Measurement of the muon and muon neutrino masses

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The measurement of the muon and muon neutrino masses is reviewed. The corrections to the energy spectrum of the muonic atom due to the finite size of the nucleus, the vacuum polarization, the Lamb shift, screening of the nuclear charge by electrons, and so on, which are needed to determine the muon mass from the experimental data on the mesonic x-ray energy, are studied. The measured mass of the negatively charged muon is compared with the mass of the positively charged muon determined from the distance between the levels of the hyperfine splitting in muonium (μ^+e^-) and by measurement of the π^--meson mass. An important contribution to the error in determining the μ^+ mass comes from the inaccuracy in calculating the radiative corrections to the hyperfine-structure levels in (μ^+e^-). The smallest upper limit on the muon neutrino mass is at present obtained from experiments in which the muon momentum is measured in the decay of a π meson at rest. The prospects of raising the accuracy of the muon and muon neutrino mass measurements are discussed.

INTRODUCTION

The muon was discovered by Anderson and Neddermeyer[1] and by Street and Stevenson[2] in investigations of interactions of cosmic-ray particles with matter in 1936–1938. During the decade following this discovery muons were identified with the Yukawa heavy mesons responsible for nuclear forces. However, the fundamental contradiction following from this hypothesis was clear even in those years: cosmic-ray muons pass through the entire atmosphere of the Earth without noticeable absorption, whereas the Yukawa mesons should interact strongly with nuclei and be rapidly absorbed in the atmosphere. This contradiction became more prominent after the work by Conversi, Pancini, and Piccioni,[3] who showed that cosmic-ray muons interact extremely weakly with target nuclei. After the studies carried out by Lattes *et al.*,[4,5] in which π mesons were discovered, it became clear that cosmic-ray muons are of secondary origin, being formed as a result of π-meson decay in the atmosphere.

The π–μ–e decay sequence has been observed in nuclear emulsion and in propane chambers. Measurement of the ranges of μ mesons arising in π–μ decay has shown that they form a group of monochromatic particles. The appearance of monochromatic muons in the decay of stopped π mesons shows that the π meson decays into two particles (owing to momentum conservation), one of which is neutral.

A curious and well known fact is that in January, 1957 the journal "The Physical Review" received during the course of two days three manuscripts devoted to three independent experiments which demonstrated the parity violation in weak interactions predicted by Lee and Yang. The β decay of the polarized ^{60}Co nucleus was described in the first study by Wu *et al.*[6] In the second study by Garwin, Lederman, and Weinrich,[7] the μSR method was used to demonstrate spatial parity nonconservation in the decays $\pi^+\to\mu^++\nu_\mu$ and $\mu^+\to e^++\nu_e+\bar{\nu}_\mu$.

The study by Friedman and Telegdi[8] reported the observation of muonium and the appearance of a polarization loss due to the hyperfine interaction and precession of the total magnetic moment of muonium (μ^++e^-) in an external magnetic field.

1. MEASUREMENT OF THE MUON MASS

In Refs. 1 and 2 it was also shown that the muon mass is roughly $200m_e$. In the 1950s a series of measurements were carried out of the muon and π-meson masses from their ranges in nuclear emulsion and their deflection in a magnetic field. The most careful measurements of this type were those performed at the Berkeley synchrocyclotron by Barkas *et al.*[9] From these experiments it was found that

$$m_{\pi^+}=(273.34\pm0.33)m_e,$$

$$m_{\pi^-}=(272.8\pm0.45)m_e,$$

$$m_{\mu^+}=(206.93\pm0.35)m_e,$$

$$P_\mu=(29.80\pm0.04)\ \mathrm{MeV}/c,$$

$$E_\mu=P_\mu^2/(2m_\mu)=(4.19\pm0.01)\ \mathrm{MeV},$$

where P_μ and E_μ are the momentum and energy of the muon produced in the decay of a π meson at rest.

The muon binding energy in the atom, assuming only the electromagnetic interaction between the muon and the nucleus and neglecting the finite size of the nucleus, is given by

$$E_n=-m_1c^2\alpha^2Z^2/2n^2, \tag{1}$$

where m_1 is the reduced muon mass, $m_1=m_\mu A/(m_\mu+A)$, n is the principal quantum number of the level, α is the fine-structure constant, and A and Z are the nuclear mass and charge.

The mesonic x-ray energy corresponding to the transition of the muon from level n_2 to level n_1 will accordingly be

TABLE I. Corrections to the energy spectrum of the muonic atom from the Pustovalov rule (Ref. 12). $E_{\text{p.n.}}$ is the muon binding energy for the pointlike nucleus, ΔE is the correction related to the nuclear size, and E is the corrected value of the muon binding energy ($E_{\text{p.n.}}$, ΔE, and E are in keV).

