Subthreshold π-meson production in ion collisions at intermediate energies

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The results of the experimental and theoretical study of pion subthreshold production and emission in collisions of heavy ions with energies from 25 to 100 MeV/nucleon are discussed. The methods of detecting charged and neutral mesons are described, and the main regularities in the experimental spectral and angular distributions are summarized. The various theoretical approaches used to study this phenomenon are classified and analyzed, and the possibilities that they offer for interpreting the experimental data are compared.

INTRODUCTION

During the last five to seven years intensive experimental and theoretical studies have been made of the physical processes accompanying ion–ion collisions at intermediate energies of the bombarding particles ($E_i/A_1 < 100$ MeV/nucleon; E_i is the relative kinetic energy of the incident ion, and A_1 is its mass number). This is facilitated both by the building of new heavy-ion accelerators (Darmstadt, Germany; Hanil, France; Michigan, USA, and others) and by the observation of many interesting physical phenomena at these energies, such as threshold production of π mesons, emission of high-energy photons, electron and positron emission, subthreshold nuclear fission, and so on.

This review is devoted to the results of experimental and theoretical studies of the subthreshold production and emission of π mesons in heavy-ion reactions at energies of $E_i/A_1 = 25$–100 MeV/nucleon. As in the case of high-energy γ emission (see, for example, Ref. 1), in this energy range in independent-particle models the one- and two-nucleon pion production mechanisms are either kinematically forbidden or have negligible probability (for example, if the internal Fermi motion in the colliding nuclei is taken into account). Therefore, an investigator of this problem is faced with a major question which must be answered: what is the physical mechanism by which a significant fraction of the nuclear collision energy is transferred to an individual particle, in this case, a π meson, which is not on the mass shell of the system?

Compared with the analogous process of high-energy photon production, the answer to this question is more complicated both from the experimental and from the theoretical point of view. From the viewpoint of experiment, the detection of π^0 mesons and the determination of their spectral and angular characteristics is difficult because the process has a relatively small probability (its cross section ranges from one to several nanobarns). Moreover, studies of the spectra of neutral or charged pions require rather different techniques, which in a number of cases make it difficult to compare the results directly (for example, there are almost no experimental data on the π-meson angular distributions).

The theoretical description of the phenomenon also encounters additional difficulties. First, the Hamiltonian of the πNN interaction leading to pion production is not as well defined as the electromagnetic one. It is well known that here there can be different types of coupling with very different coupling constants, and even the role of the individual terms of the nonrelativistic expansion is not completely clear.

Second, the π meson is a strongly interacting particle, and the nuclear medium can greatly affect its motion. In any case, in contrast to the photon emission process, this question requires additional study.

Third, the relatively large pion momentum also does not make the theoretical description any easier. From the viewpoint of the microscopic picture of the nucleus–nucleus collision, owing to its multiparticle nature it can be described by various approaches, which either explicitly or implicitly take into account the nucleon–nucleon interaction in the colliding ions and in the full system.

The complexity of the phenomenon and the possibility it offers for extracting fundamental information about the nuclear forces, the collision dynamics, and the pion production mechanism have recently generated a large number of experimental[2–15] and theoretical[16–55] studies, and their number continues to grow. Although the main experimental regularities are now known with some certainty, there are still problems, and their theoretical interpretation involves an enormous diversity of initial postulates and corresponding models. This raises the question of the classification of the various theoretical approaches, the demonstration of the fundamental nature of the basic assumptions, the adequacy of the theoretical description of the phenomenon, and the comparative evaluation of the results obtained. The present review is devoted to the theoretical aspects of the problem. However, for the sake of completeness we briefly review the experimental studies, emphasizing the methodological features of the detection of various types of π mesons and summarizing the main regularities in the pion spectra.

There are already some reviews of this nature. They have usually been given at various international confer-

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ences or schools dealing with the problem of subthreshold pion production.[4,56,57] However, for the most part these reviews have been purely informative and have not dealt with details or the comparative analysis of the various approaches. One review is that of Ref. 58, in which the experimental situation regarding this phenomenon (in 1986) is described in considerable detail, and a brief analysis of the theoretical studies carried out by that time is given.

In the present review we have not included the large amount of information which has appeared at various international conferences and seminars/schools, since often it is only preliminary; we have restricted ourselves to journal publications. We looked at all the studies known to us which were completed by 1990.

In Sec. 1 we analyze the methods of detecting neutral and charged pions and formulate the main regularities in the experimental spectral and angular distributions of emitted π mesons.

In Sec. 2 we give a detailed review of the theoretical approaches to explaining the mechanism of this reaction, and illustrate their possibilities in the description of specific experiments. Also in this section we describe a microscopic model which without the use of free parameters permits the corresponding differential cross sections to be calculated relatively easily, as in high-energy photon emission.

In the concluding section we formulate the main conclusions and suggest some problems for further study.

1. EXPERIMENTAL STUDIES OF SUBTHRESHOLD π-MESON PRODUCTION

Methods of detecting π mesons

Various methods are used to detect neutral and charged pions. π^0 mesons are identified by coincidence measurement of the two high-energy photons from the decay $\pi^0 \rightarrow \gamma + \gamma$, which occurs with 98.8% probability. The characteristics of charged mesons are determined either by a scintillation telescope or by a magnetic spectrometer.

Let us describe the experimental methods in more detail.

a. Neutral-pion detection. The π^0 meson lives a relatively short time ($\tau = 0.87 \times 10^{-16}$ sec), and it decays close to its creation point. Therefore, by measuring the characteristics of the two γ quanta in coincidence, it is possible to uniquely determine the energy and momentum direction of a created π^0 meson.

Neutral-pion measurements possess a number of advantages over the case of charged pions:[58]

1. Photons can be detected in a relatively large solid angle (5–10% of 4π), which naturally leads to the possibility of making measurements of integrated cross sections of order 1×10^{-33} cm^2.

2. The Coulomb field of the daughter nucleus does not distort the measured spectral and angular characteristics of π^0 mesons.

3. There is no lower energy threshold for π^0-meson detection. A π^0-meson creation event can be fixed practically at zero energy.

An experimental setup for π^0-meson detection is described in detail in Ref. 58; here we restrict ourselves to only a brief description.

The setup consists of 20 Čerenkov detecting telescopes made of lead glass. Each telescope, located a distance of 49 cm from the target, consists of a converter made of 5 cm-thick F2 glass, in which high-energy γ radiation is transformed into a shower, and a 35 cm-thick absorber. The entrance window of the telescope has dimensions 9.5×9.5 cm, and those of the absorber are 14.7×14.7 cm.

The telescope dimensions were chosen so that a shower propagating in the longitudinal and transverse directions is absorbed completely.

To suppress the cosmic-ray background the entrance of each telescope was covered by a 2.5 cm plastic Čerenkov radiator of area 12.5×12.5 cm.

Owing to the relatively small radiation length (about 10%), this detector was nearly unresponsive to γ quanta, and mainly detected charged particles. The plastic detector connected in anticoincidence with the telescope significantly reduces the cosmic-ray background.

The time resolution of the Čerenkov counter was 1.4×10^{-9} sec for $E_\gamma > 20$ MeV. The energy threshold for γ detection was 18 MeV per photon, and the energy resolution for $E_\gamma \geqslant 50$ MeV was $10/\sqrt{E_\gamma(\mathrm{GeV})}\%$. Measurement of the resolution and calibration of the spectrometer were carried out in a beam of tagged photons with energies of 20 to 120 MeV.

π^0-meson decay is characterized by the kinematic relation

$$m_{\mathrm{inv}} = 2(E_{\gamma 1} \cdot E_{\gamma 2})^{1/2} \cdot \sin(\varphi/2),$$

where φ is the relative emission angle of the 2 γ quanta in the lab frame.

The detection efficiency decreased with increasing π^0-meson energy, particularly for large emission angles relative to the beam axis.

In analyzing the experimental data two main sources of background are taken into account: cascade deexcitation of the nucleus and cosmic rays.

In deexcitation of the nucleus, γ quanta of relatively low energy (~ 1 MeV) are emitted, and their contribution is suppressed by the absorber–convertor threshold. In addition, the reconstruction of the invariant mass of the decayed π^0 meson makes it possible to get rid of false events.

The cosmic-ray background is induced by muons, which can generate showers in the surrounding materials. Nevertheless, their contribution is significantly suppressed. This is achieved by satisfying the following requirements: the number of coincidences in true events must be only 2, the angle must be $\varphi > 70°$, $70 \leqslant m_{\mathrm{inv}}c^2 \leqslant 200$ MeV, and there must be no signal in the plastic detector.

The results of Monte Carlo modeling showed that the response function of the detector to determining the π^0-meson energy is a Gaussian with parameter $\sigma(T_\pi)$ given by the expression

$$\sigma(T_\pi) = 2.5 + 0.233 T_\pi \ \ \mathrm{MeV}. \tag{1}$$

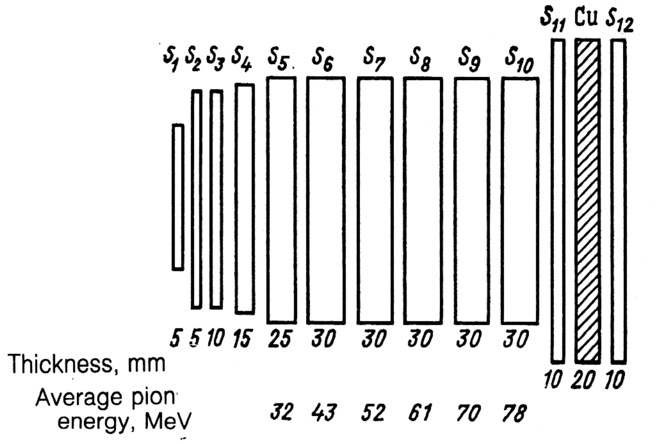

FIG. 1. Schematic view of a scintillation telescope for measuring the charged-pion energy.

The angular resolution of the γ detectors is about 8°. It can be improved by placing multiwire proportional chambers between the convertor and the absorber.[59]

In this case the energy resolution given by (1) can be increased by roughly a factor of 25. However, this leads to a decrease of several orders of magnitude in the π^0-meson detection efficiency. Therefore, this setup cannot be used to measure cross sections of order 10^{-33} cm^2.

b. Charged-pion detection. As mentioned above, charged mesons can be detected by two methods: a scintillation telescope and a magnetic spectrometer. The former is described in Refs. 9 and 10.

The scintillation telescope consists of 12 plastic scintillators encompassing a telescope angle of 0.010–0.012 sr (Fig. 1). The first four scintillators are used to suppress the intense proton background. The detectors S_5–S_{10} detect pions of energy in the range $27 \leqslant T_\pi \leqslant 82$ MeV. The pair of scintillators S_{11} and S_{12} with a copper absorber between them can measure pion energies greater than 82 MeV.

The scintillation telescope is placed in a vacuum chamber at angles of from 20 to 150° relative to the incident-beam direction.

The proton background is suppressed by comparing the amplitudes of signals in adjacent scintillators. The choice of a suitable level of signal discrimination makes it possible to get rid of 99% of the proton background. This suppression is sufficient for making measurements at angles $\theta_\pi \geqslant 27°$ with sensitivity to cross sections of order 0.1×10^{-30} cm^2/sec. The angular resolution of the telescopes was 3–4°.

To separate the π^+- and π^--meson yields we used two methods based on the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ with lifetime $\tau = 26 \times 10^{-9}$ sec. Here a μ^+ meson of energy 4.2 MeV appears.

In the first method the time between the signal from the π^+-meson stopping and the signal from detection of the μ^+ meson is fixed. This time is measured in the range $16 \times 10^{-9} \leqslant t \leqslant 100 \times 10^{-9}$ sec, with the lower limit determined by the speed of operation of the electronics. Extrapolation of the decay curve ($\sim e^{-t/\tau}$) to the time $t = 0$ gives the number of created π^+ mesons. Subtracting this number from the total number of stoppings, we find the number of π^- mesons.

The second method is based on the comparison of two integrated energy spectra, one measured immediately, and the other measured after a delay of $(10–20) \times 10^{-9}$ sec. The measurements using these methods gave consistent results.

In the analysis we introduced corrections for π-meson loss owing to decays in flight and nuclear reactions. These corrections varied from 20 to 40%, depending on the particle energy. The absolute value of the cross section was determined by measuring the current of the incident ion beam by a Faraday cylinder. The error in these measurements was of order 20%. The total error in the effective cross sections can be estimated to be 30% for π^+ mesons and somewhat higher for π^- mesons.

It should be noted that there are definite advantages in using the scintillation telescope owing to its relatively large luminosity and simplicity of construction. There are disadvantages associated with the restriction of the energy range to 27–82 MeV for π-meson detection.

Let us now consider detection by means of a magnetic spectrometer.[12]

Different groups have used different variants of the setup, some with constant and others with variable magnetic field.

The main advantage of magnetic spectrometers is the good energy resolution $\Delta p/p \sim (1–15)\%$ for momenta $p = 100–300$ MeV/c. Spectrometers with good energy resolution usually detect π mesons at an angle close to 0°, with a solid angle of order 10^{-3} sr. The energy resolution worsens with increasing pion emission angle.

Various methods are used to identify π mesons—from a scintillation four-component telescope[60] to a set of drift chambers located in front of and behind a magnet.[61] The path length can also be varied from 1.6 to 30 m. Owing to the decay time, this distance puts a lower limit of 25 MeV on the measured π-meson energy. The limit on the measured cross sections published in Ref. 12 was $\sim 10^{-36}$ cm^2/(sr·MeV·sec^{-1}).

Experimental results

The main results obtained so far are given in Table I. It should first of all be noted that practically all the measured pion spectra at energies above 30–40 MeV are described by an exponential dependence of the form $N_\pi(\varepsilon) \sim \exp(-\varepsilon/E_0)$. A maximum is seen at low energies in the neutral-pion spectra. This maximum moves to higher energies as the beam energy increases. As an example, in Fig. 2 we give the spectra of charged and neutral pions from the reaction $^{12}C + {}^{12}C$ at 85 MeV/nucleon (Ref. 11) obtained by integration over the pion emission angles. From Fig. 2 we see that the energy dependences and the total yields of charged and neutral pions are similar. This was expected, since at energies above 30 MeV reabsorption effects in light nuclei cannot significantly distort the shape of the spectrum.

The slope parameter E_0 of the spectrum, found by analyzing the neutral-pion production cross sections, lies

TABLE I. Characteristics of reactions involving intermediate- energy ions accompanied by pion emission.

