Quantum effects in extremely strong magnetic fields

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Predictions of behavior of electrons and positrons in extremely strong magnetic fields are analyzed. The solution of the problem is based on exact solutions of the Dirac equation for an electron in a homogeneous magnetic field (Furry representation). It is possible to consider the case of an extremely strong field $H \gg H_c = m^2c^3/e\hbar$; such fields are characteristic of pulsars and have also been recently achieved under laboratory conditions in collisions of heavy ions. Problems considered include that of the vacuum in an ultrastrong field, the properties of synchrotron radiation under the conditions of pulsar magnetospheres, single-photon channels of the mutual transformation of electron–positron pairs, the behavior of the simplest systems (the hydrogen atom) in ultrastrong fields, and the problem of the anomalous magnetic moment of the electron. Aspects of neutron β decay in a strong magnetic field are discussed.

It is difficult to estimate the magnitude of Bogolyubov's contribution to the development of modern quantum theory: the hypothesis of colored quarks,[1] the vacuum problem and the phenomenon of spontaneous symmetry breaking,[2] the development of the renormalization-group method,[3] and the stimulus to the development of other directions in modern high-energy physics.

In particular, Bogolyubov was very interested in the problems of the quantum theory of the motion and radiation of particles in an electromagnetic field. Investigations into the quantum theory of synchrotron radiation and the anomalous magnetic moment of the electron always enjoyed his invariable support. We dedicate this paper to the memory of N. N. Bogolyubov.

The problem of the theory of quantum phenomena observed when particles move in an extremely strong electromagnetic field arose long ago, already in early studies of Heisenberg[4] devoted to analysis of the influence of an external field on the motion of particles made in the spirit of an analysis of the possibility of developing a single-particle description. In recent years, this problem has ceased to be purely academic, and interest in it has grown in connection with new experimental possibilities—strong electromagnetic fields have become accessible for observation not only in astrophysics but also under laboratory conditions.

1. EXTREMELY STRONG ELECTROMAGNETIC FIELDS

We consider briefly the concept of an extremely strong—critical—electromagnetic field. As is well known, the uncertainty relation $\delta t \delta E \gg \hbar$ allows transient violations of the energy conservation law, and electron–positron pairs can arise from the vacuum and exist for a time of order $\tau = \hbar/\delta E \simeq \hbar/(mc^2)$. During this time, the components of the pair can move apart to distances not greater than $\delta r = c\tau = \hbar/(mc)$, i.e., distances of the order of the Compton wavelength $\lambda = \hbar/(mc) \sim 10^{-10}$ cm. This is the so-called quantum radius of the electron, and it characterizes the region of possible spatial localization of an electron in quantum theory. Now if an external electric field can do work $\sim mc^2$ on an electron over the distance δr, then the creation of a pair from the vacuum becomes a real process. For this to happen, the field must be of the order of the critical field defined by

$$e_0 \mathscr{E}_c \hbar/(mc) = mc^2, \quad \mathscr{E}_c = m^2c^3/(e_0\hbar).$$

Under these conditions, the vacuum becomes unstable, and $\mathscr{E}_c$ is the critical value of the field at which a single-particle formulation of the problem becomes impossible (see Ref. 5).

Besides a critical electric field, there also exists a critical magnetic field if the electron gyration energy $\hbar\Omega$ [$\Omega = e_0H/(mc)$ is the cyclotron frequency] is equal to the electron rest energy mc^2:

$$\hbar\Omega = \hbar e_0 H_c/(mc) = mc^2,$$

$$H_c = m^2c^3/(e_0\hbar) = 4.414 \cdot 10^{13} \text{ Oe}.$$

However, because of gyromagnetic properties the magnetic field does not do work (the Lorentz force is perpendicular to the particle trajectory). For this reason, the vacuum remains stable even when a critical field acts on it. This is particularly interesting for the investigation of processes in such an extremal field—there is a domain of ultraquantum physics in which the observed phenomena can be very exotic.

The motion of particles in an electromagnetic field, and also the quantum processes that accompany such motion (radiation, pair production, annihilation, etc.) are determined not only by the type of field configuration but also by dynamical parameters (see Ref. 6). Since the probabilities of such processes must satisfy the requirement of relativistic and gauge invariance, one must consider the well-known field invariants

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$$f_1=\frac{1}{2H_c^2}F_{\mu\nu}F^{\mu\nu}=\frac{\vec{H}^2-\vec{E}^2}{H_c^2},$$
$$f_2=\frac{1}{8H_c^2}e^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho}=\frac{\vec{E}\vec{H}}{H_c^2}, \qquad (1)$$

and also the quantum dynamical parameter

$$\chi^2=-\frac{1}{(mcH_c)^2}(F_{\mu\nu}p^\nu)^2$$
$$=\frac{1}{(mcH_c)^2}\{([\vec{p}\vec{H}]+p_0\vec{E})^2-(\vec{p}\vec{E})^2\}, \qquad (2)$$

where $F_{\mu\nu}$ is the tensor of the electromagnetic field, and p^ν is the 4-momentum of the particle. The quantum processes in an external field depend strongly on the value of this dynamical parameter. If $\chi \ll 1$ and the problem admits expansion with respect to this parameter, one can develop a general method for determining the quantum corrections to the classical expressions for the probability of the process. If, however, $\chi \to 1$ or $\chi > 1$, then we come into the domain of ultraquantum phenomena, in which the external field must be taken into account exactly outside the framework of perturbation theory.

The problems of developing quantum electrodynamics in strong electromagnetic fields have been at the center of attention in recent years.[6–10] In the process of development of this direction, it has become necessary to advance the frontiers of investigation of physical phenomena into the domain of field strengths that in the recent past were not merely inaccessible but even appeared extravagant.

Considering examples of the existence of ultrastrong fields, we must first mention pulsars, which are unique laboratories for investigating the behavior of matter under conditions of ultrahigh matter density and ultrastrong magnetic fields. During the summer of 1967, during tests of the new 76-m Cambridge radio telescope, repeated radio signals with a period of the order of a second and a duration of a few hundredths of a second were detected. This event was initially remarkable, and there was even a suggestion of modulated signals sent by other civilizations. Thus, pulsars were discovered; in particular, a pulsar was discovered at the center of the Crab Nebula (PSR 0531 +21), which also radiates in the optical and x-ray ranges. At the present time, more than 400 pulsars are known.[1)]

The most probable model of a pulsar is a neutron star that is an inclined rotator. After a significant fraction of hydrogen has been burnt in the interior of a star and transformed into helium by thermonuclear fusion, the gas pressure is no longer capable of holding up the mass of the higher layers of the star. Catastrophic contraction commences, and the star itself contracts to a radius of the order of 10 km. A neutron star appears, and its birth is accompanied by a strong growth in the magnetic field strength. Indeed, if one imagines the sun ($R=0.7\cdot 10^{11}$ cm, $H\sim 1.5$ Oe) compressed to a diameter of 10 km, then under the condition of constancy of the magnetic flux HS, $S=\pi R^2$, the magnetic field of the neutron star will reach the value $2\cdot 10^{11}$ Oe.

Observations of hard x rays from the pulsar Hercules X−1 indicate that the magnetic field at the pulsar surface is $H\sim(5\text{–}6)\cdot 10^{12}$ Oe. This field, which is "frozen" into the star, must become stronger as the center of the star is approached, and, according to the estimates of Ref. 11, the field induction in the central part of the neutron star reaches the stupendous value $\sim 10^{17}$ Oe.

The discovery of neutron stars was an outstanding event in the astrophysics of the sixties and seventies and laid the foundations for the development of the physics of the interaction of matter with the electromagnetic field under critical conditions.

It may appear somewhat unexpected that magnetic fields like the extremal fields of pulsars can be realized under laboratory conditions. Such fields can be observed in noncentral collisions of heavy ions. Suppose that two nuclei with charges z_1 and z_2 move toward each other. Then, choosing the optimum impact parameter, one can briefly obtain a field

$$H=\frac{JR^2}{2c(r^2+R^2)^{3/2}}, \quad J=\frac{(z_1+z_2)ev}{2\pi R}.$$

In these expressions, R is the radius of the "current loop" (Biot–Savart–Laplace law), r is the distance from the symmetry axis to the point of observation, J is the current of the ions, z_1 and z_2 are their charges in units of the electron charge, and v is the relative velocity. Setting $z_1+z_2=174$, $v=0.1c$, and $R=15$ fm (1 fm$=10^{-15}$ m), we find that on the symmetry axis ($r=0$) the magnetic field strength reaches $H\sim 3\cdot 10^{14}$ G (Ref. 12). At the present time, experiments are being made (in Germany) with heavy ions with the aim of detecting the spontaneous production of positrons by a Coulomb field, and recently success was achieved. In 1988, experiments made at Darmstadt in Germany on the heavy-ion linear accelerator UNILAC led to the observation of the production of electron–positron pairs in the ultrastrong electric field produced by collisions of uranium, thorium, and lead heavy ions.[13] The results of the experiments still do not yet have an unambiguous interpretation, but the attainment of critical values of the electromagnetic field under laboratory conditions is not in doubt.

Recently, the problem of the occurrence of ultrastrong magnetic fields has also been discussed in connection with possible physical processes that may accompany the motion of colliding proton–antiproton beams in colliders of the new generation (LHC and SSC).[14] Because of the high energies of the particles, the resulting field may reach values $H_c=M_W^2c^3/(e\hbar)=10^{24}$ Oe and create favorable conditions for the production of $W^\pm$- and Z^0-boson condensates, which then decay into leptons.

In the considered estimates of the extremal values of the electromagnetic field, allowance must also be made for the energy of the particles that interact with the field (see Ref. 6). It is well known[6,7] that the electrodynamic processes that take place in an external field depend on the quantum dynamical parameter (2), which in the case of the motion of an electron in a purely magnetic field has the form

$$\chi=-\frac{1}{mcH_c}F_{\mu\nu}p^{\nu}=\frac{p_{\perp}}{mc}\frac{H}{H_c}. \quad (3)$$

If the parameter χ reaches values $\chi>1$, a way is opened up to the experimental investigation of entirely new interesting effects, since in this ultraquantum region of phenomena the quantum laws begin to play a particularly fundamental role.

In this connection, we should mention the experiments made comparatively recently using the SPS accelerator at CERN,[15] in which a beam of relativistic electrons with energy 150 GeV and very small angular spread was directed along the axis of a germanium crystal. A strong macroscopic field acted along this axis in the crystal, and its strength could be estimated by considering the effect of the nuclei of the atoms of the crystal on a test particle.[16] Analysis showed that the maximum electric field on the crystal axis was $E_{\max}\cong 10^{12}$ V/cm. The equivalent magnetic field, estimated from the ability of the averaged crystal field $E_{\max}$ to deflect electrons and positrons, reached values ~ 3 GOe$=3\cdot 10^9$ Oe.

Thus, in this CERN experiment[15] the dynamical parameter ξ (2) exceeded unity: $\chi\gg 1$. Modern experimental physics has reached a new and very interesting frontier—the observation of quantum effects under conditions of critical values of the external field and high particle energies. Possibilities have been opened up for observation of entirely new nonlinear effects and a possibility of testing the theory under conditions of high energy and extreme values of the external field strength.

2. VACUUM POLARIZATION BY AN EXTERNAL FIELD IN QUANTUM ELECTRODYNAMICS

The problem of electrodynamics in a strong electromagnetic field arose in the early studies of Heisenberg and Euler[4] devoted to calculation of vacuum polarization effects, and also Sauter,[17] who analyzed the well-known "Klein paradox" associated with the production of electron–positron pairs in a strong electric field. As was first pointed out by Dirac, even if an external field does not lead to pair production it does influence the vacuum of the electrons and positrons, giving rise to a redistribution of the charges of the vacuum and changing its energy (vacuum polarization). The phenomenon arises because application of an external field shifts the energy levels of the vacuum electrons from the vacuum energy levels without an external field. Such a change of the energy leads to a change in the equations of the electromagnetic field; the Lagrangian is also changed, and the Maxwell function

$$\mathscr{L}_0=\frac{1}{8\pi}(\vec{E}^2-\vec{H}^2)$$

now becomes the first term in the expansion of a complete function with respect to a constant that characterizes the interaction of the vacuum electrons with the field: $\mathscr{L}=\mathscr{L}_0+\mathscr{L}_1+...$, where

$$\mathscr{L}_1=\frac{1}{8\pi^2}\int s^{-1}ds\,\mathrm{e}^{-im^2s}\Big[(es)^2f_2\cot(esH)-1+\frac{2}{3}(es)^2f_1\Big], \quad (4)$$

and f_1 and f_2 are determined by (1)—they are invariants of the electromagnetic field. It follows from this in particular that the correction $\mathscr{L}_1$ to the Lagrangian does not depend on the parameters of a plane electromagnetic wave—an external field consisting of plane waves does not polarize the vacuum.

In the special case of weak fields ($H/H_c\ll 1$, $E/E_c\ll 1$), Heisenberg and Euler obtained from Eq. (4) the result

$$\mathscr{L}_1=\frac{1}{360\pi^2H_c^2}\{(\vec{E}^2-\vec{H}^2)^2+7(\vec{E}\vec{H})^2\}. \quad (5)$$

These are the first terms of the correction resulting from the expansion in a series in powers of f_1 and f_2, and they characterize the shift in the energy of the classical electromagnetic field. In this opposite limiting case of an extremely strong field, we can obtain from (4) to logarithmic accuracy the result

$$(H/H_c)\gg 1,\quad E=0,\quad \mathscr{L}_1=\frac{e^2}{24\pi^2}H^2\ln(H/H_c),$$
$$(E/E_c)\gg 1,\quad H=0, \quad (6)$$
$$\mathscr{L}_1=-\frac{e^2}{24\pi^2}E^2\ln(E/E_c)+\mathrm{i}\frac{1}{8\pi^3}e^2E^2\sum_{n=1}^{\infty}\frac{1}{n^2}\mathrm{e}^{-n\pi E_c/E}$$

(see Ref. 20 for a discussion of the corrections of higher order in e^2).

Note that because $\mathscr{L}_1$ is complex the vacuum state is quasistationary. The loss of stationarity is due to the possibility of real production of electron–positron pairs from the vacuum when the strong field acts on it. As follows from the quantum decay law of a stationary state $\Psi\sim\mathrm{e}^{-iEt-\gamma t}$, $\gamma=w/2$ ($c=\hbar=1$), twice the imaginary part of the Lagrangian determines the probability w of production of an electron–positron pair by an external field per unit volume per unit time:

$$w=2\,\mathrm{Im}\,\mathscr{L}_1.$$

Retaining the leading terms of the sum (6), we find that

$$w=\frac{1}{4\pi^3}(eE)^2\mathrm{e}^{-\pi E_c/E}. \quad (7)$$

It follows from this expression that Im $\mathscr{L}_1=0$ in the limit $E\to 0$, so that the probability (7) vanishes, i.e., the vacuum in a constant and homogeneous magnetic field, and also in the field of a plane electromagnetic wave and in a crossed field is stable with respect to the spontaneous production of electron–positron pairs. Vacuum instability was already discussed in the early studies of Refs. 17 and 18 into the influence of negative-energy states on the interaction of an electron with an external electric field.

Considering the single-particle problem of motion in a homogeneous electric field, Klein[18] showed that in a clas-

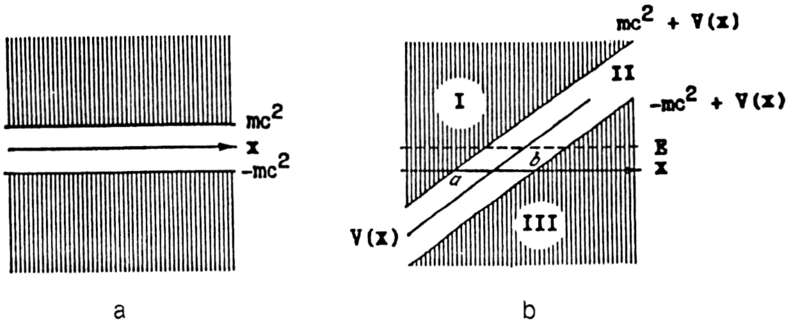

FIG. 1. Energy states of an electron: (a) free; (b) in an electric field.

sically forbidden region of motion, where the wave function should be damped, a wave that reaches to infinity exists ("Klein paradox"). Sauter gave an exhaustive solution of this problem,[17] by considering particle states with not only positive but also negative energy. He established the critical value of the electric field at which the single-particle approach to the solution of the problem becomes impossible. Omitting here the rigorous solution of the problem (see Ref. 17), we give here only a brief illustration of the motion of an electron in a homogeneous electric field that changes the configuration of the electron energy levels and the boundaries of the upper continuum ($E=mc^2$) and lower continuum ($E=-mc^2$) (see Fig. 1a):

$$V(x)=e_0\mathscr{E}x, \quad E=\pm mc^2+e_0\mathscr{E}x. \tag{8}$$

This can be represented graphically. Figure 1b shows the rotation of the forbidden energy band that separates the positive- and negative-frequency (i.e., electron and positron) states.[19] It can be seen from this graph that in an electric field there is no rigorous separation of these states—there can always be a tunneling transition from region III to region I. As is well known, the probability of such a transition in the semiclassical approximation is

$$D=\exp\left\{-\frac{2}{\hbar}\int_a^b |p(x)|\,dx\right\},$$

$$p(x)=\frac{1}{c}\sqrt{(E-e_0\mathscr{E}x)^2-m^2c^4},$$

and integration of this expression leads to the result

$$D=e^{-\pi E_c/E}, \quad E_c=m^2c^3/e_0\hbar \tag{9}$$

[see also (7)]. To rule out the possibility of spontaneous transition of electrons to the lower continuum of states, one introduces, of course, the assumption that all the states with negative energy are occupied (Dirac background), and then after the transition of an electron to the upper continuum of states a "hole" appears in the background and behaves like an antiparticle—a positron. When the electric field E reaches its critical value E_c, the electron–positron vacuum becomes unstable and electron–positron pairs are produced. Under these conditions, the single-particle approach to the solution of the problems of the quantum theory becomes impossible.

Returning to the question of vacuum polarization, we note that this polarization may also occur under the influence of individual photons, the quanta of the electromagnetic field. In Ref. 20, Ritus showed that the vacuum polarization by photons with large squares of the virtual momenta is similar to the vacuum polarization by a strong external field. Study of the Lagrangian in the limit of a strong field can, by virtue of these considerations, give us the same information as the results of investigation of the photon polarization function in the limit of large virtual momenta. In other words (see Ref. 6), the behavior of quantum electrodynamics at short distances (in the region of large momenta) can be studied by investigating the quantum corrections to the Lagrangian of a strong electromagnetic field. One can see this by considering the behavior of the ratio $\mathscr{L}_1/\mathscr{L}_0$ [see (6)] in a strong field:

$$\mathscr{L}_1/\mathscr{L}_0=-\frac{e^2}{3\pi}\ln\frac{H}{H_c}$$

and comparing it with the photon polarization function at large squares of the photon momenta:[21]

$$\mathscr{P}_0\cong-\frac{e^2}{3\pi}\ln\frac{k^2}{m^2}.$$

Thus, the Lagrangian of a strong electromagnetic field can be particularly important for the development of short-distance quantum electrodynamics.

In conclusion, we emphasize that the relativistic theory of vacuum polarization leads to a nonlinear electrodynamics characterized by a special Lagrangian (5) that is a function of the field invariants f_1 and f_2 [see (1)]. This has an interesting physical interpretation in terms of nonlinear Lagrangians introduced in order to achieve a nonlinear generalization of Maxwell's electrodynamics:

$$\mathscr{L}=\frac{E_c^2}{4\pi}\left[1-\sqrt{1-\frac{\vec{E}^2-\vec{H}^2}{E_c^2}-(\vec{E}\vec{H})^2/E_c^4}\right] \text{(Born–Infeld)}$$

$$\mathscr{L}=\frac{E_c^2}{8\pi}\ln(1+(\vec{E}^2-\vec{H}^2)/E_c^2) \quad \text{(Schrödinger)}.$$

In these expressions, the choice of the nonlinear generalization is arbitrary, and it is only Dirac's theory of the electromagnetic vacuum that opens up a physical model of the Lagrangian. The nonlinear generalization of the theory makes it possible to investigate some fundamentally new physical phenomena: light–light scattering, nonlinear scattering of light by charges, and some other effects.

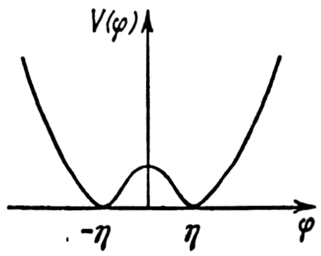

FIG. 2. Graph of the potential energy $V(\varphi)=\lambda^2(\varphi^2-\eta^2)^2/4$.

3. RESTORATION OF SPONTANEOUSLY BROKEN SYMMETRY OF THE VACUUM OF ELECTROWEAK INTERACTIONS IN A STRONG MAGNETIC FIELD

In recent years there has been interest in the effect of extremely strong magnetic fields on the vacuum state of the electroweak interactions.[9,22,23] As is well known (see, for example, Refs. 24 and 25), spontaneous symmetry breaking is the basis of the modern gauge theory of the weak and electromagnetic interactions. In accordance with this (Salam–Weinberg–Glashow) theory, the original massless particles acquire mass through the Higgs mechanism based on spontaneous breaking of the local $SU(2)/U(1)$ symmetry ($W^\pm$ and Z^0 bosons). In this connection, it is interesting to investigate the properties of the vacuum of the electroweak interactions and, in particular, the influence of external factors on the process of spontaneous symmetry breaking.

We consider the simplest Abelian Higgs model, in which the original Lagrangian has $U(1)$ symmetry and possesses the form

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+(\hat{\mathscr{P}}^*_\mu\varphi^*)(\hat{\mathscr{P}}_\mu\varphi)-V(\varphi\varphi^*), \qquad (10)$$

where

$$V(\varphi\varphi^*)=-\frac{m^2}{2}\varphi^*\varphi+\frac{\lambda^2}{4}(\varphi^*\varphi)^2,$$

$$\hat{\mathscr{P}}_\mu=\partial_\mu+ieA_\mu,\quad m^2>0,\quad \lambda>0.$$

The function φ describes a scalar charged field, A_μ describes a quantized vector field, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the tensor of the electromagnetic field, m is the mass of the particles described by the field φ, and λ is a dimensionless parameter that characterizes their interaction.

Note that the minus sign in the expression for the effective potential V opens up the possibility of spontaneous symmetry breaking. Indeed, $V(\varphi\varphi^*)$ has a minimum at the values

$$\varphi_0=\pm m/\lambda=n.$$

By virtue of this, the "equilibrium position" $\varphi=0$ corresponding to the vacuum becomes unstable—there is vacuum degeneracy, since now there are two stable positions of a minimum (see Fig. 2). The choice of one of them is explained by random factors, but it is very important that transition of the system from one equilibrium state to the other is ruled out. In quantum field theory with infinitely many degrees of freedom the probability of such a transition becomes vanishingly small.

Thus, the vacuum degeneracy leads to spontaneous symmetry breaking: The original Lagrangian (10), which is symmetric with respect to the discrete transformation $\varphi\to-\varphi$, loses this symmetry property if the negative sign in the mass term of the effective potential is chosen. This can be clearly seen from the absence of symmetry of V about the point $\varphi=\eta$. In what follows, it will be convenient to write the effective potential in the form

$$V=\frac{\lambda^2}{4}(\varphi^2\varphi-\eta^2)^2,$$

since the addition to the original expression of a constant does not affect the equation of motion. It is then convenient to go over to a new function χ, setting $\varphi=\eta+\chi$, where χ describes excited states near the stable vacuum η. We then find[24] that

$$V=\frac{\lambda^2}{4}[4\chi^2\eta^2+4\chi^3\eta+\chi^4],$$

from which it follows that the $\chi\to-\chi$ symmetry is here broken. Omitting for simplicity the nonlinear interactions of the fields, we now obtain an expression for the original Lagrangian (10) after the spontaneous symmetry breaking:

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\lambda^2\eta^2\chi^2+e^2\eta^2A_\mu^2.$$

This Lagrangian describes a scalar field χ with mass $\mu=\sqrt{2}\lambda\eta$, and the vector field A_μ has now also become massive—it has acquired a mass $m=\sqrt{2}e\eta$. Thus, the massless Maxwell field has been transformed into a Proca boson—as a result of the spontaneous symmetry breaking, the Maxwell gauge field has acquired a mass (Higgs effect).