Element	Parameter	1s	2s	3s	2p	3p
$^{9}_{4}Be$	$-E_{\text{p.n.}}$,	45,04	11,26	5,0	11,26	5,0
	$-\Delta E$	0,588	0,076	0,022	0,000	0,000
	$-E$	44,48	11,180	4,978	11,260	5,0
$^{16}_{8}O$	$-E_{\text{p.n.}}$,	180,14	45,04	20,02	45,04	20,02
	$-\Delta E$	5,362	0,728	0,216	—	—
	$-E$	174,78	44,31	19,80	45,03	20,02
$^{31}_{15}P$	$-E_{\text{p.n.}}$,	633,31	158,33	70,37	158,33	70,37
	$-\Delta E$	43,117	5,879	1,758	0,013	0,004
	$-E$	590,19	152,45	68,61	158,31	70,36
$^{91}_{40}Zr$	$-E_{\text{p.n.}}$,	4503,88	1125,88	500,39	1125,88	500,39
	$-\Delta E$	1038,755	146,074	44,699	4,402	1,532
	$-E$	3464,76	979,81	455,69	1121,48	498,86
$^{208}_{82}Pb$	$-E_{\text{p.n.}}$,	18926,04	4731,51	2102,89	4731,51	2102,89
	$-\Delta E$	8818,581	1337,552	424,877	247,024	84,609
	$-E$	10107,46	3393,96	1678,02	4484,49	2018,28

$$E(n_2\rightarrow n_1)=m_1c^2\frac{\alpha^2Z^2}{2}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right). \quad (2)$$

Therefore, the muon mass can be determined from (2) by measuring the mesonic x-ray energy in the muon transition from level n_2 to level n_1.

However, as mentioned above, Eqs. (1) and (2) do not include the finite size of the nucleus and a number of other corrections. Compared with the case of ordinary atoms, these corrections are quite large because the muon mass is roughly 200 times the electron mass and a muon in the K shell is about 200 times closer to the nucleus than a K electron is. If we compare the orbital radius of the muon in the 1s state, $r_\mu=\hbar/(\alpha c m_\mu Z)\approx(2.9/Z)\times10^{-11}$ cm, with the nuclear radius $R=1.3\times10^{-13}\cdot A^{1/3}$, it is easy to see that even in light atoms R is several percent of r_μ, and for $Z\approx30R$ it becomes comparable to r_μ. Below we shall consider the corrections to the energy spectrum of the muonic atom. In the case of the pionic atom there are no corrections related to the magnetic-moment interaction, but the nuclear interaction of the π meson must be taken into account.

To obtain the muon mass from the mesonic x-ray energy it is necessary to include the following corrections to the energy spectrum:

1) the correction for the finite size of the nucleus;

2) the radiative corrections: a) the Lamb shift and b) the vacuum polarization;

3) the correction for screening of the nuclear charge by the electrons;

4) the correction for the nuclear polarization;

5) inclusion of the fine and hyperfine level structure;

6) inclusion of the quadrupole splitting and the level shift.

The contributions of these corrections to the energy spectrum depend both on the nuclear charge and on the principal quantum number of the level for which the corrections are introduced.

1. The correction for the finite size of the nucleus

The shift of the energy levels due to the finite size of the nucleus is large, and first-order perturbation theory does not give an accurate determination of it. It is usually necessary to numerically solve the Dirac equation with potential corresponding to a reasonable finite distribution of the nuclear charge. For given Z and A the largest shift is that of the 1s level. The shift of the 2p level is very small, since the muon wave function for the 2p state vanishes in the nonrelativistic limit for $r=0$.

Various authors have obtained simple expressions[10,11] for determining the scale of the corrections owing to the finite nuclear size:

$$\frac{\delta E_s}{E}=\frac{4}{5}\frac{1}{n^3}\left(\frac{ZR}{r_\mu^B}\right)^2, \quad (3)$$

$$\frac{\delta E_{2p}}{E}=0.0018\left(\frac{ZR}{r_\mu^B}\right)^4-0.0010\left(\frac{ZR}{r_\mu^B}\right)^5+..., \quad (4)$$

where r_μ^B is the Bohr radius of the muon. Equation (3) is applicable for the 1s level for $Z\leqslant10$ and for ns levels different from 1s for $Z>10$. Equation (4) can be used to calculate the corrections for the 2p level.

Pustovalov[12] has obtained an expression for a quick estimate of the shift of the 1s, 2s, 3s, 2p, and 3p levels in various muonic atoms due to the finite nuclear size:

$$E_n=\varepsilon E_0, \quad (5)$$

where $\varepsilon=(n+\Delta n)^{-2}$, $E_0=-m_\mu c^2\alpha^2Z^2/2$ is the muon binding energy in the ground state for a pointlike nucleus, and Δn is calculated from the expression

$$\Delta n=A_0+A_1\tau+A_2\tau^2+A_3\tau^3+A_4\tau^4+A_5\tau^5+(C_0+C_2\tau^2)\times\exp[-B_1\tau-B_2\tau^2-B_3\tau^3-B_4\tau^4],\quad \tau=RZ/r_\mu^B. \quad (6)$$

The values of the parameters in (6) were calculated using perturbation theory and are given in Ref. 12.

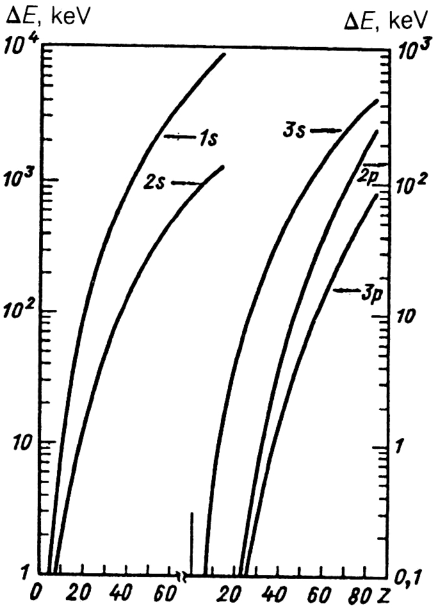

FIG. 1. Shifts of the muonic-atom levels due to the finite size of the nucleus. Calculations using the Pustovalov rule.[12]

The corrections calculated using Eq. (5) for various atoms are given in Table I (see also Fig. 1). We see that in heavy atoms the correction for the finite size of the nucleus for 1*s* levels is close to the value of the level energy calculated for the pointlike nucleus.

