Interaction of a beam with the accelerator vacuum chamber. Methods for calculating coupling impedance

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Methods for calculating the coupling impedance for the coupling of the beam with the accelerator vacuum chamber are reviewed. The definitions and properties of these impedances and their relationship with the wake potentials are discussed in detail. Attention is focused on methods which can be used at low frequencies and in the resonant region. An attempt is made to classify impedance calculation methods. Analytic formulas for calculations are collected. This paper can also be used as a guide to the literature on the subject.

1. IMPEDANCES AND WAKE POTENTIALS

As a charged-particle beam passes through the vacuum chamber of an accelerator, it excites currents in the chamber wall. The fields generated by these currents in turn act on the beam particles. This interaction of the beam with the vacuum chamber results in beam energy losses, alters the shape of the bunches, alters the shifts of betatron frequencies, and, finally, may drive instabilities of various types (see the reviews[1–3]). The beam stability problem is so important, especially in advanced high-intensity accelerators, that the beam-chamber interaction must be examined carefully. For such studies it is convenient to use the concept of coupling impedances, associated with the coupling of the beam with the vacuum chamber, or the related concept of wake potentials. [The Russian word used in the Russian-language version of this paper as the translation of the English word "wake" is *navedennyĭ* ("induced"), although *ostavlennyĭ* ("left behind") would be more accurate. There are the further equivalents *sputnyĭ* ("accompanying") and *kil'vaternyĭ* ("wake").]

1.1. Definitions and properties

Longitudinal impedance. We consider a cyclic accelerator in which the average radius of the equilibrium orbit is R. We introduce a cylindrical coordinate system (r,θ,z), as is customary in the theory of cyclic accelerators.[4] The z axis runs perpendicular to the plane of the accelerator ($z=0$), and θ is the azimuthal angle along the orbit. We assume that a "rigid" bunch with charge q is moving through the vacuum chamber and that the charge distribution is

$$\rho(r,\theta,z,t)=q\lambda(\theta-\omega_0 t)f(r,z), \tag{1.1}$$

where $\omega_0=\beta c/R$ and βc are the angular and linear velocities of an equilibrium particle, and $f(r,z)$ and $\lambda(\theta')$ are the transverse and longitudinal distributions of the charge of the bunch, normalized by the conditions

$$\iint rdrdzf(r,z)=1; \quad \int_0^{2\pi} d\theta'\lambda(\theta')=1.$$

As usual, the assumption that a bunch is "rigid" means that the shape of the bunch changes so slowly that the charge distribution can be assumed to be in a steady state in the frame of reference of the bunch: $\lambda(\theta',t)=\lambda(\theta')$, where $\theta'=\theta-\omega_0 t$. We are ignoring synchrotron oscillations of the bunch particles. Since we are discussing longitudinal effects at this point, we will also ignore betatron oscillations and assume that the bunch current density has only a longitudinal component j_θ (we omit the subscript θ below):

$$j(r,\theta,z,t)=\omega_0 r\rho(r,\theta,z,t). \tag{1.2}$$

The current in the chamber is

$$J(\theta,t)=\int rdr\int dzj(r,\theta,z,t)=\omega_0 q\lambda(\theta-\omega_0 t). \tag{1.3}$$

It is natural to switch from the longitudinal field $E(r,\theta,z,t)$ excited by the beam in the chamber to the field averaged over transverse coordinates with weight $f(r,z)$:

$$E(\theta,t)=\int rdr\int dzf(r,z)E(r,\theta,z,t). \tag{1.4}$$

We introduce a Fourier representation for average quantities as follows:

$$E(\theta,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\sum_{n=-\infty}^{\infty} e^{-in\theta}E_n(\omega), \tag{1.5}$$

with a corresponding expression for $J(\theta,t)$. From (1.3) we easily find the current harmonics:

$$J_n(\omega)=\omega_0 q\lambda_n\delta(\omega-\omega_0 n),$$

where

$$\lambda_n=\int_0^{2\pi} d\theta\exp(-in\theta)\lambda(\theta)/(2\pi)$$

and $\delta(x)$ is the Dirac δ function.

It follows from the linearity of Maxwell's equations that the relationship between the current harmonics and the harmonics of the field excited in the chamber is linear. If the accelerator chamber is azimuthally uniform (i.e., if there is no dependence on θ), the relationship is thus

$$-2\pi RE_n(\omega)=Z_n(\omega)J_n(\omega), \tag{1.6}$$

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where the function $Z_n(\omega)$ is called the *longitudinal coupling impedance of the beam with the vacuum chamber.* The concept of a beam–chamber coupling impedance was apparently introduced by Lebedev and Zhilkov,[5] although all the calculations necessary for finding the longitudinal and transverse impedances of a smooth chamber had been carried out in earlier studies.[6,7] Instead of a weighted average over transverse coordinates[8] as in (1.4) it is frequent to find authors using either the field at the center of the beam, $E(r=R,\theta,z=0,t)$ or simply the field averaged over the beam cross section.[9] That approach is equivalent to choosing $f(r,z)=\delta(z)\delta(r-R)/R$ in (1.4) or taking this function to be a product of unit step functions, e.g., $f(r,z) = \Theta(a - |z|)\Theta(\sqrt{a^2-z^2} - |r - R|)/(\pi a^2 r)$ for a beam with a circular cross section of radius a.

In the more realistic case in which the chamber has various irregularities, all harmonics of the current generally contribute to the nth harmonic of the field:

$$-2\pi RE_n(\omega)= \sum_{m=-\infty}^{\infty} Z_{nm}(\omega)J_m(\omega). \tag{1.7}$$

The quantity $Z_{nm}(\omega)$ can quite naturally be called the *matrix of the longitudinal coupling impedance.*[3,8]

Since only the nth harmonic of the field excited in the chamber has a systematic effect on the nth harmonic of a perturbation of the equilibrium beam charge distribution, it is customary to restrict Eq. (1.7) to diagonal terms:

$$-2\pi RE_n(\omega)\cong Z_{nn}(\omega)J_n(\omega), \tag{1.8}$$

with $\omega=n\omega_0$, since $E_n(\omega)$ and $J_n(\omega)$ are proportional to $\delta(\omega-\omega_0 n)$. The function $Z(\omega)\equiv Z_{nn}(n\omega_s)$ is called the *longitudinal impedance.* However, the off-diagonal elements of the impedance matrix $Z_{-n,n}(\omega)$ are also important in research on the mutual effects of oppositely directed beams in a single chamber.[10]

From the representation (1.5) we find $E_{-n}(-\omega)=E_n^*(\omega)$ and $J_{-n}(-\omega)=J_n^*(\omega)$, so that $Z_{-n,-n}(-\omega) = Z_{nn}^*(\omega)$ follows from the definition (1.7).

The calculation of a chamber impedance is a purely electrodynamic problem. An impedance, as a function characterizing an accelerator chamber, should not depend on characteristics of the beam.[1)] For a study of the beam stability we find a second relationship between the harmonics of the field and the current from a kinetic equation:

$$J_m(\omega)=-2\pi R \sum_{n=-\infty}^{\infty} \Pi_{mn}(\omega)E_n(\omega), \tag{1.9}$$

where $\Pi_{mn}(\omega)$, the "beam conductance matrix," incorporates information on the beam characteristics (Ref. 3, for example). Once the chamber impedance matrix and the beam conductance matrix have been calculated, substitution of (1.7) into Eq. (1.9) yields an eigenvalue problem for the spectrum of beam excitations. The existence of eigenvalues with $\mathrm{Im}\omega>0$ indicates an instability of the beam, and the value of $\mathrm{Im}\omega$ determines the instability growth rate.

To show that the definitions (1.6)–(1.8) are equivalent to other common definitions of the impedance, and also looking ahead to a generalization to aperiodic structures, we consider as a model chamber an infinite periodic structure with period $D=2\pi R$. We introduce a cylindrical coordinate system (r,φ,z), directing the z axis along the chamber axis (Fig. 1,a and b). Everything said above can be said about this structure if we change some notation and the normalization of the distributions:

$$\iint rdrd\varphi f(r,\varphi)=1, \qquad \int_0^{2\pi R} dz'\lambda(z')=1.$$

In the approximation of a rigid bunch we have

$$\lambda(z-\beta ct)= \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \sum_{n=-\infty}^{\infty} e^{ik_n z}\lambda_n\delta(\omega-\beta ck_n),$$

where $k_n=n/R$, $\quad \lambda_n=\int_0^{2\pi R}dz \exp(-ik_n z)\lambda(z)/(2\pi R)$, and the Fourier harmonic of the current is

$$J_n=2\pi q\beta c\lambda_n\delta(\omega-\beta ck_n).$$

All the rest of the discussion is analogous.

As a perturbation of the charge density of the equilibrium distribution, Keil and Zotter[9] consider a single mode

$$\rho(r,\varphi,z,t)=\rho_n\exp(ik_n z-i\omega_n t)\theta(a-r),$$

propagating in a chamber with phase velocity $\omega_n/k_n=\beta c$ equal to the velocity of the beam particles. Correspondingly, the perturbation current in the chamber is

$$J(z,t)=\rho_n\beta c\pi a^2\exp(ik_n z-i\omega_n t).$$

We solve Maxwell's equations in the ω domain with a current $\rho_n\beta c \exp(ik_n z)\theta(a-r)$ on the right side [we omit a time dependence $\exp(-i\omega_n t)$ from all quantities]. We denote the resulting complex amplitude of the longitudinal component of the field by $E_z(r,z)$, and the average over the beam cross section by

$$\overline{E_z(r,z)}=\frac{1}{\pi a^2}\int_0^a rdr\int_0^{2\pi} d\varphi E_z(r,z).$$

It is a straightforward matter to derive expressions for the Fourier harmonics:

$$\begin{aligned} J_m(\omega)&= \int_{-\infty}^{\infty} dt e^{i\omega t}\frac{1}{2\pi R}\int_0^{2\pi R} dz\, e^{-ik_m z}J_z(z,t) \\ &=2\pi\rho_n\beta c\pi a^2\delta(\omega-\omega_n)\delta_{mn}; \\ E_l(\omega)&=\delta(\omega-\omega_n)\frac{1}{2\pi R}\int_0^R dz\, e^{-ik_l z}\overline{E_z(r,z)}. \end{aligned} \tag{1.10}$$

Substituting (1.10) into the definition of the impedance matrix, (1.7), we find the matrix element $Z_{ln}(\omega=\omega_n)$. The expression for the longitudinal coupling impedance,

$$Z_n(\omega)\equiv Z_{nn}(\omega=\omega_n),$$

becomes

$$Z_n(\omega)=-\frac{1}{\rho_n\beta c\pi a^2}\int_0^{2\pi R} dz\, e^{-ik_n z}\overline{E_z(r,z)}, \tag{1.11}$$

which is the definition used by Keil and Zotter.[9]

We turn now to the case of an aperiodic structure. We consider a model vacuum chamber consisting of an (infi-

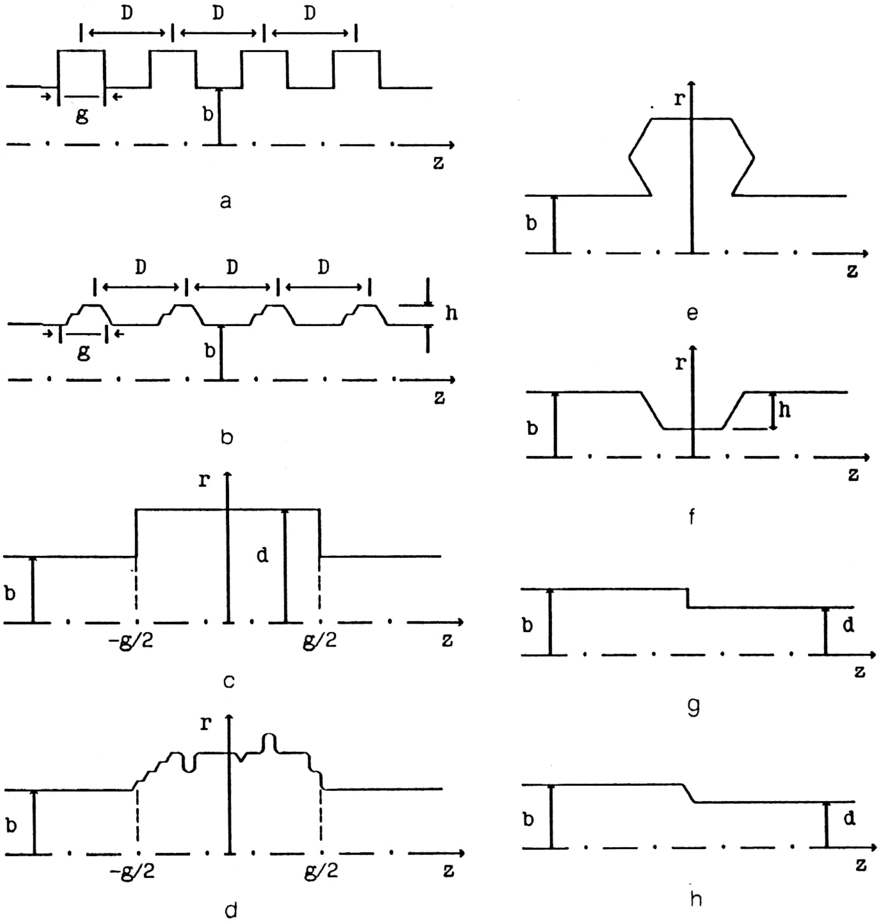

FIG. 1. Irregularities of a vacuum chamber.

nite) smooth tube on which there may be one or several irregularities (Fig. 1,c–h). The chamber of any accelerator, of course, is either closed or of finite length. Nevertheless, our idealized infinite structure is convenient for calculating the impedance of chamber elements remote from other irregularities. We assume that a rigid bunch with charge distribution

$$\rho(x,y,z,t)=qf(r,z)\lambda(z-\beta ct)$$

is moving along the z axis of the chamber with velocity βc, where the longitudinal and transverse distributions are normalized by the conditions

$$\iint dxdyf(x,y)=1;\quad \int_{-\infty}^{\infty} dz'\lambda(z')=1.$$

The total charge of a bunch is thus q. The current in the chamber is $J_z(z,t)=q\beta c\lambda(z-\beta ct)$. As before, we introduce fields which are averaged over the transverse coordinates with weight $f(x,y)$,

$$E(z,t)=\int\int dxdyf(x,y)E(x,y,z,t),$$

but instead of the Fourier representation (1.5) we use a double Fourier integral,

$$E_z(z,t)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\int_{-\infty}^{\infty} dk\, e^{ikz}E_z(k,\omega). \tag{1.12}$$

