Electrodynamics of neutrinos in a medium

V. N. Oraevskiĭ and V. B. Semikov
Institute of Terrestrial Magnetism, the Ionosphere, and the Propagation of Radio Waves, Russian Academy of Sciences, Troitsk, Moscow Province

Ya. A. Smorodinskiĭ
Kurchatov Institute Russian Scientific Center, Moscow

Fiz. Elem. Chastits At. Yadra **25**, 312–376 (March–April 1994)

A study is made of the influence of the matter density and electromagnetic fields on neutrino propagation in a medium. It is shown that in a dispersive medium the neutrino acquires new characteristics, namely, an induced electric charge and an induced magnetic moment, which depend on the parameters of the medium and the frequency spectrum and polarization of the electromagnetic field. In the low-energy region in which the neutrino de Broglie wavelength $\lambda_\nu = p^{-1}$ is greater than or equal to the mean separation $n_0^{-1/3}$ between particles the contribution of electromagnetic processes with neutrino participation can exceed the contribution due to ordinary weak interactions, i.e., the electromagnetic contribution in the medium cannot be regarded as a small contribution like the case of radiative corrections in vacuum. Many-particle equations of motion of the neutrinos and electrons—relativistic kinetic equations—are derived. By means of the solutions of these equations for the neutrino spin in random magnetic fields record upper limits on the magnetic moments of Dirac neutrinos are obtained from the constraints provided by primordial nucleosynthesis in the early universe on additional neutrino species.

INTRODUCTION

With this review we present to the reader an introduction to the macroscopic theory of neutrinos in dispersive media such as the plasma in metals and stars and ferromagnets, the lepton plasma of the early universe, etc.

When he conjectured the existence of the neutrino, Pauli imposed very stringent bounds on its parameters, including the exclusion principle and electrical neutrality, explaining in this way the fact that this hypothetical particle escaped experimental detection. A remarkable aspect of physics is that practically any hypothesis can ultimately be tested experimentally. It was no different with Pauli's hypothesis.

Subsequent experimental discoveries of decays of elementary particles, parity nonconservation in γ decays of nuclei, etc., stimulated the further development of the theory of weak interactions and neutrino physics.

The theoretical problems of neutrino physics are intimately related to experimental possibilities.

Neutrinos are currently detected in experiments with different sources of neutrino fluxes and in different energy ranges. Particular problems (the search for mass, a magnetic moment of the neutrino, a difference of the squares of the masses Δm^2, and the vacuum mixing angle Θ_ν in flavor oscillations) are solved both by means of laboratory sources of neutrino-radioactive isotopes, reactors, and accelerators as well as indirectly by studying information obtained using ground-based detectors to study natural astrophysical sources—the sun and supernovas.

In the latter cases, using laboratory data on neutrino properties, we can obtain unique information about the internal structure of stars (density, magnetic field). These aims very largely determine the program of future investigations in neutrino physics and astrophysics. The difficulties of neutrino detection are still great, but definite progress has been made in recent years in connection with the commissioning of new experimental facilities.

The physics of the free neutrino, especially in connection with the discovery of the renormalizable theory of the electroweak interactions, competes in its completeness with quantum electrodynamics (QED) and Maxwell–Lorentz theory. A natural extension of the physics of the free neutrino is the physics of neutrinos in a medium.

Interest in this subject increased significantly after the discovery by Mikheev and Smirnov[1] of the possibility of resonant enhancement of $\nu_e \leftrightarrow \nu_\mu$ neutrino oscillations in an inhomogeneous medium, which had been described earlier for the case of a medium with uniform density by Wolfenstein.[2]

The MSW effect[1,2] makes it possible to explain the strong suppression of the observed flux of electron neutrinos from the sun (Davis paradox) even in the case of small mixing angles in vacuum, $\Theta_\nu \ll 1$, when the probability of vacuum neutrino oscillations[3] is small.

In all the four on-going experiments with solar neutrinos, which differ in the methods of measurement and the detection thresholds, a deficit of the electron-neutrino flux is currently confirmed and can be explained in the MSW model.[1,2]

Another effect observed in Davis's facility (Brookhaven), the anticorrelation of the neutrino flux with the solar activity,[4,5] can be explained by assuming an anomalous magnetic moment of the neutrino that interacts with the solar magnetic field, which varies during the cycle of solar activity.[6] The resulting transformation of an active (left-handed) Dirac neutrino into a sterile (right-handed) neutrino, $\nu_{e_L} \to \nu_{e_R}$ (Ref. 6), or, with allowance for a non-

 1063-7796/94/020129-28$10.00

diagonal magnetic moment of the neutrino, transformation of the left-handed electron neutrino into a right-handed muon neutrino (active for Majorana particles),[7] explains both the neutrino-flux deficit (the Davis paradox) and the variations in time of the experimentally detected neutrino fluxes.

However, in the Kamiokande-II (Ref. 8) and GALLEX (Ref. 9) experiments such variations in time have not been observed, and this has generated a certain skepticism with regard to theories that use large neutrino magnetic moments ($\mu_\nu \sim 10^{-11}$–$10^{-12}\mu_B$). The situation with regard to both the Davis paradox and the time variations of the neutrino fluxes could be clarified in the future by accumulation of statistics at the existing facilities (increase of the neutrino flux at the next minimum of the solar activity?) and also by the commissioning of new and even larger facilities (Super Kamiokande in Japan and in the neutrino observatory at Sudbury in Canada), which have significantly higher event counting rates.

Note that the change in the helicity of the Dirac neutrino ($\nu_L^D \leftrightarrow \nu_R^D$), which has a small vacuum magnetic moment, may be insignificant in the relatively weak magnetic field of a normal star (the sun), but nevertheless significant under a extreme astrophysical or cosmological conditions with strong magnetic fields (see Sec. 4).

An important application of neutrino physics in cosmology is the hot component of the dark matter responsible for the formation of the large-scale structures in the universe in accordance with the recent COBE satellite observations.[10] Any stable light neutrino with a mass of the order of a few electron-volts is a suitable potential candidate for this component. If these ideas are to be compatible with the interpretation (by neutrino oscillations) of the flux deficits of: a) solar electron neutrinos (with small mass-square difference $\Delta m^2 \sim 10^{-5}$ eV^2) and b) atmospheric muon neutrinos[11] produced by primordial cosmic rays through decays of pions and muons in the earth's atmosphere (mass-square difference $\delta M^2 \sim 10^{-2}$–10^{-3} eV^2), it is necessary to go beyond the framework of the standard model and introduce a fourth neutrino species—the sterile ν_s.[12] Such a neutrino does not affect the unobservable part of the Γ width of the Z boson in laboratory experiments but can affect primordial nucleosynthesis with allowance for MSW resonance in oscillations with ordinary active neutrinos in the hot plasma of the early universe.[13]

The effect of a medium on neutrino oscillations,[1] neutrino spin oscillations in a magnetic field,[6] and spin–flavor oscillations[1)7] is very varied and sometimes like MSW resonance[1] depending on the parameters of the stellar medium and the vacuum properties of the neutrino. This question is discussed in detail in Sec. 1.

We emphasize that in the approach of Refs. 1, 6, and 7 the inhomogeneity scale $L \sim k^{-1}$ of the medium is large and greatly exceeds the neutrino de Broglie wavelength $\lambda \sim E^{-1}$, and this makes it possible to use the WKB condition $k \ll p$ replaced in geometrical optics by zero-angle scattering [$k = 2E\sin(\theta/2) = 0$, E is the neutrino energy].

We draw the attention of the readers to a completely new aspect of neutrino physics in a medium associated with incoherent scattering of neutrinos by charged particles through nonzero angle (momentum transfer $\mathbf{k} \neq 0$), excitation in the medium of collective degrees of freedom (neutrino Cherenkov radiation, i.e., radiation involving a change of the neutrino energy), etc.

Polarization of the medium by weak forces (by a moving neutrino) at "short" distances $k^{-1} \sim r_D$, where r_D, the Debye radius of the plasma, is appreciably less than the inhomogeneity scale of the neutral equilibrium matter, $r_D \ll L$, is the main physical process that leads to the change of the electromagnetic structure of the neutrino in a dispersive medium and, as a consequence, to an additional electromagnetic interaction of neutrinos with matter (effective long-range interaction at distance $r \sim r_D$ appreciably greater than the range of the weak forces, which is $\sim M_W^{-1}$, where M_W is the mass of the W boson).

The induced electric charge[14] and induced magnetic moment[15] of the neutrino that arise as a result can lead to interesting effects in dense media such as stars collapsing prior to a supernova explosion and the early universe. Under the ordinary conditions of an isotropic medium the other low induced multipole moments of the neutrino (electric dipole and anapole) make no contribution in the standard model.[16] In the standard model an induced neutrino anapole moment exceeding the vacuum moment (by a factor of approximately $\alpha^{-1} = 137$) appears only in a ferromagnet.[17]

In extended models with complex coupling constants, an electric dipole moment d_ν automatically arises in the single-loop approximation of the neutrino interaction with the vacuum of the leptons and vector and Higgs fields, including in an isotropic medium in interaction with real leptons.[18] In the extended models constructed in the literature, these coupling constants are assumed to be real, and therefore the replacement of the vacuum magnetic moment $\mu_\nu \to (\mu_\nu^2 + d_\nu^2)^{1/2}$ proposed in Ref. 19 is not used in the present paper.

Thus to construct the electrodynamics of neutrinos in a medium it is necessary in the first place to consider the interaction in the medium of the (anomalous) magnetic moment of the neutrino with external magnetic fields (static and varying either regularly or randomly) and the interaction of the induced charge and induced magnetic moment of the neutrino with the self-consistent electromagnetic field (in a plasma).

We give a brief outline of this review. After a discussion of the criteria for statistical averaging of the wave equation for a test neutrino in a medium in Sec. 1, we briefly consider the known effects of neutrino propagation in a nondispersive medium.

Our main attention is devoted to the effect of a medium with slowly varying density on neutrino oscillations,[1,2] spin precession,[6] and spin–flavor precession of neutrinos[7] in a magnetic field. The section concludes with a generalization of the potential of the neutrino–matter interaction to the case of a homogeneous medium containing antiparticles, including a neutrino component (plasma of the early universe).

Section 2 begins with an exposition of the most general aspects of plasma physics. We then review the calculations that have been made in the single-loop approximation of the electromagnetic neutrino vertex in various dispersive media in the framework of the standard model and the simplest extended models of electroweak interactions. We give definitions of the induced electric charge and induced magnetic moment in a dispersive medium.

In Sec. 3 we consider as applications the renormalization of the MSW potential of neutrino interaction with a dispersive medium, screening of neutrino scattering by nuclei and electrons in a plasma, radiative decay of a heavy neutrino in an electron gas, the change of the helicity of a massive neutrino, $\nu_R \to \nu_L \gamma$, in a degenerate electron gas; the energy loss of neutrinos in νe scattering and annihilation of neutrino–antineutrino pairs ($\nu\tilde{\nu} \to e^+e^-$) in an isotropic dispersive medium, polarization radiation of neutrinos in a plasma, and also the change of the helicity of the Dirac neutrino in a plasma with neglect of the vacuum magnetic moment and mass.

In Sec. 4 we consider a relativistic kinetic equation of neutrinos in a dispersive medium with allowance for the contribution of a self-consistent field. In particular we consider such an equation for the neutrino spin in an electromagnetic field; this enables us to take into account not only neutrino collisions with matter but also interaction of the vacuum magnetic moment with random magnetic fields. As application we consider the spin flip of Dirac neutrinos in giant random magnetic fields generated in the electroweak phase transition in the early universe. From the constraint provided by primordial nucleosynthesis we obtain upper limits on the magnetic moment and mass of the neutrino in the standard model, these depending on the "seed" magnetic fields in the dynamical mechanism of enhancement of the galactic magnetic field.

At the end we give a summary and discuss some prospects for the development of neutrino electrodynamics in a medium.

1. NEUTRINO PROPAGATION IN A NONDISPERSIVE CONTINUUM

Before we discuss electromagnetic phenomena for neutrinos moving in a dispersive medium, we must establish criteria of statistical averaging for the interaction of neutrinos with matter (an ensemble of particles) and compare these criteria with the ones already known in the literature and used in problems of neutrino oscillations in a medium[1,6,7] (see Sec. 1.1).

Such a comparison will make clear the origin of the qualitative differences of the dependences on the matter density, for example, in the case of the change of the helicity of neutrinos propagating in a dispersive medium or in a medium with a smoothly varying (almost uniform) density.

1.1. Criteria of statistical averaging and inhomogeneity scales

We shall give here criteria of macroscopic averaging in the wave equation for a test neutrino moving in a medium.

If we consider a medium with self-consistent electromagnetic field (plasma), then irrespective of the species of the test particle and the nature of its interaction with the matter the averaging scale L_0 must satisfy the inequalities

$$\lambda_\gamma \gg L_0 \gg n_0^{-1/3}, \tag{1}$$

where λ_γ is the wavelength of the electromagnetic wave in the matter, and n_0 is the particle number density. The inequalities (1) mean that the number of particles in the averaging volume is large, $n_0 L_0^3 \gg 1$, but the macroscopic electromagnetic fields $\mathbf{E}(\mathbf{x},t)$ and $\mathbf{B}(\mathbf{x},t)$ are local quantities that vary weakly over the averaging scale, i.e., the averaging volume L_0^3 can be replaced by a point $\mathbf{x}$.

In problems of resonant neutrino oscillations in a medium[1] or in a study of the effect of a medium on spin precession in a magnetic field,[6] the authors are interested in the changes in the state of the neutrino (the wave function) at very large distances after multiple scattering in matter (through angle $\theta=0$ in each two-particle collision event).[2)] Therefore in these problems inhomogeneities at the level of the plasma scales, i.e., of the order of and greater than the Debye radius r_D, are unimportant and are smoothed. In particular the characteristic density inhomogeneity scales on the sun that are taken into account in calculations of neutrino oscillations are of order $L \sim 10^3$–10^4 km whereas the scales that are important for plasma electrodynamics are of order $r_D \sim 10^{-7}$ cm (under the same conditions of the solar plasma).

The influence of a medium on an elementary event of two-particle interaction of a neutrino with an electrically charged particle is determined by the polarization of the medium induced by the weak forces.

As a consequence, a neutrino in a plasma cannot interact with one charged particle without influencing the nearest neighbors that are coupled to the recoil particle (in νe scattering) by Coulomb forces (on a scale $k^{-1} \sim r_D$).

If the momentum transfers are large, $k \gg r_D^{-1}$ (or even $k \gg n_0^{1/3}$, where $n_0 r_D^3 \gg 1$), i.e., close collisions are realized, then these collective effects disappear, being smaller in magnitude than the ordinary Born contribution of the two-particle interaction (through short-range weak forces), which is always taken into account additively in the total differential cross section for neutrino scattering in matter. We note that for momentum transfers as large as this the very procedure of macroscopic description of the properties of the medium with respect to a test particle becomes invalid. In other words, we must always require, as in the inequalities (1), restrictions on the allowed scale of inhomogeneity in the wave equation of the test particle:

$$k^{-1} \gg L_0 \gg n_0^{-1/3}.$$

However, in contrast to the semiclassical approximation, $k \ll p$, used in the problems of Refs. 1, 6, and 7, in which the inhomogeneity scale is much greater than the neutrino de Broglie wavelength, i.e.,

$$k^{-1} \gg \max(p^{-1}, L_0), \tag{2}$$

in the problems solved below not only the neutrino energy but also the scattering angle θ is arbitrary, i.e., the quantum case $k \sim p$ is also allowed.

To preserve the condition of influence of medium inhomogeneity on the interaction of neutrinos with matter, we introduce in this case a restriction on the neutrino energy:

$$k^{-1} \gtrsim p^{-1} \gg L_0 \gg n_0^{-1/3}, \quad p \simeq E \ll n_0^{1/3}. \tag{3}$$

We emphasize the incoherence of the scattering in the situation with strong inhomogeneity, $k^{-1} \gtrsim p^{-1}$, with scattering angle $\theta \neq 0$. This means that there is a significant influence of the inhomogeneity of the medium in the neighborhood of the neutrino on the scattering cross section with simultaneous requirement of the presence of a large number of particles in a volume with radius equal to the de Broglie wavelength: $n_0 \lambda^3 \gg 1$, $\lambda \simeq p^{-1}$.

Such a condition is important for the introduction of the very concept of a neutrino wave function in a coordinate space with high degree of inhomogeneity in view of the uncertainty principle for an ultrarelativistic particle: $\Delta x \sim p^{-1}$.

As above in (1), on the replacement of the averaging volume by a point $\mathbf{x}$ in the coordinate space the error $\Delta x \sim p^{-1}$ in the coordinate dependence of the wave function must be much greater than the scale of the macroscopic averaging, $p^{-1} \gg L_0$, in accordance with (3).

The conditions (3) are necessary if we are to write down a Dirac equation for neutrinos in a medium valid outside the framework of the semiclassical approximation. In the WKB approximation[1,6,7] for a homogeneous (or almost homogeneous) space ($k \to 0$), the error Δx is unimportant in the phase of the semiclassical wave function.

We shall use below the more general conditions (2), which include the criterion (1) without restriction on the neutrino energy.

However, it must be emphasized that the collective effects in the neutrino–matter interaction discussed here begin to compete with the known contributions of the standard model for the case of soft neutrinos (or high density of the medium): $p^{-3} n_0 \gg 1$, i.e., when the conditions (3) hold.

1.2. Influence of a homogeneous medium and a medium with slowly varying density (large-scale inhomogeneities) on neutrino oscillations and spin precession in a magnetic field

Whereas the induced electromagnetic characteristics of the neutrino (induced electric charge, etc.), which influence the processes of scattering or decay of neutrinos in a dispersive medium, are proportional to the density of the medium and vanish in vacuum, all the well-known effects described briefly in this section are realized already at the vacuum level.

This applies both to neutrino flavor oscillations[3] and to neutrino spin precession[20] or neutrino spin–flavor precession[6] in an external magnetic field.

Nevertheless the addition of a medium considerably modifies the probabilities of these processes. For example in the case of passage of neutrinos through a medium with uniform density, the probability of transition of an electron neutrino into a neutrino of a different species (for example, ν_μ) can be expressed in the same form as in the case of oscillations in vacuum,[3)]

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_m \sin^2(\pi x / l_m), \tag{4}$$

with the difference that the mixing angle in vacuum θ_v is replaced by the mixing angle θ_m in the medium:

$$\sin 2\theta_m = \frac{\sin 2\theta_v}{[(\cos 2\theta_v - l_v/l_0)^2 + \sin^2 2\theta_v]^{1/2}}, \tag{5}$$

and the oscillation length l_v in vacuum is replaced by the oscillation length in the medium:

$$l_m = \frac{l_v}{[(\cos 2\theta_v - l_v/l_0)^2 + \sin^2 2\theta_v]^{1/2}}. \tag{6}$$

In Wolfenstein's expressions (4)–(6) (Ref. 2), the vacuum oscillation length $4\pi p/\Delta m^2$ (Ref. 3) is augmented by the refraction length $l_0 = 2\pi / V^{\mathrm{MSW}}$, which depends on the MSW potential of the neutrino–matter interaction:

$$V^{\mathrm{MSW}} = G_F \sqrt{2 n_e}, \tag{7}$$

where G_F is the Fermi constant, and n_e is the electron number density. In vacuum ($l_0 \to \infty$) the probability (4) is equal to the vacuum value.[3]

The potential (7) is renormalized with allowance for the single-loop corrections in the dispersive medium;[21] it is determined by the contribution of the charged currents[4)] and is equal to the difference between the potentials of the interaction of the electron neutrino

$$V = \frac{G_F}{\sqrt{2}} (2n_e - n_n) \tag{8}$$

and the muon neutrino

$$V = \frac{G_F}{\sqrt{2}} (-n_n)$$

with ordinary matter in the absence of antiparticles and background neutrinos [see the generalization (14) below for the plasma of the early universe]. If the density $n_e(r)$ is variable and decreases toward the surface of the star ($n_e \to 0, l_0 \to \infty$), then for any neutrino energy $E \approx p$ and difference $\Delta m^2 = m_2^2 - m_1^2$ of the squares of the masses there always exists on the "trajectory" of a neutrino emitted by the star a point that corresponds to MSW resonance:[1]

$$\cos 2\theta_v = l_v / l_0, \tag{9}$$

At which the depth of the oscillations $A = \sin^2 2\theta_m$ is maximal (equal to unity) and the oscillation length increases ($l_m = l_v / \sin 2\theta_v$).

The expressions (4)–(6) are valid under adiabatic conditions, $l_m \ll L$. Here L is the thickness of the resonance layer, in which the refraction length l_0 varies weakly, i.e., the inhomogeneity of the density, which decreases smoothly toward the surface of the star, is small and the inhomogeneity scale (L) large. As a result the resonance condition (9) is realized everywhere over the length L, and the electron neutrinos can be effectively transformed (converted) into muon neutrinos. Outside the resonance layer the conversion probability $P(\nu_e \to \nu_\mu)$ (including the inverse $\nu_\mu \to \nu_e$) decreases sharply, and on escape from the star can, if we ignore the low density of the stellar wind, be described already by a vacuum expression of the type (4) with small oscillation depth $A = \sin^2 2\theta_\nu \ll 1$.

Therefore in all cases of resonance oscillations or resonance neutrino spin precession and spin–flavor precession in an external magnetic field we are dealing with resonant and therefore appreciable conversion of the neutrino state without reverse transformation of the flavors (or chirality $\nu_R \to \nu_L$) outside the resonance layer, since for each value of the neutrino energy the resonance region is traversed by the emitted neutrino once, i.e., the neutrino state is "frozen" after it has passed through the resonance layer.