2. Radiative corrections

a. The Lamb shift (the self-energy correction)

The self-energy correction arises from the fact that the muon (electron) moving in the atom can emit virtual photons which are subsequently absorbed. Since this process is possible in the motion of a charged particle, one refers to it as the self-energy correction. This correction for each atomic level is calculated from the expression[13]

$$\Delta E_i^{LS}=\frac{2}{3\pi\hbar^3c^3}\sum_j |d_{ij}|^2(E_j-E_i)^3\ln\frac{m_\mu c^2}{|E_j-E_i|}, \qquad (7)$$

where d_{ij} is the dipole moment for the muon orbital motion and $|E_j-E_i|$ is the virtual-photon energy.

b. The vacuum polarization

The Coulomb field of the nucleus slightly displaces the virtual electrons and positrons in the vacuum relative to each other. This effect is referred to as vacuum polarization. Owing to the vacuum polarization, the Coulomb field around the nucleus at a distance of the order of the electron Compton wavelength (10^{-11} cm) is changed. Therefore, a muon in an atom spends a considerable amount of time in the region where the electrostatic potential of the nucleus is changed by the vacuum polarization.

The problems of vacuum polarization in mesonic atoms and the corresponding corrections to the level spectra have been studied many times: by Cooper and Henley,[10] by Kozlov *et al.*,[14] by Mickelwait and Corben,[15] by Hill and

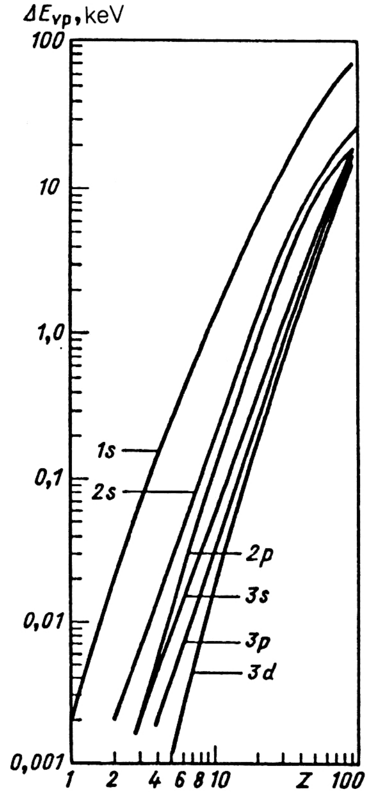

FIG. 2. Shifts of the muonic-atom levels due to the vacuum polarization.[18]

Ford,[16] by Glauber *et al.*,[17] by Pustovalov,[18] by Ford and Wills,[19] and by others.

Pustovalov obtained expressions[18] for an approximate estimate of the shifts of the first six levels owing to vacuum polarization. These expressions are quite awkward and will not be given here. We shall present only the results of the calculations carried out by Pustovalov, which give a complete idea of the magnitude of the shift due to the vacuum polarization for the first six levels of various nuclei.

The expressions obtained by Ford and Wills[19] can be used for more accurate calculations, and for each case the shifts are given by

$$\Delta E_{\text{v.p.}}=\int (f^2+g^2)V_p(r)dr, \qquad (8)$$

where $V_p(r)$ is the vacuum polarization potential for a spherically symmetric charge distribution,

$$V_p(r)=\frac{2\alpha}{3\pi}\left[V_L(r)-\frac{5}{6}V(r)\right];$$

$$V_L(r)=e^2\int \ln\left(\frac{C}{\lambda\!\!\!^{-}}|\mathbf{r}-\mathbf{r}'|\right)\frac{\rho(r')d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|},$$

$\lambda\!\!\!^{-}$ is the electron Compton wavelength, $C=1.781$, and f and g are the normalized muon wave functions from the Dirac equation.

In Fig. 2 we show the results of the Pustovalov calcu-

TABLE II. Radiative corrections to the energy spectrum of the muonic atom (Refs. 20 and 28). ΔE^{LS} is the Lamb shift in keV and $\Delta E_{\text{v.p.}}$ is the vacuum polarization in keV.

