Manifestations of QCD vacuum structure in composite quark models

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A review is devoted to a quark model in which the main feature is a quark interaction with QCD vacuum fields. A self-consistent condition for confinement of quarks through their interaction with long-wave vacuum fluctuations (condensates) is found. It is shown that the spin-dependent forces are determined by the interaction of the quarks with short-wave vacuum fluctuations—instantons. The contribution of the instanton-induced interaction to the particle masses is found for any representation of the color-flavor–spin group $SU_{\text{cfs}}(12)$. The main features of the hadron mass spectrum are explained in the framework of the model. It is shown that allowance for the interaction of quarks through the QCD vacuum changes the spectrum of multiquark states and leads to stability of the six-quark H state ($J=0$, $I=0$, $S=-2$). New QCD sum rules for the proton are derived.

1. INTRODUCTION

The dynamics of hadronic processes taking place at the characteristic scales 0.1–1 fm is determined by the strong-interaction forces. Analysis of experiments in this region and the main features of the hadron spectrum reveal three phenomena that are most important for the nature of the strong interactions: asymptotic freedom at short distances (0.1 fm), spontaneous breaking of chiral symmetry, and the confinement of quarks and gluons in color-singlet bound states at large distances (≈ 1 fm).

A significant achievement of the theory of strong interactions is the proof of renormalizability[1] and asymptotic freedom[2] of quantum chromodynamics (QCD). These properties have been rigorously established in the framework of QCD perturbation theory, which possesses in its arsenal a powerful formalism of calculations in the region of short distances, where the coupling constant α_s is small.[3] In a number of cases, proof of the fulfillment of the condition of factorization of large and small distances (separation of hard subprocesses)[4] permits calculations that are in excellent agreement with experiment.

Unfortunately, for the bound-state problem and the structure of the hadron spectrum perturbation theory is quite unsuitable. Indeed, in the limit of zero coupling constant the free theory of quarks and gluons has nothing in common with the observed hadron spectrum. Therefore, to describe low-energy hadron physics it is necessary to use essentially nonperturbative methods and approaches.

The most fundamental and direct approach is lattice QCD.[5] In this approach, one solves the equations of the theory, which is regularized in the ultraviolet region by means of the lattice-period parameter. This approach has led to significant insights into the nature of the strong interactions in the regime of strong coupling. However, the rapid growth in the number of variables with increasing size of the lattice creates difficulties even for powerful computing techniques.

Approaches that make it possible to go beyond the framework of perturbation theory (nonperturbative QCD) are a significant step in the right direction. In this approach, the large-distance effects—spontaneous breaking of chiral symmetry[6] and confinement, which govern the world of hadrons—are intimately related to the properties of the ground state of the theory: its vacuum.

It is a noteworthy fact that the QCD vacuum has a nontrivial structure. We recall that in classical mechanics the vacuum state corresponds to the state of a ball at rest at the bottom of a well. In quantum theory, the ball fluctuates around its equilibrium position. In QED, the fields also fluctuate through the emission and absorption of particle pairs, an effect that is directly observed in the Lamb shift of the levels of the hydrogen atom. However, because the coupling constant in QED is small, the vacuum state of the theory is not changed. (The situation is dramatically altered in the electrodynamics of strong fields.[7])

In the strong-coupling regime of QCD, a non-Abelian gauge theory,[8] the vacuum contains collective fluctuations of the fields that are associated with tunneling transitions between classical vacua with different topological structures. A well-known example of such collective fluctuations is the instanton solution.[9,10] The collective excitations lead to the formation of vacuum condensates of the fields and a lowering of the vacuum energy density compared with the perturbative vacuum. In phenomenological models of the vacuum as an instanton medium, these phenomena are interpreted as collective excitations of the medium.[11–13]

It was shown in Ref. 14 by means of QCD sum rules that the interaction with vacuum fields is the dominant effect in the formation of the hadron spectrum. Indeed, the mass scale of hadrons consisting of light quarks is determined by the quark condensate. This sum-rule method is based on the assumption that there exist nonvanishing vacuum expectation values of singlet combinations of the fields, and it uses dispersion relations that connect small and large distances. One calculates the hadron Green's functions in the nonperturbative vacuum, and this makes it

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possible to determine hadronic properties in terms of the vacuum condensates of the fields.[15] In this approach, confinement is implicitly postulated in the assumptions about the form of the hadron spectrum. The method based on QCD sum rules makes it possible to describe many properties of the hadrons, and it exploits economically a few input parameters (values of the vacuum condensates). Considerable interest also attaches to the inverse problem—recovery of the vacuum properties from observed quantities of low-energy physics. A problem with this method, which is based on a complicated integral transformation, is the difficulty of controlling the approximations that are necessarily made in the calculations. The convergence of the perturbation series is not always sufficiently rapid, and the choice of an adequate model of the hadron spectrum is not always uniquely determined.

The most significant achievement of QCD sum rules was the demonstration that hadron spectroscopy is governed by the interaction of quarks with the QCD vacuum. Therefore, attempts have been made in recent years to construct a microscopic model of the vacuum. These investigations often use the existence of instanton solutions of the Euclidean Yang–Mills equations.[9] Quite soon after their discovery it became clear that instantons have a direct bearing on the explanation of the nature of the spontaneous breaking of chiral symmetry and the resolution of the $U_A(1)$ problem associated with the anomalously large mass of the η' meson.[16,17] Although the interquark forces induced by the instantons gave a certain qualitative basis for phenomenological quark models, the initial attempts to estimate these effects quantitatively encountered great difficulties. For example, it was shown that in the approximation of a rarefied gas of instantons, in which the interaction between them is ignored, the path integral is dominated by large instantons, corresponding to a coupling constant that is not small.[11] Therefore, the gas approximation was internally inconsistent.

It was established subsequently that allowance for repulsion between the instantons at short distances leads to a cutoff of the large instantons. This model of the QCD vacuum became known as the instanton-liquid model.[12,13] Our main effort in this review will be concentrated on the manifestations in hadron physics of the structure of the vacuum as an instanton liquid.

Other popular models are the models of effective low-energy Lagrangians of the Nambu–Jona-Lasinio[6] and Skyrme[18] type, which are currently under active development. In their origin, these models are associated with current algebra, i.e., the use of exact or approximate symmetries of low-energy QCD (chiral and others). Great efforts have been made to establish the connection between these models and nonperturbative solutions of QCD; in particular, there have been many attempts to derive corresponding Lagrangians from the instanton model of the QCD vacuum.[20] These models usually employ the approximation of a large number of colors, $N_c \to \infty$, and have a well-developed computational formalism.[19,20]

Finally, the quark models are the traditional approaches in the description of hadron physics. In the first place, there are the nonrelativistic quark model[21] and the bag model.[22,23] The reason why quark models became popular is that they are transparent, the calculations are simple, and practically the entire hadron spectroscopy can be described by means of a small number of parameters. The basic parameters of the nonrelativistic quark model are the masses and magnetic moments of the constituent quarks. The hyperfine interaction of the quarks is described by the Breit–Fermi potential, and quark confinement is taken into account by means of a confinement potential that increases with the distance.

The basic dynamical postulate of what is probably the most popular relativistic composite quark model—the bag model—is the assumption that the quasi-independent relativistic constituents (quarks and gluons) move in a finite closed region of space—a bag. A particular attraction of the bag model was the circumstance that there did exist a hope of determining the values of the parameters directly from QCD. However, all attempts to do this were unsuccessful. The fact is that the basic hypothesis of the MIT model[23] reduces to the assumption of complete destruction of the nonperturbative vacuum fields within the bag. The bag constant $B_{\rm MIT}$, which characterizes the change of the vacuum energy density between regions inside and outside the bag, must be approximately equal to $B_{\rm QCD} = (9/32)\langle 0|(\alpha_s/\pi)G^a_{\mu\nu}G^{a\mu\nu}|0\rangle$.[14] However, analysis of the QCD sum rules showed that $B_{\rm QCD} \approx 0.55$ GeV/fm^3, whereas $B_{\rm MIT} \approx 0.06$ GeV/fm^3. Another parameter of the MIT model, the quark–gluon coupling constant $\alpha_s^{\rm MIT} = 2.2$ (which determines the mass splitting between the members of the hadron multiplets), is also in disagreement with the QCD value $\alpha_s^{\rm QCD}$ (1 GeV) $=0.7$. Therefore, in Refs. 25 and 27 we drew the conclusion that in actual fact the QCD vacuum is practically unchanged inside the bag and that the effects of the quark interaction with the nonperturbative vacuum fields must be taken into account.

Attempts to develop a version of the bag model with a nonperturbative interaction were made earlier in Refs. 24 and 26. The contribution of the interaction between the quarks induced by instantons to the masses of the nonstrange quarks was calculated in Ref. 24. In the gas approximation, the effect was a correction of the order of a few mega-electron-volts, from which the authors concluded that vacuum interactions played a small part in the hadron mass spectrum.

However, in Refs. 25 and 27, in which the more realistic model of the QCD vacuum as an instanton liquid was used, it was established that these corrections reach hundreds of mega-electron-volts and are the main contribution to the mass splitting between the hadron multiplets.

The main problem studied in Ref. 26 was that of quark confinement in the MIT model. It was shown that even if the main hypothesis of the model is given up, it is possible to obtain quark confinement self-consistently if allowance is made for the quark interaction with gluon and quark condensates. This idea was developed further in Ref. 28.

The present review is mainly devoted to a new version of the bag model in which a crucial role is played by the nonperturbative interaction between the quarks. It is conjectured that a dominant role is played by an interaction of the valence quarks with vacuum fields; this is consistent with ideas based on QCD sum rules. The model does not require additional parameters, since all the parameters are fixed in the model of an instanton liquid. As a consequence, these parameters are external to the model and are justified by the phenomenology of the QCD vacuum (QCD sum rules).

The model combines some of the most attractive features of several approaches. Namely, it takes into account confinement through the interaction of the quarks with the long-wave vacuum quark and gluon fields, which are dominant in the QCD sum rules, the perturbative interquark interaction, which is an inseparable part of the majority of composite quark models, and the short-range interaction induced by instantons that arises in realistic models of the QCD vacuum.

In Secs. 2, 3, and 4 we formulate the model. The discussion begins with the simplest Lagrangian of the bag model, which corresponds to the motion of quarks and gluons in a closed region of space, a bag. Such a theory is attractive because it realizes confinement of the color degrees of freedom; this comes about because of conservation of the vector current. However, manifest shortcomings are present in the model—the breaking of chiral symmetry and the absence of a stable state of a finite bag. Among the possible ways of overcoming these difficulties, we mention the MIT model and chiral bag models. However, these models are not completely consistent. A decisive factor for the creation of a realistic model of hadrons is that in reality the bag (hadron) is in a physical vacuum possessing a nontrivial structure. By considering a bag in an external vacuum field it is possible to formulate a model free of the difficulties mentioned above.

In Secs. 5 and 6 we obtain the mass spectrum of the ground states of the hadrons. It is shown that the model explains the main features of the spectrum: the Δ–N, Ξ–Σ, and π–ρ splittings and the nature of the spin–spin forces. Section 7 is devoted to the role of the instanton interaction in the technique of QCD sum rules. In particular, the proton sum rules are discussed in detail. It is shown that the nucleon is a bound state in the instanton field. In Sec. 8 it is shown that the interaction with the vacuum fields makes a large contribution to the electromagnetic mass differences of the hadrons and significantly improves the predictions of the quark models. Quark models usually have difficulty in describing the spin–orbit interaction of quarks. We show that the spin–orbit interaction in the baryon octet is suppressed because of a significant decrease in the constant of one-gluon exchange compared with other models. At the same time, the instanton interaction gives a good description of the excited states of the baryons. This is the subject of Sec. 9.

One of the most promising subjects for studying the properties of the vacuum is exotic hadrons. In Sec. 10, we consider in detail the situation with regard to the six-quark state of the H dihyperon. The influence of the interaction with the vacuum fields on the static properties of the hadrons is studied in Sec. 11. The main conclusions and immediate prospects are discussed at the end.

2. ZEROTH APPROXIMATION OF THE BAG MODEL

The covariant bag model (for reviews of bag models, see Refs. 29–31) in "vacuum" is described by the Lagrangian

$$\mathfrak{L}^{(0)}_{\text{bag}}(x)=\mathfrak{L}_D(x)\Theta_V(x)-\frac{1}{2}\sum_{i=1}^{N_F}\bar{q}_i(x)q_i(x)\Delta_S(x), \quad (2.1)$$

where

$$\mathfrak{L}_D(x)=\sum_{i=1}^{N_F}\bar{q}_i(x)\left(\frac{i}{2}\overleftrightarrow{\hat{\partial}}-m_i\right)q_i(x) \quad (2.2)$$

is the free Dirac Lagrangian.

In the expressions (2.1) and (2.2), $q_i(x)$ is the quark spinor field, m_i is its current mass, and the summation is over the flavors (index $i=u,d,s$) and colors (implicitly) of the quarks; $\Theta_V(x)$ and $\Delta_S(x)$ are functions that determine the shape of the bag surface S, which occupies in space the volume V:

$$\Theta_V(x)=\begin{cases}1 & \text{inside bag,}\\ 0 & \text{outside bag,}\end{cases}$$

$$\Delta_S(x)=-n^\mu(x)\partial_\mu\Theta_V(x), \quad (2.3)$$

where n^μ is the inner covariant unit 4-normal to the bag surface, with $n^2=-1$. We have also introduced the usual notation

$$f_1\overleftrightarrow{\partial}_\mu f_2\equiv f_1(\partial_\mu f_2)-(\partial_\mu f_1)f_2,\quad \hat{v}\equiv\gamma_\mu v^\mu.$$

In the bag model, it is assumed that at short distances (the region within the bag) the theory is asymptotically free, and in the zeroth approximation in the interaction the quark field within the bag V satisfies the equation for a free Dirac particle of mass m_i:

$$(i\hat{\partial}-m_i)q_i(x)\,|_V=0. \quad (2.4)$$

If color degrees of freedom are included, the Lagrangian and equations of motion take, respectively, the form

$$\mathfrak{L}_D\to\mathfrak{L}_{\text{QCD}}=-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}+\sum_{i=1}^{N_F}\bar{q}_i\left(\frac{i}{2}\overleftrightarrow{\hat{\nabla}}-m_i\right)q_i, \quad (2.5)$$

$$(i\hat{\nabla}-m_i)q\,|_V=0,\quad \mathfrak{D}^\mu_{ab}F^b_{\mu\nu}(x)\,|_V=g\bar{q}\gamma_v\frac{\lambda^a}{2}q\,|_V, \quad (2.6)$$

where we have defined the field-strength tensor of the color electromagnetic field A^a_μ:

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu$$

and the covariant derivatives

$$\nabla_\mu=\partial_\mu-igA^a_\mu\frac{\lambda^a}{2},\quad \mathfrak{D}^{ab}_\mu=\delta^{ab}\partial_\mu-f^{abc}A^c_\mu.$$

Here, f^{abc} are the structure constants for $SU(3)$; λ^a are matrix generators of $SU_C(3)$ with normalization Sp $\lambda_a^2=2$; and the color index takes the values $a=1,2,...,8$. In the framework of the model, the dynamics at large distances (confinement) is taken into account by means of boundary conditions on the bag surface, the shape of which is determined from constraint equations. These are the so-called linear and nonlinear boundary conditions. The linear boundary condition

$$i\hat{n}q(x)|_S=q(x)|_S \tag{2.7}$$

means that there is no vector flux through the bag surface:

$$\partial_\mu j^\mu(x)|_V=0,\quad n_\nu j^\mu(x)|_S=\pm i\bar{q}(x)q(x)|_S=0, \tag{2.8}$$

where the vector current is determined by the relation

$$j^\mu(x)=\bar{q}(x)\gamma^\mu q(x)\Theta_V(x). \tag{2.9}$$

In the last expression, (2.7) has been used in the second equation. To the linear boundary condition there corresponds the term in the Lagrangian (2.1) proportional to the surface Δ_S function:

$$\mathfrak{L}^{(0)}_{\rm surf}=-\tfrac{1}{2}\bar{q}(x)q(x)\Delta_S(x). \tag{2.10}$$

When color degrees of freedom are included, we have, accordingly, the boundary conditions

$$i\hat{n}q(x)|_S=q(x)|_S,\quad n^\nu F^a_{\mu\nu}(x)|_S=0, \tag{2.11}$$

or, equivalently, the condition of conservation of the color-charged vector current:

$$j^{a\mu}(x)=\left[\bar{q}(x)\frac{\lambda^a}{2}\gamma^\mu q(x)+f^{abc}F^{b\mu\nu}(x)A^c_\nu(x)\right]\Theta_V(x),$$

$$\partial_\mu j^{a\mu}(x)|_V=0,\quad n_\mu j^{a\mu}(x)|_S=0. \tag{2.12}$$

The localization of the color degrees of freedom in the region of the bag ensures color neutrality of the hadronic states. Indeed, with allowance for the equations of motion (2.6) and the linear boundary condition (2.11) we obtain

$$Q^a=g\int_V d^3x j^{a0}(x)=0, \tag{2.13}$$

where Q^a is the total color charge of the system. Therefore, the conservation of the vector current is a very attractive aspect of the complete approach of the bag model.

The behavior of the model with respect to transformations of the Poincaré and chiral symmetry groups is nontrivial. In fact, it is well known that in the framework of the model (2.1) it is not possible to preserve these symmetries for any finite bag volume. We shall consider this question in more detail.

The term (2.10) in the Lagrangian of the model, corresponding to the boundary condition (2.7), has the form of a mass term and therefore manifestly breaks the chiral invariance of the theory on the bag surface. Indeed, under the chiral transformation

$$q\to q-i(\tau\varepsilon/2)\gamma_5 q,\quad \bar{q}\to\bar{q}-i\bar{q}\gamma_5(\tau\varepsilon/2) \tag{2.14}$$

the Lagrangian acquires the "chiral-odd" correction

$$\mathfrak{L}^{(0)}_{\rm surf}\to\mathfrak{L}^{(0)}_{\rm surf}+i\bar{q}\gamma_5(\tau\varepsilon/2)q\Delta_S,$$

and for the divergence of the chiral current and its flux through the bag surface we obtain, respectively,

$$\partial_\mu \mathbf{j}^\mu_5(x)|_V=0,\quad n_\nu\mathbf{j}^\mu_5(x)|_S=-\frac{i}{2}\bar{q}(x)\gamma_5\tau q(x)\Delta_S(x), \tag{2.15}$$

where, by definition, the chiral current has the form

$$\mathbf{j}^\mu_5(x)=\bar{q}(x)\gamma^\mu\gamma_5\frac{\tau}{2}q(x)\Theta_V(x). \tag{2.16}$$

Similarly, for the energy–momentum tensor

$$T^{\mu(0)}_\nu(x)=\frac{i}{2}\bar{q}(x)\gamma^\mu\overleftrightarrow{\partial}_\nu q(x)\Theta_V(x) \tag{2.17}$$

we obtain, with allowance for (2.7),

$$\partial_\mu T^{\mu(0)}_\nu(x)|_V=0,$$

$$n_\mu T^{\mu(0)}_\nu(x)|_S=\frac{i}{2}\bar{q}(x)\hat{n}\overleftrightarrow{\partial}_\nu q(x)\Delta_S(x)$$

$$=-\tfrac{1}{2}\partial_\nu[\bar{q}(x)q(x)]\Delta_S(x). \tag{2.18}$$

The requirement of conservation of the energy–momentum tensor and the chiral current means the vanishing of the divergence of these quantities in the region within the bag and an absence of a flux of them through the bag surface:

$$\partial_\mu\mathbf{j}^\mu_5(x)|_V=0,\quad \partial_\mu T^{\mu(0)}_\nu(x)|_V=0,$$

$$n_\mu\mathbf{j}^\mu_5(x)|_S=0,\quad n_\mu T^{\mu(0)}_\nu(x)|_S=0. \tag{2.19}$$

The problem with the model (2.1) is that the nonlinear boundary condition (2.19) can be satisfied only for an infinitely distant surface. This behavior of the model with respect to the chiral transformations is organically related to the method of quark confinement, which is as if they acquired an infinite mass outside the region in which they move. Indeed, (2.18) means that for the theory with the Lagrangian $\mathfrak{L}^{(0)}_{\rm bag}$ there does not exist a state stable with respect to disintegration of the bag; for there are no factors opposing the internal pressure of the quarks on the bag walls. This is related to the fact that the model (2.1) is conformally invariant, since by virtue of the equations of motion (2.4) and (2.17) we obtain

$$T^{\mu(0)}_\mu(x)=0. \tag{2.20}$$

Note that the relation $\partial_\mu T^{\mu(0)}_\nu=-P_D n_\nu\Delta_S$ determines the internal pressure P_D of the quarks on the bag walls; however, the model (2.1) does not contain a dimensional parameter that could determine this pressure.

The nonconservation of energy and momentum and the breaking of the chiral symmetry for a bag of finite volume are the reason why a model with the Lagrangian (2.1) cannot be a realistic model of hadrons. Therefore, we need external fields to stabilize the bag and serve as sinks for the chiral flux. In this connection, in the quantum theory the boundary conditions (2.19), which determine the shape of the bag surface S, must be regarded as constraints on the physical state vectors and not as strong operator equations.

