Scattering properties of two-cluster systems produced in multiparticle nuclear reactions

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Results are presented on theoretical and experimental studies of the change of the observed parameters of two-cluster nuclear resonances formed in the final state of a multiparticle nuclear reaction. This phenomenon arises from the effect of the Coulomb and nuclear fields of the accompanying reaction products on the resonance production and decay process and is observed in the two-cluster reaction cross section as a change of the shape of the resonance peaks and a shift of their energy. Depending on the type of nuclear process, multiparticle scattering theory can be used to obtain parametrizations of the T amplitude of the reaction which can explain these variations in experiments involving light ions at low and intermediate energies.

INTRODUCTION

It is well known that data on resonance and virtual states play an important role in the study of nuclear interactions. However, the direct experimental observation of the scattering properties of even two nuclear clusters is quite difficult in some cases, particularly if one of the fragments is neutral. Therefore, in nuclear physics a great deal of work has been done on indirect methods of experimentally analyzing the scattering of several particles forming an unstable system.

In particular, the properties of two-cluster systems in a resonance or virtual state are studied using multiparticle nuclear reactions, in the final state of which the cluster system in question is formed together with other accompanying products. In developing techniques of this type for studying resonance and virtual levels in the nuclear-cluster subsystem it has usually been assumed that the scattering properties of the clusters of this subsystem are independent of the dynamics of the multiparticle process in which they are formed and of the kinematics and type of accompanying products. Consequently, it was expected that there would be a direct correspondence between the cross section for the multiparticle process and the cross section for particle scattering in this subsystem in the case where it is isolated.

In such a model the parameters of the maxima (its location and width) in the spectra of particles produced in multiparticle reactions together with resonance subsystems are directly related to the position and width of the resonance level. It should be noted that this model for analyzing the resonance interaction in the final state of multiparticle reactions was borrowed by the physics of multiparticle nuclear reactions at low energies from the analysis of meson interactions with nucleons at high energies,[1] where the fundamental assumptions of this model were completely justified.

Analysis of experiments on multiparticle nuclear reactions at low and intermediate energies carried out over several years has shown that the characteristics of the potential and resonance scattering of a group of clusters interacting in the final state, extracted from analysis of the spectra of the products of these reactions, depend considerably on the reaction energy, the type of accompanying particles, and the kinematics of the particles in the final state. This effect has been noticed both in few-nucleon nuclear reactions, reactions of deuteron breakup on nuclei, and in reactions involving clusters like d, ^{3}He, t, ^{4}He, and so on.[2] It has been noted that in a number of cases the resonance widths extracted from analysis of multiparticle reactions turned out to be smaller than the widths measured in a direct experiment on the scattering of the clusters involved in the resonance. Experiments also showed that the resonance maxima in the multiparticle reaction spectra can be shifted relative to their location predicted by the simple model of Ref. 1, and even split.[3] This phenomenon has come to be called "resonance narrowing" (Ref. 4). The experimental data collected on the change of the properties of the scattering of two and more clusters in the final state of multiparticle processes at low energies has served as the foundation for the development of a theory of particle scattering in the fields of the accompanying products and the determination, using this theory, of the laws governing the spatial anisotropy of the observed scattering parameters for two clusters in a nonisolated subsystem produced in the final state of nuclear reactions with the emission of several particles.

The first theoretical studies of multiparticle reaction spectra, in which the reasons for the distortion of the maxima corresponding to two particles interacting in the final state were analyzed, are those of Ref. 5. In these studies it was shown, first, that an important factor in resonance formation in the final state is the superposition of the am-

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plitudes corresponding to the interaction of all possible groups of clusters formed in the final state, i.e., the superposition of the amplitudes of nonorthogonal channels. Second, the effect of the reaction mechanism (whether the reaction was direct or occurred via a compound nucleus) on the properties of the parameters of nonisolated resonances in the reaction-product spectra and in their angular correlation functions was determined. The further development of the theory of multiparticle processes and comparison of its predictions with experiment made it possible to determine the fundamental laws governing the spatial anisotropy of the scattering properties of particles in nonisolated subsystems produced in the final state of the reaction. This theory can be used to study two types of effect on final-state particle scattering in the fields of the accompanying particles: the unified effect of the nuclear fields, which is important during most of the reaction, and the effect of the Coulomb fields, which are important beyond the nuclear force range.

The observed distortions of the scattering parameters of the particles in the final state due to the nuclear interaction are mainly the result of the superposition of the channel amplitudes of particle clusterings in the system where these amplitudes depend on the momentum transfer and off-shell effects of two-cluster scattering.[6,7] The asymptotic effect of the Coulomb fields of the accompanying products on the subsystem of particles scattered in the final state, which distorts the trajectories of the scattered particles, is also observed as a change of the scattering properties.[8,9] The present review contains the main results of this analysis and describes the regularities found in the change of the scattering properties of particles in nonisolated subsystems.

To study the scattering properties of particles in nonisolated subsystems, i.e., in subsystems formed together with other accompanying fragments in the final state of multiparticle reactions, we shall separately consider nuclear processes occurring via the direct interaction and via an intermediate compound nucleus. As will be shown below, this splitting of the two effects allows us to formulate the main features of the influence of the nuclear and Coulomb fields on the scattering parameters of particles in nonisolated subsystems.

To simplify our exposition, here we will discuss mainly nuclear reactions involving three particles in the final state and, consequently, the distortion of two-cluster resonances by the field of the third particle. In the Conclusion we will present the generalizations to more complicated processes.

To describe processes of the type

$$A+B\to X_1+X_2+X_3$$

we define $\mathbf{p}_0$ and E_0, the initial momentum of the relative motion of the clusters A and B and the total energy of the system. The relative coordinate of the clusters will be denoted by R. In the center-of-mass system of the three final products it is appropriate to take as the independent momenta and coordinates sets of Jacobi coordinates of the form $\{\mathbf{k}_\alpha\mathbf{p}_\alpha\}$ and $\{\mathbf{x}_\alpha\mathbf{y}_\alpha\}$, where $\mathbf{k}_\alpha$ and $\mathbf{x}_\alpha$ are the c.m. momentum and coordinate of the subsystem (α) consisting of two particles, $\alpha\in\{12,23,31\}$, and $\mathbf{p}_\alpha$ and $\mathbf{y}_\alpha$ are the momentum and coordinate of the third particle not included in the pair α relative to the center of mass of this pair. The choice of Jacobi coordinates is related to the problem of analyzing the scattering properties of the two particles in the subsystem α formed in the final state along with the third accompanying nuclear fragment. We define the internal energy of the particles in the subsystem α as $\varepsilon_\alpha=k_\alpha^2/2\mu_\alpha$, where μ_α is their reduced mass. The energy of the third particle not involved in the pair α is $E_\alpha=p_\alpha^2/2n_\alpha$, in which the parameter n_α is the reduced mass of this particle relative to the center of mass of α. In this notation the cross section of the reaction under consideration with the condition that the momentum $\mathbf{p}_\alpha$ is fixed in an infinitesimal volume determined by $d\Omega_{p_\alpha}$ and dE_α and that the momentum $\mathbf{k}_\alpha$ lies in the solid angle $d\Omega_{k_\alpha}$ has the form

$$\frac{d\sigma}{d\Omega_{k_\alpha}d\Omega_{p_\alpha}dE_\alpha}=(2\pi)^4\frac{\mu_\alpha n_\alpha k_\alpha p_\alpha}{p_0}\,|T(\mathbf{k}_\alpha\mathbf{p}_\alpha,\mathbf{p}_0,z)|^2.$$

Here $T(\mathbf{k}_\alpha\mathbf{p}_\alpha,\mathbf{p}_0,z)$ is the reaction amplitude, which must be determined in accordance with a model of the nuclear process leading to the formation of three particles and $z=E+i0$.

As is well known, if the nuclear reaction $A+B\to X_1+X_2+X_3$ occurs via a compound nucleus, then according to the Bohr model the amplitude for this process can be represented as the matrix element

$$\langle\mathbf{k}_\alpha\mathbf{p}_\alpha|\,T_3(z)G_0(z)T_{\rm comp}(z)\,|\mathbf{p}_0\rangle$$

or

$$\int d\mathbf{k}'_\alpha d\mathbf{p}'_\alpha\langle\mathbf{k}_\alpha\mathbf{p}_\alpha|\,T_3(z)\,|\mathbf{k}'_\alpha\mathbf{p}'_\alpha\rangle\times\left(z-\frac{k'^2_\alpha}{2\mu_\alpha}-\frac{p'^2_\alpha}{2n_\alpha}\right)^{-1}\langle\mathbf{k}'_\alpha\mathbf{p}'_\alpha|\,T_{\rm comp}(z)\,|\mathbf{p}_0\rangle,$$

where $\langle\mathbf{k}'_\alpha\mathbf{p}'_\alpha|\,T_{\rm comp}(z)\,|\mathbf{p}_0\rangle$ is the amplitude corresponding to the formation of the compound nucleus and the three fragments. It is well known that this amplitude is a function of energy and is independent of the initial particle momentum. The function $\langle\mathbf{k}_\alpha\mathbf{p}_\alpha|\,T_3(z)\,|\mathbf{k}'_\alpha\mathbf{p}'_\alpha\rangle$ is the scattering amplitude for three quantum objects, which can be determined on the basis of the familiar theory of the three-body problem with pair forces.[6,10]

At intermediate and high energies, when the strong coupling between the initial and final momenta is important, it is necessary to resort to direct-interaction models.

The assumption that the reaction $A+B\to X_1+X_2+X_3$ occurs via a direct interaction implies that the initial clusters of the reaction A and B consist of the particles X_1, X_2, and X_3, two of which are joined into a single cluster which is, for example, the target nucleus B with binding energy $-\kappa^2$, so that $E_0=-\kappa^2+p_0^2/2n$. The amplitude of this reaction, which is the matrix element of the operator for the transition from the initial state $|\mathbf{p}_0\rangle$ of the three-particle system to the final state $|\mathbf{k}_\alpha\mathbf{p}_\alpha\rangle$, can be represented as the sum of amplitudes corresponding to the interaction of pairs of particles in the final state,

$$\langle \mathbf{k}_\alpha \mathbf{p}_\alpha | T_3(z) | \mathbf{p}_0 \rangle = \sum_\gamma \langle \mathbf{k}_\alpha \mathbf{p}_\alpha | T_\gamma(z) | \mathbf{p}_0 \rangle, \quad \gamma \in \{12,23,13\},$$

and can be found using the scattering theory for three quantum particles (Refs. 10, 18, and 51).

We note that in both models the cross section for the multiparticle reaction depends on the scattering amplitude of three bodies pairwise interacting within the nuclear force range. Since in scattering theory the amplitude of three quantum particles, assuming that pair forces are important, is expressed in terms of two-particle amplitudes, the cross section for the reaction $A+B \to X_1+X_2+X_3$ must depend on the properties of the solutions of the problem of two bodies distorted by the effect of a third particle.

1. THE FINAL-STATE CLUSTER INTERACTION IN MULTIPARTICLE NUCLEAR REACTIONS OCCURRING VIA A DIRECT PROCESS

Most direct nuclear reactions with the production of three particles can be treated as scattering in the system of the three clusters making up the projectile nucleus and the target nucleus.

In the three-body problem the scattering amplitude for the system corresponding to the final-state interaction in the pair α can be written as[10,18]

$$T(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}_0, z) = t_\alpha(k_\alpha) F(\theta_\alpha p_\alpha, \mathbf{p}_0, z). \quad (1)$$

In (1), $t_\alpha(k_\alpha)$ is the on-shell scattering amplitude of the particles in the subsystem α (for the partial wave corresponding to a virtual state or resonance in this subsystem) and θ_α is the angle between the vectors $\mathbf{k}_\alpha$ and $\mathbf{p}_\alpha$. The nature of the function $F(\theta_\alpha p_\alpha, \mathbf{p}_0, z)$ corresponding to the reaction amplitude in the absence of the final-state interaction is related to the dynamics of the direct nuclear reaction and off-shell effects. One variant of the derivation of (1) will be given in Sec. 4.

Therefore, the function F can be viewed as a result of the effect of the nuclear field of the accompanying reaction products on the scattering of the clusters in the visible subsystem, which, after the isolation of the strongest dependence on the kinematical variables in this function, allows the dependence of the reaction cross section on the properties of the final-state interaction and the reaction dynamics to be seen. The amplitude of interest, $T(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}_0, z)$ (1), near the energy of the virtual or resonance state in the pair α can therefore be viewed as the scattering amplitude of the particles in the nonisolated system α. This makes it possible to write this amplitude as

$$T_\alpha(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}_0, z) = \tilde{t}_\alpha(\mathbf{k}_\alpha), \quad (2)$$

where the parameters of the subsystem α in the amplitude t_α (the scattering length for a virtual state or the resonance location and half-width for a resonance state) can have a noticeable dependence on the variables θ_α, $\mathbf{p}_\alpha$, and $\mathbf{p}_0$. This dependence will be strongest when the extrema in the behavior of the amplitude $t_\alpha(k_\alpha)$ coincide with the extrema of the function $F(\theta_\alpha p_\alpha, \mathbf{p}_0, z)$.

Therefore, the effect of the nuclear field on the scattering of the particles in the subsystem α leads to a change in the properties of the scattering amplitude of the particles in the nonisolated system and to a change in their observable scattering parameters from those of an isolated system. Since the factor F is a function of all the kinematical variables, we can expect that the nonisolated subsystem will have different properties in different kinematical regions of the final state of the set of products of the multiparticle process. This means that a resonance or virtual state of the isolated system can be observed in the nonisolated system as a state with parameters depending on the reaction kinematics, i.e., spatial anisotropy of the scattering properties of the particles in the nonisolated subsystem is observed.

2. DEPENDENCE OF THE PARTICLE SCATTERING PARAMETERS IN THE NONISOLATED SYSTEM ON THE MOMENTUM TRANSFER

This scheme for analyzing the features of the effect of the nuclear field on a subsystem of particles interacting in the final state can be demonstrated for the example of nucleon inelastic scattering on deuterons, $N+d \to 3N$, at low energies, which was studied in detail in Ref. 6. Assuming two-particle forces, the amplitude of this reaction, $T(z)$, is the sum of the amplitudes $T_\alpha(z)$ corresponding to the interaction of all pairs of nucleons in the final state, each of which can be represented in the form (1) with amplitude $t_\alpha(k_\alpha)$, the on-shell, two-nucleon, S-wave scattering amplitude.

There exist kinematical regions in which the effects related to the off-shell behavior of the scattering amplitude and to the reaction dynamics can be separated and for which it is possible to write down an analytic function F summing the full effect of the nuclear field of the accompanying product on the nonisolated two-nucleon subsystem. In the region of quasifree scattering of the nucleon on the deuteron, which is dominated by a single interaction of the primary nucleon with one of the nucleons of the deuteron, the cross section for the reaction $N+d \to 3N$ is mainly determined by the deuteron form factor multiplied by the on-shell nucleon–nucleon scattering cross section, i.e., by the scattering cross section of the two nucleons isolated from the third nucleon. This means that in this kinematical region the parameters of the maxima of the reaction cross section will be determined by the parameters of the virtual level of the two nucleons:

$$T(\mathbf{k}_{12}\mathbf{p}_3, \mathbf{p}_0, z) = \text{const}\, t_{12}(k_{12}) g_0(k_{12}, k'_{12}) \times \left[\kappa^2 + \frac{1}{m}\left(\mathbf{p}_3 + \frac{\mathbf{p}_0}{2}\right)^2\right]^{-1}, \quad k'^2_{12} = \left|\mathbf{p}_0 + \frac{\mathbf{p}_3}{2}\right|. \quad (3)$$

Here κ^2 is the deuteron binding energy, m is the nucleon mass, and the function $g(k,k')$ describes off-shell effects and is given by

$$g(k,k') = \frac{t(k,k',k^2/2\mu + i0)}{t(k)}.$$

As is well known, at low energies the nucleon–nucleon scattering amplitude can be treated in the linear approximation in the nuclear force range. In this case the nucleon–

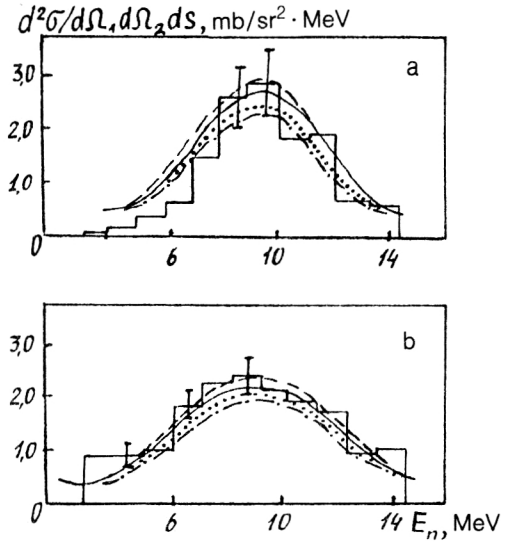

FIG. 1. Differential cross section of the reaction $N+d\to N+N+p$ for $E_N=21.5$ MeV. The neutron coincidence angles are 35° (a) and 30° (b). The histogram shows the experimental data of Ref. 11. The lines were calculated for the following values of the NN-scattering parameters: $a_{nn}=-16.4$ F, $r_{nn}=2.86$ F (solid line); $a_{nn}=-23.7$ F, $r_{nn}=2.86$ F (dashed line); $a_{nn}=-27.7$ F, $r_{nn}=3.7$ F (dotted line); $a_{nn}=-16.4$ F, $r_{nn}=3.4$ F (dot–dash line).

