The "Darmstadt effect" and related questions

Yu. N. Pokotilovskiĭ
Joint Institute for Nuclear Research, Dubna

Fiz. Elem. Chastits At. Yadra **24**, 5–80 (January–February 1993)

The present status of the observations made in experiments at the GSI, Darmstadt of narrow positron lines and electron–positron pairs in the range of effective masses 1.6–1.8 MeV/c^2 in collisions of very heavy ions at energies below the Coulomb barrier is reviewed. The results of experiments at low and high energies looking for a similar phenomenon in other processes are presented. Theoretical attempts to explain the "Darmstadt effect" are considered.

1. MOTIVATION OF INVESTIGATIONS INTO THE PRODUCTION OF POSITRONS IN HEAVY-ION COLLISIONS AT ENERGIES BELOW THE COULOMB BARRIER

The solution of the Dirac equation for the bound states of an electron in the field of a point Coulomb center becomes singular at $Z=137$.[1] The expression for the energy of the lowest electron level $1S_{1/2}$ is

$$E_1=m_ec^2(1-x^2)^{1/2}, \tag{1}$$

where $x=Ze^2/\hbar c=\alpha Z$. For nuclei of finite size, the first estimate of the energies of the electron levels was made by Pomeranchuk and Smorodinsky,[2] who showed that for $Z=137$ there is no singularity in the solution and that the charge at which the energy of the $1S_{1/2}$ level reaches $E=-m_ec^2$ (the energy of the lower continuum), i.e., the binding energy of an electron in the K shell reaches $2m_ec^2$, is approximately equal to 200 (critical charge). Later, more accurate solutions of the Dirac equation for nuclear charges near the critical value made by Werner and Wheeler,[3] Gershteĭn and Zel'dovich,[4] Popov,[5] and also Pieper and Greiner[6] and Müller *et al.*[11] led to the value $Z_c=173\pm1$ for the critical charge. Gershteĭn and Zel'dovich,[4,10] and also Pieper and Greiner[6] were the first to suggest that for a bare nucleus with $Z>Z_c$, or in the presence of a vacancy in the K shell, there is spontaneous capture of an electron from the negative continuum to the K shell and production of a hole in the negative continuum, this being observed as the production of a free positron. In a different terminology, the vacuum becomes negatively charged, and simultaneously there appears a positron; naturally, the total charge is conserved.

Thus, if in an encounter of two heavy nuclei[4,10,11] the total charge exceeds the critical value, spontaneous production of positrons can be expected. It is important that this spontaneous process does not depend on the velocities of the nuclei and is possible for an arbitrarily slow approach of the nuclei to each other. The analysis of Refs. 4–10 shows that because of the Coulomb barrier in the effective potential of the two nuclei the produced positron is initially localized near the nuclei, and then tunnels to infinity.

It follows from the above that in a heavy-ion collision one can investigate experimentally the spontaneous production of positrons and positron–electron pairs, demonstrating in this way the fundamental role of the vacuum in quantum electrodynamics.

In the middle of the seventies, a program was begun at the GSI (Darmstadt, Germany) using the UNILAC accelerator to look for the spontaneous production of positrons in collisions of very heavy ions.

As an example, let us consider the picture of the processes accompanying the collision of two ^{238}U nuclei at an energy of the incident ions near the Coulomb barrier, ~5.9 MeV/nucleon.[12] Depending on the relative orientation of the two deformed nuclei, the collision may lead to partial interpenetration of the nuclei and the formation of a compound nucleus during a time 10^{-22}–10^{-21} sec. Independently of the formation of a compound nucleus, the total charge is $Z_u=Z_1+Z_2$. During this short collision time, a superheavy atom with total charge $Z_u=184$ of the "nucleus" is formed. Since the velocity of the ions in the collision, $v\sim0.1c$, is small compared with the velocity of the orbital electrons, it can be assumed that the electron levels go over adiabatically to the levels of the new two-center potential, which varies with the changing distance between the nuclei. For $Z_u>Z_c$ and a sufficiently small distance between the nuclei, the $1S_{1/2}$ level is embedded in the negative continuum for a short time.

We list the main processes that occur in such a collision (in all that follows, we have in mind collisions of ions with energy near and below the Coulomb barrier).

1. Further, deeper ionization of the incident ion and target atom.

2. The formation of vacancies in the K shell with a probability reaching a few percent; this has been investigated theoretically[13–15] and confirmed experimentally.[16–18] At the same time, because of the strong localization of the electron wave functions in the field of the two nuclei, there is an appreciable probability for processes with large momentum transfer from the moving nuclei to the electrons—the production of δ electrons.

3. Excitation of the nuclei—both Coulomb excitation and in the transfer of one or several nucleons. When the phenomena described below were observed, the conditions were such that the inelastic energy transfer did not exceed 10–20 MeV, and the mass transfer did not exceed five nucleons.[19]

4. The deexcitation of the nuclei excited during scat-

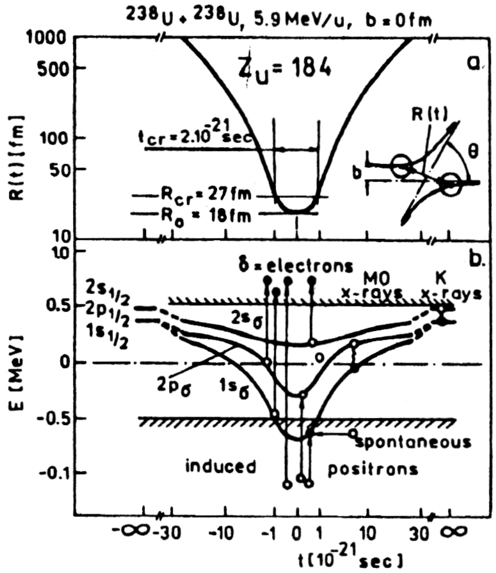

FIG. 1. Internuclear separation (a) and binding energy (b) for three quasiatomic levels as a function of the time in the process of elastic scattering of uranium ions with energy 5.9 MeV/nucleon. The main processes are shown: production of δ electrons, production of induced (quasiatomic) positrons, and production of positrons in the possible spontaneous process.[30]

tering occurs through fission, radiation of photons and conversion electrons, and internal pair conversion if the excitation energy exceeds $2m_ec^2$. The last process is one of the components of the background of positrons and positron–electron pairs present when one is looking for the spontaneous production of positrons. These nuclear positrons are emitted during the time 10^{-15}–10^{-13} sec after the collision of the ions.

5. The collision of the heavy ions gives rise to a very deep Coulomb potential (well depth $\sim$20 MeV) that varies rather rapidly in time and space. This removes electrons (as in Coulomb ionization) from the negative continuum with transition either to vacant discrete quasimolecular levels or directly to the positive continuum (Refs. 61, 60, 57, and 48). In both cases, the hole produced in the negative continuum is observed as positron production with a broad ($\sim$1 MeV) bell-shaped spectrum that depends on the collision dynamics. The emission time of such positrons, which in different publications are called quasiatomic, induced, or dynamic positrons, is $\sim 10^{-21}$ sec.

6. The most interesting process is the one already mentioned: spontaneous production of positrons. They must be produced mainly at the time of closest approach of the nuclei and have energy near

$$E_{e^+} = -(m_ec^2 + E_{1s_\sigma}), \qquad (2)$$

where E_{1s_σ} is the energy of the electron ground state in the field of the colliding nuclei at the time of encounter. For example, for a $U+U$ collision we must have $E_{e^+} \sim 300$ keV (Ref. 20). In Fig. 1, these phenomena are shown by arrows. The time of spontaneous transition of an electron from the negative continuum to a vacancy in the K shell of the "superheavy" nucleus must be $\sim 10^{-19}$ sec (the corresponding width is $\sim$10 keV),[21] and this is greater by two orders of magnitude than the characteristic ion collision time ($\sim 10^{-21}$ sec). Even before the experiments in which structure was observed in the positron spectrum it was clear that this is the greatest difficulty in the attempt to observe the spontaneous process, since the dynamic and spontaneous processes must take place coherently,[22] and there is no prescription by means of which they could be separated experimentally. This led to discussion[23,21] of the as yet unsettled question of the possible production during the nuclear interaction, in a small ($\sim 10^{-3}$) fraction of scattering events and below the Coulomb barrier, of quasimolecules with lifetime appreciably greater than the collision time. This process was subsequently discussed[25–29] in connection with attempts to explain the observed features of the positron spectrum. If T is the lifetime of such a supercritical molecule, then one can hope to observe in the positron spectrum a peak with energy in accordance with the expression (2) and with width $\Delta E(\mathrm{keV}) \cong 2\hbar/T \cong 1.32 \cdot 10^{-18}/T(\mathrm{sec})$ inversely proportional to the time of fusion (sticking) of the nuclei into a quasimolecule, and this would be an unambiguous indication of the existence of two important phenomena—spontaneous production of positrons and simultaneously the production of a long-lived quasimolecule.

2. EXPERIMENTS ON POSITRON PRODUCTION IN HEAVY-ION COLLISIONS

2.1. Experimental technique

Positron production in collisions of very heavy ions was investigated at the GSI by three independent groups by means of three magnetic spectrometers: EPOS,[31,12,18] TORI,[32] and ORANGE.[34–36] The first two groups used solenoidal transport systems in which the positrons emitted in the ion collisions moved along helical trajectories to detectors (scintillation or silicon) at quite large distances from the target ($\sim$1 m). The solid angle of positron detection was $\sim$20% of 4π. The third spectrometer used a principle[33] of focusing charged particles by means of a toroidal $1/r$ magnetic field for analysis by means of the momentum of the positrons or, for an opposite field, electrons in a large solid angle. The toroidal magnetic field with axis coinciding with the ion-beam axis is produced by means of 60 coils of a special shape placed at equal angles around the spectrometer axis. The focal region, which has approximately the shape of a cylinder, is covered with a position-sensitive detector. The solid angle of positron detection is $\sim$25% of 4π.

The main bulk of the experimental data on positron production in heavy-ion collisions at energies below the Coulomb barrier was obtained by means of the EPOS and ORANGE spectrometers. During the investigations, both spectrometers were improved—the energy and angle resolutions were raised, and the background was decreased. To investigate positron–electron coincidences, the spectrometers EPOS[37–39] and ORANGE[40] were subsequently modified by being joined to existing almost mirror-symmetric devices adjusted to electron spectrometry. For the investigation of positron production, the triple differential cross sections $d^3\sigma/(dE_e{+}d\Omega_e{+}d\Omega_p)$ were measured, and for the investigation of positron–electron collisions the fivefold differential cross sections $d^5\sigma/(dE_{e+}d\Omega_{e+}dE_{e-}d\Omega_{e-}d\Omega_p)$; here, E_{e+}, Ω_{e+}, E_{e-}, Ω_{e-} are the energies and solid angles of the positrons and electrons, and Ω_p is the ion solid angle. Both heavy ions were detected in coincidences with positrons and electrons by means of position-sensitive plane-parallel avalanche counters, which made it possible to determine the polar and azimuthal angles of both ions with angle resolution 0.5–1°. The resolution time of the ion coincidences was 0.5 nsec, and that of the positron–electron coincidences was ~ 5 nsec. The energy resolution of the positron–electron spectrometers was improved during the investigations from a few tens of kilo-electron-volts to ~ 10 keV at energy ~ 350 keV.

Simultaneously with the measurement of the differential cross sections for production of positrons or positron–electron pairs, measurements were made of the spectrum of gamma rays in coincidences with heavy ions by means of several detectors with NaI(Tl) scintillators placed around the target. The main source of the detected gamma rays was nuclei excited after scattering, and these measurements were used to take into account the contribution of the nuclear positrons to the measured positron spectrum.

The intensity of the beam of very heavy ions was 1–2 nA. The projected beam monochromatization was ~ 0.01 MeV/nucleon, i.e., $\sim 2 \cdot 10^{-3}$ of the ion energy below the Coulomb barrier; the actual monochromaticity in the beam was maintained at the level ~ 0.02 MeV/nucleon. The target thicknesses were 300–600 μg/cm^2, for which the spread of the energy losses of the beam ions in the target reached ~ 0.1 MeV/nucleon. In the experiments, there were problems associated with destruction of the targets by the beam during the measurements, and these led to an increase of the target inhomogeneity; details can be found in Refs. 72–74.

Diagrams and detailed descriptions of the spectrometers will not be given here; they can be found in Refs. 12, 18, and 34–40. From the above list of possible processes of positron production in heavy-ion collisions it is obvious that the search for new mechanisms must be preceded by the systematic investigation of the general properties of production of nuclear and quasiatomic positrons for both the above- and below-critical regions of total charges of the colliding ions.

2.2. Nuclear positrons

Nuclear positrons arise in internal pair conversion in excited nuclei in transitions with energy greater than $2m_ec^2$. There is no experimental way of separating nuclear positrons from quasiatomic positrons, and therefore one must use an indirect method based on gamma-ray spectra measured simultaneously under the same kinematic conditions of the ion collisions as for the positrons.[41] From these measurements, one can hope to obtain the positron spectrum from the internal pair conversion by using the relation

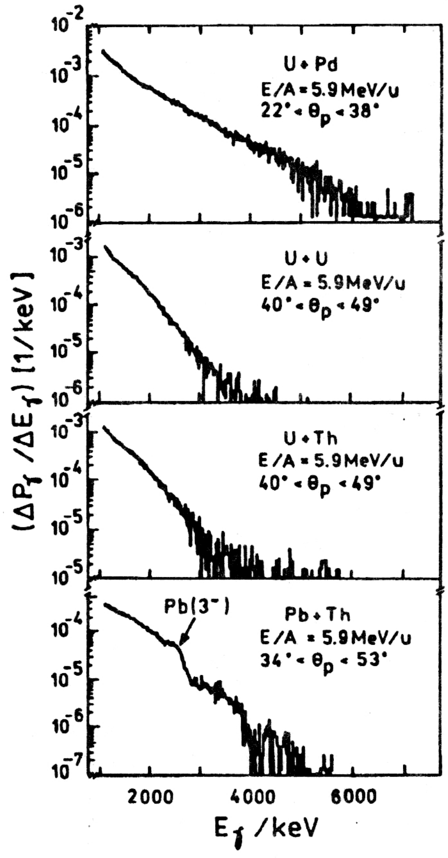

FIG. 2. Differential probability of production of γ rays with energy above 1 MeV in U+Pd, U+U, U+Th, and Pb+Th collisions at 5.9 MeV/nucleon. The γ rays were measured in coincidences with ions scattered in the indicated angles. The harder spectrum of γ rays for the U+Pd system is due to the fact that the energy of the ions in this case is above the Coulomb barrier, and the contribution from nuclear reactions is large.[44]

$$(dP_{e+}/dE_{e+})_{\rm nuc} = \int_{2m_ec^2}^{\infty} \left(\frac{dP_\gamma}{dE_\gamma}\right) \times [d\beta(E_{e+},Z,M\lambda)/dE_{e+}]\,dE_\gamma, \quad (3)$$

where $(dP_{e+}/dE_{e+})_{\rm nuc}$ is the spectrum of the nuclear positrons, dP_γ/dE_γ is the spectrum of the measured gamma rays, and $d\beta(E_{e+},Z,M\lambda)/dE_{e+}$ is the theoretical coefficient of internal pair conversion, which depends on the multipolarity $M\lambda$ of the intranuclear transition and the nuclear charge Z. The procedure (3) is quite reliable, since the process of internal pair conversion can be excellently described theoretically[42,43] if the electromagnetic properties of the nuclear transition are known. For nuclear transitions in the range of a few mega-electron-volts, the coefficient of internal pair conversion depends weakly on Z and is $\sim 10^{-4}$–10^{-3}, decreasing strongly for transitions with multipolarity greater than $E1$ or $M1$. For $E0$ transitions, the internal pair conversion competes with electron conversion and is $\sim 10^{-3}$ of it. Figure 2 shows typical gamma spectra measured in coincidence with heavy ions scattered into the corresponding angle intervals for four different

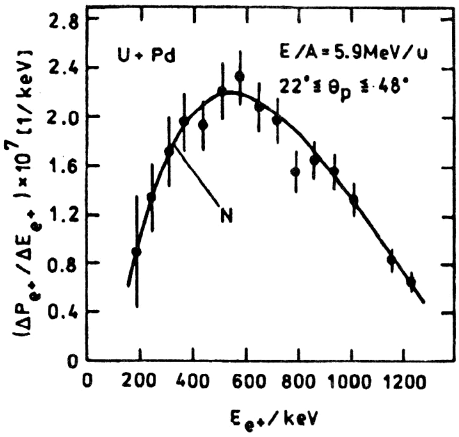

FIG. 3. Spectrum of positrons from U+Pd collisions with energy 5.9 MeV/nucleon measured in coincidences with heavy ions scattered in the indicated angles. The continuous curve N is the result of calculation of the spectrum of nuclear positrons in accordance with Eq. (3), using the measured γ spectrum under the assumption of 90% $E1$ and 10% $E2$ transitions.[44]

pairs of colliding ions. As can be seen from the figure, the spectra are continuous exponential distributions with maximum energy around 4–5 MeV.

Figure 3 shows the measured positron spectrum for the lightest of the investigated systems, U+Pd ($Z_u=138$). The experimental data are well described by the theortical curve calculated in accordance with (3), using the experimental spectrum (Fig. 2) and the theoretical conversion coefficient for $E1$ transitions[42] with a 5–10% admixture of $E2$ transitions. Fixing this ratio $E1/E2$ and assuming that it does not change appreciably in going to heavier nuclei, one can use the same procedure to determine the nuclear contribution to the positron spectrum by measuring the gamma spectrum. Figure 4 shows the yield of positrons in coincidences with scattered nuclei, normalized to the number of nuclear positrons calculated from the measured gamma spectra for collisions of uranium nuclei with various nuclei.[45,46] In the calculations, $E1$ transitions with a small (5–10%) $E2$ admixture were assumed. The main

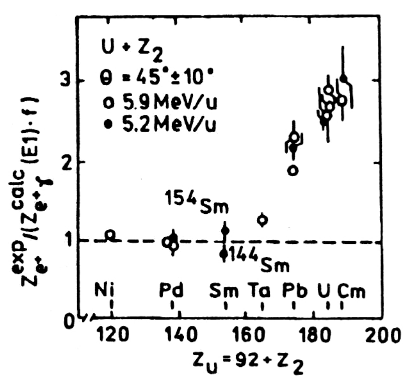

FIG. 4. Experimental positron yield, normalized to the positron yield from the process of internal pair conversion calculated from the measured spectrum in collisions of uranium ions with energy 5.9 MeV/nucleon and 5.2 MeV/nucleon with different nuclei.[45,46]

conclusion from this illustration is that the yield of positrons, normalized to the calculated nuclear contribution, is close to unity for total nuclear charge $Z_u=Z_1+Z_2<160$, i.e., all positrons have nuclear origin irrespective of Z_u, and there is a sharp increase when $Z_u>160$, indicating the appearance of a new source of positrons—the quasiatom mechanism. The yield of positrons can be extrapolated[46] into the region of large Z in accordance with the expression

$$N_{e^+}=N_{e^+}(\text{nuc.})+N_{e^+}(\text{quasiatom.})=C_{e^+}N_\gamma+P_{e^+}N_p. \tag{4}$$

Here, C_{e^+} is the effective coefficient of internal pair conversion for the colliding nuclei, which, as was shown, is relatively independent of Z_u and the details of the gamma-ray emission process, N_γ is the intensity of the gamma rays with $E_\gamma>2m_ec^2$, P_{e^+} is the probability of quasiatomic positron production, and N_p is the number of scattered nuclei. This procedure makes it possible to separate the contributions of the nuclear and quasiatomic positrons to the measured spectrum. It should be mentioned that a different relationship between the contributions of the $E1$ and $E2$ transitions was assumed to describe the spectrum of nuclear positrons[47] for systems with $Z_u<160$, namely, there was assumed to be a pure $E1$ transition for $E_{e^+}>800$ keV and a pure $E2$ transition for $E_{e^+}<600$ keV with a smooth transition between them. In this case too, good fitting of the experimental positron spectrum to the spectrum calculated in accordance with Eq. (3) was achieved.

2.3. Quasiatomic positrons

Already in the first year of operation of the UNILAC accelerator[45,46] experimental investigations revealed a new strong source of positron production associated with quasiatomic processes in the rapidly varying Coulomb field of the colliding nuclei. Quasiatomic positrons begin to make an appreciable contribution when $Z_u>160$ and with increasing Z_u form the main component in the total positron yield. Figure 5 shows the positron spectra measured by the ORANGE group for $^{208}Pb+^{232}Th$ scattering with energy 5.9 MeV/nucleon at the indicated angles. This system forms a total charge $Z_u=172$, in the field of which the $1S_{1/2}$ level does not yet reach the negative continuum. The shape of this broad continuous distribution must basically reflect the frequency distribution of the time picture of the ion collision.

The production of quasiatomic positrons obviously depends on the extent to which the negative continuum is perturbed by the total charge that is formed, and for given energy of the incident ions and given total charge of the nuclei it depends on the collision kinematics. It is interesting to investigate this dependence. We introduce $q_{\min}$, the minimum momentum transfer on scattering needed for the production of a positron–electron pair with total energy E:

$$q_{\min}=E/\hbar v, \tag{5}$$

where v is the initial velocity of the incident ion.