3. BAG MODEL WITH ALLOWANCE FOR THE QCD VACUUM STRUCTURE

The decisive factor that enables us to overcome the difficulties is that in reality the hadron bag is immersed in a physical medium saturated with collective vacuum fluctuations. Allowance for the interaction between the fields that populate the hadron with the vacuum fields radically changes the structure of the model. In such an approach, the shape of the bag surface is determined self-consistently by minimizing the total energy of the hadron as a system of Dirac quarks in an external vacuum field.

Let us consider the hierarchy of fields in the QCD vacuum and in the hadron–vacuum system. As is well known, the QCD vacuum has a fairly complicated structure. The nonperturbative fields can be nominally divided into two parts: the short-wave part, which contributes to the interaction of the quarks at short distances, and the long-wave part, which determines the confinement. In the instanton-liquid model,[12,13] the first part is associated with the fine-grain structure of the vacuum, which is dominated by a single-instanton fluctuation with effective scale $\rho_{\rm av}\propto 1.5$–$2\ \mathrm{GeV}^{-1}$, while the second part is associated with the long-wave collective excitations of the instanton liquid with wavelength $\lambda\approx R\approx R_{\rm conf}$, where $R\approx 3\rho_{\rm av}$ is the average distance between the instantons, and $R_{\rm conf}\approx 5$–$6\ \mathrm{GeV}^{-1}$ is the confinement radius.

The main assumption of our model[27] is that the QCD vacuum is practically unchanged by the color fields, namely, the interaction of the quarks and gluons localized in the bag with the vacuum fields determines the structure of the hadron. We shall regard the bag and the fields localized in it as immersed in the physical (instanton) vacuum, and we shall assume that the quarks give rise to (practically) no distortion of the local properties of this medium. We also give an analogy with the assumptions of QCD sum rules. In this case the probe, i.e., the nonlocal object that makes it possible to separate the mass of the lowest hadronic states, is not the hadron bag but the correlation function of the hadronic currents. Here too it is assumed implicitly that the local sources do not distort the properties of the physical vacuum.

In the bag–vacuum system, there arise three different scales of the field fluctuations: the small-scale instanton fluctuation with a characteristic frequency, $\varepsilon_i\propto 1/\rho_{\rm av}$, the fluctuations of the fields localized in the bag, with frequency $\varepsilon_q\propto\omega_q/R_{\rm bag}$, and the long-wave vacuum fluctuations with $\varepsilon_{\rm vac}\propto 1/R_{\rm conf}$. As will be shown in what follows, for the low-lying excitations of the quarks in the bag the condition for factorization of the large, intermediate, and short distances, $1/\rho_{\rm av}>\omega_q/R_{\rm bag}>1/R_{\rm conf}$, is satisfied, and the use of effective Lagrangians is justified. At the scale $r\leqslant\rho_{\rm av}$ the main effect is the interaction associated with a tunneling transition through an instanton, while at the scale $r\propto R$ it is confinement of the quarks in the bag. In the bag, the asymptotic states with respect to the instanton interaction are those of the valence quarks in the bag, while outside the bag they are the vacuum quarks. For such a hierarchy of interactions, it can be assumed that the interaction of the quarks in the hadron takes place mainly on one instanton on the background of the external vacuum medium.

The long-wave vacuum components of the fields $Q(x)$, $\mathfrak{G}^a_\mu(x)$ satisfy the classical Yang–Mills equations (on the state of the physical vacuum $|\underline{0}\rangle$):

$$(i\hat{\nabla}-m_i)Q^i(x)|\underline{0}\rangle=0,$$

$$\nabla^\mu_{ab}G^b_{\mu\nu}(x)|\underline{0}\rangle=g\bar{Q}^i(x)\frac{\lambda^a}{2}\gamma_\nu Q^i(x)|\underline{0}\rangle,$$

$$G^a_{\mu\nu}=\partial_\mu\mathfrak{G}^a_\nu-\partial_\nu\mathfrak{G}^a_\mu+gf^{abc}\mathfrak{G}^b_\mu\mathfrak{G}^c_\nu \tag{3.1}$$

and are vacuum singlet renormalization-invariant expectation values:[14]

$$\langle\underline{0}|\frac{\alpha_s}{2\pi}:G^a_{\mu\nu}(0)G^{a\mu\nu}(0):|\underline{0}\rangle\approx 0.012\ \mathrm{GeV}^4,$$

$$\langle\underline{0}|\alpha_s^{4/9}:\bar{Q}(0)Q(0):|\underline{0}\rangle\approx-(250\ \mathrm{MeV}^3),..., \tag{3.2}$$

where the values of the condensates are determined phenomenologically in the framework of current algebra and QCD sum rules, the ellipses represents expectation values of operators of higher dimension, and normal ordering of the operators with respect to the perturbative vacuum $|0\rangle$ of the theory is understood.

We turn to the description of the interaction of the valence quarks in the bag with the external vacuum field. We shall take into account the interaction of the quarks with the background vacuum field by means of the substitutions

$$q(x)\Theta_V(x)\to q(x)\Theta_V(x)+Q(x),$$

$$A^a_\mu(x)\Theta_V(x)\to A^a_\mu(x)\Theta_V(x)+\mathfrak{G}^a_\mu(x) \tag{3.3}$$

in the Lagrangian of the bag model,

$$\mathfrak{L}^{\rm QCD}\Theta_V(x)\to\mathfrak{L}^{\rm QCD}\Theta_V(x)+\Delta\mathfrak{L}^{\rm vac}, \tag{3.4}$$

where

$$\Delta\mathfrak{L}^{\rm vac}=[\bar{q}(x)\Theta_V(x)]\left(\frac{i}{2}\overleftrightarrow{\hat{\partial}}-m\right)Q(x)$$

$$+\bar{Q}(x)\left(\frac{i}{2}\overleftrightarrow{\hat{\partial}}-m\right)[q(x)\Theta_V(x)]$$

$$+g\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)\mathfrak{G}^a_\mu(x)\Theta_V(x), \tag{3.5}$$

Q and $\bar{Q}$ are anticommuting vacuum quark fields, and $\mathfrak{G}^a_\mu(x)$ is the external gauge field. The localized components of the fields $q(x)$ and $A(x)$ are approximated by the solutions of the equations of the bag model (2.6), which are the equations of the zeroth approximation in the interaction (3.5).

The interaction with the external long-wave vacuum field (3.5) creates an energy that increases with increasing bag size [see (5.15)]. As a result, there comes a time at which further growth becomes disadvantageous, i.e., large fluctuations of the bag size are strongly suppressed. Thus, in our model the stabilization of the bag and fulfillment of the nonlinear boundary condition (2.19), which is re-

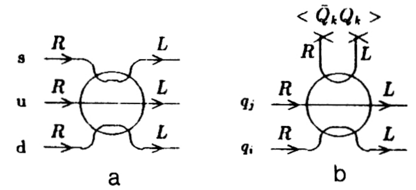

FIG. 1. Interaction of quarks (a) in the zero mode in the field of an instanton; b) the same when one of the quarks interacts with the quark condensate.

garded as a constraint on the physical hadronic states [see (3.49)], arise because of the interaction of the localized fields in the bag with the vacuum fields. As will be seen below, the bag stabilization occurs self-consistently, and the corresponding contribution of the confinement forces to the hadron masses is determined through the vacuum properties and does not depend on the number of quarks in the hadron [see (6.1)–(6.3)].

The interaction of the quarks with the short-wave component of the vacuum fields—the small instantons—is approximated by means of 't Hooft's effective Lagrangian,[10,35] which in the model of the vacuum as an instanton liquid[12,25] takes the form

$$\Delta\mathfrak{L}^{\text{inst}}=\sum_{i>j}^{i=u,d,s} n_{\text{av}}(-k')^2\{\bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL}\times[1+\tfrac{3}{32}\lambda_i^a\lambda_j^a(1-\tfrac{3}{4}\sigma_{\mu\nu}^i\sigma_{\mu\nu}^j)]+(R\leftrightarrow L)\}, \quad (3.6)$$

where

$$k'=\frac{4\pi\rho_{\text{av}}^3}{3}\,\frac{\pi}{m_*\rho_{\text{av}}}$$

characterizes the intensity of the quark–instanton interaction and is proportional to the instanton volume, n_{av} is the density of the instantons in the vacuum, ρ_{av} is the characteristic scale of an instanton in the QCD vacuum, $q_{R,L}=(1\pm\gamma_5)q/2$, $m^*=-\frac{2}{3}\pi^2\rho_{\text{av}}^2\langle 0|\bar{Q}Q|0\rangle$ is the effective mass of a quark with zero current mass in the physical vacuum[35] or in the bag, if the asymptotic states are quarks in the bag, and $\langle 0|\bar{Q}_iQ_i|0\rangle$ is the quark condensate. The effective mass takes into account the long-range correlations of the fields in the instanton vacuum. The $(R\leftrightarrow L)$ term in (3.6) corresponds to interaction through the anti-instanton. In addition, we implicitly assume averaging over the position of the instanton in space and over its orientation in the color space. The procedure of averaging over the collective coordinates of the instanton ensures translational and gauge invariance of the instanton interaction. The Lagrangian (3.6) is written down for the qq interaction of the theory corresponding to the $SU_f(2)$ sector of the flavor group. The separation of the subgroup $SU_f(2)$ corresponds to the case when one of the quarks interacts with the vacuum condensate (see Fig. 1b). On the transition to the $q\bar{q}$ system, it is necessary to make in (3.7) the substitution

$$\lambda_q\to-\lambda_{\bar{q}}^T,\quad \sigma_q\to\sigma_{\bar{q}}^T.$$

In what follows, we shall for convenience use the notation

$$\Gamma^{ij}=[1+\tfrac{3}{32}\lambda_i^a\lambda_j^a(1-\tfrac{3}{4}\sigma_{\mu\nu}^i\sigma_{\mu\nu}^j)]. \quad (3.7)$$

In the interaction (3.6), the short-distance effects are taken into account explicitly through the zero modes of the quarks in the instanton field, and the external fields correspond to slowly varying fields localized in the bag or long-wave vacuum fluctuations outside the bag. Thus, the representation (3.6) is justified if the external momenta of the quarks $p_q\propto\varepsilon_q$, ε_{vac} are much smaller than the characteristic frequency of the instanton fluctuation $\propto\rho_{\text{av}}^{-1}$, and this is true for the low-lying states of the quarks in the bag:

$$\left(\frac{\varepsilon_q}{\varepsilon_i}\right)^2\propto\left(\frac{\omega_q\rho_{\text{av}}}{R}\right)^2\propto\left(\frac{2(1.5\text{–}2)}{5\text{–}6}\right)^2\propto\frac{1}{4}\ll 1$$

[ω_q are the values of the modes of the quark excitations in the bag; see (4.18)].

The effective Lagrangian (3.6) constructed on the zero fermion modes in the instanton field has the specific feature that there is nonvanishing only of the amplitudes in which a pair of quarks is in a state with the quantum numbers of the zero mode:[10]

$$\boldsymbol{\sigma}_i\oplus\mathbf{c}_i|\ \rangle=0, \quad (3.8)$$

where $\boldsymbol{\sigma}_i$ is the spin and $\mathbf{c}_i$ is the color [subgroup $SU_c(2)$] of the ith quark. This leads to a nontrivial spin–spin and spin–orbit interaction of the interquark forces (see Secs. 6 and 9). It should be noted here that in the case of an $SU(3)_F$-symmetric vertex $\Delta\mathfrak{L}_{\text{inst}}^{(3)}$ the selection rule (3.8) prohibits a contribution of the interaction of three quarks in the zero mode in the instanton field to the energy of ordinary hadrons. In addition, in Sec. 10 we shall see that for multiquark systems the effects of taking into account the $SU_f(3)$-symmetric vertex[35] (Fig. 1a) are small.

By means of the Fierz transformation, we obtain from (3.6) an interaction expressed in terms of color-singlet bilinear combinations of the fields (meson representation):

$$\Delta\mathfrak{L}^{\text{inst}}=n_{\text{av}}k^2\{[(\bar{q}_R\tau^a q_L)^2+\tfrac{1}{20}(\bar{q}_R\sigma_{\mu\nu}\tau^a q_L)^2]+(R\leftrightarrow L)\}, \quad (3.9)$$

where $k^2=\frac{15}{256}k'^2$, $\tau^a=(1,i\tau)$. Further, we introduce the notation

$$\phi_{ij}(x)=\bar{q}_{Rj}(x)q_{Li}(x), \quad (3.10)$$

$$\sigma_q=\bar{q}(x)q(x),\quad \pi_q(x)=\bar{q}(x)i\gamma_5\tau q(x),$$

$$\eta_q=\bar{q}(x)i\gamma_5 q(x),\quad \alpha_q(x)=\bar{q}(x)\tau q(x). \quad (3.11)$$

Then, since

$$\phi=\tfrac{1}{2}(\sigma_q+i\eta_q)+\tfrac{1}{2}(\alpha_q+i\pi_q)\tau,$$

the expression for the instanton interaction can be rewritten in the form

$$\Delta\mathfrak{L}^{\text{inst}}=2n_{\text{av}}k^2(\sigma_q^2-\alpha_q^2-\eta_q^2+\pi_q^2)+\dots=2n_{\text{av}}k^2\,\text{Re}\det\phi+\dots. \quad (3.12)$$

Here, the ellipsis denotes the tensor part of the instanton interaction (which we shall sometimes omit in the expressions); in addition, we have written here part of the interaction for the $SU_F(2)$ sector of the $SU_F(3)$-symmetric theory. By means of the notation in the form (3.13) it is possible to trace the connection between our model and chiral bag models, the meson–instanton coupling constant playing the role of the constants.

To take into account the $SU_F(3)$ breaking in the instanton interaction, we must make in (3.6) the substitution

$$k'^2 \to k'^2_{ij} = k'^2 \frac{m_*^2}{m_i^* m_j^*},$$

where $m_i^* = m_i + m^*$ is the effective mass of the quark of flavor i with current mass m_i. The expression (3.12) takes in the $SU_F(3)$ theory the form

$$\Delta\mathfrak{L}^{\text{inst}} = 2n_{\text{av}}k^2(\xi_\sigma\sigma_q^2 - \xi_\delta\delta_q^2 - \xi_\varepsilon\varepsilon_q^2 - \xi_\kappa\varkappa_q^2 - \xi_{\eta'}\eta_q'^q + \xi_\pi\pi_q^2 + \xi_K K^2) + \ldots,$$

$$\xi_\sigma = \xi_\delta = \xi_\varepsilon = \xi_\pi = 1, \quad \xi_\kappa = \xi_K = \frac{m^*}{m_s^*} \approx 0.6,$$

$$\xi_\eta = \frac{m^* - m_s/3}{m_s^*} \approx 0.4, \quad \xi'_\eta = 2\frac{m^* + m_s/3}{m_s^*} \approx 1.4, \quad (3.13)$$

where the relations for the coefficients follow from the fact that the density of the instantons is proportional to the product of the effective masses of the quarks, and the probability of finding a quark in the zero mode is inversely proportional to its mass. The constants $\xi_k = \langle 0|j_k|k\rangle$ determine the behavior of the meson correlation functions at characteristic distances $\propto \rho_{\text{av}}$, where the nonperturbative effects are already important. At the same time, in the perturbative region these constants $\xi_k = \langle 0|j_k|k\rangle$ are practically independent of the quark masses and approximately equal to each other. In (3.14), the numerical values for ξ are obtained for $m^* \approx 200$–300 MeV and $m_s \approx 150$ MeV.[61]

Let us reconsider the linear boundary condition (2.7), which is taken into account by the surface term (2.10) in the Lagrangian (2.1) of the model. As we noted above, a serious defect of this interaction is the nonconservation of the axial vector current for a finite bag. To restore one of the fundamental properties of strong interactions, the chiral symmetry, the surface term $\bar{q}q\Delta_S$ is usually replaced in the Hamiltonian of the model by a chiral-invariant expression of the type $\bar{q}(\omega M\bar{\omega})q\Delta_S$.[36,29] Here, $(\omega M\bar{\omega})_{ij}$ is a functional of the additional elementary fields (π, σ,...). In such a form, the chiral bag models are intimately related to σ models.[29,10]

In our approach, it is obvious that a linear (in the fields) interaction on the bag surface having a chiral-invariant form can be written in the general case as

$$\mathfrak{L}^{\text{surf}} = -l[\bar{q}(x)MQ(x)\bar{Q}(x)Mq(x) + \bar{q}(x)Mq\bar{Q}(x)MQ(x)]\Delta_S(x), \quad (3.14)$$

where the matrices M must be such that the condition of conservation of the axial-vector isotriplet current is satisfied, and the singlet axial current has an anomaly.[38] These requirements uniquely fix the form of the interaction on the bag surface as an effective interaction of quark fields in the condensate with valence quarks in the bag resulting from instanton exchange:

$$\begin{aligned}\mathfrak{L}^{\text{surf}}_{\text{inst}} &= -l\sum_{i>j}\{[\bar{q}_{iR}q_{iL}\bar{Q}_{jR}Q_{jL} + \bar{q}_{iR}Q_{iL}\bar{Q}_{jR}q_{jL} + (i\leftrightarrow j)]\Gamma^{ij} + (R\leftrightarrow L)\}\Delta_S \\ &= -\tfrac{1}{2}l\bar{q}\{\tau^a(\bar{Q}\tau^aQ) + \gamma_5\tau^a(\bar{Q}\gamma_5\tau^aQ) + \tfrac{1}{20}[\sigma_{\mu\nu}\tau^a(\bar{Q}\sigma_{\mu\nu}\tau^aQ) + \gamma_5\sigma_{\mu\nu}\tau^a(\bar{Q}\gamma_5\sigma_{\mu\nu}\tau^aQ)]\}q\Delta_S. \quad (3.15)\end{aligned}$$

The constant Lagrange multiplier in (3.15),

$$l = \{(\bar{Q}\tau^aQ)^2 + (\bar{Q}\gamma_5\tau^aQ)^2 + \tfrac{1}{20}[(\bar{Q}\sigma_{\mu\nu}\tau^aQ)^2 + (\bar{Q}\gamma_5\sigma_{\mu\nu}\tau^aQ)^2]\}^{-1/2},$$

is determined from the condition of self-consistency of the linear boundary condition specified in (3.15). In the long-wave limit, (3.15) becomes the condition (2.10): $\langle 0|\mathfrak{L}^{\text{surf}}_{\text{inst}}|0\rangle = -\frac{1}{2}\Sigma_i\bar{q}^iq^i\Delta_S$, and the factor l is related to the decay constant f_π through the low-energy relation $l^{-1} = |\langle\bar{Q}Q\rangle| = \frac{1}{2}f_\pi^2K$, where $K = m_\pi^2/(m_u+m_d)$. These constants are related to the meson residues λ_π, which are determined by the relations $\langle 0|j_m|m\rangle = \lambda_n\delta_{nm}$, where $\lambda_\pi = Kf_\pi$. It should be recalled that the instanton interaction (3.5), (3.6) on the scale $\rho_{\text{av}} < r < R_{\text{bag}}$ associated with restoration of the chiral symmetry is small compared with the main effect—the confinement of the quarks in the bag.

In the meson variables, the expression (3.5) takes the more compact form

$$\begin{aligned}\mathfrak{L}^{\text{surf}}_{\text{inst}} &= -\tfrac{1}{2}l(\sigma_q\sigma_Q - \alpha_q\alpha_Q - \eta_q\eta_Q + \pi_q\pi_Q)\Delta_S, \\ l &= [\sigma_Q^2(x) - \alpha_Q^2(x) - \eta_Q^2(x) + \pi_Q^2(x)]^{-1/2} \\ &= [\tfrac{1}{2}\,\text{Re}\det\phi_Q]^{-1/2}, \quad (3.16)\end{aligned}$$

where the fields with index Q are defined as in (3.11) with the substitution $q\to Q$, the Lagrange multiplier is expressed in the new notation, and the linear boundary conditions become (i is the quark flavor)

$$i\hat{n}q_i(x)|_S = l[\sigma_Q(x) - \tau\alpha_Q(x) - i\gamma_5\eta_Q(x) + i\gamma_5\tau\pi_Q(x)]_{ij}q_j(x)|_S \quad (3.17)$$

for the quark fields and

$$K\bar{q}\gamma_5\tau q|_S = -2i(n\partial)\pi|_S, \quad K\bar{q}q|_S = -2(n\partial)\sigma|_S,$$
$$K\bar{q}\tau q|_S = -2(n\partial)\alpha|_S, \quad K\bar{q}\gamma_5 q|_S = -2i(n\partial)\eta|_S \quad (3.18)$$

for the external fields. By virtue of the linear boundary conditions,

$$\bar{q}_i(x)[\sigma_Q(x) - \tau\alpha_Q(x) - i\gamma_5\eta_Q(x) + i\gamma_5\tau\pi_Q(x)]q_i(x)|_S = 0, \quad q\hat{n}q|_S = 0, \quad (3.19)$$

and, in addition, the boundary conditions (3.18) are consistent with the constraint (3.16) for the external fields, since it follows directly from (3.17) and (3.18) that

$$n^\mu \partial_\mu(\sigma_q^2-\alpha_q^2-\eta_q^2+\pi_q^2)+...|_S=0. \quad (3.20)$$

The corresponding nonlinear boundary condition can be reduced to the form

$$(\partial\sigma_Q)^2-(\partial\alpha_Q)^2-(\partial\eta_Q)^2+(\partial\pi_Q)^2$$
$$=Kn\partial\bar{q}_i(\sigma_Q-\tau\alpha_Q-i\gamma_5\eta_Q+i\gamma_5\tau\pi_Q)q. \quad (3.21)$$

We consider the question of axial symmetries in more detail. Because the isotopic structure of the interaction (3.16) is proportional to $(1-\tau^i\tau^j)$, where τ^j are the Pauli matrices for the isotopic spin, the Lagrangian of the model is invariant with respect to axial isotopic transformations (2.14) augmented by appropriate transformations of the vacuum fields.

