigma_i,J_f,\sigma_f,l,j)R\mathfrak{U}_{nlj}(R),\tag{10}$$

where

$$G_{n}^{1}(J_i,\sigma_i,J_f,\sigma_f,l,j)\equiv\langle\{\Psi_{\sigma_f}^{J_fM_f}|a_{nijm}\}_{J_iM_i}|\Psi_{\sigma_i}^{J_iM_i}\rangle\tag{11}$$

is the shell fractional-parentage coefficient for separation of the nucleon.[32]

We note that the definition of the fractional-parentage coefficient used in Ref. 32 agrees with that of Ref. 33 but differs somewhat from the analogous definition of Ref. 34. The relation between the coefficients in Refs. 32 and 34 is given by the Wigner–Eckart theorem and reduces to the appearance of an additional Clebsch–Gordan coefficient in the fractional-parentage coefficient of Ref. 34.

Using the technique of Refs. 33 and 34, it is easy to obtain explicit expressions for the single-particle fractional-parentage coefficients corresponding to a transition between the ground states of the parent and daughter nuclei.[32] In the case of the simple shell model,

$$G_{n}^{1}(\text{sh.})=\begin{cases}\delta_{j,J_{Nf}}\sqrt{2n_j}, & \text{even subsystem } N_i\\ \delta_{j,J_{Ni}}\sqrt{1-n_j/\Omega_j}, & \text{odd subsystem } N_i,\end{cases}\tag{12}$$

where $2n_j(2n_j-1)$ is the number of particles in subshell j in the even (odd) subsystem N_i of the parent nucleus, and $\Omega_j=(2j+1)/2$ is the maximum number of pairs n_j in subshell j; the Kronecker symbol δ in (12) takes into account the angular-momentum selection rules.

The analogous fractional-parentage coefficients in the superfluid shell model have the form[32]

$$G_{n}^{1}(\text{su.})=\begin{cases}\delta_{j,J_{Nf}}(-1)^{l}v_j^{i}\sqrt{2\Omega_j}, & \text{even subsystem } N_i\\ \delta_{j,J_{Ni}}u_j^{f}, & \text{odd subsystem } N_i.\end{cases}\tag{13}$$

The superfluid coefficients u_j and v_j are determined by the expressions[33,35]

$$u_j^2+v_j^2=1,\tag{14}$$

$$v_j^2=\frac{1}{2}\left\{1-\frac{\varepsilon_j-\lambda}{\sqrt{(\varepsilon_j-\lambda)^2+\Delta_j^2}}\right\},\tag{15}$$

where ε_j, λ, and Δ_j are, respectively, the energy, chemical potential, and energy gap for subshell j.

In Table I we present the absolute values of the fractional-parentage coefficients for the Fermi levels $(j_n)_F$ of the neutron subsystems of parent nuclei with $115\leqslant N\leqslant 145$, $Z\approx 80$, calculated in the version of constant pairing,[32] $\Delta_j=\Delta=\text{const}$. It can be seen from Table I that the single-particle coefficients G_n^1 for filling of the subshells increase (decrease) monotonically in the case of even (odd) subsystems N_i. The superfluid coefficients $G_n^1(\text{su.})$ are close in value to the corresponding shell coefficients $G_n^1(\text{sh.})$ for a given subshell $(J_n)_F$. Elsewhere, the coefficients G_n^1 vanish. In addition, we note that the coefficients $G_n^1(\text{sh.})$ and $G_n^1(\text{su.})$ are not only numerically similar, but also satisfy the correspondence principle. According to Refs. 32, 33, and 35, the limiting transition from the superfluid shell model to the ordinary shell model can be made by means of the substitution $v_j^2\to n_j/\Omega_j$ for an even subsystem N_i of the parent nucleus, and $v_j^2\to n_j/(\Omega_j-1)$ for an odd subsystem. Clearly, in this limit the superfluid fractional-parentage coefficients reduce to the coefficients of the simple shell model.[32]

An exposition of the general theory of fractional-parentage coefficients for both the simple shell model and the translationally invariant shell model can be found in the monograph of Ref. 27.

The form factors of composite particles $(d,t,{}^3\text{He},\alpha)$ have a somewhat more complex structure, but they can also be calculated by means of the computational technique described above.[9,29–35]

In the case of separation of a deuteron, the channel function $\mathfrak{U}_{dc}(R)$ is determined by the relation[30,31]

TABLE I.

	Odd neutron subsystems			Even neutron subsystems		
nlj	N	G_n^1(sh.)	G_n^1(su.)	N	G_n^1(sh.)	G_n^1(su.)
$3p^{3/2}$	115	1,0	0,778	116	1,414	1,406
	117	0,707	0,711	118	2,0	1,566
$2f^{5/2}$	119	1,0	0,801	120	1,414	1,913
	121	0,816	0,625	122	2,0	2,160
	123	0,577	0,472	124	2,449	2,398
$2g^{9/2}$	127	1,0	—	128	1,414	1,227
	129	0,894	0,922	130	2,0	1,717
	131	0,775	0,840	132	2,449	2,068
	133	0,632	0,757	134	2,828	2,356
	135	0,447	0,667	136	3,162	2,558
$1i^{11/2}$	137	1,0	0,899	138	1,414	1,770
	139	0,913	0,860	140	2,0	2,026
	141	0,816	0,811	142	2,499	2,283
	143	0,707	0,752	144	2,828	2,522
	145	0,577	0,686	146	3,162	2,723

$$\mathfrak{U}_{dc}(\mathbf{R})=\sum_{\substack{M_f,M,l,m_l\\ M_l,M_L,M_S}}(J_f,J,M_f,M|J_c,M_c)\times(L,1,M_L,M_S|J,M)(L,1,M_L,M_S|J,M)\chi_{dl}(\mathbf{r})\times Y_{lm_l}(\mathbf{r})\chi_{1M_S}(\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n)Y_{LM_L}(\mathbf{R})\Psi_{\sigma_f}^{J_fM_f}, \quad (16)$$

where $\mathbf{x}_p=\{\mathbf{r}_p,\boldsymbol{\sigma}_p\}$ ($\mathbf{x}_n=\{\mathbf{r}_n,\boldsymbol{\sigma}_n\}$) represents the coordinate and spin of the proton (neutron) forming the deuteron, $\mathbf{R}$ is the coordinate of the motion of the centers of mass of the deuteron and daughter nucleus $(A-2)$ [for $A\gg1$, we have $\mathbf{R}=(\mathbf{r}_p+\mathbf{r}_n)/2$], $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$, $\chi_{dc}(\mathbf{r})$ is the component of the internal wave function of the ground state of the deuteron with relative angular momentum l ($l=0,2$), $\chi_{1M_S}(\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n)$ is the spin function, and L is the orbital angular momentum of the relative motion of the deuteron and residual nucleus. The antisymmetrization operator $\hat{A}$ in this case includes the distinct permutations of both neutrons (N) and protons (Z) of the daughter nucleus with the nucleons of the separated deuteron:

$$\hat{A}=\frac{1}{\sqrt{NZ}}\sum_p(-1)^p\hat{P}\{\ldots\}. \quad (17)$$

The form factors for t, ^{3}He, and α particles are determined in exactly the same way (for more details, see Refs. 9, 30, and 31).

The relation (3) makes it possible to determine the spectroscopic factor of fragment X in channel c:[9,30,31]

$$W_{Xc}=\int_0\Psi_{Xc}^2(R)dR. \quad (18)$$

From (10) it is easy to see that for closed nucleon channels, by virtue of the normalization condition for the shell wave functions, we have

$$W_{Nc}=|G_n^1(J_i,\sigma_i,J_f,\sigma_f,l,j)|^2 \quad (19)$$

(see Table I and the comments therein).

The deuteron form factors and spectroscopic factors have been studied by a number of authors.[30,35–40] In these studies, it has been shown that: 1) the deuteron spectroscopic factors W_{dc} in heavy spherical nuclei are fairly large ($W_{dc}\sim10^{-1}$–10^{-3}) and may even exceed the α-particle spectroscopic factors for favored α transitions ($\sim10^{-3}$), which have an appreciable superfluid enhancement[9] (by up to three orders of magnitude); 2) the effect of "alignment"[9,26,27] of the angular momenta of the separated np pair may enhance the deuteron spectroscopic factors by 1–2 orders of magnitude for large angular momenta L; 3) the contribution of the d component ($l=2$) to the spectroscopic factor W_{dc} is appreciable only for significantly suppressed transitions.[9,30] This means that in Eq. (16) in all the calculations one can make use of the internal wave function of the deuteron in the approximation $\chi_{dl}(\mathbf{r})=\chi_{d0}(\mathbf{r})$, discarding the term with $l=2$.

In inclusive reactions at intermediate energies, a large number of levels of the residual nucleus are excited, and its states are not detected. Therefore it seems expedient to measure the yield of fragments X by means of the quantity

$$W_X=\sum_c W_{Xc}, \quad (20)$$

where the summation is taken over all open and closed channels of decay of the nucleus A into the subsystems X and $A-X$.[9]

The cross sections for various direct processes with emission of the fragment X can be estimated on the basis of the known distributions of the spectroscopic factors W_{Xc} in the energies, momenta, etc.

The study of sum rules of the type (20) was begun by Lane,[41] who showed that the sum of the spectroscopic factors of the nucleons for a given shell is equal to the number of nucleons in that shell.

Further progress in developing the language of effective numbers was made in studying the phenomenon of clustering. The main results of such studies for light nuclei ($A \leq 20$) have been obtained by Neudachin's group. The total and partial sums of the d, t, ^{3}He, and α spectroscopic factors for nuclei of the $1p$ shell were studied in Refs. 42–44 (see the monographs of Refs. 26 and 27).

For heavy nuclei, an analogous series of investigations was carried out by Kadmenskiĭ's group.[9]

According to (8), the total nucleon spectroscopic factor W_N has the form

$$W_N=\sum_c W_{Nc}=\int_0^\infty \overline{\Psi_N^2(R)}dR, \tag{21}$$

where

$$\overline{\Psi_N^2(R)}=\sum_c \Psi_{Nc}^2(R). \tag{22}$$

Equations (8) and (9) enable us to write (22) as

$$\overline{\Psi_N^2(R)}=\sum_{\substack{M_f,M,J_f,\sigma_f,j,l\\ M_f,M',J_c,M_c,n,n'}} (J_f,j,M_f,M|J_c,M_c) \times(J_f,j,M_f',M'|J_c,M_c)\langle\Psi_{\sigma_i}^{J_iM_i}|a_{nljM}^{+}|\Psi_{\sigma_f}^{J_fM_f}\rangle \times\langle\Psi_{\sigma_f}^{J_fM_f'}|a_{nljM'}|\Psi_{\sigma_i}^{J_iM_i}\rangle\mathfrak{U}_{nlj}(R)\mathfrak{U}_{n'lj}(R)R^2. \tag{23}$$

The orthonormality and completeness of the Clebsch–Gordan coefficients

$$\sum_{J_c,M_c} (J_f,j,M_f,M|J_c,M_c)(J_f,j,M_f',M'|J_c,M_c) =\delta_{M_f'M_f}\delta_{M'M} \tag{24}$$

and the completeness of the system of wave functions $\Psi_{\sigma_f}^{J_fM_f}$ for the daughter nucleus permit a substantial simplification of Eq. (23)

$$\overline{\Psi_N^2(R)}=\sum_{n',n,j,l,m} \mathfrak{U}_{nlj}(R)\mathfrak{U}_{n'lj}(R)R^2 \times\langle\Psi_{\sigma_i}^{J_iM_i}|a_{nljM}^{+}a_{n'ljM}|\Psi_{\sigma_i}^{J_iM_i}\rangle =\sum_\nu n_\nu^iR^2\mathfrak{U}_{nlj}^2(R), \tag{25}$$

where n_ν^i is the occupation number of the state $\nu=(n,l,j,m)$ in the parent nucleus A.

The quantity $\overline{\Psi_N^2(R)}$ is related to the single-nucleon density $\rho_N(R)$ by the simple equation

$$\overline{\Psi_N^2(R)}=\sum_\nu n_\nu^iR^2\mathfrak{U}_{nlj}^2(R)=4\pi R^2\rho_N(R). \tag{26}$$

Therefore the sum of the spectroscopic factors W_N is

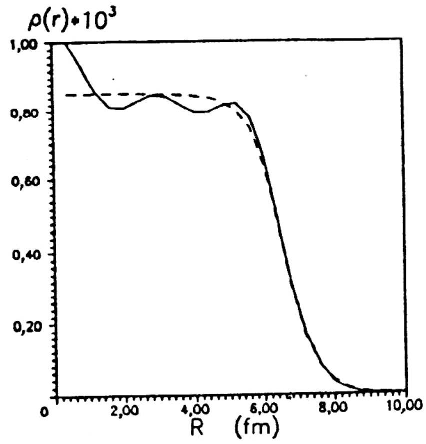


FIG. 1. Single-nucleon density for the nucleus ^{208}Pb. The dashed curve is for the Woods–Saxon parametrization, and the solid curve is the calculation in the Hartree–Fock approximation.

$$W_N=\begin{cases} N & \text{for neutrons,}\\ Z & \text{for protons.}\end{cases} \tag{27}$$

In deriving Eq. (25), we used only the properties of orthonormality and completeness of the shell basis. Therefore the final result is model-independent in the sense that it does not depend on the specific form of the basis wave functions of the shell model; i.e., the given results also hold in the case of $L-S$ coupling, in Migdal's λ representation for the theory of finite Fermi systems,[4–6] and in the approximation of a free Fermi gas, which is widely used in the physics of intermediate energies.[34]

The properties of the occupation numbers for various Fermi systems, including nuclei, have been well studied.[4–6] Therefore we note here only that Eqs. (25) and (26) correspond to the ordinary Hartree approximation.

The effective numbers of nucleons and their distributions have been calculated in various studies (see, for example, the monograph of Ref. 45 or the review of Ref. 46).

In Fig. 1 we show a typical plot of the distribution of the nucleon density in the Hartree approximation. Figure 2 shows a typical distribution of W_{Nl} as a function of the angular momentum l of the separated nucleon.

Let us consider in more detail the distribution of nucleons in the separation energies $|\varepsilon_j|$. Since the sum (21) contains comparable contributions from practically all shell states ν, and not just those near the Fermi surface, it is necessary to take into account the deviation of ε_j from the analogous value ε_j^0 given by the simple shell model.[4] In analyzing the dependence of ε_j on ε_j^0, it is necessary in principle to include effects of fragmentation of shell states far from the Fermi surface with respect to states of three, five, etc., quasiparticles, as well as the energy dependence of the shell potential $V(r,\varepsilon)$.[4,47] The first effect leads to the existence of a finite width Γ_j of the single-particle level j. In Refs. 9 and 48 it was shown that, by virtue of the condition $\Gamma_j \ll D_0 \approx 2\omega$ (where D_0 is the energy interval between neighboring shell states with given l and j), it is permissible to neglect the influence of the fragmentation effect on estimates of the

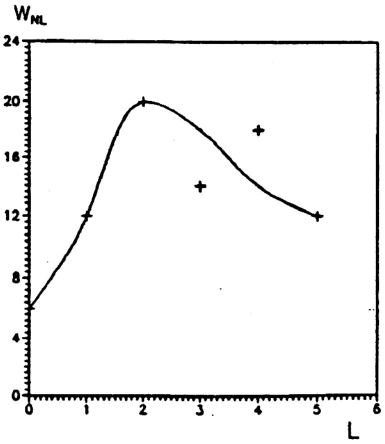


FIG. 2. Distribution of effective numbers W_{Nl} as a function of the angular momentum l of the separated nucleon for the nucleus ^{208}Pb. The solid curve is for the Fermi-gas approximation.

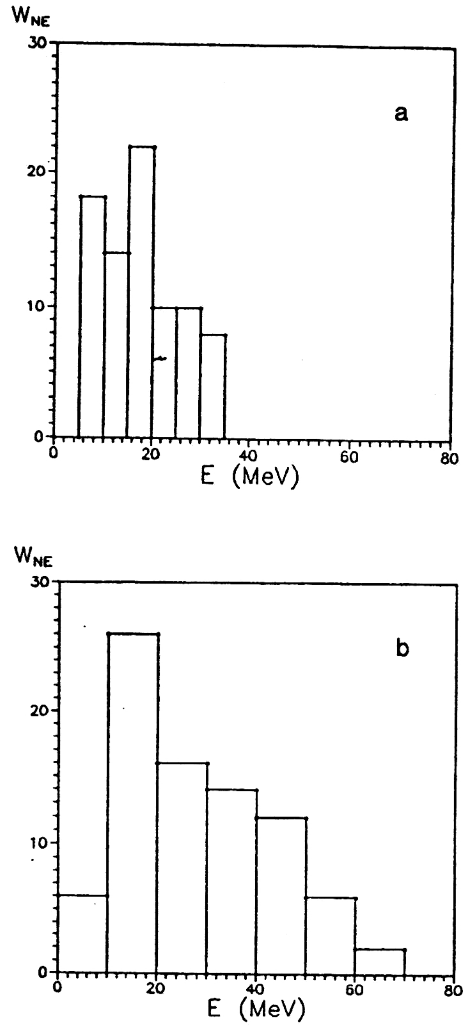


FIG. 3. Distribution of effective numbers W_{NE} in the nucleon separation energy: a) without allowance for the energy dependence of the depth of the shell potential $V(E)$; b) with allowance for this dependence.

cross sections for various reactions on the basis of the sum rule (20). The second effect is important[9] and can be taken into account by using the method of Ref. 47, in which an analysis of the $(p,2p)$ reaction led to an approximate expression for the depth of the potential $V(\varepsilon)$:

$$V(\varepsilon)=V_0+\alpha_0(\mu-\varepsilon), \tag{28}$$

where V_0 is the depth of the potential near the Fermi surface, which is the same as the depth of the analogous potential in the simple shell model, $\alpha_0=-0.52$, and μ is the chemical potential. Estimates of α_0 on the basis of optical calculations[34] give the value $\alpha_0=-0.30$, which is noticeably different from the value $\alpha_0=-0.52$ in Ref. 47. In Refs. 9 and 47 it was shown that the replacement of V_0 by $V(\varepsilon)$ leads to a renormalization of the single-particle shell energies:

$$\varepsilon_j=\varepsilon_j^0+\alpha_0\,\frac{\mu-\varepsilon_j^0}{1+\alpha_0}. \tag{29}$$

According to Ref. 9, when $|\varepsilon_j^0|\gg|\mu|$ we have $\varepsilon_j\approx 2\varepsilon_j^0$, whereas near the Fermi surface we have $\varepsilon_j\approx\varepsilon_j^0$; this leads to an "extension" of the histogram for the partial effective nucleon number in a given energy interval $\Delta E=E_2-E_1$:

$$W_{NE}=\sum_{n,j,l,m} W_{nljm}\theta(\varepsilon_{nlj}-E_2)\,\theta(E_1-\varepsilon_{nlj}). \tag{30}$$

The effect described above is demonstrated in Fig. 3.

A systematic study of sum rules for the d, t, ^{3}He, and α spectroscopic factors in the case of heavy spherical nuclei was carried out by Kadmenskiĭ and his coworkers in Refs. 30–32 and 48–50, and this work is summarized in the monograph of Ref. 9. We shall list briefly the main results of these investigations.

It was shown in the studies mentioned above that the sum rules for d, t, ^{3}He, and α are given by properties of the shell potential that are universal for all spherical nuclei and are even practically independent of the form chosen for the self-consistent field. It was established by direct calculation that the effective numbers W_x for $X=d$, t, ^{3}He, α are large ($W_x>A$ for nuclei heavier than ^{16}O) and are stable with respect to effects of configuration mixing, even-odd effects, and so forth.

The classifying possibilities of the energy distributions of the effective numbers were demonstrated, and the limits of applicability of the relations for the quantities W_X were investigated. Expressions were constructed for the inclusive cross sections for quasielastic knockout (p,pX) using the method of distorted waves. It was shown that the angular spectra for quasielastic knockout (p,pX) in the case of large momentum transfers are proportional to the analogous spectra for elastic pX scattering under similar kinematic conditions.

In Refs. 10 and 51 (see also Ref. 9) an investigation was made of the approximation of effective numbers with allowance for nuclear distortions in the Glauber–Sitenko formalism and the optical model. It was found that this approximation makes it possible to reproduce theoretically the shape of the momentum spectra for the $(p,p'd)$ reaction over a wide range of energies ($0.38\leqslant T_p\leqslant 1.4$ GeV) and target nuclei ($4\leqslant A\leqslant 64$), as well as the absolute values of $d\sigma/d\Omega_d$ in the case of low energies $T_p=0.38$ GeV. At the same time, for $T_p>0.38$ GeV the theoretical values $\bar{W}_d$ differ appreciably from the corresponding experimental values $\bar{W}_d^{\exp}$ and do not reproduce the A dependence $\bar{W}_d^{\exp} \sim A^{1/3}$. This last fact indicates a limitation of the approximation of effective numbers in the region of intermediate energies.