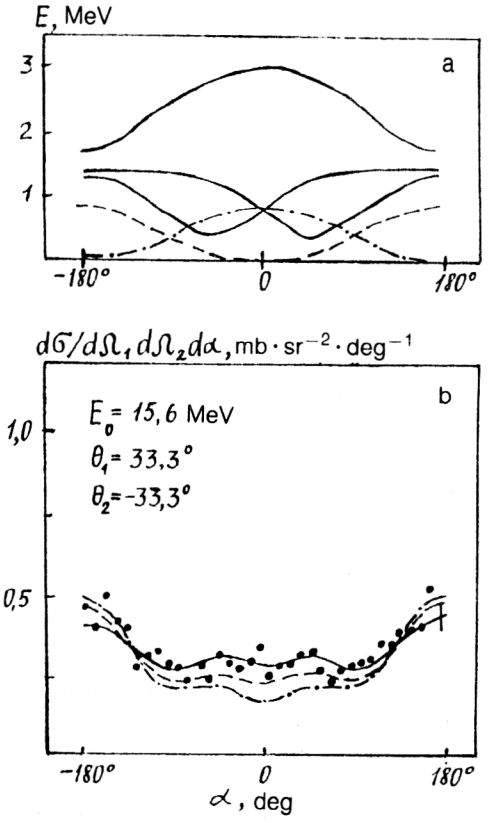

FIG. 2. Differential cross sections for the reaction $N+d\to N+N+p$ for $E_N=15.6$ MeV. The neutron coincidence angles are $\pm 33°$. The points are the experimental data of Ref. 12. The curves were calculated for the following values of the NN-scattering parameters: (a) $a_{nn}=-16.7$ F, $r_{nn}=2.7$ F (solid line); (b) $a_{nn}=-16.7$ F, $r_{nn}=3.5$ F (dashed line).

nucleon scattering cross section depends on two parameters: the scattering length a_{NN} and the effective range r_0. This means that by analyzing the cross section for the reaction $N+d\to 3N$ in the region of quasifree scattering it is possible to determine the two-nucleon scattering parameters. In several studies[11,12] it has been shown, on the basis of analysis of the complete data on the reactions $p+d\to 2p+n$ and $n+d\to 2n+p$ with numerical calculations of the integral equations for the three-body scattering amplitudes, that it is impossible to determine these two scattering parameters (a_{NN},r_0) simultaneously from the experimental data. Therefore, in such an analysis it is necessary to fix one of the parameters and determine the other. In Fig. 1 we show the sensitivity of the calculations in the quasifree neutron scattering region to the effective interaction radius[12] for fixed scattering length and compare the results with experiment.[13]

In Fig. 2 we present a similar analysis of the sensitivity of the calculations to variation of the two-nucleon scattering length for fixed effective range.[11] The analysis shows that the properties of the two-nucleon scattering in the nonisolated system can be studied meaningfully using the experimental data on three-particle reactions if, for example, the scattering length a_{NN} at fixed effective range is selected or if the experimental accuracy is improved considerably.

When studying the dependence of the scattering length on the reaction kinematics outside the quasifree scattering region it is necessary to take into account both the single interaction and rescattering processes.

We note that the amplitude for three-nucleon breakup can be written in parametrized form in the case of small and large emission angles of the nucleon not participating in the final-state interaction. In particular, in the case of backward emission of the third nucleon, the amplitude of the reaction $N+d\to 3N$ can be written in the form (3). The function

$$\left[\kappa^2+\frac{1}{m}\left(\mathbf{p}_3+\frac{1}{2}\mathbf{p}_0\right)^2\right]^{-1} \quad \text{for} \quad \theta_3=(-\mathbf{p}_0\hat{\mathbf{p}}_3)\cong 180°$$

varies weakly, so that in this region the effect of the off-shell dependence of the two-nucleon scattering amplitude in the nonisolated subsystem is most pronounced. Equation (3) for the reaction amplitude under these kinematical conditions can be written in the form (2):

$$t(\mathbf{k}_{12},\mathbf{p}_3,\mathbf{p}_0,z)=\text{const}\ \tilde{t}_{12}(k_{12},\tilde{a}_{NN}),$$

where $\tilde{t}_{12}$ is the scattering amplitude of the two nucleons in the nonisolated subsystem and $\tilde{a}_{NN}$ is the nucleon–nucleon scattering length.

Analysis of the experimental data[13] on the reaction $p+d\to 2p+n$ in this kinematical region made it possible to study the dependence of $\tilde{a}_{NN}$ in the nonisolated subsystem of the neutron and proton on the emission angle of the second proton into the backward hemisphere. It was shown that at low proton energies (10–20 MeV) $\tilde{a}_{NN}$ depends weakly on θ_3.

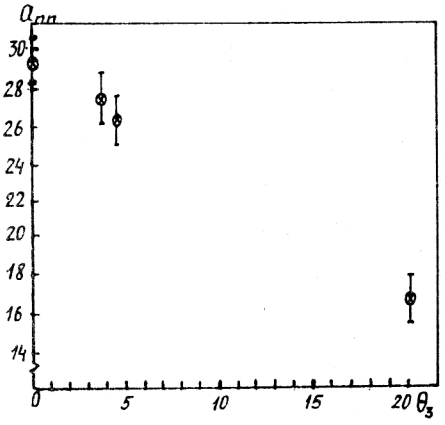

FIG. 3. Behavior of the neutron–neutron scattering length $\tilde{a}_{NN}$ for the case of a nonisolated NN subsystem produced in the reaction $n+d\to N+N+p$ at the primary neutron energy 14 MeV as a function of the proton emission angle. The points are the values obtained by analyzing the experimental data of Refs. 14–16.

In the kinematical region of the final state of the three-nucleon reaction where the final-state interaction of two nucleons ($k_{12}\cong 0$) is observed with the condition that the third nucleon travels in the forward direction, the parametrization of the amplitude for this reaction has the form

$$T(\mathbf{k}_{12}\mathbf{p}_3,\mathbf{p}_0,z)$$
$$=\text{const}\Bigg[\sum_{\alpha\neq d}\left(\kappa^2+\frac{1}{m}\left(\mathbf{p}_\alpha+\frac{1}{2}\mathbf{p}_0\right)^2\right)^{-1}t_\alpha(k_\alpha)$$
$$-2\pi^2 im^2 t(\tfrac{3}{4}p_0)\sum_\alpha t_\alpha(k_\alpha)q_\alpha^{-1}\ln\frac{i\Delta+k_\alpha+\frac{1}{2}q_\alpha}{i\Delta+k_\alpha-\frac{1}{2}q_\alpha}\Bigg],$$
$$\alpha\in\{12,23,31\},\quad \mathbf{q}_\alpha=\mathbf{p}_0-\mathbf{p}_\alpha,\quad \Delta^2=m\kappa^2. \tag{4}$$

For $p_0\gg\Delta$, Eq. (4) corresponds to the impulse approximation, and at lower energies the applicability of this approximation is related to the weak dependence on the kinematical variables of the contribution to the reaction amplitude from multiple rescattering processes.

In this kinematical region the strong dependence of the function F is imposed on the final-state interaction. Consequently, the parameters of the two-nucleon system extracted from the three-particle spectra in this kinematical region can differ strongly from the data of direct experiments. In particular, using the representation (2) with the scattering length $\tilde{a}_{NN}$ in the nonisolated subsystem, the experimental data can be used to obtain the dependence of this quantity on the momentum transfer $\bar{q}$. The result of this analysis of the experimental data[14–16] is shown in Fig. 3. We see that at small q the quantity $\tilde{a}_{NN}$ depends strongly on θ_3.

3. ENERGY DEPENDENCE OF THE DIFFERENTIAL CROSS SECTION FOR A THREE-PARTICLE REACTION NEAR A TWO-CLUSTER RESONANCE

Let us consider the interaction of three structureless particles, assuming that the pair 1–2 has a resonance characterized by orbital angular momentum l and complex energy $z_R=E_R-i\Gamma/2$. In multichannel scattering theory the reaction amplitude can be written as

$$T(\mathbf{k}_{12}\mathbf{p}_3,z)=T_0(\mathbf{k}_{12}\mathbf{p}_3,z)=\langle\mathbf{k}_{12}\mathbf{p}_3|\,[t_{12}(z)G_0(z)+1]\times(V_{13}+V_{23})\,|\tilde{\Psi}(z)\rangle. \tag{5}$$

In (5) the energy parameter z is

$$z=E+i0=\frac{k_{12}^2}{2\mu_{12}}+\frac{p_3^2}{2n_3}+i0,$$

where $m_{12}=m_1m_2/(m_1+m_2)$ is the reduced mass of the 1–2 subsystem, n_3 is the reduced mass of this subsystem and the third particle, and $\mathbf{k}_{12}$ and $\mathbf{p}_3$ are the Jacobi momentum coordinates (the c.m. motion of the system has been separated out). The amplitude $T_0(z)$ describes the quasifree scattering process. The function $|\tilde{\Psi}(z)\rangle$ is expressed in terms of the wave function of the full system, $|\Psi(z)\rangle$ (Refs. 17 and 18). The explicit expression for $|\tilde{\Psi}(z)\rangle$ will not be used below, and we do not give it here. The operator $G_0(z)$ is the free Green function of the system

$$G_0(z)=(z-H_0)^{-1}=\left(z-\frac{\hat{k}_{12}^2}{2\mu_{12}}-\frac{\hat{p}_3^2}{2n_3}\right)^{-1},$$

t_{12} is the scattering operator in the 1–2 subsystem, and V_α is the interaction operator in the pair α. Equation (5) was written down without the Coulomb interaction in the final state. The amplitude $T(\mathbf{k}_{12}\mathbf{p}_3,z)$ displays resonance behavior if the energy ε_{12} in the 1–2 pair is close to the resonance energy E_R. In order to isolate this resonance behavior explicitly, we use the well known expansion of the two-particle amplitude near an isolated resonance:[19–21]

$$t(\mathbf{k},\mathbf{k}',z)=(-1)^l(2l+1)P_l(\hat{\mathbf{k}}\hat{\mathbf{k}}')\frac{\chi(k)\chi(k')}{z-z_R}+\hat{t}(\mathbf{k},\mathbf{k}',z). \tag{6}$$

In (6), $\chi(k)$ is the resonance vertex function, $P_l(\kappa)$ is the corresponding Legendre polynomial depending on the cosine of the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$, and $\hat{t}(\mathbf{k},\mathbf{k}',z)$ is the nonresonance part of the amplitude. We note that the momentum $\mathbf{k}'_{12}$ is not fixed in this case. This is obviously related to the presence of the interaction with the third particle and can be interpreted as the result of the effect of the field of this particle on the resonance production process. Since our aim is to compare the two- and three-particle reactions, in (5) we explicitly isolate the on- and off-shell effects, for which we write the operator $G_0(z)$ as the Sochocki formula:

$$G_0(z)=-i\pi\delta(E-H_0)+\mathcal{P}\frac{1}{E-H_0}. \tag{7}$$

As a result, substituting the expansion (6) and (7) into (5), we have

$$T(\mathbf{k}_{12}\mathbf{p}_3,z)=T_1(\mathbf{k}_{12}\mathbf{p}_3,z)+T_2(\mathbf{k}_{12}\mathbf{p}_3,z). \tag{8}$$

Here $T_1(\mathbf{k}_{12}\mathbf{p}_3,z)$ is obtained from (5) by replacing the operator $t_{12}(z)$ by $\hat{t}_{12}(z)$. The amplitude $T_2(\mathbf{k}_{12}\mathbf{p}_3,z)$ displays resonance behavior, and near this resonance it is

$$T_2(\mathbf{k}_{12}\mathbf{p}_3,z)=(-1)^l 2\mu_{12}\frac{\chi(k_{12})}{k_{12}^2-2\mu_{12}z_R}\times\left[-\frac{i\pi}{2}k_{12}\chi(k_{12})B_1(\mathbf{k}_{12}\mathbf{p}_3)+B_2(\mathbf{k}_{12}\mathbf{p}_3)\right], \tag{9}$$

where

$$B_1(\mathbf{k}_{12}\mathbf{p}_3)=\int d\Omega_{k'}P_l(\widehat{\mathbf{k}_{12}\mathbf{k}'}\times\left\langle\frac{\mathbf{k}'}{\mathbf{k}}k_{12}\mathbf{p}_3\middle|V_{13}+V_{23}\middle|\tilde{\Psi}(z)\right\rangle,$$

$$B_2(\mathbf{k}_{12}\mathbf{p}_3)=2\mu_{12}\int dk'\,\mathscr{P}\frac{1}{k_{12}^2-k'^2}\chi(k')P_l(\widehat{\mathbf{k}_{12}\mathbf{k}'}\times\langle\mathbf{k}'\mathbf{p}_3|V_{13}+V_{23}|\tilde{\Psi}(z)\rangle. \tag{10}$$

Approximating the functions B_1 and B_2 by constants near the resonance energy (for fixed particle emission angles), we rewrite Eq. (9) as

$$T_2(\mathbf{k}_{12}\mathbf{p}_3,z)=-\frac{i\pi}{2}k_{12}t^l_{\rm res}(k_{12})B_1+(-1)^l 2\mu_{12}\frac{\chi(k_{12})}{k_{12}^2-2\mu_{12}z_R}B_2\,. \tag{11}$$

Here $t^l_{\rm res}(k_{12})$ is the resonance part of the amplitude $t_{12}(z)$, i.e., the first term in (6). The vertex function $\chi(k_{12})$ is usually approximated as

$$\chi(k)=\frac{i^l\Gamma^{1/2}}{(2\pi)(2\mu k)^{1/2}}, \tag{12}$$

which can be obtained either by equating the expression for $t^l_{\rm res}(k)$ in (6) to its traditional parametrization[17,18]

$$t^l_{\rm res}=\frac{1}{(2\pi)^2\cdot k}\frac{\Gamma}{k^2-2\mu z_R},$$

or from the theory of complex scale transformations of the Hamiltonian (Refs. 20 and 22–24). Substitution of (12) into (11) leads to the following parametrization for the amplitude T_2 (Refs. 8, 20, and 21):

$$T_2(\mathbf{k}_{12}\mathbf{p}_3,z)=k_{12}t^l_{\rm res}(k_{12})C_1+k_{12}^{1/2}t^l_{\rm res}(k_{12})C_2=C_1k_{12}t^l_{\rm res}(k_{12})\left(1+k_{12}^{-1/2}\frac{C_2}{C_1}\right), \tag{13}$$

where the constants C_1 and C_2 are expressed in terms of B_1 and B_2, respectively. The quantity $\lambda=C_2/C_1$ in (13) determines the ratio of the contributions to the amplitude from the terms corresponding to on- and off-shell scattering. The first terms in (11) and (13), proportional to k_{12}, correspond to the scattering of two real particles. Therefore, analysis of the multiparticle dynamics of the process leads to Eq. (13) instead of the expression of the Migdal–Watson approximation,[1] i.e., to the appearance of an additional dependence on k_{12}.