We then have $J_z(k,\omega)=2\pi q\beta c\tilde{\lambda}(k)\delta(\omega-\beta ck)$, where $\tilde{\lambda}(k)=\int dz\exp(-ikz)\lambda(z)$. Since all possible harmonics of the current contribute to the field harmonic with wave number k in a nonuniform structure, a natural generalization of Eq. (1.7), which introduces the impedance matrix, to an infinite aperiodic structure is

$$E_z(k,\omega)=-\frac{1}{2\pi}\int_{-\infty}^{\infty} dk'Z(k,k';\omega)J_z(k',\omega). \tag{1.13}$$

The kernel of the *integral longitudinal-impedance operator* in the definition (1.13) has the dimensionality of an impedance (ohms). If the structure is uniform along z (if the chamber is smooth), this kernel is diagonal:

$$Z_0(k,k';\omega)=\frac{2\pi}{L}\delta(k-k')Z_L(k,\omega),$$

where L is the length of the structure. The quantity $Z_L(k,\omega)/L$ is the longitudinal impedance per unit length of the smooth chamber (Sec. 2). Since $J_z(k',\omega)\equiv\hat{J}_z(k'=\omega/\beta c)\delta(\omega-\beta ck')$ in the rigid-bunch approximation, the integral in (1.13) can be evaluated:

$$E_z(k,\omega)=-Z\left(k,k'=\frac{\omega}{\beta c};\omega\right)\frac{1}{2\pi\beta c}\hat{J}_z\left(k'=\frac{\omega}{\beta c}\right).$$

It follows from dynamic considerations that the only harmonic of the field which has any systematic effect on a bunch is that which is synchronized with the bunch; i.e., the only quantity of interest for the dynamics is $E_z(k=\omega/\beta c,\omega)$. By analogy with (1.8), the longitudinal impedance of the structure is then

$$Z(\omega)\equiv Z\left(k=\frac{\omega}{\beta c},k'=\frac{\omega}{\beta c};\omega\right)=-2\pi\beta c\,\frac{E_z(k=\omega/\beta c,\omega)}{\hat{J}_z(k'=\omega/\beta c)}. \tag{1.14}$$

It is usually more convenient to calculate fields in the ω domain, i.e., to find the quantities

$$E(z,\omega)=\int dk\,\exp(ikz)E(k,\omega)/(2\pi),$$

rather than $E(k,\omega)$. Inverting this Fourier transformation, we rewrite (1.14) as

$$Z(\omega)=-\frac{2\pi\beta c}{\hat{J}_z(\omega/\beta c)}\int_{-\infty}^{\infty}dz\,\exp\left(-i\frac{\omega z}{\beta c}\right)E_z(z,\omega). \tag{1.15}$$

If we consider a point charge q, we have $f(x,y)=\delta(x-x_0)\delta(y-y_0)$, $\lambda(z)=-\delta(z)$, $\hat{\lambda}(k)=1$, and $\hat{J}_z(k')=2\pi q\beta c$. From (1.15) we then find

$$Z(\omega)=-\frac{1}{q}\int_{-\infty}^{\infty}dz\,\exp\left(-i\frac{\omega z}{\beta c}\right)E_z(z,\omega), \tag{1.16}$$

where $E_z(z,\omega)$ is taken at x_0, y_0—on the trajectory of the charge. The expression (1.16) is the same as the usual definition of the longitudinal impedance for an infinite tube with an irregularity.[11]

Longitudinal wake potential. The concept of a *longitudinal wake potential* is introduced in the following way. We assume that a point charge q is moving with velocity $v\simeq c$ parallel to the z axis of a vacuum chamber with a transverse displacement $\mathbf{r}'=(x',y')$ from the axis. For a point test charge Q following the leading charge q at distance s, with the same velocity and with transverse displacement $\mathbf{r}$, the energy losses can then be written

$$\Delta U\equiv QqW^{\delta}(s,\mathbf{r},\mathbf{r}'), \tag{1.17}$$

where the function $W^{\delta}(s,\mathbf{r},\mathbf{r}')$ is called the *longitudinal wake potential* of the point charge. Denoting by $\mathbf{E}(\mathbf{r},z,t)$ the field excited by the charge q in the chamber, we can calculate ΔU in (1.17) by integrating the work performed by the field E on the charge Q. We find

$$W^{\delta}(s,\mathbf{r},\mathbf{r}')=-\frac{1}{q}\int_{-\infty}^{\infty}dt\beta cE_z(\mathbf{r},z=\beta ct-s,t)$$
$$=-\frac{1}{q}\int_{-\infty}^{\infty}dzE_z\left(\mathbf{r},z,t=\frac{z+s}{\beta c}\right). \tag{1.18}$$

The dimensionality of W^{δ} is V/C, so that this quantity cannot, strictly speaking, be called a "potential."[2)] Nevertheless, this is the usage which has developed historically. We also note that several assumptions have been made in the definition (1.18):

- It is assumed that the charges are ultrarelativistic, i.e., that $\beta\to 1$, and the dependence on β is omitted in W^{δ}. Under the condition $\beta<1$, the concept of a wake potential becomes considerably less convenient.[12]
- The change in the trajectory of a test charge due to the fields induced in the structure is ignored. This assumption is usually justified in the limit $\beta\to 1$.
- Finally, it is implicitly assumed that the energy losses and thus (1.18) are independent of the absolute coordinates of the charges, z_q and z_Q, depending on only their relative positions, i.e., on $s=z_q-z_Q$. In a uniform structure (one which is translationally invariant along z), this circumstance is obvious. In other cases, one should distinguish a systematic effect; e.g., in a periodic structure, one should examine average properties over a period (more on this below).

One of the most important properties of the wake potential follows from causality considerations ($v\to c$):

$$W^{\delta}(s)=0\quad\text{for } s<0.$$

The longitudinal coupling impedance can be defined[12,11] as the Fourier transform of[3)] $W^{\delta}(s)$:

$$Z(\omega,\mathbf{r},\mathbf{r}')=\int_{-\infty}^{\infty}\frac{ds}{\beta c}\exp\left(i\frac{\omega s}{\beta c}\right)W^{\delta}(s,\mathbf{r},\mathbf{r}'). \tag{1.19}$$

Substituting $W^{\delta}(s)$ from (1.18), and making a simple change of variables, we find

$$Z(\omega)=-\frac{1}{q}\int_{-\infty}^{\infty}dz\,\exp\left(-i\frac{\omega z}{\beta c}\right)E_z(z,\omega),$$

which is the same as the result derived above [see (1.16)].

The relationship between $Z(\omega)$ in (1.19) and the wake potential $W^{\delta}(s)$ yields several analytic properties of the longitudinal impedance as a function of the frequency in the ultrarelativistic limit ($\beta\to 1\Leftrightarrow\gamma\to\infty$). We continue the analytic function $Z(\omega)$ into the complex ω plane. Then we can draw the following conclusions:

- Since $\mathrm{Im}\,W^{\delta}(s)=0$, then $Z(-\omega)=Z^*(\omega)$; i.e., $\mathrm{Re}Z(\omega)$ is an even function, and $\mathrm{Im}Z(\omega)$ is an odd function.
- Since $W^{\delta}(s)=0$ for $s<0$, all the singularities of the function $Z(\omega)$ lie in the lower half-plane.
- Since $W^{\delta}(s)=\int_0^{\infty}d\omega\,\mathrm{Re}\,Z(\omega)/\pi$ is a finite quantity, the asymptotic behavior as $\omega\to\infty$ is $\mathrm{Re}Z(\omega)\sim\omega^{-\alpha}$, $\alpha>1$.

From these properties we find dispersion relations between ReZ and ImZ:

$$\mathrm{Im}Z(\omega)=-\frac{2\omega}{\pi}\,\mathrm{P.v.}\int_0^{\infty}d\xi\,\frac{\mathrm{Re}Z(\xi)}{\xi^2-\omega^2};$$
$$\mathrm{Re}Z(\omega)=\frac{2}{\pi}\,\mathrm{P.v.}\int_0^{\infty}d\xi\,\frac{\xi\mathrm{Im}Z(\xi)}{\xi^2-\omega^2}. \tag{1.20}$$

Using the definition (1.17) for the wake potential, we easily find an expression for the energy losses per particle of an isolated bunch with a normalized charge density distribution $\rho(s\mathbf{r})$, $\int dsd\mathbf{r}\rho(s,\mathbf{r})=1$. The loss coefficient is (Ref. 11, for example)

$$k\equiv\frac{\Delta U}{q^2}=\int ds_1d\mathbf{r}_1ds_2d\mathbf{r}_2\rho(s_1,\mathbf{r}_1)\rho(s_2,\mathbf{r}_2)W^{\delta}(s_1-s_2,\mathbf{r}_1,\mathbf{r}_2). \tag{1.21}$$

If we ignore the transverse dimensions of the bunch, we have $\rho(s,\mathbf{r})=\lambda(s)\delta(\mathbf{r})$ and

$$k=\frac{1}{\pi}\int_0^\infty d\omega\,\mathrm{Re}Z(\omega)|\lambda(\omega)|^2, \tag{1.22}$$

where $\lambda(\omega)=\int_{-\infty}^{\infty}ds\,\exp(-i\omega s/c)\lambda(s)$ is a harmonic of the bunch spectrum. For a Gaussian bunch with mean-square length l we have $\lambda(s)=\exp(-s^2/2l^2)/(\sqrt{2\pi}l)$, $\lambda(\omega)=\exp[-(\omega l/c)^2/2]$, and (1.22) becomes

$$k=\frac{1}{\pi}\int_0^\infty d\omega\,\mathrm{Re}Z(\omega)\exp[-(\omega l/c)^2]. \tag{1.23}$$

In addition to the wake potential of a point charge, $W^\delta(s)$, a *bunch wake potential*[4)] is often introduced:

$$W(s)=q\int_{-\infty}^{\infty}dz\lambda(z)W^\delta(s-z)$$
$$=q\int_{-\infty}^{s}dz\lambda(z)W^\delta(s-z) \tag{1.24}$$

or

$$W(s)=q\int_0^\infty dz\lambda(s-z)W^\delta(z).$$

Here $q\lambda(z)$ is the longitudinal charge distribution of the bunch. The loss coefficient in (1.21) is expressed in terms of $W(s)$ as follows:

$$k=\frac{1}{q}\int_{-\infty}^{\infty}ds\lambda(s)W(s).$$

Warnock[8] defined the longitudinal wake potential for the case of a cyclic accelerator as the average of $W(\omega_0\tau,t)=-2\pi RE[\omega_0(t-\tau),t]$ over the orbital period. In an azimuthally uniform accelerator, the t dependence of W drops out, and we find, using (1.6),

$$W(\omega_0\tau)=\omega_0 q\sum_{n=-\infty}^{\infty}e^{-in\omega_0\tau}\lambda_n Z_{nn}(n\omega_0). \tag{1.25}$$

If the chamber has irregularities, we find, using (1.7),

$$W(\omega_0\tau,t)=\omega_0 q\sum_{n,m=-\infty}^{\infty}e^{-in\omega_0\tau}Z_{nm}(m\omega_0)\lambda_m e^{i(n-m)\omega_0 t}.$$

Averaging this expression over the period $T=2\pi/\omega_0$, we single out the systematic effect; we again find (1.25), which is analogous to (1.24).

The *longitudinal coupling impedance* introduced above [see (1.6), (1.8), (1.11), and (1.15)] means the ratio of a harmonic of the voltage induced in the structure to the magnitude of the harmonic of the current perturbation which created this voltage.

Transverse impedance and wake potential. If the beam is displaced from the chamber axis in the transverse direction, a transverse force arises and acts on the beam by virtue of currents induced in the chamber wall. The ratio of the harmonic of the integral transverse force acting on an individual particle of a bunch to the magnitude of the harmonic of the dipole moment of the beam current which created this force is called the *transverse (dipole) coupling impedance*.