The further logic of the investigations carried out in the studies mentioned above is connected with the conception of short-range nucleon-nucleon correlations.

The use of the hypothesis[52] that the shell wave function of a few-nucleon group X, $\Psi_{\nu 1 \ldots \nu x}=\hat{A}\{\{\Pi_{j=1}^{x}\Pi_{i<j}^{x} f(\mathbf{r}_i - \mathbf{r}_j)\varphi_\nu(\mathbf{x}_i)\}$ (where f is the Jastrow factor[53]), is proportional (in the case of small relative distances between the nucleons) to the wave function of the corresponding free fragment X, $\chi_x(\xi)$, according to the relation

$$\Psi_{\nu 1 \ldots \nu x} \to (C_x)^{x-1}\chi_x(\xi)\prod_{i=1}^{x} \varphi_{\nu_i}(\mathbf{R},\boldsymbol{\sigma}_i) \tag{31}$$

(where ξ is the set of internal coordinates of the fragment X, and $\mathbf{R}$ is the position of its center of mass) makes it possible to represent the effective number $\bar{W}_x$ for the (p,pX) reaction in the form

$$\bar{W}_x = F_x(C_x^2)^{x-1}\int d\mathbf{R} f_x^2(b_x,z)\rho^x(R), \tag{32}$$

in which F_x is a combinatorial factor $F_d=3/4NZ$; $F_t=(N(N-1)/4)Z$; $F_{^3\mathrm{He}}=(Z(Z-1)/4)N$; $F_\alpha=1/16N(N-1)Z(Z-1)$, $\rho(R)$ is the single-nucleon density, normalized by the condition $\int d\mathbf{R}\rho(R)=1$, b is the impact parameter, and $f_x^2(b,z)$ is the nuclear distortion factor, calculated on the basis of the Glauber–Sitenko method.[10,14,51,54] The authors of Refs. 9, 10, and 51 made use of a phenomenological approach, in which they obtained the approximately constant parameter value $C_d^2=180\mp 50$ fm^3 within the error corridor.

In a number of studies (see, for example, Ref. 55) it was stated that excitation of a virtual Δ isobar can lead to a significant enhancement of the cross section for free pd backward scattering in a narrow range of kinetic energies. Using this fact, and also the results of Refs. 23 and 24 on $(p,n)_\Delta$ charge-exchange reactions in the region of Δ-isobar excitation, we construct the dependence of C_d^2 from (32) on the incident-proton energy on the same scale with the cross section for excitation of the Δ isobar in the reaction $p+p\to n+\Delta^{++}$. The results of this comparison are shown in Fig. 4, from which it can be seen that excitation of baryon resonances and short-range NN correlations can in principle imitate each other.

2. INTEGRAL METHODS OF ANALYZING INCLUSIVE NUCLEAR REACTIONS ON THE BASIS OF EFFECTIVE NUMBERS

In the Introduction we have already spoken about problems associated with going beyond the DWIA and about the method of effective numbers. This section is devoted to the application of the method of effective numbers for the studied reactions.

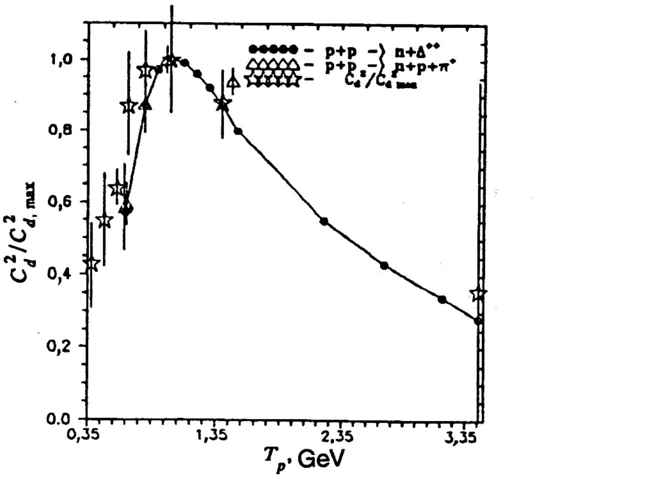


FIG. 4. Ratios $C_d^2/C_{d\max}^2$ and cross sections for the reactions $p+p\to n+p+\pi^+$ and $pp\to n\Delta^{++}$ (normalized with respect to their maximum values) as functions of the kinetic energy of the incident proton.

Our discussion will be based systematically on the results obtained in Refs. 9, 10, and 27 from the analysis of inclusive (p,pX) reactions involving knockout of clusters, which are kinematically analogous to the $(p,n)_\Delta$ process. We shall use the system of units in which $\hbar=c=1$.

In Refs. 2 and 56–59 it was argued in detail that the $(p,n)_\Delta$ reaction can be studied by the method of distorted waves. On this basis, we write the invariant cross section for the $(p,n)_\Delta$ reaction in the form[28]

$$d\sigma=\frac{2E_iE_A}{\lambda^{1/2}(S_{NA},m_N^2,M_A^2)}\frac{1}{2}\frac{1}{2J_i+1}\sum(2\pi)^4\delta^{(4)} \times(P_i+P_A-P_n-P_f)|T_{B+\Delta,A}^{np}|^2 d\mathbf{P}_n, \tag{33}$$

where the summation goes over the quantum numbers m, M_i, and f. In the first Born approximation, the matrix element of the T matrix has the form

$$T_{B+\Delta,A}^{np}=\left\langle \hat{A}\left\{\chi_n^{(-)}(\mathbf{k}_n,\mathbf{r})\Psi_{B+\Delta}(\mathbf{r}_1,\ldots,\mathbf{r}_A)\right.\right. \times\left|\sum_{j=1}^{A}V_{N\Delta}(\mathbf{r}_j,\mathbf{r})\right|\hat{A}\{\chi_p^{(+)} \times(\mathbf{k}_p,\mathbf{r})\Psi_{a_iJ_iM_i}(\mathbf{r}_1,\ldots,\mathbf{r}_A)\}\Big\rangle. \tag{34}$$

In Eq. (33) and in what follows, we use the following notation: $E_i=(\mathbf{P}_i^2+m_N^2)^{1/2}$ is the energy of the incident proton with momentum $\mathbf{P}_i$ and mass m_N, $E_A=(\mathbf{P}^2+M_A^2)^{1/2}$ is the energy of the target nucleus A, S_{NA} is the square of the invariant mass of the $p+A$ system, $\lambda(x,y,z)=(x-y-z)^2-4yz$ is the kinematic or triangle function,[16] and P_i, P_A, P_n, and P_f are, respectively, the 4-momenta of the incident proton, the detected neutron, and the undetected fragments $f=B+\Delta$. Equation (34) is written in the $p+A$ center-of-mass system. It contains the wave function $\Psi_{\alpha JM}$ of the target nucleus A with spin J, spin projection M, and other quantum numbers α, the wave function $\Psi_{B+\Delta}$ of the unobserved system f consisting of the nucleus $B=A-1$ and the Δ isobar, the distorted waves $\chi_p^{(+)}$ and $\chi_n^{(-)}$ of the proton and neutron, the operator $V_{N\Delta}$ of the transition $NN\to N\Delta$, and the antisymmetrization operator $\hat{A}$. The factor $[2(2J_i+1)]^{-1}$ in Eq. (33) arises because of the average over the spin projections of the incident proton and target nucleus, i.e., both the beam and the target are assumed to be unpolarized.

In Refs. 9–11 it was shown that the usual method of taking into account nuclear distortions in calculating $\chi_p^{(+)}(\chi_n^{(-)})$ in the optical model leads to underestimated values of the theoretical cross sections, since it does not include the contribution from processes involving incoherent rescattering of the proton (neutron) by the nucleons of the target nucleus A (residual nucleus $B+\Delta$). This effect can be taken into account by using Glauber distorted waves:[9–17]

$$\hat{\chi}_p^{(+)}(\mathbf{k}_p,\mathbf{r})=(2\pi)^{-3/2}\exp(i\mathbf{k}_p\mathbf{r})\prod_{j=1}^{A}[1-\Gamma \times(\mathbf{b}-\mathbf{b}_j)\theta(z_j-z)]\chi_m(\boldsymbol{\sigma}), \qquad (35)$$

$$\hat{\chi}_m^{(-)}(\mathbf{k}_m,\mathbf{r})=(2\pi)^{-3/2}\exp(i\mathbf{k}_n\mathbf{r})\prod_{j=1}^{A-1}[1-\Gamma \times(\mathbf{b}-\mathbf{b}_j)\theta(z-z_j)]\chi_{m_n}(\boldsymbol{\sigma}), \qquad (36)$$

where $\Gamma(\mathbf{b})$ is the profile function

$$\Gamma(\mathbf{b})=\frac{1}{2\pi ik}\int e^{i(\mathbf{q}\cdot\mathbf{b})}A_{NN}(\mathbf{q})d^2q. \qquad (37)$$

In Eqs. (35)–(37), $\mathbf{b}$ is the impact parameter, $\mathbf{q}$ is the momentum transfer, $A_{NN}(\mathbf{q})$ is the NN scattering amplitude, $\chi_m(\boldsymbol{\sigma})$ is the spin function of a nucleon with spin $\boldsymbol{\sigma}$ and spin projection m, and $\theta(z)$ is the step function

$$\theta(z)=\begin{cases}1 & \text{for } z\geqslant 0\\ 0 & \text{for } z<0.\end{cases} \qquad (38)$$

In Refs. 9 and 10, for the example of (p,pX) inclusive reactions, a study was made of the approximation of effective numbers, which differs from the standard DWIA in that the optical distorted waves are replaced by Glauber waves. In this approximation, which makes essential use of the condition of completeness of the final states of the undetected fragments, the $A(p,n)_\Delta B$ reaction cross section can be represented in the form (see the monographs of Refs. 9, 11, 26, and 28, and also the references therein)

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n}=\int d\mathbf{Q}[\Phi_N^A(\mathbf{Q})]^2\,\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i,\mathbf{Q})}{d\Omega_n}, \qquad (39)$$

where $[\Phi_N^A(\mathbf{Q})]^2$ is the momentum distribution in the nucleus A of the nucleons participating in the charge-exchange reaction.

The nucleon momentum distribution $[\Phi_N^A(\mathbf{Q})]^2$ can be readily calculated in terms of the partial proton $(|\Phi_p^A(\mathbf{Q})|^2)$ and neutron $(|\Phi_n^A(\mathbf{Q})|^2)$ momentum distributions:

$$[\Phi_N^A(\mathbf{Q})]^2=|\Phi_p^A(\mathbf{Q})|^2+\tfrac{1}{3}|\Phi_n^A(\mathbf{Q})|^2, \qquad (40)$$

where the factor 1/3 in (40) is the isotopic weight factor for production of the Δ^+ isobar on the neutron. The effective number of protons (neutrons) taking part in the $(p,n)_\Delta$ process is determined by the integral of the momentum distribution:

$$\tilde{N}_{p(n)}=\int d\mathbf{Q}|\Phi_{p(n)}^A(\mathbf{Q})|^2. \qquad (41)$$

In the plane-wave approximation, the effective numbers are $\tilde{N}_p^A(\mathrm{PW})=Z$ $(\tilde{N}_p^A(\mathrm{PW})=N)$, where $Z(N)$ is the number of protons (neutrons) in the nucleus A. A detailed analysis of the properties of the effective numbers of nucleons and clusters was carried out in the monographs of Refs. 6, 9, and 27, and in the review of Ref. 22. The presence of the momentum $\mathbf{Q}$ of the intranuclear nucleon among the arguments of the cross section $d\sigma_{p+p\to n+\Delta^{++}}(P_i,\mathbf{Q})/d\Omega_n$ for charge exchange on a free nucleon indicates the need to take into account off-mass-shell effects. It is customary to calculate the off-mass-shell effects in the optical approximation.[60] However, in the studied region of energies $T_p>0.6$ GeV the influence of off-mass-shell effects on the integrated cross sections can be neglected, since the momentum $\mathbf{P}_i$ of the incident nucleon and the momentum transfer $\mathbf{q}$ satisfy the conditions $|\mathbf{P}_i|\gg P_F$ and $|\mathbf{q}|\sim P_F$, where P_F is the Fermi momentum. The point is that the strength of the interaction depends on the momentum $\mathbf{P}_i$ of the incident proton and on the momentum $\mathbf{Q}$ of the intranuclear nucleon as $(P_i^2+Q^2)^{1/2}$, so that the overall correction due to the Fermi motion of the nucleons and their binding does not exceed 3–5%. Moreover, the off-mass-shell effects have little influence on the A dependence of the integrated $A(p,n)_\Delta B$ reaction cross section in which we are interested. Because of these circumstances, at this stage of the investigations we can make use of the approximation

$$\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i,\mathbf{Q})}{d\Omega_n}\approx\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i,\mathbf{Q}=0)}{d\Omega_n}\equiv\left.\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i)}{d\Omega_n}\right|_{\mathrm{b}}. \qquad (42)$$

In this approximation, the expression (39) factorizes:

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n}=\tilde{N}\left.\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i)}{d\Omega_n}\right|_{\mathrm{b}}; \qquad (43)$$

in the approximation of effective numbers, the expression for $\tilde{N}$ has the simple structure[9,10]

$$\tilde{N}=\int d\mathbf{Q}[\Phi_N^A(\mathbf{Q})]^2=\left(Z+\frac{N}{3}\right)\int d\mathbf{r}\rho(\mathbf{r})f^2(b,z)\equiv\left(Z+\frac{N}{3}\right)\langle f^2\rangle. \qquad (44)$$

In Eq. (44), $\langle f^2\rangle$ is an effective absorption factor, $\rho(\mathbf{r})$ is the single-nucleon density, normalized by the condition $\int d\mathbf{r}\rho(\mathbf{r})=1$, and $f^2(b,z)$ is the Glauber absorption factor

$$f^2(b,z)=\left[\left(1-\frac{\sigma_{PN}^{\rm tot}-\sigma_{PN}^{\rm el}}{A}T_-(b,z)\right)\right.$$
$$\left.\times\left(1-\frac{\sigma_{nN}^{\rm tot}-\sigma_{nN}^{\rm el}}{A}T_+(b,z)\right)\right]^A$$
$$=\sum_{\lambda_p=0}^{A}\sum_{\lambda_n=0}^{A}\binom{A}{\lambda_p}$$
$$\times\left(1-\frac{\sigma_{PN}^{\rm tot}}{A}T_-(b,z)\right)^{A-\lambda_p}\left(\frac{\sigma_{PN}^{\rm el}}{A}T_-(b,z)\right)^{\lambda_p}$$
$$\times\binom{A}{\lambda_n}\left(1-\frac{\sigma_{nN}^{\rm tot}}{A}T_+(b,z)\right)^{A-\lambda_n}$$
$$\times\left(\frac{\sigma_{nN}^{\rm el}}{A}T_+(b,z)\right)^{\lambda_n}\equiv\sum_{\lambda_p=0}^{A}\sum_{\lambda_n=0}^{A}f^2_{\lambda_p\lambda_n}(b,z),\quad(45)$$

where $\sigma_{PN}^{\rm tot}(\sigma_{nN}^{\rm tot})$ is the total cross section for proton-nucleon (neutron-nucleon) scattering, and $\sigma_{PN}^{\rm el}(\sigma_{nN}^{\rm el})$ is the analogous elastic cross section. The thickness functions $T_\mp$ can be written in the standard form

$$T_+(b,z)=A\int_z d\xi\rho(\{b^2+\xi^2\}^{1/2}),\quad(46)$$

$$T_-(b,z)=A\int^z d\xi\rho(\{b^2+\xi^2\}^{1/2}).\quad(47)$$

Equation (45) is an expansion of the absorption factor in the number $\lambda_p(\lambda_n)$ of quasielastic collisions of the incident proton (outgoing neutron) with the nucleons of the nucleus $A(B)$. It enables us to represent the effective numbers $\tilde{N}$ in the form

$$\tilde{N}=\left(Z+\frac{N}{3}\right)\int d\mathbf{r}\rho(\mathbf{r})\sum_{\lambda_p=0}^{A}\sum_{\lambda_n=0}^{A}f^2_{\lambda_p\lambda_n}(b,z).\quad(48)$$

We define the partial sum $\tilde{N}_{v_pv_n}$ as

$$\tilde{N}_{v_pv_n}=\left(Z+\frac{N}{3}\right)\sum_{\lambda_p=0}^{v_p}\sum_{\lambda_n=0}^{v_n}\int d\mathbf{r}\rho(\mathbf{r})f^2_{\lambda_p\lambda_n}(b,z).\quad(49)$$

Each partial sum (49) describes the contribution to the total cross section from a certain group of final states of the nucleus $B+\Delta$. For example, $\tilde{N}_{00}$ corresponds to the fact that the state (ΔN^{-1}) was formed as a result of the reaction, and there are no other excitations in the nucleus B; the quantity $\tilde{N}_{10}$ corresponds to a process in which the incident nucleon first excited the $1p-1h$ state in the nucleus A, after which, following charge exchange, the (ΔN^{-1}) excitation was formed. In general, if $v_p+v_n=k$, this means that the excitation $(kp-kh)+(\Delta N^{-1})$ was formed in the reaction $A(p,n)_\Delta B$.

Charge symmetry enables us to simplify the expression (45) somewhat, since this expression implies equality of the cross sections: $\sigma_{PN}=\sigma_{nN}=\sigma_{NN}$; therefore, instead of the two thickness functions $T_\mp(b,z)$, we can introduce the single function

$$T(b)=T_-(b,z)+T_+(b,z)=A\int dz\rho(\{b^2+z^2\}^{1/2}).\quad(50)$$

In this case, in the limit of large mass numbers $A\gg1$ we obtain the well-known eikonal approximation

$$f^2_{\rm eik}(b)=\exp\{-(\sigma_{NN}^{\rm tot}-\sigma_{NN}^{\rm el})T(b)\}.\quad(51)$$

We note that in going from (45) to (51) we made use of the fact that the cross sections $\sigma_{PN}(T_p)$ and $\sigma_{nN}(T_n)$ depend weakly on the energy $T_p(T_n)$, since, strictly speaking, $\sigma_{PN}(T_p)=\sigma_{nN}(T_n)$ only when $T_p=T_n$.

If the nucleus is detected in the ground state, as was the case, for example, in the experiment of Ref. 61, then in the absorption factors (45) and (51) we must formally put $\sigma_{NN}^{\rm el}=0$. In this case, Eq. (51) corresponds to the eikonal approximation for the optical model of elastic scattering.

Substituting the expression (51) for $f^2(b)$ into Eq. (48), we represent the effective absorption factor $\langle f^2\rangle$ in the eikonal approximation in the form

$$\langle f^2\rangle=\frac{2\pi}{A}\int dbbT(b)e^{-\sigma T(b)},\quad(52)$$

where $\sigma=\sigma_{NN}^{\rm tot}$ for exclusive and $\sigma=\sigma_{NN}^{\rm tot}-\sigma_{NN}^{\rm el}$ for inclusive reactions. The integral (52) can be estimated by the saddle-point method:

$$\langle f^2\rangle=\{(2\pi)^{3/2}b_0\}/\{A\sigma^2e|T'(b_0)|\},\quad(53)$$

where b_0 is the root of the equation

$$T(b_0)=\sigma^{-1}.\quad(54)$$

For energies $T_p>0.6$ GeV, the value of σ lies in the interval $\sigma\approx20-40$ mb ($\sigma_{NN}^{\rm tot}\approx40$ mb, $\sigma_{NN}^{\rm el}\approx10-20$ mb. In this case, $b_0=R_A$ and $|T'(b_0)|\approx1/\sigma a$, where R_A is the radius of the nucleus, a is the diffuseness of its boundary, and the approximate expression for $\langle f^2\rangle$ can be written in the form

$$\langle f^2\rangle\approx\{(2\pi)^{3/2}\}R_Aa/(\sigma Ae).\quad(55)$$

It follows at once from Eq. (55) that $\langle f^2\rangle\sim A^{-2/3}$, and therefore the A dependence of $\tilde{N}$ has the form

$$\tilde{N}=\kappa_1A^{1/3}.\quad(56)$$

In Tables II and III we give the A and T_p dependences of the effective numbers $\tilde{N}$. It can be seen from these tables that the effective numbers have the form $\tilde{N}=\kappa_1A^\alpha$, where α is a slowly increasing function of λ_p and λ_n: $\alpha_{00}=0.31$ for $\tilde{N}_{00}$ and $\alpha_{33}=0.35$ for $\tilde{N}_{33}$. The series (48) then converges very rapidly—about 90% of the total value of $\tilde{N}$ comes from the partial sum $\tilde{N}_{11}$. This result in fact justifies our use of the approximation of completeness and confirms the validity of the relations (50) and (51). However, we note here that the somewhat overestimated value $\alpha_{33}=0.38$ for $\tilde{N}$ arises because of the neglect of absorption of the Δ isobar in the nucleus (for example, emission of a Δ isobar in the channel of mesonless deexcitation). Allowance for this absorption reduces the value of α to 0.36.