The expressions (13) are easily generalized to the case where particles 1 and 2 are composite, i.e., the resonance in the 1–2 pair can decay via different channels. Using α to denote the observed decay channel, for T_2 we obtain

$$T_2(\mathbf{k}_{12}\mathbf{p}_3,z)=\frac{\Gamma_\alpha^{1/2}}{E_{12}-R_R+i\Gamma/2}\left[\sum_\beta\left(\frac{k_\beta\Gamma_\beta}{k_\alpha}\right)^{1/2}\times C_\beta+\frac{1}{k_\alpha^{1/2}}C_0\right], \tag{14}$$

where the momentum k_β describes the entrance channel β for the resonance interaction in the 1–2 subsystem (this momentum is on-shell), and Γ_β is the partial width of the resonance decay via this channel.

As an example of the use of the parametrizations derived above, we studied the experimental data on the excitation of the states ^{8}Be(2^+,E^*=2.9 MeV) and ^{8}Be(4^+,E^*=11.4 MeV) in the final state of three-particle reactions. The choice of the ^{8}Be nucleus was motivated by the fact that the observed values of the half-width of the excited states for it can differ by 20% in going from binary to three-particle reactions. In the analysis we used the data of the kinematically complete experiments performed at the Institute of Nuclear Research, Ukrainian Academy of Sciences (Refs. 25 and 26) and the JINR Nuclear Physics Laboratory (Ref. 3), together with the results of other studies.[27–29] In all these studies the experimental data were obtained by projecting the information from the locus corresponding to a definite three-particle final state onto the energy axis of one of the detectors.

Since all the reactions studied are characterized by kinetic energy of the final products which is not very high, off-shell effects were neglected in the parametrization (13), i.e., we took $\lambda=0$. Therefore, compared to the Migdal–Watson model of Ref. 1, our parametrization of the cross section leads to an additional energy factor E_{12} which is independent of the type of third particle (in the reactions studied it was d, t, or α). The approximation thus obtained is determined by the descent of the operator $G_0(z)$ to the mass shell and, accordingly, it satisfies the three-particle unitarity condition.

The calculations carried out on the basis of the parametrization (13) revealed systematically better agreement with experiment than in the Migdal–Watson model.[1]

An example of the analysis of the experimental data using our approximation is given in Fig. 4 and Table I, where we present the data of Ref. 7 for various three-particle reactions with the final-state formation of short-lived ^{8}Be nuclei in the 2^+ and 4^+ states at excitation energies of $\sim$3 and 11 MeV, respectively [the points are the experimental results, and the solid lines are the calculation using Eq. (13)]. In Table I we also give the results of the calculation using the Migdal–Watson model (the parameters with subscript 1) and with the modified energy dependence obtained in the present study (subscript 2). It can be seen that in the overwhelming majority of three-particle

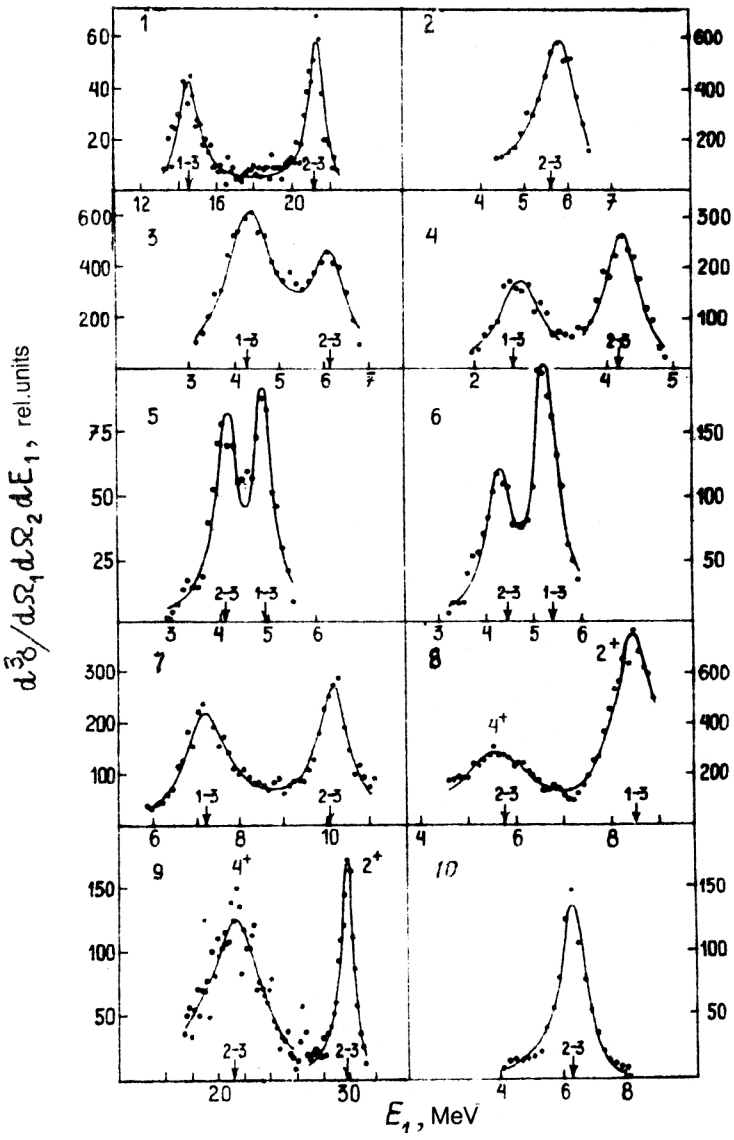

FIG. 4. Fragments of the correlation energy spectra of three-particle reaction products in the region where the first (2^+) and second (4^+) excited states of the ^{8}Be nucleus contribute. The kinematical conditions and types of reaction are given in Table I.

reactions studied, the total rms deviation of the calculated values of the differential cross sections in the region where excited states of the ^{8}Be nucleus contribute satisfies the inequality $\chi_2^2 < \chi_1^2$.

4. EFFECT OF INTERFERENCE PHENOMENA AND RESCATTERING PROCESSES ON THE SHAPE OF THE ENERGY SPECTRA OF TWO-CLUSTER RESONANCE DECAY PRODUCTS IN THREE-PARTICLE REACTIONS

As noted above, in reactions with the production of three particles, even in the kinematical region of final states where the resonance interaction of one pair of particles is important, the amplitude of the process is the sum of the amplitudes of three nonorthogonal channels, each of which corresponds to the scattering of one of the three possible pairs of nuclear clusters. If in some kinematical region of final states a resonance interaction in two or even three nonorthogonal channels is possible, the addition of the amplitudes of these channels can either enhance (by almost an order of magnitude) the resonance in question or weaken it almost to the level of the background.

A considerable amount of attention has been paid to the study of interference phenomena in the cross sections for three-particle reactions in the region where the two-cluster resonance contributes. Destructive interference is clearly seen in the data on the reaction $p+{}^{11}\mathrm{B}\to 3\alpha$ shown in Fig. 5. Here, along with the experimental data (the points) from Ref. 31, we give the calculated spectra including the interference of the amplitudes of nonorthogonal channels (dashed line) and neglecting this interference (dot–dash line). The arrows indicate the expected location of the maximum contribution of the first excited state of the ^{8}Be nucleus in the 2–3 and 3–1 pairs. We see from Fig. 5 that at $E_p=2.65$ MeV the experimental spectra have a deep minimum at the location of the expected maxima. An interesting detail of the spectrum for $\theta_{\alpha_2} = 40°$ is the strong asymmetry of the right-hand peak ($E_{\alpha_2} \sim 5$ MeV), which is characteristic for destructive interference of the amplitudes of nonorthogonal channels.

TABLE I. Results of determining the parameters E_1^R, Γ_1, and χ_1^2 (Migdal–Watson formula) and E_2^R, Γ_2, and χ_2^2 [Eq. (13)] using the data of kinematically complete experiments on the first (2^+) and second (4^+) excited states of the ^{8}Be nucleus.

Number of spectrum in Fig. 4	Reaction	Incident particle energy, MeV	V_{rel}, 10^9 cm/s	l–j	E_1^R, MeV	E_2^R, MeV	Γ_1, MeV	Γ_2, MeV	χ_1^2	χ_2^2
				2^+, first state						
1	^{9}Be(^{3}He,α)2α	9,94	4,11	1—3	2,92±0,08	2,79±0,08	1,26±0,12	1,03±0,10	1,78	1,24
				2—3	3,02±0,08	2,83±0,08	1,42±0,13	1,24±0,12	–"–	–"–
2	^{11}B(p,α)2α	1,98	2,32	2—3	2,89±0,07	2,67±0,07	1,66±0,09	1,49±0,08	7,08	2,49
3	^{11}B(p,α)2α	2,62	2,41	1—3	3,15±0,07	3,02±0,07	1,30±0,8	1,30±0,08	7,95	4,66
				2—3	3,18±0,07	3,05±0,07	1,32±0,10	1,20±0,10	–"–	–"–
4	^{11}B(p,α)2α	0,680	2,13	1—3	3,12±0,05	3,05±0,05	0,92±0,07	0,93±0,07	4,06	3,11
				2—3	3,01±0,05	2,92±0,05	0,90±0,06	0,95±0,06	9,58	5,41
5	^{11}B(p,α)2α	0,680	2,13	1—3	2,92±0,07	2,85±0,07	0,90±0,12	0,88±0,10	1,40	1,89
				2—3	2,94±0,07	2,86±0,07	0,98±0,12	0,95±0,10	–"–	–"–
6	^{11}B(p,α)2α	0,680	2,13	1—3	2,72±0,06	2,63±0,06	1,00±0,08	0,94±0,07	2,06	2,96
				2—3	3,29±0,06	3,21±0,06	0,93±0,10	0,96±0,08	–"–	–"–
7	^{10}B(d,α)2α	2,50	3,50	1—3	3,10±0,06	2,91±0,06	1,46±0,08	1,37±0,08	3,98	2,10
				2—3	2,93±0,06	2,75±0,06	1,54±0,09	1,33±0,09	–"–	–"–
8	^{10}B(d,α)2α	3,00	3,55	1—3	2,95±0,05	2,89±0,05	1,05±0,06	1,18±0,07	6,72	4,14
9	^{7}Li(α,t)2α	50	4,82	2—3	3,21±0,10	3,02±0,10	1,28±0,11	1,26±0,05	4,35	1,32
10	^{9}Be(p,d)2α	9,0	2,62	2—3	2,99±0,10	2,87±0,10	1,02±0,11	1,03±0,11	1,06	1,94
				4^+, second state						
8	^{10}B(d,α)2α	3,00	2,54	2—3	11,81±0,06	11,78±0,06	2,38±0,11	2,68±0,12	6,72	4,14
9	^{7}Li(α,t)2α	50	3,97	2—3	11,62±0,11	11,18±0,11	4,30±0,18	4,59±0,20	3,17	2,67

An example of constructive interference is illustrated in Fig. 6, where we show correlation data for the reaction ^{7}Li(d,$\alpha\alpha$)n, taken from Ref. 32. Superposition of the contributions from two-cluster states of ^{5}He (dot–dash line) and ^{8}Be (dashed line) on the same region of the spectrum at 70° leads to a considerable increase of the cross section at the maximum for $E_{\alpha_2} \sim 10$ MeV.

Interference phenomena can significantly change both the shape of the differential cross sections[33,35] and their absolute values, and this must be incorporated in the analysis of the experimental data. In Fig. 7 we show fragments of the α-particle spectra for the reaction $^7\text{Li}+d\rightarrow\alpha+\alpha+n$ at the deuteron energies 1.9, 2.1, and 2.2 MeV (Ref. 37). It has been found that the rescattering of particles from the decay of the nucleus $^5\text{He}_{\text{g.s.}}$ on the accompanying α particle plays an important role in this reaction. At E_d=1.9 MeV excitation of the 16.6-MeV level of the ^{8}Be* nucleus is impossible, and therefore the data at this energy (light points) were used to test for the appearance of the rescattering process in the allowed kinematical region (shown by the horizontal arrows in Fig. 7 and corresponding to the α-particle detection angles $\theta_{\alpha_1} = 45°$ and $\theta_{\alpha_2} = 120°$). We see that the inclusion of the interference of the cluster channel amplitudes causes the yields at E_d=2.1 and 2.2 MeV to exceed the data at E_d=1.9 MeV (light points) precisely where the rescattering contribution is allowed.

An analogous rescattering phenomenon has been discovered in the ^{10}B(d,3α) reaction in a study of the 19.9-MeV (2^+) resonance of the ^{8}Be nucleus (Ref. 36). The channel-rescattering phenomenon has been used in a num-

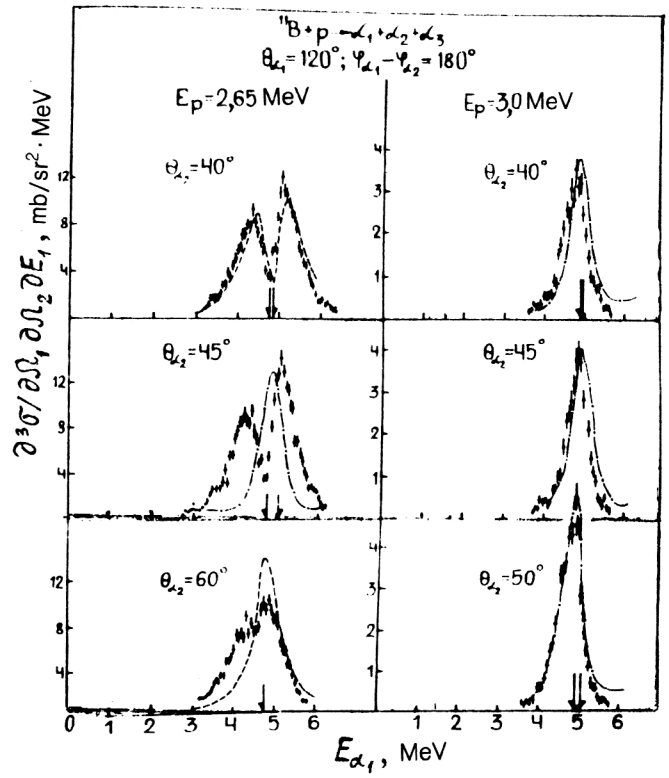

FIG. 5. The reaction $^{11}\text{Be}+p\rightarrow\alpha_1+\alpha_2+\alpha_3$ at the proton energies 2.65 and 3.0 MeV. Spectra of α particles emitted at 120°. The emission angle of the second particle is given in each figure.

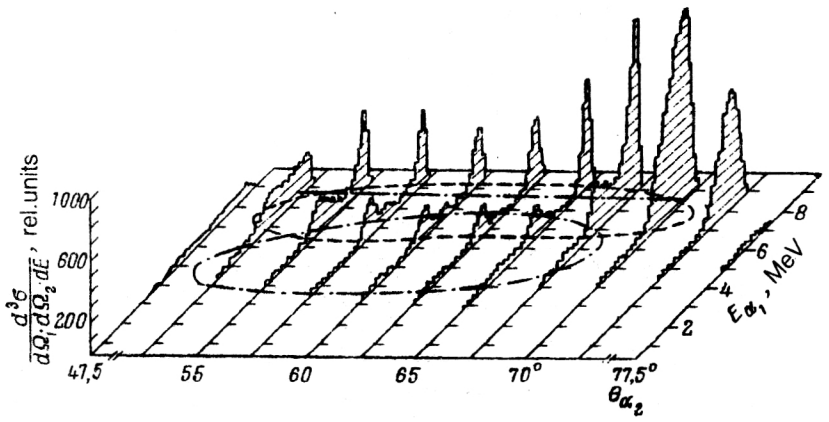

FIG. 6. The reaction $^7Li+d\rightarrow\alpha_1+\alpha_2+n$ at the deuteron energy E_d=4.0 MeV. Spectra of α particles emitted at 90° coincident with another α particle emitted at various angles.

ber of studies (Refs. 38–40) to determine the lifetime of unbound two-cluster states. This analysis offers the unique possibility of studying nuclear lifetimes in the range 10^{-20}–10^{-21} s.