We now consider neutrino spin precession and spin–flavor precession in a medium with regular external magnetic fields. Although recently the applications of these effects in solar-neutrino physics have been questioned in connection with the recent Kamiokande-II (Ref. 8) and GALLEX (Ref. 9) experiments, the electromagnetic interaction of neutrinos with an external magnetic field may be important in the strong magnetic fields of evolved stars, namely, neutron supernova remnants, white dwarfs, or in giant primordial magnetic fields in the early universe (see Sec. 4).

For neutrino spin precession in a homogeneous and static magnetic field **B**, the probability of conversion of active left-handed Dirac neutrinos into sterile right-handed neutrinos:[6]

$$P(\nu_{e_L} \to \nu_{e_R}) = \frac{(2\mu_\nu B_\perp)^2}{V^2 + (2\mu_\nu B_\perp)^2} \sin^2[V^2 + (2\mu_\nu B_\perp)^2]^{1/2} \frac{r}{2} \tag{10}$$

decreases with increasing density of the medium ($\sim V^{-2}$) where the potential V is proportional to the matter density [see Eq. (8)], μ_ν is the neutrino magnetic moment, and $B_\perp$ is the component of the magnetic field perpendicular to the direction of motion of the neutrino.

In the standard model the anomalous magnetic moment of the neutrino, $\mu_\nu = 3.1 \cdot 10^{-19}\ \mu_B m_\nu/\mathrm{eV}$, is small in view of the experimental bounds on the neutrino mass m_ν. Here $\mu_B = e/2m_e$ is the Bohr magneton,[5)] e is the charge of the electron, and m_e is its mass. As a consequence the bound $m_{\nu_e} < 10$ eV from tritium decay[22] leads to the impossibility of interpreting the observed solar neutrinos by means of neutrino spin precession described by the probability (10) for an acceptable extrapolation of the magnetic field strengths into the interior of the sum (to $\sim 10^5$ G at the bottom of the convective zone).

However weak laboratory bounds on the same magnetic moment ($\mu_\nu < 1.08 \cdot 10^{-9}\ \mu_B$ from $\tilde{\nu} e$ scattering of reactor antineutrinos[23]) make it possible to use extended models of the electroweak interactions in which for a small mass the neutrino magnetic moment is large [a value $(10^{-11}$–$10^{-12})\mu_B$ would be sufficient to explain by neutrino spin precession not only the observed deficit of the flux of electron neutrinos but also for manifestation of variations of it in time].

The arbitrariness in the choice of the corresponding parameters of the extended models can be strongly restricted by means of several astrophysical observations or the observed helium abundance converted to the time of the primordial nucleosynthesis in the early universe. For example from the observation of the rate of cooling of white dwarfs one can deduce the inequality $\mu_{\nu_e} < 10^{-11}\ \mu_B$ (Ref. 24), while from the luminosity of red giants it follows that $\mu_{\nu_e} < 2 \cdot 10^{-12}\ \mu_B$ (Ref. 25). In Ref. 26 the similar bound $\mu_{\nu_e} \leqslant 3 \cdot 10^{-12}\ \mu_B$ was deduced from the abundance of primordial helium.

The most stringent limits on the diagonal and transition (μ_{12}) magnetic moments of Dirac neutrinos would follow from the same cosmological restriction associated with primordial nucleosynthesis if one used the additional hypothesis of a fossil origin of the seed magnetic field B_{seed} in the dynamo mechanism of enhancement of the galactic magnetic field. In particular from the inequality obtained in Ref. 27,

$$\mu_{\nu_e} \leqslant \frac{10^{-30}\mu_B}{B_{\mathrm{seed}}},$$

after substitution of the astrophysical limit on the seed field obtained in the modern nonlinear magnetic dynamo theory, $B_{\mathrm{seed}} \geqslant 10^{-11}$ G (Ref. 28), there would follow a record bound on the intrinsic (diagonal) moment of the electron neutrino,

$$\mu_{\nu_e} < 10^{-19}\mu_B,$$

or on its mass in the standard model: $m_{\nu_e} \leqslant 0.3$ eV. The generation of the corresponding giant random magnetic fields in the electroweak phase transition at temperature $T \sim T_{\mathrm{EW}} \sim 10^5$ MeV (Ref. 29) would not contradict the observed isotropy of the universe. The upper limits on the magnetic moment obtained in Refs. 27 and 30 rule out an explanation of the data on solar neutrinos by spin precession or spin–flavor precession of Dirac neutrinos.

We note that the bound $\mu_{\nu_e} < 10^{-12}\ \mu_B$ from the observations of the neutrinos from the supernova SN1987A (Ref. 31) may be uncritical if one takes into account in the expression (8) the change of the signs of the potential V within dense matter with a large number of neutrons n_n (Ref. 32) or a more correct position of the "resonance" layer $V=0$ with inclusion of $\nu\nu$ scattering[33] in the stage of a neutrino opaque star.

The absence of resonance and the suppression of the $\nu_L \to \nu_R$ conversion probability (10) in a medium in which the potential V is nonzero are changed into the opposite properties when the static homogeneous (one-dimensional) field $B_\perp = \mathrm{const}$ is replaced by a two-dimensional magnetic field $\mathbf{B} = \mathbf{B}_\perp e^{i\Phi(t)}$ rotating in the plane perpendicular to the neutrino momentum with nonvanishing geometric phase velocity $\dot{\Phi} \neq 0$, more precisely, $\dot{\Phi} > 0$ (Refs. 34).

In this case the probability of conversion of left-handed neutrinos into right-handed neutrinos[35] can have a resonant and therefore irreversible nature only when allowance is made for the matter density,[34] and in the probability

$$P(\nu_L\to\nu_R)=\frac{(2\mu_\nu B_\perp)^2}{(V-\dot{\Phi})^2+(2\mu_\nu B_\perp)^2}\sin^2\left(\sqrt{(V-\dot{\Phi})^2+(2\mu_\nu B_\perp)^2}\,\frac{r}{2}\right) \tag{11}$$

resonance $V=\dot{\Phi}$ is realized both for transitions of Majorana neutrinos, $\nu_{e_L}\to\nu^c_{\mu_R}$, when there is a shift of the resonance of neutrino spin–flavor precession $[V=\sqrt{2}G_F(n_e-n_n)-\Delta m^2/2E=0]$ when $\Delta m^2=m_2^2-m_1^2\neq0$, as well as for Dirac neutrinos, when $m_{\nu_L}=m_{\nu_R}=m_{\nu_e}$, i.e., $\Delta m^2=0$. Note that the case of neutrino spin precession with an electron Dirac neutrino ($\nu_{e_L}\to\nu_{e_R}$) is of most interest in the problem of Ref. 34, but if the Dirac neutrino has a nondiagonal magnetic moment, $\mu_{12}\neq0$, there can also be a neutrino spin–flavor precession transition $\nu_{e_L}\to\nu_{\mu_R}$ to the sterile muon neutrino with $V=\sqrt{2}G_F(n_e-n_n/2)-\Delta m^2/2E$.

In the absence of matter ($n_e=n_n=0$) in a static and homogeneous magnetic field ($\dot{\Phi}=\Phi=0$), neutrino spin–flavor precession is suppressed compared with ordinary precession[20] because of the relatively large level splitting $\Delta E=\Delta m^2/2E$ (Ref. 6). However, in a medium, as can be seen from the expression (11) in the case $\dot{\Phi}\neq0$, resonant enhancement of the precession is possible.

The potential V in this formula can be expressed in the form[7,36]

$$V=c_L-c_R-\Delta m^2/2E,$$

where $\Delta m^2=m_2^2-m_1^2>0$ and $c_L=G_F(2n_e-n_n)/\sqrt{2}$ is the potential of the electron (left-handed) neutrinos, $c_R=0$ for the right-handed muon Dirac neutrinos ν_{μ_R}, and $c_R=G_Fn_n/\sqrt{2}$ in the Majorana case with active $\nu^c_{\mu_R}$.

Physically neutrino spin–flavor precession is, as it were, a combined effect of neutrino oscillations (i.e., it also depends on the neutrino energy) and neutrino spin precession in an external magnetic field but with the possibility of using all the advantages of the flavor-nondiagonal magnetic moment, $\mu_{12}\neq0$, in contrast to the intrinsic (diagonal) magnetic moment of the Dirac neutrino.

In particular Majorana neutrinos, which possess only a nondiagonal (transition) magnetic moment, are not subject to the criterion that restricts their magnetic moment as a result of the appearance in cosmology of additional degrees of freedom (through the electromagnetic channel in νe scattering,[26,37] through neutrino spin–flavor precession in random magnetic fields,[30] and through the production of an unobserved excess helium abundance in the early universe). Right-handed Majorana antineutrinos as truly neutral particles are the same left-handed active neutrinos, and the total number of degrees of freedom for the three flavors is not increased ($3\cdot2=6$).

Thus the dependence of the state of the neutrinos on the matter density in the process of propagation through a medium with large inhomogeneity scale can take very different forms. The medium can suppress neutrino spin precession of Dirac neutrinos in a static magnetic field[6] and can enhance resonantly neutrino oscillations[1] or neutrino spin–flavor precession in the same magnetic field.[7] It is characteristic that in vacuum all the effects persist and are only changed quantitatively.

We note that for Dirac antineutrinos in the lowest order in the constant G_F the sign of the potential V in the medium is changed ($V\to-V$) and accordingly so is the sign of the diffraction length $l_0=2\pi/V$: $l_0\to-l_0$. Therefore in the case of antineutrinos for the same sign of the difference $\Delta m^2=m_2^2-m_1^2>0$ there do not exist the resonant neutrino oscillations (5) or neutrino spin–flavor precession in a magnetic field; instead, the medium suppresses these effects for antineutrinos.

To conclude this section, in which we have made a brief review of the literature on neutrino propagation in a nondispersive medium, we give more general expressions for the potential V that follow from calculation of the neutrino self-energy in a thermal reservoir of background particles[38] and lead to subtle effects in the early universe.[13,38]

The main contribution to the potential V is determined by the shift of the neutrino energy in the medium, which can be obtained by statistical averaging of locally point (in the employed low-energy approximation) microscopic interaction Lagrangian of the standard model:

$$L_{\rm int}=G_F\sqrt{2}\sum_j[\bar{\Psi}_j\gamma_\mu(g_L^{(j)}(1-\gamma_5)+g_R^{(j)}(1+\gamma_5))\Psi_j]\times\left[\bar{\Psi}_\nu\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi_\nu\right], \tag{12}$$

where $g^{(j)}_{L,R}$ are known constants for the fermions of the medium ($j=e,\ \nu,\ u,\ d$), including the quarks that constitute the nucleons.[39] In particular, for electrons with allowance for the contribution of the charged currents the constants are

$$g_L^{(e)}=\xi+\frac{1}{2},\quad g_R^{(e)}=\xi,\quad \xi=\sin^2\theta_W.$$

In accordance with the criterion (discussed above) for homogeneous equilibrium media, the statistical averaging of the current of the background particles in (12) corresponds to the forward-scattering amplitude or the self-energy diagram (Fig. 1c) for a test neutrino immersed in a thermal reservoir.[38]

Use of the local Lagrangian (12) in the Dirac equations leads to the spectra of massive Dirac neutrinos and antineutrinos:

$$E=p+V(1-r)/2\quad\text{for } \nu,\qquad E=p-V(1+r)/2\quad\text{for } \tilde{\nu}. \tag{13}$$

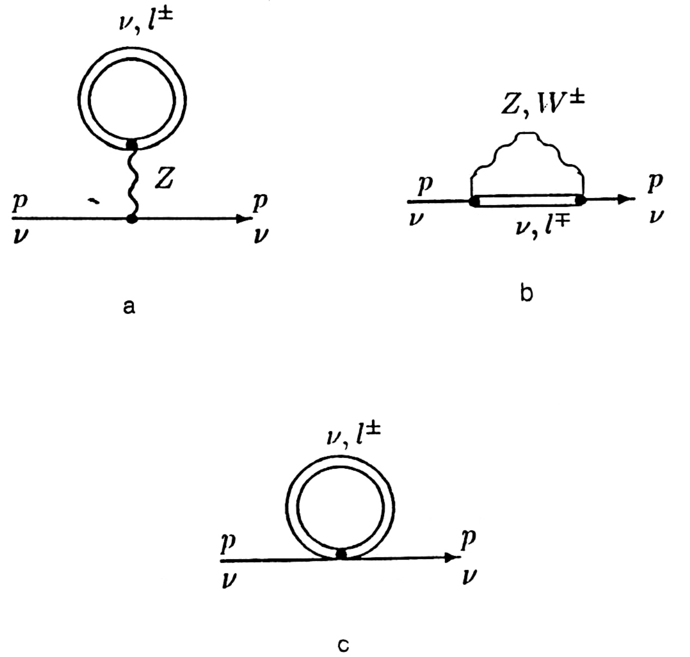

FIG. 1. Self-energy diagrams for a test neutrino in a medium. The double lines denote the electron–positron propagators in the medium.

In Eqs. (13), $r=\pm 1$ is the neutrino helicity, which is conserved in a homogeneous medium. It is readily seen from (13) that the right-handed Dirac neutrinos ($r=+1$) and left-handed Dirac antineutrinos ($r=-1$) are sterile.

Further, following the authors of Refs. 13 and 38, we restrict our attention to low energies $E<100$ MeV of the neutrinos and to the corresponding temperatures in an equilibrium medium, when, on the one hand, we can ignore the presence of muons in the medium and, on the other, sum the contributions of the quarks in the interaction of the neutrinos with the nucleons.

Under these conditions, the spectrum of the electron neutrinos (13) is determined by the potential

$$V=G_F n_\gamma \sqrt{2} L_0, \tag{14}$$

which depends on the asymmetry L_0, or the differences of the number densities of the particles and antiparticles, normalized by the photon number density n_γ in the early universe:

$$L_0=L_e-\frac{L_n}{2}+2L_{\nu_e}+L_{\nu_\mu}+L_{\nu_\tau}, \tag{15}$$

where $L_i=(n_i-\tilde{n}_i)/n_\gamma$.

Note the absence in (15) of a contribution of the protons (respectively, antiprotons), which is exactly canceled by the contribution of the neutral current of the electrons (positrons) [see (12) and Ref. 39].

From the expressions (14) and (15), in the absence of a contribution of antiparticles and with neglect of the contribution of the neutrino background, we obtain the potential $V=G_F\sqrt{2}(n_e-n_n/2)$ (8) used above in (4) and (10) for resonant neutrino oscillations[1] and neutrino spin precession.[6] If for degenerate dense matter of a collapsing star, in the absence of antiparticles and on their equilibrium conditions of the neutrino opacity, we retain the first three terms of the sum (15), then the potential (14) becomes identical to the expression (1) of Ref. 33.

As we already noted above, for the Dirac antineutrino the potential (14) changes sign.

However, if in the same single-loop approximation in the diagram of Fig. 1b we calculate the corrections due to the finite mass of the gauge bosons [going beyond the approximation of the point Lagrangian (12)], then the potential of the interaction of the electron neutrinos with the thermal reservoir,[38]

$$V_L=\sqrt{2}G_F n_\gamma(T)[L_0-AT^2/M_W^2], \quad A\sim 55, \tag{16}$$

will depend on an additional correction ($T\ll M_W$) that does not change sign on the transition from a test neutrino to a test antineutrino.

As is well known, baryon asymmetry persists in the process of evolution of the universe at the level $(n_B-\tilde{n}_B)/n_\gamma\sim 10^{-9}$. With allowance for the conservation in the standard model of the difference $B-L$ of the baryon and lepton numbers, the lepton asymmetry must be at the same level $L_0\sim 10^{-9}$. This means that for the temperatures $T\gg m_e$ considered here the correction in the potential (16) is important.

The difference between the absolute values of the potential (16) for neutrinos and antineutrinos leads to different depths of neutrino spin precession in the primordial magnetic field of the early universe (10) or to different refractive indices.

As a consequence of the difference between such left–right transitions ($\nu_L\rightarrow\nu_R, \tilde{\nu}_R\rightarrow\tilde{\nu}_L$), there may be an unacceptably large neutrino assymetry $L_{\nu_e}=(n_\nu-\tilde{n}_\nu)/n_\gamma$ at the onset of nucleosynthesis of the light elements. From the bounds on their abundances it is possible to obtain cosmological upper limits on the product $\mu_\nu B_\perp$.

In the absence of a magnetic field, the same arguments applied to the analysis of resonant neutrino oscillations ($\nu_e \leftrightarrow \nu_x$) with the participation of a sterile, $\nu_x \neq \nu_{e_R}$ neutrino in a thermal reservoir with the potential (16) lead to bounds on the mixing angle of active and sterile neutrinos.[13]

The calculation of bounds on the fundamental parameters Δm^2, θ_ν, m_ν, μ_ν that follow from consideration of all neutrino propagation effects in a medium goes beyond the scope of the present review (see, for example, Ref. 40).

In the above we have restricted ourselves to an analysis of the studies of other authors with a single aim—to show the difference of our approach to the problem of neutrino electrodynamics in a medium, above all in a dispersive one.

2. NEUTRINO ELECTRODYNAMICS IN A DISPERSIVE MEDIUM

2.1. Normal modes in an isotropic plasma

We begin by giving some general results from plasma physics, the knowledge of which will be sufficient for the reader, a specialist in high-energy physics, to understand this section of the review.

The choice of a plasma as medium simplifies the illustration of the general properties of all dispersive media. For example in an anisotropic substance such as a ferromagnet we have in place of the emission of plasmons by moving neutrinos (photons in a medium) the emission of magnons,[41] etc.

A characteristic property of all dispersive media is the presence in them of a self-consistent electromagnetic field $\delta A_\mu(\mathbf{x},t)$, which is described by Maxwell's equations with an induced (by the field itself) current. In a plasma, the induced current has in the Fourier representation the form

$$\delta j_\mu(\omega,\mathbf{k}) = \Pi_{\mu\nu}(\omega,\mathbf{k})\delta A^\nu(\omega,\mathbf{k}), \tag{17}$$

where in the isotropic case the polarization tensor $\Pi_{\mu\nu}(\omega,\mathbf{k})$ is determined by two linear-response functions—the longitudinal, $\varepsilon_l(\omega,k)$, and transverse, $\varepsilon_{\rm tr}(\omega,k)$, permittivities. Note that when allowance is made for the spin of the electrons of the medium the average vector current $\delta j_\mu(\omega,\mathbf{k}) = e\delta\langle\bar{\Psi}_e\gamma_\mu\Psi_e\rangle$ (17) contains not only a conduction current but also a magnetization current $\delta\mathbf{j} = \mathrm{curl}\,\delta\mathbf{M}$, which depends on the magnetic permeability $[\sim(\mu^{-1}(\omega,k)-1)]$. In accordance with the Bohr–Van Leeuwen (see, for example, Ref. 42), this third linear-response function is in the classical limit ($\hbar\to 0$) a characteristic of the medium that depends on $\varepsilon_{l,\rm tr}$ and is not important for a plasma ($\mu\approx 1$). However for a ferromagnet ($\mu \gg 1$) the magnetization current is important in the calculation of the anapole moment of the neutrino.[17]

Nontrivial solutions of Maxwell's equations ($\delta A_\mu \neq 0$) exist for waves that satisfy the well-known dispersion relations

$$\varepsilon_l(\omega,k) = 0, \tag{18}$$

$$\varepsilon_{\rm tr}(\omega,k) - k^2/\omega^2 = 0, \tag{19}$$

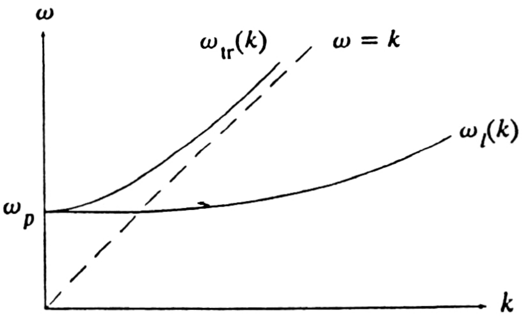

FIG. 2. Spectra of longitudinal, $\omega_l(k)$, and transverse, $\omega_{\rm tr}(k)$, plasmons in an isotropic plasma.

which have solutions (plasmon spectra) that are different from the spectra of vacuum photons ($\omega \neq k$, see Fig. 2).

For example transverse plasmons in a cold plasma, $\omega_{\rm tr} = (\omega_p^2 + k^2)^{1/2}$ possess a nonvanishing mass $m_\gamma \neq 0$ equal to the frequency of the Langmuir plasmon:

$$m_\gamma = \omega_p = (4\pi\alpha n_0/m_e)^{1/2}, \quad \alpha = e^2.$$

In vacuum, after they have left the plasma ($n_0 \to 0$), such plasmons are transformed into ordinary photons ($q^2=0$), but in the medium the gauge invariance of the second kind for the "photons" determined by the dispersion relations (18) and (19) is broken, i.e., $q^2 \neq 0$.

This lifts the kinematic prohibition on the emission of such plasmons by a free charge moving in the medium. However, in an isotropic plasma there exists only polarization radiation of longitudinal waves, for which there can be Cherenkov resonance of the phase velocity of the wave ($v = \omega_l/k$) and the velocity of the radiating charge (see Fig. 2, where $\omega_l/k < 1$ below the bisector $\omega = k$).