With respect to the axial singlet transformations

$$\begin{aligned}
&q\to q+i\varepsilon\gamma_5 q, && \bar{q}\to\bar{q}+i\varepsilon\bar{q}\gamma_5,\\
&Q\to Q+i\varepsilon\gamma_5 Q, && \bar{Q}\to\bar{Q}+i\varepsilon\bar{Q}\gamma_5,\\
&\sigma\to\sigma-2\varepsilon\eta', && \eta\to\eta+2\varepsilon\sigma,\\
&\pi_a\to\pi_a+2\varepsilon\alpha_a, && \alpha_a\to\alpha_a-2\varepsilon\pi_a,\\
&\phi\to\phi+2i\varepsilon\phi, && \det\phi\to\det\phi+2N_F i\varepsilon\det\phi
\end{aligned} \quad (3.22)$$

the instanton Lagrangian acquires, through the axial anomaly, a correction:

$$\mathfrak{L}_{\rm inst}\to\mathfrak{L}_{\rm inst}+\delta\mathfrak{L}_A, \quad (3.23)$$

where

$$\delta\mathfrak{L}_A=-\varepsilon 2N_F(2n_{\rm av}k^2)\,{\rm Im}\det\phi\,\Theta_V$$
$$=-\varepsilon 2N_F(2n_{\rm av}k^2)2(\pi_q\alpha_q-\eta_q\sigma_q)\Theta_V. \quad (3.24)$$

Thus, within the bag the singlet axial current

$$j_\mu^5(x)=\bar{q}(x)\gamma_\mu\gamma_5 q(x)$$

has an anomalous divergence:

$$\partial_\mu j_\mu^5|_V=-4in_{\rm av}k'^2\sum_{i>j}\{\bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL}\Gamma^{ij}-(R\leftrightarrow L)\}\Theta_V$$
$$=4N_F 2n_{\rm av}k^2(\pi_q\alpha_q-\eta_q\sigma_q)\Theta_V$$
$$=-N_F Q(x)\Theta_V. \quad (3.25)$$

Here, $Q(x)=8n_{\rm av}k^2[\eta_q(x)\sigma_q(x)-\pi_q(x)\alpha_q(x)]$ is the density of the topological charge in the bag. We see directly from (3.17) that on the bag surface

$$n_\mu j_\mu^5|_S=-il\sum_{i>j}\{[\bar{q}_{iR}q_{iL}\bar{Q}_{jR}Q_{jL}+\bar{q}_{iR}Q_{iL}\bar{Q}_{jR}q_{jL}]$$
$$\times\Gamma^{ij}-(R\leftrightarrow L)\}\Delta_S$$
$$=4N_F\frac{l}{2}(\sigma_q\eta_Q+\eta_q\sigma_Q-\alpha_q\pi_Q-\pi_q\alpha_Q)\Delta_S. \quad (3.26)$$

Outside the bag we expand the long-wave meson fields in the effective theory in the neighborhood of the vacuum solution $\langle\sigma_Q\rangle=\langle\bar{Q}Q\rangle\neq 0$, $\langle\eta_Q\rangle=\langle\alpha_Q\rangle=\langle\pi_Q\rangle=0$. In this region, the fluctuation meson fields satisfy the free equations

$$(\partial^2-m_m^2)\phi_Q(x)=0, \quad (3.27)$$

and in the chiral limit $m_m=0$, except $m_{\eta'}\neq 0$. By current algebra we have for the singlet current

$$j_\mu^{5m}(x)=f_{\eta'}\partial_\mu\eta'_Q(x),\quad \partial^\mu j_\mu^{5m}(x)=f_{\eta'}m_{\eta'}^2\eta'_Q(x).$$

The requirement of continuity on the bag surface leads to the relations

$$m_{\eta'}^2=4N_F 2n_{\rm av}(k\xi_{\eta'})^2,$$
$$f_{\eta'}^2=2K^{-1}\langle\sigma_Q(x)\rangle|_S=2K^{-1}|\langle\bar{Q}Q\rangle|, \quad (3.28)$$

and the divergence

$$j_\mu^{5A}(x)=2[\eta_Q(x)\partial_\mu\sigma_Q(x)-\sigma_Q(x)\partial_\mu\eta_Q(x)$$
$$+\pi_Q(x)\partial_\mu\alpha_Q(x)-\alpha_Q(x)\partial_\mu\pi_Q(x)] \quad (3.29)$$

of the singlet axial current outside the bag is indeed proportional in the long-wave limit to the singlet η:

$$j_\mu^5|_{\rm out}=-2\sigma_Q\partial_\mu\eta_Q\Theta_{\rm out}, \quad (3.30)$$

$$\partial_\mu j_\mu^5|_{\rm out}=-4N_F n_{\rm av}k^2\langle\bar{Q}Q\rangle\eta_Q\Theta_{\rm out}, \quad (3.31)$$

$$n_\mu j_{\mu q}^5|_S=n_\mu j_{\mu Q}^5|_S. \quad (3.32)$$

The equations of motion (2.4) and (3.27) and the boundary conditions (3.17) and (3.21) determine the equations of the chiral bag model, the basis of which is the interaction of the quarks with the QCD vacuum.

The axial charge of the flavor-singlet current

$$Q_5=\int_V d^3x j_0^5(x)=\int_V d^3x(\bar{q}_R q_L-\bar{q}_L q_R) \quad (3.33)$$

is anomalously not conserved:

$$\dot{Q}_5=\int_V d^3x\partial_0 j_0^5(x)$$
$$=4N_F n_{\rm av}k^2\left[\int_{\rm in}d^3x(\pi_q\alpha_q-\eta_q\sigma_q)\right.$$
$$\left.+\int_{\rm out}d^3x(\pi_Q\alpha_Q-\eta_Q\sigma_Q)\right]. \quad (3.34)$$

The matrix element of the axial charge (3.33) with respect to a proton state with definite spin determines the contribution of the quarks to the proton helicity:

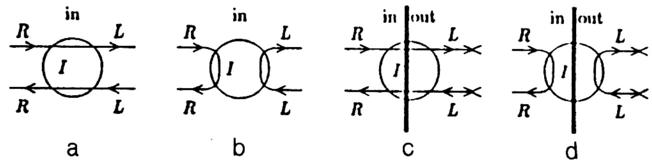

FIG. 2. Interaction of a quark–antiquark pair with an instanton in the bag (in) (a, b) and on its surface (in–out) (c, d). Diagrams a and c represent scattering, and diagrams b and d are for annihilation. For anti-instantons the substitution $R \leftrightarrow L$ must be made.

$$\left\langle P \left| \int d^3x j_3^5(x) \right| P \right\rangle = \left\langle P \left| - \int d^3x x_3 [\nabla \mathbf{j}^5(x)] \right| P \right\rangle = -4N_F n_{\rm av} k^2 \left\langle P \left| \int_{\rm in} d^3x x_3 (\pi_q \alpha_q - \eta_q \sigma_q) \right| P \right\rangle - \int_{\rm out} d^3x x_3 (f_{\eta'} m_{\eta'}^2 \eta'), \tag{3.35}$$

where we have used the relation $\int_V d^3x \mathbf{j} = -\int_V d^3x \mathbf{x}(\nabla \mathbf{j}) + dV \mathbf{j}|_S$ and the boundary conditions (3.32). The coefficient in (3.35), which determines the contribution of the anomaly, shows that on each of the instantons, which are distributed with density $n_{\rm av}$, there is a change of the helicity by two units for each of the N_F quarks in the zero mode, and the same process takes place on the anti-instanton.[54,55] It should be noted that the contribution to this matrix element from the meson field outside the bag is strongly suppressed by the exponential decay of the η' form factor. The nonconservation of the axial charge is directly related to the "spin crisis." We shall return to this question below, in Sec. 10. In the above we have ignored the isotopic mass differences. Consideration of their influence would give us relations for the meson decay constants, and also the anomalous contributions to the meson–nucleon coupling constants.

For completeness, we also note that if we were to take into account the tensor part of the interaction, then in this case the requirement of conservation of the angular-momentum tensor would give a further nonlinear boundary condition of the form

$$(t_\mu n_\nu - t_\nu n_\mu) \bar{q} [\sigma_{\rho\sigma}, \sigma_{\mu\nu}] (T_Q^\sigma - \tau \mathbf{T}_Q^\alpha - i\gamma_5 T_Q^\eta + i\gamma_5 \tau \mathbf{T}_Q^\pi) q(x) \Delta_S(x) = 0, \tag{3.36}$$

where

$$T_Q^\Gamma = \tfrac{1}{2} (t_\mu n_\nu - t_\nu n_\mu) \bar{Q} \Gamma \sigma_{\mu\nu} Q,$$

and t_μ is the unit vector tangent to the bag surface: $t^2 = 1$, $tn = 0$.

In fact, the interaction (3.15) specifies the structure of the boundary as an instanton "smeared" over the bag surface with "thickness" of the transition layer of order $\rho_{\rm av}$ (Fig. 2). Since the distance between the instantons is $R \approx 3\rho_{\rm av} \approx R_{\rm bag}$, allowance for the contribution to the quark–quark interaction of the effective single-instanton vertex (3.9) is justified.

Thus, our model is a generalization of the chiral bag models in which the interaction through instantons breaking the $U_A(1)$ symmetry is taken into account. Moreover, in our model the meson fields are interpreted as fluctuations over the nonperturbative vacuum, and therefore the parameters of the interaction of the quarks with the meson fields are fixed.

Summarizing everything that has been said above, we write down the Lagrangian of the model:

$$\mathfrak{L}_{\rm bag} = [\mathfrak{L}_{\rm QCD}(x;q) + \Delta\mathfrak{L}^{\rm inst}(x;q)\Theta_V(x) + \Delta\mathfrak{L}^{\rm vac}(x;q,Q)] + \mathfrak{L}^{\rm inst}_{\rm surf}(x;q,Q)\Delta_S(x) + \Delta\mathfrak{L}^{\rm meson}(x;Q)\Theta_{\bar{V}}(x), \tag{3.37}$$

where the corresponding expressions are determined in (2.3), (2.5), (3.6), (3.5), and (3.16) and were motivated in the preceding text (see Fig. 3).

It is also important to note the following. We assume that the interaction in (3.37) with the external field does not lead to a strong disturbance of the vacuum. If this were not the case, perturbation theory near the solution of the bag model (2.1) would be unjustified.[37]

We consider the equations of motion and the symmetries of the model in the framework of perturbation theory. As a free theory, we choose a model with Lagrangian $\mathfrak{L}^{(0)}_{\rm bag}$. Under the assumption of factorization of the large and small distances, the vacuum $|\underline{\tilde{0}}\rangle$ of the model is the direct product of the vacuum of the bag model (2.1), $|{\rm bag}\rangle$, and the physical QCD vacuum $|\underline{0}\rangle$:

$$|\underline{\tilde{0}}\rangle = |{\rm bag}\rangle \otimes |\underline{0}\rangle, \tag{3.38}$$

which satisfies the relations (a bag not filled with valence constituents has zero energy):

$$H_{\rm bag}|\underline{\tilde{0}}\rangle = 0, \quad \langle \underline{\tilde{0}} | \underline{\tilde{0}} \rangle = 1. \tag{3.39}$$

Accordingly, we define the hadron state as[1)]

$$|h_0\rangle = |{\rm bag\ Hadron}\rangle \otimes |\underline{0}\rangle, \tag{3.40}$$

where $|{\rm bag\ Hadron}\rangle$ is the state formed by applying the creation operators of the free theory to the vacuum $|{\rm bag}\rangle$ of the bag model.

The construction of the effective interaction reduces to the perturbative construction of the evolution operator $U(-\infty,0)$ and the explicit interpretation of the rapidly oscillating fields q, $\bar{q}$ localized in the bag: $(\varepsilon_{\rm vac}/\varepsilon_q)^2 \propto (1/\omega_q)^2 \leqslant 1/4 \ll 1$. The evolution operator $U_\varepsilon(-\infty,0)$ is defined as

$$U_\varepsilon(-\infty,0) = T \exp\left[-i \int_{-\infty}^0 dt e^{\varepsilon t} \int d^3x \Delta\mathscr{H}^{\rm res}_{\rm int}(x) \right], \tag{3.41}$$

where the adiabatic parameter ε serves to regularize (3.41). The residual interaction $\Delta\mathscr{H}^{\rm res}_{\rm int}(x)$ has the form

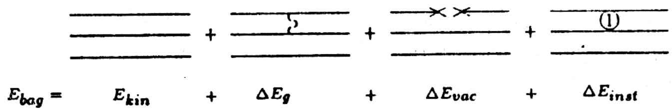

FIG. 3. Different contributions to the hadron bag energy $E_{\rm bag}$: $E_{\rm kin}$ is the kinetic energy of the quarks, $\Delta E_{\rm g}$ is the energy of one-gluon exchange, $\Delta E_{\rm vac}$ is the energy of the interaction with the condensates, and $\Delta E_{\rm inst}$ is the energy of the instanton interaction.

$$\Delta\mathscr{H}^{\rm res}_{\rm int}(x)\equiv\mathscr{H}_{\rm bag}-\mathscr{H}^{(0)}_{\rm bag}$$
$$=\Delta\mathscr{H}_{\rm vac}(x)+\Delta\mathscr{H}_{\rm inst}(x)+\Delta\mathscr{H}_{\rm OGE}(x)$$
$$+[\mathscr{H}^{\rm surf}_{\rm inst}(x)-\mathscr{H}^{\rm surf}_{0}(x)], \qquad (3.42)$$

where

$$\Delta\mathscr{H}_{\rm vac}(x)=\left[\frac{i}{2}\bar{Q}(x)\gamma_0\partial_t q(x)-\frac{i}{2}\partial_t\bar{q}(x)\gamma_0 Q(x)\right]\Theta_V(x), \qquad (3.43)$$

and the expressions for $\Delta\mathscr{H}_{\rm inst}(x)$, $\mathscr{H}^{\rm surf}_{\rm inst}(x)$, and $\mathscr{H}^{\rm surf}_{0}(x)$ are given in (3.6), (3.15), and (2.10), respectively. The residual perturbative QCD interaction (2.5) is approximated by the one-gluon exchange potential

$$\Delta\mathscr{H}_{\rm OGE}(x)=\frac{1}{2}\sum_{i>j}\int d^3x\mathbf{B}^a_i\mathbf{B}^a_j, \qquad (3.44)$$

where $\mathbf{B}^a_i$ is the color-magnetic field induced in quark i. In addition, Dyson's time ordering is understood in (3.41).

By means of the evolution operator, the physical state of the hadron is determined by the expression

$$|h\rangle=\sqrt{Z^h_2}\lim_{\varepsilon\to 0}U_\varepsilon|h_0\rangle(\langle h_0|U_\varepsilon|h_0\rangle)^{-1} \qquad (3.45)$$

with normalization $\langle h|h\rangle=1$. The physical hadron state $|h\rangle$ is a superposition of states of the system of valence quarks with different numbers of pairs of sea quarks. The constant $Z^h_2=|\langle h_0|h\rangle|^2$ determines the probability of a purely valence component in the hadron. The state $|h\rangle$ is an eigenstate for the total Hamiltonian H with eigenvalue equal to the total energy of the ground state: $H|h\rangle=E_h|h\rangle$, where E_h is the total energy of the system, and the difference between the perturbed and unperturbed energy of the interaction is

$$\Delta E=E-E_0=\langle h_0|\Delta H|h\rangle^f_{\rm av}. \qquad (3.46)$$

The Lagrangian of the model (3.37) is formally symmetric with respect to transformations of the Poincaré group, the isospin group $SU(2)$, the charge group $U(1)$, and the chiral isovector symmetry and anomalously breaks the chiral singlet symmetry, i.e., it satisfies all the symmetry properties of low-energy QCD. However, the Lagrangian of the zeroth approximation (2.1) does not have a stable state for which the energy–momentum tensor $T^0_{\mu\nu}$ and axial current j^μ_5 are conserved. We define the operator densities in the theory with interaction as

$$T_{\mu\nu}=U^{-1}_\varepsilon T^0_{\mu\nu}U_\varepsilon,\quad \mathbf{J}^\mu_5=U^{-1}_\varepsilon\mathbf{j}^\mu_5U_\varepsilon \qquad (3.47)$$

and require conservation of the corresponding quantities self-consistently on physical states by virtue of an interaction that restores the corresponding symmetries:

$$\langle h_0|\partial_\mu[U^{-1}_\varepsilon T^0_{\mu\nu}(x)U_\varepsilon]|h_0\rangle=0, \qquad (3.48)$$

$$\langle h_0|\partial_\mu[U^{-1}_\varepsilon\mathbf{j}^\mu_5(x)U_\varepsilon]|h_0\rangle=0, \qquad (3.49)$$

where the tensor $T^{\mu\nu}_0$ and the current j^μ_5 are defined in (2.16) and (2.17). We understand the equations of motion and the canonical commutation relations in precisely the same weak sense (on vectors of physical states):

$$(i\hat{\partial}-m_i)q_i(\mathbf{x},t=0)|h\rangle=0, \qquad (3.50)$$

$$\{q_i(\mathbf{x},t=0),q^+_j(\mathbf{y},t=0)\}|h\rangle=i\delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ij}. \qquad (3.51)$$

The conditions (3.48)–(3.51) relate the hadron spectrum, i.e., the form of the quark–quark potential, to the characteristics of the QCD vacuum, which are fundamental for our model. This is a natural consequence of the self-consistent approach that we have adopted.

Another method making it possible to take into account the effects of vacuum structure and to stabilize the bag against disintegration was proposed by the MIT group.[23] It consists of adding to the Lagrangian of the model (2.1) a phenomenological potential energy with constant density [cf. (3.4)]:

$$\mathfrak{L}_D\to\mathfrak{L}_D-B. \qquad (3.52)$$

This leads to the inhomogeneous nonlinear boundary condition

$$B=\tfrac{1}{2}n^\mu\partial_\mu[\bar{q}(x)q(x)]|_S, \qquad (3.53)$$

which guarantees energy–momentum conservation:

$$T^{\mu\nu}_{\rm MIT}=\left[\frac{i}{2}\bar{q}(x)\gamma^\mu\overleftrightarrow{\partial}_\nu q(x)+Bg^{\mu\nu}\right]\Theta_V,$$

$$\partial_\mu T^{\mu\nu}_{\rm MIT}|_V=0,$$

$$n_\mu T^{\mu\nu}_{\rm MIT}|_S=[B-\tfrac{1}{2}n^\mu\partial_\mu(\bar{q}(x)q(x))]n^\nu\Delta_S(x)=0. \qquad (3.54)$$

The last equation follows from the condition (3.53). The MIT hypothesis actually means that the potential energy with density proportional to B used to confine the quarks in the bag arises from the complete displacement of the vacuum fields by the color fields from the region of the bag. This assumption corresponds to the picture of a phase transition between two QCD vacua: the perturbative phase within the bag and the nonperturbative phase outside it.[32] It is the point of departure for many soliton-like hadron models. Thus, it is assumed that the introduction of quark sources into the region of the bag leads to a significant rearrangement of the vacuum within the bag. However, as was noted in the Introduction, such an assumption is not self-consistent, and it also contradicts the results of QCD sum rules. In addition, the constant is fundamental for the MIT model and, therefore, does not depend on the structure of the hadron, and this is not a natural assumption.

Our version of the model of Ref. 27 is based on an assumption opposite to the MIT hypothesis (that the vacuum is destroyed). In the instanton-liquid model, the effective Lagrangian contains the energy density of the instanton vacuum, which compensates the contribution of B: $B-2n_{\rm av}\approx 0$.

4. APPROXIMATION OF A STATIC SPHERICALLY SYMMETRIC CAVITY

To advance further, we consider the approximation[22,33,34] of a static spherical cavity for the equations of the zeroth approximation of the model (2.1). In this approximation, we fix a spherical shape of the bag surface with radius $R_{\rm bag}$, the distribution of the density of which is proportional to $\delta(R-R_{\rm bag})$, and the position of the bag is space with center at the point $x_0=(0,x_0)$. The functions that specify the shape of the bag surface take the form

$$\Theta_V(r)=\Theta_V(R-r),$$
$$\Delta_S(r)=\delta(r-R)n_\mu$$
$$\mathbf{r}=(0,\mathbf{r}/|\mathbf{r}|). \qquad (4.1)$$

In the cavity approximation and in the zeroth approximation in the interaction, the Hamiltonian of the model is

$$\mathscr{H}^{(0)}_{\rm bag}(x)=\sum_{i=1}^{N_F}\bar q_i(x-x_0)\left(\frac{i}{2}\gamma^0\overleftrightarrow{\partial}_0+m_i\right)q_i(x-x_0)\Theta_V(x-x_0)+\tfrac{1}{2}\bar q(x-x_0)q(x-x_0)\Delta_S(x-x_0). \qquad (4.2)$$

In what follows, we assume that x_0 is chosen at the origin.