The kinematic dependence of the positron production probability can be simply described by means of the dependence[48–50]

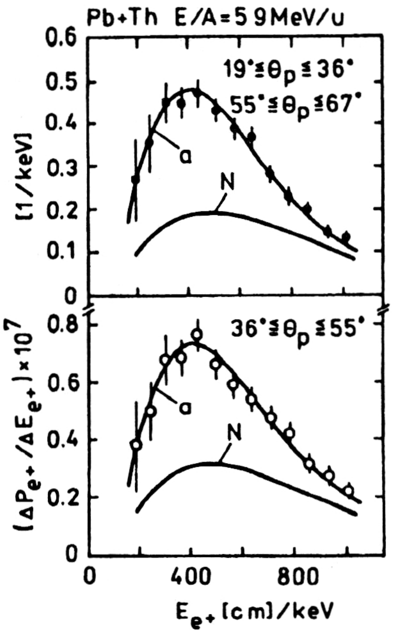

FIG. 5. Positron spectra from Pb+Th collisions at energy 5.9 MeV/nucleon, measured in coincidences with heavy ions scattered in the indicated angles. The background of nuclear positrons (N has been subtracted). Curves a give the calculation of the spectrum of induced (quasiatomic) positrons.[34]

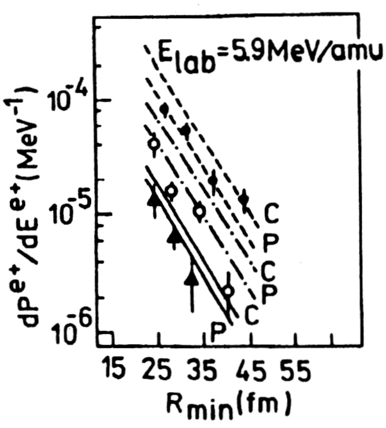

FIG. 6. Probability for the production of quasiatomic positrons with energy in the interval 424–530 keV as a function of $R_{\min}$, the distance of closest approach of the ions in U+U (points), U+Pb (open circles), and Pb+Pb (triangles) scattering.[45,46] The straight lines are the calculation of the yield of quasiatomic positrons[57] for the corresponding systems, made with different degrees of completeness.

$$P_{e^+}(b)=P_0 \exp(-mq_{\min}b), \tag{6}$$

where b is the collision impact parameter.

Such a treatment was used earlier in Ref. 49 to describe Coulomb ionization in collisions with light ions and subsequently was extended to heavy-ion collisions and the description of the production of vacancies in inner shells and pair production.

The expression (6) must be valid for $b>q_{\min}^{-1}$ with a constant $m\cong 2$ for $Z>137$ (Refs. 49 and 51), a value that differs appreciably from the experimental value $m=2.73\pm 0.20$.[44]

For close collisions, i.e., small impact parameters, a better approximation is achieved[51,52,16] by expressing the probability of production of induced positrons in terms of $R_{\min}$, the distance of closest approach of the nuclei during scattering:

$$R_{\min}=a+\sqrt{a^2+b^2}=a\left(1+1/\sin\frac{\theta}{2}\right),$$

$$2a=\frac{Z_1Z_2e^2}{\mu(E_1/A_1)}, \quad b(\theta)=a\cot\frac{\theta}{2}, \tag{7}$$

where θ is the c.m.s. scattering angle, $\mu=A_1A_2/(A_1+A_2)$ is the reduced mass, E_1 is the energy of the incident ion, and $2a$ is the minimum distance in a head-on collision. The corresponding scaling has the form

$$P_{e^+}\sim\exp(-\lambda R_{\min}), \tag{8}$$

where the parameter λ for each total charge Z_u is found as an adjustable parameter.

Generally speaking, the experimental data agree fairly well with this simple picture[44–46] (see Fig. 6). The dynamic nature of the production of the induced positrons can be parametrized[53] by using a critical time τ, defined as the time interval between the two extrema of $\dot{R}(t)/R(t)$, the logarithmic derivative, with respect to the time, of the distance $R(t)$ (Fig. 1). This time interval determines the amplitude of the dynamic process. The corresponding scaling law has the form

$$P_{e^+}\sim\exp(-2\tau/c\lambda_e), \tag{9}$$

where τ can be expressed in terms of measured quantities:

$$2\tau=(2a/v)(\varepsilon+1.16+0.45/\varepsilon), \tag{10}$$

in which $v=(2E_1/A_1)^{1/2}$ is the velocity of the incident ion, ε is the eccentricity, and λ_e is the electron Compton wavelength.

From the expression (6) it is possible to obtain the dependence of the probability of dynamic production of positrons on the energy of the incident ion. For fixed nuclear charges and scattering angle, it follows from (7) that $b\sim E_1^{-1}$, $R_{\min}\sim E_1^{-1}$, and from that (5) $q_{\min}\sim E_1^{-1/2}$. Therefore, we may expect that

$$P_{e^+}(E_1)\sim\exp(-sE_1^{-3/2}), \tag{11}$$

i.e., growth of the production of dynamic positrons with energy of the incident ion. The experiments with the ORANGE spectrometer confirm this dependence (Fig. 7), but the result of the calculations[28] gives a probability systematically higher (by 30%) (see below). The listed models, while giving a good description of the probability of production of induced positrons as a function of the collision parameters, nevertheless do not predict the absolute magnitude of the probability, and they also deviate from the experimental data in closer collisions ($b\ll 10$ fm).[54,15] For a more rigorous quantitative description, it would be necessary to consider a two-center problem for a time-dependent

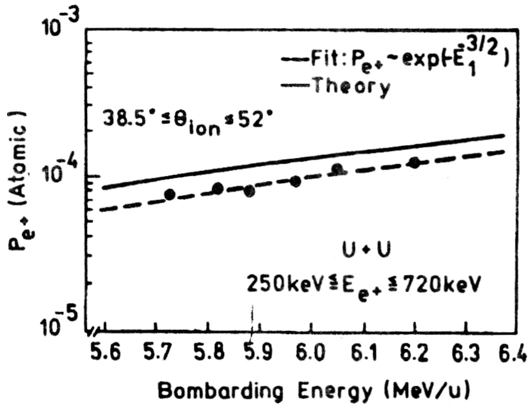

FIG. 7. Probability of production of quasiatomic positrons as a function of the energy of the colliding U+U ions. The positron spectra are summed in the indicated energy interval, and the range of scattering angles of the ions is also given. The continuous curve is the result of the theoretical calculation of Ref. 28, and the broken curve was obtained by fitting in accordance with Eq. (11) of Ref. 44.

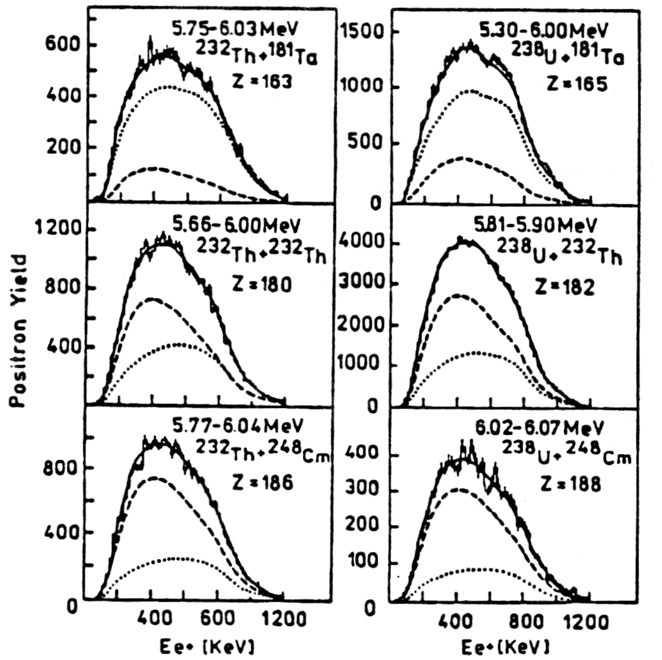

FIG. 8. Positron spectra measured for different pairs of colliding ions. The curves correspond to calculated positron spectra: the broken curves for quasiatomic positrons, the dotted curves for nuclear positrons, and the continuous curves for the sum of the quasiatomic and nuclear positrons.[39]

Dirac equation (Refs. 57, 22, 12, and 28) in which a dependence of the electromagnetic potentials on the time arises from the motion of the colliding nuclei. Because of the relativistic contraction of the electron wave functions in the strong quasiatomic Coulomb potential, there is localization of the induced transitions of the electrons from the negative continuum at the turning point of the scattering trajectory. This simplifies the problem, making it possible to use the monopole approximation, i.e., to make a restriction in the monopole expansion of the two-center potential to the spherically symmetric part. Solutions of the stationary two-center problem for the Dirac equation have been studied for bound states[58] and for continuum states.[59] It was found that the corrections to the monopole approximation are small for $S_{1/2}$ and $P_{1/2}$ states. In Fig. 8 we compare the theoretically calculated spectra with the experimental ones measured by the EPOS group. It can be seen that there is good agreement of the experimental data and the calculations as regards both the shape of the spectrum and the amplitude.

The dependence of the yield of dynamic positrons on the total charge of the ions is illustrated in Fig. 9.[44] Analysis of the experimental points gives the dependence

$$d\sigma/d\Omega \sim Z^{19.5\pm1}. \qquad (12)$$

This result is consistent with the earlier measurements[46] of the ORANGE group and with the recent results of the group working with the TORI spectrometer.[62] A Z dependence of this type, which was obtained experimentally in a fairly wide range of charges of the colliding nuclei, was predicted by the theoretical calculations[61,57] that we have mentioned ($\sim Z^{20}$).

A somewhat lower yield of dynamic positrons in experiments as compared with calculations was already noted in the early measurements of Ref. 34. Subsequent experiments with more accurate allowance for the contribution of the nuclear positrons and production resulting from external pair conversion in the target (Refs. 35, 55, 56, and 20) confirmed the discrepancy. As was shown by a careful analysis[44] recently made by the ORANGE group, it can be described by a single coefficient $\bar{f}=0.76\pm0.04$ for several combinations of charges of the colliding ions from $Z_u=164$ (Pb+Pb) to $Z_u=184$ (U+U). A similar general discrepancy between the theory of dynamic positron production and the experiments was found by the TORI group[63] in an investigation of positron production in elastic U+U collisions at energies near and slightly above the

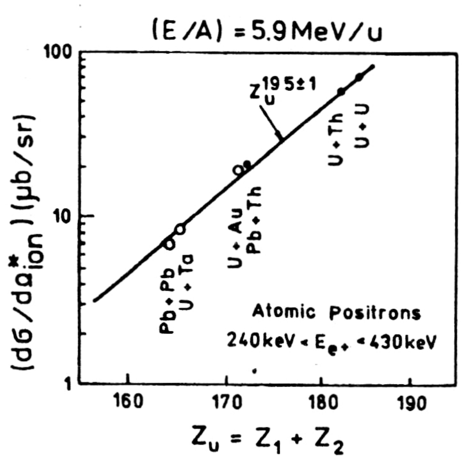

FIG. 9. Differential cross section for production of quasiatomic positrons as a function of the total charge of the colliding nuclei. The positron spectra are summed in the indicated energy interval. The positrons were detected in coincidences with ions scattered in the interval of angles 76–104°. The points and open circles correspond to the results obtained for different modifications of the ORANGE spectrometer.[44]

Coulomb barrier. However, as was noted, the investigations of the EPOS group[39] (Fig. 8) indicate good (to within 20%) agreement of the experimental data and the calculations. An even larger discrepancy between experiment and theory was found by the TORI group in the measurements of Ref. 62, in which for Pb+Pb collisions at energy 8.6 MeV/nucleon it was found, in contrast to the data on collisions below the Coulomb barrier, that the experimental positron yield was about twice the theoretically calculated yield. The discrepancy could be due to a number of approximations and uncertainties in the solution of this complicated problem (besides the ones already mentioned), namely, uncertainty about the electron structure of the colliding ions (it was assumed in the calculations that the Fermi level is $n=3$, i.e., all states above $3S$ and $4P$ in the unified quasiatom are ionized), the extent to which electron–electron interactions are taken into account,[15] neglect of possible strong asymmetry in the electron configuration of the colliding atoms and a broad initial distribution of the charge states during the collision time, etc. However, considered in totality, the calculations (Refs. 60, 57, 55, 52, 28, 29, 22, and 21) of the quasiatom positron-production processes describe well the general shape of the spectra of the dynamic positrons and the dependence of the positron yield on the collision parameters.

2.4. Positron peaks

As we have already noted, the main motive was to look for evidence of a resonance spontaneous transition between the negative continuum and the most bound quasiatomic states when $Z_u > Z_c$. However, in elastic Rutherford scattering, owing to the short collision time ($\sim 10^{-21}$ sec), the expected peak of spontaneously produced positrons must be smeared in accordance with the Heisenberg uncertainty principle to a width of several hundred kilo-electron-volts, so that the process of spontaneous production would become indistinguishable from the process of dynamic positron production. In above-critical collisions with a time delay due to nuclear contact, one could expect enhancement of the spontaneous process, leading to the appearance of peaks in the positron spectrum.

Narrow positron lines with energy in the range 200–400 keV were indeed found on the background of the broad spectrum of dynamic positrons in collisions of heavy ions with above-critical total charge in experiments of the EPOS group[31] (Fig. 10) and the ORANGE group[34] (Fig. 11).[1)] Analysis of the gamma spectra measured in collisions with the scattered ions showed that the observed peaks cannot be associated with pair conversion for transitions with multipolarity $E1$ and above in excited nuclei after scattering. To eliminate $E0$ transitions, an investigation was made of the spectrum of conversion electrons in a search for a strong line that would indicate the presence of a distinguished $E0$ transition that, after internal pair conversion, could give the observed peak in the positron spectrum. Another possible cause was the order-of-magnitude weaker process of internal pair conversion in which an emitted electron is captured by a vacancy in the electron shell, so that the emitted positron is monochromatic. On

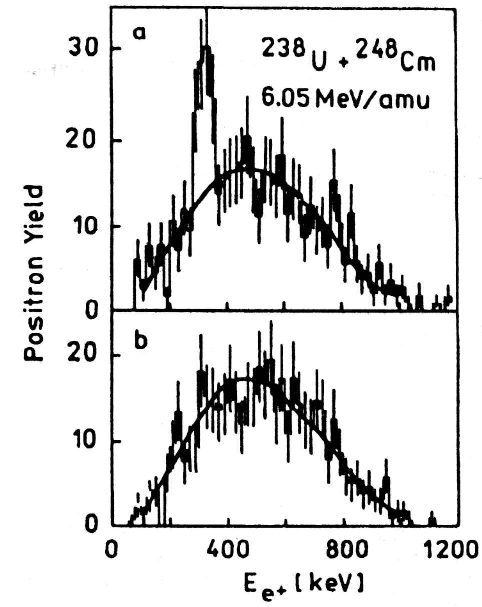

FIG. 10. Spectra of positrons from U+Cm at energy 6.05 MeV/nucleon, measured in coincidences with ions scattered in angles $100° \leqslant \theta_{cms} \leqslant 130°$ (a) and in angles $50° \leqslant \theta_{cms} \leqslant 80°$ (b). The continuous curves are the calculated spectra of quasiatomic positrons under the assumption of elastic scattering, added to nuclear positrons calculated from the measured γ spectrum.[31]

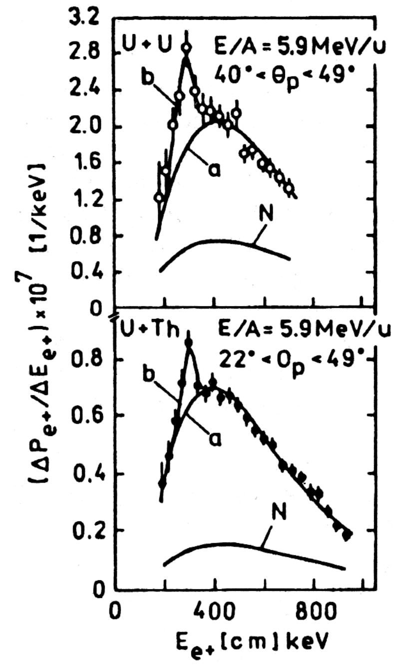

FIG. 11. Spectra of positrons from U+U and U+Th collisions with energy 5.9 MeV/nucleon, measured in coincidence with heavy ions scattered in the indicated angles. The background of nuclear positrons (N) is subtracted. The curves a are the calculated spectra of quasiatomic positrons with coefficient 0.80. The peak b is calculated for a δ function in accordance with the spectrometer resolution.[34,35]

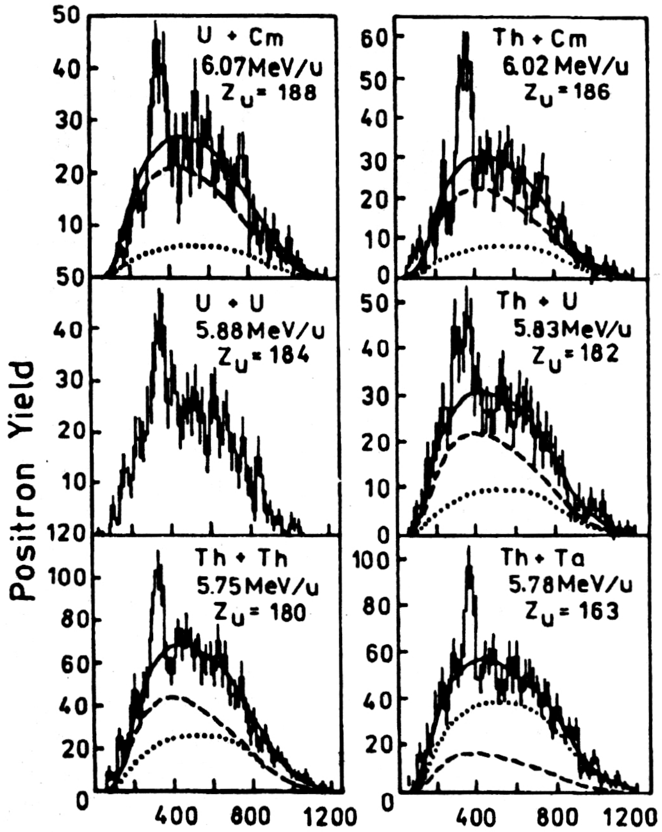

FIG. 12. Isolated positron peaks observed by the EPOS group in collisions of very heavy ions at the indicated energies in the range of total charges $163 \leqslant Z_u \leqslant 188$ of the nuclei of the colliding ions. The broken curves are the calculated spectra of quasiatomic positrons, the dotted curves are the calculated spectra of nuclear positrons, and the continuous curves are the sum of the two calculated contributions.[65]

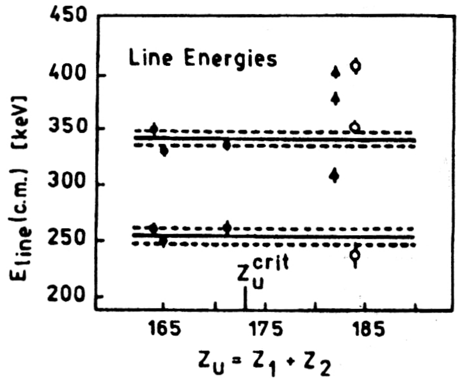

FIG. 13. Energies of positron lines as functions of the total charge Z_u of the colliding nuclei. The data from Ref. 55 are represented by open circles, from Ref. 36 by points, and from Ref. 67 by triangles. One can assume that there are two common lines for the four systems with mean energies 255 ± 7 keV and 342 ± 6 keV. The mean energies are shown by the continuous lines; the broken lines are the statistical deviations.[66]

the other hand, analysis of the width of the positron current precludes pair conversion as the reason for its appearance. The observed width of the peak (~ 80 keV in both experiments), which is almost entirely due to instrumental resolution, is nevertheless significantly less than would be expected in the case of a process of pair conversion in a nucleus moving in accordance with the known kinematics; this width is 150 keV.

These first results on the spectra of the positrons from U+Th ($Z_u = 182$), U+U ($Z_u = 184$), and U+Cm ($Z_u = 188$) collisions already indicated that the energies of the positron peaks appeared to be independent of Z_u. This immediately indicated a contradiction with the possible mechanism of spontaneous positron production in the above-critical Coulomb field,[21] for which a strong dependence $\sim Z_u^{20}$ of the positron energy on the total charge was expected. To verify the Z independence, the EPOS group[65] made a systematic investigation of the positron spectra for different combinations of colliding nuclei. Figure 12 shows the positron spectra for five Z_u (180–188). All the spectra exhibit clearly defined peaks concentrated around $E = 320$–340 keV. In this connection, it is natural to ask whether a similar positron line is emitted for $Z_u < Z_c = 172$. An early experiment with poor statistics[34] (Fig. 5) did not reveal in the spectrum any features for the Pb+Th ($Z_u = 172$) system at the intensity level characteristic of the systems with greater charge. Measurements with better resolution and statistics were made by the ORANGE group[36] for the systems Pb+Pb ($Z_u = 164$), U+Ta ($Z_u = 165$), and U+Au ($Z_u = 171$). As in the case of higher charges, the positron spectra were found to contain a peak at energy 315 ± 4 keV with width 33 keV and an additional peak at energy 240 ± 3 keV with width 28 keV. The same ORANGE group, repeating the investigations with greater accuracy and better resolution for above-critical nuclear systems,[55] found for them too the additional peak.

In Fig. 13,[66] we give a compilation of the results of the EPOS[67] and ORANGE[55,36] groups that illustrates the positions of the positron peaks for different combinations of colliding ions. One can see the two groups of positron peaks with energies near 255 and 340 keV in all the investigated systems. However, the overall picture does not create an impression of complete consistency. The third peak at 395 keV is not manifested for the below-critical systems, and the peaks at 310 and 360 keV found by the EPOS group for U+Th do not fit the general pattern.