For transverse waves, for which the phase velocity $\omega_{\rm tr}/k$ is always greater than the speed of light, $\omega_{\rm tr}/k > 1$, such radiation (in the isotropic case) is impossible; it is only in a magnetoactive plasma that the spectra of the transverse waves admit refractive indices $n = k/\omega_{\rm tr} > 1$, i.e., Cherenkov emission of transverse waves becomes possible.

The normal modes described by Eqs. (18) and (19) are weakly damped in an equilibrium medium, i.e., following emission by moving neutrinos, such waves propagate for a long distance, involving many particles of the medium in the interaction. Moreover, for longitudinal waves there can be both weak collisionless damping (Landau damping, or the inverse Cherenkov effect of absorption by electrons of the medium) and collisional damping (virtual plasmon in electromagnetic νl scattering).

In an isotropic plasma, transverse plasmons are damped only collisionally, this being again because of the absence of Cherenkov resonance ($\omega_{\rm tr}/k > 1$) with radiating (absorbing) particles having velocities $v \leqslant 1$.

Knowledge of these results from plasma physics will facilitate understanding of the specific features of the electromagnetic interaction of neutrinos in dispersive media.

We give below a list of results of single-loop calculations of the neutrino electromagnetic vertex in various media and in various electroweak models. The details of the calculations can be found in the cited references. All the different electromagnetic vertices were calculated by two different methods that gave identical results: the diagram method using thermodynamic Green's functions and the method using the quantum relativistic equation for neutrinos.

2.2. Neutrino electromagnetic vertex in a medium

We give the results of exact calculations of the neutrino electromagnetic vertex made in the single-loop approximation for different media and electroweak models.

In the simplest case of an isotropic plasma, the neutrino electromagnetic vertex was calculated for the first time by Adams, Ruderman, and Woo[43] already in 1963 in the framework of the old $V-A$ model with charged currents.

They calculated the energy losses or the rate of cooling of a dense star in the process of the decay of plasmons $\gamma_{l,\mathrm{tr}}$ into a pair consisting of an active neutrino and an active antineutrino, $\gamma_{l,\mathrm{tr}} \rightarrow \nu\tilde{\nu}$, this making, in particular, the decisive contribution to the cooling rate of the degenerate electron gas of white dwarfs. Of course, they used the value of the QED polarization tensor of the plasma known at that time[44] and showed that the contribution of the mean axial current to the $\gamma\rightarrow\nu\tilde{\nu}$ decay probability is much less than the polarization contribution of the mean vector current.

Some numerical errors of Ref. 43 were taken into account in several subsequent studies that used the $V-A$ model; for example, the correct result was obtained by Zaidi in Ref. 45.

The probabilities of the various electroweak processes that are important in the evolution of stars were determined more accurately in numerous studies after the discovery of the standard model.[46] For the same $\gamma\rightarrow\nu\tilde{\nu}$ process we mention here one of the investigations of Dicus.[47]

However an axial electromagnetic form factor was not introduced in the standard model before the publication of Ref. 15, apparently because of the existence of a single (until the publication of our papers) example of a collective process in a plasma with the participation of neutrinos. This was the plasmon decay $\gamma\rightarrow\nu\tilde{\nu}$, in which the contribution of the axial current is indeed unimportant.[43]

Standard model

In an *isotropic* dispersive medium, the electromagnetic vertex of the Dirac neutrino in the rest frame of the medium, $\Omega_\mu = \delta_{\mu_0}$, has the form[48]

$$\Gamma_\mu^{(D)}(\omega,\mathbf{k})=\alpha^{-1}[G_V\Pi_{\mu\nu}(\omega,\mathbf{k})\gamma^\nu + i\varepsilon_{\mu\nu\sigma 0}A(\omega,k)q^\nu\gamma^\sigma]L, \quad (20)$$

where $A(\omega,k)$ is the magnetic axial form factor,[15] which arises from the mean axial current of the particles of the medium and is proportional to the weak constant G_F and the constant $c_A=g_R-g_L$ ($c_A=+0.5$ for the electron neutrino and $c_A=-0.5$ for $\nu_{\mu,\tau}$); $L=(1-\gamma_5)/2$ is the projection operator for the left-handed neutrino[6)] (see Fig. 3). For a QED plasma, the polarization tensor $\Pi_{\mu\nu}$, which

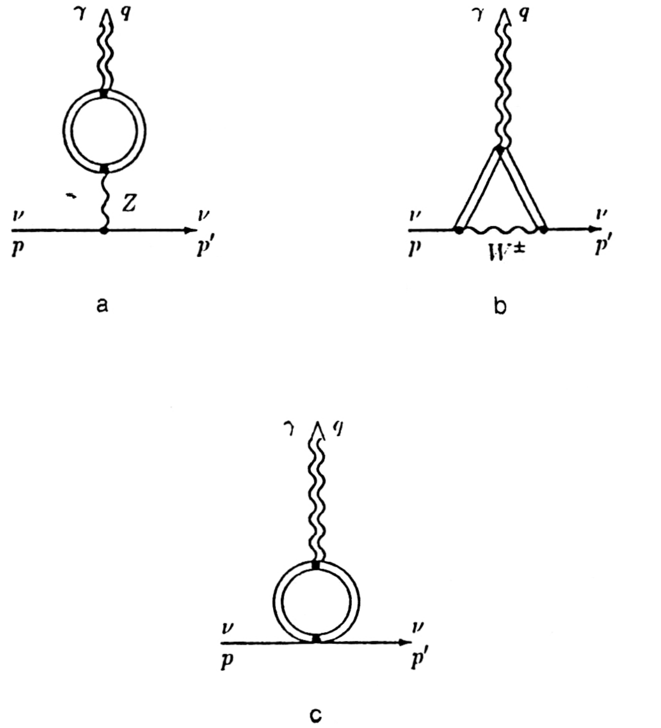

FIG. 3. Electromagnetic vertex Γ_μ of a neutrino in a medium. The double lines denote the electron–positron propagators in the medium; the double wavy line represents a photon in the medium (plasmon) with momentum $q_\mu = p'_\mu - p_\mu$ and polarization e_μ. In the employed low-energy approximation, $|q^2| \ll M_W^2$, the contribution of the self-interaction of the Yang–Mills fields is omitted.

does not depend on G_F, was calculated earlier in Ref. 50 in an arbitrary frame $\Omega_\mu \neq \delta_{\mu_0}$. As in Ref. 51, the tensor $\Pi_{\mu\nu}$ can be written down in the point approximation:

$$G_V\Pi_{\mu\nu}(\omega,\mathbf{k})/\alpha=F_l(\omega,k)\hat{e}_\mu\hat{e}_\nu+F_{\mathrm{tr}}(\omega,k)\delta_{\mu i}\delta_{\nu j}\times(\delta_{ij}-\hat{k}_i\hat{k}_j),$$

where $G_V=2G_Fc_V$; $i, j=1, 2, 3$; $F_{l,\mathrm{tr}}$ are the longitudinal and transverse neutrino electromagnetic form factors, which in an isotropic plasma (in the limit of temperatures $T\ll M_W$) has the form[16,48]

$$F_l(\omega,k)=G_F\sqrt{2}c_V(\varepsilon_l(\omega,k)-1)q^2/\alpha, \quad (21)$$

$$F_{\mathrm{tr}}(\omega,k)=G_F\sqrt{2}c_V(\varepsilon_{\mathrm{tr}}(\omega,k)-1)\omega^2/\alpha. \quad (22)$$

Here the vector constant is $c_V=2\xi+1/2$ for the electron neutrino and $c_V=2\xi-1/2$ for the neutrinos $\nu_{\mu,\tau}$. We ignore the contribution of the heavy hadrons of the medium to the electromagnetic vertex (20), assuming for simplicity that the background is fixed.

In a somewhat different form, identical to (20) after renotation of the variables,[18] the electromagnetic vertex of the Dirac neutrino in an isotropic electron gas (plasma with fixed ions) was also obtained by D'Olivo *et al.* in Ref. 52 [Eq. (3.1)].

In an external electromagnetic field A_μ^{ext}, a plasma becomes anisotropic, and the number of linear-response functions is increased.

If the total field A_μ is decomposed into a sum $A_\mu = A_\mu^{\text{ext}} + \delta A_\mu$ with weak ($\delta A_\mu \ll A_\mu^{\text{ext}}$) self-consistent electromagnetic field, the functions of the linear (in δA_μ) response depend nonlinearly on A_μ^{ext}. If exact solutions of the Dirac equations for the electrons in the external field A_μ^{ext} are known, the density matrix of the electrons in the medium can be calculated using these solutions for an arbitrarily strong field A_μ^{ext}.

In the semiclassical approximation, for example, in a relatively weak homogeneous and static field $B_0 \ll m_e^2/e \sim 4\cdot 10^{13}$ G, the problem is simplified by the possibility of taking into account only the classical Lamor gyration of the electrons of the medium.

The electromagnetic vertex of the Dirac neutrino in a magnetic plasma (anisotropic dispersive medium) was calculated using such a WKB condition in Ref. 53 and found to be

$$\Gamma_\mu^{(D)}(\omega,\mathbf{k},B_0) = \left[\frac{G_F\sqrt{2}c_V}{\alpha}\Pi_{\mu\nu}(\omega,\mathbf{k},B_0)\gamma^\nu + i\delta_{\mu i}G_F\sqrt{2}e_{ikl}k_l A_k^\nu(\omega,k,B_0)\gamma_\nu\right]L. \tag{23}$$

The spatial components of the classical polarization tensor $\Pi_{ij}(\omega,\mathbf{k},B_0)$ are known from the literature on plasma physics,[53] and the time and mixed components are determined from the conservation of the induced electromagnetic current: $\Pi_{00}=\Pi_{ij}k_ik_j/\omega^2$, $\Pi_{i0}=\Pi_{0i}=\Pi_{ij}k_j/\omega$

The additional dispersion characteristic $A_k^\nu(\omega,\mathbf{k},B_0)$ in the expression (23) arises in the standard electroweak model from the averaging of the axial current of the electrons and is[53]

$$A_k^\nu(\omega,\mathbf{k},B_0) = \int \frac{d^3p}{\varepsilon_p} a_k^\nu(\mathbf{p}) \frac{\partial f_0^e(\varepsilon_p)}{\partial \varepsilon_p} \sum_{n=-\infty}^{\infty} J_n^2(b_e) \times \left[\omega\left(P\frac{1}{\omega - k_z v_z - n\Omega_e/\gamma} - i\pi\delta \times \left(\omega - k_z v_z - \frac{n\Omega_e}{\gamma}\right)\right) - 1\right], \tag{24}$$

where the tensor $a_k^\nu(\mathbf{p})$ has components $a_k^0(\mathbf{p}) = p_k/m_e$, $a_{ik} = \delta_{ik} + p_ip_k/(m_e(\varepsilon_p + m_e))$ and corresponds to the polarization 4-vector $a^\nu = a_k^\nu \zeta_k$ in the density matrix $(\hat{p}+m_e)(1-\hat{a}\gamma_5)$ of the electron, which in the rest frame has polarization $(\zeta_i\zeta_i \leqslant 1)^{1/2}$. The expression (24) also contains $f_0^e(\varepsilon_p)$, the equilibrium Fermi distribution function of the electrons, and $J_n(b_e)$, a Bessel function with argument $b_e = k_\perp v_\perp \gamma/\Omega_e$, where $\Omega_e = eB_0/m_e$ is the cyclotron frequency in the magnetic field B_0 directed along the z axis, and $\gamma = \varepsilon_p/m_e$.

Finally, we mention one further result for the standard model. This is the neutrino electromagnetic vertex *in a ferromagnet*, and it can be used to calculate the induced magnetic[41] and anapole[17] moments of the neutrino in this medium. To calculate these moments, it is sufficient to know the interaction of the neutrino with a static transverse field. In this case the vertex Γ_μ has 3-vector part[17,41]

$$\Gamma_n^{(D)}(0,\mathbf{k}) = \left[\frac{G_F\sqrt{2}c_V}{\alpha}\gamma_i e_{imp} e_{kln} k_m k_l (\mu_{kp}^{-1}(0,k) - \delta_{kp}) + \frac{G_F 2\sqrt{2}m_e}{\alpha} c_A \gamma_i e_{nkp} k_p (\mu_{ik}^{-1}(0,k) - \delta_{ik})\right] L, \tag{25}$$

where $\mu_{ik}(\omega,\mathbf{k})$ is the tensor of the magnetic permittivity.

We emphasize that the expressions (20), (23), and (25) describe the neutrino electromagnetic vertex in the same standard model of the electroweak interactions but in different media. A common term in these vertices would be the omitted and here unimportant (see the beginning of this section) vacuum part $\Gamma_\mu^{\text{vac}}(q^2)$, in which the anomalous magnetic moment disappears altogether in the limit of zero neutrino mass ($m_\nu \to 0$).

Extended models

We recall that it is necessary to go beyond the framework of the standard model if one is to reconcile all the currently known experiments with natural sources of neutrino fluxes, in particular to explain the deficits in the fluxes of solar (electron) and atmospheric (muon) neutrinos, and also the hot component of the dark matter (HDM) in the interpretation of the data of the COBE experiment.

Generalizations of the simplest vacuum models, the symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model[55,56] and the model with charged scalar $SU(2)L$ singlet,[57] to the case of an isotropic medium were considered in Refs. 18 and 58, and generalization of the model with additional horizontal $SU(2)_H$ symmetry[59,60] was considered in Ref. 61. A common property of these models is the appearance already at the tree level of additional Lorentz structures in the interaction of the neutrinos with the charged leptons: S (scalar)[18,58,61] and T (tensor),[18,61] which lead to changes of the neutrino helicity.

These helicity changes are not due to the existence of a neutrino rest mass or vacuum magnetic moment. The corresponding additional tree diagrams (with exchange of additional Higgs particles) lead to an increase of the total cross section for neutrino scattering by charged leptons, $\nu_L l^- \to \nu_R l^-$, by five orders of magnitude compared with the contribution of the electromagnetic scattering channel through the vacuum magnetic moment in the same model.[26,62]

The generalization of the above vacuum extended models to the case of an isotropic medium automatically leads to the appearance of an effective Lorentz-invariant neutrino mass, which is proportional to the matter density.[18,61,63]

In the framework of neutrino electrodynamics, it is interesting to consider the contribution of the above additional structures S and T to the electromagnetic vertex calculated in the same single-loop approximation as in the standard model and resulting from the polarization of the dispersive medium by the moving neutrino.

One of the consequences of the modified electromagnetic vertex can be a screening of the vacuum magnetic moment in a dense medium due to the sign change of the total magnetic moment and vanishing of $\mu_\nu^{(\mathrm{tot})}$ at a certain critical density.[18,61]

For the example of the simplest minimal symmetric model, we consider below in Sec. 3 a different electromagnetic effect in a dispersive medium, which is the change in the helicity of the Dirac neutrino in a plasma with neglect of the vacuum magnetic moment.

Minimal symmetric model $SU(2)_L \otimes SU(2)_R \otimes U(1)$ (Refs. 55 and 56)

The appearance in this model of a second (heavy) Z boson does not lead to a qualitative change of the electromagnetic interaction of the neutrino with matter, since the additional diagram in the model[56] will be analogous to Fig. 3a, i.e., will not lead to a change of the helicity, and is a small correction for $m_{Z_R} \gg m_{Z_L}$. In contrast, the corrections from the heavy right-handed W boson will, despite their relative smallness due to the bounds on the mixing angle ζ for the $W_{L,R}$ bosons,[56,64] lead to a qualitative effect—the neutrino helicity change $\nu_L \to \nu_R$.

In the electromagnetic vertex (20) of the Dirac neutrino for an isotropic dispersive medium, there appears in the model of Ref. 56 an additional term from the contribution of the charged right-handed currents (from the mixing of the $W_{L,R}$ bosons):[65]

$$\Gamma_\mu^{\mathrm{add}}(\omega,\mathbf{k}) = 2\sqrt{2}\,\frac{G_F}{\alpha}\sin 2\zeta m_e A(\omega,k)\left[\Omega_\mu - \frac{(q\Omega)q_\mu}{q^2}\right], \tag{26}$$

where $A(\omega,k)$ is the same magnetic form factor[15] that was introduced in the standard model for the vertex (20), and Ω_μ is the 4-velocity of the medium.

As regards its origin, the vertex (26) is an obvious result of averaging in the dispersive medium the vacuum product of scalars $(\bar{\Psi}_\nu \Psi_\nu)$, which automatically arises when one uses Fierz's rule for an interaction Lagrangian[56] that includes charged right-handed currents. Besides the appearance of the vertex (26), the use of the Lagrangian of Ref. 56 leads to the appearance of a finite neutrino mass in a homogeneous medium.[63]

In the same vacuum model the use of Fierz's rule leads to the appearance of one further additional contribution; it is proportional to the pseudoscalar $\bar{\Psi}_e \gamma_5 \Psi_e$ and is not important for the isotropic case, i.e., $\langle \bar{\Psi}_e \gamma_5 \Psi_e \rangle = 0$.

However in a ferromagnet such a term leads in the vertex (25) to an additional contribution, which is also proportional to the mixing parameter $\sin 2\zeta$, if one takes into account the connection between the expectation value of the pseudoscalar and the magnetization $\mathbf{M}(\mathbf{x},t)$:[18]

$$\langle \bar{\Psi}_e \gamma_5 \Psi_e \rangle = \operatorname{div} \mathbf{M}(\mathbf{x},t)/e.$$

2.3. Induced neutrino electric charge and magnetic moment

The expression "electric charge" must be used with some care. First, in contrast to the electron charge, this characteristic is not universal, the neutrino charge being dependent on the density and temperature of the medium. Second the neutrino has different "electric charges" for the same temperature and density of the medium depending on the space—time characteristics of the external electromagnetic fields that act on the neutrino or on the modes of the electromagnetic self-radiation of the moving neutrino.

Thus the induced electric charge[14]

$$e_\nu^{\mathrm{ind}} = -\frac{|e|G_F c_V}{2\pi\alpha\sqrt{2}r_D^2} \tag{27}$$

should be called a quasistatic charge. Through this charge the neutrino can interact with an appropriate electric field.

The neutrino interaction with a high-frequency electromagnetic field $(\omega \gg k\langle v\rangle)$ is described by the charge[66]

$$e_\nu^{\mathrm{ind}} = -\frac{|e|G_F c_V \omega_p^2}{2\pi\alpha\sqrt{2}}, \tag{28}$$

which depends on a further characteristic of the medium—the Langmuir (plasma) frequency $(\omega_p = \langle v\rangle / r_D)$.

For antineutrinos, the attraction of electrons is replaced by repulsion, and the induced electric charge (27)–(28) changes sign.

Of course, when a test neutrino is introduced into a dispersive medium, the global gauge invariance (of the first kind) remains unbroken, i.e., for a system that overall (with allowance for the fixed background of heavy ions) is electrically neutral the total electric charge is zero.

The collective interaction of the neutrinos ν_μ and ν_τ with matter was first considered in Refs. 67, 68, and 69, in which a restriction was made to the contribution of the neutral currents. We mention that in Ref. 67 the polarization losses of neutrinos in a plasma were calculated classically from the Poynting flux. In the case of a small neutrino charge, it is valid to use the Bethe approximation ($e_\nu^{\mathrm{ind}} e/\hbar c \ll 1$, cf. the relation $Ze^2/\hbar v \ll 1$ in Bethe's case), and the necessary quantum calculation was made in Ref. 70.

The electromagnetic vertex and induced electric charge of neutrinos in a medium were also obtained in a somewhat different way by Altherr and Kainulainen.[71] For a degenerate electron gas the induced charge calculated in Ref. 71 differs from (28) only by a factor 4/3.

It is easy to see that the induced charge (27) is small—of order $\sim 10^{-16}e$ in a metal or $\sim 10^{-8}e$ under the conditions of a collapsing star with density $\rho \sim 10^{12}$ g/cm^3.

However, despite this small value, the neutrino induced electric charge has important consequences: polarization[70] and Cherenkov[14] emission of left-handed neutrinos in a dispersive medium, $\nu_L \to \nu_L \gamma$, or to screening of elastic scattering of neutrinos by nuclei in the dense plasma of a collapsing star,[72] which has a significant influence on the collapse dynamics[73] (see Sec. 4).

The correct determination of the charges (27) and (28) in an anisotropic dispersive medium follows directly from the normalization of the longitudinal and transverse electromagnetic form factors (21) and (22), which determine the vector part of the neutrino electromagnetic vertex (20) in the standard model. For example the static charge (27) follows from the normalization of the form factor (21), i.e., $e_\nu^{\text{ind}}/|e| = \lim_{k\to 0} F_l(0,k)$.

It is important to emphasize that the observable quantities—the cross sections and neutrino energy losses in the dispersive medium—contain, of course, form factors. Exactly as for the electron, all the neutrino multipole electromagnetic moments are obtained only for a static situation with a massive neutrino ($m_\nu \neq 0$) at rest in the dispersive medium.[16] The nonuniformity of the electron concentration produced by the polarization of the dispersive medium by the weak forces in the originally homogeneous (before the introduction of the test neutrino) medium is precisely the induced charge of the neutrino.

A good illustration of the analogy with an ordinary charge is the part of the cross section for elastic scattering of a neutrino by a fixed Coulomb center that would correspond to the modification of the diagram of Fig. 3c with replacement of the free plasmon by a virtual plasmon. When the neutral currents are ignored, this is the only diagram in νA scattering and corresponds to the cross section[72]

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2(e_\nu^{\text{ind}}/e)^2[1 - v^2 \sin^2\theta/2 - m_\nu^2/2E^2]}{2v^2p^2[\sin^2\theta/2 + (2pr_D)^{-2}]^2}, \quad (29)$$

where the neutrino charge is equal to (27), and m_ν is the neutrino rest mass, which may be equal to zero ($v=1$).

