Two-quantum cascades of radiative neutron capture
1. Spectroscopy of excited states of complex nuclei in the neutron binding-energy region

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The possibilities of revealing the excited states in complex (primarily, deformed) nuclei and ordering them in a decay scheme are analyzed for energies up to the excitation energy $E^* \lesssim 4$–6 MeV and above. An algorithm for constructing the decay scheme of states excited by two-step cascades with intensities $i_{\gamma\gamma} \gtrsim 5 \times 10^{-4}$ per decay is described. The construction of a decay scheme for a number of high-lying states in combination with analysis of the spectroscopic data on the (n,γ) and $(n,2\gamma)$ reactions is shown to be possible, and the opportunities that it offers are discussed. It is concluded that in the decay of compound states of nuclei in the vicinity of the $4S$ maximum of the neutron strength function an important role is played by "channeling"—the presence of strong cascades which excite groups of several dozen levels in each of the nuclei studied. Such cascades account for up to half of the total intensity of all the primary transitions in the decay of compound states.

INTRODUCTION

By now the excited states of complex (heavy) nuclei have been studied in detail in two relatively narrow excitation-energy ranges—one near the ground state, 2 MeV, and the other near the neutron binding energy, where neutron time-of-flight spectroscopy, which makes it possible to work with an energy resolution of a fraction of an eV near 10 keV, has been used to detect tens and hundreds of neutron resonances and to study their decay channels.

In the region near the ground state nuclear spectroscopy has been used to accumulate very rich information both about the excited-state scheme and about the nature of these states. These data have been used to develop various theoretical models such as the interacting-boson model, the quasiparticle–phonon model, and others.

Neutron-spectroscopy studies have given information on hundreds of states at excitations of 6–8 MeV. These data in a first approximation are reproduced well by the statistical theory based on the concept of the compound nucleus with parameters averaged over a large number of neutron resonances. However, this theory does not give any definite information about the nature of an individual compound state.

The intermediate range of excitation energies which is discussed below has been studied rather less, owing to the experimental difficulties. On the one hand, the present setups do not have sufficient energy resolution to excite individual states in complex nuclei lying above 2–3 MeV. On the other hand, in the case of γ decay (for example, after neutron capture) the population of these states cannot be determined uniquely using an ordinary single-detector gamma spectrometer, since there are no criteria for selecting primary decay quanta with energies of the order of, or less than, half the neutron binding energy. Moreover, owing to the complex nature of these states, their population probability will fluctuate fairly weakly according to the Porter–Thomas law.

Nevertheless, several attempts have been made to obtain information about the population of intermediate states. In particular, from the shape of the spectrum of α particles from the two-step $(n,\gamma\alpha)$ reaction on resonance neutrons[1] it has proved possible to determine the radiative strength functions of primary soft γ transitions. This led to refinement of the shape of the energy dependence of the radiative strength function (the "tail" of the giant electric dipole resonance), which together with extrapolation to the region of intermediate states[2] (according to the density of low-lying states and neutron resonances) made it possible, for example, to fairly accurately reproduce the total radiative widths Γ_γ for the studied nuclei and to predict them for nuclei which have not been studied, in particular, for radioactive nuclei. Such data are of interest both for astrophysics[3] and for reactor construction.[4]

However, new experimental techniques are needed which will allow systematic studies of states at intermediate excitation energies, where the nuclear level structure becomes more complicated: there is a transition from very simple states of known nature near the ground state ("order") to extremely complex compound states ("chaos"). The dynamics of this transition process can be important in selecting the nuclear models which are most effective for describing the properties of the excited nucleus. From the practical point of view study of the properties of intermediate states of complex nuclei can ensure the needed improvement in the accuracy of the predictions of the total radiative widths of neutron resonances and, accordingly, of the neutron cross sections of nuclei far from the beta-stability band, including short-lived nuclei, which are practically impossible to study experimentally.

