Influence of fission dynamics on formation of fragment charge distribution

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The results of calculations of the isobaric charge distribution of fission fragments are reviewed. The calculations were made in the framework of dynamical approaches based on the Fokker–Planck equation for the distribution function of the collective variables and a Schrödinger equation with friction for a harmonic oscillator with time-dependent coefficients. Three collective coordinates were taken into account: the main fission coordinate, i.e., the distance between the centers of mass of the nascent fragments (or elongation parameter), the neck parameter, and a collective coordinate that determines the charges of the future fragments (charge coordinate). The use of the dynamical model of the formation of the charge distribution shows that statistical equilibrium with respect to the charge mode can be established at each time during almost the entire descent from the saddle point to scission, except for the very final stage directly preceding the breaking of the neck. This explains the successful use of the statistical model for calculations of the fission-fragment charge distribution. The deviation of the variance of the charge distribution from its equilibrium value in the final stage of the process before fission is due to the sharp growth of the inertia and friction coefficients of the charge mode. This growth "freezes" the variance of the charge distribution. The study of the variances of the fragment charge distribution as a function of the excitation energy indicates that the fluctuations of the charge mode in fission are mainly of a quantum nature. The possibility of estimating the magnitude and mechanism of nuclear viscosity in fission from the observed even–odd effect in the yields of the fragment charges is discussed.

INTRODUCTION

Among the unresolved problems of fission physics, those of the fission dynamics and the formation of the experimentally observed fragment distributions occupy a central position. The two problems are intimately related—if the dynamics is unknown, one can hardly hope for full and detailed understanding of the complicated picture of formation of the observed fragment distributions.

In recent years, fairly extensive experimental investigations have been made of the mass–energy[1–3] and charge[4] fission-fragment distributions. The new information provided by these extremely laborious experiments is reviewed in Ref. 5.

The mechanism of formation of the mass–energy distributions is very complicated and is determined by the dynamics of the descent of the fissioning nucleus from the saddle point to the fission point. The dependence of the descent dynamics on a large number of different factors (relief of the energy surface, the nuclear viscosity of the collective motion manifested in fission, the configuration of the fission shapes of the nucleus, the influence of fluctuations of the collective variables, etc.) has the consequence that, as was shown in Ref. 6, the formation of the mass–energy distributions cannot be explained in the framework of the statistical[7,8] and dynamical[9,10] fission models traditionally used for this.

Some progress in understanding the formation of the mass–energy distributions was achieved in Refs. 11 and 12 in the framework of a new approach (diffusion model) to fission dynamics based on a multidimensional Fokker–Planck equation. A physically equivalent formulation of this approach for the description of fission dynamics, called the diffusion model,[11–14] can also be realized[15,16] on the basis of a system of stochastic Langevin equations, which is equivalent to the multidimensional Fokker–Planck equation.

Detailed and systematic comparison of the predictions of the diffusion model with experimental data makes it possible to obtain valuable information on the dynamic and static characteristics of the fissioning nucleus that, ultimately, determine the nature of the dynamics.

With regard to the charge distribution of the fission fragments, until recently the situation appeared much simpler compared with the mass–energy distributions, and it seemed that the charge distribution was not of great interest for understanding the dynamics of the process.

The extensive experimental information on the isobaric charge distribution of fission fragments (the distribution of fragments of fixed mass with respect to charge) can be briefly summarized in the following terms. The fragment charge distribution has the form of a simple single-hump curve which can be approximated to good accuracy by a Gaussian distribution with mean value Z_p and variance σ_Z^2 (Refs. 5, 6, and 17). The mean value—the most probable charge of the fragments—differs appreciably from $Z_{\rm UCD}$, the charge corresponding to unchanged charge density of the fissioning nucleus.[5,17] For actinide fission, $\Delta_p = Z_p - Z_{\rm UCD}$ varies from 0.4 to 0.8 (Ref. 18) for the fragment with the most probable mass, and for a light fragment $\Delta_p > 0$. The variances of the charge distribution (measured by various methods) are in the range from 0.3 to 0.5 (Refs. 18 and 19) and remain constant or depend weakly on the excitation energy[5,6,17] up to about

$E^*=40$–50 MeV. This weak dependence of σ_Z^2 on E^* is often regarded[20] as an indication of a quantum nature of the fluctuations of the collective coordinate that describes the redistribution of the charge on fission.

An estimate of the characteristic times of the charge mode—the vibration period and relaxation time[6,21,22]—and comparison of them with the typical descent times of the nucleus from the saddle point to scission support the conclusion that the descent is a quasiequilibrium process with respect to the charge mode, so that the statistical approach can be used to describe the formation of the charge distribution.

The recently achieved progress in experimental methods of measuring the fragment charges led to new and rather unexpected experimental data (for more details see Ref. 5). Detailed study of the fragment charge distribution and use of the dynamical approach to study its formation showed[5,6] that it, like the other distributions, can be a valuable source of information about the dynamics of the process, particularly in its final stage directly preceding separation of the nucleus into fragments and the actual breaking of the neck joining the incipient fragments. The experimental data on the fragment charge yields offer hope[4,5] of estimating the viscosity of the collective nuclear motion that leads to fission.

The aim of this review is to present and analyze the main results of study of the formation of the isobaric charge distribution of the fragments in the framework of dynamical approaches. The main attention will be devoted to the influence of the fission dynamics on the parameters of the charge distribution predicted in the calculations. In a certain sense, this paper can be regarded as a continuation of the review of Ref. 11, in which a dynamical approach—the diffusion model—was used to consider the formation of the mass and energy fission-fragment distributions of heated nuclei. Many dynamical aspects of the formation of the isobaric charge distribution of fission fragments and the products of deep inelastic collisions of heavy ions were discussed earlier in Refs. 6 and 23.

Section 1 describes a model of independent separation of the mass and charge on fission. This makes it possible to consider the division of the charge between the fragments for all continuous shapes of the fissioning nucleus on its descent from the saddle point to scission. In the model it is assumed that the charge density is different in the nascent fragments but constant within each of them. Such a distribution of the charge density in the fissioning nucleus is a first approximation to the real distribution, but it nevertheless reflects well the consequences of the polarization of the nuclear matter on fission.

The model is justified by comparing its predictions with results of the variational problem of the distribution of the charge density in the hydrodynamic model and the droplet model.

It has already been noted that the estimates of the characteristic times of the charge mode in fission—the vibration period and the relaxation time—indicate that the statistical approach can be used to calculate the charge distribution. Section 2 presents the results of statistical calculations of a charge distribution in which the level density was calculated in a superfluid model using single-particle spectra in a finite potential of Woods–Saxon type.

In Sec. 3, we consider the formation of the fragment charge distribution in the diffusion model based on the multidimensional Fokker–Planck equation.

At the end, we summarize the main conclusions and discuss unresolved problems of dynamical calculations of the fission-fragment charge distribution.

1. MODEL FOR DESCRIBING INDEPENDENT DIVISION OF MASS AND CHARGE ON FISSION

Parameters describing separation into fragments with different charge densities

The fragment charge distribution results from the redistribution of the charge distribution of the fissioning nucleus into the charge densities of the fragments formed at separation. Therefore, if the formation of the charge distribution is described in terms of collective variables, the parameters of the charge density of the fissioning nucleus must be regarded as corresponding collective coordinates in addition to those that characterize, for example, the shape of the nucleus.

If the division of the nucleus with respect to the charge mode is a quasiequilibrium process and the statistical approach can be used to describe the charge distribution, then to determine the fragment charges one can use[8,24] a criterion of minimum potential energy or some other thermodynamic potential. The first requires solution of a variational problem for the distribution of the charge density in the framework of a macroscopic model. According to experimental data on the fission-fragment charge distributions,[17,18] the polarizability of the nuclear matter in fission is small but a well-established fact, and as a first, rough approximation it can therefore be assumed that the charge densities are different in the nascent fragments but constant within each of them:

$$\rho(\mathbf{r})=\begin{cases}\rho_L^p, & \mathbf{r}\in v_L,\\ \rho_H^p, & \mathbf{r}\in v_H.\end{cases} \tag{1}$$

Here and in what follows, the subscripts L and H refer to the light and heavy fragments, respectively, and v_L and v_H are the cluster volumes.

Various parameters can be used to characterize the deviation of the fragment charge densities from the unchanged charge density (UCD) of the original nucleus.[25–28] In our model,[25] we characterize the variation of the charge density by a parameter ζ introduced as follows. For unchanged charge distribution of the nucleus, the excess of neutrons over protons per nucleon in the light and heavy fragments,

$$\delta_L=\frac{N_L-Z_L}{A_L}=\frac{\rho_L^n-\rho_L^p}{\rho_L},\quad \delta_H=\frac{N_H-Z_H}{A_H}=\frac{\rho_H^n-\rho_H^p}{\rho_H}, \tag{2}$$

is equal to the corresponding quantity for the initial nucleus:

$$\delta=\frac{N-Z}{A}=\frac{\rho_0^n-\rho_0^p}{\rho_0}, \tag{3}$$

where ρ^p, ρ^n, and $\rho_0=\rho_0^n+\rho_0^p$ are the proton, neutron, and total densities in the original fissioning nucleus. If it is assumed that $\delta_L=\delta+\zeta$, then use of the relations between the numbers of nucleons of the fragments and the original nucleus, $Z_L+Z_H=Z$, $A_L+A_H=A$, leads, for given fragment mass ratio $k=A_L/A_H$, to the expression $\delta_H=\delta-k\zeta$.

The fragment charge densities can be expressed in terms of ζ and the charge density ρ^p of the original nucleus by

$$\rho_L^p=\rho_0^p\left(1-\frac{\zeta}{1-\delta}\right),\quad \rho_H^p=\rho_0^p\left(1+\frac{k\zeta}{1-\delta}\right). \tag{4}$$

The total density $\rho_L^n+\rho_L^p=\rho_H^n+\rho_H^p=\rho_0$ is constant over the entire nucleus. For given mass asymmetry, ζ determines the ratio of the numbers of protons and neutrons in the light and heavy fragments, $k_Z=Z_L/Z_H$ and $k_N=N_L/N_H$, which for unchanged charge distribution are k. If $\zeta\neq0$, the relations between k_Z, k_N, and k are

$$k_Z=k\frac{1-\delta-\zeta}{1+\delta+k\zeta},\quad k_N=k\frac{1+\delta+\zeta}{1+\delta-k\zeta}. \tag{5}$$

We can express ζ in terms of

$$\Delta=(Z_L-Z_{L\,\mathrm{UCD}})=-(Z_H-Z_{H\,\mathrm{UCD}}), \tag{6}$$

the deviation of the fragment charges from the charges that one would expect for unchanged charge distribution (UCD) in the fissioning nucleus. It is customary to use Δ in discussions of experimental data on fragment charges. Indeed, using (5) and expressing δ and δ_L in terms of the isospins of the light fragment and the original nucleus, we have

$$\zeta=\frac{2T_L}{A_L}-\frac{2T}{A}=\frac{2(T_L-T_L^{\mathrm{UCD}})}{A_L}=\frac{2\Delta T_L}{A_L}, \tag{7}$$

where T_L^{UCD} is the light-fragment isospin for unchanged charge distribution. Since the change of the fragment isospins is related to the redistribution of the charge without change of the nucleon number, $\Delta T_L=-\Delta$. Thus,

$$\zeta=-\frac{2\Delta}{A_L}=-\frac{2(1+k)\Delta}{kA}. \tag{8}$$

Although Δ and ΔT_L are transparent parameters, they are of order unity, and it is therefore more convenient to use ζ or Δ/A, which are much less than unity.

In Refs. 26–28, the mass and charge asymmetry parameters are used:

$$\eta_A=\frac{A_H-A_L}{A_H+A_L},\quad \eta_Z=\frac{Z_H-Z_L}{Z_H+Z_L}. \tag{9}$$

It is easy to relate these parameters to the parameters k and ζ introduced above:

$$\eta_A=\frac{k-1}{k+1},\quad \eta_Z=\frac{k_Z-1}{k_Z+1}=\frac{(k-1)(1-\delta)-2k\zeta}{(k+1)(1+\delta)}. \tag{10}$$

All the listed parameters characterize only the deviations of the mean proton (neutron) densities in the fragments from the densities of the original fissioning nucleus. Further parameters must be introduced in order to describe different spatial distributions of the protons and neutrons in the fragments, for example,[29] in a discussion of the fragment charge polarization.

It should be noted that the model proposed in Refs. 25–28 allows a description of independent division of the mass and charge during fission not only for the separated fragments but also for all continuous shapes of the fissioning nucleus directly prior to separation. In this connection, we note that calculations of the deformation energy surface in the liquid-drop model[30] and by the Hartree–Fock method[31] suggest that fission occurs at a critical deformation at which the nucleus has a continuous shape with a neck that is still quite thick. Therefore, our model allows statistical calculations that match better the real scission situation than what was done hitherto. In addition, the model for independent division of the charge and mass for all continuous shapes with a significant neck allows a natural treatment of the formation of the fragment charge distribution in dynamical approaches,[32] including the Fokker–Planck approach. Despite its simplicity, the model was found to work well in the description of the charge and mass distributions in fission[25,33] and of the products of reactions with heavy ions.[27,28]

Potential energy

The potential energy can be calculated as a function of the shape parameters $(\mathbf{q})$ of the fissioning nucleus and the parameter ζ by Strutinsky's shell-correction method,[30] according to which it is made up of the shell correction, a pairing correction, and the macroscopic energy, which was calculated in the liquid-drop model in a study of the charge distribution.

The shell and pairing corrections were calculated in accordance with Strutinsky's method, which is described in detail in Ref. 30, using single-particle spectra of finite Woods–Saxon potentials for a fissioning system with nonuniform density. The method of constructing such potentials is described below.

We shall assume that the deformation liquid-drop energy, which depends on the change in the parameters $\{\mathbf{q}\}$ and ζ, is made up of symmetry energy, Coulomb energy, and, in the general case, surface energy. As usual, the deformation energy will be measured from the energy of the original spherical nucleus in the liquid-drop model.