The result (29) differs from the case of electron scattering by the same screened potential[74] by the unimportant term $m_\nu^2/2E^2$ in the numerator of (29), which is a consequence of the presence of the γ_5 matrix in the neutrino electromagnetic vertex, and by the important factor $(e_\nu^{\text{ind}}/e)^2$, which is determined by the expression (27).

Whereas in the case of electron scattering by a Coulomb center in vacuum ($r_D \to \infty$) we obtain from (29) without allowance for the factor $(e_\nu^{\text{ind}}/e)^2$ the standard Mott–Rutherford cross section, in the neutrino case we find that in the limit $r_D \to \infty$ (vacuum) the charge e_ν^{ind} itself vanishes [see (27)].

It is therefore worth comparing neutrino Coulomb scattering with electron scattering in the same dispersive medium. Note that in the medium the total cross section $\sigma(E)$ will not have a dependence $\sim E^{-2}$ in either case (electron or neutrino) because of the Debye screening of the potential, which eliminates the Coulomb divergence.

The analogous cross section for scattering of the Majorana neutrino by a Coulomb center differs from (29) by the replacement in the numerator of the factor in the square brackets by $[1 - v^2\sin^2\theta/2 - m_\nu^2/E^2]$, which vanishes for a stopped neutrino ($v=0$), and this means that the Majorana neutrino has no induced electric charge, in contrast to the Dirac neutrino.[16]

However, in contrast to the vacuum, a moving Majorana neutrino radiates in a dispersive medium electromagnetic waves, like the Dirac neutrino ($\nu \to \nu\gamma$), including a massless neutrino, for which the electromagnetic characteristics (form factors in the medium) are the same for both neutrinos (Dirac and Majorana).

The induced neutrino magnetic moment $\mu_k^{\text{ind}} = 1/2 \int d^3r[r\mathbf{J}]_k$, which is determined by the neutrino electromagnetic current $J_\mu = e\bar{\nu}_L(p')\Gamma_\mu \nu_L(p)$ with vertex Γ_μ (20), does not lead to helicity change, i.e., in this sense it resembles the normal magnetic moment of a charged Dirac particle

$$\mu_\nu^{\text{ind}} = \frac{eG_F}{4\pi\alpha\sqrt{2}} \lim_{k\to 0} A(0,k) \quad (30)$$

in the case of interaction with a static magnetic field or

$$\tilde{\mu}_\nu^{\text{ind}} = \frac{eG_F}{4\pi\alpha\sqrt{2}} \lim_{\omega\to\infty} A(\omega,k) \quad (31)$$

in the case of interaction of a neutrino with a high-frequency transverse electromagnetic field. Using the explicit expression[15] for the magnetic form factor $A(\omega,k)$, we can readily verify that (30) yields the result

$$\mu_\nu^{\text{ind}} = \frac{e_\nu^{\text{ind}}}{2m_e c_V} \quad (32)$$

and exactly the same expression for the moment (31) with replacement of the induced charge e_ν^{ind} (27) by (28). For the parameter $\xi = 0.25$ ($c_V = 1$) of the standard model, the magnetic moment (32) has the canonical form except that in the denominator we have in place of the neutrino mass the mass m_e of the real electrons of the medium attracted to the neutrino.

Despite the large value of the magnetic moment (32) ($\mu_\nu^{\text{ind}} \sim 10^{-11}\mu_B$ for a density $\rho \sim 10^{12}$ g/cm^3 of an isotropic dispersive medium), the contribution of the axial form factor in, for example, the $\gamma \to \nu\bar{\nu}$ plasmon decay process is small because the mean axial current is proportional to the small ratio $k/p \ll 1$, where k is the momentum transfer, and $p \approx E$ is the energy of an ultrarelativistic neutrino [cf. the first and second terms of the vertex (20)].

3. PROPAGATION OF NEUTRINOS IN DISPERSIVE MEDIA

This section is devoted to the description of the various effects of interaction of neutrinos with a dispersive medium that results from the electromagnetic vertex (20) in the standard model and the vertex (26) in the simplest extended model of the electroweak interactions. For completeness of the exposition, we include in this section the results of some foreign authors that confirm our concept of the important role of the collective electromagnetic interaction of neutrinos with matter.

The vertex (20) is calculated in the single-loop approximation and, at the first glance, its contribution to the cross section for neutrino scattering by the charges in the dispersive medium can be expected to be as small as the contribution from the vacuum radiative corrections in the same low-energy approximation. However because the dispersive medium has a high polarizability ($|\varepsilon^{(\text{med})} - 1| \gg 1$) compared with vacuum ($|\varepsilon^{(\text{vac})} - 1| \approx 0$), the partial contribution of the electromagnetic amplitudes with the vertex (20) to the cross sections of the elementary neutrino processes ($\sim G_F^2$) competes with the contribution of the tree diagrams of the standard model (see below). Moreover, we are justified in expecting that already in the first order in the weak constant ($\sim G_F$) the amplitude of forward neutrino scattering with allowance for the electromagnetic channel of the reactions through the vertex (20) will differ strongly from the standard value, leading to a modification of the MSW potential (7).

3.1. Renormalization of the MSW potential in an electron gas

It was shown in Ref. 21 for the example of a plasma with fixed uniformly distributed ions (electron gas) that by virtue of the conservation of the electric charge the single-loop corrections to the forward scattering amplitude, described by means of the electromagnetic vertex (20), do not, despite their large value, change the conclusions in MSW theory.

We recall the connection between the potential V of the interaction of a left-handed neutrino with matter and the amplitude $f_{\nu a}(0) = -\langle pp_a|M|pp_a\rangle/8\pi E_a$ for forward scattering by particles of species a having density n_a:

$$V=-\frac{2\pi}{p}\sum_a f_{\nu a}(0)n_a, \tag{33}$$

where p is the neutrino momentum. The single-loop corrections to neutrino scattering in matter in the employed 4-fermion point approximation are described by the matrix elements

$$\langle p'p'_{(e)}|M_{\nu e}^{(\mathrm{em})}|pp_{(e)}\rangle=\bar{\nu}(p')(+ie\Gamma_\mu)\nu(p)D^{\mu\nu}\times(-ie)\bar{u}(p'_e)\gamma_\nu u(p_e) \tag{34}$$

for νe scattering by electrons and

$$\langle p'p'_{(i)}|M_{\nu i}^{(\mathrm{em})}|pp_{(i)}\rangle=\bar{\nu}(p')(ie\Gamma_\mu)\nu(p)D^{\mu\nu}(+iZe)\times(p'_{(i)}+p_{(i)})_\nu \tag{35}$$

for neutrino scattering by massless nuclei with charge $+Ze(e=|e|=\sqrt{a})$.

For fixed ions we substitute $(p'_{(i)}+p_{(i)})_\nu = 2M\delta_{\nu 0}$. Here the electromagnetic vertex Γ_μ is given by (20); the propagator $D_{\mu\nu}(\omega,\mathbf{k})$ of the photons in the medium has the form

$$D_{\mu\nu}(\omega,\mathbf{k})=\frac{\hat{e}_\mu\hat{e}_\nu}{q^2\varepsilon_l(\omega,k)}+\frac{\delta_{\mu i}\delta_{\nu j}(\delta_{ij}-\hat{k}_i\hat{k}_j)}{\omega^2\varepsilon_{\mathrm{tr}}(\omega,k)-k^2}, \tag{36}$$

where $\varepsilon_{l,\mathrm{tr}}$ are the longitudinal and transverse permittivities, and $\hat{e}_\mu=(k,\omega\hat{k})/(q^2)^{1/2}$ is a unit polarization vector ($q^2=\omega^2-k^2$, $\varepsilon^2=-1$).

For the forward scattering amplitude we substitute in (34)–(35) the static limit $\omega = E_{p_a} - E_{\mathbf{p}_a-\mathbf{k}} = 0$, $\mathbf{k}\to 0$, which leads to the single-loop corrections

$$f_{\nu e}^{(\mathrm{em})}(0)=+\frac{G_F E_p\sqrt{2}c_V}{2\pi} \tag{37}$$

for νe scattering and

$$f_{\nu i}^{(\mathrm{em})}(0)=-\frac{G_F E_p\sqrt{2}Zc_V}{2\pi} \tag{38}$$

for scattering by ions; these are comparable in magnitude with the Born contributions of the tree approximation used in the derivation of the MSW potential (7):

$$f_{\nu e}^{(B)}(0)=-\frac{G_F E_p\sqrt{2}c_V}{2\pi} \tag{39}$$

and

$$f_{\nu i}^{(B)}(0)=-\frac{G_F E_p}{2\pi\sqrt{2}}[Z(1-4\xi)-N]. \tag{40}$$

We recall that substitution of the Born amplitudes (39) and (40) in the definition of the neutrino interaction potential (33) with subsequent subtraction of $V^{\mathrm{MSW}} = V_{\nu_e} - V_{\nu_\mu}$ determines the MSW potential (7) in terms of the contribution of the charged currents with allowance for the different values $c_V=2\xi\pm 0.5$ of the vector constant for ν_e and ν_μ.

If we now use the condition of electrical neutrality of the medium,

$$Zn_i=n_e,$$

then we can readily show [using Eq. (33)] that the large single-loop contributions (37) and (38) exactly cancel (by virtue of the conservation of electric charge) and do not change the value of the MSW potential (7).

3.2. Screening of elastic scattering of neutrinos by nuclei and electrons in a plasma

One of the consequences of the presence of the induced electric charge (26) is screening of the cross section for elastic scattering of the electron neutrino by nuclei in the dense plasma of a collapsing star.[72]

It is here necessary to distinguish two situations corresponding to the value of the nonideality parameter $\Gamma_i=\langle U\rangle/T_i$ of the medium. This is equal to the ratio of the mean interaction energy of the particles to the temperature T_i of the nondegenerate ions.

A degenerate electron Fermi gas is denser the more ideal it is ($\Gamma_e \ll 1$), but simultaneously the medium of nondegenerate ions can be in the liquid or even the solid state if the corresponding nonideality parameter is large, $\Gamma_i=Z^2e^2n_i^{1/3}/T_i \gg 1$, for sufficiently high densities n_i. For iron nuclei ($Z=26$) at density $\rho\sim 10^{12}$ g/cm^3 and ion density $n_i\sim 6\cdot 10^{33}$ cm^{-3} (Ref. 73), the parameter Γ_i is comparable with unity ($\Gamma_i - 1$) at temperature $T_i\sim 15$ MeV.

We consider below the case of high temperatures T_i when the nonideality parameter is small, $\Gamma_i \ll 1$ (ideal gas of ions), and the structure factor that takes into account the influence of neighboring ions (nuclei) on the cross section for scattering of neutrinos by an isolated nucleus can be set equal to unity ($\Lambda=1$, see Ref. 72).

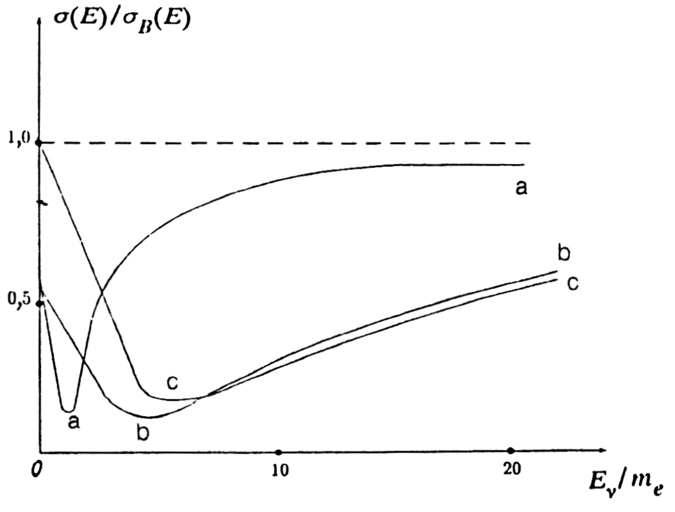

FIG. 4. Cross section for elastic scattering of a neutrino by spinless nuclei, $\nu A \to \nu A$, in the dense plasma of a collapsing star: a) by a $^{56}_{26}$Fe nucleus for density $\rho = 10^{10}$ g/cm^3; b) by a $^{56}_{26}$Fe nucleus for density $\rho = 10^{12}$ g/cm^3; c) by a $^{16}_{8}$O nucleus for density $\rho = 10^{12}$ g/cm^3.

Leinson (see, for example, Ref. 75) has also considered the important opposite case of an ion liquid ($\Gamma_i \gg 1$), for which the cross section for νA scattering depends strongly on the structure factor $\Lambda \neq 1$.

Thus, with allowance for interaction of a neutrino with an ion through the electrons that surround the ion and are coupled to it by the self-consistent electrostatic field [in a plasma, the electrons are free, $\varepsilon_p = (m_e^2 + p^2)^{1/2}$], the cross section for elastic νA scattering of a massless electron neutrino by one spinless nucleus,

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E^2 (1+\cos\Theta)}{16\pi^2} \left[Z(1-4\xi) - N + \frac{Z(1+4\xi)}{1+k^2 r_D^2} \right]^2, \quad (41)$$

contains not only the well-known contribution of the neutral currents but also an additional term described by an analog of the diagram of Fig. 3c, in which the real plasmon is replaced by a virtual plasmon with vertex Ze. This term vanishes in vacuum ($r_D \to \infty$) but makes a significant contribution in a medium for small momentum transfers $k = 2E \sin(\theta/2)$ comparable with the reciprocal Debye radius: $k \ll r_D^{-1}$.

Naturally, for large momentum transfers $k \gg r_D^{-1}$ the neutrino does not interact with the environment of the nucleus, and at the corresponding short distances from it ($r \sim k^{-1}$) only the contribution of the nucleons of the nucleus is important.

Noting that the contribution of the protons for the well-known parameter $\xi \sim 0.23$ of the standard model is insignificant, we draw attention in the expression (41) to the opposite signs of the contributions of the electrons and neutrons, this corresponding to the screening effect.

Figure 4 gives the dependence of the total cross section $\sigma(E)$ on the neutrino energy E calculated by means of (41) and normalized by the Born cross section σ_B (by the standard contribution of the neutral currents). The straight line $\sigma/\sigma_B = 1$ corresponds to the cross section of $\nu_{\mu,\tau}A$ scattering of muon and τ neutrinos for chosen parameter $\xi = 0.25$.

Indeed, it can be seen from the expression (27) that for the neutrinos $\nu_{\mu,\tau}$ ($c_V = 2\xi - 0.5$) the induced charge vanishes, $e_\nu^{\text{ind}} = 0$, i.e., there is no screening effect. Therefore the diffusion of the neutrinos $\nu_{\mu,\tau}$ to the edge of a radiating neutrino sphere lasts much longer, and if it were possible in underground experiments to detect neutrino pulses (from a supernova) separately for the different neutrino flavors, then the electron neutrinos should be detected first, and then the neutrinos $\nu_{\mu,\tau}$ (at the same energy E, see Fig. 4).[65] We note that a neutral-current channel (of inelastic scattering $\nu_{e,\mu,\tau} + D \to \nu_{e,\mu,\tau} + n + p$) is foreseen in future experiments at Sudbury (Canada). This will make it possible to distinguish the contributions of the different species of neutrino with allowance for the remaining channels of the reactions on deuterium.

In Ref. 73, the screening effect of Ref. 72 was taken into account in a calculation of the diffusion coefficient of electron neutrinos in the framework of relativistic kinetic theory. It was shown that allowance for the additional electromagnetic interaction[72] reduces the diffusion coefficient by a factor of order ~ 1.4 at low density ($\rho < 10^{11}$ g/cm^3) and by a factor ~ 1.8 at density $\rho < 10^{12}$ g/cm^3, and it is argued in Ref. 73 that this must have a strong effect on the collapse dynamics of the supernova precursor.

Using the vertex (20), we can readily obtain an analogous screening effect for quasielastic scattering of electron neutrinos by electrons in a dense equilibrium electron gas. At higher density, the cross section for $\nu_e e$ scattering is reduced by 20–30% compared with Born scattering (calculated in the absence of the dispersive medium) for intermediate energies $E \gg 10$ MeV of the emitted neutrinos.[76]

Using the relativistic generalization of the random-phase approximation, Horowitz and Wehrberger[76] also confirmed the result of Ref. 72 for the scattering of electron neutrinos by ions in a dispersive medium.

3.3. Increase in the probability of radiative decay of a massive neutrino in an electron gas

The problem of the radiative decay $\nu_2 \to \nu_1 \gamma$ of massive neutrinos is important for the clarification of the stability of the known neutrino species, in particular in connection with searches for monochromatic (nonequilibrium) photons from decays of fossil galactic neutrinos in the halo of the Galaxy (see, for example, the review of Ref. 77) or for the explanation of possible anomalies in the Wien region of the spectrum of the microwave background.[78,79]

In the latter case it is important to require that the neutrino lifetime τ_ν for which one is seeking must not exceed a period $\sim 10^{12}$ sec, corresponding to the time of decoupling of matter and radiation in the early universe (after hydrogen recombination). If this is not the case, photons from the neutrino decays could not come into equilibrium with matter.

In the standard model, the mechanism of compensation of the contributions of the virtual leptons in the vacuum probability of $\nu_2 \to \nu_1\gamma$ decay [the Glashow–Iliopoulos–Maiani (GIM) mechanism] leads to a very long lifetime $\tau_\nu \sim 2 \cdot 10^{34}\ (100\ \mathrm{eV}/m_{\nu_2})^5$ sec, which appreciably exceeds the age of the universe $t_0 \sim 3 \cdot 10^{17}$ sec and corresponds in the limit $m_{\nu_1} \to 0$ to a probability Γ equal to[80,81]

$$\Gamma = \frac{\alpha G_F^2 m_{\nu_2}^5}{2^{11}\pi^4} \left| \sum_{l=e,\mu,\tau} \frac{m_l^2 u_{l\nu_2}^* u_{l_1}}{M_W^2} \right|^2 . \qquad (42)$$

Here the square of the sum is the GIM factor, which is basically determined by the ratio of the masses of the τ lepton and the W boson and the square of the mixing matrix $|u_{\tau\nu_2}^* u_{\tau\nu_1}|^2 = \sin^2 2\theta_\nu/4$.

Attempts to reduce the neutrino lifetime by lifting the suppression of the GIM mechanism in the lepton sector were made earlier in various modifications of the standard model. In particular, introduction of a fourth generation of fermions or the addition of an isosinglet neutrino[82] makes it possible to increase the probability (42) by a factor of about $(M_W/m_\tau)^4 \sim 4 \cdot 10^6$, but this still leads to an insufficient reduction of the lifetime to explain the anomalies of the background radiation or to observe ultraviolet lines above the background radiation from the halo of the Galaxy.[77]

In the study of Ref. 83 considered here, D'Olivo, Nieves, and Pal consider in the framework of the standard model $\nu_2 \to \nu_1\gamma$ neutrino decay in an electron gas, for which one can use the neutrino electromagnetic vertex (20), or, rather, its part[7)] corresponding to the diagram of Fig. 3b (to the contribution of the charged currents) with allowance for the mixing matrix of the leptons $u_{\nu_{1,2}e}$ (mixing of neutrinos of two species with electrons). A contribution of muons and τ leptons is assumed to be absent, i.e., one does not consider, for example, the stage of the early universe $T \gg m_{\mu,\tau}$, when the GIM mechanism will occur both in vacuum and in a medium.

We shall proceed from the matrix element corresponding to the diagram of Fig. 3b for the vertex (20) when in it the contribution of the charged currents is separated $[\omega_{tr} = (\omega_p^2 + k^2)^{1/2} \approx k]$ and we consider the emission of a transverse high-frequency ($\nu_2 \to \nu_1\gamma_{tr}$) plasmon:

$$M_{\nu_2 \to \nu_1\gamma} = \frac{e[\nu_1(p')\Gamma_\mu(\omega,\mathbf{k})\nu_2(p)]e^\mu}{[2\varepsilon_{p'}\varepsilon_p(\omega(2\varepsilon_{tr} + \omega\partial\varepsilon_{tr}/\partial\omega))_{\omega=\omega_{tr}}]^{1/2}} . \qquad (43)$$

Here, $e^\mu = (0,\mathbf{e})$ is the polarization vector of the transverse plasmon, and in the normalization of the amplitude, taking into account the high-frequency limit $\varepsilon_{tr} \approx 1 - \omega_p^2/\omega^2$ of the permittivity, a factor $[\omega(2\varepsilon_{tr} + \omega\partial\varepsilon_{tr}/\partial\omega)]_{\omega=\omega_{tr}} = 2\omega_{tr} \approx 2k$ is also substituted. As in Ref. 52, we assume that the neutrino in the final state is massless ($\varepsilon_{p'} = p'$), and from (43) we obtain the probability for decay of a massive neutrino with emission of a plasmon with frequency $\omega = \omega_{tr} \approx k$:

$$\Gamma = \frac{G_F^2 \omega_p^4 m_{\nu_2}^2 |u_{e\nu_2}^* u_{e\nu_1}|^2}{8(2\pi)^3 \alpha \varepsilon_p} \int \frac{d^3k}{\varepsilon_{p'}\omega} \left[\frac{\varepsilon_{p'} + \varepsilon_p}{\omega} - \frac{m_{\nu_2}^2}{2\omega^2} \right] \times \delta(\varepsilon_{p'} + \omega - \varepsilon_p). \qquad (44)$$

The integral in (44) is readily calculated, and in Ref. 52 the final result

$$\Gamma = G_F^2 \omega_p^4 m_{\nu_2} |u_{e\nu_2}^* u_{e\nu_1}|^2 F(V)/32\pi^2\alpha \qquad (45)$$

is compared with the vacuum probability (42) for two asymptotic limits of the temperature of the medium: $T \ll m_e$, corresponding to the nonrelativistic plasma of stars with Langmuir plasma frequency $\omega_p = (4\pi\alpha n_e/m_e)^{1/2}$, and $T \gg m_e$, corresponding to the ultrarelativistic plasma of a hot universe in the lepton stage (in the absence of muons and τ leptons, $T \ll m_\mu$). In the expression (45), $V = p/\varepsilon_p$ is the velocity of the massive neutrino, and $F(V) = (1 - V^2)^{1/2}[(2/V)\ \ln|(1+V)/(1-V)| - 3]$ is a function of the velocity that in a wide range of masses m_{ν_2} and neutrino energies has a value of order unity.