In the standard non-Abelian model of the electroweak interactions, the expression for the original Lagrangian is more complicated, but the general concept of spontaneous symmetry breaking is not changed, and as a result of the breaking the initially massless gauge $W^\pm$ and Z^0 bosons acquire mass.

It is also interesting to note that the problem of spontaneous symmetry breaking has a well-known analogy with superconductivity theory, to the development of which Bogolyubov made a fundamentally important contribution.[26] The analogy becomes particularly transparent if we consider the structure of the expression

$$F=\int d^3x\left\{\frac{1}{4m}(\hat{\mathscr{P}}^*\Psi^*)(\hat{\mathscr{P}}\Psi)+\frac{H^2}{2}+\Lambda\left(-\frac{T_c-T}{4T_c}|\Psi|^2+\frac{1}{2n}|\Psi|^4\right)\right\}$$

for the free energy of a superconductor and compare this expression with the Lagrangian (10). In this expression, $\Lambda=\Lambda(T_c,n)$ is a constant, T_c is the critical temperature of the transition to the superconducting state, $\vec{\hat{\mathscr{P}}}=(-i\vec{\nabla}-2e\vec{A})$, and Ψ is the superconducting wave function that describes the condensate of Cooper pairs (the charge of a pair is $2e$). Further, it is well known that the superconducting state is destroyed when the temperature T is raised, and also when the magnetic field strength is increased, the strong magnetic field being capable of destroying the condensate of Cooper pairs.

It turns out that the behavior of the vacuum of the electroweak interactions has an analogy with the superconducting state of a metal, namely, temperature (Kirzhnits and Linde[27]) and an external magnetic field (Salam and Strathdee[28]) can have a strong influence on the spontaneous symmetry breaking.

We consider again the Abelian Higgs model, in which the original Lagrangian (10) must be augmented by an external magnetic field. This can be done by replacing the vector potential A_μ of the gauge field by the new function $A'_\mu = A_\mu + A^{\rm cl}_\mu$. Study of the problem of spontaneous symmetry breaking in this case shows[23] that the external field restores the symmetry partly or completely, the restoration depending on the strength of the field. In the limiting case of a critical value of the magnetic field capable of destroying the spontaneous symmetry breaking, we can, using the Lagrangian (10), write

$$\frac{H_c^2}{2}-\frac{m^2}{2}\varphi^*\varphi+\frac{\lambda^2}{4}(\varphi^*\varphi)^2=\frac{H_c^2}{2}-\frac{\lambda^2\eta^4}{4}$$

(in the state with a stable minimum, or vacuum). It is obvious that the spontaneously broken symmetry will be restored if $H_c=(1/\sqrt{2})\lambda\eta^2\sim M^2$, where M is the mass acquired by the particles of the gauge field. Thus, in the strong magnetic field there can be absorption of the condensate of the scalar field like the absorption of the condensate of Cooper pairs.

The critical magnetic field $H_c=M^2c^3/e\hbar$ can be estimated by assuming that the mass in this expression is the mass of the bosons ($W^\pm,Z^0$) in the Salam–Weinberg–Glashow theory ($\sim$90 GeV). Then for the field $H_{\rm cr}$ we obtain the very large value $H_{\rm cr}\sim 10^{24}$ Oe. Such fields have not yet been observed in nature, but the academic interest shown in them is entirely justified, since we are concerned here with the foundation of the theory.

Here, it should be emphasized that the problem of the influence of an external field on the vacuum of the electroweak interactions is rather complicated, but we do not have the possibility of going into this question in more detail and we refer the reader to the review of Ref. 22.

4. ASPECTS OF THE MOTION OF A PARTICLE IN A STRONG MAGNETIC FIELD

Investigation of processes occurring in a strong electromagnetic field do not permit perturbative treatment of the interaction of the electron with this field. One of the strong methods that can be used independently of the strength of the electromagnetic field is the method of "exact solutions" (see Refs. 5, 7, and 29), in accordance with which the state of a particle in an external electromagnetic field is described by a wave function that is an eigenfunction for the Dirac equation:

$$i\hbar\frac{\partial}{\partial t}\Psi=\{c(\vec{\alpha}\vec{\mathscr{P}})+\rho_3mc^2+e\Phi\}\Psi,$$
$$\vec{\mathscr{P}}=-i\hbar\vec{\nabla}-\frac{e}{c}\vec{A}. \tag{11}$$

Although not very many exact solutions of the Dirac equation exist (they do for the problem of the motion of an electron in a Coulomb field, in homogeneous magnetic and electric fields, in the field of a plane electromagnetic wave, and in some cases of combinations of fields), exact solutions are still of great interest, not only from the point of view of testing the theory by comparing its results with experimental data but also for the prediction of new physical effects.

At the end of the forties, A. A. Sokolov laid the foundations for the study of the quantum theory of a radiating electron moving in a magnetic field (1949), and also M. M. Al'perin solved the problem of the radiation of an electron in the field of a plane wave (1944). This was the early period in the development of the method of exact solutions, which later became known as the Furry picture, after Furry, who showed that the Feynman–Dyson formalism could be generalized to the case when the electron is not free but in a bound state (1951). This method can be accommodated in the solution of the Schrödinger picture if for the unperturbed function one takes the exact solution of the Dirac equation for a particle in an external field in a bound state. The 4-potential of the external field is decomposed into two parts: $A_\mu=A_\mu^{\rm ext}+A_\mu^{\rm qu}$, where $A_\mu^{\rm ext}$ is taken into account exactly by solving the Dirac equation, and the quantized part of the field is treated perturbatively. Thus, exact solutions provide a foundation for a complete technique of solving problems involving the interaction of a bound electron with a radiation field. As we have already mentioned, the method of exact solutions proved to be very interesting and important, since on its basis it was possible to predict some new physical effects of a quantum nature long before their experimental observation. The method was used also to investigate physical phenomena in the case of critical values of the magnetic field, since the vacuum remains stable under these conditions and the electron retains its individuality.

Nonlinear electron energy spectrum

We consider some aspects of the motion of an electron in a constant and homogeneous magnetic field,[7,29,30] above all the energy spectrum of an electron obtained by exact solution of the Dirac equation (see Ref. 7):

$$E=mc^2\left\{1+\left(\frac{p_z}{mc}\right)^2+2n\frac{H}{H_c}\right\}, \tag{12}$$

where $n=0,1,2,...$ is the energy quantum number, and $H_c=m^2c^3/e\hbar$ is the critical value of the magnetic field $H=(0,0,H)$. Note that the electron energy spectrum (12) is degenerate with respect to the spin. If one uses the generalized Dirac–Pauli equation and takes into account the electron anomalous magnetic moment $\mu=-\mu_0(1+a_e)$, then the spin degeneracy is lifted, and the expression (12) becomes[31]

$$E=mc^2\left\{\left[1+2\left(n+\frac{1+\zeta}{2}\right)\frac{H}{H_c}\right]^{1/2}+\frac{1}{2}a_e\zeta\frac{H}{H_c}\right\}, \quad (13)$$

where $\zeta=\pm 1$ corresponds to projection of the spin onto the direction of the magnetic field, and a_e is the anomalous part of the magnetic moment. It can be seen from these expressions that in a homogeneous magnetic field there exists a state with minimum energy, for which $n=0$ and the spin of the particle is in the opposite direction to the magnetic field: $\zeta=-1$. At the same time

$$E=mc^2\left[1-\frac{1}{2}a_e\frac{H}{H_c}\right].$$

Restoring the sign of the energy and setting $a_e=\alpha/(2\pi)$, we obtain the intriguing result

$$E=\pm mc^2\left[1-\frac{\alpha}{4\pi}\frac{H}{H_c}\right],$$

from which it follows that in the limit $(\alpha/4\pi)(H/H_c)\to 1$ the electromagnetic vacuum loses its stability, since the energy gap $2mc^2$ between the states with positive and negative energies can collapse. However, as will be shown in what follows, the anomalous magnetic moment of the electron (a_e) manifests a dynamical nature and becomes dependent on the field strength and the energy of the electron (see Sec. 9).

In the case of a spinless particle, we can set $\zeta=0$ in (13), and in the nonrelativistic approximation we then obtain Landau levels with an equidistant spectrum:

$$E\cong mc^2+p_z^2/(2m)+\hbar\Omega(n+1/2),$$

where $\Omega=eH/(mc)$ is the cyclotron frequency.

Note that in the case of macroscopic semiclassical motion the quantum number n is very large; for example, for a storage ring ($R=1$ m, $H=10^4$ Oe) it is of order 10^{20}–10^{21}, and the electron energy spectrum in this case is quasicontinuous, since the distance between neighboring energy levels is vanishingly small. However, in the case of small $n\sim 1$ and strong fields the magnetic field becomes a "quantizing" field, the energy spectrum is essentially nonlinear (nonequidistant), and the energy remains relativistic for all values of n. Such a situation can be encountered in astrophysical problems, namely, an excited electron ($n\gg 1$) loses its energy through radiation, and since, in contrast to a synchrotron, the radiative energy losses are not compensated under astrophysical conditions, the energy of such an electron corresponds to the region of small quantum numbers. Under these circumstances, the problem becomes essentially a quantum one, and the semiclassical method is invalid.

Ritus[8] analyzed the possible application of the results of calculations obtained by means of the model of a crossed field to describe processes in an arbitrary constant electromagnetic field. He showed that under the condition of smallness of the field invariants

$$\varepsilon,\eta\ll 1;\quad \varepsilon,\eta\ll\chi, \quad (14)$$

where

$$\varepsilon=\frac{1}{\sqrt{2}}\left[(f_1^2+f_s^2)^{1/2}-f_1\right]^{1/2},$$

$$\eta=\frac{1}{\sqrt{2}}\left[(f_1^2+f_2^2)^{1/2}+f_1\right]^{1/2},$$

and f_1, f_2, and χ are determined by the expressions (1) and (2), the probability of the process in an arbitrary constant electromagnetic field, which is a function of the invariants, $w=w(\chi f_1,f_2)$, can be approximated to good accuracy by the expression $w(\chi,0,0)$ in a crossed field, for which $f_1=f_2=0$.

We now consider a homogeneous magnetic field. Then $f_1=H/H_c$, $f_2=0$, and the dynamical parameter is $\chi=(H/H_c)(p_\perp/mc)=(H/H_c)^{3/2}\sqrt{2n}$, where n is the energy quantum number. At the same time, we have taken into account the energy spectrum of the electron in the magnetic field, for which

$$E=mc^2\left(1+\left(\frac{p_\perp}{mc}\right)^2\right)^{1/2}=mc^2\left(1+2n\frac{H}{H_c}\right)^{1/2}.$$

If now $n\gg 1$ (the energy spectrum is quasicontinuous), then the case of semiclassical motion is realized, and the condition $\chi\gg(f_1)^{1/2}$ holds. Then the results obtained in the crossed-field model must also be valid for the case of a homogeneous magnetic field in the semiclassical approximation. However, such an approach is restricted—if one considers the region of small quantum numbers and strong fields, the parameter χ can be conveniently represented in the form $\chi=(f_1)^{3/2}(2n)^{1/2}$, from which it follows that if n is bounded and $f_1>1$, then the condition $\chi\gg(f_1)^{1/2}$ is not satisfied. An important factor here is the discreteness of the parameter χ, although it is still greater than unity: $\chi>1$ ($H>H_c$). In this connection, we note that the energy spectrum of an electron in a crossed field is continuous: If $\vec{E}=(0,E,0)$, $\vec{H}=(0,0,H)$, and $|\vec{E}|=|\vec{H}|\sin\eta$, then the energy is

$$E=cp_1\sin\eta+\cos\eta\ mc^2\left[1+\left(\frac{p_3}{mc}\right)^2+2n\frac{H}{H_c}\cos\eta\right]^{1/2}$$

(see Ref. 32), and in the limit $\eta\to\pi/2$ the discrete part of the spectrum disappears. Therefore, the processes in a crossed field will be valid for the case of a magnetic field only under the conditions of semiclassical motion, when the quantizing properties of the field can be completely ignored. In the case of an extremely strong field, $H\to H_c$, such an assumption is unjustified, and the problem must be treated separately.

Spatial localization of an electron

If the magnetic field approaches the critical value H_c, then a process of strong spatial localization of the electron occurs. Indeed, as follows from the exact solutions of the Dirac equations, the square of the radius of the particle orbit has the form[7,31]

$$\langle R^2\rangle=(n+s+1/2)\,\frac{2c\hbar}{e_0H}=(n+s+1/2)2\,\frac{H}{H_c}\left(\frac{\hbar}{mc}\right)^2.$$

The radial quantum number $s=0, 1, 2,...$ in this expression characterizes the position of the center of the orbit of electron gyration, $\langle R^2\rangle-\langle R\rangle^2=s2(H/H_c)(\hbar/mc)^2$, and, taking the center of the orbit at the origin, we can set $s=0$. It then follows from (15) that in a strong field $H\gg H_c$

$$\langle R^2\rangle=\left(\frac{\hbar}{mc}\right)^2 2(n+1/2)\,\frac{H_c}{H}.$$

In the region of macroscopic motion (the semiclassical case), the quantum number n is, as we have already mentioned, very large, $n\sim10^{20}$, and the radius of the orbit of electron gyration has a macroscopic value. However, in the case of small quantum numbers, $n\to0$, the electron is strongly localized near the Compton radius of the orbit ("ground" state):

$$R_H=\frac{\hbar}{mc}\,\sqrt{H_c/H}. \tag{15}$$

This quantity is analogous to the radius of the Bohr orbit in a Coulomb field, $R_B=\hbar^2/(me^2)$, the radius of the electron orbit in the ground (unexcited) state. The strong localization of the electron in a critical magnetic field has great importance for understanding the behavior of the simplest quantum systems in such a field.

5. HYDROGEN ATOM IN A STRONG MAGNETIC FIELD

As we have already noted, strong magnetic fields $H\gg H_c$ can exist at the surface of neutron stars,[11] where, by virtue of the strong localization, the electrons move in a restricted region with characteristic dimensions $\Delta R_H\sim(\hbar/(mc))\sqrt{H_c/H}$ (15). The size of the region of motion in the direction of the field H is determined by the interaction of the electron with other fields. Near the surface of a pulsar, as such fields one should consider the Coulomb field, since the magnetosphere of the star is filled with plasma. It is therefore of interest to investigate the behavior of the simplest quantum-mechanical systems associated with the Coulomb interaction in a strong magnetic field (see Refs. 33 and 34).

Following Ref. 33, we consider first of all the problem of a hydrogen atom in a strong magnetic field, the strength of which can be determined on the basis of the requirement that the critical field be greater than the intra-atomic field:

$$\hbar\Omega>e^2/a_B,\quad a_B=\hbar^2/(me^2),\quad H_{cr}=H_c\alpha^2,\quad \alpha=\frac{e^2}{\hbar c}. \tag{16}$$

To find solutions of the Schrödinger equation for an electron moving in the Coulomb field of a nucleus and an external magnetic field, we shall assume that the strong field localizes the motion of the particle so much that the radius r_B of the Bohr orbit greatly exceeds the radius ΔR_H: $r_B\gg\Delta R_H$. Then for motion in the plane at right angles to the direction of the external magnetic field the Coulomb field can be treated as a perturbation that does not change the energy spectrum of the electron in the magnetic field even when allowance is made for its interaction with the Coulomb field of the nucleus. Therefore, the radial wave functions that describe the motion of the electron in the plane perpendicular to the magnetic field can be taken from the problem of the motion of an electron in a homogeneous magnetic field (see Ref. 7), namely, if the magnetic field is expressed in a cylindrical coordinate system in the form $A=(A_\rho,A_\varphi,A_z)=(0,H\rho/2,0)$, then

$$\Psi=(2\gamma/(2\pi L))^{1/2}\exp(i(-Et/\hbar+l\varphi+p_z/\hbar)I_{n,s}(x), \tag{17}$$

where $x=\gamma\rho^2$, $\gamma=eH/(2c\hbar)$, and $I_{n,s}(x)=(n!s!)^{-1/2}\times\exp(-x/2)x^{(n-s)/2}Q_s^{n-s}(x)$ is a Laguerre function associated with the Laguerre polynomials $Q_s^{n-s}(x)$. Here, $n=l+s=0, 1,...$ is the principal quantum number, s is the radial number, $s=0, 1, 2,...$, and $-\infty<l\leqslant n$. The quantum number $n=0, 1,...$ labels the Landau energy levels

$$E=\hbar\Omega(n+1/2),\quad \Omega=e_0H/(mc),$$

and $s=0, 1,...$ determines the distance from the origin to the center of the orbit of the electron gyration in the magnetic field. If there is no Coulomb field, then any energy level with fixed n is infinitely degenerate with respect to s.

We consider further longitudinal motion of the electron and restrict ourselves to studying the energy levels of this motion that are obtained in a Coulomb field. At the same time, we shall consider the transverse (with respect to the magnetic field) motion in the state $n=s=0$. Then the solution of the Schrödinger equation

$$\hat{\mathscr{H}}\Psi=\left\{\frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}\right)^2-\frac{ze^2}{r}\right\}\Psi=\mathscr{E}\Psi \tag{18}$$

can be sought in the form

$$\Psi=I_{0,0}(x)\Phi(z)$$

and from Eq. (18) we find that

$$\left\{-\frac{h^2}{2m}\frac{\partial^2}{\partial z^2}-\frac{ze^2}{r}\right\}\Phi(z)I_{0,0}(x)=\mathscr{E}\Phi(z)I_{0,0}(x),$$

where

$$\mathscr{E}=E-\hbar\Omega/2.$$

Integrating this equation with respect to $\rho d\rho$ with allowance for the normalization

$$2\gamma\int_0^\infty I_{0,0}(x)\rho d\rho=1,$$

we arrive at the equation

$$\frac{d^2\Phi}{dz^2}+\frac{2m}{\hbar^2}(\mathscr{E}-\bar{U}(z))\Phi=0, \tag{19}$$

in which

$$\frac{1}{2\gamma}\bar{U}(z)=-e^2\int_0^\infty(\rho^2+z^2)^{-1/2}I_{0,0}^2(x)\rho d\rho. \tag{20}$$

Thus, in accordance with our assumption the motion of the electron in the direction of the magnetic field (the z axis) is characterized by Eq. (19) with the "effective" Coulomb potential (20). In the strong magnetic field, the potential $U(z)$ is cut off at distances z of order $\gamma^{-1/2}$, as one can see by calculating the integral (20):

$$\bar{U}(z) = -e^2\sqrt{\pi\gamma}\exp(\gamma z^2)(1-\Phi(\sqrt{\gamma}z),$$

where $\Phi(z)$ is the error function:

$$\Phi(z) \cong (2/\sqrt{\pi})e^{-z^2}z \quad (z \ll 1),$$

and $\bar{U}(0) = -e^2(\pi\gamma)^{1/2}$. For $\sqrt{\gamma}z \gg 1$, the effective potential $\bar{U}(z)$ can approximate (21) by a one-dimensional Coulomb potential, since for $z \gg 1$

$$1-\Phi(z) = \frac{1}{\pi}e^{-z^2}\sum_{m=0}^{\infty}(-1)^m\frac{\Gamma(m+1/2)}{z^{(m+1/2)}} \sim \frac{1}{\sqrt{\pi}}e^{-z^2}\frac{1}{\sqrt{z}}.$$

Thus, the magnetic field effectively cuts off the interaction of the electron with the Coulomb field at $z \sim (\gamma)^{-1/2}$. Therefore, it is convenient to choose the effective potential from the very beginning in the form[35]

$$\bar{U}(z) = -\frac{e^2}{|z|+a_H}, \tag{21}$$

where

$$a_H \cong \gamma^{-1/2} = \frac{\hbar}{mc}\sqrt{H_c/H}.$$

The solution of the Schrödinger equation (19) with effective potential in the form (21) can be found exactly. We consider the region of values $z>0$. In this region,

$$\frac{d^2\Phi}{dt^2} + \frac{2m}{\hbar^2}\left(\mathscr{E} + \frac{e^2}{t}\right)\Phi = 0, \tag{22}$$

where $t = z + a_H$. Setting $c = 2me^2/\hbar$, $a = -2m\mathscr{E}/\hbar^2$, $x = 2\sqrt{at}$, we rewrite (22) in the form

$$\frac{d^2\Phi}{dt^2} + \left(-\frac{1}{4} + \frac{\lambda}{x}\right)\Phi = 0, \quad \lambda = c/(2\sqrt{a}). \tag{23}$$

The solution of this equation that is bounded as $z \to \infty$ is the Whittaker function

$$\Phi = NW_{\lambda,1/2}(x). \tag{24}$$

To construct a solution for $z<0$, the function (24) must be continued into the region $z<0$. This can be done by continuing the solution (24) either as $\Phi(-z) = \Phi(z)$ or as $\Phi(-z) = -\Phi(z)$. Therefore, the excited levels corresponding to these solutions have a twofold degeneracy.

To determine the energy spectrum, i.e., the energy levels of the longitudinal motion that correspond to the zeroth Landau level, we turn to Eq. (19) and use the results of the study of the problem of finding the lowest bound-state level under the assumption that $\hbar\Omega \gg me^4/\hbar^2$ (see Ref. 36). Then

$$\mathscr{E}_0 = -\frac{2m}{\hbar^2}\left(\int_{a_H}^{a_B} U(z)dz\right)^2,$$

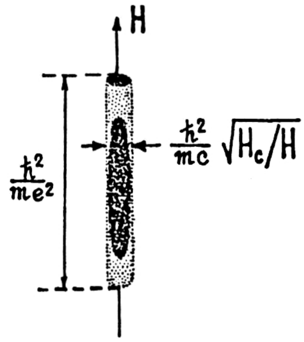

FIG. 3. Needle shape of the electron cloud in a hydrogen atom in a strong magnetic field $H \to H_c$.

and the logarithmically occurring integral is "cut off" above and below at distances $|z| \sim a_B$ and $|z| \sim a_H$. We then find that to logarithmic accuracy

$$\mathscr{E}_0 = -\frac{me^4}{2\hbar^2}\ln^2\frac{H}{H_c} = -Ry\ln^2\frac{H}{H_c}, \quad H_c' = H_c\alpha^2,$$
$$\mathscr{E}_n = -\frac{me^4}{2\hbar^2 n^2}, \quad n = 1,2,..., \tag{25}$$

and the corresponding wave function is

$$\Phi_0 = \sqrt[4]{a}W_{\lambda_0,1/2}(x), \quad x = \frac{2me^2}{\hbar^2}\ln\frac{H}{H_c'}(z+a_H),$$
$$\lambda_0 = (a_B\sqrt{a})^{-1} = \ln\frac{H_c'}{H}, \tag{26}$$

$a = (me^2/\hbar^2)\ln(H/H_c')$. We restrict consideration to the ground state. Note that in the approximation $\ln(H/H_c') \gg 1$ the wave function can be expressed in terms of a Macdonald function:

$$\Phi_0 = \sqrt{\frac{x}{\pi}}K_{1/2}(x/2). \tag{27}$$

Note that the ground-state energy of the atom in the strong magnetic field, $\mathscr{E}_c$, is less than $\mathscr{E}_n$ in the absence of the field H [see Eq. (25)]. The energy spectrum of the atom is changed.

We now say a few words about the "deformation" of the atom. Qualitatively, the structure of the hydrogen atom in the magnetic field can be estimated from the critical values of the region of localization of the electron. The localization in the direction of the magnetic field is determined by the Coulomb law [the radius of the Bohr orbit is $a_B \cong \hbar^2/(me^2)$], while in the plane perpendicular to the field the localization is proportional to the Compton wavelength: $a_H \cong (\hbar/mc)\sqrt{H_c/H}$. In other words, the atom loses spherical symmetry and acquires the shape of a needle (see Fig. 3). The dimensions of the atom can be estimated more rigorously by means of the wave function (26). In particular, we find

$$\langle z^2\rangle = 2\int_0^\infty a^{1/2}z^2W_{\lambda_0,1/2}(x)dz = a_B^2\bigg/\left(8\ln^2\frac{H}{H_c'}\right).$$

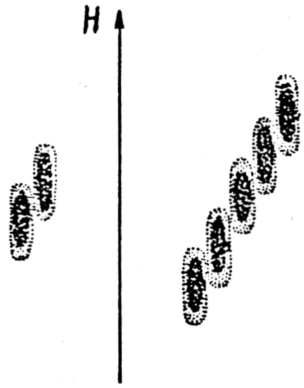

FIG. 4. Scheme of formation of molecules and polymer chains from hydrogen atoms in a strong magnetic field $H \to H_c$.

