Theory of the gravitational field

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The basic principles of the relativistic theory of gravitation are presented; energy–momentum conservation laws are formulated; equations for the gravitational field are derived; dark matter is predicted to exist in the universe.

INTRODUCTION

Einstein's general theory of relativity, the basic equations of which were constructed by Hilbert and Einstein in 1915, opened a new era in the study of gravitational phenomena. However, from the time of its creation this theory encountered not only successes but also fundamental difficulties in the definition of the physical characteristics of the gravitational field and, as a consequence, in the formulation of the energy–momentum conservation law. Einstein clearly understood the fundamental importance of the energy–momentum conservation laws; moreover, he assumed that the source of the gravitational field must be the total tensor of the matter and the gravitational field taken together. Thus, in 1913 he wrote that the "gravitational-field tensor $\vartheta_{\mu\nu}$ is the source of the field on an equal footing with the tensor $\Theta_{\mu\nu}$ of the material systems. A distinguished role of gravitational-field energy compared with all other forms of energy would lead to unacceptable consequences." In the same paper, Einstein reached the conclusion that "in the general case, the gravitational field is characterized by ten space–time functions," the components of the metric tensor $g_{\mu\nu}$ of the Riemannian space. However, adopting this approach to the construction of the theory, Einstein did not succeed in making a tensor of the matter and the gravitational field the source of the field, since a pseudotensor and not a tensor arose in general relativity for the gravitational field. In 1918, Schrödinger showed that for an appropriate choice of the coordinate system all components of the energy–momentum pseudotensor of the gravitational field can be made to vanish outside a spherically symmetric source. In this connection, Einstein wrote: "With regard to Schrödinger's considerations, they appear plausible by analogy with electrodynamics, in which the stresses and energy density of any field are nonzero. However, I can find no reason why this must also be the case for gravitational fields. Gravitational fields can be specified without introducing stresses and an energy density." As we see, Einstein abandoned for the case of the gravitational field the concept of a classical field of Faraday–Maxwell type possessing an energy–momentum density, although he did make an important step in associating the gravitational field with a tensor quantity. Einstein took this to be the metric tensor $g_{\mu\nu}$ of a Riemannian space. For Einstein, this approach was evidently entirely natural, since his ideas about the gravitational field were formed under the influence of the principle of equivalence of inertial and gravitational forces that he himself had introduced: "For an infinitesimally small region, coordinates can always be chosen in such a way that the gravitational field will be absent in that region." He emphasized this idea repeatedly; for example, in 1923 he wrote: "For any infinitesimally small neighborhood of a point in an arbitrary gravitational field, one can find a local coordinate system in a state of motion such that with respect to this local coordinate system no gravitational field exists (local inertial system)." Thus arose the idea that the gravitational field cannot be localized. In Einstein's opinion, the existence of the energy–momentum pseudotensor is in complete agreement with the equivalence principle.

However, in reality Einstein's assertion is not true in general relativity, since in this theory the curvature tensor of the Riemannian space must be regarded as the physical characteristic of the field. We owe the clear recognition of this fact to Synge, who wrote: "If we accept the idea that space–time is a Riemannian four-space (and if we are relativistic we must), then surely our first task is to get the feel of it just as early navigators had to get the feel of a spherical ocean. And the first *thing* we have to get the feel of is the Riemann tensor, for it *is* the gravitational field—if it vanishes, and only then, there is no field. Yet, strangely enough, this most important element has been pushed into the background." Later he noted: "In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observed world-line." Thus, in accordance with general relativity, matter (all matter fields except the gravitational field) is characterized by an energy–momentum tensor, while the gravitational field is characterized by the Riemann curvature tensor. Moreover, whereas the former has second rank, the latter has fourth rank, i.e., a fundamental difference between the characteristics of matter and of the gravitational field arose in general relativity. The introduction into general relativity of the energy–momentum pseudotensor of the gravitational field did not enable Einstein to preserve energy–momentum conservation laws in his theory. This was very clearly recognized by Hilbert, who in this connection wrote in 1917: "...I assert that for the general theory of relativity, i.e., in the case of general invariance of the Hamilton function, energy equations that...correspond to the energy equations in orthogonally-invariant theories do not exist at all, and I could even note this circumstance as the characteristic feature of the general theory of relativity." In general relativity, because there is no ten-parameter group of motions of space–time,

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it is in principle impossible to introduce energy, momentum, and angular-momentum conservation laws like those that hold in any other physical theory. The conservation laws for energy, momentum, and angular momentum are fundamental laws of nature. It is these laws that introduce common universal physical characteristics for all forms of matter, making it possible to study quantitatively the transformation of certain forms of matter into other forms. In this connection, it is natural to attempt to construct a theory of gravitation in which all the energy, momentum, and angular-momentum conservation laws hold and in which the gravitational field possesses an energy–momentum density like the one that holds for the Faraday–Maxwell electromagnetic field. In general relativity, the Lagrangian scalar density of the gravitational field contains second derivatives of the field, in contrast to all other physical theories. However, 50 years ago Rosen[1] showed that if in addition to the Riemannian metric $g_{\mu\nu}$ one introduces a Minkowski-space metric $\gamma_{\mu\nu}$, then one can always construct a Lagrangian density of the gravitational field that is a scalar with respect to arbitrary coordinate transformations and contains derivatives of not higher than the first order. In particular, he constructed such a Lagrangian density that leads to the Hilbert–Einstein equations. The bimetric formalism arose in this manner. However, such an approach immediately complicated the problem of constructing a theory of gravitation, since, using the tensors $\gamma_{\mu\nu}$ and $g_{\mu\nu}$, one can form rather a large number of densities that are scalars with respect to arbitrary coordinate transformations, and it is not at all clear which scalar density must be chosen as the Lagrangian density for constructing the theory of gravitation. Following this route, Nathan Rosen chose as Lagrangian densities various different scalar densities and on their basis constructed different theories of gravitation, which in general, naturally, give different predictions for the same gravitational effects. We shall see below that in the framework of the special theory of relativity, which describes phenomena in both inertial and noninertial frames of reference, we can succeed, using the geometrization principle, which reflects the universality of the gravitational field interaction with matter, in unifying Poincaré's idea[2] of the gravitational field as a physical field in the spirit of Faraday and Maxwell with Einstein's idea of a Riemannian space–time geometry. Namely, the geometrization principle makes it possible to find an infinite-dimensional noncommutative gauge group that permits the construction of a Lagrangian density of the gravitational field itself. All this led to the relativistic theory of gravitation (RTG),[3] which possesses all conservation laws as in all other physical theories. We give below a detailed exposition of the basic principles and equations of the theory. The relativistic theory of gravitation is a field theory to exactly the same extent as classical electrodynamics, and therefore one could call it classical gravidynamics.