Projectile	Energy E/A, MeV/nucleon	Target	Pion	Pion energy, MeV	Emission angle, deg	σ_π, 10^{-33} cm^2	E_0, MeV	Shape of the angular distribution	Reference
^{16}O	25	Al Ni	π^0	0—100	0—180	1,3±0,3 2,5±0,5	20,6±4,2	Maximum at 40–60° Forward–backward asymmetry about 90°	(Ref. 5)
^{14}N	35	Al Ni W	π^0	0—150	0—180	70±10 115±13 159±20	23±3	Maximum at 40–60° Nearly isotropic Minimum at 90°, forward–backward symmetry	(Ref. 6)
^{16}O	38	^{27}Al ^{197}Au	π^0	0—80	0—180	60±10 240±40	—	Raised at large angles Nearly isotropic	(Ref. 7)
Ar	44	Ca Sn U	π^0	0—100	0—180	$2,4\cdot10^3$ $3,7\cdot10^3$ $6\cdot10^3$	16	—	(Ref. 2)
^{12}C	60	^{12}C Ni U	π^0	0—150	0—180	$1,7\cdot10^3$ $7\cdot10^3$ $13\cdot10^3$	22 22 15	—	(Ref. 3)
	74	^{12}C Ni U	π^0	0—200	0—180	$8,5\cdot10^3$ $31\cdot10^3$ $64\cdot10^3$	25 27 26	—	(Ref. 3)
	84	^{12}C Ni U	π^0	0—200	0—180	$18,9\cdot10^3$ $72\cdot10^3$ $174\cdot10^3$	28 27 26	—	(Ref. 3)
^{12}C	60	^{7}Li ^{12}C ^{208}Pb	$\pi^{+,-}$	20—80	27 60 90 120	—	—	—	(Ref. 10)
	75	^{7}Li ^{12}C ^{208}Pb	$\pi^{+,-}$	20—80	27 60 90 120	—	—	—	(Ref. 10)
	85	^{7}Li ^{12}C ^{208}Pb	$\pi^{+,-}$	20—80	27 60 90 120	—	—	Sharp forward maximum	(Ref. 10)
	85	^{12}C ^{116}Sn ^{124}Sn	$\pi^{+,-}$	20—80	70	—	15	—	(Ref. 11)
	85	^{12}C Au	π^+	20—80	55 90 130	—	—	Cross section decreases with increasing pion angle and energy	(Ref. 9)
	86	^{6}Li ^{12}C ^{27}Al Cd Pb	π^-	60—180	0	—	—	—	(Ref. 12)
		^{12}C	π^+	60—180	0	—	—	—	(Ref. 12)
^{16}O	93	^{7}Li ^{12}C ^{27}Al ^{58}Ni Ag ^{197}Au ^{232}Th	$\pi^{+,-}$	—	0	—	—	—	(Ref. 15)
	94	^{27}Al	π^+	—	—	—	—	—	(Ref. 14)

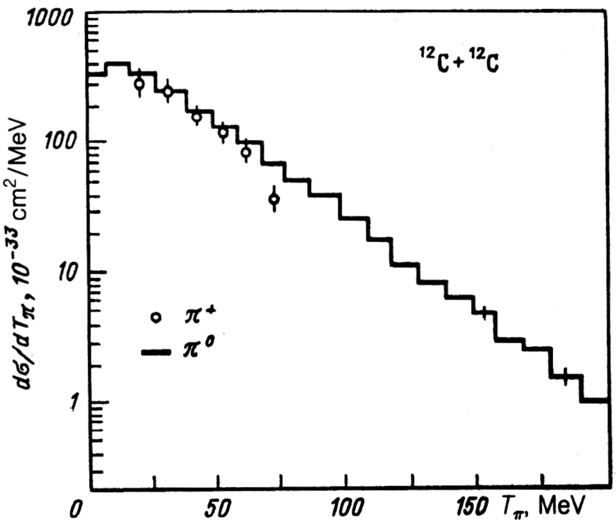

FIG. 2. Energy spectra of neutral (histogram) and charged (circles) pions for the reaction $^{12}C + ^{12}C$ at the energy 85 MeV/nucleon (Ref. 11).

mainly in the range from 20 to 25 MeV. In Fig. 3 we show the values of the parameter E_0 obtained from various reactions.

Some peculiarities were observed in the spectrum of π^0 mesons from the reaction $^{14}N + Ni$ at 35 MeV/nucleon. In addition to the exponentially falling dependence, a peak was observed at a π^0-meson energy of 140 MeV (Ref. 6). However, we note that the energy resolution of the spectrometer for detecting π^0 mesons of energy 140 MeV was about 25–30%, and it is possible to judge the reliability of the presence of this peak only after improving the apparatus resolution.

The angular distributions of both charged and neutral π^0 mesons created in collisions of ions of energy 85 MeV/nucleon have a sharp forward maximum. In the coordinate system with origin at the nucleon–nucleon center of mass the angular distribution becomes symmetric about 90°.

As the energy of the colliding ions decreases the situation becomes much less clear. For example, in studying the reaction $^{14}N + ^{27}Al \rightarrow \pi + X$(35 MeV/nucleon) the maximum yield is observed in the forward hemisphere,[6,7] while for the reaction $^{16}O + ^{27}Al \rightarrow \pi^0 + X$(38 MeV/nucleon) it is in the backward hemisphere. The data on the angular distribution of the pion from the reactions $^{16}O + ^{27}Al$ and $^{16}O + ^{58}Ni$ at 25 MeV/nucleon are just as contradictory. In Fig. 4 we show some π^0-meson angular distributions. It is difficult to see the source of the different angular behavior of the π^0-meson yields for these reactions. The only thing that is certain is that the range of nuclei studied must be extended significantly to obtain more reliable information.

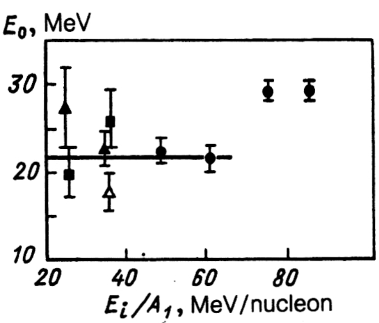

FIG. 3. Dependence of the slope parameter E_0 on the beam energy E_i/A_1 (from Ref. 58).

The behavior of the total cross section for pion production as a function of the mass numbers and energy of the colliding nuclei has been studied in Refs. 3, 4, 6, and 58.

To approximate the cross section as a function of mass number we used the expression $\sigma_\pi \sim (A_1/A_2)^{1/2}$. Analysis showed that a dependence of the form $(A_1/A_2)^{2/3}$ becomes preferable as the beam energy increases.

It follows from the experimental data that the integrated pion production cross section grows sharply with increasing energy of the colliding ions. At relatively low energies, near threshold, this dependence is quite nonlinear even on a logarithmic scale. However, as the energy increases (for $E_i/A_1 > 50$ MeV/nucleon) it can definitely be assumed that $\sigma_\pi \sim e^{\alpha\varepsilon}$. This behavior indicates that the systematics and predictions of the cross sections will apparently be fairly reliable at high beam energies, but will become quite uncertain near threshold.

2. THEORETICAL DESCRIPTION OF THE SUBTHRESHOLD PION EMISSION PROCESS

When analyzing this problem theoretically it is necessary to answer the question of what is the physical mechanism leading to π-meson emission in an ion–ion collision. One could start from the fact that the form of the πNN interaction Hamiltonian and the basic laws governing pion production in one- and two-nucleon reactions are fairly well known (see, for example, Refs. 62–66). However, a special feature of the present situation is that in the range of colliding- ion energies under study ($E_i/A_1 = 25$–100 MeV/nucleon) the one- and two-nucleon processes are deeply subthreshold. This unavoidably leads to the problem of transfer of the ion–ion collision energy to individual nucleons in order to make the π-meson production process above-threshold. Naturally, any microscopic approach and its solution must be based on some method of taking into account the internucleon interaction.

The manifestations of the nucleon–nucleon interaction in this phenomenon can be quite diverse and in some cases obscure. Examples of such cases are the production of a compound nucleus or "hot spots" of smaller size at the moment the systems combine. Evidence for this might be nucleon redistribution in energy or phase space owing to the inclusion of collision terms in the kinetic equation. This interaction leads to the formation during the collision process of a time-varying mean nuclear field or an internuclear optical potential, the excitation of certain coherent nuclear states, and so on.

However, it is far from always possible to uniquely relate the observed characteristics to the internucleon pa-

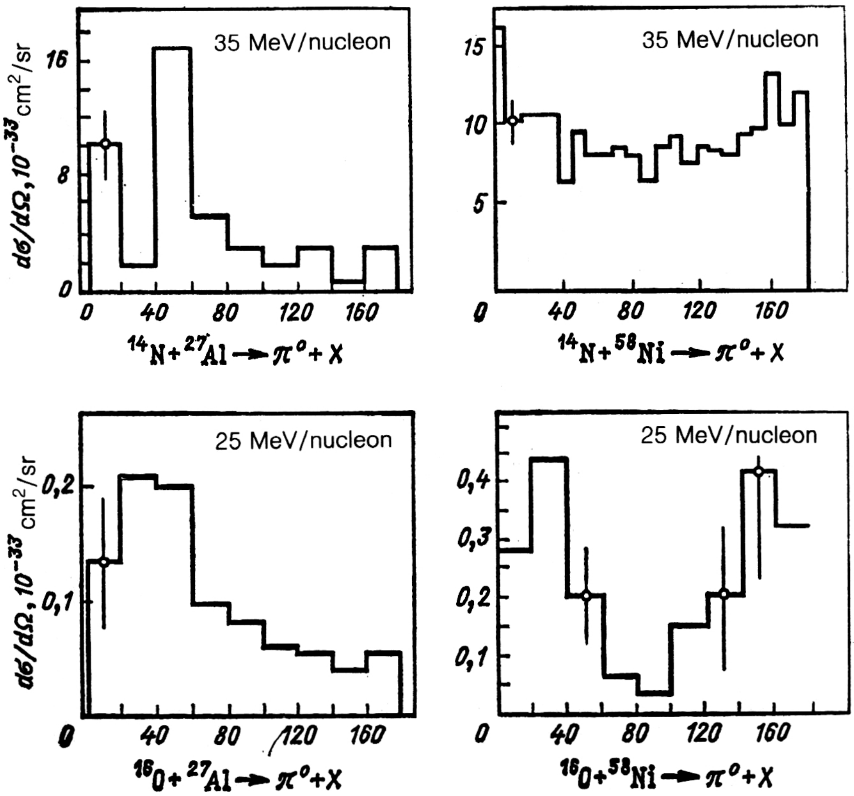

FIG. 4. Experimental angular distributions of π^0 mesons for the reactions $^{16}O + ^{27}Al$, ^{58}Ni ($E_i/A_1 = 25$ MeV/nucleon) and $^{14}N + ^{27}Al$, ^{58}Ni ($E_i/A_1 = 35$ MeV/nucleon) from Ref. 6.

rameters, and even more so to carry out a sufficiently accurate microscopic calculation. The fundamental multiparticle nature of the problem and the complexity of the quantum-mechanical description force the investigator to introduce models with varying degrees of microscopic nature and with some sort of parametrization of the observed phenomenon.

The models can be split into the following groups, depending on the basic postulates on which the model description of pion emission is based:

1) thermodynamical models;[16–23]

2) models based on individual nucleon–nucleon collisions;[24–30]

3) models based on the time-dependent mean-field approximation;[31–35]

4) coherent-source models;[36–51]

5) optical models.[52–55]

Of course, this classification of studies is rather arbitrary; some can also fall in other groups. For us the key feature was the means by which the observed pion characteristics are related to the nuclear structure parameters. We note that similar groups of models can also be distinguished in the theoretical analysis of the problem of high-energy electromagnetic radiation in such reactions.[1]

Below, we analyze the basic ideas and give the main results of each of these approaches. In the equations we use the system of units in which $\hbar = c = 1$.

Thermodynamical models

This group contains the approaches based on the assumption that heavy-ion reactions go through a compound- nucleus stage.[16–23] It is assumed that in this state there is thermodynamical equilibrium either for the entire system as a whole or for individual parts of it. This provides the basis for using the concepts of temperature and chemical potential, and for introducing various equilibrium distributions. The emission of high-energy particles (γ quanta, pions, nucleons, light fragments, and so on) is viewed as the result of the subsequent evaporation process. This description of nuclear reactions is widely used at low energies, and has been quite successful in explaining and parametrizing the main regularities of such phenomena.

The possibility of using the evaporation reaction mechanism at intermediate energies of the colliding ions is mainly justified by two experimental facts (see Sec. 1): the exponential falloff with increasing pion energy of the cross section for the subthreshold reaction and the nearly isotropic angular distributions of the emitted particles (the latter, however, is not firmly established, owing to the contradictions still inherent in the rather sparse experimental data). However, there is a significant difference from the low-energy region: for the compound nuclear system to be able to emit a π meson, the temperature of a hot spot must be much higher than that required for nucleon evaporation at low energies of the colliding nuclei. Naturally, the question arises of the possibility in principle for such quasiequilibrium states to appear during times on the nuclear scale. At the end of this section we discuss in more detail the main difficulties of the thermodynamical approach to the description of π-meson emission in nuclear reactions at intermediate energies.

A fundamental study in this area is that of Ref. 16, in which the evaporation mechanism was first applied to the subthreshold pion emission process at intermediate collision energies. Assuming that statistical equilibrium is reached very quickly in the compound system, and that the time needed for particle evaporation (estimated to be of order 5 F/c, where c is the speed of light) is much smaller than the characteristic times for various dissipative processes, the following expression (in the nonrelativistic limit) is used for the probability $W_{if}(\varepsilon)$ of emission of a particle of energy ε:

$$W_{if}(\varepsilon)d\varepsilon=\frac{(2s+1)M}{\pi^2}\frac{\rho(U)}{\rho(E)}\sigma_{ji}(\varepsilon)d\varepsilon. \quad (2)$$

Here s and M are the spin and mass of the emitted particle, $\rho(E)$ is the level density in the compound system, $\rho(U)$ is the level density in the system after particle evaporation, and $\sigma(\varepsilon)$ is the cross section for the inverse process leading to compound-nucleus formation via the same channel. Accordingly, the differential cross section for the process has the form

$$d\sigma/d\varepsilon=\sigma_0 W_{if}(\varepsilon)\left\{\sum_j\int W_{if}(\varepsilon_j)d\varepsilon_j\right\}^{-1},$$

where σ_0 is the cross section for compound-nucleus formation in the incoming channel, and the summation runs over the various types of evaporated particles.

For this type of description of the emission process, the main problem is to calculate the nuclear level density ρ and to parametrize the cross section $\sigma_{fi}(\varepsilon)$. Since in the reaction in question ($^{12}C + {}^{12}C \to \pi + X$ for $E_i/A_1 = 60$, 74, and 84 MeV/nucleon) the temperature of the compound nucleus is too high to use the standard expressions for the level density from the low-energy region, the level density is calculated directly in the Fermi-gas model. To determine the chemical potential of the system and its temperature and entropy we used the well known expressions of statistical thermodynamics at given values of the excitation energy, mass number, and nuclear density (the latter is taken to be that of the normal state, equal to 0.15 F^{-3}). In calculating the cross section for subthreshold pion production we also took into account the competing processes: evaporation of one or several nucleons at the initial stages. Of course, here the fulfillment of the general energy balance was taken into account.