It follows from (2.4) and (2.7) that a quark with mass m in the cavity of the sphere of radius R and placed at the origin satisfies the Dirac equation

$$(i\hat\partial-m_q)q(x)=0 \qquad (4.3)$$

with boundary condition at $r=R$:

$$-i\gamma\hat{\mathbf{r}}q(x)=q(x). \qquad (4.4)$$

Only the solutions of (4.3) with $j=1/2$ satisfy the linear boundary conditions (4.4). The noninteracting normal modes in the cavity form a basis for the quark field:

$$q_\alpha(\mathbf{x},t)=\frac{1}{\sqrt{4\pi}}\sum_{\rm modes}\left[u_m(\mathbf{x})b(m,\alpha)\exp\left(-\frac{i\omega_\alpha t}{R}\right)+v_m(\mathbf{x})d^+(m,\alpha)\exp\left(\frac{i\omega_\alpha t}{R}\right)\right] \qquad (4.5)$$

and they determine canonical creation and annihilation operators b (d) for the quarks (respectively, antiquarks), which satisfy the anticommutation relations

$$\{b^+(m,\alpha),b(m',\alpha')\}=\{d^+(m,\alpha),d(m',\alpha')\}=\delta_{m,m'}\delta_{\alpha,\alpha'}. \qquad (4.6)$$

These operators are used to define a Fock space for the cavity, the states of which are safe states for the bag model. The wave functions u_m and v_m are normalized by the condition

$$\frac{1}{4\pi}\int_V d^3x u_m^+(\mathbf{x})u_{m'}(\mathbf{x})=\frac{1}{4\pi}\int_V d^3x v_m^+(\mathbf{x})v_{m'}(\mathbf{x})=\delta_{mm'}, \qquad (4.7)$$

so that the conserved operator of the quark number has the form

$$\hat N=\sum_{\rm modes}\frac{\omega_\alpha}{R}[b^+(m,\alpha)b(m,\alpha)-d^+(m,\alpha)d(m,\alpha)]. \qquad (4.8)$$

The Hamiltonian

$$H^{(0)}_{\rm bag}=\sum_{\rm modes}\frac{\omega_\alpha}{R}[b^+(m,\alpha)b(m,\alpha)+d^+(m,\alpha)d(m,\alpha)] \qquad (4.9)$$

of the model of the zeroth approximation is obtained from (4.2) after substitution of the expression for the field (4.5) and use of the normalization condition (4.7). In (4.5) and (4.6), m denotes the set of quantum numbers of the spin projection m and the Dirac quantum number k, and the set α determines the color, flavor, and excitation mode of the quark.

The S-wave solution ($k=-1$, $j^P=1/2^+$) of Eqs. (4.3) and (4.4) has the form

$$u_m^0(\mathbf{x})=\begin{pmatrix} ig\left(\frac{\omega r}{R}\right)\chi_m \\ g\left(\frac{\omega r}{R}\right)\sigma\hat{\mathbf{r}}\chi_m\end{pmatrix},\quad v_m^0(\mathbf{x})=\begin{pmatrix} ig\left(\frac{\omega r}{R}\right)\sigma\hat{\mathbf{r}}\chi_m^c \\ f\left(\frac{\omega r}{R}\right)\chi_m^c\end{pmatrix}, \qquad (4.10)$$

where the wave function v_m in (4.10) and (4.5) is the charge conjugate of u_m : $v=\gamma_2u^*$. In (4.10), χ_m is the Pauli spinor for mass m:

$$\chi_\uparrow=\begin{pmatrix}1\\0\end{pmatrix},\quad \chi_\downarrow=\begin{pmatrix}0\\1\end{pmatrix},\quad \chi_\uparrow^c=\begin{pmatrix}0\\-1\end{pmatrix},n\ \chi_\downarrow^c=\begin{pmatrix}1\\0\end{pmatrix}. \qquad (4.11)$$

The expressions (4.10) correspond to the choice of the Feynman representation of the Dirac matrices ($\gamma_0^+=\gamma_0$, $\gamma^+=-\gamma$, $\gamma_5^+=\gamma^5$):

$$\gamma_0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad \gamma=\begin{pmatrix}0&\sigma\\-\sigma&0\end{pmatrix},$$
$$\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3=\begin{pmatrix}0&1\\1&0\end{pmatrix}. \qquad (4.12)$$

The energy and momentum of a quark in the cavity have, respectively, the form $\varepsilon_n=\omega_n/R$ and $p_n=(\omega_n^2-m_n^2R^2)^{1/2}/R$ where the eigenfrequencies ω_n satisfy the equation

$$\tan(p_nR)=\frac{k(p_nR)}{\omega_n-km_nR+k} \qquad (4.13)$$

and

$$f\left(\frac{\omega r}{R}\right)=N_0 j_0(pr),$$

$$g\left(\frac{\omega r}{R}\right)=-N_0\left(\frac{\omega-mR}{\omega+mR}\right)^{1/2} j_1(pr), \qquad (4.14)$$

where $j_l(r)$ are spherical Bessel functions, and the normalization coefficients N_l, determined by the condition $\int d^3x q_\alpha^+(x) q_\alpha(x)=1$, are

$$N_{nk}^{-2}\equiv\bar{N}_{nk}^{-2}R^3$$
$$=R^3 j_0^2(p_n)[2\varepsilon_n(\varepsilon_n+k/R)+m/R]/\varepsilon_n(\varepsilon_n-m). \qquad (4.15)$$

The corresponding P-wave solution ($k=+1$, $j^P=1/2^-$) has the form

$$u_m^1(\mathbf{x})=\begin{pmatrix} -i\tilde{g}\left(\frac{\omega r}{R}\right)\boldsymbol{\sigma}\hat{\mathbf{r}}\chi_m \\ \tilde{f}\left(\frac{\omega r}{R}\right)\chi_m \end{pmatrix},$$

$$v_m^1(\mathbf{x})=\begin{pmatrix} i\tilde{f}\left(\frac{\omega r}{R}\right)\chi_m^c \\ -\tilde{g}\left(\frac{\omega r}{R}\right)\boldsymbol{\sigma}\hat{\mathbf{r}}\chi_m^c \end{pmatrix} \qquad (4.16)$$

with

$$\tilde{f}\left(\frac{\omega r}{R}\right)=N_1 j_0(pr),$$

$$\tilde{g}\left(\frac{\omega r}{R}\right)=-N_1\left(\frac{\omega+mR}{\omega-mR}\right)^{1/2} j_1(pr). \qquad (4.17)$$

For massless quarks, the lowest modes are determined from (4.13) and have the values $X=\omega$ ($m_q=0$):

$$X_{0_{1/2}}=2.043, \quad X_{1_{1/2}}=3.812. \qquad (4.18)$$

5. PERTURBATION THEORY IN AN EXTERNAL FIELD IN THE CAVITY APPROXIMATION

We consider the theory with the interaction (3.42), treating it as a small perturbation. Since the cavity approximation is not relativistically covariant, it is convenient to use time-independent perturbation theory in the bag model. The correction to the hadron energy resulting from the interaction of the quarks with the external fields is determined by the expression[47]

$$\Delta E\equiv E-E_0=\frac{\langle h_0|H_{\text{int}}|h\rangle}{\langle h_0|h\rangle}=\langle h_0|H_{\text{int}}|h\rangle_c^f, \qquad (5.1)$$

where $|h_0\rangle$ is the unperturbed hadron wave function (3.41) corresponding to the Hamiltonian (4.2): $H_0=\int d\mathbf{x}\mathscr{H}_{\text{bag}}^{(0)}(\mathbf{x})$. In general form, the n-particle wave function (3.41) can be written

$$|h_0\rangle=\left(\sum \alpha_i M_{\alpha_1,\ldots,\alpha_n} c_{\alpha_1}^+ \ldots c_{\alpha_n}^+|\text{bag}\rangle\right)\otimes|\underline{0}\rangle, \qquad (5.2)$$

where c are the operators of creation (respectively, annihilation) of a quark (respectively, antiquark) with quantum numbers α_i, and the coefficients M determine the internal structure of the hadron. The index c means that only connected diagrams are taken into account, and the index f means allowance for the finite part of the expression for the energy. A diagram is said to be connected if all the operators H_{int} acting at earlier times are related to H_{int} at $t=0$. The state $|h_0\rangle$, the ground state for the free theory, evolves adiabatically from the time $t=-\infty$ to the time $t=0$ under the influence of the perturbation by the external field (3.43): $H_{\text{int}}(t)=\int d\mathbf{x}\Delta\mathscr{H}_{\text{int}}^{\text{res}}(\mathbf{x},t)$:

$$|h\rangle=U_\varepsilon(-\infty,0)|h_0\rangle \qquad (5.3)$$

with evolution operator (3.42):

$$U_\varepsilon(-\infty,0)=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{0}dt_1 e^{\varepsilon t_1}\ldots$$
$$\times\int_{-\infty}^{0}dt_n e^{\varepsilon(t_1+\ldots+t_n)}T[H_{\text{int}}(t_1)\ldots H_{\text{int}}(t_n)]. \qquad (5.4)$$

The physical hadron state $|h\rangle$ is a superposition of states of valence quarks with different numbers of sea quark pairs.

The expression (5.1) has the compact form[48]

$$\Delta E=\left\langle h_0\left|H_{\text{int}}\sum_{k=0}^{\infty}[(E_0-H_0)^{-1}H_{\text{int}}]^k\right|h_0\right\rangle_c^f, \qquad (5.5)$$

where the interaction operator H_{int} is expressed in the Schrödinger representation. In Ref. 49, a procedure for separating the finite part of the expression (5.5) was found. It reduces to the following. If among the intermediate states in the expressions

$$\sum_n \langle h_0|H_{\text{int}}|n_1\rangle\langle n_1|(E_0-H_0)^{-1}H_{\text{int}}|n_2\rangle$$
$$\ldots\langle n_k|(E_0-H_0)^{-1}H_{\text{int}}|h\rangle_c \qquad (5.6)$$

the initial state $|h_0\rangle$ is encountered, then the finite part is separated by the substitution

$$\langle n_k=h_0|(E_0-H_0+i\varepsilon)^{-1}$$
$$\to-\sum_{j=1}^{k-1}\langle n_j|(E_0-H_0+i\varepsilon)^{-1},$$

where j is the set of states that stand to the left of the kth state in Eq. (5.6). If there are no preceding states, then the finite part of such a divergent diagram is zero.

We shall use this formalism to calculate the corrections to the energy and the other static charactersitics of hadrons resulting from the interaction of the quarks with the external vacuum fields.[51] We first consider the allowance for the long-wave vacuum fluctuations. In the calculations, it is convenient to use a fixed-point gauge: $x_\mu^0 A^\mu(x_0)=0$.[50] In this gauge, the fields can be represented in the form of a series ($x_\mu^0=0$):

$$Q^{\text{vac}}(x)=Q^{\text{vac}}(0)+x^{\sigma}\nabla_{\sigma}Q^{\text{vac}}(0)+\frac{x^{\sigma}x^{\rho}}{2}\nabla_{\sigma}\nabla_{\rho}Q^{\text{vac}}(0)+...,$$

$$\mathfrak{G}_{\mu}^{\text{vac}\,a}(x)=\frac{x^{\rho}}{2}G^{a}_{\rho\mu}+\frac{x^{\rho}x^{\sigma}}{3}\mathscr{D}_{\sigma}G^{a}_{\rho\mu}+.... \quad (5.7)$$

In addition, in practical calculations it is convenient to take into account the interaction with the external field by expressing the total quark propagator in the form

$$S_q(x)=S_q^{\text{bag}}(x)+S_q^{\text{ext}}(x;Q), \quad (5.8)$$

where

$$iS_q^{\text{bag}}(\mathbf{x}_1,t_1,\mathbf{x}_2,0)^{ab}=\langle\text{bag}|T[q^a(\mathbf{x}_1,t_1)\bar{q}^b(\mathbf{x}_2,0)]|\text{bag}\rangle, \quad (5.9)$$

$$S_q^{\text{ext}}(x,0;Q)^{ab}=\langle\underline{0}|:Q^a(x)\bar{Q}^b(0):|\underline{0}\rangle=\chi^a(x)\bar{\chi}^b(0)-ig(\lambda^k)^{ab}\times[x_{\mu}G^k_{\mu\nu}(0)\gamma_{\nu}]\gamma_5+.... \quad (5.10)$$

In (5.8), the first term is the "free propagator" of the quark in the bag (5.9). The second term in (5.8) is due to the external quark fields and corresponds to allowance for the $\langle\bar{Q}Q\rangle$ condensate. The final term in (5.10) takes into account the external long-wave gluon field. The gluon vacuum field arises in the higher orders of perturbation theory and does not play a significant role in the physics of the light hadrons.

Going further, we represent the propagator in the external quark field with allowance for one-pion exchange (5.7) and the equations of motion in the form

$$\langle Q^{ia}_{\alpha}(x)\bar{Q}^{jb}_{\beta}(0)\rangle=-\tfrac{1}{12}\delta^{ij}\delta^{ab}\delta_{\alpha\beta}\langle\bar{Q}^iQ^i\rangle+..., \quad (5.11)$$

$$\langle Q^{ia}_{\alpha}(x)\bar{Q}^{jb}_{\beta}(y)Q^{kc}_{\gamma}(z)\bar{Q}^{ld}_{\delta}(0)\rangle=\tfrac{1}{144}\langle\bar{Q}^iQ^i\rangle\langle\bar{Q}^kQ^k\rangle[\delta^{ab}\delta_{\alpha\beta}\delta_{ij}\delta^{cd}\delta_{\gamma\delta}\delta_{kl}-\delta^{ad}\delta_{\alpha\delta}\delta_{il}\delta^{cb}\delta_{\gamma\beta}\delta_{kj}]+..., \quad (5.12)$$

where the condensates of higher dimensions are omitted, since they make a small contribution.

Ignoring the interaction with the gluon fields, which is proportional to the small α_s, we find that the Hamiltonian of the interaction of the quarks in a bag of radius R with the external vacuum fields (3.44) takes the form

$$H_{\text{vac}}(t)=\int_V d^3x\mathscr{H}_{\text{vac}}(t,\mathbf{x})=\int_V d^3x\sum_i\frac{\varepsilon_i}{2}[\bar{Q}_i(t,\mathbf{x})\gamma_0q_i(t,\mathbf{x})+\bar{q}_i(t,\mathbf{x})\gamma_0Q_i(t,\mathbf{x})], \quad (5.13)$$

where q_i are the normal modes of the quark field (4.5) with single-particle energy ε_i.

We formulate the rules for calculating the various coefficients in the expression for ΔE. For this we must draw various Feynman diagrams (Fig. 4); substitute the expression for the quark propagator (5.8) in all the quark lines;

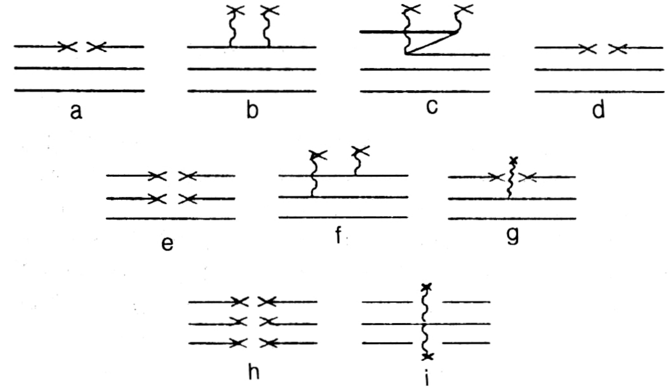

FIG. 4. Diagrams of single-particle (a–d), two-particle (e–g), and three-particle (h and i) contributions to the energy of hadrons in the external field.

and decompose the expectation value of the energy operator into products of the external fields and average the resulting products over the state of the hadron bag to ensure that these products have the same quantum numbers as the hadron. The expectation values of any other operator are calculated similarly.

We consider a simple example of calculations, in accordance with these rules, of the contribution from the diagram shown in Fig. 4a. From (5.4), we obtain in the first order of perturbation theory

$$\Delta E=-i\int_{-\infty}^{0}dte^{\varepsilon t}\langle h_0|:H_{\text{int}}(0)H_{\text{int}}(t):|h_0\rangle. \quad (5.14)$$

For massless quarks in the first order in the interaction (5.13), using (5.11), we obtain a correction to the energy proportional to the quark condensate:

$$\Delta E^a_{\text{vac}}=-i\frac{\varepsilon^2}{4}\int_{-\infty}^{0}dt\int_V d\mathbf{x}\int_V d\mathbf{y}e^{-i\omega t+\varepsilon t}\times\langle\underline{0}|\langle h_0|q^+(\mathbf{x})Q(\mathbf{x})\bar{Q}(\mathbf{y})\gamma_0q(\mathbf{y})|h_0\rangle|\underline{0}\rangle=-\frac{\langle\bar{Q}Q\rangle\varepsilon_0}{48}\left\langle h_0\right|\int d\mathbf{x}\int d\mathbf{y}\bar{q}(\mathbf{x})q(\mathbf{y})\left|h_0\right\rangle=-N_q\frac{\pi\langle\bar{Q}Q\rangle R^2}{24(X_0-1)}, \quad (5.15)$$

where N is the number of quarks in the hadron, and ε_0 is the lowest energy of a quark in the S state [fo. X_0, see (4.18)]. Other contributions are shown in the diagrams in Fig. 4. In the calculations, we shall not take into account the gluon condensate $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ because, as calculations show, its contribution does not have a significant influence on the final values of the masses. We calculate similarly the contribution of the interaction with the high-frequency vacuum fluctuations ($\omega_{\text{vac}}\gg\omega_q$) induced by the instantons.

This expansion (Fig. 3), which has the structure

$$E=E_{\text{kin}}+E_{\text{vac}},$$

where

$$E_{\rm kin}=\frac{N_q\mathcal{E}}{R},\quad E_{\rm vac}=-A\langle\bar{Q}Q\rangle R^2+B\langle\bar{Q}Q\rangle^2R^4+\ldots,\tag{5.16}$$

resembles the formulas of the QCD sum rules, in which the dimensional parameter that determines the scale of hadron formation is the Euclidean momentum Q (instead of R in our approach). It is well known that in the QCD sum rules self-consistency of the calculations requires the existence of a "window of stability," i.e., a region of "virtualities" Q^2 in which, on the one hand, the contributions of the interactions with the nonperturbative fields to the polarization operator are still small but, on the other, the contribution of the continuum of excited states is already small. In the bag model, a similar role is played by the condition of dynamical stability (5.26), which is a consequence of the nonlinear boundary condition (2.19). To the stable state with equilibrium radius $R_{\rm bag}$ there corresponds a state in which $R_{\rm bag}$ is not too large and, as a consequence, the interaction with the vacuum fields is not too strong. At the same time, $R_{\rm bag}$ is fairly large, since otherwise, if the quark kinetic energy were too great, degrees of freedom associated with excitations of the bag surface, i.e., the continuum of states of the bag model, would be important. These fluctuations are suppressed as $1/R$. Thus, the interaction takes place effectively in the range of distances in which the perturbative and nonperturbative corrections are simultaneously small.

The mass formula corresponding to the Hamiltonian of the model (3.38) has the form[2)] (Ref. 34)

$$M_H^2=E^2-\langle P^2\rangle,\tag{5.17}$$

where the energy of the quarks in the bag

$$E=E_{\rm kin}+\Delta E_{\rm OGE}+\Delta E_{\rm inst}+E_{\rm vac}\tag{5.18}$$

is the sum of the kinetic energy of the quarks and the energy of their interaction with the vacuum field and due to one-gluon exchange, and $\langle P^2\rangle$ is the correction for the center-of-mass motion; N_i is the number of valence quarks of the definite flavor in the given hadronic state.

Usually,[40] the energy of the center-of-mass motion is estimated by means of the expression

$$\langle \mathbf{P}^2_{\rm c.m.}\rangle=\sum_i^{u,d,s}N_i(p_i)^2,\tag{5.19}$$

where for massless quarks $\langle p_i^2\rangle=\omega_i^2/R^2$.