To calculate the T_p dependence of $\tilde{N}$ and $\langle f^2\rangle$, we used the data on the cross sections $\sigma_{NN}^{\rm tot}$ and $\sigma_{NN}^{\rm el}$, which were systematized in Refs. 62 and 63. It can be seen from Table II

TABLE II. Effective numbers of nucleons $\tilde{N}$ as a function of the mass number A at incident-proton energy T_p=6 GeV.

A	$\tilde{N}_{opt}$	$\tilde{N}_{00}$	$\tilde{N}_{11}$	$\tilde{N}_{22}$	$\tilde{N}_{33}$	$\tilde{N}_{eik}$
12	1,79	1,69	2,23	2,32	2,34	2,43
16	1,79	1,71	2,28	2,40	2,42	2,51
27	2,66	2,58	3,46	3,65	3,68	3,78
40	3,07	3,00	4,08	4,33	4,38	4,47
58	3,38	3,32	4,54	4,85	4,92	5,02
118	3,90	3,86	5,31	5,69	5,74	5,88
208	4,35	4,33	5,98	6,43	6,55	6,63

The subscript "opt" means that in the calculation of $\tilde{N}$ the absorption factors were computed in the optical model of elastic scattering. The meaning of the subscripts "eik" and "jk" ($j, k=0, 3$) is described in detail in the text. The same notation is used in all subsequent tables and figures. Absorption factors (48) without allowing the Δ isobar to go into the $\Delta N \to NN$ channel are used in the calculations.

that in the studied region of energies the T_p dependence of the effective numbers $\tilde{N}$ is weak, and this justifies the validity of the approximation

$$\sigma_{NN}(T_p) \approx \sigma_{NN}(T_n).$$

In the analysis of the effective numbers of nucleons and of the absorption factors, we used the following parametrization of the single-nucleon density $\rho(r)$. For $A \geqslant 20$ the density $\rho(r)$ was represented in the form of the Woods–Saxon distribution

$$\rho(r) = \rho_0 \left\{ 1 + \exp\left(\frac{r - R_A}{a} \right) \right\}^{-1}, \tag{57}$$

where a=0.54 fm and $R_A = 1.12 A^{1/3} - 0.86 A^{-1/3}$ (Ref. 34), while for $A<20$ we used the Hartree approximation for the single-particle density, calculated in the oscillator shell basis:

$$\rho(r) = \frac{4}{A(a_0 \pi)^{3/2}} \left[1 + \frac{A-4}{6} (r/a_0)^2 \right] \exp\{ -(r/a_0)^2 \}, \tag{58}$$

where a_0=1.6 fm.[26,34]

3. ELEMENTARY PROCESSES INVOLVING PIONS, NUCLEONS, AND DELTA ISOBARS, AND THE CORRESPONDENCE PRINCIPLE IN THE PHYSICS OF INTERMEDIATE ENERGIES

The contemporary formalism for describing strong processes at intermediate energies involving pions, nucleons, and low-lying nonstrange baryon resonances was outlined in detail in the monograph of Ref. 7. However, a number of questions of both technical and physical character remained beyond the scope of that study.

For example, in the region of intermediate energies specific manifestations of the first principles of quantum theory, such as the complementarity principle, the correspondence principle, and the superposition principle, have been investigated to a much lesser extent than in low-energy nuclear physics. It is fairly rare that use is made of the formalism of the quantum theory of angular momentum, which has been well developed in the physics of low energies. The question of the identifiability of specific models of strong processes at intermediate energies remains practically open. The space-time description of strong interactions is abandoned unjustifiably early (in the energies).

Bearing in mind the remarks made above and also the problem of considering in detail the general physical conception of a resonance, we shall present a modified formalism for describing processes involving pions, nucleons, and Δ isobars for the example of the simplest $(p,n)_\Delta$ charge-exchange reaction.

For definiteness, we consider the process $p+p \to n+p+\pi^+$. In the notation of Bjorken and Drell,[64] the cross section for this process has the form

$$d\sigma = \frac{2m^2}{\lambda^{1/2}(S,m^2,m^2)} \frac{m}{E_n} \frac{d\mathbf{p}_n}{(2\pi)^3} \frac{m}{E_p} \frac{d\mathbf{p}_p}{(2\pi)^3} \frac{d\mathbf{p}_\pi}{2E_\pi (2\pi)^3} \times (2\pi)^4 \delta(p_1 + p_2 - p_n - p_p - p_\pi) S_F \left\langle \left| \sum_i \mathfrak{M}_i \right|^2 \right\rangle, \tag{59}$$

where S_F is the statistical factor taking into account the identity of the particles.

The notation $\langle |\Sigma_i \mathfrak{M}_i|^2 \rangle$ stands for an average over the spin projections in the initial state and a summation over the spins in the final state:

$$\left\langle \left| \sum_i \mathfrak{M}_i \right|^2 \right\rangle = \frac{1}{4} \sum_{m_1 m_2 m_3 m_4} \left| \sum_i \mathfrak{M}_i \right|^2. \tag{60}$$

In the P-wave approximation, the sum of amplitudes $\Sigma \mathfrak{M}_i$ includes the connected diagrams giving the main contribution to the studied process[65,66] (see Fig. 5).

Let us calculate the cross section (59) in the laboratory system. Using the notation of Refs. 65 and 66, we represent the double differential cross section for the inclusive reaction $p+p \to n+p+\pi^+$ in the form

TABLE III. Effective numbers $\tilde{N}$ for the nucleus ^{12}C as a function of the energy T_p.

T_p, GeV	$\tilde{N}_{opt}$	$\tilde{N}_{00}$	$\tilde{N}_{11}$	$\tilde{N}_{22}$	$\tilde{N}_{33}$	$\tilde{N}_{eik}$
1,0	1,79	1,69	2,67	2,98	3,06	3,12
6,0	1,79	1,69	2,23	2,32	2,34	2,43
10,0	1,83	1,74	2,24	2,33	2,34	2,43
14,0	1,83	1,74	2,30	2,40	2,41	2,51
20,0	1,93	1,84	2,27	2,33	2,34	2,43

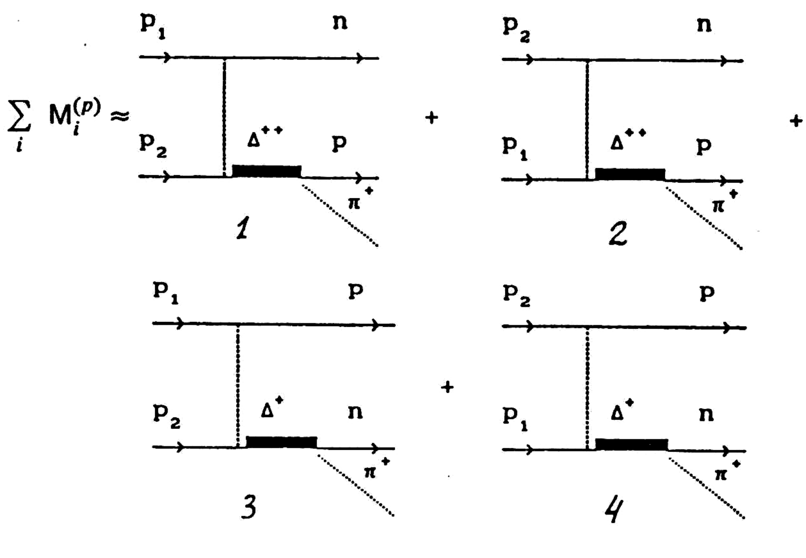


FIG. 5. The P-wave diagrams giving the main contribution to the studied process (see the text).

$$\frac{d^2\sigma}{d\Omega_n dE_n}=F_{\text{kin}}(S,p_n)\int d\Omega_\pi \rho_{\text{phase}}(\theta_\pi,\varphi_\pi)\times\left\langle\left|\sum \mathfrak{M}_i\right|^2\right\rangle, \quad (61)$$

where the kinematic factor F_{kin} is given by

$$F_{\text{kin}}(S,p_n)=\frac{m^4 p_n}{\lambda^{1/2}(S,m^2,m^2)}\frac{S_F}{(2\pi)^5}, \quad (62)$$

the density of final states (apart from a normalization) is

$$\rho_{\text{phase}}=\frac{p_\pi^2}{|E_\pi(p_\pi-q_0\xi)+p_\pi E_p|}, \quad (63)$$

$$q_0=|\mathbf{p}_n-\mathbf{p}|, \quad (64)$$

ξ is the cosine of the angle between the vectors $\mathbf{q}_0=\mathbf{p}-\mathbf{p}_n$ and $\mathbf{p}_\pi$, given by

$$\xi=\frac{p\cos\theta_\pi+p_n\cos\theta_{n\pi}}{\sqrt{p^2+p_p^2-2pp_p\cos\theta_n}}, \quad (65)$$

$\cos\theta_n$ is the cosine of the emission angle of the neutron, and $\cos\theta_{n\pi}$ is the cosine of the angle between the directions of the outgoing pion and neutron.

The amplitude M_i on the mass shell is related to the analogous amplitude $\mathfrak{M}_i$ in (59) by the simple equation

$$\mathsf{M}_i=(2\pi)^4\delta(p_1+p_2-p_p-p_n-p_\pi)\mathfrak{M}_i. \quad (66)$$

According to Ref. 64, to find the invariant amplitude M_i corresponding to diagram i of Fig. 5, it is necessary to associate a certain factor with each element of the diagram. In Refs. 65–67, the following conditions are adopted.

1) Each external fermion line corresponds to a spinor χ_m and an isospinor Φ_τ.

2) Each internal pion line corresponds to a propagator

$$iG_\pi(q)=\frac{i}{q^2-m_\pi^2+i\varepsilon}. \quad (67)$$

3) Each internal thick fermion line corresponds to a Δ propagator

$$iG_\Delta(S)=\frac{i2M_\Delta}{M_\Delta^2-S-iM_\Delta\Gamma_\Delta(S)}. \quad (68)$$

4)The π mesons in the initial and final states correspond to the isotopic functions

$$|\pi^+\rangle=-\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\\ 0\end{bmatrix}, \quad (69)$$

$$|\pi^0\rangle=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}, \quad (70)$$

$$|\pi^+\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -i\\ 0\end{bmatrix}. \quad (71)$$

5) Each internal momentum k that is not fixed by the conservation laws at the vertices corresponds to an integral

$$\int\frac{d^{(4)}k}{(2\pi)^4}. \quad (72)$$

6) The $\pi N\Delta$ decay vertex corresponds to the factor

$$\langle\pi^\lambda|\mathbf{T}^+\rangle\frac{f_{\pi N\Delta}}{m_\pi}(\mathbf{S}^+\cdot\mathbf{p}_\pi)(2\pi)^4\delta(p_{\text{in}}-p_{\text{out}}) \quad (73)$$

where $\lambda=0,+,-$:

$$\langle\pi^+|\mathbf{T}^+\rangle=-T_{-1}^+, \quad (74)$$

$$\langle\pi^0|\mathbf{T}^+\rangle=T_0^+, \quad (75)$$

$$\langle\pi^-|\mathbf{T}^+\rangle=-T_{+1}^+. \quad (76)$$

The annihilation operators of the spin ($\mathbf{S}^+$) of the Δ isobar are determined by the matrix elements

$$\langle\chi_m^N|S_{-\mu}^+|\chi_{m_\Delta}^\Delta\rangle=(-1)^\mu\left(1\ \frac{1}{2}\ \mu m\middle|\frac{3}{2}\ m_\Delta\right), \quad (77)$$

$$\langle\chi_{m_\Delta}^\Delta|S_\mu|\chi_m^N\rangle=\left(1\ \frac{1}{2}\ \mu m\middle|\frac{3}{2}\ m_\Delta\right). \quad (78)$$

The isospin operators T are determined by exactly the same relations. The vector $\mathbf{p}_\pi$ is the momentum of the pion in the rest system of the Δ isobar with invariant mass $\sqrt{S_\Delta}$:

$$p_\pi(S_\Delta)=\left[\frac{(S_\Delta-m^2+m_\pi^2)^2}{4S_\Delta}-m_\pi^2\right]^{1/2}. \qquad (79)$$

7) The vertex of Δ-isobar production is given by

$$-\langle \mathbf{T}|\pi^\lambda\rangle \frac{f_{\pi N\Delta}}{m_\pi}(\mathbf{S}\cdot\mathbf{q})(2\pi)^4\delta(p_{\text{in}}-p_{\text{out}}). \qquad (80)$$

8) The πNN vertex is given by

$$\langle \pi^\lambda|\mathbf{T}\rangle \frac{f_{\pi NN}}{m_\pi}(\boldsymbol{\sigma}\cdot\mathbf{q})(2\pi)^4\delta(p_{\text{in}}-p_{\text{out}}), \qquad (81)$$

where $\boldsymbol{\sigma}(\boldsymbol{\tau})$ are the usual Pauli spin (isospin) matrices.

9) The quantity $\mathbf{q}$ in items 7 and 8 has the meaning of the momentum of the virtual pion in the Breit system. For diagrams 1, 2, 3, and 4 in Fig. 5 its values in the laboratory system are, respectively,

$$\mathbf{q}_1=\sqrt{\frac{E_n+m}{E+m}}\,\mathbf{p}-\sqrt{\frac{E+M}{E_n+m}}\,\mathbf{p}_n, \qquad (82)$$

$$\mathbf{q}_2=\sqrt{\frac{2m}{E_n+m}}\,\mathbf{p}_n, \qquad (83)$$

$$\mathbf{q}_3=\sqrt{\frac{E_p+m}{E+m}}\,\mathbf{p}-\sqrt{\frac{E+m}{E_p+m}}\,\mathbf{p}_p, \qquad (84)$$

$$q_4=\sqrt{\frac{2m}{E_p+m}}\,\mathbf{p}_p, \qquad (85)$$

with

$$\mathbf{p}=\mathbf{p}_p+\mathbf{p}_n+\mathbf{p}_\pi, \qquad (86)$$

$$E+m=E_p+E_n+E_\pi. \qquad (87)$$

10) The contribution of the ρ meson to the charge-exchange reaction amplitude is taken into account by means of the replacement[67]

$$V^{\text{eff}}_{NN\to N\Delta}\equiv\frac{f_{\pi NN}}{m_\pi}(\boldsymbol{\sigma}\cdot\mathbf{q})\frac{f_{\pi N\Delta}}{m_\pi}(\mathbf{S}\cdot\mathbf{q})G_\pi(q)$$
$$\to\frac{f_{\pi NN}}{m_\pi}\frac{f_{\pi N\Delta}}{m_\pi}\{(\boldsymbol{\sigma}\cdot\mathbf{q})(\mathbf{S}\cdot\mathbf{q})G_\pi(q)+C_\rho[\boldsymbol{\sigma}\cdot\mathbf{q}]$$
$$\times[\mathbf{S}\cdot\mathbf{q}]G_\pi(q)\}, \qquad (88)$$

where G_ρ is the ρ-meson propagator, defined by analogy with G_π, and C_ρ is the ρ-meson coefficient.

11) The effective transition potential $V^{\text{eff}}_{NN\to N\Delta}$ in item 10 must be modified to take into account two important physical effects.

First, in each of the virtual vertices it is necessary to insert a form factor $F_\pi(F_\rho)$ in order to take into account off-mass-shell effects:

$$V^{\text{eff}}_{NN\to N\Delta}=\frac{f_{\pi NN}f_{\pi N\Delta}}{m_\pi^2}\{F_\pi^2(q)q^2G_\pi(q)\tfrac{1}{3}[(\mathbf{S}\cdot\boldsymbol{\sigma})$$
$$+S_{12}(\hat{q})]+C_\rho F_\rho^2(q)q^2G_\rho(q)\tfrac{1}{3}[2(\mathbf{S}\cdot\boldsymbol{\sigma})$$
$$-S_{12}(\hat{q})]\}, \qquad (89)$$

where $S_{12}(\hat{q})=3(\boldsymbol{\sigma}\cdot\hat{q})(\mathbf{S}\cdot\hat{\mathbf{q}})-(\mathbf{S}\cdot\boldsymbol{\sigma})$, $\hat{q}$ is a unit vector parallel to $\mathbf{q}$, the form factors $F_\pi(q)$ and $F_\rho(q)$ have the structure

$$F_\pi(q)=\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-t}, \qquad (90)$$

$$F_\rho(q)=\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-t}, \qquad (91)$$

$t=-\mathbf{q}^2$ in the Breit system, and Λ_π and Λ_ρ are cutoff constants. Then the expression (89) can be written in the form

$$V^{\text{eff}}_{NN\to N\Delta}=V_C(q)(\mathbf{S}\cdot\boldsymbol{\sigma})+V_{NC}(q)S_{12}(\hat{q}). \qquad (92)$$

The central (V_C) and noncentral (V_{NC}) components of the transition potential are related to the longitudinal (V_L) and transverse (V_T) components by the standard equations

$$V_C(q)=\tfrac{1}{3}[V_L(q)+2V_T(q)], \qquad (93)$$

$$V_{NC}(q)=\tfrac{1}{3}[V_L(q)-V_T(q)], \qquad (94)$$

$$V_L(q)=\frac{f_{\pi NN}F_{\pi N\Delta}}{m_\pi^2}F_\pi^2(q)q^2G_\pi(q), \qquad (95)$$

$$V_T(q)=C_\rho\frac{f_{\pi NN}f_{\pi N\Delta}}{m_\rho^2}F_\rho^2(q)q^2G_\rho(q). \qquad (96)$$

Second, the expression (89) must be supplemented with a g' term of the Landau–Migdal type in order to take into account short-range correlations:

$$V_C(q)\to V_C(q)+g'_{N\Delta}\frac{f_{\pi NN}F_{\pi N\Delta}}{m_\pi^2}F_\pi^2(q). \qquad (97)$$

12) To perform actual calculations, it is convenient to express V_C and V_{NC}, the components of the potential $V^{\text{eff}}_{NN\to N\Delta}$, in terms of the new quantities

$$\hat{V}(0,q)=\sqrt{12\pi}V_C(q), \qquad (98)$$

$$\hat{V}(2,q)=-\sqrt{\frac{24\pi}{5}}V_{NC}(q). \qquad (99)$$

Then the transition potential is

$$V^{\text{eff}}_{NN\to N\Delta}=-\sum_{L=0,2}\hat{V}(L,q)\sum_{\mu,\nu,M}(11\mu\nu|LM)Y^*_{LM}(\hat{q})\sigma_\nu S_\mu. \qquad (100)$$

The representation (100) is convenient for using the standard technique of angular momentum. In fact, if we take into account the fact that the matrix elements of the Pauli matrices have the form

$$\langle\chi_{m_n}|\sigma_\nu|\chi_{m_1}\rangle=\sqrt{3}\left(\frac{1}{2}\,1m_1\nu\,\middle|\,\frac{1}{2}\,m_n\right), \qquad (101)$$

and introduce the abbreviated notation

$$W(S_\Delta)=\frac{f_{\pi N\Delta}}{m_\pi}\sqrt{4\pi}P_\pi(S_\Delta)G_\Delta(S_\Delta), \qquad (102)$$

then the P-wave amplitude $\mathfrak{M}_j^{(P)}$ ($j=1,2,3,4$) will be determined by the expression

$$\mathfrak{M}_j^{(P)}=(-1)^j W(S_\Delta(j)) ISF(j) \times \sum_{L=0,2} \hat{V}(L,q_j) \sum_{\mu,\nu,M,\lambda,m_\Delta} (11\mu\nu|LM) Y^*_{LM}(\hat{q}_j) \times\left(\frac{1}{2}\,1 m_i(1,j)\nu\middle|\frac{1}{2}\,m_f(1,j)\right) \times\left(1\,\frac{1}{2}\,\mu m_i(2,j)\middle|\frac{3}{2}\,m_\Delta\right) Y_{1\lambda}(\hat{P}_\pi(j)) \times\left(1\,\frac{1}{2}\,\lambda m_f(2,j)\middle|\frac{3}{2}\,m_\Delta\right), \quad (103)$$

where (i and f indicate initial and final states)

$$\begin{cases} m_i(1,1)=m_1 \\ m_i(2,1)=m_2 \\ m_f(1,1)=m_n \\ m_f(2,1)=m_p \end{cases} \begin{cases} m_i(1,3)=m_1 \\ m_i(2,3)=m_2 \\ m_f(1,3)=m_p \\ m_f(2,3)=m_n \end{cases}$$

$$\begin{cases} m_i(1,2)=m_2 \\ m_i(2,2)=m_1 \\ m_f(1,2)=m_n \\ m_f(2,2)=m_p \end{cases} \begin{cases} m_i(1,4)=m_2 \\ m_i(2,4)=m_1 \\ m_f(1,4)=m_p \\ m_f(2,4)=m_n, \end{cases} \quad (104)$$

and the isospin factor $ISF(j)$ is given by the simple formula

$$ISF=(-1)^{\tau_v}\sqrt{3}\left(\frac{1}{2}\,1\tau_i(1)-\tau_v\middle|\frac{1}{2}\,\tau_f(1)\right) \times\left(1\,\frac{1}{2}\,\tau_v\tau_i(2)\middle|\frac{3}{2}\,\tau_\Delta\right)\left(1\,\frac{1}{2}\,\tau_r\tau_f(2)\middle|\frac{3}{2}\,\tau_\Delta\right), \quad (105)$$

in which $\tau_i(1)[\tau_i(2)]$ is the isospin projection of the initial nucleon 1(2), $\tau_f(1)[\tau_f(2)]$ is the isospin projection of the final nucleon 1(2), $\tau_{v(r)}$ is the isospin projection of the virtual (real) pion, and τ_Δ is the isospin projection of the Δ isobar.