An important problem in the calculations is the choice of cross section of the inverse process $\sigma_{fi}(\varepsilon)$. Whereas for nucleon evaporation the corresponding cross section can be assumed to be close to the geometrical one, the large Compton wavelength of the π meson makes this approximation inapplicable. In the present study we chose the parametrization in the form $\sigma(\varepsilon) = a(\Gamma/2)^2[(\varepsilon-\varepsilon_0)^2 + (\Gamma/2)^2]^{-1}$, and to find the parameters a, ε_0, and Γ we used the experimental data for the absorption reaction $\pi^0 + {}^{12}C$ (Ref. 67). Finally, the parameter σ_0 was treated as a fitted parameter, and its value was determined from the total cross section of the above reaction, and also that of the reaction $^{14}N + {}^{27}Al \to \pi^0 + X$ at 35 MeV/nucleon.

The results of calculations of the differential cross sections for pion emission are shown in Fig. 5. The good agreement between the theoretical results and experiment

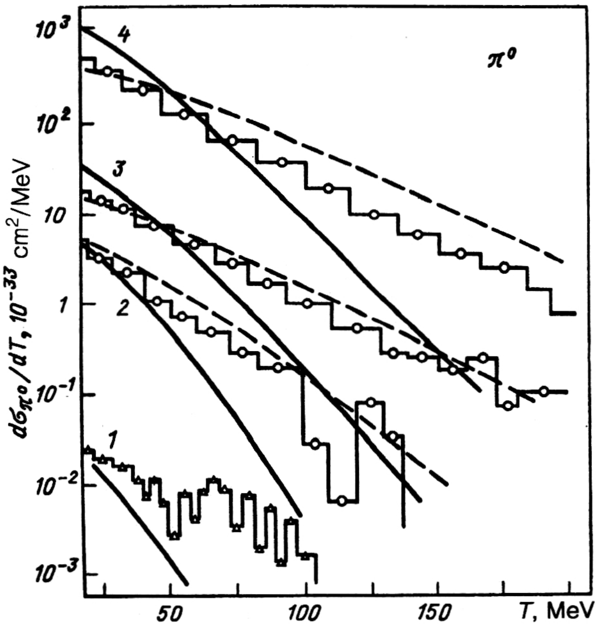

FIG. 5. Values of $d\sigma_{\pi^0}/dT$ for the reactions $^{14}N + {}^{58}Ni$ at $E_i/A_1 = 35$ MeV/nucleon (1) and $^{12}C + {}^{12}C$ for $E_i/A_1 = 60$ MeV/nucleon (2), 74 MeV/nucleon (3), and 84 MeV/nucleon (4). In cases 1 and 2, 3 the theoretical and experimental values are divided by 10^2 and 10, respectively. The dashed lines are the results of calculations from Ref. 16, and the solid lines are from Ref. 20.

leads to the conclusion that the evaporation mechanism can successfully compete with the other approaches to this problem.

In Ref. 17 the pion emission probability was estimated using a thermodynamical method somewhat different from that described above, but close to it in spirit. It has been used earlier to study nuclear reactions at high collision energies.[68,69] The method is based on the idea that the probability for emitting a particle or a fragment is mainly determined by only the available phase space and the conservation laws. Here an important component is the assumption that all states in the available phase space are equally probable. However, here again it is necessary to introduce the concepts of temperature and chemical potential, mainly to simplify the calculations. As was shown in Ref. 70, in many respects the standard thermodynamical approach and the evaporation variant of the model give similar final results. This is essentially a consequence of the equivalence of the canonical and microcanonical distributions for sufficiently large systems.

The authors of Ref. 17 used the thermodynamical method to calculate the total and differential cross sections for the π^0-meson emission process in the reaction $^{12}C + {}^{12}C$ and for the most part confirmed the conclusions of Ref. 16. However, they had to decrease the value of the uncalculated parameter σ_0 by roughly a factor of 3.

In Ref. 18 the statistical model developed for describing the multifragmentation process[71,72] was used to answer the question of whether or not pions must really be emitted during an early stage of the reaction, as is explicitly or implicitly assumed in the overwhelming majority of models. Owing to the large number of outgoing channels corresponding to the emission of various fragments in addition to the π meson, the statistical sums were calculated by the method of statistical sampling. The authors concluded that the pion must really be created in an early stage of the ion–ion collision, since this process and the multifragmentation process are incompatible: pions and fragments cannot coexist within the interaction volume.

The success of the first steps in the description of pion emission using the evaporation mechanism (and the similar thermodynamical approach) stimulated further development of the model formulated in Ref. 16. In Ref. 19 it was extended to the more complicated situation where a heavy nucleus is involved in the reaction. In this case the time needed for a nucleon to pass through the massive nucleus is several times greater than the decay time of the system. This makes it impossible to speak of a quasiequilibrium state of the compound nucleus as a whole. However, the ideology of Ref. 16 can be preserved if it is assumed that the combined system contains a strongly heated "hot source" (a hot spot or fireball) of limited size. As in Ref. 16, it was assumed that the nuclear density at this hot spot corresponds to that of the normal phase, and that the nucleon absorption cross section is given geometrically as πR_a^2, where R_a is the radius of the hot spot, and that the π-meson absorption cross section $\sigma_{fi}(\varepsilon)$ (the cross section for the inverse process) is parametrized so as to agree with the experimental data on π^+- and π^--meson absorption by nuclear matter in the normal state. As before, the only uncalculated parameter σ_0 (now the cross section for formation of the hot spot) was used as the fitted parameter in comparing the theoretical and experimental values of the total cross section σ_{π^0}.

The reactions $^{12}C + {}^{12}C$, Ni, U at energies of 60, 74, and 84 MeV/nucleon were studied. The spectral distributions were described well for the reaction $^{12}C + {}^{12}C$, but for the heavier targets the pion yield was much too high at the high-energy end of the spectrum.

The next step in the development of the model was taken in Ref. 20. The idea there was the following. In a collision of nuclei of greatly differing masses it is improbable to have only a single hot spot. The large cross-sectional area makes it possible in principle to form hot spots in various places in a cross section of the target nucleus, and the actual location of a hot spot will be determined by the values of the impact parameter. As a result, the total pion yield will be represented as the sum of yields from different hot spots corresponding to all possible impact parameters b:

$$\sigma_\pi = \int_0^{b_{\max}} N_\pi(b)\cdot 2\pi b db.$$

Here $N_\pi(b)$ is the number of π mesons emitted by an individual fireball:

$$N_\pi(b) = \int d\varepsilon W_{if}(b,\varepsilon)\left\{\sum_j \int W_{if}(b,\varepsilon_j)d\varepsilon_j\right\}^{-1}$$

and the probability $W_{if}(b,\varepsilon)$ is given by the same expression (2), but now the quantities U, E, and σ_{fi} are functions of the impact parameter b. The value of $b_{\max}$ was determined from the condition that the total energy of the fireball is $E(b) \geqslant m_\pi$ (m_π is the pion mass).

For each value of the impact parameter the number of nucleons in the fireball was determined by fixing the geometrical volume of the intersection of the two spheres corresponding to the interacting nuclei and by using the normal nuclear density. In fact, this approach made it possible to do without the fitted parameter σ_0. The further calculation of the nuclear level density was carried out using the degenerate Fermi-gas model by a scheme completely analogous to that of Ref. 16.

The method of parametrizing the pion absorption cross section (the cross section for the ineverse process σ_{fi}) was also changed somewhat. The expression obtained using the optical model in the plane-wave approximation[73] was used, together with the experimental data on π^+- and π^--meson absorption.

The total cross sections were calculated for ten reactions with incident-ion energies in the range from 25 to 84 MeV/nucleon. There was found to be satisfactory agreement with experiment when the masses of the colliding nuclei were close, but for strongly asymmetric systems the pion yield was too low. The authors attribute this to the calculated fireball temperatures being lower than required by experiment.

The spectral distributions for π-meson energies above 50 MeV were much worse, especially for large differences between the nuclear masses (the calculated pion yields are roughly an order of magnitude smaller than the experimental ones; see Fig. 5). The authors take this as evidence for a direct (nonstatistical) pion production mechanism in addition to evaporation mechanisms.

This scheme was modified further in Ref. 21 and also used to describe the high-energy photon emission process. The modifications were mainly made in the fireball creation and decay process, in particular, in its dimensions and energetics. Whereas in Ref. 20 the fireball dimensions were defined purely geometrically as the region of overlap of two spherical spatial volumes, in Ref. 21 the part of this volume corresponding to the overlap of the two spheres in momentum space was excluded. The Pauli principle was thereby taken into account. Its role was also incorporated in the definition of the fireball energy. The authors assume that the fireball can be formed only where nucleon–nucleon collisions play the dominant role. Therefore, when the energy of the hot spot is calculated it is necessary to exclude the contribution of those nucleons (or those single-particle dissipation processes) which in momentum space lie in the region forbidden by the Pauli principle. This cuts off small momenta of the nucleons in the incident nucleus and large momenta in the target nucleus. The rest of the calculation of the cross section for the process and its parametrization are the same as in Ref. 20.

The reactions $^{12}C + {}^{12}C$, ^{58}Ni, ^{238}U at energies between 48 and 84 MeV/nucleon accompanied by π-meson emission were studied. The authors think that reasonable agreement between the theoretical and experimental values was obtained for σ_{π^0} and $d\sigma_{\pi^0}/d\varepsilon$, although they can differ by a factor of 2–4. Meanwhile, comparison of the results of Refs. 20 and 21 shows that for heavy targets (Ni and U) the absolute values of the cross sections are increased by a factor of 3–4, which worsened the agreement with experiment. As before, in the spectral distributions the pion yield is observed to be too high at low energies and too low at high energies.

We recall that although fitted parameters are not introduced formally in Refs. 19–21, the calculations are still based on rather strong initial assumptions: the nucleon density in the fireball is assumed to be the same as that in normal nuclear matter, and the data for "cold" targets are used to parametrize the absorption cross sections (the inverse process).

Equation (2) was used in Ref. 22 to estimate the role of charge effects in the π^+- and π^--meson production process. According to (2), the probability ratio is

$$R_{\pi^-/\pi^+}=\frac{\rho^-(U)}{\rho^+(U)}\frac{\sigma_{\pi^-}}{\sigma_{\pi^+}}=\frac{\sigma_{\pi^-}}{\sigma_{\pi^+}}\exp\left(-2\frac{\mu_p-\mu_n}{T}\right),$$

where $\sigma_{\pi^{+(-)}}$ is the total cross section for $\pi^{+(-)}$-meson absorption by the compound nucleus and $\rho^{+(-)}(U)$ is the level density of the compound system after $\pi^{+(-)}$-meson emission; μ_p and μ_n are the chemical potentials. Their difference in the compound nucleus was found to be (for a collision of identical nuclei)

$$\mu_p-\mu_n=1.44\frac{Z}{r_0A^{1/3}}\cdot(\sqrt[3]{4}-1).$$

Here $r_0 = 1.25$ F, and Z and A are the charge and mass numbers of the target nucleus. The value obtained exactly coincides with the difference of the Coulomb energies of the two nuclei before colliding and the compound system, calculated for normal nuclear matter density. The temperature of the compound nucleus was determined from the slope of the experimental curves, and the cross sections σ_{π^+,π^-}, were taken from experiment and extrapolated in the corresponding manner to the required mass numbers.

Estimates of R_{π^-/π^+} were made for ^{12}C + ^{12}C reactions at 85 MeV/nucleon and La + La reactions at 138 and 246 MeV/nucleon. Satisfactory agreement with the experimental data was obtained.

A new method of applying the statistical approach to this process (see Refs. 17 and 18) was recently suggested in Ref. 23. Since pion production significantly decreases the excitation energy and temperature of the system, the main assumption of the method—that the energy per degree of freedom is much smaller than the total excitation energy of the system—is not satisfied. In order to overcome this difficulty, the authors of Ref. 23 suggested that two subsystems be distinguished in the statistical ensemble. One of them, described as a microcanonical ensemble, contains only those particles whose creation significantly changes the characteristics of the ensemble (π mesons, Δ resonances, α particles, and so on). The other is treated as a large Gibbs canonical ensemble and contains only the nucleon component. Its energy, momentum, and electric and baryon charges can be found from the corresponding conservation laws. In this approach the number of emitted π mesons with energy ε is given by the expression

$$\frac{d^3N_\pi}{dp_\pi^3}=\frac{V}{(2\pi)^3}\,\Omega_A(E^*-\varepsilon)/\Omega_{\rm tot}(E^*),$$

where V and E^* are the volume and excitation energy of the ensemble, $\Omega_A(E) = \rho(E)dE$ is the number of possible states of a system of A nucleons in the energy interval $E,E+dE$, and

$$\Omega_{\rm tot}(E^*)=\Omega_A(E^*)+\frac{3\cdot4\pi}{(2\pi)^3}\int\Omega_A(E^*-\varepsilon)p_\pi^2dp_\pi.$$

In this case ε is comparable with E^*, and the last term can be neglected. In this approximation the problem of the statistical description of pion emission reduces to that of determining the density of states of a two-component nucleon gas. The latter is solved using the usual statistical approach, which requires determination of the entropy of the system.

The pion spectral distributions were calculated for the reactions ^{12}C + ^{12}C, Ni, U at 84 MeV/nucleon, ^{40}Ar + Ca, Sn at 44 MeV/nucleon, and ^{14}N + Ni at 35 MeV/nucleon. Except for the latter, the agreement with the experimental data is good. However, the calculations involved a fitted parameter—the coefficient describing the transformation of the collision energy into thermal energy—and the nuclear density was taken to be 0.21 F^{-3}, higher than that of the normal state.

In summarizing the approach as a whole, we must state the following. The approach is based on the hypothesis of the fast development and equally fast pion decay (during the characteristic nuclear times) of a strongly heated and therefore unstable zone in a compound nucleus (or the entire compound nucleus). The thermodynamics of the development and subsequent evolution of essentially nonequilibrium states of this type is almost completely undeveloped at the present time. Therefore, the use, in the calculations, of the equations of equilibrium thermodynamics and also the characteristics of the normal state of nuclear matter appears unjustified. In view of this, the possibility of describing the spectral characteristics of pion emission for individual reactions in the studies described above can, in our opinion, be viewed only as a convenient parametrization of the process which unfortunately does not significantly enhance our understanding of its physical nature. Moreover, as was noted also in Ref. 21, the evaporation mechanism is completely incapable of explaining pion production in reactions with energy of the incident light ion (^{12}C, ^{14}N) of up to 35 MeV/nucleon, while from the viewpoint of experiment the pion spectra in this ion energy range differ little from the same spectra for $E_i/A_1>35$ MeV nucleon.

Models based on individual nucleon–nucleon collisions

Study of the kinematics of the collision of two free nucleons leading to π-meson production shows that in the c. m. frame the collision energy must exceed 135–140

MeV, or in the lab frame it must be at least 280–290 MeV. However, if the nucleons are components of a nucleus, as in ion–ion collisions, the picture of the NN collision becomes more complicated.

First, each nucleon now moves in the mean nuclear field, and its maximum momentum will be determined by the addition of the Fermi momentum and the momentum associated with the translational motion (the latter for a single particle). As was shown in Ref. 24, this redefinition significantly lowers the energy threshold of the reaction $N+N \to N+N+\pi$.

Second, in the collision of two nuclei the mean field acting on the nucleons changes with time, which, in turn, also changes the nucleon momentum distributions.

Third, for NN collisions in a nuclear medium it is necessary to take into account the Pauli principle when determining the particle final states.