Thus, the weak dependence (or its complete absence) of the energies of the positron lines on Z_u casts doubt on the mechanism of spontaneous positron production in above-critical systems. Analysis of the shape and width of the peaks with allowance for the instrumental resolution leads to a width of the positron lines less than 40 keV. Therefore, during the collision there would have to be the formation of a relatively long-lived quasiatom (quasimolecule) with lifetime $\sim 10^{-19}$–10^{-20} sec. Such a possibility was discussed both purely schematically, by the formal introduction of a certain delay time,[23,21,28] and also more comprehensively, with an attempt to justify a model of the formation of such a molecule (Refs. 24, 25, 27, and 29). The absence of a Z_u dependence of the energies of the positron peaks makes the situation very confusing, especially if it is borne in mind that spontaneous positron emission should not occur for $Z_u < Z_c$.

Figure 14 shows the cross section for production of

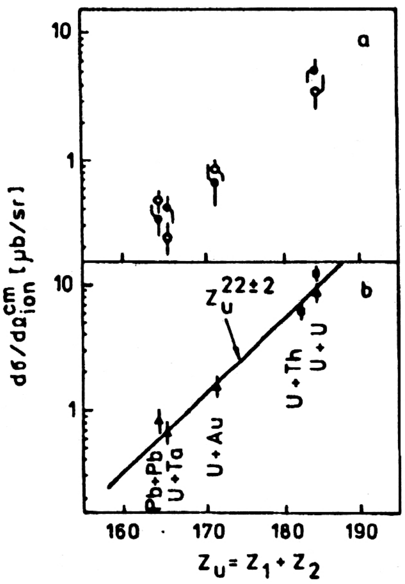

FIG. 14. Cross section for production of two positron lines (a) at $E_{e^+} \approx 258$ keV (points) and at $E_{e^+} \approx 340$ keV (open circles) as a function of the total charge of the nuclei. Fitting in accordance with the law $\sigma \sim Z^n$ gives $n=24\pm2$ and $n=20\pm2$, respectively, for each of the peaks. Mean cross section (b) for the two lines in (a) (triangles) together with the cross section for the line $E_{e^+} \approx 280$ keV (squares) obtained for U+Th and U+U scattering.[35] Analysis of all the data gives $n=22\pm2$ (continuous line).[36]

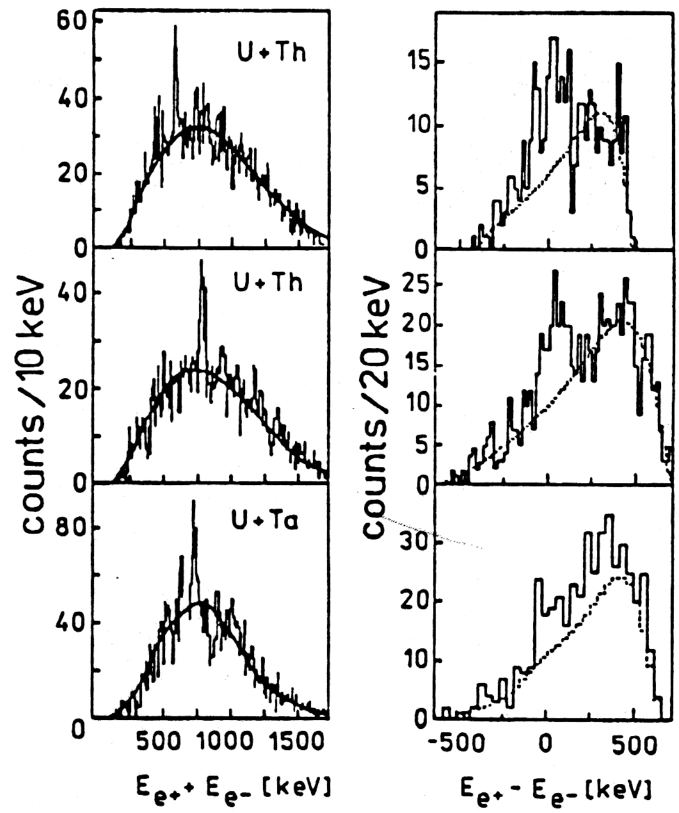

FIG. 15. Spectra of sums and differences of positron and electron energies for the indicated pairs of colliding ions for U+Th at 5.83–5.9 MeV/nucleon and for U+Ta at 5.93–6.13 MeV/nucleon. The continuous curve in the total spectrum and the broken curve in the difference spectrum are the calculation of the total contribution from internal pair conversion based on the measured γ spectrum, quasiatomic positrons, and internal-conversion electrons.[38,71]

positron lines, integrated over the energy, for the two energies 255 and 340 keV (Ref. 36) as a function of Z_u. Analysis of the dependence gives $d\sigma_{e^+}/d\Omega_{\rm ion} \sim Z_u^{22\pm2}$ for both lines. This is close to the dependence $\sigma \sim Z_u^{20}$ for the cross section for positron production in the induced process.

Thus, the width and constancy of the positron lines indicate that they are independent of the details of the nuclear and quasiatomic processes during the collisions of the nuclei and indicate a common source. An obvious possibility is that the source is a two-particle decay of a previously unknown light particle into a positron–electron pair.[65,68–70] The lifetime of such a particle could already be estimated[65,36] from analysis of the positron peaks, and was found to be $5\cdot10^{-20} < \tau < 10^{-10}$ sec. The lower limit follows from consideration of the width of the positron peaks, and the upper one from the circumstance that for motion of the decaying system (at rest in the center-of-mass system) in the laboratory system with the c.m.s. velocity ($\beta \cong 0.05$) for a time greater than 10^{-10} sec the decay products would escape from the region of effective detection. Computer Monte Carlo modeling of the decay process of such a particle under the experimental conditions of both facilities and comparison of the calculations with the measured positron spectra led to the conclusion of a relatively low velocity of the positron source in the center-of-mass system ($\beta < 0.03$). Thus, a remarkable picture was obtained—decay into a positron–electron pair of an object at rest in the system of the colliding ions during a time two orders of magnitude greater than the collision time, when the ions are already separated by $\gtrsim 10^2$ fm.

2.5. Positron–electron coincidences

The first measurements of spectra of positron–electron coincidences were made by the EPOS group.[67] The facility detected positrons and electrons emitted in opposite hemispheres, and there was simultaneous measurement of the time of flight of both particles along helical trajectories in the solenoidal magnetic field from the target to the detectors. Figure 15 (Refs. 38 and 71) shows the spectra of the total energy of the coincident positrons and electrons and the corresponding spectra of the differences of their energies for different flight times of the positrons and electrons, corresponding to different angles of emission of the particles relative to the beam, for U+Th and U+Ta collisions. It is noted in Ref. 71 that the observed peaks in the spectrum of the total energy for both investigated ion pairs belong to a common group of three lines: ~610, ~750, and ~810 keV. It was noted that the line intensities depend on the choice of the beam energy of the incident ions. There is no doubt about the connection between the positron lines observed in the previous investigations and the positron–electron events. The cross section for production of the positron–electron peaks is 5–20 μb/sr. The energies of the positron–electron peaks in the spectrum of the total energy correspond well to twice the energy of the isolated positron lines. The peaks in the spectra of the total energy are much narrower than the individual lines of the positrons and electrons and the width of the spectrum of the energy differences of the two particles. Narrow total peaks and broad difference peaks agree with the assumption of decay, into a positron and an electron, of a free particle at rest in the center-of-mass system. Indeed, the appreciable Doppler broadening associated with the center-of-mass motion must disappear in the first order for the total spectrum and be doubled in the difference spectrum. The energy of the total peak determines in this case the invariant mass of the decaying particle. Analysis of the data in Ref. 67 leads to the conclusion of a $\sim 3\%$ contribution to the total positron spectrum from the decay of particles with mass 1.6–1.8 MeV/c^2. In an investigation of U+U and Pb+Pb collisions, the ORANGE group[40] found in the total spectrum of the coincidence positrons and electrons a peak of energy 815 ± 5 keV at separation angle $180° \pm 18°$. This result confirms the conclusions of the EPOS group, but the cross section was found to be an order of magnitude less than in Ref. 65. There is also evidence for the existence of a peak near ~ 600 keV for both systems, but there are no indications of a line at 750 keV. However, such an unambiguous conclusion about decay of a universal particle is undermined by some experimental observations. As was shown in Ref. 71, the use of more rigorous criteria in the analysis of the two-dimensional (E_{e^+}, E_{e^-}) and time (t_{e^+}, t_{e^-}) distributions leads to the conclusion that the decay of a free particle into a positron and an electron emitted in opposite directions can be associated, at the least, only with one of the observed peaks, the one at 819 keV, the other two exhibiting some features that cast doubt on the hypothesis. In this connection, it must be noted that reliable measurement of the angle correlations is made difficult by the strong nuclear scattering of low-energy leptons (~ 300 keV) in comparatively thick (~ 300 $\mu\mathrm{g/cm}^2$) targets. The line with energy ~ 610 keV exhibits preferential emission of both leptons in the forward hemisphere relative to the direction of the ion beam. The experiments show that whereas for U+Th collisions the energy differences of the positrons and electrons are distributed on the average around zero, the same distributions for U+Ta collisions are much broader and shifted to higher energies of the positrons and lower energies of the electrons. Such a shift is qualitatively correlated in the given case with interaction in the final state with the Coulomb field of the ions.

The question of the dependence of the cross sections for production of positron lines and positron–electron pairs on the energy of the incident ions is unclear. Indications of the existence of such a dependence were already found in the investigation of the isolated positron lines.[31,65] Subsequently, it was more clearly noted[71] that the cross section for production of positron–electron pairs in U+Ta collisions changes appreciably with the beam energy (Fig. 16) if, for example, one considers the positron–electron peak at 748 keV. It is necessary to mention in this connection the rapid destruction of the targets by the beam during the experiments,[72–74] which leads to an increase of the target inhomogeneity and, therefore, to a spread of the energy of

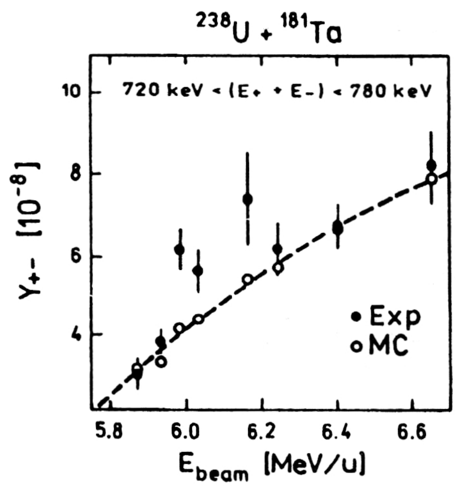

FIG. 16. Yield of positron–electron line with total energy 748 keV (points) in U+Ta collisions as a function of the ion energy. The open circles are the result of a Monte Carlo calculation of the background.[71]

the ions in the target. Therefore, there is no prescription for exactly relating the energy dependences of the cross sections for different combinations of colliding nuclei. In contrast to the earlier experimental results of the TORI group, which at energy above the Coulomb barrier did not observe features in the positron spectrum,[76] the more recent data of the ORANGE group have demonstrated for the first time[77] a clear peak in the total spectrum of positron–electron coincidences at 635 keV (confidence level 6.5σ) in U+Ta collisions. The measurements were made at 6.3 MeV/nucleon under conditions of deep inelastic collisions, this being verified by the fact of fission of uranium nuclei, which served as a trigger. Compared with the elastic scattering of the same ions, when this peak was also observed, the cross section in the given case was larger by a factor of 20. Analysis of the angle correlations of the positrons and electrons for this peak also strongly contradict the situation with 180° separation.

It is interesting to investigate the dependence of the cross section for the production of positron peaks and positron–electron lines on the momentum transfer for scattering of heavy ions. As for quasiatomic positrons, one can parametrize the dependence in various ways. If it is done in a form analogous to (8),

$$P_{e^+e^-} \sim \exp(-\alpha R_{\min}), \qquad (13)$$

then for U+Ta scattering for the three lines of the total energy of the positron–electron pair the following results are obtained for the slope parameter α: for the line at 625 keV, $\alpha = 0.42 \pm 0.08\ \mathrm{fm}^{-1}$; for the line at 748 keV, $\alpha = 0.35 \pm 0.07\ \mathrm{fm}^{-1}$; and for the line at 805 keV, $\alpha = 0.21 \pm 0.07\ \mathrm{fm}^{-1}$ (Ref. 39). Such exponential behavior of the probability of production of narrow positron–electron lines can be compared with the corresponding dependence for the production of quasiatomic positrons, $\alpha = 0.13\ \mathrm{fm}^{-1}$ (Refs. 48 and 39), or with the similar dependence of the probability of transfer during scattering of one neutron: $\alpha = 1.08\ \mathrm{fm}^{-1}$ for U+Au.[75]

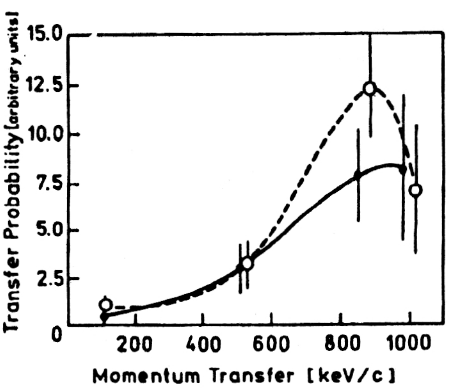

FIG. 17. Distribution of probability for momentum transfer to a third body obtained from angular distributions of the positron in the peaks at total energy 635 keV (points and continuous curve) in U+Ta scattering and at total energy 555 keV (open circles and broken curve) in U+U scattering.[77]

Narrow resonances in the total positron–electron spectrum were observed by the ORANGE group[77] for the U+U and U+Ta systems—in the first case with peak energy 555 keV, in the second with 635 keV, and in both cases with angle correlation of the leptons contradicting decay of a free particle. The small width of the resonances provides a strong argument for a process in which a heavy third particle participates in the decay of a particle; it could be a target nucleus that takes up recoil momentum. However, the leptons were observed in coincidences with both scattered ions, so that one can assume a two-step process in which a particle produced during the time of collision of the ions then decays into leptons near a third nucleus. Figure 17 shows the dependence of the probability of the positron–electron peaks on the momentum transfer to the leptons. The mean momentum transfer is 800–1000 keV/sec, which corresponds to a characteristic range of order of the electron Compton wavelength and may be associated with the scale of the decaying particle.

Concluding the review of the data on the production of positrons and positron–electron pairs in collisions of very heavy ions at energies below the Coulomb barrier, we can state that the existence of the effect itself is not in doubt. The regularity with which narrow lines are manifested in independent experiments, in conjunction with their statistical significance [$(5–6)\sigma$], makes their interpretation as a statistical fluctuation impossible. However, as has been noted, the hypothesis of two-particle decay into a positron and an electron of a free elementary or composite particle at rest in the center-of-mass system of the colliding ions cannot be recognized as valid for the description of all the considered phenomena. It appears that more complicated scenarios, including additional assumptions about the velocities of the positron sources, their lifetime, and the presence of an interaction in the final state, which could explain some of the experimental data, can hardly give a self-consistent general picture. As will be seen below, this makes it necessary to put forward very exotic hypotheses to explain the phenomena.

3. SEARCH FOR LOW-ENERGY POSITRON–ELECTRON RESONANCES

3.1. Search for neutral particles with mass 1.6–1.8 MeV/c^2 that decay into a positron–electron pair in nuclear transitions

Light neutral particles with mass $m > 2m_e$ were also sought in experiments before the discovery of the narrow positron and positron–electron lines in heavy-ion collisions. There were various initial assumptions behind such searches.

First, in the early stage of the creation and development of a unified model of the weak and electromagnetic interactions the mass of the fundamental scalar of the theory, the Higgs boson, was not *a priori* limited.[78] Therefore, it did not seem unjustified to look for light scalars. At the same time, there were found to be discrepancies (subsequently shown to be incorrect) between the theoretical and experimental energies of the x-ray lines in heavy μ-mesic atoms. These discrepancies could be attributed to the possible existence of a light scalar particle that interacted weakly with muons and nucleons; the inclusion, in the muon–nucleon interaction, of exchange of this scalar eliminated the discrepancy between the theory and experiment.[79,80] An experiment to look for emission of light penetrating particles in $0^+ \to 0^+$ transitions in ^{16}O (E=6.05 MeV) and ^4He (E=20.2 MeV) through their decay into a positron–electron pair ruled out their existence for lifetimes longer than 10^{-10} sec.

Second, the axion problem, which appeared at the end of the seventies (and is still not yet solved), gave rise to many unsuccessful experimental searches for it. In the original "standard" version, the axion,[82] a neutral ($J^\pi = 0^-$) particle with mass not predicted by the theory, was expected to be most probably light ($m_a < 2m_e$) and to decay into two photons. A review of laboratory searches for the standard axion up to 1987 can be found, for example, in Ref. 83. Interest in the search for axions, but with the accent on an axion mass $m_a > 2m_e$ and with dominant decay $a \to e^+e^-$, became more acute after the observation of the narrow positron lines at the GSI. None of the previous experiments to look for the "standard" axion (in nuclear transitions) with $m_a > 2m_e$ and short lifetime $\tau < 10^{-10}$ sec was capable of detecting it. However, in decays of the heavy vector mesons, $J/\psi \to \gamma a$, $\Upsilon \to \gamma a$, such a particle must necessarily manifest itself.[83] Therefore, after the observation of the "Darmstadt peaks" the "standard" axion model was modified with respect to the constants of its interaction with heavy quarks,[84,85,88] and this made experiments on the decay of heavy vector mesons insensitive to the existence of such an axion. This attempt to "revive" the axion (viable axion model[85,199]) stimulated a new series of experiments to look for the process $A^* \to A + a$, $a \to e^+e^-$ with axion mass m_a=1.5–2.0 MeV/c^2 and short lifetime $\tau_a < 10^{-10}$ sec in $M1$ nuclear transitions. The idea behind these new experiments was to compare the total yield of positrons and positron–electron pairs and their angle correlations with a calculation in accordance with the theory of internal pair conversion.[42,43] An observed excess in the

TABLE I. Experimental data on the search for pseudoscalar particles emitted in nuclear transitions with subsequent decay $a \to e^+e^-$.

Nucleus	E, MeV	$(J^\pi, T)^* \to (J^\pi, T)$	m_a, MeV/c^2	τ_a, sec	Γ_a/Γ_γ	Upper limit on coupling constants for $m_a = 1.7$ MeV/c^2	Experiment
^{10}B	3,59	$(2^+, 0) \to (3^+, 0)$	1,7	$< 10^{-9}$	$7,2 \cdot 10^{-3}$	$g^{(0)} < 1,4 \cdot 10^{-2}$	[89]
^{14}N	9,17	$(2^+, 1) \to (1^+, 0)$	1,02—2,2	$< 10^{-11}$	$4 \cdot 10^{-4}$	$g^{(1)} < 1,4 \cdot 10^{-2}$	[90]
^{13}C	3,68	$(3/2^-, 1/2) \to (1/2^-, 1/2)$	1,7—2,0	$< 10^{-11}$	$7 \cdot 10^{-5}$	$\lvert g^{(0)} + \frac{g^{(1)}}{\sqrt{3}} \rvert < 3 \cdot 10^{-3}$	[91]
^{6}Li	3,56	$(0^+, 1) \to (1^+, 0)$	1,7	$< 2 \cdot 10^{-11}$	$9 \cdot 10^{-5}$	$g^{(1)} < 1,2 \cdot 10^{-2}$	[87]
^{10}B	3,59	$(2^+, 0) \to (3^+, 0)$	1,7	$< 2 \cdot 10^{-11}$	$2,6 \cdot 10^{-3}$	$g^{(0)} < 5,1 \cdot 10^{-3}$	[87]
^{14}N	7,03	$(2^+, 0) \to (1^+, 0)$	1,7	$< 2 \cdot 10^{-11}$	$2,7 \cdot 10^{-3}$	$g^{(0)} < 4,1 \cdot 10^{-3}$	[87]
^{12}C	15,1	$(1^+, 1) \to (0^+, 0)$	1,02—2,5	10^{-13}—10^{-8}	$1,5 \cdot 10^{-4}$	$g^{(1)} < 1,25 \cdot 10^{-2}$	[92]
^{14}N	9,17	$(2^+, 1) \to (1^+, 0)$	1,8	$< 10^{-11}$	$4 \cdot 10^{-4}$	$g^{(1)} < 2 \cdot 10^{-2}$	[93]
^{8}Be	18,15	$(1^+, 0) \to (0^+, 0)$	1,8	$< 10^{-11}$	$4 \cdot 10^{-4}$	$g^{(0)} < 1,6 \cdot 10^{-2}$	[93]
^{8}Be	15,1	$(1^+, 0) \to (2^+, 0)$	1,8	$< 10^{-11}$	$4 \cdot 10^{-4}$	$g^{(0)} < 1,6 \cdot 10^{-2}$	[93]
^{8}Be	17,6	$(1^+, 1) \to (0^+, 0)$	1,8	$< 10^{-11}$	$1 \cdot 10^{-3}$	$g^{(1)} < 2 \cdot 10^{-2}$	[93]
^{8}Be	18,15	$(1^+, 0) \to (0^+, 0)$	4—15		$4 \cdot 10^{-4}$		[94]
^{2}H	2,23	$(0^+, 1) \to (1^+, 0)$	1,7	$< 10^{-10}$	$3,4 \cdot 10^{-5}$	$g^{(1)} < 10^{-2}$	[97]

Note: E is the transition energy, (J^π, T) are the quantum numbers of the levels, m^a is the mass of the particle, τ_a is the lifetime of the particle, and Γ_a/Γ_γ is the upper limit of the probability for emission of the particle in the given transition (90% confidence level).

total yield or a deviation in the angle correlation would be interpreted as the presence of an additional source of positron–electron pairs. The old experiments[86] aimed at investigating internal pair conversion in nuclear transitions of different multipolarities were reexamined from this point of view.[87]

The effective Lagrangian of the axion–nucleon interaction can be written in the form

$$\mathscr{L}_{aNN} = \frac{1}{2} i\bar{\psi}\gamma_5 (g^{(0)} + \hat{\tau}^3 g^{(1)}) \psi a, \tag{14}$$

where ψ and a are the nucleon and axion operators, respectively, and $g^{(0)}$ and $g^{(1)}$ are the isoscalar and isovector effective axion–nucleon coupling constants. A theoretical analysis[98] shows that the relative probability for emission of an axion in nuclear transitions has the form

$$\Gamma_a/\Gamma_\gamma = \frac{1}{2} \left(\frac{k_a}{k_\gamma}\right)^3 \left(\frac{g^{(1)(0)}}{e_\mu^{(1)(0)}}\right)^2. \tag{15}$$

Here, k_a and k_γ are the momenta of the axion and photon, $g^{(1)(0)}$ are the isovector or isoscalar axion–nucleon coupling constants, and $e_\mu^{(1)(0)}$ are the corresponding isospin magnetic moments of the nucleons.