Element	Parameter	$1s_{1/2}$	$2s_{1/2}$	$2p_{1/2}$	$2p_{3/2}$	$3d_{3/2}$	$3d_{5/2}$	$4f_{7/2}$
^{31}P	ΔE^{LS}	—	—	—	$0{,}55\cdot 10^{-3}$	$0{,}04\cdot 10^{-3}$	—	—
	$\Delta E_{\text{v.p}}$	—	—	—	0,401	0,07	—	—
^{40}Ca	ΔE^{LS}	0,21±0,04	0,05±0,01	—	—	—	—	—
	$\Delta E_{\text{v.p}}$	6,93	1,06	0,92	0,90	0,19	0,18	0,05
^{120}Sn	ΔE^{LS}	1,55±0,15	0,39±0,06	0,02±0,02	0,10±0,01	0,01±0,01	0,01	—
	$\Delta E_{\text{v.p}}$	35,48	7,71	10,45	9,73	2,69	2,61	0,88
^{209}Bi	ΔE^{LS}	2,99±0,16	0,72±0,15	0,26±0,16	0,62±0,12	0,05	0,05	0,01
	$\Delta E_{\text{v.p}}$	68,17	19,80	33,20	30,60	10,88	10,19	3,80

lations of $\Delta E_{\text{v.p.}}$ and E^{LS} (Ref. 18), and in Table II we give the results of the more accurate calculations of Ref. 20 for several nuclei.

3. The correction for electron screening of the nuclear charge

The electrons in a muonic atom screen the nuclear charge, making the effective nuclear charge that the muon sees somewhat less than the true charge, which leads to a shift of the muon energy levels. Up to about the level $n=14$ these shifts should be small, since the muon orbits in levels with $n<14$ lie inside the electron K shell. This effect has been estimated in many studies. Recently, Fricke[21] calculated the influence of electron screening on the muon energy levels using the fully self-consistent Hartree–Fock–Slater procedure. The Coulomb interaction between the muon and electrons and the electron exchange potential were also taken into account. Auger electrons are emitted by the atom in transitions between high-lying levels, and at the start of a radiative transition it is not known exactly how many electrons the atom contains. Therefore, in Ref. 21 the muon binding energy was calculated for the 1s to 8k states in ^{208}Pb with 2, 10, 28, 46, and 80 electrons. The results are given in Table III. The shifts for various muon levels are close when the number of screening electrons is the same. We see from this table that K electrons give the dominant contribution to the level shifts. From Table IV we see how the energies of various mesonic x-ray transitions are changed owing to electron screening.

4. The correction for the nuclear polarization (the dispersion effect)

This effect is the least well studied, and the calculations of the level shifts related to nuclear polarization are not very accurate. By the term "nuclear polarization" in this case we mean the possible change of the nuclear charge distribution due to the presence of the muon (meson) in the K shell of the atom, which, in turn, can affect the muonic levels in the atom. The calculation of the corresponding level shifts involves summation over all the excited states of the nucleus, which requires the use of a specific nuclear model. Therefore, the calculations carried out by different authors vary within wide limits. Without dwelling on the details of these calculations, we present the results obtained by Chen[22] for various levels in ^{208}Pb:

$1s-(6.0\pm0.6)$ keV,

TABLE III. Binding energy and level shift (in keV) of the muon in lead for various numbers of electrons (Ref. 21).

Level	Level energy without the shift, keV	Level shift for a given number of electrons				
		2	10	28	46	80
$1s_{1/2}$	−10525,456	5,383	10,275	14,132	15,804	17,190
$2p_{1/2}$	−4782,313	5,381	10,272	14,128	15,800	17,185
$3d_{3/2}$	−2162,653	5,361	10,249	14,105	15,776	17,162
$4d_{3/2}$	−1213,989	5,300	10,177	14,031	15,702	17,086
$4f_{5/2}$	−1197,504	5,320	10,201	14,055	15,726	17,112
$4f_{7/2}$	−1188,483	5,319	10,200	14,053	15,725	17,107
$5f_{5/2}$	−766,442	5,220	10,083	13,933	15,604	16,985
$5g_{7/2}$	−761,806	5,255	10,124	13,976	15,646	17,032
$6g_{7/2}$	−529,156	5,110	9,954	13,801	15,471	16,848
$6h_{9/2}$	−527,603	5,160	10,013	13,861	15,531	16,918
$7i_{11/2}$	−387,085	5,036	9,866	13,710	15,379	16,765
$8k_{13/2}$	−296,137	4,881	9,684	13,523	15,190	16,576

TABLE IV. Change of the energy (in keV) of mesonic x-ray transitions due to electron screening in lead for various numbers of electrons (Ref. 21).

Transition	Transition energy, keV	Shift for a given number of electrons				
		2	10	28	46	80
$2p_{1/2} \to 1s_{1/2}$	5784,143	–0,002	–0,003	–0,004	–0,004	–0,005
$3d_{3/2} \to 2p_{1/2}$	2619,660	–0,020	–0,023	–0,023	–0,024	–0,023
$4f_{5/2} \to 3d_{3/2}$	956,144	–0,041	–0,048	–0,050	–0,050	–0,050
$5g_{7/2} \to 4f_{5/2}$	435,698	–0,065	–0,077	–0,079	–0,080	–0,080
$6h_{9/2} \to 5g_{7/2}$	234,203	–0,095	–0,111	–0,115	–0,115	–0,114
$7i_{11/2} \to 6h_{9/2}$	140,518	–0,124	–0,147	–0,151	–0,152	–0,153
$8k_{13/2} \to 7i_{11/2}$	90,948	–0,155	–0,182	–0,187	–0,189	–0,189

$2s-(1.2\pm0.2)$ keV,

$2p-(1.9\pm0.2)$ keV.