A more consistent way of taking into account relativistic motion of the bag was proposed in Refs. 41–43. This method is based on Bogolyubov canonical transformations[44,45] or dynamical symmetries, as a consequence of which the new dynamical variables include collective variables corresponding to the motion of the system as a whole. As a result, the collective variables can be expressed in terms of the original fields of the system. Thus, the operators of the displacement and velocity of the center of mass of the bag–quark system can be written in the form

$$\mathbf{X}_0(t)=\frac{i}{2}\frac{1}{E_{\rm bag}}\int_{\rm bag}d^3x\mathbf{x}[q^+(x)\dot{q}(x)-\dot{q}^+(x)q(x)],\tag{5.20}$$

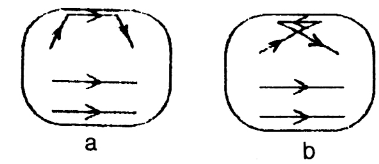

FIG. 5. Diagrams representing the corrections to the energy due to recoil effects.

$$\dot{\mathbf{X}}_0(t)=-\frac{1}{2E_{\rm bag}}\int_{\rm bag}d^3x[q^+(x)i\nabla q(x)-i\nabla q^+(x)q(x)],\tag{5.21}$$

and the corresponding correction to the energy becomes (see Fig. 5)

$$\begin{aligned}\mathbf{P}^2_{\rm c.m.}&=(E_{\rm bag}\dot{\mathbf{X}}_0)^2\\&=\sum_{m,n}\Bigg[\Bigg(\Big|\int d^3xu_m^+(x)i\nabla u_{n\neq m}(x)\Big|^2\\&\quad-\Big|\int d^3xu_m^+(x)i\nabla v_n(x)\Big|^2\Bigg)\\&\quad\times b^+(m)b(m)+(b\leftrightarrow d,u\leftrightarrow v)\Bigg]\\&\quad+\Bigg[-2\Big|\int d^3xu_m^+(x)i\nabla u_{n\neq m}(x)\Big|^2\\&\quad\times b^+(m)b(m)b^+(n)b(n)\\&\quad+2\Big|\int d^3xu_m^+(x)i\nabla v_n(x)\Big|^2b^+(m)b(m)\\&\quad\times d^+(n)d(n)+(b\leftrightarrow d,u\leftrightarrow v)\Bigg].\end{aligned}\tag{5.22}$$

It is easy to estimate the contribution of the energy of the center-of-mass motion for a quark–antiquark system in the ground state (meson):

$$\begin{aligned}\langle M|\mathbf{P}^2_{\rm c.m.}|M\rangle=2\Bigg[\Bigg(\Big|\int d^3xu_s^+(x)i\nabla u_p(x)\Big|^2\\-\Big|\int d^3xu_s^+(x)i\nabla v_s(x)\Big|^2\Bigg)\\+2\Big|\int d^3xu_s^+(x)i\nabla v_s(x)\Big|^2\Bigg],\end{aligned}\tag{5.23}$$

and for a baryon:

$$\begin{aligned}\langle B|\mathbf{P}^2_{\rm c.m.}|B\rangle=3\Bigg[\Big|\int d^3xu_s^+(x)i\nabla u_p(x)\Big|^2\\-\Big|\int d^3xu_s^+(x)i\nabla v_s(x)\Big|^2\Bigg],\end{aligned}\tag{5.24}$$

where for the intermediate states we have restricted ourselves to the lowest modes: s for the antiquark and p for the quark. The values of the matrix elements for the spherical modes of the massless quarks are

$$\left|\int d^3xu_s^+(x)i\nabla u_p(x)\right|^2=0.16\left(\frac{\omega_p}{R}\right)^2,$$

$$\left|\int d^3xu_s^+(x)i\nabla v_s(x)\right|^2=0.08\left(\frac{\omega_s}{R}\right)^2. \quad (5.25)$$

Thus, it can be seen that Eq. (5.19) overestimates the energy of the center-of-mass motion for the baryons. For the light pseudoscalar mesons, these corrections are comparable with the value of the bare mass, and therefore in this case these estimates are, generally speaking, invalid. In addition, the corrections (5.22), in contrast to (5.19), are not additive; they depend on the occupation number of the quarks and are even not positive definite, a property characteristic of fermionic degrees of freedom.[46]

The bag radius is determined from the condition of its stability:

$$\left.\frac{\partial M_H^2}{\partial R}\right|_{R_{\text{bag}}}=0. \quad (5.26)$$

It must be emphasized once more that, as follows from (5.15), the energy of a quark in the bag increases with increasing size of the bag. Thus, in our model quark confinement is achieved self-consistently by interaction of the quarks with the fields that populate the physical vacuum. Indeed, the Hamiltonian of the zeroth approximation did not have a stable state, but the interaction with the external field stabilized the system.

We recall for completeness that in the MIT model the Hamiltonian

$$\mathscr{H}_{\text{MIT}}(x)=[\mathscr{H}_{\text{QCD}}(x)-B]\Theta_v(x) +\frac{1}{2}\sum_{i=1}^{N_F}\bar{q}_i(x)q_i(x)\Delta_S(x) \quad (5.27)$$

corresponds to the mass formula of the model (5.18), in which the bag energy E_{MIT} is the sum of the bag potential energy E_{bag}, the kinetic energy E_{kin} of the quarks, and also the energy of their interaction through one-gluon exchange, E_{OGE}, and the energy of the zero-point vibrations E_0 (Casimir effect):[39]

$$E_{\text{MIT}}=\frac{4}{3}\pi R^3B+\sum_i^{u,d,s}\frac{N_i\varepsilon_i}{R}+\frac{\alpha_s}{R} \times\sum_{i\neq j}\mu_{ij}(\sigma\lambda^a)_i(\sigma\lambda^a)_j-\frac{Z_0}{R}. \quad (5.28)$$

In the expression for the energy, the bag constant B, the strong-interaction constant α_s, Z_0, and the quark masses m_q are fitted parameters.

The form of the Hamiltonian (5.27) corresponds to the assumption that the quarks within the bag are in the perturbative vacuum and that their interaction is realized through the one-gluon exchange (5.28). The potential energy, proportional to the constant B and responsible for the confinement of the quarks in the bag, arises because the vacuum fields are displaced from the volume of the bag. In contrast to the MIT model, in our model there is no need to introduce in the mass formula *ad hoc* geometric terms like the volume (respectively, surface) energy BR^3 (σR^3) and the Casimir energy Z_0/R. Such contributions cannot be well justified theoretically, and, as calculations of Z_0 show, are numerically unimportant.[53]

We shall make calculations in accordance with Eq. (5.17) in the first order of perturbation theory in the interaction with the vacuum fields. How important is such an approximation? As regards the interaction with the condensates in the following orders, appropriate allowance is readily made. Such a procedure signifies allowance for condensates of higher dimensions or, equivalently, allowance for fluctuations of the instanton liquid with even greater wavelength. In our approach, the rapid convergence of the series that takes into account such contributions is ensured by the following factors: Each successive term contains an extra small parameter $\omega_{\text{vac}}/\omega_q \ll 1$, and therefore the expansion coefficients c_n, which are proportional to the overlap of the wave functions, decrease with increasing order n of the series: $c_n \propto (1/n!)(\omega_{\text{vac}}/\omega_q)^n$. This is the reason why the expansion is dominated by the condensate of lowest dimension $\langle\bar{Q}Q\rangle$, which determines the scale of masses of the hadron ground states. The contributions of the condensates of higher dimensions must have an important influence on the spectrum of excited states of the hadrons.

Allowance for multi-instanton exchanges is a more complicated and as yet unresolved problem. As we shall show below, the instanton effects are most important for the ground states of the scalar and pseudoscalar mesons and, in particular, lead to a practically massless π meson already in the first order in the interaction. Clearly, in these channels it is necessary to take into account all orders in the instanton interaction. In the model of an instanton liquid, such allowance reduces effectively to a formal renormalization of the parameters of the model: the effective instanton scale ρ_{av} and the effective instanton density n_{av}. However, the selection of these parameters to prove the Goldstone nature of the π meson is dangerous, since in this region of parameters the theory is unstable, and the corresponding series are singular.

Thus, in the framework of the approximations that we have used we do not claim a quantitative description of very high excitations of the hadrons and the light scalar and pseudoscalar mesons.

6. SPECTRUM OF GROUND STATES OF HADRONS AND SPIN-DEPENDENT FORCES

By means of the mass formula (5.17), we can calculate the mass spectrum of the hadronic ground states. The energy of a quark in the bag with allowance for the interaction with the vacuum field has the form (5.18) (Fig. 3):

$$E=E_{\text{kin}}+\Delta E_{\text{OGE}}+\Delta E_{\text{inst}}+E_{\text{vac}},$$

where the separate contributions to the energy are determined in (5.16), (5.28), (5.15), and (3.6) and in the given form are

$$E_{\text{vac}}=-\sum_i N_i\langle\underline{0}|\bar{Q}_iQ_i|\underline{0}\rangle A_iR^2,$$

TABLE I. Matrix elements of one-gluon exchange M and of the instanton interaction K for the light hadrons.

Hadrons	M_{00}	M_{0s}	M_{ss}	k_0	k_s
π	−6	0	0	−1	0
K	0	−6	0	0	−1
η	−2	0	−4	1/3	−4/3
η'	−4	0	−2	2/3	4/3
ρ/ω	2	0	0	0	0
K^*	0	2	0	0	0
Φ	0	0	2	0	0
N	−3	0	0	−3/2	0
Λ	−3	0	0	−3/2	−1
Σ	1	−4	0	0	−3/2
Ξ	0	−4	1	0	−3/2
Δ	3	0	0	0	0
Σ'	1	2	0	0	0
Ξ'	0	2	1	0	0
Ω	0	0	3	0	0

$$\Delta E_{\rm inst}=\left(\frac{\rho_c}{R}\right)^2\frac{[k_0\lambda_0+k_s\lambda_s]}{R}, \tag{6.1}$$

with constants having the values

$$A_i=\frac{\pi}{12}\frac{(y+2)^2y}{X_i^2[2y(y-1)+a]},$$

$$\lambda_0=\pi\bar{N}_0^4R^6I_0,\quad \lambda_s=\lambda_0\frac{m^*I_s}{m_s^*I_0}, \tag{6.2}$$

where $\gamma=\varepsilon R$, $a=mR$,

$$I_0=\int_0^1 dxx^2[\,j_0^2(X_0x)+j_1^2(X_0x)\,]^2,$$

$$I_s=\frac{N_s^2}{\bar{N}_0^2}\int_0^1 dxx^2\Big\{[\,j_0^2(X_0x)-j_1^2(X_0x)\,]^2\Big[\,j_0^2(X_sx)\times\left(1+\frac{m_s}{\varepsilon}\right)-j_1^2(X_sx)\left(1-\frac{m_s}{\varepsilon}\right)\Big]+\frac{4X_s}{\varepsilon R}j_0(X_0x)j_1(X_0x)j_0(X_sx)j_1(X_sx)\Big\},$$

$\bar{N}_0$ and $\bar{N}_s$ are the normalizations of the wave functions in the bag model for massless, $m_0=0$, and massive m_s quarks, $X_0=\varepsilon_0(m_0)R$, and $X_s=\varepsilon_0(m_s)R$ [see (4.15)].

The values of the matrix elements with respect to the color–spin–flavor wave function of the hadrons, k_0 and k_s, are given in Table I. Also given there are the matrix elements of one-gluon exchange, which must be substituted in the approximate expression for the color-magnetic interaction[57] in the bag model:

$$\Delta E_{\rm OGE}=\frac{0.117\alpha_s}{R}[M_{00}+(1-0.13m_sR)M_{0s}+(1-0.25m_sR)M_{ss}]. \tag{6.3}$$

We note that for the vector mesons and baryons of the decuplet the coefficients k_0 and k_s are zero. The absence of an instanton contribution in these channels is due to the fact that the Lagrangian (3.6) is constructed by using fermion zero modes in the instanton field. Therefore, the matrix elements k_i are nonzero for the hadrons in which a pair of quarks can be in a state with the quantum numbers of the zero mode (3.8).[10] It is obvious that for massless mesons in the baryon decuplet Eq. (3.8) cannot hold, and this explains the absence of instanton contributions for these states.

The results of the calculation are given in Table II. The values of the parameters are taken close to the values that are used in the QCD sum rules[14] and in the model of an instanton liquid:[12,13]

$$\alpha_s=0.4;\quad \rho_c^2=3.6\ \text{GeV}^{-2};$$

$$m_s=250\ \text{MeV};\quad \langle\bar{Q}Q\rangle=-(250\ \text{MeV})^3. \tag{6.4}$$

The calculation was made in the first order in the instanton interaction, i.e., the procedure for finding the equilibrium radius (5.17) does not take into account the contribution of the instantons (as was done in Ref. 58, in which arguments for such a procedure are given).

It can be seen from Table II that the model satisfactorily describes the mass spectrum of the hadron ground states. The entire hadron spectrum can be described with a significantly smaller value of the quark–gluon constant, $\alpha_s=0.4$, compared with the $\alpha_s=2.2$ in the MIT model. Note that a large value of α_s would lead not only to problems associated with the applicability of QCD perturbation theory in the bag[79,49] but also to the impossibility of describing the spectrum of excited hadronic states.[59,60]

It follows from Table II that the spin–spin splitting between the hadron multiplets is determined by the instanton-induced interaction. Thus, in the baryons the interaction (3.6) gives a significant attraction in channels in which there is a scalar diquark and determines, in particular, the N–Δ splitting. A similar picture is observed in

TABLE II. Masses and radii of the hadronic ground states.

Hadrons	M_0, MeV	E_G, MeV	E_I, MeV	M, MeV	R, GeV^{-1}	$M_{\exp}$, MeV
π	765	−48	−183	466	5,83	140
K	901	−40	−137	684	5,71	498
η	942	−38	−120	750	5,66	550
η'	850	−43	306	1150	5,74	960
ρ/ω	765	16	0	784	5,83	770/783
K^*	901	13	0	917	5,71	896
Φ	1093	11	0	1052	5,59	1020
N	1219	−22	−223	941	6,25	940
Λ	1360	−23	−201	1100	6,17	1116
Σ	1360	−17	−162	1160	6,17	1192
Ξ	1501	−20	−169	1292	6,09	1315
Δ	1219	22	0	1243	6,25	1236
Σ'	1360	20	0	1381	6,17	1386
Ξ'	1501	17	0	1520	6,09	1532
Ω	1644	14	0	1660	6,02	1672

QCD sum rules,[61,66] in which allowance for the so-called direct instantons is extremely important in the scalar and pseudoscalar channels (see below).

A further achievement of the model is the explanation of the large mass of the η' meson [the $U_A(1)$ problem]. In the MIT model, as in perturbative QCD, this problem was not resolved. As we show in the Appendix, the instantons lead to a mixing of different quark species in the scalar channels. As a result, this mechanism gives practically pure octet and singlet states for the pseudoscalar mesons, whereas the vector mesons are almost perfectly mixed states, since the transitions $s\bar{s}\to u\bar{u}$, $d\bar{d}$ resulting from (3.6) are absent in accordance with the selection rule (3.8). A similar result is obtained by the QCD sum rules.[61,62] The quantitative agreement with the experimental values for the pseudoscalar octet is not entirely satisfactory. At the same time, it follows from Table II that allowance for only the first order in the interaction with the instantons leads to a decrease of the octet mass by about 1/3. Therefore, as noted above, in the problem of the masses of the particles of the pseudoscalar octet it is necessary to take into account the higher orders in the instanton interaction. A similar conclusion was drawn in Refs. 12 and 13, in which the polarization operator of the pseudoscalar currents in the instanton liquid was considered.

A further achievement of the model, of no slight importance, is that it is actually close to the nonrelativistic model. Thus, the results of Table II are practically unchanged if the calculations are made in the framework of the Isgur and Karl model[64] with addition of the nonrelativistic limit of the Lagrangian (3.7):

$$V_{ij}^{\text{inst}}=-\frac{\pi^2\rho_c^2}{6}\sum_{i\neq j}\eta_{ij}\frac{m^{*2}}{m_i^*m_j^*}\left[1+\frac{3}{32}\,(1+3\sigma^i\sigma^j)\lambda_i^a\lambda_j^a\right]\times\left[\frac{1-\tau_i^f\tau_j^f}{2}\right]\delta(r_{ij}), \qquad (6.5)$$

where τ_i^f is the matrix of flavors of quark i, and $r_{i,j}$ is the distance between the quarks, and with a corresponding decrease of α_s by about 5 times. This last fact is of fundamental importance for the nonrelativistic model,[64] since it makes it possible to reduce significantly the spin–orbit interaction arising from allowance for one-gluon exchange, the large value of which leads to difficulties in the description of the excited states of hadrons[65] (see below).

7. CONTRIBUTION OF INSTANTONS TO HADRONIC QCD SUM RULES

It is important to emphasize that the fundamental interactions on which the model developed above is based must also be taken into account in the framework of QCD sum rules.[14] As is well known, this method is based on the first principles of QCD and makes it possible to relate the phenomenology of the QCD vacuum, information about which is expressed through the vacuum expectation values of the fields, to the physical characteristics of hadrons. The success of the method is due to the fact that in the operator expansion of the current correlation function it is usually sufficient to retain only a few operators of the lowest dimension, such as the quark, $\langle\bar{Q}Q\rangle$, and the gluon, $\langle G^2\rangle$, condensates.

However, there are channels that require more detailed information about the structure of the QCD vacuum. A typical example is the correlation function of the (pseudoscalar) currents, for which the contribution of the direct instantons is important.[61,62] In the cited studies it was shown that the contribution of the direct instanton makes it possible to explain the main features in the spectrum of the pseudoscalar mesons, in particular, to obtain an almost massless pion, and it gives a qualitative solution of the $U_A(1)$ problem.

It can be seen from Table II that in the quark model the instanton-induced interaction is extremely important for the nucleon because the nucleon wave function has a component in which two quarks are in a state with spin zero (scalar diquark). We shall show below that, as for the sum rules for pseudoscalar mesons, only an instanton contribution to the nucleon sum rules makes it possible to

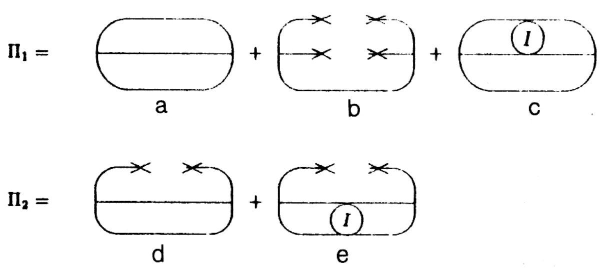

FIG. 6. Contributions of the quark loop (a), the four-quark condensate $\langle 0|\bar{q}q|0\rangle^2$ (b), direct instantons (c, e), and the quark condensate $\langle 0|\bar{q}q|0\rangle$ (d) to the nucleon sum rules.

obtain stable sum rules that reproduce the experimental nucleon masses.[66]

The most general expression for the nucleon current has the form[56,67,68]

$$O(x)=aO_1(x)+bO_2(x), \tag{7.1}$$

where

$$O_1(x)=\varepsilon^{abc}(u^aCd^b)\gamma_5u^c,\quad O_2(x)=\varepsilon^{abc}(u^aCd^b)\gamma_5u^c,$$

and a and b are arbitrary real parameters.

The current correlation function (7.1) has two Dirac structures, with respect to which we can write the dispersion relation ($Q^2=-q^2$)

$$i\int d^4xe^{iqx}\langle \underline{0}|T[O(x)\bar{O}(0)]|\underline{0}\rangle=\hat{q}\Pi_1(Q^2)+\Pi_2(Q^2), \tag{7.2}$$

where, obviously, the function $\Pi_2(Q^2)$ is related to the spontaneous breaking of the chiral symmetry and is proportional to the nucleon mass.

The standard analysis of the sum rules for baryons[56,67,68] is restricted to the contribution of the operators of dimension 6 in $\Pi_1(Q^2)$ and $\Pi_2(Q^2)$ (the others make a negligible contribution[67]). Here, in addition, we take into account the contribution of the direct instantons (Fig. 6).