1. FUNDAMENTAL PROPOSITIONS OF THE RELATIVISTIC THEORY OF GRAVITATION

Turning to the construction of the theory of the gravitational field, we shall base it on the following main propositions.

Proposition I. The RTG is based on the special theory of relativity. This means that Minkowski space (a pseudo-Euclidean geometry of space–time) is the fundamental space for all physical fields, including the gravitational field. This proposition is necessary and sufficient if we are to obtain conservation laws for energy, momentum, and angular momentum for matter and the gravitational field taken together. In other words, the Minkowski space reflects the dynamical properties inherent in all forms of matter. It guarantees for them the existence of common physical characteristics that permit quantitative description of the transformation of certain forms of matter into other forms. Minkowski space cannot be regarded as existing *a priori*, since it reflects properties of matter and, therefore, is inseparable from matter. Minkowski space has deep physical content, since it determines universal properties of matter such as energy, momentum, and angular momentum. The gravitational field is described by a symmetric second-rank tensor $\Phi^{\mu\nu}$ and is a real physical field that possesses energy and momentum densities, a rest mass m, and polarization states corresponding to spins 2 and 0. The elimination from the states of the field $\Phi^{\mu\nu}$ of the representations corresponding to the spins 1 and 0′ is made by making the components satisfy the field equation

$$D_\mu \Phi^{\mu\nu}=0, \qquad (1)$$

where D_μ is the covariant derivative in the Minkowski space. Besides eliminating the unphysical states of the field, Eq. (1) introduces into the theory the Minkowski-space metric $\gamma_{\mu\nu}$, and this makes it possible to separate inertial forces from the effect of the gravitational field. By choosing a diagonal metric $\gamma_{\mu\nu}$, one can completely eliminate the effect of the inertial forces. The Minkowski-space metric makes it possible to introduce the concepts of a standard length and standard time interval in the absence of a gravitational field. We shall see later that the interaction of the tensor gravitational field with matter can be introduced in such a way that it appears to deform the Minkowski space, changing the metrical properties without violating causality.

Proposition II. *The geometrization principle.* Since the gravitational field is described by the symmetric second-rank tensor $\Phi^{\mu\nu}$, and its interaction with other fields can be assumed to be universal, a unique possibility is opened up of "adjoining" this field in the matter Lagrangian density directly to the tensor in accordance with the rule

$$L_M(\tilde{\gamma}^{\mu\nu},\Phi_A)\to L_M(\tilde{g}^{\mu\nu},\Phi_A), \qquad (2)$$

where

$$\tilde{g}^{\mu\nu}=\tilde{\gamma}^{\mu\nu}+\tilde{\Phi}^{\mu\nu}, \quad \tilde{g}^{\mu\nu}=\sqrt{-g}g^{\mu\nu},$$
$$\tilde{\gamma}^{\mu\nu}=\sqrt{-\gamma}\gamma^{\mu\nu}, \quad \tilde{\Phi}^{\mu\nu}=\sqrt{-\gamma}\Phi^{\mu\nu}, \qquad (3)$$

Φ_A are the matter fields, and $g=\det g_{\mu\nu}$, $\gamma=\det \gamma_{\mu\nu}$, $\tilde{g}^{\mu\nu}\tilde{g}_{\sigma\nu}=\delta^{\mu}_{\sigma}$. The field indices are raised and lowered by means of $\gamma_{\mu\nu}$, and those of the tensor $g^{\mu\nu}$ by means of the metric tensor of the Riemannian space. By matter, we understand all forms of matter except the gravitational field. Such a form of interaction of the gravitational field with matter introduces the concept of an effective Riemannian space, in which the matter moves, and is called the geometrization principle. In accordance with this principle, the motion of matter under the influence of the gravitational field $\Phi^{\mu\nu}$ in the Minkowski space with metric $\gamma_{\mu\nu}$ is identical to its motion in the effective Riemannian space with metric $g_{\mu\nu}$. The effective Riemannian space has, in the literal sense of the word, a field origin that is due to the presence of the gravitational field $\Phi^{\mu\nu}$. Since the metric properties in the presence of a gravitational field are determined by the tensor of the effective Riemannian space, and without the gravitational field by the Minkowski-space tensor $\gamma_{\mu\nu}$, the RTG can answer this question: How do the sizes of a body and the rate of a clock change under the influence of the gravitational field? If a theory does not contain the tensor $\gamma_{\mu\nu}$ in the field equations, it is in principle incapable of answering such questions. In general relativity, the gravitational field is characterized by the metric tensor $g_{\mu\nu}$; in our theory, it is determined by the tensor $\Phi^{\mu\nu}$, and the effective Riemannian space is constructed by means of the field $\Phi^{\mu\nu}$ and also the Minkowski-space metric tensor $\gamma^{\mu\nu}$, which fixes a definite choice of the coordinate system. In our theory, there exist Galilean (inertial) coordinate systems, and therefore acceleration has an absolute nature. The motion of a test body in the effective Riemannian space takes place along a geodesic of this space, but it is not a free motion, since it is governed by the influence of the gravitational field. If the test body were charged, it would emit electromagnetic waves, since its motion in the field would be accelerated. Since the effective Riemannian space is created by the gravitational field $\Phi^{\mu\nu}$, which is in the Minkowski space, it can always be specified (and this is very important) in a single coordinate system. This means that we shall consider only Riemannian spaces that can be specified in a single chart. From our point of view, Riemannian spaces with complicated topology are completely ruled out, since they do not have a field origin. It should be noted that since matter moves in the effective Riemannian space, the matter equations of motion do not contain the Minkowski-space metric tensor $\gamma_{\mu\nu}$. The Minkowski space influences the motion of matter only through the metric tensor $g_{\mu\nu}$ of the Riemannian space; this tensor is determined from equations that contain the metric tensor $\gamma_{\mu\nu}$.

2. GAUGE GROUP OF TRANSFORMATIONS

Since the matter Lagrangian density has the form

$$L_M(\tilde{g}^{\mu\nu},\Phi_A), \tag{4}$$

it is easy to find a gauge group of transformations under which the matter Lagrangian density changes by only a divergence. For this purpose, we use the invariance of the action

$$S_M=\int L_M(\tilde{g}^{\mu\nu},\Phi_A)d^4x \tag{5}$$

under an arbitrary infinitesim change of the coordinates:

$$x'^{\alpha}=x^{\alpha}+\xi^{\alpha}(x), \tag{6}$$

where ξ^{α} is an infinitesim displacement 4-vector.