One definition of transverse impedance is given in Ref. 13. Let us consider a uniform circular beam of radius a in a cylindrical chamber of radius b. We direct the z axis along the chamber axis. The period of the structure is $2\pi R$. Transverse oscillations of the beam with a small amplitude d in the vertical direction (along the y axis; the azimuthal angle is $\varphi=\pi/2$) are simulated by introducing a surface perturbation of the beam charge density:

$$\rho=\rho_n d\sin\varphi\delta(a-r)\exp(ik_n z-i\omega_n t),$$

which propagates through the chamber with phase velocity $\omega_n/k_n=\beta c$, where $\omega_n=n\omega_0$ and $k_n=n/R$. The corresponding current density is $\mathbf{j}=(j\sin\varphi,\ j\cos\varphi,\ \rho\beta_p c)$, where $j=i\rho_n dk_n c(\beta_p-\beta)\theta(a-r)\exp(ik_n z-i\omega_n t)$. The quantity $\beta_p c$ is the longitudinal velocity of the beam particles, which is generally different from the phase velocity βc of the perturbation wave. The further arguments are analogous to those in the introduction of the longitudinal impedance: Eqs. (1.10) and (1.11). Finding the field amplitudes from Maxwell's equations in the ω domain, we find the average transverse component of the Lorentz force, averaged over the beam cross section:[5)]

$$\frac{1}{e}\mathbf{F}_\perp(z,t)=\overline{[\mathbf{E}(r,\varphi,z)+\beta_p c\times\mathbf{B}(r,\varphi,z)]_\perp}\exp(-i\omega_n t).$$

By analogy with (1.10), we switch to Fourier harmonics of the dipole moment,

$$J_n(\omega)d=2\pi\rho_n\beta_p c\pi a^2 d\delta(\omega-\omega_n),$$

and of the integral force,

$$\frac{2\pi R}{e}\mathbf{F}_{\perp n}(\omega)=2\pi\delta(\omega-\omega_n)\int_0^{2\pi R}dz\,e^{-ik_n z}$$
$$\times\overline{[\mathbf{E}(r,\varphi,z)+Z_0\beta_p\times\mathbf{H}(r,\varphi,z)]_\perp},$$

where $Z_0=\sqrt{\mu_0/\varepsilon_0}=120\pi\ \Omega$. The *transverse impedance* is defined as

$$Z_\perp(\omega)\equiv Z_{\perp n}(\omega_n)=-i\frac{2\pi RF_{\perp n}(\omega)}{e\beta_p J_n(\omega)d}.$$

The factor i is introduced because the phase of the deflecting field and that of the dipole moment differ by $\pi/2$. We thus have

$$\mathbf{Z}_\perp(\omega)=-\frac{i}{\beta_p^2 cd\rho_n\pi a^2}\int_0^{2\pi R}dz\,e^{-ik_n z}$$
$$\times\overline{[\mathbf{E}(r,\varphi,z)+Z_0\beta_p\times\mathbf{H}(r,\varphi,z)]_\perp}. \tag{1.26}$$

The dimensionality of $Z_\perp$ is Ω/m. In addition to the explicit dependence on β_p in (1.26), $Z_\perp$ also depends on $\beta=\omega/kc$ through the fields. The dependence on β drops out of the expression for the transverse impedance of a smooth chamber [see (2.3)]. However, if one calculates the resonant impedance of an irregularity, the result depends on the ratio of β and β_p through the combinations $(\beta-\beta_p)$

and $(1-\beta\beta_p)$. Incorporating betatron oscillations, it becomes possible to impose the relationship $\beta=(1\pm Q/n)\beta_p$, where Q is the betatron frequency.

Again in the transverse case, one can define an analog of the impedance matrix (or integral operator). We consider a point charge q moving with velocity $\beta_p c$ along the z axis of the chamber with displacement r' from the axis. It is a straightforward matter to find a harmonic of the dipole moment of the current:

$$J(k,\omega)r'=2\pi q\beta_p cr'\delta(\omega-\beta_p ck).$$

The transverse deflecting force acting on a unit charge at the point $(\mathbf{r}',z)$ at time t is written as a double Fourier integral, by analogy with (1.12):

$$\frac{1}{e}\mathbf{F}_\perp(\mathbf{r},z,t)=[\mathbf{E}+\mathbf{v}\times\mathbf{B}]_\perp(\mathbf{r},z,t)$$

$$=\frac{1}{(2\pi)^2}\int d\omega\, e^{-i\omega t}\int dk\, e^{ikz}\frac{1}{e}\mathbf{F}_\perp(\mathbf{r},k,\omega).$$

Since generally all harmonics of the dipole moment contribute to a harmonic of the deflecting field with wave number k, we introduce a *transverse impedance operator* $\mathbf{Z}_\perp(k,k',\omega)$ as follows:

$$-\frac{i}{\beta_p}\frac{1}{e}\mathbf{F}_\perp(\mathbf{r},k,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk'\mathbf{Z}_\perp(k,k',\omega)r'J(k',\omega) \tag{1.27}$$

[cf. the definition (1.13)]. Substituting $J(k',\omega)$, and expressing $F_\perp(\mathbf{r},k,\omega)$ in terms of $F_\perp(\mathbf{r},z,\omega)$, we find

$$\mathbf{Z}_\perp(k,\omega)=\mathbf{Z}_\perp(k,k'=\omega/\beta_p c,\omega)$$

$$=\frac{-i}{\beta_p qr'}\int_{-\infty}^{\infty}dz\, e^{-ikz}[\mathbf{E}(\mathbf{r},z,\omega)+Z_0\beta_p \times\mathbf{H}(\mathbf{r},z,\omega)]_\perp. \tag{1.28}$$

The arguments k and ω in $Z_\perp$ are generally independent. They can be related on the basis of dynamic considerations, with allowance for betatron oscillations: $k=(\omega\pm\omega_\beta)/\beta_p c$.

Another way to define the transverse impedance is in terms of the wake potential. The transverse momentum acquired by a charge Q moving along the z axis of the chamber with displacement $\mathbf{r}=(x,y)$ from the axis, at a distance s from the leading charge q (with displacement $\mathbf{r}'$ and velocity $v=\beta c\sim c$), can be written

$$\mathbf{p}_\perp=\frac{Qq}{c}\mathbf{W}_\perp^\delta(s,\mathbf{r},\mathbf{r}'). \tag{1.29}$$

Denoting by $E(\mathbf{r},z,t)$ the field excited by the charge q in the chamber, we write the "transverse wake potential" (more precisely, the "point function") as

$$\mathbf{W}_\perp^\delta(s,\mathbf{r},\mathbf{r}')\equiv\frac{1}{q}\int_{-\infty}^{\infty}dz[\mathbf{E}+\mathbf{v}\times\mathbf{B}]_\perp[\mathbf{r},z,t=(z+s)/v)], \tag{1.30}$$

i.e., as the ratio of the integral deflecting field to the magnitude of the charge.[6)] For a periodic structure or for an element of the accelerator chamber, the range of the integration along z is finite. The assumptions are the same as in the introduction of the longitudinal wake potential.

The *transverse impedance* can be defined as the Fourier transform of $-i\mathbf{W}_\perp^\delta/r'$:

$$\mathbf{Z}_\perp(\omega,\mathbf{r},\mathbf{r}')=-\frac{i}{r'}\int_{-\infty}^{\infty}ds\frac{1}{\beta c}\exp\left(i\frac{\omega s}{\beta c}\right)\mathbf{W}_\perp^\delta(s,\mathbf{r},\mathbf{r}'). \tag{1.31}$$

After substituting (1.30) into (1.31), we find

$$\mathbf{Z}_\perp(\omega,\mathbf{r},\mathbf{r}')=-\frac{i}{qr'}\int_{-\infty}^{\infty}dz\exp\left(-i\frac{\omega z}{\beta c}\right)\times[\mathbf{E}+\mathbf{v}\times\mathbf{B}]_\perp(\mathbf{r},z,\omega), \tag{1.32}$$

which is the same as (1.28) in the limit $\beta\to1$, $\beta_p\to1$, $k=\omega/\beta c$, aside from changes in notation.

The Panofsky–Wenzel theorem[14] relates the transverse momentum $\mathbf{p}_\perp$ which a test charge Q acquires as it moves through a closed cavity with the field excited in the cavity by a charge which passed through it previously or by an rf oscillator:

$$\mathbf{p}_\perp=Q\int_0^L dz\nabla_\perp A_z(\mathbf{r},z,t=z/c),$$

under the condition $\mathbf{A}_\perp=0$ at $z=0$, L. The integration here is over the trajectory of the charge Q in the cavity, $\mathbf{A}$ is the vector potential of the field in the cavity, and $\nabla_\perp$ is the 2D gradient operator in the $z=$const plane. We can use this equation to relate the longitudinal and transverse wake potentials [(1.18) and (1.30), respectively]:

$$\frac{\partial\mathbf{W}_\perp^\delta(s,\mathbf{r},\mathbf{r}')}{\partial s}=\nabla_\perp W^\delta(s,\mathbf{r},\mathbf{r}'). \tag{1.33}$$

Equation (1.33) is valid for many cases of practical importance,[12] in particular, in a cylindrically symmetric structure and for a cavity of arbitrary shape in the case in which the approximation of a "closed" cavity is applicable.

The Fourier transformations (1.19) and (1.31) of the wake potentials into impedances can be used to find the relationship between the transverse and longitudinal impedances in the limit $\gamma\to\infty$ from (1.33):

$$\mathbf{Z}_\perp(\omega,\mathbf{r},\mathbf{r}')=\frac{c}{\omega r'}\nabla_\perp Z(\omega,\mathbf{r},\mathbf{r}'). \tag{1.34}$$

Higher multipoles. In addition to the longitudinal and transverse impedances and potentials, one introduces impedances and wake potentials of higher multipolarities (Refs. 1 and 12, for example). Let us consider an axisymmetric structure in which a leading charge q is moving parallel to the z axis with velocity $v\simeq c$, and a test charge Q is following at a distance s. We assume that the transverse coordinates of q are $\mathbf{r}'=(r',0)$, and those of the test charge are $\mathbf{r}=(r,\varphi)$. The field created by the first charge can be expanded in azimuthal harmonics $\cos m\varphi$. For the δ function we use the expansion $2\pi\delta(\varphi)=1+2\Sigma_{m=1}^{\infty}\cos m\varphi$. The boundary conditions in an axisymmetric structure do not mix azimuthal harmonics $E^{(m)}$, $H^{(m)}$—the solutions of Maxwell's equations for fixed

m—with different values of m. On this basis we define *m-pole wake potentials* by analogy with (1.18) and (1.30):

$$W^{\delta}_{\parallel\ m}(s,\mathbf{r},\mathbf{r}')=-\frac{1}{q}\int_{-\infty}^{\infty}dz E_z^{(m)}\left(r,z,t=\frac{z+s}{c}\right)\cos m\varphi; \tag{1.35}$$

$$\mathbf{W}^{\delta}_{\perp\ m}(s,\mathbf{r},\mathbf{r}')=\frac{1}{q}\int_{-\infty}^{\infty}dz\{\mathbf{e}_r(E_r^{(m)}-Z_0H_\varphi^{(m)})\cos m\varphi + \mathbf{e}_\varphi(E_\varphi^{(m)}+Z_0H_r^{(m)}\sin m\varphi\}.$$

As before, we are assuming that in the case of an infinite periodic structure we deal with quantities averaged over a period, and the fields depend on z and t only through the combination $(z-ct)$. In (1.35) we have singled out the dependence on the angle φ which follows from Maxwell's equations in the case ρ, $j_z \propto \cos m\varphi$. It can be shown[15,16,12] that in an axisymmetric structure we have

$$W^{\delta}_{\parallel\ m}(s,\mathbf{r},\mathbf{r}')=r^m r'^m F'_m(s)\cos m\varphi;$$
$$\mathbf{W}^{\delta}_{\perp\ m}(s,\mathbf{r},\mathbf{r}')=mr^{m-1}r'^m F_m(s)\{\mathbf{e}_r\cos m\varphi-\mathbf{e}_\varphi\sin m\varphi\}. \tag{1.36}$$

For $m=0$ we have $F_0(s)\equiv 0$ and $F'_0(s)\equiv W^{\delta}(s,0,0)$ [cf. (1.18)], while for $m\geqslant 1$ we have $F'_m(s) = (d/ds)F_m(s)$ (Ref. 12, for example). Furthermore, since we have $\beta\to 1$, causality requires $F'_m(s) \sim \Theta(s)$. It is easy to verify that the m-pole potentials defined in (1.36) satisfy the relation (1.33). It can be seen from the definition (1.36) that $\mathbf{W}^{\delta}_{\perp\ 0}\equiv 0$ and that the longitudinal wake potential $\mathbf{W}^{\delta}(s,\mathbf{r},\mathbf{r}')\equiv \mathbf{W}^{\delta}_{\parallel\ 0}(s,\mathbf{r},\mathbf{r}')$ is independent of $\mathbf{r}$ and $\mathbf{r}'$. Furthermore, the dipole transverse potential satisfies $\mathbf{W}^{\delta}_{\perp}(s,\mathbf{r},\mathbf{r}')\equiv\mathbf{W}^{\delta}_{\perp\ 1}(s,\mathbf{r},\mathbf{r}')=\mathbf{r}'F_1(s)$; i.e., the direction of the transverse deflecting force is the same as the direction in which the leading charge is deflected and is independent of the radial position of the test charge. We also note that in the case $\mathbf{r}=\mathbf{r}'$, i.e., in the case in which the two charges are moving along the same trajectory, we have $W^{\delta}_{\varphi m}=0$, since $\varphi=0$, and the vector of the m-pole transverse potential is directed radially.

It follows from dimensionality considerations that $F'_m(s) \sim b^{-2m}$ in (1.36), where b is a characteristic transverse dimension, e.g., the radius of a smooth chamber. Since the transverse deflections r and r' in accelerators are usually considerably smaller than b, the m-pole ($m\geqslant 1$) wake potentials are suppressed:

$$W^{\delta}_{\parallel\ m}\sim\left(\frac{r}{b}\right)^m\left(\frac{r'}{b}\right)^m,$$

$$W^{\delta}_{\perp\ m}\sim\left(\frac{r}{b}\right)^{m-1}\left(\frac{r'}{b}\right)^m.$$

We can thus restrict the analysis of the beam dynamics to the lower-index wake potentials—the monopole ($m=0$) longitudinal potentials and the dipole ($m=1$) transverse potentials—and thus the same impedances.

We now define *m-pole longitudinal and transverse impedances* by analogy with (1.9) and (1.31) in terms of the Fourier transforms of W^{δ}_m, separating out the dependence on the transverse coordinates [(1.36)]:[7)]

$$Z_{\parallel\ m}(\omega)=\int_{-\infty}^{\infty}\frac{ds}{c}\exp\left(i\frac{\omega s}{c}\right)F'_m(s),$$
$$Z_{\perp\ m}(\omega)=-i\int_{-\infty}^{\infty}\frac{ds}{c}\exp\left(i\frac{\omega s}{c}\right)F_m(s). \tag{1.37}$$

For this definition of the multipole impedances, the dimensionality of $Z_{\parallel\ m}$ is Ω/m^{2m}, and that of $Z_{\perp\ m}$ is Ω/m^{2m-1}. Working from the Panofsky–Wenzel theorem, (1.33), the relationship between F_m and F'_m, and the relationship between impedances of identical multipolarity at $m\geqslant 1$, we find

$$Z_{\parallel\ m}(\omega)=\frac{\omega}{c}Z_{\perp\ m}(\omega). \tag{1.38}$$

Finally, we note a relationship between the impedances of different multipolarities, specifically, a monopole longitudinal impedance and a dipole transverse impedance:[17]

$$Z_{\perp}(\omega)=\frac{2c}{\beta b^2}\frac{Z(\omega)}{\omega}=\frac{2R}{b^2}\frac{Z(\omega)}{n}, \tag{1.39}$$

where R is the average radius of the accelerator, b is the cross-sectional radius of the vacuum chamber, and $n=kR=\omega R/\beta c$ is the harmonic of the orbital frequency. This approximate relation is often used to estimate $Z_\perp$, since the latter is more difficult to calculate than the longitudinal impedance. Equation (1.39) can be found from (1.38) in the $m=1$ case, in which one assumes, on the basis of qualitative considerations, $Z_{\parallel\ 1}\simeq Zb^2$. The coefficient of $2/\beta$ is introduced so that (1.39) will hold exactly for the low-frequency impedance of a smooth chamber stemming from a finite conductivity of the wall [see (2.2) and (2.3)].