The cross section $\sigma(NN \to NN\pi)$ was estimated in a simplified picture of the collision in Refs. 25 and 26. In Ref. 25 this was done using a model used for knockout reactions: a π meson and two free nucleons are formed as the result of a collision of an incident nucleon with a given momentum and a nucleon of the target nucleus located off the mass shell. It was also assumed that the pion production cross section in the nucleus coincides with that in empty space. It was shown that reasonable cross sections are obtained only for incident-ion energies above 150 MeV/nucleon. At lower energies the values of $\sigma(NN \to NN\pi)$ turned out to be several orders of magnitude too small compared with the experimental values.

This conclusion was essentially confirmed and even strengthened by the calculations of Ref. 26, where the authors studied a more realistic situation. In particular, the cross section was estimated in two models—the degenerate Fermi gas and the shell model with a three-dimensional oscillator potential for each nucleus. In both cases the cross section proved to be too small for the internucleon interaction to be neglected.

The next step in the realization of a picture of individual NN collisions was related to the inclusion of the time evolution of the nucleon one-particle distributions in the colliding nuclei. This was done using various modifications of the Boltzmann kinetic equation.[27–30] Its solution leads to time-dependent energy or momentum distributions taking into account various elastic and inelastic collision processes occurring in the nuclear medium.

In Ref. 27 the π^0-meson yield in heavy- ion reactions at intermediate energies was determined by solving the kinetic equation in the form (for the two-component Fermi-gas model)

$$\frac{dn_i^x}{dt}=\sum_y\Bigg\{\sum_{jkl}\big[\omega_{kl\to ij}^{xy}g_k^x g_l^y g_j^y n_k^x n_l^y(1-n_i^x)(1-n_j^y) - \omega_{ij\to kl}^{xy}g_j^y g_k^x g_l^y n_i^x n_j^y(1-n_k^x)(1-n_l^y)\big]\delta(\varepsilon_i^x+\varepsilon_j^y - \varepsilon_k^x-\varepsilon_l^y) - n_i^x\omega_{i\to i'}^x g_{i'}^x\delta(\varepsilon_{i'}^x-\varepsilon_i^x+\varepsilon_F^x+B_x)\Bigg\}. \quad (3)$$

The indices x and y refer to the type of nucleon (p or n), g_i^x is the density of one-particle states in an energy interval of 1 MeV having number i and central point ε_i^x (the numbering of the intervals begins from the bottom of the potential well); n_i^x is the number of levels filled at time t in the i-th energy interval, $\omega_{i\to i}^x$ is the probability per unit time for a transition of the nucleon x from the state i to the continuum state i', the energy $\varepsilon_{i'}^x$ is measured from the Fermi level ε_F^x; $i'=1,\ldots, E^*-B_x$; B_x is the binding energy of a nucleon of type x, and E^* is the excitation energy of the compound nucleus. The factor $\delta(\varepsilon_i^x+\varepsilon_j^y-\varepsilon_k^y-\varepsilon_l^y)$ ensures energy conservation in the elementary processes and is equal to unity when satisfied and zero otherwise. The quantity $\omega_{kl\to ij}^{xy}$ is the probability per unit time for the transition of nucleons of types x and y, respectively, from the states k,l to the states i, j. This was defined as

$$\omega_{kl\to ij}^{xy}=\sigma_{kl}[2(\varepsilon_k^x+\varepsilon_l^y)/m]^{1/2}\Bigg[V\sum_{q,r}g_q^x g_r^y\delta(\varepsilon_k^x+\varepsilon_l^y - \varepsilon_q^x-\varepsilon_r^y)\Bigg]^{-1},$$

where V is the nuclear volume, m is the nucleon mass, and σ_{kl} is the cross section for free NN scattering, which was parametrized in the standard manner in accordance with Ref. 74.

The presence of a time-dependent nucleon source was also taken into account in the solution of Eq. (3) as applied to an ion collision. It was assumed that some (or all) of the nucleons of a sufficiently light incident ion can be captured by the massive target. Here the approximation that the energy transferred by these nucleons to the compound system is distributed statistically among n_0 excitons at the initial stage was very important. It turned out that the results of the calculations were very sensitive to the value of this parameter of the theory. From analysis of the preequilibrium neutron spectra in ion–ion reactions and also from the results of the study of the pion spectra it was concluded that $n_0 \approx A_1$.

In determining the π^0-meson yield the following possible reaction channels were taken into account:

$$\begin{array}{ll} n+n\to n+n+\pi^0, & p+p\to p+p+\pi^0, \\ p+n\to p+n+\pi^0, & p+n\to d+\pi^0 \end{array} \quad (4)$$

The corresponding differential cross sections were parametrized on the basis of expressions obtained from analysis of the experimental data in Ref. 75. The number of π^0 mesons emitted per unit time was defined as

$$\frac{dN^\pi}{dt}=\sum_{ijklm}\{\omega_{ij\to klm}^{pp\pi^0}g_m^\pi g_i^p g_j^p n_i^p n_j^p(1-n_k^p)(1-n_l^p) + \omega_{ij\to klm}^{pn\pi^0}g_m^\pi g_i^p g_j^n n_i^p n_j^n(1-n_k^p)(1-n_l^n) + \omega_{ij\to klm}^{nn\pi^0}g_m^\pi g_i^n g_j^n n_i^n n_j^n(1-n_k^n)(1-n_l^n)\}.$$

Here $\omega_{ij\to klm}^{xy\pi^0}$ is the probability per unit time for a transition of nucleons of type x and y, respectively, from states i, j to states k, l with the simultaneous emission of a π^0 meson in a state m with energy ε_m^π:

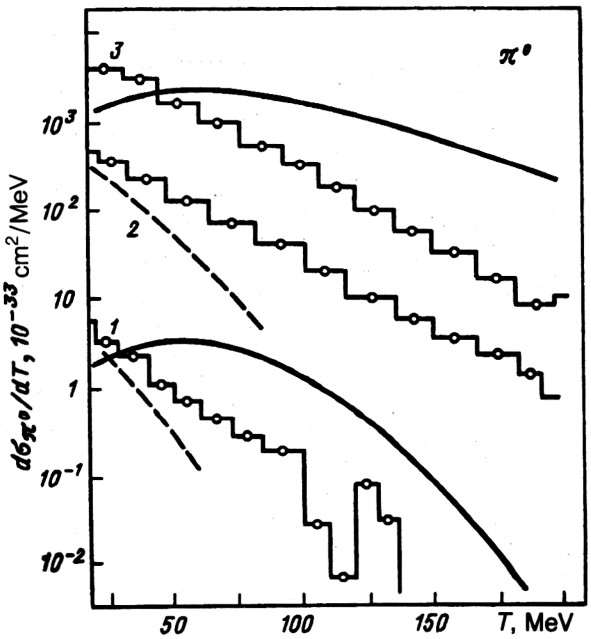

FIG. 6. Values of $d\sigma_{\pi^0}/dT$ for the reactions ^{12}C + ^{12}C at 60 MeV/nucleon (1), 84 MeV/nucleon (2), and ^{12}C + ^{238}U at 84 MeV/nucleon (3). In case (1) the theoretical and experimental values are divided by 10. The solid lines are the result of calculations from Ref. 27, and the dashed lines are from Ref. 30.

$$\omega^{xy\pi^0}_{ij\to klm}=\sigma^{xy\pi^0}_{kl}[2(\varepsilon^x_k+\varepsilon^y_l)/m]^{1/2}\left[V\sum_{q,r,p}g^x_q g^y_r g^\pi_p\delta(\varepsilon^x_i+\varepsilon^y_j-\varepsilon^x_q-\varepsilon^y_r-\varepsilon^\pi_p)\right]^{-1};$$

g^π_m is the density of final states of the π^0 meson.

This scheme was used to calculate the total cross sections for π^0-meson production for the reactions ^{12}C (60, 74, and 84 MeV/nucleon) + ^{12}C, ^{58}Ni, ^{238}U; ^{14}N (35 MeV/nucleon) + ^{27}Al, ^{58}Ni, ^{148}W, and ^{40}Ar (44 MeV/nucleon) + ^{40}Ca, ^{119}Sn, ^{238}U. It was found that the theoretical values of σ_π differ by a factor of about 2–3 from the experimental values, with the difference not being systematic.

The spectral distributions $d\sigma_{\pi^0}/dT$ (where T is the meson kinetic energy) were also calculated for the reactions ^{12}C (84 MeV/nucleon) + ^{12}C, ^{238}U, and ^{14}N (35 MeV/nucleon) + ^{58}Ni (Fig. 6). We see that $(d\sigma_{\pi^0}/dT)_{\rm theor}$ depends very weakly on the pion energy, as required by experiment, and that it exceeds the experimental value by more than an order of magnitude at the high-energy end of the spectrum. Since the fitted parameter n_0 had already been fixed by the total cross sections σ_{π^0}, the author attributed the observed discrepancy to enhancement of the role of Δ-resonance states when the pion energy increases.

The total cross sections σ_{π^0} for the reactions ^{12}C + ^{12}C, ^{58}Ni, ^{238}U and the energies 60, 74, and 84 MeV/nucleon were also calculated in Ref. 29 on the basis of the semiclassical variant of the Boltzmann kinetic equation with the introduction into it of the self-consistent nuclear field (the so-called Vlasov–Uehling–Unlenbeck equation). The one-particle phase-space distribution $f(\mathbf{r},\mathbf{p},t)$ taking into account the Pauli principle was found from the equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}-\frac{\partial U}{\partial \mathbf{r}}\cdot\frac{\partial f}{\partial \mathbf{p}}=(2\pi)^{-9}\int d^3p'd^3p_1d^3p_1'\cdot\sigma|\mathbf{v}-\mathbf{v}'|\times[ff'(1-f_1)(1-f_1')-f_1f_1'(1-f)(1-f')]\times(2\pi)^3\delta(\mathbf{p}+\mathbf{p}'-\mathbf{p}_1-\mathbf{p}_1'). \quad (5)$$

In the calculations the angular dependence of the nucleon–nucleon scattering cross section σ was neglected, and the cross section was taken to be 40×10^{-27} cm^2. The self-consistent nuclear field was given by the expression

$$U(\rho)=-356(\rho/\rho_0)+303(\rho/\rho_0)^{7/6},$$

calculated with Skyrme forces depending on the nuclear density ρ, and the initial nucleon momentum distribution was found in the Thomas–Fermi model (with U expressed in MeV).

The distributions obtained from Eq. (5) were used to calculate the total cross sections σ_{π^0}, and here the same elementary processes (4) as in Ref. 27 were taken into account. In addition, the possibility of excitation and decay of the $\Delta(3,3)$ resonance, which is not blocked by the Pauli principle, was taken into account. The analytic expressions from Ref. 62 and recent experimental data were used to parametrize the elementary cross sections.

The possible absorption of the π^0 meson and the Δ resonance by the nuclear medium was also taken into account phenomenologically. For this the absorption probability $P(x)=1-e^{-x/\lambda_0}$ was introduced, where $\lambda_0\approx3$ F is the mean free path of the meson or the Δ resonance and x is the path traveled inside the nuclear system. This effect turned out to be important. Whereas for the reactions listed above the theoretical values of σ_{π^0} are 2–3 times greater than the experimental values, the inclusion of absorption made it possible to decrease the pion yield by a factor of 2 on the average. As far as the angular and spectral distributions of π^0 mesons are concerned, they were not calculated, as noted in Ref. 29, owing to serious technical difficulties.

The problem of calculating the pion spectral distributions was solved in Ref. 30 using a scheme close to that of Ref. 29. The practical modifications involved the choice of functional form for the self-consistent nuclear field $[U(\rho)=-218(\rho/\rho_0)+164(\rho/\rho_0)^{4/3}$ (MeV)], a different choice of experimental elementary NN processes leading to π^0-meson production,[75] and the method of describing the production and decay of the $\Delta(3,3)$ resonance. In the latter case the cross sections were parametrized in accordance with Ref. 76. The same scheme was also used to take into account the possible absorption of π^0 mesons and Δ resonances by the nuclear matter, and the conclusion of Ref. 29 about the quantitative role of this effect (decrease of the total cross sections σ_{π^0} by roughly a factor of 2) was confirmed.

The author concluded that it is important to include the $\Delta(3,3)$ resonance in NN processes in calculating the pion spectra, but not the total cross sections. There are several reasons for this: the fact that the spin–isospin state is specific to the Δ resonance and therefore that the role of the Pauli principle in the nucleon medium is weakened, and the fact that the kinematics allowing a significant part of the available NN-collision energy to be transformed into the Δ mass with subsequent pion emission is favored. As a result, when the Δ resonance is included the π^0-meson yield grows markedly with increasing meson kinetic energy, approaching the experimental values.

However, careful study of the results of calculating $d\sigma_{\pi^0}/dT$ for the reactions ^{12}C (60, 74, and 84 MeV/nucleon) + ^{12}C (Fig. 6) does not lead to the conclusion that they are in satisfactory agreement with experiment. In spite of the fact that in calculations of the pion spectrum the author restricted himself to values $T<80$ MeV (in experiment $T=0$–200 MeV), even in this restricted interval the discrepancy between the theoretical and experimental yields at the high-energy end reaches an order of magnitude. In the opinion of the author, this is due to the limitations of the semiclassical nucleon momentum distribution in reproducing the high-energy components in the pion spectra, when purely quantum effects play an important role.

In giving an overall assessment of the approaches based on kinetic equations suggested in Refs. 27 and 30, we note the following. On the one hand, the solution of the time equations makes it possible to study the dynamics of the collision process and to determine the stage at which the emission of high-energy γ quanta or π mesons is most probable. In addition, in this approach it becomes possible to avoid the introduction of fitted parameters, and to use the accumulated information about the cross sections for elementary processes and the properties of the nuclear mean field.

However, on the other hand, the classical approach to the microscopic problem (as in Refs. 29 and 30) or the rejection of the actual nuclear field in favor of parameters without any clear physical justification (the Fermi-gas model and the number of excitons n_0 of the compound system in Refs. 27 and 28) actually do not permit the true depth of understanding of the physics of the process to be gauged. Moreover, the proposed approaches make it possible to distinguish only the incoherent component of the π-meson emission process, since they involve the summation of only the cross sections and not the amplitudes of the elementary processes. Another nontrivial feature is the complexity of realizing the computational scheme. All of this seriously hinders calculations of the spectral and angular distributions, on the one hand, and the study of reactions with incident ions more massive than ^{12}C and ^{14}N, on the other.

Models based on the time-dependent mean-field approximation

The models of the preceding section are closely related to approaches in which the time dynamics of the collision is treated differently, using the time-dependent Hartree–Fock approximation.[31–35] Its use makes it possible either to include dynamical distortions of the momentum and energy distributions of the nucleons in the varying self-consistent field of the two nuclei, or to directly calculate the amplitude of the pion production process with Hartree–Fock wave functions. The latter case, in principle, makes it possible to go beyond the inclusion of only incoherent NN collisions, while preserving the possibility of studying the time dynamics, as in approaches based on the kinetic equation.