In the ratios of the probabilities of $\nu_2 \to \nu_{1\gamma}$ decay in the medium and in vacuum[52] for the two asymptotic limits,

$$\frac{\Gamma^{(NR)}}{\Gamma} = 1.3 \cdot 10^{19} r F(V) \left(\frac{n_e}{10^{24}\ \mathrm{cm}^{-3}} \right) \left(\frac{\mathrm{eV}}{m_{\nu_2}} \right)^4, \qquad \frac{\Gamma^{(ER)}}{\Gamma} = 1.5 \cdot 10^9 r F(V) \left(\frac{T}{m_{\nu_2}} \right)^4, \qquad (46)$$

the ratio

$$r = \frac{|u_{e\nu_2}^* u_{e\nu_1}|^2}{|u_{\tau\nu_2}^* u_{\tau\nu_1}|^2}$$

of the squares of the mixing matrices can be set equal to unity for the purpose of estimates.

Using the expression (46), we can make some estimates of the possible decays of light ($m_{\nu_2} < 2m_e$) neutrinos[8)] in the hot plasma of the early universe. It is obvious from (42) and (46) that for a mass of an unstable neutrino (τ neutrino?) of order $m_{\nu_2} \leqslant 1$ MeV and temperature $T \gg 1$ MeV the vacuum lifetime $\tau_\nu \sim 2 \cdot 10^{14}$ sec of the massive neutrino is replaced by $\tau_\nu \sim 10^5$ sec in the hot electron plasma, and this value is much less than the time of decoupling of matter and radiation ($\sim 10^{12}$ sec), i.e., the photons from the neutrino decays have sufficient time to be thermalized. The question of the possibility of interpreting the anomaly in the Wien part of the background radiation[78] by means of the mechanism of neutrino decay in a medium can be finally resolved by considering the kinetics of the decays and calculating the spectra of the photons with allowance for the kinetics of their thermalization process.

This is already obvious from the estimate (46) with temperature ($\geqslant 1$ MeV) corresponding to the lepton stage of expansion to the time $t \leqslant 1$ sec and with lifetime $\tau_\nu \sim 10^5$ sec corresponding to a later time of the Big Bang with much lower temperature $T \ll m_e$.

3.4. Change of neutrino chirality in the degenerate electron gas of a supernova precursor

Before we present the result of Ref. 71 on the change of neutrino helicity in a dispersive medium (in the standard model of the electroweak interactions considered here), we must recall the known mechanisms of helicity change at the microscopic level.

In vacuum and homogeneous matter, there exist two such mechanisms.

First, it is well known that a massive Dirac particle has two helicity states, $r = \pm 1$, which persist in vacuum or in homogeneous matter up to the time of collision with an initial particle of the medium, when the initial helicity state, for example, left-handed $r = -1$, goes over either into the left-handed state $r = -1$ again or into the right-handed state $r = \pm 1$. The last effect is weak in the ultrarelativistic case, of order $O((m/E)^2) \ll 1$ relative to the transition without change of helicity.

Note that throughout the review we discuss transition of states with chirality change $\Psi_L \leftrightarrow \Psi_R$, i.e., change of the eigenvalue of the γ_5 matrix ($\lambda = -1 \leftrightarrow \lambda = +1$), which commutes, as the helicity operator $\Sigma_j p_j$, with the Hamiltonian of a free (in vacuum and in a homogeneous medium) massless particle. For an ultrarelativistic but massive particle ($m \neq 0, m \ll E$) only the helicity operator commutes with the same Hamiltonian, and the change of its eigenvalues ($r = -1 \leftrightarrow r = +1$) are adequate to describe the change of the helicity up to corrections that are $O((m/E)^2) \ll 1$.

One further possibility of changing the helicity is associated with the presence for a Dirac particle of an anomalous magnetic moment that interacts with an external macroscopic electromagnetic field or with an electrically charged particle of the medium.

The last helicity-changing channel, like the spin-flip result due to the presence of a neutrino mass, has a low probability because of the large mean free path of neutrinos with respect to collisions, this being comparable only with the radius of the core of a supernova.

However even on the sun collisions are a seed mechanism for the appearance of a right-polarized state Ψ_R that is absent at the time $t=0$ of neutrino production. For a massless neutrino, the initial left-handed helicity $r = -1$ can be changed (with some probability corresponding to a range $1 \gg R_\odot$) by the presence of a large anomalous magnetic moment μ' in the presence of right-handed currents in the interaction of the neutrino with the vacuum of vector bosons and leptons. The helicity is changed more effectively in a macroscopic magnetic field of a star that has a component $\mathbf{H} \perp \mathbf{n}$ transverse to the neutrino direction of motion $\mathbf{n} = \mathbf{p}/p$ (see above).

We recall that a consequence of the relative equation of motion of the spin in an external electromagnetic field is a time dependence of the projection $\zeta_{11}(t)$ of the polarization onto the direction of motion $\mathbf{n} = \mathbf{p}/p$. It is determined by the equation[85]

$$\frac{d\zeta_{11}}{dt} = 2\mu'((\zeta_\perp)_j[\mathbf{Hn}]_j) + \frac{2}{v}\left(\frac{\mu m^2}{\varepsilon^2} - \mu'\right)(\zeta_\perp)_j E_j,$$

where $\mu = e/2m_l + \mu'$ is the total magnetic moment of the spin or particle with charge e, and the total polarization vector $\zeta_i = n_i\zeta_{11} + (\zeta_\perp)_i$ includes a transverse part $(\zeta_\perp)_i$ that is a function of both the right- and left-polarized states: $\zeta_\perp^2 = |\Psi_R^* \Psi_L|^2$. For a neutriono in an external magnetic field, as for an electron, the helicity changes only with allowance for the radiative corrections, i.e., the anomalous part of the magnetic moment μ'.

We must emphasize the semiclassical nature of the spin equation of motion that is obtained for relatively slowly varying electromagnetic fields: ω, $k \ll p$, where ω and $\mathbf{k}$ are the frequency and wave vector of the electromagnetic field.[85]

In dispersive media, we still have the same sources of helicity change (mass and magnetic moment of the neutrino) but now the role of mass or magnetic field in the extended electroweak models can be played by certain effective characteristics that, like the neutrino induced electric charge and magnetic moment, depend on the density or temperature of the medium and make a much greater contribution than the corresponding vacuum characteristics (see below). In the standard electroweak model, Altherr and Kainulainen[71] found in a dispersive medium a possibility of helicity change in the $\nu_L \to \nu_{R\gamma}$ process of plasmon emission by taking into account in the electromagnetic vertex (20) an additional term that depends on the neutrino mass m_ν:

$$2\sqrt{2}G_F(1+4\xi)\frac{[(q\Omega)I_\beta - q^2K_\beta]}{(q\Omega)^2 - q^2} m_\nu \Omega^\mu \gamma_5, \qquad (47)$$

where Ω_μ is the 4-velocity of the medium and the integrals I_β and K_β are

$$I_\beta = \int d^4k\,\delta(k^2 - m_e^2) f_0(k)/8\pi^3,$$

$$K_\beta = \int d^4k\,\delta(k^2 - m_e^2) f_0(k)\left(\frac{k\Omega}{kq}\right)/8\pi^3.$$

Here

$$f_0(k) = \Theta(k_0)[\exp\beta(k_0 - \mu_e) + 1]^{-1} + \Theta(-k_0) \times [\exp\beta(-k_0 + \mu_e) + 1]^{-1}$$

is the Fermi distribution function, so that the contribution (47) vanishes not only for a massless neutrino ($m_\nu = 0$) but also in vacuum $[f_0(k) = 0)]$.

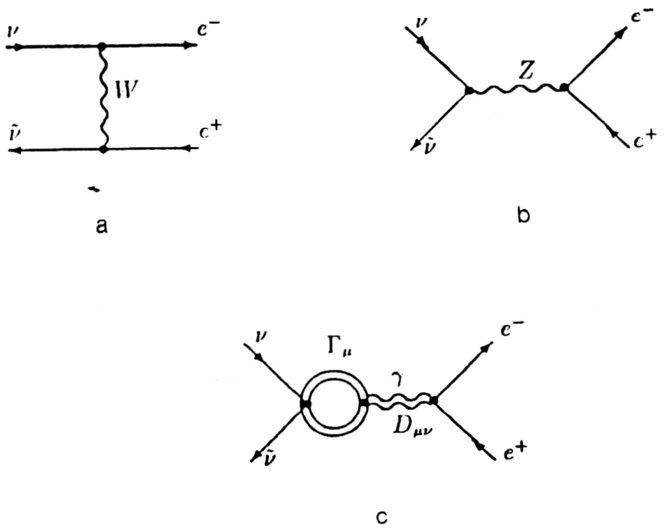

FIG. 5. Diagrams of $\nu\tilde{\nu}$ annihiliation with exchange of: a) W boson, b) Z boson, c) plasmon γ.

Naturally, the probability of emission of a plasmon with energy E in a degenerate ultrarelativistic electron gas with plasma frequency $\omega_0=(4\alpha/3\pi)^{1/2}\mu_e$ (Ref. 71),

$$\Gamma(E)\approx\frac{\alpha^3(4+\xi^{-1})^2}{128\pi^3}\left(\frac{m_\nu}{E}\right)^2\left(\frac{\mu_e}{M_W}\right)^4\omega_0 f_0(E), \tag{48}$$

is small [of order $O((m_\nu/E)^2)\ll 1$] compared with the probability of emission without helicity change in the same standard model and in the same medium ($\nu_L\to\nu_L\gamma$, see Sec. 3.6).

On the other hand, the qualitative change, i.e., the neutrino steralization $\nu_L\to\nu_R$ in the channel (48), is stronger by two orders of magnitude (in energy losses through right-handed neutrinos) than as a result of electromagnetic scattering through interaction of the magnetic moment (which in the standard model is proportional to the neutrino mass m_ν) with the electrons in the same degenerate gas.[71]

Unfortunately, this does not in any way affect the rate of cooling of the supernova precursor star, which loses through the radiation of active (left-handed) neutrinos much more energy (by $\approx$ ten orders of magnitude). It is only in extended electroweak models that allowance for the processes with production of right-handed neutrinos significantly changes the collapse dynamics.

3.5. Density effect in neutrino energy losses and in $\nu\tilde{\nu}\to e^-e^+$ annihilation

In this section we consider the processes of elastic νe scattering and $\nu\tilde{\nu}\to e^-e^+$ annihilation of neutrino pairs in a dispersive medium that are described by the diagrams of Fig. 5. The crossing-symmetric process of νe scattering is obtained by turning the time axis in Fig. 5 upward and making the substitutions $\tilde{\nu}\to\nu$, $e^+\to e^-$.

Besides the obvious Born diagrams of the standard model, an important contribution is made by the electromagnetic process corresponding to the pole diagram of Fig. 5c with vertex $\Gamma_\mu(\omega,\mathbf{k})$ (20) for neutrinos and with virtual plasmon described by the Green's function (36).

In the s channel ($q^2<0$) there is a pole for emission (absorption) of a longitudinal plasmon: $\varepsilon_l=0$ [see (18) in Fig. 2], and the differential cross section for νe scattering has resonance form in the neighborhood of momentum transfers and energies corresponding to the dispersion curve (see Fig. 2).

Indeed, the part of the cross section for νe scattering corresponding to the electromagnetic contribution will, in accordance with (36), be proportional to the Breit–Wigner factor:[48]

$$|\varepsilon_l(\omega,k)|^2\approx\frac{(E-E')^4}{[(E-E')^2-\omega_p^2]^2+(E-E')^4(\mathrm{Im}\,\varepsilon_l)^2},$$

if we use the high-frequency approximation for the permittivity, $\mathrm{Re}\,\varepsilon_l(\omega,k)\approx 1-\omega_p^2/\omega^2$, where $\omega=E-E'$ is the energy transfer (to the moving neutrino). In a transparent medium ($\mathrm{Im}\,\varepsilon_l\ll\mathrm{Re}\,\varepsilon_l$), the resonance is narrow, and the integration over the energy transfer leads to a small correction in the total cross section or in the neutrino energy losses.[49]

At low neutrino energies, these last have the order of the collisional limit:[86]

$$Q_0=n\int_0^{\omega_{\max}}\omega\,\frac{d\sigma_B(\omega)}{d\omega}\,d\omega, \tag{49}$$

which is determined by the Born contribution of the diagrams of the standard model (of the type of Figs. 5a and 5b), where $\omega_{\max}=2E^2/(2E+m)$, and m and n are the mass and concentration of the particles of the medium. Here $E\ll(mE_0)^{1/2}$, where E_0 is the characteristic energy of the particles of the medium; for example, in a metal for degenerate electrons $E_0\approx p_F^2/2m_e$. For high energies ($E\gg p_F$), the collective effects are unimportant, and the neutrino losses are determined exactly by the limit (49). The Fermi density effect[87] would be important (for the conditions of nonrelativistic particles of the medium) only for soft neutrinos, the detection of which in a laboratory experiment is at present impossible.

Kirzhnits, Losyakov, and Chechin[88] showed that the neutrino energy losses in a medium are determined by the contraction of the trace of the product of the neutrino currents (including the axial part) and the correlation function $D_{\mu\nu}^{ab}$:

$$D_{\mu\nu}^{ab}=i\Theta(t)\langle[I_\mu^a(x),I_\nu^b(0)]\rangle,$$

which in the electroweak model in the low-energy approximation has three components (a, $b=V$, A) in accordance with the presence of axial and vector currents I_μ^a of the particles of the medium.

The pseudotensor $D_{\mu\nu}^{VA}(q)$ depends on one characteristic of the medium, since from the variables q_μ and Ω_ν, where $q=(\omega,\mathbf{k})$ is the momentum transfer and Ω_ν is the 4-velocity of the medium, one can construct[88] only the one pseudovector $D_{\mu\nu}^{VA}(q)=D_{\nu\mu}^{AV}(-q)=i\varepsilon_{\mu\nu\rho\sigma}q^\rho\Omega^\sigma R_3/2m$ where the linear-response function R_3 is proportional to the form factor $A(\omega,k)$.[15] The tensors $D^{VV}_{\mu\nu}$ and $D^{AA}_{\mu\nu}$, as in statistical QED,[50] in which there is a response only to a vector perturbation (only $D^{VV}_{\mu\nu}$), each depend on two linear-response functions.

In the model of electroweak interactions of neutrinos with matter, the medium can be characterized overall by five linear-response functions[86] instead of the two in statistical QED that correspond to the usual permittivities $\varepsilon_{l,\mathrm{tr}}(\omega,k)$ in the same isotropic dispersive medium.

This in no way contradicts our assertion[48] of the existence of three electromagnetic form factors (F_l,F_{tr},A) in the vertex (20); for in the electromagnetic channel of neutrino interaction with the medium there is only vector excitation of the dispersive medium through the electromagnetic potential A_μ or the current j_μ of the charged particles, i.e., in this channel we have the participation of the correlation functions $D^{VV}_{\mu\nu}$ and $D^{VA}_{\mu\nu}$, which depend on the same characteristics of the medium.

For the hot plasma of the early universe, the contribution of the axial currents is altogether insignificant because of the weak asymmetry of the particle number $[(n_i-\tilde{n}_i)/n_\gamma\sim 10^{-9}]$. Therefore the total cross section for the annihilation of neutrino pairs, $\nu\tilde{\nu}\to e^+e^-$, which is important for the processes of establishment of equilibrium and quenching of the neutrinos, depends only on the two response functions $\varepsilon_{l,\mathrm{tr}}$ of an isotropic dispersive medium and in the rest frame of the medium as a whole has the form[89]

$$\sigma(\nu\tilde{\nu}\to e^+e^-)$$
$$=\frac{G_F^2q^2}{3\pi}\left\{c_V^2+c_A^2+\frac{c_V^2}{2}\left[1-\frac{(\omega-2F_1)^2}{k^2}\right]\left[\left|\frac{\varepsilon_l-1}{\varepsilon_l}\right|^2\right.\right.$$
$$\left.-2\,\mathrm{Re}\left(\frac{\varepsilon_l-1}{\varepsilon_l}\right)\right]+\frac{c_V^2}{2}\left[1+\frac{(\omega-2E_1)^2}{k^2}\right]$$
$$\left.\times\left[\left|\frac{\varepsilon_{\mathrm{tr}}-1}{\varepsilon_{\mathrm{tr}}-k^2/\omega^2}\right|^2-2\,\mathrm{Re}\left(\frac{\varepsilon_{\mathrm{tr}}-1}{\varepsilon_{\mathrm{tr}}-k^2/\omega^2}\right)\right]\right\}. \quad (50)$$

Here $\omega=E_1+E_2$ is the energy of the $\nu\tilde{\nu}$ pair, and the constants $c_{V,A}$ were determined above by the expression (20). Inclusion of a blocking factor and averaging over the final state of the e^+e^- pairs does not make it possible to obtain an analytic expression for the total cross section (50), although they change little in the qualitative picture of the resonance contribution of the diagram of Fig. 5c analyzed here.[89]

The cross section (50) takes an even simpler form in the center-of-mass system that coincides with the rest frame of the medium as a whole, i.e., for $\mathbf{k}=\mathbf{p}_1+\mathbf{p}_2=0$.

In this situation we have the simple expressions $\varepsilon_{\mathrm{tr}}=\varepsilon_l=1-\omega_p^2/\omega^2$ and $\omega_p^2=4\pi T^2\alpha/9$, and the contribution of all three diagrams of Fig. 5 corresponds to the total cross section

$$\sigma_{\mathrm{cms}}(\nu\tilde{\nu}\to e^+e^-)=\frac{G_F^2q^2}{3\pi}\left[\frac{c_V^2}{|\varepsilon_{\mathrm{tr}}|^2}+c_A^2\right]. \quad (51)$$

In vacuum ($T=0$, $\varepsilon_{\mathrm{tr}}=1$) the cross section (51) reduces to the standard form,[39] while in a hot plasma, $T\gg m_e$, in which the permittivity has a small imaginary part $\mathrm{Im}\,\varepsilon_l=\mathrm{Im}\,\varepsilon_{\mathrm{tr}}=\alpha\omega/12T$ (Ref. 89), the cross section (51) acquires a narrow peak of a Breit–Wigner resonance:

$$|\varepsilon|^{-2}=1+\frac{2x^2x_0^2-x_0^4-\alpha^2x^6/36}{(x^2-x_0^2)^2+\alpha^2x^6/36}=1+F(q^2)$$

with strong enhancement $F(\omega_p^2)\approx 324/\alpha^3\pi\approx 2.7\cdot 10^8$ but very small width of order $\pi\alpha^2/108$. In the last expression we have introduced dimensionless quantities: $x=\omega/2T=(q^2/4T^2)^{1/2}$ and the resonance position $x_0^2=\pi\alpha/9\approx 2.5\cdot 10^{-3}$ corresponding to low pair energies $q\approx\omega_p\approx 0.1T$.

As for the case of νe scattering, the narrow resonance in the annihilation has practically no effect after averaging over the equilibrium states of all the particles, i.e., there is no influence of the collective mechanisms of neutrino interaction in a dispersive medium in the given application to a hot universe if one uses averaging over the initial states of the equilibrium neutrinos.[89]

However an analysis has recently been made of a different situation, in which the $\nu\tilde{\nu}\to e^+e^-$ process is important for the dynamics of the outer layer of the neutrino sphere of a supernova.[90] The assertion of a nonequilibrium neutrino distribution function[90] requires additional analysis using (50) in the calculation of the energy release of $\nu\tilde{\nu}$ pairs in a hot plasma.

Besides deviation of the neutrino distribution function from equilibrium, an important role in increasing the electromagnetic interaction of neutrinos in a dispersive medium and, as a consequence, in the additional heating of a supernova shell can be played by broadening of the resonance peak, for example, by Doppler shift of the cyclotron resonance in an anisotropic dispersive medium such as a magnetoactive plasma [see the electromagnetic vertex (23) above]. The possible presence of strong magnetic fields for supernova remnants is well known, and this question warrants further study.

3.6. Polarization and Cherenkov radiation of neutrinos in a plasma

The fact that an electrically neutral particle can radiate photons in a medium is not a novelty in physics. In papers written long ago, Ginzburg[91] and Frank[92] studied the Cherenkov radiation of a magnetic dipole (i.e., a neutron with an anomalous magnetic moment) in a medium without spatial dispersion.

However, in the standard model of the electroweak interactions a massless neutrino does not have any dipole electromagnetic moments ($\mu_\nu=d_\nu=0$), and therefore it is not meaningful to speak of radiation such as that considered in Refs. 91 and 92. Even if the neutrino does have a rest mass, the effect will be small.