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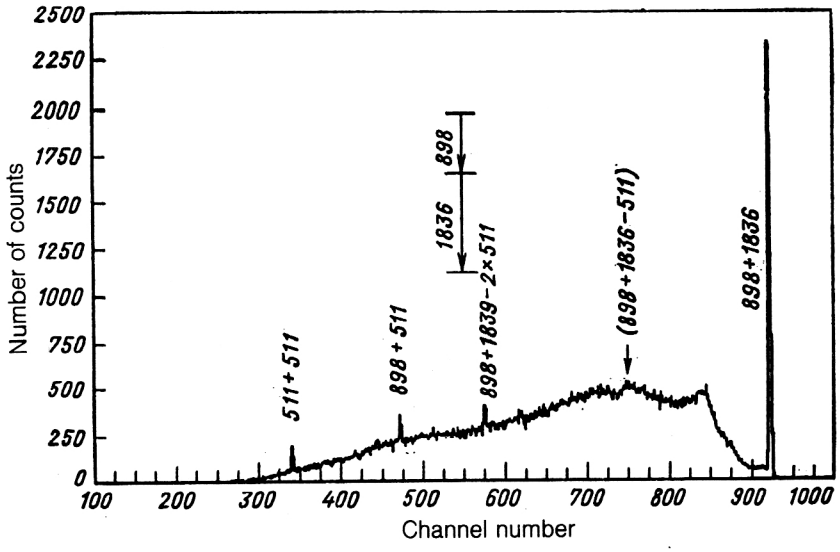

FIG. 1. Spectrum of summed amplitudes of coincident pulses for ^{88}Y. The peaks are labeled by the values of the recorded energy in keV.

1. SOME FEATURES OF THE TECHNIQUE OF STUDYING THE $(n,2\gamma)$ REACTION

The possibility of direct experimental study of intermediate states is offered by the sum-coincidence (SC) method, realized using Ge(Li) detectors connected to a permanent device for storing information about the coincident γ energy measured in each detector.

This method, which is referred to below as the "$(n,2\gamma)$ reaction," made it possible for the first time to begin systematic experimental studies in an excitation range hitherto unstudied. The method is based on the fact that the semiconductor germanium detector linearly transforms the γ energy into the amplitude of a measured signal, and the sum of the energies E_1 and E_2 of two successive transitions, $E_1 + E_2 = E_\lambda - E_f$, is determined only by the energies E_λ and E_f of the decaying level (λ) and of the level populated by the two-quantum cascade (f) and is independent of the excitation energy of the intermediate state. Those cases of cascade detection for which there is simultaneous complete absorption of the energy of the two photons in the two detectors lead to the appearance of a coincident-pulse peak in the spectrum of the summed amplitudes. Incomplete absorption of the energy of at least one of the photons shifts the amplitude sum toward lower energies and forms the corresponding continuum.

In Fig. 1 we show the spectrum of the amplitudes of coincident pulses from the radioactive source ^{88}Y. The peak corresponding to total absorption of the cascade energy $E_1 + E_2 = 898 + 1836$ keV is marked, as are the peaks in the continuum related to the formation and detection of pairs of quanta of energy 511 + 511 keV.

When writing the coincident-pulse amplitude codes for a permanent information storage device (magnetic tape, for example) using spectra analogous to that shown in Fig. 1, it is easy to isolate from the mass of γ–γ coincidences only those cases in which the entire energy of the cascade is completely absorbed in the two detectors.

Although the acceptance of this method is rather small—usually no more than 10 useful events are obtained from 10^6 decays—the possibility of eliminating the background distribution related to incomplete absorption of the γ energy means that the $(n,2\gamma)$ method gives more information than the traditional method of the (n,γ) reaction.

The sum-coincidence method is traditional in classical nuclear spectroscopy. It has been used to "compress" the information[5] obtained using Ge(Li) detectors to study the γ decay of unstable nuclei, to study the decay schemes of light and near-magic nuclei[6] [NaI(Tl) detectors], and to determine the spins of neutron resonances[7] [NaI(Tl) detectors].

The fundamentally new feature in our application of this technique is that we use semiconductor detectors to isolate cascades from two γ transitions when their total energy is equal to or less than the neutron binding energy in complex compound even–odd and even–even nuclei with large level density.

Our use of such detectors with an efficiency of 5% and later 10% of the efficiency of detecting the 1332-keV quantum by a NaI(Tl) crystal of dimensions 76×76 mm allowed us to select useful γ–γ coincidences at a rate of up to 1000 events per day at the peak corresponding to total absorption of cascades of a given total energy in thermal-neutron capture by the sample.

2. SHAPE OF AN INDIVIDUAL CASCADE LINE

The crucial feature responsible for the high efficiency of the $(n,2\gamma)$ reaction in studying the properties of states in the range $0 < E^* < B_n$, where B_n is the neutron binding energy, proved to be the nearly ideal shape of the line formed by one cascade in their intensity spectrum distribution. The selection of only cases of total absorption of the cascade energy (which form peaks similar to those in Fig. 1) in the absence of background makes it possible to obtain only the total-absorption peaks in the intensity distributions. Here the background component is completely eliminated.