There are two types of contribution from the direct instanton: The diagram of Fig. 6c clearly violates the hypothesis of factorization of the four-quark operator $\langle 0|\bar{q}\Gamma q\bar{q}\Gamma q|0\rangle$ (Ref. 14), while the diagram of Fig. 6e gives an exponential contribution. The latter is associated with a process in which all the large momentum Q passes through the vacuum fluctuation, and therefore it is proportional to the exponential instanton form factor. As in the quark model, the contribution of the gluon condensate $\langle \underline{0}|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ is unimportant in the determination of the mass and residue of the nucleon.[50]

The contribution of the direct instantons is calculated by substituting in the polarization operator (7.2) the quark Green's function in the external field of the instanton:

$$S(x,y)=S_0(x,y)+S_\pm(x,y), \tag{7.3}$$

where

$$S_0(x,y)=\frac{i}{2\pi^2}\frac{(x-y)^\mu\gamma_\mu}{(x-y)^4} \tag{7.4}$$

is the free propagator approximating the contribution of the nonzero modes, and

$$S_\pm(x,y)=\langle Q^a_\alpha(x)\bar{Q}^b_\beta(0)\rangle=\int d^4z\,\frac{[\Psi^0_z(x)\bar{\Psi}^0_z(0)]^{ab}_{\alpha\beta}}{m^*} \tag{7.5}$$

is the Green's function of the quark in the zero mode:[10]

$$\psi^\pm(x)=\varphi(x-z)\,\frac{(1\pm\gamma_5)}{2}\,\gamma_\mu(x-z)_\mu U \tag{7.6}$$

with

$$\varphi(x)=\frac{\sqrt{2}}{\pi}\frac{\rho_{av}}{[\rho_{av}^2+x^2]^{3/2}|x|}. \tag{7.7}$$

In (7.6), ρ_{av} is the mean instanton dimension, z is its position, U is the matrix of the orientation in the color–spin space ($U^+U=1$), which satisfies (3.9), and the minus and plus signs correspond to the anti-instanton and the instanton, respectively;

$$m^*=-\tfrac{2}{3}\pi^2\langle \underline{0}|\bar{Q}Q|\underline{0}\rangle\rho_{av}^2 \tag{7.8}$$

is the effective mass of the quark in the instanton liquid.[35,12]

Substituting (7.3)–(7.6) in (7.2) and averaging over the position of the instanton in space and its orientation in the color space, we obtain

$$\Pi_1=\frac{1}{(4\pi)^4}\left\{-Q^4\ln\frac{Q^2}{\mu^2}\chi+\frac{k^2}{6Q^2}\eta-\frac{3(4\pi)^4n_{av}}{5\pi^2(m^*\rho_{av})^2(\rho_{av}Q)^2}\right.$$
$$\times\left[1-\frac{24}{7}\frac{1}{(\rho_{av}Q)^2}+\frac{5(\rho_{av}Q)^3}{8}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\alpha d\beta\right.$$
$$\left.\left.\times\frac{\exp(\alpha+\beta)}{[\exp(-\alpha)+\exp(-\beta)]^3}\exp(-2Q\rho_{av}t)\right]\phi\right\}, \tag{7.9}$$

$$\Pi_2=\frac{k}{(4\pi)^4}\left\{Q^2\ln\frac{Q^2}{\mu^2}\eta-\frac{(4\pi)^2n_{av}(\rho_{av}Q)^2}{3m_*^2}K_1^2(\rho_{av}Q)\right\}, \tag{7.10}$$

where $t=(\cosh\alpha+\cosh\beta)/2$, $K_1(z)$ is a Macdonald function, and

$$k=-(4\pi)^2\langle \underline{0}|\bar{Q}Q|\underline{0}\rangle,$$

$$\chi=\frac{5(a^2+b^2)+2ab}{8},\quad \varphi=\frac{13(a^2+b^2)+10ab}{16},$$

$$\eta=\frac{7b^2-5a^2-2ab}{4},\quad \phi=b^2-a^2. \tag{7.11}$$

Further, proceeding from the assumption that in all the vacuum expectation values the instantons are dominant, we can express them in terms of the parameters of the instanton liquid[12]—the effective ρ_{av} and the instanton density $n_{av}\approx 0.8\cdot 10^{-3}$ GeV^4:

$$\langle 0|\bar{Q}Q|0\rangle = -\frac{(3n_{av})^{1/2}}{\pi\rho_{av}}. \tag{7.12}$$

Applying a Borel transformation to (7.9) and (7.10), as was done in Ref. 14, we arrive at the nucleon sum rules

$$\Pi_1(\tau) = F\exp(-M_N^2\tau^2) \tag{7.13}$$

$$= 2\tau^{-6}\left\{E_2(\tau)\chi + 64z^{-6}\left[f\eta + 0.9\left(1 - \frac{24}{7}z^{-2} + \frac{5\sqrt{\pi}}{32}z^3\exp(-z^2)\right)\phi\right]\right\}, \tag{7.14}$$

$$\Pi_2(\tau) = FM_N\exp(-M_N^2\tau^2) \tag{7.15}$$

$$= k\tau^{-4}\{E_1(\tau)\eta + 2\sqrt{\pi}z\exp(-z^2)\varphi\}, \tag{7.16}$$

where τ is the Borel parameter, $F = (4\pi)^4\lambda_N^2$ [λ_N is the nucleon residue, defined as $\langle 0|j_N|N_{k\lambda}\rangle = \lambda_N u_N(k,\lambda)$], $z = \rho_{av}/\tau$;

$$E_n(\tau) = 1 - \exp(-s_0\tau^2)\sum_{k=0}^{n}(s_0\tau^2)^k;$$

$f = 2n_{av}(\pi^2\rho_{av}^4/2) \propto 1/20$ is the packing parameter in the instanton-liquid model.[12]

The standard procedure then consists[14,56] of seeking the stability plateau, i.e., the region in τ in which the right- and left-hand sides of the sum rules agree with each other to a certain accuracy. Then in this region not only the contributions from the operators of higher dimension but also the contribution of the continuum threshold S_0 will be small.

We note that the direct instantons contribute to the sum rules (7.14) and (7.16) for any choice of the current (7.1). In addition, it turns out that the sum rule for the structure Π_1 is very sensitive to the choice of the current (7.1). If $|a| = |b|$,[56] then the contribution of the direct instantons is zero, but already for a small change in the values of the coefficients a and b the contribution of the instantons in the region $\tau \propto 1$ GeV^{-1} becomes appreciably greater than the loop contribution (Fig. 6a). Thus, for any $a \neq b$ the sum rule for Π_1 does not satisfy the criterion of smallness of the nonperturbative contributions, and it becomes unstable, and therefore in what follows we shall not consider it.

On the other hand, the exponential contributions of the diagram of Fig. 6e to Π_2 stabilize the sum rule and are practically independent of the choice of the current (7.1). Note also that the α_s corrections to Π_2 (Ref. 69) are also appreciable and narrow the region of convergence of Π_1, while the α_s corrections to Π_2 are small.

Figures 7 and 8 show graphs of the right-hand sides of the expressions

$$M_N^2(\tau) = -\frac{\partial_{\tau^2}\Pi_2(\tau)}{\Pi_2(\tau)}; \tag{7.17}$$

$$F(\tau) = \exp(M_N^2\tau^2)\Pi_2(\tau) \tag{7.18}$$

for various choices of the current.

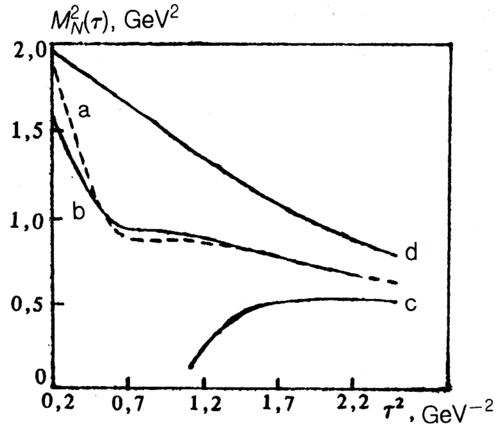

FIG. 7. Right-hand side of the sum rule (7.17) for various choices of the nucleon current: a) the Ioffe current, $a = -b = 1$; b) current with a scalar "diquark," $b = 0$; c) expression without a continuum, $b = 5$, $a = -7$; d) expression without instantons, $\varphi = 0$.

It can be seen from the figures that without allowance for the direct instantons a bound state does not arise. Further, the practically exact value of the nucleon mass is obtained with the choice of the nucleon current either in the form with a vector diquark ($a = -b = 1$) or in the form with a scalar diquark ($b = 0$):

$$\text{a) } M_N = 0.94 \text{ GeV}, \quad F = 7.5 \text{ GeV}^6, \quad s_0^2 = 3.1 \text{ GeV}^2,$$

$$\text{b) } M_N = 0.96 \text{ GeV}, \quad F = 5.5 \text{ GeV}^6, \quad s_0^2 = 2.5 \text{ GeV}^2. \tag{7.19}$$

For the current ($b = 5$, $a = -7$) when the continuum contribution is eliminated, a bound state arises only through the instanton contribution. In addition, for practically all a and b the square of the residue of the nucleon current is almost twice as large as the values given in Ref. 67. It should be noted that this result is practically insensitive to the parameters of the instanton liquid, since the corresponding coefficient in Π_2 (7.16) is a number. Thus, allow-

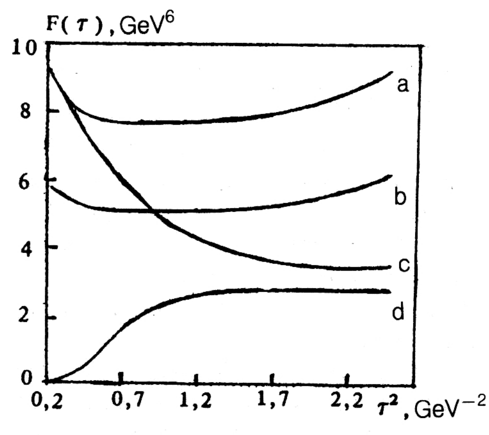

FIG. 8. Right-hand side of the sum rule (7.18). The notation is as in Fig. 7.

TABLE III. Electromagnetic mass differences.

Particles	R	ΔM_{kin}	ΔM_{EM}	ΔM_{gl}	ΔM_{vac}	ΔM_{inst}	ΔM_{tot}	ΔM_{exp}	ΔM_{MIT}	ΔM_{NR}
$P-n$	4,42	−1,77	0,57	0,09	−0,18	0,00	−1,29	−1,2933±0,00002	−1,20	−1,3
$\Sigma^+ - \Sigma^0$	4,25	−1,77	0,37	−0,02	−0,15	−1,84	−3,41	−3,09±0,07	−1,80	−3,5
$\Sigma^0 - \Sigma^-$	4,25	−1,77	−1,51	−0,02	−0,15	−1,84	−5,29	−4,88±0,06	−3,40	−4,5
$\Xi^0 - \Xi^-$	4,44	−1,77	−1,67	−0,13	−0,18	−1,43	−5,18	−6,4±0,6	−4,00	−6.3
$\Delta^0 - \Delta^{++}$	6,04	3,54	−2,22	−0,16	0,89	0,00	2,05	2,70±0,30	1,10	3,0
$\Delta^0 - \Delta^+$	6,04	1,77	−0,38	−0,08	0,45	0,00	1,76		1,30	2,3
$\Delta^0 - \Delta^-$	6,04	−1,77	−0,96	0,08	−0,45	0,00	−3,10		−2,80	−3,9
$\Sigma^{*0} - \Sigma^{*+}$	5,68	1,77	−0,37	−0,07	0,37	0,00	1,70	1,1±1,4	1,40	2,0
$\Sigma^{*0} - \Sigma^{*-}$	5,68	−1,77	−1,06	0,07	−0,37	0,00	−3,13	−3,5±1,5	−2,90	−3,7
$\Xi^{*0} - \Xi^{*-}$	5,51	−1,77	−1,13	0,06	−0,34	0,00	−3,18	−3,2±0,92	−3,00	−3.8
$\pi^\pm \to \pi^0$	2,00	0,00	1,94	0,00	0,00	0,00	1,94	4.5936±0,0005	1,60	
$K^+ - K^0$	2,00	−1,77	1,87	−0,21	−0,02	0,00	−2,91	−4,024±0,032	−1,60	−6,0
$\rho^\pm - \rho^0$	6,16	0,00	0,72	0,00	0,00	−2,78	0,72	−0,3±2,2	0,90	−0,6
$K^{*+} - K^{*0}$	5,68	−1,77	0,60	0,00	−0,37	0,00	−1,47	−4,51±0,52	−1,10	−2,7

ance for the direct instantons in the correlation function of the nucleon current makes it possible to obtain a stable result for the nucleon mass and residue.

Note also that the nucleon mass is effectively inversely proportional to the instanton scale ($1/\rho_{av}$). The agreement between the calculated and experimentally observed values of the nucleon mass justifies the estimate of the mean instanton scale in the QCD vacuum, $\rho_{av} \approx 1.6\ \mathrm{GeV}^{-1}$, which corresponds to the independent estimates of Refs. 12 and 13. It also follows from our results that the nucleon is formed as a bound state in the instanton field when two quarks are in zero modes in a state with spin zero and the third quark acquires an effective mass by entering the condensate (Fig. 6e).

Thus, the results obtained for the two-point QCD sum rules confirm the conclusion obtained above in the quark model, namely, that the instanton-induced interaction plays the dominant role in hadron spectroscopy. It must be emphasized that this interaction must also be important in the three-point QCD sum rules in the description of the magnetic moments, axial charges, form factors of the hadrons, and other quantities.

8. ELECTROMAGNETIC MASS DIFFERENCES AND EFFECTS OF THE QCD VACUUM

To describe the mass spectrum of the ground states, it is actually necessary to determine two quantities: the scale of the hadronic masses and the scale of the spin–spin splitting. Then practically the entire spectrum can be recovered by symmetry. This is the reason for the successful description of the ground states in the majority of hadron models.

However, there are more subtle effects such as, for example, the electromagnetic mass differences and the mass spectrum of the excited and exotic (multiquark and hybrid) states, which, taken together, represent a good proving ground for selection of the best approach. Here, as a rule, the predictions of the different models reveal strong differences. In this section, we turn to a discussion of such effects in the framework of the quark model that we have proposed.

In quark models,[70,71] the mass differences between members of an isotopic-spin multiplet are usually due to three factors: 1) electromagnetic interaction of the quarks in the hadron, ΔM_{em}; 2) dependence of the kinetic energy ΔM_{kin} of the quarks on their masses; 3) the influence of the mass difference of the u and d quarks on the strong-interaction potential ΔM_{strong}.

Thus, the problem reduces to allowance for the electromagnetic interaction of the particles in the presence of the strong interaction. In the framework of the bag model, these contributions can be calculated successively. For example, the authors of Ref. 72 calculated ΔM_{em} and ΔM_{kin} for the ground states. In Ref. 71, the dependence of the one-gluon potential ΔM_g on the quark masses was taken into account. However, comparison of the theoretical results of Ref. 73 with experiment[74] (see Table III) shows that the MIT bag model used in Refs. 72 and 73 does not describe the electromagnetic mass differences of the hadrons.

We consider the influence of the interaction with the QCD vacuum on the isotopic mass differences. The total difference of the energies between two states of a multiplet is given by a sum:

$$\Delta M_{tot} = \Delta M_{em} + \Delta M_{kin} + \Delta M_{strong},$$

$$\Delta M_{strong} = \Delta M_g + \Delta M_{vac} + \Delta M_{inst}, \qquad (8.1)$$

where ΔM_g is due to the hyperfine interaction of the quarks in the bag, while ΔM_{vac} and ΔM_{inst} arise from the interaction of the quarks with the vacuum.

Detailed calculations of the contributions ΔM_{em}, ΔM_{kin}, and ΔM_g were made in Refs. 72 and 73, in which the standard (for the MIT model) values of the mass of the strange quark, $m_s = 280$ MeV, and the coupling constant,

$\alpha_s = 2.2$, were used. We shall use these formulas and choose the values of the parameters in accordance with our model: $m_s = 220$ MeV and $\alpha_s = 0.4$.

From (6.3), we readily obtain, in the first order of the expansion in the quark mass m_q,

$$\Delta M_{\rm vac} = 0.035 \sum_{i=u,d} N_i m_i \langle \bar{Q}Q \rangle R^3,$$

$$\Delta M_{\rm inst} = -\sum_{i=u,d} \frac{m_i}{m^*} E_{\rm inst}. \qquad (8.2)$$

As above, the instanton interaction is taken into account in the first order of perturbation theory. Therefore, in the calculation we use the value of the equilibrium radius R_0 which arises without allowance for the instanton interaction (see Table III). To calculate the other contributions $\Delta M_{\rm em}$, $\Delta M_{\rm kin}$, $\Delta M_{\rm g}$, and $\Delta M_{\rm vac}$, we use

$$R = R_0 - \frac{3E_{\rm inst}(R_0)}{M_0(R_0)R_0}, \qquad (8.3)$$

where M_0 is the hadron energy without the contribution $E_{\rm inst}$. Equation (8.3) is analogous to the one obtained in Ref. 58, where the change in the bag radius is due to the interaction of the quarks with the pion field.

The results of calculations of the various contributions to the electromagnetic mass differences are presented in Table III. In the calculations, we have used the values from Table II and the results of Refs. 72 and 73. Table III also contains the results of the nonrelativistic model[70,71] and of the bag model.[72,73] The vacuum parameters as given above in (6.4) were used, and the mass difference

$$m_d - m_u = 3.7 \text{ MeV} \qquad (8.4)$$

was chosen on the basis of the most accurately measured difference $M_p - M_n$.

Comparing the calculated values with the experimental values and with those calculated in the bag model, we see that the interaction through the vacuum makes a significant contribution to the isotopic mass differences of the hadrons belonging to the baryon octet and the pseudoscalar mesons.

On the general background of the satisfactory description of the data, exceptions are the differences $\pi^\pm - \pi^0$ and $K^{*+} - K^{*0}$. In the first case, the deviation from experiment can be attributed to mixing with η, η' states, but in the second case the deviation appears surprising.

9. EXCITED STATES OF BARYONS IN THE NONRELATIVISTIC QUARK MODEL

As was already mentioned in the Introduction, in the description of excited states of hadrons in the standard approach,[64] which uses a one-gluon exchange potential, there is a problem associated with the large spin–orbit interaction induced by this potential. Because of this interaction, the center of the masses of the two states $N^*1^-/2$ is lowered by almost 400 MeV. On the other hand, splitting of $\Delta^*1^-/2$ and $\Delta^*3^-/2$ can arise only through LS forces, since, the spin symmetry being the same, spin–spin forces do not contribute to the mass difference, and tensor forces

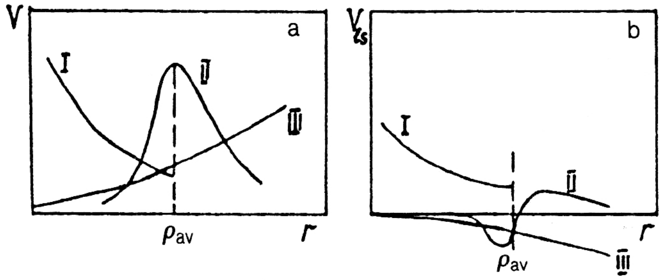

FIG. 9. Potential of interaction between quarks (a) induced by one-gluon exchange (I), instanton exchange (II), and long-wave fluctuations (III). The spin–orbit interaction (b) induced by the corresponding interactions.

are forbidden for $S = 1/2$ states. Thus, there is a problem for quark models—the spin–orbit part of the interaction must be strongly suppressed in some channels.

In the framework of our approach, this problem, which is serious for the quark models, finds a natural solution.[75] There are three contributions to the spin–orbit interaction. The perturbative contribution is determined by short distances and is proportional to α_s. The contribution associated with the instanton interaction has the opposite sign to that of the first contribution, since this interaction begins to dominate at intermediate distances (Fig. 9). The first contribution arises from the confinement potential.

The instanton contribution is dominant in the baryon octet, and the one-gluon contribution is dominant in the decuplet. Therefore, our model predicts a change in the sign of the spin–orbit splitting for the excited states of the nucleon and Δ isobar.

It is important to emphasize that the considered mechanism cannot be imitated by considering only the one-gluon exchange potential. Such phenomena include the solution of the $U_A(1)$ problem and the explanation of some weak-interaction processes in which the quark chirality changes.[75]

Note that the spectrum of excited states is practically independent of the form of the confinement potential. However, to simplify the calculations we shall use[63] a potential quark model of the type of Refs. 64 and 65 with the instanton interaction (6.5). As an effective Hamiltonian, we choose

$$H^{\rm eff} = \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \frac{k}{2} \sum_{i<j} (\mathbf{r}_{ij}^2 + U' + H^{\rm inst}), \qquad (9.1)$$

where $\mathbf{r}_{ij}$ is the distance between the quarks, and U' is the anharmonic part of the confining potential. We ignore the mass difference of the u and d quarks:

$$m_u = m_d = m_1 = m_2 = m, \quad m_s = m_3 = m', \qquad (9.2)$$

If we define

$$\mathbf{R} = [m(\mathbf{r}_1 + \mathbf{r}_2) + m'\mathbf{r}_3]/M, \quad M = 2m + m',$$

$$\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}, \quad m_\rho = m,$$

$$\boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}, \quad m_\lambda = \frac{3mm'}{2m + m'}, \qquad (9.3)$$

TABLE IV. Baryon mass spectrum in the nonrelativistic model with allowance for instantons.