The more or less final form of the microscopic approach to the description of the subthreshold π-meson production process on the basis of the time-dependent Hartree–Fock approximation was first formulated in Refs. 31 and 32. The proposed model was based on two main assumptions: a) the π meson is created in the process $N+N\to N+\Delta\to N+N+\pi$ in the first collision of two nucleons possessing sufficient energy; b) the initial and final states of the nucleons are determined by the time-dependent total nuclear field of the two nuclei.

The number of emitted ions N_π was defined as

$$2\varepsilon\frac{dN_\pi}{dk^3}=(2\pi)^{-3}\sum_n\left|\int d^3r dt e^{i(\varepsilon t-\mathbf{kr})}\times\langle\Phi_n(t)|\mathbf{j}(\mathbf{r})|\Phi_0(t)\rangle\right|^2. \tag{6}$$

Here ε and $\mathbf{k}$ are the meson energy and momentum, $\Phi_0(t)$ is the initial wave function of the system of two colliding nuclei, and $\Phi_n(t)$ are the corresponding multiparticle wave functions of the system, which are orthogonal to $\Phi_0(t)$. All these functions at the initial time are represented as Slater determinants for each nucleus separately, with the arguments in the single-particle wave functions pertaining to nucleons from different nuclei located along the relative internucleon separation $\mathbf{R}(t)$. The value of $\mathbf{R}(t)$ changes with time at a rate corresponding to the velocity of the relative motion of the colliding nuclei. Its minimum value is determined by the collision parameter b. The further evolution of the one-particle states is determined by the Hartree–Fock, time-dependent equation.

The total cross section for the process is obtained by integrating over collision parameters b:

$$\frac{d^3\sigma_\pi}{dk^3}=\int_0^\infty\frac{dN_\pi(b)}{dk^3}\cdot 2\pi b\,db.$$

The quantity $\mathbf{j}(\mathbf{r})$ in (6) is a function of the pion source. The results of Ref. 63, where reactions of the type (p,π^+) were studied in detail, were used to determine it. Of the two possible physical mechanisms leading to π^+-meson production—the one-nucleon mechanism of proton transition to a bound neutron state and the two-nucleon mechanism involving excitation and decay of a Δ-resonance state via the scheme $N+N\to N+\Delta\to N+N+\pi$, in the present study we consider only the latter. This is motivated by the more favorable kinematics of the process, in particular, the large momentum transfer. Naturally, such a momentum is more easily redistributed in a system of two nucleons.

An important component of the determination of $\mathbf{j}(\mathbf{r})$ was the approximation of the one-pion exchange potential by a two-dimensional δ function for the process $N+N\to N+\Delta$:

$$V_{NN\to N\Delta}(\mathbf{r},\mathbf{r}')\sim\int\frac{d^3q}{(2\pi)^3}\cdot e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')}(\mathbf{S}\cdot\mathbf{q})$$
$$\times(\sigma\cdot\mathbf{q})/(\mathbf{q}^2+\mu^2)$$
$$\approx S_z\sigma_z\delta(z-z')\Omega_\perp. \quad (7)$$

Here $\mathbf{S}$ is the spin matrix corresponding to the transition to the Δ resonance and $\Omega_\perp$ is the accessible volume of momentum space in the direction perpendicular to the beam axis [$\Omega_\perp \approx (2\pi R_0^2)^{-1}$ was used, where R_0 is the nuclear radius]. This same approximation was also used in Ref. 63, where higher energies of the colliding nucleons were considered. There it was justified by the fact that the momentum transfer in the process $NN\to N\Delta$ is almost parallel to the beam direction and much larger than the pion mass.

In order to carry out an actual calculation, the authors of that study restricted themselves to only a central collision with two-dimensional (axial) geometry for the model reaction ^{16}O + ^{16}O at $E_i^{(\mathrm{c.m.})}/A_1$ equal to 20, 30, and 40 MeV/nucleon. The calculation was methodological in nature and was not carried out for the purpose of comparison with any experiments. Nevertheless, the authors compared their results with the data on the forward pion yield obtained for the reaction Ne + NaF at $E_i/A_1 = 80$ and 164 MeV/nucleon, and found that the theory on the whole reproduces the experimental shape of the spectrum.

The model was later improved in Ref. 33 by doing away with the axial geometry and replacing the expression $\delta(z - z')\Omega_\perp$ in the one-pion exchange potential (7) by a three-dimensional δ function. This in turn made it possible to consider noncentral collisions and to study the dependence of the pion yield on the collision parameter b. It was found that the number of π mesons created in a collision is proportional to the volume of the geometrical intersection of the two nuclei. Moreover, analysis of the time picture showed that pion emission occurs at an early stage of the collision and, of particular importance, only a small number of nucleons participate in this process. This conclusion thereby does not support the initial premise of models using the classical bremsstrahlung mechanism for pion production. In those models it is assumed that all the nucleons of the colliding nuclei participate coherently in the process (see the following section).

The authors calculated the spectral distribution of π mesons at $\theta_\pi = 0°$ for collisions of oxygen ions with $E_i/A_1 = 80$ MeV/nucleon and compared it with the experimental result obtained for the reaction ^{12}C + ^{12}C for $E_i/A_1 = 84$ MeV/nucleon. It was found that the agreement is completely satisfactory in the hard part of the spectrum (for $k > 100$ MeV).

The Hartree–Fock method has also been used in the theoretical study of Ref. 34. The authors did not deal with the problem of describing any particular experiments on pion emission. They attempted to answer two questions: what is the microscopic picture of pion production in an NN collision, and can the nuclear collision dynamics be described sufficiently reliably without resorting to the full, very complicated Hartree–Fock calculations?

In studying the microscopic π-meson production process, the main terms of the πNN interaction Hamiltonian are singled out in the nonrelativistic expansion of the pseudoscalar coupling:

$$H_{\pi NN}\approx\frac{g}{2m}(\boldsymbol{\sigma}\cdot\mathbf{k})(\tau\Phi)+\frac{g^3}{2m}\Phi\Phi+\frac{ig^2}{8m^2}\tau(\Phi\times\dot{\Phi}).$$

Here g is the pseudocalar coupling constant, Φ is the pion field, $\mathbf{k}$ is the pion momentum, and $\boldsymbol{\sigma}$ and τ are the spin and isospin Pauli matrices. The first term corresponds to p-wave production of the meson, and the second and third terms to direct and charge-exchange s-wave pion rescattering. In addition, the Δ-resonance term $H_{\pi N\Delta} \sim (\mathbf{S}\cdot\mathbf{k}) \times(T\cdot\Phi)$ was also considered, where $\mathbf{S}$ and T are generalized (3/2) matrices. Quantitative calculations of the pion yield made for the case of a central collision of ^{16}O ions at energies of 40 and 80 MeV/nucleon showed that s-wave rescattering is important at relatively low π-meson energies and can even give the dominant contribution to the cross section for the process at $T_\pi < 30$ MeV.

An original method has been developed for studying the collision dynamics of multiparticle nuclear systems. The mean field of the colliding nuclei is modeled using two three-dimensional harmonic oscillators transformed into coordinate systems moving toward each other. The time dependence of the collective and single-particle characteristics which are introduced is taken from comparison with the exact calculation in the Hartree–Fock approximation with Skyrme forces. But the special feature of this scheme is the fact that such self-consistent calculations are carried out only once. This greatly simplifies the problem, since it allows the initial conditions to be varied. Study of the reaction with ^{16}O ions has shown that all the pion production mechanisms considered above lead to exchange processes along the collision axis. This serves as the justification for the approximations which were used earlier in Refs. 31 and 32 [see the discussion of Eq. (7)].

Whereas direct microscopic calculations of the amplitude of the subthreshold π-meson production process were carried out in Refs. 31–34, in Ref. 35 the Hartree–Fock wave functions were used to find the time-dependent single-particle density matrix $\rho(\mathbf{r},\mathbf{r}',t,t')$. Then it was used to calculate the nucleon single-particle phase-space distribution $f(\mathbf{r},\mathbf{p},\omega,t)$:

$$f(\mathbf{r},\mathbf{p},\omega,t)=\int d\tau\int d^3s e^{-i(\mathbf{p}\cdot\mathbf{s}-\omega\tau)}\rho(\mathbf{r},\mathbf{r}',t,t').$$

Here $\mathbf{s}=\mathbf{r}-\mathbf{r}'$ and $\tau=t-t'$. This function was then averaged over the volume of the reaction zone and the reaction time (which are parmeters of the theory), after which the momentum distribution $f(\mathbf{p})$ obtained in this way was used to determine the π-meson yield:

$$\frac{d^2N_\pi}{d\varepsilon d\Omega_\pi}=\int d^3p_1d^3p_2f(\mathbf{p}_1)f(\mathbf{p}_2)$$
$$\times\int\frac{d\Omega_q}{4\pi}\frac{|\mathbf{k}|}{|\mathbf{k}'|}\left(\frac{d^2\sigma_\pi}{d\varepsilon' d\Omega'_\pi}\right)_{\text{c.m.}}$$
$$\times[\sigma_{NN}(E_{\text{c.m.}})]^{-1}(1-f(\mathbf{p}'_1))$$
$$\times(1-f(\mathbf{p}'_2)), \tag{8}$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the pion momenta in the lab and c.m. frames, respectively, σ_{NN} is the NN-collision cross section, and $(d^2\sigma_\pi/d\varepsilon' d\Omega_\pi)_{\text{c.m.}}$ is the differential cross section for π-meson production in an elementary NN collision in the c.m. frame. It was assumed that it is isotropic, and the dependence on the meson energy was neglected. The possible absorption of pions by the nuclear medium was also taken into account phenomenologically in the calculations. As in Refs. 29 and 30 (see above), this was done by introducing the pion absorption probability for a path x: $P(x) = 1 - e^{-x/\lambda_0}$, where λ_0 is the mean free path.

As is easily seen from (8), the proposed model can be used to study only the mechanism of π-meson production via incoherent NN collisions. In this regard the approach is closely related to those considered above (see Refs. 27–30) and differs from them only in how the momentum distribution is obtained.

Calculations of the total cross sections σ_π for a number of reactions have shown that the experimental dependence of σ_π on the incident- ion energy can be satisfactorily reproduced for $\lambda_0 = 5$ F. From this fact the author concludes that the contribution of coherent pion production processes is unimportant. However, it seems to us that theoretical study of only the total cross sections without analyzing the spectral and angular distributions is still insufficient for such categorical conclusions.

In evaluating this approach as a whole, we note that it is fundamental both for studying the microscopic nature of the π-meson production process in an elementary NN collision and for the quantum-mechanical calculation of the time picture of an ion–ion collision. However, on the other hand, this fundamental nature has a serious drawback: it leads to enormous computational difficulties in its practical realization. It is therefore not surprising that it has not proved possible in even a single study to make further progress in the model problem of a collision of oxygen ions having closed nucleon shells. However, also in this case the numerical calculations of the cross sections have for the most part been carried out with a simplified geometry.

COHERENT-SOURCE MODELS

Let us consider several models which differ widely in the method of solving this problem, but which are based on a common initial assumption: that pion production is a collective event in which several (or all) nucleons of the colliding nuclei participate coherently.

In Refs. 36 and 37 the "cooperative" model was proposed. It occupies an intermediate position between single-nucleon collision models and models of compound-state formation. It is assumed that the kinetic energy of colliding nuclei of A and B nucleons is redistributed as a result of the interaction among individual clusters consisting of M and N nucleons. The collision of the clusters also leads to pion production.

In this model the cross section for production of a particle of type λ with energy E_λ and momentum $\mathbf{p}_\lambda$ is given by

$$E_\lambda\frac{d^3\sigma_\lambda}{d^3\mathbf{p}_\lambda}=\sum_{M,N}\sigma_{AB}(M,N)F_{MN}^{(\lambda)}(\mathbf{p}_\lambda).$$

Here $\sigma_{AB}(M,N)$ is the relative probability of forming virtual clusters, and $F_{MN}^{(\lambda)}(\mathbf{p}_\lambda)$ is the partial probability for production of a particle of momentum $\mathbf{p}_\lambda$ by the clusters M and N.

The quantity $\sigma_{AB}(M,N)$ is calculated using the cascade approach. The cross section of an individual NN collision is taken to be $\sigma_0 = 120 \times 10^{-27}$ cm^2. The statistical limit is essentially used: all states leading to cluster formation and satisfying the energy, momentum, etc. conservation laws are assumed to be equally probable. In turn, the spectral factor $F_{MN}^{(\lambda)}(\mathbf{p}_\lambda)$ is determined by the number of final states, with the level density depending on the volume V_{MN} of the system. It is assumed that the volume V_{MN} is related to the number of nucleons as $V_{MN}\cdot\rho = M + N$, where ρ is a parameter of the theory. It is taken to be

$$\rho=\rho_0=0.17\ \text{F}^{-3}.$$

In addition to π-meson production, the model allows production of any complex compound systems.

The spectral factor $F_{MN}^{(\pi)}(\mathbf{p}_\pi)$ given by

$$F_{MN}^{(\pi)}(\mathbf{p}_\pi)=\int d^3p_Ad^3p_B\omega_M(\mathbf{p}_M)\cdot\omega_N(\mathbf{p}_N)\Phi^{(\pi)}(\mathbf{p}_\pi),$$

where $\mathbf{p}_{A(M)}$ and $\mathbf{p}_{B(N)}$ are the momenta of the nuclei (clusters) A (M) and B (N), $\omega(\mathbf{p})$ is the momentum distribution function, and $\Phi^{(\pi)}(\mathbf{p}_\pi)$ is the number of states leading to π-meson production relative to the total number of final states. The function $\omega(\mathbf{p})$ is taken to be a normal distribution of width

$$\sigma^2_{AM}=\tfrac{1}{5}(p_F^{(A)})^2M(A-M)/(A-1);$$

$$\sigma^2_{BN}=\tfrac{1}{5}(p_F^{(B)})^2N(B-N)/(B-1),$$

where $p_F^{(A)}$ and $p_F^{(B)}$ are the single-nucleon Fermi momenta of the nuclei A and B.

The model proposed in Ref. 36 was used to calculate the doubly differential cross sections for π-meson production in the interaction of the ^{12}C nucleus at energies of 75–85 MeV/nucleon with various targets, and to compare them with experiment. It proved possible to give a good reproduction of both the absolute yield and the shape of the π^0-meson energy spectrum for various values of the angles. The results of the calculations for lower incident-ion energies (35–44 MeV/nucleon) indicate a significant decrease of the theoretical π-meson yields, especially with increasing energy. As was noted in Ref. 37, for $E_i/A_1 = 30$–40 MeV/nucleon it is necessary to take into account the effect of the nuclear mean field, which was neglected in Ref. 36.

The calculations carried out by the same authors[37] for the reaction $^{16}O + {}^{27}Al \rightarrow \pi + X$ at 25 MeV/nucleon also led to a pion yield smaller than experiment (by roughly a factor of 5).

The authors of Ref. 38 attempted to analyze the reactions $^{16}O + {}^{27}Al$, ^{197}Au for $E_i/A_1 = 38$ MeV/nucleon and $^{27}Al + {}^{20}Ne$ for $E_i/A_1 = 200$ MeV/nucleon on the basis of the collective-motion model, in which the nuclei were treated as indivisible particles. However, the absolute π^0-meson yields could not be calculated, owing to the large uncertainty in the parameters characterizing the population of the nuclear final states after pion production.