Using Eq. (103) and taking into account what we have said above, we can calculate the P-wave amplitude for a reaction of the type $p+p\rightarrow n+p+\pi^+$ in the tree approximation.

The contribution of S-wave πN scattering can be taken into account according to the methods of Refs. 7 and 65–69 by adding to the P-wave diagrams analogous S-wave diagrams, which differ from them by the replacement of the Δ propagator (the thick lines in Fig. 5) by an effective $\pi\pi NN$ vertex (the dark squares in Fig. 6). This amplitude can be readily calculated by using the S-wave Hamiltonian of the πN interaction, which contains isoscalar (1) and isovector (2) components (in the notation of Ref. 7):

$$H^{(1)}_{\pi N}=\frac{4\pi}{m_\pi}\lambda_1\bar{\psi}(x)\mathbf{\Phi}(x)\mathbf{\Phi}(x)\psi(x), \quad (106)$$

$$H^{(2)}_{\pi N}=\frac{4\pi}{m_\pi^2}\lambda_2\bar{\psi}(x)\boldsymbol{\tau}[\mathbf{\Phi}(x)\partial_t\mathbf{\Phi}(x)]\psi(x). \quad (107)$$

According to Ref. 66, it has the form

$$\mathfrak{M}_j^{(S)}=(-1)^{j+1}\frac{f_{\pi N\Delta}}{m_\pi}\sqrt{4\pi}q_j\sum_\lambda Y_{1\lambda}(\hat{q}_j)\left(\frac{1}{2}\,1 m_i(1,j) -\lambda\middle|\frac{1}{2}\,m_f(1,j)\right) G_\pi(q_j)\frac{8\pi}{m_\pi}\delta_{m_i(2,j),m_f(2,j)}\mathrm{K}^{\pi N}, \quad (108)$$

where

$$\mathrm{K}^{\pi N}=(-1)^{\tau_v}\sqrt{3}\left(\frac{1}{2}\,1\tau_i(1)-\tau_v\middle|\frac{1}{2}\,\tau_f(1)\right) \times\Bigg[\lambda_1\delta_{\tau,\tau}\delta_{\tau(2),\tau(2)}+\lambda_2\sum_\nu (-1)^\nu\sqrt{6}(11\tau_v\nu|1\tau_r)\left(\frac{1}{2}\,1\tau_i(2) -\nu\middle|\frac{1}{2}\,\tau_f(1)\right)\Bigg], \quad (109)$$

in which τ denotes the isospin projection, the subscript $v(r)$ indicates a virtual (real) pion, $i(f)$ indicates an initial (final) nucleon, and the bracketed numbers (1) and (2) correspond to the upper and lower nucleon lines of the skeleton diagram. The values of the parameters λ_1 and λ_2 are taken from Ref. 67:

$$\begin{cases} \lambda_1=\lambda_1^0+0.222[\mathrm{GeV}^{-1}](\sqrt{S}-m_\pi-m) \\ \lambda_1^0=0.0075 \\ \lambda_2=0.0528, \end{cases} \quad (110)$$

where $\sqrt{S}$ is the invariant mass, and for diagrams 1 and 2 we have

$$S(1)=S(2)=(E_1+E_2-E_n)^2-(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_n)^2, \quad (111)$$

while for diagrams 3 and 4 we have

$$S(3)=S(4)=(E_1+E_2-E_p)^2-(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_p)^2. \quad (112)$$

From the formal point of view, Eqs. (103) and (108) completely solve the problem of finding the reaction amplitude for charge exchange of nucleons with pion production, in the tree approximation with inclusion of S and P waves. On the one hand, each of the amplitudes is relativistically invariant (in spite of its noncovariant form); on the other hand, the expressions (103) and (108) are readily interpreted in terms of low-energy nuclear physics, since they are constructed on the basis of the standard quantum theory of angular momentum.[70]

In fact, the formalism outlined above permits a smooth transition from low to intermediate energies, and from the practical point of view it provides a fairly convenient modification of the relativistic Δ-isobar model (see the monograph of Ref. 7 and references therein).

Obviously, a field-theory formalism satisfying the correspondence principle is of definite practical interest in its own right, both from the point of view of a correct low-energy limit and in the sense of possible direct application of the nonrelativistic computational technique in the region of intermediate energies. However, a much more urgent problem

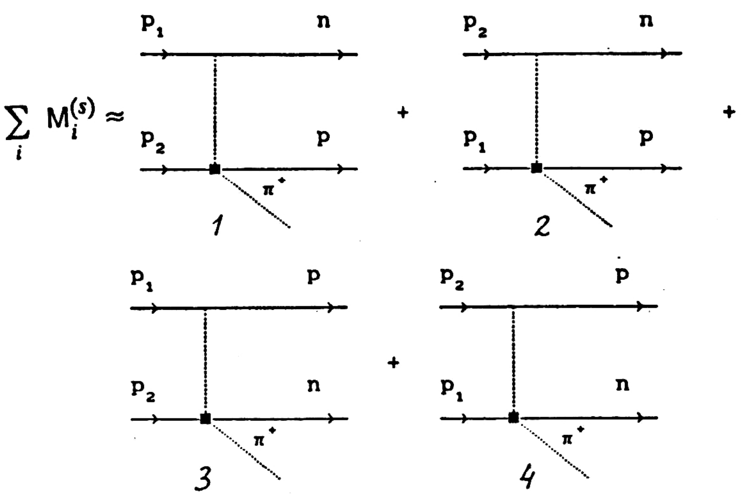


FIG. 6. The S-wave diagrams with an effective $\pi\pi NN$ vertex (see the text).

is to seek unusual physical effects which could be discovered experimentally and analyzed in terms of the formalism presented here.

4. THE PROBLEM OF DISCRETE AMBIGUITIES FOR THE $\pi N\Delta$ AND $\rho N\Delta$ VERTEX FUNCTIONS

We have already noted in the Introduction that a consistent scheme for analyzing inclusive reactions at intermediate energies must include methods of calculating elementary processes [i.e., cross sections of the type $d\sigma_{a+X\to X+a}/d\Omega_X$ from Eq. (1)]. In the preceding section, we presented the appropriate formalism, containing a number of phenomenological constants whose numerical values must be found experimentally. In this connection, there arises the question of whether this set of constants is unique, or whether there can exist several equivalent sets, and, in that case, the problem of choosing an "optimal" set from among them.

In our $\pi+\rho+g'$ model, the phenomenologically determined constants are the Landau–Migdal parameter $g'_{N\Delta}$ and the cutoff parameters Λ_π and Λ_ρ in the monopole form factors

$$F_b(t)=\frac{\Lambda_b^2-m_b^2}{\Lambda_b^2-t}, \tag{113}$$

where $b=\pi$, ρ (b is a boson) and t is the square of the 4-momentum transfer. The parameters of this model are established on the basis of the criterion of the best fit to the available experimental data for a large class of processes (charge exchange, π-atom processes, photoexcitation, etc.).

In this sense, the problem of finding the parameters of the $\pi+\rho+g'$ model is completely analogous to the problem of finding the parameters of the optical model on the basis of the condition of the best description of the experimental phase shifts in the analysis of elastic scattering of pions by nuclei. In principle, this problem can be formulated as follows. We consider the problem of elastic scattering of a particle with fixed energy E by a potential $V(r)$:

$$(T+V)\psi=E\psi. \tag{114}$$

As usual, boundary conditions for the wave function ψ must be specified at both limits:

1) $\psi(r)$ is regular for $r\to 0$;

2) $\psi(r)$ is determined by the set of experimental phase shifts $[\delta_L(E)]_{\rm exp}$ in the limit $r\to\infty$.

Thus, we arrive at a problem of the Sturm–Liouville type for finding the natural depth, range, and diffuseness of the potential $V(r)$.

As a result of a formal solution of this problem, we obtain a fixed set of "phase-equivalent" optical potentials, from which it is necessary to select one "physical" potential on the basis of additional criteria. We note also that the energy and L dependence of the potentials obtained in this way corresponds to the many-body character of the problem and to the contribution of field-theory effects.

At energies $0.8\leqslant T_p\leqslant 1.5$ GeV, the main contribution to the cross section for the reaction $p+p\to n+p+\pi^+$ with detection of neutrons in the region of the first diffraction maximum in the angles and the Δ-isobar peak in the momenta is given by diagrams 1–4 in Fig. 5. To these diagrams it would also be necessary to add the diagrams shown in Fig. 6, corresponding to the contribution of S-wave πN scattering to the charge-exchange reaction. However, in the studied range of proton energies and neutron emission angles and momenta the contribution of the S-wave diagrams is relatively small (see Fig. 7).

Before presenting the results of actual calculations, we make a number of remarks of a general character.

It should be emphasized that our technique is based not on perturbation theory in the interaction representation, but on an expansion of the total amplitude in a series of renormalized diagrams. This means that the effects of renormalization and vacuum polarization, the contribution of isobars heavier than the $\Delta(1232)$, off-mass-shell effects, etc., are included in the form factors $F_B(t)$ and in the g' term. The *a priori* accuracy of the $\pi+\rho+g'$ model described above is similar to the accuracy of the optical model of low-energy nucleon-nucleus scattering. From the conceptual point of view, the most complete and convincing justification of this approach to the problem of strong interactions can be found in the studies of Migdal and his collaborators (see Refs. 4–6 and references therein).

To avoid misunderstandings, we shall describe our ter-

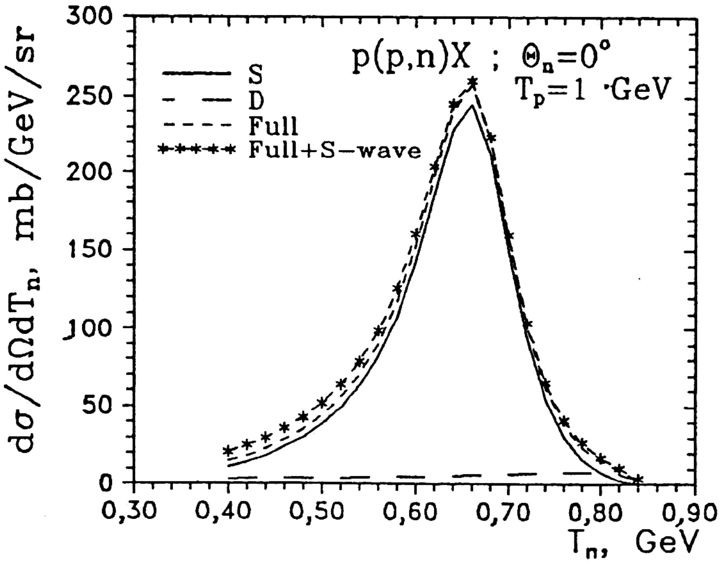


FIG. 7. Partial contributions of the spectator and decay modes to the cross section for the "forward" $p(p,n)X$ reaction with allowance for interference and corrections from nonresonance S-wave πN scattering.

minology in detail. In a number of studies (see, for example, Ref. 67) diagrams 1 and 3 in Fig. 5 are designated as DET (Delta Excitation in Target) and DEP (Delta Excitation in Projectile), respectively, while diagrams 2 and 4, which correspond to Pauli exchange terms, are absent, since the objects ^{3}He and t investigated in Ref. 67 are not identical to the nucleon. In the $(p,n)_\Delta$ reaction studied here, the additional exchange diagrams 2 and 4 appear. Therefore it seems reasonable to employ a more flexible terminology[71] according to which the DET diagrams will be called spectator diagrams, and the DEP diagrams will be called decay diagrams. Then it is natural to refer to the first diagram in Fig. 5 as the spectator direct (SD) diagram, the second as the spectator exchange (SE) diagram, the third as the decay direct (DD) diagram, and the fourth as the decay exchange (DE) diagram.

A calculation of the cross sections for the $(p,n)_\Delta$ reaction in the framework of the modified relativistic $\pi+\rho+g'$ model was carried out for three essentially different sets of parameters of the vertex functions:

OSET ($\Lambda_\pi=1.3$ GeV, $\Lambda_\rho=1.4$ GeV,

$C_\rho=3.96$, $g'_{N\Delta}=0.6$) (Ref. 67);

JAIN ($\Lambda_\pi=1.2$ GeV, $\Lambda_\rho=2.0$ GeV,

$C_\rho=2.00$, $g'_{N\Delta}=0.3$) (Ref. 58);

DMIT ($\Lambda_\pi=0.65$ GeV, $\Lambda_\rho=0.0$ GeV,

$C_\rho=0.0$, $g'_{N\Delta}=0.9$) (Ref. 72).

The coupling constants $f_{\pi NN}$ and $f_{\pi N\Delta}$ were set equal to $f^2_{\pi NN}/4\pi=0.081$ and $f^2_{\pi N\Delta}/4\pi=0.36$, respectively, i.e., the generally accepted values.[7]

As was shown in Fig. 8, all three sets have similar noncentral parts of the potentials, $V_{NC}(q)$, whereas the central potentials $V_C(q)$ have approximately the same shape and form an almost equidistant spectrum of depths in the order JAIN, OSET, DMIT (from bottom to top).

In Fig. 9 we show the theoretical and experimental inclusive neutron spectra for the reaction $p+p\rightarrow n+X$ at $T_p=1$ GeV for the case of small neutron detection angles ($<15°$). The small discrepancy between the theory and experiment in this particular calculation is due to the fact that effects of the energy resolution of the experimental setup were not taken into account. Nevertheless, we can say that the theory and experiment are in satisfactory agreement, and for angles $\theta_{\rm lab}\in[0°,15°]$ there is practically no difference between the results of the calculations for the potentials JAIN, OSET, and DMIT.[65]

The significant difference between the depths of these potentials enables us to conclude that there is a discrete ambiguity in the determination of the parameters of the $\pi+\rho+g'$ model. Neither the angular nor the momentum spectra of the neutrons allow us to assign a preference to any one of the sets. We note that, within a single family of parameters, small differences in the results of a calculation (or fit) may be due to continuous ambiguities in the parameters Λ_π, Λ_ρ, C_ρ, and $g'_{N\Delta}$. Theoretical estimates of Λ_π made in

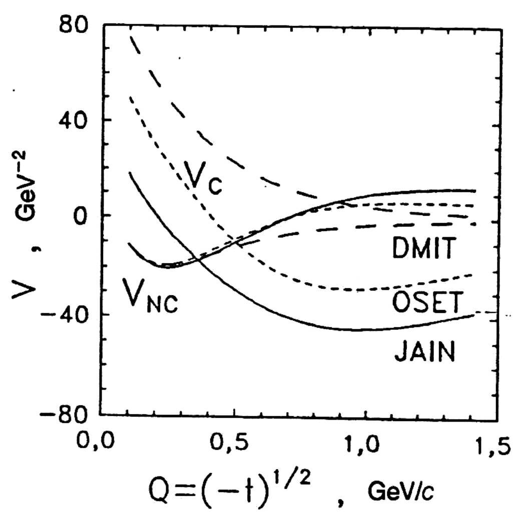


FIG. 8. Effective $NN\rightarrow N\Delta$ transition potentials.

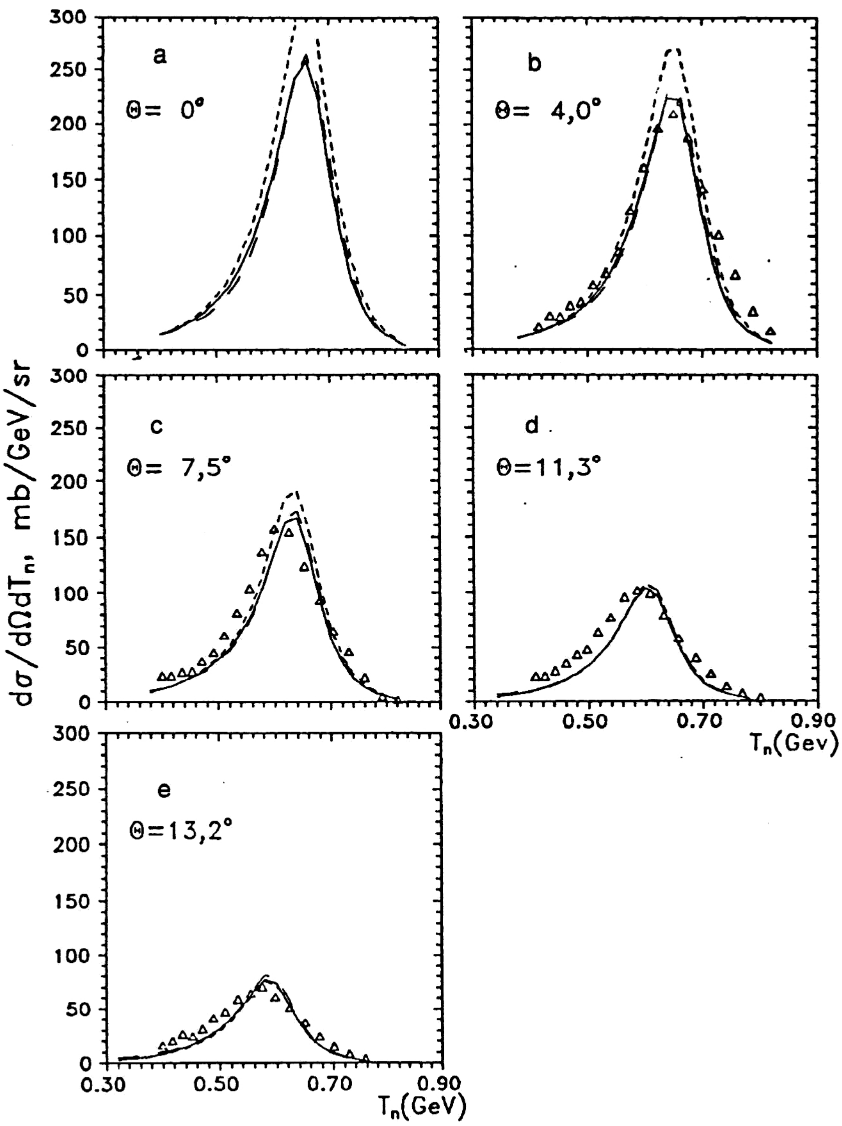


FIG. 9. Cross sections for the $p(p,n)X$ reaction for various neutron detection angles, calculated for three sets of parameters: JAIN (solid curves), OSET (dashed curves), and DMIT (curves with long dashes); the triangles are experimental data.

Refs. 73 and 74 on the basis of the dispersion approach are close to analogous phenomenological values of Λ_π for the sets OSET (Ref. 67) and JAIN.[75]

The fact that these estimates differ somewhat from the DMIT value $\Lambda_\pi=0.65$ (Ref. 72) is not fatal and does not allow us to reject this set solely on the basis of the first principles of quantum theory.