Once the initial concepts have been defined, it is natural to take up the question of just why we need to know impedances and/or wake potentials.

1.2. Effects caused by the interaction of the beam with the vacuum chamber

Beam energy losses. The energy lost by a single particle bunch due to the interaction with a certain element of the chamber is $\Delta U=kq^2$, where q is the charge of bunch, and k is the loss coefficient for the given element, given by Eqs. (1.21)–(1.23) above. If a periodic train of bunches passes through an element, and if the fields excited by the leading bunches have not yet decayed by the time at which subsequent bunches arrive, the interference of fields can substantially increase the losses; see, for example, the review of Wilson[12] and the bibliography there. It can be seen from Eqs. (1.22) and (1.23) that the losses are due primarily to the behavior of ReZ at frequencies below the characteristic bunch frequency $\omega\approx c/l$. For the typical transverse chamber dimension b, the cutoff frequency is $\omega_c\approx c/b$. The high-frequency ($\omega\gg\omega_c$) behavior of ReZ thus affects the losses only in accelerators with very short bunches ($l\ll b$), e.g., in linear electron accelerators.

• The low-frequency reactive part of the longitudinal impedance, Im$Z(\omega)$, determines the *distortion of the potential well* created by the accelerating voltage, leading to a distortion of the bunch shape.[1] The "negative-mass" instability and the increase in bunch length are related to $\mathrm{Im}Z(\omega)$ (Ref. 1).

- The reactive parts of the longitudinal and transverse impedances cause shifts in the *synchrotron and betatron frequencies*, respectively. The shift of the betatron frequency in a cyclic accelerator, for example, is

$$\delta Q_{x,y}=-\frac{r_0N}{2\pi\gamma Q_{x,y}}\frac{\mathrm{Im}\,Z_{\perp\ x,y}}{Z_0}, \tag{1.40}$$

where $r_0=q_0^2/m_0c^2$ is the electromagnetic radius of the particles being accelerated, N is the total intensity, $\gamma=1/\sqrt{1-\beta^2}$, and $Q_{x,y}$ is the betatron frequency.

- Finally, a study of the beam stability conditions in accelerators leads to *constraints on the magnitude of the coupling impedances*.[18,19] As an example, we write the condition for the stability of an unbunched beam in a cyclic accelerator:[18]

$$\left|\frac{Z_n}{n}\right|\leqslant F\frac{\beta^2|\eta|E}{eI_0}\left(\frac{\Delta p}{p}\right)^2. \tag{1.41}$$

Here $n=\omega/\omega_0$ is the index of the longitudinal mode of the perturbation, $Z_n=Z(n\omega_0)$, I_0 is the average current, $\eta=\alpha-1/\gamma^2$, $\alpha=1/\gamma_c^2$ is the orbit expansion coefficient, $E=\gamma m_0c^2$, p is the equilibrium momentum, Δp is the momentum spread, and the factor F depends on the shape of the momentum distribution. For real-world distributions we would have $F\simeq 1$. The real part of the impedance determines the instability growth rate. There are also constraints like (1.41) for the transverse impedance.

Information on the impedances or wake potentials thus makes it possible to calculate effects resulting from the interaction of the beam with the accelerator vacuum chamber. An analysis of the chamber–beam interaction in terms of impedances and an analysis in terms of wake potentials are, in principle, equivalent. The latter approach is taken more often for linear accelerators, while for cyclic machines it is often more convenient to work in the frequency domain, i.e., with impedances.

The problem of determining an impedance (or wake potential) for a given geometry of the vacuum chamber or of elements thereof has been under study for more than two decades. Methods for solving this problem have been developed extensively. There are both theoretical methods, which yield impedances, and experimental methods, which make it possible to find impedances from experiments on prototypes or working accelerators. The purpose of the present review is to examine existing methods for calculating the coupling impedances of structures or individual elements of an accelerator. We focus on those methods which are most important for large proton synchrotrons, in particular, the UNK.

We should cite several reviews on this subject.[12,20,11] Wilson's review[12] of wake potentials is a good introduction to the subject. It contains references to earlier papers (see also Refs. 21 and 22). Ng's lecture[20] on impedances is of interest, although it deals basically with that author's own results. A good review was recently published by Heifets and Kheifets.[11] It discusses in detail questions concerning the behavior of the impedance at high frequencies—questions of importance for the design of new linear e^+e^- colliders with very short bunches. Palumbo and Vaccaro[23] have reviewed methods for measuring impedances.

2. IMPEDANCES OF TYPICAL ELEMENTS OF ACCELERATOR CHAMBERS

2.1. Curvature effects

In a synchrotron, the chamber as a whole is toroidal, if various irregularities are ignored. In an analysis of the interaction of a beam with a chamber, on the other hand, it is customary to ignore the curvature of the chamber and to deal with a rectilinear infinite periodic structure. It is intuitively clear that this approach is justified if the radius of curvature is large. This approximation was recently studied quantitatively in Refs. 24–27, which also contain references to earlier papers. According to Ref. 26, the increment in the longitudinal impedance due to curvature is[8)]

$$\frac{Z}{n}=iZ_0\left(\frac{h}{\pi R}\right)^2\left[A-3B\left(\frac{\nu}{\pi}\right)^2\right], \tag{2.1}$$

where h is the height of the chamber (a chamber with a rectangular cross section is assumed), R is the radius of curvature of the orbit, $\nu=\omega h/c$ is a dimensionless frequency, and the coefficients A and B are on the order of unity. Equation (2.1) is valid under the condition $\nu<\sqrt{R/h}$. It describes a very small increment in the impedance of accelerators in which the condition $h/R\ll 1$ holds. At frequencies $\nu>\sqrt{R/h}$ there are some resonances due to the curvature of the chamber. According to Ref. 25, for the SSC they contribute R_{sh}/n values on the order of 10^{-3} Ω at $\sim$5000 GHz. For the tevatron the values are 10^{-2} Ω at $\sim$500 GHz. Estimates for the UNK, with $R=3306$ m and $h\simeq 6$ cm, lead to $R_{sh}/n\simeq 10^{-2}$ Ω at frequencies $\sim$700 GHz. Clearly, such impedance levels pose no danger. Furthermore, the irregularities in the ring of a real accelerator—primarily the rectilinear sections—can strongly suppress these "curvature resonances," making the figures given above upper estimates.

2.2. Impedance of a smooth chamber

Knowing the effects which stem from the curvature of the chamber, we can treat the vacuum chamber of a cyclic accelerator as an infinite structure with a period equal to the perimeter of the accelerator. We first consider the contribution to the impedance from the smooth part of the chamber, which is a waveguide with a constant cross section. To calculate the fields excited in the smooth vacuum chamber by the current, we solve Maxwell's equations with boundary conditions at the chamber wall and with the periodicity condition. We use the definition of the longitudinal impedance, (1.11) [or (1.26) for the transverse impedance]. There is no difficulty in solving such a problem in the axisymmetric case. The longitudinal impedance per unit length of a cylindrical chamber is[9)] (Ref. 9, for example)

$$\frac{Z(\omega)}{L}=Z_0\frac{\omega}{2\pi c}\left[\frac{i}{(\beta\gamma)^2}\left(\ln\frac{b}{a}+\frac{1}{4}\right)+(1-i)\frac{\delta}{2b}\right], \quad (2.2)$$

where $L=2\pi R$ is the perimeter of the accelerator, $\delta = \sqrt{2/(\mu_0\sigma\omega)}$ is the skin thickness at frequency ω, σ is the conductivity of the wall material, a is the radius of the beam, and b is the radius of the chamber cross section. Equation (2.2) was derived through the use of Leontovich boundary conditions. It is valid under the following conditions: (i) $\delta\leqslant\Delta$, where Δ is the thickness of the chamber wall, and (ii) $\omega b/c\ll\gamma$. The latter condition stems from a simplification of the expression of which (2.2) is a limiting case; this condition is not of fundamental importance. The first term in square brackets in (2.2) stems from space-charge effects. Its rapid decrease ($\propto\gamma^{-2}$) with increasing energy stems from a cancellation of the effects of the electric and magnetic fields on the beam in the relativistic limit. The second term arises from the finite conductivity of the wall material. At low frequencies, under the condition $\delta\geqslant\Delta$, it is modified by the substitution[28] $\delta\to\delta\cot[(1-i)\Delta/\delta]\cong\delta^2/[(1-i)\Delta]$. On occasion, the real part of the impedance is written

$$\mathrm{Re}\,Z(\omega)=\frac{L}{2\pi b}\mathscr{R},$$

where $\mathscr{R}$ is the surface impedance of the chamber wall, which is $Z_0\delta\omega/2c = \sqrt{\mu_0\omega/2\sigma}$ in the case $\delta\leqslant\Delta$ and $1/(\Delta\sigma)$ in the case $\delta>\Delta$. As an example, we can look at the numbers for the vacuum chamber of the second stage of the UNK, where $\sigma\simeq2\cdot10^6$ S/m is the conductivity of stainless steel at 4 K. We use $\Delta=2.5$ mm and an energy range 400–3000 GeV. According to (2.2), the space charge contributes $i(5.8$–$0.14)$ mΩ. The contribution from the resistive wall is much larger: $(1 - i)2.3/\sqrt{f(\mathrm{MHz})}$ Ω in the frequency range from 20 kHz to 500 GHz. At lower frequencies, it becomes purely resistive and independent of the frequency: 18.9 Ω. Such a value of Re Z would lead to a large heat evolution in the chamber wall: ~0.1–1 W/m, depending on the operating conditions of the UNK. This situation would be tolerable in an ordinary accelerator, while in a superconducting accelerator it would significantly increase the load on the cryogenic system. In order to reduce the ohmic losses in the walls, it has therefore been suggested that the inner surface of the UNK-2 vacuum chamber be plated with a thin layer of copper. The conductivity of copper, σ_c, differs from that of stainless steel in that it increases greatly at liquid-helium temperatures: by a factor of 50–5000 from the value $\sigma_c\simeq6\cdot10^7$ S/m at 300 K, depending on the purity of the copper. As a result, the ohmic losses are lowered by two orders of magnitude. On the other hand, there is an increase in the losses due to eddy currents excited in the walls by the magnetic field, which varies during the acceleration cycle. As a compromise, the thickness of the copper plating would be from 0.3 to 3 μm, depending on the conductivity of the copper.

The transverse impedance (the dipole mode) of a smooth axisymmetric chamber is[17]

$$Z_\perp(\omega)=Z_0R\left[\frac{i}{(\beta\gamma)^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+(1-i)\frac{\delta}{b^3}\right]. \quad (2.3)$$

The first and second terms in (2.3) have the same meaning as in (2.2): the space charge and the impedance of the resistive wall. The values for the UNK-2 are $i(0.91$–$0.12)$ MΩ/m with $\gamma=426$–3200 and $(1 - i)0.43/\sqrt{f(\mathrm{MHz})}$ MΩ/m, respectively.

The primary effects of the interaction of a beam with a smooth chamber are the beam energy losses and, at low energies, the negative-mass instability. Since electromagnetic waves with phase velocities below the velocity of light cannot exist in a rectilinear smooth chamber, there are no resonances in such a structure.

2.3. Resonances

Various irregularities in a vacuum chamber (cavities, inserts, adapter sections connecting sections with different cross sections, etc.) raise the possibility of slow waves in the chamber, which can interact resonantly with the beam. At low frequencies, the irregularities make a basically reactive contribution to the impedance.

At higher frequencies, resonances begin. Two cases are possible here. First, there can be narrow-band resonances due to elements with substantial resonator properties at frequencies below the cutoff frequency ω_c of the smooth part of the vacuum chamber. In this case the system has a memory, and it acts on the coupled oscillations of the bunches. As an example we cite the resonators of the accelerating stations of the UNK, in which it is necessary to damp several high-index oscillation modes,[29] since the longitudinal impedance which they create is above the stability threshold of multipole oscillations of the bunches. The second case is that of elements whose resonator properties are poorly expressed or for which the frequencies of the resonances are well above ω_c. Such resonances have large widths, due to radiation into ports of the energy of oscillations excited in the interior of the resonator. These resonances affect an individual bunch and may thereby change the shape of a bunch and drive single-bunch instabilities. The sources of these resonances might be various small irregularities (a corrugation, bellows, points at which vacuum equipment is connected, etc.) or regions of a large flaring of the chamber.

The discussion above dealt with an individual element of a chamber. The calculation methods which are reviewed in Sec. 3 or impedance measurements usually make it possible to find the impedance of an individual element or a periodic structure of identical elements. However, how do the elements affect each other? If we know the impedance of the individual elements, can we find the impedance which the set of elements creates? For elements which are far apart (i.e., waves excited in one do not reach another), the impedances are evidently additive at any frequency. For neighboring elements, the interaction may be important, especially at frequencies near resonances (more on this in Sec. 3).

Since the resonant frequencies of the various chamber elements are different, the frequency dependence of the chamber impedances may be exceedingly complex. Under certain conditions, the impedance of the real chamber is replaced by a model impedance in order to study the effect of the chamber on the beam, in particular, on an individual bunch. For example, the model of a wide-band resonator[30] with the parameter values[10)] $Q=1$, $f_r=1.3$ GHz, $R_{sh}/n_r=|\mathrm{Im}Z/n|=0.9\ \Omega$ is used in calculations on the dynamics of a beam in the LEP. This substitution is legitimate, since the distances between bunches in the LEP are very large (there are 4–8 bunches over the orbit with a perimeter of 27 km in the LEP), and the bunches themselves are short. As a result, the detailed structure of the impedance (narrow peaks) does not affect the dynamics, especially single-bunch effects.