In the model proposed in Refs. 39–43 π-meson production is treated as a classical bremsstrahlung emission process of one nucleus in the field of the other. The number N_π of π mesons emitted by a nucleus moving along the trajectory $\mathbf{R}(t)$ is given by the expression

$$\frac{d^2N_\pi}{d\varepsilon d\Omega} = \frac{g^2k}{36\pi^3m}\left|\sum_{i=1,2}\int_{-\infty}^{+\infty} dt \mathbf{j}_\pi(\mathbf{v}_i,\mathbf{k},t)e^{-i(\varepsilon t-\mathbf{k}\mathbf{R}_i)}\right|^2. \quad (9)$$

Here $\mathbf{k}$ and ε are the pion momentum and energy, $g^2/4\pi = 14$, m is the nucleon mass, and $\mathbf{v}_i = \dot{\mathbf{R}}_i(t)$. The index $i = 1$ corresponds to the incident ion and $i = 2$ corresponds to the ion at rest; $\mathbf{j}_\pi$ is the nucleon current leading to π-meson production:

$$\mathbf{j}_\pi\langle S\rangle\langle T\rangle\rho_i(\mathbf{p}_i)\left\{\gamma_i\mathbf{v}_i\left[\varepsilon-\frac{\gamma_i}{1+\gamma_i}\mathbf{k}\cdot\mathbf{v}_i\right]-\mathbf{k}\right\}\gamma_i^{-1},$$

where $\mathbf{p}_i^2 = \mathbf{k}^2 - (\mathbf{k}\cdot\mathbf{v}_i)^2$, $\gamma_i = (1 - v_i^2)^{-1/2}$, $\rho_i(\mathbf{p}_i)$ is a Fourier component of the nuclear density, and $\langle S\rangle$ and $\langle T\rangle$ are the effective spin and isospin of the nuclei participating in the reaction. They are calculated using several assumptions. The point is that the spins and isospin of the ground states of the colliding nuclei are either zero or quite small. In the collision various values of S and T become energetically allowed. The distributions $S(t)$ and $T(t)$ are, in general, complicated functions of the time, but as a first approximation in Ref. 43 they were replaced by the average values calculated in the statistical model:[44]

$$\langle S\rangle = \langle T\rangle = (A_{\text{eff}}/3)^{1/2};\quad A_{\text{eff}} = \begin{cases} A_1 + A_1^{2/3}A_2^{1/3} & \text{or} \\ (2+\alpha)A_1, \end{cases}$$

where A_1 and A_2 are the mass numbers of the incident particle and the target nucleus; $\alpha \ll 1$ is a parameter of the model.

The cross section for π-meson production is obtained by integrating (9) over all possible values of the impact parameter:

$$\frac{d^2\sigma_\pi}{d\varepsilon d\Omega} = \int_0^\infty \frac{d^2N_\pi(b)}{d\varepsilon d\Omega}2\pi b db. \quad (10)$$

To calculate the cross section it is necessary to find the equations of motion of the centers of mass $\mathbf{R}_i(t)$ of each of the ions. For this the classical equations of motion are solved, taking into account the nuclear and Coulomb ion–ion interactions. The internuclear potential was parametrized as in Ref. 77.

To obtain effective bremsstrahlung leading to π-meson emission, in Ref. 43 it was assumed that between the two nuclei there acts the friction force

$$\mathbf{F}(\mathbf{R},\mathbf{v}) = -k\int d^3r\rho_1(\mathbf{r})\rho_2(\mathbf{r}-\mathbf{R})[\mathbf{v}_1(\mathbf{r}) - \mathbf{v}_2(\mathbf{r}-\mathbf{R})],$$

where ρ_i and $\mathbf{v}_i$ are, respectively, the nuclear density and velocity, and k is a free parameter of the theory. It was taken to be 5000 MeV$\cdot$F^2.

Equations (9) and (10) were used to calculate the doubly differential spectra of π mesons created in ion collisions at energies between 60 and 84 MeV/nucleon.

It proved possible to obtain agreement with the values of the total cross sections for the reaction $^{12}C + X$, where X are various nuclei, as functions of the beam energy and the target mass. However, the calculated shape of the spectrum for the reaction $^{12}C + {}^{12}C$ turned out to be steeper than that measured experimentally. It should be noted that to explain the experimentally observed angular distribution it was necessary to make additional assumptions about the interference of the amplitudes for π-meson emission by the two colliding nuclei.

The proposed model gives a good reproduction of the yield of pions with energy $T_\pi \lesssim 100$ MeV. At higher energies the authors of the mechanism proposed in Ref. 43 think that it is necessary to include not only friction, but also other possible energy-dissipation channels.

The authors of Refs. 45–49 developed a microscopic model of π-meson production based on excitation of isobar-analog resonances. The principal intermediate state was taken to be the $\Delta(3,3)$ resonance of energy 1232 MeV. The process is described by the diagram shown in Fig. 7.

The isobar-analog resonances are constructed in the particle–hole model using the Tamm–Dankcoff approximation.

States of both the incident nucleus and the target nucleus are excited simultaneously in the reaction. It is assumed that the nuclei interact with each other only at their surfaces, without penetrating each other. The peripheral interaction can excite collective states like the $M1$ resonance.

The pion production process is calculated using second-order perturbation theory. In this approximation the probability for the process is given by[47]

$$W_{if} = 2\pi\frac{|\sum_n\langle f|V|n\rangle\langle n|V|i\rangle|^2}{(E_0-E_f)^2+(\Gamma_0/2)^2}\rho(E_f), \quad (11)$$

where $\rho(E_f)$ is the density of final states and V is the internuclear interaction potential. In obtaining (11) it was assumed that all the energies $E_n = E_0$ and the widths $\Gamma_n = \Gamma_0$ of the intermediate states are identical.

Taking the Δ isobar as the intermediate state and remembering that there are three particles in the final state, the differential cross section can be transformed to

$$\frac{d^3\sigma_\pi}{dk^3}=\frac{2\pi}{v_1}\int\frac{|\sum n\langle f|V_2|n\rangle\langle n|V_1|i\rangle^2}{(\varepsilon+m+m_\Delta)^2+(\Gamma_\Delta/2)|^2}\frac{d^3p_2}{(2\pi)^3}$$
$$\times\delta(E_f-E_i).$$

The intermediate nuclear state $|n\rangle$ was constructed as the sum of particle–hole states of the incident and target nuclei. Here there was assumed to be production of an isobar–hole pair in the incident nucleus and a nucleon–hole pair in the target nucleus.

The interaction V_1 leading to excitation of nuclear states was taken to be

$$V_1=\sum_{k=1}^{A_1}\sum_{l=1}^{A_2}v_{kl}(|\mathbf{r}+\xi_k-\xi_l|)(\mathbf{S}_k\cdot\boldsymbol{\sigma}_l)(\mathbf{T}_k\cdot\boldsymbol{\tau}_l),$$

where $\mathbf{r}$ is the distance between the centers of mass of the colliding nuclei, ξ_k and ξ_l are the internal coordinates of the nucleons, and $\mathbf{S}$ and $\mathbf{T}$ are the Δ-isobar spin and isospin operators. In the calculation of the matrix elements it was assumed that the spatial matrix elements are identical for all single-particle quantum numbers. In this approximation the integrations over $\mathbf{r}$ and ξ are carried out independently:

$$\langle|v_{kl}(|\mathbf{r}+\xi_k-\xi_l|)|\rangle\sim v(\mathbf{q})F(\mathbf{q}),$$

where $\mathbf{q}$ is the momentum transfer, $v(\mathbf{q})$ is a Fourier component of the NN-interaction potential, and $F(\mathbf{q})$ is the nuclear form factor.

The interaction V_2 leads to Δ-isobar decay with pion emission. The expression for V_2 was obtained using the nonrelativistic variant of the πNN interaction:

$$H_{\pi NN}=2\sqrt{4\pi}(f/\mu)\int d^3\xi_\pi\rho(|\xi_\pi-\xi_N|)\mathbf{T}\cdot(\mathbf{S}\cdot\nabla\Phi(\xi_\pi)),$$

where $f^2=0.08$, ξ_π and ξ_N are the pion and nucleon coordinates, and $\rho(\xi)$ is the nuclear density distribution. Finally, the pion production cross section is expressed in terms of the product of three form factors: $v(\mathbf{q})$, $F_1(\mathbf{k})$, and $F_2(\mathbf{q})$.

Numerical calculations of the cross sections for neutral- pion production in this model were carried out for the reactions $^{12}C + {}^{16}O$ and $^{14}N + {}^{27}Al$. Excitation of the $M1$ resonance, which was constructed as the superposition of particle–hole configurations of the $1p$ shell of the light nucleus, was taken into account. The one-pion exchange potential was used as the interaction potential.

The results of calculations of the π-meson spectral and angular distributions showed that for these reactions it is possible to obtain agreement with experiment for E_i/A_1 from 35 to 84 MeV/nucleon without the use of fitted parameters. Unfortunately, this theoretical approach is applicable in practice only for light nuclei, when it is possible to restrict oneself to a small number of particle–hole configurations in the formation of the excited states, and the model approximation is justified. Naturally, for heavier nuclei other model approaches are preferable.

In Ref. 50 a model similar to the Weizsäcker–Williams method (see, for example, Ref. 78) was developed. In it the electromagnetic interaction is replaced by the sum of photon cross sections averaged over the virtual-photon spectra. The model of Ref. 50 is based on the kinematic relations between the energy ω and the momentum $\mathbf{q}$ transferred from one nucleus to the other in the π-meson emission process. The authors assume that ω and $\mathbf{q}$ determine the characteristics of the virtual pion which arises in the excitation of the isobar-analog state. The applicability of the method requires that the perturbation be sufficiently small and that the incident particle not break up during the reaction time. Therefore, the role of peripheral collisions is singled out.

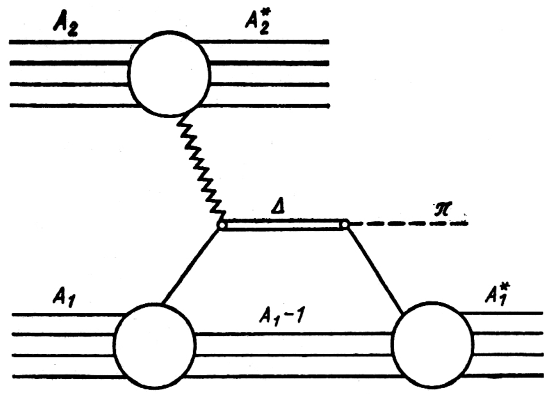

FIG. 7. Diagram describing pion production via excitation of an isobar-analog state (from Ref. 47).

It is assumed that both the target nucleus and the incident nucleus can emit π-mesons. Nevertheless, if the dimensions of the colliding nuclei differ considerably, the smaller nucleus generates more virtual pions, owing to the large nuclear density gradient at the surface of the nucleus.

According to (50), virtual pions appear in a transition of a nucleus from the ground state to an isobar-analog state. The wave functions of these states are written as products:

$$\Psi(\mathbf{R},\xi_1,\xi_2,\ldots,\xi_A,t)=e^{i(\mathbf{PR}-Et)}\Phi(\xi_1,\ldots,\xi_A)\chi_S\chi_T,$$

where $\mathbf{R}$, $\mathbf{P}$, and E are the coordinate, momentum, and energy of the center of mass of the nucleus, ξ_i is the set of internal variables, and χ_S and χ_T are the spin and isospin components of the wave functions.

The pion current written in the nonrelativistic approximation has the form

$$j_\alpha(\omega,\mathbf{q})=(f/\mu)\int dtd^3r\cdot e^{-i(\mathbf{qr}-\omega t)}\times\left\langle\Psi'\sum_{i=1}^{A}(\boldsymbol{\sigma}_i\cdot\nabla_i)\tau_i^{(\alpha)}\delta(\mathbf{r}-\mathbf{r}_i)\times\theta(|b|-R)\Psi\right\rangle.$$

Here $f/\mu=1.4$ F, b is a parameter of the theory determining the degree to which the reaction is peripheral, α is the type of pion, and $\theta(x) = 1$ if $x > 0$, and $\theta(x) = 0$ if $x < 0$. Determining the current j_α and solving the Klein–Gordon equation

$$[\omega^2 - \mathbf{q}^2 - \mu^2 - \Pi(\omega,\mathbf{q})]\Phi_\alpha(\omega,\mathbf{q}) = j_\alpha(\omega,\mathbf{q}),$$

it is possible to find the virtual-pion field. It depends strongly on the pion polarization $\Pi(\omega,\mathbf{q})$, which was found as in Ref. 79.

The final expression for the cross section of the process was obtained assuming that virtual pions scatter incoherently on the nucleus. The cross sections were calculated using the data on πN scattering and taking into account the recoil effect. Coherent calculations were carried out for the following reactions: $^{12}C + {}^{12}C$, $^{208}Pb \to \pi + X$ at 85 MeV/nucleon, and $^3He + {}^{12}C \to \pi + X$ at 97 MeV/nucleon.

In virtual- pion production by the ^{12}C nucleus it was assumed that the nucleon from the state $1p_{3/2}$ makes a transition to the state $1p_{1/2}$, and the nucleon in the state $1s_{1/2}$ does not contribute to this process. The wave functions of the $1p$ state were constructed in the harmonic-oscillator model. It was shown that the inclusion of the polarization operator increases the virtual-pion yield up to energies $\varepsilon_\pi \sim 250$ MeV.

The proposed model proved to be very sensitive to the parameter b. The absolute pion yield varies sharply with varying b, while the slope of the spectrum remains practically constant.

Analysis of the reaction $^{12}C + Pb$ led to the conclusion that the contribution of the heavy nucleus to the pion production process is negligible.

Comparison of the theoretical and experimental cross sections at various energies showed that it is not possible to explain the yield of pions with energy greater than 60 MeV in this approach. Apparently, the use of the restricted basis of excited states for virtual-pion production led to a significant decrease of the theoretical cross section. In addition, the exclusive variant of the calculation is what was actually carried out, with fixed definite reaction channels. Since so far only the inclusive cross sections have been measured experimentally, it can hardly be hoped that the results of the calculations will agree with experiment.

The authors of Ref. 51 have proposed an original method which allows the inclusive cross sections for π-meson production to be expressed in terms of the nuclear structure function, with the latter determined using the results of experiments on proton emission. For $E_i/A_1 \leqslant 100$ MeV/nucleon in the c.m. frame of the two nucleons this energy is insufficient for pion production in a free NN collision. The idea of the authors was to describe the nuclei as systems of interacting nucleons and to relate their internal degrees of freedom to the relative motion. This is achieved by introducing the dynamical structure function of the colliding nuclei:

$$S(\mathbf{q},\omega) = \sum_f \left| \left\langle f \left| \sum_{i=1}^{A} e^{i\mathbf{q}\mathbf{r}_i} \right| 0 \right\rangle \right|^2 \delta(\omega - E_f + E_0).$$

where $|0\rangle$ and $|f\rangle$ are the ground and excited state of a nucleus with energies E_0 and E_f, respectively. The structure function describes the response of the nucleus to an excitation induced by the process accompanying the transfer of energy ω and momentum $\mathbf{q}$.