Further, bearing in mind that the linear dimension of the atom in the plane perpendicular to H has the order a_H, we obtain

$$\langle z^2 \rangle^{1/2}/a_H = (H_c'/H)^{1/2} \ln(H_c'/H). \tag{28}$$

In other words, the estimate that we gave earlier is confirmed.

It is interesting to note that the deformation of atoms in a strong magnetic field has decisive importance in the process of formation of molecules. Indeed, in the case that we consider of extremely strong magnetic fields, $H \sim H_c$, a neutral molecule cannot be formed by a homeopolar bond; for the spin of each electron is oriented in the opposite direction to the magnetic field, and therefore the spins are parallel to each other. Therefore, the total coordinate wave function, which depends on the coordinates of the two electrons in the field of the nuclei of the atoms, can only be antisymmetric, and this (in the case of the absence of a field H) does not lead to the formation of a molecule: A homeopolar bond should not be realized in a strong field.

However, by virtue of the loss of spherical symmetry (the atom is like a needle), the nucleus of the atom is practically unscreened by the electrons, and therefore such a nucleus can strongly attract the electron cloud of a different atom (see Fig. 4). As a result, there are effective forces of attraction, and even polymer molecules can be formed, these being aligned along a force line of the magnetic field (Ruderman's filamentary crystals; see Ref. 37). Thus, in a strong magnetic field there exists a fundamentally new mechanism of formation of molecules, due physically to deformation of the atoms.

6. SYNCHROTRON RADIATION IN A STRONG MAGNETIC FIELD

The investigation of the quantum properties of synchrotron radiation proved to be very interesting from the point of view of the development of the quantum theory of macroscopic motion. It led to the discovery of fundamental, at times unexpected effects of great theoretical and practical significance.[30,38] The original ideas about the possible manifestation of quantum effects in synchrotron radiation were not obvious, since synchrotron radiation had been well described in its basic features by the methods of the classical electrodynamics of Maxwell and Lorentz. Indeed, the usual criterion for the applicability of the classical methods of describing the radiation of a relativistic charge is that the energy $\mathscr{E}_\phi$ of the emitted photon must be small compared with the electron energy. At the same time, as was shown by Vladimirskiĭ[39] and Schwinger,[40] quantum effects can be manifested only if $\mathscr{E}_\phi = (\hbar c/R)(E/mc^2)^3 \cong E$. Thus, the criterion for the possibility of a classical description is

$$E \ll E_{1/2} = mc^2 \left(\frac{mcR}{\hbar} \right)^{1/2} \tag{29}$$

(the estimate of the photon energy is taken at the maximum of the spectral curve). This criterion was also confirmed by arguments relating to the invariance of the power of the synchrotron radiation, by virtue of which the power depends only on the invariant parameter

$$\chi = \frac{1}{H_c} [-(F_{\mu\nu} p^\nu)^2]^{1/2} = \frac{H}{H_c} \frac{E}{mc^2} = \left(\frac{E}{E_{1/2}} \right)^2. \tag{30}$$

(We ignore the motion along the field: $cp_\perp \cong E$.) However, it was found that the $E_{1/2}$ criterion did not cover all features associated with the discrete properties of synchrotron radiation, above all the influence of discreteness of the radiation on the motion of the particle. Indeed, since the energy $\mathscr{E}_\phi = (\hbar c/R)(E/(mc^2))^3$ of the emitted photon is fairly high, the number of such high-energy photons emitted during one gyration of the electron is finite and equal to $N_{\rm gyr} \cong W/\mathscr{E}_\phi = (4\pi/3)(e^2/\hbar c)(E/mc^2)$, where $W = \frac{2}{3}(e^2c/R^2)(E/mc^2)^4$ is the power of the radiation. To get a clearer estimate of the influence of the discreteness of the radiation, one can find the length L (in centimeters) of the path L traversed by the electron without the emission of high-energy photons (the "mean free" path):

$$L = \frac{c\mathscr{E}_\phi}{W} = \frac{3}{2} \frac{\hbar^2}{me^2} \frac{H_c}{H} = 34.9 \cdot 10^4/H. \tag{31}$$

For ordinary field strengths for accelerators and storage rings, $H \sim 10^4$ and $L \sim 30$ cm, i.e., on the average one photon is emitted on a path of length 30 cm.

The discreteness of the radiation, which appears here as an important factor, can influence the particle trajectory, producing quantum fluctuations of it as a consequence of the electron recoil on photon emission. To estimate the critical value of the energy at which the radiation begins to affect the particle trajectory, we note that

$$\bar{R}^2 = \frac{n+s+1/2}{\gamma}, \quad \overline{\Delta R^2} = \frac{s}{2\gamma}, \quad \gamma = eH/(2c\hbar),$$

where $\overline{\Delta R^2} = \overline{R^2} - (\bar{R})^2$ is the mean-square fluctuation of the radius, and $s = 0, 1, 2, \ldots$ is the radial quantum number, which characterizes the position of the center of the orbit of electron gyration. It is obvious that excitation of radial fluctuations commences if the photon energy is

$$\mathscr{E}_\phi = \frac{\hbar c}{R} \left(\frac{E}{mc^2} \right)^3 = \sqrt{\overline{\Delta E^2}} = \sqrt{e^2 H^2 \overline{\Delta R^2}} = \sqrt{eHc\hbar s},$$

from which we obtain for the critical value of the energy the value

$$E_{\rm cr}=E_{1/5}=mc^2\left(\frac{mcR}{\hbar}\right)^{1/5}. \tag{32}$$

The $E_{1/5}$ criterion, as the condition for excitation of radial fluctuations of the electron trajectory, was first found by Sokolov,[41] who began the development of the quantum theory of synchrotron radiation.

Thus, the limits of applicability of the classical theory of synchrotron radiation are determined by three quantities.

1. $E_{1/2}=mc^2(mcR/\hbar)^{1/2}$, which is the limit of applicability of the classical theory of radiation and means physically that the energy of the radiated photon must be less than the energy of the electron: $\hbar\omega\ll E$.

2. $E_{1/5}=mc^2(mcR/\hbar)^{1/5}$, which is the limit of applicability of the equations of motion with allowance for the radiative friction force. Physically, this means that the energy of the emitted photon must be less than the energy of the quantum excitation of the radial degrees of freedom: $\hbar\omega \ll \sqrt{\Delta E^2} = \sqrt{eHc\hbar}$. Otherwise, when $E\sim E_{1/5}$, the radiation reaction force must be modified to take into account the discreteness of the radiation, i.e., the effect of quantum fluctuations of the trajectory. Note that $E_{1/5}$ occurs only in the expression for the probability of quantum transitions and does not occur when one considers the power of the synchrotron radiation. Thus, the number $E_{1/5}$ essentially distinguishes a "quasiquantum" range of energies: The classical description of the radiation is still valid, but the discrete nature of the radiation is already beginning to be manifested in the form of quantum fluctuations of the electron trajectory (quantum "broadening" of the orbit[42]).

3. One must consider especially the region of small quantum numbers, when the quantizing properties of the magnetic field begin to be manifested most strongly. Under these conditions, the dynamical parameter $\chi = (H/H_c)\times(p_\perp/mc) = (H/H_c)^{3/2}\sqrt{2n}$ takes an essentially discrete series of values, and although for $H/H_c\ll 1$ it remains small, a classical description of synchrotron radiation is impossible (see also Ref. 43 for an analysis of the criteria of applicability of the classical theory).

The quantum theory of synchrotron radiation was developed by our group on the fundamental basis of the quantum description of the state of an electron in a homogeneous magnetic field (solution of the Dirac equation) and the rigorous method of quantum electrodynamics (Furry picture). This method, the method of "exact solutions,"[5,7,38] did not lead to any restrictions on the strength of the magnetic field and made it possible to treat the problem even in the range of the fields above the critical Schwinger value ($H\gg H_c$).

This approach and choice of model (homogeneous magnetic field) opened up the possibility of predicting and establishing some essentially new physical effects: quantum fluctuations of the electron trajectory under the conditions of macroscopic motion (Sokolov and Ternov, 1953, Ref. 42), radiative polarization of electrons and positrons moving in storage rings (Sokolov and Ternov, 1963, Ref. 44), and a dynamical nature of the anomalous magnetic moment of the electron (Ternov, Bagrov, Bordovitsyn, and Dorofeev, Ref. 45). The method of exact solutions also made it possible to consider the exotic region of magnetic fields $H\gg H_c$, which is of particular interest in astrophysics, where fields of order $H_c\cong 4\cdot 10^{13}$ Oe are reliably observed. As was shown by Klepikov in 1954,[46] under these conditions new reaction channels are opened: single-photon annihilation and single-photon production of electron–positron pairs.

In order to analyze spin effects, we developed and applied a method of separating the solutions of the Dirac equation (11) with respect to the spin states (see Refs. 7 and 38), in accordance with which the electron wave function satisfies not only the equation $\hat{H}\Psi=E\Psi$ but also the subsidiary equation $\hat{S}\Psi=\zeta\Psi$, where $\hat{S}$ is a polarization operator that commutes with the Hamiltonian. To describe longitudinal polarization (spin projection along the direction of motion of the electron), this operator can be taken to be

$$\hat{T}_0=\vec{\sigma}\hat{\vec{\mathscr{P}}}/|\vec{\mathscr{P}}|, \tag{33}$$

where $\vec{\sigma}$ are the Pauli matrices, and $\hat{\vec{\mathscr{P}}}$ is the kinetic momentum: $\hat{\vec{\mathscr{P}}} = \hat{\vec{p}} - (e/c)\vec{A}$. Then the wave function will satisfy besides (11) the equation $\hat{T}_0\Psi=\zeta_\parallel\Psi$, where $\zeta_\parallel=\pm 1$ corresponds to the two possible orientations of the spin along the direction of motion of the electron or in the opposite direction. To investigate the projection of the spin onto the direction of the magnetic field, it is expedient to introduce the polarization operator

$$\hat{\vec{O}}=\rho_3\vec{\sigma}+\rho_1 c\hat{\vec{\mathscr{P}}}/E-\rho_3 mc^2\hat{\vec{\mathscr{P}}}(\vec{\sigma}\hat{\vec{\mathscr{P}}})/(E(E+mc^2)), \tag{34}$$

which in the electron rest frame becomes the Pauli spin operator. The projection of the spin onto the magnetic field is an integral of the motion [for field $\vec{H}=(0,0,H)$, it is the component $\hat{O}_3$), and the wave function must satisfy the equation $\hat{O}_3\Psi=\zeta_\perp\Psi$, where $\zeta_\perp=\pm 1$ corresponds to the two possible spin orientations: along and opposite to the external field $\vec{H}$.

Quantum theory of radiation at small χ

Using the general methods of the quantum theory of radiation,[7] it is possible to obtain the following expression for the power of the synchrotron radiation:

$$W_i=\frac{e^2c}{2\pi}\sum_{n',s',\zeta',k_3'}\int d\kappa^3\delta(\kappa-\kappa_{nn'})\Phi_i, \tag{35}$$

where $c\hbar\kappa_{nn'}=E_n-E_{n'}$ is the frequency of the radiation; n, s, k_3, ζ are the quantum numbers that determine the state of the electron; and i corresponds to the two polarization components of the photon. The expressions for the function Φ_i ($i=\sigma$, $i=\pi$ are the two components of the linear polarization) have the form

$$\Phi_\sigma=\bar{\alpha}_1^*\bar{\alpha}_1,\quad \Phi_\pi=(\bar{\alpha}_2\cos\vartheta,\bar{\alpha}_3\cos\vartheta),$$

and the matrix elements of the Dirac matrices $\bar{\alpha}_i=\langle n,s,k_3,\zeta|\alpha_i|n',s',k_3',\zeta'\rangle$ are related to the Laguerre functions $I_{nn'}(x)$ and $I_{ss'}(x)$:

$$\left.\begin{matrix}-i\bar{\alpha}_1\\ \bar{\alpha}_2\end{matrix}\right\} = [AI_{n,n'-1}(x) \mp BI_{n-1,n'}(x)]I_{ss'}(x)\delta_{k_3'-\kappa\cos\vartheta}, \quad (36)$$

$$\bar{\alpha}_3 = [CI_{n-1,n'-1}(x) + DI_{nn'}(x)]I_{ss'}(x)\delta_{k_3',k_3-\kappa\cos\vartheta}. \quad (37)$$

in which A, B, C, D are spin coefficients that depend on the choice of the polarization operator. Here, we are not able to dwell on the calculations in more detail (see Ref. 7) and turn to the conclusions.

We first consider the semiclassical approximation.

Semiclassical approximation

In this case $n \gg 1$, the energy spectrum is quasicontinuous, and the dynamical parameter $\chi = (H/H_0)(P_\perp/mc)$ is a continuous quantity. Under these conditions, the sum over n' can be replaced by an integral, $\Sigma_{\nu=0}^{n} \to \int_0^n d\nu$ $(\nu = n - n')$, and after the necessary change of the variable of integration we obtain for the total power of the radiation the expression

$$W = \frac{27}{16\pi}\frac{e^2c}{R^2\varepsilon_0^{9/2}}\int_0^\infty dy\, y^2(1+\xi_0 y)^{-6}\oint d\Omega \sum_i \Phi_i. \quad (38)$$

Note that this expression for the power is valid only in the semiclassical approximation $(n \gg 1)$. In the derivation of this expression, a very important factor was the relation[48]

$$\sum_{s'=0}^{\infty} I_{ss'}^2(x) = 1, \quad (39)$$

by virtue of which the number $E_{1/5}$ disappears from the expression for the radiation power. In the semiclassical region, we may approximate the Laguerre function (36) by means of Euler or Macdonald functions:[7]

$$I_{nn'}(x) = \frac{1}{\pi\sqrt{3}}(1 - x/x_0)^{1/2}K_{1/3}\left(\frac{2}{3}\sqrt[4]{x_0^2 nn'}(1 - x/x_0)^{3/2}\right),$$

where $x_0\sqrt{n} - (n')^{1/2}]$. Summing now over the polarization states of the electron and photon, we obtain

$$W = W^{\mathrm{cl}}\frac{9\sqrt{3}}{8\pi}\int_0^\infty \frac{y\,dy}{(1+\xi y)^3}\left(\int_y^\infty K_{5/3}(x)dx + \frac{\xi^2 y^2}{(1+\xi y)}K_{2/3}(y)\right). \quad (40)$$

Here $W^{\mathrm{cl}} = \frac{2}{3}(e^2c/R^2)(E/(mc^2))^4$, and $\xi = (3/2)\chi$. Thus, the radiation power depends on just the one parameter χ. Integrating over the spectrum, we obtain

$$W = W^{\mathrm{cl}}\left[1 - \frac{55\sqrt{3}}{16}\chi + 48\chi^2 - \ldots\right], \quad \chi \ll 1. \quad (41)$$

The correction in χ was first found by our group[49] and was later confirmed for a spinless electron by Schwinger.[50] It is interesting to note that the quantum corrections have by now been experimentally confirmed. If in the expression (40) we retain the terms that depend on the initial spin orientation, then we obtain[7]

$$W = W^{\mathrm{cl}}\frac{9\sqrt{3}}{8\pi}\int_0^\infty y\,dy\left(\int_y^\infty K_{5/3}(x)dx - \zeta\xi y K_{2/3}(y)\right). \quad (42)$$

We have here retained only the terms linear in ξ. In 1977, a method was proposed at the Institute of Nuclear Physics of the Siberian Branch of the USSR Academy of Sciences for measuring the spin dependence of synchrotron radiation.[51] As follows from (42), the additional power of the synchrotron radiation associated with orientation of the spin relative to the direction of the magnetic field $(\zeta_\perp = \zeta)$ has the form

$$\Delta = \frac{|\zeta|\xi y K_{1/3}(y)}{\int_y^\infty dx K_{5/3}(x)}. \quad (43)$$

The most favorable possibility for observation is opened up in the short-wavelength region of the spectrum: $y \gg 1$ (since $y \to y^{\mathrm{cl}} = \frac{2}{3}\nu(mc^2/E)^3$). Then, remembering the asymptotic behavior $K_\mu(x) = \sqrt{\pi/2x}e^{-x}$ of the Macdonald function, we readily find that

$$\Delta = |\zeta|\xi y, \quad \xi = \frac{3}{2}\chi = \frac{3}{2}\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^2.$$

In the case of observation of the radiation of a beam of electrons, $|\zeta|$ in this expression characterizes the mean polarization of the beam that it acquires in the process of radiation. Therefore, observing the power of the synchrotron radiation at a fixed frequency in the spectrum, one can determine the polarization characteristics. It is particularly advantageous to carry out the experiment in a storage ring, since the radiative energy losses of the electrons are compensated in the ring by the external sources of the rf field. As a result, the particle energy remains on the average constant. Thus, the quantum corrections to the synchrotron power [see (42)] can be regarded as an experimentally established fact, and the actual method of observing them is one of the interesting ways of determining the polarization properties of electron and positron particle beams.

Macroscopic quantum fluctuations of an electron trajectory

Thus, in the case when the energy of a radiated photon is sufficient for excitation of radial degrees of freedom of the electron, $\mathscr{E}_\phi = \sqrt{\Delta E^2} = eH\sqrt{R^2 - \bar{R}^2}$, i.e., $E \sim E_{1/5}$, radial transitions are excited, and the classical theory of the motion of the particle becomes invalid, since it cannot explain the fluctuating nature of the radiation reaction.

Returning to the expression (40) for the power of the synchrotron radiation, we divide the integrand by the photon energy $\hbar\omega$ and find that the probability of quantum transitions has the form

$$\frac{dw_{ss'}}{dy} = \frac{\sqrt{3}}{2\pi}\frac{c^2}{\hbar R}\frac{E}{mc^2}\frac{1}{(1+\xi y)^2}\left(\int_y^\infty K_{5/3}dx + \frac{\xi^2 y^2}{(1+\xi y)}K_{2/3}(y)\right)I_{ss'}^2\frac{\xi_1 y^2}{(1+\xi y)^2}, \quad (44)$$

where $\xi_1 = \frac{9}{8}(E/E_{1/5})^5$ (see Refs. 7 and 30).[2)] We have here restored the radial factor $I^2_{ss'}(z)$, which on summation over s' becomes unity and does not contribute to the synchrotron power. The radial factor $I^2_{ss'}$, reflects the stochastic nature of the excitation of the radial degrees of freedom of the electron. This becomes particularly clear if one assumes that in the initial state the center of the orbit of electron gyration is at the origin ($s=0$). Then

$$I^2_{ss'}(x) = \frac{e^{-x}x^{s'}}{s'!}, \quad x = \frac{\xi_1 y^2}{(1+\xi_1)^2}, \quad \xi_1 = \frac{9}{8}\frac{\hbar}{mcR}\left(\frac{E}{mcr}\right)^5$$

takes the form of an ordinary Poisson distribution.

In the approximation of the classical theory ($\hbar \to 0$), the argument in this formula also vanishes, and therefore in the limit

$$\lim_{x\to 0} I^2_{0,s'}(x) = \delta_{0s'},$$

in other words, the radial number s is not subject to change in the classical theory. If the orbit at the initial time was circular, it remains such at subsequent times too. Therefore, from the point of view of the classical theory the radiation will have as a consequence only a reduction in the radius of the orbit, and in accelerators and storage rings this is compensated by the external rf electric field, and fluctuations of the radius of the orbit do not occur:

$$s = \frac{eH}{c\hbar}\,(\overline{R^2} - (\bar{R})^2) = \text{const.}$$

The problem of the change in the squared fluctuation of the radius was solved for the first time by the methods of quantum theory by Sokolov and one of the present authors[42] on the basis of the method of exact solutions of the Dirac equation. The result was the effect of quantum "broadening" of the trajectory.

Using the expression (44), it is readily seen that the change in the radial quantum number s has the form

$$\frac{ds}{dt} = \int_0^\infty dy \sum_{s'} (s'-s) w_{ss'}.$$

Then, bearing in mind that $\Sigma^\infty_{s'=0} I^2_{ss'}(x)(s' - s) = x$, we obtain[39]

$$\frac{ds}{dt} = \frac{d}{dt}\left\{\frac{eH}{c\hbar}\,(\overline{R^2} - (\bar{R})^2)\right\}. \tag{45}$$

In the case of semiclassical motion, when $\xi \ll 1$, it follows from this that

$$\frac{ds^{\text{semicl}}}{dt} = \frac{55}{48\sqrt{3}}\frac{e^2}{mcR^2}\left(\frac{E}{mc^2}\right)^5 \tag{46}$$

or

$$\overline{\Delta R^2} = \frac{55}{48\sqrt{3}} = \frac{e^2}{mc^2}\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^5 t.$$

Thus, for a constant mean energy of the electron the stochastic process of radiation leads to an expression characteristic of Brownian motion and reflecting the effect on the particle of random forces: $\overline{x^2} = 2Dt$, where D is the diffusion coefficient. It should be noted here that this effect is similar to the influence of a vacuum electromagnetic field on an electron. Vacuum fluctuations of the field, as was particularly clearly emphasized by Bogolyubov and Tyablikov,[52] induce a certain smearing or effective radius of an electron. At the same time, the electron executes a Brownian motion with a definite square of the displacement.

The change of the squared fluctuation of the radius can be regarded as a macroscopic manifestation of quantum fluctuations of the radiation—a consequence of the discrete nature of photon emission and the recoil effect to which the electron is subject. The electrons move in the radial direction in accordance with quantum laws, whereas the motion in the circle remains on the average classical ("macroscopic atom"[7]).

However, the situation is changed if the radiative energy losses of the electron are not compensated. This occurs, for example, under astrophysical conditions and, in particular, in a pulsar magnetosphere. In this case, it is interesting to emphasize that by virtue of two processes—the radiative reduction in the radius of the electron orbit and the growth of the squared fluctuation—a certain equilibrium is established. Since $R^2 = (2c\hbar/(eH))(n+s+1/2)$, we find that

$$\frac{dR^2}{dt} = \frac{2c\hbar}{eH}\left[\frac{dn}{dt} + \frac{ds}{dt}\right], \tag{47}$$

where ds/dt is determined by (46), and the change of the principal quantum number n is related to the change in the energy:

$$\frac{dn}{dt} = -\frac{H_c}{H}\frac{EW}{(mc^2)^2}, \quad W = \frac{2}{3}\frac{e^2c}{R^2}\left(\frac{E}{mc^2}\right)^4.$$

Then it follows from (47) that $dR^2/dt = 0$ under the condition of a minimum value of the radius:

$$R_{\min} = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}\left(\frac{E}{mc^2}\right)^2.$$

This expression is analogous to the case of electron motion in the focusing field of a storage ring,[7] when besides the quantum broadening of the particle trajectory forces of radiative damping act, and the amplitude of the electron fluctuation reaches a certain minimum value. It should be said that the quantum excitation of radial fluctuations is important in engineering practice in the construction of storage rings, determining the condition of focusing of beams. Quantum fluctuations were observed in the United States (Sands[53]) and in the USSR (Korolev[54]), in complete agreement with the prediction of the theory (for more details, see Ref. 30). The $E_{1/5}$ criterion for the onset of their excitation was also confirmed.