5. DEPENDENCE OF THE ANGULAR CORRELATION FUNCTIONS OF THREE-PARTICLE REACTION PRODUCTS ON THE MECHANISM FOR THE EXCITATION AND DECAY OF TWO-CLUSTER SUBSYSTEMS

The angular distribution of the decay products of a two-cluster resonance and its correlation with an accompanying third particle are among the sensitive features of this resonance from which useful information can be obtained. A formalism applicable for analyzing the angular distributions should distinguish the dynamical effects of the three-particle process from the effects arising from the angular-momentum conservation laws.

The angular correlations are used to determine the spins and parities, the degree to which the decaying nuclei are polarized, the lifetimes of two-cluster nuclear systems, the degree of deformation of the state studied, and so on. In Ref. 41 it was suggested that the correlation data be used to determine the parameters of the nuclear interaction between one of the three-particle reaction products and the resonance.

The series of studies of Refs. 42–45 was devoted to investigation of the regularities of the decay of the state $^7Li^*$(4.63 MeV) as a function of the conditions under which it is excited in the $^9Be(d,t)\alpha\alpha$ reaction at the deuteron energies 3.0 and 13.6 MeV and in the $^7Li(\alpha,\alpha')\alpha t$ reaction at the α-particle energy 27.2 MeV. In Fig. 8 we show the angular correlation functions of tritons from the decay of this state and of the accompanying α particles for identical kinematical decay conditions in the exit channels of the two reactions. The fact that the exit channels are identical allows us to assume that there are large differences both in the absolute value and in the triton emission-angle dependence of the angular correlation functions owing to differences in the mechanisms for the processes, and also owing to the different spin structures of the exit channels. The dashed line shows the results of the calculation assuming a sequential reaction mechanism with formation of the compound nucleus $^{11}B^*$ in the first stage. The solid line is the optical-model calculation for the reaction with excitation of $^7Li^*$(4.63 MeV) in the first stage and the subsequent statistical decay of this state in the second stage. The dot–dash line shows the calculation using the distorted-wave method, which gives the results closest to experiment (points).

A great deal of attention has been paid to the study of the decay properties of the $^5He_{g.s.}$ nucleus excited in vari-

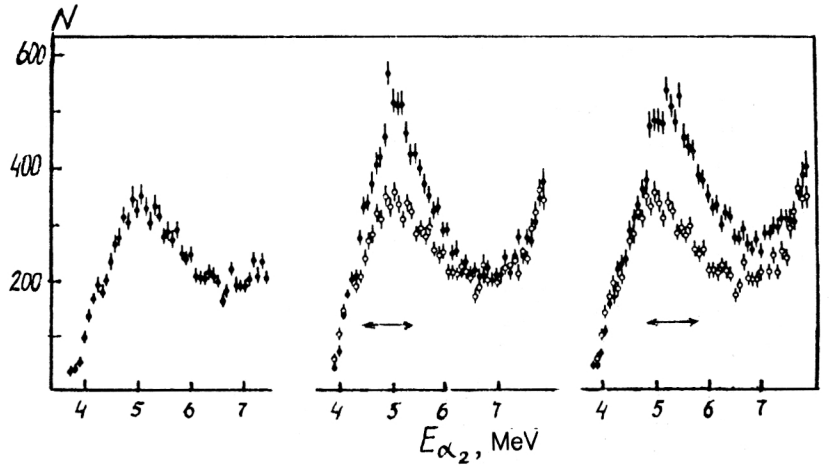

FIG. 7. The reaction $^7Li+d\rightarrow\alpha_1+\alpha_2+n$. The regions of a possible contribution from a second scattering of the α particles from the decay of ^8Be(16.6 MeV) on neutrons are shown by the horizontal arrows for E_d=2.1 and 2.2 MeV.

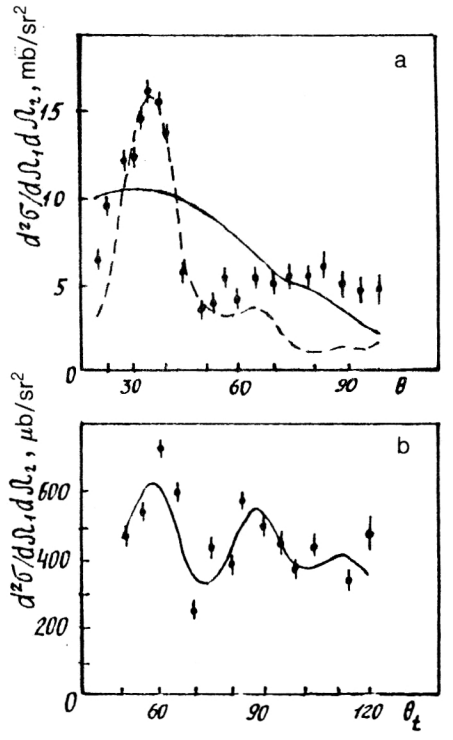

FIG. 8. Angular correlation functions for the reactions $\alpha+{}^7\text{Li}^*(4.63\ \text{MeV})\to\alpha+t+\alpha$ (a); $d+{}^9\text{Be}\to\alpha+{}^7\text{Li}^*(4.63\ \text{MeV})\to\alpha+t+\alpha$ (b).

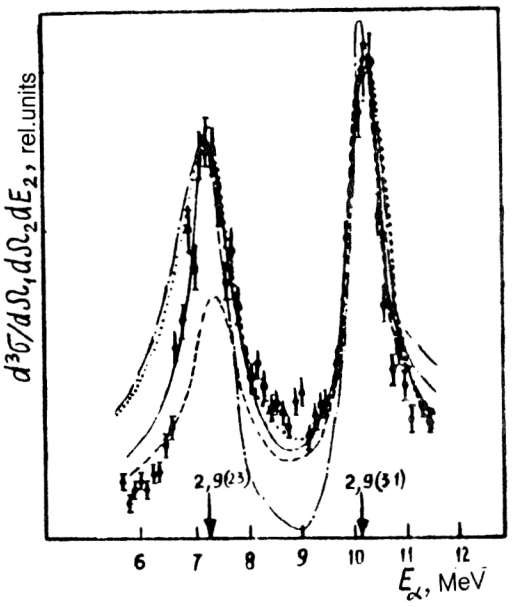

FIG. 9. Fragment of the spectrum of α particles emitted at 117.5° coincident with α particles emitted at 57°5′. The curves show the result calculated by summing nonrelativistic graphs.

ous three-particle reactions.[46–49] The average characteristics of the ${}^5\text{He}_{\text{g.s.}}$ nucleus coincide with the data obtained from free scattering of neutrons on α particles. However, the angular distributions of the products of ${}^5\text{He}_{\text{g.s.}}$ decay, for example, in the reactions ${}^6\text{Li}(\alpha,\alpha n){}^5\text{Li}_{\text{g.s.}}$ and ${}^7\text{Li}(\alpha,\alpha n){}^6\text{Li}$ (Ref. 49), display marked differences. The asymmetry of the α-particle angular distribution relative to the direction of emission of the ${}^5\text{He}_{\text{g.s.}}$ center of mass is attributed to the ${}^5\text{He}_{\text{g.s.}}$ polarization, and the above-noted differences are due to the effect on the angular correlation of the field of the ${}^5\text{Li}$ nucleus formed in ${}^6\text{Li}(\alpha,\alpha n){}^5\text{Li}_{\text{g.s.}}$ reactions.

6. SHAPE CORRELATIONS OF THE ENERGY SPECTRA OF THREE-PARTICLE REACTION PRODUCTS WITH THE SPIN AND PARITY OF THE INTERMEDIATE COMPOUND NUCLEUS

In this section we shall study multiparticle nuclear reactions occurring via a compound nucleus with the condition that there are overlapping resonances in the particle subsystems in the final state.

As an example, let us consider a nuclear reaction with the production of three particles in the final state with a resonance pairwise interaction. Let the reaction occur with the formation of a compound nucleus in the state with energy E^* and quantum characteristics J^π. Then the amplitude has the form

$$T(E^*,J^\pi)=\text{const}\sum_\alpha\sum_{l_\alpha}\sum_{L_\alpha} g_\alpha(l_\alpha,L_\alpha,k_\alpha p_\alpha). \qquad (15)$$

Here $g_\alpha(l_\alpha,L_\alpha,k_\alpha p_\alpha)$ is the amplitude of the probability for the compound nucleus to decay via the channel corresponding to the final-state interaction of the particles α, obtained from the amplitude $T_\alpha(\mathbf{k}_\alpha\mathbf{p}_\alpha)$ by the partial-wave expansion, and l_α and L_α are the orbital angular momenta of the sybsystem α and the third particle.

In the case where overlap of the resonances of all pairs of particles in the final state is observed in the spectrum of one of the particles, the amplitude of the process is given by[4]

$$T(J^\pi,E^*,l_{12}^*,l_{13}^*,l_{23}^*)=\text{const}\sum_\alpha\sum_{L_\alpha}[g_\alpha(l_\alpha^*,L_\alpha)+\sum_{\gamma\neq\alpha}\sum_{l_\gamma}\sum_{L_\gamma}B_{l_\gamma L_\gamma}^{l_\alpha^* L_\alpha}\cdot g_\gamma(l_\gamma L_\gamma)], \qquad (16)$$

where $B_{l_\gamma L_\gamma}^{l_\alpha L_\alpha}$ are the coefficients of the transition from the basis $|k_\alpha p_\alpha,l_\alpha L_\alpha\rangle$, determined by the final-state wave function, to the bases $|k_\gamma p_\gamma,l_\gamma L_\gamma\rangle$, determined by all possible states of the other pairs of particles. If the characteristics of the state in which the compound nucleus R^* decays into three particles are unknown, then it is possible to use the width and position of the peaks of the resonance two-particle interaction in the measured spectrum of the emitted particles to determine the spin and parity J^π of the given state of the nucleus R^* by choosing the values of the transition coefficients such that these coefficients depend on the values of J^π (Ref. 4).

In Fig. 9 we show a fragment of the spectrum of α particles emitted at the angle 117° from the reaction ${}^{11}\text{Be}+p\to 3\alpha$ at the energy 2.5 MeV coincident with α particles emitted at 57° (Ref. 31). The different curves (the

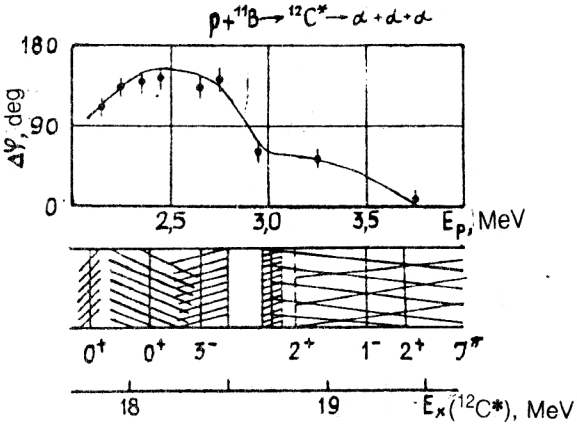

FIG. 10. Dependence of the optimal phase shifts of the pair interaction amplitudes $(\alpha_1-\alpha_3)$ and $(\alpha_2-\alpha_3)$ on the incident-proton energy for the reaction $^{11}B+p\to\alpha_1+\alpha_2+\alpha_3$. The corresponding fragment of the level scheme of the $^{12}C^*$ nucleus is given.

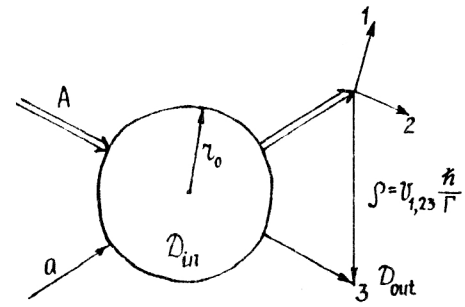

FIG. 11. Schematic representation of the regions of formation and decay of a two-cluster nuclear resonance.

notation is explained in Fig. 9) correspond to the calculations using the method described above for different spins and parities of the compound nucleus ^{12}C. We see that the calculation for $J^\pi=2^+$ gives results closest to experiment (points). A detailed study of the dependence of the shape of the correlation spectra on the total energy in a system of three α particles has been carried out in Ref. 50. The spectra have different shapes for different incident-proton energies, similar to what is seen in Fig. 5 for $E_p=2.65$ and 3.0 MeV. A correlation in the phase shifts of the scattering of three α particles with the spin and parity of the levels of the compound nucleus $^{12}C^*$ has been established. In Fig. 10 we show the dependence of the optimal (from the viewpoint of agreement between the calculated cross sections and the experimental data) values of the phase differences in the amplitudes of the interaction of two pairs of α particles in the reaction $^{11}B(p,3\alpha)$ on the incident-proton energy. We see that the proton energy range of interest can be split into two segments: $2.15<E_p<2.8$ MeV (destructive interference) and $2.8<E_p<4.0$ MeV (constructive interference). Two levels can contribute in the first region: the 3^- level ($E^*=18.36$ MeV) and the 0^+ level ($E^*=18.1$ MeV). In the second region the dominant contribution comes from the 0^+ (18.71 MeV), 1^- (19.2 MeV), and 2^+ (19.39 MeV) levels. Since the energies of the interacting pair of α particles and the third α particle differ little in the two regions of Fig. 10, it can be assumed that the nature of the interference phenomena determines the stage of formation of the compound nucleus $^{12}C^*$ in different excited states. Analysis shows that this interference is destructive when these states have negative parity and constructive when they have positive parity. The phase shifts shown in Fig. 10 as a function of the proton energy indicate that the α-particle widths of the 0^+ (18.1 MeV) and 3^+ (18.51 MeV) levels are small. This is the reason why the contribution of the 18.36-MeV level with negative parity ($J^\pi=3^-$) dominates in the first segment of the proton energy range under study and destructive interfrence is observed. Constructive interference in the high-energy part of the proton energy range implies, first, that the 18.71-MeV level of $^{12}C^*$ has positive parity, while the α-particle width of the 2^+ level at 19.39 MeV is considerably larger than that of the 1^- level at 19.2 MeV. Similarly, the nonrelativistic-graph summation method has been used to determine the spin and parity of the ^{12}C nucleus at $E^*=26.9$ MeV (Ref. 4), the existence of which was proved in a series of experimental studies. Therefore, the proposed method has proved to be an effective means of determining the spectroscopic characteristics of highly excited states of light nuclei.

7. EFFECT OF THE COULOMB FIELD OF THE THIRD PARTICLE ON THE DECAY OF A TWO-CLUSTER NUCLEAR RESONANCE

As already noted in the Introduction, if the clusters formed in the final state of the reaction are charged, the resonance decay never becomes free, owing to the long-range nature of the Coulomb potential. Assuming that the resonance formation process is determined by the nuclear forces, let us divide all space into two regions: the inner region D_{in} of the nuclear interaction of the system and the outer region D_{out} where the Coulomb force becomes important. Therefore, the role of the Coulomb forces reduces to the asymptotic deviation of the particle trajectories from straight lines. Accordingly, whereas the nuclear field of the accompanying reaction products mainly affects the resonance production process, the Coulomb field affects the resonance decay process.

The decay of a two-cluster nuclear resonance formed in a three-particle final state of a reaction is shown schematically in Fig. 11. The inner region of the reaction $a+A\to1+b^*\to1+2+3$ has characteristic dimension r_0 corresponding to the nuclear force range in the system. The average distance between the resonance at the time of its decay and the accompanying particle is denoted by

$$\rho_0=v_{1,2,3}\tau=v_{1,2,3}\frac{\hbar}{\Gamma},$$

where $v_{1,2,3}=p_1/n_1$ is the relative rate of separation of the resonance and the accompanying particle.