A problem with this is the transfer of γ radiation between detectors. In Fig. 2 we show the working geometry of an experiment using Ge(Li) detectors of 5% efficiency. "Forward" Compton scattering and radiation transfer by annihilation quanta in this geometry are suppressed by

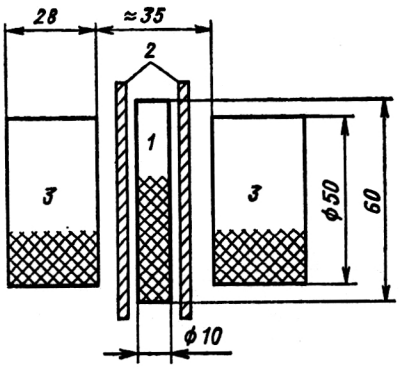

FIG. 2. Working geometry of the experiment: 1—sample; 2—lead filters, each 3 mm thick; 3—detectors. Dimensions in mm.

means of two lead filters having a total thickness of 6 g/cm^2.

The filtering efficiency is illustrated in Fig. 3, where we show the cascade-transition distribution for a ^{60}Co source without filters (Fig. 3a) and with them (Fig. 3b). In Fig. 3a we observe, in addition to the peaks at 1173 and 1332 keV, additional peaks related to the fact that in one of the detectors there is energy loss of the quantum undergoing backward Compton scattering, while the other records an additional transition of the cascade plus the backward-scattered quantum. The total area associated with this distribution process here is only 12%. The lead filters decrease its fraction to several tenths of a percent in the cascade energy range 1785–4123 keV (Ref. 8).

In practical studies of specific nuclei using the $(n,2\gamma)$ reaction, as a rule, the peak corresponding to cascade detection is observed on a larger or smaller background. This background can be eliminated from the analyzed cascade intensity distributions by means of γ–γ coincidences, the sums of which lie in a range equal to the width of the peak at its base. As a rule, useful events are selected from an interval of total energies ΔE whose width is related to the cascade energy E_c as $\Delta E/E_c \lesssim 0.005$. It can be shown that in the first approximation both the difference of the counts in the range ΔE and in the two adjacent intervals each of width $0.5\Delta E$, and the channel-by-channel difference of the cascade intensity distributions which were obtained have a practically normal distribution with zero mean and dispersion determined by only the counting statistics if in the range $E_c \pm 0.5\,\Delta E$ of the spectrum analogous to that of Fig. 1 there is no peak related to detection of a cascade from two transitions to a different final state.

In Fig. 4 we show the intensity distribution of "cascades" with total energy $E_c \approx 6.7$ MeV, obtained from an interval of width $\Delta E \approx 20$ keV, and in Fig. 5 we show the analogous distribution obtained from the same interval minus the distributions accumulated in the two adjacent intervals, each of width about 10 keV. These are typical