We make a remark concerning the surface energy. The liquid-drop symmetry energy contains not only a volume but also a surface term. Both depend on the coordinate ζ. The parameters of these two terms are intimately related, and the division of the symmetry energy into purely volume and surface terms is rather arbitrary and difficult. Therefore, it was subsequently assumed that the surface energy does not depend on the parameter ζ as, for example, in the liquid-drop model.[34]

The symmetry energy for arbitrary shape of the nucleus can be calculated under the assumption that it is uniformly distributed over the complete nucleus and determined[35] by the expression

$$V_{\rm sym}=a_{\rm sym}\int \frac{(\rho_n-\rho_p)^2}{\rho_0}\,dv, \tag{11}$$

where $a_{\rm sym}$ is the coefficient of the symmetry energy. Under the assumption of constant proton and neutron densities within each fragment, the symmetry energy can be expressed in the form

$$V_{\rm sym}=\frac{a_{\rm sym}}{\rho}\,[(\rho_L^n-\rho_L^p)^2 v_L+(\rho_H^n-\rho_H^p)^2 v_H], \tag{12}$$

where v_L and v_H are the volumes of the light and heavy fragments. The use of the parameters δ and ζ in (12) leads to the formula

$$V_{\rm sym}(\mathbf{q},\zeta)=a_{\rm sym}(A\delta^2+kA\zeta^2). \tag{13}$$

For $\zeta=0$, the symmetry energy is naturally independent of the deformation parameters $\mathbf{q}$ of the nucleus.

The calculation of the Coulomb energy for a nonuniform charge distribution is difficult, but when the individual parts of the nucleus have constant charge density the calculation reduces to calculation of the Coulomb energy of the complete nucleus and its individual parts corresponding to nascent fragments with charge density of the original nucleus.

The Coulomb energy of the fissioning nucleus with constant charge density in the nascent fragments can be represented as the sum

$$V_c(\mathbf{q})=V_{cL}(\mathbf{q})+V_{cH}(\mathbf{q})+V_{cLH}(\mathbf{q}), \tag{14}$$

where $V_{cL}(\mathbf{q})$ and $V_{cH}(\mathbf{q})$ are the Coulomb self-energies of the light and heavy fragments, and $V_{cLH}(\mathbf{q})$ is the energy of their Coulomb interaction. The Coulomb energy with constant initial density over the complete nucleus can be expressed in exactly the same form:

$$V_c^0(\mathbf{q})=V_{cL}^0(\mathbf{q})+V_{cH}^0(\mathbf{q})+V_{cLH}^0(\mathbf{q}). \tag{15}$$

Bearing in mind that

$$\frac{V_{cH}}{(\rho^p)^2}=\frac{V_{cL}^0}{(\rho^p)^2},\quad \frac{V_{cH}}{(\rho^p)^2}=\frac{V_{cH}^0}{(\rho^p)^2},\quad \frac{V_{cLH}}{\rho_L^p\rho_H^p}=\frac{V_{cLH}^0}{(\rho^p)^2} \tag{16}$$

and eliminating $V_{cLH}(\mathbf{q})$ from (14) by the relation that follows from (15) and (16),

$$V_{cLH}(\mathbf{q})=[V_c^0(\mathbf{q})-V_{cLH}^0(\mathbf{q})]\,\frac{\rho_L^p\rho_H^p}{(\rho^p)^2}, \tag{17}$$

we obtain an expression for the Coulomb energy of the nucleus with different density in each separating fragment in terms of the Coulomb energies of the parts of the nucleus with the original charge density:

$$V_c(\mathbf{q})=\left[\frac{(\rho_L^p)^2}{(\rho^p)^2}-\frac{\rho_L^p\rho_H^p}{(\rho^p)^2}\right]V_{cL}^0(\mathbf{q})+\left[\frac{(\rho_H^p)^2}{(\rho^p)^2}-\frac{\rho_L^p\rho_H^p}{(\rho^p)^2}\right]V_{cH}^0(\mathbf{q})+\frac{\rho_L^p\rho_H^p}{(\rho^p)^2}\,V_c^0(\mathbf{q}). \tag{18}$$

Expressing the charge densities in terms of the charges and volumes of the emerging fragments and using the parameters k, k_Z, and ζ, we obtain a relation for the Coulomb energy that was used directly in the calculation:

$$\begin{aligned}V_c(\mathbf{q},\zeta)=&\left(1-\frac{\zeta}{1-\delta}\right)\left(1+\frac{k\zeta}{1-\delta}\right)E_c^0B_c+\left[\left(1-\frac{\zeta}{1-\delta}\right)^2\right.\\&\left.-\left(1-\frac{\zeta}{1-\delta}\right)\left(1+\frac{k\zeta}{1-\delta}\right)\right]E_c^0B_{cL}\\&+\left[\left(1+\frac{k\zeta}{1-\delta}\right)^2-\left(1-\frac{\zeta}{1-\delta}\right)\right.\\&\left.\times\left(1+\frac{k\zeta}{1-\delta}\right)\right]E_c^0B_{cH}.\end{aligned} \tag{19}$$

Here, E_c^0 is the Coulomb energy of the original spherical nucleus, and B_c, B_{cL}, and B_{cH} are the Coulomb energies of the complete nucleus and the nascent fragments with original charge density in units of the Coulomb energy of the original spherical nucleus.

Effective methods for calculating the Coulomb energies B_c, B_{cL}, and B_{cH} are described in detail in Refs. 36 and 37; a list of formulas is also given in Ref. 30. The Coulomb energies B_{cL} and B_{cH} of the nascent fragments can also be calculated by the layer method developed by Beringer.[38] Thus, it follows from (13) and (19) that the liquid-drop energy as a function of the parameter ζ is described by an oscillator dependence characterized by a coefficient of rigidity with respect to variations of the parameter ζ:

$$C_\zeta=2a_{\rm sym}kA+\frac{2E_c^0}{(1-\delta)^2}\,[(1+k)(B_{cL}+kB_{cH})-kB_c]. \tag{20}$$

Having in mind the relation between the parameters ζ and Δ, we can also readily write down an expression for C_Δ, the rigidity coefficient with respect to Δ, which can be directly used to estimate the variance of the charge distribution:

$$C_\Delta=\frac{8(1+k)^3E_c^0}{k^2(1-\delta)^2\Delta_0^2}\,C_\zeta. \tag{21}$$

The estimate $C_\Delta\approx 3.2\pm 0.3$ MeV is obtained in Ref. 4. The symmetry energy makes the dominant contribution to C_ζ (and C_Δ). The coefficients C_ζ and C_Δ are not very sensitive to the geometrical configuration at scission. This justifies the estimates usually made for the rigidity[5,6] for a simplified scission configuration in the form of spherical fragments. The rigidity C_ζ does not change strongly, and during the descent of the fissioning nucleus from the saddle point to scission it increases by not more than 20%.[32] For symmetric ($k=1$) fission, $V_{\rm LDM}(\zeta)$ has a minimum at $\zeta=0$; for asymmetric ($k\neq 1$) fission, the minimum, situated at

$$\zeta_{\min} = \frac{E_c^0[B_c(k-1)+(1+k)(B_{cH}-B_{cL})]/(1-\delta)^2}{2E_c^0[kB_c-(1+k)(B_{cL}+kB_{cH})]/(1-\delta)^2+2a_{\rm sym}kA}, \tag{22}$$

is displaced with increasing asymmetry ever further into the region of negative ζ. Using the relation between ζ and Δ, we can determine from (22) the most probable fragment charge by using the criterion of minimum potential energy.[24]

Figure 1 gives typical results of liquid-drop calculations, which determine the main behavior of the energy as a function of ζ. The shape of the fissioning nucleus was parametrized by the well-known triplet $q \equiv (c,h,\alpha)$.[30] Here, c is the elongation parameter, h is the thickness of the neck for given elongation of the nucleus, and the parameter of the "mirror" (mass) asymmetry α determines the ratio of the volumes (masses) of the nascent fragments. Figure 1a shows the gain in the deformation energy $[V_{\rm LDM}(\mathbf{q},0)-V_{\rm LDM}(\mathbf{q},\zeta_{\min})]$ resulting from minimization with respect to ζ, while Fig. 1b gives the amount of charge $\Delta_p = Z_p - Z_{\rm UCD}$ that must be transferred from the heavy to the light fragment if the drop energy is to be a minimum. It can be seen that the gain in the energy resulting from the minimization increases with increasing mass-asymmetry parameter.

The construction of the single-particle potentials for a fissioning nucleus with inhomogeneous density presents considerable difficulties and is associated with great uncertainty. For two-center potentials, it can be done[26,28] by introducing different frequencies for the nascent fragments. A different approximation can be used for finite Woods–Saxon potentials. For a fissioning nucleus with different proton and neutron densities in the nascent fragments one can consider the protons and neutrons in potentials characterized by different asymmetric deformations for the proton and neutron systems. Accordingly, for given elongation parameter c and given neck h, the potential for the proton system is characterized by the parameters $\{c,h,\alpha_Z\}$, and for the neutron system by $\{c,h,\alpha_N\}$, where α_Z and α_N are determined from the equations $k_Z = k_Z(c,h,\alpha_Z)$ and $k_N = k_N(c,h,\alpha_N)$. It was this approximation that was used in the calculations in Ref. 39 with a finite Woods–Saxon potential.[30]

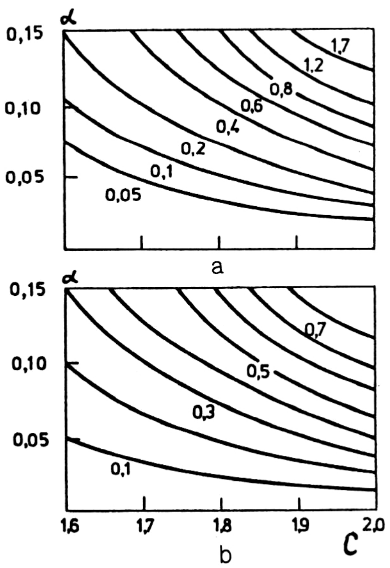

FIG. 1. Gain in energy (in mega-electron-volts) resulting from minimization of the liquid-drop energy with respect to the charge coordinate ζ for the nucleus ^{238}U (a). The values of $\Delta_p = Z_p - Z_{\rm UCD}$, the deviation of the most probable fragment charge from the value of the charge corresponding to unchanged charge density (b). At each point of the plane (c,α), the values of the neck parameter h correspond to the minimum liquid-drop energy.

Variational solution to the problem of the charge-density distribution in the fissioning nucleus

The use of different charge densities in the fragments, but constant within each of them,[25–28] is a first, rough approximation to the real situation, although it does reflect well the consequences of the polarization of the nuclear matter on fission. Therefore, it is necessary to compare the predictions of our model with the results of a model corresponding to a more realistic density distribution within the fissioning nucleus.

As we have already noted, the calculation of a physically reasonable density distribution in the nascent fragments using the criterion of a minimum potential energy requires solution of a variational problem in the framework of some macroscopic model. The simplest variational solution of the problem is obtained in the hydrodynamic model.

In accordance with this model,[35] the nucleus consists of two interpenetrating liquids—a proton and a neutron liquid—and has sharp boundaries. It is also assumed that the total density of the nucleus is constant over the complete volume. The potential energy is expressed as a sum of two terms: the Coulomb energy V_c and the symmetry energy $V_{\rm sym}$, which gives rise to forces that hinder separation of the proton and neutron liquids on a deviation of the densities from the equilibrium values:

$$V = \tfrac{1}{2}\int \Phi_c'(\mathbf{r})\rho_p(\mathbf{r})d\mathbf{r} + \frac{a_{\rm sym}}{\rho_0}\int [\rho_p(\mathbf{r})-\rho_n(\mathbf{r})]^2 d\mathbf{r}, \tag{23}$$

where $\Phi_c'(\mathbf{r})$ is the Coulomb potential in energy units. A constraint on the density in the form of the condition of conservation of the particle number is imposed:

$$\int \rho_p(\mathbf{r})d\mathbf{r} = Z. \tag{24}$$

We consider the variational problem for the functional V. For this, it is necessary to solve the Euler equation for the function

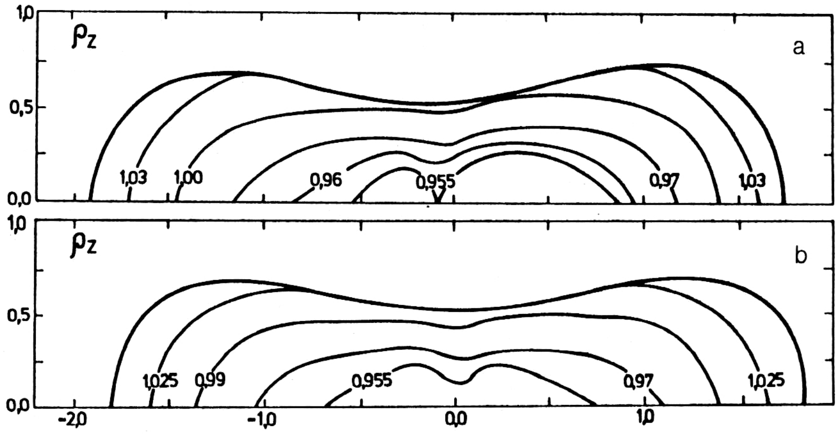

FIG. 2. Spatial distribution of the charge density in the fissioning nucleus ^{238}U, represented in cylindrical coordinates. The numbers on the lines of constant density are the density in units of ρ_0^p. The outermost curve corresponds to the nuclear surface for the corresponding deformation. The unit of length is the radius R_0 of the equilibrium sphere. The deformation parameters have the values: a) $c=1.8$, $\alpha=0$, $h=0$; b) $c=1.8$, $\alpha=0$, $h=0.03$. The distribution was obtained by the variational solution in the hydrodynamic model.

$$\widetilde{V}=V+\lambda\varphi, \tag{25}$$

where λ is an undetermined Lagrange multiplier, and $\varphi=0$ is the constraint equation, in this case Eq. (24). On the basis of (23)–(25), we obtain a system consisting of Euler's equation and the constraint:

$$\int\left\{\Phi_c'(\mathbf{r})+\frac{4a_{\text{sym}}}{\rho_0}\left[\rho_p(\mathbf{r})-\rho_n(\mathbf{r})\right]-\lambda\right\}\delta\rho_p(\mathbf{r})d\mathbf{r}=0, \tag{26}$$

$$\int\rho_p(\mathbf{r})d\mathbf{r}-Z=0. \tag{27}$$

From Eq. (26),

$$\lambda=\Phi_c'(\mathbf{r})+\frac{4a_{\text{sym}}}{\rho_0}\left[\rho_p(\mathbf{r})-\rho_n(\mathbf{r})\right]. \tag{28}$$

Finding $\rho_p(\mathbf{r})$ from (28) and substituting it in (27), we obtain

$$\lambda=4a_{\text{sym}}\frac{Z-N}{A}-\bar{\Phi}_c', \tag{29}$$

where $\bar{\Phi}_c'$ is the Coulomb potential averaged over the volume of the nucleus. From the expressions (28) and (29) we find the distribution of the charge density of the fissioning nucleus:

$$\rho_p(\mathbf{r})-\rho_n(\mathbf{r})=\frac{Z-N}{A}\rho_0-\frac{\rho_0}{4a_{\text{sym}}}\left[\Phi_c'(\mathbf{r})-\bar{\Phi}_c'\right]. \tag{30}$$

Bearing in mind that $\rho_0=\rho_p(\mathbf{r})+\rho_n(\mathbf{r})$, we have

$$\rho_p(\mathbf{r})=\frac{Z}{A}\rho_0-\frac{\rho_0}{8a_{\text{sym}}}\left[\Phi_c'(\mathbf{r})-\bar{\Phi}_c'\right],$$

$$\rho_n(\mathbf{r})=\frac{N}{A}\rho_0-\frac{\rho_0}{8a_{\text{sym}}}\left[\Phi_c'(\mathbf{r})-\bar{\Phi}_c'\right]. \tag{31}$$

Iterating, using Eqs. (31), we can determine the spatial distribution of the nuclear density within the nucleus.