In the extended electroweak models, the magnetic moment of the neutrino (including a massless one) can be large, $\mu_\nu\leqslant 10^{-11}\,\mu_B$, but in a dispersive medium the extended models lead, as will be shown below, to a much larger contribution of the polarization effects associated with the existence of an effective (induced) magnetic moment μ_{eff} of the neutrino.

We consider here the polarization radiation of a massless neutrino in an isotropic dispersive medium ($\nu_L \to \nu_L \gamma$) in the standard model with the vertex (20) with no change of helicity or flavor. The probability of this radiation process is much greater than in the decay $\nu_2 \to \nu_1 \gamma$ through the nondiagonal magnetic moment when $m_{\nu_2} > m_{\nu_1}$.

The matrix element of polarization radiation of longitudinal plasmons by a massless neutrino corresponding to the diagram shown in Fig. 3c has the form[70]

$$\langle p'q|S|p\rangle = -i(2\pi)^4 \left(4E_{p'}E_p \left| q^2 \frac{\partial \operatorname{Re} \varepsilon_l}{\partial \omega} \right| \right)^{-1/2} \times \delta^{(4)}(p'+q-p) J_\mu e^\mu. \quad (52)$$

Here the conserved neutrino electromagnetic current ($J_\mu q^\mu = 0$) can, with allowance for the definition of the longitudinal polarization tensor $\Pi^{(l)}_{\mu\nu} = -q^2(\varepsilon_l(\omega,k) - 1)e_\mu e_\nu$ of statistical QED and the unit polarization 4-vector $e_\mu = (k, \omega\hat{\mathbf{k}})/(q^2)^{1/2}$ of the longitudinal plasmon, be expressed in the form

$$J_\mu = eF_l(\omega,k)\bar{\nu}(\mathbf{p}')\gamma_\mu \frac{(1-\gamma_5)}{2} \nu(\mathbf{p}),$$

where the form factor $F_l(\omega,k)$ is defined in (21).

The polarization losses of the massless ($E_p = p$) neutrino, determined by means of (52), reduce to the integral

$$\frac{dE_p(\text{polar.})}{dl} = \frac{G_F^2 c_V^2}{4\pi^2 \alpha} \int_{k_{\min}}^{k_{\max}} \frac{\omega_j(k) q_j^4 dk}{k|\partial \operatorname{Re} \varepsilon_l/\partial\omega|_{\omega=\omega_j}} \times \left[1 - \frac{\omega_j(k)}{p} + \frac{q_j^2}{2p^2}\right], \quad (53)$$

in which the radiation frequencies of the longitudinal plasmon [the solution of the dispersion equation $\varepsilon_l(\omega,k) = 0$ (18)] satisfy an energy conservation law that in the semiclassical approximation ($k \ll p$) reduce to the condition $\omega \approx \mathbf{kc}$ of Cherenkov resonance. For a Langmuir plasmon ($\omega_j = \omega_p$), the lower limit in the integral (53) is $k_{\min} = \omega_p$. The upper limit for relatively hard neutrinos ($p > r_D^{-1}$) is $k_{\max} \approx r_D^{-1}$, and for soft neutrinos ($p < r_D^{-1}$), i.e., $E \ll 1$ keV in a metal or $E \ll a$ few mega-electron-volts in the shell of a supernova precursor star, the conservation laws give the limit $k_{\max} = 2p - \omega_p$.

Of course, the threshold energy of the neutrino exceeds the plasmon energy ($p \gg \omega_p$).

Thus, we can give two asymptotic forms of the result (53):

$$\frac{dE_p(\text{polar.})}{dl} \approx \frac{G_F^2 c_V^2 \omega_p^2}{32\pi^2 \alpha r_D^4}, \quad \text{if } p > r_D^{-1},$$
$$\frac{dE_p(\text{polar.})}{dl} \approx \frac{G_F^2 c_V^2 \omega_p^2 p^4}{6\pi^2 \alpha}, \quad \text{if } \omega_p \ll p < r_D^{-1}. \quad (54)$$

Comparing the polarization losses (54) with the collisional losses in the scattering of soft neutrinos by degenerate electrons in a metal,

$$\frac{dE_{\text{Born}}(\nu e \to \nu e)}{dl} = \frac{G_F^2 \omega_p^2 p^4}{12\pi^2 \alpha} (3 + 4\xi + 8\xi^2),$$

we see that the collective losses are of the same order as the collisional losses in the range of low energies but negligibly small for hard neutrinos, $p \gg r_D^{-1}$, for which the losses are determined by binary collisions described by the Born diagrams of νe scattering in the standard model.

This agrees with the conclusions of the previous section and, in particular, with the assertion made by the group at the P. N. Lebedev Physics Institute,[86,88] namely, that there is no significant enhancement of the neutrino energy losses in a medium compared with the collisional losses. We note that in the elegant method of calculation proposed in Ref. 88, the neutrino energy losses include the polarization losses previously studied in our paper of Ref. 70.

The density effect could be significant in the laboratory for the low neutrino energies as yet unaccessible to measurement by modern detectors.

On the other hand, plasmon emission by soft neutrinos in a collapsar ($E_p \ll 1$ MeV) may compete with collisional losses in νe scattering by degenerate electrons in the dense shell of a collapsing star when the neutrino mean free path is less than the diameter of the star. The problem of the correct calculation of the neutrino energy losses in a relativistic medium is still apparently unsolved (see some comments about this in Ref. 88).

It appears to us that the neutrino energy losses in an anisotropic medium may be much greater. For example for a magnetoactive plasma estimates by means of the first term in the vertex (23) show[14] that in a strong magnetic field the neutrino losses through Cherenkov radiation of extraordinary waves can be significantly greater than the collisional losses.

3.7. Change of neutrino helicity in a plasma with neglect of the vacuum mass ($m_\nu^{\text{vac}} = 0$) and vacuum dipole moments ($\mu_\nu = d_\nu = 0$)

We considered above helicity-changing scattering processes in vacuum and in a homogeneous medium due to the presence of a neutrino vacuum magnetic moment or vacuum rest mass. In a dispersive medium, a moving neutrino polarizes the medium by the weak forces, and therefore the additional terms in the interaction Lagrangian due to the right-handed currents lead to a modification of the magnetic vertex Γ_μ as a result of separation of electric charges (polarization), as in the case of the standard model.

Having no interest in the corrections to the terms in Γ_μ of the standard model that do not lead to helicity change, we concentrate attention on the partial contribution (26) corresponding to change of the neutrino helicity in the model of Ref. 56.

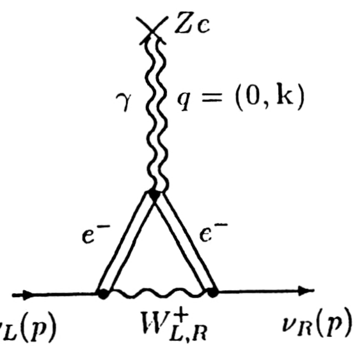

FIG. 6. Feynman diagram of neutrino elastic scattering by a Coulomb center in a medium with change of helicity (left→right). In the ultrarelativistic approximation ($m_\nu \to 0$) the notation is the same for the helicity change ($r=-1 \to r=+1$). The double lines denote the propagators of the electrons and a photon in the medium.

We consider the matrix element of neutrino scattering by a nucleus in a plasma with helicity change $[\nu_L+(Z,A) \to \nu_R+(Z,A)]$ in the model of Ref. 55 (see Fig. 6).

In the rest frame of the medium as a whole ($\Omega_\mu = \delta_{\mu 0}$) we can, using the electromagnetic vertex (26), write down such a matrix element:[58]

$$M_{f_i} = \frac{2\sqrt{2}G_F \sin 2\zeta Z r_D^2 m_e}{1+(kr_D)^2} A(0,k) \bar{\nu}_f(\mathbf{p}') \nu_i(\mathbf{p}) \times \frac{2\pi\delta(E-E')}{2\sqrt{EE'}}, \tag{55}$$

where $A(\omega,k)$ is the neutrino magnetic form factor obtained in Ref. 15 [see the vertex (20)], which is equal in the static case to

$$A(0,k) = -\frac{16\pi^2\alpha}{k} \int_0^\infty p dp [f^-(E_p) - f^+(E_p)] \ln\left|\frac{2p+k}{2p-k}\right|, \tag{56}$$

in which $f^\pm(E_p)$ are the equilibrium distribution functions of the electrons and positrons in the plasma.

We draw attention to an important difference between the electrodynamic effects in a dispersive medium and the effects of neutrino oscillations in a medium in which only the forward scattering amplitude ($\mathbf{k}=0$) is taken into account. In the given case, the matrix element (55) would simply vanish for a massless neutrino ($m_\nu = 0$), i.e., it is the incoherence of the neutrino scattering process in matter ($\mathbf{k} \neq 0$) that is important.

In the special case of a degenerate electron gas, for example, in the shell of a collapsing star, the form factor (56) can be readily calculated, and, using the amplitude (55), we can calculate the cross section of the process $\nu_L \to \nu_R$ with helicity change:

$$\sigma(E) = Z^2 \mu_{\rm eff}^2 \alpha \left[\ln\left|1+4(Er_D)^2\right| - \frac{4(Er_D)^2}{1+4(Er_D)^2} \right], \tag{57}$$

which recalls the Schwinger cross section for the scattering of the neutron (a particle with anomalous magnetic moment) by a Coulomb center.

However, in the cross section (57) the role of the effective magnetic moment $\mu_{\rm eff}$ is played by[58]

$$\mu_{\rm eff} = 8\sqrt{\pi} \frac{p_{f_e}}{E} \mu_\nu^{\rm vac}, \tag{58}$$

which is significantly larger than the vacuum moment $\mu_\nu^{\rm vac}$, which is determined in the given model by the expression[93]

$$\mu_{\nu_e} = \frac{eG_F m_e \sin 2\zeta}{2\sqrt{2\pi^2}}.$$

This is due to the fact that the neutrino energy E is always less than the energy of the ultrarelativistic electrons ($E \leqslant p_{F_e}$) that produce left-handed neutrinos in the URCA process of electron capture by nuclei.

We can also use the astrophysical bound $\mu_\nu^{\rm vac} < 10^{-12}\,\mu_B$ on the neutrino magnetic moment;[31] this determines the maximum admissible flux of sterile high-energy neutrinos that freely escape a supernova and are transformed in the intergalactic magnetic field (for SN1987A) into left-handed active neutrinos. The absence of an additional number of events from such high-energy neutrinos in detectors with known (lower) detection threshold is what leads to the bound we have given. We recall that there are arguments[32,33] for a lifting of the stringent bound $\mu_\nu^{\rm vac} < 10^{-12}\,\mu_B$ (with replacement by $\mu_\nu^{\rm vac} < 10^{-11}\,\mu_B$, see the discussion in Sec. 2), but there still remains the rather stringent astrophysical bound $\mu_\nu^{\rm vac} \leqslant 2\cdot 10^{-12}\,\mu_B$ (Ref. 25) discussed in Sec. 2, or the even more stringent bound $\mu_\nu^{\rm vac} < 10^{-19}\,\mu_B$ (Ref. 27).

Restricting the effective magnetic moment (58) in accordance with the above limit ($\mu_\nu^{\rm vac} < 10^{-12}\mu_B$), we obtain an astrophysical bound on the mixing parameter of the left- and right-handed bosons $W_{L,R}$ in the model of Ref. 58:

$$\sin 2\zeta \leqslant 0.4 \frac{E}{p_{F_e}},$$

which for intermediate neutrino energies $\langle E\rangle \sim 10$ MeV in the initial stage of neutrino transparency of a star ($\rho \geqslant 10^{12}$ g/cm^3, $p_{F_e} \sim 40$ MeV) is comparable with the laboratory bound sign $2\zeta \leqslant 0.1$ (Ref. 56).

To conclude the section, we note the absence of a singularity in the cross section (57) in the limit $E \to 0$ despite the dependence of the effective magnetic moment (58) on the energy $E(\sigma_{E\to 0} \sim G_F^2 E^2)$.

4. RELATIVISTIC KINETIC ENERGY FOR NEUTRINOS WITH ALLOWANCE FOR A SELF-CONSISTENT FIELD IN THE STANDARD MODEL OF ELECTROWEAK INTERACTIONS

Up to now we have considered the single-particle problem of the propagation of a neutrino through a dispersive medium. To describe the transport of neutrino fluxes in the plasma of stars, in particular collapsing stars, or in the plasma of the hot early universe, it is necessary to introduce many-particle transport equations—relativistic kinetic equations for the neutrinos and particles of the medium.

4.1. Covariant equation for the νe system. Parity violation. The radiation damping force

In the early studies (see the bibliography in the book of Ref. 94), a relativistic kinetic equation for neutrinos was derived by various methods for sufficiently rarefied media in which a restriction can be made to classical relativistic distribution functions of the type of Jüttner[95] equilibrium distribution $f \sim \exp[(\mu - p_\mu \Omega^\mu)/T]$, where μ and T are the Lorentz-invariant chemical potential and temperature.

As a result the authors of Ref. 94 obtained on the right-hand side of the relativistic kinetic equation Boltzmann collision integrals that are not valid for degenerate media described quantum statistically such as a supernova precursor star or the dense plasma of the early universe, for which it is already important to take into account the blocking factor $(1-f)$.

In Ref. 96, one of the present authors obtained a collision integral for arbitrary density of the electrons and neutrinos, generalizing in this way the result of the Dutch physicists,[94] who also did not take into account the contribution of the self-consistent field (see below). The same result for the νe collision integral as in Ref. 96 was obtained in Ref. 97.

We recall that the relativistic kinetic equation for spinor particles describes the evolution of single-particle Wigner distribution functions

$$f^{(a)}_{r'r}(\mathbf{p},\mathbf{x},t) = f^{(a)}(\mathbf{p},\mathbf{x},t)\,\frac{\delta_{r'r}}{2} + S^{(a)}_j(\mathbf{p},\mathbf{x},t)\,\frac{(\sigma_j)_{r'r}}{2}. \tag{59}$$

Here, $f^{(a)}(\mathbf{p},\mathbf{x},t)$ is the Lorentz-invariant distribution function of the number of particles of species a (in the phase space of the variables x and p, which in general do not commute), which is related to the concentration by the equation $n^{(a)}(\mathbf{x},t) = \int d^3p f^{(a)}(\mathbf{p},\mathbf{x},t)$; $\mathbf{S}^{(a)}(\mathbf{p},\mathbf{x},t)$ is the spin distribution function.

For the simplest νe system in the standard model,[46] the relativistic kinetic equation was derived in Ref. 96 from the quantum Lioville equation.

In contrast to Ref. 96, we present here in covariant form a complete system of kinetic equations, retaining the terms that arise from the pseudovector weak current of the electrons. These terms in the kinetic equations reflect explicitly the parity violation effects that have been actively investigated in recent years in various macroscopic phenomena.[98] The covariant (in a flat metric) equation for the distribution function of the number of electrons has in the statistical electroweak model the form

$$p_\mu \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial x_\mu} + eF_{k\mu}(\mathbf{x},t)p^\mu \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_k} + \frac{G_F}{\sqrt{2}} \int \frac{d^3p'}{\varepsilon^e_{p'}} \Big\{ (pp')\,\frac{(1-4\xi)^2}{\sqrt{2}}\,\frac{\partial f^{(e)}(\mathbf{p}',\mathbf{x},t)}{\partial x_j}\,\frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j}$$

$$-(1-4\xi)p_\mu \frac{\partial a^\mu(\mathbf{p}',\mathbf{x},t)}{\partial x_j}\,\frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} \Big\} + G_F\sqrt{2}c_V \int \frac{d^3p'}{\varepsilon^\nu_{p'}}\,(p'_\nu)\,\frac{\partial f^{(\nu)}(\mathbf{p}',\mathbf{x},t)}{\partial x_j}\,\frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} = J^{(\nu e)}_{\text{coll}}(\mathbf{p},\mathbf{x},t), \tag{60}$$

where $k=1, 2, 3$; $\mu=0, 1, 2, 3$. We emphasize the conservation of the 4-current $j^a_\mu(\mathbf{x},t) = \int (d^3p/\varepsilon_p)p_\mu f^a(\mathbf{p},\mathbf{x},t)$, $\partial j^a_\mu(x,t)/\partial x_\mu = 0$, which follows from the form of the collision integral[96] with allowance for the contribution of the self-consistent field of electroweak origin ($\sim G_F$) [see the relativistic kinetic equation (65) below for neutrinos]. The second covariant equation for the square $[\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)]^2$ of the spin distribution function has the form

$$p_\mu \frac{\partial [\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)]^2}{\partial x_\mu} + \frac{m_e G_F}{\sqrt{2}} \int \frac{d^3p'}{\varepsilon^{(e)}_{p'}} \Big\{ \frac{m_e a_\mu(\mathbf{p},\mathbf{x},t)}{4}\,\frac{\partial a^\mu(\mathbf{p},\mathbf{x},t)}{\partial x_j} - \frac{(1-4\xi)}{2}\,p'_\mu a^\mu(\mathbf{p},\mathbf{x},t)\,\frac{\partial f^{(e)}(\mathbf{p}',\mathbf{x},t)}{\partial x_j} \Big\}$$

$$\times \frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} - m_e G_F\sqrt{2} \int \frac{d^3p'}{\varepsilon^\nu_{p'}}\,[p'_\mu a^\mu(\mathbf{p},\mathbf{x},t)]\,\frac{\partial f^{(\nu)}(\mathbf{p}',\mathbf{x},t)}{\partial x_j}\,\frac{\partial f^{(e)}(\mathbf{p},\mathbf{x},t)}{\partial p_j} = J_{\text{coll}}(\mathbf{S}^{(e)},\mathbf{x},t). \tag{61}$$

We have here introduced the 4-vector $a_\mu(\mathbf{p},\mathbf{x},t)$ of the spin distribution that is the statistical generalization[96] of the Pauli–Lubański 4-vector and has the components

$$a^\mu(\mathbf{p},\mathbf{x},t) = \left[\frac{\mathbf{p}\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t)}{m_e};\ \mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t) + \frac{\mathbf{p}(\mathbf{p}\mathbf{S}^{(e)}(\mathbf{p},\mathbf{x},t))}{m_e(\varepsilon_p + m_e)} \right]. \tag{62}$$

From (62) the Lorentz invariance of the magnitude of the 3-vector of the spin distribution, $\{\mathbf{S}^e(\mathbf{p},\mathbf{x},t)]^2 = -a_\mu a^\mu$ is obvious.

We note that in quantum kinetics one usually considers a simpler equation for the vector $\mathbf{S}^e(\mathbf{p},\mathbf{x},t)$ (with lower nonlinearity). The invariant expression (61) is more convenient for demonstrating parity violation effects.

The system of self-consistent equations is closed by Maxwell's equations for the electric field $F_{\mu\nu}(\mathbf{x},t)$ and

Wigner distribution functions for the neutrinos:

$$q_\mu \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_\mu} + \frac{G_F c_V}{\sqrt{2}} \int \frac{d^3p'}{\varepsilon_{p'}^{(e)}} (p'q) \frac{\partial f^{(e)}(\mathbf{p}',\mathbf{x},t)}{\partial x_j} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial p_j} + G_F\sqrt{2} \int \frac{d^3q'}{\varepsilon_{q'}^{(\nu)}} (q'q) \frac{\partial f^{(\nu)}(\mathbf{q}',\mathbf{x},t)}{\partial x_j} \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial q_j} = J_{\text{coll}}^{(\nu e)}(\mathbf{q},\mathbf{x},t), \qquad (63)$$

$$\frac{\partial S_i^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial t} + n_k \frac{\partial S_i^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial x_k} - G_F\sqrt{2} n_\gamma(T) L_0 e_{ikl} n_k S_l^{(\nu)}(\mathbf{q},\mathbf{x},t) + 2\mu_\nu((\mathbf{n}\mathbf{S}(\mathbf{q},\mathbf{x},t))E_i(\mathbf{x},t) - n_i(\mathbf{S}^{(\nu)} \times (\mathbf{q},\mathbf{x},t)\mathbf{E}(\mathbf{x},t)) + [\mathbf{S}^{(\nu)}(\mathbf{q},\mathbf{x},t)\times\mathbf{B}(\mathbf{x},t)]_i - (\mathbf{n}\mathbf{B}(\mathbf{x},t))[\mathbf{S}^{(\nu)}(\mathbf{q},\mathbf{x},t)\times\mathbf{n}]_i) = \frac{J_i^{\text{coll}}(\mathbf{q},\mathbf{x},t)}{\varepsilon_q}. \qquad (64)$$

The relativistic kinetic equation (64) represented here in noncovariant form, describes the rotation of the neutrino spin in the total electric field (**E**,**B**), including the self-consistent field in the plasma, and it also takes into account collisions of the neutrinos with the particles of the medium.[99] The electromagnetic terms in this equation depend on the anomalous magnetic moment μ_ν of the neutrino; $\mathbf{n}=\mathbf{q}/\varepsilon_q$ is the neutrino velocity. The third term on the left-hand side of (64) is due to neutrino forward scattering on all the particles of the background and depends on the total asymmetry $L_0=\Sigma_a(n_a-n_{\tilde{a}})/n_\gamma(T)$ of the particles normalized by the density $n_\gamma(T)$ (15) of the equilibrium photons.