Since nuclear forces are responsible for resonance formation in these processes, for the reaction amplitude we can use the familiar expression taking into account the

Coulomb interaction of the reaction products on the background of their nuclear interaction (Refs. 51–53 and 62):

$$T(\mathbf{k}_\alpha\mathbf{p}_\alpha,\mathbf{p}^0,z)=\langle\Psi_\beta|\langle\mathbf{p}^0-|(z-H_\beta)G(z)\times(V_S^\alpha+V_L^\alpha-U_L^\alpha)|\Psi_C^+(\mathbf{k}_\alpha\mathbf{p}_\alpha)\rangle, \quad (17)$$

where henceforth, according to Fig. 11, $\mathbf{k}_\alpha=\mathbf{k}_{23}$ and $\mathbf{p}_\alpha=\mathbf{p}_1$. Here V_S^α and V_L^α, respectively, denote the short- and long-range parts of the potential V^α coupling particles from different clusters in the channel α, U_L^α is the long-range part of the interaction potential of the clusters, Ψ_C^+ is the Coulomb wave function of the final state, and H_β is the channel Hamiltonian of the channel β. To separate the resonance behavior of the transition operator

$$T^{\beta\alpha}(z)=(z-H_\beta)G(z)(V_S^\alpha+V_L^\alpha-U_L^\alpha)$$

we proceed as follows. For $G_\alpha(z)$ we use the expansion of formal resonance theory:[20,63]

$$G_\alpha(z)=R(z)+\sum_{ij}[I+R(z)W]|\Psi_i\rangle\times\frac{A_{ij}(z)}{\omega(z)}\langle\Psi_j|[WR(z)+I]. \quad (18)$$

Here the states $|\Psi_i\rangle$ form an orthogonal projector $P=\Sigma_i|\Psi_i\rangle\langle\Psi_i|$ and $R(z)$ is the resolvent of the Hamiltonian $QH_\alpha Q$:

$$R(z)=(zQ-QH_\alpha Q)^{-1}\cdot Q,$$

where $Q=I-P$. The function $\omega(z)$ is the determinant of the formal resonance theory, the zeros of which specify the locations of the bound states in the system and (in the case of analytic continuation to the unphysical sheet) the resonances. The elements of this determinant have the form

$$\omega_{ij}(z)=z\cdot\delta_{ij}-\langle\Psi_i|W+WR(z)W|\Psi_j\rangle,$$

where $W=H_\alpha-QH_\alpha Q$ and $A_{ij}(z)$ are the corresponding minors. In the case in question the determinant $\omega(z)$ is assumed to have resonance behavior:

$$\omega(z)=\omega_0(z)(z-z_R),$$

where z_R is the energy of the resonance state. The explicit form of the function ω_0 is given in, for example, Refs. 63b and 71. For $G(z)$ we then obtain

$$G(z)=[I+G(z)V^\alpha[R(z)+|\Psi_1(z)\rangle P_R(z)\langle\Psi_2(z)|]. \quad (19)$$

Here $P_R(z)$ is a propagator of the form

$$P_R(z)=\left(z-z_R-\frac{\hat{p}_1^2}{2n_1}\right)^{-1},$$

and the states $|\Psi_i(z)\rangle$, $i=1,2$, are given by the relations

$$|\Psi_1(z)\rangle=\omega_0^{1/2}(z)[I+R(z)W]|\Phi\rangle$$

$$\langle\Psi_2(z)|=\omega_0^{1/2}(z)\langle\Phi|[WR(z)+I],$$

where for simplicity we have restricted ourselves to the case where $i=j=1$ in (18). Applying the Veselova procedure[51,61] to (13), for the complete Green function of the system we obtain an expression of the form

$$G(z)=\tilde{G}(z)+G_R(z),$$

where the resonance part of the Green function of interest below is

$$G_R(z)=[I-R(z)V^\alpha]^{-1}\times[I+P_R(z)X(z)]|\Psi_1(z)\rangle P_R(z)\langle\Psi_2(z)|. \quad (20)$$

The operator $X(z)$ is the solution of the integral equation

$$X(z)=U(z)+X(z)P_R(z)U(z),$$

whose kernel $U(z)$ is interpreted as the optical potential of the interaction in the system of resonance plus accompanying particle:

$$U(z)=\langle\Psi_2(z)|V^\alpha[I-R(z)V^\alpha]^{-1}|\Psi_1(z)\rangle.$$

Therefore, the properties of the transition operator $T^{\beta\alpha}(z)$ and the reaction amplitude (17) are determined by the properties of the potential $U(z)$ and its Green function $g_R(z)=P_R(z)+P_R(z)X(z)P_R(z)$. We shall use the coordinate representation. Then the parameter ρ_0 introduced in Fig. 11 will correspond to the characteristic size of the range of integration over the relative coordinate ρ in the system of resonance plus accompanying particle, owing to the exponential falloff in ρ of the matrix elements $\langle\rho'|P_R(z)|\rho\rangle$ for $\rho>\rho'$:

$$\langle\boldsymbol{\rho}'|P_R(z)\boldsymbol{\rho}\rangle=-\frac{n_1}{2\pi}\frac{\exp(ik_R|\boldsymbol{\rho}-\boldsymbol{\rho}'|)}{|\boldsymbol{\rho}-\boldsymbol{\rho}'|},$$

where $k_R^2=2n_1(z-z_R)$, $\operatorname{Im}k_R(z)>0$. It is easily shown that

$$\operatorname{Im}k_R\cong\frac{\Gamma}{4(E_{\text{tot}}^C-E_R)}\sqrt{2n_1(E_{\text{tot}}^C-E_1)}\cong\frac{\Gamma}{2v_1}=\frac{1}{2\rho_0}$$

with the condition that the total energy of the products of the final state of the reaction, E_{tot}^C, satisfy

$$E_{\text{tot}}^C-E_R\gg\frac{\Gamma}{2}.$$

The properties of the interaction operator $\langle\rho|U(z)|\rho\rangle$ have been studied in Refs. 68–70 for the example of the system of bound cluster plus third particle. It was shown that this interaction operator is nonlocal at distances of order r_0, and for ρ' and ρ considerably larger than the nuclear force range the potential $U(z)$ becomes local and equal to the sum of the Coulomb potential for the system of resonance plus accompanying particle and the polarization potential. Therefore, the local part of the operator $U(z)$ can be written as

$$\langle\boldsymbol{\rho}'|U(z)|\boldsymbol{\rho}\rangle_{\text{local}}=\delta(\boldsymbol{\rho}'-\boldsymbol{\rho})\left[V^S(\rho)+\frac{Q_\alpha q_\alpha}{\rho}f(\rho)\right],$$

where $V^S(\rho)$ is the local nuclear part of the potential $U(z)$, Q_α is the resonance charge, and q_α is the charge of the accompanying particle. The function $f(\rho)$ tends to zero for $\rho\to0$, but at distances much larger than r_0 it approaches the asymptotic value $f(\rho)\xrightarrow[\rho\gg\rho_0]{}1$.

Our analysis shows that when the matrix element (17) for the scattering amplitude is analyzed in the coordinate representation it is necessary to distinguish the following limiting cases.

1. The resonance in the subsystem α is short-lived ($\Gamma \sim 1$ MeV), so that the parameter ρ_0 is of the order of the size of the region where the nuclear interaction dominates in the system.

2. The resonance in the subsystem α is long-lived, so that $\rho_0 \gg r_0$.

Therefore, in the case of the short-lived resonance the inclusion of the long-range part of the operator $U(z)$ has practically no effect on the resonance component of the amplitude for the process, while in the case of a long-lived resonance the properties of the matrix element $\langle \rho' | g_R(z) | \rho \rangle$ in most of the range of integration over ρ are determined by the purely Coulomb terms. This implies that in the second case, which is simpler to analyze, the matrix elements $\langle \rho' | g_R(z) | \rho \rangle$ in most of the range of integration over ρ can be replaced by the matrix elements of the purely Coulomb resolvent:

$$g_R^C(z) = \left(z - \frac{\hat{p}_\alpha^2}{2n_\alpha} - V^C(\rho) \right)^{-1} = \left(z - \frac{\hat{p}_\alpha^2}{2n_\alpha} - \frac{Q_\alpha q_\alpha}{\rho} \right)^{-1}.$$

The case of a short-lived resonance will be considered in the following section. Here we restrict ourselves to the case of a long-lived one.

In this case for the resonance part of the scattering amplitude (17) we have

$$T^R(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}^0, z) = T_1^R(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}^0, z) + T_2^R(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}^0, z),$$

where the term

$$T_1^R(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}^0, z) = \langle \Psi_\beta | \langle \mathbf{p}^0 - | (z - H_\beta) | \Psi_1(z) \rangle g_R^C(z) \times \langle \Psi_2(z) | V_\alpha^S | \Psi_C^+(\mathbf{k}_\alpha \mathbf{p}_\alpha) \rangle (1 - \delta_{\alpha,\beta})$$

corresponds to the quasielastic knockout process and

$$T_2^R(\mathbf{k}_\alpha \mathbf{p}_\alpha, \mathbf{p}^0, z) = \langle \Psi_\beta | \langle \mathbf{p}^0 - | B(z) g_R^C(z) | \Psi_C^+(\mathbf{k}_\alpha \mathbf{p}_\alpha) \rangle,$$

$$B(z) = (z - H_\beta) V^\alpha [I - R(z) V^\alpha]^{-1} | \Psi_1(z) \rangle \langle \Psi_2(z) | V_\alpha^S.$$

To calculate T_2^R we take into account the fact that the matrix element $\langle \Psi_\beta | \langle \mathbf{p}^0 - | B(z) | \mathbf{k}_\alpha \mathbf{p}_\alpha \rangle$ near momenta $\mathbf{p}_\alpha$ such that $E_{\text{tot}}^C - E_R - p_\alpha^2/2n_\alpha \cong 0$ can be approximated by a constant. Transforming to the coordinate representation, we obtain

$$\langle \Psi_\beta | \langle \mathbf{p}^0 - | B(z) g_R^C(z) | \mathbf{r}_\alpha \boldsymbol{\rho}_\alpha \rangle = \chi_\alpha(\mathbf{r}_\alpha) f_\alpha(\hat{\boldsymbol{\rho}}_\alpha) \langle 0 | g_R^C(z) | \boldsymbol{\rho}_\alpha \rangle, \tag{21}$$

where $\chi_\alpha(\mathbf{r}_\alpha)$ is the vertex function of the resonance decay and $f_\alpha(\hat{\boldsymbol{\rho}}_\alpha)$ is a function depending only on the angular variables. The matrix element $\langle 0 | g_R^C(z) | \rho \rangle$ is known explicitly[51] and is

$$\Gamma(1 + i\nu_\alpha) W_{-i\nu,1/2}(-2ik_R \rho),$$

where $W_{a,c}(x)$ is the Whittaker function, the parameter k_R was introduced earlier, and $\nu_\alpha = (Q_\alpha q_\alpha n_\alpha)/k_R$ is the Coulomb parameter for the interaction in the system of resonance plus third particle. Using the representation (21), we arrive at integrals of the form[54]

$$I_{lm} = \Gamma(1 + i\nu_\alpha) \int d\mathbf{r}_\alpha \int d\mathbf{p}_\alpha \chi_\alpha(\mathbf{r}_\alpha) W_{-i\nu,1/2}(-2ik_R \rho_\alpha) \times Y_{lm}^*(\hat{\boldsymbol{\rho}}_\alpha) \langle \mathbf{r}_\alpha \boldsymbol{\rho}_\alpha | \Psi_C^+(\mathbf{k}_\alpha \mathbf{p}_\alpha) \rangle, \tag{22}$$

which correspond to resonance behavior of the amplitude T_2^R. Replacing $\chi_\alpha(\mathbf{k}_\alpha)$ by a suitable constant, in (22) we use the approximation of zero nuclear force range in the pair α, and for the function Ψ_C^+ we use its asymptotic expansion in terms of two-particle Coulomb wave functions (which is equivalent to replacing Ψ_C^+ by an asymptotic operator like the Muhlerin–Cinnes operator:[59,60]

$$\langle \mathbf{r}_\alpha \boldsymbol{\rho}_\alpha | \Psi_C^+(\mathbf{k}_\alpha \boldsymbol{\rho}_\alpha) \rangle = \frac{1}{(2\pi)^3} \exp(i\mathbf{k}_\alpha \mathbf{r}_\alpha + i\mathbf{p}_\alpha \boldsymbol{\rho}_\alpha) \prod_\gamma \exp\left(-\frac{\pi}{2} \eta_\gamma \right) \times \Gamma(1 + i\eta_\gamma) \Phi(-i\eta_\gamma, 1; ik_\gamma r_\gamma - i\mathbf{k}_\gamma \mathbf{r}_\gamma)$$

$$\left(\eta_\gamma = \frac{q_i q_j k_\alpha}{\mu_\alpha}, \quad i,j \in \alpha \right).$$

As a result, the integrals over $\mathbf{r}_\alpha$ and $\boldsymbol{\rho}_\alpha$ in (22) decouple, and the integral over $\mathbf{r}_\alpha$ gives the vertex function of resonance decay in the pair α, $\chi_\alpha^c(\mathbf{k}_\alpha)$. The integral over $\boldsymbol{\rho}_\alpha$ can be calculated by the Nordsieck method,[54] where, as was shown in Refs. 72 and 73, the resonance behavior of these integrals is identical for all the partial waves. For this reason we restrict ourselves to consideration of the case with $l=0$:

$$I_0 = \int d\boldsymbol{\rho} \frac{\exp(ik_R \rho)}{\rho} \Psi(1 + i\nu, 2; -2ik_R \rho) \times \Gamma(1 + i\nu) \exp(i\mathbf{p}\boldsymbol{\rho}) \Phi(-i\eta_{12}, 1; ik_{12}\rho - i\mathbf{k}_{12}\boldsymbol{\rho}) \times \Phi(-i\eta_{13}, 1; ik_{12}\rho - i\mathbf{k}_{13}\boldsymbol{\rho}).$$

We use the integral representation for $\Psi(a,c;z)$:

$$\Psi(a,c;z) = \frac{1}{\Gamma(a)} \int_0^\infty d\nu \nu^{a-1} (1+\nu)^{c-a-1} e^{-\nu z}$$

and integrate over ρ, taking into account the results of Ref. 58. In the end we find

$$I_0 = \int_0^\infty d\nu \nu^{i\nu} (1+\nu)^{-i\nu} 2\pi \frac{\gamma^{i\eta_{12}} (\alpha+\beta)^{i\eta_{13}}}{\alpha^{1+i\eta_{12}+i\eta_{13}}} \times {}_2F_1\left(-i\eta_{12}, -i\eta_{13}, 1; -\frac{\alpha\delta - \beta\gamma}{\gamma(\alpha+\beta)} \right),$$

where

$$\alpha = \tfrac{1}{2}(k_R'^2 - p_1^2), \quad \gamma = \mathbf{p}_1 \mathbf{k}_{12} + k_R' k_{12} + \alpha,$$

$$\beta = \mathbf{p}_1 \mathbf{k}_{13} + k_R' k_{13}, \quad \delta = k_{12} k_{13} - \mathbf{k}_{12} \mathbf{k}_{13} + \beta, \tag{23}$$

with $k_R' = k_R(1 + 2\nu)$. The integral over ν can be calculated by an asymptotic expansion in the small parameter $\varepsilon = (k_R - p_1)/2k_R$, since $\alpha = k_R(k_R' + p_1)(\nu + \varepsilon)$. Then[58]

for the resonance behavior of the amplitude T_2^R we obtain an expression of the form

$$T_2^R(\mathbf{k}_{23}\mathbf{p}_1,\mathbf{p}^0,z)$$

$$=\frac{\exp[-(\pi/2)(\eta-\nu)]\Gamma(1+i\eta_{12})\Gamma(1+i\eta_{13})}{\Gamma(1+i\eta_{12}+i\eta_{13})}$$

$$\times\Gamma(1+i\eta-i\nu)\frac{\gamma^{i\eta_{12}}(\alpha+\beta)^{i\eta_{13}}}{\alpha^{1+i\eta_{12}+i\eta_{13}}}$$

$$\times{}_2F_1\left(-i\eta_{12},-i\eta_{13},1;-\frac{\alpha\delta-\beta\gamma}{\gamma(\alpha+\beta)}\right)$$

$$\times T_0(\mathbf{p}_1,\mathbf{p}^0,z)\chi^c(\mathbf{k}_{23}). \qquad (24)$$

Here the parameters α, β, γ, and δ are taken from (23) with k_R' replaced by k_R, and $\eta=\eta_{12}+\eta_{13}$ has the meaning of the resonance production amplitude. The parametrization (24) is simplified considerably when $|\mathbf{k}_{13}-\mathbf{p}_1|\ll p_1$ and $|\mathbf{k}_{12}-\mathbf{p}_1|\ll p_1$. These conditions mean that in the decay process, from the viewpoint of the accompanying particle the resonance looks like a single object. Then the argument z of the hypergeometric function ${}_2F_1(a,b,c;z)$ is close to unity. Taking into account the weak effect of factors of the type $\eta^{i\eta_{12}}$ on the shape of the resonance curve, we obtain

$$T_2^R(\mathbf{k}_{23}\mathbf{p}_1,\mathbf{p}^0,z)=\exp\left[-\frac{\pi}{2}(\eta-\nu)\right]\Gamma(1+i\eta-i\nu)$$

$$\times\frac{T_0(\mathbf{p}_1,\mathbf{p}_0,z)\chi^c(k_{23})}{\alpha^{1+i\eta-i\nu}}, \qquad (25)$$

so that the main difference from the Breit–Wigner formula is in the appearance of the additional factor

$$\exp\left[-\frac{\pi}{2}(\eta-\nu)\right]\Gamma(1+i\eta-i\nu)\alpha^{-i\eta+i\nu}.$$

It can be shown that a similar result is also valid for the amplitude of the quasielastic knockout process.[73]

It is useful to note that the parametrization (25) can be obtained by replacing the Whittaker function by its asymptote

$$W_{-i\nu,1/2}(-2ik_R\rho)\to\frac{\exp(ik_R\rho)}{\rho}\rho^{-i\nu},$$

or by replacing Ψ_C^+ by any other suitable Coulomb asymptotic operator. For example, when the three-particle Muhlerin–Cinnes operator is used for Ψ_C^+, the parameter γ is replaced by $\gamma'=\mathbf{k}_{12}\mathbf{p}_1+k_Rp_1$, and the main resonance factor in the final-state interaction amplitude is preserved. This is consistent with the arguments at the beginning of this section.