Radiative polarization of electrons and positrons in storage rings

Radiative polarization of electrons and positrons takes the form of a directed process of orientation of the particle spins under the influence of synchrotron radiation during prolonged circulation in storage rings. This effect was predicted by one of the present authors (Ternov[55]) and was rigorously established on the basis of exact solutions of the Dirac equation separated with respect to polarization states in a collaboration with Sokolov.[56]

Calculation of the probability of quantum transitions per unit time[7,30] accompanied by spin flip (spin-flip transitions) leads to a characteristic dependence of the probability on the initial orientation of the spin ζ with respect to the direction of the magnetic field:

$$\frac{dw^{\uparrow\downarrow}}{d\Omega\, dy}=\frac{3}{32\pi}\frac{e^2}{\hbar R}\frac{\varepsilon}{\varepsilon_0^3}\xi^2y^3[\cos^2\vartheta K_{1/3}^2(z)+(\sqrt{\varepsilon}K_{2/3}(z)$$
$$+\zeta\sqrt{\varepsilon_0}K_{1/3}(z))^2], \qquad (48)$$

where

$$\varepsilon=1-\beta^2\sin\vartheta,\quad \varepsilon_0=1-\beta^2,\quad z=\tfrac{1}{2}y(\varepsilon/\varepsilon_0)^{3/2},$$
$$y=\tfrac{2}{3}\nu\varepsilon_0^{3/2},\quad \omega=\nu\omega_0.$$

An explicit dependence on the spin orientation also remains in the integrated probability (see Refs. 50, 7, and 27):

$$w_\zeta^{\uparrow\downarrow}=\frac{1}{2\tau}\left(1+\zeta\frac{8\sqrt{3}}{15}\right), \qquad (49)$$

where the polarization time has the form

$$\tau=\frac{8\sqrt{3}}{15}\frac{\hbar^2}{mce^2}\left(\frac{mc^2}{E}\right)^2\left(\frac{H_c}{H}\right)^3=\frac{15\sqrt{3}}{16}\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^2\frac{W}{E}\xi^{-1}.$$

It follows from this that because of the radiation the electrons will have a tendency to go over to states with preferred spin orientation in the opposite direction to the magnetic field.[55] For positrons, the spin orientation will be in the other direction. States with the preferred spin orientation correspond to the minimum value of the potential energy of particles possessing magnetic moment $\mu=(e_0\hbar/(2mc))\zeta$ in the magnetic field:

$$U=-\frac{e}{|e|}\mu H.$$

Here, we shall not dwell in detail on the kinetics of the polarization process (see Refs. 56 and 57) and, turning to an ensemble of electrons, we shall characterize the polarization of a beam of particles by the mean value $\zeta(t)=\langle\zeta\rangle$, bearing in mind that the interaction of an electron with the electromagnetic field leads to transition of the particle to a mixed state. We then find

$$\frac{d}{dt}\zeta(t)=\sum_{\zeta'}(\zeta'-\zeta)w_\zeta^{\uparrow\downarrow}=-2\sum_\zeta \zeta w_\zeta^{\uparrow\downarrow},$$

from which it follows that

$$\zeta(t)=-\frac{8\sqrt{3}}{15}(1-e^{-t/\tau}), \qquad (50)$$

and the limiting degree of polarization (when $t\gg\tau$) reaches values

$$P(\infty)=8\sqrt{3}/15=0.924.$$

The estimate of the polarization time τ shows that for the magnetic field strengths typical of accelerators ($H\sim 10^4$ Oe) the polarization effect is accessible to observation only in the case of prolonged circulation of the particles in the magnetic field (of order 1 h). Such a possibility is realized in storage rings, and although under actual conditions of a storage ring there are phenomena that depolarize the particle beam, the effect of radiative polarization does exist in them and provides a unique basis for creating polarized electrons and positrons of high energy. Radiative polarization has been observed in the USSR, France, Germany, the United States, and Japan, in good agreement with the Bonet theory; these questions are considered in detail in our review of Ref. 57.

7. ASPECTS OF SYNCHROTRON RADIATION IN EXTREMELY STRONG MAGNETIC FIELDS ("MAGNETIC BREMSSTRAHLUNG")

We now consider the limiting ultraquantum case when the dynamical invariant parameter $\xi=(3/2)\chi$ takes large values: $\chi=(H/H_c)(p_\perp/mc)\gg 1$. This region is of great interest in connection with astrophysical problems associated with radiation of high-energy electrons in an ultrastrong magnetic field $H\sim H_c=4\cdot 10^{13}$ Oe.

We return to the expression (40) for the power of the synchrotron radiation. After integration by parts, this expression for the "magnetic bremsstrahlung" takes the form

$$W=W^{\rm cl}\frac{9\sqrt{3}}{8\pi}\int_0^\infty dy\left[\frac{1}{2}\frac{y^2}{(1+\xi y)^2}K_{5/3}(y)\right.$$
$$\left.+\frac{\xi^2y^2}{(1+\xi y)^4}K_{2/3}(y)\right]. \qquad (51)$$

We have here deliberately introduced the expression "magnetic bremsstrahlung," since in the case $\xi\gg 1$ the properties of this radiation are very different from the ordinary properties of synchrotron radiation.

Indeed, let us find an expression for $W^{\rm ulqu}$ in the ultraquantum region, when $\xi\gg 1$. We note that the functions $y^2K_{5/3}(y)$ and $y^3K_{2/3}(y)$ reach a maximum at $y\sim 1$, after which they decrease exponentially. In the ultraquantum case, by virtue of the factor $(1+\xi y)^2$ in the denominator, the spectrum is truncated at $y\ll 1$. Then for the Macdonald functions $K_\mu(y)$ we can use the asymptotic expression $K_\mu(y)\cong 2^{\mu-1}\Gamma(\mu)/y^\mu$, and, integrating in (51) by means of the integral

$$\int_0^\infty dy\, y^{p-1}(1+\xi y)^{-q}=\frac{1}{\xi^p}\frac{\Gamma(p)\Gamma(q-p)}{\Gamma(q)},$$

we find[47,58,7]

$$W^{\rm ulqu}=\frac{16}{27}\frac{\Gamma(2/3)}{3^{1/3}}\frac{W^{\rm cl}}{\chi^{4/3}}. \qquad (52)$$

Thus, in the ultraquantum case the power of the magnetic bremsstrahlung is strongly suppressed ($\chi\gg 1$). In contrast to the semiclassical case ($\chi\ll 1$), when the quantum effects yielded only small corrections to the classical expressions

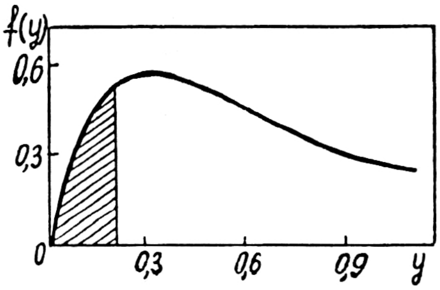

FIG. 5. Schematic spectral distribution of synchrotron power.

[see (41)], in (52) already the main term is a quantum one, and transition to the classical approximation is here impossible.

To obtain a more transparent interpretation of our result, we consider the form of the radiation spectrum. By means of the standard methods of quantum theory, we can readily obtain an expression for the frequency of the radiation:

$$\omega_{\max}=\frac{3}{2}\frac{c}{R}\frac{\beta y_{\max}}{\varepsilon_0^{3/2}}\left(1+\frac{3}{2}\beta y_{\max}\chi\right)^{-1}$$
$$\cong\frac{3}{2}\frac{c}{R}\frac{1}{\varepsilon_0^{3/2}\left(1+\frac{3}{2}\chi\right)}, \qquad (53)$$

where $\beta y_{\max}$ is of the order of unity. From this there follow two limiting cases:

$$\omega_{\max}=\begin{cases}\frac{3}{2}\frac{c}{R}\left(\frac{E}{mc^2}\right)^3, & \chi\ll 1,\\ E/\hbar, & \chi\gg 1.\end{cases} \qquad (54)$$

Thus, in the ultraquantum case the frequency of the radiation terminates at values $\omega'=E/\hbar$ much less than the characteristic frequency of synchrotron radiation $\omega'_{\max}\ll\omega^{\text{cl}}_{\max}$ (see Ref. 58), since $\omega'\cong\omega^{\text{cl}}_{\max}/\chi$. In other words, the electron cannot radiate an energy greater than its self-energy E. Figure 5 shows the graph of the spectral distribution of the power of the synchrotron radiation $W=W^{\text{cl}}\int_0^\infty f(y)dy$ in the classical approximation, when

$$f(y)=\frac{9\sqrt{3}}{8\pi}y\int_y^\infty K_{5/3}(x)dx.$$

The characteristic features of this curve are determined by its asymptotic behavior:

$$f(y)=\frac{9\sqrt{3}}{8\pi}\begin{cases}2^{2/3}\Gamma(2/3)y^{1/3} & \text{for } y\ll 1,\\ \sqrt{\pi/2}\sqrt{y}e^{-y} & \text{for } y\gg 1.\end{cases}$$

Since

$$f^{\text{ulqu}}(y)=\frac{9\sqrt{3}}{8\pi}\frac{\Gamma(2/3)}{2^{1/3}}\left[\frac{2}{3}\frac{y^{1/3}}{(1+\xi y)^2}+\frac{\xi^2y^{7/3}}{(1+\xi y)^4}\right]$$

[this follows when $y\ll 1$ from (51)], we can see that there is a similarity of these curves in the region of small values of y. For comparison, we find the energy of the radiation in accordance with the classical theory, restricting the integral to the area of the curve up to a certain $y_{\max}=\frac{2}{3}(R/c)(E/\hbar)\varepsilon_0^{3/2}$. Then we obtain

$$W^{\text{semicl}}=\frac{3\sqrt{3}}{4\pi}2^{2/3}\Gamma(2/3)\frac{e^2c}{R^2\varepsilon_0^2}\int_0^{y_{\max}}y^{1/3}dy$$
$$=\frac{3\cdot 3^{7/6}}{8\pi}\Gamma(2/3)\frac{W^{\text{cl}}}{\chi^{4/3}}.$$

If we compare this with the exact expression (52), the difference is slight:

$$\frac{W^{\text{semicl}}}{W^{\text{ulqu}}}=\frac{3^{7/6}\cdot 3\cdot 3^{1/3}\cdot 3^3}{8\pi\cdot 16}=1.04.$$

Thus, in this model the decrease in the power of the synchrotron radiation in the ultraquantum case acquires an admittedly formal but very transparent interpretation.

Thus, the spectrum of the radiation and its power are very different from those of ordinary synchrotron radiation. We now consider the angular distribution of the power and compare it with the corresponding characteristics of synchrotron radiation, which, as is well known,[7] possesses a "projector effect": The radiation is directed forward along the motion of the electron and is concentrated in a narrow cone with opening angle $\Delta\vartheta\sim mc^2/E$.

However, in the ultraquantum region, with the parameter $\chi\gtrsim 1$, the position is changed greatly. Indeed, let us return to the expression (40) for the radiation power and restore in it the integration over the angle ϑ. Then for the spectral–angular distribution of the radiation power we obtain

$$W=\frac{9}{8\pi^2}\frac{e^2c}{R^2}\frac{1}{\varepsilon_0^{9/2}}(J_1+J_2),$$

$$J_1=\int_0^\infty dyy^2(1+\xi y)^{-3}\int_0^\pi\sin\vartheta\, d\vartheta(\varepsilon^2K^2_{2/3}(z)$$
$$+\varepsilon\cos^2\vartheta K^2_{1/3}(z)), \qquad (55)$$

$$J_2=\frac{\xi^2}{2}\int_0^\infty dyy^4(1+\xi y)^{-4}\int_0^\pi\sin\vartheta\, d\vartheta(\varepsilon^2K^2_{2/3}(z)$$
$$+\varepsilon\cos^2\vartheta K^2_{1/3}(z)+\varepsilon\varepsilon_0K^2_{1/3}(z)).$$

Here $z=(y/2)(\varepsilon/\varepsilon_0)^{3/2}$, $\varepsilon=1-\beta^2\sin^2\vartheta$, $\varepsilon_0=1-\beta^2$, $\xi=\frac{3}{2}\xi$. To analyze the angular distribution of the radiation power, we introduce a new variable, setting $x=(y/2)(\varepsilon/\varepsilon_0)^{3/2}$ or $y=2x(\varepsilon_0/\varepsilon)^{3/2}$. Further, introducing the angle $\vartheta=\pi/2-(\varepsilon_0)^{1/2}\Psi$, we have for small $(\varepsilon_0)^{1/2}\Psi\varepsilon=\varepsilon_0(1+\Psi^2)$, $\cos^2\vartheta=\varepsilon_0\Psi^2$, and we then find that

$$J_1=8\varepsilon_0^5\int_0^\infty dxx^2\int_{-\infty}^\infty\frac{d\Psi}{\left[1+\frac{2\xi x}{(1+\Psi^2)^{3/2}}\right]}\left[\frac{K^2_{2/3}(x)}{(1+\Psi^2)^{5/2}}+\frac{\Psi^2K^2_{1/3}(x)}{(1+\Psi^2)^{7/2}}\right],$$

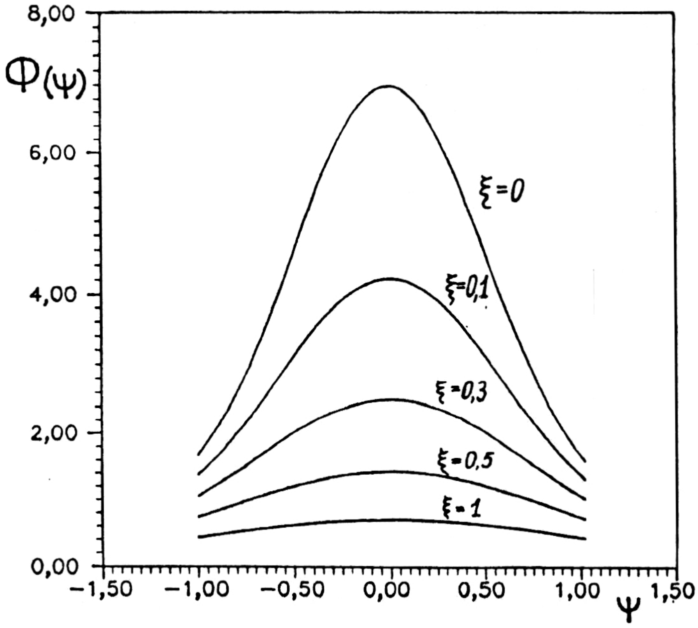

FIG. 6. Scheme of the projector effect in synchrotron radiation and transition to the quantum case.

$$J_2=16\varepsilon_0^8\xi^2\int_0^\infty x^4dx\int_{-\infty}^\infty \times\frac{d\Psi}{\left[1+\frac{2\xi x}{(1+\Psi^2)^{3/2}}\right]^4}\frac{(K_{2/3}^2(x)+K_{1/3}^2(x))}{(1+\Psi^2)^{11/2}}. \quad (56)$$

Since the Macdonald functions $K_{1/3}$ and $K_{2/3}$ are nonzero only for an argument x of order unity, and at larger values of x these functions decrease exponentially, the limits of integration on the transition to an integration over Ψ, which varies in the range from $-\beta/\varepsilon_0$ to β/ε_0, can be extended to infinity. Unfortunately, exact integration over the spectrum is difficult, and therefore for the qualitative analysis of the angular distribution of the radiation power we estimate the integrals, making the assumption that at the maximum of the integrand in (56), i.e., for $\chi\sim 1$, we can set $x=1$ in the denominator, and then the integrals over the spectrum can be calculated. By means of the integral

$$\int_0^\infty x^{\mu-1}K_\rho^2(x)dx=\frac{2^{\mu-3}}{\Gamma(\mu)}\Gamma\left(\rho+\frac{\mu}{2}\right)\Gamma\left(\frac{\mu}{2}-\rho\right)\Gamma^2\left(\frac{\mu}{2}\right)$$

we find that

$$W=\frac{e^2c}{16R^2\varepsilon_0^2}\int_{-\infty}^\infty \Phi(\Psi)d\Psi, \quad (57)$$

where

$$\Phi(\Psi)=\frac{\left\{7(1+\Psi^2)^2+5\Psi^2(1+\Psi^2)+\frac{35\xi^2\sqrt{1+\Psi^2}}{[(1+\Psi^2)^{3/2}+2\xi]}\right\}}{[(1+\Psi^2)^{3/2}+2\xi]^3}.$$

In the classical approximation, when $\xi\to 0$, we obtain from this the well-known formula in the theory of synchrotron radiation for the angular distribution of the power characterized by the projector effect with all the radiation concentrated in a narrow cone, and the deflection of the radiation cone from the direction of the velocity vector of the particle is very small: $\Psi\sim(\varepsilon_0)^{1/2}=mc^2/E$,

$$\Phi^{\text{cl}}(\Psi)=\frac{7}{(1+\Psi^2)^{5/2}}+\frac{5\Psi^2}{(1+\Psi^2)^{7/2}}$$

(see Ref. 7). However, in the quantum case the sharpness of the radiation peak is strongly smoothed, and for $\xi\sim 1$ the pronounced directionality of the radiation disappears altogether (Fig. 6). Thus, under the conditions of the ultraquantum case the dependence on the angle ϑ of the radiation becomes weakly expressed.

Further, we consider the polarization properties of the radiation and compare them with the case of semiclassical motion of an electron, when the synchrotron radiation has a pronounced linear polarization. As is well known (Refs. 7 and 31), $\frac{7}{8}W^{\text{cl}}$ is the so-called σ component of the radiation, characterized by the fact that the electric vector of the radiation lies in the plane of the orbit of gyration of the particle and is directed along the radius to the center of the orbit, while $\frac{1}{8}W^{\text{cl}}$ corresponds to the π component, for which the electric vector is directed almost along the direction of the external magnetic field.

We now return to the expression (55) for the spectral–angular distribution of the radiation power. After integration over the angle with allowance for the components of the linear polarization,[7] this takes the form

$$W_{\sigma,\pi}^{\text{qu}}=W^{\text{cl}}\int_0^\infty dy\, f_{\sigma,\pi}(y), \quad (58)$$

$$f_{\sigma,\pi}(y)=\frac{9\sqrt{3}}{16\pi}\frac{y}{(1+\xi y)^3}\left\{\int_y^\infty K_{5/3}(x)dx\pm K_{2/3}(y)+\frac{\xi^2y^2}{1+\xi y}K_{2/3}(y)\right\}.$$

In the approximation of classical theory, $\xi\to 0$, we obtain the well-known result, in accordance with which $W_\sigma=\frac{7}{8}W^{\rm cl}$, $W_\pi=\frac{1}{8}W^{\rm cl}$. We now consider this expression in the ultraquantum case, when $\xi=(3/2)\chi\gg 1$. Then the Macdonald functions K_μ must be taken in the asymptotic limit $y\ll 1$:

$$W_{\sigma,\pi}^{\rm ulqu}=W^{\rm cl}\frac{9\sqrt{3}}{16\pi}\frac{\Gamma(2/3)}{2^{1/3}}\int_0^\infty dy\left[\frac{2}{3}\frac{y^{1/3}}{(1+\xi y)^2}\pm\frac{y^{1/3}}{(1+\xi y)^3}+\frac{\xi^2y^{7/3}}{(1+\xi y)^4}\right].$$

After integration, we obtain[58]

$$W_\sigma^{\rm ulqu}=\tfrac{41}{64}W^{\rm ulqu},\quad W_\pi^{\rm ulqu}=\tfrac{23}{64}W^{\rm ulqu},\tag{59}$$

where $W^{\rm ulqu}=2^{8/3}\Gamma(/3)W^{\rm cl}/(9\xi^{4/3})$. Thus, the preferred polarization of the radiation is maintained, although the difference between the radiation powers of the σ and π components is somewhat smoothed out: $W_\sigma/W_\pi\sim 5/3$ instead of 7/1.

The case of circular polarization is very interesting. One finds[58] that

$$W^{\rm ulqu}=\tfrac{1}{2}W^{\rm ulqu}(1+\tfrac{11}{16}l\zeta),\tag{60}$$

where $l=\pm 1$ corresponds to the sign of the circular polarization of the photons, and $\zeta=\pm 1$ to the sign of the longitudinal polarization of the electron. We see that electrons with longitudinal spin orientation will emit photons predominantly with the same circular polarization. For unpolarized electrons, circular polarization will be absent.

The properties of the radiation considered here require essentially the case of macroscopic motion, when the quantum number $n=\gamma R^2=eHR^2/(2c\hbar)$ is very large. In this case of semiclassical motion of the electron, the dynamical parameter $\chi=(H/H_c)(p_\perp/mc)$ takes continuous values, since the electron energy spectrum is quasicontinuous:

$$\chi=\left(\frac{H}{H_c}\right)^{3/2}\sqrt{2n},\quad n\gg 1.$$

However, it is obvious that one can also consider a different range of energy values in which the quantum effects will have a strong influence. This is the region of small quantum numbers $n=0$, 1, 2,..., i.e., electron states for which the magnetic field manifests its quantizing properties. If we assume an extremely strong magnetic field, $H\gg H_c$, then we have the parameter $\chi>1$, remaining discrete, and the electron has the relativistic spectrum

$$E=mc^2\sqrt{1+2nH/H_c}$$

for any $n\neq 0$, and in this connection it is not valid to treat the radiation as a continuous process. This confirms once more that the model of an alternating field can give a description of the radiation of the electron only in the semiclassical approximation. The expression

$$W^{\rm ulqu}=2^{2/3}\Gamma(2/3)W^{\rm cl}/(9\xi^{4/3})$$

for the radiation power obtained for the case of a purely magnetic field was also found by considering the radiation of an electron in a crossed field, but this method cannot bring out the quantizing properties of the field.

We consider some results relating to the calculation of the intensity of the radiation of an electron in a weakly excited state in a strong magnetic field. This problem was solved by the present authors in collaboration with Bagrov[59,60] by means of the method of exact solutions. We first of all consider the weak-field case: $H<H_c$. Then for the radiation power we obtain the expression

$$W=\frac{2n-1}{2n}W^{\rm cl},\quad W^{\rm cl}=\frac{2}{3}\frac{eH^2}{m^2c^3}\frac{\beta^2}{1-\beta^2},\quad \beta<1,\tag{61}$$

which differs more strongly from the classical expression, the lower the number n of the energy level.

It is not difficult to calculate the lifetime of the excited state. Indeed, since the quantum transitions in the considered case ($\beta<1$) have a dipole nature, the emission process is accompanied by transitions to a neighboring level. For the total lifetime of the electron in the state n, we obtain

$$T(n)=\sum_{k=1}^n 1/w(n),$$

where $w(n)=\frac{2}{3}[(2k-1)/T_0](H/H_c)^2$, and $T_0=\hbar^2/(mce^2)=1.7\cdot 10^{-19}$ sec. Then

$$T_n(\bar{n})=\frac{3}{2}\frac{m^3c^5}{e^4H^2}\sum_{k=1}^n\frac{1}{2k-1}=\frac{3}{2}\frac{m^3c^5}{e^4H^2}\left\{\frac{C}{2}+\ln 2\sqrt{n}+\frac{1}{48n^2}\cdots\right\},$$

where C is Euler's constant. Thus, the lifetime of the electron in the excited state does not differ strongly from the classical value $T^{\rm cl}=\frac{3}{2}(m^3c^5/e^4H^2)$. This expression can be obtained by integrating the classical result

$$W^{\rm cl}=-\frac{\partial E}{\partial t}=\frac{2}{3}\frac{e^4H^2}{m^4c^7}(E^2-m^2c^4)\quad\frac{E}{mc^2}=\coth\left(\frac{t+t_0}{T^{\rm cl}}\right).$$

For fields $H\sim 10^4$ Oe, the lifetime of the excited state is approximately 5 sec.

In the case of an ultrastrong field, $H>H_c$, the electron is relativistic already when it is in the first excited level, and therefore the energy of the emitted photon in the dipole transition $n=1\to 0$ is practically equal to the energy of the particle. Unfortunately, in the case of an ultrastrong field $H>H_c$ the calculations can be made only numerically (see Refs. 59, 60, and 61). The following expressions are obtained:

$$W_\sigma=0.742W, \quad W_\pi=0.258W,$$

$$W=0.302\frac{e^2m^2c^3}{\hbar^2}\left(\frac{E}{mc^2}\right)^2.$$

It is important to emphasize that these expressions agree neither with the classical expression (61) nor with the radiation power (52) in the ultraquantum case. Thus, the radiation of electrons in the first excited states has important properties that can be described neither by the classical theory nor even by the semiclassical method. The results presented here may be of interest in astrophysical problems. In particular, in pulsar magnetospheres, in contrast to storage rings, the radiative energy losses of the electrons are not compensated. Therefore, the radiation of weakly excited charges in extremely strong magnetic fields can be particularly interesting, and we shall return once more to this question.

8. SPIN EFFECTS IN A STRONG MAGNETIC FIELD

Evolution of the spin of a relativistic particle

As is well known (see, for example, Ref. 30), the spin of a particle can, from the point of view of a physical interpretation, be described in a nonrelativistic theory, since the spin angular momentum possesses the property of an integral of the motion—it is conserved in time. This opens up the basic possibility of investigating spin effects independently of the orbital motion of particles. We recall in this connection that in accordance with the hypothesis of Uhlenbeck and Goudsmit an electron possesses a spin mechanical angular momentum not associated with displacement of the particle in space.