Table I lists the main results of some searches for pseudoscalar particles. Generally, excited nuclear states were produced by bombarding appropriate targets with charged particles in electrostatic accelerators. An exception was the studies of Refs. 95–97, in which the emission of positron–electron pairs following capture of thermal reactor neutrons by protons was investigated.

In none of the experiments were there found to be deviations from the theory of internal pair conversion, and this established upper limits on the probability of emission of pseudoscalar particles in the corresponding nuclear transitions, and also on the pseudoscalar–nucleon coupling constants.

The given restrictions on the existence of the axion (0^-), in support of which there are strong arguments in the theory,[82,83,98] are also correct if the Darmstadt peaks are due to the decay of a massive "magnetic photon" (axial vector 1^+), since the selection rules in nuclear transitions are the same for them.

We now consider experimental studies devoted to the search for scalar particles (0^+), which could also be the reason for the observation of the positron–electron peaks at the GSI. Light scalars appear in supersymmetric extensions of the standard model.[176,177]

In nuclear transitions, scalars are best sought in $0^+ \to 0^+$ transitions, which mainly take place through internal pair conversion or electron conversion (in heavy nuclei). Both of these processes are of order α^2, whereas scalar emission can take place as a first-order process; this enhances the sensitivity of an experiment to look for scalars. The Lagrangian of the interaction of the scalars with the nucleons is

$$\mathscr{L}_{\varphi NN} = g_{\varphi NN} \bar{\psi}\psi\varphi, \tag{16}$$

where φ and ψ are the operators of the scalar and the nucleon, and $g_{\varphi NN}$ is the scalar–nucleon coupling constant.

TABLE II. Experimental data on the search for massive scalar particles emitted in $0^+ \to 0^+$ nuclear transitions.

Nucleus	E, MeV	m_φ, MeV/c^2	τ, sec	$\Gamma_\varphi/\Gamma_{e^+e^-}$	Experiment
^{16}O	6,05	1,03–5,84	$7\cdot10^{-10}<\tau<10^{-5}$	10^{-4}	[81,102]
^{4}He	20,2	1,03–18,2	$7\cdot10^{-10}<\tau<10^{-5}$	30	[81,102]
^{16}O	6,05	1,8	$<10^{-11}$	10^{-2}	[90]
^{16}O	6,05	1,25–3,2	$<10^{-11}$	$5\cdot10^{-3}$	[99]
^{16}O	6,05	1,5–3,5	$<10^{-11}$	$2\cdot10^{-4}$	[100]
^{4}He	20,2	3–14	$>10^{-9}$	0,05*	[101]

*For mass $m_\varphi \sim 3$ MeV/c^2

Note: E is the transition energy, m_φ is the mass of the particle, τ is the lifetime of the particle, and $\Gamma_\varphi/\Gamma_{e^+e^-}$ is the upper limit of the ratio of the widths for emission of a particle and e^+e^- is the upper limit of the ratio of the widths for emission of a particle and e^+e^- conversion in the given transition (90% confidence level) for mass $m \sim 1.7$ MeV/c^2.

The relative probability of emission of a scalar particle in a $0^+ \to 0^+$ transition was obtained in Ref. 80:

$$\Gamma_\varphi/\Gamma_{e^+e^-} = \frac{15\pi}{2} \frac{g^2_{\varphi NN}}{4\pi\alpha^2} \left(1 - \frac{m^2_\varphi}{E^2}\right)^{3/2}, \qquad (17)$$

where m_φ is the mass of the scalar, and E is the transition energy.

An experimental search for scalar particles in the interval of masses of the Darmstadt peaks was made in Refs. 81, 93, and 99–102. The $0^+ \to 0^+$ transitions are rather rare, and therefore only two such transitions were investigated: in ^{16}O (E=6.05 MeV) and in ^4He (E=20.2 MeV). In the early studies (Refs. 81, 101, and 102), the experiments made it possible to detect only a relatively long-lived ($\tau > 10^{-9}$ sec) scalar particle, since the point of the presumed production of the particle was separated from its decay into a positron–electron pair by a rather thick (> 10 cm) shield. In the later studies of Refs. 93, 99, and 100, made in connection with the results obtained at Darmstadt, the measurements were made with emphasis on the search for short-lived particles, and there was no shield separating the point of production from the decay region. However, because of the restricted volume of the decay region effective for detection, such experiments lost sensitivity for long lifetimes of the scalar particles.

Table II gives the main results of experiments to look for scalar particles in $0^+ \to 0^+$ nuclear transitions.

Significant bounds on the scalars can be obtained from experiments on neutron–nucleus scattering by making measurements of the angular distribution of the scattered neutrons in the range of energies of the order of tens of kilo-electron-volts[105] and from precision measurements of the energy dependence of the total cross sections.[106] The motivation for such experiments was quite different, namely, to determine the fundamental properties of the neutron: the polarizability, electromagnetic radius, and strength of the neutron–electron coupling. However, internucleon exchange of a massive boson must make an additional and quite definite contribution to the angular distribution[103,104] or to the dependence of the total cross section.[107] The scalar–nucleon interaction Lagrangian (16) corresponds to an exchange interaction potential of Yukawa type:

$$V_{nN} = -A \frac{g^2_{\varphi NN}}{4\pi} \frac{e^{-m_\varphi r}}{r}, \qquad (18)$$

where A is the number of nucleons in the nucleus. In the Born approximation, the scattering amplitude

$$f_\varphi = \frac{g^2_{\varphi NN}}{4\pi} \frac{2M_n A}{4M_n E(1-\cos\theta) + m^2_\varphi}, \qquad (19)$$

where M_n and E are the mass and energy of the neutron, makes it possible to separate the part in the total amplitude containing the term obtained as a result of the interference of f_φ with the nuclear scattering amplitude.

The decay of a scalar particle into a positron–electron pair is described by a Lagrangian analogous to (16) with replacement of $g_{\varphi NN}$ by $g_{\varphi ee}$, which determines the rate of decay of the scalar into a pair and some other effects capable of experimental testing. By virtue of $g_{\varphi ee}$, there may exist a contribution to the Lamb shift in hydrogen[108] proportional to the product $g^2_{\varphi NN} g^2_{\varphi ee} \sim g^2_{\varphi NN}/\tau(\varphi \to e^+e^-)$. From the known agreement between theory and experiment for the measurement of the anomalous magnetic moment of the electron it is also possible to obtain a bound on $g_{\varphi ee}$.[108] However, if there exist several particles with different types of coupling, then their contributions to the anomalous magnetic moment may cancel each other, and the corresponding bounds will be accordingly weakened. Figure 18 gives the bounds on the constants of the coupling of scalar (and, in part, pseudoscalar) particles with nucleons and electrons obtained from various experiments. The bounds are represented in terms of $\alpha_N = g^2_{\varphi NN}/4\pi$ and the particle lifetime τ:

$$\frac{1}{\tau} = \frac{\Gamma_{ee}}{\hbar} = \frac{1}{2} m_\varphi \frac{g^2_{\varphi ee}}{4\pi} F, \qquad (20)$$

where F is a function of the mass, spin, and parity of the particle.[108]

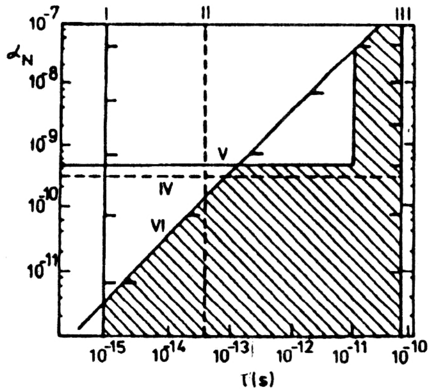

FIG. 18. Bounds on the constants of the coupling of a hypothetical scalar (or pseudoscalar) particle with mass $m_\varphi = 1.7\ \mathrm{MeV}/c^2$ to nucleons, $g_{\varphi NN}$, and electrons, $g_{\varphi ee}$. The allowed region is hatched. The values of $\alpha_N = g^2_{\varphi NN}/4\pi$ and τ are given in terms of $g_{\varphi ee}$ by the expression (20). The lines of the bounds are as follows: I corresponds to the hyperfine splitting in positronium (only for a pseudoscalar) (Ref. 109); II corresponds to the anomalous magnetic moment of the electron (Ref. 108); III corresponds to heavy-ion collisions (GSI); IV corresponds to neutron–nucleus scattering[104] (only for a scalar); V corresponds to production in a $0^+ \to 0^+$ transition in ^{16}O (Ref. 100); VI corresponds to the Lamb shift in hydrogen, and the absence of an anomaly in ne scattering gives the bound on the product $g_{\varphi ee} g_{\varphi NN}$.

3.2. Search for resonances in elastic positron–electron scattering at low energies

The most natural way of obtaining a neutral particle that decays into a positron–electron pair is to use the inverse reaction and look for a resonance in low-energy positron–electron scattering. The resonance must be observed at energy $E_{\text{cms}} = m_\varphi c^2$ in the center-of-mass system, or at positron kinetic energy $E_{\text{kin}} = m_\varphi^2/2m_e - 2m_e = m_e(\rho^2-4)/2$, $\rho = m_\varphi/m_e$ in the laboratory system. For $m_\varphi = 1.7$ MeV, $E_{\text{kin}} \cong 1.8$ MeV. In the first order, scattering of positrons by electrons can be described by direct single-photon exchange together with the annihilation diagram. If there is additional exchange of a massive particle, then the matrix element consists of many terms, including interference between different diagrams. A detailed analysis[108–110] shows that the annihilation diagram with participation of a massive particle plays the main role in the formation of the resonance in the scattering cross section. The $\varphi \to e^+e^-$ decay width is determined by the interaction Lagrangian

$$\mathscr{L}_i = g_i(\bar{\psi}\Gamma_i\psi)\varphi, \tag{21}$$

where ψ is the positron–electron field, φ is the neutral particle with spin 0 or 1 responsible for the resonance in the scattering, i denotes the type of coupling (S,P,V,A), and g_i is the corresponding coupling constant. The expressions for the widths have different forms for different types of coupling.[108–110] For example, for the case of a pseudoscalar particle

$$\Gamma_{e^+e^-}(0^-) = \tfrac{1}{2}\alpha^P_{ee} m_e(\rho^2-4)^{1/2}, \tag{22}$$

with interaction strength $\alpha^P_{ee} = (g^P_{ee})^2/4\pi$. In Refs. 109 and 110, general expressions were obtained for the differential cross sections for resonance scattering of positrons by electrons for different types of coupling. Integrated over the polarizations of the particles that participate in the scattering and over the scattering angles, the cross section has the usual form

$$\sigma = \sigma_0 \frac{(\rho\Gamma/2)^2}{(E-E_R)^2 + (\rho\Gamma/2)^2}, \tag{23}$$

where

$$\sigma_0 = (2J+1)\frac{4\pi}{\rho^2-4}\left(\frac{\hbar}{m_e c}\right)^2\left(\frac{\Gamma_{ee}}{\Gamma}\right)^2 = (2J+1)\pi\lambda\!\!\!^{-}{}^2\rho^2\left(\frac{\Gamma_{ee}}{\Gamma}\right)^2 \tag{24}$$

is the unitary cross section, in which J is the spin of the resonance, Γ is the total width (containing in addition to the elastic channel other possible processes), and $\lambda\!\!\!^{-}$ is the wavelength of the incident positron in the center-of-mass system. For $\rho \sim 3.5$, the unitary cross section is $\sigma_0 \sim 2000$ b (for $\Gamma_{ee}/\Gamma = 1$). The angular distribution of the scattering for spin-0 particles is isotropic in the center-of-mass system.

The ordinary quantum-electrodynamic scattering

$$d\sigma/d\Omega = \alpha^2\left(\frac{\hbar}{mc}\right)^2 f(\theta) \tag{25}$$

has an angular distribution that is strongly forward-peaked and several orders of magnitude smaller than the resonance cross section.

Bounds on the expected widths follow from comparison of the experimental and theoretical electron g factor.[108,109] For example, for the case of exchange of a pseudoscalar particle $\alpha^P < 10^{-8}$, and accordingly $\Gamma_P < 7\cdot 10^{-3}$ eV, and the corresponding lifetime to decay is $\tau > 10^{-13}$ sec. Of course, the main problem in the experiment is the very small expected width of the resonance. In a real experiment, two factors come into play that by many times broaden the resonance in the scattering cross section and, accordingly, lower the amplitude. First, the electrons in the target are not at rest, but have a momentum distribution that depends on the strength of the coupling of the electrons to the nucleus. For a hydrogen-like atom with charge Z, the effective resonance width has the form[109]

$$\Delta E_{1s} \approx 0.5\rho(\rho^2-4)^{1/2} Z\alpha m_e, \tag{26}$$

and for a resonance mass ~ 1.7 MeV this width is

$$\Delta E_{1s} \approx 20\ \text{keV}\cdot Z. \tag{27}$$

Obviously, to increase the sensitivity in the search for the resonance it is best to use targets with small Z. The second factor that broadens the resonance is the instrumental line width in the experiment, which is also greater by several orders of magnitude than the expected natural width of the resonances. With allowance for these resonance-broadening factors, the shape of the resonance in the cross section takes the form

$$\tilde{\sigma} = \tilde{\sigma}_0 \frac{(\Delta E/2)^2}{(E-E_R)^2 + (\Delta E/2)^2}, \tag{28}$$

where ΔE is the width of the resonance determined by the momentum distribution of the electrons in the target and by the experimental resolution, and

$$\tilde{\sigma}_0=\sigma_0 \frac{\rho\Gamma}{\Delta E}. \tag{29}$$

Thus, for good experimental resolution (~ 10 keV) and a beryllium target ($Z=4$) the amplitude of the resonance in the positron–electron scattering under the assumption that $\alpha_\varphi \sim 10^{-8}$ is lowered and, accordingly, its width is increased by $\Delta E/\Gamma$ times, i.e., by approximately seven orders of magnitude.

The experimental results on the search for resonances in the scattering cross section can be represented in the form of an integral over the width of the resonance, $\int \sigma dE = \bar{\sigma}_0 \Delta E$, or in the form of the width Γ or its reciprocal, the particle lifetime. For the above limits obtained from the $g-2$ experiments, we expect $\sigma_0 \Delta E < 15$ b · eV. The early experimental investigation[112] was devoted to measurement of the differential cross section for positron scattering by electrons with the aim of verifying the quantum-electrodynamic calculations (Bhabha's formula[111]). The measurements were made with low accuracy ($\sim 10\%$) in the range of positron energies 0.6–1.0 MeV. Soon after the observations of the Darmstadt positron–electron peaks, several laboratories began to look for corresponding resonances in the scattering.

The experiments to look for resonances in positron–electron scattering can be divided into two groups—those with low and high atomic numbers of the target element. For targets with low atomic number, one can hope for a relatively smaller broadening of the resonance due to the momentum distribution of the electrons in the atom. Targets with high atomic number were used because the production of positron–electron pairs at the GSI requires the presence of the strong electromagnetic field produced by the nucleus of a heavy atom. However, such experiments definitely have a bad energy resolution because of the broad momentum distribution of the inner electrons of a heavy atom. In experiments on targets with small atomic number, beryllium and polymer films were used, and the heavy element was mainly thorium. We shall consider first the experiments with targets having small atomic number.

In the first experiment,[113] beryllium was used and the limit 1000 b · eV was obtained for the integrated resonance cross section. To accelerate the positrons, the Stuttgart group (Refs. 114–116 and 125) used an electrostatic accelerator with high monochromatization (~ 1 keV). In their first study,[114] they established an upper limit of ~ 65 b · eV on the resonance cross section. In the next study,[115] the group found a resonance at the effective mass 1.83 MeV with width ~ 8 keV, which agreed well with the width predicted from the Compton profile of the bound electrons in beryllium. The cross section in the resonance was 30 b · eV at the 5σ confidence level. In the following study of Ref. 125, in which measurements were made with better statistical accuracy, the result of Ref. 115 could not be repeated. In later work of this group, an experimental device permitting a significant increase of the ratio of the

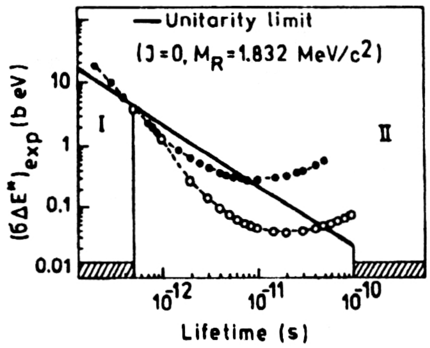

FIG. 19. Upper limit (confidence level 95%) on the integrated total cross section for production in e^+e^- scattering of the hypothetical neutral particle with invariant mass 1.832 MeV/c^2 as a function of the lifetime of the particle. The points are from Ref. 123; the open circles are from Ref. 124. The continuous line shows the model-independent total cross section for a resonance with $J=0$ and mass 1.832 MeV/c^2 in the unitarity limit.[24] Region I is excluded by the earlier experiments on positron–electron scattering,[121] and region II is excluded by data from observation of the production of positron peaks in heavy-ion collisions.[36]

effect to the background from nonresonance scattering was used. The target was a thin scintillator (0.38 mm), and selection was based on the pulse height in this trigger. It was assumed that if in the first layer of the target a neutral particle was formed, then it would pass through the remaining thickness of the scintillator without energy release and would not decay. Elastic scattering would be accompanied by a much greater energy release, and these events were rejected. Thus, in the interval of resonance masses 1.78–1.92 MeV it was possible to establish a limit on the width from 0.22 to 2.74 meV, corresponding to a lifetime $(0.24–3.0) \cdot 10^{-12}$ sec. In one further study[126] a positive effect was found in the search for a positron–electron resonance—at mass 1730 keV with width 30 keV, using a polyethylene target. The result, 1400 ± 600 b · eV, was criticized in Ref. 127. In Ref. 128, in contrast to other experiments, the particles after scattering were analyzed in energy by two magnetic spectrometers in coincidence, and the limit 20 b · eV was achieved with 10^{-12}–10^{-10} sec for the lifetime. A large series of experiments[118–124] was carried out by a joint GSI–ILL group, using the reactor at Grenoble (spectrometer BILL). The positrons were obtained from the process of pair conversion after neutron capture in a titanium target placed near the core of the reactor. Simultaneously with positron–electron scattering, positron scattering by nuclei (Mott scattering) was detected. In the last experiments[123,124] of this group the "active shadow" method was used to suppress the nonresonance background, namely, a scintillator switched to anticoincidence shut off possible trajectories of particles after scattering in the target but made it possible to detect the decay products of a long-lived resonance outside the target. Figure 19 shows the results of the last studies[123,124] of the GSI–ILL group in the form of bounds on the cross section at resonance as a function of the lifetime of the hypothetical particle φ. It can be seen that these data rule out a resonance with $J=0$ with lifetime less than $5 \cdot 10^{-11}$ sec. For a state

TABLE III. Results of experiments to look for resonances in low-energy positron–electron scattering using targets containing elements with low atomic number; m_φ is the range of energies in the center-of-mass system.

m_φ, MeV	Target	$\int \sigma_{res} dE$ or τ for $m_\varphi = 1.8$ MeV	Experiment
1,5–2,0	Be	< 1000 b · eV	[113]
1,62–1,67	mylar	< 65 b · eV	[114]
1,80–1,87	Be	30b · eV(5σ)	[115], withdrawn in [125]
1,80–1,87	Be	$\tau > 10^{-13}$ sec	[125]
1,78–1,92	NE 102A	$\tau > 3 \cdot 10^{-12}$ sec	[116]
1,62–1,89	Be	< 62 b · eV	[117]
1,36–1,87	CH_2	1400±600 b · eV	[126], criticism in [127]
1,830	lucite	< 20 b · eV	[128]
1,78–1,86	Be	< 12,6 b · eV	[118]
1,79–1,86	Be	< 6,3 b · eV	[119]
1,50–1,65	mylar	< 30 b · eV	[121]
1,77–1,86	Be	$\tau > 7{,}5 \cdot 10^{-12}$ sec	[123]
1,77–1,86	Be	$\tau > 5 \cdot 10^{-11}$ sec $(J = 0)$ $\tau > 10^{-10}$ sec $(J = 1)$	[124]
1,9–2,5	Li	$\tau > 5 \cdot 10^{-13}$ sec $(J = 0)$	[277]

with $J=1$, the bound is $1 \cdot 10^{-10}$ sec. Thus, with allowance for the results on the observation of positron peaks in heavy-ion collisions, from which it followed that the lifetime of the source of the peaks is less than 10^{-10} sec,[36] the complete admissible interval of lifetimes of the hypothetical particle is ruled out. A summary of the results of the experiments on light targets is given in Table III.