As will be shown below, parametrizations of the type (24)–(25) lead to a shift of the location and broadening of the resonance peak.

8. INCLUSION OF THE POLARIZATION POTENTIAL. PASSAGE TO THE CASE OF A SHORT-LIVED RESONANCE

According to the results of the preceding section, in general the resonance behavior of the final-state interaction amplitude is given by integrals of the form

$$I_{lm}=\int d\boldsymbol{\rho}\,Y_{lm}^*(\hat{\boldsymbol{\rho}})\langle 0|g_R(z)|\boldsymbol{\rho}\rangle\exp(i\mathbf{p}_1\boldsymbol{\rho})$$

$$\times\Phi(-\eta_{12},1;ik_{12}\rho-i\mathbf{k}_{12}\rho)\cdot\Phi(-i\eta_{13},1;ik_{13}\rho$$

$$-i\mathbf{k}_{13}\rho)$$

$$=\int d\boldsymbol{\rho}\,Y_{lm}^*(\hat{\boldsymbol{\rho}})\langle 0|\left(z-z_R-\frac{\hat{p}_1^2}{2n_1}-\frac{Q_1q_1}{\rho}-V_{\rm pol}(\boldsymbol{\rho})\right)^{-1}$$

$$\times|\boldsymbol{\rho}\rangle\Phi(-i\eta_{12},1;ik_{12}\rho-i\mathbf{k}_{12}\boldsymbol{\rho}_{12})$$

$$\cdot\Phi(-i\eta_{13},1;ik_{13}\rho-i\mathbf{k}_{13}\boldsymbol{\rho}). \qquad (26)$$

In (26) the effective interaction potential in the system of resonance plus accompanying particle, $U(z)$, is replaced by a local potential which is the sum of Coulomb and polarization terms. Before calculating the integrals (26), let us discuss the role of the polarization potential $V_{\rm pol}(\boldsymbol{\rho})$. Since this potential is attractive, the parameter z_R in (26) is effectively replaced by $z_R+V_{\rm pol}$ with $E_R'=E_R+\mathrm{Re}\,V_{\rm pol}(\boldsymbol{\rho})<E_R$, and the polarization potential leads to an additional [compared with Eqs. (24)–(25)] shift of the location of the resonance peak. In addition, since $U(z)$ is analogous to an optical potential, its imaginary part leads to an effective increase of $\Gamma/2$.

At this stage let us study in more detail the behavior of the kernel of the operator $U(z)$ in coordinate space. For this we rewrite the expression for $U(z)$ as

$$U(z)=\langle\Psi_2(z)|V^\alpha|\Psi_1(z)\rangle+\langle\Psi_2(z)|V^\alpha$$

$$\cdot G_Q(z)V^\alpha|\Psi_1(z)\rangle, \qquad (27)$$

where $G_Q(z)=R(z)[Q-QV^\alpha QR(z)]^{-1}$. It is easily seen that $G_Q(z)=(zQ-QH_\alpha Q-QV^\alpha Q)^{-1}$, from which the analogy between $U(z)$ and the optical potential can be seen. The main difference from the optical interaction is that the states Ψ_i are not normalized and they contain the continuum contribution, the size of which increases with the width of the resonance state considered:

$$\Gamma=-\frac{2\,\mathrm{Im}\langle\Phi|WR(E_R+i0)W|\Phi\rangle}{\omega_0(E_R)},$$

$$\omega_0(E_R)=1-\frac{d}{dE}\mathrm{Re}\langle\Phi|WR(E_R+i0)W|\Phi\rangle.$$

In the case of long-lived resonances the quantity $\omega_0(E_R)$ differs only slightly from unity, and the continuum contribution to the vertex functions for resonance formation and decay can be neglected. Then

$$U(z)\cong\langle\Phi|V^\alpha+V^\alpha G_Q(z)V^\alpha|\Phi\rangle$$

is the usual optical potential for the system of resonance (quasistationary state) plus third particle. As is well

known, in the case of short-lived resonances there is a large probability to find the particles outside the potential well. Separating the contributions of the discrete and continuous spectra in $R(z)$, we write Ψ_i as the sum

$$\Psi_i=\Psi_i^d+\Psi_i^C,$$

where the first term is normalizable and the second describes the motion of particles in the continuum. Here the ratio of the amplitudes of these contributions to Ψ_i at the boundary of the nuclear interaction region is estimated from $((1/r_0)(2\hbar v_0/\pi\Gamma))^{-1/2}$ with $v_0=\sqrt{2E_R/\mu_\alpha}$ (Ref. 74). It is easily seen that this ratio is typically of order unity if $\Gamma\sim 1$ MeV ($E_R\sim 10$ MeV, $\mu_\alpha\sim m_N$, $r_0\sim 5$ F). For this reason, for short-lived resonances the contribution of Ψ_i^C to $U(z)$ is important, so that the local part of $\langle\rho'|U(z)|\rho\rangle$ corresponding to the first term in (27) differs noticeably from the purely Coulomb interaction at distances of the order of the size of the unstable cluster. Therefore, the local part of $U(z)$ related to the electromagnetic interaction takes the form $[(Q_\alpha q_\alpha)/\rho]f(\rho)$ in coordinate space, and $f(\rho)=1$ only for $\rho\gg r_0$. This implies that in the case of short-lived resonances decaying directly at the boundary of the region where the nuclear interaction dominates, the contribution of the polarization potential to the final-state interaction amplitude is of the same order as that of the Coulomb interaction in the center of mass in the system of resonance plus third particle. For this reason, in contrast to the case of a long-lived resonance, the contribution of the polarization term to (26) cannot be neglected, as was done in Ref. 74, for example.

To estimate the expected additional shifts and broadening of the resonance peak we proceed as follows. Since, as noted above, the asymptotic properties of the Green functions and the scattering states play the dominant role in this effect, to include the polarization potential in the partial Green function $g_R^l(\rho'=0,\ \rho,\ z)$ it is sufficient to restrict ourselves to the eikonal approximation. Then we have

$$\begin{aligned}g_R^l(0,\rho,z)=&\text{const}\exp(ik_R\rho)\Psi(l+1+i\nu,2l+2,\\&-2ik_R\rho)\rho^l\exp\left(\frac{n_1}{ik_R}\right)\int_\infty^\rho d\rho' V_{\text{pol}}(\rho')\\\approx&\text{const}\exp(ik_R\rho)\rho^{-i\nu-1}\\&\times\exp\left(\frac{in_1}{k_R}\right)\int_\rho^\infty d\rho' V_{\text{pol}}(\rho').\end{aligned}$$

In the limiting case where $|\mathbf{k}_{12}-\mathbf{p}_1|\ll p_1$ and $|\mathbf{k}_{13}-\mathbf{p}_1|\ll p_1$, for I_{lm} we obtain a representation of the form

$$\begin{aligned}I_{lm}=&\text{const}\int_0^\infty d\rho\rho^2\exp(ik_R\rho)\rho^{-i\nu-1}\\&\times\exp\left(\frac{in_1}{k_R}\right)\int_\rho^\infty d\rho' V_{\text{pol}}(\rho')\\&\times\frac{\exp(-\pi/2\eta)\Gamma(l+1+i\eta)}{(2l+1)!}\rho^l\\&\times\exp(ip_1\rho)\Phi(l+1+i\eta,2l+2;-2ip_1\rho),\end{aligned}\qquad(28)$$

where, as before, $\eta=\eta_{12}+\eta_{13}$. Writing the polarization potential as

$$V_{\text{pol}}(\rho)=U(\rho)-\frac{Q_1q_1}{\rho},$$

we bring (28) to the form

$$\begin{aligned}I_{lm}=&\text{const}\frac{\exp(-\pi/2\eta)\Gamma(l+1+i\eta)}{(2l+1)!}\int_0^\infty d\rho\\&\times\exp(ik_R\rho)\rho\exp\left[-\frac{in_1}{k_R}\int_0^\rho d\rho' V_{\text{pol}}(\rho')\right]\rho^l\\&\times\exp(ip_1\rho)\Phi(l+1+i\eta,2l+2;-2ip_1\rho).\end{aligned}\qquad(29)$$

Since the dominant contribution to the integral over ρ comes from the range $0\leqslant\rho\leqslant 2\rho_0$, from the properties of $U(\rho)$ for the short-lived resonance it follows that the integrand in (29) no longer contains the term r^{-iv}. Therefore, the limiting case of a short-lived resonance is obtained if the parameter ν in (24)–(25) is taken to be zero:

$$\begin{aligned}&T_2^R(\mathbf{k}_{23}\mathbf{p}_1,\mathbf{p}^0,z)\\&=\exp\left(-\frac{\pi}{2}\eta\right)\Gamma(1+i\eta)\frac{\chi^c(\mathbf{k}_{23})}{\alpha^{1+i\eta}}T_0(\mathbf{p}_1,\mathbf{p}^0,z).\end{aligned}\qquad(30)$$

As expected, the shift and broadening of the resonance grow, owing to the increase of the Coulomb parameter of the problem. This is obviously a consequence of the properties of the effective potential $U(z)$ in the system of resonance plus accompanying particle. In the case of a long-lived resonance the effective Coulomb parameter of the problem is $\eta-\nu=\eta_{12}+\eta_{13}-\nu$, owing to Coulomb rescattering in the intermediate state of the resonance as a whole on the accompanying particle. There is actually no such rescattering in the case of a short-lived resonance, since the resonance decays directly near the nuclear interaction region. Here the potential $U(z)$ at the typical distance ρ traveled by the resonance from its creation point to its decay point ($\rho\sim\rho_0\sim r_0$) differs significantly from the Coulomb interaction, so that the Green function $g_R(z)$ can no longer be replaced by $g_R^c(z)$. Now, since the long-range Coulomb tail of $U(z)$ does not appear, $g_R(z)$ can be effectively parametrized using the original Migdal–Watson model. This was first noticed in Refs. 8 and 9, where the parametrization (30) was obtained for the decay amplitude of a short-lived resonance.

9. CHANGE OF THE PARAMETERS OF TWO-CLUSTER NUCLEAR RESONANCES DECAYING IN THE COULOMB FIELD OF THE ACCOMPANYING NUCLEAR REACTION PRODUCTS

In this section we shall analyze the expression for the differential cross section of the reaction $a+A\to 1+2+3$ with the formation of an unstable cluster in the 2–3 pair and obtain approximate expressions for the change of the observable parameters of this cluster (its location and half-width) under the effect of an external Coulomb field. The parametrizations (24), (25), and (30) for the T amplitude of the reaction near the resonance energy in the 2–3 pair

lead to the following expression for the squared modulus of the amplitude of the process:

$$|T_R(\mathbf{k}_{23}\mathbf{p}_1,\mathbf{p}^0,z)|^2$$
$$=A_\xi A_{12}A_{13}A_\eta^{-1}\left[\left(\frac{k_{23}^2}{2\mu_{23}}-E_R\right)^2+\frac{\Gamma^2}{4}\right]^{-1}$$
$$\times\exp 2[\xi\arg\alpha-\eta_{12}\arg\gamma-\eta_{13}\arg(\alpha+\beta)]$$
$$\times\left|{}_2F_1\left(-i\eta_{12},-i\eta_{13},1;-\frac{\alpha\delta-\beta\gamma}{\gamma(\alpha+\beta)}\right)\right|^2$$
$$\times|\tilde{T}(\mathbf{k}_{23}\mathbf{p}_1,\mathbf{p}^0,z)|^2. \quad (31)$$

Here $\xi=\eta_{12}-\eta_{13}-\nu'=\eta-\nu'$, with $\nu'=\nu$ for a long-lived resonance and $\nu=0$ for a short-lived one, $A_\xi=\pi\xi(\sinh\pi\xi)^{-1}\exp(\mp\pi\xi)$, $A_\eta=\pi\eta(\sinh\pi\eta)^{-1}$, $A_{ij}=A_{\eta_{ij}}$, $ij=12,13$, and $|\tilde{T}|^2$ denotes the (nonresonance) squared modulus of the T amplitude independent of the external Coulomb field. To calculate the Coulomb parameters it is convenient to use the expression

$$\eta_\alpha=0.158q_iq_j\left(\frac{\mu_\alpha(\text{rel. units})}{E_\alpha(\text{MeV})}\right)^{1/2}, \quad (32)$$

where the reduced mass is in nucleon masses and $E_\alpha=k_\alpha^2/2\mu_\alpha$, $\alpha=ij$. As already noted, these parametrizations are valid when the kinetic energy of the reaction products in the c.m. frame, E_{tot}^C, and the resonance excitation energy and half-width satisfy the inequality

$$E_{\text{tot}}^C-E_R\gg\frac{\Gamma}{2}.$$

Then for the real and imaginary parts of k_R we have the following estimates:

$$\text{Re}\,k_R=[2n_1(E_{\text{tot}}^C-E_R)]^{1/2},$$
$$\text{Im}\,k_R=\frac{\Gamma}{4(E_{\text{tot}}^C-E_R)}\text{Re}\,k_R$$

or

$$\text{Im}\,k_R(\text{F}^{-1})=0.055\left[\frac{\Gamma^2(\text{MeV})n_1(\text{rel. units})}{(E_{\text{tot}}^C-E_R)(\text{MeV})}\right]^{1/2}.$$

The relative momenta in pairs of charged particles, in terms of the moduli of which the corresponding Coulomb parameters are expressed, can be written as

$$\mathbf{k}_{12}=a_1\mathbf{p}_1-a_2\mathbf{k}_{23},\quad \mathbf{k}_{13}=a_3\mathbf{p}_1+a_4\mathbf{k}_{23}, \quad (33)$$

where

$$\begin{bmatrix}a_1\\a_3\end{bmatrix}=\frac{M}{(m_1+m_2)(m_2+m_3)}\begin{bmatrix}m_2\\m_3\end{bmatrix},$$
$$\begin{bmatrix}a_2\\a_4\end{bmatrix}=m_1\begin{bmatrix}(m_1+m_2)^{-1}\\(m_1+m_3)^{-1}\end{bmatrix}.$$

When the additional conditions

$$\frac{a_2}{a_1}\frac{k_{23}}{p_1}\ll 1,\quad \frac{a_4}{a_3}\frac{k_{23}}{p_1}\ll 1$$

are satisfied, the following approximations can be used for the moduli of the relative momenta k_{12} and k_{13}:

$$k_{12}\cong a_1p_1\left(1-\frac{a_2}{a_1}\frac{k_{23}}{p_1}\cos\theta_{1,23}\right),$$
$$k_{13}\cong a_3p_1\left(1+\frac{a_4}{a_3}\frac{k_{23}}{p_1}\cos\theta_{1,23}\right),$$

where $\theta_{1,23}$ is the angle between the vectors $\mathbf{p}_1$ and $\mathbf{k}_{23}$. These additional conditions are certainly satisfied sufficiently far from the resonance production threshold and for excitation energy E_R which is not too high, for example, if the following equivalent conditions are satisfied:

$$\frac{m_1m_3}{m_2M}\frac{E_R}{E_{\text{tot}}^C-E_R}\ll 1,\quad \frac{m_1m_2}{m_3M}\frac{E_R}{E_{\text{tot}}^C-E_R}\ll 1. \quad (34)$$

In this approximation the parameters α, β, γ, and δ satisfy the equations

$$\left|\frac{\alpha}{\gamma}\right|<1,\quad \left|\frac{\alpha}{\alpha+\beta}\right|<1,\quad \left|\frac{\delta'}{\alpha+\beta}\right|<1,$$

where $\delta'=k_{12}k_{13}-\mathbf{k}_{12}\mathbf{k}_{13}$. If we now write the argument of the hypergeometric function z as

$$z=-\frac{\alpha\delta'-\beta\gamma}{\gamma(\alpha+\beta)}=1-\frac{\alpha}{\gamma}-\frac{\alpha}{\alpha+\beta}+\frac{\alpha^2}{\gamma(\alpha+\beta)}-\frac{\alpha\delta'}{\gamma(\alpha+\beta)},$$

then, using the conditions (34), we find that in a typical experiment the parameter z has real part close to unity and small imaginary part. This means that the conditions (34) can be viewed as a criterion for the applicability of the simplified variant of a parametrization of the type (28) and (32).