The results of experimental studies of mesonic x-ray emission in heavy mesonic atoms are viewed as experimental proof of the presence of the nuclear polarization effect. For example, in the study by Anderson *et al.*,[23] where the mesonic x-ray spectra of ^{206}Pb were measured, it was found that the energy of the $2p_{1/2}\to 1s_{1/2}$ transition is 7 keV larger than the calculated value (including all the necessary corrections except the nuclear polarization). Attempts have also been made to attribute this shift to other causes.[24]

The level splitting and shift due to the hyperfine interaction are calculated quite accurately. These effects have been thoroughly studied also in ordinary atoms. Quadrupole splitting (and level shift) occurs only in deformed nuclei and is of interest in experiments on nuclear properties. There is no need to discuss these corrections here.

In the 1950s and later experiments were carried out to determine the π^- and μ^- masses from measurements of the energy of the characteristic x-ray emission of π and μ mesons. Since the γ-ray detectors used in the 1950s did not have good enough resolution, the so-called absorption method was used to determine the mesonic x-ray energy in the earliest experiments. The absorption method is based on the fact that the absorption coefficient of mesonic x-rays in matter decreases sharply when the mesonic x-ray energy is less than the energy needed to eject an electron from the K shell of the absorber.

In these experiments various absorbers with monotonically varying Z were placed between the target and the spectrometer. A sharp variation of the line intensity of the mesonic x-rays was observed. It turned out that judicious choice of transitions and absorbers permitted the mesonic x-ray energy to be determined with a relative error of about 10^{-4}. After the introduction of the necessary corrections, the muon and π-meson masses were determined from these data (Table V). In Table VI we give the results of measurements of the energy of some mesonic x-ray transitions carried out in recent years with record accuracy. We note that the relative accuracy of determining the muon mass is close to the relative accuracy with which the mesonic x-ray energy is measured. As seen from Tables V and VI, the accuracy of measuring the mesonic x-ray energy in recent years is much improved compared with the 1960s. However, since the beginning of the 1960s this energy has been measured in order to check the accuracy of quantum-electrodynamics calculations and for studying nuclear properties (to determine the nuclear radius, the nuclear quadrupole moment, and so on). Here the μ^- mass is taken equal to the μ^+ mass, which is measured with an accuracy roughly an order of magnitude greater.

The μ^+ mass is determined from experiments to measure the ratio of the μ^+ and proton magnetic moments. The most accurate data were obtained in measurements of the frequency of transitions between hyperfine-structure states in muonium (see Fig. 3). Since the μ^+ and e^- have spin equal to 1/2, muonium $\mathrm{Mu}=(\mu^+e^-)$ can be produced in two different states with total angular momentum $F=0$ and $F=1$. The splitting of these states can be estimated as

$$\Delta W=-\frac{32\mu_e\mu_\mu}{3r_{\mu e}^3}\approx 1.84\cdot 10^{-5}\ \mathrm{eV}$$

or

$$\Delta\nu=\Delta W/h\approx 4463\ \mathrm{MHz},$$

TABLE V. Masses of negative muons and pions determined from the mesonic x-ray energy.

Atom	m_μ/m_e	m_π/m_e	Reference
C, P, Si	206,93 ± 0,13	—	[14]
P	206,76 ± 0,02	—	[26,27]
C, P, Si	—	273,34 ± 0,13	[25]

TABLE VI. Precision measurement of the mesonic x-ray energy.

Nucleus	Transition	E, eV	$\Delta E/E$, ppm	Method	Reference, year
$^{31}P_\mu$	$3d_{5/2} \to 2p_{3/2}$	88013,3 ± 2,8	32	BCS	[28], 1979
$^{13}C_\mu$	$2p_{3/2} \to 1s_{1/2}$	75322,6 ± 1,0	13	BCS	[29], 1985
Mg_π	$4f \to 3d$	25436,133 ± 0,058	2,3	BCS	[30], 1986

where $r_{\mu e}$ is the Bohr radius of muonium, and μ_e and μ_μ are the electron and muon magnetic moments.

Calculations taking into account the radiative and relativistic corrections lead to the following theoretical value for $\Delta\nu$ (see, for example, Ref. 31 for more details):

$$\Delta\nu_{\text{theor}} = (\mu_\mu/\mu_p)(1.40207991 \pm 0.6ppm) \cdot 10^6 \text{ kHz}.$$

The quantities ν_{12}, the distance between levels 1 and 2, and ν_{34}, the distance between levels 3 and 4, are measured directly in experiment for a given value of the external magnetic field (see Fig. 3). For example, in the experiment of Ref. 32 the external magnetic field was close to 13.6 kG. The transitions corresponding to ν_{12} and ν_{34} were induced by microwave radiation of known wavelength. The values of ν_{12} and ν_{34} are used to determine $\Delta\nu$. The most accurate value of $\Delta\nu$ so far is that obtained in Ref. 32:

$$\Delta\nu_{\text{exp}} = 4463302.88(16) \text{ kHz},$$

i.e., the relative error is 0.036 ppm.

If the ratio of the muon and proton magnetic moments is known, then m_μ/m_e can be found from the following expression:

$$m_\mu/m_e = (g_\mu/2)(\mu_p/\mu_\mu)(\mu_e^{\text{B}}/\mu_p).$$

The value of g_μ is determined from independent experiments.

In Table VII we give the data on the measured μ^+ mass. There for comparison we also give m_π. We see that the accuracy of measuring μ^+ is about 8 times higher than that for m_π. It is expected that the accuracy of measuring μ^+ will be improved by a factor of about 1.5 in the near future. No significant improvement in the accuracy with which the π mass is measured is expected in the near future.