The majority of the experiments on targets with high atomic number were carried out by a method different from those with light targets. They used a continuous spectrum from radioactive sources irradiating a target situated right next to the source and investigated the spectrum of positrons and electrons in coincidences or the electron spectrum. With such a formulation of the experiment, one could only speak of the presence or absence of peaks in the spectrum of charged particles emitted from such a target, but one could not make a reliable estimate of the resonance cross section. In the first experiment,[129] with thorium and tantalum targets, in which two magnetic spectrometers in coincidence were used, a weak peak with mass 1360 keV was found for the thorium target. In Ref. 130, this result was not confirmed. It should be noted that in experiments with heavy targets several positive results indicating the presence of resonances were obtained, but there is no interpretation of these results. In heavy targets, various background processes may play an important part. For example, it was noted in Ref. 130, in a criticism of Ref. 129, that the gamma background from positron annihilation in the target could give rise to peaks in the electron spectrum because of the Compton effect. One may also expect subtle processes in a heavy atom leading to features in the spectrum of emitted electrons. The only study of positron scattering on a heavy target made under stringent kinematic conditions[121] gave a bound on the existence of a resonance, but it was weaker than the one that follows from the value of the electron anomalous magnetic moment.[108,109] Nevertheless, Sakai's group[133,135,136] continues to insist on the reliability of their experiments, in which they observed very narrow (2.5 keV) electron peaks on the background of a continuous spectrum in an experiment in which thorium targets were irradiated by positrons from ^{68}Ga decay. The results of all these experiments are given in Table IV.

3.3. Search for pairs of correlated gamma rays in heavy-ion collisions and in other processes

The conjecture that in heavy-ion collisions there is production of a neutral long-lived particle, possibly in several excited states, naturally led to the conclusion[137,138] that such a system may decay into two or three photons, depending on the spin and parity of the particle.

The first experimental search[138] for such pairs of gamma rays in heavy-ion collisions was made at Berkeley (LBL), using the SUPERHILAC accelerator. Uranium and thorium ions were scattered at energy ~6 MeV/nucleon. An upper limit on the production of pairs of gamma rays in the range of effective masses 1.5–1.8 MeV equal to $3 \cdot 10^{-10}$ per incident ion was established, and the corresponding cross section did not exceed 30 μb.

The next experiment[139] of this group had a higher sensitivity. It used 14 germanium detectors measuring 5×5 cm, surrounded by an anti-Compton BGO shield and connected pairwise in coincidence. There was found to be a narrow resonance in the spectrum of the total energy release for pairs of germanium detectors detecting pairs of

TABLE IV. Results of experiments to look for resonances in low-energy positron–electron scattering using targets containing heavy elements.

Target	Kinetic energy of peak, keV	σ or $\int \sigma dE$, b·eV	Experiment
Th	$E_{e^-} = 340$		[129]
Th	—		[130]
Th	—		[131]
Th	$E_{e^-} = 330$	1400±600	[132]
Th	$E_{e^-} = 330{,}8 \pm 1{,}0$ width 3,7±0,5	28±10 mb/sr	[133], critism in [122], answer in [135]
U	$E_{e^-} = 330{,}8 \pm 1{,}0$ width 3,7±0,5	34±13 mb/sr	[133], critism in [122], answer in [135]
Th	$E_{e^+} = 320$	1900±600 b·eV	[134]
Th	—	< 120 b·eV	[121]
Th	$E_{e^-} = 330{,}58 \pm 0{,}40$ width 2,3±0,4	180(1±0,18±0,25) mb	[136]
Th	$E_{e^-}' = 410{,}3 \pm 0{,}3$	212(1±0,23±0,25) mb	[136a]

gamma rays emitted in opposite directions from an object moving with the center-of-mass velocity of the U+Th ions. The total energy of the gamma rays was 1062±1 keV, and the width was less than 2.5 keV. The cross section for production of this peak was 50±25 μb. At higher energies of the gamma rays, the bound was 3 μb.

This observation stimulated enthusiasm among the investigators of the problem of the narrow positron–electron resonances in heavy-ion collisions. Indeed, from the point of view of the phase space a neutral system possessing several excited states can readily decay into a positron–electron pair when it is at a higher energy level, but near the threshold this decay channel is strongly suppressed, and therefore there is decay into a pair of gamma rays. The resonance in the positron–electron system at energy 1062 keV must influence the lifetime of parapositronium. We mention in this connection the well-known experiment to measure the lifetime of orthopositronium,[140] in which there was an appreciable (10σ) discrepancy between the experiment and the calculation, which included radiative corrections up to order $\alpha^2 \ln(1/\alpha)$. Another interesting anomaly bearing on the question is the strong discrepancy (5σ) in the cross section for production of positron–electron pairs by gamma rays with energy 1077 keV (experiment in Ref. 141, calculation in Ref. 155). Neither of these discrepancies between experiment and theory has yet been explained, and therefore it seems natural to include in the consideration virtual quasistationary states in the positron–electron system. For the case of the anomaly in the experimental cross section for production of pairs near the threshold, the inclusion of an additional source of decays into pairs could explain the observed discrepancy. However, the calculated contribution to the cross section for pair production resulting from a resonance in the positron–electron system increases with the energy of the gamma rays[156] (admittedly, more slowly than the ordinary process in the framework of QED). At the same time, there is good agreement between the experiment and theory for the cross section for pair production at gamma-ray energies above 1.2 MeV. To resolve this contradiction, it is necessary to assume that the contribution to the cross section due to the resonance is in some manner suppressed at relatively large momentum transfers $\Delta p > 0.5$ MeV/c to the particle. Such a suppression may arise for a larger resonance state ($\sim 10^{-11}$–10^{-10} cm). Such a model of an extended object has many arguments in its favor and will be discussed further.

The experimental observation of pairs of gamma rays in heavy-ion collisions[139] stimulated several experiments to look for such pairs in other processes. The motivation for these searches was in the first place intuitive and proceeded from the assumption that a sudden change of the electromagnetic field near a heavy nucleus could be the reason for the appearance of both positron–electron peaks and pairs of gamma rays.

In Ref. 142 a limit was established on the emission at angle 180° of pairs of gamma rays in the range of total energies 1.4–2.0 MeV in spontaneous fission of ^{252}Cf. The probability was found to be less than $8 \cdot 10^{-7}$ (95%). In Ref. 143, the limit $\sim 3 \cdot 10^{-12}$ on the same process was established in alpha decay of ^{239}Pu.

Soon afterwards,[144] the authors of Ref. 139, having repeated their experiment with better statistics and for different pairs of colliding nuclei, found an adequate interpretation for the peak that they had found at 1062 keV. It was found that this peak and a second peak found at total energy 1043 keV of the gamma rays in collisions of uranium and thorium nuclei at $\sim$6 MeV/nucleon were due to cascades from high-spin states of ^{238}U. The peak at 1062 keV is determined by a $32^+ \to 30^+$ cascade in coincidence with a $30^+ \to 28^+$ cascade, and the peak at 1043 keV is determined by coincidence of $32^+ \to 30^+$ and $28^+ \to 26^+$ transitions. The observed peaks are very narrow because of the mutual compensation of the Doppler shifts of the energies of photons emitted in opposite directions from a

TABLE V. Summary of results of experiments to look for two- and three-photon resonance annihilation in low-energy positron–electron scattering; E is the c.m.s. energy.

E, keV	Bound on cross section or width	Experiment
1830	$\int \sigma_{\gamma\gamma} dE < 167$ b·eV	[146]
1062–1100	peaks not detected	[147]
1062	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 0{,}11$ MeV	[148]
1830	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 10$ MeV	[152]
1500–2000	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 20$ MeV	[149]
1600–2000	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 6{,}6$ MeV	[150]
1830	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 2{,}5$ MeV	[116]
1830	$\Gamma_{ee}\Gamma_{\gamma\gamma}/\Gamma < 5$ MeV	[152]
1500	$\int \sigma_{\gamma\gamma} dE < 11{,}5$ b·eV $\Gamma_{3\gamma}\Gamma_{e^+e^-}/\Gamma_t < 3{,}8$ MeV	[152,a]
1540	$\int \sigma_{\gamma\gamma} dE < 11{,}7$ b·eV $\Gamma_{3\gamma}\Gamma_{e^+e^-}/\Gamma_t < 4{,}3$ MeV	[152,a]
1640	$\int \sigma_{\gamma\gamma} dE < 16{,}7$ b·eV $\Gamma_{3\gamma}\Gamma_{e^+e^-}/\Gamma_t < 7{,}5$ MeV	[152,a]

moving nucleus. The same study established a new and lower limit on the production of 180°-correlated pairs of gamma rays with total energy in the range 1.2–2.0 MeV. The cross section was found to be less than 0.6 μb, a value approximately two orders of magnitude less than the cross section for production of positron–electron peaks in the GSI experiments (of the EPOS group) and an order of magnitude less than the results of the ORANGE group.

3.4. Experiments to look for resonance two- and three-photon annihilation in low-energy scattering of positrons by electrons

The hypothetical resonance manifested in the form of positron–electron pairs in heavy-ion collisions could, in principle, decay into two or three photons, depending on the spin and parity of the resonance. As was shown in the previous section, in neither heavy-ion collisions nor in other experiments were two-photon events detected, and three-photon events were not sought.

If the reason for the phenomena observed in the GSI experiments is a complex composite object with internal excited states, then radiative transitions between them are possible. If their contribution to the total decay width of the excited states is appreciable, then the resonance may not be noted in the elastic positron–electron channel. The widths for decay of scalar and vector particles into two and, respectively, three photons are given in Ref. 110:

$$\Gamma_{\gamma\gamma}^{S,P} = \tfrac{1}{16}\alpha_\gamma^{S,P} m_{S,P}^3, \tag{30}$$

where $\alpha_\gamma^{S,P} = (g_\gamma^{S,P})^2/4\pi$ for scalar and pseudoscalar particles, and

$$\Gamma_{3\gamma}^{V} \cong \frac{(g_\gamma^V)^4}{92160\pi^3} m_V^3. \tag{31}$$

With allowance for possible experimental broadening of the observed resonance, it is possible, as in the case of resonance scattering, to obtain a formula for the cross section for resonance annihilation into two photons:

$$\left(\frac{d\sigma}{d\Omega}\right)\Delta E = \left(\frac{\hbar}{m_e c}\right)^2 \frac{\Gamma_{\gamma\gamma}\Gamma_{e^+e^-}}{\Gamma} \frac{\rho}{\rho^2-4} \frac{1}{\gamma^2(1-\beta\cos\theta)^2}, \tag{32}$$

where Γ is the total width, $\rho = m_S/m_e$, $\gamma = \rho/2$, θ is the photon emission angle in the laboratory system, and $\beta = (\rho^2-4)^{1/2}/\rho$ is the c.m.s. velocity.

This cross section must be compared with nonresonance annihilation, which is $\sim\alpha^2(\hbar/mc)^2$. Exact expressions are given, for example, in Ref. 110. Bounds on the widths $\Gamma_{\gamma\gamma}$ were obtained from the value of the electron anomalous magnetic moment,[153] and more stringent bounds from the cross section for Delbrück scattering:[154]

$$g_{\gamma\gamma}^S < 0.04 \text{ GeV}^{-1}, \quad g_{\gamma\gamma}^P < 0.5 \text{ GeV}^{-1}.$$

If these limits are used, and also the previously given bounds $\Gamma_{e^+e^-} < 7$ meV from the $g-2$ experiments, one can obtain the expected limiting ratio of the cross sections for resonance and nonresonance annihilation: $\int \sigma_{\gamma\gamma} dE/\sigma_{\gamma\gamma}^{\text{nonres}} < 0.2$ (0.1) for masses $m = 1.8$ and 1.062 MeV.

The early study of Ref. 145, devoted to measurement of the cross section for annihilation of positrons in flight, confirmed the agreement of QED theory with experiment to 5% accuracy at several energy points. Recent experiments[146–152] to look for resonances in annihilation, like experiments to look for resonance scattering, were made by different methods: both with and without monochromatization of the incident positrons, the hope being to see a peak in the total spectrum of the gamma rays. The attention in these experiments was drawn to two ranges of the total energy of the gamma rays: 1.8 MeV, where the GSI data most convincingly indicated a 180° correlation of positron–electron pairs in heavy-ion collisions, and the point 1062 keV, where the LBL group found the $\gamma\gamma$ peak in collisions of uranium and thorium nuclei.[139,144] Almost all experiments sought two-photon resonance annihilation.

In the studies of Ref. 152 (Heidelberg–Darmstadt Crystal Ball) and Ref. 152a, three-photon annihilation was also sought.

The experimental data are summarized in Table V. The joint analysis of the data on the positron–electron and photon widths made in Ref. 116 leaves open practically the entire region $\Gamma_{\gamma\gamma} > \Gamma_{ee}$ up to widths $\Gamma \sim 10$ meV. It is true that the result of the experimental search for a $\gamma\gamma$ channel in heavy-ion collisions ($\sigma < 0.6\ \mu$b)[144] indicates $\Gamma_{\gamma\gamma} \ll \Gamma_{ee}$ if one uses the data of the EPOS group, who obtained a cross section for production of positron peaks of 5–10 μb/sr (the results of the ORANGE group were approximately an order of magnitude lower). If it is assumed that $\Gamma_{\gamma\gamma} \ll \Gamma_{ee}$, then the best bound, $\Gamma_{\gamma\gamma} < 2.5$ meV at mass 1.8 MeV, corresponding to coupling constant $g_{\gamma\gamma} < 0.3$ GeV^{-1}, is still significantly weaker than the bound[154] that follows from the experiments to investigate the Delbrück scattering. From this it follows that we need more sensitive experiments to look for resonance positron annihilation. In conclusion, we mention Ref. 151, in which an attempt was made to obtain a bound on the coupling constant $g_{\gamma\gamma}$ and, correspondingly, the width $\Gamma_{\gamma\gamma}$ from analysis of the process $e^+ + e^- \to e^+ + e^- + \varphi \to e^+ + e^- + e^+ + e^-$. The calculation showed that the cross section of the process is $\sigma \sim (2J+1)\Gamma_{\gamma\gamma}/m_\varphi^3$. From the cross section $\sigma_{\exp} = 1.61 \pm 0.12\ \mu$b the authors obtained the bound $\Gamma_{\gamma\gamma}\Gamma_{ee}/\Gamma < 2$ eV (90%) and, if it is assumed that $\Gamma_{\gamma\gamma} \ll \Gamma_{ee}$, the bound is as yet rather weak.

3.5. Experiments to look for new light neutral particles in decays of heavy vector mesons, kaons, and pions

During the last 10–15 years, quite a large number of experiments to look for light neutral particles have been performed at high and intermediate energies. These searches were mainly motivated by the predictions of various extensions of the standard model. The indication of the possible existence of axions[82] stimulated a number of experimental investigations of processes in which produced axions could decay into positron–electron pairs or two gamma rays. The discovery of the Darmstadt peaks made the problem more acute and stimulated modifications of the axion models[84,85] and new experiments to look for the positron–electron decay channel. A further important argument for seeking light neutral particles was the prediction of possible light Higgs bosons in supersymmetric extensions of the standard model.[176,177]

Searches for light neutral particles are made at high energies in experiments of two types: in particle decays and in beam-dump experiments. One of the possible ways of testing the existence of a new neutral particle in the mass range of the Darmstadt peaks is to look for decays $V \to \gamma + \alpha$, where V is a heavy vector meson (J/ψ or Υ), and a is the new particle, for example, an axion. Since the axion model includes two Higgs doublets with different *a priori* vacuum expectation values θ_1 and θ_2, the axion coupling can "rotate" between the upper and lower quark states, depending on the ratio of these expectation values, $x = \theta_2/\theta_1$, which is the fundamental parameter of axion theory. Bounds that do not depend on this parameter can be obtained by using the product of the experimental limits on the probability of radiative decays of heavy vector mesons, the result being compared with the theoretical estimate

$$P = \mathrm{BR}(J/\psi \to \gamma a)\mathrm{BR}(\Upsilon \to \gamma a) \cong 1.6 \cdot 10^{-8}$$

for the standard axion. In the first experiments to look for axions in such decays, events with observation of one photon in the final state were sought, and the limit $P < 10^{-9}$ was obtained.[157–159] In accordance with the experimental conditions, this was a real limit for axion lifetimes greater than 10^{-9} sec. However, if the lifetime is much less, then the axion decays inside the detector, violating the condition for detection of one photon. Subsequent experimental investigations of the decay of heavy quarkonium[160–162] closed this possibility too for a standard axion with mass in the interval 1.6–1.8 MeV with a margin of more than two orders of magnitude.

Calculations of the decays of charged kaons with emission of new pseudoscalar particles, $K^+ \to \pi^+ a$, depend very strongly on the models, but in a number of experiments rather low limits on the probability of such processes were established. The early experiment of Ref. 163, in which a limit on the decay $K^+ \to \pi^+ e^+ e^-$ was measured, was weakly sensitive to small invariant masses for the system (e^+e^-). An estimate given in a later analysis[171] of this experiment gave a limit on the emission of a neutral particle with mass corresponding to the Darmstadt peaks and decaying rapidly into a positron–electron pair: $\sim 10^{-4}$. The experiments of a Japanese group[165,166] looking for the processes $K^+ \to \pi^+ \nu\tilde{\nu}$ and $K^+ \to \pi^+ \gamma$, $K^+ \to \pi^+ \gamma\gamma$, $K^+ \to \pi^+ \gamma\gamma\gamma$ were also sensitive to $K^+ \to \pi a$ decays. The first of these studies looked for missing mass in the kaon decay; this corresponded to a long lifetime of the neutral particle ($\tau > 10^{-11}$ sec); the second study made it possible to detect the decay products ($\tau < 10^{-11}$ sec). The analysis of Ref. 171 showed that, depending on the lifetime, the bound is $\sim 10^{-7}$ ($\tau > 10^{-11}$ sec) and $\sim 10^{-4}$ ($\tau < 3 \cdot 10^{-13}$ sec) with an intermediate bound in the interval between these limits. An early experiment at Berkeley[164] was sensitive to fast decays of the neutral particle and gave[171] a bound $\sim 10^{-4}$ on $K^+ \to \pi^+ a$. The analysis of Ref. 168 of the same experiment gave the limit $1.5 \cdot 10^{-5}$ (see also the analysis in Ref. 169). For the model of the standard axion, the production probability must be $5.7 \cdot 10^{-3}$. The most sensitive experiment to investigate the possible decay $K^+ \to \pi^+ a$ was performed at Brookhaven.[170] Its result for small masses is shown in Fig. 20.

The decays $\pi^+ \to e^+ \nu a$ are also important in establishing bounds on a short-lived axion, since this decay is enhanced in relation to the ordinary pion beta decay $\pi^+ \to \pi^0 e^+ \nu$ on account of the larger phase space at small

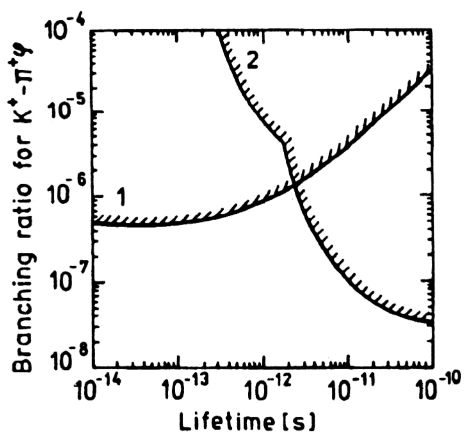

FIG. 20. Limits on the probability of the process $K^+\to\pi^+\varphi$, $\varphi\to e^+e^-$ for $m_\varphi=1.8$ MeV/c^2 as a function of the lifetime of the φ particle. Curve *1* is from Ref. 170; curve *2* represents the data of the analysis in Ref. 171 of the experiments of Ref. 166.

axion mass. A Dubna group[172] established a limit $\sim 5\cdot 10^{-9}$ on the probability of the decay $\pi^+\to e^+e^-e^+\nu$ (in the commentary on this experiment given in Ref. 173 it is suggested that the limit is somewhat higher). The later study of Ref. 174 established a limit of 10^{-10} on such a decay. This value should be compared with the calculation of Ref. 173 for the probability of the process $\pi^+\to e^+\nu a$ for a short-lived axion in the model of Refs. 84, 85, and 88: $\mathrm{BR}\sim 3\cdot 10^{-6}(m_u/m_d)(m_a/1.8\ \mathrm{MeV})$. The result of the experiment of Ref. 174 is shown in Fig. 21.

In Ref. 175, the decay $\pi^0\to e^+e^-$ was considered. The experimental result, $\mathrm{BR}(\pi^0\to e^+e^-)/\mathrm{Br}(\pi^0\to\gamma\gamma) = (1.8\pm 0.7)\cdot 10^{-7}$ is in disagreement with the expected value for a short-lived axion[84,85] with mass 1.8 MeV, which is approximately an order of magnitude greater.