Hadrons	Strangeness	SU(6) state	E_s^0, MeV	$\delta E^{\text{inst}}(p)$	M_{theor}
$N(939)1/2^+$	0	$^2 8$	1230	$-1/2$	940
$\Delta(1232)3/2^+$	0	$^4 10$	1230	0	1230
$\Lambda(1116)1/2^+$	-1	$^2 8$	1380	$(2+x)/6$	1119
$\Sigma(1193)1/2^+$	-1	$^2 8$	1380	$x/2$	1177
$\Sigma(1385)3/2^+$	-1	$^4 10$	1380	0	1380
$\Xi(1318)1/2^+$	-2	$^2 8$	1530	$x/2$	1327
$\Xi(1530)1/2^+$	-2	$^4 10$	1530	0	1530
$\Omega(1672)3/2^+$	-3	$^4 10$	1680	0	1680

then for the two sums in (9.1) we obtain the expressions

$$H_{\text{HO}}=m_1+m_2+m_3+\frac{2}{3}k(\rho^2+\lambda^2)+\frac{\mathbf{P}_R^2}{2M}+\frac{\mathbf{P}_\rho^2}{2m_\rho}+\frac{\mathbf{P}_\lambda^2}{2m_\lambda}. \tag{9.4}$$

Separation of the center-of-mass motion with respect to the variable R leads to two uncoupled equations for an oscillator with respect to the variables ρ and λ. If the quark masses are equal, $m_1=m_2=m_3=m_u$, then H_{HO} becomes

$$H_{\text{HO}}^0=-(\Delta_\rho+\Delta_\lambda)/2m_d+\tfrac{2}{3}k(\rho^2+\lambda^2)+\text{const}. \tag{9.5}$$

The classification of the baryonic states in the quark model with Hamiltonian H_{HO}^0 is well known.[76,64] Because of the antisymmetry with respect to the color, the wave function must be completely symmetric with respect to permutation of the $SU(6)$ indices and the spatial coordinates. For the ground states, the $O(3)$ part is completely symmetric:

$$\Psi^{00}=\frac{\alpha^3}{\pi^{3/2}}\exp[\alpha^2(\rho^2+\lambda^2)/2];\quad \alpha^2=\omega m=\sqrt{km}. \tag{9.6}$$

We denote H_{HO} in (9.4) for systems with one, two, or three strange quarks as H_{HO}^1, H_{HO}^2, and H_{HO}^3, respectively, so that

$$H_{\text{HO}}^s=H_{\text{HO}}^0+H_{\text{HO}}^{\text{corr}}, \tag{9.7}$$

where $H_{\text{HO}}^{\text{corr}}$ breaks the $SU(3)_f$ symmetry. Therefore, the Λ, Σ, Ξ, and Ω in the ground state are obtained as

$$E_0^s=E_0+\langle\Psi^{00}|H_s^{\text{corr}}|\Psi^{00}\rangle, \tag{9.8}$$

and give

$$E_1^0=E^0+\delta_s,\quad \text{for } \Lambda,\Sigma,$$
$$E_2^0=E^0+2\delta_s,\quad \text{for } \Xi,$$
$$E_3^0=E^0+3\delta_s,\quad \text{for } \Omega, \tag{9.9}$$

where $\delta_s=\delta m+\omega(1-x)/2$, $x=m_u/m_s$, $\delta m=m_s=m_u$. Allowance should also be made for the fact that, generally speaking, the confinement potential is not harmonic. This leads to the following assertions:[76,64] In the first order of the theory, any potential $U'(r_{ij})$ always splits the $N=2$ oscillator level in such a way that the five degenerate multiplets, $56^+(L=0,2)$, $70^+(L=0,2)$, $20^+(L=1)$, are split as

$$E^2(56,0^+)=E'-\Delta,\quad E^2(70,0^+)=E'-\Delta/2,$$

$$E^2(56,2^+)=E'-2\Delta/5,\quad E^2(70,2^+)=E'-\Delta/5,$$

$$E^2(0,1^+)=E', \tag{9.10}$$

and, in addition, we have the relation

$$E^2(20,1^+)-E^1(70,1^-)=E^1(70,1^-)-E^0(56,0^+). \tag{9.11}$$

In accordance with (3.7), we choose for the Hamiltonian of the hyperfine interaction the expression

$$H^{\text{inst}}=-\frac{\eta}{m_i m_j}\delta^3(\rho_i-\rho_j)\left(\frac{1-\sigma_i\sigma_j}{2}\right)\left(\frac{1-\tau_i^a\tau_j^a}{2}\right), \tag{9.12}$$

which is the nonrelativistic limit of the Lagrangian induced by the instantons. In (9.12), σ^a and τ^a are the spin and flavor operators ($\text{Sp}\,\sigma^2=\text{Sp}\,\tau^2=2$).

With allowance for this interaction, it is easy to explain the structure of the ground states of the baryons in the framework of the potential model. The Hamiltonian (9.12) projects onto states antisymmetric with respect to the spin and flavor. Thus, the decuplet states, which are not perturbed by the interaction (9.12), are well described by the expressions (9.11). Because of (9.12), the octet states are split and have less mass (N–Δ splitting).

There is also a natural explanation for the Λ–Σ splitting, since the contribution H_{inst} for Λ is greater than for Σ and has a negative sign (see Table I):

$$m(\Sigma)-m(\Lambda)=P\frac{(1-x)}{3}>0,$$

where $P=3\eta\alpha^3/m_d^2\pi^{3/2}$ is the scale of the instanton interaction. In Table IV, we give the masses of the ground states of the baryon octet. In the calculations we used the parameters

$$E^0=1230\text{ MeV},\quad P=580\text{ MeV},$$

$$\delta=150\text{ MeV},\quad x=0.7. \tag{9.13}$$

In the potential model, the low-lying P-wave baryons belong to the 70-plet of the first excited level of the system (9.5):

$$^2N(70,1^-)J^P=1^-/2,3^-/2, \tag{9.14}$$

$$^4N(70,1^-)J^P=1^-/2,3^-/2,5^-/2, \tag{9.15}$$

$$^2\Delta(70,1^-)J^P=1^-/2,3^-/2, \tag{9.16}$$

TABLE V. Matrix elements of the instanton interaction for excited nucleon states.

States	Instanton contributions (P)	
$^2N\,(56,2)5/2^+$	$-1/4$	$\sqrt{2}/8$
$^2N\,(70,2)5/2^+$	$\sqrt{2}/8$	$-1/8$
$^2N\,(56,2)3/2^+$	$-1/4$	$\sqrt{2}/8$
$^2N\,(70,2)3/2^+$	$\sqrt{2}/8$	$-1/8$
$^2N\,(56,2)1/2^+$	$-5/8$	$\sqrt{2}/8$
$^2N\,(70,2)1/2^+$	$\sqrt{2}/8$	$-5/16$

where we have listed only the nonstrange representatives. For (9.15) and (9.16), there are no instanton contributions and these states are degenerate. For (9.14), we have the contribution

$$\langle {}^2NJ^- | H^{\text{inst}} | {}^2NJ^- \rangle = -P/4 = -145 \text{ MeV}. \qquad (9.17)$$

Thus, we have a resolution of the paradox that arises when the one-gluon potential is used, i.e., there arises a splitting between the doublet $N^+_{1/2}(1535)$, $N^+_{3/2}(1520)$ and the N, Δ states with masses around 1660 MeV.

Therefore, we choose

$$E'(70,1^-) = 1665 \text{ MeV}. \qquad (9.18)$$

In addition, as was noted above, the second excited level of the model (9.5) contains five $SU(6)$ multiplets that are not perturbed by H^{inst}, and their masses depend on the two parameters $E^2(20,1^+)$ and Δ'. By virtue of this, we conclude from (9.10), (9.13), and (9.18) that $E^2(20,1^+) \approx 2000$ MeV. The best fit is obtained if we choose $\Delta' = 200$ MeV.

Table V gives the instanton contributions for the states N of positive parity.

Table VI gives the spectrum of excited states in the N–Δ system. It can be seen that the results of the calculations satisfactorily describe the masses of these states. Moreover, for the second excited level some calculated values are very close to the observed ones: $N^+_{1/2}(1440)$, $N^+_{5/2}(1680)$, $N^+_{1/2}(1710)$, $N^+_{3/2}(1710)$, which are well established. In Table VI, we distinguish states of the 20-plet and the states with high orbital angular momentum—the former do not arise in the products of baryon–meson scattering, and the masses of the latter can be reduced by relativistic corrections to the kinetic-energy operator.[76] In

TABLE VI. Mass spectrum of excited states of the nucleon and the Δ isobar.

States	Theoretical values	Experimental masses	Status
$N\,5/2^-$	1665	1670—1685 (1675)	****
$N\,3/2^-$	1665	1650—1750 (1700)	***
	1520	1515—1530 (1520)	****
$\Delta 3/2^-$	1665	1670—1770 (1700)	****
$N\,1/2^-$	1665	1640—1680 (1650)	****
	1520	1520—1555 (1535)	****
$\Delta 1/2^-$	1665	1615—1675 (1620)	****
$N\,7/2^+$	1960[b]	(1990)	**
$\Delta 7/2^+$	1920[b]	1940—1960 (1950)	****
$N\,5/2^+$	1960[b]	(2000)	**
	1945[b]		
	1715[b]	1675—1690 (1680)	****
$\Delta 5/2^+$	1960[b]	2000—2200 (2000)	****
	1920[b]	1870—1920 (1805)	****
$N\,3/2^+$	2000[a]		
	1960[b]		
	1945[b]		
	1900		
	1715[b]	1650—1750 (1720)	****
$\Delta 3/2^+$	1960[b]		
	1920[b]	1900—1970 (1920)	***
	1800	1550—1700 (1600)	***
$N\,1/2^+$	2000[a]		
	1960[b]		
	1750	1680—1740 (1710)	***
	1410	1430—1470 (1440)	****
$\Delta 1/2^+$	1920[a]	1870—1920 (1910)	****
	1900	......	

all the calculations only the constant of the instanton interaction is used in addition to the standard parameters of the potential model. Thus, the results show how the problem of the spin–orbit interaction can be solved by taking into account the instanton mechanism. Practically the same results and conclusions that we published in Ref. 63 were simultaneously reproduced in Ref. 77.

10. MULTIQUARK STATES. STABILITY OF THE *H* DIBARYON

In accordance with the predictions of the quark models with confinement, one must observe experimentally exotic objects—glueballs, hybrid states of quarks and gluons, and also multiquark hadrons. However, it has been found that these are not readily observed.

In the quark bag model, the study of multiquark states[78,79] is one of the most interesting of its applications. In these systems, vacuum effects must be manifested most strikingly. It is important to emphasize that the existence of stable multiquarks is a necessary consequence of our notions of the QCD vacuum and has fundamental importance for understanding the properties of matter at both the microscopic and cosmological levels.

Investigations in the field of search for stable multiquark states is a subject of sharp competition, and such investigations are included in the plans for the major scientific centers such as the JINR, CERN, BNL, KEK, IHEP, and others. However, the specific conditions of the experiments limit the number of productive investigations. Thus, in the first experiments to look for the H dihyperon, made soon after the pioneering studies of Jaffe (1977), the question of the existence of the H dihyperon was not resolved, and all that was obtained was an upper limit 40 nb for the production of the H in the $pp \to K^+K^+H$ reaction.[85]

In the MIT model, the hadron spectrum is determined by the one-gluon exchange potential

$$\Delta E_g = -\frac{\alpha_s}{4R} \sum_{i>J}^{N} \mu_{ij} (\sigma\lambda^a)_i (\sigma\lambda^a)_j , \qquad (10.1)$$

where N is the total number of quarks, and μ_{ij} determines the intensity of the color–spin interaction. In the limit of $SU(3)_f$ symmetry ($m_i=0$), the contribution (10.1) to the energy of the hadronic n-quark state can be expressed in terms of the Casimir operators of the symmetry group:

$$\Delta E_g = \frac{\alpha_s\mu}{4R} [8N + \tfrac{4}{3} J(J+1) - C_6^{CS} + \tfrac{1}{2} C_3^C], \qquad (10.2)$$

where J is the total spin, C_6^{CS} is the Casimir operator of the color–spin group $SU(6)_{CS}$, and C_6^C is the Casimir operator of $SU(3)_C$ ($C_3^C=0$ for color singlets).

The corresponding expression for the instanton interaction can be expressed in terms of the operators of the color–spin–isospin group $SU(12)_{CSF}$ [with obvious notation; see also (3.7) and (6.2)]:

$$\Delta E_{\text{inst}} = -\frac{4\pi^2\rho_C^2}{3R^3} I_0\{N(N-1) + 13N - \tfrac{9}{2} I(I+1) - \tfrac{3}{2} J(J+1) - \tfrac{3}{8} C_3^C + \tfrac{9}{8} C_6^{CS} + \tfrac{3}{8} C_6^{CT} + \tfrac{3}{8} C_4^{ST} - \tfrac{9}{8} C_{12}\}. \qquad (10.3)$$

Analysis of the theoretical predictions leads to the following conclusion: The multiquark systems have a very high density of energy levels that is comparable in order of magnitude with the expected lifetimes of the systems themselves.

In this connection, particular interest attaches to the strongly bound state of the H dihyperon[78]—a six-quark state with vacuum quantum numbers ($IJ^P=00^+$) enriched with strangeness -2. This state is a special one because of the form of the interactions (10.2) and (10.3), namely, the lightest states are those in which the quarks are in the most symmetric (antisymmetric) states with respect to the color–spin (flavor) representation. This so-called Hund rule for quark systems[78] is a consequence of the fundamental Pauli principle.

To calculate the matrix elements of the operators in ΔE_g, ΔE_{vac}, and ΔE_{inst},[3)] it is necessary to know the cluster expansion (dissociation) of the six-quark wave function of the H: $|q^6\rangle \to |q^3\rangle|q^3\rangle$, $|q^6\rangle \to |q^4\rangle|q^2\rangle$. The method of expansion[81] and the corresponding wave functions are given in the Appendix.

Using the H wave function,[80] for the matrix elements we obtain[4)]

$$\Delta E_g^H = \alpha_s(-5\mu_{00} - 22\mu_{0s} + 3\mu_{ss})/R,$$

$$\Delta E_{\text{inst}}^H = -\tfrac{27}{4} n_{\text{av}} (I_0\eta_{ud} + 2I_s\eta_{us})/R^3, \qquad (10.4)$$

$$\Delta E_g^{H^*} = \tfrac{1}{3}\alpha_s(-\tfrac{11}{6}\mu_{00} - 41\mu_{0s} + \tfrac{67}{6}\mu_{ss})/R,$$

$$\Delta E_{\text{inst}}^{H^*} = -\tfrac{43}{8} n_{\text{av}} (I_0\eta_{ud} + 2I_s\eta_{us})/R^3, \qquad (10.5)$$

where the coefficients are determined in (6.3), $I_s\eta_{us} \approx 0.65 I_0\eta_{ud}$, $uu \to ud$, and $\eta_{ud} = (4\pi^2\rho_{\text{av}}^4/(3m_*^2)$. Using these relations, we obtain for the binding energy the estimate

$$\Delta E^H = 2E^\Lambda - E^H \qquad (10.6)$$

and for the dihyperons ($R_\Lambda = 6.17\ \text{GeV}^{-1}$, $M_\Lambda = 1115$ MeV)

$$\Delta E^H = +140\ \text{MeV}, \quad R_H = 5.2\ \text{GeV}^{-1},$$

$$\Delta E^{H^*} = -110\ \text{MeV}, \quad R_{H^*} = 5.3\ \text{GeV}^{-1}.$$

It is to be expected that the error in the calculation of the binding energy is of the order of the masses. One can then estimate the masses by using the values of the energies of the physical threshold. Thus, according to our estimate, the H mass is less than $2M_\Lambda$ but above the $N\Lambda$ threshold; in its turn, the vector dihyperon H^* is absolutely unstable: $M_{H^*} > 2M_\Lambda$. In accordance with the estimate of the H lifetime in $\Delta T=1$ weak decays established in Ref. 82, the state with $M_H = 2.09$ GeV is long-lived: $\tau_H \approx 10^{-7}$ sec.

It should be recalled that any model has a definite accuracy of predictions of order 10–30%, and it is therefore important that in our model a tendency to stabilization of the dihyperon can be seen. This is actually a system of three scalar diquarks in each of which strong attraction arises because of the instanton interaction. Clearly, this is a manifestation of the same forces that split the η–η' states, i.e., the stability of H is directly associated with the solution of the $U_A(1)$ problem.

We have also proved Hund's spectroscopic rule for quark systems.[78–80] At the same time, in our approach its origin is quite different from the one in Ref. 78. The nonperturbative interaction between the quarks through the instantons gives a strong attraction in the color- and spin-symmetric representation and is completely absent for antisymmetric states. The instanton interaction takes into account the strong attraction at the intermediate scale ρ_{av} and forms the quasibound diquark state.[83,84] Thus, the Hund rule is associated with the existence of diquarks.

Some time ago, an experiment using an emulsion chamber made by the group of B. A. Shakhbazyan at Dubna provided data that confirm the existence of H in an interval of masses close to the predicted one.[86,87] The group found two events that can be interpreted as weak decay of the H dihyperon in accordance with the scheme of Ref. 86: $H\to p+\Sigma^-$, $\Sigma^-\to n+\pi^-$. A kinematic analysis showed that the corresponding masses of the H state were

$$M_H=2174.6\pm 13.0\ \text{MeV}/c^2,$$

$$M_H=2218.0\pm 12.0\ \text{MeV}/c^2.$$

Two events of intranuclear conversion $H+p\to p+2\Lambda$ were also found.

Later, in experiments on the production of the H dihyperon by neutrons on nuclei, two events were found that had mass values close to those of Ref. 86 and split in accordance with the scheme $H\to p+\pi^-+\Lambda$, $\Lambda\to p+\pi^-$.

These observations are a serious confirmation of the results that were obtained first at Dubna. However, it is necessary to continue independent tests at CERN, KEK, and BNL, which are being currently made. A direct observation of H in decays would be a critical success. Strong restrictions on the H mass are obtained in experiments in nuclear emulsions made to observe decays of double hypernuclei.[88] An upper bound has been obtained, namely, $M_H<2204$ MeV, which is below the threshold upper bound: $M_H<M_{2\Lambda}=2230$ MeV.

To conclude this part, we should also note that the H is of great theoretical interest and is an object on which one can test models. The H mass was estimated in various forms of the bag model (Refs. 78, 79, 89, and 90), in the lattice approach,[91,92] in QCD sum rules,[93] and in the Skyrme model.[94,95] The results give contradictory estimates. It should be said here that some of the approaches (for example, the early lattice calculations and the QCD sum rules) probably do not possess sufficient accuracy. Other approaches differ significantly in the choice of the model assumptions concerning the structure of the strong interactions. As a rule, the instanton interaction, which, from our point of view, is dominant in H, is not taken into account, leading to an overestimate of the H mass. Attention should be drawn to the lattice calculations of Ref. 92, in which a strongly bound H state was obtained. These calculations used a lattice with a small period, and this appears to have been sufficient to take into account the influence of topologically nontrivial vacuum fluctuations.

In low-energy QCD (the Skyrme model) there exists a representative of a state with quantum numbers of the H dihyperon that differs appreciably from the type of states in quark models and is associated with nontrivial topological properties of the QCD vacuum. A feature of such a representation is a new type of stability of states with respect to decays, namely, one expects suppression of the decay $H\to\Lambda\Lambda$ (even if $M_H>2M_\Lambda$) because the field configuration of the H dihyperon is topologically very different from the product of the two configurations corresponding to the field of the Λ hyperon. The experiment of Ref. 96 identified one event associated with the weak decay $H\to p+\Sigma^-$, $\Sigma^-\to n+\pi^-$ with mass

$$M_H=2408.9\pm 11.2\ \text{MeV}/c^2,$$

and this confirms the theoretical possibility noted above.

In Ref. 97 there was a report of observation of an event interpreted as intranuclear conversion of a stable dibaryon A ($I=1/2$, $Y=-1$, $B=2$, $S=-3$) into 3Λ with mass

$$M_A=2450\text{–}2500\ \text{MeV}/c^2.$$

The possible existence of a particle with such quantum numbers is associated in the theory with rather subtle effects of the interquark interaction, when forces associated with delocalization of quarks in a multiquark object are dominant.[98]

All this indicates that, despite the poor statistics of the data, there is an unflagging interest in the exotic states that are predicted by the majority of quark models.

11. STATIC PROPERTIES OF HADRONS

It is well known that the quarks in the proton have a complicated structure that depends strongly on the depth of penetration into the quark. Thus, the quarks that are considered in the parton model of QCD are the quark components of the proton wave function in the infinite-momentum frame and are bare quarks, at least at the Q^2 in a scattering experiment by means of which they are described. With increasing Q^2, for example, in deep inelastic scattering, the quark structure is manifested as an evolution described in terms of the Gribov–Lipatov–Altarelli–Parisi equation,[99] but this structure can be understood only on the basis of perturbation theory. Quarks that are used as basic objects in the quark models are constituent quarks and have a nontrivial and nonperturbative representation in terms of bare quarks and gluons.

One of the most important applications of the constituent-quark model is the calculation of the static matrix elements of the currents, expressed in terms of the operators of the bare quarks, using phenomenological currents formed from constituent quarks. We shall consider in the framework of our model the various static properties of the hadrons: the charges, mean-square radii, and magnetic moments. We shall take into account only the single-particle corrections to these quantities. We shall discuss the extent to which this approximation is violated by the instanton interaction in the following section.