We shall consider simultaneously the reactions $p+p\rightarrow n+X$ and $n+p\rightarrow p+X$ in the region of excitation of the Δ isobar.

For the reaction $p+p\rightarrow n+X$, the decay amplitudes (DD and DE) have a small isospin weight, and, accordingly, the contribution of the decay modes to the charge-exchange reaction cross section is small (see Fig. 10, in which we show the relative contributions of the S and D modes to the cross section for the reaction $p+p\rightarrow n+X$ at $T_p=1$ GeV and $\theta=0°$ for the JAIN set). To estimate the contributions of the various components to the total amplitude for the reaction $p+p\rightarrow n+X$, we show in Fig. 10 the partial (hypothetical) cross sections $d^2\sigma/d\Omega_n dT_n$ as functions of the energy of the detected neutron. The total physical amplitude for the process is the result of the interference of the partial amplitudes. It can be seen from Fig. 10 that the cross section for the process $p+p\rightarrow n+X$ is determined mainly by the spectator mode, and for $T_p\sim1$ GeV, i.e., far from the threshold, this conclusion holds for all three sets of potentials (JAIN, OSET, DMIT) over practically the entire range of studied angles and energies.

In order to ascertain the relative role of a partial superposition of the individual amplitudes (SD+SE=S and DD+DE=D), we consider the reaction $n+p\rightarrow p+X$, for which the isospin weights of the S and D modes are commensurate. In this case, the decay modes give an appreciable contribution to the total cross section. Nevertheless, even this additional information is insufficient to assign a preference to one of the three sets discussed above (see Fig. 11).

It can be seen from Fig. 11 that the decay and spectator amplitudes are practically orthogonal at $T_p=1$ GeV and $\theta=0°$, and therefore the total cross section is approximately equal to the sum of the hypothetical partial spectator and decay cross sections. This result holds for all three sets

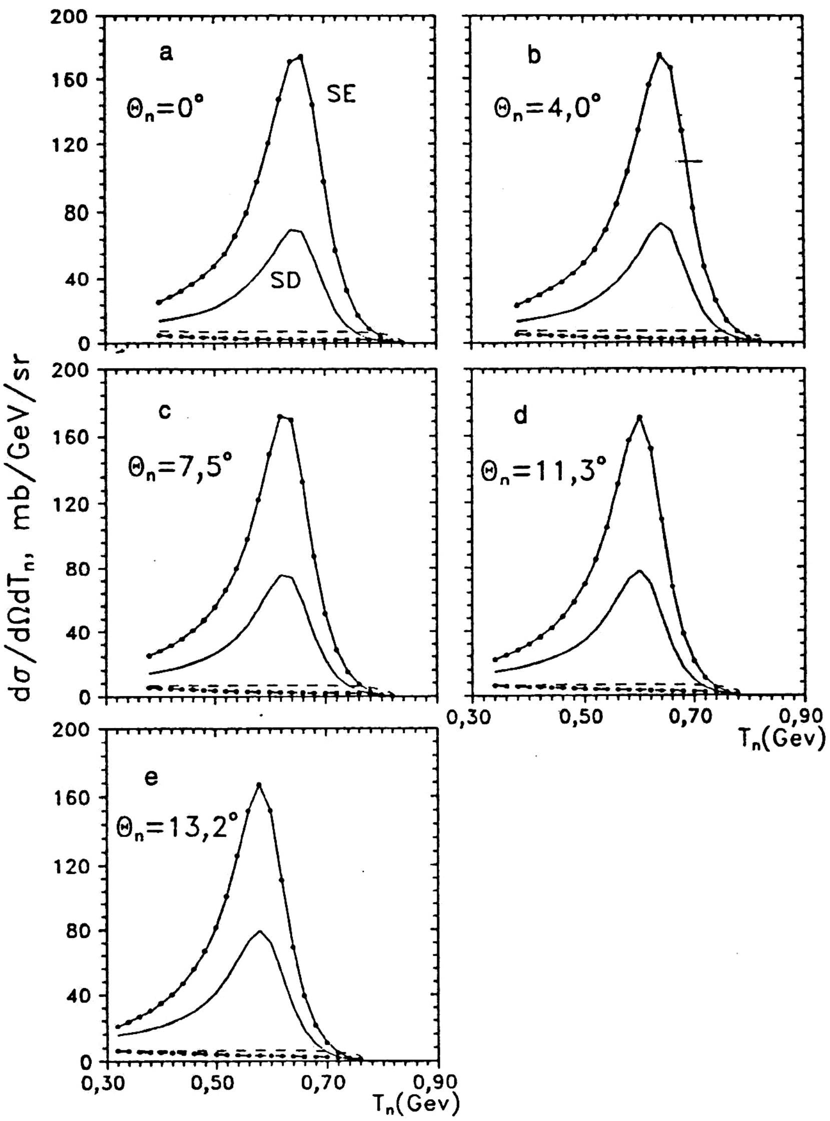


FIG. 10. Partial contributions of the diagrams shown in Fig. 5 to the cross section for the $p(p,n)X$ reaction, calculated with the JAIN set: SD (solid curves), SE (solid curves), DD (dashed curves), and DE (dashed curves).

(OSET, JAIN, DMIT) and in this sense is model-independent.

The maxima of the S and D modes are separated by an amount $\Delta E \approx 140$ MeV$\approx m_\pi$, which indicates that this shift has a kinematic character.

We shall consider in more detail the individual SD, SE, DD, and DE amplitudes and their features for all three sets.

In Figs. 12a–12c, corresponding to the JAIN, OSET, and DMIT sets, respectively, we show the results of calculations of the partial cross sections for $\theta_{lab}=0°$, 4°, 7.5°, 11.3°, 13.2°. It can be clearly seen that the partial cross sections for all three sets have a number of common properties. For example, all the spectator components have the maximum at practically the same position; the decay partial cross sections have a smooth energy dependence and a sharp cutoff at the hard boundary of the spectrum. The total contributions of the spectator and decay components are practically the same for all three sets (see Fig. 11), and the spectator term always dominates in the neighborhood of the resonance maximum. This concludes our list of the similarities of the analogous partial cross sections for the various sets.

We now consider the SD and SE partial cross sections. Figures 12a–12c clearly show significant differences between the angular and momentum partial spectra for the three sets that we have investigated.

For the JAIN set, the SE term is much larger than the SD term for all the investigated angles, and the corresponding amplitudes are practically orthogonal for $\theta=0°$, i.e., $\sigma(S) \approx \sigma(SE)+\sigma(SD)$. With increasing neutron detection angle (θ), the SE and SD amplitudes begin to interfere destructively, and this leads to a monotonic decrease of $\sigma(S)$.

For the OSET set, the SD term decreases monotonically with increasing θ, remaining dominant up to $\theta=7.5°$. The contribution of the SE term is practically independent of the angle θ. For $\theta \leqslant 4°$, the interference of the SD and SE amplitudes is constructive. At large angles, it becomes destructive.

For the DMIT set, $\sigma(S) \approx \sigma(SD)$ for all the investigated cases.

The contributions of the individual components to the decay mode for the JAIN, OSET, and DMIT sets differ just as much as in the case of the spectator modes considered above (see Figs. 12a–12c).

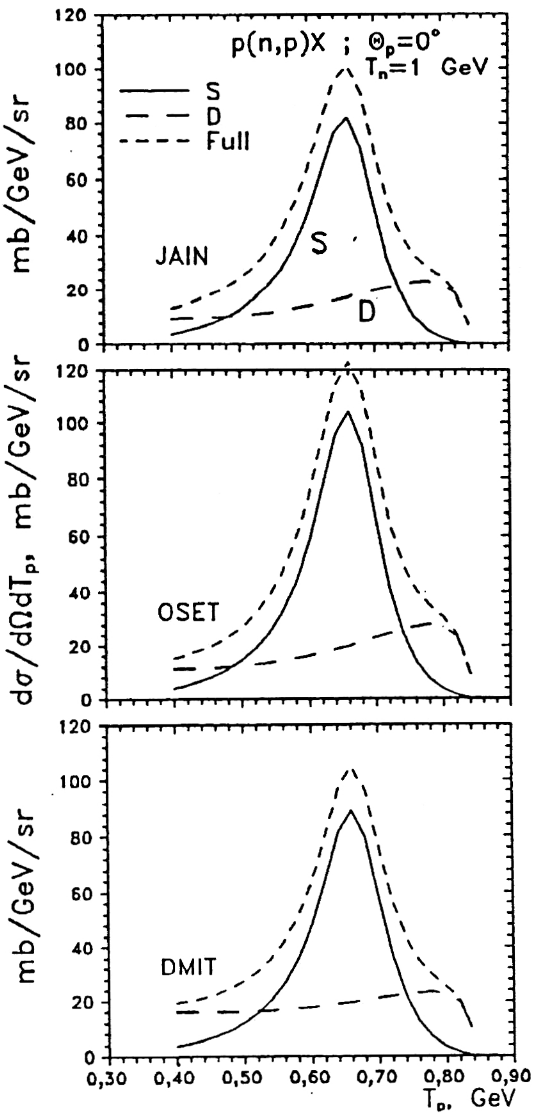


FIG. 11. Partial contributions of the spectator and decay modes to the cross section for the "forward" $p(p,n)X$ reaction for various sets of phenomenological constants (see the text).

The observed characteristics of the formation of the total $(p,n)_\Delta$ reaction amplitude from the partial amplitudes cast doubt on various intuitive arguments in favor of one or another mechanism for the process. In order to resolve the observed discrete ambiguity described above, we require additional arguments that are not based on the intrinsic features of our diagrammatic technique.

One of the most attractive possibilities for testing the JAIN, OSET, and DMIT sets is to study the T_p dependence of the reduced cross sections for the charge-exchange reaction.[1,66] It is well known (see, for example, Ref. 1 and the references cited in that review) that the Δ-isobar mechanism of the reaction $p+p\to n+p+\pi^+$ dominates in the region of energies $0.8\leqslant T_p\leqslant 3$ GeV. The nonresonance background begins to have an appreciable influence for $T_p>1.5$ GeV. As has been shown by one of the present authors (E.A.S.), it is in the region of energies $0.8\leqslant T_p\leqslant 3$ GeV that one finds approximate scaling of the invariant reduced cross sections,[1] i.e., the invariant cross sections normalized to the total cross section of the corresponding "elementary" reaction at the same initial projectile energy and, if one is considering charge exchange of relativistic nuclei, to the corresponding transition form factor and Glauber absorption factor.

In Fig. 13 we show the results of calculations of σ_{red}. Clearly, there is acceptable agreement between the theoretical and experimental data for the DMIT and OSET sets, whereas the JAIN set reproduces the T_p dependence of the reduced cross sections much worse. Thus, the use of the JAIN set is at least limited to the region of energies $T_p\approx 1$ GeV. Moreover, owing to the small value of the constant $g'=0.3$, it corresponds to a strong effective transition potential $V^{\text{eff}}_{NN\to N\Delta}$ representing an attraction (see Fig. 8).

It is interesting to note that the analysis of the experimental data on π-mesic atoms leads to a constant $g'_{N\Delta} = 0.4 \mp 0.2$, in complete agreement with $g'_{N\Delta}$ for the JAIN and OSET sets. The existence of a fairly wide error corridor in the determination of $g'_{N\Delta}$ may indicate a possible dependence of $g'_{N\Delta}$ on the excitation energy.

The set of experimental data on coherent pion production[76] and also the fact that the experimental values of the cross sections for the reaction $\pi+N\to\Delta+\pi\to N+\pi+\pi$, which takes place through the mechanism of ρ-meson exchange,[77–79] are not small enable us to assign a preference to the value $g'_{N\Delta} = 0.6$ and, thus, to give a definitive solution to the problem in favor of the OSET set.

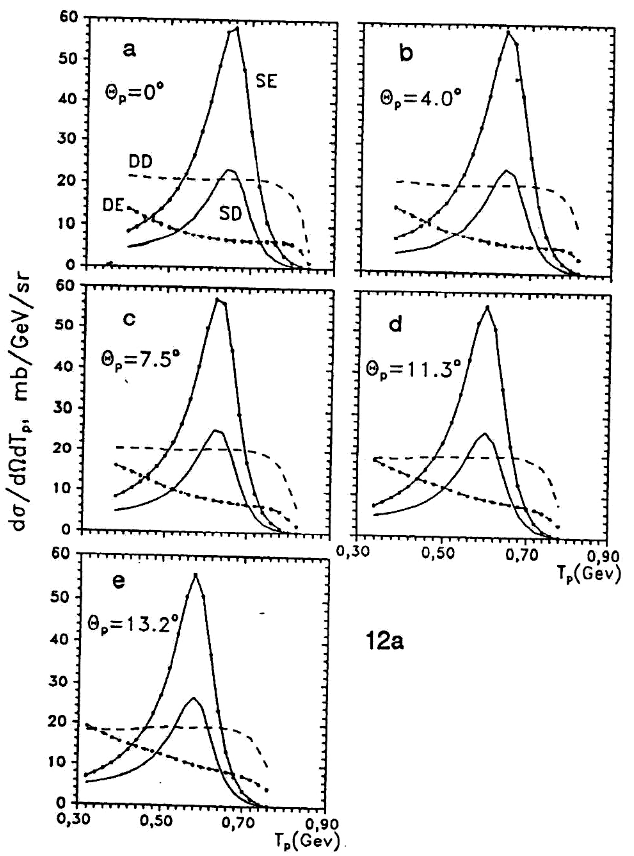
A solution of the inverse scattering problem on the basis of an analysis of the energy dependence of observed quantities is not something new. This problem is well known, both in classical mechanics (nonrelativistic[80] and relativistic[81]) and in quantum mechanics (see, for example, Ref. 82). However, as we showed above, in going to intermediate energies in quantum field theory this problem acquires an unusual formulation; namely, for a given analytic form of the parametrization of the vertex functions it is formulated as a generalized Sturm–Liouville problem for finding the eigenvectors of the model.

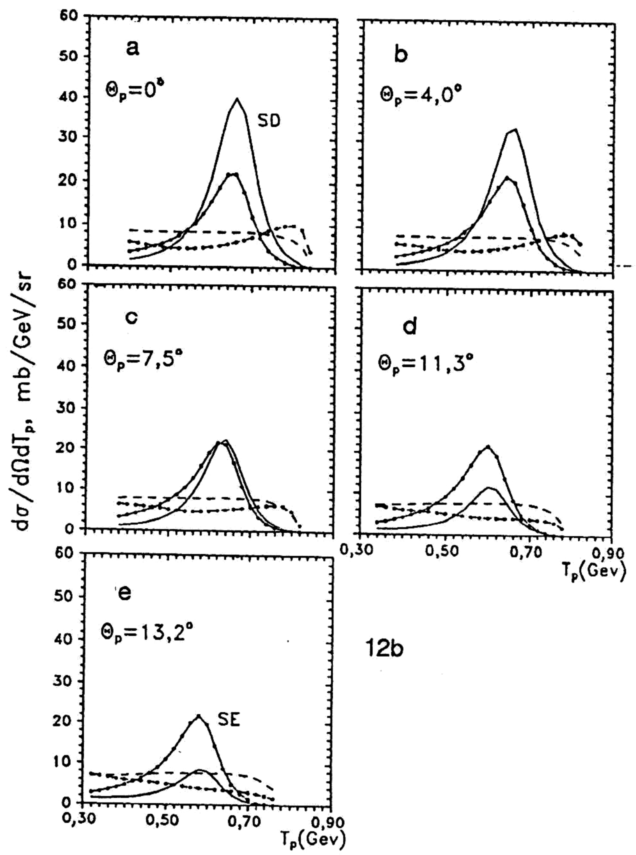
We summarize the foregoing discussion as follows.

1) Roughly equivalent descriptions of the momentum and angular spectra of the neutrons from the charge-exchange reaction $p+p\to n+p+\pi^+$ in the energy interval $0.8\leqslant T_p\leqslant 1.5$ GeV are given by the JAIN, OSET, and DMIT sets of parameters λ_π, λ_ρ, C_ρ, and $g'_{N\Delta}$.

2) These sets (and the transition potentials $V^{\text{eff}}_{NN\to N\Delta}$) reproduce the total amplitude $T_{NN\to N\Delta}$ in fundamentally different ways, leaving the value of its squared modulus $|T_{NN\to N\Delta}|^2$ practically unchanged. This means that in going from one set to another the amplitude $T_{NN\to N\Delta}$ acquires an additional unimodular factor of the type $\exp(i\alpha)$, and the agreement of the relativistic $\pi+\rho+g'$ model with the results of the phase-shift analysis of pion-nucleon scattering[7] has the consequence that the additional phase α is a multiple of π: $\alpha_n=\pi n$ $(n=0,1,2,\ldots)$. In its turn, the discontinuous change of α_n in going from one set to another is related to the equidistant spacing of the spectrum of transition potentials.[65]

3) The scaling of the "reduced" invariant cross sections (Fig. 13), in conjunction with additional data[76,83] on the





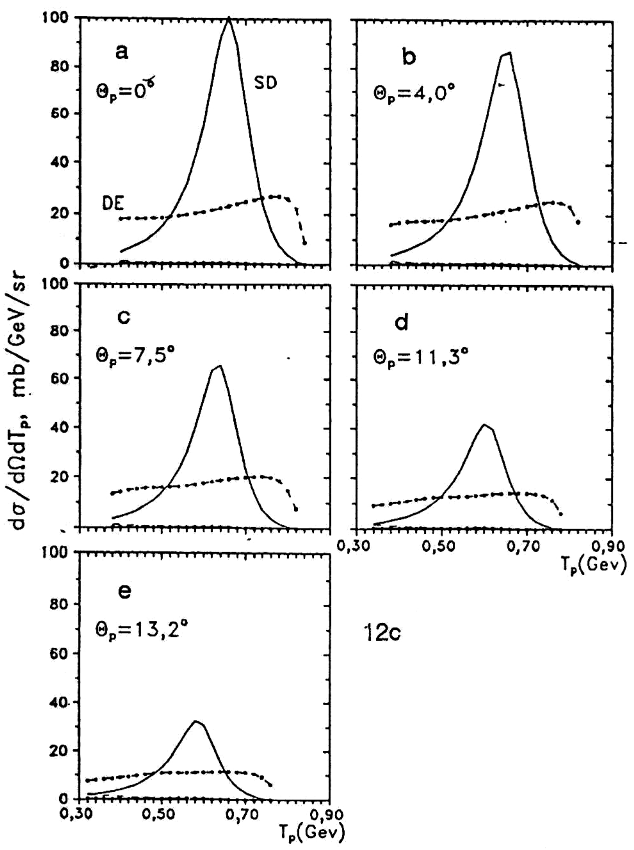


FIG. 12. a. Partial contributions of the graphs shown in Fig. 5 to the cross section for the $p(n,p)X$ reaction, calculated with the JAIN set: SD (solid curves), SE (solid curves), DD (dashed curves), and DE (dashed curves). (b) The same as in Fig. 12a, but for the OSET set. (c) The same as in Fig. 12a, but for the DMIT set.

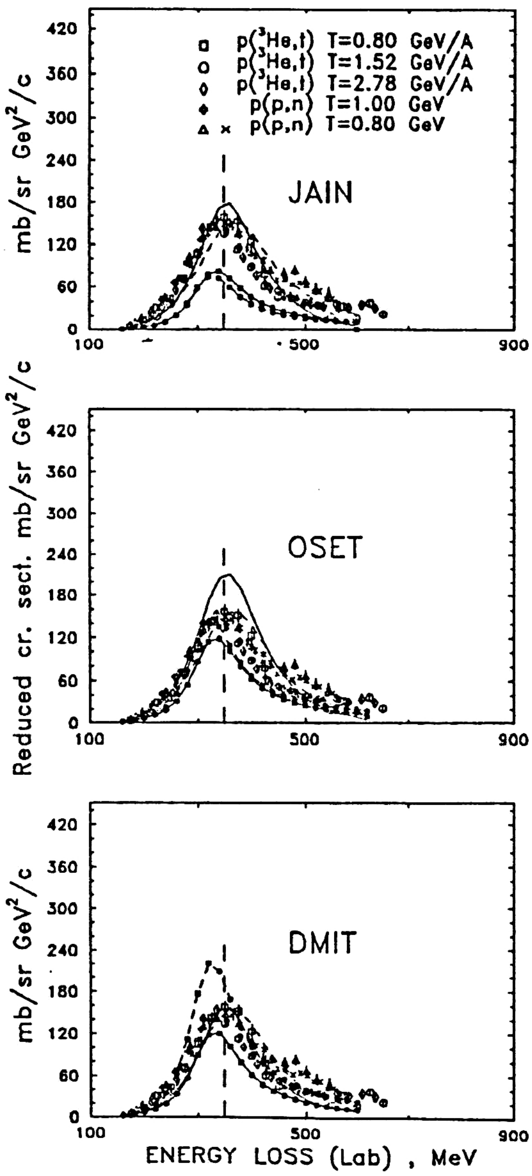


FIG. 13. Reduced invariant cross sections for the "forward" $p(p,n)$ and $p(^3\mathrm{He},t)$ reactions (see the review of Ref. 1 and references therein to the experimental data). The theoretical curves were calculated for the energies $T=0.8$ GeV (solid), 1.0 GeV (dashed), 1.52 GeV/A (solid with points), and 2.78 GeV/A (dot–dash); allowance was also made for the Glauber absorption factor for the $p(^3\mathrm{He},t)$ reaction.