However, the situation is changed on transition to relativistic motion of a particle, since in Dirac's relativistic theory[62] there is conservation of only the total angular momentum

$$\vec{J}=\vec{L}=\vec{S}=[\vec{r}\vec{p}]+\frac{\hbar}{2}\vec{\sigma}.$$

The vectors $\vec{L}$ and $\vec{S}$ acquire a transparent physical meaning only in the nonrelativistic approximation. In this sense, Dirac's relativistic theory reveals an inseparability of the properties of orbital and spin motion—the spin angular momentum is not conserved independently of the orbital angular momentum.

The reason for such behavior of the orbital and spin angular momenta in Dirac's theory resides in the special oscillating nature of the motion of relativistic particles. This leaves an imprint on all the operator quantities, endowing them with unusual properties that make the physical interpretation of the operators and an understanding of their connection with the classical dynamical variables difficult.

In this connection, it is sensible to introduce in Dirac's single-particle theory operators with definite parity, making the assumption that any operator can be represented as a sum of an even, $|\hat{F}|$, and an odd, $\{\hat{F}\}$, part:

$$\hat{F}=[\hat{F}]+\{\hat{F}\},$$

where the even and odd parts are determined by means of the sign operator

$$\hat{\Lambda}=\hat{\mathscr{H}}/|\mathscr{H}|=\hat{\mathscr{H}}/E,$$

where $\hat{\mathscr{H}}=c(\vec{\alpha}\vec{p})+\beta_3mc^2$ is the operator of the Hamiltonian in Dirac's theory.

Then the even part of the operator does not mix states of the particle belonging to different signs of the energy,

$$[\hat{F}]=\tfrac{1}{2}(\hat{F}+\hat{\Lambda}\hat{F}\hat{\Lambda}), \tag{62}$$

commutes with the Hamiltonian, and is an integral of the motion, while the odd part

$$\{\hat{F}\}=\tfrac{1}{2}(\hat{F}-\hat{\Lambda}\hat{F}\hat{\Lambda}) \tag{63}$$

anticommutes with the Hamiltonian and oscillates in time, reflecting in these oscillations the interference of charge-conjugate states:

$$\frac{d}{dt}\{F\}=\frac{i}{\hbar}(\hat{\mathscr{H}}\{\hat{F}\}-\{\hat{F}\}\hat{\mathscr{H}})=-\frac{2i}{h}\{\hat{F}\}\hat{\mathscr{H}},$$

whence

$$\{\hat{F}\}=\{F(0)\}\exp(-2iEt/\hbar).$$

The oscillation frequency is $\Omega=2E/h$.

By means of these expressions, we readily obtain

$$[\vec{\alpha}]=c\vec{p}/E, \quad \{\vec{\alpha}\}=c\{\vec{\alpha}(0)\}\exp(-i\Omega t),$$

and then

$$\begin{aligned}\frac{d\vec{L}}{dt}&=c[\vec{\alpha}\vec{p}]=c[\vec{\alpha}(0)\vec{p}]\exp(-i\Omega t),\\ \frac{d\vec{S}}{dt}&=-c[\vec{\alpha}(0)\vec{p}]\exp(-i\Omega t).\end{aligned} \tag{64}$$

Thus, both the orbital and spin angular momenta oscillate rapidly (zitterbewegung) but on the average are conserved in time. Since, further, $\hat{x}=c\alpha_x$, integrating the expression for the operators $|\vec{\alpha}|$ and $\{\vec{\alpha}\}$, we obtain the characteristic (Schrödinger) interpretation of zitterbewegung:

$$\langle x\rangle=\frac{c^2p_1t}{E}=\frac{\hbar c}{2E}\{\alpha_1(0)\}\sin\Omega t,$$

in accordance with which the "macroscopic coordinate" $|x|=c^2p_1t/E$ characterizes the classical trajectory and the "microscopic coordinate" $\{x\}$ describes the zitterbewegung around this trajectory. In all this discussion, we have considered only the free motion of a particle. We now consider the behavior of the spin in an external magnetic field; to this end, we shall characterize the spin states by means of a unit three-dimensional spin vector

$$\hat{\vec{O}}=\rho_3\vec{\sigma}+\rho_1c\vec{p}/E-\rho_3c^2\vec{p}(\vec{\sigma}\vec{p})/(E(E+mc^2)), \tag{65}$$

which admits a simple generalization to the case of an electron in a magnetic field. For this it is sufficient to make the substitution $\vec{p}\to\hat{\vec{\mathscr{P}}}=\vec{p}-(e/c)\vec{A}$. The operator $\hat{\vec{O}}$ is one of the possible polarization operators by means of which one can separate a solution of the Dirac equation with respect to the spin states (for more detail, see Refs. 7, 30, and 31). The operator $\hat{\vec{O}}$ in the electron rest frame goes over into the Pauli spin operator, by virtue of which it retains the same properties in the laboratory coordinate system:

$$\hat{O}_i\hat{O}_j-\hat{O}_j\hat{O}_i=\varepsilon_{ijk}\hat{O}, \quad (\hat{\vec{O}}\vec{S})^2=1.$$

The introduction of this operator corresponds to Darwin's idea of describing the electron spin by the requirement that in the electron rest frame this operator should become the Pauli spin.

As is well known, the evolution of the spin of an electron in an external field can be described by means of the Bargmann–Michel–Telegdi (BMT) equation (see Refs. 63–65). We succeeded[65] in generalizing this equation to take into account the electron zitterbewegung. We consider now the evolution of the spin operator in the Heisenberg representation, noting first of all that in the Heisenberg representation the variation of any operator $\hat{F}$ in time can be represented in the form[55,65]

$$\frac{d\hat{F}}{dt}-\left[\frac{d\hat{F}}{dt}\right]=\frac{i\hbar}{2E}\frac{d^2\hat{F}}{dt^2}. \tag{66}$$

Making now an identification with the operator of the three-dimensional spin, $\hat{F}=\hat{\vec{O}}$, and taking into account the anomalous part of the magnetic moment, we obtain

$$\hat{\mathscr{H}}=c(\vec{\alpha}\vec{\mathscr{P}})+\rho_3mc^2+\frac{\alpha}{2\pi}\mu_0\rho_3(\vec{\sigma}\vec{H}),$$

$$\frac{d\hat{\vec{O}}}{dt}=\frac{ec}{E}\left(1+\frac{\alpha}{2\pi}\gamma\right)[\vec{O}\vec{H}]-\frac{\alpha}{2\pi}\frac{e}{mc}\frac{c^2[\vec{O}\vec{\mathscr{P}}](\vec{\mathscr{P}}\vec{H})}{E(E+mc^2)}+\frac{i\hbar}{2E}\frac{d^2\vec{O}}{dt^2}. \tag{67}$$

In the limit $\hbar\rightarrow0$, this exact Heisenberg equation for the even part of the operator of the derivative is functionally identical to the classical BMT equation. Further, going over to the expectation values with respect to a wave packet, $\langle[\vec{O}]\rangle=\zeta$, $\langle c\vec{\mathscr{P}}\rangle=\vec{\beta}E$, we obtain a generalization of the BMT equation in the form (Ref. 65; see also Ref. 55)

$$\dot{\vec{\zeta}}=\frac{ec}{E}\left(1+\frac{\alpha}{2\pi}\gamma\right)[\vec{\zeta}\vec{H}]-\frac{\alpha}{2\pi}\frac{e}{mc}\frac{\gamma}{1+\gamma}[\vec{\zeta}\vec{\beta}](\vec{\beta}\vec{H})+\frac{i\hbar}{2E}\ddot{\vec{\zeta}}, \tag{68}$$

where $\gamma=E/mc^2$. This equation gives a complete description of the evolution of the electron spin, including zitterbewegung. Moreover, as in the Dirac representation, one can consider a magnetic field, including one that is extremely strong. Omitting here consideration of the anomalous magnetic spin moment, and writing the evolution equation in the form

$$\ddot{\vec{\zeta}}+\frac{2iE}{\hbar}\dot{\vec{\zeta}}-\frac{2ie_0c}{\hbar}[\vec{H}\vec{\zeta}]=0 \tag{69}$$

(here, $e=-e_0$ is the electron charge), we obtain in the case of a homogeneous magnetic field a solution of (69) in the form

$$\zeta_1+i\zeta_2=C_1\exp[i(\omega_1-\omega)t]+C_2\exp[-i(\omega_1+\omega)t],$$

$$\zeta_1-i\zeta_2=C_3\exp[i(\omega_2-\omega)t]+C_4\exp[-i(\omega_2+\omega)t],$$

$$\zeta_3=\zeta_{\parallel}+\dot{\zeta}_3(0)\sin 2\omega t/2\omega,$$

where $\hbar\omega_1=\sqrt{E^2+2e_0c\hbar H}$, $\hbar\omega_2=\sqrt{E^2-2e_0c\hbar H}$, $\hbar\omega=E$. It is important to emphasize that in the general case the precession of the spin in the magnetic field and the Schrödinger zitterbewegung cannot be separated—the spin projections vary in accordance with a periodic law with frequencies $\omega_1\pm\omega$, $\omega_2\pm\omega$. This becomes particularly clear when the strength of the magnetic field approaches the critical value $H_c=m^2c^3/e\hbar$. In this case, the frequency is

$$\hbar\omega_2=(m^2c^4+2e_0\hbar cHn-2e_0\hbar cH)^{1/2}$$

and approximate extraction of the root is ruled out. In the ground state ($n=0$), the frequency becomes imaginary. Only if the spin precession frequency $\Omega=e_0Hc/E$ is much less than the frequency $\omega=2E/\hbar$ of the Schrödinger zitterbewegung ($H\ll H_c$) can the solution of Eq. (69) be written approximately in the form

$$\zeta_1(t)-\zeta_{\perp}\cos(\Omega t+\alpha)=\frac{\Omega}{2\omega}\zeta_{\perp}\sin\alpha\sin 2\omega t,$$

$$\zeta_2(t)-\zeta_{\perp}\sin(\Omega t+\alpha)=-\frac{\Omega}{2\omega}\zeta_{\perp}\cos\alpha\sin 2\omega t,$$

from which it follows that the Schrödinger zitterbewegung is superimposed on the macroscopic motion of the spin—the precession in the magnetic field. Thus, an extremely strong magnetic field has a special influence on the spin behavior—it becomes impossible to separate the spin motion from the orbital motion.

We also emphasize one further feature associated with an ultrastrong magnetic field, which is that the interaction energy of the spin magnetic moment in the ultrastrong magnetic field loses the usual property of small quantum corrections

$$|U|=\mu_0|(\vec{\sigma}\vec{H})|=\mu_0\frac{m^2c^3}{e\hbar}=\frac{1}{2}mc^2 \quad \text{as } H\rightarrow H_c,$$

i.e., it reaches the ordinary values characteristic of orbital motion.

Spin effects in the synchrotron radiation of an electron in an extremely strong magnetic field

We have seen that in an extremely strong magnetic field, under conditions of relativistic motion of the electron, the spin and orbital properties of the particle can no longer be separated. This contradicts the hypothesis of Uhlenbeck and Goudsmit, in accordance with which it is postulated that the spin characteristics of a particle are not related to its displacement in space. It is characteristic that in Dirac's theory, in its quasirelativistic approximation, a spin–orbit interaction is already manifested in terms that are $\sim\beta^2$, and if the motion is ultrarelativistic, the coupling of the spin and orbital properties of the electron becomes extremely relevant.

We consider above all in this connection the probability of quantum transitions of an electron accompanied by spin flip. We shall assume that the electron is polarized, $\zeta=\pm 1$ characterizing the projection of the polarization operator onto the direction of the magnetic field. We recall that the projection operator $\hat{O}$,

$$\frac{\hat{O}H}{H}\Psi=\zeta\Psi,$$

possesses the property of an integral of the motion (transverse polarization).

If the invariant dynamical parameter satisfies $\xi=(3/2)\chi\ll 1$, then for the probability of quantum transitions we obtain the expression[7,39,58]

$$\underset{(\chi\ll 1)}{w}=\frac{5\sqrt{3}}{6}\frac{e^2}{c\hbar}\frac{c}{R}\frac{E}{mc^2}\left\{\frac{1+\zeta\zeta'}{2}\left[1-\left(\frac{55\sqrt{3}}{24}+\zeta\right)\xi+\xi^2\left(20-\frac{245\sqrt{3}}{18}\zeta\right)\right]+\frac{(1-\zeta\zeta')}{2}\frac{4}{3}\xi^2\left(1+\frac{35\sqrt{3}}{64}\zeta\right)\right\}.$$

We note first of all that the probability of spontaneous spin-flip transitions is proportional to ξ^2, i.e., to the square of Planck's constant:

$$\omega^{\uparrow\downarrow}\sim\xi^2=\left[\frac{3}{2}\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^2\right]^2.$$

These transitions make a small contribution to the classical theory of synchrotron radiation, but nevertheless they lead to observable effects and, in particular, explain the phenomenon of radiative polarization of electrons and positrons in storage rings.[7,57] As is well known, the mean energy of an electron remains constant in a storage ring, the radiative losses being compensated by the external source of the rf electric field. For this reason, the motion of a particle may last for a long time (tens of hours), and it is this that creates the necessary conditions for the realization of radiative polarization, since the polarization time is of the order of an hour.

We consider further in the same approximation $\xi\ll 1$ the power of the synchrotron radiation:

$$W=W^{\rm cl}\left\{\frac{1+\zeta\zeta'}{2}\left[1-\left(\frac{55\sqrt{3}}{24}+\zeta\right)\xi+\xi^2\left(20-\frac{245\sqrt{3}}{18}\zeta\right)...\right]+\frac{1-\zeta\zeta'}{2}\frac{4}{3}\xi^2\left(1+\frac{35\sqrt{3}}{64}\zeta\right)\right\}. \quad (70)$$

As one would expect, the spin-flip transitions have the order ξ^2. This can be given a qualitative explanation, namely, if we consider the radiation power of a classical magnetic moment precessing around a magnetic field (Bordovitsyn[66]),

$$W=\frac{2}{3}\frac{\mu^2}{c^3}\overline{(\ddot{\zeta})^2}=\frac{2}{3}\frac{\mu_0^2}{c^3}\omega^4\zeta_1^2,$$

and assume that the magnetic moment is the spin magnetic moment $\vec{\mu}=-\vec{\mu}_0\zeta$ ($\mu_0=e_0\hbar/(2mc)$) of an electron, i.e., a Bohr magneton, then with allowance for the precession law

$$\dot{\vec{\zeta}}=-\frac{2\mu_0}{\hbar}[\vec{\zeta}\vec{H}_c],\quad \omega_0=e_0H_c/(mc)$$

we can obtain in the laboratory coordinate system

$$W=W^{\rm cl}\tfrac{1}{9}\xi^2,$$

i.e., in the total radiation power there is a contribution of the radiation of the magnetic spin moment of the electron, and this radiation is accompanied by spin flip (the rigorous quantum problem of the radiation of a neutral particle possessing a magnetic moment in a homogeneous magnetic field was considered in Ref. 67). At the same time, it should be noted that in accordance with (70) the power of the synchrotron radiation accompanied by spin-flip transitions depends on the initial spin orientation, in full agreement with the considered qualitative model of the radiation of a magnetic moment. This interesting fact was first noted by one of us in Ref. 55, and it was used in the development of the theory of radiative polarization of electrons.

Also noteworthy is the dependence of the synchrotron power on the spin orientation in the first order of the quantum corrections in ξ. As we have already noted, measurement of one dependence made it possible to estimate the degree of polarization of a beam of electrons moving in a storage ring.[51]

Averaging over the initial spin state ζ and summing over the final state ζ', we obtain an expression for the total power of the radiation in the case when $\xi=(3/2)\chi\ll 1$:

$$W=W^{\rm cl}\left\{1-\frac{55\sqrt{3}}{24}\xi+\frac{64}{3}\xi^2-...\right\}. \quad (71)$$

This expression is obtained for an electron; if one considers a spinless electron (the wave function satisfies the Klein–Gordon equation), one finds a difference in the terms proportional to $\hbar$ (i.e., ξ^2), as was pointed out by Matveev.[69] Indeed, we observe the following difference between the powers of the synchrotron radiation of an electron (el) and of a spinless electron (KG) (Fig. 7):

$$W=W^{\rm cl}\int_0^\infty f(y)dy,$$

$$f^{\rm el}(y)=\frac{9\sqrt{3}}{8\pi}\frac{y}{(1+\xi y)^3}\left[\int_y^\infty K_{5/3}(x)dx+\frac{\xi^2y^2}{1+\xi y}K_{2/3}(y)\right],$$

$$f^{\rm KG}(y)=\frac{9\sqrt{3}}{8\pi}\frac{y}{(1+\xi y)^3}\int_y^\infty K_{5/3}(x)dx.$$

(These are exact expressions, valid for any ξ.)

It is obvious that the difference

$$\Delta f=9\sqrt{3}\xi^2\frac{y^3}{(1+\xi y)^4}K_{2/3}(y) \quad (72)$$

of these expressions is explained by the different statistics satisfied by electrons and bosons. This difference also contributes to the synchrotron power: If $\xi\ll 1$, then $\Delta W=(8/3)W^{\rm cl}\xi^2$. As we have already noted, the first-order terms are the same for the electron and the boson (the spinless electron).

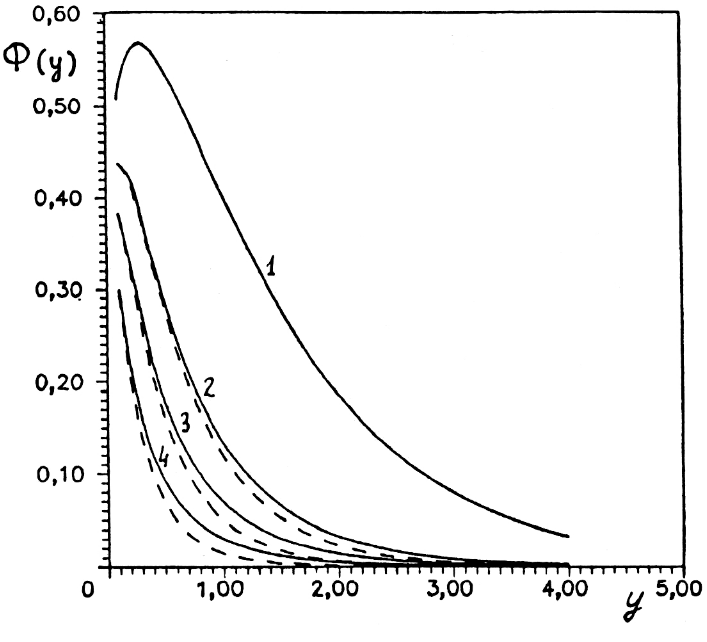

FIG. 7. Graph of the spectral distribution of synchrotron radiation. The broken curves correspond to the Klein–Gordon solution. The numbers in the graph correspond to the following values of ξ: 1) $\xi=0$; 2) $\xi=0.1$; 3) $\xi=0.5$; 4) $\xi=1$; 5) $\xi=2$.

We now turn to the case of an ultrastrong field, for which the dynamical parameter satisfies $\xi=(3/2)\chi>1$. Under these conditions, spin-flip transitions have the same order as those without spin flip:

$$W=W^{\rm ulqu}\left\{\frac{25}{32}\frac{1+\zeta\zeta'}{2}+\frac{7}{32}\frac{1-\zeta\zeta'}{2}\right\}, \tag{73}$$

and $W^{\rm ulqu}$ is determined by (52). It is interesting to note that the difference between the electron and boson radiation powers contributes to the basic expression for W:

$$\Delta W=\frac{7}{16}W^{\rm ulqu}.$$

Thus, in the case of an ultrastrong field the spinless electron radiates $(9/16)W^{\rm ulqu}$, i.e., only about half the value given by Dirac's theory.

We now consider spin effects in the synchrotron radiation of electrons in low levels. As is shown by the analysis of Refs. 59, 60, and 68, the spin plays a particularly interesting role on a transition of an electron from the first excited level $n=1$ to the ground state $n=0$. In the ground state [see (13)], the electron spin can be directed only in the opposite direction to that of the magnetic field: $\zeta=-1$. If now in the first excited state $(n=1)$ the spin of the particle is oriented along the field direction, then a transition to the ground state $n=0$ can occur only with reorientation of the spin, but the time of such a transition ($\xi\ll1$, unexcited energy levels) is very long. For a field of strength 10^4 Oe, it is $\sim10^{10}$ sec. If, however, the electron in the excited state $n=1$ has spin along the direction opposite to the field, then transition to the state $n=0$ will occur without spin flip and in a short time.

Therefore, the state with $n=1$ and $\zeta=1$ is metastable, and the lifetime of this state is very long, $\sim10^{10}$ sec. Effectively, the electron does not go over to the ground state. Thus, one can make the following picture of the spontaneous transitions of electrons in the magnetic field. The electrons go over to a lower state effectively without change of the spin orientation. After a time $T=\frac{3}{2}(H_c/H)^2(\hbar^2/me^2c)$, all electrons with spin oriented in the direction opposite to the magnetic field go over to the ground state $n=0$. Electrons whose spin is oriented along the direction of the field go over to the first excited state $n=1$ and remain there for a very long time $\sim10^{10}$ sec (for a field of strength 10^4 Oe).

Thus, a system of noninteracting and nonradiating electrons in a magnetic field must consist of two subsystems: electrons with spin oriented along the field direction, in the state $n=1$, and electrons with spin in the opposite orientation in the state $n=0$. The energy gap $\Delta E=mc^2H/H_c$ between these states for a field $H=10^4$ Oe is 10^{-4} eV.

In an ultrastrong magnetic field, this picture is radically changed: For $H>H_c$, the probability of spontaneous transitions does not depend on the spin orientation, and spin-flip transitions occur with the same probability[70]

$$w=\frac{0.421}{T_0}\left(\frac{E}{mc^2}\right),\quad T_0=\frac{\hbar^2}{mce^2}=1.7\cdot10^{-19}\ \text{sec},$$

as those without spin flip. At the same time, it is characteristic that the expression for the probability for a "spinless" electron differs numerically from the above expression:

$$w^{\rm KG}=\frac{0.159}{T_0}\left(\frac{E}{mc^2}\right).$$

Thus, the spin makes only a statistical contribution to the probability of the quantum transitions.

We consider the radiative polarization of electrons and positrons under the conditions of a strong magnetic field. A closed expression for the degree of radiative polarization, valid for the complete range of the parameter χ, was first obtained by Bagrov[71] (see also Refs. 72 and 6). Here, we shall consider only the limiting cases. It follows from the general expressions of the quantum theory of synchro-

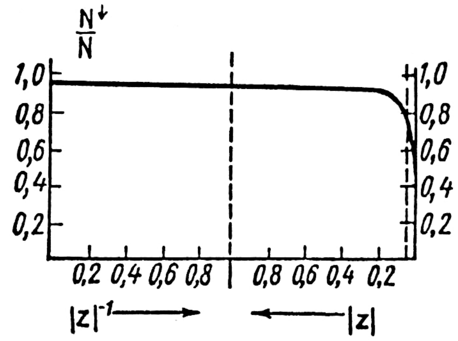

FIG. 8. Degree of self-polarization of the electron spin as function of the parameter $z=2\chi/3$.

tron radiation that the exact expression for the probability of quantum transitions accompanied by a change in the transverse polarization of the electron can be represented in the form

$$w^{\uparrow\downarrow}=\frac{\sqrt{3}}{4\pi}\frac{e^2}{\hbar R}\left(\frac{E}{mc^2}\right)\xi^2\int_0^\infty\frac{y^2dy}{(1+\xi y)^3}(K_{2/3}+\zeta K_{1/3}), \tag{74}$$

where $\xi=\frac{3}{2}(\hbar/mcR)(E/mc^2)=\frac{3}{2}\chi$, and $\zeta=\pm1$ characterizes the projection of the spin onto the direction of the magnetic field. We consider this expression in the two limiting cases $\chi\ll1$ and $\chi\gg1$. For $\chi\ll1$ we obtain the well-known result (see Ref. 7)

$$w^{\uparrow\downarrow}=\frac{\sqrt{3}}{4\pi}\frac{e^2}{\hbar R}\left(\frac{E}{mc^2}\right)\xi^2\int_0^\infty y^2dy(K_{2/3}+\zeta K_{1/3})$$
$$=\frac{1}{2\tau}\left(1+\zeta\frac{8\sqrt{3}}{15}\right), \tag{75}$$

where $1/\tau=(5\sqrt{3}/8)(W^{\mathrm{cl}}/E)\xi$ is the reciprocal of the polarization time. In the other limiting case $\chi\gg1$,

$$w^{\uparrow\downarrow}=\frac{\sqrt{3}}{4\pi}\frac{e^2}{\hbar R}\left(\frac{E}{mc^2}\right)\xi^2\int_0^\infty dy\left[\frac{2^{-1/3}\Gamma(2/3)y^{4/3}}{(1+\xi y)^3}+\frac{2^{-2/3}\Gamma(1/3)y^{5/3}}{(1+\xi y)^3}\right]$$
$$=\frac{1}{2\tau^*}\left[1+\frac{5}{2}\zeta\frac{\Gamma(1/3)}{\Gamma(2/3)(3\chi)^{1/3}}\right],\quad \frac{1}{\tau^*}=\frac{9}{16}\frac{W^{\mathrm{ulqu}}}{E}. \tag{76}$$

The degree of polarization of the beam in an ultrastrong magnetic field is greatly reduced:

$$\underset{\chi\ll1}{p}=8\sqrt{3}/15,\quad \underset{\chi\gg1}{p}=\frac{5}{2}\frac{\Gamma(1/3)}{\Gamma(2/3)}(3\chi)^{-1/3}. \tag{77}$$

The general behavior of the curve that characterizes the change in the degree of polarization (Fig. 8) is interesting.[6] It can be seen from Fig. 8 that in the case of prolonged circulation of a beam of electrons in a magnetic field a large proportion of the electrons acquires spin orientation in the direction opposite to the field. The percentage of polarized electrons is very high ($8\sqrt{3}/15=0.92$) and varies weakly with variation of the parameter z. It is only for very large values of the parameter that $p\to0$ and $N^{\downarrow}=N/2$ and the beam is unpolarized. The polarization effect in an ultrastrong field may be of interest in the physics of pulsar magnetospheres.