3.6. Search for new neutral particles with mass in the range 1.6–1.8 MeV/c^2 in beam-dump experiments

In experiments of this type, electrons or protons, reaching a heavy target, cause a cascade of bremsstrahlung

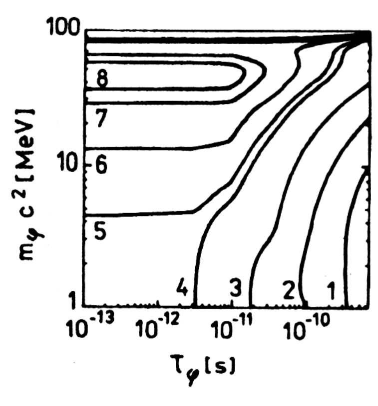

FIG. 21. Upper limit on the probability of the decay $\pi^+\to e^+\nu\varphi$ (90%) as a function of the mass and lifetime of a neutral particle φ that decays into a positron–electron pair: *1*) $3\cdot 10^{-9}$; *2*) 10^{-9}; *3*) $3\cdot 10^{-10}$; *4*) 10^{-10}; *5*) $8\cdot 10^{-11}$; *6*) $5\cdot 10^{-11}$; *7*) $3.5\cdot 10^{-11}$; *8*) $3\cdot 10^{-11}$ (Ref. 174).

processes in which not only photons but also new neutral particles must be emitted. These particles can also be produced by the photons through the Primakoff process. Leaving the target, some of these particles decay in flight into positron–electron pairs or a pair of gamma rays, and these can be detected. Calculation of the intensity with which the neutral particles are generated in the target on the basis of the assumed coupling constants and calculation of the decay rate on the basis of the same constants make it possible, after comparison with an experiment, to obtain bounds on these constants. If the coupling constant $g_{\varphi ee}$ is large, corresponding to a short lifetime of the particle, then the fraction of these decays within the target or in a shield placed after the target is increased, and this leads to a two-sided bound on the coupling constants in beam-dump experiments. Because of the large particle fluxes in the targets, these experiments have a very high sensitivity. Depending on the length of the target and shield, the experiment can be adjusted to different lifetimes of the neutral particles until decay and, accordingly, different coupling constants. Small coupling constants and long lifetimes were ruled out already in the early beam-dump experiments[178] with low energy of the particles in the beam. Measurements with short targets were made to look for fast decays of the particles ($\tau\lesssim 10^{-13}$ sec). Table VI summarizes the results of the experimental searches for penetrating neutral particles that decay into positron–electron pairs in beam-dump experiments.

4. THEORETICAL MODELS FOR EXPLAINING THE DARMSTADT EFFECT

The unusual features of the experimental observations at the GSI stimulated a considerable number of theoretical attempts to explain the phenomenon in its entirety or some of its properties.

Several authors tried to stay within the framework of the well known models and attempted to describe the experimental data by means of standard quantum electrodynamics. In many other publications, the authors postulated unusual bound states (in the standard model), localized regions with modified QED, with an unusual Higgs or QCD vacuum. In theoretical calculations, previously unknown bound states in the positron–electron system were found; in some studies, a completely new interaction was postulated. Overall, the GSI peaks stimulated a considerable number of ideas, calculations, experiments, and considerations having a more general nature and not merely the utilitarian aim of explaining a specific experimental fact. Chronologically, the suspicion that a new particle, the axion, had been discovered at the GSI was the first proposal made. We begin our consideration of the theoretical approaches with this conjecture.

4.1. A new point particle

The main problem in postulating any new particle is to explain why it was not observed earlier and also to avoid contradictions with numerous experiments in atomic and nuclear physics and in elementary-particle physics that are in good agreement with the standard model. Here, there

TABLE VI. Summary of results of beam-dump experiments to look for neutral pseudoscalar particles in the mega-electron-volt range of masses that decay into a positron–electron pair.

Beam, energy	Target, minimum range	Eliminated range of particle lifetimes, sec	Eliminated range of coupling strength $\alpha = g^2_{\varphi ee}/4\pi$ for m_φ=1.8 MeV/c^2	Experiment
Electrons, 45 MeV	Ta, 11,8 / cm^2	$> 10^{-11}$	$10^{-13} < \alpha < 10^{-10}$	[178]
Electrons, 2.5 GeV	W+Fe, 2,4 m	$5\cdot 10^{-13}$—10^{-7}	$10^{-14} < \alpha < 4\cdot 10^{-8}$	[180]
Electrons, 1.5 GeV	W, 10 cm	$6\cdot 10^{-14}$—$9\cdot 10^{-11}$	$10^{-11} < \alpha < 10^{-8}$	[182]
Electrons, 9 GeV	W, 10–12 cm	10^{-14}—$5\cdot 10^{-11}$	$10^{-12} < \alpha < 10^{-7}$	[181, 183]
Electrons, 20 GeV	~ 200 m	$<3\cdot 10^{-6}$ for m_φ=1.8 MeV/c^2	$\alpha > 3\cdot 10^{-16}$	[184]
Electrons, 20 GeV	H, bubble chamber	$6\cdot 10^{-16}$—$6\cdot 10^{-13}$ for m_φ=1.8 MeV/c^2	$2\cdot 10^{-9} < \alpha < 2\cdot 10^{-6}$	[185]
Protons, 590 MeV	Cu, 7,5 m	< 0,1	$\alpha > 10^{-20}$	[186]
Protons, 400 GeV	Cu	$<3\cdot 10^{-6}$	$> 3\cdot 10^{-16}$	[187]
Protons, 800 GeV	Cu, 5,5 m	10^{-14}—10^{-11}	$10^{-10} < \alpha < 10^{-7}$	[188]
Protons, 590 GeV	Cu, 8 m	< 0,05	$\alpha > 2\cdot 10^{-20}$	[189]
Protons, 800 GeV	repetition and an improvement of Ref. 188			[190]

are two possibilities—to assume that the coupling constants of the new particle are sufficiently small to explain why it was not observed in earlier experiments or to endow the new particle with such special properties that it can be produced only when very special conditions are satisfied, for example, in the strong electromagnetic field in the GSI experiments. Since the standard model is so successful, any hypothesis that not merely postulates a new particle but also leads to new physics at low energies that does not agree with the standard model is doubly implausible. However, if the hypothetical particle fits naturally into the standard model or a "soft" extension of it, then such a particle is more acceptable for consideration. Such a possibility was in fact considered from the very beginning.[68–70] We must immediately make a distinction between a point particle and a composite extended object. In the framework of the standard model, an extended particle can be a composite system consisting of electrons, quarks, photons, and gluons, or some complicated excited state of the Higgs vacuum, etc. Such possibilities will be considered later. Point particles that do not contradict the standard model can have various natures. Vector or axial-vector particles are barely possible, since they must be identified with a new gauge field with a corresponding extension of the gauge group and, therefore, a new interaction. On the other hand, scalars and pseudoscalars can be introduced into the Higgs sector, and, moreover, such a particle—the axion—was very desirable independently of the GSI results and before their appearance.[82,98] A detailed justification of the axion hypothesis has been given in several publications[82,83] and is not reproduced here. We merely mention that the main argument was the need to solve the problem of *CP* violation in QCD: Why is it so small? The currently most gentle and natural device is to extend the Higgs sector, namely, in place of one doublet of Higgs fields (φ_0,φ_+) one assumes in the hypothesis of Ref. 82 two independent doublets (φ_0,φ_+) and (φ_0',φ_+') with different vacuum expectation values. One doublet generates the mass of the upper quarks, and the other gives the mass of the lower quarks. Such a form of the theory enables one to make the *CP* violation arbitrarily small, but the price that has to be paid is the appearance of a new Goldstone boson—an axion with undetermined mass but, for the first, "standard," form of the axion model, fixed constants of the coupling with the quarks and leptons. Numerous experiments quite quickly ruled out this standard version of the axion model.[83] In connection with the GSI results, it was noted that the standard axion model does not use the most general form of coupling of the axion to the quarks and leptons. New forms were proposed[84,85,88] in order to avoid the experimental difficulties for masses in the range 1.6–1.8 MeV. These new models had the following properties: 1) For an axion with mass 1.6–1.8 MeV there were no experimental prohibitions existing for a light axion with mass less than $2m_e$; 2) fast decay $a\to e^+e^-$ with time $\lesssim 10^{-13}$ sec made it possible to avoid the bounds given at that time by the beam-dump experiments; 3) weak coupling to the heavy quarks made it possible to avoid bounds that followed from decays of heavy vector mesons. Moreover, independently of the GSI experimental data, it was interesting to elucidate the stability of the axion idea under the pressure of experimental bounds. Models of new pseudoscalar particles in which, in contrast to the standard axion, universality of the coupling to the different quarks and leptons is violated can never be definitely ruled out, since one can assume that the axion is coupled only to heavy, as yet undiscovered quarks.

If we return to the GSI results, then the axions in such models must have appreciable coupling to the first quark doublet (u,d). Let us consider, following, for example, Refs. 68–70 and 191–193, the production of a scalar or pseudoscalar particle in a collision of heavy ions under the conditions of the GSI experiments and using the known bounds on the coupling constants. From the very beginning, the subject of the analysis was this question: From what source can these new particles arise? From electron transitions in a quasiatom or from interaction with nucleons or with the electromagnetic field? If it is assumed that the source of the particles is the electron shell of a quasiatom, then the probability of their emission can be determined by using the data of calculations of x-ray emission associated with transitions between molecular orbits and of electron bremsstrahlung.[194] In order of magnitude, the cross section for production of the neutral particle φ as a result of interaction with electrons in a collision of heavy ions can be represented as the product of the cross section for the production of x rays in such a process with $E_\gamma > m_\varphi$ and the ratio of the coupling constants:

$$\sigma_\varphi \approx \frac{\alpha_\varphi}{\alpha}\,\sigma_\gamma(E > m_\varphi) < 1.4 \cdot 10^{-6} 400\ \mu\text{b} = 6 \cdot 10^{-10}\ \text{b}.$$

We have here used the bound on the coupling of the hypothetical particles to electrons obtained from the $g-2$ experiments, $\alpha_\varphi < 10^{-8}$ (in the original study,[68] the stronger bound $\alpha_\varphi < 1.6 \cdot 10^{-9}$, which was subsequently revised, was obtained), and the cross section for the emission of x rays in Pb+Pb collisions at 5.8 MeV/nucleon.[194] As we noted, the cross section for the production of positron lines is greater by several orders of magnitude, so that the assumption that the particle is produced as a result of interaction with the positron–electron field is ruled out. If it is assumed that the particle is produced as a result of interaction with a nuclear current, then one must use the bound obtained from measurements and calculations of electron levels in atoms. In this case, the contribution to the interaction of the electrons with the nucleus due to the exchange of the new particle between the electrons and nucleons of the nucleus would cause a shift of the levels of atomic electrons compared with the calculation in the framework of QED. The Lagrangian of the electron–nucleon interaction includes constants for coupling of the hypothetical particle both to electrons (21) and to nucleons (quarks) (14). Bounds are then obtained on the product of coupling constants $g^i_{\varphi ee} g^i_{\varphi NN}$, where i signifies the type of coupling. In the nonrelativistic limit, a Yukawa term with interaction range m_φ^{-1} appears in the interaction potential of the nucleons with the electrons (this is analogous to the case, considered above, of the additional interaction (18) of neutrons with nuclei as a result of exchange of a massive scalar particle). In the case of S and V interactions, the electrons react to the nuclear density; for A and T interactions, they react to the spin density; in the case of a pseudoscalar (axion), the interaction disappears in the nonrelativistic limit. The bound from the Lamb shift in hydrogen gives $g_{\varphi ee} g_{\varphi NN} < 2 \cdot 10^{-8}$, and precision measurements of K_α transitions in heavy nuclei (Fm) give $g_{\varphi ee} g_{\varphi NN} < 10^{-6}$. The mechanism of production of φ particles as a result of interaction with nucleons can be regarded as similar to nuclear bremsstrahlung. The cross section of this process in heavy-ion collisions under the conditions of the GSI experiments is known: $\sigma(E_\gamma > 1.6\ \text{MeV}) \approx 500$ μb.[195] Since the cross section for the production of positron–electron peaks in the GSI experiments is approximately two orders of magnitude smaller, we may expect $\alpha_{\varphi NN} = g^2_{\varphi NN}/4\pi \gtrsim 10^{-2}\alpha \approx 10^{-4}$, $g_{\varphi NN} \approx 0.03$. Allowance for the fact that the GSI results give evidence against a too long lifetime of the hypothetical particle ($\tau < 10^{-10}$ sec) indicates that the coupling constant $g_{\varphi ee}$ in (20) is not too small. This gives $g_{\varphi ee} g_{\varphi NN} > 4 \cdot 10^{-7}$, a value that exceeds by more than an order of magnitude the bound from the Lamb shift in hydrogen given above. We emphasize that the bound on the production of a neutral particle due to a nucleon current does not go through for a pseudoscalar axion. The bounds from decays of vector mesons given above established the upper limits $g_{\varphi cc} < 4 \cdot 10^{-7}$ and $g_{\varphi bb} < 2 \cdot 10^{-5}$. Thus, if it is assumed that the axion was observed in the GSI experiments, then such an axion is "nonstandard"—it has different coupling constants with different quarks and must interact more strongly with the u and d quarks than with the remaining quarks and leptons. It was this that led to the creation of more "flexible" axion models.

We consider the possibility of production of a hypothetical particle as a result of interaction with the electromagnetic field.[191–193] The interaction Lagrangian depends on the type of particle and has the form

$$\mathscr{L}_S = g^S \varphi \mathbf{E}^2 = -\tfrac{1}{2} g^S \varphi F_{\mu\nu} F^{\mu\nu} \tag{33}$$

for a scalar particle and

$$\mathscr{L}_P = g^P \varphi \mathbf{E}\mathbf{B} = -\tfrac{1}{4} g^P \varphi F_{\mu\nu} \widetilde{F}^{\mu\nu} \tag{34}$$

for a pseudoscalar particle. The probability for emission of a neutral particle in a collision of ions is given by

$$\beta = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_k}\, |\widetilde{j}(\mathbf{k},\omega_k)|^2, \tag{35}$$

where $\omega_k = (\mathbf{k}^2 + m_\varphi^2)^{1/2}$, with Fourier transform of the current

$$\widetilde{j}(\mathbf{k},k_0) = \int d^4x e^{-ikx} g^S \mathbf{E}^2(x,t), \tag{36}$$

for example, for the case of a scalar particle. If, further, we specify a model of the motion of the nuclei in the complex in the scattering process—rectilinear,[192,193] rotation,[70,191] or vibration,[191,197]—then calculation of the momentum distribution of the produced neutral particles unavoidably leads to a large width $\Delta k \sim m_\varphi$ of the distribution. This also follows from qualitative considerations:[192] Since the strong electromagnetic field of the nuclei is concentrated in a small region with $R_0 \approx 10$ fm and varies rapidly in time, there are no natural restrictions for the width of the momentum distribution of the produced particles all the way to $k \approx R_0^{-1}$. Such an argument is of a general nature and independent of the particular mechanism of production of the neutral object if the source is a rapidly varying strong field concentrated in a small region. For example, the given considerations can serve as an additional argument against subsequently discussed new magnetic resonances in the positron–electron system and nucleating centers of a new special phase of the QED vacuum. In any such model, the momentum distribution of the produced neutral objects in the system of the colliding ions must be broad: $\Delta k \sim m$. Returning to the direct production of particles by the electromagnetic field, we recall that the GSI experiments yielded $\beta \sim 10^{-5}$, from which it follows (Refs. 70, 191, 193, and 197) that $g^S \approx 0.2$ MeV^{-1} ($g^P \approx 0.3$ MeV^{-1}), contradicting both the bound from Delbrück scattering[154] and the arguments that follow from the data on the relative probability of decay of the hypothetical neutral objects into positron–electron pairs or pairs of gamma rays. Indeed, for $g^{S,P} \approx 0.2$–0.3 MeV^{-1} the lifetime of the particle until decay into two gamma rays is $\sim 10^{-19}$–10^{-18} sec according to (30). It follows from the bounds obtained from the $g-2$ experiments for the electron that the lifetime until decay into a positron–electron pair is greater than 10^{-13} sec. Comparison of these quantities, and also the experimental data on the relative probability for the production of pairs of gamma rays and electron–positron pairs in heavy-ion collisions decisively rules out production of the neutral particles by interaction with the electromagnetic field.

We also mention here the exotic scenario of Ref. 70, according to which collision of the ions results in the formation of a rapidly rotating quasimolecule ($\omega \sim 850$ keV) for a time $\sim 10^{-19}$ sec. The frequency spectrum of the current (36) contains δ-function peaks, and particles are produced with a discrete set of energies. To obtain monochromatic positrons, it was postulated in Ref. 70 that the mass of the produced neutral particle is near $2m_e$, the momentum in the system of the ions is $p = 1.35$ MeV/c, and $g_{\varphi\gamma\gamma} \sim 0.3$ MeV^{-1}. This proposal was criticized in Ref. 197, where it was shown that even if one does not worry about the never proven possibility of fusion of nuclei into a quasimolecule for a time $\sim 10^{-19}$ sec and the unattainably large angular momentum of the molecule ($\sim 10^3 \hbar$) the proposed mechanism fails on account of the coupling constant ($g_{\varphi\gamma\gamma} < 10^{-5}$ MeV^{-1} from Delbrück scattering). One might suggest that the large momentum spread of the produced particle in the center-of-mass system of the ions obtained from the calculation could be avoided by assuming that the produced object that emits the positron–electron pair has a mass much greater than $2m_e$. This also agrees with the angular distribution of the positron–electron pairs, which does not always correspond to separation angle 180°. Such an assumption is not supported by the experimental facts. On the one hand, it was shown[198] that if the strong Coulomb field of the colliding ions is responsible for the particle production, then the angular distribution of the ions after scattering depends strongly on the mass of the produced particle. The corresponding experiment of the ORANGE group[200] showed that the measured angular distribution of the heavy ions in events with the production of narrow positron resonances agrees with the assumption that the produced object has mass $2m_e + E^{\text{kin}}_{e^+e^-}$, and the energy distribution of the ions indicates a very small energy loss of 1.5 MeV, admittedly with a 10-MeV uncertainty. Finally, in such a scenario one must ask this question: What kind of heavy particle remains after emission of the positron–electron pair?

Despite the "danger" noted above of introducing new vector particles, it was suggested[201] that the source of the positron–electron resonances in the GSI experiments could be a new pseudovector (1^-) particle with mass ~ 1.7 MeV produced in the strong electromagnetic field of the colliding ions as a result of interaction with three photons:

$$\mathscr{L} = \frac{\lambda}{4} F_{\mu\nu} F^{\mu\nu} F_{\lambda\sigma} G^{\lambda\sigma}, \tag{37}$$

where $F_{\mu\nu} = \partial A_\mu / \partial x_\nu - \partial A_\nu / \partial x_\mu$ is the electromagnetic field tensor, and $G_{\lambda\sigma} = \partial G_\lambda / \partial x_\sigma - \partial G_\sigma / \partial x_\lambda$ is the tensor of the new massive pseudovector field. In such an interaction, particles can be produced at rest in the center-of-mass system of the ions without appreciable dynamic suppression at small momenta, as in the case of production as a result of interaction with two photons (33)–(34).[70,191–193] From the known cross section for the production of positron–electron pairs at the GSI (~ 100 μb), the effective coupling constant was found[201] to be $\lambda \sim 3 \cdot 10^{-3}$ MeV^{-4}, which gives a lifetime until decay into three gamma rays of $\tau(G \to 3\gamma) \cong 10^{-12}$ sec; at the same time, one can achieve a fit with $\tau(G \to e^+e^-) \approx 3 \cdot 10^{-13}$ sec. This proposal was criticized in Ref. 202 because the unavoidable mixing of G bosons and photons in an external electric field in accordance with

$$\mathscr{L} = \frac{\lambda}{2} \mathbf{E}^2 F_{\mu\nu} G^{\mu\nu} + 2\lambda F^{0j} E_j E_k G^{0k} \tag{38}$$

would lead to an enhanced yield of positron–electron pairs in nuclear transitions (for example, an $E3$ transition ($3^- \to 0^+$) with energy 2.61 MeV in the ^{208}Pb nucleus) as a result of emission of a G boson and its subsequent decay. Comparison of a calculation in accordance with the theory of internal pair conversion[42] with experiment showed that the discrepancy in the yield of positron–electron pairs in ^{208}Pb does not exceed 10^{-4}, giving a bound $\lambda < 3 \cdot 10^{-9}$ MeV^{-4}.

4.2. Explanations of the effect in the framework of electrodynamics without invoking essentially new hypotheses

A characteristic feature of these models (and, incidentally, practically all the others) is the absence of a rigorous systematic treatment of the production of narrow positron–electron resonances. The papers merely indicate some circumstance that in principle is capable of generating structure in the spectrum of positrons or in the spectrum of positron–electron pairs. One can describe some features of the phenomenon, but no attempt is made to explain everything, this evidently being due to the great difficulty of describing such a complicated process. Giving almost no formulas, to say nothing of calculations, we shall merely note the aspects on which the authors of the various hypotheses concentrate, the assumptions used, and the aspects of the Darmstadt effect that can be described.

We mention first the attempts to explain the resonances in the positron spectrum by the formation, in heavy-ion collisions, of a quasimolecule with spontaneous positron production. A possible way of explaining the fusion of ions into a quasimolecule can be found if one assumes that in the potential of the nucleus–nucleus interaction there are one or several pockets (wells) that lead to the existence of resonance states in scattering. The calculations of Ref. 203 agree with the possible existence of such wells in U+U and U+Cm interactions. Because of the delay in scattering, there is enhanced spontaneous positron production. In Ref. 29, the coupled-channel method was used to calculate positron spectra, and resonances were obtained. The parameters of the potential were not calculated in Ref. 29; rather, they were fitted to the experimental results, using the positron peaks. There was also no attempt to explain the Z dependence of the energies of the positron peaks.