$p(\alpha,\alpha)$ $(N+\pi)$ reaction and on π-mesic atoms, and also the combined information on the reaction $\pi+N\to\Delta+\pi\to N+\pi+\pi$, enables us to assign a preference to the OSET set.

5. INTERFERENCE EFFECTS IN THE REACTIONS $p+p\to n+p+\pi^+$, $n+p\to p+n+\pi^0$, AND $n+p\to p+p+\pi^-$

The comparison of different inclusive charge-exchange reactions (in particular, $p+p\to n+X$ and $n+p\to p+X$) at intermediate energies is of considerable interest for many reasons.

First, such an analysis makes it possible, with a high degree of confidence, to obtain information about the relative contributions of different isospin components of the nuclear forces. Second, there is an additional possibility of studying various effects of the final-state interaction. Third, the reactions $p+p\to n+X$ and $n+p\to p+X$ serve as a test for the appearance of dibaryon resonances.[84]

Finally, as we shall show below, on the basis of the analysis of these reactions it is possible to determine the limits of applicability of the approximation of transition potentials and of the formalism of effective numbers at intermediate energies.

The purpose of the present section is to study the energy and angular dependence of the ratio of the cross sections for the charge-exchange reactions $p+p\to n+X$ and $n+p\to p+X$ in the region of excitation of the $\Delta(1232)$ resonance.

In most recent theoretical studies (with the possible exception of those of Refs. 65–67, 85, and 86, which were based on the formalism of quantum field theory), inclusive charge-exchange reactions in the region of Δ-isobar excitation have been described by means of the meson-exchange model (the OPE model or the $\pi+\rho+g'$ model) in conjunction with the approximation of a transition potential (for more details, see Refs. 57, 58, 71, 75, and 87–89).

From the point of view of the diagrammatic technique, this approximation corresponds to inclusion of direct and exchange diagrams of the types shown in Fig. 5, and in the momentum representation it is described by the $NN\to N\Delta$ transition potential[57,58]

$$V_{\sigma T}(\omega,\mathbf{q})=\{V_L(q)(\mathbf{S}^+\cdot\hat{q})(\boldsymbol{\sigma}\cdot\hat{q})+V_T(q)([\mathbf{S}^+\times\hat{q}]\times[\boldsymbol{\sigma}\times\hat{q}])\}(\mathbf{T}\cdot\boldsymbol{\tau})$$

[cf. Eq. (88)].

For the charge-exchange reactions $p+p\to n+\Delta^{++}$ and $n+p\to p+\Delta^0$, the corresponding isospin matrix elements are equal to the Clebsch–Gordan coefficients

$$\left(\frac{1}{2}\,1\,\frac{1}{2}\,1\,\middle|\,\frac{3}{2}\,\frac{3}{2}\right)=1 \quad\text{and}\quad \left(\frac{1}{2}\,1\,\frac{1}{2}\,1\,\middle|\,\frac{3}{2}\,\frac{1}{2}\right)=1/\sqrt{3}. \tag{115}$$

Therefore, in the potential approximation described above, the cross sections for the kinematically equivalent charge-exchange reactions $p+p\to n+\Delta^{++}$ and $n+p\to p+\Delta^0$ are proportional, and

$$R=\frac{d\sigma_{p+p\to n+\Delta^{++}}/d\Omega_n dE_n}{d\sigma_{n+p\to p+\Delta^0}/d\Omega_p dE_p}=3. \tag{116}$$

The relation (116), which was obtained for the charge-exchange reaction on free nucleons, is widely used to analyze analogous inclusive processes on nuclei (see, for example, Refs. 89–91 and others). In most studies, the charge-exchange cross section on a nucleus is expressed in terms of the cross section for the corresponding free process and the effective number of nucleons participating in the reaction. For example, in the distorted-wave impulse approximation without the effects of Fermi motion and Pauli blocking, the (p,n) reaction cross section has the form[89,91]

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n} \approx \left(Z + \frac{N}{\langle R \rangle} \right) \langle f^2 \rangle \frac{d\sigma_{p+p\to n+\Delta^{++}}}{d\Omega_n}, \qquad (117)$$

where $\langle R \rangle = 3$ is the integrated value of the ratio R, and $\langle f^2 \rangle$ is the effective absorption factor calculated in the Glauber approximation or on the basis of some similar model taking into account the influence of nuclear distortions in the entrance and exit channels of the reaction. Analogous formulas, having the same degree of accuracy as (117), have been used by many authors (see, for example, Ref. 20).

The formal basis for the use of the impulse approximation is the smallness of the binding energy of a nucleon in the target nucleus in comparison with the energy of the incident particle ($T_p \sim 1$ GeV). In this case, two fundamental problems traditionally drop out of the analysis.

1. The approximation of effective numbers presupposes the validity of the RPA, i.e., the exclusion of interference effects from the results of calculations.

To be more specific, in the case of exclusive reactions a necessary condition for the validity of factorized relations of the type (117) is the absence of interference of channels.

In an inclusive experiment, when the detector is designed to detect one type of particle (X) and the energy of the incident particles is sufficient to satisfy the approximation of completeness (the closure approximation, according to the terminology of Ref. 28), the number of open reaction channels is extremely large. Formally, this provides a basis for the use of the principle of random phases for the interference terms.[26,27] For knockout reactions, this approximation is rather good (see Refs. 9, 22, 26, and 27). In the case of excitation of baryon resonances, the physics of the process is complicated by the fact that, besides intranuclear degrees of freedom, much harder intranucleon degrees of freedom are also excited. The number of channels of deexcitation of these degrees of freedom is small at $T_p \sim 1$ GeV/nucleon, the principle of random phases for these reaction channels is completely inapplicable[65,66] (at least in the energy region studied here), and in a number of cases interference of channels begins to play a decisive role in the formation of the cross sections for the process. We note that interference effects of this kind were first studied on the basis of the formalism of effective numbers in Ref. 22.

2. In studies making use of the potential approach (see, for example, Refs. 57, 58, 71, 75, 87–89, and 92–95), there is no discussion of the degree of applicability, in the region of energies ~1 GeV, of the relations for the transition potentials obtained in the low-energy approximation without inclusion of field-theory effects other than a trivial retardation.

It can be seen from the following discussion that it is not always permissible to ignore these problems.

The experimental data on the ratio $R(T_n, \theta_n)$ for $T=0.8$ GeV and $\theta=0°$ are more or less close to the theoretical value $R_{\text{theor}}=3$ only near the maximum of the Δ peak, and near the upper kinematic limit of the spectrum one observes a suppression of R by practically an order of magnitude (Fig. 14). In Ref. 84 this effect was explained qualitatively by the existence of a constructive interference of the virtual Δ^+ and Δ^0 isobars for the reaction $n+p\to n+X$, and a destructive interference of the Δ^{++} and Δ^+ isobars for the process

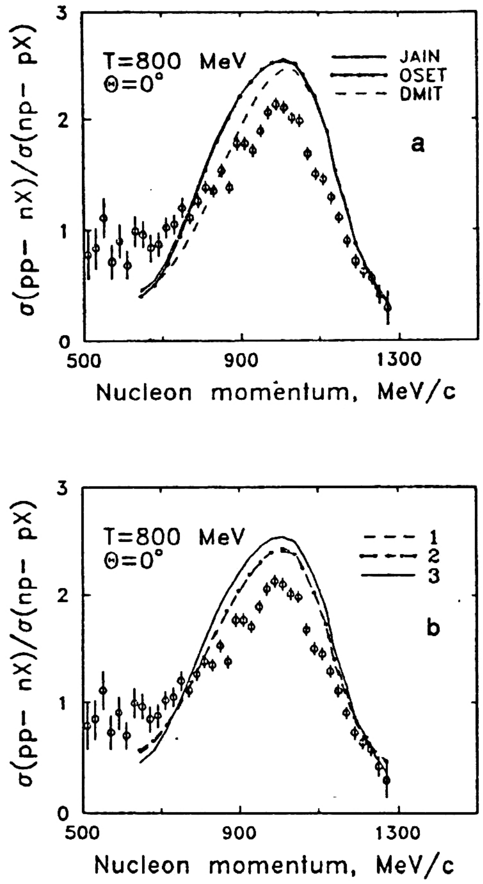


FIG. 14. Ratio R as a function of the momentum of the detected nucleon: a) calculation for various sets of constants of the one-boson model with a g' term; b) calculation for the JAIN set with only the P-wave contribution (curve *1*), with the S- and P-wave contributions but without allowance for the resolution of the spectrometer (curve *2*), and with allowance for this resolution (curve *3*).

$p+p\to n+X$. However, no consistent calculations making it possible to reconstruct a sufficiently complete picture of this effect were carried out in that study.

In Ref. 66 a detailed investigation was made of effects of the interference of the Δ^{++} and Δ^+ isobars (the Δ^+ and Δ^0 isobars) excited in the reactions $p+p\to n+X(n+p\to p+X)$ by nucleons with energy in the range $0.8 \leq T \leq 10$ GeV. Calculations were performed for the OSET, JAIN, and DMIT sets of parameters of the vertex functions.

As can be seen from Figs. 15 and 16, the ratio $R(T,\theta)$ has a similar angular and energy dependence for all three sets, despite the fact that the numerical values of the theoretical cross sections for the reactions $p+p\to n+X(n+p\to p+X)$ for $T_p(T_n)>1.5$ GeV are very different [σ(DMIT)>σ(OSET)>σ(JAIN)] (see Fig. 13). The angular dependence of the integrated ratios $\langle R \rangle$ is weak (see Fig. 17) and changes by less than 10% in going from the JAIN set to the OSET or DMIT set. With increasing energy T_n, the effects of interference of the Δ^+ and Δ^0 isobars in the reaction $n+p\to p+X$ become weaker, and $\langle R \rangle \to 3$ (see Fig. 18). We note that for the entire studied region of energies and angles the integrated quantity $\langle R \rangle$ lies in the interval $2 \leq \langle R \rangle \leq 3$, and this enables us to understand the success of estimates of the

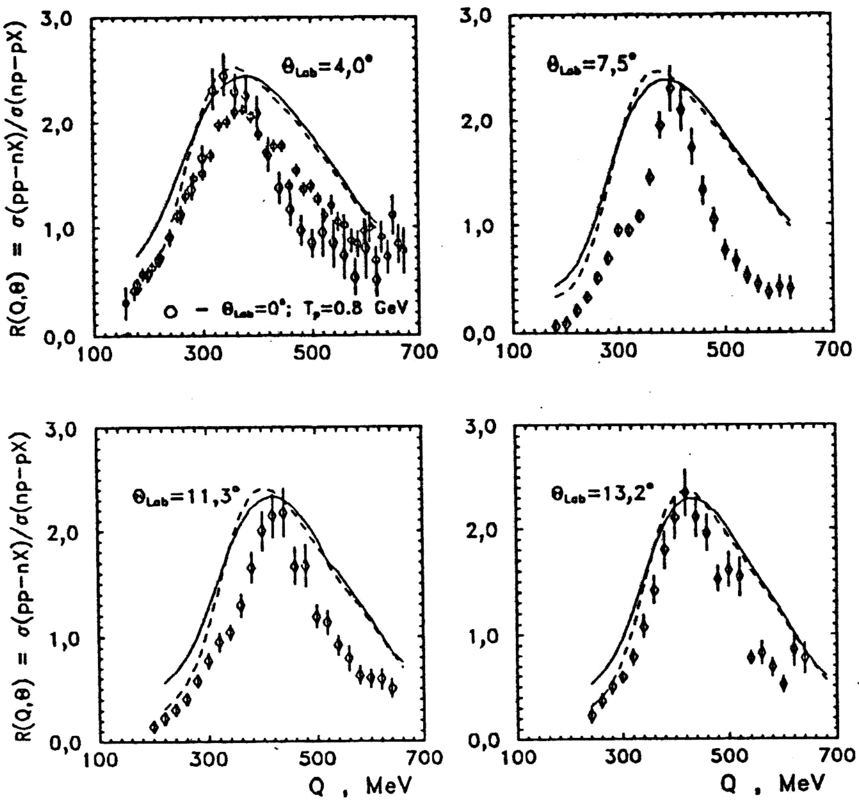


FIG. 15. Calculation of the dependence of the ratio R for the JAIN set at various nucleon emission angles, with allowance (solid curves) and without allowance (dashed curves) for the resolution of the spectrometer; in both cases, only the isobar contribution is taken into account.

type (117) for charge-exchange differential cross sections integrated over the momentum spectrum, while these spectra themselves can be noticeably distorted as a result of interference of the spectator and decay modes in the total amplitude of the resonance charge-exchange reaction.

The mechanism of influence of the D modes on the momentum spectrum of the detected nucleons is shown in Fig. 19. It can be clearly seen from this figure that (contrary to the assertion in Ref. 84) the interference of the Δ^{++} and Δ^{+} isobars gives practically no contribution to the process

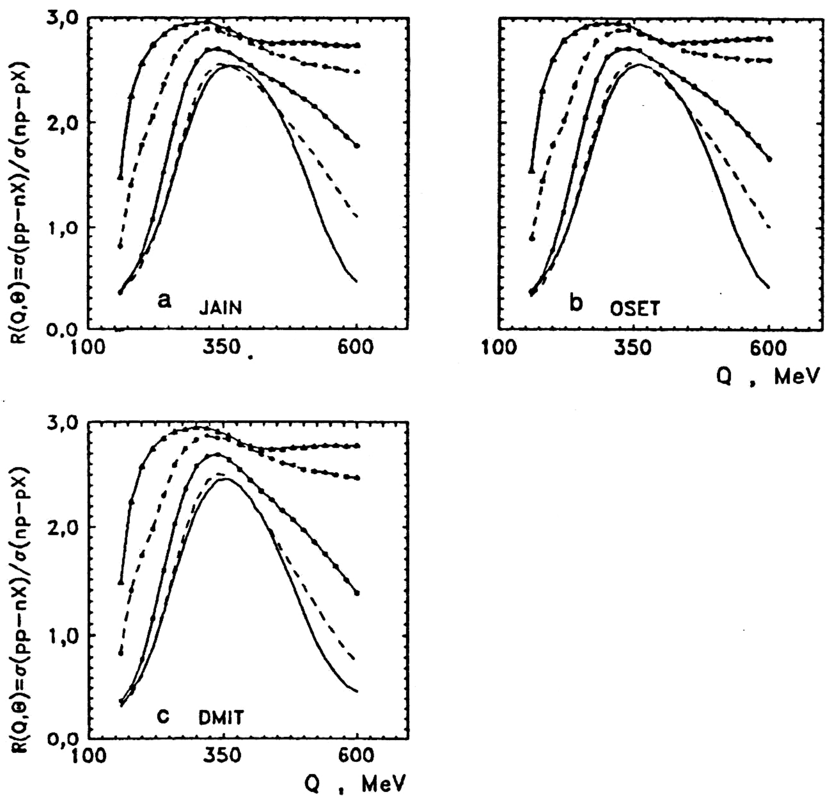


FIG. 16. The same as in Fig. 15, but the ratio is shown for a fixed nucleon emission angle at various initial energies.

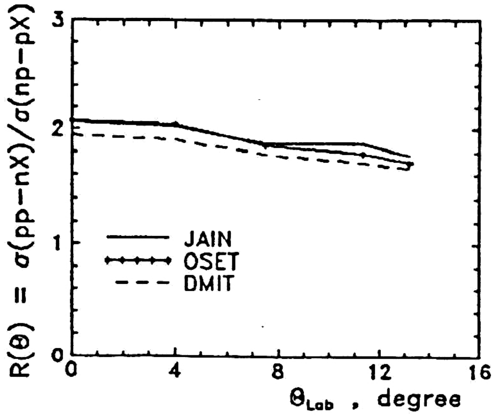


FIG. 17. Angular dependence of the averaged ratio R at kinetic energy 1 GeV.

$p+p\rightarrow n+X$, since the decay diagrams have a small isospin weight:

$$\mathrm{ISF}(1)=\mathrm{ISF}(2)=-\sqrt{2},\quad \mathrm{ISF}(3)=\mathrm{ISF}(4)=\sqrt{2/3}, \qquad (118)$$

where the phases are chosen according to Ref. 70. For the process $n+p\rightarrow p+X$, the decay modes are not small. In the case of Δ-isobar decay with emission of a π^0 meson we have

$$\mathrm{ISF}(1)=\mathrm{ISF}(2)=-\mathrm{ISF}(3)=-\mathrm{ISF}(4)=2/3, \qquad (119)$$

while for production of a π^- meson we have

$$\mathrm{ISF}(1)=\mathrm{ISF}(2)=\mathrm{ISF}(3)=\mathrm{ISF}(4)=\sqrt{2/3}. \qquad (120)$$

Thus, for the process $n+p\rightarrow p+X$ there is no isospin suppression of the decay components, and we observe constructive interference of the S and D amplitudes (Fig. 20). It can be clearly seen from Fig. 20 that the decay peak is shifted into the hard part of the spectrum by an amount $\omega_{SD}\approx m_\pi\approx 140$ MeV; this is explained by the difference between the S and D kinematics (the positions of the Δ-propagator singularities for the S and D diagrams). It is interesting to note that the size of this displacement is practically independent of the neutron (proton) laboratory detection angle $\theta_{\rm lab}$ for $\theta_{\rm lab}\leqslant 15°$. However, as $\theta_{\rm lab}$ increases from 0° to 15°, despite the constancy of the displacement $\omega_{SD}\approx m_\pi$, one observes an enrichment of the hard part of the spectrum by the decay modes (see Figs. 19 and 20). It is this effect that is associated with the decrease of $R(T,\theta)$ with increasing angle θ at small energy transfers $\omega=E_p-E_n\leqslant 0.4$ GeV in the laboratory system (Fig. 21). This effect does not

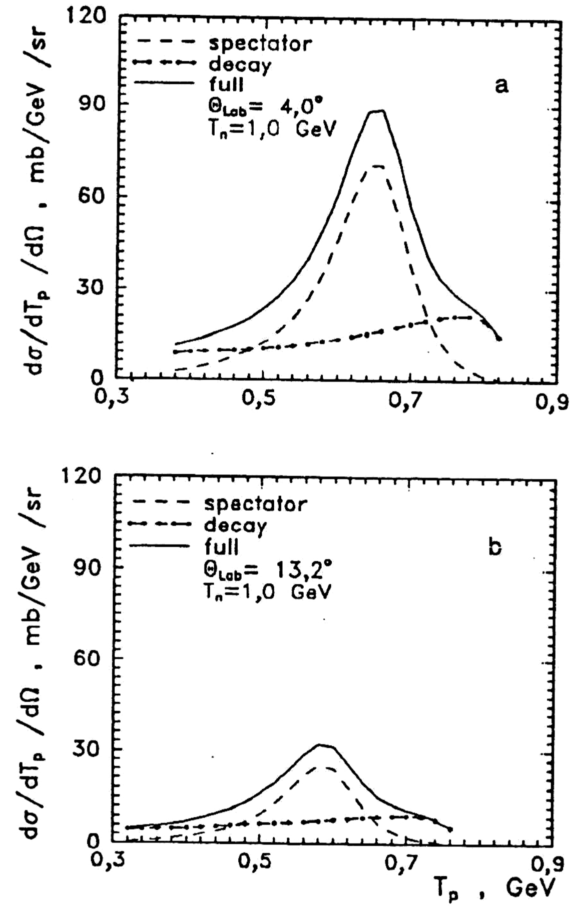


FIG. 19. Partial cross sections for the "forward" reaction $n+p\rightarrow p+X$, calculated with the JAIN set.