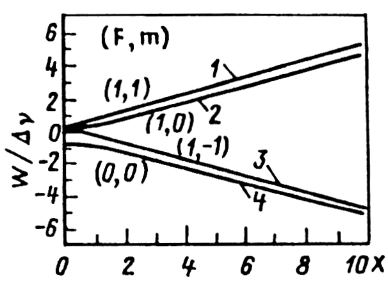

FIG. 3. Breit–Rabi diagram for the ground state of muonium in an external magnetic field. Along the horizontal axis is the magnetic field strength $x = H/H_0$ (H_0 is the typical field created by the muon at the electron in the $1s$ state).

The measurement of the distance between the hyperfine levels in the system $(He^{++}\mu^-e^-)$ is analogous to the measurement of $\Delta\nu$ in muonium, since $(He^{++}\mu^-)$ has charge equal to $+1$ and spin 1/2, the same as the μ^- spin. However, the theoretical calculations of $\Delta\nu$ for $(He^{++}\mu^-e^-)$ are complicated, owing to the extent of the pseudonucleus $(He^{++}\mu^-)$. At present the accuracy of these calculations is about 10^{-3} (see, for example, Ref. 31). The accuracy of measuring $\Delta\nu$ is about the same. Therefore, from the viewpoint of determining the μ^- mass, measurement of $\Delta\nu$ for $(He^{++}\mu^-e^-)$ is not interesting.

It should be noted that the value of $\Delta\nu$ for muonium is at present measured roughly an order of magnitude more accurately than the theoretical calculation.

2. MEASUREMENT OF m_{ν_μ}

Interest in the measurement of the $\bar{\nu}_e$ mass grew considerably after the well known work of the ITEP group.[39] Okun' writes:[40] "From the theoretical point of view, at present there is no reason to assume that the neutrino mass is zero. The views on this subject dominating among theoretical physicists have radically changed during the last decade. Earlier it was considered more natural to expect that the neutrino mass is zero rather than some small value. Now the widespread view is that the existence of a massless particle requires an exact local symmetry, and since there is no such symmetry in the case of the neutrino its mass need not be zero."

On the other hand, the existence of a neutrino mass could solve (a) the problems in the theory of galaxy formation and (b) the problems of massive invisible galactic coronas and galactic clustering.

There exists a cosmological limit on the masses of all types of neutrino, according to which $m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau}$ certainly cannot exceed 100 eV. However, this limit on the ν_μ and ν_τ masses ceases to operate if these particles decay sufficiently rapidly into $\nu_e + \gamma$. The experiment of Ref. 41, which was carried out at the Brookhaven accelerator at the suggestion of Pontecorvo, showed that the ν_e and ν_μ are not identical particles.

These facts indicate how important is the measurement of the ν_μ mass. In principle, the ν_μ mass can be determined from the following processes:

a) Decay of a π meson at rest $[\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)]$. Monochromatic muons with momentum of about 30 MeV/c are produced in the decay of a π meson at rest. Since the π and μ masses are measured in independent experiments, for determining the neutrino mass it is sufficient to measure the momentum of the muon from this

TABLE VII. The μ^+ and π^- masses.

Quantity	Data	Relative error, ppm	Reference
m_{μ^+}/m_e	206,766±0,005	24	[33], 1961
	206,765±0,002	10	[34], 1963
	206,7682±0,0005	2,4	[35], 1972
	206,76859±0,00029	1,4	[36], 1977
	206,76835±0,00011	0,53	[37], 1982
	206,768259±0,000062	0,3	[32], 1982
m_{μ^+}, MeV/c^2	105,658386±0,000044	0,42	[32], 1982
	105,658389±0,000034	0,32	[38], 1988*
m_{π^-}, MeVc^2	139,657550±0,000330	2,4	[38], 1988*
m_{π^-}/m_e	273,126770±0,000710	2,6	[30], 1986

*Preferred values.

reaction. From energy and momentum conservation ($m_\pi = \sqrt{m_\mu^2+P_\mu^2} + \sqrt{m_{\nu_\mu}^2+P_\nu^2}$, $P_\nu=P_\mu$, and $c=1$) for m_{ν_μ} we obtain

$$m_{\nu_\mu}=(m_\pi^2+m_\mu^2-2m_\pi\sqrt{m_\mu^2+P_\mu^2})^{1/2}.$$

In Table VIII we give the results of measurements of the muon momentum from π-meson decay and the data on m_π, m_μ, and P_μ used to find the upper limit on the neutrino mass. During the last 25 years the upper limit on m_{ν_μ} has been decreased by more than a factor of 18. As seen from the data on m_π, m_μ, and P_μ, at present it is P_μ that is measured with the largest error. The absolute value of the errors of measuring P_μ is about 2.5 times greater than that of m_π, i.e., to significantly improve the accuracy of deter-

TABLE VIII. Results of measurements of the momentum of the μ^+ from π^+ decay and data on m_μ and m_π used to determine the upper limit on the ν_μ mass.