Introducing the analogous simplification into (33), we obtain

$$|T_R(\mathbf{k}_{23}\mathbf{p}_1)|^2=2\pi\xi[(\exp(2\pi\xi)-1)(1+y^2)]^{-1}$$
$$\times\exp(2\xi\,\text{arccot}\,y)|\tilde{T}(\mathbf{k}_{12}\mathbf{p}_3)|^2, \quad (35)$$

where $y=2(E_{23}-E_R)/\Gamma$.

Since under the conditions where the model is applicable the Coulomb parameter is a fairly slowly varying function of the variable y, from (35) we find that the part of the squared modulus of the T amplitude depending on the Coulomb parameter must have a maximum when the energy in the 2–3 pair is

$$E_R^*=E_R-\frac{\Gamma}{2}\eta. \quad (36)$$

The quantity E_R^* can obviously be viewed as the value of the resonance energy modified by the external field. According to (36), the resonance curve is asymmetric. Nevertheless, if we use the definition of the observed resonance width analogous to the definition of a purely Breit–Wigner curve, we can find the dependence of this quantity Γ^* on the Coulomb parameter ξ. In Fig. 12 we show a graph of this dependence in the range $0\leqslant\xi\leqslant 10$ of greatest practical interest. We see that the Coulomb field always leads to broadening of the resonance curve, and this broadening increases with the intensity of the external Coulomb field.

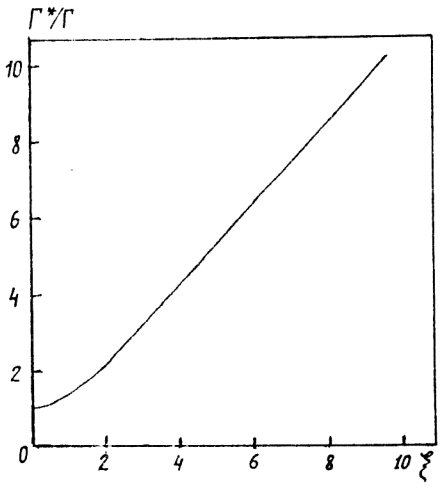

FIG. 12. Dependence of the observed width Γ* on the Coulomb parameter ξ.

The value of the observed resonance width can be determined using the following analytic expressions approximating the curve in Fig. 12:

$$\Gamma^*=\Gamma\begin{cases}1+0.025|\xi|+0.425\xi^2, & 0\leqslant|\xi|\lesssim 1,\\ 0.25+1.05|\xi|, & 1.5\leqslant|\xi|\leqslant 10.\end{cases} \qquad (37)$$

We note that when the parametrizations (24)–(25) and (30) are applicable, the value of Im ξ is vanishingly small compared with Re ξ, so in (31)–(35) we assume that the Coulomb parameter ξ is purely real. We see from Eqs. (36) and (37) that in going from a long- to a short-lived resonance the shift and broadening of the resonance are increased, owing to the increase of the Coulomb parameter of the problem, $|\xi|=|\eta-\nu|<\eta$. This fact, as noted above, is a consequence of the influence of the polarization potential, which is manifested the more strongly, the less stable (i.e., the more short-lived) the excited 2–3 cluster is. From (35) we see that in the case of a short-lived resonance the location of the resonance peak is always shifted toward lower energies of the relative motion in the pair of resonance particles (which is consistent with the attractive nature of V_{pol}), whereas in the case of a long-lived resonance the quantity $\Delta E_R = E_R^* - E_R$ can be either positive (in certain regions of phase space) or negative.

This model has been used to analyze the experimental data on the reaction ^{10}Be$(d,3\alpha)$ at the deuteron energy 13.6 MeV. In studies of this reaction,[64] broad peaks at the ^{8}Be* excitation energy near 19 MeV were discovered in the α-particle correlation spectra. The closest of the known levels is $E^*=18.9$ MeV, but its quantum numbers are $J^\pi=2^-$. By repeated careful measurements it has been found that depending on the α-particle detection angles, the location of the unidentified peaks is shifted in the range 18.8–19.7 MeV (the error in determining the centroid of the peak is ±0.1 MeV). It turned out that these are precisely the shifts to be expected for states of ^{8}Be* at an excitation energy of 19.9–20.2 MeV with positive parity.

As an example illustrating this phenomenon, in Fig. 13 we show fragments of the correlation energy spectra of α

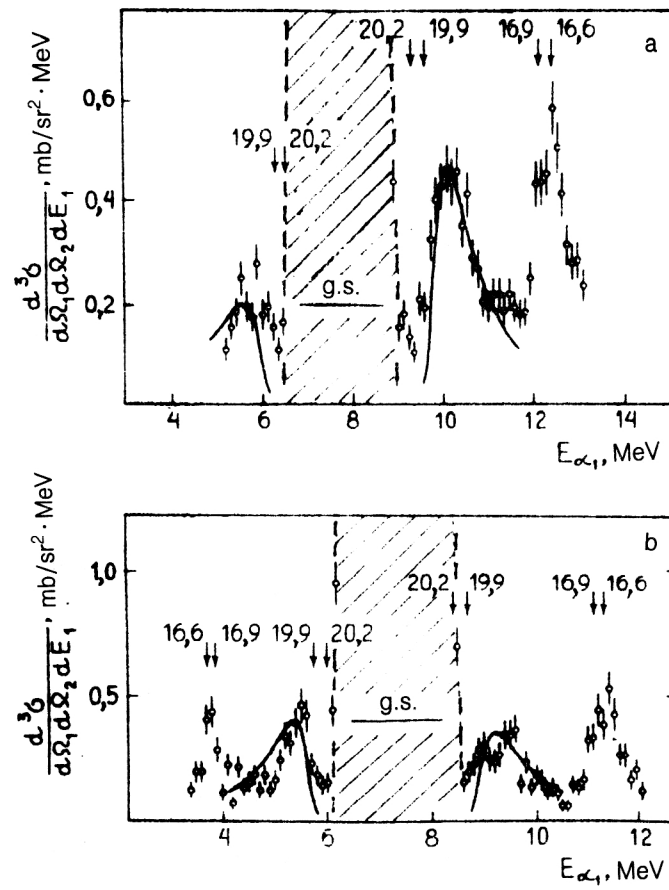

FIG. 13. The reaction $^{10}B+d\to\alpha_1+\alpha_2+\alpha_3$. The deuteron energy is 13.6 MeV. Fragments of α-particle spectra for $\theta_{\alpha_2}=110°$ and (a) $\theta_{\alpha_1}=40°$, (b) $\theta_{\alpha_1}=50°$ ($\varphi_1-\varphi_2=180°$). The arrows indicate the locations of the maximum contributions from various states of the ^{8}Be* nucleus, neglecting Coulomb effects.

particles from the reaction ^{10}B$(d,3\alpha)$ at the deuteron energy 13.6 MeV in the region where highly excited states of the ^{8}Be* nucleus ($E^*>16$ MeV) contribute. The arrows indicate the locations of the kinematically expected maxima of the contribution of the known states of the ^{8}Be* nucleus. The shaded region is the intense peak from the ground state of ^{8}Be. We see that the agreement with the kinematical calculations for the doublet of relatively narrow states at 16.6–16.9 MeV ($\Gamma=108$ keV) is good. At the same time, the maximum contribution of the broad states at $E^*\sim 19.9$–20.2 MeV ($\Gamma\sim 1000$ keV) is shifted by 300–400 keV in the lab frame [500–600 keV in the c.m. frame of the ^{8}Be* nucleus]. According to (30), the expected shifts due to the Coulomb interaction of the final products of this reaction are 570 keV for the 19.9-MeV state of the ^{8}Be* nucleus. The solid line in Fig. 13 shows the calculated energy spectrum, taking into account the Coulomb interaction according to (30) and (35) for the 19.9-MeV (2^+) state of the ^{8}Be* nucleus. Here the choice of the model for the short-lived resonance was motivated by the fact that the width of this state is of order 1 MeV.

The degree of deformation of the energy spectra in the region where the two-cluster resonance contributes de-

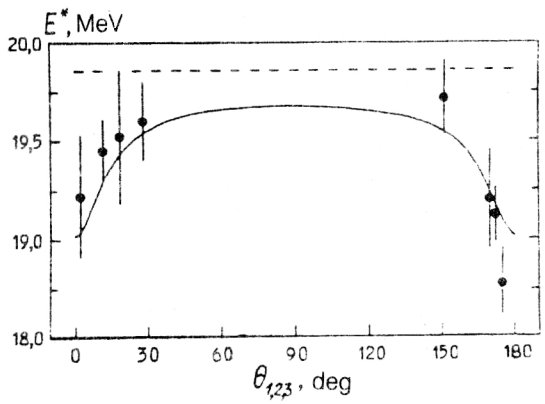

FIG. 14. Dependence of the location of the maximum contribution of the state ^{8}Be* (19.9 MeV) observed in the reaction ^{10}B(d,3α) on the decay angle $\theta_{1,23}$. The deuteron energy is 13.6 MeV.

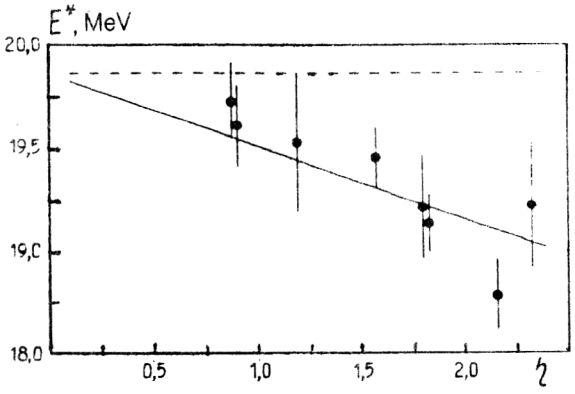

FIG. 15. Dependence of the location of the maximum contribution of the state ^{8}Be* (19.9 MeV) observed in the reaction ^{10}B(d,3α) on the Coulomb parameter η. The solid line is the calculation using (36); the dashed line is the location of the resonance line of the ^{8}Be* (19.9 MeV) nucleus known from the excitation function of α-particle elastic scattering on ^{4}He nuclei (Ref. 66).

pends on the emission angles of its decay products and of the third particle, $\theta_{i,jk}$. In Fig. 14 we show the calculated position of the resonance of the ^{8}Be* (19.9 MeV) nucleus, shifted by the Coulomb field of the third particle as a function of the angle $\theta_{i,jk}$ (solid line). The experimental data (points) correspond to the location of the maxima in the correlation spectra for the ^{8}Be* excitation energy in the range 19–20 MeV. The dashed line is the location of the 19.9-MeV level of ^{8}Be* treated as an isolated system.[65] We see from Fig. 14 that the largest shifts of the resonance location are observed for angles $\theta_{i,jk}$ close to 0° and 180°, corresponding to motion of one of the decay fragments in the direction of the accompanying particle. The symmetry of the angular dependence of the resonance position about the angle $\theta_{i,jk}=90°$ is due to the fact that the decay fragments (α particles) are identical. When the final products have different masses, there is no such symmetry.

It follows from the kinematics of these processes that when the decay angle $\theta_{i,jk}$ in the ij and ik pairs of final reaction products formed after the decay of a two-cluster state R^*_{jk} is changed, the speed of the relative motion and, accordingly, the effective Coulomb parameter $\eta=\eta_{ij}+\eta_{ik}$ are changed. In Fig. 15 we show the experimental data (points) and calculated position of the maximum contribution of the 19.9-MeV state of the ^{8}Be* nucleus as a function of the Coulomb parameter η, which varied between 0.5 and 2.4 for the kinematical conditions studied, as explained above. It should be noted that the experimental data given in Figs. 13–15 are also consistent with the results of the calculations for two other levels of the ^{8}Be* nucleus close in excitation energy (20.1 MeV, 0^+, $\Gamma=1.1$ MeV and 20.2 MeV, 4^+, $\Gamma\sim1$ MeV). For $0.5\leqslant\eta\leqslant2.4$ these resonances should be shifted in the ranges 19.8–18.8 MeV and 19.95–19.0 MeV, respectively, i.e., close to the calculated values for the 19.9-MeV state, shown by the solid line in Figs. 13–15. We chose the 19.9-MeV (2^+) state as an illustration, since it has the largest partial decay width ($\Gamma_\alpha/\Gamma=0.96$; Ref. 66). We also note that in this case the use of the long-lived resonance model, as in Ref. 74, does not explain the experimental data.

When certain additional conditions are satisfied, the parametrizations (31) and (35) possess the property of probability conservation: the probability for resonance decay via a fixed channel α is independent of the external Coulomb field. Since this probability is, up to multiplication by the width of the partial decay via the channel in question, equal to the area under the resonance curve, to prove this statement it is sufficient to calculate the integral

$$W_\alpha=\Gamma_\alpha\int_{-\infty}^{\infty}dE\left|\exp\left(-\frac{\pi}{2}\xi_\alpha\right)\Gamma(1+i\xi_\alpha)\times\left(E-E_R+i\frac{\Gamma}{2}\right)^{-1-i\xi_\alpha}\right|^2.$$

In the definition of W_α we have neglected the factor for the density of final states and also have assumed that the Coulomb parameter of the problem ξ_α depends weakly on the energy $E+E_\alpha$ and that the integration can be extended to the entire real axis. These simplifications are valid if the resonance peak in the α pair is not located too close to the production threshold of the unstable cluster in question.

With these assumptions we have

$$W_\alpha=2\pi\frac{\Gamma_\alpha}{\Gamma},$$

i.e., it is independent of ξ_α. Therefore, in this case the external Coulomb field leads only to redistribution of the probability and a shift of the maximum of the probability density by an amount $\Delta E_R=E^*_R-E_R=-(\Gamma/2)\xi_\alpha$. Since the values of ξ_α are different for resonance decay via different channels, the maxima in the energy spectrum of the third particle are shifted relative to each other. This implies that the Coulomb field can lead to a change of the probability ratios for resonance decay via different channels compared to resonance decay in the vacuum. In particular, if the shift of the resonance maximum in the function $|T^2|$ is sufficiently large for one of the reaction channels, so that the location of this maximum turns out to

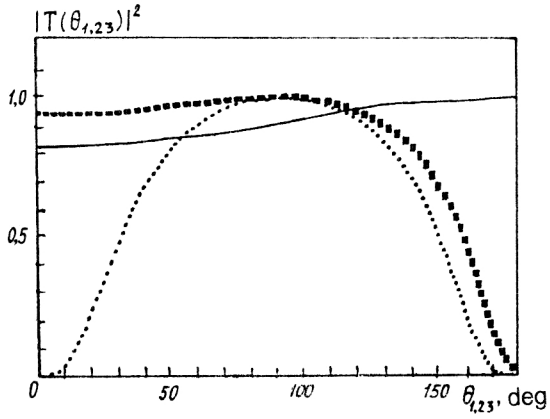

FIG. 16. Dependence of the probability for the decay of the nucleus ^{8}Be* (20.2 MeV) via various channels on the angle between the momentum of the third α particle and the momentum of the relative motion of the α particles from the decay of this nucleus for the reaction $^{10}B+d\to 3\alpha$. The deuteron energy is 13.6 MeV. (●) $d+^{10}B\to\alpha+\alpha+\alpha$; (–) $d+^{10}B\to\alpha+^7Be+n$; (□) $d+^{10}Be\to\alpha+^7Li+p$.

be close to the edge of the phase space, suppression of the resonance decay via this channel will be observed.