Under these coordinate transformations, the field functions $\tilde{g}^{\mu\nu}$ and Φ_A change as follows:

$$\tilde{g}'^{\mu\nu}(x')=\tilde{g}^{\mu\nu}(x)+\delta_{\xi}\tilde{g}^{\mu\nu}(x)+\xi^{\alpha}(x)D_{\alpha}\tilde{g}^{\mu\nu}(x),$$

$$\Phi'_A(x')=\Phi_A(x)+\delta_{\xi}\Phi_A(x)+\xi^{\alpha}(x)D_{\alpha}\Phi_A(x), \tag{7}$$

where the expressions

$$\delta_{\xi}\tilde{g}^{\mu\nu}(x)=\tilde{g}^{\mu\alpha}D_{\alpha}\xi^{\nu}(x)+\tilde{g}^{\nu\alpha}D_{\alpha}\xi^{\mu}(x)-D_{\alpha}(\xi^{\alpha}\tilde{g}^{\mu\nu}),$$

$$\delta_{\xi}\Phi_A(x)=-\xi^{\alpha}(x)D_{\alpha}\Phi_A(x)+F^{B;\alpha}_{A;\beta}\Phi_B(x)D_{\alpha}\xi^{\beta}(x) \tag{8}$$

are the Lie variations.

The operators δ_{ξ} satisfy the conditions of a Lie algebra, i.e., the commutation relation

$$[\delta_{\xi_1},\delta_{\xi_2}](\cdot)=\delta_{\xi_3}(\cdot) \tag{9}$$

and the Jacobi identity

$$[\delta_{\xi_1},[\delta_{\xi_2},\delta_{\xi_3}]]+[\delta_{\xi_3},[\delta_{\xi_1},\delta_{\xi_2}]]+[\delta_{\xi_2},[\delta_{\xi_3},\delta_{\xi_1}]]=0,$$

where

$$\xi^{\nu}_3(x)=\xi^{\mu}_1D_{\mu}\xi^{\nu}_2-\xi^{\mu}_2D_{\mu}\xi^{\nu}_1=\xi^{\mu}_1\partial_{\mu}\xi^{\nu}_2-\xi^{\mu}_2\partial_{\mu}\xi^{\nu}_1. \tag{10}$$

If (9) is to hold, we require fulfillment of the conditions

$$F^{B;\mu}_{A;\nu}F^{C;\alpha}_{B;\beta}-F^{B;\alpha}_{A;\beta}F^{C;\mu}_{B;\nu}=f^{\mu\alpha;\tau}_{\nu\beta;\sigma}F^{C;\sigma}_{A;\tau}, \tag{11}$$

where the structure constants f are

$$f^{\mu\alpha;\tau}_{\nu\beta;\sigma}=\delta^{\mu}_{\beta}\delta^{\alpha}_{\sigma}\delta^{\tau}_{\nu}-\delta^{\alpha}_{\nu}\delta^{\mu}_{\sigma}\delta^{\tau}_{\beta}. \tag{12}$$

It is easy to show that they satisfy the Jacobi identity

$$f^{\alpha\nu;\sigma}_{\beta\mu;\tau}f^{\tau\rho;\omega}_{\sigma\varepsilon;s}+f^{\nu\rho;\sigma}_{\mu\varepsilon;\tau}f^{\tau\alpha;\omega}_{\sigma\beta;s}+f^{\rho\alpha;\sigma}_{\varepsilon\beta;\tau}f^{\tau\nu;\omega}_{\sigma\mu;s}=0 \tag{13}$$

and possess the antisymmetry property

$$f^{\alpha\nu;\rho}_{\beta\mu;\sigma}=-f^{\nu\alpha;\rho}_{\mu\beta;\sigma}.$$

Under the coordinate transformation (6), the variation of the action is zero:

$$\delta_c S_M=\int_{\Omega'}L'_M(x')d^4x'-\int_{\Omega}L_M(x)d^4x=0. \tag{14}$$

The first integral in (14) can be written in the form

$$\int_{\Omega'}L'_M(x')d^4x'=\int_{\Omega}JL'_M(x')d^4x,$$

where

$$J=\det\left(\frac{\partial x'^{\alpha}}{\partial x^{\beta}}\right).$$

In the first order in ξ^{α}, the determinant J is

$$J = 1 + \partial_\alpha \xi^\alpha(x). \tag{15}$$

Taking into account the expansion

$$L'_M(x') = L'_M(x) + \xi^\alpha(x) \frac{\partial L_M}{\partial x^\alpha},$$

and also (15), we can represent the expression for the variation in the form

$$\delta_c S_M = \int_\Omega [\delta L_M(x) + \partial_\alpha(\xi^\alpha L_M(x))] d^4x = 0.$$

Because the volume of integration Ω is arbitrary, we have the identity

$$\delta L_M(x) = -\partial_\alpha(\xi^\alpha(x) L_M(x)), \tag{16}$$

where the Lie variation δL_M is

$$\delta L_M(x) = \frac{\partial L_M}{\partial \tilde{g}^{\mu\nu}} \delta \tilde{g}^{\mu\nu} + \frac{\partial L_M}{\partial(\partial_\alpha \tilde{g}^{\mu\nu})} \delta(\partial_\alpha \tilde{g}^{\mu\nu}) + \frac{\partial L_M}{\partial \Phi_A} \delta \Phi_A + \frac{\partial L_M}{\partial(\partial_\alpha \Phi_A)} \delta(\partial_\alpha \Phi_A). \tag{17}$$

It follows from this in particular that if the scalar density depends only on $\tilde{g}^{\mu\nu}$ and on its derivatives $\partial_\alpha(\tilde{g}^{\mu\nu})$, then under the transformation (8) it also changes only by a divergence:

$$\delta L(\tilde{g}^{\mu\nu}(x)) = -\partial_\alpha(\xi^\alpha(x) L(\tilde{g}^{\mu\nu}(x))), \tag{16a}$$

where the Lie variation δL is

$$\delta L(\tilde{g}^{\mu\nu}(x)) = \frac{\partial L}{\partial \tilde{g}^{\mu\nu}} \delta \tilde{g}^{\mu\nu} + \frac{\partial L}{\partial(\partial_\alpha \tilde{g}^{\mu\nu})} \delta(\partial_\alpha \tilde{g}^{\mu\nu}). \tag{17a}$$

The Lie variations (8) were established in the context of the coordinate transformations (6). However, one can adopt a different point of view, in accordance with which (8) can be regarded as gauge transformations. In this case, the arbitrary infinitesim 4-vector $\xi^\alpha(x)$ will be a gauge vector and not a coordinate displacement vector. In what follows, to emphasize the difference between the gauge group and the group of coordinate transformations, for the group parameter we shall use the notation $\varepsilon^\alpha(x)$, and we shall call the transformations

$$\tilde{g}^{\mu\nu}(x) \to \tilde{g}^{\mu\nu}(x) + \delta \tilde{g}^{\mu\nu}(x),$$

$$\Phi_A(x) \to \Phi_A(x) + \delta \Phi_A(x) \tag{18}$$

of the field functions with increments

$$\delta_\varepsilon \tilde{g}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha} D_\alpha \varepsilon^\nu(x) + \tilde{g}^{\nu\alpha} D_\alpha \varepsilon^\mu(x) - D_\alpha(\varepsilon^\alpha \tilde{g}^{\mu\nu}),$$

$$\delta_\varepsilon \Phi_A(x) = -\varepsilon^\alpha(x) D_\alpha \Phi_A(x) + F^{B;\alpha}_{A;\beta} \Phi_B(x) D_\alpha \varepsilon^\beta(x) \tag{19}$$

gauge transformations.