Figure 2 shows the spatial distributions of the proton density calculated for ^{238}U in the parametrization $\{c,h,\alpha\}$.[30] At a certain stage of the fission process, the nascent fragments are formed, the heavy before the light one in the case of asymmetric fission. The proton density increases with increasing distance from the fragment centers, the density increasing more rapidly at the edge of the nucleus than in the middle. It attains maximum values at the equatorial ends of the figure. This behavior of the charge density is due to the Coulomb repulsion of the protons.

The hydrodynamic model is rather simple and only approximately reflects the real properties of nuclear matter, and therefore the variational problem of the charge-density distribution was solved in the droplet model,[40,41] which is an improved form of the drop model. Generalized to arbitrary deformations,[42] the droplet model gives a convenient parametrized solution of the variational problem, modeling many characteristic features in the behavior of real nuclear matter.

In the droplet model, the actual distribution of the proton and neutron densities in the nucleus is divided into two regions: the interior with densities ρ_p, ρ_n, and $\rho=\rho_n+\rho_p$, and the surface region, where the density mainly falls to zero. The extrapolation of ρ_p,ρ_n,ρ into the surface region determines the effective surfaces S_Z,S_N,S with radii R_Z, R_N, and R, respectively, which are such that all the Z protons, N neutrons, and A nucleons are within the corresponding surfaces. These densities and surfaces defined in this manner are the degrees of freedom of the droplet model. However, instead of the densities ρ_p and ρ_n, it is more convenient to use as degrees of freedom two functions that are related to them and have small absolute magnitude:

$$\varepsilon(\mathbf{r})=-\tfrac{1}{3}(\rho(\mathbf{r})-\rho_0)/\rho_0, \tag{32}$$

which characterizes the deviation of ρ in the interior region from the standard value $\rho_0=(4/3\pi r_0^3)^{-1}$, and

$$\delta(\mathbf{r})=\frac{\rho_n(\mathbf{r})-\rho_p(\mathbf{r})}{\rho(\mathbf{r})}, \tag{33}$$

which is the difference between the neutron and proton densities in the interior region. Then ρ_p and ρ_n can be expressed in terms of ε and δ as follows:

$$\rho_p=\tfrac{1}{2}\rho_0(1-3\varepsilon)(1-\delta),$$

$$\rho_n=\tfrac{1}{2}\rho_0(1-3\varepsilon)(1+\delta). \tag{34}$$

Further, it is convenient to represent the functions $\varepsilon(\mathbf{r})$ and $\delta(\mathbf{r})$ as the sum of their mean (over the volume) values and the deviations from these means:

$$\varepsilon(\mathbf{r})=\bar{\varepsilon}+\tilde{\varepsilon}(\mathbf{r}), \quad \delta(\mathbf{r})=\bar{\delta}+\tilde{\delta}(\mathbf{r}). \tag{35}$$

To find the most probable charge of the fragments, an integration is performed using equilibrium distributions of the protons and neutrons, which are determined by the equilibrium parameters $\bar{\varepsilon}$, $\bar{\delta}$, $\tilde{\delta}$, and $\tilde{\varepsilon}$ obtained by solving the variational problem of the nuclear energy in the droplet model.[41,42] The expressions for $\bar{\varepsilon}$ and $\bar{\delta}$ include[42] a dependence on the energy functionals B_s, B_c, and B_v, while $\tilde{\varepsilon}$ and $\tilde{\delta}$ are proportional to $\tilde{\Phi}(\mathbf{r})=\Phi(\mathbf{r})-\bar{\Phi}$, the deviation of the Coulomb potential $\Phi(\mathbf{r})$ from its mean $\bar{\Phi}$ over the volume.

An extremely important requirement for solving the problem of determining the most probable fragment charge is fulfillment of the conditions of conservation of the particle number. These conditions for the protons and neutrons in the droplet model were formulated with a good degree of accuracy in Ref. 43.

Knowledge of the charge-density distribution for arbitrary shapes of the fissioning nucleus and fulfillment of the conservation conditions make it possible to calculate the distribution of the charge between the fragments, the method of dividing the fissioning nucleus into parts corresponding to the nascent fragments being chosen at the same time. Then, integrating over the volumes of the parts, the charges and masses of the light and heavy fragments are found.

To estimate the variance of the charge distribution in the statistical model, it is necessary to calculate the second derivative of the nuclear energy with respect to the variation of the charge density. The droplet model developed in Ref. 42 for arbitrary nuclear shape permits this, since the nuclear energy is given for all and not only the equilibrium values of $\bar{\varepsilon}$, $\bar{\delta}$, $\tilde{\varepsilon}$, and $\tilde{\delta}$. The distributions of the proton and neutron densities in the droplet model are shown in Fig. 3, from which it can be seen that they are similar to the corresponding distributions of the hydrodynamic model. Knowing the distribution of the charge density in the fissioning nucleus, one can calculate the most probable charge of the fragments, or $\Delta_p=Z_p-Z_{\mathrm{UCD}}$. Figure 4 gives Δ_p as a function of the light-fragment mass obtained in our model and by solving the variational problem in the framework of the hydrodynamic model and the droplet model. The behavior of the curves shows that there is good qualitative agreement between the results obtained in the three models. Curve *3* is somewhat below the first two, but this behavior can be explained by a dependence on the set of model parameters. The parameter set that we used[42] was regarded as provisional.

The qualitative agreement for the results on the most probable fragment charge and its dependence on the separation parameter obtained in the three models establishes the validity of our model[25,26] and shows that it can be used to calculate the fragment charge distribution.

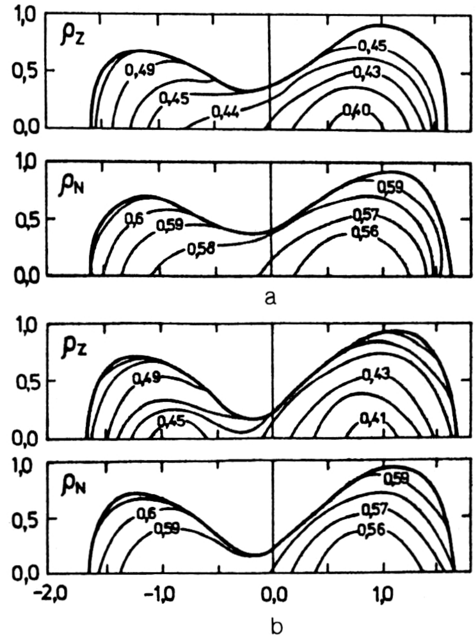

FIG. 3. Spatial distributions of the proton (a) and neutron (b) densities in the fissioning nucleus ^{238}U, represented in cylindrical coordinates. The numbers on the lines of constant density are the densities in units of ρ_0. The outermost curve corresponds to the nuclear surface for the corresponding deformation. The unit of length is the radius R_0 of the equilibrium sphere. The calculation was made for two sets of deformation parameters, using the Lawrence parametrization (Ref. 44): a) $c=1.9$, $s=1.0$, $\alpha=0.2$; b) $c=1.98$, $s=1.0$, $\alpha=0.2$. The distributions were obtained by the variational solution in the droplet model.

2. STATISTICAL CALCULATIONS OF THE FRAGMENT CHARGE DISTRIBUTION

The conditions for using a statistical model to describe the fragment distributions are best satisfied for the charge distribution. Estimates[6,21,22] show that the fluctuations of the charge density leading to redistribution of the charge between the fragments (dipole isovector vibrations) relax

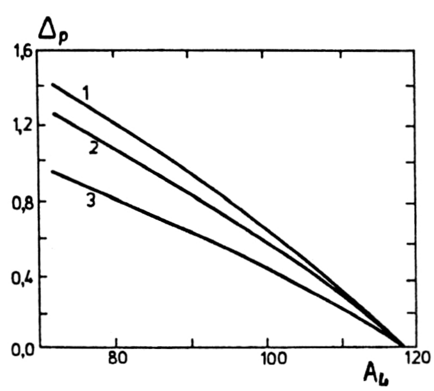

FIG. 4. Dependences of Δ_p, the deviation of the most probable fragment charge from the charge corresponding to unchanged density, on the light-fragment mass for ^{238}U obtained in the model with different densities in the fragments (but constant density within each of them) (curve *1*), in variational calculations in the hydrodynamic model (curve *2*), and in the droplet model (curve *3*). The curves were calculated for the same elongation parameter.

very rapidly compared with the time of descent of the nucleus from the saddle point to scission. A statistical model in Fong's original formulation[7] but with density of excited states calculated on the basis of single-particle level schemes was successfully used to calculate the parameters of the charge distribution by Ignatyuk.[8] To calculate the charge distributions, we[39] used a form of the statistical model developed by Moretto.[45,46] The advantage of this approach is that it can be used for an arbitrary deformation of the fissioning nucleus, and not only when there is a well-defined scission point, a necessary condition of Fong's original statistical model. The statistical approach of Refs. 45 and 46 enables one to follow the evolution of the various distributions of the fission products during the entire descent from the saddle point to the scission point. The formalism that Moretto developed for statistical calculations as a function of the deformation was used in Refs. 47 and 48 to describe the fragment mass distributions.

Under the assumption of statistical equilibrium among all degrees of freedom, the relative probability of finding the system with deformation $(\mathbf{q},\zeta)$ is

$$P(\mathbf{q},\zeta,E^*)=FT^2\rho(E^*-V(\mathbf{q},\zeta)), \tag{36}$$

where $E_T=E^*-V(\mathbf{q},\zeta)$ is the local excitation energy in the collective coordinates $(\mathbf{q},\zeta)$, T is the temperature corresponding to E_T, and F is a function that depends on the inertia parameters, which, in their turn, depend on the coordinates. The strongest dependence on the collective coordinates is in the density $\rho(E^*-V(\mathbf{q},\zeta))$ of excited states, and therefore the dependence of FT^2 on the coordinates is usually ignored.

The density of excited states was calculated in the superfluid model[49,50] on the basis of the same single-particle spectrum in a finite Woods–Saxon potential that was used to calculated the shell correction.

In the description of the fragment charge distribution, the collective coordinates q include not only the nuclear deformation parameters $\mathbf{q}$ but also ζ. The yield of fragments with given masses and charges is assumed to be proportional to $P(E^*,\mathbf{q},\zeta)$, i.e.,

$$Y(Z/A)\sim P(E^*,\mathbf{q},\zeta). \tag{37}$$

The shape parameters $\{\mathbf{q}\}$ are free, and the calculation is made for the complete region of $\{\mathbf{q}\}$, beginning at the saddle point and ending in the scission region.

The condition of a maximum of $P(E^*,\mathbf{q},\zeta)$ was used to determine the most probable fragment charge. For weak dependence of the temperature on the parameter ζ, this condition is practically equivalent to the corresponding equation of Fong's statistical model.[7,8] At low excitation energy, this condition reduces to a minimum of the potential energy calculated with allowance for shell corrections, while at high excitations, when the shell effects disappear, it reduces to the condition of a minimum of the energy in the liquid-drop model.

Figure 5 shows Δ_p as a function of the fragment mass ratio for ^{236}U, ^{240}Pu, ^{252}Cf. The almost straight line in the diagrams is obtained from the condition of minimum potential energy in the drop model with $c=1.9$. The values of

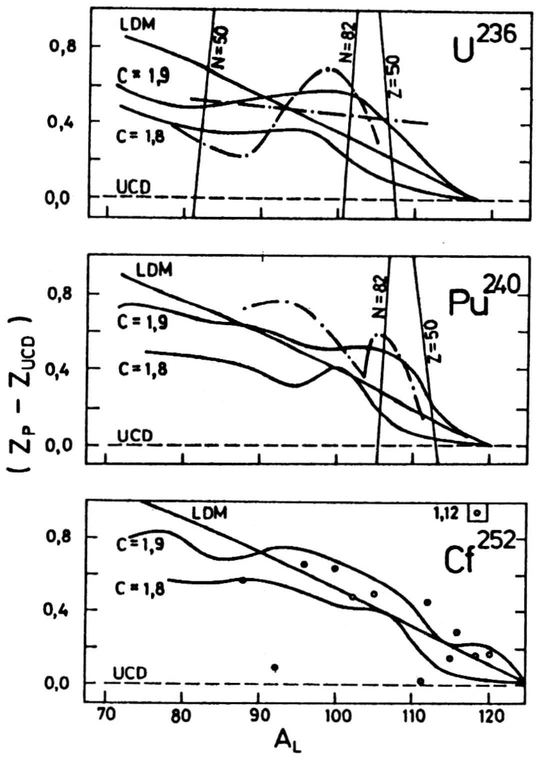

FIG. 5. Dependences of $\Delta_p=Z_p-Z_{\rm UCD}$, the deviation of the most probable fragment charge from the charge corresponding to unchanged density, on the light-fragment mass for ^{236}U, ^{240}Pu, ^{252}Cf. The continuous curves are the calculated values of Δ_p with allowance for shell corrections (the values of the elongation parameter are given next to the curves). The almost straight continuous lines are the values calculated in the liquid-drop model for elongation parameter $c=1.9$. The chain curves are the experimental data from Refs. 51–53.