We note that this effect was discovered in the reaction ^{7}Li$(d,\alpha\alpha)n$ at $E_d=6.8$ MeV with excitation of a long-lived state of the ^{5}He* nucleus ($E^*=16.76$ MeV, $\Gamma=76$ keV; Ref. 75).

Since the parameter ξ_α depends on the angle θ_α between the vectors $\mathbf{k}_\alpha$ and $\mathbf{p}_\alpha$, the effect of the Coulomb field of the accompanying particle can be observed in the angular dependence of the reaction cross section.

In Fig. 16 we show the dependence, calculated from (35), of the squared modulus of the amplitude $|T^2|$ on $\theta_{1,23}$ for different reaction channels,

$$d+^{10}B\to\alpha^8Be(E^*=20.2\ \text{MeV})\to\begin{cases}\alpha+^7Be+n,\\ \alpha+^7Li+p,\\ \alpha+\alpha+\alpha,\end{cases}$$

at the same incident-deuteron energy as before. The transition to the state with $E^*=20.2$ MeV is related to the fact that the level with $E^*=19.9$ MeV decays via the 3α channel with 98% probability, whereas in this case the decay via different channels is characterized by a sizable probability. Since the width of the state with $E^*=20.2$ MeV is larger than that for $E^*=19.9$ MeV, the short-range resonance model is used, as before. All the curves are normalized to their maximum value, taken to be unity.

10. NUCLEAR REACTIONS WITH THE PRODUCTION OF FOUR PARTICLES IN THE FINAL STATE

The above analysis of the effect on a final-state resonance of the nuclear and Coulomb fields of the accompanying particles can be generalized to the case of reactions with the production of four particles. This generalization can be based on the integral equations of the theory of the scattering of four nonrelativistic particles obtained in Refs. 16 and 20. Here, in contrast to three-particle reactions, there are significantly more different particle clustering channels, and the interference between them can significantly change the two- or three-particle resonance in the final state of such a process.

In the case of direct four-particle processes the dependence of the resonance parameters of the two- and three-particle subsystems on the momentum transfer can be studied on the basis of analysis of the first iterations of the integral equations obtained in Ref. 6.

Let us consider the case of the nuclear reaction $\alpha+A\to 1+2+3+4$ leading to the production of four charged particles in the final state, three of which (2,3,4) form a resonance decaying in the Coulomb field of particle 1.

The nonisolated resonance consisting of three particles (2,3,4) in the final state of a four-particle reaction can be studied as in Secs. 7 and 8 above. We assume that $\mathbf{p}_1$, the momentum of particle 1 in the c.m. frame, is considerably larger than $\mathbf{k}_\alpha$, the relative momentum of any pair of particles α of the subsystem (2,3,4). The inclusion of the Coulomb rescattering of particle 1 with charge q_1 on each of the particles 2,3,4 leads to the following parametrization of the four-particle reaction amplitude:

$$T(\{\mathbf{k}_\alpha\},\mathbf{p}_1,z)=\exp\left(-\frac{\pi}{2}\xi\right)\Gamma(1+i\xi)\times\left(E-\frac{p_1^2}{2n_1}-E_R+i\frac{\Gamma}{2}\right)^{-1-i\xi},\qquad(38)$$

where the parameter ξ is defined as in Sec. 9:

$$\xi=\sum_{i=2,3,4}\frac{q_1q_i\mu_{1i}}{k_{1i}}-\nu,\quad \nu=\frac{q_1(q_2+q_3+q_4)n_1}{k_R},$$

$$k_R=[2n_1(E-E_R)]^{1/2},\quad n_1=\frac{m_1(m_2+m_3+m_4)}{m_1+m_2+m_3+m_4},$$

and, as before, for a short-lived resonance the parameter ν must be set to zero. Here $\{k_\alpha\}$ is the set of momenta of the relative motion of the particles of the 2,3,4 subsystem. It can be shown that the parametrized expression (38) becomes

$$T(\{\mathbf{k}_\alpha\},\mathbf{p}_1,z)\cong\exp\left(-\frac{\pi}{2}\xi\right)\Gamma(1+i\xi)\left(E-\frac{p_1^2}{2n_1}-E_R^*+i\frac{\Gamma^*}{2}\right)^{-1},$$

where E_R^* and Γ^* are the parameters of the nonisolated resonance, related to the corresponding parameters of the isolated resonance as

$$E_R^*=E_R-\frac{\Gamma}{2}\xi,\quad \Gamma^*\cong\Gamma\exp(0.425|\xi|).$$

It follows from this analysis that in the case of the scattering of particles of like charge the observed maximum in the spectrum of the accompanying particle 1, corresponding to

a nonisolated three-particle resonance, is broader and shifted to higher energies compared with the maximum corresponding to an isolated resonance.

Let us now consider the mutual Coulomb effect of two independent resonances formed in the final state of a four-particle reaction. For this analysis it is convenient to use the integral equations for the scattering amplitude as in Refs. 6 and 20. On the basis of the results of these studies, the amplitude of the reaction involving four particles for the region of the final-state configuration space where the resonance interaction in the independent pairs α and α' is important can be written as

$$T_{\alpha,\alpha'}(\mathbf{k}_\alpha \mathbf{k}_{\alpha'} \mathbf{K}_{\alpha\alpha'}, z)$$

$$= \langle \Psi^{\alpha,\alpha'}(z) | G_{\alpha,\alpha'}(z) V^S_{\alpha,\alpha'} | \Psi^+_C(\mathbf{k}_\alpha \mathbf{k}_{\alpha'} \mathbf{K}_{\alpha\alpha'}) \rangle. \quad (39)$$

Here the function $|\Psi^+_C(\mathbf{k}_\alpha \mathbf{k}_{\alpha'} \mathbf{K}_{\alpha\alpha'})\rangle$ corresponds to the Coulomb scattering of the particles in the final reaction channel, the operator $V^S_{\alpha,\alpha'}$ is the nuclear potential acting between the particles of the α and α' pairs, and $G_{\alpha,\alpha'}(z)$ is the Green function of the independent α and α' subsystems, which can be expressed in terms of the convolution of the Green functions for these subsystems:

$$G_{\alpha,\alpha'}(E+i\pi) = \int_{-\infty}^{\infty} \frac{d\varepsilon}{-2\pi i} g_\alpha(\varepsilon + i\tau_1) \otimes g_{\alpha'}(E-\varepsilon+i\tau_2)$$

$$(\tau = \tau_1 + \tau_2, \quad \tau_1, \tau_2 > 0), \quad (40)$$

where the operator $g_\alpha(z)$, the Green function of the α subsystem, has the form

$$g_\alpha(z) = \left(z - \frac{\hat{k}^2_\alpha}{2\mu_\alpha} - V^S_\alpha - V^C_\alpha \right)^{-1}.$$

Here $\otimes$ denotes the tensor product of operators, and the operators V^S_α and V^C_α are the nuclear and Coulomb interactions in the subsystem α. The state $\langle \Psi^{\alpha,\alpha'}(z)|$ in (39) is expressed in terms of the full Green function of the system $G(z) = (z-H)^{-1}$,

$$\langle \Psi^{\alpha,\alpha'}(z)| = \langle \Psi_\beta | (z-H_\beta)[I + G(z) V^{\alpha,\alpha'}],$$

where H_β is the asymptotic Hamiltonian of the initial channel β, $|\Psi_\beta\rangle$ is the initial state of the system, and $V^{\alpha,\alpha'} = \Sigma_{\alpha \neq \beta \neq \alpha'} V_\beta$ is the operator for the interaction between the particles with the condition that one of them belongs to the pair α and the other to the pair α'. In (39), $\mathbf{K}_{\alpha,\alpha'}$ denotes the momentum of the relative motion of the centers of mass of the pairs α and α'.

In this notation the resonance part of the amplitude $T_{\alpha,\alpha'}(z)$ corresponding to the production of two resonance sybsystems has singularities related to the singularities of the Green function, the expression for which contains terms of the form

$$B_\alpha(z)(z - z^\alpha_R - H_{0\alpha,\alpha'} - h^C_{\alpha'})^{-1}, \quad (41)$$

$$B_{\alpha,\alpha'}(z)(z - z^\alpha_R - H_{0\alpha,\alpha'})^{-1}. \quad (42)$$

In Eqs. (41) and (42) the coefficients $B_\alpha(z)$ and $B_{\alpha,\alpha'}(z)$ are nonsingular functions, $H_{0\alpha,\alpha'} = \hat{K}^2_{\alpha,\alpha'}/2n_{\alpha,\alpha'}$ is the operator for the kinetic energy of the relative motion of the centers of mass of the pairs α and α', the operator h^C_α corresponds to the internal energy of the subsystem, taking into account the Coulomb interaction, and z^α_R and $z^{\alpha'}_R$ are the complex energies of the resonances in the pairs α and α'.

We note that if the particles in the subsystems are neutral, the amplitude $T_{\alpha,\alpha'}$ contains a term proportional to a product of the form $a_\alpha a_{\alpha'}$, where

$$a_\alpha = \left(\frac{k^2_\alpha}{2\mu_\alpha} - z^\alpha_R \right)^{-1}, \quad a_{\alpha'} = \left(\frac{k^2_{\alpha'}}{2\mu_{\alpha'}} - z^{\alpha'}_R \right)^{-1}$$

In the case of charged particles the parametrization of the resonance part of the amplitude $T_{\alpha,\alpha'}(z)$ takes the form[76,77]

$$T_{\alpha,\alpha'}(\mathbf{k}_\alpha \mathbf{k}_{\alpha'} \mathbf{K}_{\alpha,\alpha'}, z) = \exp\left(-\frac{\pi}{2}\xi_\alpha\right)\Gamma(1+i\xi_\alpha) a_\alpha^{1+i\xi_\alpha} B_{\alpha'}$$

$$+ \exp\left(-\frac{\pi}{2}\xi_{\alpha'}\right)\Gamma(1+i\xi_{\alpha'}) a_{\alpha'}^{1+i\xi_{\alpha'}} B_\alpha$$

$$+ \exp\left(-\frac{\pi}{2}\xi_{\alpha,\alpha'}\right)\Gamma(1+i\xi_{\alpha,\alpha'})$$

$$\times (a_\alpha^{-1} + a_{\alpha'}^{-1})^{-1-i\xi_{\alpha,\alpha'}} B_{\alpha,\alpha'}. \quad (43)$$

Here the parameters ξ_α, $\xi_{\alpha'}$, and $\xi_{\alpha,\alpha'}$ are defined as[77]

$$\xi_\alpha = \sum_{i\in\alpha, j\in\alpha'} \eta_{ij} - \sum_{j\in\alpha'} \nu_{\alpha j},$$

where the parameter $\nu_{\alpha j}$ is defined for the corresponding three-particle sybsystem, as in Sec. 7. The parameter $\xi_{\alpha'}$ is defined analogously, and

$$\xi_{\alpha,\alpha'} = \sum_{i\in\alpha, j\in\alpha'} \eta_{ij} - \nu_{\alpha,\alpha'}, \quad \nu_{\alpha,\alpha'} = \frac{q_\alpha q_{\alpha'} n_{\alpha,\alpha'}}{K_{R\alpha,\alpha'}},$$

where $K_{R\alpha,\alpha'} = [2n_{\alpha,\alpha'}(E - z^\alpha_R - z^{\alpha'}_R)]^{1/2}$. As before, if the resonance in the pair α is short-lived, in the final state it does not undergo Coulomb rescattering as a whole on the accompanying particles, and therefore the parameters $\nu_{\alpha j}$ must be set equal to zero.

Analysis of the parametrization (43) leads to a change of the parameters of nonisolated resonance subsystems analogous to that of the case of a three-particle resonance subsystem in the field of a fourth particle. The third term in Eq. (43) contains an expression of the form $(a_\alpha^{-1} + a_{\alpha'}^{-1})^{-1}$, which leads to the summation of the half-widths

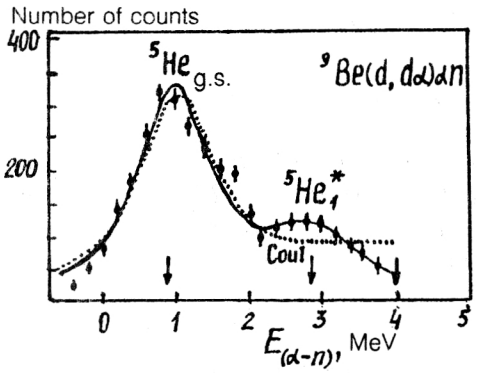

FIG. 17. Projection of the two-dimensional energy distribution $(E_{d'} \times E_\alpha)$ on the excitation energy axis in the $n\alpha$ system. The reaction is $^9\text{Be}+d \to d'+\alpha+\alpha+n$ and the deuteron energy is 13.6 MeV.

of the resonance subsystems and thereby additional broadening of the resonance peaks compared with the behavior of a two-particle nonisolated resonance in the field of a third particle.

In Fig. 17 we show the data on the energy dependence of the cross section for the reaction $^9\text{Be}(d,d\alpha)\alpha n$ at the deuteron energy 13.6 MeV in the region where the ground and first excited states of the ^{5}He nucleus contribute. The data (points) were obtained by projecting events in the $(E_d \times E_\alpha)$ plane on the excitation-energy axis for the $n\alpha$ pair for the same effective phase space corresponding to excitation energy of the ^{5}He nucleus in the range 4.7–6.7 MeV. A satisfactory description of the experimental data can be obtained only when the effect of the Coulomb field of the accompanying α particle is included (solid line). The dotted line corresponds to the calculation neglecting the Coulomb field. The arrows indicate where the contribution of the $^2P_{1/2}$ (g.s.) and $^3P_{3/2}$ (first excited state) levels of the ^{5}He nucleus is expected to be largest in accordance with the kinematical calculations.[67] The arrow labeled "Coul" shows the calculated location of the first excited state of the ^{5}He nucleus, shifted by the Coulomb field in accordance with (40), written for a short-lived ($\Gamma \sim 4$ MeV) resonance.

CONCLUSION

In this review we have discussed the main reasons for the appearance of spatial anisotropy of the observed scattering parameters of charged nuclear clusters produced together with other particles in the final state of multiparticle reactions. We have shown that this phenomenon is related to the effect of the nuclear and Coulomb fields of the accompanying products on the interaction of the particles in the nonisolated subsystem. The effect of the nuclear fields of the accompanying particles on the group of clusters is due to the important two-particle nuclear rescattering of the clusters of the multiparticle process in the interaction volume. This effect is manifested, first, as a dependence on the momentum transfer or the initial energy and scattering angle in the case of direct nuclear reactions and, second, as a dependence on the relative location in the final state, i.e., the correlation angles determining the degree of interference of the cluster channel amplitudes and the particle rescattering probability in each channel.

The spatial anisotropy of the effect of the Coulomb field of the accompanying particles on the scattering of clusters in nonisolated subsystems is determined by both the particle charge and the relative location, i.e., the correlation angles.

The effect of the field of the accompanying products on the final-state interaction is manifested in the cross section for the multiparticle reaction as a change of the parameters of this interaction (the resonance energy and half-width for a resonance and the scattering length for a virtual state). Compared with the interaction in the isolated system, these parameters, extracted from the multiparticle reaction spectra, turn out to be functions of the kinematical variables of the particles participating in the process and their charges. As shown in this review, in some kinematical regions this dependence can be found analytically.

For multiparticle processes involving four or more particles, and also in the case where resonance scattering in more than one nonisolated particle subsystem is observed in the final state, the dependence of the observed scattering parameters in each of these subsystems becomes more complicated. However, in any specific case this dependence can be found by numerical analysis.

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