In complete agreement with Eqs. (9) and (10), the operators satisfy the same Lie algebra, i.e., the commutation relation

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}](\cdot) = \delta_{\varepsilon_3}(\cdot) \tag{20}$$

and the Jacobi identity

$$[\delta_{\varepsilon_1}, [\delta_{\varepsilon_2}, \delta_{\varepsilon_3}]] + [\delta_{\varepsilon_3}, [\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]] + [\delta_{\varepsilon_2}, [\delta_{\varepsilon_3}, \delta_{\varepsilon_1}]] = 0. \tag{21}$$

Here, as before, we have

$$\varepsilon_3^\nu(x) = \varepsilon_1^\mu D_\mu \varepsilon_2^\nu - \varepsilon_2^\mu D_\mu \varepsilon_1^\nu = \varepsilon_1^\mu \partial_\mu \varepsilon_2^\nu - \varepsilon_2^\mu \partial_\mu \varepsilon_1^\nu.$$

The gauge group arose from the geometrized structure of the matter scalar Lagrangian density $L_M(\tilde{g}^{\mu\nu}, \Phi_A)$, which by virtue of the identity (16) changes only by a divergence under the gauge transformations (19). Thus, the geometrization principle, which determined the universal nature of the interaction of the matter and the gravitational field, has given us the possibility of formulating the noncommutative infinite-dimensional gauge group (19). The essential difference between gauge and coordinate transformations appears in a decisive place in the theory, in the construction of the Lagrangian scalar density of the gravitational field by itself. The difference arises because of the fact that under a gauge transformation the metric tensor $\gamma_{\mu\nu}$ does not change; therefore, by virtue of (3),

$$\delta_\varepsilon \tilde{g}^{\mu\nu}(x) = \delta_\varepsilon \tilde{\Phi}^{\mu\nu}(x).$$

On the basis of (19), we obtain the transformation for the field:

$$\delta_\varepsilon \tilde{\Phi}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha} D_\alpha \varepsilon^\nu(x) + \tilde{g}^{\nu\alpha} D_\alpha \varepsilon^\mu(x) - D_\alpha(\varepsilon^\alpha \tilde{g}^{\mu\nu}),$$

but this transformation for the field is very different from the transformation of it under a displacement of the coordinates:

$$\delta_\xi \tilde{\Phi}^{\mu\nu}(x) = \tilde{\Phi}^{\mu\alpha} D_\alpha \xi^\nu(x) + \tilde{\Phi}^{\nu\alpha} D_\alpha \xi^\mu(x) - D_\alpha(\xi^\alpha \tilde{\Phi}^{\mu\nu}).$$

Under the gauge transformations (19), the equations of motion for the matter are unchanged, since under such transformations the matter Lagrangian density changes only by a divergence.

3. LAGRANGIAN DENSITY AND EQUATIONS OF MOTION FOR THE GRAVITATIONAL FIELD BY ITSELF

As is well known, if only the tensor $g_{\mu\nu}$ is used, it is impossible to construct a Lagrangian density of the gravitational field by itself that is a scalar with respect to arbitrary coordinate transformations in a quadratic form containing derivatives of not higher than the first order. Therefore, such a Lagrangian density must necessarily contain the metric $\gamma_{\mu\nu}$ in addition to the metric $g_{\mu\nu}$. Since, however, the metric $\gamma_{\mu\nu}$ does not change under the gauge transformation (19), so that under this transformation the Lagrangian density of the gravitational field by itself changes only by a divergence, strong restrictions on the structure of this density must arise. It is here that we have the fundamental difference between gauge and coordinate transformations. Whereas coordinate transformations impose practically no restrictions on the structure of the Lagrangian scalar density of the gravitational field by itself, the gauge transformations enable us to find the Lagrangian density. A direct general method of constructing the Lagrangian was given in the monograph of Ref. 3. Here, we choose a simpler method to construct the Lagrangian. On the basis of (16a), we conclude that the simplest scalar densities $\sqrt{-g}$ and $\tilde{R} = \sqrt{-g}R$, where R is the scalar curvature of the effective Riemannian space, change as follows under the gauge transformation (19):

$$\sqrt{-g} \to \sqrt{-g} - D_\nu(\varepsilon^\nu \sqrt{-g}), \tag{22}$$

$$\tilde{R} \to \tilde{R} - D_\nu(\varepsilon^\nu \tilde{R}). \tag{23}$$

The scalar density $\tilde{R}$ can be expressed in terms of the Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{24}$$

as follows:

$$\tilde{R} = -\tilde{g}^{\mu\nu}(\Gamma^\lambda_{\mu\nu}\Gamma^\sigma_{\lambda\sigma} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\lambda}) - \partial_\nu(\tilde{g}^{\mu\nu}\Gamma^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma}\Gamma^\nu_{\mu\sigma}). \tag{25}$$

Since the Christoffel symbols are not tensors, each term in (25) is not a scalar density. However, if we introduce the tensors $G^\lambda_{\mu\nu}$ given by

$$G^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}), \tag{26}$$

then the scalar density can be written identically in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda}) - D_\nu(\tilde{g}^{\mu\nu}G^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^\nu_{\mu\sigma}). \tag{27}$$

Note that in (27) each group of terms behaves separately as a scalar density under arbitrary coordinate transformations. With allowance for (22) and (23), the expression

$$\lambda_1(\tilde{R} + D_\nu Q^\nu) + \lambda_2\sqrt{-g} \tag{28}$$

changes only by a divergence under arbitrary gauge transformations. Taking the vector density Q^ν equal to

$$Q^\nu = \tilde{g}^{\mu\nu}G^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^\nu_{\mu\sigma},$$

we eliminate from the previous expression the terms with derivatives of higher than the first order, and we obtain the Lagrangian density

$$-\lambda_1\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda}) + \lambda_2\sqrt{-g}. \tag{29}$$