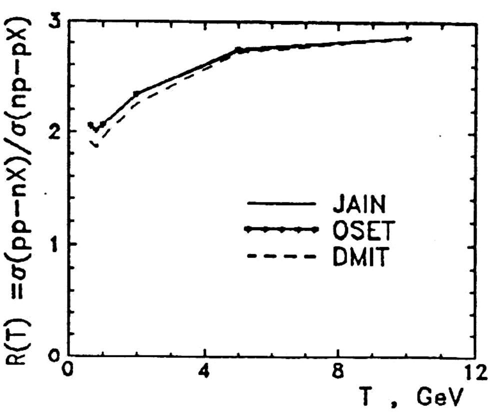


FIG. 18. The same as in Fig. 17, but for emission angle 0° and as a function of the initial energy of the detected neutron.

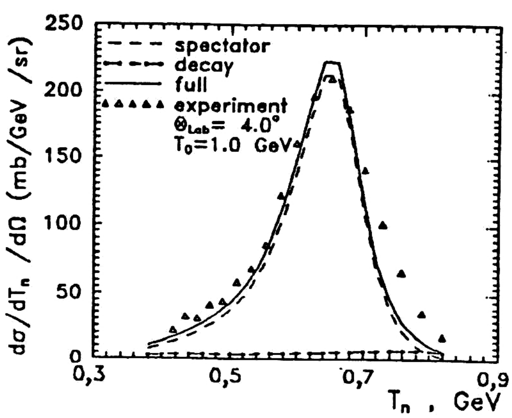


FIG. 20. The same as in Fig. 19, but for the reaction $p+p\rightarrow n+X$.

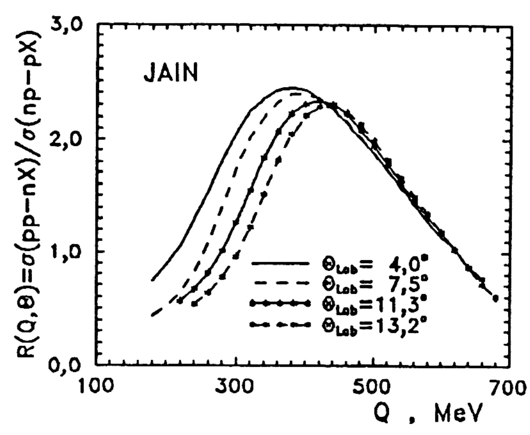


FIG. 21. Calculated ratio R at energy 1 GeV as a function of the energy transfer Q at various detection angles; the calculations include a convolution of the theoretical cross sections with the resolution function for the Gatchina experiment.[97–100]

influence the soft part of the spectrum, where the change in the momentum due to the deflection of the particle is small against the background of inelastic losses.

By comparing Eqs. (119) and (120), we can draw an important conclusion. We introduce the notation $M_S(p)$ for the matrix element corresponding to the sum of the resonance spectator diagrams 1 and 2, and $M_D(p)$ for the sum of the resonance decay diagrams 3 and 4 in Fig. 5. It is obvious that

$$|M_S(p)+M_D(p)|^2=|M_S(p)|^2+|M_D(p)|^2 +2\,\mathrm{Re}(M_S(p)M_D^*(p)). \qquad (121)$$

It can be seen from (119) and (120) that the absolute value of each of the terms in (121) is twice as large for the exclusive reaction of charge exchange with production of a neutral π^0 meson as for the kinematically equivalent reaction with production of a π^- meson, and the off-diagonal term in (121) has opposite signs for the processes $n+p\to p+p+\pi^-$ and $n+p\to n+p+\pi^0$. This means that for the analogous inclusive reaction $n+p\to p+X$ the corresponding off-diagonal elements will be subtracted from one another.

Physically, this means that, in accordance with the completeness of the system of final states, the virtual Δ^0 isobars decaying into the channels $p+\pi^-$ and $n+\pi^0$ interfere destructively. If one attempts to perform a kinematically equivalent exclusive experiment, this interference disappears. Thus, in this case we encounter a nontrivial example of the Feynman approach to quantum mechanics (see Ref. 96 and the description of the thought experiments on which the path-integral approach is based). The correction terms describing the contribution of S-wave πN scattering have practically no influence on the magnitude of the effect described above, leaving unchanged the qualitative conclusion about the possible existence of coherent states $p+\pi^-=\Delta^0$ and $n+\pi^0=\Delta^0$ with isospin 3/2.

We now compare the results of the calculation with experiment. It can be seen from Fig. 14 that by including the decay graphs in the calculation it is possible to give a qualitative description of the experimental data,[84] and allowance for the contribution of S-wave πN scattering gives somewhat better agreement of the theory with experiment.

In Fig. 15 we show the results of a calculation together with data extracted from an analysis of the reaction $p+d\to n+X$.[97–100] The value of the ratio $R(Q,\theta_{\mathrm{lab}})$ was calculated under the assumption that

$$\sigma_{p+d\to n+X}\approx\langle f^2\rangle(\sigma_{p+p\to n+X}+\sigma_{n+d\to p+X}), \qquad (122)$$

where $\langle f^2\rangle$ is the screening factor ($\langle f^2\rangle=0.7$ for $\theta=4°$, $\langle f^2\rangle=0.75$ for $\theta=7.5°$ and 11.3°, and $\langle f^2\rangle=0.8$ for $\theta=13.2°$).

It is evident from Fig. 15 that the theory is in qualitative agreement with experiment. Some quantitative discrepancy between the theory and experiment can be explained fairly readily. The rough approximation (122) does not take into account the fact that the hard part of the spectrum of neutrons from the reaction, $\sigma_{p+d\to n+X}$, is enriched by the contribution of quasielastic processes, while the soft part of the spectrum is enhanced by the final-state interaction.[84] It is only in the vicinity of the maximum of the Δ peak that both of these effects are small. We note that the values used above for $\langle f^2\rangle$ (Ref. 66) are in good agreement with other analogous calculations in the Glauber approximation and demonstrate clearly the well-known trend toward a decrease of the screening effects with increase of the emission angle of the detected particle.

The anomalous relation $R(T,\theta)_{\mathrm{exp}}>R(T,\theta)_{\mathrm{theor}}$ for the angle $\theta=4°$ (Fig. 14) near the upper kinematic limit of the spectrum is explained by the effects of the finite energy resolution of the spectrometers. Therefore the convolution of the theoretical cross section with the resolution function of the experimental setup[1,124] eliminates this anomaly and significantly improves the agreement between the theory and experiment.

The results presented above are confirmed by theoretical calculations for the (^{3}He,t) reaction performed by the group of Oset.[67,101] A fundamental difference in our approach is the fact that we choose a special class of experiments in which the effect of interference of the virtual Δ isobars manifests itself most clearly.

The existence of pronounced interference of the Δ^+ and Δ^0 isobars in charge-exchange reactions imposes additional limits on the applicability of the impulse approximation,[28] the ideology of effective numbers,[9,26,27] the Glauber approximation,[14,102–104] cascade calculations,[45] and the model of successive decays for reactions of multiparticle production.[16]

6. ANALYSIS OF THE REACTIONS $A(p,n)_\Delta B$ AND $A(^3\mathrm{He},t)_\Delta B$ IN THE LANGUAGE OF EFFECTIVE NUMBERS

The experience of working with effective numbers shows that for many direct reactions they give a good description[9,10,26,27] of the experimental data. In all the cases that have been investigated, the deviation of the theory from experiment has been related to a breakdown of the DWIA.[9,10,26] For example, for the (p,pd) deuteron-knockout reaction at large momentum transfers q, the application of the DWIA is incorrect because of the short-lived NN

TABLE IV. Mass and angular dependences (in the laboratory system) of the experimental cross sections[97–100] (at T_p=1 GeV) and of the effective numbers $\tilde{N}^{\exp}$. The value $\tilde{N}^T$ is calculated theoretically for θ_n=0°.

Target	$\tilde{N}^T$	$\frac{d\sigma[A(p,n)_\Delta B]^{\exp}}{d\Omega_n}$ mb/sr			$\tilde{N}^{\exp}$		
	θ_n = 0°	θ = 4°	θ = 7,5°	θ = 11,3°	θ = 4°	θ = 7,5°	θ = 11,3°
H	1,000	42,7	31,0	20,7	1,00	1,00	1,00
D	0,605	52,1	44,7	29,6	1,22	1,44	1,43
^{7}Li	2,150	123,8	90,5	—	2,90	2,92	—
^{9}Be	2,500	177,1	132,4	81,2	4,15	4,27	3,92
^{10}B	2,770	165,8	118,3	73,2	3,88	3,82	3,54
^{11}B	2,740	159,8	117,5	74,3	3,74	3,79	3,59
^{12}C	2,960	162,3	122,9	81,3	3,80	3,96	3,92
^{16}O	3,150	220,6	159,9	118,0	5,17	5,16	5,07
^{19}F	3,110	246,6	169,4	117,9	5,78	5,46	5,91
^{24}Mg	4,650	254,3	153,6	128,4	5,96	4,95	5,91
^{25}Mg	4,650	243,6	179,6	122,5	5,70	5,79	5,92
^{26}Mg	4,650	263,6	207,1	128,8	6,17	6,68	6,22
^{27}Al	4,840	255,8	191,1	133,7	5,99	6,16	6,46
^{40}Ca	5,900	331,5	228,2	162,1	7,76	7,36	7,83
^{44}Ca	5,860	344,0	245,1	—	8,06	7,90	—
Cu	6,790	400,2	297,9	209,9	9,37	9,61	10,14
^{116}Sn	8,130	554,7	—	—	12,99	—	—
^{124}Sn	8,060	533,2	—	—	12,49	—	—
^{181}Ta	9,020	611,5	451,8	303,3	14,32	14,67	14,66
Pb	9,270	588,4	481,6	324,5	13,78	15,53	15,68

correlations,[10] and at small q the cross section for the process is enhanced in comparison with the DWIA as a result of the contribution of excited clusters.[26]

It should be noted that in the case of light nuclei of the $1p$ shell the discrepancy between $\tilde{N}^{\exp}$ and $\tilde{N}^T$ is mainly due to the existence of pronounced structural characteristics of these nuclei (for example, ^{6}Li has a quasimolecular αd structure), and for $A>16$ the deviation of $\tilde{N}^T$ from $\tilde{N}^{\exp}$ is a direct indication of the existence of additional reaction mechanisms.

From this standpoint, we shall analyze the experimental data on $A(p,n)_\Delta B$ reactions in the language of effective numbers. In Table IV we show the dependence of the effective numbers $\tilde{N}^{\exp}(\theta_n)$ on the neutron detection angle θ_n in the laboratory system, calculated according to Eq. (1) on the basis of the experimentally measured cross sections[97–100] at T_p=1 GeV for the angles θ_n=4°, 7.5°, 11.3°, and 13.2°. It is clear from this table that within the experimental errors the approximation of effective numbers reproduces the angular dependence of the cross sections for the charge-exchange reaction. Thus, in general, the angular dependences of the cross sections $d\sigma^{\exp}[A(p,n)_\Delta B]/d\Omega_n$ favor the DWIA or the approximation of effective numbers. Physically, this means that the process takes place at the periphery of the nucleus, i.e., in the region in which the nucleon density is small, and therefore all NN and $N\Delta$ interactions in the nucleus are close to the vacuum interactions.

The situation is more complicated in the case of the A dependence of the cross sections. In Tables V and VI we show the results of an analysis of experiments carried out in Refs. 61 and 105 (T_p=0.8 GeV, θ_n=0°) and in Refs. 97–100 (T_p=1 GeV, θ_n=4°). It can be seen from these tables that the values of $\tilde{N}^{\exp}$ are systematically greater than $\tilde{N}^T$ by approximately a factor of 1.5; this indicates clearly that the approxi-

TABLE V. The A dependence of the cross sections and effective numbers for T_p=0.8 GeV at θ_n=0° according to the data of Refs. 61 and 105. The values $\tilde{N}^T$ were calculated in Ref. 20.

Target	A	Z	$\frac{d\sigma[A(p,n)B]_\Delta}{d\Omega_n}$	$N^{\exp}$	$\tilde{N}^{\exp}$	$\Delta\tilde{N}=\tilde{N}^{\exp}-\tilde{N}^T$	$\tilde{K}=\frac{\tilde{N}^{\exp}}{\tilde{N}^T}$	$\langle f_{33}^2\rangle$
H*	1	1	33,0∓3,0	1,0	1,0	0	1,0	—
Al	27	13	271,4∓2,0	8,2	6,0	2,2	1,36	0,34
Ti	47,9	22	372,1∓2,7	11,3	7,9	3,4	1,43	0,26
Cu	63,5	29	425,0∓3,2	12,9	8,9	4,0	1,45	0,22
W	183,9	74	695,5∓5,6	21,1	12,3	8,8	1,71	0,11
Pb	207,2	82	695,4∓5,5	21,1	12,6	8,5	1,70	0,10
U	238	92	767,9∓6,3	23,3	13,0	10,3	1,80	0,09

*Data of Ref. 106, $\theta\to0°$.

TABLE VI. The same as in Table V, but for $T_p=1$ GeV at $\theta_n=4°$ (Refs. 97–100).

Target	A	Z	$\frac{d\sigma[A(p,n)_\Delta B]^{exp}}{d\Omega_n}$	$\tilde{N}^{exp}$	$\tilde{N}^T$	$\Delta\tilde{N}=\tilde{N}^{exp}-\tilde{N}^T$	$\tilde{K}=\frac{\tilde{N}^{exp}}{\tilde{N}^T}$	$\langle f_{33}^2\rangle$
H	1	1	42,7∓4,3	1,0	1,0	0	1,00	—
D*	2	1	52,1∓5,2	1,2	0,6	0,62	2,02	0,45
^{7}Li	7	3	123,8∓3,8	2,9	2,2	0,75	1,35	0,50
^{9}Be	9	4	117,1∓5,3	4,2	2,5	1,65	1,66	0,44
^{10}B	10	5	165,8∓5,0	3,9	2,8	1,11	1,40	0,42
^{11}B	11	5	159,8∓4,7	3,7	2,7	1,00	1,36	0,39
^{12}C	12	6	162,3∓4,8	3,8	3,0	0,84	1,28	0,37
^{16}O	16	8	220,6∓11,0	5,2	3,2	2,02	1,64	0,30
^{19}F	19	9	246,6∓12,4	5,8	3,1	2,67	1,86	0,25
^{24}Mg	24	12	254,3∓17,5	6,0	4,7	1,37	1,29	0,29
^{25}Mg	25	12	243,6∓17,0	5,7	4,7	1,05	1,23	0,28
^{26}Mg	26	12	263,6∓17,6	6,2	4,7	1,52	1,33	0,28
^{27}Al	27	13	255,8∓7,5	6,0	4,8	1,15	1,24	0,27
^{40}Ca	40	20	331,5∓23,2	7,8	5,9	1,86	1,38	0,22
^{44}Ca	44	20	344,0∓24,2	8,1	5,9	2,20	1,38	0,21
Cu	64	29	400,2∓16,1	9,4	6,8	2,58	1,38	0,17
^{116}Sn	116	50	554,7∓28,0	13,0	8,1	4,86	1,60	0,11
^{124}Sn	124	50	533,2∓26,9	12,5	8,1	4,43	1,55	0,11
^{181}Ta	181	73	611,5∓26,1	14,3	9,0	5,30	1,59	0,08
Pb	208	82	588,4∓23,8	13,8	9,3	4,51	1,49	0,08

*For the deuteron, we give the factor $\langle f_{01}^2\rangle$.

mation of effective numbers is inadequate for the description of the $A(p,n)_\Delta B$ reaction. The use of "optical" absorption factors instead of Glauber factors (i.e., the transition to the DWIA) only worsens the relationship between the theory and experiment. In this case, the ratio $\tilde{K}=\tilde{N}^{exp}/\tilde{N}^T\approx 2$ only increases in comparison with the analogous values of $\tilde{K}$ from Tables V and VI.

This situation can be understood qualitatively if we consider the A dependence of the quantities $\Delta\tilde{N}=\tilde{N}^{exp}-\tilde{N}^T$. It follows from Tables V and VI and from Fig. 25 that on the whole the values of $\Delta\tilde{N}$ increase according to the law A^α, where $\alpha=0.6$–0.7. If we assume that the off-mass-shell effect in the cross section $d\sigma[p+p\to n+\Delta^+]/d\Omega_n$ is not too large (estimates made in Ref. 57 confirm this), we can assume that the expression (117) describes the part of the cross section associated with the $(p,n)_\Delta$ reaction in which a real Δ^{++} or Δ^+ isobar is produced (see Fig. 22).

By definition, the diagram in Fig. 22 includes only the direct charge-exchange process. However, in our formalism the cross section contains both a direct and an exchange term. In the terminology of Ref. 67, the latter corresponds physically to excitation of a Δ isobar in the incident particle and, as was shown in that same study, plays an important role. An analogous remark applies also to the diagram of Fig. 23 discussed below.

Bearing in mind the foregoing remarks, the observed effective number $\tilde{N}$ can be represented as a sum

$$\tilde{N}=\kappa_1 A^{1/3}+\kappa_2 A^{2/3}, \tag{123}$$

whose first term is reproduced completely in the approximation of effective numbers (see Fig. 23). The second term is associated with the process of mesonless deexcitation involving a virtual Δ isobar (see the diagram in Fig. 24).

In fact, by virtue of crossing symmetry, the cross section for the process described by the diagram of Fig. 24 can be represented in the form

$$\frac{d\sigma^{(2)}_{A(p,n)_\Delta B}}{d\Omega_n}\approx\int d\mathbf{P}\int d\mathbf{Q}'\int d\mathbf{Q}\varphi(\mathbf{P},\mathbf{Q},\mathbf{Q}')$$

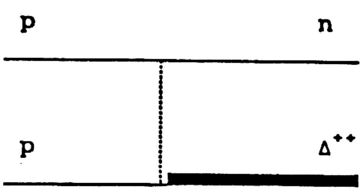


FIG. 22. Diagram describing the $(p,n)_\Delta$ process in the approximation of a transition potential.

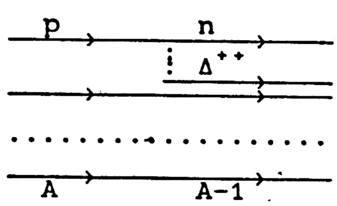


FIG. 23. The $(p,n)_\Delta^{\pi N}$ process: the produced Δ isobar is deexcited through decay into a pion and a nucleon.

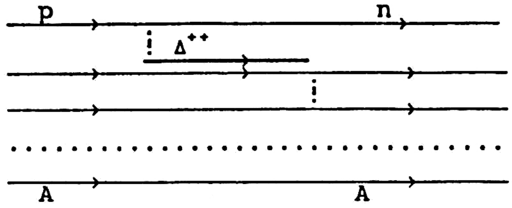


FIG. 24. The $(p,n)_\Delta^{NN}$ process: the virtual Δ isobar is deexcited as a result of charge exchange on one of the intranuclear nucleons. In Ref. 91 this process was called "mesonless Δ-isobar deexcitation."

$$\times[\Phi_N^{A-1}(\mathbf{Q}')]^2\,\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P},\mathbf{Q}')}{d\mathbf{P}}$$

$$\times|G(E_\Delta)|^2[\Phi_N^A(\mathbf{Q})]^2\,\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i,\mathbf{Q})}{d\Omega_n}. \qquad (124)$$

In Eq. (124), corresponding to the process represented in Fig. 24, $\varphi(\mathbf{P},\mathbf{Q},\mathbf{Q}')$ is a kinematic factor, $G(E_\Delta)$ is the Green's function describing propagation of a Δ isobar with energy E_Δ in the intermediate state, **P** is the momentum of the fast nucleon produced as a result of the second charge exchange, and **Q** and **Q**′ are, respectively, the momenta of the Fermi motion of the target nucleons on which the first and second charge exchanges took place. In deriving Eq. (124), we have made use of the completeness approximation, as well as an average over the spin projections of the Δ isobar. As in the case of single charge exchange, the relation $|\mathbf{P}_i|\gg P_F$ enables us to neglect the **Q** dependence of the cross section $d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i,\mathbf{Q})$. Consequently, the cross section for the $(p,n)_\Delta^{NN}$ reaction also factorizes:

$$\frac{d\sigma^{(2)}_{A(p,n)_\Delta B}}{d\Omega_n}=\Delta\tilde{N}\,\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P}_i)}{d\Omega_n}\bigg|_{\rm b}, \qquad (125)$$

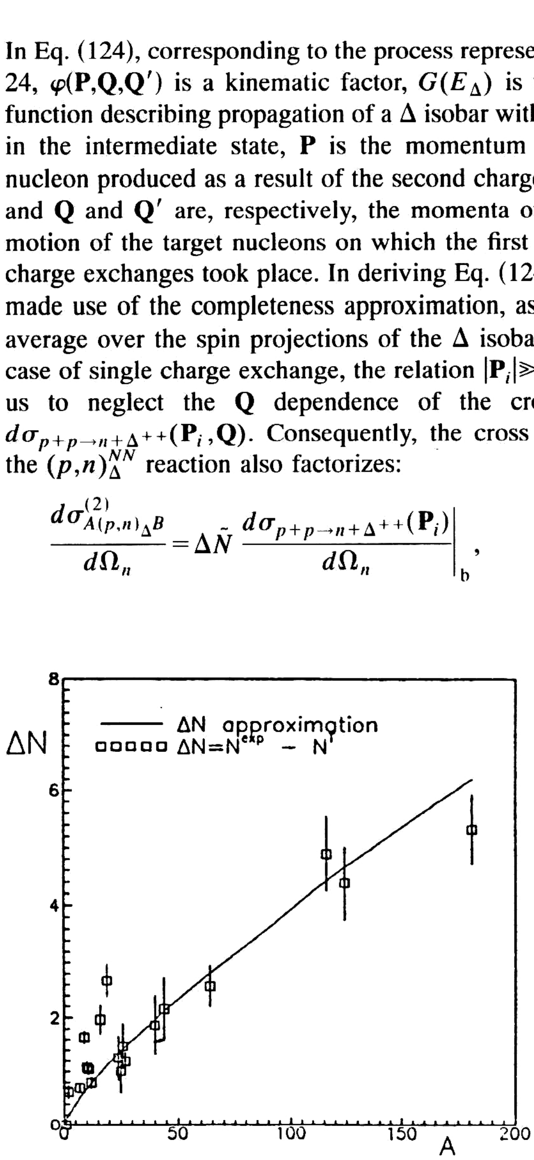


FIG. 25. Calculated dependence of $\Delta N=N^{\rm exp}-N^T$ on the mass number of the target nucleus, and comparison with the experimental data (see the text).