Δ_p calculated with allowance for the shell corrections are raised or lowered compared with the liquid-drop calculations in different regions of fragment mass numbers. In fact, it is difficult to say what shell structure causes these changes relative to the values obtained in the liquid-drop model, but it appears improbable that this shell structure is associated with magic numbers of already formed fragments. To show the dependence of the results on the separation parameter, the theoretical curve for $c=1.8$ is also shown in the figure. It is interesting to note that for deformation $c=1.9$, where there is reasonable agreement between the calculated Δ_p and experimental data,[51–53] the Coulomb-repulsion energy of the fragments is approximately equal to the mean kinetic energy of the fragments. Thus, it follows from these statistical calculations that the most probable fragment charge, being a sensitive function of the main fission deformation, which determines the distance between the centers of mass of the nascent fragments, reflects approximately the behavior of the experimental values in the region of the physical scission point.

The calculated charge distributions for ^{236}U fission with fragment masses 133/103 are shown for two excitation energies (10 and 15 MeV) in Fig. 6 together with the experimental points.[54] The agreement is quite good. The curves of the charge distributions reflect the behavior of the total deformation energy as a function of ζ at low very rapidly compared with the time of descent of the nucleus from the saddle point to scission. A statistical model in Fong's original formulation[7] but with density of excited states calculated on the basis of single-particle level schemes was successfully used to calculate the parameters of the charge distribution by Ignatyuk.[8] To calculate the charge distributions, we[39] used a form of the statistical model developed by Moretto.[45,46] The advantage of this approach is that it can be used for an arbitrary deformation of the fissioning nucleus, and not only when there is a well-defined scission point, a necessary condition of Fong's original statistical model. The statistical approach of Refs. 45 and 46 enables one to follow the evolution of the various distributions of the fission products during the entire descent from the saddle point to the scission point. The formalism that Moretto developed for statistical calculations as a function of the deformation was used in Refs. 47 and 48 to describe the fragment mass distributions.

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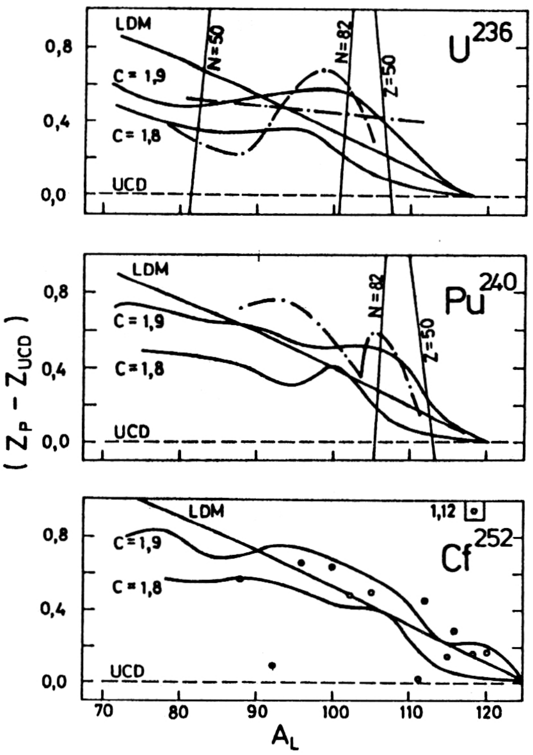

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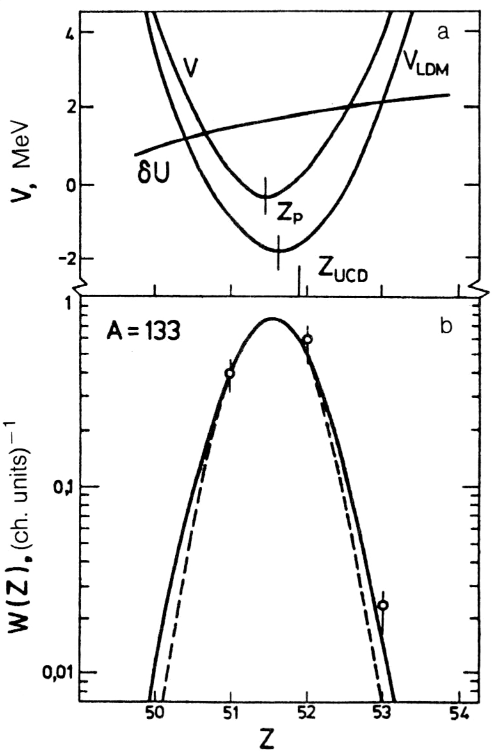

FIG. 6. Potential energy (a) calculated in the liquid-drop model (lower curve) and with allowance for shell corrections, and also the shell correction as a function of the heavy-fragment charge. The charge distribution of the fission fragments (b) for fragments with mass ratio $A_H/A_L = 133/103$ for fission of ^{236}U. The continuous curve corresponds to excitation energy 15 MeV, and the broken curve to 10 MeV. The experimental data are taken from Ref. 54. The values of $Z_{\rm UCD}$ and Z_p are indicated by the vertical lines.

excitation energies and the drop energy at $E^* > 30$ MeV (see Fig. 6a). As a rule, the charge distributions have a Gaussian form, and the calculated widths of the charge distributions depend on the excitation energy approximately as $E^{*1/4}$ in accordance with the statistical theory (Fig. 6b).

Our dependence of the charge-distribution variances on the excitation energy also agrees with the results of Ref. 33, in which a dependence on the angular momentum was also taken into account in the calculation of the density of excited states. However, these results do not agree with experiment, according to which the widths of the distributions do not change when the excitation energy is increased to 40 MeV. The discrepancy could be due to the influence of quantum states associated with dipole isovector vibrations on the width of the charge distribution, as was assumed in Ref. 20. An expression for the deformation probability valid at all excitation energies and for which (37) is an approximation has the form[46]

$$P(E^*,\mathbf{q},\zeta) = \sum_n \rho(E^* - E_n(\mathbf{q}))\,|\Psi_n(\mathbf{q},\xi)|^2, \tag{38}$$

where $E_n(q)$ is the energy of the dipole isovector vibrations, and $\Psi_n(\zeta)$ are the corresponding wave functions.

Using a model with constant temperature to calculate the density of excited states, we can write

$$P(E^*,\mathbf{q},\zeta) = \sum_n N_n \exp\left(-\frac{E_n}{T}\right)|\Psi_n(\mathbf{q}\zeta)|^2. \tag{39}$$

To use (39) in calculations, it is necessary to know the solution of the Schrödinger equation for the ζ motion, and this involves laborious calculations. Such an approach to the calculation of the charge distribution was realized in Ref. 26. However, one can make an estimate of the use of (39) for calculating the relative deformation probability with parameter ζ. If harmonic-oscillator wave functions are used, then the sum in (39) can be calculated exactly, and $P(E^*,\mathbf{q},\zeta)$ is represented by a Guassian curve with variance

$$\sigma_\zeta^2 = \frac{\hbar\omega_\zeta}{2C_\zeta} = \coth\frac{\hbar\omega_\zeta}{2T}, \tag{40}$$

which for $T \gg \hbar\omega_\zeta$ (high-temperature limit) is

$$\sigma_\zeta^2 = \frac{T}{C_\zeta} \tag{40a}$$

and represents the magnitude of the statistical fluctuations, while in the opposite limit $T \ll \hbar\omega_\zeta$ it tends asymptotically to a temperature-independent value

$$\sigma_\zeta^2 = \frac{\hbar\omega_\zeta}{2C_\zeta} = \frac{\hbar}{2C_\zeta}\left(\frac{C_\zeta}{m_\zeta}\right)^{1/2}, \tag{40b}$$

which is the variance of the coordinate of the quantum oscillator in its ground state.

The energies of the dipole isovector vibrations corresponding to charge separation in fission are in the scission region (see Fig. 3) 2.5–30 MeV, for which for temperatures $T \cong 1$–1.5 MeV (corresponding to excitation energies $E^* \cong 40$–50 MeV of the fissioning nucleus) the condition of the low-temperature limit is no longer satisfied.

3. DESCRIPTION OF FORMATION OF ISOBARIC CHARGE DISTRIBUTION OF THE FRAGMENTS ON THE BASIS OF THE MULTIDIMENSIONAL FOKKER–PLANCK EQUATION

Various approximations of the scission point have been used (Refs. 7, 8, 33, and 39) in statistical calculations of the charge-distribution parameters. It was found that the variance of the charge distribution and, especially, the most probable fragment charge depend strongly on the choice of the scission configuration. However, there is no unambiguous criterion for its determination (see, for example, Refs. 11 and 12). For all determinations of the scission line in the space of the collective coordinates, the scission configuration will depend on the dynamics of descent of the nucleus from the saddle point to scission. Therefore, it is necessary to study the dynamical evolution of the collective coordinates that determine the parameters of the charge distribution even when the assumption of statistical equilibrium with respect to the charge mode is made. It would also be interesting from the methodological point of view to establish the extent to which the statistical approach really is valid for studying the charge distribution in fission.

At the present time, the charge distribution of the reaction products of deep inelastic collisions between heavy ions is studied under the assumption that the main mechanisms of its formation are either stochastic exchange of nucleons between the reaction products being formed[55,56] or collectivized motion of the nucleons manifested through isovector vibrations of the nucleon density in the double nuclear system.

We consider the evolution of the parameters of the fragment charge distribution during fission in terms of a collective mode. For the parameter of this mode, we shall use Δ defined by the expression (6); it is directly related to the fragment charge.

We considered[32] a multidimensional Fokker–Planck equation for a three-dimensional case: Besides the collective coordinate Δ, we also took into account the dynamics of the main fission coordinate x (elongation parameter c or distance ρ between the centers of mass of the fragments) and the neck parameter h.[30] The multidimensional Fokker–Planck equation that is the dynamical equation of our approach can be obtained from a rather general approach (linear response theory[57,58] or perturbation theory[59]) to the interaction of several distinguished collective degrees of freedom with single-particle variables under the assumption that at any time the latter are in a state of statistical equilibrium, forming a thermal reservoir with temperature T. The point of departure[60] is the general quantum-mechanical equation for the reduced density matrix. Subjecting this equation to a Wigner transformation, we obtain in the case of harmonic motion an equation analogous to a classical Fokker–Planck equation with frequency-dependent transport coefficients

$$\frac{\partial f}{\partial t}=-\frac{p}{m}\frac{\partial f}{\partial x}+m\omega^2 x\frac{\partial f}{\partial p}+\gamma\frac{\partial}{\partial p}\left[p\frac{f}{m}\right]+\gamma T^*\frac{\partial^2 f}{\partial p^2}, \tag{41}$$

where $T^*=(\hbar\omega/2)\coth(\hbar\omega/2T)$ is the effective temperature corresponding to the considered harmonic mode.

Equation (41) takes into account both the statistical and the quantum fluctuations of the harmonic motion. It is written down for the one-dimensional case. The generalization to the multidimensional case is obvious (see the brief exposition of the general formalism in Ref. 61).

The potential energy with respect to the charge mode is described by a harmonic oscillator whose rigidity depends both on the basic fission coordinate and on the neck parameter. The inertia (m_Δ) and friction (γ_Δ) coefficients of this mode also depend on these collective coordinates. It is in fact sufficient to consider the influence of only one of them (the basic fission coordinate is most convenient) on the evolution of the parameters of the charge mode, following along the trajectory of the descent from the saddle point to scission.

Thus, we have a problem of the vibration dynamics of a harmonic oscillator whose parameters vary with the time. The fluctuation dynamics of such an oscillator is described by the following system for the second moments of the distribution function $f(\Delta,p_\Delta,t)$, which satisfies the Fokker–Planck equation (41):

$$\frac{d\sigma_\Delta^2}{dt}=\frac{2\sigma_{\Delta p_\Delta}}{m_\Delta(t)},$$

$$\frac{d\sigma_{\Delta p}^2}{dt}=-2\left[C_\Delta(t)\sigma_{\Delta p_\Delta}+\frac{\gamma_\Delta(t)\sigma_{p_\Delta}^2}{m_\Delta(t)}-D_\Delta(t)\right],$$

$$\frac{d\sigma_{\Delta p_\Delta}}{dt}=C_\Delta(t)\sigma_\Delta^2+\frac{\sigma_{p_\Delta}^2}{m_\Delta(t)}-\gamma_\Delta(t)\sigma_{\Delta p_\Delta}/m_\Delta(t). \tag{42}$$

The time dependence of the parameters C_Δ, γ_Δ, and D_Δ of the system (42) is determined by the time dependence of the first moments $\langle x\rangle$ and $\langle h\rangle$ of the distribution function. The diffusion coefficient in (42) is related to the friction coefficient by Einstein's equation $D_\Delta=\gamma_\Delta T_\Delta^*$.

We note that the system for the variances of the coordinate, its conjugate momentum, and their correlation moment obtained in Ref. 62 by using for the problem a Schrödinger equation with friction[63] differs from (42) only in the form of the last term in the equation for $d\sigma_{p_\Delta}^2/dt$.

For the solution of the system (42), the initial distribution with respect to the charge mode was assumed to be an equilibrium distribution at the saddle point:

$$\sigma_\Delta^2(0)=\frac{T_{sd}^*}{C_{\Delta sd}},\quad \sigma_{p_\Delta}^2(0)=T_{sd}^* m_{\Delta sd},\quad \sigma_{\Delta p_\Delta}(0)=0. \tag{43}$$

The solution of (42) and of the system for the first moments $\langle x\rangle$ and $\langle h\rangle$ constitutes the solution of the multidimensional Fokker–Planck equation by the global moment approximation,[64] which is a fairly good[65] approximation to the exact solution of the equation in the case of nearly harmonic motion.

Before we discuss the calculations of the variances of the isobaric charge distribution by the diffusion model, we consider the choice of the coefficients of the Fokker–Planck equation (41). The determination of the coefficients corresponding to the charge mode in fission is, except for the rigidity coefficient, a complicated and essentially incompletely solved problem. Therefore, one often uses[32,61] estimates of the coefficients; this applies particularly to the inertia and friction coefficients of the charge mode.

The potential energy was represented in the form

$$V(x,h,\alpha,\Delta)$$

$$=V(x,h,\alpha,\Delta=0)+\frac{C_\Delta(x,h,\alpha)}{2}(\Delta-\Delta_p(x,h,\alpha))^2, \tag{44}$$

where $V(x,h,\alpha,\Delta=0)$ was calculated in some version of the liquid-drop model,[34,66] and C_Δ and Δ_p were determined by the Δ dependence of the potential energy. The Coulomb and symmetry energy parameters were $a_c=0.72$ MeV (Ref. 67) and $a_{\rm sym}=27.62$ MeV (Ref. 66). For symmetric fission, $\Delta_p(x,h)=0$.