Thus, we see that the requirement that the Lagrangian density of the gravitational field by itself change only by a divergence under the gauge transformation (19) uniquely determines the structure of the Lagrangian density (29). However, if we limit ourselves to this density alone, the equations of the gravitational field will be gauge-invariant, and the Minkowski-space metric $\gamma_{\mu\nu}$ does not occur in the system of equations determined by the Lagrangian density (29). Since in such an approach the Minkowski-space metric disappears, we lose the possibility of representing the gravitational field as a physical field of Faraday–Maxwell type in Minkowski space. For the Lagrangian density (29), the introduction of the metric $\gamma_{\mu\nu}$ of Eqs. (1) does not save the situation, since the physical quantities—the interval and the curvature tensor of the Riemannian space, and also the gravitational-field tensor $t^{\mu\nu}_g$—will depend on the choice of the gauge, and this is physically unacceptable. To preserve the notion of a field in Minkowski space and eliminate such ambiguity, it is necessary to add to the Lagrangian of the gravitational field a term that breaks the gauge group. At the first glance, it might appear that there would arise here a large arbitrariness in the choice of the Lagrangian density of the gravitational field, since a group can be broken in many different ways. However, it turns out that this is not the case, since our physical requirement imposed by Eqs. (1) on the polarization properties of the gravitational field, as a field with spins 2 and 0, has the consequence that the term breaking the group (19) must be chosen in such a way that Eqs. (1) are consequences of the system of equations of the gravitational field and the matter fields, since it is only in this case that we do not obtain an overdetermined system of differential equations. To this end, we introduce in the Lagrangian scalar density of the gravitational field the term

$$\gamma_{\mu\nu}\tilde{g}^{\mu\nu}, \tag{30}$$

which in the presence of the conditions (1) also changes only by a divergence under the transformations (19), but only on the class of vectors that satisfy the condition

$$g^{\mu\nu}D_\mu D_\nu\varepsilon^\sigma(x) = 0. \tag{31}$$

A completely analogous situation occurs in electrodynamics with a nonvanishing photon rest mass. With allowance for (28)–(30), the general Lagrangian scalar density has the form

$$L_g = -\lambda_1\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda}) + \lambda_2\sqrt{-g} + \lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu} + \lambda_4\sqrt{-\gamma}. \tag{32}$$

We have introduced the final term in (32) in order to use it to make the Lagrangian density vanish in the absence of the gravitational field. The restriction of the class of gauge vectors through the introduction of the term (30) automatically has the consequence that Eqs. (1) become consequences of the gravitational-field equations. We shall show this directly below. In accordance with the principle of least action, the equations for the gravitational field by itself have the form

$$\frac{\delta L_g}{\delta\tilde{g}^{\mu\nu}} = \lambda_1 R_{\mu\nu} + \frac{1}{2}\lambda_2 g_{\mu\nu} + \lambda_3\gamma_{\mu\nu} = 0, \tag{33}$$

where $R_{\mu\nu}$ is the Ricci tensor:

$$R_{\mu\nu} = D_\lambda G^\lambda_{\mu\nu} - D_\mu G^\lambda_{\nu\lambda} + G^\sigma_{\mu\nu}G^\lambda_{\sigma\lambda} - G^\sigma_{\mu\lambda}G^\lambda_{\nu\sigma}. \tag{34}$$

Since in the absence of a gravitational field Eqs. (33) must be satisfied identically, we obtain the relation

$$\lambda_2 = -2\lambda_3. \tag{35}$$

We now find the energy–momentum tensor density of the gravitational field in the Minkowski space:

$$t^{\mu\nu}_g = -2\frac{\delta L_g}{\delta\gamma_{\mu\nu}} = 2\sqrt{-\gamma}\left(\gamma^{\mu\alpha}\gamma^{\nu\beta} - \frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\right)\frac{\delta L_g}{\delta\tilde{g}^{\alpha\beta}} + \lambda_1 J^{\mu\nu} - 2\lambda_3\tilde{g}^{\mu\nu} - \lambda_4\tilde{\gamma}^{\mu\nu}, \tag{36}$$

where

$$J^{\mu\nu}=D_\alpha D_\beta(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}+\gamma^{\alpha\nu}\tilde{g}^{\beta\mu}-\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}-\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}). \quad (37)$$

If in the expression (36) we take into account the dynamical equations (33), we obtain for the gravitational field by itself the equation

$$\lambda_1 J^{\mu\nu}-2\lambda_3\tilde{g}^{\mu\nu}-\lambda_4\tilde{\gamma}^{\mu\nu}=t_g^{\mu\nu}. \quad (38)$$

If this equation is to be satisfied identically in the absence of a gravitational field, we must set

$$\lambda_4=-2\lambda_3. \quad (39)$$

Since for the gravitational field by itself we always have the equation

$$D_\mu t_g^{\mu\nu}=0, \quad (40)$$

it follows from Eq. (38) that

$$D_\mu\tilde{g}^{\mu\nu}=0. \quad (41)$$

Thus, Eqs. (1), which determine the polarization states of the field, follow directly from Eqs. (38). With allowance for Eqs. (41), the field equations (38) can be written as follows:

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}-\frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu}=-\frac{1}{\lambda_1}t_g^{\mu\nu}. \quad (42)$$

In Galilean coordinates, this equation has the simple form

$$\Box\tilde{\Phi}^{\mu\nu}-\frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu}=-\frac{1}{\lambda_1}t_g^{\mu\nu}. \quad (43)$$

It is natural to interpret the numerical factor $m^2=-\lambda_4/\lambda_1$ as the square of the graviton mass, and in accordance with the correspondence principle $-1/\lambda_1$ must be taken equal to 16π. Thus, all the unknown constants in the Lagrangian density have been determined:

$$\lambda_1=-\frac{1}{16\pi},\quad \lambda_2=\lambda_4=-2\lambda_3=\frac{m^2}{16\pi}. \quad (44)$$

The constructed Lagrangian scalar density of the gravitational field by itself has the form

$$L_g=\frac{1}{16\pi}\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma}-G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda})$$
$$-\frac{m^2}{16\pi}\left(\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu}-\sqrt{-g}-\sqrt{-\gamma}\right). \quad (45)$$

The corresponding dynamical equations for the gravitational field by itself can be written in the form

$$J^{\mu\nu}-m^2\tilde{\Phi}^{\mu\nu}=-16\pi t_g^{\mu\nu} \quad (46)$$

or

$$R^{\mu\nu}-\frac{m^2}{2}(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=0. \quad (47)$$

These equations significantly restrict the class of gauge transformations, leaving only trivial ones that satisfy Killing conditions. Such transformations are a consequence of the Lorentz invariance and hold in any theory.

The Lagrangian density constructed above leads to Eqs. (47), from which it follows that Eqs. (41) are their consequences, and therefore outside matter we have ten equations for the ten unknown field functions. By means of Eqs. (41), we can readily express the unknown field functions $\Phi^{0\alpha}$ in terms of the field functions Φ^{ik}, where the superscripts i and k take the values 1, 2, 3. Thus, in the Lagrangian density of the gravitational field by itself the structure of the mass term, which breaks the gauge group, is uniquely determined by the polarization properties of the gravitational field.