3. METHODS FOR CALCULATING IMPEDANCES

A calculation of impedances or wake potentials ultimately reduces to solving an electrodynamic problem: that of finding the fields (electric fields, magnetic fields, and force fields) excited in a chamber by a given beam current. There are various ways to solve this problem. The various calculation methods can be classified (extremely arbitrarily) on the basis of the following characteristics:

- the approach taken (electronic-circuit or electrodynamic),
- the solution method (analytic, semianalytic, or direct numerical), and
- the initial formulation of the problem (in the time domain or the frequency domain).

Clearly, different methods may be convenient, depending on the geometry of the element of a vacuum chamber and on the particular frequency range or, in the case of wake potentials, on the particular interval of lag times which is of interest in the problem at hand.

3.1. Electronic-circuit methods

The element of the vacuum chamber, which usually has a complex geometry, is replaced by some equivalent electronic element (a long line, an RLC circuit, etc.). The parameters of the equivalent element, e.g., the resonant frequency, the Q, or the length of the line, are determined from additional considerations. The model of a broad-band resonator mentioned above can be put in this category. The studies which have taken the electronic-circuit approach are characterized by the following:

- The (complex) geometry is replaced by equivalent electronic circuits, and the parameters of these circuits are estimated on the basis of heuristic considerations.
- The results are derived in the form of fairly simple expressions (which should be classified as analytic on this basis).
- The frequency interval in which the replacement is successful is limited (the frequencies are usually low, $\omega \ll \omega_c$).

A typical example of this approach is Ng's calculation of the low-frequency impedance of beam separators.[32] In that study, the system of separator plates with external circuits is modeled by a circuit with lumped parameters: a resistance R, an inductance L, and a capacitance C. The values of these parameters are related to the geometric dimensions of the irregularity and to the resistance of the external circuit. As a result, one finds an expression for the longitudinal impedance at low frequencies:

$$Z(\omega)=4\sin^2\left(\frac{\omega l}{2c}\right)\frac{R-i\omega L}{1-\omega^2 LC-i\omega RC}, \tag{3.1}$$

where l is the length of the separator plates.

The following formula is used to calculate the impedance of strip-line beam-position sensors at low frequencies:

$$Z(\omega)=Z_c\left(\frac{\varphi}{2\pi}\right)^2\left[\sin^2\left(\frac{\omega l}{c}\right)-i\sin\left(\frac{\omega l}{c}\right)\cos\left(\frac{\omega l}{c}\right)\right], \tag{3.2}$$

where Z_c is the characteristic impedance of an electrode, l is the length of the sensor, and φ is the angle in the cross section subtended by the electrodes. This formula was derived in Ref. 33; see also Ref. 34, and for more details, Ref. 35.

The impedances of traveling-wave kicker magnets at low frequencies were calculated in Refs. 36 and 37 by replacing the magnet by an equivalent long line with a certain characteristic impedance, loaded at its ends.

3.2. Electrodynamic methods

Electrodynamic methods constitute another, larger group. By directly solving Maxwell's equations with a given current, one finds the electromagnetic field excited in the chamber and determines the coupling impedances or wake potentials. Here we should distinguish between two subgroups on the basis of the initial formulation of the problem: in the *time domain* or in the *frequency domain*. In methods which use the *frequency domain*, one calculates the fields created by a given harmonic of the current $I\propto\exp(-i\omega t)$. An integration of the field directly yields the coupling impedance (Sec. 1). In the *time domain*, one calculates the wake potentials created during the passage of a given charge distribution through the structure. One then finds the impedances by means of Fourier transformations. The Fourier transformation (1.19) relates the impedance to the wake potential $W^\delta(s)$ of a point charge, i.e., the response function. When the wake potential is instead calculated for a bunch of finite dimensions (this is the case for most of the numerical methods which work in the time domain), the Fourier transformation yields not the impedance itself but its convolution $Z(\omega)\tilde{\lambda}(\omega)$, where $\tilde{\lambda}(\omega)$ is the spectrum of the bunch. For high frequencies, the quantity $\tilde{\lambda}(\omega)$ is usually small; for a Gaussian bunch, for example, it is $\tilde{\lambda}(\omega)=\exp(-\omega^2\sigma^2/2c^2)$. The unavoidable errors in the convolution calculation lead to errors at high frequencies.

In order to solve Maxwell's equations it is necessary to impose boundary conditions on the fields. At the wall of the vacuum chamber, this condition can be the condition $\mathbf{E}_t=0$ in the limit of ideally conducting walls, or it can be

TABLE I. Electrodynamic methods in the ω domain.

Method	Solution method	Basic studies	Type of irregularity	Comment
Eigenfunction method	Anal.	e.g., Refs. 39, 49, and 50	Fig. 1, a and c	$b\to 0$
	Numer.	see Refs. 45, 47, 48, 52, and 51	Fig. 1, c–e	I
			Fig. 1, a and b	P
Field joining method	Semianal.	[75, 76, 77, 78, 79]	Fig. 1c	I
		[9, 57, 56, 59]	Fig. 1a	P
		[81, 82, 80]	Fig. 1g	I
Matrix method	Semianal.	[66, 67]	Fig. 1b	P
			Corrug.	
Perturbation method	Anal.	[61, 62, 65, 63, 51]	Fig. 1b	$h\ll b$
				$D\gg h$
Equivalent boundary conditions	Anal.	[72, 73, 74]	Fig. 1a	$h\ll b$
			Corrug.	P
Integral equations	Anal.	[83, 84, 85, 86]	Fig. 1, a–d	P and I
	Semianal.	[87]	Fig. 1, d and f	I
Direct numerical methods	Numer.	[88, 89]	Fig. 1, c–e	I
Diffraction method	Anal.	[90, 50, 91, 11]	Fig. 1, a, c, g, h	P and I
				$\omega\gg\omega_c$
Energy method	Anal.	[97, 98]	Fig. 1, a and g	Approx. method

the Leontovich condition for a metal. In addition, to "close" the problem it is necessary to impose some further conditions:

- *the periodicity condition*—for an infinite periodic structure which can serve as a model of a linear accelerator, for an iris-loaded waveguide, etc., or a cyclic accelerator with curvature effects ignored;
- *the radiation condition*—for open aperiodic structures (a resonator with infinite ports or a semi-infinite waveguide);
- *the "closure" condition*—the boundary condition at some imaginary surface which closes the region in which the fields are being calculated. For example, imaginary metal mufflers in feed ports transform a resonator with ports into a closed resonator, making it possible to study the impedance at frequencies below the cutoff frequency.

Electrodynamic methods in the frequency domain. Methods for calculating impedances in the frequency domain are listed in Table I. The papers cited here are papers in which the method was proposed or papers with examples of the use of a method and the basic types of irregularities to which it applies.

The "Comment" column gives limitations on the method and states whether the method is applicable to periodic structures (P) or to an individual irregularity separated from other chamber elements (I). There are of course some methods which do not conform to the classification of this table. For example, an elegant proof[38] that the impedance of irregularities which are axisymmetric but asymmetric in the longitudinal direction is independent of the direction of the beam is based exclusively on Maxwell's equations and on the formal definition of the impedance in (1.16). The assignment of Refs. 88 and 89 to the "direct numerical methods" row is also arbitrary. Those papers use approximate boundary conditions which simulate the conditions of radiation into open ports for structures of the type in Fig. 1, d and e, making it possible to use known numerical methods.

Eigenfunction method. In the frequency domain, in the approximation of a "closed resonator," the calculation of an impedance can be reduced to a standard internal eigenvalue boundary-value problem. Since the solution of the inhomogeneous equation is expressed as a series in eigenfunctions, the impedance can be expressed as a formal series. In practice, however, this approach yields only resonant values of the impedance (for the modes whose eigenfunctions have been found) or the behavior of the impedance at low frequencies for very simple models, in which it is possible to find all the eigenfunctions analytically and to sum the series.

We denote by $\mathbf{E}_s$ and $\mathbf{H}_s$ the complete set of eigenfunctions of a boundary-value problem with ideal boundary conditions at the walls of a closed resonator. Here $s=(s_1,s_2,s_3)$ is a generalized index. The eigenfunctions satisfy orthogonality conditions $\mu_0\int dVH_m^*H_n=\varepsilon_0\int dVE_m^*E_n=\delta_{mn}N_n$, where δ_{mn} is the Kronecker delta, N_n is a normalization coefficient, and the integration is over the volume of the resonator.[39] We seek a solution of the inhomogeneous Maxwell's equations with a current $\mathbf{j}$,

$$\begin{aligned} \operatorname{curl}\mathbf{H}&=-i\omega\varepsilon_0\mathbf{E}+\mathbf{j},\\ \operatorname{curl}\mathbf{E}&=i\omega\mu_0\mathbf{H}, \end{aligned} \tag{3.3}$$

with the same ideal boundary conditions, in the form $\mathbf{E}=\Sigma\alpha_s\mathbf{E}_s$, $\mathbf{H}=\Sigma\beta_s\mathbf{H}_s$, where α_s and β_s are undetermined coefficients.[39] Substituting the series in eigenfunctions into Eqs. (3.3), we easily find

$$\begin{aligned} \alpha_s&=\frac{i\omega}{\omega_s^2-\omega^2}\frac{1}{N_s}\int_V dV\mathbf{j}\mathbf{E}_s^*,\\ \beta_s&=\frac{\omega_s}{\omega}\alpha_s. \end{aligned} \tag{3.4}$$

The Fourier transform of the current of a point charge q is

$$j_z(\omega)=\frac{q}{r}\,\delta(r-a)\delta(\varphi-\varphi_0)\exp(i\omega z/\beta c).$$

From the definition (1.16) of the longitudinal impedance we then find

$$Z(\omega)=-\frac{1}{q}\int_L dz\,\exp\left(-\frac{i\omega z}{\beta c}\right)E_z(r=a,z)$$
$$=\sum_s \frac{i\omega}{\omega_s^2-\omega^2}\frac{|I_s(\omega)|^2}{N_s}, \tag{3.5}$$

where $I_s(\omega)=\int_L dz\,\exp(-i\omega z/\beta c)E_{sz}(r=a,z)$.

From (3.5) we easily find an expression for the low-frequency limit of $Z(\omega)$ at $\omega\ll\min(\omega_s)$:

$$Z(\omega)=-i\omega\sum_s \frac{|I_s(\omega)|^2}{\omega_s^2 N_s}. \tag{3.6}$$

It can also be seen from (3.5) that in the limit $\omega\to\omega_s$ there is a resonant increase in the sth term in the $Z(\omega)$ series. We introduce a nonzero but small absorption in the resonator walls, adding an imaginary part to the eigenvalues: $\omega_s\to\omega_s'-i\omega_s''=\omega_s'(1-i/2Q_s)$ (Ref. 71). The Q of the sth mode in this case is $Q_s=\omega_s' W_s/P_s\gg 1$, where $W_s=N_s/2$ is the energy of the sth oscillation mode in the resonator, and P_s is the average power dissipated in the resonator walls. The power loss is defined as

$$P_s=\frac{1}{2\sigma\delta}\int_S ds|H_{s\tau}|^2, \tag{3.7}$$

where σ is the conductivity of the wall material, δ is the skin thickness at the frequency ω_s', $H_{s\tau}$ is the tangential component of the magnetic field near the wall, and the integration is over the inner surface of the resonator. At $\omega\simeq\omega_s'$, the sth term in the expression (3.5) for the longitudinal impedance is dominant:

$$Z(\omega\cong\omega_s')\cong R_s=\frac{Q_s}{\omega_s N_s}\frac{|I_s(\omega_s')|^2}{\int_S ds|H_{s\tau}|^2}; \tag{3.8}$$

alternatively, using the expression for the Q, we find

$$R_s=\frac{\sigma\delta\left|\int_L dz\,\exp\left(-i\frac{\omega_s z}{\beta c}\right)E_{sz}(r=a,z)\right|^2}{\int_S ds|H_{s\tau}|^2}. \tag{3.9}$$

We can write expressions for a cylindrical resonator in the limit in which the feed ports have a small radius ($b\to 0$). We denote the length of the resonator by L and the radius by d. We use the index $s=(m,n,p)$, where m is the number of azimuthal variations [$\exp(im\varphi)$], m is the number of radial variations, and p is the number of longitudinal variations of the E field. We denote by μ_{mn} the nth zero of the Bessel function of the first kind, $J_m(x)$. The (m,n,p) resonance has a frequency approximately equal to the resonant frequency of the resonator, $\omega_{mnp}=\sqrt{\mu_{mn}^2+(\pi pd/L)^2}c/d$. The longitudinal impedance is

$$R_{0np}=\frac{Z_0}{2\pi\beta^2}\frac{L^3}{d^2\delta}\frac{\mu_{0n}^2}{J_1^2(\mu_{0n})}\frac{c}{\omega_{0np}d}\frac{1}{1+\delta_{p0}+2d/L}$$
$$\times\left[\left(\frac{\omega_{0np}L}{2\beta c}\right)^2-\left(\frac{\pi p}{2}\right)^2\right]^{-2}\begin{Bmatrix}\sin^2\\ \cos^2\end{Bmatrix}\left(\frac{\omega_{0np}L}{2\beta c}\right). \tag{3.10}$$

The upper line in the curly brackets corresponds to even values of p, and the lower line to odd values.