Isovector vibrations of the charge density in the fissioning nucleus and the inertia coefficient of the charge mode

We determined[32] the inertia coefficient of the charge mode from the calculated frequencies of dipole isovector vibrations and the rigidity coefficients C_Δ of the charge mode. A detailed study of the isovector vibrations of the density in a fissioning nucleus and their role in forming the charge distribution had not been made until recently. As the only exception, we mention Ref. 68, in which isovector vibrations were considered for a restricted class of shapes of the fissioning nucleus. In Ref. 69, we made a systematic study of the isovector density vibrations in the hydrodynamic model[35] for shapes of the fissioning nucleus characteristic of its descent from the saddle point to scission. The frequencies of the isovector vibrations were calculated in the hydrodynamic approximation, in which the deviations of the proton and neutron densities from their equilibrium values ρ_0^p and ρ_0^n were described by a local fluctuation $g(\mathbf{r},t)$ as follows:

$$\rho_p(\mathbf{r},t)=\rho_0^p[1+g(\mathbf{r},t)],\quad \rho_n(\mathbf{r},t)=\rho_0^n\left(1-\frac{Z}{N}g(\mathbf{r},t)\right). \tag{45}$$

The fluctuation $g(\mathbf{r},t)$ satisfies a wave equation[35] with phase velocity $u=8a_{\mathrm{sym}}ZN/(m^*A)$, where m^* is the effective nucleon mass, and a_{sym} is the coefficient of the symmetry energy in the von Weizsäcker formula. For harmonic density vibrations, $g(\mathbf{r},t)=g(\mathbf{r})\exp(-i\omega t)$, we find that $g(\mathbf{r})$ satisfies the Helmholtz equation

$$\Delta g(\mathbf{r})+k^2 g(\mathbf{r})=0 \tag{46}$$

with a homogeneous boundary condition on the surface of the nucleus:

$$(\mathbf{n}\nabla g)_s=0, \tag{47}$$

where $\mathbf{n}$ is the vector of the normal to the surface of the nucleus. The wave number k is related to the coefficient of the symmetry energy as follows:

$$k^2=\frac{\omega^2}{u^2}=\omega^2\left(\frac{m^*A}{8a_{\mathrm{sym}}ZN}\right)^2. \tag{48}$$

In the calculations, m^* was taken to be 0.7 of the nucleon mass.[35]

The eigenfrequencies ω_l of the isovector vibrations are determined by the boundary conditions (47). In the calculations we represented the solution as an expansion with respect to spherical harmonics with subsequent truncation of the series (for more details, see Ref. 68). The results of the calculations of the energies of the isovector vibrations of different multipolarities taking place along the symmetry axis of the fissioning system are shown in Fig. 7, which gives only values for the isovector vibrations corresponding to large fission deformations, which are characteristic for the interval from the saddle point to the scission point. For the actinide nuclei, we note that for spherical configurations ($\rho=0.375$) the calculated energies of the isovector vibrations correspond to the energy of the giant dipole resonance in the hydrodynamic model, $\omega_1=uk_1=2.08u/R_0$, and this leads to $\hbar\omega_1\cong 80A^{-1/3}$ MeV.[35,70]

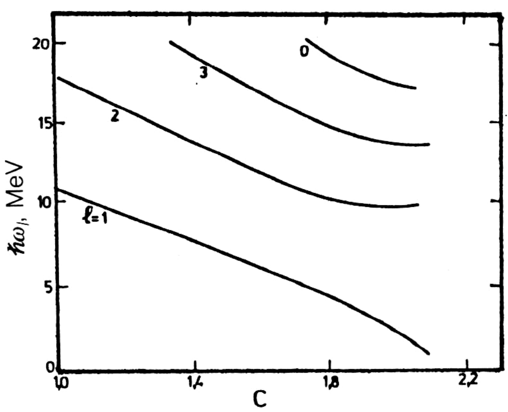

FIG. 7. Energy of isovector vibrations as functions of the elongation parameter c for the nucleus ^{236}U. The value of the neck parameter is zero. The value of l next to the curves indicates the multipolarity of the given vibration mode.

It can be assumed that the main role in the formation of the charge distribution is played by dipole isovector vibrations along the symmetry axis of the fissioning nucleus, since it is they that lead to the most effective redistribution of the charge between the fragments and are most readily excited as the nucleus divides. The lowest-energy mode corresponds to dipole isovector vibrations, and in what follows we shall use the notation $\omega_1=\omega_\Delta$. Figure 8 shows, in an enlarged scale, the energy of the dipole isovector vibrations as a function of the deformation parameters. It follows from the calculations that the frequency of the dipole isovector vibrations decreases with decreasing neck radius, which can come about because of increase of ρ or h. The h dependence of ω_Δ, which is weak at small ρ, becomes stronger at larger ρ. For a very small neck radius, the energy of the dipole vibrations tends to zero; this corresponds to rapid growth of the inertia parameter corresponding to charge exchange between the nascent frag-

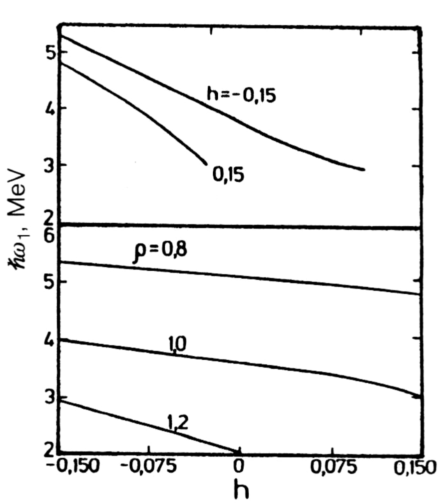

FIG. 8. Energies of dipole isovector vibrations as functions of ρ and h for $\alpha=0.0$ for ^{236}U.

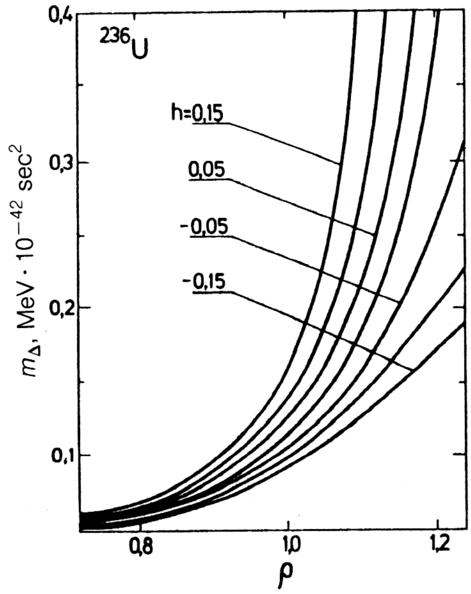

FIG. 9. Dependence of the inertia coefficient of the charge mode on the basic fission coordinate ρ. The values of the neck parameter h for which they were calculated are given.

ments. Note that in the scission region ($\rho=1.61$–1.24) the energies of the dipole vibrations are of order 2–3 MeV, and this is appreciably less than the estimate $\hbar\omega_1=80/(A_L^{1/3}+A_H^{1/3})$ MeV, which for symmetric fission with ^{236}U leads to $\hbar\omega_1\cong 8$ MeV, and is also much less than the values of $\hbar\omega_1$ that follow from the condition of constancy of the variances σ_Z^2 in the interval of excitation energies $E^*=20$–50 MeV.

The inertia parameter m_Δ of the charge mode was found from the formula

$$m_\Delta=\frac{C_\Delta(\rho,h)}{\omega_1^2(\rho,h)},$$

where the rigidity C_Δ of the charge mode is related to C_ζ by $C_\Delta=C_\zeta(\partial\zeta/\partial\Delta)^2$. The dependence of m_Δ on ρ and h is shown in Fig. 9. It can be seen that a characteristic feature of the change of m_Δ is the sharp increase of this parameter as the scission point is approached, i.e., with increasing ρ or h. Note also that m_Δ depends strongly on the geometry of the fissioning system, especially on the neck radius. This agrees with the estimates obtained earlier.[62,71]

In Ref. 71, the following expression was obtained for the inertia coefficient of the charge mode for flow of an inviscid liquid through a circular opening of radius r_n connecting two contiguous spherical fragments:

$$m_\Delta=\frac{2\pi}{3}r_0^3 m\frac{A^2}{NZ}\frac{1}{r_n}, \tag{49}$$

where r_0 is the nuclear radius parameter, m is the nucleon mass, and A, Z, and N are the mass number and numbers of protons and neutrons of the fissioning nucleus. In the derivation of (49), the fact that the neck joining the nascent fragments has a definite length l was completely ignored.

For a flow of viscous liquid through a cylindrical neck with radius r_n and length l it was shown in Ref. 62 that the inertia coefficient of the charge mode is determined in this case by

$$m_\Delta=\frac{4}{3\pi}\frac{m^*A^2}{\rho_0 NZ}\frac{l+2r_n}{r_n^2}. \tag{50}$$

Here, $\rho_0=0.145$ fm^{-3} is the standard nuclear density, and m^* is the effective nucleon mass.

The use of this result[72] to estimate the variance of the charge distribution in the statistical limit [the expression (40b)] leads to agreement with the experimental variances of the charge distribution for $r_n\approx 2.4$ fm, which is close to the neck radius obtained from the static calculations of Refs. 30 and 73.

Friction coefficient of the charge mode

At the present time, there is no well-developed approach for describing the dissipation of dipole isovector vibrations in fissioning nuclei. Therefore, γ_Δ is often[32,61,74] taken to be a varied parameter that does not depend on the coordinates. Of course, this is a rough approximation. However, to estimate the friction parameter one can use experimental data on the widths of giant dipole resonances, and the dependence of the dissipation on the geometry of the fissioning nucleus can be described by simple hydrodynamic models. In Ref. 62, the dissipation of the isovector vibrations was estimated by means of a model of steady flow of a viscous liquid along a cylindrical tube,[75] which for the considered model plays the role of a neck connecting the nascent fragments.

The kinetic energy of the liquid in the cylindrical tube of radius r_n and length l is

$$E_{\rm kin}=\frac{\pi\rho(\Delta p)^2 r_n^6}{96\nu_0 l}, \tag{51}$$

where Δp is the pressure difference across the ends of the tube.

The energy dissipated by the liquid in such a cylinder per unit time is

$$\dot{E}_{\rm kin}=-\frac{\pi(\Delta p)^2 r_n^4}{8\nu_0 l}. \tag{52}$$

Therefore, the reduced friction coefficient of the charge mode is

$$\beta_\Delta=\frac{\gamma_\Delta}{m_\Delta}=-\frac{\dot{E}_{\rm kin}}{2E_{\rm kin}}=\frac{\sigma\nu_0}{\rho_0 r_n^2}\equiv\frac{\sigma\nu}{r_n^2}, \tag{53}$$

where ν_0 and ν are the coefficients of dynamic and kinematic viscosity. For a good description of the widths of the giant dipole resonances, $\nu=0.135\cdot 10^{-21}$ fm$^2\cdot$sec^{-1} (Ref. 76).

As for m_Δ, a characteristic feature of the variation of β_Δ is a rapid growth on approach to the scission point on account of the proportionality to r_n^{-2}. Therefore, the friction parameter $\gamma_\Delta=\beta_\Delta m_\Delta$ will increase on the descent from the saddle point to scission even more rapidly. This must be understood when analyzing the results of the calculations of Refs. 32, 61, and 74, in which γ_Δ was taken to be a varied parameter independent of the coordinates.

Variances of the charge distribution

In a good approximation,[32,77,78] the variance σ_Z^2 of the charge distribution is equal to the variance of the parameter of the charge mode fixed at the moment of scission:

$$\sigma_Z^2 \approx \sigma_\Delta^2(t_{sc}). \tag{54}$$

The point and time t_{sc} of scission were determined from the intersection of the descent trajectory of the fissioning nucleus with the scission line,[11] which was found from the condition of equality of the forces of the Coulomb repulsion of the nascent fragments and of the nuclear attraction between them.[11]

In an analysis of the results of calculations of the variances of the charge distribution, it is helpful to have in mind three limiting cases of solution of the Fokker–Planck equation.

a) For sufficiently large friction with respect to the fission coordinate statistical equilibrium (instantaneous statistical limit) with respect to the charge mode can be established for $t \gg \beta_\Delta^{-1}, \omega_\Delta^{-1}$. In this case

$$\sigma_{\Delta_{st}}^2 = \frac{T_\Delta^*(t)}{C_\Delta(t)}, \tag{55}$$

where $T_\Delta^*(t)$ is the effective temperature of the charge mode.

b) For sufficiently large friction with respect to the charge mode, there is a memory ("freezing") of the values of the charge mode that the system possessed when entering this regime.

c) Vanishingly small friction with respect to all the degrees of freedom corresponds to the so-called dynamic limit.[32,77] The expression for $\sigma_\Delta^2(t)$ in this limit in the case of adiabatic variation of the parameters of the charge mode has the form[32]

$$\sigma_\Delta^2(t) = \sigma_\Delta^2(0)\,[m_\Delta(0)C_\Delta(0)/m_\Delta(t)C_\Delta(t)]^{1/2}. \tag{56}$$

The maximum vibration period of the charge mode is $(1\text{–}1.5)\cdot 10^{-21}$ sec, while the descent time is more than $5\cdot 10^{-21}$ sec, so that the adiabatic approximation for the charge mode is valid.

The results of the calculations of Ref. 69 with varied γ_Δ are given in Figs. 10 and 11. As a general result, we note that for a wide range of values of the friction parameter, $(0.1\text{–}0.75)\cdot 10^{-21}$ MeV · sec, statistical equilibrium with respect to charge exchange between the fragments can be established at any time of the descent from the saddle point. The values of γ_Δ in this interval are close to the estimates obtained for the friction parameter of the charge mode in Ref. 62. For anomalously large values of γ_Δ—of order $(5\text{–}10)\cdot 10^{-21}$ MeV · sec—statistical equilibrium cannot be established, and the system "remembers" the initial conditions. At the same time, the variance σ_Z^2 tends to its limit of frozen initial conditions. For $\gamma_\Delta \leqslant 0.12\cdot 10^{-21}$ MeV · sec, σ_Z^2 tends to the dynamic limit.