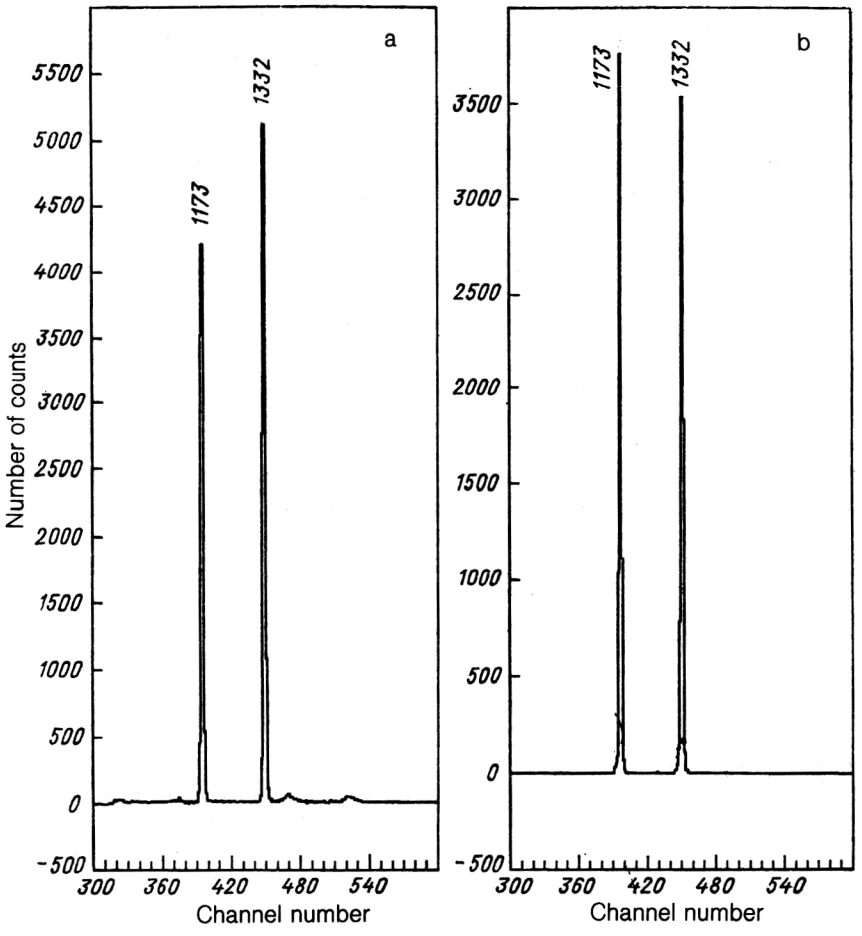

FIG. 3. Cascade intensity distribution for ^{60}Co: a—without lead filters; b—with filters.

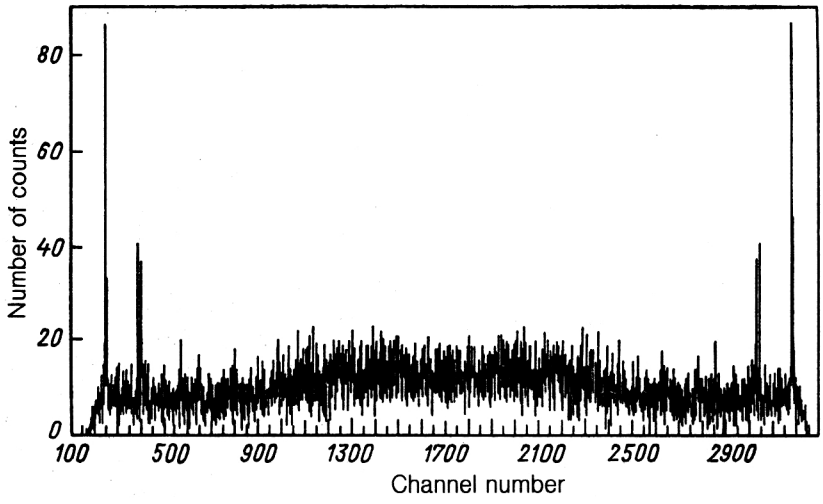

FIG. 4. Cascade intensity distribution from the background part of the SC spectrum of the reaction ^{167}Er$(n,2\gamma)$. The amplitude sum corresponds to the energy $E_1 + E_2 \approx 6.7$ MeV.

conditions for selecting useful information on cascade decay using the peaks of the SC spectrum. In this particular case the ratio of the area of the distribution in Fig. 5 to the area of the distribution in Fig. 4 is 150/155 02, while the average ratio of the square of the channel-by-channel count in Fig. 5 to the channel-by-channel count in Fig. 4 (χ^2 per degree of freedom) is 1.02.

The conclusion that the background distribution is practically normal when it is subtracted (see Fig. 5) using the coincidences forming the background in the SC spectrum is valid only when the distributions do not contain any intensities from strong cascades. This follows from analysis of the components forming the background component in the SC spectrum. In the region of the maximum observed cascade energies the background component of this spectrum arises primarily from the momentum addition in the spectrometric part of the coincidence scheme, and also from the capture of direct and scattered neutrons in the experimental geometry shown in Fig. 2 if the resulting cascade energy is larger than the neutron binding energy in the isotope in question. For smaller values of the recorded sums of cascade transition energies the dominant role in creating the background begins to be played by the incomplete absorption of the cascade energy in the active volume of the detectors. This process, as seen from Fig. 6, considerably worsens the ratio of the areas of the peak and the background beneath it when the excitation energy of the final cascade level is above 300–500 keV.