4. EQUATIONS OF MOTION FOR THE GRAVITATIONAL FIELD AND MATTER

The total Lagrangian density of the matter and the gravitational field is

$$L=L_g+L_M(\tilde{g}^{\mu\nu},\Phi_A), \quad (48)$$

where L_g is determined by the expression (45).

On the basis of (48) and the principle of least action, we obtain the complete system of equations for the matter and gravitational field:

$$\frac{\delta L}{\delta\tilde{g}^{\mu\nu}}=0, \quad (49)$$

$$\frac{\delta L_M}{\delta\Phi_A}=0. \quad (50)$$

Since under an arbitrary infinitesim change of the coordinates the variation δS_M of the action is zero,

$$\delta S_M=\delta\int L_M(\tilde{g}^{\mu\nu},\Phi_A)d^4x=0,$$

we can obtain the identity (see Ref. 3) in the form

$$g_{\mu\nu}\nabla_\lambda T^{\lambda\nu}=-D_\nu\left(\frac{\delta L_M}{\delta\Phi_A}F^{B;\nu}_{A;\mu}\Phi_B(x)\right)-\frac{\delta L_M}{\delta\Phi_A}D_\mu\Phi_A(x). \quad (51)$$

Here, $T^{\lambda\nu}=-2\delta L_M/\delta g_{\lambda\nu}$ is the matter tensor in the Riemannian space, and ∇_λ is the covariant derivative in this space with the metric $g_{\lambda\nu}$. It follows from the identity (51) that if the matter equations of motion (50) are satisfied, then

$$\nabla_\lambda T^{\lambda\nu}=0. \quad (52)$$

In this case, if the number of equations (50) for the matter is equal to four, we can use instead of them the equivalent equations (52). Since in what follows we shall consider only such equations for the matter, we shall always use equations for the matter in the form (52). Thus, the complete system of equations for the matter and gravitational field will have the form

$$\frac{\delta L}{\delta\tilde{g}^{\mu\nu}}=0, \quad (53)$$

$$\nabla_\lambda T^{\lambda\nu}=0. \quad (54)$$

The matter will be described by the velocity **v**, the matter density ρ, and the pressure p. The gravitational field is determined by the ten components of the tensor $\Phi^{\mu\nu}$. Thus, we have 15 unknowns. To determine them, we must add to the 14 equations (53)–(54) the equation of state of the matter. If we take into account the relations

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}}=-\frac{1}{16\pi}R_{\mu\nu}+\frac{m^2}{32\pi}(g_{\mu\nu}-\gamma_{\mu\nu}), \tag{55}$$

$$\frac{\delta L_M}{\delta \tilde{g}^{\mu\nu}}=\frac{1}{2\sqrt{-g}}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right), \tag{56}$$

then the system of equations (53) and (54) can be represented in the form

$$\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)+\frac{m^2}{2}\left[g^{\mu\nu}+\left(g^{\mu\alpha}g^{\nu\beta}-\frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right]$$
$$=\frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \tag{57}$$

$$\nabla_\lambda T^{\lambda\nu}=0. \tag{58}$$

By virtue of the Bianchi identity

$$\nabla_\mu\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)=0$$

we obtain from Eqs. (57)

$$m^2\sqrt{-g}(g^{\mu\alpha}g^{\nu\beta}-\tfrac{1}{2}g^{\mu\nu}g^{\alpha\beta})\nabla_\mu\gamma_{\alpha\beta}=16\pi\nabla_\mu T^{\mu\nu}. \tag{59}$$

Taking into account the expression

$$\nabla_\mu\gamma_{\alpha\beta}=-G^\sigma_{\mu\alpha}\gamma_{\sigma\beta}-G^\sigma_{\mu\beta}\gamma_{\sigma\alpha}, \tag{60}$$

where $G^\sigma_{\mu\alpha}$ is determined by the expression (26), we find

$$(g^{\mu\alpha}g^{\nu\beta}-\tfrac{1}{2}g^{\mu\nu}g^{\alpha\beta})\nabla_\mu\gamma_{\alpha\beta}=\gamma_{\mu\lambda}g^{\mu\nu}(D_\sigma g^{\sigma\lambda}+G^\beta_{\alpha\beta}g^{\alpha\lambda}), \tag{61}$$

but since

$$\sqrt{-g}(D_\sigma g^{\sigma\lambda}+G^\beta_{\alpha\beta}g^{\alpha\lambda})=D_\sigma\tilde{g}^{\lambda\sigma}, \tag{62}$$

the expression (61) takes the form

$$\sqrt{-g}(g^{\mu\alpha}g^{\nu\beta}-\tfrac{1}{2}g^{\mu\nu}g^{\alpha\beta})\nabla_\mu\gamma_{\alpha\beta}=\gamma_{\mu\lambda}g^{\mu\nu}D_\sigma\tilde{g}^{\lambda\sigma}. \tag{63}$$

Using (63), we can represent the expression (59) in the form

$$m^2\gamma_{\mu\lambda}g^{\mu\nu}D_\sigma\tilde{g}^{\lambda\sigma}=16\pi\nabla_\mu T^{\mu\nu}.$$

This expression can be rewritten as follows:

$$m^2D_\sigma\tilde{g}^{\lambda\sigma}=16\pi\gamma^{\lambda\nu}\nabla_\mu T^\mu_\nu. \tag{64}$$

By means of this relation, we can replace Eq. (58) by the equation

$$D_\sigma\tilde{g}^{\nu\sigma}=0. \tag{65}$$

Therefore, the system of equations (57) and (58) reduces to the system of gravitational equations

$$\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)+\frac{m^2}{2}\left[g^{\mu\nu}+\left(g^{\mu\alpha}g^{\nu\beta}-\frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right]$$
$$=\frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \tag{66}$$

$$D_\mu\tilde{g}^{\mu\nu}=0. \tag{67}$$

If we introduce the tensor

$$N^{\mu\nu}=R^{\mu\nu}-\frac{m^2}{2}[g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}],\quad N=N^{\mu\nu}g_{\mu\nu},$$

the system of equations (66) and (67) can be written as

$$N^{\mu\nu}-\frac{1}{2}g^{\mu\nu}N=\frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \tag{66a}$$

$$D_\mu\tilde{g}^{\mu\nu}=0. \tag{67a}$$

The system can also be represented in the form

$$N^{\mu\nu}=\frac{8\pi}{\sqrt{-g}}\left(T^{\mu\nu}-\frac{1}{2}g^{\mu\nu}T\right), \tag{68}$$

$$D_\mu\tilde{g}^{\mu\nu}=0 \tag{69}$$

or

$$N_{\mu\nu}=\frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right), \tag{68a}$$

$$D_\mu\tilde{g}^{\mu\nu}=0. \tag{69a}$$

It must be emphasized especially that both (68) and (69) contain the Minkowski-space metric tensor. Coordinate transformations that leave the Minkowski-space metric form invariant relate physically equivalent frames of reference. The simplest of them will be inertial systems. Therefore, the possible gauge transformations satisfying the Killing conditions $D_\mu\varepsilon_\nu+D_\nu\varepsilon_\mu=0$ do not take us out of the class of physically equivalent reference systems. If we suppose that it is possible to measure experimentally the characteristics of the Riemannian space and the motion of the matter with arbitrarily high accuracy, then on the basis of Eqs. (68a) and (69a) we can determine the Minkowski-space metric and find Galilean (inertial) coordinate systems. Thus, Minkowski space is in principle observable.