We are interested in the matrix elements of operators that have different Lorentz structures: vector densities

$$g_V=\int d\mathbf{x}\bar{\psi}(\mathbf{x})\gamma_0 Q\psi(\mathbf{x}), \quad (11.1)$$

$$\langle r^2\rangle_{\rm em}=\int d\mathbf{x}x^2\bar{\psi}(\mathbf{x})\gamma_0 Q\psi(\mathbf{x}), \quad (11.2)$$

axial-vector densities

$$g_A=\int d\mathbf{x}\bar{\psi}(\mathbf{x})\gamma_3\gamma_5\begin{pmatrix}1\\ \tau_3\end{pmatrix}Q\psi(\mathbf{x}), \quad (11.3)$$

$$\langle r^2\rangle_{ax}=\int d\mathbf{x}x^2\bar{\psi}(\mathbf{x})\gamma_3\gamma_5\begin{pmatrix}1\\ \tau_3\end{pmatrix}Q\psi(\mathbf{x}), \quad (11.4)$$

scalar densities (σ term)

$$\sigma=m_u\int d\mathbf{x}[\bar{u}(\mathbf{x})u(\mathbf{x})+\bar{d}(\mathbf{x})d(\mathbf{x})], \quad (11.5)$$

$$y=\left[2\int d\mathbf{x}\bar{s}(\mathbf{x})s(\mathbf{x})\right]\left[\int d\mathbf{x}[\bar{u}(\mathbf{x})u(\mathbf{x})+\bar{d}(\mathbf{x})d(\mathbf{x})\right], \quad (11.6)$$

and magnetic moments

$$\mu=\tfrac{1}{2}\int d\mathbf{x}\bar{\psi}(\mathbf{x})[\mathbf{r}\times\gamma]\psi(\mathbf{x}). \quad (11.7)$$

The procedure for calculating form factors and charges in the bag model is as follows. Suppose that the quark current is specified in the general form $J_\Gamma(x)=\bar{q}(x)\Gamma q(x)$, where Γ is the appropriate combination of spin–flavor–color matrices, and we consider the single-nucleon matrix element

$$\langle N(p)|J_\Gamma|N(p')\rangle=N_N\bar{u}(p)G_\Gamma(k^2)u(p'),$$

where $N(p_i)$ is the nucleon state with 4-momentum p_i, $N_N=[(M_N/p_0)(M_N/p_0)]^{1/2}$ is the covariant normalization of the nucleon state, and $k=p-p'$. For the wave functions of the static approximation, it is convenient to work in the Breit system:

$$\mathbf{p}'=-\tfrac{1}{2}\mathbf{k},\quad \mathbf{p}=\tfrac{1}{2}\mathbf{k},\quad p_0'=p_0=[M_N^2+\tfrac{1}{4}\mathbf{k}^2]^{1/2}, \quad (11.8)$$

in which we have the relation

$$N_N\bar{u}(p)G_\Gamma(k^2)u(p')=\left\langle N_{\rm bag}\middle|\sum_{\rm quarks}M_\Gamma(\mathbf{k})\middle|N_{\rm bag}\right\rangle \quad (11.9)$$

with matrix M_Γ in the spin–flavor–color space given by the expression

$$M_\Gamma(\mathbf{k})=\int_{\rm bag}d^3x\,\exp(i\mathbf{kx})\bar{q}(x)\Gamma q(x). \quad (11.10)$$

Expanding $\exp(i\mathbf{kx})$ in a Taylor series and equating the coefficients of the zeroth, first, and second powers of the momentum transfer $\mathbf{k}$ on the left- and right-hand sides of (11.9), we obtain expressions for the charges in terms of integrals of the quark wave functions. The vector constant g_V determines the normalization of the wave function and, therefore, is always equal to unity. This factor, like the baryon magnetic moments, which are close to the experimental values (see below), indicates that the quark model works well in the vector channel. The reason for this is that the flavor charge of the bare and the constituent quark are equal by virtue of charge conservation. Therefore, in a measurement with a certain degree of resolution the flavor currents in the proton appear to be directly attached to the constituent quark. The sea quark–antiquark pairs, which form a "cloud" around the quark, do not contribute to the flavor currents at a scale of order 1 fm.

TABLE VII. Electromagnetic radii of the hadrons (fm^2).

Hadrons	$\mu_{\rm th}$	$\mu_{\rm MIT}$	$\mu_{\rm exp}$
P	0,69	0,53	0,68
N	0	0	−0,116
π	0,61	0,24	0,44

As in the MIT bag model, the formula for the mean-square radius in the case of massless quarks has the form

$$\langle r^2\rangle_{\rm em}=R^2\frac{2\omega^2(\omega-1)+4\omega-3}{6\omega^2(\omega-1)}\sum_i e_i. \quad (11.11)$$

The results are given in Table VII. It can be seen that the model agrees well with experiment, whereas the standard MIT model gives poorer agreement.

Using the spin-unitary symmetry of the baryon octet for the magnetic moment (11.7), we obtain

$$\mu_p=\mu_0,\quad \mu_n=-\tfrac{2}{3}\mu_0,\quad \mu_\Lambda=\tfrac{1}{3}\mu_s,\quad \mu_{\Sigma^+}=\tfrac{8}{9}\mu_0+\tfrac{1}{9}\mu_s,$$

$$\mu_{\Sigma^-}=-\tfrac{4}{9}\mu_0+\tfrac{1}{9}\mu_s,\quad \mu_{\Xi^0}=-\tfrac{2}{9}\mu_0-\tfrac{4}{9}\mu_s,$$

$$\mu_{\Xi^-}=\tfrac{1}{9}\mu_0-\tfrac{4}{9}\mu_s,\quad \mu_{\Lambda\Sigma^0}=-\frac{1}{\sqrt{3}}\mu_0, \quad (11.12)$$

where the magnetic moment of a quark of definite flavor is, as in the bag model, equal to

$$\mu_i=\frac{R(4y_i+2a_i-3)}{6[2y_i(y_i-1)+a_i]},\quad y_i=\omega_iR,\quad a_i=m_iR. \quad (11.13)$$

The results of calculations with $m_u=m_d=0$, $m_s=220$ MeV and the values of R from Table II are given in Table VIII together with the results of the bag model and of the nonrelativistic model.[64] Note that in the nonrelativistic calculations better agreement with experiment is achieved if two additional parameters are used (the magnetic moments of the nonstrange and strange quarks), whereas our model and the MIT model do not have additional parameters. At the same time, our model gives a better description of the baryon magnetic moments than the MIT model.

In our opinion, the apparent mean-square radius of the pion is a result of the large mixing of its wave function with radial excitations and the quark sea which arises as a consequence of the instantons. On the other hand, the diquark mechanism can explain the deviation from the predictions

TABLE VIII. Magnetic moments of baryons (in Bohr magnetons μ_N). (The underlined values were used to fit the parameters of the model.)

Hadrons	μ_{th}	μ_{MIT}	μ_{NR}	μ_{exp}
P	2,20	1,90	<u>2,79</u>	2,7928
N	−1,47	−1,27	−1,86	−1,9130
Λ	−0,60	−0,49	<u>−0,61</u>	−0,613±0,004
Σ^+	2,08	1,84	2,68	2,42±0,05
Σ^-	−0,74	−0,68	−1,04	−1,04±0,025
Ξ^0	−1,21	−1,06	−1,44	−1,250±0,014
Ξ^-	−0,54	−0,44	−0,51	−0,6507±0,00025
$\Lambda\Sigma^0$	−1,23	−1,10	−1,61	−1,60±0,07

of the $SU(6)$-symmetric quark model and obtain, in particular, a nonzero mean-square radius of the neutron.[102]

We now consider the leptonic decay constants of the baryons ($B\to B'e^-\nu$). The ratio of the decay constants is

$$\frac{g_A}{g_V}=\frac{\langle B'\uparrow|\int d\mathbf{x}\bar{\psi}(\mathbf{x})\lambda^+\gamma_3\gamma_5 Q\psi(\mathbf{x})|B\uparrow\rangle}{\langle B'\uparrow|\int d\mathbf{x}\bar{\psi}(\mathbf{x})\lambda^+ Q\psi(\mathbf{x})|B\uparrow\rangle}, \qquad (11.14)$$

where $\lambda^+=\lambda_1+i\lambda_2$ ($\lambda^+=\lambda_4+i\lambda_5$) for decays without (respectively, with) a change in the strangeness, and λ_i are the generators in the flavor space. The right-hand side of (11.14) is readily calculated in the symmetric basis of nonrelativistic wave functions.[101] However, as was first shown by P. N. Bogolyubov,[22] an important role in the calculation of the axial-vector constants is played by relativistic effects, which reach 30% for ultrarelativistic quarks. In the general case ($m\neq 0$) we have

$$\frac{g_A}{g_V}=\frac{5}{3}\langle\sigma_Z\rangle,$$

$$\langle\sigma_Z\rangle_{ij}=\int d\mathbf{x}q_i^+(\mathbf{x})\sigma_Z q_j(\mathbf{x})$$

$$=\frac{\omega_i\omega_j[2(y_i-y_j)+a_i-a_j]}{(\omega_i^2-\omega_j^2)[2y_i(y_i-1)+a_i]^{1/2}[2y_j(y_j-1)+a_j]^{1/2}}. \qquad (11.15)$$

The results of calculations of the transition constants with and without change in the strangeness are given in Table IX.

It can be seen that the model gives practically the same values as the MIT model. This is because g_A/g_V depends on the dimensionless parameter $m_S R$, which is practically the same for the two models. Moreover, all charges in models with spherical symmetry can be expressed solely in terms of a few constants—integrals of the components of the Dirac spinor. As a result, most charges are connected by general, model-independent relations.[104] It is also important to note that the results of these relativistic models agree much better with experiment than the results of the nonrelativistic model.

In contrast to the situation for the vector currents, the axial charges of the bare quark and the constituent quark are not equal. The reason for this is that the axial charge corresponds to the helicity carried by the quark, and by virtue of the spontaneous breaking of the chiral symmetry the quark helicity can change in an interaction of the quark with the quark condensate $\langle\bar{Q}Q\rangle$ even though the axial current is conserved in the chiral limit. Overall, as can be seen from the results, the model works well in the axial-vector channel, and the relation between the parton (bare quarks) method of describing the static properties of the nucleon and the description in terms of the constituent quarks is practically identical.

We consider in more detail the question of the relation between the two methods of description for the singlet axial-vector current and compare the situation that arises in this case with the situation for the isotriplet current, which is the best studied. The nucleon matrix element of the spatial component of the axial-vector current J_μ^5 is described by two form factors G_1 and G_2:

TABLE IX. Axial-vector decay constants of members of the baryon octet.

Decay	$(g_A/g_V)_{theor}$	$(g_A/g_V)_{MIT}$	$(g_A/g_V)_{NR}$	$(g_A/g_V)_{exp}$
$n\to pe^-\nu$	1,088	1,088	5/3	1,254±0,006
$\Lambda\to pe^-\nu$	0,713	0,709	1	0,694±0,025
$\Sigma^-\to ne^-\nu$	−0,238	−0,236	1/3	−0,362±0,043
$\Xi^-\to\Lambda e^-\nu$	0,238	0,236	1/3	0,25±0,043
$\Xi^-\to\Sigma^0 e^-\nu$	1,188	1,181	5/3	
$\Xi^0\to\Sigma^+ e^-\nu$	1,188	1,181	5/3	

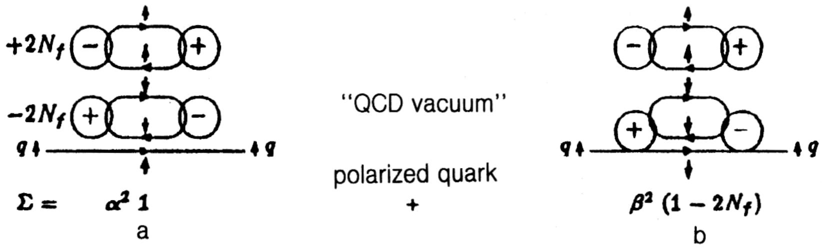

FIG. 10. Contributions to the "quark-spin screening": a) polarized quark in a topologically neutral instanton vacuum; b) anomalous change of the helicity of a quark interacting with the vacuum.

$$\langle N(p_2)|\mathbf{J}^{(0,j)5}|N(p_1)\rangle$$
$$=N_N\bar{u}(p)[G_A(k^2)\gamma\gamma_5+H_A(k^2)\mathbf{k}\gamma_5]\left(I,\frac{\tau_j}{2}\right)u(p'),\qquad(11.16)$$

this being a consequence of Lorentz invariance. In the nonrelativistic limit $k^\mu\to 0$, this relation goes over in the nucleon rest frame into

$$\lim_{k^\mu\to 0}\langle N(p)|\mathbf{J}^{(0,j)5}|N(p')\rangle$$
$$\equiv\mathbf{J}^{(0,j)5}(0)=u_0^+\left[G_A(0)\boldsymbol{\sigma}+\frac{D_A(0)}{2M_N}(\boldsymbol{\sigma}\mathbf{k})\mathbf{k}\right]\left(I,\frac{\tau_j}{2}\right)u_0.\qquad(11.17)$$

In the isotriplet channel in the chiral limit, the form factor $H_A(k^2)$ has a pole $H_A(k^2)=[D_A(k^2)]/k^2$ at $k^2=0$ with residue $D_A(0)=-2f_\pi g_{\pi N}$, since in this limit the pions are massless Goldstone bosons. The requirement of conservation of the isotriplet current, $\lim_{k^\mu\to 0}\mathbf{k}\mathbf{J}^{(3)5}(k)=0$, leads to the well-known Goldberger–Treiman relation between the axial constant $G_A^{(3)}$ and the residue D_A:

$$G_A^{(3)}=-\frac{D_A(0)}{2M_N}\equiv\frac{g_{\pi N}f_\pi}{M_N}.\qquad(11.18)$$

On the other hand, in the isosinglet channel, even in the chiral limit, the form factor $H_A(k^2)$ does not have a pole, since, by virtue of the axial anomaly, the η' acquires a mass, which in the instanton-liquid model is, in accordance with (3.28), $m^2_{\eta'}=4N_F 2n_{\rm av}(k\xi_{\eta'})^2$, and the corresponding current is not conserved. Therefore, the axial constant $G_A^{(0)}$ is related to the forward matrix element of the current divergence:

$$\lim_{k^\mu\to 0}\langle N(p)|\nabla\mathbf{J}^{(0)5}|N(p')\rangle=iG_A^{(0)}u_0^+\boldsymbol{\sigma}\mathbf{k}u_0.\qquad(11.19)$$

In the isotriplet channel, adding the quark and pion contributions in the chiral bag model, we obtain

$$\mathbf{J}^{a5}(0)_{\rm chiral\ bag}=\frac{5}{6}\frac{\omega}{6\omega-1}u_0^+[\boldsymbol{\sigma}-(\boldsymbol{\sigma}\hat{k})\hat{k}]\frac{\tau^a}{2}u_0.\qquad(11.20)$$

It follows immediately from this expression that the isotriplet axial current is conserved, as a consequence of which the Goldberger–Treiman relation (11.18) holds, and for (11.20) and (11.18) the nucleon isotriplet axial-vector charge is

$$G_A^{(3)}=\frac{5}{6}\frac{\omega}{\omega-1}.\qquad(11.21)$$

These results are a significant achievement of the chiral bag model.[37]

In the case of the isosinglet axial-vector current, the corresponding current in the chiral bag model has the form

$$\mathbf{J}^5(0)_{\rm bag}=\lim_{k^\mu\to 0}\int d^3r e^{i\mathbf{kr}}\mathbf{J}^5(r)\qquad(11.22)$$
$$=\lim_{k^\mu\to 0}-i\frac{\partial}{\partial\mathbf{k}}\int d^3r e^{i\mathbf{kr}}\nabla\mathbf{J}(r)$$
$$=\mathbf{J}^5(0)_{\rm quark}+\mathbf{J}^5(0)_{\eta'}.\qquad(11.23)$$

As was shown above, the anomalous contribution of the quarks to the matrix element in the instanton-liquid model has the form

$$\nabla\mathbf{J}^5(0)_{\rm quark}=-4in_{\rm av}k'^2\sum_{i>j}\int d^3r e^{i\mathbf{kr}}\{\bar{q}_{iR}q_{iL}\bar{q}_{jR}q_{jL}\Gamma^{ij}-(R\leftrightarrow L)\}\Theta_V$$
$$=(4N_F 2n_{\rm av}k^2)\int d^3r e^{i\mathbf{kr}}(\pi_q\alpha_q-\eta_q\sigma_q)\Theta_V$$
$$=-N_F\int d^3r e^{i\mathbf{kr}}Q_q(x)\Theta_V\qquad(11.24)$$

and is calculated by means of the diagrams shown in Figs. 10 and 11. In effective meson theory, the η' field is expressed in the form

$$\langle N|\eta'|N\rangle=-\frac{g_{\eta'N}}{8\pi M_n}u_0^+\boldsymbol{\sigma}\nabla\left(\frac{e^{-m_{\eta'}r}}{r}\right)u_0$$
$$=\frac{g_{\eta'N}}{8\pi M_n}u_0^+\frac{\boldsymbol{\sigma}\hat{r}}{r^2}e^{-m_{\eta'}r}(1+m_{\eta'}r)u_0$$
$$=\left\langle N\left|\frac{(\mathbf{P}_\eta\boldsymbol{\sigma})}{r^2}\right|N\right\rangle,\quad r\geqslant R_{\rm bag},\qquad(11.25)$$

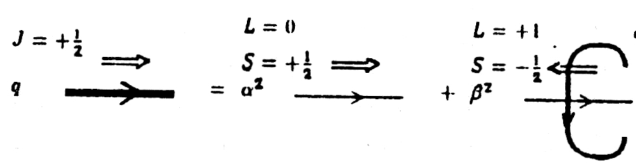

FIG. 11. Spin structure of a valence quark.

and, in addition,

$$\nabla \mathbf{J}^5(r)_{\eta'} = -f_{\eta'}(r)\Theta_{\bar{V}}(r). \tag{11.26}$$

To calculate the contribution of the divergence of the meson field, we first calculate the constant $g_{\eta' N}$, for which we use the boundary condition (3.30). We then have

$$\hat{n}\mathbf{J}^5_{\text{quark}}|_S = \frac{1}{8\pi}\frac{\omega}{\omega-1} u_0^+ \sigma \hat{r} u_0$$

$$= \frac{g_{\eta' N}}{8\pi M_N} f_{\eta'} f(y_\eta) u_0^+ \frac{\sigma}{R^3} u_0 = \hat{n}\mathbf{J}^5_{\eta'}|_S,$$

$$f(y_\eta) = [2(1+y_\eta)+y_\eta^2]e^{-y_\eta}, \quad y_\eta = m_\eta R. \tag{11.27}$$

As a result, in the chiral bag model we have for the constant of the coupling between the η' and the nucleon the relation[100]

$$g_{\eta' N} = \frac{M_N}{f_{\eta'} f(y_\eta)} \frac{\omega}{\omega-1}. \tag{11.28}$$

Further, using the local boundary condition (11.27), we obtain

$$\mathbf{J}^5(0) = \frac{3(1+y_\eta)+y_\eta^2}{2(1+y_\eta)+y_\eta^2}\frac{1}{3}\frac{\omega}{\omega-1} - N_F \int_V d\mathbf{r}\mathbf{r} Q(x). \tag{11.29}$$

The total contribution to the matrix element of the isosinglet axial-vector current is

$$G_A^{(0)} = 0.08, \tag{11.30}$$

which should be compared with the result of the EMC experiment:

$$G_A^{(0)} = 0.12 \pm 0.2. \tag{11.31}$$

Thus, in the single-particle approximation the proposed model describes the static properties with accuracy $\leqslant 20\%$.

12. CONCLUSIONS

We have proposed a quark model in which the most important role is played by the interaction of the quarks with the QCD vacuum. It is possible to describe the hadron mass spectrum with a much smaller quark–gluon coupling constant, $\alpha_s = 0.4$, than the value $\alpha_s = 2.2$ in the MIT model. The large coupling constant led not only to problems in the application of QCD perturbation theory in the bag but also to the impossibility of describing the spectrum of excited hadronic states. In the review, we have shown that the spin–spin mass splitting between the hadron multiplets is determined by the interaction through instanton exchange. Thus, the pseudoscalar octet loses a third of its mass through this interaction. In the baryons, this interaction gives a strong attraction in the channels in which there is a scalar diquark and, in particular, determines the N–Δ splitting.

Interaction through instantons has a direct bearing on the resolution of the $U_A(1)$ problem, leading in the flavor-singlet channel to repulsion. This exceptional fact must play a crucial role in the formation of multiquark states. In particular, there is a strong tendency to the existence of a stable dilambda H, for which there are some experimental indications.

A similar situation is observed in the QCD sum rules, in which allowance for direct instantons is extremely important in the pseudoscalar and scalar channels.

In the review we have proposed for the first time a self-consistent formulation of the bag model based on a microscopic model of the QCD vacuum. It is a good basis for investigating many static and dynamical characteristics of hadrons. On the other hand, the development of a better model of the vacuum would provide a basis for constructing a more realistic model of hadrons.

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[1)] It may be assumed that the hadron is a superposition of a nontopological component (3.40) and a configuration with nontrivial topology (Skyrmion):

$$|\text{Nucleon}\rangle = \alpha|\text{bag Nucleon}\rangle + \beta|\text{Skyrmion}\rangle + ... \tag{*}$$

It is probable that one of the components must be suppressed for baryons. Both representations describe the static and dynamical properties of the baryons and explain the EMC polarization experiments.[103] However, the nature of the spin for these components has a different origin. This fact will probably make it possible in future experiments that measure gluon structure functions to establish which of the components is dominant. The representation (*) can be even more interesting for multiquark systems (see below), in which states that exceed the strong-decay threshold can be stable for topological reasons. In this case, the quark component is completely absent.

[2)] If one proceeds naively and in (5.17) retains only the terms E_{kin} and E_{vac}, then one obtains the "nucleon" mass $M_N = \frac{9}{2}[-(X^2/(X-1))(\pi/12)\langle\bar{Q}Q\rangle]^{1/3} \approx [-88\langle\bar{Q}Q\rangle]^{1/3}$; if one proceeds as naively in sum rules,[56] then one obtains a numerically very similar result: $M_N = [-2(2\pi)^2\langle\bar{Q}Q\rangle]^{1/3} = [-80\langle\bar{Q}Q\rangle]^{1/3}$.

[3)] We consider the contribution of only the two-particle operator ΔE_{inst}. As the estimates of Ref. 80 showed, the contribution of the three-particle operator is negligibly small: $\Delta E_{\text{inst}} \approx 5$ MeV.

[4)] In Ref. 49, the $O(\alpha_s)$ corrections to the hadron masses and, in particular, to the mass of the H dihyperon were calculated. The H mass was found to be 2.19 GeV, corresponding to a stable state. However the corrections were large. Therefore, there is doubt about the applicability of perturbation theory in α_s.

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