We also consider the correlation of the longitudinal electron polarization (the spin projection onto the direction of the velocity: $[(\vec{\sigma}\mathscr{P})/\mathscr{P}]\Psi=\zeta\Psi$) with the circular polarization of the radiated photons.

As is well known (see Ref. 7), the operator of the longitudinal polarization is an integral of the motion of an electron in a magnetic field and describes the particle helicity. To this end, we consider the power of the synchrotron radiation with allowance for the circular polarization of the photons:[58]

$$W_l=\frac{9}{64\pi}\frac{l^2c}{R^2\varepsilon_0^{9/2}}\int_0^\infty\frac{y^2dy}{(1+\xi y)^4}\int_0^\pi\sin\vartheta\, d\vartheta\,\Phi_l,$$

where $\varepsilon_0=1-\beta^2$, $l=\pm1$ ($l=1$ and $l=-1$ correspond to right and left circular polarizations, respectively), and

$$\Phi_l=\frac{1+ll'}{2}[2+\xi y(1+l\zeta)]^2(\cos\vartheta\sqrt{\varepsilon}K_{1/3}+l\varepsilon K_{2/3})^2$$
$$+\frac{1-ll'}{2}\varepsilon_0\varepsilon\xi^2y^2(1+l\zeta)K_{1/3}^2. \tag{78}$$

Here, the functions K_μ depend on the argument $z=\frac{1}{2}y(\varepsilon/\varepsilon_0)^{3/2}$, $\varepsilon=1-\beta^2\sin^2\vartheta$. We note that radiation with spin flip, $\zeta\zeta'=-1$, will always possess circular polarization agreeing with longitudinal polarization of the electron, since the expression (78) contains the factor $(1+l\zeta)$.

In the ultrarelativistic case ($\xi\gg1$), it is particularly interesting to investigate the circular polarization for high harmonics, when $y\xi\gg1$. Then

$$\underset{\xi y\gg1}{\Phi_l}=\xi^2y^2(1+l\zeta)\left[\frac{1+ll'}{2}(\cos\vartheta\sqrt{\varepsilon}K_{1/3}+l\varepsilon K_{2/3})^2+\frac{1-ll'}{2}\varepsilon_0\varepsilon K_{1/3}^2\right]. \tag{79}$$

For these harmonics, the circular polarization of the radiation not only with but also without spin flip is strictly correlated with the initial direction of the longitudinal polarization of the electrons.

It also follows from the expression (78) that in the classical approximation the circular polarization disappears. This can be seen by integrating (78) over the angle ϑ. Then

$$W_l=W^{\mathrm{cl}}\int_0^\infty dy\, f_l(y),$$

where

$$f_l(y)=\frac{9\sqrt{3}}{16\pi}\frac{y}{(1+\xi y)^4}\left\{\frac{1+ll'}{2}\left[1+(1+l\zeta)\left(\xi y+\frac{1}{2}\xi^2y^2\right)\right]\times\int_y^\infty K_{5/3}dx+\frac{1-ll'}{2}(1+l\zeta)\frac{\xi^2y^2}{2}\int_y^\infty K_{1/3}dx\right\}.$$

In the limit $\xi \to 0$, the circular polarization vanishes. However, the situation is entirely different on the transition to the ultraquantum case, when the parameter $\xi = (3/2)\chi$ takes large values. In this case

$$W_l^{\text{ulqu}} = \tfrac{1}{2} W^{\text{ulqu}} (1 + \tfrac{11}{16} l \xi), \tag{80}$$

i.e., electrons with longitudinal spin orientation will radiate photons predominantly with the same circular polarization. For unpolarized electrons, circular polarization of the radiation will be absent. The polarization will be partial:

$$P = \frac{W_1^{\text{ulqu}} - W_{-1}^{\text{ulqu}}}{W_1^{\text{ulqu}} - W_{-1}^{\text{ulqu}}} = \frac{11}{16} \xi.$$

Thus, in an ultrastrong magnetic field it is possible to observe circular polarization of synchrotron radiation independently of the angle of observation. However, this requires an initial longitudinal polarization of the electrons, and therefore observation of circular polarization of the synchrotron radiation in this case would make it possible to obtain information about the longitudinal spin orientation of the particles.

We have already mentioned the possibility that has now been realized of observing transverse polarization of electrons through the properties of the synchrotron radiation, namely, its dependence on the spin projection of the particles onto the direction of the magnetic field.[51] This dependence occurs in the quantum corrections (proportional to χ) to the classical expression for the radiation power. In an ultrastrong magnetic field, the longitudinal polarization occurs not in the form of corrections but in the main expression, and this opens up special possibilities for observation. Quite generally, the polarization properties are a very characteristic indicator of synchrotron radiation, and they are associated with great successes of radio astronomy, since, in particular, polarization can serve as a criterion in an experimental verification of hypotheses about the nature of radio radiation.

Observation of circular polarization of synchrotron radiation in an ultrastrong magnetic field (for example, in the case of the magnetic field of pulsars) opens up a way of investigating longitudinal polarization of electrons. However, this problem is not simple because of physical considerations. The fact is that the longitudinal polarization of electrons is an integral of the motion only approximately, when one does not take into account the anomalous magnetic moment (AMM) of the electron:

$$\mu = -\mu_0 (1 + a_e), \quad a_e = \frac{g-2}{2},$$

with the Schwinger value $a_e = \alpha / 2\pi$ of the anomaly. The anomalous magnetic moment of the electron plays a very special role in the particle spin dynamics. Indeed, using the evolution equation for the spin, one can find that the change in the spin projection onto the electron velocity, $\vec{\mathscr{P}} + \vec{p} - (e/c)\vec{A}$, has the form

$$\frac{d}{dt} \vec{\zeta} \vec{\mathscr{P}} = \frac{eH}{mc} a_e [\vec{\mathscr{P}} \vec{\zeta}] = \frac{eH}{mc} \frac{\alpha}{2\pi} [\vec{\mathscr{P}} \vec{\zeta}],$$

where $\zeta = [O]$ is the even part of the spin operator O, from which it follows that the electron AMM destroys the longitudinal polarization. A more detailed analysis[55] shows that

$$\frac{\vec{\zeta} \vec{\mathscr{P}}}{|\mathscr{P}|} = \cos \frac{\alpha}{2\pi} \frac{eH}{mc} = \cos \left(a_e \frac{eH}{mc} t \right). \tag{81}$$

Thus, under ordinary conditions the longitudinal polarization of an electron is not conserved in time, and observation of it is ruled out.

However, as will now be shown (see Ref. 45), in an ultrastrong magnetic field the anomalous magnetic moment manifests its dynamical nature, and its value decreases with increasing field and even vanishes at certain values of the field strength H. At the same time, the integral-of-the-motion properties for the longitudinal polarization are restored, since $a_e \to 0$. Thus, observation of circular polarization of synchrotron radiation, which does not depend on the angle of observation (in an ultrastrong field), could give interesting information on the spin properties of particles.

9. DYNAMICAL NATURE OF THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

We first of all consider briefly the question of why it is regarded as important to study the deviation of the value of the anomalous magnetic moment from the Bohr magneton: $\mu = \mu_0 (I + a_e)$, $a_e = (g-2)/2$. Unquestionably important here is the strong, and sometimes decisive, influence of the AMM on the dynamics of the spin of a particle in an external field. This follows directly from the Bargmann–Michel–Telegdi spin evolution equation. In particular, under the influence of the AMM there arise depolarization resonances capable of destroying the transverse polarization of a beam of electrons acquired while they move in a storage ring. At the same time, if the spin precession frequency is a multiple of the frequencies characteristic of the orbital motion (the frequencies of the betatron radial, ω_r, vertical, ω_z, and phase, ω_φ, oscillations) the transverse polarization is destroyed and because of the depolarization the radiative polarization effect does not lead to a preferred electron spin orientation. The resonance condition[73]

$$a_e \frac{eH}{mc} = N_1 \omega_r + N_2 \omega_z + N_3 \omega_\varphi, \quad N_i \text{ are integers,}$$

depends strongly, as can be seen from the expression, on a_e.

The AMM has an even stronger influence on the longitudinal polarization of the electron—the spin projection of the particle onto the direction of its motion. As we have just shown, as a result of this influence the longitudinal polarization is no longer conserved in time. Physically, this is due to the fact that the frequency of the gyration of the particle in the magnetic field, $\dot{\vec{\mathscr{P}}} = (ec/E)[\vec{H} \vec{\mathscr{P}}]$, and the frequency of the spin precession, $\dot{\vec{\zeta}} = (ec/E)(1 + a_e E / mc^2)[\vec{\zeta} \vec{H}]$, are not equal, and this leads to the expression (81).

As we have already noted, the vacuum AMM of the electron also plays an important role in the stability of the electromagnetic vacuum. The solution of the generalized Dirac–Pauli equation

$$\left\{i\gamma^{\mu}\left(\frac{\partial}{\partial x_{\mu}}+ieA_{\mu}^{\text{ext}}\right)-m+a_{e}\mu H/m\right\}\Psi=0, \qquad (82)$$

where $a_e=\alpha/2\pi$ is the anomalous part of the magnetic moment, leads to a solution in which the spin degeneracy is lifted:[31]

$$E=\pm mc^{2}\left\{1+\frac{H}{H_{c}}(2n+\zeta+1)^{1/2}-\zeta a_{e}\mu H/mc^{2}\right\}.$$

(We have omitted here the motion of the particle along the field.) If, now, one takes the Schwinger value of the anomalous moment, then in the ground state ($n=0$, $\zeta=-1$) the vacuum becomes unstable if $(\alpha/4\pi)(H/H_c)=1$, since

$$E=\pm mc^{2}\left(1+\frac{\alpha}{4\pi}\frac{H}{H_{c}}\right).$$

However, a more detailed analysis shows that in a strong magnetic field ($H\rightarrow H_c$) the energy gap $E=\pm mc^2$ between the states not merely does not decrease but actually increases. The fact is that the electron AMM manifests its dynamical nature and $a_e=f(E,H)$ becomes dependent on the magnetic field—in an ultrastrong external magnetic field the electron AMM decreases strongly.

The anomalous moment of the electron is also of great interest in connection with the development of a high-precision measuring technique, opening up the possibility of investigating not only the terms of order $\alpha=e^2/\hbar c$ but also the higher approximations in α:

$$\mu=-\mu_{0}\left(1+a_{1}\frac{\alpha}{H}+a_{2}\left(\frac{\alpha}{H}\right)^{2}+a_{3}\left(\frac{\alpha}{H}\right)^{2}+\dots\right).$$

The dynamical nature of the AMM may lead to a contribution accessible to observation, since the influence of the magnetic field is appreciable on the background of the high powers of the expansion with respect to the square of the charge. In the single-loop approximation, the electron AMM was first calculated by Schwinger,[74] but in the general case the magnetic moment of the electron is a rather complicated function that depends on the strength of the external magnetic field. Attention was first drawn to this circumstance by Gupta,[75] who in the first order of perturbation theory with respect to α considered not only the linear but also the higher terms in the expansion of the magnetic moment with respect to the characteristic parameter a^{-1}, where $a=H_c/2H$, and H_c is the critical field.

It should be emphasized that in all the early studies[74,75] the nonrelativistic approximation of the problem was considered. This comes out especially clearly in the work of Luttinger[76] in which an electron in an external field is taken in a "ground" state in which the energy of the orbital motion of the particle and the energy of the interaction of the magnetic moment with the external field compensate each other.

In this connection, one can expect that in the case of relativistic motion of the electron this magnetic moment is not only a function of the field strength but may also depend on the electron energy; finally, we note that in all the cited studies only the case of a weak field, with the strength H restricted to the interval $0<H<H_c$, was considered. In the light of the recent discoveries of extremely strong magnetic fields near pulsars, and also under laboratory conditions (as we have already briefly mentioned), it is of interest to investigate the value of the AMM in a strong magnetic field.

We now consider the problem of the anomalous magnetic moment of the electron in the first order of perturbation theory in α by a method that is free of the limitations just noted. We note that the problem of the dynamical nature of the electron AMM was posed and solved for the first time in our collaboration with Bagrov and Bordovitsyn.[45]

We base our study of the electron anomalous magnetic moment on the integrodifferential Dirac–Schwinger equation

$$\left[i\gamma^{\mu}\left(\frac{\partial}{\partial x_{\mu}}+ieA_{\mu}^{\text{ext}}\right)-m\right]\Psi(x)=\int M(x,y)\Psi(y)dy, \qquad (83)$$

the right-hand side of which contains the interaction of the electron with the electromagnetic vacuum and includes self-energy effects and corrections associated with vacuum polarization by the external field. The mass operator in the first order in e^2 of perturbation theory for quantum electrodynamics,

$$M(x,y)=-ie^{2}\gamma^{\mu}S^{c}(x,y)\gamma^{\nu}D_{\mu\nu}^{c}(y-x),$$

contains the electron propagator S^c with allowance for the external magnetic field,

$$\left[i\gamma^{\mu}\left(\frac{\partial}{\partial x_{\mu}}+ieA_{\mu}^{\text{ext}}\right)-m\right]S^{c}(x,y)=-\delta^{4}(x-y),$$

and also the photon propagator $D_{\mu\nu}^c(y-x)$.

We shall use the Feynman representation for the electron and photon propagators:

$$S^{c}=\begin{cases} i\sum\limits_{E_{n}>0}\Psi_{n}(\vec{r})\bar{\Psi}_{n}(\vec{r}')\exp[-iE_{n}(t-t')], & t>t', \\ -i\sum\limits_{E_{n}<0}\Psi_{n}(\vec{r})\bar{\Psi}_{n}(\vec{r}')\exp[-iE_{n}(t-t')], & t<t', \end{cases}$$

where

$$\bar{\Psi}_{n}=\Psi_{n}^{+}\rho_{3},$$

$$D_{\mu\nu}^{c}=(x-x')$$

$$=-\frac{i}{2}\frac{\delta\mu\nu}{(2\pi)^{3}}\int\exp[in(\vec{r}-\vec{r}')-i\omega(t-t')]\frac{d^{3}\kappa}{\omega}.$$

In these expressions, $\Psi_n=\Psi_n(\vec{r})\exp(-iE_nt/\hbar)$ are stationary electron functions satisfying the Dirac equation unperturbed by the radiative corrections. The sum over n for $t>t'$ includes states with only positive energy, while for

$t > t'$ the causal function is equal to the same sum with the opposite sign, taken over the states with negative energy (positron states).

Integration over time leads to the time-independent equation

$$(E-\hat{\mathscr{H}})\Psi=\int K(\vec{r},\vec{r}')\Psi(\vec{r}')d^3\vec{r}=\hat{R}\Psi(\vec{r}), \qquad (84)$$

where the kernel $K(\vec{r},\vec{r}')$ has the form

$$K(\vec{r}-\vec{r}')=\frac{e^2}{4\pi^2}\sum_{n',\varepsilon}\int\frac{d^3\kappa}{\kappa}\frac{1}{K_n-\varepsilon(K_{n'}+\kappa)} \times e^{i\vec{\kappa}\vec{r}}\alpha_\mu\Psi_{n'}(\vec{r})\Psi^+_{n'}(\vec{r}')e^{i\vec{\kappa}\vec{r}}\alpha_\mu,$$

$E=c\hbar K$ is the electron energy, and $\varepsilon=\pm 1$ is the sign of the energy in the intermediate states, with $\varepsilon=1$ for the electron states and $\varepsilon=-1$ for the positron states.

We mention here that the simplest way of calculating the mass operator is the linear approximation in the magnetic field strength and the nonrelativistic approximation. One can then readily find that

$$\int M(x,y)\Psi(y)dy=-\frac{\alpha}{2\pi}\mu_0\sigma^{\mu\nu}F_{\mu\nu}\Psi(x),$$

and we arrive at the Dirac–Pauli equation (82). In this equation, the additional interaction of the magnetic moment with the field is manifested locally, while the dynamical nature of μ remains hidden. To reveal the dynamical origin of the electron AMM, and also to reveal the nonlocality of the interaction that generates it, we must return to Eq. (84) in its full form, giving up the limit of a quasistatic field. In this way we will also establish the limit of applicability of the Dirac–Pauli equation for an electron in the approximation linear in the field, since sometimes the dynamical nature of the AMM is ignored and the AMM is automatically included in the Dirac–Pauli equation even in the case of a strong electromagnetic field. This then leads to difficulties in explaining the vacuum stability.

We return to the Dirac equation (84). Regarding the right-hand side of this equation as a perturbation, we can see that the radiative corrections to the energy are completely determined by the matrix elements of the operator $\hat{R}$, which characterizes the effective energy of the vacuum interaction:

$$W_{\zeta\zeta'}=\int\Psi^+_{n\zeta'}(\vec{r})\hat{R}\Psi_{n\zeta}(\vec{r})d^3r,$$

where the indices ζ, $\zeta'=\pm 1$ characterize the initial and final spin orientations. We must consider the spin-dependent field correction to the electron mass, since this part of the mass arises because the electron possesses a vacuum magnetic moment.[45] As a result of calculations, we obtain

$$\Delta m^{Sp}=W^{Sp}_{\zeta\zeta'}=\frac{e^2}{4\pi}\sum_{n,\varepsilon=\pm 1}\int_0^\infty\int_0^\pi\frac{\kappa d\kappa\sin\vartheta d\vartheta}{K-\varepsilon(K'+\kappa)} \times(D_{-1}D'_{-1}-D_1D'_{-1})\Phi, \qquad (85)$$

where D are spin coefficients determined by wave functions that are exact solutions of the Dirac equation separated with respect to the spin states by means of the operator $\hat{O}_3$:

$$\hat{O}_3\Psi=\zeta\Psi,\quad \zeta=\pm 1,$$

$$\Phi=\frac{k_0}{K}\left[\left(1-\varepsilon\frac{K}{K'}\right)\frac{n'-n}{y}-\varepsilon\frac{K}{K'}\right][I^2_{nn'}(y)-I^2_{n-1,n'-1}(y)],$$

and the Laguerre function is related to the Laguerre polynomials $Q_n^{n-n'}(y)$ by

$$I_{nn'}(y)=\frac{1}{\sqrt{n!n'!}}e^{-y/2}y^{(n-n')/2}Q_n^{n-n'}(y).$$

The part of the vacuum operator $\hat{R}$ associated with the anomalous magnetic moment is usually replaced in the nonrelativistic approximation by the operator $\hat{R}'=\mu\rho_3(\vec{\sigma}\vec{H})$, where the constant $\mu=-\mu_0\alpha/2\pi$ is interpreted as the anomalous magnetic moment of the electron. As Pauli noted, such a generalization of the Dirac equation is covariant, and therefore it can be assumed that in the general case too the operator replacement $\hat{R}\to\hat{R}'$ remains valid, though now μ may depend on the strength of the magnetic field and on the electron energy. This will then mean that the anomalous magnetic moment of the electron has a dynamical nature.

Comparing further the matrix elements of the operator $\hat{R}'$,

$$W'_{\zeta\zeta'}=\int\Psi^+_{\zeta'}(r)\hat{R}'\Psi_\zeta(r)d^3x=\mu H(D_1D'_1-D_{-1}D'_{-1}),$$

with the energy Δm^{Sp} of the vacuum interaction [see (85)], we may conclude that the operator substitution $\hat{R}\to\hat{R}'$ is indeed possible if μ is chosen in the form

$$\mu=\frac{\mathrm{Re}\,\Delta m^{Sp}}{H(D_1D'_1-D_{-1}D'_{-1})}=-\frac{\alpha}{2\pi}\mu_0\zeta f(n,a). \qquad (86)$$

Summing now in (85) over the sign factor $\varepsilon=E_n/|E_n|=\pm 1$, we obtain for the function $f(n,a)$ the expression[55]

$$f(n,a)=-8a\sum_{n'}\int_0^\infty\int_0^{2\pi}\frac{xdx\sin\vartheta d\vartheta}{(\sqrt{\eta+x^2\cos^2\vartheta}+x)^2-1} \times\left\{1+\frac{\eta-1+x^2\sin^2\vartheta}{x\sin^2\vartheta\sqrt{\eta+x^2\cos^2\vartheta}}\right\}\{I^2_{n,n'}(z) -I^2_{n-1,n'-1}(z)\}, \qquad (87)$$

where $a=k_0^2/4\gamma=H_c/2H$, $\eta=(n'+a)/(n+a)$, $z=(n+a)x^2\sin^2\vartheta$. This expression for the function $f(n,a)$ does not contain divergences, is finite in the complete range of variation of the energy of the external field, and is exact in the framework of Dirac's theory.

Weak field

We consider the case of a weak magnetic field, when $a=H_c/2H\gg 1$. At the same time, we can also assume that $(n+a)\gg 1$, by virtue of which we can make an expansion with respect to $(n+a)^{-1}$ and, integrating and summing in (87), we obtain[45]

$$f(n,a)=1-\frac{7}{3a^2}\left(\ln a-\frac{576\ln 2-83}{420}\right),$$

as a result of which the electron AMM acquires a correction to the Schwinger value in the form of a dependence that is quadratic in the field:

$$\mu=-\mu_0(1+a_e),$$
$$a_e=\frac{\alpha}{2\pi}\left[1+\frac{28}{3}\left(\frac{H}{H_c}\right)^2\ln\frac{H}{H_c}+...\right]. \quad (88)$$

However, as we showed in Ref. 45, this is valid only for not too strongly excited quantum states of the electron, when $n\ll a$. This corresponds to the nonrelativistic approximation because, since the principal quantum number is given by $n=(p_\perp/mc)^2a$, the requirement $n\ll a$ is equivalent to the case $\beta\ll 1$. We note that the case considered here corresponds to the approximation $\chi=\frac{1}{2}(n/a^3)^{1/2}=\frac{1}{2}(p_\perp/mc)(H/H_c)\ll 1$, but under the condition of small numbers n (discrete variation of the parameter χ). In this case, the corrections in the field do indeed have a quadratically logarithmic nature and do not depend on the electron energy. Nevertheless, even in this simplest case the dynamical nature of the AMM is manifested. If, however, the field remains weak ($a\gg 1$) but the electron is relativistic, $n/a=(p_\perp/mc)^2\gg 1$, then the behavior of the anomalous magnetic moment is strongly changed. Indeed, let us return to the original expression (87) and analyze it under the assumption $a\gg 1$ and $n\gg 1$. The main contribution in this case will be made by transitions to excited intermediate levels with $n'\gg 1$, and therefore we can again go over from a summation over n' to an integration, though the variable of integration must be chosen in the form $u=(\sqrt{n}-\sqrt{n'})/\sqrt{n'}$, i.e., one must introduce

$$n'=n/(1+u)^2,\quad dn'=-2ndu/(1+u)^3.$$

At the same time, for the Laguerre functions $I_{n,n'}(z)$ we can use the approximation by means of the Macdonald functions $K_{1/3}$ and $K_{2/3}$, and then Re $f(n,a)$ can be represented in the form[6,7]

$$\mathrm{Re}\, f(n,a)=1\int_0^\infty\frac{du}{(1+u)^3}\int_0^\infty dt\,\sin\left(t+\frac{\chi^2}{3u^2}t^3\right),$$

where χ is the invariant dynamical parameter. Approximate integration leads to the results

$$a_e=\begin{cases}\frac{\alpha}{2\pi}\left[1-12\chi^2\left(\ln\frac{1}{\chi}+...\right)\right], & \chi\ll 1,\\ \alpha\Gamma(1/3)/[9\sqrt{3}(3\chi)^{2/3}], & \chi\gg 1\end{cases} \quad (89)$$

(see also Refs. 8 and 7). Thus, in the region $\chi\ll 1$ (quasiquantum region) the corrections to the Schwinger value of the AMM are as before small, but for a parameter $\chi\gg 1$ there is a quite different behavior of the AMM—its value decreases sharply with increasing field or with increasing energy of the electron and is very different from the Schwinger value $\alpha/2\pi$ (ultraquantum case).