It is obvious that in the absence of collisions the Lorentz-invariant magnitude of the total neutrino spin $[S^{(\nu)}(\mathbf{p},\mathbf{x},t)]^2$ is conserved. [This can be verified by multiplying the relativistic kinetic equation by the spin vector $S_i(\mathbf{q},\mathbf{x},t)$, cf. (61).]

We consider below applications of Eq. (64) associated with change of the helicity of a Dirac neutrino, $\nu_L\rightarrow\nu_R$, in an external magnetic field $\mathbf{B}_0$.

The Lorentz-invariant νe collision integrals on the right-hand sides of Eqs. (60), (61), and (63) were obtained in Ref. 96.

In the kinetic equation (60), the third term does not conserve parity, and in the kinetic equation (61) nor do the third and fourth terms, which arise from the allowance for the weak pseudovector current of the electrons in the matrix elements of ee and νe scattering. Below we shall omit these terms [and Eq. (61) entirely], being interested only in the case of unpolarized media.

Retaining in the kinetic equation (60) only the standard Vlasov part with self-consistent electromagnetic field $F_{\mu\nu}(\mathbf{x},t)$, and in the kinetic equation (63) the first two terms, we obtain, after eliminating the electron distribution function $\delta f^{(e)}(\mathbf{p},\mathbf{x},t)$ from (63), a relativistic kinetic equation for the neutrinos in the rest frame of the medium as a whole in the form[96] [here the right-hand side $J_{\text{coll}}^{(\nu e)}/\varepsilon_q\equiv(\partial f/\partial t)_{st}^{(\nu e)}$]

$$\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial t} + \mathbf{n}\frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{x}} + e\int \frac{d^4Q e^{-iQx}}{(2\pi)^4} [F_l(\omega,k)\mathbf{E}_\parallel(\omega,\mathbf{k}) + F_{\text{tr}}(\omega,k) \times(\mathbf{E}_\perp(\omega,\mathbf{k}) + [\mathbf{n}\mathbf{B}(\omega,\mathbf{k})])] \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{q}} = \left(\frac{\partial f}{\partial t}\right)_{st}^{(\nu e)}, \qquad (65)$$

where $F_l(\omega,k)$ and $F_{\text{tr}}(\omega,k)$ are the neutrino electromagnetic form factors defined above in (21) and (22); $Q_\mu=(\omega,\mathbf{k})$; $\mathbf{E}=\mathbf{E}_\parallel+\mathbf{E}_\perp$; $\mathbf{B}$ are the electromagnetic fields in the dispersive medium, and $E_\parallel=\mathbf{k}(\mathbf{kE})/k^2$.

It is obvious that the third term on the left-hand side of Eq. (65) is proportional to the force of electromagnetic origin. Whereas for a point charge e (when the form factors are equal to unity: $F_l=F_{\text{tr}}=1$) this term is determined by the Lorentz force, i.e., is equal to the standard expression $e(\mathbf{E}(\mathbf{x},t)=[\mathbf{n}\mathbf{B}(\mathbf{x},t)])\partial f(\mathbf{q},\mathbf{x},t)/\partial\mathbf{q}$, for neutrinos with electromagnetic structure (20), with allowance for the constant of the weak coupling to the electric charge, $G_F\sim e^2/M_W^2$, the third term in (65) is proportional to the radiation damping force ($\sim e^3$).

The polarization origin of such a force becomes obvious after simple manipulations in (65) using the explicit expressions (21) and (22) for the form factors F_l and F_{tr} in an isotropic dispersive medium, for which the Fourier integrals can be completely calculated, and the considered term[96]

$$\frac{\sqrt{2}G_F c_V}{e}\left(\frac{\partial^2}{\partial x_j \partial x_n} + n_n \frac{\partial^2}{\partial t \partial x_j}\right) 4\pi P_n(\mathbf{x},t) \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial q_j} \qquad (66)$$

is proportional to the second derivative of the polarization vector of the dispersive medium:

$$4\pi P_n(\mathbf{x},t) = D_n(\mathbf{x},t) - E_n(\mathbf{x},t),$$

which is equal to the difference between the vectors of the electric displacement, $D_n(\mathbf{x},t)\int d^4x'\varepsilon_{nj}(\mathbf{x}-\mathbf{x}',t-t')E_j(\mathbf{x}',t)$, and the electric field intensity $E_n(\mathbf{x},t)$.

Note that the expression (66) for the force is also valid for anistropic media when the permittivity tensor depends, for example, on an external magnetic field. In vacuum ($\mathbf{D}=\mathbf{E}$), the effect corresponding to plasmon emission by a moving neutrino disappears, i.e., there is no damping force in (66).

Finally, in special cases of the excitation in a dispersive medium of electrostatic waves ($\omega\ll k\langle v\rangle$) or the propagation of a high-frequency transverse wave ($\omega \ll k\langle v \rangle$), the term (66) can be represented in the form of the effective Lorentz force

$$e_\nu^{\text{ind}} \mathbf{E}_\parallel \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{q}}$$

or

$$\tilde{e}_\nu^{\text{ind}} (\mathbf{E}_\perp + [\mathbf{nB}]) \frac{\partial f^{(\nu)}(\mathbf{q},\mathbf{x},t)}{\partial \mathbf{q}},$$

which is proportional to the neutrino induced electric charges (27) and (28), respectively.

4.2. Collision integrals with allowance for dynamical polarization of the medium

The analogy with plasma electrodynamics becomes complete if in the collision integral $(\partial f/\partial t)_{st}^{(\nu e)}$ on the right-hand side of the relativistic kinetic equation (65) we retain in the total cross section only the contribution of electromagnetic scattering of neutrinos by electrons (described by the diagram that is crossing symmetric with respect to the annihilation diagram in Fig. 5c).

Indeed it is necessary to retain in the cross section all the terms, including the Born contributions, i.e., it is necessary to sum the matrix elements of the diagrams of the standard model with the amplitude of an electromagnetic scattering. The importance of the last is obvious from the case of scattering by nuclei discussed above (Sec. 3.2). Therefore the approximate result (67) obtained here is methodological and derived only to elucidate the connection with the electrodynamics of dispersive media. The collision integral of Refs. 96 and 97 based solely on the Born diagrams of the same standard model would be just as incomplete.

Proceeding now from the covariant integral of elastic collisions of particles with arbitrary interaction law obtained earlier by Chernikov,[100] we can write down in the WKB approximation of small momentum transfers ($k \ll p$) the standard Fokker–Planck representation of the collision integral:

$$\left(\frac{\partial f}{\partial t}\right)_{st}^{\nu e} = \frac{\partial}{\partial q_i} \int d^3p I_{ij}(\mathbf{p},\mathbf{q}) \left\{ \frac{\partial f^\nu(\mathbf{q})}{\partial q_j} f^e(\mathbf{p}) \times \left[1 - \frac{(2\pi)^3}{2} f^e(\mathbf{p})\right] - f^\nu(\mathbf{q}) \times [1-(2\pi)^3 f^\nu(\mathbf{q})] \frac{\partial f^e(\mathbf{p})}{\partial p_j} \right\}, \quad (67)$$

where $f^e(\mathbf{p}) \equiv f^e(\mathbf{p},\mathbf{x},t)$, etc.

The kernel $I_{ij}(\mathbf{p},\mathbf{q})$, determined by the amplitude of electromagnetic νe scattering,[48] has in the classical approximation the form

$$I_{ij}(\mathbf{p},\mathbf{q}) = (4\pi\alpha)^2 \int \frac{d^3k}{(2\pi^3)} \frac{k_i k_j}{k^4} \pi\delta(\mathbf{kn} - \mathbf{kv}) \times \left| \frac{F_l(\mathbf{kn},k)}{\varepsilon_l(\mathbf{kn},k)} + \frac{[k^2\mathbf{vn} - (\mathbf{kn})^2] F_{\text{tr}}(\mathbf{kn},k)}{(\mathbf{kn})^2 \varepsilon_{\text{tr}}(\mathbf{kn},k) - k^2} \right|^2. \quad (68)$$

Here, $F_l(\omega,k)$ and $F_{\text{tr}}(\omega,k)$ are the same form factors (21) and (22) that determine the force term on the left-hand side of the relativistic kinetic equation (65). Note that in the case of a point electron, for which the electromagnetic form factors are equal to unity ($F_l = F_{\text{tr}} = 1$), the expression (68) would be identical to the classical kernel of a relativistic collision integral of Lenard–Balescu type that Silin obtained.

In the case of neutrinos, for which the form factors F_l and F_{tr} are proportional to the density n_0 of the medium, the transition $n_0 \to 0$ to the vacuum limit differs from the passage to the limit in the relativistic Coulomb collision integral.[101]

Substitution of the vacuum values of the permittivities, $F_l = F_{\text{tr}} = 1$ in the integral obtained by Silin[101] leads to the Belyaev–Budker result,[102] whereas in the νe collision integral (67) the kernel (68) vanishes (for $n_0 = 0$). In this case, the remaining νe integral is determined by the matrix elements of the Born diagrams (cf. Figs. 5a and 5b) and has already been obtained in Ref. 96.

Thus, in the complete neutrino collision integral, including (68), allowance is made for the dynamic polarization of the medium, i.e., the influence on the collision event of the self-consistent electromagnetic field, or, ultimately, the influence on the frequency of νe collisions of the polarization perturbations of the electron density in the neighborhood of the neutrino "trajectories."

Unfortunately, the calculation of the total collision integrals for neutrinos in a dispersive medium is still a problem that has not been completely solved. In general form, such a collision integral is presented in Ref. 103.

The importance of including in the neutrino relativistic kinetic equation the contribution of the self-consistent field (linear in the coupling constant G_F) was also noted by Rudzsky,[104] who considered a relativistic kinetic equation for two flavors of neutrino (with mixing) propagating in a medium of electrons, nucleons, and nuclei.

Apart from the already considered polarization terms on the left-hand side of the relativistic equation (65), which are proportional to the interaction of the inhomogeneity of the particles of the medium with the neutrino flux described by the flavor-diagonal distribution function (59), Rudzsky[104] obtained additional terms linear in the interaction and, in contrast to (65), not containing derivatives but already in the relativistic kinetic equation for auxiliary (mixed with respect to the flavor) components of the distribution function $f^{(\nu_e\nu_\mu)}(\mathbf{p},\mathbf{x},t)$. It is these terms, determined by the difference between the (forward) scattering amplitudes of muon and electron neutrinos, that lead to fluctuations in the densities of the $\nu_{e,\mu}$ neutrinos in the MSW effect.[104]

The system of relativistic kinetic equations obtained in Ref. 104, including the collision integrals, completely describes the neutrino oscillations in a medium for arbitrary inhomogeneity of the medium and goes beyond the approximation of adiabaticity and coherence of the neutrino scattering by the particles of the medium (constancy of the energy).

4.3. Kinetics of neutrino spin in a medium with variable external magnetic field

If the time required for establishment of thermodynamic equilibrium for the neutrino component in a medium or the characteristic time of variation of the energy spectrum of a nonequilibrium neutrino flux is much greater than the neutrino spin precession period in the external electromagnetic field, then it is possible to use a factorization of the spin distribution function, $\mathbf{S}^{(\nu)}(\mathbf{p},t) = f^{(\nu)}(\mathbf{p})\mathbf{S}(t)$, corresponding to allowance for rapid rotation of the spin $\mathbf{S}(t)$ in a uniform flux with neutrino density $n^{(\nu)}=\int d^3 f^{(\nu)}(\mathbf{p})$.

As a result, the diagonal components of the total Wigner distribution function (59), $f_{--}=f_L$ and $f_{++}=f_R$, corresponding to left- and right-handed neutrinos, can be written in the form

$$f_L(\mathbf{p},t)=f^{(\nu)}(\mathbf{p})\frac{(1-S_z(t))}{2},$$

$$f_R(\mathbf{p},t)=f^{(\nu)}(\mathbf{p})\frac{(1+S_z(t))}{2},$$

where the factor $F_R=(1+S_z(t))/2$ is equal to the probability of a change of the neutrino helicity:

$$P_{\nu_L\leftrightarrow\nu_R}(t)=\frac{1+S_z(t)}{2}. \quad (69)$$

In the collisionless approximation for neutrinos propagating along the z axis, interaction with time-dependent electromagnetic fields can be described by means of combinations of the transverse field components

$$\widetilde{H}_\perp e^{\pm i\alpha(t)}=\mu_\nu(B_x(t)+E_y(t)\pm i(B_y(t)-E_x(t)),$$

$$\alpha(t)=\tan^{-1}\left(\frac{B_y(t)-E_x(t)}{B_x(t)+E_y(t)}\right),$$

$$\widetilde{H}_\perp(t)=\mu_\nu(B_x^2(t)+B_y^2(t)+E_x^2(t)+E_y^2(t)+2[E_y(t)B_x(t)-B_y(t)E_x(t)])^{1/2},$$

which occur in the relativistic kinetic equation (64) with vanishing right-hand side. It was shown in Ref. 105 that Eq. (64) reduces to a differential equation of third order:

$$(\dot{\alpha}-V)\widetilde{H}_\perp^2\frac{d^3S_z(t)}{dt^3}-[\ddot{\alpha}\widetilde{H}_\perp^2+2\dot{\widetilde{H}}_\perp\widetilde{H}_\perp^2(\dot{\alpha}-V)]\frac{d^2S_z(t)}{dt^2}+[(\dot{\alpha}-V)^3\widetilde{H}_\perp^2+4\widetilde{H}_\perp^4(\dot{\alpha}-V)+\ddot{\alpha}\dot{\widetilde{H}}_\perp\widetilde{H}_\perp-(\ddot{\widetilde{H}}_\perp\widetilde{H}_\perp-2(\dot{\widetilde{H}}_\perp)^2)(\dot{\alpha}-V)]\frac{dS_z(t)}{dt}+[4\widetilde{H}_\perp^3\dot{\widetilde{H}}_\perp(\dot{\alpha}-V)-4\ddot{\alpha}\widetilde{H}_\perp^4 S_z(t)=0, \quad (70)$$

where the potential of the interaction of the neutrinos with the matter V is determined by the expression (16), and the initial conditions correspond to the absence of right-handed neutrinos:[105]

$$S_z(0)=-1,\quad \dot{S}_z(0)=0,\quad \ddot{S}_z(0)=4\widetilde{H}_\perp^2(0). \quad (71)$$

In the special case of a magnetic field of constant amplitude ($\widetilde{H}_\perp=\text{const}$) that rotates with constant velocity $\dot{\alpha}=\text{const}$ in the plane perpendicular to the neutrino direction of propagation, the relativistic kinetic equation (70) reduces to the simple equation

$$\frac{d^3S_z(t)}{dt^3}+\omega_0^2\frac{dS_z(t)}{dt}=0, \quad (72)$$

in which the angular velocity of rotation of the neutrino spin is

$$\omega_0=\sqrt{(\dot{\alpha}-V)^2+4\widetilde{H}_\perp^2}. \quad (73)$$

Solving Eq. (72) with the initial conditions (71), we obtain from the definition (69) the well-known expression for the probability of spin flip of Dirac neutrinos in a medium with rotating magnetic field:[34]

$$P_{\nu_L\leftrightarrow\nu_R}=\frac{4\widetilde{H}_\perp^2}{\omega_0^2}\sin^2(\omega_0 t/2). \quad (74)$$

The resonance condition $\dot{\alpha}=V$ can be readily satisfied, for example, in the interior of the sun,[34] but such resonance can hardly be related to the anticorrelation of the neutrino flux with the solar activity in view of the smallness of the magnetic moment μ_ν. The kinetic equation (70), obtained from (64), contains all currently known solutions for the motion of neutrino spin in an external magnetic field, including the cases of a static homogeneous field[6] [i.e., $\dot{\alpha}=0$ in (73) and (74)] or the linearized field of an Alfvén wave in a plasma.[99]

In connection with applications for the cosmological bound on the neutrino magnetic moment from primordial nucleosynthesis[27] in the presence of giant random magnetic fields in the early universe,[29] it is interesting to consider the motion of the neutrino spin in random electromagnetic fields. Such an investigation was made in the recent study of Ref. 105, in which the method of statistical averaging (70) was used to obtain the same simple equation (72), but with different spin precession frequency:

$$\omega_0=\sqrt{V^2+8\langle\widetilde{H}_\perp^2\rangle+\tfrac{6}{5}L^{-2}}, \quad (75)$$

which depends on the mean square field

$$\langle \widetilde{H}_\perp^2 \rangle = \tfrac{2}{3}\mu_\nu^2 \langle \mathbf{B}^2 \rangle_{\mathbf{x}=0}$$

in an isotropic medium and on the scale L of this random field, defined by the ratio

$$L^{-2} = \frac{\int k^2 \langle \mathbf{B}^2 \rangle_k d^3k/(2\pi)^3}{\langle \mathbf{B}^2 \rangle_{\mathbf{x}=0}}.$$

Here $\langle \mathbf{B}^2 \rangle_{\mathbf{x}=0}/8\pi = \int \langle \mathbf{B}^2 \rangle_k d^3k/(8\pi(2\pi)^3)$ is the mean energy density of the random magnetic field, determined by the correlation $\langle B(\mathbf{x}_1) B(\mathbf{x}_2) \rangle$ of the fields at coincident space points $\mathbf{x}_1 = \mathbf{x}_2$ ($\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2 = 0$), this being a consequence of the locality of the interaction of the neutrino spin with the magnetic field in the original relativistic kinetic equation (64).

The strength of the random field on the scale L, $B(T) = \sqrt{\langle \mathbf{B}^2 \rangle}$, is proportional to the critical field $B_c = M_W^2/e \sim 10^{24}$ G for the W bosons that is generated by the inhomogeneities of the Higgs condensate near the temperature $T_{EW} \sim M_W$ of the electroweak phase transition[29] and is equal to

$$B(T) = \frac{c 10^{24} G}{\sqrt{N}} \left(\frac{T}{T_{EW}} \right)^2. \tag{76}$$

Here $N = (L/L_0)^3$ is the number of domains for which the unit cell (domain) has side L_0, and $c \geqslant 1$ is the phenomenological field enhancement parameter introduced by Olesen to explain the fossil origin of the seed field B_{seed} in the dynamo mechanism of enhancement of the galactic magnetic field. Assuming that the critical process for the subsequent nucleosynthesis is the left-handed–right-handed conversion $\nu_L \to \nu_R$ of neutrinos at the temperature $T_{\text{QCD}} \sim 200$ MeV, which determines the hadronization of quarks with subsequent decay of pions and muons into left-handed neutrinos, we can eliminate the unknown parameter c in Eq. (76), using the ratio

$$\frac{B_{200}}{B_{\text{seed}}} = \left(\frac{T_{200}}{T_{\text{now}}} \right)^{1/2} \left(\frac{L_{\text{now}}}{L(200)} \right)^{3/2}. \tag{77}$$

Here $T_{\text{now}} \approx 3 \cdot 10^{-10}$ MeV is the temperature of the microwave background, $L_{\text{now}} \sim 100$ kpc $\approx 3 \cdot 10^{23}$ cm is the mean intergalactic separation, and T_{200} and $L(200)$ are the temperature and scale of the random field at the time of the QCD phase transition.

Using the ratio (27), we can replace the bound

$$\mu_\nu \leqslant \frac{4.5 \cdot 10^2 \ \mu_B}{B_{200}/1G}$$

on the neutrino magnetic moment from nucleosynthesis in the early universe[77] by the inequality

$$\mu_\nu \leqslant \frac{10^{-30} \ \mu_B}{B_{\text{seed}}/1G}. \tag{78}$$

If in addition we use the bound $B_{\text{seed}} \geqslant 10^{-11}$G of the modern magnetic-dynamo theory,[28] then from (78) we deduce a record upper limit for the magnetic moment of the Dirac neutrino:

$$\mu_\nu \leqslant 10^{-19} \ \mu_B,$$

or a bound on the mass of the electron neutrino in the standard model:

$$m_\nu \leqslant 0.3 \text{ eV}.$$

We must emphasize the presence in the expression (75) used above of a certain minimum possible domain scale $L_{\min} \geqslant V^{-1}$; this makes it possible to avoid suppression of the neutrino spin precession in the random magnetic field:[105]

$$L_0 \geqslant L_{\min} = 10^{-2} l_H \left(\frac{\text{MeV}}{T} \right)^3, \tag{79}$$

where $l_H(T) \sim T^{-2}$ is the horizon scale. For smaller scales, when the precession length $l_{\text{osc}} = (V^2 + 8\langle \widetilde{H}_\perp^2 \rangle)^{-1/2}$ becomes less than the domain scale L, the neutrino spin cannot follow the change in the direction of the random field on transition to a new domain [in the simplest model, we assume that the field within a domain is uniform and that the field (76) has the same value for each domain with $L = L_0$].

Thus a topical problem is the construction of relativistic magnetohydrodynamics for random fields with macroscopic scale (79) that develop from microscopic giant random fields described by the standard model of the electroweak interactions.[29,106]

CONCLUSIONS

We draw some conclusions. The resonant enhancement of neutrino oscillations in a medium[1] predicted in 1985 is apparently confirmed in the on-going underground experiments with solar neutrinos (see the Introduction and Sec. 1). This has stimulated a huge number of studies on the problem of neutrino oscillations in the interior of the earth, in various astrophysical objects, and in the early universe. Basically, MSW theory is a completed direction in the physics of neutrinos in a medium. The continuing development in breadth and the numerous new studies are essentially of an applied nature with the aim of obtaining from the already known solutions of the wave equation of neutrinos in a medium new additional bounds on the vacuum mixing angle or on the difference of the squares of the neutrino masses.