Data	Relative error ppm	Reference, yr
P_μ = (29,80±0,04) MeV/c	1342	[42], 1964
m_π = (273,27±0,11) m_e	402	
m_μ = (206,76±0,02) m_e	97	
m_{ν_μ} < 4,6 MeV/c^2		
P_μ = (29,7873±0,0014) MeV/c	470	[45], 1978
m_π = (139,5667±0,0017) MeV/c^2	12	
m_μ = (105,65948±0,00035) MeV/c^2	3,3	
m_{ν_μ} < 0,56 MeV/c^2		
P_μ = (29,79139±0,00083) MeV/c	28	[46], 1984
m_π = (139,56761±0,00077) MeV/c^2	5,5	
m_μ = (105,65932±0,00029) MeV/c^2	2,6	
m_{ν_μ} < 0,25 MeV/c^2		
P_μ = (29,79139±0,00083) MeV/c	28	[46], 1984
m_π = (139,56752±0,00033) MeV/c^2	2,6	[30], 1986
m_μ = (105,658386±0,000044) MeV/c^2	0,4	
m_{ν_μ} < 0,26 MeV/c^2		

mining m_{ν_μ} (to decrease the upper limit on m_{ν_μ}) from this reaction it is necessary to measure P_μ and m_π more accurately.

b) The decay of a muon at rest, $\mu^\pm \to e^\pm + \nu_e + \nu_\mu$, can also be used to find m_{ν_μ}. Since three particles are produced in the final state, the electrons from the reaction can have energy in the range from zero to some value $E_e^{\max}$, which is realized when the two neutrinos are emitted in the direction opposite to that of the electron. In this case, assuming that the two neutral particles are identical, from energy and momentum conservation it follows that

$$m_{\nu_\mu} = \tfrac{1}{2}(m_\mu^2 + m_e^2 - 2m_\mu E_e^{\max})^{1/2}.$$

Using the measured result $E_e^{\max} = (53.00 \pm 0.32)$ MeV, in the early 1960s it was found that $m_{\nu_\mu} < 5m_e$ (see Ref. 42). A significant improvement of the measurement of m_{ν_μ} from this reaction cannot really be expected.

c) In principle, the reaction of nuclear capture of the muon from ^{3}He can be used to determine the neutrino mass. Helium does not form a molecule, and no difficulties arise from mesic molecular phenomena. A charged particle is produced in the reaction, which is convenient for detection and precise measurement of its energy. The tritium nucleus produced in this reaction must be monochromatic and have energy close to 1.90 MeV, which has been confirmed in the experiments of Zaïmidoroga *et al.*[43] However, the author does not know of more accurate measurements of the tritium energy using this reaction.

3. PERSPECTIVES FOR IMPROVING THE ACCURACY OF MEASURING THE π^-, μ^-, AND ν_μ MASSES

At the present time no experiments are being carried out to measure m_π in which there would be a significant improvement in the accuracy of m_π. There is an idea of how the accuracy of m_μ and m_{ν_μ} can in principle be improved. Since a new channel for obtaining low-energy muon beams with unique parameters is now being constructed at PSI in Switzerland,[44] improvement of the accuracy of measuring the μ^- mass by an order of magnitude or more does not seem as unlikely as it did a few years ago.

We see from the data quoted below that the expected beam will have small dispersion and transverse size regulated by the energy, and the intensity will be about 10^4 particles/s:

Intensity	$10^4 \mu^-$/s and $2 \cdot 10^4 \mu^+$/s;
Energy (variable)	10 eV–100 keV;
$\Delta P/P$	10^{-3};
Beam size at the output	Several micrometers at 100 keV; Several centimeters at 10 eV;
Emittance at 50 keV	$\pi \cdot 10^{-2}$ cm · mrad.

Although there are not yet any specific plans to measure m_μ and m_{ν_μ} in low-energy muon beams, this question is already being discussed in the literature and possible variants of experiments are being studied. There is a proposal[44] to obtain $(p\mu^-)$ atoms using beams of 15-eV muons and fluxes of atomic hydrogen with temperature 80 K and pressure 0.01 mm Hg in the region of intersection with the muon beam. Laser radiation collinear with the muon beam would be used to excite some of the muons captured at the level $(n,j) = (14,12\tfrac{1}{2})$ to the level $(n,j) = (16,14\tfrac{1}{2})$. The fact that the muons undergo a transition to the level $(16,14\tfrac{1}{2})$ induced by the laser radiation can be ascertained from the delay of the mesonic x-rays from the $2p \to 1s$ transition, since the lifetime of the $(16,14\tfrac{1}{2})$ level under the conditions of that experiment is estimated to be 1400 ns, while for the $(14,12\tfrac{1}{2})$ level it is close to 650 ns. This measurement makes it possible to determine m_μ with a relative error of no more than 10^{-8}. There is also discussion of a determination of the ν_μ mass with an accuracy of 30 keV using μ^- capture by ^{3}He in a thin layer of a cold target with subsequent accurate measurement of the tritium energy.

[1] C. D. Anderson and S. N. Neddermeyer, Phys. Rev. **50**, 263 (1936); **51**, 884 (1937); **54**, 88 (1938).

[2] J. S. Street and E. C. Stevenson, Phys. Rev. **52**, 1003 (1937).

[3] M. Conversi, E. Pancini, and O. Piccioni, Phys. Rev. **68**, 232 (1945).

[4] C. M. G. Lattes, H. Muirhead, G. P. S. Occhialini, and C. F. Powell, Nature **159**, 694 (1947).