Writing the effective interaction of the nuclei in the collision accompanied by pion emission in the simplest possible form,

$$H_{\text{int}} = \sum_{i=1}^{A_1} \sum_{j=1}^{A_2} v_0 \delta(\mathbf{r}_{1i} - \mathbf{r}_{2j}) e^{-i\mathbf{k}\mathbf{r}_{1i}} / \sqrt{2\varepsilon}, \tag{12}$$

the authors obtained the following expression for the differential cross section of the process $d\sigma_\pi/d^3k$:

$$\frac{d\sigma_\pi}{d^3k} = \frac{v_0^2}{2\varepsilon(2\pi)^5} \frac{M}{k_i} \int d^3k_f d\omega_1 d\omega_2 \cdot \delta(\omega - \omega_1 - \omega_2) \mathscr{P} S_1(\mathbf{q}_1,\omega_1) S(\mathbf{q}_2,\omega_2).$$

Here v_0 is the force constant (a parameter of the theory) characterizing both the interaction strength of the two nucleons and the coupling to the pion field; k_i and k_f are the initial and final momenta of the relative motion of the two nuclei, and M is their reduced mass;

$$\omega = \frac{1}{2M}(k_i^2 - k_f^2) - \varepsilon - \frac{k^2}{2(A_1 + A_2)m};$$

$$\mathbf{q}_1 = -\frac{A_1}{A_1 + A_2}\mathbf{k} - \Delta\mathbf{K};$$

$$\mathbf{q}_2 = -\frac{A_2}{A_1 + A_2}\mathbf{k} + \Delta\mathbf{K};$$

$$\Delta\mathbf{K} = \mathbf{k}_f - \mathbf{k}_i.$$

The factor $\mathscr{P}$ is introduced to take into account the Pauli principle in the collision process. It selects only those final states which are allowed.

In the asymptotic limit corresponding to large momentum transfer the authors used an approximation[80] allowing the structure function to be related to the nucleon momentum distribution $n(\mathbf{p})$:

$$S(\mathbf{p},\omega) = \sum_i \int \frac{d^3p_i}{(2\pi)^3} n(\mathbf{p}_i) \delta\left(\omega - \frac{(\mathbf{p}_i + \mathbf{q})^2}{2m} + \frac{\mathbf{p}_i^2}{2m}\right).$$

Writing the momentum distribution as $n(\mathbf{p}) = N \cosh^{-2}(\gamma p)$ (where N is the normalization constant), as obtained in Ref. 81 in the large-momentum limit, it proved possible to determine the possible values of the parameter γ using the experimental data on proton–nucleus scattering in the energy range $E_p = 0.6$–1.0 GeV. The distributions $n(\mathbf{p})$ found in this manner made it possible to calculate the nuclear structure functions and the corresponding differential cross sections.

Numerical calculations of the pion spectra were carried out for the reaction $^{12}C + {}^{12}C$ for $E_i/A_1 = 44$, 60, 74, and 84 MeV/nucleon. The fitted parameter v_0 was determined from the total cross section for this reaction σ_{π^0} at 74 MeV/nucleon. The authors consider that, although the results depend quite strongly on γ, good agreement with the experimental spectra can be obtained for some values of it. This fact is viewed as evidence that it is necessary to take into account nucleon correlations in the ground states of the colliding nuclei.

From our side, we note that in spite of its attractiveness, this method has serious deficiencies: the microscopic picture of π-meson production is almost completely ignored [see the Hamiltonian (12)], the extrapolation of the data obtained at high nucleon energies to the low-energy region is poorly justified, and there are ambiguities in the parametrization of the momentum distributions.

THE OPTICAL MODEL OF SUBTHRESHOLD π-MESON PRODUCTION

The authors of Refs. 52–55 proposed a microscopic model for describing subthreshold pion production in nucleus–nucleus collisions with energies of the relative motion of the nuclei of up to 100 MeV/nucleon. In it both pion production in the nuclear field and the interaction of the colliding nuclei are treated using quantum mechanics. Since an important feature of the description of the process is the representation of the internuclear field in terms of single-nucleon optical potentials, this model of subthreshold π-meson production is referred to as the optical model. In contrast to other microscopic approaches (for example, those based on the time-dependent mean-field approximation), the computational scheme of the optical model is quite simple and allows the pion spectral and angular distributions to be obtained for arbitrary masses and energies of the colliding ions. Earlier, a similar model was shown to be effective in calculating the spectra of high-energy γ radiation.[1,82]

The initial assumptions are the following: a) The meson emission occurs in the initial stage of the collision and can be treated as a peripheral process; b) the amplitude of the process is calculated in second-order perturbation theory (in first order in the internuclear interaction potential); c) the internuclear field is represented as the sum of one-nucleon optical potentials.

Assumption (a) is based on the fact that in the proposed mechanism, which is analogous to nuclear bremsstrahlung in the microscopic formulation of the problem, the surface regions of the colliding nuclei, where the gradient of the nuclear potential is a maximum, are most important for the emission of high-energy particles. Assumption (b) is based on the fact that reactions with sufficiently large energy of the translational motion are considered. Moreover, the internuclear field is modeled using phenomenological optical potentials, the parameters of which to some degree take into account the effect of distortion of the nuclear field. In the end the model contains no free parameters, since in calculations of the differential cross sections of specific reactions optical potentials found from analysis of nucleon–nucleus scattering experiments are used for the corresponding nuclei and collision energies.

The amplitude of the process is written as

$$M=\sum_{\lambda=1,2}\sum_{n^{(\lambda)}}\{\langle f^{(\lambda)}|H_\pi^{(\lambda)}|n^{(\lambda)}\rangle(E_f^{(\lambda)}-E_n^{(\lambda)}+\varepsilon)^{-1}\langle n^{(\lambda)}|U^{(\lambda)}|i^{(\lambda)}\rangle+\langle f^{(\lambda)}|U^{(\lambda)}|n^{(\lambda)}\rangle\times(E_i^{(\lambda)}-E_n^{(\lambda)}-\varepsilon)^{-1}\langle n^{(\lambda)}|H_\pi^{(\lambda)}|i^{(\lambda)}\rangle\}.$$

The indices $i^{(\lambda)}$, $n^{(\lambda)}$, and $f^{(\lambda)}$ label the initial, intermediate, and final states of the colliding nuclei ($\lambda=1$ refers to the incident nucleus and $\lambda=2$ to the target nucleus); $E_{i,nf}^{(\lambda)}$ are the corresponding energies, ε is the total pion energy, and $H_\pi^{(\lambda)}$ is the operator leading to pion production. We used the "static" form[83] for it, since in this energy range $\varepsilon/2m\ll 1$:

$$H_\pi^{(\lambda)}=-\frac{f}{\mu}\left(\frac{2\pi}{\varepsilon}\right)^{1/2}\sum_{j=1}^{A_\lambda}\left(\mathbf{k}-\hat{\mathbf{p}}_j^{(\lambda)}\frac{\varepsilon}{2m}\right)\sigma_j\cdot\tau_j\times\exp(-i\mathbf{k}\mathbf{r}_j^{(\lambda)}),$$

where $f^2=0.088$, $\mathbf{k}$ and μ are the π-meson momentum and mass, A_λ is the mass number of the nucleus λ, $\mathbf{r}_j^{(\lambda)}$ is the coordinate of the jth nucleon in the nucleus λ, and $\hat{\mathbf{p}}_j^{(\lambda)}=-i\partial/\partial\mathbf{r}_j^{(\lambda)}$.

The matrix elements were calculated with the wave functions

$$|s^{(\lambda)}\rangle\equiv|\mathbf{k}_s^{(\lambda)},\beta_s^{(\lambda)}\rangle=\Psi_{\beta_s^{(\lambda)}}(\xi^{(\lambda)})\exp(i\mathbf{k}_s^{(\lambda)}\cdot\mathbf{R}_\lambda),\quad s=i,n,f,$$

where $\mathbf{R}_\lambda$ is the coordinate of the center of mass of the nucleus λ, $\xi\equiv(\xi_1,...,\xi_A)$, $\xi_j^{(\lambda)}=\mathbf{r}_j^{(\lambda)}-\mathbf{R}_\lambda$, $\mathbf{k}_s^{(\lambda)}$ is the momentum of the nucleus, $\beta_s^{(\lambda)}$ are quantum numbers describing the internal states of the ion, and $U^{(\lambda)}$ is the potential energy of the interaction of the two colliding nuclei, averaged over the ground state of the nonemitting nucleus (it is assumed that when one of the partners in the collision emits a pion the other does not change its internal state):

$$U_f^{(1,2)}=\langle\mathbf{k}^{(2,1)},\beta_i^{(2,1)}|\sum_{j,l}W(\mathbf{r}_j^{(1)}-\mathbf{r}_l^{(2)})|\mathbf{k}_i^{(2,1)},\beta_i^{(2,1)}\rangle,$$

$W(\mathbf{r}_j^{(1)}-\mathbf{r}_l^{(2)})$ is the potential energy of the interaction of the jth nucleon of nucleus 1 with the lth nucleon of nucleus 2. Then, using assumption (a) about the peripheral nature of the emission process, the matrix elements $\langle s'|U|s\rangle$ were expressed in terms of the Fourier components in the momentum transfer of the one-nucleon optical potentials of the colliding nuclei.

The following expression was obtained for the doubly differential π-meson production cross section:

$$\frac{d^2\sigma_\pi}{d\varepsilon d\Omega}=f^2\frac{kA_1m^2}{(2\pi)^4k_i^{(1)}\mu^2}\int d\Omega_f^{(1)}k_f^{(1)}|U(\mathbf{q})|^2\times\sum_{\lambda=1,2}A_\lambda\mathbf{B}_\lambda^2(1+F_\lambda(\mathbf{k})).$$

Here $d\Omega_f^{(1)}=\sin\theta_f^{(1)}d\theta_f^{(1)}d\varphi_f^{(1)}$, $\theta_f^{(1)}$, $\varphi_f^{(1)}$ are angles defining the direction of the vector $\mathbf{k}_f^{(1)}$, $\mathbf{B}_\lambda$ is a kinematic factor

$$\mathbf{B}_\lambda=(\mathbf{k}-\eta_\lambda\mathbf{k}_i^{(\lambda)})[\varepsilon-(2\mathbf{k}_i^{(\lambda)}\cdot\mathbf{k}-\mathbf{k}^2)/2A_\lambda m]^{-1}-[\mathbf{k}-\eta_\lambda(\mathbf{k}_f^{(\lambda)}+\mathbf{k})][\varepsilon-(2\mathbf{k}_f^{(\lambda)}\cdot\mathbf{k}+\mathbf{k}^2)/2A_\lambda m]^{-1};$$

$$\eta_\lambda=\varepsilon/2A_\lambda m;\quad U(\mathbf{q})=\sum_{j=1}^{\widetilde{A}} V_j(\mathbf{q}),$$

where $\widetilde{A}$ is the mass number of the heaviest of the two colliding nuclei, $V_j(\mathbf{q})$ is the Fourier component of its one-nucleon optical potential $V_j(\mathbf{r})$ [the definition $V(\mathbf{q}) = \int V(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d^3\mathbf{r}$], is used and with $\mathbf{q} = \mathbf{k}_i^{(1)} - \mathbf{k}_f^{(1)} - \mathbf{k}$. The correction factor due to nucleon–nucleon correlations is

$$F_\lambda(\mathbf{k})=G_\lambda(\mathbf{k}) - \zeta_\lambda|f_\lambda(\mathbf{k})|^2,$$

where

$$G_\lambda(\mathbf{k})=\frac{\zeta_\lambda}{2}\left\{[G_\lambda^{(a)}(\mathbf{k}) - G_\lambda^{(s)}(\mathbf{k})]\frac{\zeta_\lambda A_\lambda}{4} - [G_\lambda^{(a)}(\mathbf{k}) + G_\lambda^{(s)}(\mathbf{k})]\right\}.$$

Here $\zeta = 1$, N_λ/A_λ, or Z_λ/A_λ, respectively, for π^0-, π^--, or π^+-meson emission,

$$f_\lambda(\mathbf{k})=\int \rho_\lambda(\xi)e^{-i\mathbf{k}\xi}d^3\xi;$$

$$G_\lambda^{(\beta)}(\mathbf{k})=\int\int \rho_\lambda(\xi)\rho_\lambda(\xi')G_\lambda^{(\beta)} \times(\xi,\xi')e^{-i\mathbf{k}(\xi-\xi')}d^3\xi d^3\xi',$$

$\rho_\lambda(\xi)$ and $G_\lambda^{(\beta)}(\xi,\xi')$ for the nucleus λ are, respectively, the mass density and pairing correlation function with isolated nucleon pairs symmetric ($\beta=s$) or antisymmetric ($\beta=a$) in the spatial state. The function $\rho_\lambda(\xi)$ is normalized by the condition $\int\rho_\lambda(\xi)d^3\xi = 1$, and the pairing correlation function $G_\lambda(\xi,\xi')$ is given by the expression

$$\rho_\lambda(\xi,\xi')=\rho_\lambda(\xi)\rho_\lambda(\xi')[1+G_\lambda(\xi,\xi')],$$

where $\rho_\lambda(\xi,\xi')$ is the particle-pair distribution density.

The spectral and angular distributions $d^2\sigma/dTd\Omega$, $d\sigma_\pi/dT$ (T is the pion kinetic energy), and $d\sigma_\pi/d\Omega$ neglecting correlation corrections [$F_\lambda(k)=0$] were calculated for the following reactions: ^{16}O (25 MeV/nucleon) + ^{27}Al, ^{58}Ni; ^{14}N (35 MeV/nucleon) + ^{27}Al, ^{58}Ni; ^{16}O (38 MeV/nucleon) + ^{27}Al; ^{197}Au; ^{40}Ar (44 MeV/nucleon) + ^{40}Ca, ^{119}Sn; ^{12}C (60, 74, and 84 MeV/nucleon) + ^{12}C; ^{18}O (84 MeV/nucleon) + ^{7}Li; ^{12}C (84 MeV/nucleon) + ^{7}Li, ^{58}Ni, ^{116}Sn, ^{124}Sn, ^{197}Au, ^{208}Pb, ^{238}U. The following definition[84] was used for the one-nucleon optical potentials:

$$V(r)=V_c - Vf(x_0)\alpha V_{SO}\sigma\mathbf{l}\frac{d}{dr}f(x_{SO}) - i \times\left[Wf(x_w) - 4W_D\frac{d}{dx_D}f(x_D)\right].$$

Here

$$V_c=\begin{cases} Ze^2/r, & r\geqslant R_c;\\ (Ze^2/2R_c)(3-r^2/R_c^2), & r\leqslant R_c;\end{cases}$$

$$R_c=r_cA^{1/3},\quad f(x_i)=(1+e^{x_i})^{-1},$$

$$x_i=(r-R_i)/a_i,\quad R_i=r_iA^{1/3},$$

$\alpha = 2.0$ F^2, and Z and A are the charge and mass numbers of the nucleus in question. The values of the parameters r_c, r_i, a_i ($i=SO, O, W, D$), V, V_{SO}, W, and W_D, determined from analysis of nucleon–nucleus scattering, were taken from the review of Ref. 84. In practice, in most cases a significant role is played by only the volume component $Vf(x_0)$.