Ultrastrong magnetic fields

An entirely new situation with regard to the field corrections to the anomalous magnetic moment arises in the region of superstrong magnetic fields $H\gg H_c$. We return to the expression (87) and, making the change of variable $x\to x/\sin^2\vartheta$ and integrating exactly, reduce it to the form

$$f(n,a)=\frac{a\ln a}{n+a}-8a\int_0^\infty\sum_{h'=0}^{\infty}\frac{x^2(1+\eta)-(1-\eta)^2}{x\sqrt{s}}$$
$$\times\{I^2_{n,n'}[(n+a)x^2]-I^2_{n-1,n'-1}[(n+a)x^2]\}\Phi(x,\eta)dx,\quad \eta=(n'+a)/(n+a),$$

where $s=[(x-\sqrt{\eta})^2-1][(x+\sqrt{\eta})^2-1]$, and the discontinuous function Φ is defined by

$$\Phi_{x,\eta}=\begin{cases}\tan^{-1}[\sqrt{s}/[(x+\sqrt{\eta})^2-1]] & \text{for } |\sqrt{\eta}-1|\leqslant x\leqslant|\sqrt{\eta}+1|,\\ \frac{1}{2}\ln\left|\frac{x^2+\eta-1+\sqrt{\eta}}{2x\sqrt{\eta}}\right| & \text{for other } x.\end{cases}$$

In the asymptotic limit $a\to 0$, only the single term $n'=0$ remains from the complete sum (localization of the Green's function), and its integration gives

$$f(1,a)=-2a(\ln(1/a)-1.08),$$
$$f(n,a)=2a\ln a/n,\quad a_e=(a/2\pi)f. \quad (90)$$

Thus, in the limit $H\gg H_c$ intermediate states with $n'=0$ make a real contribution to the electron AMM. This circumstance emphasizes once more that both the real and the virtual states in a superstrong magnetic field are strongly localized in the direction at right angles to the magnetic field with localization within a circle defined by $\pi R^2\cong\pi(\hbar/mc)^2H_c/H$. As follows from (90), the function $f(n,a)$ in the limit $a\to 0$ is negative, and, therefore, the value of the AMM tends to zero with decreasing a (with increasing field) from the side of negative values. However, since for $a\gg 1$ the function $f(n,a)$ is positive, and, in addition, it is continuous in the complete range of its argument a (for fixed value of n), this region contains at least two characteristic points, at one of which $f(n,a)$ vanishes, while at the other it reaches its minimum value, which, obviously, is negative. It is also clear that both points are near $a\sim 1$, i.e., $H\sim H_c$. Figure 9 shows the graph of the function f for the state $n=1$ calculated in accordance with Eq. (90). The change in the sign of the electron AMM was first noted in our study of Ref. 45, and later it was noted in Ref. 77. Thus, the anomalous magnetic moment of the electron manifests a clear dynamical nature, being a nonlinear function of the particle energy and of the magnetic field strength.

It is also interesting to consider the behavior of the electron AMM in a combined field consisting of a homogeneous magnetic field and a plane wave propagating along it (Redmond configuration). The wave can be assumed to be circularly polarized, so that the vector potential has the form

$$A_\mu = a e_\mu(nx) + \sqrt{2} b [e_\mu^{(1)} \sin\psi \cos(kx) + g e_\mu^{(2)} \cos\psi \sin(kx)].$$

It was shown[78] that in the ultraquantum case $\chi \gg 1$ the expression (89) is generalized to

$$a_e = \frac{\alpha\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \left[1 - \frac{2}{9}\left(\frac{b}{a}\right)^2 \left(\frac{1}{2} + \cos^2\Psi\right)\right], \qquad (91)$$

in accordance with which the electron AMM also exhibits a dependence on the polarization of the plane wave.

We now briefly consider the problem of the anomalous magnetic moment of the electron in the Weinberg–Salam–Glashow theory, which unifies the electromagnetic and weak interactions.

Compared with electrodynamics, in this theory electrons of sufficiently high energy can emit not only photons but also Z^0 and $W^\pm$ bosons, this leading to a change of the radiative effects. In particular, the weak interactions contribute to the value of the electron AMM.

The general nature of the dynamical dependence of the electron AMM on the particle energy and on the field strength remains the same,[79] being expressed, in particular, in a nonlinear dependence on the invariant dynamical parameter χ. The investigation of Ref. 79 made for the crossed-field model ($E=H$, $\mathbf{E}\cdot\mathbf{H}=0$) showed that in the case of small values of this parameter the corrections to the static value of the AMM are, like (89), quadratic, whereas in the case of very large values of the parameter the contribution of the weak interactions to the a_e anomaly decrease in inverse proportion to $\chi^{2/3}$. The magnitude of the contribution depends on the constants $G_W = g_V/2\sqrt{2}$ and $G_Z = (g_V^2 - 3g_A^2)/4(g_V^2 + g_A^2)^{1/2}$, which are related to the gauge constants g_V and g_A of the vector and axial-vector interactions. In particular, for very large values of the parameter χ we obtain

$$a_e^W(\chi) = \frac{G_W^2}{2\pi} 11 \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}, \quad \text{if } \chi \gg \left(\frac{M_W}{m}\right)^3 \sim 4\cdot 10^{15},$$

$$a_e^Z(\chi) = \frac{G_Z^2}{2\pi} \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{1}{2} [1 - 11\alpha_0^2], \quad \text{if } \chi \gg \left(\frac{M_Z}{m}\right)^3 \sim 6\cdot 10^{15}, \qquad (92)$$

where $\alpha_0 = (g_V^2 + g_A^2)/(g_V^2 - 3g_A^2) = (1 - 4\sin^2\theta_W)$, θ_W is the Weinberg angle, and χ in these expressions depends on the electron mass. The criterion $\chi \gg (M/m)^{3/2}$ can be replaced by the simpler one $\chi \gg 1$ if we introduce a critical field for the W and Z bosons:

$$H_c^W = 10^{24}\ \text{G}, \quad H_c^Z = 1.2\cdot 10^{24}\ \text{G}.$$

It is most interesting to compare these contributions in the case when the parameter is $\chi \gg (M/m)^3$. Expressing all values of the constants G_W and G_Z in terms of the universal Fermi constant G_F, and taking the Weinberg angle to satisfy $\sin^2\theta_W = 0.23$, we find that

$$a_e^W = 11 a_e^\gamma, \quad a_e^Z = -\frac{11}{3} a_e^\gamma,$$

$$\chi \gg \left(\frac{M_Z}{m}\right)^3, \quad a_e^\gamma = \frac{\alpha\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}. \qquad (93)$$

Thus, under these conditions the weak contributions exceed the photon contribution by almost an order of magnitude; moreover, the W-boson contribution is dominant.[79]

It should be noted here that the crossed-field model gives a simple solution to the problem of the electron AMM only in the semiclassical ultrarelativistic region of particle motion if the model is applied under the conditions of a homogeneous magnetic field. The realization of the crossed-field model in nature is very problematic. If we now consider electron motion in a magnetic field, then the expressions (92), and also all results for $\chi \ll 1$, give a correct description only in the semiclassical approximation, in which the quantizing properties of the field can be ignored (the crossed field is characterized by a continuous electron energy spectrum). Thus, the crossed-field model does not give complete information about the behavior of the AMM of an electron in an external field: Weakly excited electron states (the region of small numbers) make a fundamental contribution to the dynamical nature of the anomalous magnetic moment [see (90)]. It needs to be especially emphasized that weakly excited electron states (small transverse momentum) may be particularly characteristic of electrons in a pulsar magnetosphere, since the energy quantum number decreases rapidly on account of the uncompensated radiative losses.

It should also be pointed out that the critical fields (93) which characterize the weak interaction, $H_c^W \sim 10^{24}$ Oe, differ from the Schwinger field $H_c = 4\cdot 10^{13}$ Oe in an important respect—in fields of such strengths, the vacuum becomes unstable and one must question the possibility of using the single-particle approach to solve the problem. It is well known that the exact solutions of the equations of the quantum theory for vector particles in a magnetic field have a singularity, namely, at $H \sim H_0^W \sim 10^{24}$ Oe the square of the energy of a boson in the ground state in the magnetic field becomes negative (tachyonic field modes). Indeed, in the Weinberg–Salam–Glashow model the solution of the Proca equation for a boson in a magnetic field has the energy spectrum

$$E^W = \pm Mc^2 \sqrt{1 + (2n-1)\frac{H}{H_c^W}}, \quad H_c^W = \frac{M_W^2 c^3}{e\hbar},$$

from which it follows that in the ground state $n=0$ when $H>H_c^W$ the spectrum becomes imaginary. This difficulty does not exist in the electron energy spectrum, since $E^e = \pm mc^2\sqrt{1+2nH/H_c}$. Therefore, the conditions (93) require an analysis of their validity on the basis of the fundamental propositions of the theory.

To this it should be added that the question of the vacuum stability of the weak interactions is also related, as we have already noted, to the problem of the restoration of the spontaneously broken symmetry in extremely strong fields. This is also related to our incomplete knowledge about the constants g_A and g_V and also their possible dependence on the external field. It may be that in an ultrastrong field g_A and g_V have a "floating" nature.

Thus, the expressions obtained for the contribution of the weak interactions to the electron AMM, and also the possibility of their experimental observation must be used with a certain caution. In the light of this, it appears that the contribution to the electron AMM of the electromagnetic interactions is most important.

The value of the electron AMM has already for a long time been of considerable interest from the point of view of its experimental investigation. The anomalous part of the g factor of free electrons was first measured by Dehmelt (see Ref. 80), using an rf field that caused spin-resonance transitions of polarized electrons moving in a magnetic field. Since the electron AMM makes a decisive contribution to the dynamics of the particle spin, this method opened up a possibility of obtaining direct information about the magnitude of the anomaly $a_e=(g-2)/2$ ("attack on the anomalous magnetic moment"). Under Dehmelt's lead, a method has been developed in recent years at Washington University[81] for confining individual electrons to a trap—an artificial atom (geonium) has been created. In the experiments, observations were made of $g-2$ transitions of individual electrons with spin flip,[81] and one of the most accurate values of the magnetic anomaly was obtained:

$$a_e^{\exp}=(1\ 159\ 652\ 410\pm 200)\cdot 10^{-12}$$

(see also Ref. 29). It should be mentioned that the results of the experiment agree well with the theoretical investigations of the higher approximations in the square of the charge in the expression for the AMM (Ref. 82).[3)]

Thus, a possibility is opened up for investigating the dynamical nature of the electron AMM, since the corrections to the Schwinger value of the AMM that depend on the field strength and are calculated in the e^2 order of QED can become significant on the background of the higher powers of the perturbation expansion with respect to the square of the charge (see Ref. 6).

Great interest attaches to observation of the dynamical nature of the electron AMM under the conditions of an extremely strong field with dynamical parameter satisfying $\chi>1$.

We have already mentioned the experiments made at the SPS accelerator at CERN,[15] in which electron beams accelerated to energy 150 GeV were directed with very small opening angle along the axis of a germanium crystal cooled to low temperatures. The strong macroscopic fields acting along the crystal axis, together with the high electron energies, led to an estimate for the value of the parameter χ of order unity. Then the energy of the spin interaction of the beam electrons with the field becomes equal in order of magnitude to the particle rest energy (in the electron rest frame). This can lead to a number of interesting quantum effects and, in particular, opens up the possibility of observing a dependence of the electron AMM on the field strength. The possibility of observing the dynamical nature of the electron AMM was also noted by Baryshevskiĭ,[16] who proposed an investigation of a_e using the precession of a particle channeled in a bent single crystal.

In conclusion, we should mention the high-precision experiments to compare the electron and positron anomalous magnetic moments at high energies. At the Institute of Nuclear Physics of the Siberian Branch of the USSR Academy of Sciences at Novosibirsk, under the leadership of A. N. Skrinsky,[84] the two magnetic moments were compared at 625 MeV in the same storage ring. Because of the identity of the experimental conditions, a high accuracy was guaranteed. The experiments made in the VÉPP-2M storage ring were based on the radiative polarization effect discovered by Sokolov and one of the present authors (Ternov). As a result of this effect, a preferred polarization of beams is established after circulation of the particles in the ring for about an hour; the electrons are polarized in the direction opposite to the field, and the positrons along the field. After this, using an additional oscillating longitudinal field, the experiment gave rise to resonant depolarization at the frequency

$$\omega_r=(\gamma a_e-1)\omega_H/\gamma\cong 0.4\omega_H/\gamma,$$

$$\omega_H=\frac{eH}{mc}\ \text{is the cyclotron frequency.}$$

This corresponds to tuning the frequency of the external alternating field at the fixed point in the orbit in which the depolarizer is situated to the spin precession frequency. The experimental result established equality of the anomalous moments of the electron and positron at a good confidence level (95%):

$$\left|\frac{a_{e^+}-a_{e^-}}{a_{e^-}}\right|<1.0\cdot 10^{-5}$$

(see Ref. 84). Investigation of the values of the magnetic moments of particles and antiparticles is important for testing the basic propositions of quantum electrodynamics—the equality of these magnetic moments has an intimate relationship to the CP invariance of the theory (see the monograph of Isaev[85]).

10. SINGLE-PHOTON ANNIHILATION AND CREATION OF ELECTRON–POSITRON PAIRS IN AN EXTREMELY STRONG MAGNETIC FIELD

As is well known,[7] a single photon cannot be transformed spontaneously into an electron–positron pair in the vacuum—the formation of a pair requires the presence of an external field, which takes up the excess momentum. The creation and annihilation of electron–positron pairs in a magnetic field was first considered by Klepikov,[46] and after several years it became clear that the channel of single-photon production of e^+e^- pairs is the main mechanism of generation of relativistic plasma in pulsar magnetospheres.

Single-photon pair production

The probability of electron–positron pair production can be calculated by the standard methods of quantum electrodynamics in the Furry representation by means of the exact solutions of the Dirac equation for a particle in a homogeneous magnetic field.[7] The probability of single-photon production of an e^+e^- pair is found to be

$$w(\zeta)=\frac{e^2}{\hbar c}\frac{mc^2}{\hbar}\frac{H}{H_c}\frac{\sqrt{3}}{4\pi}q\int_0^\infty dy\,S,$$

where $q=\frac{4}{3}(H_c/H)(mc^2/E_\gamma)$, $a=2qc\hbar^2y$, E_γ is the energy of the photon, and

$$S=\frac{1+\zeta^-\zeta^+}{2}[\tanh^2 yK_{2/3}(a)+2q\sinh^2 yK_{1/3}(a)$$
$$-\zeta^-\tanh yK_{1/3}(a)]$$
$$+\frac{1-\zeta^-\zeta^+}{2}[K_{2/3}(a)-\zeta^-K_{1/3}(a)]. \quad (94)$$

Here, as before, $K_\mu(a)$ is a Macdonald function, spin projection onto the field direction ζ^- corresponds to the electron, and ζ^+ corresponds to the positron. It is, unfortunately, difficult to integrate the expression (94). Therefore, we shall consider the asymptotic behavior of (94) in two limiting cases.

1. In the case of low photon energies, when $q\gg 1$,

$$E_\gamma\ll\frac{4}{3}\frac{H_c}{H}mc^2.$$

Using the asymptotic behavior $K_\mu(z) \underset{z\to\infty}{=} \sqrt{\pi/2z}e^{-z}$ of the Macdonald function, we find that

$$w(\zeta)=w_\infty\left\{\frac{1+\zeta^-\zeta^+}{12}+\frac{(1-\zeta^-)(1-\zeta^-\zeta^+)}{6}\right\},$$

where the total probability w_∞, summed over the spins, has the form

$$w_\infty=\frac{3\sqrt{3}}{16\sqrt{2}}\frac{e^2}{\hbar c}\frac{mc^2}{2}\frac{H}{H_c}e^{-2q} \quad (95)$$

(see Refs. 46 and 47). It follows from this that pair production with oppositely oriented spins is most probable; moreover, the electron spin must be oriented in the direction opposite to the magnetic field ($\zeta^-=-1$):

$$w^{\uparrow\downarrow}=\frac{1-\zeta^-}{3}w_\infty=\frac{2}{3}w_\infty.$$

2. In the opposite limiting case, when $g\ll 1$ (ultrastrong magnetic field or high photon energies), we obtain, taking into account the asymptotic behavior $K_\mu(a) \underset{q\to 0}{\cong} 2^{\mu-1}\Gamma(\mu)/q^\mu$:

$$w(\zeta)=w_0\left\{\frac{3}{40}(1+\zeta^-\zeta^+)+\frac{7}{40}(1-\zeta^-\zeta^+)\right\},$$

and, summing over the particle spins, we obtain the expression

$$w_0=\frac{5}{7}\frac{\Gamma(5/6)}{\Gamma(7/6)}\frac{1}{2^{4/3}}\frac{e^2}{\hbar c}\frac{mc^2}{\hbar}\frac{H}{H_c}q^{1/3} \quad \text{for } q\ll 1. \quad (96)$$

It follows from this that in the case of an ultrastrong field (high photon energies) too production of a fermionic pair with oppositely oriented spins is the most probable.

In conclusion, we note that the exponential factor in (95) strongly suppresses the probability of pair creation when $q\gg 1$, i.e., at comparatively low photon energies (or in the case of a weak field, when $q=\frac{4}{3}(H_c/H)(mc/E_\gamma)\gg 1$) the probability is effectively equal to zero. However, in the case of a strong field, when $H>H_c$, the probability becomes measurable. As was shown in Ref. 46, the maximum of the probability occurs at $q=0.1$.

Thus, in an extremely strong magnetic field the absorption of photons as a result of pair creation can be appreciable.

Single-photon annihilation of electrons and positrons in a magnetic field

The process of single-photon annihilation of a pair can be treated in complete analogy with the creation process.[46,7] The annihilation probability is found to be

$$w=\frac{2}{3}\frac{e^2}{\hbar}\frac{1}{\gamma}\left(\frac{mc^2}{E}\right)^4\Phi(\zeta,q),$$

where $q=\frac{4}{3}(H_c/H)(mc^2/E_\gamma)=\frac{2}{3}(H_c/H)(mc^2/E)$, since $E_\gamma=2E$, $\gamma=e_0H/2\hbar c$.

The function $\Phi(\zeta,q)$ has the form

$$\Phi(\zeta,q)=\frac{1-\zeta^-\zeta^+}{2}[\zeta^-K_{1/3}(q)+K_{2/3}(q)]^2$$
$$+\frac{1+\zeta^-\zeta^+}{2}K_{1/3}(q). \quad (97)$$

Note that if we multiply the probability by the number of electrons of the medium and introduce the electron density $P=N/L^3$, then we can obtain the reciprocal lifetime of an electron with respect to annihilation with positrons of the medium:

$$\frac{1}{\tau}=\frac{2}{3}\frac{e^2}{\hbar\gamma}P\left(\frac{mc^2}{E}\right)^4\Phi(\zeta,q).$$

If $q\gg 1$ (weak field),

$$\frac{\Phi^{\uparrow\downarrow}}{\Phi^{\uparrow\uparrow}}=(1+\zeta^-)^2\left[1+\frac{1}{6q}-\frac{5}{72q^2}+\ldots\right]+\frac{1}{36q^2}.$$

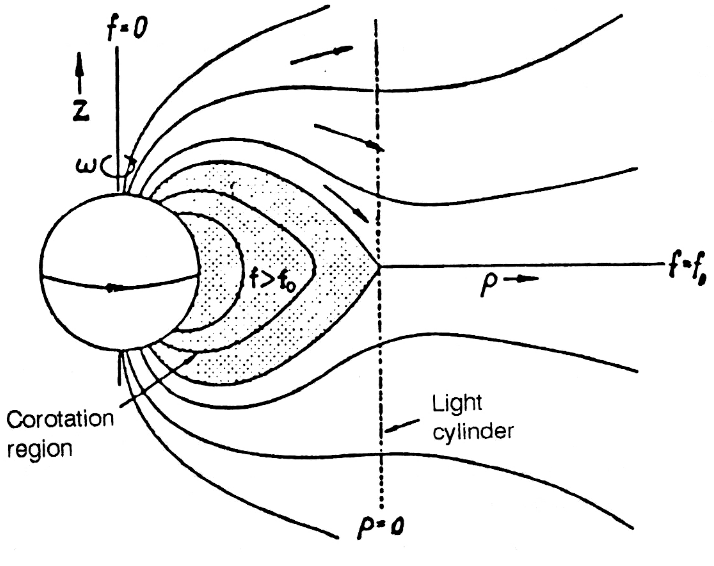

FIG. 10. Standard model of the plasma of a pulsar magnetosphere.

In the case of antiparallel spins, the pair annihilation is enhanced if the electron spin is oriented along the direction of the magnetic field.

In the opposite limiting case $q \ll 1$ (ultrastrong field), we have

$$\frac{\Phi^{\uparrow\downarrow}}{\Phi^{\uparrow\uparrow}} = \left[\frac{2^{4/3}\pi}{\sqrt{3}\Gamma^2(1/3)} + \zeta^- \right].$$

In other words, the probability of annihilation of a pair with oppositely oriented spins is much greater than in the case of parallel spins. If we now average (97) over the spins of both particles, we find that

$$w = \frac{e^2}{3\hbar\gamma L^3} \left(\frac{mc^3}{E} \right)^4 \Phi(q), \qquad (98)$$

where

$$\Phi(q) = 2K_{1/3}^2(q) + K_{2/3}^2(q) = \begin{cases} \dfrac{3\pi}{2q} e^{-2q}, & q \gg 1, \\ \dfrac{\Gamma^2(2/3)}{2^{2/3} q^{4/3}}, & q \ll 1. \end{cases}$$

Thus, in an extremely strong magnetic field, channels associated with single-photon processes, in particular, pair production, are opened. The investigation of pair production in an extremely strong magnetic field is of interest in connection with processes in strong magnetic fields in space, in particular in the field of pulsars. When a photon propagates in such a field, pairs must be produced, and, in their turn, they will generate photons, etc. Thus, a cascade process similar to what is observed when a cosmic ray passes through matter can begin. In a macroscopic ultrastrong magnetic field, such a cascade process will be associated with the electron–positron and electromagnetic vacua.

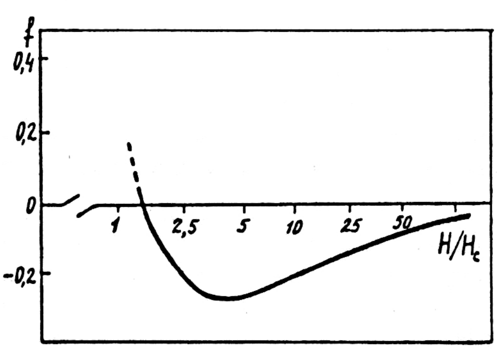

FIG. 9. Asymptotic behavior of the anomalous magnetic moment in an ultrastrong magnetic field for $n=1$ [see Eq. (90)].