Our method of subtracting the background using coincidences falling in regions of the SC spectrum adjacent to the peak leads, when the numerical method of improving the resolution[9] (without decreasing the detection efficiency) is used, to the appearance of characteristic "alternating-sign" structures in the two-photon cascade intensity distributions. These are related to events in which the soft transition of a sufficiently intense cascade is completely absorbed in one of the detectors, while the other absorbs the part of the energy of the hard photon equal to the difference between the energies of the cascade with the smallest total energy and the soft transition of the cascade to a lower-lying level. Examples of such cases are given in Fig. 7 for ^{144}Nd and ^{163}Dy. In practice, in the absence of anti-Compton shielding of the spectrometer "alternating-sign" structures should be taken into account when collecting statistics of more than 500–1000 pulses at any peak of the distributions of cascades with the maximum total energies.

This effect distorts the results most when the negative part of the "alternating-sign" structure is superimposed on a possible intense cascade.

The last problem of the method described here is the determination of the energies and intensities of cascade γ transitions and the use of these data as the basis for constructing the most reliable possible γ-decay scheme. The latter, in turn, can be used to determine the energy dependence of the cascade intensity in the range of excitation energies E^* accessible in practice: $520 \lesssim E^* \lesssim B_n - 520$ keV.

The possible systematic errors which can distort the cascade intensities which are found are determined by the physics of the emission process and detection of the cascade γ radiation. A general check of the experimental tech-

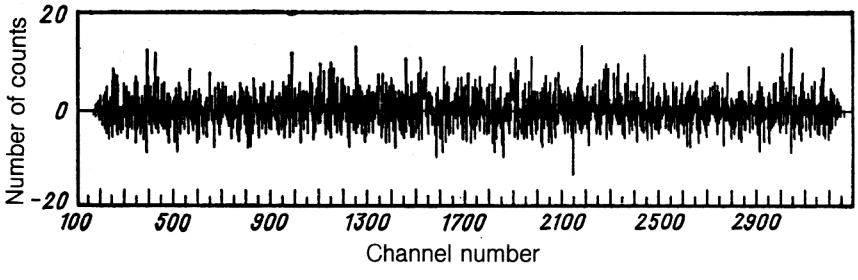

FIG. 5. Example of a background distribution for the total energy $E_1 + E_2 \approx 6.7$ MeV in the reaction ^{167}Er$(n,2\gamma)$.

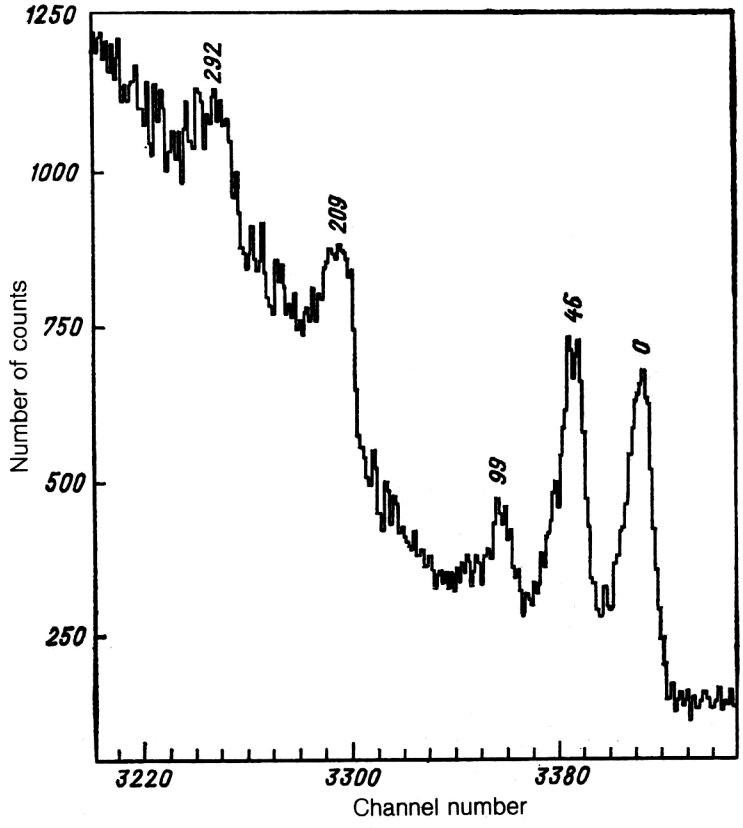

FIG. 6. Part of the spectrum of summed amplitudes of coincident pulses from the reaction $^{182}W(n,2\gamma)$. The energies of the final cascade levels are given in keV.