The existence of Minkowski space is reflected in conservation laws, and therefore the verification of them in physical phenomena is at the same time a verification of the space–time structure.

The system of gravitational equations can also be given the different equivalent form

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}+m^2\tilde{\Phi}^{\mu\nu}=16\pi t^{\mu\nu}, \tag{70}$$

$$D_\mu\tilde{\Phi}^{\mu\nu}=0, \tag{71}$$

where $t^{\mu\nu}=-2\delta L/\delta\gamma_{\mu\nu}$ is the energy–momentum tensor density of the matter and the gravitational field in the Minkowski space. The form of these equations is reminiscent of the equations of electrodynamics with photon mass μ in the absence of gravitation:

$$\gamma^{\alpha\beta}D_\alpha D_\beta A^\nu+\mu^2A^\nu=4\pi j^\nu, \tag{72}$$

$$D_\nu A^\nu=0. \tag{73}$$

Whereas in electrodynamics the source of the vector field A^ν is the conserved electromagnetic current j^ν produced by the charged bodies, in the RTG the source of the tensor field is the conserved total energy–momentum tensor of the matter and the gravitational field. Therefore, the gravitational equations will be linear even for the gravitational field by itself. We note especially that in Eqs. (66) there have appeared not only the well-known cosmological term but also a term containing the Minkowski-space metric $\gamma_{\mu\nu}$; moreover, both of these terms enter with a common constant, which is equal to the graviton mass, and therefore is very small. The second term in Eqs. (66), which contains the metric $\gamma_{\mu\nu}$, leads to the appearance of repulsive forces that are very strong in strong gravitational fields. This fact changes the nature of collapse and the evolution of the universe. As we saw earlier, the presence of a graviton rest mass has fundamental significance for the construction of a field theory of the gravitational field. Indeed, because of the presence of a graviton mass it follows from the theory that a homogeneous and isotropic universe can be only flat.

5. CAUSALITY PRINCIPLE IN THE RTG

The RTG is constructed in the framework of the special theory of relativity in the same way as the theories of other physical fields. In accordance with special relativity, any motion of any point test body always takes place within the causality light cone of the Minkowski space. Therefore, the noninertial frames of reference realized by test bodies must also be within the causality cone of the pseudo-Euclidean space–time. This determines the complete class of possible noninertial frames of reference. Local equivalence of inertia and gravity acting on a material point will hold if the light cone of the effective Riemannian space does not pass outside the causality light cone of the Minkowski space. It is only in this case that a gravitational field acting on a test body can be locally eliminated by going over to an allowed noninertial frame of reference associated with this body. If the light cone of the effective Riemannian space were to pass outside the causality light cone of the Minkowski space, this would mean that for such a "gravitational field" there does not exist an allowed noninertial frame of reference in which the action of this "field" on the material point may be eliminated. In other words, local "equivalence" of inertia and gravity is possible only when the gravitational field, as a physical field acting on particles, does not carry their world lines outside the causality cone of the pseudo-Euclidean space–time. This condition is to be regarded either as a causality principle or as an equivalence principle that makes it possible to choose solutions of the system of equations (66) and (67) that have physical meaning and correspond to gravitational fields. The causality principle is not satisfied automatically. The reason for this is that the gravitational interaction occurs in the coefficients of the second derivatives in the field equations, i.e., it changes the original space–time geometry. Only the gravitational field possesses this property. The interaction of all other known physical fields usually does not affect the second derivatives of the field equations and therefore does not change the original pseudo-Euclidean geometry of space–time.

We now give an analytic formulation of the causality principle in the RTG. Since in the RTG the motion of matter under the influence of a gravitational field in the pseudo-Euclidean space–time is equivalent to motion of the matter in the corresponding effective Riemannian space–time, for causally connected events (world lines of particles and light) we must, on the one hand, have the condition

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu\geqslant 0, \tag{74}$$

and, on the other, for such events we must have fulfillment of the inequality

$$d\sigma^2=\gamma_{\mu\nu}dx^\mu dx^\nu\geqslant 0. \tag{75}$$

For a chosen frame of reference realized by physical bodies, we have the condition

$$\gamma_{00}>0. \tag{76}$$

In the expression (75), we separate the timelike and spacelike parts:

$$d\sigma^2=\left(\sqrt{\gamma_{00}}dt+\frac{\gamma_{0i}dx^i}{\sqrt{\gamma_{00}}}\right)^2-S_{ik}dx^i dx^k. \tag{77}$$

Here, the indices i and k take the values 1, 2, 3, and

$$S_{ik}=-\gamma_{ik}+\frac{\gamma_{0i}\gamma_{0k}}{\gamma_{00}}, \tag{78}$$

where S_{ik} is the metric tensor of three-dimensional space in the four-dimensional pseudo-Euclidean space–time.

The square of the spatial distance is determined by the expression

$$dl^2=S_{ik}dx^i dx^k. \tag{79}$$

We represent the velocity $u^i=dx^i/dt$ in the form $u^i=ue^i$, where u is the magnitude of the velocity, and e^i is an arbitrary unit vector in the three-dimensional space:

$$S_{ik}e^i e^k=1. \tag{80}$$

In the absence of the gravitational field, the speed of light in the chosen coordinate system is readily determined from the expression (77) by setting it equal to zero:

$$\left(\sqrt{\gamma_{00}}dt+\frac{\gamma_{0i}dx^i}{\sqrt{\gamma_{00}}}\right)^2=S_{ik}dx^i dx^k.$$

From this we find

$$u=\frac{\sqrt{\gamma_{00}}}{1-\dfrac{\gamma_{0i}e^i}{\sqrt{\gamma_{00}}}}. \tag{81}$$

Thus, an arbitrary four-dimensional isotropic vector u^ν in the Minkowski space is

$$u^\nu=(1,ue^i). \tag{82}$$

For the simultaneous fulfillment of the conditions (74) and (75) it is necessary and sufficient that any isotropic vector,

$$\gamma_{\mu\nu}u^\mu u^\nu=0, \tag{83}$$

satisfies the causality condition

$$g_{\mu\nu}u^\mu u^\nu\leqslant 0, \tag{84}$$

which means that the light cone of the effective Riemannian space does not pass outside the causality light cone of the pseudo-Euclidean space–time. The causality conditions can be written as follows:

$$g_{\mu\nu}v^{\mu}v^{\nu}=0, \tag{83a}$$

$$\gamma_{\mu\nu}v^{\mu}v^{\nu}\geqslant 0. \tag{84a}$$

In general relativity, the solutions of the Hilbert–Einstein equations that have physical meaning satisfy at each point of space–time the inequality

$$g<0,$$

and also the requirement imposed by the dominant energy condition, which is formulated as follows: For any nonspacelike vector K_ν, the inequality $T^{\mu\nu}K_\mu K_\nu\geqslant 0$ must hold, and $T^{\mu\nu}K_\nu$ for any vector K_ν must form a nonspacelike vector.