[5] C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Nature **160**, 453, 486 (1947).

[6] C. S. Wu, E. Ambler, R. W. Hayward *et al.*, Phys. Rev. **105**, 1413 (1957).

[7] R. L. Garwin, L. M. Lederman, and M. Weinrich, Phys. Rev. **105**, 1415 (1957).

[8] J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

[9] W. H. Barkas, W. Brinbaum, and F. M. Smith, Phys. Rev. **101**, 778 (1956).

[10] L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).

[11] S. W. Flugge and F. T. Adler, Naturwiss. **40**, 601 (1953).

[12] G. E. Pustovalov, Zh. Eksp. Teor. Fiz. **36**, 1806 (1959) [Sov. Phys. JETP **9**, 1288 (1959)].

[13] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed. (Pergamon Press, Oxford, 1977) [Russian original, Nauka, Moscow, 1972, Vol. 2].

[14] S. Kozlov, V. Fitch, and J. Rainwater, Phys. Rev. **95**, 291 (1954).

[15] A. B. Mickelwait and C. H. Corben, Phys. Rev. **96**, 1145 (1954).

[16] L. D. Hill and K. W. Ford, Phys. Rev. **94**, 1617 (1954).

[17] R. Glauber, W. Rarita, and P. Schwed, Phys. Rev. **120**, 609 (1960).

[18] G. E. Pustovalov, Zh. Eksp. Teor. Fiz. **32**, 1519 (1957) [Sov. Phys. JETP **5**, 1234 (1957)].

[19] K. W. Ford and J. G. Wills, Nucl. Phys. **35**, 295 (1962).

[20] R. C. Barrett, Phys. Lett. **28B**, 93 (1968).

[21] B. Fricke, Lett. Nuovo Cimento **2**, 859 (1969).

[22] M. Chen, Phys. Rev. C **1**, 1167, 1176 (1970).

[23] H. L. Anderson, C. K. Hargrove, E. P. Hinks *et al.*, Phys. Rev. **187**, 1565 (1969).

[24] K. W. Ford and J. G. Wills, Phys. Rev. **185**, 1429 (1969).

[25] M. Stearns, M. B. Stearns, S. DeBenedetti, and L. Leipuner, Phys. Rev. **97**, 240 (1955).

[26] J. Lathrop, R. A. Lundy, V. L. Telegdi *et al.*, Nuovo Cimento **17**, 113 (1960).

[27] S. Devons, G. Gidal, L. M. Lederman, and G. Shapiro, Phys. Rev. Lett. **5**, 330 (1960).

[28] B. Aas, W. Beer, I. Beltrami *et al.*, Nucl. Phys. **A329**, 450 (1979).

[29] F. W. N. DeBoer, B. Aas, P. Baertschi *et al.*, Nucl. Phys. **A444**, 589 (1985).

[30] B. Jeckelmann, W. Beer, G. DeChambrier *et al.*, Nucl. Phys. **A457**, 709 (1986).

[31] V. W. Hughes, in *Exotic Atoms '79: Fundamental Interactions and Structure of Matter*, edited by K. Crowe, J. Duclos, G. Fiorentini, and G. Torelli (Plenum Press, New York, 1980), p. 3.

[32] F. G. Mariam, W. Beer, P. R. Bolton *et al.*, Phys. Rev. Lett. **49**, 993 (1982).

[33] D. P. Hutchinson, J. Menes, G. Shapiro *et al.*, Phys. Rev. Lett. **7**, 129 (1961).

[34] G. McD. Bingham, Nuovo Cimento **27**, 1352 (1963).

[35] K. M. Crowe, J. F. Hague, A. Schenck *et al.*, Phys. Rev. D **5**, 2145 (1972).
[36] D. E. Casperson, T. W. Crane, A. B. Denison *et al.*, Phys. Rev. Lett. **38**, 956 (1977).
[37] E. Klempt, R. Schulze, H. Wolf *et al.*, Phys. Rev. D **25**, 652 (1982).
[38] Particle Data Group, Phys. Lett. **239B**, VI.10 (1990).
[39] V. A. Lubimov, E. G. Novikov, V. Z. Nozik *et al.*, Phys. Lett. **94B**, 266 (1980).
[40] L. B. Okun', *Particle Physics*, Vols. 1 and 2, transl. of the 1st Russian ed. (Harwood, Chur, 1985) [Russian original, 2nd ed., Nauka, Moscow, 1988].
[41] G. Danby, J.-M. Gaillard, K. Goulianos *et al.*, Phys. Rev. Lett. **9**, 36 (1962).
[42] A. O. Vaĭsenberg, *Mu Mesons* [in Russian] (Nauka, Moscow, 1964).
[43] O. A. Zaĭmidoroga, M. M. Kulyukin, B. M. Pontecorvo *et al.*, Zh. Eksp. Teor. Fiz. **41**, 1804 (1961) [Sov. Phys. JETP **14**, 1283 (1962)].
[44] D. Taqqu, Nucl. Instrum. Methods A **247**, 288 (1986).
[45] M. Daum, G. H. Eaton, R. Frosch *et al.*, Phys. Lett. **74B**, 126 (1978).
[46] R. Abela, M. Daum, G. H. Eaton *et al.*, Phys. Lett. **146B**, 431 (1984).

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