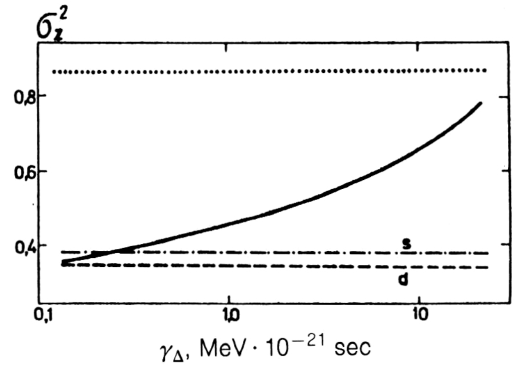

FIG. 10. Variance of the charge distribution as a function of the friction coefficient with respect to the charge degree of freedom. The chain line corresponds to the statistical limit, the broken line to the dynamic limit, and the dots to the limit of frozen initial conditions. The calculation was made for ^{236}U.

Figure 11 shows the variance as a function of the time. The behavior of the variance during descent is mainly due to the growth of the inertia parameter m_Δ, which depends strongly on the geometry of the fissioning system, especially on the neck parameter.[62,70,71] The rigidity with respect to the charge degree of freedom influences the evolution of the variance weakly, since it remains almost constant, increasing on descent from the saddle to fission by not more than 20%.

Figure 12 shows the time evolution of the variance of the charge coordinate for the nucleus ^{236}U, calculated in the diffusion model (with parameter γ_Δ independent of the coordinates and determined in accordance with (53)). In the calculations using a coordinate-dependent γ_Δ we observe a "freezing" of the variances of the charge mode due to the sharp growth of γ_Δ on the approach to scission. In the region of the scission point, the variance of the charge distribution calculated in the diffusion model is close to its statistical value (curve *1* and the chain curve in Fig. 12). The figure shows $\sigma_{\Delta d}^2$ calculated in accordance with the approximate expression (56) and the exact dynamic limit. It can be seen that in the two cases the values of $\sigma_{\Delta d}^2$ are practically the same. This means that the adiabatic approximation for the Δ oscillator is satisfied with high accuracy.

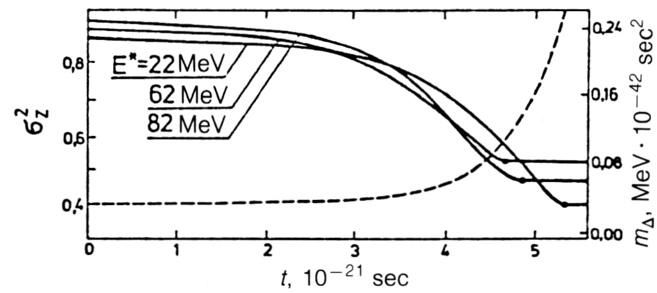

FIG. 11. Dependence of the variance of the charge coordinate (continuous curves) and inertia coefficient of the charge mode (broken curve) on the time during the descent from the saddle point to scission. The calculation was made for ^{236}U.

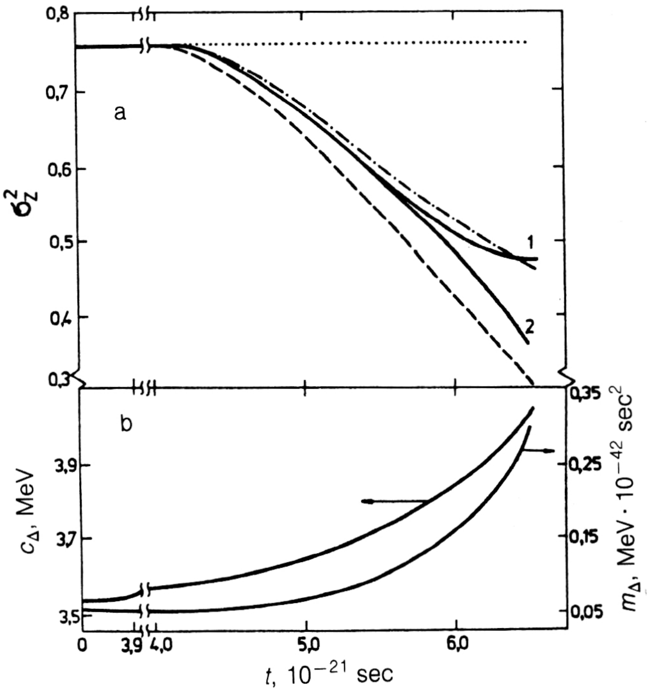

FIG. 12. Variance of the charge coordinate as a function of the time (a) during descent from the saddle point to scission for ^{236}U. The continuous curves were calculated in the diffusion model, curve *1* corresponds to a coordinate-dependent friction coefficient [Eq. (53)], and curve *2* to the value $\gamma_\Delta = 2\pi R_0^3 \cdot 10^{-25}$ MeV · sec · fm^{-3}. The chain curve corresponds to the statistical limit, the broken curve to the dynamic limit, and the dotted line to the limit of frozen initial conditions. The time dependence of the rigidity coefficient and the inertia coefficient of the charge mode are shown in (b).

Thus, analysis of the dynamical evolution of the variances of the charge distribution shows that for reasonable values of the friction parameter σ_Z^2 is close to its statistical limit. On the other hand, the statistical limit of σ_Z^2 is determined by the geometry of the nucleus at the time of scission. Therefore, the charge distribution is exceptionally informative about the scission configurations, which, in its turn, is determined by the descent dynamics. Thus, it appeared that the statistically equilibrated charge distribution could carry unique information about the configuration of the fissioning nucleus at the time of scission.

It is well known (Refs. 6, 11, 12, and 79) that the different mechanisms of nuclear viscosity used in dynamical calculations lead to very different trajectories of the descent of the fissioning nucleus from the saddle point to scission and, as a consequence, to different shapes of the scission configurations.

The two-body mechanism of nuclear viscosity,[79] hindering the development of deformations of high multipolarity, leads to elongated scission configurations. On the transition from light to heavier fissioning nuclei the elongation of the scission configurations is significantly increased.[11,79] In contrast, the single-body viscosity mechanism[80,81] and its modifications,[82,83] which suppress the development of deformations of low multipolarity, lead to compact scission configurations that for all fissioning nuclei from Pt to Fm remain practically the same as what follows from the static calculations in the liquid-drop model.[30]

The variances σ_M^2 and $\sigma_{E_k}^2$ of the mass–energy distributions calculated in the diffusion model were found[11,12,16] to be insensitive to the viscosity mechanism used in the calculations. Therefore, the elucidation of the nuclear viscosity mechanism in fission is at the present time one of the main problems of both experimental and theoretical investigations of this unique nuclear process (numerous references can be found in Ref. 84).

The existing differences in the shapes of the scission configurations could lead to different variances of the charge distribution and their dependence on the fissility parameter. Calculations show[85] that this conclusion is not justified. In the case of the single-body mechanism, in which the shape of the scission configurations is almost the same for all fissioning nuclei, σ_Z^2 exhibits with increasing A a weak dependence of the type $A^{2/3}$, which reflects the dependence of the energy of the isovector dipole vibrations and the nuclear rigidity with respect to these vibrations on A. Although for the two-body mechanism the shape of the scission configurations changes appreciably with increasing A, it changes in such a way that the resulting increase of ρ_{sc} and decrease of h_{sc}, compensating their influence on $\hbar\omega_\Delta$, leave it practically unchanged (see Fig. 9). Therefore, for the two-body mechanism too the variances and the A dependence of σ_Z^2 are practically the same as for the one-body mechanism (see Table I).

The two- and one-body viscosity mechanisms lead not only to different descent trajectories and shapes of the scission configurations but also to different descent velocities, which can differ by more than an order of magnitude.[11,83] Therefore, one could hope that the "freezing" of the charge mode that occurs in the actual descent stage because of the sharp growth of the inertia and friction coefficients would permit discrimination of the viscosity mechanism realized in fission.

This question was investigated in Ref. 86 on the basis of the Schrödinger equation with friction for a harmonic oscillator whose parameters depend on the time. The calculations showed that the estimates under the assumption of statistical equilibrium of this mode (expression (40b)) are in good agreement with the results of the dynamical treatment. It also follows from the results of Ref. 86 that nonadiabaticity of the charge mode and freezing of its variance occur very late, shortly before ($(1–2) \cdot 10^{-22}$ sec) the breaking of the neck joining the nascent fragments. This fact (the very late "freezing" of the charge mode) makes it impossible to choose between the two fundamentally different viscosity mechanisms that could be realized in fission.

In Ref. 87, a study was made of the fluctuation dynamics of the charge mode in the very final stage of the process, when the neck breaks, on the basis of the Schrödinger equation for an oscillator with time-dependent inertia coefficient.

The dependence of σ_Z^2 on the rate of breaking of the neck, which can be characterized by the change of the neck

TABLE I. Parameters that characterize the descent dynamics (values of the collective coordinates at scission $\{\rho_{sc}, h_{sc}\}$) and variances (σ_Z^2) of the charge distribution for the two-body (top line for each nucleus) and "surface" single-body viscosity mechanisms.

Nucleus	ρ_{sd}	h_{sd}	ρ_{sc}	h_{sc}	σ_Z^2 (charge units)^2
^{216}Rn	0,810	−0,041	1,316	−0,513	0,44
			1,217	−0,083	0,42
^{230}Th	0,720	−0,027	1,335	−0,175	0,47
			1,220	−0,086	0,44
^{240}Pu	0,642	−0,016	1,380	−0,205	0,49
			1,222	−0,090	0,45
^{247}Cm	0,613	−0,013	1,400	−0,217	0,49
			1,226	−0,091	0,45
^{252}Cf	0,583	−0,011	1,415	−0,229	0,49
			1,231	−0,093	0,46
^{258}Fm	0,558	−0,010	1,439	−0,233	0,50
			1,234	−0,094	0,47

radius r_n, is shown in Fig. 13. It can be seen from the figure that for a change of the neck radius equal to 1–2 fm/sec the calculated σ_Z^2 agree with the experimental data. Significantly lower and more rapid fissioning lead to data that differ significantly from the experimental data.

Figure 14 shows the dependence of the charge distribution on the excitation energy, calculated on the basis of the Fokker–Planck equation.[32] For frequencies of the isovector dipole vibrations characteristic of the scission region and equal to 2.5–3.0 MeV, a dependence of the variance on the excitation energy is observed in the complete considered interval 20–70 MeV. However, we note that it is significantly enhanced when $E^* > 40$ MeV $((\partial\sigma_Z^2/\partial E^2) > 0)$ and does not weaken $((\partial\sigma_Z^2/\partial E^2) < 0)$ if $\sigma_Z^2 - \sqrt{E^*}$, as would be the case if thermal fluctuations played the main role. Such an energy dependence of σ_Z^2 indicates that the charge fluctuations in fission are mainly quantum in nature.

Relaxation times of the charge mode

Characteristic times in fission have long[1,6,21] been used to estimate the role of the memory of the fissioning system with respect to the corresponding mode of its prehistory during the descent from the saddle point to scission. In Ref. 21, the characteristic times were taken to be the vibration periods of the system with respect to the mass-asymmetry and charge modes, for which the values $(1\text{–}3)\cdot 10^{-21}$ and 10^{-22} sec, respectively, were chosen. Comparing these estimates with the time of descent of the nucleus from the saddle point to scission, the authors of Ref. 21 concluded that the statistical model of Refs. 7 and 8 could not be used to study the mass distribution of the fragments but was valid for the charge distribution.

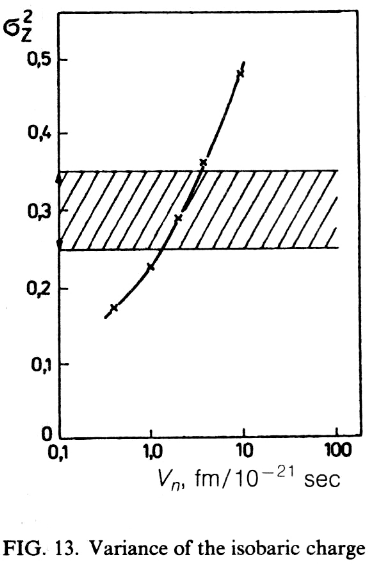

FIG. 13. Variance of the isobaric charge distribution of the fragments on the rate of breaking of the neck. The hatched region corresponds to the experimental values with allowance for the errors (the figure is taken from Ref. 87).

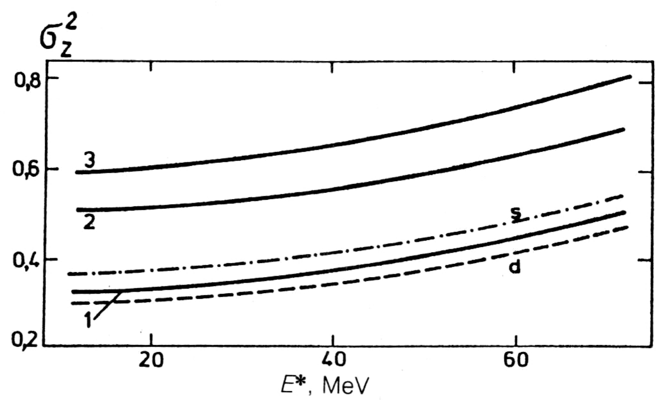

FIG. 14. Variances of the charge distribution as a function of the excitation energy. Curves *1*, *2*, and *3* correspond to different values of the friction coefficient of the charge mode: $0.1\cdot 10^{-21}$, $5\cdot 10^{-21}$, $10\cdot 10^{-21}$ MeV · sec; the chain curve corresponds to the statistical limit, and the broken curve to the dynamic limit.

For a system with dissipation, it is more natural to use the relaxation time as a characteristic time. We estimate the relaxation time of the charge mode in the framework of the diffusion model.[22] The system (42) can be reduced to the two equations

$$\ddot{\sigma}_\Delta+\widetilde{\beta}\dot{\sigma}_\Delta+\omega_\Delta^2\sigma_\Delta=U/m^2\sigma_\Delta^3, \tag{57}$$

$$\dot{U}=-2\beta\dot{U}+2D\sigma_\Delta^2, \tag{58}$$

where $U = \sigma_\Delta^2\sigma_{p_\Delta}^2 - \sigma_{\Delta p_\Delta}^2$, and $\beta_\Delta=\gamma_\Delta/m_\Delta$ and $\widetilde{\beta}=\beta_\Delta+\dot{m}_\Delta/m_\Delta$ are the damping coefficient and generalized damping coefficient of the charge mode, respectively.