In our theory, the solutions of Eqs. (68a) and (69a) that have physical meaning must, in addition to these requirements, also satisfy the causality conditions (83a) and (84a). On the basis of Eq. (68a), the latter can be written in the form

$$R_{\mu\nu}K^{\mu}K^{\nu}\leqslant\frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right)K^{\mu}K^{\nu}+\frac{m^2}{2}g_{\mu\nu}K^{\mu}K^{\nu} \tag{85}$$

or

$$\sqrt{-g}R_{\alpha\beta}v^{\alpha}v^{\beta}\leqslant 8\pi T_{\alpha\beta}v^{\alpha}v^{\beta}. \tag{85a}$$

To conclude this section, we note that although the gravitational equations (66) and (67) with graviton mass were already obtained several years ago, the logic of our construction gradually led to the conclusion of the existence of a graviton rest mass, since it alone makes it possible to construct a tensor field theory in Minkowski space leading to an effective Riemannian geometry of space–time.

6. SOME PHYSICAL PREDICTIONS OF THE RTG

The system of equations (66) and (67) of the RTG leads to qualitatively new physical predictions quite different from those of general relativity. For example, the notion of collapse is changed completely. It turns out that in the collapse of a spherically symmetric body of arbitrary mass the process of contraction comes to a stop in a region close to the Schwarzschild sphere and is replaced by a subsequent expansion. This means that in nature there must exist not only contracting objects but also expanding objects. Thus, in accordance with the RTG the existence in nature of "black holes" (objects that do not have material boundaries and are "cut off" from the external world) are completely ruled out. Another important physical prediction relates to the evolution of a homogeneous and isotropic universe. It follows from Eqs. (66) and (67), and also from the causality conditions (83) and (84), that a homogeneous and isotropic universe exists for an infinite time, and its three-dimensional geometry is Euclidean. The universe evolves cyclically from a maximum finite density to a minimum finite density, then again to a maximum value (if there is no dissipation), etc. The theory predicts the existence in the universe of a large amount of dark matter, since in accordance with Eqs. (66) and (67) the total matter density at the present time is

$$\rho=\rho_c+\frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2. \tag{86}$$

It can be seen from this that the matter density, even for a sufficiently small graviton mass, is close to the critical density ρ_c determined by the Hubble constant H and is equal to

$$\rho_c=\frac{3H^2}{8\pi G}. \tag{87}$$

The RTG explains all known gravitational experiments in the solar system and makes it possible, as we have seen earlier, to introduce for the gravitational field, as for all other physical fields, the concept of an energy–momentum tensor. Since the source of the gravitational field in the theory is the field energy–momentum tensor of the matter and the gravitational field, the inertial mass of a static body is exactly equal to its active gravitational mass. In the framework of general relativity, such a conclusion cannot be drawn, although Einstein did attempt to take it as the basis of the theory. The experimental verification of this prediction is at the same time a verification of our theory. The energy–momentum tensor $-2\delta L_g/\delta g_{\mu\nu}$ of the gravitational field in the Riemannian space outside matter is, in accordance with Eqs. (66), equal to zero. However, this does not mean the absence of gravitational radiation, since a gravitational wave, carrying energy, moves on an effective gravitational background. With regard to the gravitational emission of massive gravitons, this question was considered in Ref. 4, in which Loskutov showed that the calculations that had been made earlier were based on an incorrectly derived general expression for the intensity. Its derivation did not take into account the important fact that in reality gravitons propagate, not in the Minkowski space, but in the effective Riemannian space. Allowance for this circumstance led Loskutov to the conclusion that the intensity of the gravitational emission of massive gravitons is a positive-definite quantity. The expression for the intensity is given in Ref. 4. The system of gravitational equations (66) and (67) of the RTG opens up new possibilities for investigations both of a fundamental nature and in concrete situations, in the study of particular gravitational phenomena.

In conclusion, we must make some important remarks. Can the graviton mass be set equal to zero? Since the graviton mass in our theory lifts the degeneracy with respect to the gauge group, its vanishing directly in Eqs. (66) and (67) is not correct. In our theory, it must not be equal to zero. The system of gravitational equations (66) and (67) is hyperbolic, and the causality principle ensures the existence in the whole of space of a spacelike surface that is crossed by every nonspacelike curve in the Riemannian space only once, i.e., in other words, there exists a global Cauchy surface on which initial physical conditions can be specified for particular problems. Under certain general conditions, Penrose and Hawking[5] proved a theorem that establishes the existence of a singularity in general relativity. Now on the basis of Eqs. (68a) isotropic vectors of the Riemannian space outside matter satisfy by virtue of the causality conditions (85) the inequality

$$R_{\mu\nu}v^{\mu}v^{\nu}\leqslant 0, \quad (88)$$

and it follows from this that in the RTG the conditions of the singularity theorems are not satisfied, and therefore they do not hold for the RTG. In the RTG, events that are spacelike in the absence of the gravitational field can never become timelike under the influence of the gravitational field. By virtue of the causality principle, the effective Riemannian space–time in the RTG will possess isotropic and timelike geodesic completeness. On the basis of the complete exposition given above, we may draw the following general conclusion: If by virtue of the universality of gravitation it is assumed that the source of the gravitational field in Minkowski space is the conserved energy–momentum tensor of the matter and a massive gravitational field, then this field itself will be manifested as a second-rank tensor field. By analogy with electrodynamics, it is natural to write the field equations in the form

$$\Box\Phi^{\mu\nu}+m^2\Phi^{\mu\nu}=\lambda t^{\mu\nu}, \quad \partial_\mu\Phi^{\mu\nu}=0.$$

However, such a system of equations follows from the Lagrangian formalism only if the interaction of matter and the gravitational field is realized in accordance with the geometrization principle, and this reduces the action of this field to an effective space–time geometry. Thus, the adoption of a conserved energy–momentum tensor of matter as a universal source of the gravitational field necessarily leads to an effective Riemannian geometry. Since the field theory of gravitation requires the introduction of a graviton mass, and in its structure the theory is very close to electrodynamics, it is very probable that the photon rest mass is also nonzero.

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