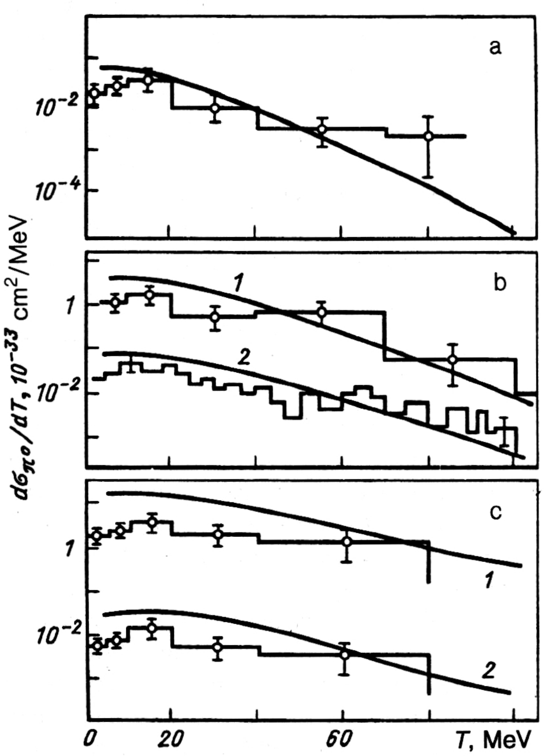

FIG. 8. Values of $d\sigma_{\pi^0}/dT$ for the reactions (a) ^{16}O + ^{27}Al at E_i/A_1 = 25 MeV/nucleon; (b) ^{14}N + ^{27}Al (1) and ^{14}N + ^{58}Ni (2) for E_i/A_1 = 35 MeV/nucleon; (c) ^{16}O + ^{197}Au (1) and ^{16}O + ^{27}Al (2) for E_i/A_1 = 38 MeV/nucleon. The experimental and theoretical values (curve 2) were divided by 10^2.

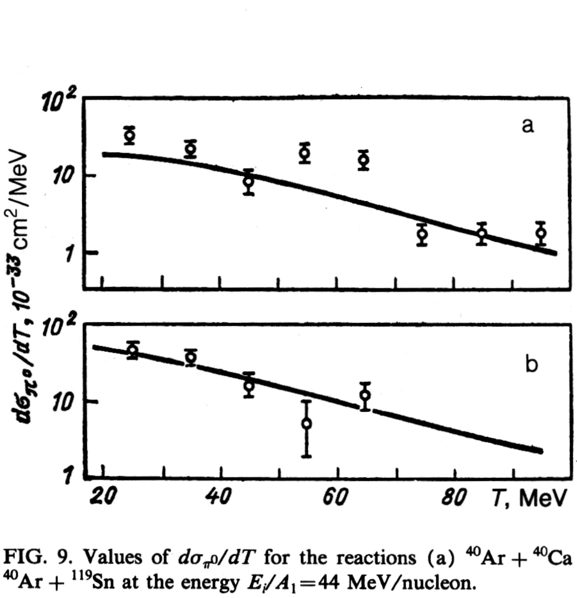

FIG. 9. Values of $d\sigma_{\pi^0}/dT$ for the reactions (a) ^{40}Ar + ^{40}Ca and (b) ^{40}Ar + ^{119}Sn at the energy E_i/A_1=44 MeV/nucleon.

In Figs. 8–11 we show the results of calculations of $d\sigma_\pi/dT$ and $d^2\sigma_\pi/dTd\Omega$ for some of the reactions listed. Since there is an ambiguity, rather significant in some cases, in the values of the parameters of the optical potentials, different sets of them were used in the calculations, and in the figures we give the best results. The sensitivity of the differential cross sections to the parameters of the one-nucleon optical potentials should be noted. This was also observed earlier in the study of similar reactions accompanied by emission of high-energy photons.[1,82] Satisfactory agreement of the cross sections $d\sigma_\pi/dTd\Omega$ at fixed pion emission angle with experiment was also obtained for reactions in which emission of charged π mesons was observed. Here again the model gives a good reproduction of the dependence of the cross sections on both the incident-ion energy and the target-nucleus mass. However, for reactions on target ions with large mass the pion yield is somewhat too high at relatively small values of the angles θ [for example, in the reactions ^{12}C (85 MeV/nucleon) + ^{124}Sn, ^{197}Au, ^{208}Pb]. In addition, the π-meson yield in the backward hemisphere is systematically smaller than the experimental yield. The possible reasons for this are the neglect of pion absorption by the nuclear medium, which can be important, especially for massive nuclei, and the neglect of the contribution from incoherent terms in calculations of the probability for the process. As was shown in Ref. 85, bremsstrahlung emission from the undirected motion of nucleons incident upon an external nuclear field significantly increases the total yield of high-energy photons in the backward hemisphere. Since such terms are present in the pion emission amplitude, a similar effect can also be expected for the backward π-meson yield.

The role of the correlation factor $F_\lambda(\mathbf{k})$ was estimated for some of the reactions listed. In view of the obvious difficulties in calculating the pairing correlation function exactly, it was determined using a simplified model of the nucleus as a degenerate Fermi gas. Only nucleon–nucleon correlations due to the Pauli principle were thereby actually taken into account. In this model

$$G_\lambda(\xi,\xi')=-\frac{9\pi}{2}\left[J_{3/2}(k_F|\xi-\xi'|)\right]^2(k_F|\xi-\xi'|)^{-3}$$

[$J_{3/2}(x)$ is the Bessel function of order 3/2 and k_F is the Fermi momentum] and

$$G_\lambda^{(s)}(\mathbf{k})=-G_\lambda^{(a)}(\mathbf{k})=V_\lambda^{-1}\int_{V_\lambda}G_\lambda(\xi)e^{i\mathbf{k}\xi}d^3\xi$$

(V_λ is the volume of the nucleus). Calculations of the pion spectral distributions showed that even though there is some improvement of the agreement between theory and experiment, the contribution of the correlation factor to the cross section is no greater than 30%.

The study leads to the following conclusions.

First, even though the model uses a simple computational scheme and does not introduce fitted parameters, it gives a completely satisfactory description of the spectral and angular distributions of π mesons in a wide range of energies and masses of the colliding ions.

Second, taking into account the results of studies on high-energy γ radiation obtained using the same model,[1,82] it can be seen that the reaction mechanism is similar in the two cases. Namely, photon and pion emission is the result of the collective "bremsstrahlung" of the nucleons of one ion at the periphery of the nuclear field of the other. In any case, the actual "nuclear bremsstrahlung" emission mechanism plays the leading role in processes of this type, and, being peripheral, it operates mainly during the early stage of the reaction.

Third, the differential cross sections are quite sensitive to the parameters of the one-nucleon optical potentials, which makes it possible to use the experimental data to test them independently. At the same time this also gives additional information about the nature of the internuclear interaction of the colliding ions. As far as the experimentally observed exponential decay of the cross sections with increasing ion energy is concerned, it is mainly a consequence of the exponential dependence on $\mathbf{r}$ of the optical potentials $V(\mathbf{r})$ near the nuclear boundary and is largely determined by the diffusivity parameter a_0.

CONCLUSION

This analysis of the experimental and theoretical situation regarding the problem of subthreshold pion production in ion–ion collisions allows us to formulate several conclusions and a wish list.

In experiment:

a) The experimental data on the angular distributions of the emitted pions are still insufficient, and those that are available are contradictory and have a large spread. From the viewpoint of theory it is especially important to have information on the differential cross sections of π mesons emitted into the backward hemisphere, since this could solve the problem of the ratio of the contributions from coherent and incoherent pion-emission sources.

b) It is of great interest to carry out experiments, for example, to study the spectral distribution while simultaneously fixing the emission angles of the pion and the scattered ion. A possible diffraction structure of such a distribution could indicate a π-meson emission mechanism involving "nuclear bremsstrahlung" and give precise information on the shape of the internuclear potential. Various exclusive cross sections would also be valuable, although we understand how difficult it is to obtain them. Experiments of this type would go a long way toward answering the question of the actual pion production mechanism.

c) It is desirable to study the energy and mass dependence of the threshold in pion emission; this would also help to specify the physical reaction mechanism.

d) It would be useful to accurately study the pion spectra in the high-energy region and to go to kinetic energies above 200 MeV. The discovery of a relative increase in the π-meson yield with increasing energy would make it possible to more accurately assess the role of the Δ-resonance mechanism of pion production.

In theory:

a) It can be assumed that π-meson emission mainly occurs during an early stage of the ion collision. This is

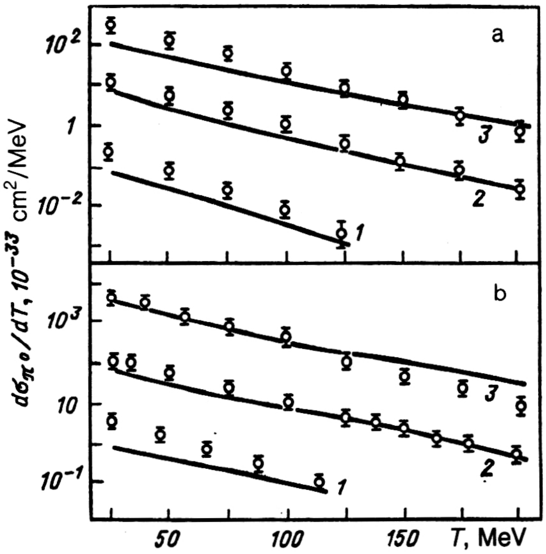

FIG. 10. Values of $d\sigma_{\pi^0}/dT$ for the reactions (a) $^{12}C + ^{12}C$ [(1) $E_i/A_1 = 60$ MeV/nucleon, (2) 74 MeV/nucleon, (3) 84 MeV/nucleon]; (b) $^{18}O + ^7Li$ (1), $^{12}C + ^{58}Ni$ (2), and $^{12}C + ^{238}U$ (3); for all reactions $E_i/A_1 = 84$ MeV/nucleon. The experimental and theoretical values are divided by 10^2 (curve 1) and 10 (curve 2).

indicated by the results of both direct calculations focused on the time dynamics (using the time-dependent Hartree–Fock approximation or the kinetic equations; Refs. 27–35) and other microscopic and semimicroscopic approaches

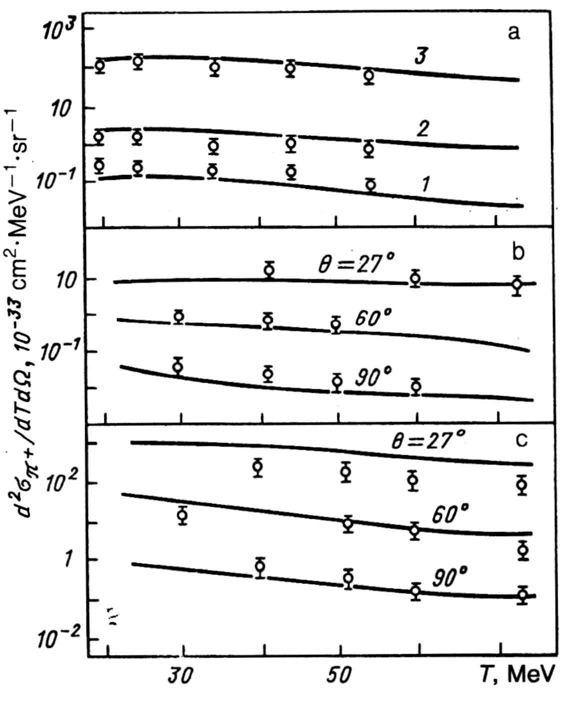

FIG. 11. Values of $d^2\sigma_{\pi^+}/dTd\Omega$ for the reactions: (a) $^{12}C + ^{12}C$ (1), $^{12}C + ^{116}Sn$ (2), $^{12}C + ^{124}Sn$ (3), where for all of them $\theta = 70°$, $E_i/A_1 = 85$ MeV/nucleon; (b) ^{12}C(75 MeV/nucleon) + ^{12}C; ^{12}C(85 MeV/nucleon) + ^{208}Pb. In all the figures the experimental and theoretical values for the lower curves are divided by 10^2, and for the middle curves they are divided by 10.

explicitly or implicitly containing the assumption of emission at an early stage.

b) An explanation of the exponential nature of the pion spectra has been found only in Refs. 16–23 and 52–55; in the first case this explanation is conditional, since the evaporation model itself is based on this fact and took it into account by using the canonical distribution or in the determination of the values of the model parameters; in the second case the exponential slope is explained by the corresponding behavior of the one-nucleon optical potentials near the nuclear boundary. As far as the other studies are concerned, either they were used to calculate only the total cross sections, or the computational scheme did not permit the source of this behavior to be revealed.

c) Comparison of the features of the spectral or angular distributions of high-energy γ quanta with the pion ones reveals that they are quite similar. This fact leads to the approximation that the physical mechanisms of these reactions are similar. Simultaneous calculations of the distributions for π mesons and γ quanta using the same model were carried out in Refs. 53 and 82 for a large number of reactions, and the results indicate that the "nuclear bremsstrahlung" emission mechanism does actually operate in both cases. As far as alternative approaches are concerned, the results obtained using them are still insufficient to draw any conclusions.

d) There is no unity in the conclusions about the role of any of the microscopic π-meson production mechanisms at the level of one- and two-nucleon processes. If we judge from the final result on the calculated values of the cross sections, an important role might be played by both the one-nucleon[51–55] and the two-nucleon[31–35] mechanisms, and also by the transition of the nucleon into the $\Delta(3,3)$ state with its subsequent pion decay.[45–49] However, it should be noted that the most extensive calculations of the pion spectra have been carried out only in Ref. 53. There are also indications that, in addition to the p-wave mechanism, the s-wave mechanism of pion production can also play an important role, at least in the low-energy part of the spectrum.[34]

e) It is also necessary to further study the dynamics of the nucleus–nucleus collision from the viewpoint of evaluating the role and the ratio of the coherent and incoherent pion-production mechanisms. The conclusion of predominance of the incoherent component drawn in Refs. 27–30 and 35, where the theoretical scheme permitted the calculation of the pion yield from only incoherent NN collisions, contradicts the results of Refs. 45–55, where the role of coherent effects was estimated. In our opinion, both sources can contribute to the total cross section for the process, as is also indicated by the results of studies of high-energy γ emission.[85,86]

f) A satisfactory method of including rescattering and possible pion absorption in the nuclear medium is almost completely lacking. The semiempirical estimates of the absorption effect made in Refs. 29, 30, and 35 indicate only that it should be taken into account.

g) The main goal of the theoretical studies which have been carried out has been to try to understand and explain the behavior of the inclusive characteristics of the reaction. However, for assessing the role of the various initial theoretical assumptions and the possibility of using them to model the real situation it would be quite valuable to have calculations of the exclusive cross sections and correlation factors. So far, no such theoretical schemes have been worked out, although they would be useful for determining the area in which future experimental studies should be carried out.

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