If in the original system (42) we ignore the diffusion and friction, then Eq. (57) becomes identical to the equation for σ obtained earlier in Ref. 86, in which a study was made of the dynamics of formation of the charge distribution in deep inelastic heavy-ion collisions and in fission. Here, $\widetilde{\beta}=\dot{m}/m$ and the dependence of the inertia parameter on the time leads to the appearance of "friction" in the system. For rapid variation of m, the system "remembers" the value of the variance σ_y^2 even if friction with respect to this coordinate is completely absent. In the absence of friction, $U=U_0=\hbar^2/4$ (Ref. 86).

The approximate solution of Eq. (57) has the form

$$\sigma_\Delta=\sigma_{\Delta_0}^2\exp(-2t/\tau_\Delta)+\sigma_{\Delta st}^2[1-\exp(2t/\tau_\Delta)] \tag{59}$$

and is very similar to the usual law of a transient process used to analyze experimental material with respect to the mass distributions of the products of quasifission in heavy-ion reactions:[88,89]

$$\langle\Delta A\rangle=\langle\Delta A\rangle_0\exp(-t/\tau_\Delta)$$
$$+\langle\Delta A\rangle_{st}[1-\exp(-t/\tau_\Delta)], \tag{60}$$

except that $\sigma_{\Delta 0}$, $\sigma_{\Delta st}$, and τ_Δ are functions of the time.

The oscillator that possesses the equation of motion (57) is in a regime of damped vibrations if $\omega_\Delta>\widetilde{\beta}_\Delta/2$ and in a regime of aperiodic damping if $\omega_\Delta>\widetilde{\beta}_\Delta/2$. In the case of damped vibrations, the solution (59) is the result of averaging over a period. The form of the solution (59) follows from the form of Eq. (57): The left-hand side of (57) is the equation of damped vibrations, the solution of which corresponds to the first term in (59), while the inhomogeneity [right-hand side of (57)] generates the second term in (59). The parameter τ_Δ in (59) is determined by

$$\tau_\Delta=\begin{cases}2\beta_\Delta^{-1}, & \omega_\Delta>\widetilde{\beta}_\Delta/2\\ [\widetilde{\beta}_\Delta/2-(\widetilde{\beta}_\Delta^2/4-\omega_\Delta^2)^{1/2}]^{-1}, & \omega_\Delta<\widetilde{\beta}_\Delta/2.\end{cases} \tag{61}$$

From the expression (60) for $\langle\Delta\rangle$ it is obvious that τ_Δ is the relaxation time of the mean value of the collective coordinate (it is this parameter that is used and discussed in all that follows), while the variance of this coordinate relaxes twice as fast.

The results of a calculation of the relaxation time of the charge mode are given in Figs. 15 and 16 for both viscosity mechanisms. Over almost the entire descent time $\tau_\Delta\approx(0.40.3)\cdot10^{-21}$ sec, and it is only just before scission, when the Δ oscillator enters the regime of aperiodic damping ($\widetilde{\beta}_\Delta/2>\omega_\Delta$) that τ_Δ increases sharply, reaching the value $(0.81.0)\cdot10^{-21}$ sec. The comparison of the calculated τ_α and τ_Δ made in Fig. 16 shows that the charge mode is indeed much more rapid than the mass-asymmetry mode. The values of the relaxation time and of the vibration period $2\pi/\omega_\Delta$ (Fig. 15) confirm that during the entire descent the statistical approach is valid for the charge distribution, and it is only just before the actual scission that there is a memory ("freezing") of the conditions in which the fissioning system found itself before the transition to this regime.

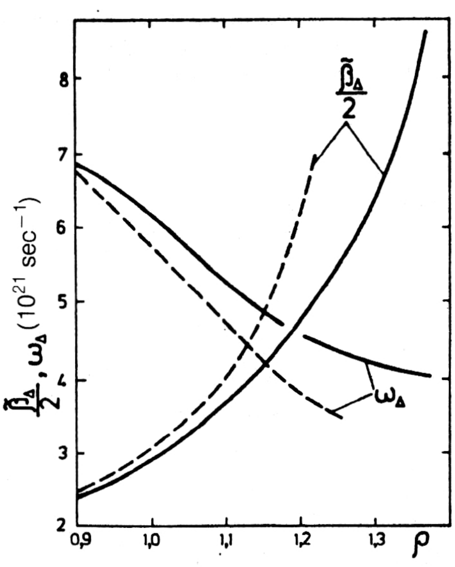

FIG. 15. Dependence of the frequency ω_Δ and the reduced friction coefficient (53) of the charge mode on the basic fission coordinate ρ during descent from the saddle point to scission for ^{236}U. The continuous curves are the results of a calculation with the two-body viscosity mechanism, and the broken curves the results with "surface" single-body viscosity.

Even–odd effects in fragment charge yields

In recent years (see Refs. 4, 5, 90, and 91) it has been pointed out that the observed fine structure of the charge

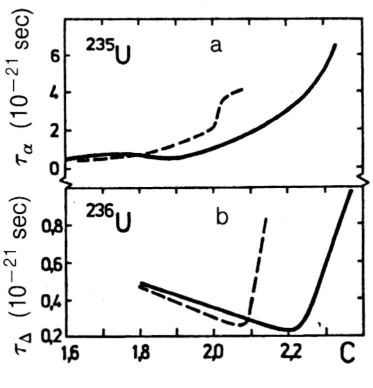

FIG. 16. Dependence of the relaxation time τ_Δ of the charge mode (b) on the elongation parameter during descent from the saddle point to scission for ^{236}U. The continuous curve is the calculation with the two-body viscosity mechanism, while the broken curve is made with "surface" single-body viscosity. For comparison (a) the corresponding relaxation times of the mass-asymmetry mode for ^{235}U are given.

TABLE II. Parameters characterizing the descent dynamics (values of the collective coordinates at the saddle point (ρ_{sd}, h_{sd}) and at the scission point (ρ_{sc}, h_{sc}) and difference ΔV of the deformation energy between the saddle point and scission point) with the use of the macroscopic liquid-drop model[99] (top line for each nucleus), the liquid-drop model with $\Gamma = -0.1$ (Ref. 34) (bottom line for each nucleus), and the droplet model.[41] The calculations were made using the "surface" single-body viscosity mechanism (first row for each nucleus) and the two-body mechanism with $V_0 = 1.5 \cdot 10^{-23}$ MeV · sec · fm^{-3}.

Nucleus	ρ_{sd}	h_{sd}	ρ_{sc}	h_{sc}	ΔV, MeV
^{211}Po	0,877	−0,053	1,212	−0,082	3,1
	0,877	−0,053	1,279	−0,135	4,1
	0,884	−0,062	1,290	−0,142	2,8
	0,977	−0,088	1,281	−0,133	0,2
^{223}Ra	0,762	−0,033	1,212	−0,085	7,0
	0,762	−0,033	1,319	−0,163	8,8
	0,729	−0,033	1,336	−0,176	7,8
	0,772	−0,036	1,327	−0,169	3,6
^{230}Th	0,720	−0,027	1,220	−0,086	9,7
	0,720	−0,027	1,335	−0,175	11,6
	0,674	−0,024	1,361	−0,194	11,3
	0,731	−0,029	1,343	−0,181	6,4
^{240}Pu	0,642	−0,016	1,222	−0,089	15,6
	0,642	−0,016	1.380	−0,205	19,6
	0,597	−0,014	1,414	−0,227	19,7
	0,633	−0,014	1,398	−0,217	12,8
^{252}Cf	0,583	−0,011	1,233	−0,093	23,0
	0,583	−0,011	1,415	−0,229	28,3
	0,535	−0,010	1,464	−0,259	29,8
	0,575	−0,009	1,440	−0,244	20,4
^{258}Fm	0,558	−0,009	1,234	−0,010	29,2
	0,558	−0,009	1,439	−0,234	33,5
	0,508	−0,010	1,485	−0,273	35,4
	0,549	−0,008	1,464	−0,257	24,8

distributions, or, more correctly, analysis of the even–odd effect in the fission-fragment yields of even–even compound nuclei, makes it possible, using model representations,[92,93] to estimate the dissipated energy $E_{\rm dis}$ during the descent of the fissioning nucleus from the saddle point to scission and, thus, can help to elucidate the mechanism of nuclear viscosity in fission. The basic idea of such an approach[4,5,90] is that the dissipative forces lead to internal excitations manifested in the breaking of nucleon pairs. The number of broken pairs is a measure of the dissipated energy, on the other hand, and determines the observed even–odd effects, on the other.

It was found[94–97] that the dissipated energy $E_{\rm dis}$ increases smoothly with increasing parameter Z^2/A in neutron-induced fission of all the studied actinides from thorium to californium, varying from 3 to 11 MeV. This tendency in the $E_{\rm dis}$ dependence is similar to the dependence of the difference of the deformation energy between the saddle point and the scission point for these nuclei on the same parameter of the calculated energy in the study of the descent dynamics.[11,12,98] The ratio of the energy dissipated (on the descent from the saddle point to scission, $E_{\rm dis}$) to ΔV remains practically constant and is 0.3–0.5 for the complete range of studied nuclei. The uncertainty in this estimate for the ratio $E_{\rm dis}/\Delta V$ can be rather large, since one cannot rule out the possibility of a twofold overestimation of $E_{\rm dis}$. Approximately the same uncertainty is inherent in ΔV. There are two reasons for the uncertainty in the estimate of ΔV: the determination of the scission point[11,12] and also the much larger uncertainty in the choice of the coefficient of the curvature energy in fission. In the "canonical" set of parameters of the liquid-drop model[66,99] and in the droplet model (Refs. 41 and 42), $a_{\rm curv}=0$, while in other approaches,[73,100,101] and also in the liquid-drop model[34] with $\Gamma = -0.1$, the value of $a_{\rm curv}$ corresponds to approximately 10 MeV. Allowance for such a contribution of the curvature energy to the deformation energy leads, in the first place, to a much flatter landscape of the energy surface between the saddle point and the scission point and, accordingly, to a smaller value of ΔV (Table II).

Despite the uncertainty in the absolute value of $E_{\rm dis}/\Delta V$, the constancy of this ratio for all the studied nuclei appears well established.[90] From this it may be concluded that 30–50% of the energy ΔV available for dissipation during descent goes over, depending on the macroscopic model used in the dynamical calculations (see Table II), into internal excitation. This indicates, in its turn, that during the descent the dissipative forces are comparatively small and correspond better to the two-body[79] than to the single-body[80–83] mechanism of nuclear viscosity.[1)]

Thus, the experimental data on the even–odd effect in the fragment charge yields in conjunction with a dynamical analysis of the formation of the fission fragment distributions can also be a unique source of information on the viscosity of nuclear matter in fission, like the data on the mass–energy distributions (Refs. 1, 5, 11, and 12). A detailed description of the analysis of even–odd effects in the fragment charge yields is given in Refs. 90 and 91.

CONCLUSIONS

In the review, we have considered the influence of the dynamical characteristics of fission on the formation of the isobaric charge distribution of the fission fragments under the assumption that the main mechanism of formation is collectified motion of nucleons manifested through dipole isovector vibrations. The role of other isovector modes is not considered, although their contribution to the variance of the charge distribution can be very appreciable if one bears in mind the results obtained for heavy-ion deep inelastic collisions.[104,105]

We note also that the distributions of the reaction products of heavy-ion deep inelastic collisions have also been successfully analyzed[55,56,106] by the alternative mechanism of formation of the distributions of the products of these reactions through stochastic exchange of nucleons between the product nuclei that are being formed. The role of noncollective excitations and their contribution to the relaxation of the neutron-excess degree of freedom in heavy-ion deep inelastic collisions were studied in Refs. 105 and 107.

In the formation of the isobaric fission-fragment distribution and the equilibration of the neutron-excess degree of freedom in fission both mechanisms, the collective and the noncollective, apparently occur. Unfortunately, at the present time there is not a single estimate of the role of the noncollective mechanism in the equilibration of the neutron-excess degree of freedom in fission. The study of these questions in fission theory lags seriously behind the understanding of the analogous approaches for heavy-ion deep inelastic collisions.

The dynamical approach to the formation of the charge distribution shows that statistical equilibrium with respect to the charge mode can be established at each instant of time throughout almost the entire descent from the saddle point to scission, except for its very last stage directly preceeding the breaking of the neck. This explains the successful use of the statistical model for calculations of the fission-fragment charge distribution. The deviation of the charge-distribution variance from its equilibrium value during the final stage of the process before scission is due to the nonadiabaticity of this stage with respect to the charge mode resulting from the rapid growth of the inertia and friction coefficients of the charge mode, leading to freezing of the charge-distribution variance. Therefore, we mention especially that the problem of reliable determination of the inertia and friction coefficients of the charge mode, which determine the details of the dynamical formation of the charge distribution and, ultimately, the values of its observed parameters, is today an essentially unresolved problem.

In the framework of the employed model, we have shown that the variance of the charge distribution is mainly due to quantum dipole isovector vibrations. The energies of these vibrations in the fissioning nucleus are 2.5–3.0 MeV for the scission configurations, and these are much smaller than the values that one could expect from the condition of constancy of the charge-distribution variances up to 40 MeV. The energy dependence of the variances is quite weak and becomes significantly stronger when $E^* > 40$ MeV, and not weaker, as would be the case for statistical fluctuations. This energy dependence of the charge-distribution variances indicates that in fission the charge fluctuatiions are largely quantum in nature.

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[1)]The assumption that the dissipation of the collective motion during the descent from the saddle point to scission is responsible for the breaking of nucleon pairs and, therefore, the magnitude of the even–odd effect is not the only assumption currently made. In Ref. 102 there is a detailed discussion of and argument for the hypothesis that most of the quasiparticle excitations result from the rapid breaking of the neck.[103]

[1] M. G. Itkis, V. N. Okolovich, A. Ya. Rusanov, and G. N. Smirenkin, Fiz. Elem. Chastits At. Yadra **19**, 702 (1988) [Sov. J. Part. Nucl. **19**, 301 (1988)].

[2] M. G. Itkis, S. M. Luk'yanov, V. N. Okolovich *et al.*, Yad. Fiz. **52**, 23 (1990) [Sov. J. Nucl. Phys. **52**, 15 (1990)].

[3] S. M. Luk'yanov, Yu. É, Penionzhkevich, V. S. Salamatin *et al.*, *International Seminar School on Heavy Ion Physics*, D7-90-142 [in Russian] (JINR, Dubna, 1990), p. 225.

[4] F. Gonnenwein, *Physics and Chemistry of Fission*, ZFK-732 (Dresden, 1991), p. 12.

[5] F. Gonnenwein, *Nuclear Fission Process*, edited by C. Wagemans (CRC Press, Boca Raton, USA, 1991), p. 288.