In contrast to this direction, neutrino electrodynamics in a medium, the foundations of which were formulated above, is in a stage of development. The well-studied interaction of the neutrino vacuum magnetic moment with a regular external magnetic field is the basis of the corresponding single-particle wave equation (Sec. 1) or the relativistic kinetic equations that describe neutrino fluxes in a medium (Sec. 4). Note that the motion of neutrinos in homogeneous and weakly inhomogeneous external electromagnetic fields was also studied in Refs. 107–109 on the basis of a calculation of the neutrino single-loop mass operator in a given external field. What is still uninvestigated is the effect of neutrino collisions in dense matter on processes with helicity change in such external fields. Outside the scope of neutrino physics, but particularly important for determining a cosmological bound on the neutrino magnetic moment from primordial nucleosynthesis, would be the solution of the problem of the evolution of random magnetic fields generated in the electroweak phase transition at temperature $T_{\rm EW}\approx M_W$. This problem is also fundamentally important for the solution of the problem of the origin of the magnetic fields in galaxies.

A direction that appears promising is investigation of the interaction of neutrinos with nonequilibrium media when perturbations of the particle number density or the particle flux density of the medium increase. This can lead to enhancement of the interaction (current × current in the employed low-energy approximation of the weak interaction). In such a study, one can use the known results of the nonlinear theory of plasmas and other dispersive media, in particular one can use in the first stage the well-developed theory of weak turbulence.[110]

One can also expect an increase of the cross sections and the neutrino energy losses in anisotropic media. In particular, it would be interesting to calculate the cross sections of the various elementary processes that the present authors have already made for isotropic dispersive media (Sec. 3) for a magnetoactive (anisotropic) plasma with the electromagnetic vertex (23) (Sec. 2).

Calculations of neutrino transport in a dense plasma with strong regular (in a supernova) or stochastic (in the early universe) magnetic fields would permit in this case verification of the possibility of existence of collective mechanisms of neutrino interaction with matter and their influence on the collapse dynamics of a supernova precursor star and the cooling of the core remnant and also on the neutrino quenching temperature (of decoupling from matter) and primordial nucleosynthesis in the early universe.

Finally, it is in principle important to go beyond the low-energy approximation, $|q^2|/M_W^2\ll 1$, that we have used above, i.e., it is important to study the role of collective effects in a high-density dispersive medium $(n\leqslant M_W^3, T\leqslant M_W)$ up to the appearance in the background either of the effects of Bose condensation of W bosons[111] or the high-temperature restoration of symmetry.[112] In this case, it is already necessary to take into account the contribution of the self-interaction of the Yang–Mills fields to the dispersion characteristics of the medium, and this in itself is a more complicated problem than the single-loop calculation of the vacuum radiative corrections in the standard model and of the linear-response functions of the medium in the low-energy point-interaction approximation.

One of the authors (V. B. S.) is grateful to the Kosmion Science Education Center for partial support with initiative grants.

[1)]Spin–flavor neutrino precession is the established expression for simultaneous oscillations of the spin and flavor of the type $\nu_{e_L}\leftrightarrow\nu_{\mu_R}$.

[2)]We consider an unbounded medium irrespective of the mean free path of the test particle, which is determined by the form of the interaction and the matter density. Addition of the coherent elastic-scattering amplitudes with the same final momenta of the neutrinos and target particles (there is no momentum transfer, $\mathbf{k}=0$) is valid for any neutrino energy, since for such summation the important thing is not the number of particles in a volume corresponding to the neutrino de Broglie wavelength $\lambda=E^{-1}$ $(v\sim\lambda^3)$ but rather the number of particles that corresponds to volumes with the scales of the collision impact parameter $b\sim k^{-1}\sim L\to\infty$. Under these conditions and with allowance for the spectrum of massless neutrinos in an electron gas, $E=p+G_Fn_0\sqrt{2}$ (see below), the wave refractive index $n=p/E(p,n_0)$ contains the finite mean density $n_0=N/v$ in the volume $v=L^3\to\infty$ $(N\to\infty)$. With allowance for the normalization of the forward scattering amplitude $f(0)=-G_F\sqrt{2}E/2\pi$, we obtain by a series expansion the standard expression for the refractive index of the neutrino wave: $n=1+2\pi f(0)n_0/p^2$.

[3)]Here the probability of flavor change is determined by the equation

$$P(\nu_e\to\nu_\mu)=1-P(\nu_e\to\nu_e).$$

[4)]The oscillation probability (4) does not depend on the contribution of the neutral currents to the total phase of the wave function $\Psi=(\psi_{\nu_e}\psi_{\nu_\mu})$ for the two neutrino species.

[5)]In all that follows below, we use a system of units with $\hbar=c=1$, the Feynman metric $qx=q_\mu x^\mu=\omega t-\mathbf{kx}$, $\mu=0, 1, 2, 3$, and the standard representation of the Dirac γ matrices with $\gamma_5=\gamma_5^+=i\gamma^0\gamma^1\gamma^2\gamma^3$, so that the left-handed bispinor is defined as $\nu_L(p)=[(1-\gamma_5)/2]\nu(p)$.

[6)]A correction (be a factor 2) of the coefficient of the magnetic (axial) form factor was made by the authors of Ref. 16, and a correction to the calculation in Ref. 48 of the νe scattering cross section in a plasma was taken into account in Ref. 49. The appearance of the factor α^{-1} in the first term of (20) is due to the definition of the electromagnetic current $e\bar{\nu}(p')\Gamma_\mu\nu(p)$ and the desire to retain here the standard form of the QED polarization tensor $\Pi_{\mu\nu}$. In the second term of (20), the factor α^{-1} may include the form factor and with allowance for the definition of $A(\omega,k)$,[15] the magnetic form factor will not depend at all on the constant a (like the ratio $\Pi_{\mu\nu}/\alpha$ calculated in the same single-loop approximation).

[7)]The authors of Ref. 83 calculated in their previous study[52] the electromagnetic vertex Γ_μ, which, after renotation,[18] agrees with the result (20) that we have obtained earlier in Ref. 48.

[8)]Neutrinos with mass $m_\nu>2m_e$ decay much more rapidly than in the channel with probability (42).[84]

[1]S. P. Mikheev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; S. P. Mikheev and A. Yu. Smirnov, Usp. Fiz. Nauk **153**, 3 (1987) [Sov. Phys. Usp. **30**, 759 (1987)].

[2]I. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[3]S. M. Bilenky and B. M. Pontecorvo, Phys. Rep. **41C**, 225 (1978); S. M. Bilen'kiĭ and B. M. Pontecorvo, Usp. Fiz. Nauk **123**, 181 (1977) [Sov. Phys. Usp. **20**, 776 (1977)]; B. M. Pontecorvo, Usp. Fiz. Nauk **141**, 675 (1983) [Sov. Phys. Usp. **26**, 1087 (1983)]; S. M. Bilen'kiĭ, Fiz. Elem. Chastits At. Yadra **18**, 449 (1987) [Sov. J. Part. Nucl. **18**, 188 (1987)].

[4]G. A. Bazilevskaya, Yu. I. Stozhkov, and T. N. Charakhch'yan, JETP Lett. **35**, 341 (1982).

[5]R. Davis, *Talk Presented at the 13th Intern. Conf. on Neutrino Physics and Astrophysics "Neutrino 88"* (Boston, USA, 1988).

[6]M. V. Voloshin, M. I. Vysotskiĭ, and L. B. Okun', Yad. Fiz. **44**, 677 (1986) [Sov. J. Nucl. Phys. **44**, 440 (1986)]; M. V. Voloshin and M. I. Vysotskiĭ, Yad. Fiz. **44**, 847 (1986) [Sov. J. Nucl. Phys. **44**, 546 (1986)]; M. V. Voloshin, M. I. Vysotskiĭ, and L. B. Okun', Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)].

[7]E. Kh. Akhmedov, Phys. Lett. **213B**, 64 (1988); E. Kh. Akhmedov, *Contributed Paper Presented at the 13th Intern. Conf. on Neutrino Physics and Astrophysics, "Neutrino 88"* (Boston, USA, 1988); C. Lim and W. J. Marciano, Phys. Rev. D **37**, 1368 (1988).

[8]K. S. Hirata *et al.*, Phys. Rev. D **44**, 2241 (1991).

[9]P. Anselmann *et al.*, GALLEX Collab., Phys. Lett. **285B**, 390 (1992).

[10]A. Albrecht and A. Stebbins, Phys. Rev. Lett. **69**, 2615 (1992).

[11]K. S. Hirata *et al.*, Phys. Lett. **205B**, 416 (1988).

[12]J. T. Peltoniemi and J. W. F. Valle, Preprint IFIC/93-04 (1993) (sub. to Nucl. Phys. B).

[13]J. Enqvist, K. Kainulainen, and J. Maalampi, Phys. Lett. **244B**, 186 (1990).

[14]V. N. Oraevsky and V. B. Semikoz, in *Plasma Astrophysics. Proc. of the Joint Varenna Abastumani Intern. School and Workshop* (Sukhami, USSR, 1986), p. 321; Physica **A142**, 135 (1987).

[15]V. B. Semikoz, Yad. Fiz. **46**, 1592 (1987) [Sov. J. Nucl. Phys. **46**, 987 (1987)].

[16] V. B. Semikoz and Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. **95**, 35 (1989) [Sov. Phys. JETP **68**, 20 (1989)].
[17] V. B. Semikoz and Ya. A. Smorodinskiĭ, JETP Lett. **48**, 399 (1988).
[18] V. N. Oraevsky, A. I. Rez, and V. B. Semikoz, Preprint IZMIRAN, No. 5a(952) (1991).
[19] L. B. Okun', Yad. Fiz. **44**, 847 (1986) [Sov. J. Nucl. Phys. **44**, 546 (1986)].
[20] K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45**, 963 (1980).
[21] V. B. Semikoz, Phys. Lett. **284B**, 337 (1992).
[22] R. G. H. Robertson *et al.*, Phys. Rev. Lett. **67**, 957 (1991).
[23] D. A. Krakauer *et al.*, Phys. Rev. D **44**, R6 (1991).
[24] S. I. Blinnikov, Preprint ITEP, No. 19 (1988).
[25] G. Raffelt, Phys. Rev. Lett. **64**, 2856 (1990).
[26] A. I. Rez and V. B. Semikoz, Phys. Lett. **249B**, 125; A. I. Rez and V. B. Semikoz, in *Proc. of the Intern. School "Low Energy Weak Interactions" (LEWI 90)* (Dubna, 1990), p. 136.
[27] K. Enqvist, V. Semikoz, A. Shukurov, and D. Sokoloff, Preprint NORDITA-92/91, Phys. Rev. D (1993) (accepted).
[28] A. Poezd, A. Shukurov, and D. Sokoloff, Mon. Not. R. Astron. Soc. (1993) (sub.).
[29] T. Vachaspati, Phys. Lett. **265B**, 258 (1991); P. Olesen, Phys. Lett. **281B**, 300 (1992).
[30] K. Enqvist, P. Olesen, and V. Semikoz, Phys. Rev. Lett. **69**, 2157 (1992).
[31] D. Nötzold, Phys. Rev. **380**, 1658 (1988).
[32] M. B. Voloshin, Preprint ITEP, No. 45 (1988).
[33] S. I. Blinnikov and L. B. Okun, Preprint ITEP, No. 123 (1988).
[34] A. Yu. Smirnov, JETP Lett. **53**, 291 (1991).
[35] J. Vidal and J. Wudka, Phys. Lett. **249B**, 473 (1990); C. Aneziris and J. Schechter, Int. J. Mod. Phys. **A6**, 2375 (1991).
[36] E. Kh. Akhmedov and E. Kh. Bychuk, Zh. Eksp. Teor. Fiz. **95**, 442 (1989) [Sov. Phys. JETP **68**, 250 (1989)].
[37] M. Fukugita and S. Yazaki, Phys. Rev. **360**, 3817 (1987).
[38] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988).
[39] L. B. Okun', *Leptons and Quarks*, transl. of 1st Russ. ed. (North-Holland, Amsterdam, 1982) [Russ. original, 2nd ed., Nauka, Moscow, 1990].
[40] G. Raffelt, Phys. Rep. **198**, 1 (1990).
[41] L. B. Leinson and V. N. Oraevskiĭ, JETP Lett. **48**, 60 (1988)].
[42] A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin, 1984) [Russ. original, Vysshaya Shkola, Moscow, 1988].
[43] J. B. Adams, M. A. Ruderman, and C. H. Woo, Phys. Rev. **129**, 1383 (1963).
[44] E. S. Fradkin, Zh. Eksp. Teor. Fiz. **36**, 1286 (1959) [Sov. Phys. JETP **9**, 912 (1959)]; V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **40**, 1775 (1961) [Sov. Phys. JETP **13**, 1249 (1961)].
[45] M. H. Zaidi, Nuovo Cimento **A40**, 502 (1965).
[46] S. A. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, *Stockholm*, edited by Swartholm, Almquest, and Weacsell (1968), p. 367.
[47] D. A. Dicus, Phys. Rev. D **6**, 941 (1972).
[48] V. N. Oraevskiĭ, A. Yu. Plakhov, V. B. Semikoz, and Ya. A. Smorodinskiĭ. Zh. Eksp. Teor. Fiz. **93**, 1557 (1987) [Sov. Phys. JETP **66**, 890 (1987)].
[49] V. N. Oraevskiĭ, V. B. Semikoz, and Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. **95**, 2288 (1989) [Sov. Phys. JETP **68**, 1309 (1989)]
[50] E. S. Fradkin, Tr. Fiz. Inst. Akad. Nauk SSSR **29**, 7 (1965).
[51] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).
[52] J. C. D'Olivo, J. F. Nieves, and P. B. Pal, Phys. Rev. D **40**, 3679 (1989).
[53] V. N. Oraevsky and V. B. Semikoz, Phys. Lett. **263B**, 455 (1991).
[54] A. I. Akhiezer *et al.*, Plasma Electrodynamics [in Russian] (Nauka, Moscow, 1974).
[55] C. Patti and A. Salam, Phys. Rev. D **10**, 275 (1974).
[56] M. A. Beg, R. V. Budny, R. Mohapatra, and A. Sirlin, Phys. Rev. Lett. **38**, 1250 (1977).
[57] M. Fukugita and T. Yanagita, Phys. Rev. Lett. **58**, 1807 (1987).
[58] V. B. Semikoz, JETP Lett. **49**, 254 (1989) [JETP Lett. **49**, 288 (1989)].
[59] M. Leurer and N. Marcus, Phys. Rev. D **43**, 2259 (1991).
[60] K. S. Babu and R. N. Mohapatra, Phys. Rev. D **42**, 3778 (1990).
[61] V. Semikoz, Preprint NORDITA-92/44 (1992).
[62] A. I. Rez and V. B. Semikoz, JETP Lett. **51**, 492 (1990).
[63] V. N. Oraevsky, V. B. Semikoz, and Ya. A. Smorodinsky, Phys. Lett. **227B**, 255 (1989).
[64] P. Langacker and S. Sankar Uma, Phys. Rev. D **40**, 1569 (1989).
[65] V. B. Semikov, in *Proc. of Fifth All-Union School: Particles and Cosmology* [in Russian] (VNO-IYaI (Institute of Nuclear Research), 1989).
[66] V. N. Oraevsky and V. N. Ursov, Phys. Lett. **209**, 83 (1988).
[67] V. N. Oraevskiĭ and V. B. Semikoz, Zh. Eksp. Teor. Fiz. **86**, 796 (1984) [Sov. Phys. JETP **59**, 465 (1984)].
[68] V. N. Oraevsky and V. B. Semikoz, Phys. Lett. **139B**, 90 (1984); in *Proc. of the Second International Symposium: Problems of Nonlinear and Turbulent Processes in Physics* [in Russian] (Kiev, 1983), p. 369.
[69] V. N. Oraevskiĭ and V. B. Semikoz, Yad. Fiz. **42**, 702 (1985) [Sov. J. Nucl. Phys. **42**, 446 (1985)].
[70] V. N. Oraevskiĭ, V. B. Semikoz, and Ya. A. Smorodinskiĭ, JETP Lett **43**, 709 (1986).
[71] T. Altherr and K. Kainulainen, Phys. Lett. **262B**, 79 (1991).
[72] B. Leinsen, V. N. Oraevsky, and V. B. Semikoz, Phys. Lett. **209B**, 80 (1988).
[73] B. Banarjee and S. M. Chitre, Preprint TIER/TH/91-36 (1991).
[74] R. N. Dalitz, Proc. R. Soc. A: **206**, 59 (1951).
[75] L. B. Leinson, JETP Lett. **51**, 269 (1990).
[76] C. J. Horowitz and K. Wehrberger, Phys. Rev. Lett. **66**, 272 (1991); Phys. Lett. **266B**, 2366 (1991).
[77] T. M. Aliev and M. I. Vysotskiĭ, Usp. Fiz. Nauk **135**, 709 (1981) [Sov. J. Nucl. Phys. **24**, 1008 (1981)].
[78] T. Matsumoto *et al.*, Astrophys. J. **329**, 567 (1988).
[79] Z. G. Berezhiani *et al.*, Yad. Fiz. **51**, 1614 (1990) [Sov. J. Nucl. Phys. **51**, 1020 (1990)].
[80] T. M. Aliev and M. I. Vysotsky, Preprint ITEP-37, Moscow (1981).
[81] B. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982).
[82] A. De Rujula and S. L. Glashow, Phys. Rev. Lett. **45**, 942 (1980).
[83] J. C. D'Olivo, J. F. Nieves, and P. B. Pal, Phys. Rev. Lett. **64**, 1088 (1990).
[84] G. Börner, *The Early Universe* (Springer-Verlag, Berlin, 1988), p. 245.
[85] V. V. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics*, 2nd ed. (Pergamon Press, Oxford, 1982) [Russ. original, Nauka, Moscow, 1980].
[86] D. A. Kirzhnits, V. V. Losyakov, and V. A. Chechin, JETP Lett. **50**, 9 (1989).
[87] E. Fermi, *Scientific Works*, Vol. 2 [Russian translation] (Nauka, Moscow, 1972).
[88] D. A. Kirzhnits, V. V. Losyakov, and V. A. Chechin, Zh. Eksp. Teor. Fiz. **97**, 1089 (1990) [Sov. Phys. JETP **70**, 609 (1990)].
[89] K. Enqvist, K. Kainulainen, and V. Semikoz, Nucl. Phys. **B374**, 392 (1992).
[90] H.-Th. Janka, Astron. Astrophys. **244**, 378 (1991).
[91] V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **10**, 589 (1940).
[92] I. M. Frank, Izv. Akad. Nauk SSSR, Ser. Fiz. **6**, 3 (1942).
[93] M. J. Dunkan, J. A. Grifols, A. Mendeza, and S. Sankar Uma, Phys. Lett. **191B**, 304 (1987).
[94] S. R. de Groot, W. A. van Leeuwen, and C. G. van Weert, *Relativistic Kinetic Theory: Principles and Applications* (North-Holland, Amsterdam, 1980) [Russ. transl., Mir, Moscow, 1983].
[95] F. Jüttner, Z. Phys. **47**, 5452 (1928).
[96] V. B. Semikoz, Physica **A142**, 157 (1987).
[97] S. W. Bruenn, Astrophys. J. Suppl. Ser. **58**, 771 (1985).
[98] L. N. Labzovskiĭ, Zh. Eksp. Teor. Fiz. **89**, 1921 (1985) [Sov. Phys. JETP **62**, 1108 (1985)].
[99] V. Semikoz, Preprint NORDITA-92/60 (1992); Phys. Rev. D **48** (accepted).
[100] N. A. Chernikov, Dokl. Akad. Nauk SSSR **114**, 530 (1957) [Sov. Phys. Dokl. **2**, 248 (1957)]; Doctoral Dissertation [in Russian], JINR, Dubna (1963).
[101] V. P. Silin, Zh. Eksp. Teor. Fiz. **40**, 1768 (1961) [Sov. Phys. JETP **13**, 1244 (1961)].
[102] S. T. Belyaev and G. N. Budker, Dokl. Akad. Nauk SSSR **107**, 807 (1956) [Sov. Phys. Dokl. **1**, 218 (1956)].
[103] V. N. Oraevsky and V. N. Ursov, Phys. Lett. **221B**, 357 (1989).
[104] M. A. Rudzsky, Astrophys. Space Sci. **165**, 65 (1989).
[105] K. Enqvist and V. Semikoz, Phys. Lett. **312B**, 310 (1993).
[106] J. Ambjörn and P. Olesen, Nucl. Phys. **B315**, 606 (1989).
[107] A. V. Borisov, V. Ch. Zhukovskiĭ, A. V. Kurilin, and I. M. Ternov, Yad. Fiz. **41**, 743 (1985) [Sov. J. Nucl. Phys. **41**, 473 (1985)].

[108] A. V. Borisov, V. Ch. Zhukovskiĭ, and A. I. Ternov, Yad. Fiz. **46**, 564 (1987) [Sov. J. Nucl. Phys. **46**, 310 (1987)].
[109] A. V. Borisov, V. Ch. Zhukovskiĭ, and A. I. Ternov, Dokl. Akad. Nauk SSSR **308**, 841 (1989) [Sov. Phys. Dokl. **34**, 884 (1989)].
[110] A. A. Galeev and R. Z. Sagdeev, *Fundamentals of Plasma Physics* [in Russian] (Energoatomizdat, Moscow, 1983), p. 590.
[111] A. D. Linde, Phys. Lett. **86B**, 39 (1979).
[112] D. A. Kirzhnits and A. D. Linde, Phys. Lett. **42B**, 471 (1972).

Translated by Julian B. Barbour