It was pointed out in Ref. 204 that rapid rearrangement of the electron shells in the two-center potential in a collision of ions may, under certain assumptions, give rise to oscillations in the energy spectrum of the positrons and electrons (the idea of rapid rearrangement of the electron shell is introduced *ad hoc*). In Ref. 205, the same authors, calculating the spectra of positrons and δ electrons for the below-critical colliding-ion systems Ta+Th and Au+U in the framework of the quasimolecular theory, did not obtain the required features. To ensure spontaneous emission of positrons for below-critical systems, they considered[205] the very hypothetical possibility of production of a long-lived spherical nucleus with radius $R_0 \sim (A_1+A_2)^{1/3}$. For the combination Au+U, the condition of criticality is ensured, and spontaneous positron emission must occur. However, the Z dependence of the energy of the positron peaks in this model does not agree with experiment.

In Ref. 206 calculations were made of the spectra of quasiatomic positrons in heavy-ion collisions with allowance for the dipole term in the two-center Coulomb potential of the colliding nuclei, in contrast to the earlier studies of Refs. 57 and 22, in which only the monopole part was taken into account in the expansion of the potential. In accordance with the result of Ref. 206, allowance for the magnetic dipole term can, because of interference with the monopole term, significantly change the spectrum, while at the same time having almost no influence on the total yield of quasiatomic positrons. The study obtained a resonance structure in the positron spectrum that agreed with the experiments (for example, a peak at energy 380 keV), and the peak energy depended weakly on Z. In Ref. 207, the result of taking into account the dipole term is the opposite—in the positron spectrum, the calculation gives no resonances, and the influence of the magnetic dipole term on the spectrum is very small. In the important range of internuclear separations the solutions of the two-center problem with the best possible accuracy, including the magnetic contribution,[208] do not give significant deviations from the monopole approximation. It was suggested in Ref. 209 that in heavy-ion collisions there is formed an open resonator for a positron between two Coulomb centers if it moves along the internuclear axis. On separation of the nuclei, the particle trapped in this expanding resonator is cooled. Estimates were obtained semiclassically and in the nonrelativistic approximation. The authors of Ref. 209 themselves remark that both assumptions are hardly sufficient to obtain correct results. From the approximate calculation, a peak in the positron spectrum was obtained for such a model problem. The production of monochromatic electrons correlated with positrons was not calculated in Ref. 209; it was merely noted that quasistationary states for electrons in the field of two separating Coulomb centers must also exist at nearly equal energies.

It was shown in Ref. 210 that vibrations of the nuclei in a long-lived quasimolecule can (through higher-order QED processes) give narrow positron–electron peaks with a number of interesting features, namely, peaks are possible in below-critical systems, there is a weak Z dependence, and there can be several resonances in the spectra. The intensity of the gamma transitions observed experimentally can be made fairly small in the model of Ref. 210. In the result of the calculations there is also no angular correlation of the emitted positrons and electrons. However, the model of Ref. 210 is based on the need for the production of a quasimolecule for the long time 10^{-16} sec; in addition, this version does not ensure the equality of the positron and electron energies observed in the GSI experiments. In the brief note of Ref. 211, attention is drawn to the possible production of positron–electron pairs in heavy-ion collisions as a result of electron excitations. If vacancies are formed in the K shell, the total excitation energy of the electrons in the unified quasiatom reaches ~ 2 MeV (as estimated in the Thomas–Fermi model). The electron density in each of the ions is significantly lower than in the ground state of the quasiatomic system with charge $Z_u = Z_1 + Z_2$ of the nucleus. Then it can be assumed that in the unified system there is a form of collective electron discharge with a change of the density. The electromagnetic field of such a transition has a monopole nature and wavelength of the order of the system's diameter. In accordance with the proposed model, the pair production takes place at large distances from the nucleus. The amplitude of pair production in an electron transition has the form

$$u_{\mathrm{if}} = \int \psi_\delta^+(\mathbf{r}_1)\psi_{1S}(\mathbf{r}_1)\,\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}\,\psi_{e^+}(\mathbf{r}_2)\psi_{e^-}(\mathbf{r}_2)\,d\mathbf{r}_1 d\mathbf{r}_2, \tag{39}$$

and the large distances give a small momentum of the positron–electron pair in the center-of-mass system of the quasiatom, i.e., the positron and electron move with high probability in opposite directions. Narrow resonances in the positron–electron spectrum can be obtained by virtue of the fact that, obviously, there is a shell electron structure in the quasiatom and, in addition, the interference of the resonance and nonresonance transitions may give oscillations in the pair-production cross section. Unfortunately, this interesting idea has hardly been developed at all in the quantitative aspect, and there are no answers to a number of questions, especially concerning the Z dependence of the energy and the cross section for production of positron–electron pairs in this model. In Ref. 212 it is noted that there is an analogy between the processes that occur in heavy-ion collisions and other phenomena observed in strong external fields, and it is pointed out that the cross section of various elementary processes oscillates in a strong external field, for example, in the case of two-photon production of positron–electron pairs. The oscillating part of the pair-production probability is represented in Ref. 212 in the form of the product $W=W_0W_1W_{e+}W_{e-}$, where W_0 is the probability of production in an ion collision of photons capable of producing a pair, W_1 is the probability of production of a virtual pair, and $W_{e+}W_{e-}$ is the probability that the produced leptons leave the region of the field; the oscillations are due to the last factors. It is found that the energies of the peaks in the positron spectra depend weakly on Z but strongly on the degree of ionization of the colliding ions. Note that this last quantity must have a statistical distribution with large spread, so that one must expect the peaks to be smoothed.

In the studies of Ref. 216 it was shown, initially for one dimension, and then for three dimensions with some important approximations, that in the two-center Coulomb problem for the Dirac equation there is a periodicity effect, namely, the positron transmission coefficient depends resonantly on the energy (resonance transparency). In accordance with this, the authors proposed a qualitative scenario according to which the positron–electron pairs are produced by "heating of the vacuum" already when the ions have approached to a separation of ~ 300 fm, and the pairs subsequently leave the region in which the nuclei collide in accordance with the resonance transparency of the two-center potential.

The authors of Ref. 213 considered a model of colliding ions with an interaction potential containing shallow (of depth 1 MeV) pockets, so that the compound nucleus which is produced has resonances. As additional arguments for the existence of such a potential, the authors invoke data on below-barrier fission and the observation of anomalons. The resonances in the nuclear system lead to a structure in the spectrum of the produced positrons that in accordance with the model reflects the details of the spectrum of the compound nucleus. Thus, in Ref. 213 two model potentials are introduced—the potential of the internuclear interaction and the potential of the electric field for the electron system in the quasiatom. Numerical calculations were made for the U+U system, and these exhibited irregularities in the spectrum of the produced positrons at near-resonance energies of the nuclear system.

Without claiming to explain the Darmstadt effect, the authors of Ref. 214 point out the possibility of resonance production of pairs of particles with spin 1/2 in a spatially homogeneous electric field that varies rapidly in time:

$$A_\mu=(0,0,A_3,0),\quad A_3(t)=\begin{cases} A, & 0\leqslant t\leqslant t_0 \\ 0, & \text{at the remaining time.}\end{cases}$$

It is obvious that this observation has an intimate connection with the Klein paradox on account of the mathematical analogy between the Dirac equation with a time-dependent electric field and scattering by a potential that is constant in time but depends on the coordinate. One can establish a connection between the transmission, T, and reflection, R, coefficients for the time-independent problem and the amplitudes for the production of lepton pairs in the time-dependent problem, namely, R can be related to the pair-production amplitude. The phenomenon must occur in fields greater than the critical field $E_c=m^2c^4/Ze^3$. In ion collisions, the electric field along the line joining the ions has a behavior in time like that of the idealized examples considered in Ref. 214. The result of the calculation shows that the energy and width of the resonances varies with Z, but not so strongly as predicted for spontaneous production, $\sim Z^{20}$. In this model, there is no attempt to explain why the positron and electron have equal energies and opposite momenta. In Ref. 215, the idea was tested for a spatially inhomogeneous field of the specific form $A_\mu=(0,0,A_3,0)$, $A_3=\delta(t)b(x_3)$ with certain discontinuities of the function $b(x)$, which, as is assumed in Ref. 215, are needed to obtain the effect of resonance pair production.

4.3. Magnetic quasibound states of a positron and an electron

It should be said immediately that here no attempt is made in the problem to discuss the extremely complicated relativistic two-body problem, which is far from solution. Because of the impossibility of finding an exact solution to the problem, various simplified approaches are often used. Several studies have been made on the basis of an approach first proposed by Kemmer, and also Fermi and Yang,[217] in accordance with which the Hamiltonian for several interacting particles is written in the form

$$H=\sum_i \alpha_i\mathbf{p}_i+\beta_i m_i+\sum_{ij} V_{ij}, \tag{40}$$

i.e., as the sum of single-particle Dirac Hamiltonians for free particles and terms that describe an interaction between them. Such an approach leads to satisfactory results in calculations in atomic and nuclear physics and in elementary-particle physics. In the general case, if the spinor nature of the interacting particles in the case of two interacting fermions is taken into account rigorously, the potential must be a 16×16 matrix in the spinor space. In a simplified approach, the interaction potential is chosen phenomenologically in accordance with the nature of the problem, most often in the form of one-boson exchange. The Coulomb interaction does not give quasibound states of the positron and electron in the range of masses observed in the GSI experiments. For the energy scale of atomic physics (distances $\sim 10^{-8}$ cm) magnetic interactions are small. However, at short distances the interaction due to the magnetic moment of the particles is large and for some quantum states is attractive. A particle with magnetic moment μ creates a vector potential $\mathbf{A}=\boldsymbol{\mu}\times\mathbf{r}/r^3$ that acts on another particle. Predictions of possible quasibound positron–electron states based on model introduction of a magnetic interaction have a certain history. We mention the attempt of Ref. 218 to explain the J/ψ particles discovered at that time in the framework of an electromagnetic model that considered the Dirac equation for an electron (e_2) with anomalous magnetic moment a in the Coulomb field of the positron (e_1) (or vice versa):

$$\left[c\boldsymbol{\alpha}\mathbf{p}-\left(E-\frac{e_1e_2}{r}\right)+\beta mc^2/2\right]\psi=-a\frac{\hbar e_2}{2mc}\frac{1}{r^2}i\beta\alpha_r\psi, \tag{41}$$

where $a=\alpha/2\pi$, $\alpha_r=\boldsymbol{\alpha}\mathbf{r}/r$. After some approximations, a radial effective potential for the electron was obtained in the form

$$V(r)=-A/r+B/r^2-C/r^3+D/r^4, \tag{42}$$

which has a very narrow and high barrier at positive energies and gives quasibound states with a mass of several giga-electron-volts, scale $\sim\alpha r_0$ ($\sim 10^{-15}$ cm), and a long lifetime. There subsequently appeared numerous studies that developed this idea, making it much more complicated, with the aim (in view of the impossibility of solving the problem "head-on") of finding an equation that describes the interaction of fermions at short distances (Refs. 218–220 and 222–224). Particularly great efforts in this direction were made by Barut and collaborators; a list of their publications is contained, for example, in Ref. 222. In a number of cases, small quasibound states were obtained in a mass range corresponding to the Darmstadt positron–electron peaks—"superpositronium" in the terminology of Refs. 218, 223, and 224 or "micropositronium" in the terminology of Ref. 221. In Ref. 220 there was derived from QED an exact, in the opinion of the authors, one-time completely relativistic equation of the type (40) for the description of the interaction of two leptons. The Lagrangian was obtained from variation of an action that, from the very beginning, contained, besides the minimal interaction $\Sigma_i e_i\bar{\psi}_i\gamma^\mu\psi_i A_\mu$, a Pauli interaction term $\Sigma_i a_i\bar{\psi}_i\sigma_{\mu\nu}\psi_i F^{\mu\nu}$ to take into account phenomenologically an anomalous magnetic moment a_i. The radial part of the interaction energy contained all terms of the Coulomb, spin–orbit, and spin–spin interactions:

$$\begin{aligned}V(r)=&\frac{e_1e_2}{r}(1-\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)-e_1a_2(\beta\sigma)_1\left(\boldsymbol{\alpha}_2\times\frac{\mathbf{r}}{r^3}\right)-e_2a_1(\beta\sigma)_2\\&\times\left(\boldsymbol{\alpha}_1\times\frac{\mathbf{r}}{r^3}\right)-ie_1a_2(\beta\alpha)_1\frac{\mathbf{r}}{r^3}+ie_2a_1(\beta\alpha)_2\frac{\mathbf{r}}{r^3}\\&-a_1a_2\beta_1\beta_2\left[\frac{3(\boldsymbol{\sigma}_1\mathbf{r})(\boldsymbol{\sigma}_2\mathbf{r}_2)}{r^5}-\frac{(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)}{r^3}\right.\\&\left.+\frac{8\pi}{3}\delta(\mathbf{r})(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\right]\\&+a_1a_2\beta_1\beta_2\left[\frac{3(\boldsymbol{\alpha}_1\mathbf{r})(\boldsymbol{\alpha}_2\mathbf{r}_2)}{r^5}-\frac{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}{r^3}\right.\\&\left.-\frac{4\pi}{3}\delta(\mathbf{r})(\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)\right].\end{aligned} \tag{43}$$

In Ref. 219, it was proposed to describe a positron–electron pair by a two-particle equation of Kemmer–Fermi–Yang type:

$$[\boldsymbol{\alpha}_1(\mathbf{p}_1-e_1\mathbf{A}_1)+\boldsymbol{\alpha}_2(\mathbf{p}_2-e_2\mathbf{A}_2)+\beta_1 m+\beta_2 m+V-E]\psi(\mathbf{r})=0, \tag{44}$$

where $V=e_1e_2/r$, $r=|\mathbf{r}_1-\mathbf{r}_2|$ is the distance between the particles, and the magnetic interaction was taken into account by introducing $\mathbf{A}_i=\boldsymbol{\mu}_j\times(\mathbf{r}_i-\mathbf{r}_j)/r^3$, the vector potential for interaction of a particle at the point $\mathbf{r}_i$ with a particle at the point $\mathbf{r}_j$; $\mu=e\hbar\sigma/2mc$. Thus, the magnetic interaction is manifested here as the static interaction of two magnetic dipoles. Solution of this equation led to an effective radial part of the potential for the $^3P_0$ state with a barrier of positive energy at distance $r\sim 10$ fm and a deep well at $r<1$ fm. In accordance with the calculation, the energy of the magnetic resonance in such a potential was found to be $E=1.579$ MeV, and the lifetime of the state was estimated at $\sim 4\cdot 10^{-19}$ sec. The decay of such a resonance gives a positron and electron each with energy 279 keV in the center-of-mass system. Although relatively rigorous quantitative predictions are made in this study, the authors themselves note that their treatment is too simplified.

In Ref. 227, the model proposed in Ref. 219 was investigated to find the behavior of quasistationary levels in the external Coulomb field of two colliding nuclei. Because of strong localization of the resonance ($\sim$5–10 fm), the energy of the resonance depends weakly on the external field, whereas the width increases when the field is included by more than an order of magnitude.

In Ref. 221, the conclusions of Refs. 220 and 219 were criticized. Using the same equations, (40) and (43), the authors of Ref. 221 did not obtain quasibound states. According to Ref. 221, the magnetic part of the effective attractive potential at small r becomes very large and contains terms up to $\sim r^{-5}$. If the anomalous magnetic moment is assumed to be constant, i.e., independent of the distance, then such a singular behavior of the potential leads to a divergence of the wave function at the origin and to non-normalizability. This must lead to fall to the center—instability of the e^+e^- system—like the behavior of an electron in an atom with $Z>137$ and a point nucleus. It is emphasized in Ref. 221 that the anomalous magnetic moment is a dynamical property which arises from the interaction of a charge with its own radiation field. Since the radiative interaction of a charge with the self-field depends on the momentum transfer, the anomalous magnetic moment is not constant but varies with the distance from the electron, approaching when $r\gg\hbar/m_ec$ the asymptotic value $\mu^0_{\text{anom}} = e\hbar/2mc[\alpha/2\pi + O(\alpha^2)]$. Accordingly, an investigation was made in Ref. 221 of the behavior of the effective radial potentials and the solutions under the assumption that the anomalous magnetic moment has a form factor $F(r)$:

$$\mu_{\text{anom}} = \mu^0_{\text{anom}} F(r), \quad F(r) = 1 - \exp(-r/r_0). \tag{45}$$

The cases of S and P states were investigated for several values of r_0 between 2 and 386 fm. The investigation showed that the radial effective potential does not contain barriers with $U>0$ that could give quasibound states and gives a strong repulsion at short distances. The introduction of the form factor of the magnetic moment[225] eliminated the singular behavior of the magnetic terms in the potential at short distances, so that the Coulomb repulsion remained the main interaction. At large distances ($\sim 10^{-8}$ cm) there appears a characteristic shallow potential well describing positronium. Another reason for the criticism in Ref. 221 of the approach proposed in the studies of Ref. 220 was the failure of the positronium spectrum to match the experimental data if the calculations were made to an accuracy higher than the first order in perturbation theory. The general conclusion of Ref. 221 is that the approach based on (40) is semiclassical in the sense that the electromagnetic field produced by the particles is described as a classical field of currents, the radiative corrections are taken into account by the introduction of an anomalous magnetic moment of the particles, and the particles themselves are described by single-particle wave functions of the Dirac equation. In the opinion of the authors of Ref. 221, such an approach leads to a number of unsatisfactory features and unphysical terms in the Hamiltonian, so that the equation of Ref. 220 is true only in the first order of perturbation theory, i.e., in the nonrelativistic limit of a static interaction. The answer to this criticism in Ref. 222, which contains no concrete calculations relating to the existence of a quasistationary state, considers very general questions of quantum electrodynamics at short distances and goes far beyond the subject of this review. We merely mention that a conclusion opposite to that of Ref. 221 is drawn concerning the behavior of the anomalous magnetic moment of a particle at short distances. It is asserted in Ref. 222 that, because of the self-interaction, which is concentrated in a region $\sim r_0 = e^2/m_e c^2$, it is this region, and not $r \sim \hbar/m_e c$, that plays the decisive role. In the presence of localization of an electron in a bound state, the self-interaction term increases, and the value of the anomalous magnetic moment increases with decreasing distance, so that at $r \sim r_0$ a value of order μ_0 is reached, whereas the normal magnetic moment is reduced by relativistic effects. With regard to the production of quasibound states in the scattering of very heavy ions, Barut assumes that the most important thing is a phase transition which occurs because of the presence of a self-interaction term in the two-body problem (which, nevertheless, is absent in the Lagrangian of Ref. 220). The possible existence of magnetic quasibound states in the positron–electron system is evidently still an open question.

In Ref. 223 there is an investigation of the problem of bound states of a positron and an electron on the basis of a "one-time Bethe–Salpeter equation":[226]

$$[E - (\boldsymbol{\alpha}_1 \mathbf{p} + \beta_1 m) - (-\boldsymbol{\alpha}_2 \mathbf{p} + \beta_2 m) - \Gamma(\mathbf{p}) V(\mathbf{r})]\psi(\mathbf{r}) = 0 \tag{46}$$

with potential

$$V(\mathbf{r}) = -\frac{e^2}{r}(1-B); \quad B = \frac{1}{2}\left[\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2 + \frac{(\mathbf{r}\boldsymbol{\alpha}_1)(\mathbf{r}\boldsymbol{\alpha}_2)}{r^2}\right] \tag{47}$$

and

$$\Gamma(\mathbf{p}) = \frac{1}{2}\left(\frac{\boldsymbol{\alpha}_1\mathbf{p} + \beta_1 m}{\sqrt{\mathbf{p}^2 + m^2}} + \frac{-\boldsymbol{\alpha}_2\mathbf{p} + \beta_2 m}{\sqrt{\mathbf{p}^2 + m^2}}\right), \tag{48}$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. The effective radial potential obtained from (46)–(48) for the $^1S_0$ and $^3P_0$ states contains, besides the Coulomb term, some others that are associated with the magnetic interaction and lead to a high maximum in the potential near $r_0 = e^2/m_e c^2$. The cross section for positron scattering by electrons calculated on the basis of this potential exhibits many strong resonances in the range of c.m.s. energies 100–1000 keV. The width of the resonances in the range of masses of the Darmstadt peaks reaches hundreds of kilo-electron-volts. It is argued in Ref. 223 that calculations made there agree with the GSI experimental data and predict a broader spectrum of resonances than the one found in the Darmstadt experiments.

In their second study of Ref. 224, the same authors take as their point of departure Barut's equation[220] and arrive at the same results. We note here that experiments on positron–electron scattering do not indicate the presence of resonances in the mass range of the Darmstadt peaks and can serve as a good test of the various theoretical approaches to the relativistic two-body problem alongside the high-precision data on the energy levels of the hydrogen atom, positronium, and muonium. We also mention the strong restrictions on the existence of resonances in the positron–electron system that follow from the data on the electron g factor.