Correspondingly, singling out the dipole ($\propto\cos\varphi$) component of the current $j_z(\omega)$, we find the resonant value of the transverse ($m=1$) impedance for the E modes:

$$R_{1np}=\frac{Z_0}{8\pi\beta}\frac{L^3}{d^3\delta}\frac{\mu_{1n}^2}{J_0^2(\mu_{1n})}\Bigg(1+\delta_{p0}$$
$$+\frac{2d}{L}\left(1+\frac{1}{\mu_{1n}^2}-\frac{1}{\mu_{1n}^2J_0^2(\mu_{1n})}\right)\Bigg)^{-1}$$
$$\times\left[\left(\frac{\omega_{1np}L}{2\beta c}\right)^2-\left(\frac{\pi p}{2}\right)^2\right]^{-2}\begin{Bmatrix}\sin^2\\ \cos^2\end{Bmatrix}\left(\frac{\omega_{1np}L}{2\beta c}\right). \tag{3.11}$$

This expression for R_{1np} corresponds to a radial transverse force which is acting on the beam, i.e., $Z_{\perp\,r}$. In general, the transverse impedance $\mathbf{Z}_\perp$ [see (1.26) and (1.28)] is a two-dimensional vector which has two independent components $Z_{\perp\,r}$ and $Z_{\perp\,\varphi}$ (or $Z_{\perp\,x}$ and $Z_{\perp\,y}$). In an axisymmetric chamber, however, we would have $Z_{\perp\,\varphi}=0$.

More than a few methods have been developed for numerically solving the spectral problem with boundary conditions at an ideally conducting surface. These methods make it possible to calculate the lower eigenvalues and eigenfunctions. The best-known pieces of software for axisymmetric structure are SUPERFISH (Ref. 40, axisymmetric modes), ULTRAFISH [Ref. 41, modes with a variation $\exp(im\varphi)$], MULTIMODE,[42] PRUD,[43] and LANS.[44] These pieces of software use the method of finite elements. The URMEL package[45] uses the so-called FIT algorithm. Where the geometry is definitely three-dimensional, one can use the URMEL-3D program, which is part of the MAFIA package.[46] By introducing a finite wall conductivity σ, one can determine the quality factor Q_j for the given mode and the value of $\mathrm{Re}\,Z(\omega_j)$ at resonance for the jth eigenfunction which is the subject of the calculations. The approximation of a closed resonator in the eigenfunction method thus makes it possible to find resonant values of the impedance at frequencies below the cutoff frequency of the chamber. This comment applies to large irregularities of complex shape, e.g., the resonators of accelerating stations[47] and regions where the chamber is flared in experiment zones.[48]

If the irregularity has a simple geometry, the eigenvalues and eigenfunctions can be calculated analytically. For example, a cylindrical resonator with small exit apertures was studied in Refs. 49 and 50. The eigenfunctions and eigenvalues were approximated by the eigenfunctions and frequencies of a closed resonator. In Ref. 49, the effect of ports was evaluated by calculating the energy dissipated in them. In Ref. 50, approximate analytic expressions for the longitudinal and transverse impedances in the low- and high-frequency limits were calculated by summing a series of eigenfunctions.

A minor modification of the *eigenfunction method* makes it possible to calculate resonances at higher frequencies also, but for *periodic structures*: the "closure" conditions are replaced by periodic boundary conditions. For example, the MULTIMODE package[42] was used for this purpose in Ref. 51. The modified URMEL-P program[52] can be used for the same purpose. In that case, however, it is necessary to calculate dispersion characteristics of the periodic structure. Such calculations are extremely lengthy, especially for structures with a long period.

Joining method. To calculate the fields in structures which have axial or planar symmetry, with a boundary of fairly simple shape (Fig. 1, a and c), the *method of partial regions*, also called the *joining method*, is used. The volume inside the chamber is partitioned into simple subregions within each of which variables can be separated in the wave equations. The solutions in each subregion are expressed as series with undetermined coefficients. Substitution of the solutions into the continuity equations at the boundaries of the subregions and into the boundary conditions at the chamber walls leads to an infinite system of linear equations for these coefficients. Strictly speaking, in order to select unambiguously the solution for an infinite linear system it is necessary to impose some additional conditions, e.g., so-called conditions at an edge.[53] Usually, however, the system can be truncated at a finite size on the basis of physical considerations, and simply a few coefficients are sufficient to calculate impedances. Among the papers which have used the joining method to calculate longitudinal impedances, that by Keil and Zotter[9] is well known. The period of the structure is a cylindrical chamber of a cyclic accelerator of length $D=2\pi R$ with a flaring region of length g, a chamber radius b, and a flaring of d (Fig. 1a). The nth harmonic of a perturbation, $j_z(z,r,t) = \rho_n\beta c\exp(izn/R - i\omega t)\theta(a-r)$, propagates along the z axis of the chamber; here $\omega=\beta cn/R$, and a is the beam radius. The fields are joined at the boundary between the cavity and tube at $r=b$. The matrix equation found as a result of this joining process can be solved analytically in the low-frequency limit. For example, under the conditions $\omega\ll c/g$ and $\omega\ll c/d$, a short ($g\ll\pi b$) flaring section in the chamber makes an inductive contribution to the longitudinal impedance:

$$\frac{Z}{n}=-iZ_0\beta\frac{g}{2\pi R}\ln\frac{d}{b}. \qquad (3.12)$$

Approximate expressions for resonant values of the impedance and the frequencies are also derived. In general, the truncated system of equations is solved numerically. The elements of the kernel (matrix) of the equation are slowly convergent series. In some later papers by other authors,[54,55] the range of applicability of the method was expanded. In particular, a transformation was used which made it possible to improve the convergence of these series and thus to simplify the calculations. Expressions for resonant values under the condition $b\ll d$ were given in Ref. 55, and a program was developed for numerically calculating the impedance for a broad range of parameter values.

A similar approach was taken to calculate the transverse impedance in Ref. 56 for the same geometry. A system of equations was derived for the field coefficients. Approximate expressions were found for the low-frequency region. For example, a short flaring region in a chamber under the restrictions on the frequency given above leads to the following increment in the transverse impedance:

$$Z_\perp=-iZ_0\frac{g}{\pi b^2}\frac{d^2-b^2}{d^2+b^2}. \qquad (3.13)$$

Several programs for calculating fields in an axisymmetric periodic structure with the geometry of Fig. 1a have been developed on the basis of joining methods. The KN7C program[57] can find the parameters of azimuthally symmetric modes: the frequencies ω_n, the field pattern, and the loss factor k_n. From these results it is a simple matter to find the wake potential, since we have[12] $W(s)=\Sigma_{n=1}^{\infty}k_n\cos(\omega_n s/c)$. A Fourier transformation yields the impedance, whose real part consists of δ functions at the points ω_n with weights πk_n. The discrete peaks are usually "smeared out," and an average quantity $\bar{Z}(\omega)=2\pi^2/\Delta\omega\Sigma_n k_n$ is dealt with, where the summation is over the modes which fall in the interval[58] $(\omega-\Delta\omega/2,\omega+\Delta\omega/2)$. The TRANSVRS program[59] works in the same way as KN7C for modes with an azimuthal variation $\cos(m\varphi)$.

Aperiodic structures. For irregularities remote from other elements of the vacuum chamber it is convenient to treat the idealized problem in which the irregularity is assumed to be positioned on an infinite smooth chamber. It is of course not possible to formulate an internal boundary-value eigenvalue problem and then use various programs for solving it in this geometry without resorting to the approximation of a closed resonator. It is necessary to consider the radiation conditions at infinity: There is only an outgoing wave in an output port, while there are incident and reflected waves in an input port.

The *method of partial regions (the joining method)* is used in this problem when the irregularity has a simple geometry. Expressions for the fields in the open region are written with allowance for the radiation boundary conditions at infinity. For a simple cylindrical irregularity (Fig. 1c), the joining at the longitudinal cavity-tube boundary leads to either an integral equation or a matrix equation for the coefficients of an expansion of the field in the cavity (the longitudinal impedance[75] and the transverse impedance[76]). The truncated linear system is solved numerically (the ICYRP program). This method has also been generalized to the case of several equidistant identical irregularities of this type, and the mutual effects of these irregularities have been studied.[77]

Another version of the joining method is possible: a joining at (port)-irregularity end joints (Ref. 78, for example). A joining of the fields in the transverse plane was used in Ref. 79 to calculate the longitudinal impedance of an arbitrary axisymmetric structure. The irregularity was approximated by a sequence of stepped transitions. Matrix equations for field coefficients arose as a result of the joining at the common boundaries of neighboring regions. The approach of joining in the transverse plane is a natural one for calculating the impedance created by collimators.[80] This joining method is also used when the radii of the input and output ports are different and also to study an abrupt transition between waveguides of different dimensions.[81,82]

Perturbation-theory methods. The joining method is suitable for only the simplest shapes of irregularities of the vacuum chamber. If there is a small variation (of arbitrary shape) in the transverse dimensions of the chamber, *perturbation-theory methods* are used to calculate impedances. We denote by D the period of the perturbation of the wall of an axisymmetric chamber, and we describe the shape of the boundary in a longitudinal cross section, $\varphi=\text{const}$, by means of the function $r=b(z)=b[1+\varepsilon s(z)]$, where $s(z)=\Sigma_{p=-\infty}^{\infty} c_p \exp(i2\pi pz/D)$, $c_0=0$, $c_{-p}=c_p^*$, $\text{Var}[s(z)]=2$. The fields excited in the chamber by a given current are written as a series in spatial harmonics with a known radial profile, with undetermined coefficients. The coefficients can be found from the boundary conditions at the chamber wall:

$$\left[\frac{E_z+b'(z)E_r}{\sqrt{1+[b'(z)]^2}}+(1-i)\frac{\delta\omega}{2c}Z_0H_\varphi\right]_{r=b(z)}=0, \qquad (3.14)$$

where δ is the skin thickness. The quantity $\varepsilon=h/2b\ll 1$ is the small parameter here, where h is the height of the irregularity, and b is the mean radius of the chamber. Substituting the fields into this boundary condition, and expanding the resulting equation in powers of ε, we find a recurrence sequence of boundary conditions at $r=b$. From them we find the field coefficients and the impedance as a series in ε: $Z=Z^{(0)}+\varepsilon^2Z^{(2)}+\varepsilon^3Z^{(3)}+...$, where $Z^{(0)}$ is the impedance of the smooth chamber, and $Z^{(1)}=0$. The approach of expanding in a small parameter in the boundary conditions, which we will call the *ε expansion*, is referred to in the mathematical literature as a transfer of the boundary condition to a regular boundary.[60] The ε expansion was originally used in accelerator theory to calculate the energy losses of a beam in a cylindrical waveguide with a periodic variation of the radius.[61] The method was used in Ref. 62 to calculate the longitudinal and transverse impedances of a chamber with a small periodic perturbation of an ideally conducting wall ($\sigma\to\infty$) in the ultrarelativistic limit ($\gamma\to\infty$). The increment in the longitudinal impedance of the smooth chamber is

$$\frac{Z}{n}=-iZ_0\varepsilon^2G^2\sum_{p=-\infty}^{\infty}p^2|c_p|^2\frac{I_1(x_p)}{x_pI_0(x_p)}+O(\varepsilon^3)$$
$$=-iZ_0\varepsilon^2G^2 2\sum_{p=-\infty}^{\infty}p^2|c_p|^2\sum_{r=1}^{\infty}\frac{1}{j_{0r}^2+x_p^2}+O(\varepsilon^3), \qquad (3.15)$$

where $G=2\pi b/D$, $x_p^2=2\xi pG+p^2G^2$, $J_m(j_{mr})=0$, and $J_m(x)$ and $I_m(x)$ are the ordinary and modified Bessel functions of the first kind. It is then a simple matter to find Z/n at low frequencies ($\xi=\omega b/c\to 0$, i.e., $x_p\to|p|G$). In addition, one can derive simple formulas for the resonant frequencies $f_{p,r}$ of the rth radial mode, the width of the resonances $(2\Delta f)_{p,r}$, and the values of $Z_{p,r}$ of the resonances:

$$f_{p,r}=\frac{c}{4\pi b}\left(pG+\frac{j_{0r}^2}{pG}\right), \qquad (3.16)$$

$$(2\Delta f)_{p,r}=f_{p,r}\frac{\delta}{2b}\left[1+\left(\frac{j_{0r}}{pG}\right)^2\right], \qquad (3.17)$$

$$\frac{Z_{p,r}}{n}=Z_0\varepsilon^2|c_p|^2\frac{8b}{\delta}\left[1+\left(\frac{j_{0r}}{pG}\right)^2\right]^{-2}+O(\varepsilon^3). \qquad (3.18)$$

The increment in the transverse impedance in the case $\sigma\to\infty$, $\gamma\to\infty$ is

$$Z_\perp=-iZ_0\varepsilon^2G^2\frac{2R}{b^2}\sum_{p=-\infty}^{\infty}p^2|c_p|^2$$
$$\times\left[\frac{I_1'(x_p)}{x_pI_1(x_p)}-\frac{I_1(x_p)}{x_p^3I_1'(x_p)}\right]+O(\varepsilon^3). \qquad (3.19)$$

The first term in square brackets here is the E-wave contribution, and the second is the H-wave contribution. Correspondingly, there are two families of transverse resonances.

The ε-expansion method was used in Ref. 63 to derive a general expression for the longitudinal impedance at an arbitrary beam energy and for a finite wall conductivity. The effects of random deviations from periodicity in the positions of the irregularities were taken into account by a statistical approach. The parameters of the resonances were compared with the results of numerical calculations,[51] and it was shown that the agreement is good for small perturbations and sufficiently long periods. Higher orders of perturbation theory in ε were studied in Ref. 65. The ε expansion is valid only for perturbations whose boundary $b(z)$ is a single-valued function of z, and for which the derivative $|b'(z)|$ is bounded. A more general method was used in Ref. 66, where an additional limitation on the applicability of the ε expansion at low frequencies was pointed out: $D\gg h$, where D is the period of the structure.[63,64,67] The ε expansion can thus be used to carry out calculations on structures with small and smooth variations of the boundary under the condition that the period is significantly larger than the variation of the transverse dimension, e.g., for transition sections and joints.[51,68] For corrugations, i.e., for $D\sim h$, the ε expansion does not, strictly speaking, work.