[6] Yu. Ts. Oganessian and Yu. A. Lazarev, *Treatise on Heavy Ion Science*, Vol. 4, edited by D. A. Bromley (Plenum Press, New York, 1985), p. 1.

[7] P. Fong, *Statistical Theory of Nuclear Fission* (Gordon and Breach, New York, 1969).

[8] A. V. Ignatyuk, Yad. Fiz. **9**, 357 (1969) [Sov. J. Nucl. Phys. **9**, 208 (1969)]; A. V. Ignatyuk and V. P. Kraĭnov, "Statistical description of fission-fragment yields," in *Topical Problems of Fission Physics, Ninth School of the Moscow Engineering Physics Institute* [in Russian] (Moscow Engineering Physics Institute, 1983), p. 3.

[9] J. R. Nix and W. J. Swiatecki, Nucl. Phys. **71**, 1 (1965); J. R. Nix, Nucl. Phys. **A130**, 241 (1969).

[10] R. W. Hasse, Nucl. Phys. **A128**, 609 (1969); Phys. Rev. C **4**, 572 (1971).

[11] G. D. Adeev, I. I. Gonchar, V. V. Pashkevich *et al.*, Fiz. Elem. Chastits At. Yadra **19**, 1229 (1988) [Sov. J. Part. Nucl. **19**, 529 (1988)].

[12] G. D. Adeev and V. V. Pashkevich, Nucl. Phys. **A502**, 405 (1989).

[13] H. A. Kramers, Physica **7**, 284 (1940).

[14] P. Grange, Li Jun-Qing, and H. A. Weidenmuller, Phys. Rev. C **27**, 2063 (1983); H. A. Weidenmuller and Zhang Jing-Shang, Phys. Rev. C **29**, 879 (1984).

[15] Y. Abe, G. Gregoire, and H. Delagrange, J. Phys. (Paris) Colloq. **47**, 329 (1986); P. Frobrich and S. Y. Xu, Nucl. Phys. **A477**, 143 (1988).

[16] T. Wada, N. Carjan, and Y. Abe, Nucl. Phys. **A538**, 283 (1992); G. R. Tillack, Phys. Lett. **278B**, 403 (1992); G. I. Kosenko, I. I. Gonchar, N.

I. Pischasov, and O. I. Serdyuk, Yad. Fiz. **55**, 920 (1992) [Sov. J. Nucl. Phys. **55**, 514 (1992)].
[17] E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear Properties of the Heavy Elements*, Vol. 3. *Fission* (Prentice-Hall, Englewood Cliffs, N.J., 1964); R. Vandenbosh and J. R. Huizenga, *Nuclear Fission* (Academic Press, New York, 1973).
[18] D. C. Hofman and M. H. Hofman, Ann. Rev. Nucl. Sci. **24**, 151 (1974).
[19] H.-G. Clerc, W. Lang, H. Wohlfarth *et al.*, *Physics and Chemistry of Fission*, Vol. 11 (IAEA, Vienna, 1980), p. 65.
[20] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[21] S. A. Karamyan, Yu. Ts. Oganesyan, Yu. É. Penionzhkevich, and B. I. Pustyl'nik, Yad. Fiz. **9**, 715 (1969) [Sov. J. Nucl. Phys. **9**, 414 (1969)].
[22] G. D. Adeev, I. I. Gonchar, V. V. Pashkevich, and O. I. Serdyuk, Yad. Fiz. **50**, 1242 (1989) [Sov. J. Nucl. Phys. **50**, 774 (1989)].
[23] H. Freiesleben and J. V. Kratz, Phys. Rep. **106**, 1 (1984).
[24] R. D. Present, Phys. Rev. **72**, 7 (1947).
[25] G. D. Adeev, L. A. Filipenko, and P. A. Cherdantsev, Yad. Fiz. **23**, 30 (1976) [Sov. J. Nucl. Phys. **23**, 15 (1976)].
[26] R. K. Gupta, W. Schneid, and W. Greiner, Phys. Rev. Lett. **35**, 353 (1975).
[27] R. K. Gupta, Fiz. Elem. Chastits At. Yadra **8**, 717 (1977) [Sov. J. Part. Nucl. **8**, 289 (1977)].
[28] W. Greiner, M. Ivascu, D. Poenaru, and A. Sandulescu, *Treatise on Heavy Ion Science*, Vol. 8, edited by D. A. Bromley (Plenum Press, New York, 1989), p. 641.
[29] M. G. Mustafa, H. W. Schmitt, and U. Mosel, Nucl. Phys. **A178**, 9 (1971).
[30] M. Brack, J. Damgaard, A. S. Jensen *et al.*, Rev. Mod. Phys. **44**, 320 (1972); V. M. Strutinskiĭ, V. V. Pashkevich, M. Brak *et al.*, Preprint 73-I.Ya [in Russian] (Kiev Institute of Nuclear Research, Kiev, 1973).
[31] J. F. Berger, M. Girod, and J. Gogny, J. Phys. Lett. (Paris) **42**, L509 (1981).
[32] G. D. Adeev, I. I. Gonchar, and L. A. Marchenko, Yad. Fiz. **42**, 42 (1985) [Sov. J. Nucl. Phys. **42**, 25 (1985)].
[33] N. G. Volkov, V. M. Emel'yanov, and V. P. Kraĭnov, Yad. Fiz. **29**, 1171 (1979) [Sov. J. Nucl. Phys. **29**, 603 (1979)].
[34] V. M. Strutinskiĭ, Zh. Eksp. Teor. Fiz. **45**, 1891 (1963) [Sov. Phys. JETP **18**, 1298 (1963)]; Yad. Fiz. **1**, 821 (1965) [Sov. J. Nucl. Phys. **1**, 588 (1965)].
[35] I. Eisenburg and W. Greiner, *Nuclear Models. Collective and Single-Particle Phenomena* [in Russian] (Atomizdat, Moscow, 1975).
[36] V. M. Stutinsky, N. Ya. Layshchenko, and N. A. Popov, Nucl. Phys. **46**, 639 (1963).
[37] K. T. R. Davies and A. J. Sierk, J. Comput. Phys. **18**, 311 (1975).
[38] R. R. Beringer, Phys. Rev. C **131**, 1402 (1963).
[39] G. D. Adeev and T. Dossing, Phys. Lett. **66B**, 11 (1977).
[40] W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) **55**, 395 (1969).
[41] W. D. Myers, *Droplet Model of Atomic Nuclei* (Plenum, New York, 1977).
[42] W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) **84**, 186 (1974).
[43] G. D. Adeev and L. A. Philipenko, Phys. Lett. **72B**, 30 (1977).
[44] K. Albrecht, Nucl. Phys. **A207**, 225 (1973).
[45] L. G. Moretto, Nucl. Phys. **A180**, 337 (1972).
[46] L. G. Moretto, Nucl. Phys. **A182**, 641 (1972).
[47] A. S. Jensen and J. Damgaard, Nucl. Phys. **A210**, 282 (1973).
[48] A. S. Jensen and T. Dossing, *Physics and Chemistry of Fission*, Vol. 1 (IAEA, Vienna), p. 409.
[49] A. V. Ignatyuk and Yu. N. Shubin, Yad. Fiz. **8**, 1135 (1968) [Sov. J. Nucl. Phys. **8**, 660 (1969)].
[50] T. Dossing and A. A. Jenson, Nucl. Phys. **A222**, 493 (1974).
[51] A. C. Wahl, R. L. Ferguson, D. R. Nethaway *et al.*, Phys. Rev. **126**, 1112 (1962).
[52] W. Reisdorf, J. P. Unik, H. C. Griffin *et al.*, Nucl. Phys. **A177**, 337 (1971).
[53] S. M. Qaim and H. O. Denchlag, J. Inorg. Nucl. Chem. **32**, 1767 (1972).
[54] P. O. Strom, A. E. Greendale, D. E. Deluechi *et al.*, Phys. Rev. **144**, 983 (1966).
[55] W. U. Schröder and J. R. Huizenga, *Treatise on Heavy Ion Science*, Vol. 2 (Plenum Press, New York, 1985), p. 113.
[56] J. Randrup, Nucl. Phys. **A307**, 319 (1978).
[57] H. Hofmann and P. J. Siemens, Nucl. Phys. **A275**, 464 (1977).
[58] H. Hofmann and A. S. Jensen, Nucl. Phys. **A428**, 1c (1984).
[59] R. Hasse, Nucl. Phys. **A318**, 480 (1979).
[60] H. Hofmann, C. Gregoire, R. Lucas, and C. Ngo, Z. Phys. A **293**, 229 (1979).
[61] K. Pomorski, *International School-Seminar on Heavy Ion Physics*, D7-83-644 (JINR, Dubna, 1983), p. 441.
[62] E. S. Hernandez, W. D. Myers, J. Randrup, and B. Remaud, Nucl. Phys. **A361**, 483 (1981).
[63] K. K. Kan and J. J. Griffin, Phys. Lett. **503**, 241 (1974).
[64] U. Brosa and W. Cassing, Z. Phys. A **307**, 167 (1982).
[65] C. Ngo and H. Hofmann, Z. Phys. A **282**, 83 (1977).
[66] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966).
[67] H. C. Pauli, Phys. Rep. **7**, 35 (1973).
[68] W. E. Updegraff and D. S. Onley, Nucl. Phys. **A161**, 191 (1971).
[69] G. D. Adeev, I. I. Gonchar, and L. A. Marchenko, in *Proceedings of the Sixth All-Union Conference on Neutron Physics* [in Russian] (Central Institute of Atomic Information, Moscow, 1984), Part 1, p. 392.
[70] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975) [Russ. transl., Mir, Moscow, 1977].
[71] U. Brosa and H. J. Krappe, Z. Phys. A **287**, 65 (1978); U. Brosa and D. H. E. Gross, Z. Phys. A **294**, 217 (1980).
[72] M. Asghar, Z. Phys. A **296**, 79 (1980).
[73] M. Asghar and R. W. Hasse, J. Phys. C **6**, 455 (1984).
[74] E. Martschev and K. Pomorski, Acta Phys. Pol. **B13**, 747 (1982).
[75] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1987) [Russ. original, Nauka, Moscow 1986].
[76] R. W. Hasse and P. Nerud, J. Phys. G **2**, L101 (1976).
[77] G. D. Adeev and I. I. Gonchar, Z. Phys. A **320**. 451 (1985).
[78] G. D. Adeev, I. I. Gonchar, and L. A. Marchenko, in *Proceedings of the Sixth All-Union Conference on Neutron Physics* [in Russian] (Central Institute of Atomic Information, Moscow, 1984), Part 2, p. 14.
[79] K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C **13**, 2385 (1976).
[80] J. Blocki, Y. Boneh, J. R. Nix *et al.*, Ann. Phys. (N.Y.) **113**, 330 (1978).
[81] J. Randrup and W. J. Swiatecki, Ann. Phys. (N.Y.) **125**, 193 (1980).
[82] J. R. Nix and A. J. Sierk, Preprint LA-UR-86-698, Los Alamos (1986).
[83] J. R. Nix and A. J. Sierk, Preprint LA-UR-87-133, Los Alamos (1986); J. R. Nix and A. J. Sierk, *International School-Seminar on Heavy Ion Physics*, D7-87-68 (JINR, Dubna, 1986), p. 453.
[84] J. O. Newton, Fiz. Elem. Chastits At. Yadra **21**, 821 (1990) [Sov. J. Part. Nucl. **21**, 349 (1990)].
[85] G. D. Adeev, V. V. Pashkevich, and O. I. Serdyuk, Brief Communication 4[24], JINR (1987), p. 51.
[86] W. D. Myers, G. Mantzouranis, and J. Randrup, Phys. Lett. **98B**, 1 (1981).
[87] H. Nifenecker, J. Phys. Lett. **41**, L47 (1980).
[88] W. Q. Shen, J. Albinsky, R. Bock *et al.*, Europhys. Lett. **1**, 113 (1989).
[89] W. Q. Shen, J. Albinsky, A. Gobbi *et al.*, Phys. Rev. C **36**, 115 (1987).
[90] F. Gennenvaĭn, Vopr. At. Nauki Tekh. Ser. Yad. Konst. No. 1, 14 (1988).
[91] N. A. Gundorin, A. Duka-Zoĭomi, Ya. Kliman *et al.*, Communication R15-88-386 [in Russian], JINR, Dubna (1988).
[92] G. Mantzouranis and J. R. Nix, Phys. Rev. C **25**, 918 (1982).
[93] H. Nifenecker, G. Mariolopoulos, J. P. Bocquet *et al.*, Z. Phys. A **308**, 39 (1982).
[94] C. Schmitt, A. Guessous, J. Bocquet *et al.*, Nucl. Phys. **A430**, 21 (1987).
[95] H.-G. Gleer, W. Lang, M. Mutterer *et al.*, Nucl. Phys. **A452**, 277 (1986).
[96] M. Djebara, M. Asghar, J. P. Bocquet *et al.*, Nucl. Phys. **A425**, 120 (1984).
[97] A. C. Wahl, At. Data Nucl. Data Tables **39**, 1 (1988).
[98] G. D. Adeev, V. V. Pashkevich, N. I. Pischasov, and O. I. Serdyuk, Vopr. At. Nauki Tekh. Ser. Yad. Konst. No. 1, 49 (1988).
[99] W. D. Myers and W. J. Swiatechi, Ark. Fys. **36**, 343 (1967).
[100] J. Treiner, R. W. Hasse, and P. Schuck, J. Phys. Lett. (Paris) **44**, 2733 (1983).
[101] M. Brack, C. Guet, and H. B. Hakansson, Phys. Rep. **123**, 275 (1985).
[102] M. Asghar, N. Boucheneb, M. Djebara *et al.*, "Dynamical aspects of nuclear fission," E7-92-95 [in English], JINR, Dubna (1992), p. 215.
[103] A. R. W. Fuller, Phys. Rev. **126**, 684 (1962).
[104] L. G. Moretto, C. R. Albiston, and G. Mantzouranis, Phys. Rev. Lett. **44**, 924 (1980).
[105] P. N. Isaev, Yad. Fiz. **35**, 1417 (1982) [Sov. J. Nucl. Phys. **35**, 627

(1982)]; Preprint 81-13 [in Russian], Institute of Nuclear Physics, Novosibirsk (1981).
[106] W. U. Schröder, J. K. Huizenga, and J. Randrup, Phys. Lett. **98B**, 355 (1981).
[107] S. K. Samaddar and M. I. Sobel, Phys. Lett. **82B**, 191 (1979).

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