11. ASPECTS OF PROCESSES IN THE ULTRASTRONG MAGNETIC FIELDS OF PULSARS

A pulsar (neutron star) is surrounded by a magnetosphere, which exists because of the strong electric fields generated by the magnetic field of the star rotating in the vacuum. The electric field arises in the vacuum gap at the surface of the pulsar; it is concentrated in the gap, the width r_0 of which is of order 100 m (Refs. 86 and 87), and the strength of the field depends on Ω, the rotation frequency of the star, r_0, the width of the gap, and the magnetic field strength H:

$$E \cong \frac{\Omega r_0 H}{c}.$$

The strong electric field strips charged particles (electrons, protons, positrons) from the star, and these move in the pulsar magnetosphere along the magnetic lines of force, which close within the light cylinder that rotates synchronously with the pulsar.[88] Thus, the plasma formed by the electrons, positrons, and protons rotates synchronously with the pulsar in the region within the light cylinder with radius R_c, which determines the distance from the cylinder axis at which the velocity of synchronous rotation of the plasma reaches the velocity of light (see Fig. 10). The electric field E that arises at the surface of the pulsar can communicate a fairly high energy to the particles. In particular, for the pulsar in the Crab Nebula, for which we estimate $H \sim 4 \cdot 10^{12}$ Oe, $\Omega = 200$ sec^{-1}, $R = 10^6$ cm, and $r_0 = 10^3$ cm, the electric field E can accelerate electrons to energy $\gamma = E/mc^2 = 3 \cdot 10^6$, i.e., $E \sim 10^3$ GeV. Thus, the physics of the particles in a pulsar magnetosphere involves not only an extremely strong magnetic field but also high energies, and pulsars are important sources of cosmic rays.

Properties of the radiation

The energy of a rotating neutron star is transformed into the energy of electromagnetic radiation and into energy of particles. With this transformation of the energy there is associated an evolution of the star, since the energy losses lead to a spin-down of the star:

$$\frac{d}{dt}\left(\frac{I\Omega^2}{2}\right) = I\Omega\dot{\Omega} = -\frac{2}{3}\frac{\mu^2\Omega^4}{c^3},$$

where μ is the magnetic moment of the dipole.

Unfortunately, at the present time we are not completely clear in our understanding of the true mechanism of pulsar radiation. There exist several models, but none of them can be regarded as completely convincing. At the same time, the properties of the radiation of different pulsars are the same, and this in itself is a weighty argument in support of a common radiation model.

The mechanisms usually considered are cyclotron, synchrotron, and curvature radiation. Curvature radiation is associated with a feature of electron motion in a dipole magnetic field when the magnetic lines of force have a strongly expressed curvature, and this radiation has a synchrotron nature, since the motion of the electron takes place along the curved lines of force of the magnetic field at very small pitch angles (small compared with γ^{-1}).[89,90] Here, the pitch angle is the angle between the velocity vector and the direction of the magnetic field.

When estimating the contribution of synchrotron radiation to the energy losses, one must take into account the properties of such radiation under the conditions of an ultrastrong magnetic field, i.e., when the invariant parameter χ takes values $\chi \gg 1$. Then both the radiation spectrum and the power are different:

$$\omega_{\max} = \begin{cases} \dfrac{3}{2}\dfrac{c}{R}\left(\dfrac{E}{mc^2}\right)^3, & \chi \ll 1 \\ E/\hbar, & \chi \gg 1 \end{cases}; \quad W^{\text{ulqu}} = \frac{16}{9}\frac{\Gamma(2/3)\,W^{\text{cl}}}{(3\chi)^{4/3}}.$$

In astrophysical applications, this important aspect of the theory of synchrotron radiation has not always been taken into account.

We must also mention one further type of radiation that can be realized in pulsars—undulator radiation. This type of radiation is observed if an electron has a transverse (relative to the magnetic field) momentum $\beta_\perp$ and simultaneously a momentum $\beta_\parallel$ along the field ($\beta^2 = \beta_\perp^2 + \beta_\parallel^2$), the two momenta satisfying $\beta_\perp \ll \beta_\parallel$. In the simplest case, the electron motion is motion along a helix.

The expression for the radiation power is[7]

$$W = \frac{e^2\Omega^2}{c}\sum_{\nu=1}^{\infty}\nu^2\int_0^\pi \frac{\sin\vartheta\, d\vartheta}{(1-\beta_\parallel\cos\vartheta)^3} \times\left[\beta_\perp^2 J_\nu'^2(x) + \frac{(\cos\vartheta - \beta_\parallel)^2}{\sin^2\vartheta}J_\nu^2(x)\right], \quad (99)$$

where

$$\Omega = \frac{ecH}{E}, \quad x = \frac{\nu\beta_\perp}{1-\beta_\parallel\cos\vartheta}.$$

The frequency of the radiation is now different from the frequency of circular motion on account of the Doppler effect:

$$\omega = \frac{\nu\Omega}{1-\beta_\parallel\cos\vartheta}.$$

In the undulator regime, the electron velocity in the direction at right angles to the magnetic field is relatively small, while the velocity along the field is close to the velocity of light. Then, as follows from (99), the maximum of the radiation is at the fundamental tone, and the angle ϑ must be fairly small (small angle relative to the undulator axis).

If we introduce standard notation in the form $\beta_\parallel = \beta[1-(k/\gamma)^2]$, $\beta_\perp/\beta_\parallel = k/\gamma$, $\gamma = E/mc^2$, $k = (E/mc^2)(\beta_\perp/\beta)$, where k is the undulator constant, then with allowance for the polarization properties of the undulator radiation we can rewrite the expression (99) in the form

$$\frac{dW}{d\Omega} = \frac{\beta^2 e^2 k^2 \omega^3}{2\pi c\gamma^2\nu\Omega}\left[l_\sigma J_\nu'(x) + l_\pi \frac{\cos\Theta - \beta_\parallel}{\beta_\perp \sin\Theta}J_\nu(x)\right],$$

where $x = k\omega\sin\vartheta/\gamma\Omega$, and l_σ and l_π characterize the components of the linear polarization. In the approximation of small angles ϑ (observation along the undulator axis)

$$\frac{dW_\nu}{d\Omega} = \frac{\beta^2 e^2 k^2 \omega^3}{2\pi c\gamma^2\nu\Omega}J_\nu'^2(x)\left[l_\sigma + l_\pi\frac{1+k^2-\gamma^2\vartheta^2}{1+k^2+\gamma^2\vartheta^2}\right]^2.$$

The undulator case that we are considering ($k<1$) differs from the situation in which the transverse momentum $\beta_\perp$ is not a very small quantity ($k>1$, the wiggler regime). In the wiggler regime, the radiation of the electron, which moves in a circle, in contrast to the undulator motion, for which $k<1$, does not have a dipole nature and exhibits fully the properties of synchrotron radiation—higher harmonics of the fundamental frequency Ω are radiated. At the same time, the angular distribution of the power is radically changed (Fig. 11).

Cascade processes in a pulsar magnetosphere

In accordance with the investigations of Goldreich and Sturrock,[91,92] cascade processes can begin and develop in a pulsar magnetosphere. Indeed, electrons that have been accelerated by the strong electric field will, as they move in the magnetic field of the star, radiate hard γ rays. The γ rays, in their turn, can be transformed through the single-photon channel into e^+e^- pairs, and, thus, a cascade process develops (Fig. 12).

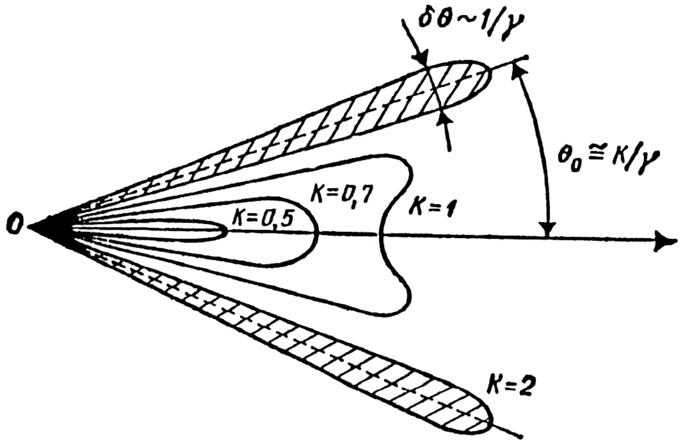

FIG. 11. Diagram of the angular distribution of undulator radiation for different values of the undulator constant k.

We shall not dwell on this in detail but refer the reader to the review of Ref. 86. However, we shall consider some aspects of the radiation and production of pairs in an ultrastrong field.

We consider the expressions for the probabilities of synchrotron radiation and single-photon pair production.

For synchrotron radiation when $\chi \ll 1$

$$w_0^\gamma = \frac{5}{2\sqrt{3}} \frac{e^2}{\hbar c} \frac{mc^2}{\hbar} \frac{H}{H_c},$$

and for production of an e^+e^- pair

$$w_0^{e^+e^-} = \frac{3\sqrt{3}}{16\sqrt{2}} \frac{e^2}{\hbar c} \frac{mc^2}{\hbar} \frac{H}{H_c} \exp(-4/\xi^\phi),$$

where

$$\xi^\phi = \frac{3}{2} \frac{H}{H_c} \frac{E_\gamma}{mc^2} = \frac{3}{2} \chi^\phi.$$

These relations are usually taken[86] as a basis for studying the cascade processes. However, in an extremely strong magnetic field, when χ and $\chi^\phi \gg 1$, the probabilities of radiation and single-photon pair production are changed:

$$w_\infty^\phi = \frac{7}{9} 2^{2/3} \Gamma(2/3) \frac{mc^2}{\hbar} \frac{H}{H_c} \frac{1}{\xi^{1/3}},$$

$$w_\infty^{e^+e^-} = \frac{5}{14} \frac{\Gamma(5/6)}{\Gamma(7/6)} \frac{e^2}{\hbar c} \frac{mc^2}{\hbar} \frac{H}{H_c} \frac{1}{(\xi^\phi)^{1/3}}.$$

This circumstance can introduce corrections into the calculation of the cascade processes.

12. INFLUENCE OF A MAGNETIC FIELD ON NEUTRON BETA DECAY

Supernovas are the most impressive explosions of stars[93,94] in which a potential energy of $\sim 10^{46}$ J is released. In 1934, Baade and Zwicky[95] showed that only one percent of this energy is sufficient to eject a shell and produce a supernova explosion. At the beginning of the forties, Gamow and Schoenberg[96,97] proposed a mechanism of energy release in the collapse of a star due to emission of neutri-

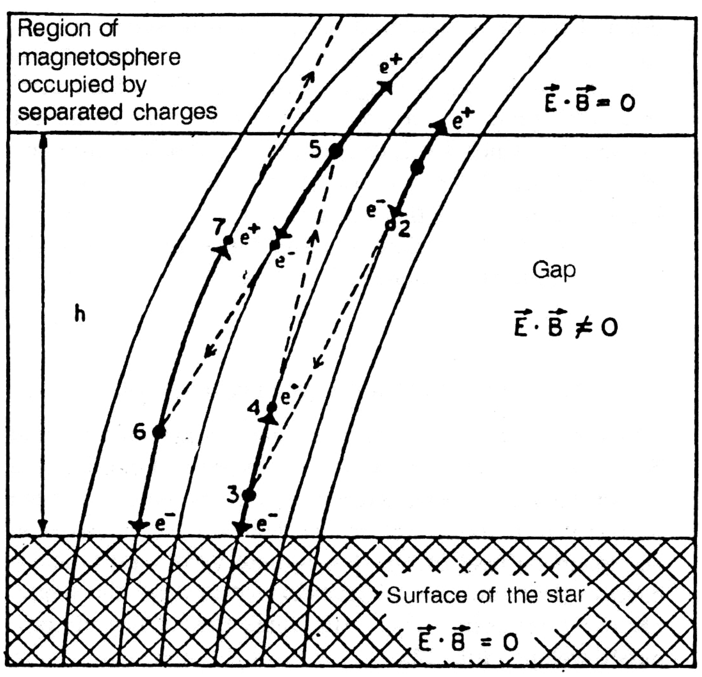

FIG. 12. Development of a cascade process.

nos. A huge amount of energy is carried away by neutrinos during a time $\sim 10^{-3}$ sec, which is determined by the rate of the elementary weak interaction.

High densities, temperatures, and magnetic field strengths are characteristic of the collapse of stars. Modern model scenarios of supernovas (see, for example, Refs. 98 and 99) make it possible to take into account an ever wider set of factors, but discussion of the role of the magnetic field is still only just beginning.[100]

Among the multifactor problems, we can identify the fundamentally important problem of the influence of an extremely strong magnetic field on neutron β decay. Processes related to this reaction are sources of neutrino fluxes during collapse.

Investigation of the influence of an external electromagnetic field on neutron β decay is associated with the need to use exact solutions of the Dirac equation for all the particles that participate in the reactions in this field. It is obvious that allowance for all the electromagnetic characteristics of the particles, including their anomalous magnetic moments, leads to a cumbersome expression for the reaction probability. To clarify the role of each of the electromagnetic properties of the particles during β decay and their possible significance for given parameters of the external electromagnetic field, it is helpful to make the usual asymptotic expansions in the so-called interesting cases in which these expansions are possible and to investigate graphical sections of the complete set of potential information, bearing in mind the multidimensional structure of, for example, the spectral–angular distribution of the produced particles. In particular, one can establish correlations between the momenta and angular momenta of the reaction products and their asymmetry with respect to the direction distinguished by the external electromagnetic field.

In the first order of perturbation theory in the constant of the weak universal interaction, the amplitude of neutron β decay in the $(V-A)$ form can be written as

$$M=(G/(2)^{1/2})\{\bar{\Psi}_p\gamma_\mu(1+\alpha_0\gamma_5)\Psi_n\bar{\Psi}_e\gamma^\mu(1+\gamma^5)\Psi_{\tilde{\nu}}\},$$

where Ψ_n, $\bar{\Psi}_p$, $\bar{\Psi}_e$, $\Psi_{\tilde{\nu}}$ are the wave functions of the neutrino, proton, electron, and antineutrino, respectively; $\sigma_0=G^A/G^V$ is the ratio of the axial and vector coupling constants, and the constant

$$G=e^2/(8M_W^2\sin\Theta_W)=1.414\cdot 10^{-49}\ \mathrm{erg}\cdot\mathrm{cm}^3$$

(without radiative corrections) is related to the mass M_W of the intermediate vector boson and the Weinberg angle θ_W.

In the simplest case of an allowed β transition, for which the form factor contains just two energy-independent β moments,[24]

$$S_0=G^V\langle 1\rangle,\quad S_1=G^A\langle\sigma\rangle,$$

one can consider not only neutron β decay but also β transitions between mirror nuclei, for example, ${}^3\mathrm{H}\to{}^3\mathrm{He}$, in which the neutron and proton numbers are exchanged. This is associated with a significant change in the energy-release parameter ε_0.

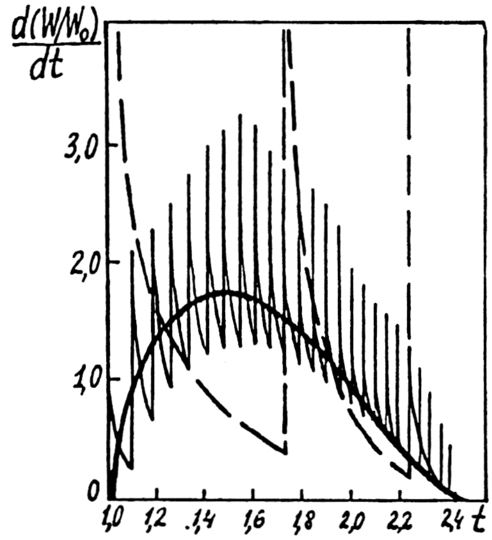

FIG. 13. Energy spectrum of electrons from β decay of neutrons in a magnetic field with strength $H=0$ (heavy curve), $H=0.1H_c$ (continuous curve), and $H=H_c$ (broken curve).

The influence of an external electromagnetic field on neutron β decay was first considered for the example of a constant and homogeneous magnetic field,[101,102] whose strength can be taken to be arbitrarily large, including values greater than the critical value H_c.

In the first studies, it was natural to investigate above all the influence of the magnetic field on the state of the electron produced in the reaction. Indeed, it was the continuity of the electron spectrum that suggested the idea of the existence of neutrinos.

In a magnetic field, the continuous electron spectrum is modified, becoming a resonance spectrum (see Fig. 13) when the electron is produced in a quantized gyration orbit. The number of resonances is

$$N=H_c(\varepsilon_0^2-1)/(2H),$$

and there may also be the case of a unique level for an electron in a magnetic field of strength

$$H\geqslant H_c(\varepsilon_0^2-1)/2$$

for given energy release ε_0.

For the proton, the spectrum will also have a resonance nature, but a unique level will be obtained at a much higher magnetic field strength:[103]

$$H\geqslant H_c(m_p/m_e)(\varepsilon_0-1).$$

The strength of the magnetic field that quantizes the transverse motion of a proton, $\sim 10^{17}$, is greater than the field strengths at which G^A and G^V become equal.

In the light of these comments, we need to consider the partial probabilities for chosen values of the quantum numbers n and n' as functions of the magnetic field and the energy release, and then, if necessary, sum them in order to obtain information about the total probability. Fortunately, all the probabilities pass through a resonance value

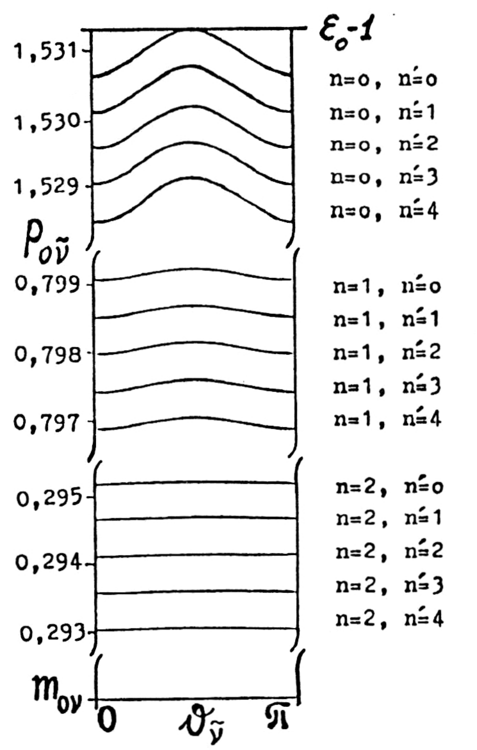

FIG. 14. Dalitz plot with respect to the antineutrino energy $p_{0\tilde{\nu}}$ and antineutrino emission angle $\theta_{\tilde{\nu}}$ for different energy levels of the electron (n) and proton (n') in a magnetic field with strength $H=H_c$.

in a fairly narrow range of magnetic field strengths, except for the case of the partial probability for $n=0$ and $n'=0$.

Using a Dalitz plot (Fig. 14) in the variables $p_{0\tilde{\nu}}$, the energy, and $\vartheta_{\tilde{\nu}}$, the emission angle of the antineutrino $e_{\tilde{\nu}}$ relative to the direction of the magnetic field for different energy levels of the electron (n) and proton (n'), one can represent the behavior of the partial probabilities.[104]

The results of calculations of the partial contributions of the angular distributions exhibit a strong nonlinear dependence on the magnetic field strengths (Fig. 15). In the graphs of the angular distributions, there is a clear asymmetry of the antineutrino emission relative to the direction of the magnetic field.[105]

As we noted above, the partial probabilities are complicated functions of the magnetic field. Investigating these dependences, we calculated neutron β decay with production of the electron in the ground state and the proton in the states with $n'=0$, 1, 2, 3 (Fig. 16). It was more convenient to represent graphically the partial probabilities for different values of H/H_c, since in this case one can clearly see the region of their resonance behavior. Note that the results given here also take into account the anomalous magnetic moments of the particles except for the antineutrinos; in addition, G^A was taken to be a constant. Returning to the stellar-collapse scenario, we note the asymmetry of the neutrino fluxes in a magnetic field. The rate of the weak processes can change[106,107] for many reasons—under the influence of the electron density, the temperature, the matter density, the magnetic field, etc.—but the asymmetry of the neutrino flux relative to the magnetic axis of the

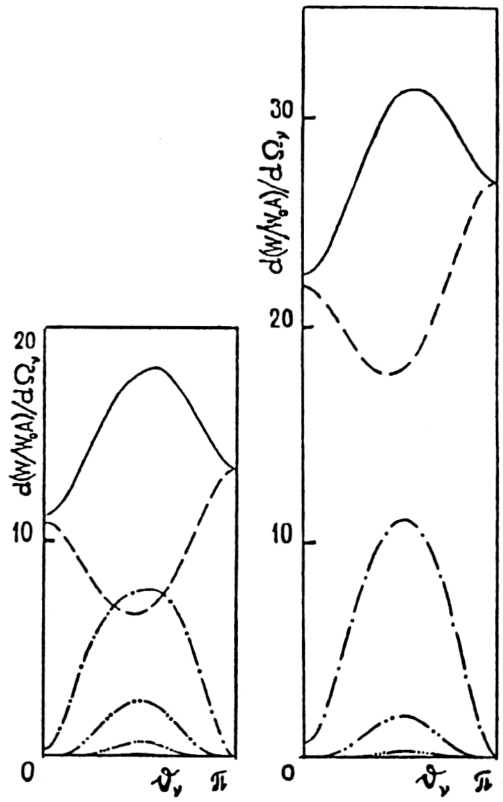

FIG. 15. Partial angular distributions in states with $n=0$ and $n'=0$, 1, 2, 3. The continuous curve corresponds to the total angular distribution, and the broken curves show the contributions (which become less asymmetric with increasing n') of the partial distributions. The ratios of the magnetic field strengths are: a) $H/H_c=1.35$; b) $H/H_c=2.7$.

pulsar due to the magnetic field of the pulsar can be reflected globally in the pulse of the produced pulsar (Refs. 105 and 108–110). This suggestion for explaining the high pulsar velocities stimulated a lively discussion (Refs. 99, 111, and 112) and extended the approaches to the problem

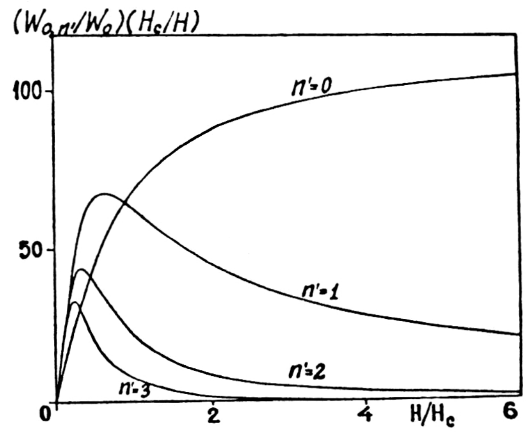

FIG. 16. Partial probabilities of β decay of a neutron to an electron state $n=0$ and of a proton to the states $n'=0$, 1, 2, 3 as functions of H/H_c. The partial probabilities are normalized to the probability W_0 of free β decay of the neutron and the ratio H/H_c.

of the asymmetry of neutrino fluxes (see, for example, Refs. 113–115).

On the basis of observational material on pulsar radiation in the radio range, using a model of pulsar acceleration due to neutrino ejection, it proved to be possible to establish an observational test[116,117] to determine the modulus of the pulsar spatial velocity and thereby augmenting data on the observed tangential velocities of pulsars.

We should also mention the elongation of supernova shells, which is a significant observed characteristic of supernova remnants. Elongation of a supernova shell along the direction that coincides with the pulsar rotation axis can be explained by asymmetry of the fluxes of radiation and particles due to the magnetic field of the pulsar through the region of its magnetic poles.[118] Here, one must also bear in mind that the rotation axis and the magnetic axis of the pulsar do not coincide, this leading to a wider range of angles of escape of the particles and radiation.

In conclusion it should be noted that the quantum processes that develop in extremely strong magnetic fields are a new interesting field of physics investigations. We hope and are confident that the development of this direction will not only have great applied importance but will also contribute to quantum field theory, to which Bogolyubov devoted many years of his life.

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[1)]For the discovery of pulsars, A. Hewish was awarded the Nobel Prize in 1974.

[2)]Note that the critical energy $E_{1/5}=mc^2(mcR/\hbar)^{1/5}$ can be expressed in terms of a field invariant, $E_{1/5}=mc^2[\frac{2}{3}(H_c/H)]^{1/4}$, and the estimate of the energy leads to a value $\sim$300–500 MeV.

[3)]Besides investigations to measure the electron g factor, analogous investigations were made with positrons and muons, in which high accuracies of the measurement were also achieved.[79,83]

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