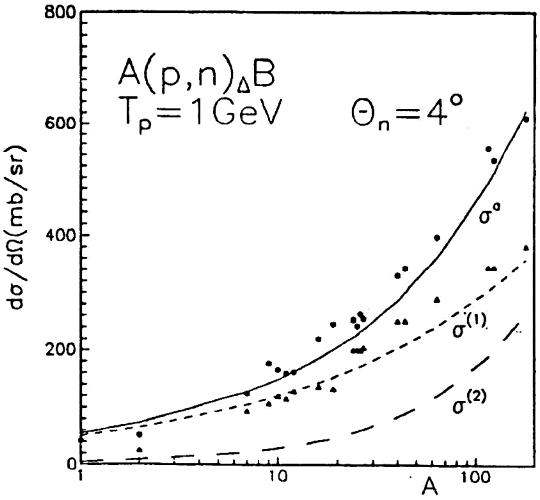


FIG. 26. Analysis of the contributions $(p,n)_{\Delta-\pi N}$ and $(p,n)_{\Delta-NN}$ (mesonless deexcitation) for the Gatchina data; the labeling of the curves is explained in the text. The stars are measured cross sections at the maximum of the Δ peak, and the triangles are calculated by the method of effective numbers without allowance for the channel of mesonless deexcitation (see the text).

$$\Delta\tilde{N}=\int d\mathbf{P}\int d\mathbf{Q}'\int d\mathbf{Q}\,\varphi(\mathbf{P},\mathbf{Q},\mathbf{Q}')$$

$$\times[\Phi_N^{A-1}(\mathbf{Q}')]^2\,\frac{d\sigma_{p+p\to n+\Delta^{++}}(\mathbf{P},\mathbf{Q}')}{d\mathbf{P}}$$

$$\times|G(E_\Delta)|^2[\Phi_N^A(\mathbf{Q})]^2. \qquad (126)$$

In Eq. (126) it is not permissible to neglect off-mass-shell effects, since the momentum $|\mathbf{P}|$ is commensurate with the Fermi momentum P_F. We note also that the region of integration in Eqs. (124) and (126) is determined by the width of the Δ peak and by the Fermi momentum, i.e., in fact by the properties of the Green's function $G(E_\Delta)$ and of the momentum distributions $[\Phi_N^A(\mathbf{Q})]^2$.

Equations (124) and (125) indicate that the angular spectra of the neutrons from the $(p,n)_\Delta^{\pi N}$ charge-exchange channel have the same shape as the analogous spectra of the neutrons from the $(p,n)_\Delta^{NN}$ charge-exchange channel. This result makes it possible to understand the experimental data of Refs. 97–100, and it also indicates the fundamental impossibility of separating the contribution of the $\Delta\to\pi N$ channel in the cross section for the charge-exchange reaction from the contribution of the $\Delta N\to NN$ channel on the basis of an analysis of only the angular spectra of the neutrons.

Direct calculations have shown that the functions $\varphi(\mathbf{P},\mathbf{Q},\mathbf{Q}')$ and $G(E_\Delta)$ depend weakly on A; the dependence of the distributions $[\Phi_N^A(Q)]^2\sim A^{1/3}$ has been well studied (see the discussion of effective numbers). Thus, $\Delta\tilde{N}\sim A^{2/3}=A^{1/3}A^{1/3}$, in complete agreement with the experimental data from Refs. 97–100, 105, and 106.

Estimates made in the approximation of plane waves (for the nucleon produced as a result of the second charge exchange) have shown that the foregoing considerations concerning the role of the $\Delta N\to NN$ charge-exchange channel make it possible to describe the $(p,n)_\Delta$ reaction cross section

TABLE VII. Separation of the $(p,n)_{\Delta}^{\pi N}$ and $(p,n)_{\Delta}^{NN}$ contributions of the charge-exchange channels on the basis of an analysis of the experimental data of Refs. 97–100 ($T_p=1$ GeV, $\theta_n=4°$) by the method of least squares: $\{d\sigma[A(p,n)_{\Delta}B]/d\Omega_n\}^a=d\sigma^{(1)}/d\Omega_n+d\sigma^{(2)}/d\Omega_n\equiv\bar{\kappa}_1A^{\alpha}+\bar{\kappa}_2A^{2\alpha}$ [analog of Eq. (123)] with $\bar{\kappa}_1=50$, $\bar{\kappa}_2=5.1$, $\alpha=0.38$. The quantity $\{d\sigma[A(p,n)_{\Delta}B]/d\Omega_n\}^T$ is calculated according to Eq. (118). The corresponding cross sections are given in the table in an obvious abbreviated notation.

A	σ^{exp}	σ^{a}	$\sigma^{(1)}$	$\sigma^{(2)}$	σ^{T}
7	123,8	127,2	104,7	22,4	93,9
9	177,1	142,3	115,2	27,1	106,8
10	165,8	149,2	119,9	29,3	119,6
11	159,8	156,0	124,4	31,6	115,3
12	162,3	162,2	128,5	33,7	128,1
16	220,6	185,3	143,4	41,9	136,6
19	246,6	200,9	153,1	47,8	132,4
24	254,3	224,4	167,3	57,1	200,7
25	243,6	228,8	169,9	58,9	200,7
26	263,6	233,0	172,4	60,6	200,7
27	255,8	237,3	174,9	62,4	205,0
40	331,5	287,3	203,1	84,2	251,9
44	344,0	311,1	210,6	90,5	290,4
116	554,7	493,4	304,4	189,0	345,9
124	533,2	511,2	312,2	199,0	345,9
181	611,5	625,6	360,5	265,1	384,3
208	588,4	674,6	380,0	294,6	397,1

not only qualitatively, but also quantitatively [see Fig. 26 and Table VII (Ref. 107)]. An analogous situation was described in Ref. 108 in an analysis of the (e,e') reaction.

The present formalism was used to calculate the cross sections for the $A(p,n)B$ reaction in the quasielastic region of excitation of the nucleus under the assumption that the incident proton knocks out a neutron or undergoes charge exchange on neutrons of the target nucleus without excitation of a Δ isobar. In this case, it is necessary to replace the factor $(Z+N/3)$ by N in Eq. (118) and to perform all the calculations according to the prescription described above. It can be seen from Tables VIII and IX that the values of $\tilde{N}^T_{CEX}$ at $T_p=1$ GeV agree with experiment with reasonable accuracy, while at $T_p=0.8$ GeV the deviations of $\tilde{N}^T_{CEX}$ from $\tilde{N}^{exp}_{CEX}$ become quite substantial. This situation can be understood qualitatively if we bear in mind two circumstances, owing to which the values of $\tilde{N}^T_{CEX}$ and $\tilde{N}^{exp}_{CEX}$ in Tables VIII and IX are overestimated. The quantity $\tilde{N}^{exp}_{CEX}$ requires a correction because it is calculated according to the formula

$$\tilde{N}^{exp}_{CEX}=[d\sigma_{A(p,n)B}/d\Omega_n]^{exp}_{CEX}[d\sigma_{d(p,n)2p}/d\Omega_n]^{exp}_{CEX}, \quad (127)$$

TABLE VIII. Effective numbers $\tilde{N}^T_{CEX}$ for the quasifree region ($T_p=1$ GeV; the experimental data are taken from Refs. 97–100). The values $\tilde{N}^T_{CEX}$ are calculated without allowance, and $\hat{N}^T_{CEX}$ with allowance, for energy discrimination of the cross sections σ^{cl}_{NN}.

Target	A	Z	$\tilde{N}^{exp}_{CEX}$			$\tilde{N}^T_{CEX}$	$\hat{N}^T_{CEX}$
			$\theta=4°$	$\theta=7{,}5°$	$\theta=11{,}3°$		
^{7}Li	7	3	2,25	2,39	—	2,03	1,69
^{9}Be	9	4	2,88	3,01	3,20	2,21	1,84
^{10}B	10	5	2,17	2,17	2,35	2,10	1,72
^{11}B	11	5	2,45	2,44	2,51	2,31	1,93
^{12}C	12	6	2,23	2,28	2,56	2,25	1,80
^{16}O	16	8	2,72	2,81	3,10	2,40	1,88
^{19}F	19	9	3,18	3,40	3,63	2,51	1,98
^{24}Mg	24	12	2,99	2,86	2,87	3,53	2,77
^{25}Mg	25	12	3,19	3,11	3,17	3,74	2,93
^{26}Mg	26	12	3,72	3,47	3,80	3,95	3,08
^{27}Al	27	13	3,22	3,26	3,45	3,80	3,02
^{40}Ca	40	20	3,39	3,60	4,04	4,42	3,41
^{44}Ca	44	20	4,26	4,45	—	5,06	3,86
Cu	64	29	5,20	5,23	5,40	5,85	4,40
^{116}Sn	116	50	6,62	—	—	7,42	5,53
^{124}Sn	124	50	7,29	—	—	8,03	5,93
^{181}Ta	181	73	9,37	8,66	8,90	8,92	6,58
Pb	208	82	9,79	9,23	9,83	9,45	6,92

TABLE IX. Effective numbers for the quasifree region at T_p=0.8 GeV and θ_n=0° (the experimental data are borrowed from Refs. 105 and 106; the notation is the same as in Table VIII).

Target	A	Z	$\tilde{N}^{exp}_{CEX}$	$\tilde{N}^{T}_{CEX}$	$\hat{N}^{T}_{CEX}$
Al	27	13	1,87	4,75	2,75
Ti	47,9	22	2,85	6,70	3,58
Cu	63,5	29	3,40	7,66	3,98
W	183,9	74	7,16	12,22	5,95
Pb	207,2	82	7,28	12,74	6,18
U	238	92	8,14	13,49	6,49

since it is impossible to obtain experimental data on scattering of protons by free neutrons [at best, one can study the inverse reaction $p(n,p)n$, when a neutron beam is scattered by a hydrogen target].

As a result of screening effects, we have $[d\sigma_{d(p,n)2p}/d\Omega_n]^{exp}_{CEX}<[d\sigma_{p(n,p)n}/d\Omega_n]^{exp}_{CEX}$; consequently, the values of $\tilde{N}^{exp}_{CEX}$ in Tables VIII and IX must be reduced by 10–20%.[42] The problem of correctly taking into account the influence of screening on the value of the cross section $[d\sigma_{p(n,p)n}/d\Omega_n]_{CEX}$ in extracting it from $[d\sigma_{d(p,n)2p}/d\Omega_n]^{exp}_{CEX}$ requires further investigation.

The theoretical values $\tilde{N}^{T}_{CEX}$ in Tables VIII and IX are overestimated, owing to the use of the NN cross sections σ^{el}_{NN} in the absorption factors without allowance for energy discrimination. The point is that incoherent rescattering of the $p(n)$ by the nucleons of the target nucleus leads to excitation of the residual nucleus and to a deflection of the proton p (neutron n) from the angle $\alpha_p(\alpha_n)$ of its original direction of motion. Emission of a neutron into the region of the quasielastic peak is kinematically allowed, provided that the angle α_p is not too large. The constraints on α_p imposed by the width ΔE_n of the spectrum $d^2\sigma_{A(p,n)B}/d\Omega_n dE_n$ ($\Delta E_n\approx 100$ MeV at half-height of the experimental quasielastic peak) have the consequence that a contribution to the cross section $\tilde{N}^{T}_{CEX}$ can come only from protons that are quasielastically rescattered at an angle $\alpha_p<\alpha^0_p$, where the value of α^0_p is determined by the kinematics of the reaction $A(p,n)B$. At energies $T_p<1$ GeV, the elastic scattering cross section is not sharply peaked in the forward direction. In particular, $\tilde{\sigma}^{el}_{NN}\approx 0.5\sigma^{el}_{NN}$ at T_p=0.8 GeV, where $\tilde{\sigma}^{el}_{NN}$ denotes the part of the cross section σ^{el}_{NN} associated with scattering of the proton at angles $\alpha_p<\alpha^0_p$ (see Table X), corresponding to energy losses $\Delta E<140$ MeV (i.e., below the threshold for pion production).

It can be seen from Tables VIII and IX that the replacement of σ^{el}_{NN} by $\tilde{\sigma}^{el}_{NN}$ (energy discrimination) significantly improves the agreement of the theory with experiment at T_p=0.8 GeV, and a reduction of the values of $\tilde{N}^{T}_{CEX}$ at T_p=1 GeV does not lead to an appreciable deviation from experiment if allowance is made for the effect of screening in extracting the cross section $[d\sigma_{n(p,n)p}/d\Omega_n]_{CEX}$ from $[d\sigma_{d(p,n)X}/d\Omega_n]_{CEX}$.

We note that all the results presented above are also applicable to the $(^3\mathrm{He},t)_\Delta$ reaction. Allowance for the $\Delta N\rightarrow NN$ charge-exchange channel makes it possible to explain qualitatively the broadening of the Δ peak, its displacement with respect to the Δ peak on free protons, the doubled value (in comparison with Glauber or DWIA calculations) of the cross section, and the A dependence of these cross sections, which were investigated in Refs. 87, 88, and 109–123. In this connection, an estimate of collective effects in the excitation of broad baryon resonances in nuclei[124,125] must be made with allowance for the contribution of mesonless deexcitation. The point is that in the region of energies under consideration the wavelength of the Δ isobar in the nucleus is smaller than the mean internucleon distance. Consequently, collective effects must be analyzed together with direct mechanisms of the type $(p,n)^{\pi N}_{\Delta}$ and $(p,n)^{NN}_{\Delta}$.

CONCLUSIONS

The theoretical analysis of data on charge-exchange reactions with excitation of Δ isobars has already been reviewed in part above; here we shall select the main results.

1. There is no doubt that the process of charge exchange with excitation of a Δ isobar in a nucleus is fairly peripheral, though the "degree of peripherality" is not too high, since both the phenomenological analysis (see Ref. 1) and the re-

TABLE X. Cross sections for elastic NN scattering without allowance (σ^{el}_{NN}) and with allowance ($\tilde{\sigma}^{el}_{NN}$) for energy discrimination.

T_p, GeV	α^0_p	σ^{el}_{NN}, mb	$\tilde{\sigma}^{el}_{NN}$, mb
0,67	23°	25	11
0,80	21°	23	11
1,00	18°	19	14,5
1,40	14°	18	16

Here $\tilde{\sigma}^{el}_{NN}$ is the part of the cross section σ^{el}_{NN}, corresponding to scattering at an angle $\alpha_p<\alpha^0_p$, for which the energy transfer is $\Delta E<m_\pi\approx 140$ MeV. The values of σ^{el}_{NN} and $\tilde{\sigma}^{el}_{NN}$ were obtained on the basis of the data of Refs. 42 and 63.

sults of calculations indicate that the $\Delta N \to NN$ channel of charge exchange is important in the reactions $A(p,n)_\Delta B$.

2. The experimental angular neutron spectra, both in the region of the quasielastic peak and in the region of Δ-isobar excitations, are reproduced quite satisfactorily. The differential cross sections for the reactions $A(p,n)_\Delta B$ $[A(p,n)B]$ of charge exchange on nuclei are proportional to the cross sections for the corresponding process on a free proton (deuteron), and the coefficients of proportionality are equal to the effective numbers $\tilde{N}$ and are independent of θ_n. It has been shown that the shape of the angular distribution $d\sigma[A(p,n)_\Delta B]/d\Omega_n$ in the region of energies $|\mathbf{P}_i| \geqslant P_F$ is the same for $(p,n)_\Delta^{\pi N}$ and $(p,n)_\Delta^{NN}$ processes.

3. The method of effective numbers makes it possible to explain the A dependences of the integrated inclusive cross sections for (p,n), $(p,n)_\Delta$, and $(^3\mathrm{He},t)_\Delta$ reactions on nuclei, just as for the total $(t,^3\mathrm{He})$ and $(^7\mathrm{Li},^7\mathrm{Be})$ reaction cross sections.[126,127]

4. The momentum and angular spectra of the neutrons from the charge-exchange reaction $p+p \to n+p+\pi^+$ in the energy interval $0.8 \leqslant T_p \leqslant 1.5$ GeV are described in approximately the same way by the JAIN, OSET, and DMIT sets of parameters λ_π, λ_ρ, C_ρ, and $g'_{N\Delta}$. However, these sets (and the transition potentials $V^{\mathrm{eff}}_{NN\to N\Delta}$) reproduce the total amplitude $T_{NN\to N\Delta}$ in fundamentally different ways, giving practically the same value for the square of its modulus, $|T_{NN\to N\Delta}|^2$. In other words, in going from one set to another the amplitude $T_{NN\to N\Delta}$ acquires an additional unimodular factor of the type $\exp(i\alpha)$, and the agreement of the relativistic $\pi+\rho+g'$ model with the results of a phase-shift analysis of pion-nucleon scattering[7] leads to the conclusion that the additional phase α is a multiple of π: $\alpha_n = \pi_n$ ($n = 0,1,2,...$). In its turn, the discontinuous change of α_n in going from one set to another is related to the equidistant spacing of the spectrum of transition potentials, and this provides an example of the existence of a discrete ambiguity in the vertex functions.

6. Scaling of the reduced cross sections, together with additional data on the $p(\alpha,\alpha)(N+\pi)$ reaction and on π-mesic atoms, as well as the combined information on the reaction $\pi+N \to \Delta+\pi \to N+\pi+\pi$, makes it possible to eliminate the indicated ambiguity and to assign a preference to the OSET phenomenological set.

7. Owing to the existence of interference between the virtual Δ^{++} and Δ^+ isobars in the inclusive charge-exchange reactions $p+p \to n+X$ and $n+p \to p+X$, the rules for calculating isospin weights must be used with caution. This interference has a different character for each of the reactions (destructive or constructive) and leads to a substantial deviation from the naive estimates, depending on the initial energy, the energy transfer, and the emission angle of the detected particle. This may be especially important for reactions on nuclei.

8. However, at the present time there is no complete picture of the process of charge exchange on nuclei with excitation of Δ isobars on the basis of a "microscopic" description. In particular, the existing approaches do not make it possible to separate correctly the effects of nuclear structure from the mechanism of the $(p,n)_\Delta$ reaction at intermediate energies. The analysis of the process is greatly complicated by collective excitations of the type (ΔN^{-1}), i.e., a Δ-isobar-nucleon "hole." For intermediate energies, the problem of renormalization of the $NN-N\Delta$ interaction in going from the vacuum to a nucleus has not been adequately studied. Even a number of technical aspects of the theoretical analysis still remain debatable. We have considered only some of the studies devoted to charge-exchange reactions with excitation of baryon resonances and with the simplest projectiles; reactions with fairly complex nuclei or with polarized particles (nuclei) have not yet been exhaustively analyzed.

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