Matrix method. To calculate the impedance of a corrugated vacuum chamber at low frequencies, Kheifets and Zotter[66] proposed a method which we will call here the "matrix method." A modification of this method proposed in Ref. 67 can be used to calculate the longitudinal and transverse impedances in the resonant region also. The *matrix method* can be used for periodic structures with axial or planar symmetry; the chamber boundary $b(z)$ must be a smooth, single-valued function of z (Fig. 1b). Structures of the "pill-box" type (Fig. 1a), for example, do not fall in this category. In this case it is not required that the variation of the boundary, $h = \max[b(z) - b]$ be small. The solution of the wave equations with a given current, written as a series in spatial harmonics with undetermined coefficients, is substituted into the boundary condition (3.14). The resulting equation is then expanded in the complete system of spatial harmonics. As a result, one finds an (infinite) matrix equation for the coefficients. After truncation, this equation is solved numerically: by the IMPASS[69] and NM[67] programs. As a result, the matrix method, like the joining method, should be classified as semianalytic. In the case $h \ll b$, the matrix equation can be expanded in the small parameter ε, and an analytic solution can be found; this solution is the same as the ε expansion discussed above. The additional conditions under which the expansion of the matrix equation is valid are sufficient conditions for the applicability of the ε expansion.[66,67] The size at which the matrix is truncated ($P \times P$) in the numerical solution is proportional to the length of the period. Values as low as $P=5$ or 7 are sufficient for corrugated structures.[67] The matrix method is thus convenient for structures with a short period.

Method of equivalent boundary conditions. The impedance of chambers with a corrugated wall is also calculated by the *method of equivalent boundary conditions*, which, like the ε expansion, transfers the boundary conditions to a smooth boundary. In a calculation of the fields in a corrugated waveguide, a so-called impedance condition is imposed. This condition imposes a linear relationship between the components of the electric and magnetic fields on some smooth surface. For example, in the case of a cylindrical waveguide of radius b, with a rectangular corrugation with period D and depth h ($h \ll b$), we have

$$E_z = iZ_0 \frac{D-g}{D} \tan\left(\frac{\omega h}{c}\right) H_\varphi, \quad E_\varphi = 0, \tag{3.20}$$

at $r=b$, where g/D is the corrugation filling factor.[70] Another example of this approach is the study by Balbekov,[72] in which an "effective" boundary condition at a smooth boundary arises from a conformal mapping of a cell of the corrugation into a rectangle. The corrugated surface is replaced by an "equivalent" smooth surface with $r = b_{\text{eff}}$ and with a dielectric coating whose dielectric constant depends on the longitudinal coordinate z. In the long-wave approximation $\lambda \gg D$, an average over z can be taken. As a result, one finds a boundary condition of the impedance type at a smooth surface. This method was used in Ref. 72 for an analytic calculation of a resonance of the longitudinal impedance with a frequency of about 6 GHz caused by a wave slowed by a corrugation in the U-70 accelerator. The transverse impedance of the corrugated U-70 chamber has been calculated in a corresponding way.[74] Interestingly, the effect of irregular inserts in an accelerator (sections of smooth chamber, etc.) which disrupt the periodicity of the corrugated structure reduces the size of the resonance by more than an order of magnitude. The result agrees well with measurements at the U-70. The impedance of this corrugated structure was calculated in Ref. 73 with the help of the standard impedance boundary condition stated above. The matrix method, applied to the same problem in the case of an undisrupted periodicity,[67] leads to the same parameter values for the slow-wave resonance as in Refs. 72–74. In addition, it indicates that there is a series of resonances of the longitudinal and transverse impedance at higher frequencies, ~ 14 GHz. There are two series of high-frequency transverse resonances, corresponding to E and H waves. For the slow-wave resonance and for the E series, the empirical relation (1.39) for the longitudinal and transverse impedances holds well. The same is true of the low-frequency increment caused in the impedance by the corrugation.

Method of integral equations. For an irregularity of fairly general shape, the impedance calculation can be reduced to the problem of solving an integral equation. An integral equation in which the unknown is the field at $r=b$, and the kernel is a series in the eigenfunctions of the cavity, was constructed in Refs. 83 and 84 for an axisymmetric structure. The integral of the solution of this equation yields the longitudinal impedance. For small irregularities, the kernel can be simplified, and the results can be derived explicitly.[86] The same possibility holds in the high-frequency limit for irregularities of a particular type.[84] The results are generalized to several identical inserts and, in a limiting case, to a periodic structure.[85]

To calculate the impedance of one or several equidistant irregularities of a fairly general type [under the restriction that $b(z)$ is a single-valued function of z; i.e., Fig. 1d but not Fig. 1e], another integral equation was found in Ref. 87. The longitudinal impedance is expressed in terms of the solution of this equation at a selected point. For an irregularity of simple shape, the kernel can be evaluated analytically, but in general it is a definite integral. A simple numerical procedure for solving the integral equation makes it possible to find the impedance at both low and high cutoff frequencies. At low frequencies, irregularities of the chamber which are regions of flaring or contraction generate a primarily inductive increment in the longitudinal impedance: $\operatorname{Im} Z(\omega) = -\omega \mathscr{L}$. The dependence of the inductance $\mathscr{L}$ on the geometric dimensions was studied in Ref. 87 by fitting the results of a numerical solution of the integral equation for certain types of perturbations. For a flaring of a cylindrical chamber whose axial cross section is a triangle with base L and height h, for example, the following result was found:

$$\mathscr{L} = \frac{Z_0}{2\pi c} \frac{Lh}{\pi b} \frac{2h}{\sqrt{L^2 + 4h^2}}. \tag{3.21}$$

If the inductance of an insert of this sort is calculated in the approximation of an unperturbed field of the beam as the ratio of the magnetic flux to the current,[9] the result $\mathscr{L} = Z_0 Lh/(4\pi cb)$ is found. The result (3.21) differs from this approximation, since it incorporates the distortion of the field inside the insert.

For a contraction of a chamber with the same triangular shape we have

$$\mathscr{L} = \frac{Z_0}{2\pi c} \frac{h^2}{b}. \tag{3.22}$$

For perturbations with a trapezoidal cross section—a pair of transition sections of length l which make the transition from the radius b of the smooth chamber to a radius $b+h$ (or $b-h$; Fig. 1f)—and under the condition $l \ll L$, b, the quantity Im Z is nearly independent of the distance between transition sections, L. Under the condition $L > b$, the inductances created by the flaring and contraction of the chamber are identical for equal values of h. For $L \gg 2b$, they are given approximately by

$$\mathscr{L} = \frac{Z_0}{\pi c} \frac{h^2}{b} \sqrt{\theta}, \tag{3.23}$$

where $\tan \theta = h/l$.

The same method has been used to study the interaction of N identical equidistant small perturbations in the resonant region. An extremely interesting result emerged: Even if the distances between irregularities are large, $D \gg b$, the size of a resonance in the longitudinal impedance, divided by the number of these resonances, oscillates in the interval $[0, N^* \text{Re}Z_1]$, where $\text{Re}Z_1$ is the resonant impedance of one irregularity. The frequencies of the resonances are nearly independent of N and D. In the limit of a periodic structure ($N \to \infty$), the integral equation becomes an infinite linear system—the same as that which arises in the matrix approach.[67]

Diffraction methods. To calculate the impedance of an irregularity at high frequencies, *diffraction-theory methods* are used. The first study in this direction was carried out by Lawson.[90] A picture of the present situation in this field is drawn by Heifets,[91] who studied the high-frequency behavior of the impedances of some simple irregularities. Let us take a closer look at the situation at frequencies $\omega b/c \gg 1$.

For a long time, the results derived on the behavior of the impedance at high frequencies were afflicted by a contradiction. The asymptotic behavior $\text{Re}\, Z(\omega) \sim \omega^{-1/2}$ was found for the longitudinal impedance in several studies,[90,50,92,93] while the numerical results of Ref. 15 and the results of several studies (e.g., Ref. 94) using the model of an optical resonator[95] indicated an $\omega^{-3/2}$ asymptotic behavior. The contradiction was successfully resolved by Heifets and Kheifets,[96] who showed, by a joining method, that for an isolated pill-box irregularity the asymptotic behavior is $\text{Re}Z(\omega) \sim \omega^{-1/2}$, while that for an infinite periodic structure is $\text{Re}Z(\omega) \sim \omega^{-3/2}$. In the case of a finite number of irregularities M, the transition from the $\omega^{-3/2}$ regime, which prevails in the frequency range $L/b \ll \omega b/c \ll M^{2/3} L/b$, to the $\omega^{-1/2}$ regime occurs at $\omega b/c \gg ML/b$. Here L is the distance between the elements, and b is the radius of the smooth part of the chamber. Corresponding results have been derived in the diffraction model.[91] The asymptotic behavior of ImZ was also studied in Refs. 84 and 85. These results are important for electron accelerators with a very short bunch.

The review[11] by Heifets and Kheifets gives a detailed description of results on the impedance at high frequencies, including studies by both the diffraction and joining methods. We will cite only one result here: The longitudinal impedance of a collimator consisting of a diaphragm of radius d on a smooth chamber of radius b ($d < b$) takes the constant value

$$Z(\omega) = \frac{Z_0}{\pi} \ln \frac{d}{b} \tag{3.24}$$

at $1 \ll \omega d/c < \gamma$. An abrupt change in chamber radius ($b > d$; Fig. 1g) has the same impedance at high frequencies.

Energy method. This approach occupies a somewhat special position among electrodynamic methods, since it allows one to avoid a rigorous solution of the electrodynamic problem of the excitation of a waveguide by a given current in calculating impedances (or wake potentials). It is similar in this regard to the electronic-circuit approach. It can be classified as an electrodynamic method because the field patterns in the individual parts of the structure are studied, as in the joining method. In particular, the fields are expanded in the system of waveguide modes. On the basis of simple qualitative considerations regarding the field characteristics—the field excited by a relativistic particle is localized in the half-space behind the particle, etc.—the joining conditions can be replaced by an equation expressing the energy balance between the radiated waves and the field of the charge in some volume (Refs. 97 and 98, for example). The energy method is ordinarily used for calculations on slow-wave structures, either uniform (with a dielectric) or periodic, of the iris-loaded-waveguide type. Along this approach one can also study the impedance of the irregularity which arises at a joint between two waveguides differing in size.

Aperiodic structures. Direct numerical methods in the frequency domain are used to calculate the impedance of a resonator of arbitrary shape with infinite ports at frequencies $\omega > \omega_c$. The boundary conditions in a field-calculation problem were formulated in Ref. 88 in such a way that the SUPERFISH program[40] could be used. The radiation conditions are replaced by the introduction of a medium, which fills the smooth chamber. The medium has a small, purely imaginary dielectric constant. However, the calculations must be repeated for each new value of the frequency, and much processor time is required. Another program, URMEL-I,[89] uses approximate boundary conditions at open ports. It then becomes possible to formulate the problem in a closed region and to calculate the longitudinal impedance numerically at frequencies above ω_c.

A semi-infinite waveguide is an irregularity of a different type. The problem of calculating the longitudinal impedance of a semi-infinite cylindrical waveguide with ideally conducting walls was solved in Refs. 99 and 100. The problem reduces to a system of two integral equations of a special type, which can be solved by a factorization method (Ref. 101, for example).

To conclude this section of the paper, we cite some recent studies[102–104] in which the impedance created by holes or slits in chamber walls (used for evacuation) was calculated. Such irregularities are typical elements of the chamber of large ring colliders; furthermore, there are usually a lot of them. The method proposed can be classified as a version of the method of *equivalent boundary conditions* according to Table I. On the other hand, it is similar to the *energy* approach in that it allows one to avoid a rigorous solution of a complex three-dimensional problem of calculating fields. This method is based on the theory of wave diffraction by small holes which was derived by Bethe.[105] The basic idea of the method is to replace a hole excited by an incident electromagnetic wave by effective "magnetic" currents in order to satisfy the boundary conditions at the walls. If the holes or slits are small, the fields generated by these currents can be calculated as the fields of effective electric and magnetic dipoles, whose magnitudes are expressed in a simple way in terms of the unperturbed fields in the chamber. The fields excited by the holes are conveniently expanded in eigenmodes of the chamber. After an integration, one obtains simple expressions for the impedance. The problem has been solved for a small hole of arbitrary shape in an ideally conducting wall in a circular chamber[102,103] and in a chamber with arbitrary cross section.[104] The range of applicability of the results is determined by the conditions $\omega h/c \ll 1$, where h is a characteristic dimension of the hole, and $h \ll b$, where b is a characteristic transverse dimension of the chamber. Bethe's theory is valid under these conditions.

We will reproduce the results for a circular chamber, which have a simple analytic form.

We assume that a cylindrical chamber has a cross section of radius b, ideally conducting walls, and a small hole in its wall whose center is at the point $(r=b,\ \varphi_h,\ z)$. The longitudinal impedance of a single small hole of arbitrary shape in the wall of this type of circular chamber is

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_e+\alpha_m)}{4\pi^2 b^2}, \tag{3.25}$$

where the polarizabilities of the hole, α_e and α_m, depend on the shape of the hole. If this shape is simple, the polarizabilities can be calculated analytically (this is true of an ellipse,[106] for example). Otherwise, they can be measured on a prototype.

For a **circular hole** of radius h, the polarizabilities are $\alpha_e = -2h^3/3$ and $\alpha_m = 4h^3/3$, and the impedance is

$$Z(\omega) = -i \frac{Z_0}{6\pi^2} \frac{\omega h^3}{cb^2}. \tag{3.26}$$

As was shown in Ref. 103, allowance for a finite wall thickness reduces the impedance introduced by the hole (by a factor of 0.56 when the wall thickness is larger than the hole diameter).