4.4. Composite extended particle consisting of new constituents

As the previous discussion has shown, it did not prove possible in theory and experiment to find a point particle whose production in heavy-ion collisions could explain the Darmstadt effect. On the one hand, such a particle must be produced in ion collisions with a large cross section, several tens of microbarns, but on the other it is not manifested in a host of other experiments, and its existence is strongly restricted by comparison of high-precision experiments and calculations in QED—for the electron g factor, etc. Because of the difficulties with a point particle, consideration was given to the possibility of constructing a composite extended model satisfying the listed requirements. It is obvious that the beam-dump experiments lose their restrictive power if the composite object is sufficiently large, so that the range of the particle is small. A composite particle possesses an internal wave function with a form factor. In the momentum space, this usually means that the effective coupling constant is suppressed at large momentum transfers. This helps to explain the low velocity in the center-of-mass system for e^+e^- events in the GSI experiments. However, to obtain momenta $\lesssim 0.1$ MeV/c, the objects must be extremely large, $\sim 10^3$ fm. In addition, such objects with internal structure must possess a complicated excitation spectrum; this does not contradict the set of peaks observed in the GSI experiments. It was found to be impossible to construct a composite particle in the framework of the standard model, but if one allows the existence of new, relatively light fermions as composite parts of the putative extended objects, then many restrictions can be avoided. It is found that the previous searches with negative outcome had a loophole, namely, the possible existence of particles on the scale 10^{-11}–10^{-10} cm.[228] A model of an extended particle consisting of new fermions f^+ and f^-, developed in detail, is described in Refs. 229 and 230. It uses an analogy with the structure of mesons, and the properties of (f^+f^-) states are investigated in the framework of the model of the spherical MIT bag. The motion of the fermions in the bag is described by the Dirac equation

$$(\hat{p}-m_f)\psi=0, \quad r<R_X, \tag{49}$$

where R_x is the bag radius, m_f is the mass of the new fermion, and there is a linear boundary condition:

$$-i\gamma_r\psi=\psi, \quad r=R_x. \tag{50}$$

Stability of the object is ensured by the boundary condition

$$\tfrac{1}{2}\partial_r(\bar{\psi}\psi)=B_x \quad \text{at } r=R_x, \tag{51}$$

where B_x is the unknown vacuum pressure. The solution gives a spectrum of states. In such a model, there are three free parameters: the bag radius R_x, the vacuum pressure B_x, and the constituent mass m_f of the fermion. The experimental data give a state energy $E_{\text{bag}}\cong 1.8$ MeV. It remains to vary m_f in the range $0<m_f<M_x/2=900$ keV, and it is found that in this range the vacuum pressure is very small, $B_x^{1/4}<240$ keV, so that the mechanism of particle confinement in such a bag can hardly be associated with any known mechanism. An energy spectrum that satisfies the experimental requirement that the separation between the levels be ~ 60 keV requires a mass $m_f\approx 830$ keV. The corresponding radius is $R_x\gtrsim 10^3$. Such a mass of the fermions constitutes an appreciable fraction of the total mass of the extended particle, and this makes it possible to describe this state in terms of a nonrelativistic potential model with a potential of the following form chosen in Ref. 230:

$$V(r)=a_x r, \quad r>r_0$$
$$V(r)=V_0, \quad r<r_0. \tag{52}$$

Here, V_0 describes a repulsive core, and a_x is the tension coefficient of the confinement potential, found here to be $a_0\cong 0.05$–0.1 keV/fm. Such a model made it possible to describe the bound states of an extended object in the field of the colliding ions and to calculate the cross section for the production in ion collisions by analogy with the dynamical production of e^+e^- pairs. Because the mechanisms of production of e^+e^- pairs and (f^+f^-) states are the same, the production cross section behaves as $\sigma_x\sim Z^{20}$, as is observed experimentally (Fig. 14). The dynamical behavior of the produced (f^+f^-) object during the collision process is not described in Ref. 230. Depending on the quantum number of the state, the decay must take place either through the channel $(f^+f^-)\to\gamma\gamma$ for 0^{-+} states or through $(f^+f^-)\to e^+e^-$ for the state 1^{--} ($\Gamma_{ee}\approx 10^{-4}$ eV) by analogy with charmonium decays. Because of the large size, the scattering cross section in a medium, $\sim\pi R_x^2$, is very large, so that the range is $\sim 10^{-6}$ cm. The presence in this model of a hard core, which leads to repulsion of the fermions at short distances within the extended particle, saves this model from the restrictions that follow from the electron g factor. Indeed, the contribution of the virtual states to the anomalous magnetic moment of the electron and muon could be made much smaller than the possible difference between experiment and theory without allowance for this contribution. This last requirement led to an important restriction on the parameters of the potential (52): $V_0>1$ GeV, $r_0<1$ fm. An even more stringent requirement arose from the need to match to experiment the cross section for the production of the (f^+f^-) objects in the GSI experiments: For $m_f=850$ keV, $V_0>100$ GeV, $r_0<10^{-2}$ fm. It is suggested in Ref. 230 that an achievement of the model is the possibility of calculating a production cross section in agreement with experiment and also the possibility that the particle can have a significant probability for remaining at rest in the center-of-mass system or being captured by one of the ions.[231] In the latter case, there is a shift of the difference spectrum of the positrons and electrons to higher positron energies and a violation of the exact kinematics of 180° decay. The most serious problem for such a model comes from the results of the experiments on e^+e^- scattering (see Fig. 19), in which no indications of resonances with width $\Gamma>10$ μeV were found. A possible scenario for saving the idea of an extended (f^+f^-) object is the following:[231] It can be assumed that the ground state is stable with respect to decay into an e^+e^- pair, and for such decay one requires a strong external field to induce the decay $(f^+f^-)\to e^+e^-$. For example, this is possible for the state $1^{++}(^3P_1)$. In an external field, this decay is enhanced in proportion to the Z^2 value of the heavy target. The excited (f^+f^-) objects produced in the GSI experiments emit photons on descent to the $^3P_1$ ground state. In the experiments on e^+e^- scattering, (f^+f^-) can be produced through the annihilation diagram by one virtual photon, i.e., in the 1^{--} state with the main decay mode being photon emission and transition to a lower level. The 1^{++} and 1^{--} states are separated in energy, and in a target with small Z decay of the 1^{++} state to an e^+e^- pair is not enhanced. Estimates of the degree of enhancement of the decay to an e^+e^- pair in a heavy target[231] do not as yet confirm this idea quantitatively.

In the studies of Ref. 232 the authors advanced a no less far-reaching model of a composite particle based on a theory of leptons as composite objects possessing color excitations. According to this model, the Darmstadt e^+e^- pairs are a consequence of decays of "leptopions" confined by color-like forces; the "leptopions" are formed from color excitations of e^+ and e^-. The mass of the color lepton in this model is 0.3 MeV.

4.5. New phase in quantum electrodynamics

In this subsection, we consider qualitatively, and very briefly, the idea of Ref. 235, which is that the strong electromagnetic field produced by the heavy ions may give rise to a phase transition in the QED vacuum. The coupling constant in this phase is ~ 1, and the narrow e^+e^- states observed in the GSI experiments can be regarded as the analogs of positronium in ordinary QED. Such a scenario has a number of attractive features, since it can explain, at least qualitatively, many experimental data. First, in the framework of this picture one can understand why a phenomenon like the one observed at the GSI in collisions of very heavy ions is absent in other experiments in which an extremely strong and rapidly varying electromagnetic field $E\sim 10^{16}$ V/cm (corresponding to a concentration of field energy ~ 1 MeV in a region measuring 10^2–10^3 fm) is not created. After the heavy ions have separated and the strong fields have disappeared, the new vacuum becomes unstable and decays in time with emission of an e^+e^- pair. If there are no fields, the particles separate with different energies in opposite directions. In such a bubble of the new phase, several excited states can be realized, and this is confirmed by experiment.

The Z_u independence of the energies of the states is also explained, since the spectrum of states is determined by the properties of the new phase itself, and the ion charge plays only the role of a trigger facilitating the formation of a nucleating center of the new phase. An important success of this approach can be seen in the possibility of calculating the mass spectrum of the "electromesons" that arise in the new phase; this spectrum agrees quite well with the GSI experiments. The calculation is made in the framework of several approaches: in the bag model,[243] in a potential model,[236] and by lattice calculations.[246]

What rigorous grounds are there for such a scenario? One can here pose several questions to which there are as yet no clear answers. First, what is the mechanism in accordance with which the field of the colliding ions can facilitate the transition to the new phase? The existing calculations[240] made to investigate QED in the presence of an external static Coulomb field suggest that the opposite effect is more probable. Second, it appears doubtful that the new phase will be metastable, i.e., that it can in principle exist in the absence of special external conditions, the nature of which is also as yet unclear. Third, why must the new phase, having been formed, decay into an e^+e^- pair and, for example, not into many photons? Many studies[233–252] have been devoted to various aspects of this exceptionally complicated problem; references to earlier investigations of a possible new nonperturbative phase of QCD, which have continued already for more than 25 years and are not associated with the Darmstadt peaks, can be found, for example, in the review of Ref. 245. It has been argued purely qualitatively[252] that in principle a new QED phase may also exist around static heavy nuclei. Scattering of heavy ions, which creates a powerful electromagnetic perturbation,[237] expels the previously formed nucleating centers of the phase from the neighborhood of the nuclei, after which they decay and are manifested in the form of an e^+e^- pair. The experiments of Refs. 142 and 143, which were inspired by such considerations and had the aim of seeking $\gamma\gamma$ pairs from spontaneous fission and α decay of heavy nuclei, did not give any positive signals. Only in the experiments of Ref. 136 on positron scattering by Th nuclei were, in the opinion of the authors of these studies, very narrow electron peaks observed.

4.6. Relativistic Coulomb resonances in the continuum

In a series of studies, Arbuzov *et al.*[253–258] drew attention to the possible existence of quasistationary states with positive energy in a system of two charged particles with equal masses.

Stationary states in nonrelativistic quantum mechanics embedded in the continuum were found in an early study of von Neumann and Wigner.[259] They found a special case of a central potential with decrease at infinity as $U(r)\sim\sin(2r/r)$ in which a level with positive energy $E=1$ has a norm, i.e., there is an isolated positive stationary level embedded in the continuum. In Ref. 260, this phenomenon was considered from a more general point of view, and a larger class of such nonrelativistic potentials was investigated.

In the relativistic two-body problem, investigated in Refs. 253–258 by the Logunov–Tavkhelidze quasipotential method, the same phenomenon was found, first for scalar particles exchanging scalar "photons" (Wick–Cutkosky model),[253–256] and then for fermions as well.[257,258]

The quasipotential equation in the momentum space used in this approach has the form

$$2\omega(M-2\omega)\varphi(\mathbf{p})=\frac{1}{(2\pi)^3}\int\frac{d\mathbf{p}'}{2\omega'}V(\mathbf{p},\mathbf{p}'|M)\varphi(\mathbf{p}'), \tag{53}$$

where M is the mass of the state, $\omega=(\mathbf{p}^2+m^2)^{1/2}$, $\omega'=(\mathbf{p}'^2+m^2)^{1/2}$, and the quasipotential is

$$V(\mathbf{p},\mathbf{p}'|M)=\frac{(2me)^2}{|\mathbf{q}|(M-\Omega+io)}, \tag{54}$$

in which $\mathbf{q}=\mathbf{p}'-\mathbf{p}$, $\Omega=\omega+\omega'+|\mathbf{q}|$.

In the nonrelativistic case $|\mathbf{p}|$, $|\mathbf{p}'|\ll m$, the potential has the form

$$V(\mathbf{p},\mathbf{p}'|M)=\frac{(2me)^2}{|\mathbf{q}|(\varepsilon-|\mathbf{q}|+io)}, \tag{55}$$

where $\varepsilon=M-2m$ is the binding energy, and in the coordinate representation

$$V(r)=-\frac{2\alpha}{\pi}\frac{1}{r}\{\cos(\varepsilon r)[\mathrm{si}(\varepsilon r)+\pi] -\sin(\varepsilon r)\mathrm{ci}(\varepsilon r)+i\pi(\varepsilon r)\}. \tag{56}$$

As $r \to \infty$, the real part of the potential has the asymptotic behavior

$$\mathrm{Re}[V(r)] \xrightarrow[r\to\infty]{} -2\alpha\cos(\varepsilon r)/r, \tag{57}$$

which is similar to the von Neumann–Wigner potential.

Stationary levels arise in such a potential because of the coherent destructive interference at infinity of waves reflected from the crests of the potential.

Numerical calculations by the method of splines gave a spectrum of very often irregularly situated levels for the (e^+e^-) and (e^-e^-) systems with separation between the levels from a few kilo-electron-volts, beginning with a level at ~ 1023 keV. Some of the levels coincided with the GSI observations. The same approach, applied to a system of two protons, also gave a rich spectrum of levels, some of which agree well with experimental data[264] in which narrow diproton resonances have been observed. In subsequent calculations,[257] in which a quasipotential equation was derived and solved for two spinor particles interacting with the electromagnetic field, it was found that the position of the levels is not changed, i.e., the phenomenon is completely determined by the Coulomb part of the interaction.

It was proposed further in Ref. 258 that Coulomb resonances in the continuum can also be realized in a system of two colliding ions, this being confirmed, in the opinion of the authors, by evidence for a resonance behavior of the cross section for the production of the positron peaks as a function of the energy of the incident ions. In the same study, the idea of Coulomb resonances is extended to three particles, for example, one of the heavy ions, a positron, and an electron; in this case, the decay of the resonance takes place under kinematic conditions for which the heavy ion takes up the recoil momentum, so that the positron and electron do not necessarily separate at 180°, and the spectrum of particles is shifted to higher positron energies—a scenario analogous to the one proposed in Ref. 231.

This conclusion that there are resonances is in part confirmed by calculations made in Ref. 261 without the use of the quasipotential approach on the basis of various initial relativistic equations and approximations. In Ref. 261, six resonances with zero width were obtained, some of them coincident with the GSI resonances. In that study too, numerical results were obtained by using the method of cubic splines. There are some important differences in the physical meaning of the resonances obtained in Refs. 253–258 and in Ref. 261. The former, obtained from an oscillating Coulomb quasipotential, occupy a region in space of the order of several oscillation periods of this potential, $\sim 5\cdot 10^{-10}$ cm, i.e., they are very extended. The latter, according to the estimate of Ref. 261, have a characteristic scale $(\hbar/mc)\alpha^{1/2} \approx 30$ fm, i.e., are fairly compact. In the later study of Ref. 262 resonances were obtained in the electron–proton system in the electron-volt range of positive energies.

The authors of Ref. 263, as in Refs. 253–256, investigating the same scalar QED model, nevertheless did not find resonances with positive energy. In Ref. 263, this contradiction was explained by the details of the computational procedure. In Ref. 265, Zastavenko noted that the analogy between the resonance levels obtained in Refs. 253–258 and 261 and the stationary states in the continuum discovered by von Neumann and Wigner in nonrelativistic quantum mechanics[259,260] can hardly be close. His result is that the potentials of Refs. 259 and 260 give in the first order of perturbation theory singular scattering amplitudes at the resonances, whereas the quasipotentials of the relativistic Coulomb problem (Refs. 253–258 and 261) do not possess this property.

4.7. Many-electron complexes

In a number of studies it has been suggested that there may exist many-electron complexes, i.e., composite objects consisting of three, four, or more electrons and positrons that are produced in heavy-ion collisions and then decay with emission of a monochromatic positron or electron[266,271] or with emission of a positron–electron pair with definite energy.[268,270]

The point of departure for Wong[266] was the identity of the positron energy in annihilation of the weakly bound complex in $e^+e^+e^- \to e^+\gamma$, $E_{e^+} = 340$ keV, and the energy of one of the GSI positron peaks (before the discovery of e^+e^- pairs). In his scenario two e^+e^- pairs are produced spontaneously when the ions collide; one of them is annihilated near the positron (or electron) from the other pair in accordance with the considered process. In contrast to the long predicted and in part experimentally investigated[267] "atomic" complexes $e^+e^+e^-$ and $e^+e^-e^-$ of large size ($\sim 10^{-8}$ cm) and small binding energy (~ 0.3 eV), in the mechanism described in Ref. 266 the complex is more compact, with scale 10^{-11}–10^{-10} cm on account of the relativistic contraction of the Coulomb orbits of the positron and electron near nuclei with $Z > 150$. Because of the compactness, the process is also possible with strict correlation of the positron and photon. For the "atomic" complex, the main decay mode is $(e^+e^+e^-) \to \gamma\gamma e^+$.

In Refs. 268 and 269 there was a discussion of the possible existence of $(e^+e^-)^n$, $n=2,3,\ldots$, complexes. To bind the particles into a complex, the authors of Refs. 269 introduced a nonlinear interaction:

$$\mathscr{L}^S = \lambda_S(\bar\psi\psi)^n \text{ or } \mathscr{L}^V = \lambda_V(\bar\psi\gamma_\mu\psi)^n. \tag{58}$$

In their subsequent study,[269] these authors showed that their Lagrangian leads to a contradiction with QED, requiring an anomalously large coupling constant.

In a series of studies, Griffin[270] developed the hypothesis of quarkonium $e^+e^-e^+e^-$, a fairly "strongly" bound object with binding energy $\sim m_ec^2$ (this value was not calculated but assumed).

Because the binding energy is so large, the object can certainly be produced in heavy-ion collisions when $Z < Z_c$ (the production cross section was not calculated). After separation of the ions, the $e^+e^-e^+e^- \to e^+e^-$ decay process agrees kinematically quite well with the GSI data.

In Ref. 271, Baldin suggested that there could be a connection between results of experiments quite widely separated in time and, at the first glance, in the nature of the processes that occur, namely, the GSI experiments on the production of e^+e^- pairs in the collision of heavy ions and the old experiments of Skobel'tsyn,[272,273] who investigated in a Wilson chamber the scattering of electrons with energy 1–3 MeV from RaC(^{214}Bi) decay. Investigating the angular distribution of the scattered electrons and simultaneously measuring their energy before and after scattering using the curvature in a magnetic field, Skobel'tsyn observed anomalously large and inelastic scattering of the electrons through large angles. In his later study of Ref. 274, devoted to analysis of the previous experiments, he interpreted his observations, not as scattering, but as production in the RaC decay with probability 7–12% and decay in flight of a particle with mass $\sim 3m_e$ and lifetime $(2\text{–}5)\cdot 10^{-10}$ sec into an electron and a neutral particle (a neutrino in accoradance with Skobel'tsyn's hypothesis).

It must be said that these remarkable results have as yet found neither unambiguous refutation nor confirmation. According to the hypothesis of Ref. 271, Coulomb resonances of the type of Refs. 253–258 can be formed not only from two but also from three particles, in agreement with Skobel'tsyn's interpretation.[274]

This hypothesis was tested in two experimental studies.[275,276] The first investigated the β decay of ^{90}Y, which emits an electron spectrum with limiting energy 2.29 MeV practically without emission of γ rays. Under the assumption that the hypothetical $e^+e^-e^-$ particle is stopped before decay in the absorber and decays into an electron and a monochromatic γ ray, this made it possible to establish a very strong limit $\sim 10^{-5}$ on the probability of emission in ^{90}Y decay of an $e^+e^-e^-$ particle with mass $\sim 3m_e$. In the same study, an analysis was made of the γ spectra (in accordance with the literature) of the β^- decays of 57 nuclei (including ^{214}Bi) with the aim of identifying in them a distinguished γ line that could be responsible for the decay $e^+e^-e^- \to e^-\gamma$ after the stopping of the decaying particle. A similar analysis was made for β^+ decays of 42 nuclei with the aim of finding the decay $(e^+e^+e^-) \to e^+\gamma$. This statistical analysis established a 0.5% limit on the probability of appearance of a three-electron cluster in β decay of a nucleus.

In Ref. 276 the authors proposed a direct experiment to investigate the possible production in Ra decay of $e^+e^-e^-$ particles in the range of masses 1.5–2.0 MeV/c^2 and lifetimes $5\cdot 10^{-11}\text{–}3\cdot 10^{-9}$ sec with subsequent decay $e^+e^-e^- \to e^-\gamma$ in flight. The measurements gave a bound on the probability of such a process at the level 10^{-3} on RaC decay.

CONCLUSIONS

Thus, the main features of the Darmstadt effect present a serious problem for interpretation and have not been explained. We list these features.

1. The energies of the e^+ and e^+e^- peaks, which are concentrated in the range of masses 1.5–1.8 MeV, are practically independent of the total charge $Z_u = Z_1 + Z_2$ of the colliding heavy nuclei, and some resonances coincide with good accuracy for different Z_u.

2. The cross section for the production of the resonances in collisions of very heavy nuclei is, according to different data, from fractions of to a few μb/sr and behave, according to the data of the ORANGE group, as $\sim Z_u^{20}$.

3. The widths of the e^+ peaks, $\sim$40 keV, and of the e^+e^- peaks, up to 10 keV, indicate a long ($\sim 10^{-19}$ sec) lifetime of the system whose decay could give the observed phenomenon.

4. The data indicate that the center of mass of the possible decaying e^+e^- system moves in the center-of-mass system of the colliding ions with low velocity not exceeding $(0.03\text{–}0.05)c$.

5. The phenomenon also occurs when $Z_u < Z_c = 173$.

6. Some of the e^+e^- peaks indicate 180° separation of the particles, while others, having a very narrow width of the peaks, do not correspond to 180° separation.

7. There is an indication that, as a function of the energy of the incident ions, the cross section of the process has a resonance nature.

8. The upper limit for the probability of production of pairs of γ rays in this range of invariant masses is smaller than the e^+e^--pair production probability by 1–2 orders of magnitude.

In numerous experiments in nuclear physics and elementary-particle physics analogous phenomena have not been found. Not one of the large number of attempts to find a complete theoretical scenario for the phenomena observed in the GSI experiments has as yet achieved success.

It is a pleasant duty to note that the idea of writing this review was insistently suggested by Professor V. A. Khalkin. I am very grateful to Academician A. M. Baldin for his stimulating interest in the problem and to L. I. Zastavenko for some comments.

[1)]The first observation of a peak in the positron spectrum in the ORANGE spectrometer was noted in Ref. 64 in U+U and U+Th scattering.

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Translated by Julian B. Barbour