In the limit of a **narrow longitudinal slit** of length l and width w, with $w \ll l$, the polarizabilities approach $\alpha_e = -\pi l w^2/24$ and $\alpha_m = \pi l w^2/24$. It follows from Eq. (3.25) that under the condition $\omega l/c \ll 1$ the low-frequency longitudinal impedance is zero in the first approximation. This result is not surprising, since a longitudinal slit causes almost no disruption of currents induced in the wall. Inclusion of succeeding terms leads to the following expression under the conditions $w \ll l < b$:

$$Z(\omega) = -i \frac{Z_0}{96\pi} \frac{\omega}{c} \frac{w^4}{b^2 l} \left(\ln \frac{4l}{w} - 1 \right). \tag{3.27}$$

A **narrow transverse slit**, in contrast to a longitudinal one, greatly disrupts the structure of the currents induced in the wall by the beam. The impedance introduced by an irregularity of this sort is naturally much greater. Substituting $\alpha_e = -\pi l w^2/24$ and $\alpha_m = \pi l^3/24/[\ln(4l/w) - 1]$ into (3.25), and ignoring the contribution of the electric dipole, which is small in comparison with that of the magnetic dipole in this case, we find

$$Z(\omega) \cong -i \frac{Z_0}{96\pi} \frac{\omega}{c} \frac{l^3}{b^2[\ln(4l/w) - 1]}. \tag{3.28}$$

This formula is applicable under the condition $w \ll l \ll b$.

Small holes in chamber walls make an inductive contribution to the impedance.

The transverse impedance of an isolated small hole of arbitrary shape in the wall of a circular chamber is

$$\mathbf{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \mathbf{a}_h \cos(\varphi_h - \varphi_b), \tag{3.29}$$

where $\mathbf{a}_h$ is a unit vector directed toward the hole, and φ_b is the azimuthal angle through which the beam is deflected. We recall that the problem is not axisymmetric, and the deflecting force is not necessarily directed along the transverse displacement of the beam. Equation (3.29) shows that this force is directed toward the hole (or away from it). In a chamber of arbitrary cross section, this would generally not be true.[104] The magnitude of the impedance depends on the angle between the direction to the hole and the vector transverse displacement of the beam.

For certain particular shapes of the hole, the following transverse impedances are found from (3.29):

$$\mathbf{Z}_\perp(\omega) = -iZ_0 \frac{2h^3}{3\pi^2 b^4} \mathbf{a}_h \cos(\varphi_h - \varphi_b) \tag{3.30}$$

for a **circular hole** of radius h;

$$\mathbf{Z}_\perp(\omega) = -iZ_0 \frac{w^4}{24\pi b^4 l} \left(\ln \frac{4l}{w} - 1 \right) \mathbf{a}_h \cos(\varphi_h - \varphi_b) \tag{3.31}$$

for a **narrow longitudinal slit** of width w and length l ($w \ll l$); and

$$\mathbf{Z}_\perp(\omega) \cong -iZ_0 \frac{l^3}{24\pi b^4[\ln(4l/w) - 1]} \mathbf{a}_h \cos(\varphi_h - \varphi_b) \tag{3.32}$$

for a **narrow transverse slit** of width w and length l under the condition $w \ll l \ll b$.

If there are M holes ($M \geqslant 3$), distributed uniformly in one cross section of the chamber, the impedance is found through a vector addition of expressions of type (3.29) to be

$$\mathbf{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \frac{M}{2} \mathbf{a}_b, \tag{3.33}$$

where $\mathbf{a}_b$ is a unit vector in the direction of the transverse displacement of the beam. We see that the deflecting force is now directed along the beam displacement; i.e., the axial symmetry is restored. The maximum value of $\mathbf{Z}_\perp$ is larger by a factor of only $M/2$ than that in the $M=1$ case. In addition, the empirical relation $Z_\perp=(2R/b^2)Z/n$ holds; see (1.39), which holds only in axisymmetric structures.

An expression for the real part of the longitudinal impedance has been derived[102,104] by calculating the energy radiated by a hole. That calculation essentially reproduces the result found by Sands,[107] who calculated the total energy radiated by effective dipoles. The real part of the impedance of small holes is much smaller than the imaginary part in the frequency range considered.

Electrodynamic methods in the time domain. A "calculation of the impedance in the time domain" implies the following steps: One first calculates the wake potential. The impedance is then found by means of a relationship in terms of a Fourier representation, (1.19) or (1.31). Methods for calculating wake potentials are discussed in detail in Wilson's review.[12] We will go into detail here on only those methods which are substantially different from the ω-domain methods and which are used to calculate impedances. We are thinking primarily of the use of the TBCI program.[113]

As a general comment we note that a calculation of fields in the time domain is usually more complicated than one in the frequency domain. For this reason, it is customary to find first the Fourier components of the fields, i.e., to solve the frequency-domain problem, even in calculations of wake potentials. This is especially true in cases in which the vacuum chamber has a simple geometry.

Analytic methods. Analytic methods for calculating fields in the t domain have much in common with the methods discussed above. For example, the "ε expansion" was originally used[61] to calculate the energy losses of a beam in terms of a longitudinal wake potential, rather than in terms of an impedance [see (1.21)]. Transverse wake potentials in a chamber with a small periodic variation of its boundary were calculated by means of an ε expansion in Ref. 108. Wake potentials for a cylindrical resonator with ports were derived in Ref. 50 through an analytic *approximation of the eigenfunctions*. Reference 109, an example of another approach, essentially uses the time domain. In that study, the longitudinal wake potential in an axisymmetric chamber with a simple irregularity (a diaphragm or resonator) was calculated in a limited interval of lag times, $\tau<L/c$, where L is a characteristic dimension of the structure. For this purpose, the electrodynamic equations in Kirchhoff form were integrated approximately.

Several semianalytic methods for calculating wake potentials are based on a joining of fields. The KN7C[57] and TRANSVRS[59] programs, which we mentioned earlier, calculate the parameters of eigenmodes of the periodic structure in Fig. 1a (up to several hundred modes). By a simple summation, those programs make it possible to find the wake potential $W(s)=\Sigma_{n=1}^{\infty}k_n\cos(\omega_n s/c)$ (Ref. 15). For example, the 450 lowest modes were taken into account in numerical calculations for the SLAC accelerator; the contribution of higher modes was summed analytically in the model of an optical resonator.[15,94]

A comparison of "direct numerical methods" in the ω and t domains reveals some important differences. For an axisymmetric structure consisting of a resonator with infinite ports (Fig. 1, c and d), so-called open boundary conditions at the ports have been formulated. They simulate propagation of the wave front in the structure as time elapses in the case $\gamma\to\infty$ (Ref. 110). These boundary conditions make it possible to carry out calculations on a mesh covering a finite region. Maxwell's equations in integral form are replaced by a discrete finite-difference scheme along the time as well as the coordinates with the help of the FIT algorithm.[111] The TBCI program[112,113] carries out a numerical solution of a discrete problem. This is the most versatile of the programs using the t domain and also the one most commonly used. The TBCI program can calculate the fields excited in a resonator by a bunch with a given distribution of charge per unit length in uniform ultrarelativistic motion. A lower limit is imposed on the bunch size by the step of the mesh along the longitudinal coordinate; for a Gaussian distribution, for example, this limit is $\sigma\geqslant(4\text{–}5)\Delta z$. There is one more dimension—the time—here than in the eigenvalue problem in the frequency domain, but this complication is partially offset on the computational side, since it is not necessary to invert large matrices. The capabilities of TBCI are reviewed in a recent paper.[114]

The applications of TBCI are extremely numerous. The most common one is that of calculating the response of the resonators of the accelerating stations as a bunch passes through them (Ref. 112, for example). This software has been used to calculate the longitudinal and transverse impedances of bellows and corrugations[115–117,93] and flaring regions of the chamber in experiment zones.[48]

A specific feature of TBCI is that this program works well for resonatorlike structures with feed ports of identical radii. In this case the range of integration of the fields is finite only along the surface which is an imaginary continuation of the tube in the resonator ($r=b$, where b is the tube radius), since the condition $E_t=0$ holds on the ideally conducting wall of the tube outside the resonator. For structures with feed ports of unequal size, of the adapter-section type, this simple approach fails, as it does for irregularities such as collimators and diaphragms. An improved algorithm for the general case, which is also suitable for a modification of TBCI, was recently proposed by Napoly.[118] In that approach, the wake potentials are expressed in terms of integrals of the magnetic field over the surface of the irregularity, without a contribution from the smooth part of the chamber.

There are some other distinctive features of impedance calculations by the TBCI program which deserve mention. First, since the bunch is not a point entity, a Fourier transformation of the wake potential yields a convolution of the impedance with the bunch spectrum $\tilde{\lambda}(\omega)$: $Z(\omega)\tilde{\lambda}(\omega)$. Since the condition $\tilde{\lambda}(\omega)\ll1$ holds at high frequencies [for a Gaussian bunch we would have $\tilde{\lambda}(\omega)=\exp(-\omega^2\sigma^2/2c^2)$], errors in the calculation of the convolution rule out calculations of the impedance at high frequencies (at $\omega>c/\Delta z$, since $\sigma\geqslant\Delta z$). Second, TBCI generates the wake potential in a limited interval of lag distances, typically up to a few tens of centimeters[119] or a few meters,[48] since numerical instabilities set in at this point (they set in at even smaller distances in the transverse case). As a result, there are errors in the calculations of the impedance at low frequencies $\omega < c/L$, where L is the boundary of the interval of lag distances.

Impedances are sometimes calculated[93,121] by choosing the parameter values for the broad-band resonator (ω, R_{sh}, and Q) by fitting a plot of the loss coefficient $k(\sigma)$ against the bunch length σ. There is also the original method of Ref. 119 for calculating the low-frequency impedance of very simple irregularities. The inductance is found by fitting the formula $\mathscr{L} = -V_{\rm ind}/(dI/dt)$, where $I(t)$ is the current of the bunch passing the element, to the results of a time-domain calculation of the voltage induced at an element of the chamber, $V_{\rm ind}$. Empirical formulas like (3.21)–(3.23) have been obtained for the inductance of simple irregularities of the types in Fig. 1, c and f, under the condition $h \ll b$.

A three-dimensional version of TBCI is now available: the T3 program,[122] which is part of the MAFIA package.[46] The use of this program to calculate the impedance of chamber elements of fairly complex geometry was demonstrated in Refs. 119 and 121. Calculations of the wake fields in a chamber with a small hole in its wall by the T3 program recently yielded estimates of the low-frequency impedance generated by this hole.[123] A fit of the numerical results yielded a formula similar to the analytic expression (3.25).

A basic advantage of the TBCI (or T3) approach in impedance calculations is its versatility; disadvantages are the extensive processor time required and the circumstance that the method does not always yield accurate results (see the discussion above).

In addition to TBCI, there are other programs for calculating wake potentials and impedances which use the t domain. The ABCI program[124] is extremely similar to TBCI. The TWA program[125] solves an inhomogeneous wave equation for the vector potential on a rectangular mesh. The CLS program,[126] which solves an equation for a Hertz vector, can be used to study chambers with coaxial conductors. Novokhatski's program[127] uses a mesh of arbitrary quadrangles. The recently developed AMOS program[128] performs a finite-difference modeling of the time evolution of the fields generated by a source with a user-specified spatial and temporal modulation in axisymmetric structures.

4. CONCLUSION

Despite the long list of references attached, numerous studies lie outside the scope of this review. These are primarily studies which develop classical electrodynamic methods for field calculations, in particular, methods using Green's functions. Many interesting studies by authors in our country on calculations of fields and the radiation by particles in waveguides have also been omitted from this list. These studies are close to the studies which we have discussed here, but they do not use the concepts of coupling impedances and wake potentials. Their basic thrust is to calculate other electrodynamic characteristics of the interaction of a beam with a vacuum chamber, e.g., the radiative loss of a bunch. Without attempting to cover all the literature pertinent to this subject, we refer the reader to some monographs[129,130] which contain fairly extensive bibliographies.

As we mentioned in the Introduction, this review has focused on those methods for calculating the beam-chamber coupling impedance which are the most important ones for large proton synchrotrons. In this application, minimization of impedances is an important weapon for combating instabilities. Other approaches may prove convenient for other applications. It is nevertheless my hope that this review will be of assistance in understanding the many papers on this topic and that it will serve, if not as a reference book, at least as a guidebook.

It can be seen from the discussion above that there are a large number of different methods for calculating coupling impedances for beams in accelerator vacuum chambers. On the other hand, we also see that there is not yet a *universal method*, suitable for all cases, for all structures, etc. Perhaps one of the strongest candidates for such a role is the TBCI program, especially the T3 program in the MAFIA package. As more-powerful computers become available, this approach will acquire even more advantages. Nevertheless, the (semi)analytic methods remain extremely useful. In the first place, they make it possible to develop the physical intuition necessary for reaching a better understanding of electrodynamic problems which arise in accelerator theory. Second, they are occasionally the simplest and fastest way to get a result. It thus appears necessary to develop the existing methods and to seek new and better ones, both numerical and analytic.

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[1)] This is not exactly true. The expressions for the impedance of a smooth chamber [Eqs. (2.2) and (2.3)], for example, contain the beam radius, in the space-charge term.

[2)] For this reason, some authors[11,1] prefer to call $W^\delta(s)$ the "point wake function."

[3)] The assumption $\mathbf{r}' = \mathbf{r} = 0$ is often used.

[4)] The quantity defined in (1.24) has the dimensionality of volts, justifying the term "potential."

[5)] Since the field components satisfy E_z, E_r, $H_\varphi \propto \sin\varphi$, and E_φ, H_z, $H_r \propto \cos\varphi$ under the conditions ρ, $j = \sin\varphi$, it is easy to verify that after we take an average we have $\bar{F}_x = 0$; i.e., we are left with only the vertical component of the force.

[6)] For small r' the relation $W_\perp^\delta \propto r'$ holds (more on this below), and therefore a factor $1/r'$ is sometimes introduced on the right side of the definition (1.29) (Ref. 11).

[7)] The dipole transverse impedance $\mathbf{Z}_\perp$ defined earlier [see (1.31)] is related to the impedance introduced here by $\mathbf{Z}_\perp = Z_{\perp 1}\mathbf{r}'/r'$.

[8)] A different result was derived in Ref. 27; in particular, the second term had the opposite sign.

[9)] If we use (1.16) instead of the definition (1.11), i.e., if we take $E_z(r=0)$ in place of the average field over the beam cross section, the term 1/4 in (2.2) is replaced by 1/2.

[10)]Measurements at a working accelerator have yielded the value[31] $|\mathrm{Im}Z/n|=0.25\ \Omega$.

[11)]An English-language version of this review has been published as a CERN preprint.[131]

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