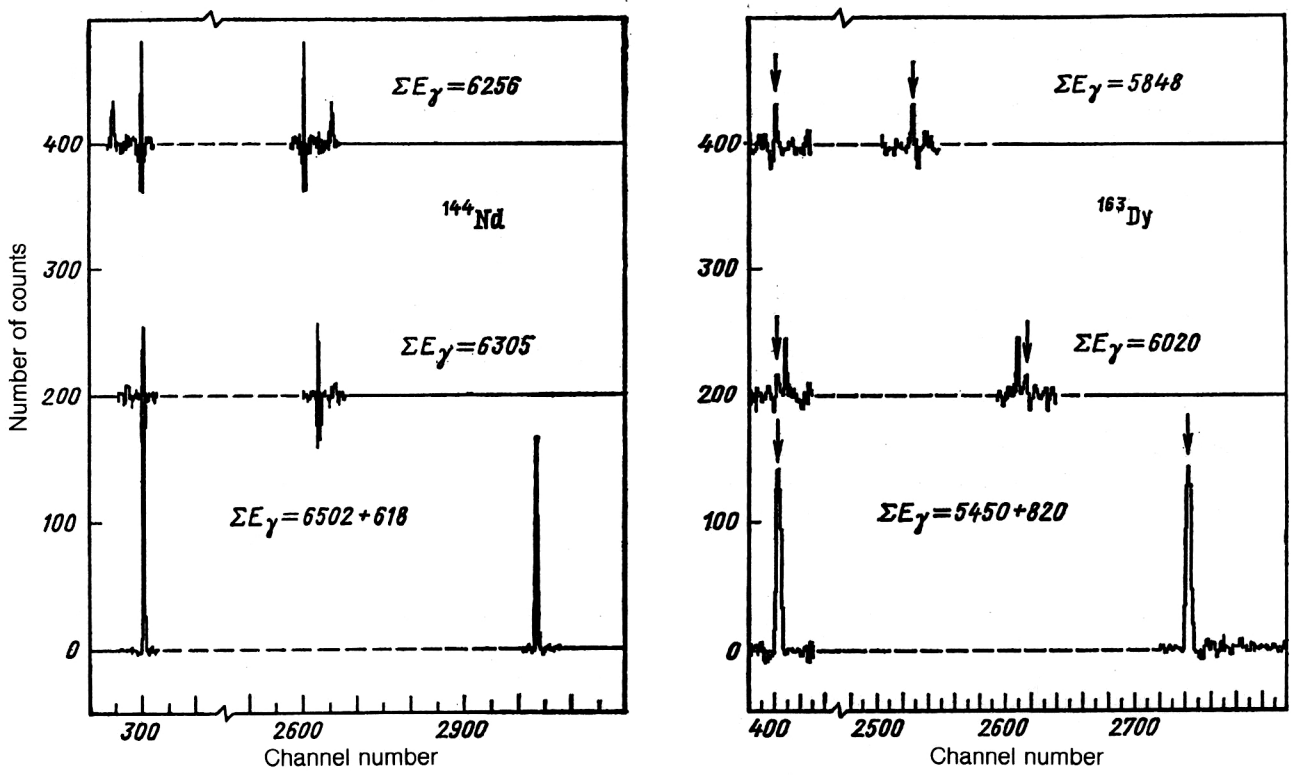

FIG. 7. Transformation of the shape of an intense cascade when it is recorded in intensity distributions of cascades with smaller total energy. The spectra are labeled by the value ΣE_γ of their total energy. Each successive spectrum is shifted upward by 200 counts.

TABLE I. Intensities of γ-transition cascades with total energy $E_1 + E_2 = 8579$ keV in ^{36}Cl.

Cascade $E_1 + E_2$, keV	Intensity, %	
	Reaction $(n, 2\gamma)$	From ^{36}Cl decay scheme (Ref. 10)
7791+788	8,1±0,8	8,5±0,4
7414+1165	8,5±0,8	10,3±0,5
6978+1600	1,8±0,4	1,8±0,1
6628+1950 }	10,6±0,9	10,3±0,4
6620+1959 }		
5716+2864	6,7±0,9	5,3±0,3
3062+5517	2,5±0,5	2,1±0,1
Total	38,3*	38,3±0,8

*Normalized to the summed intensity of the cascades from Ref. 10.

nique was made by measuring the intensities of cascades in the reaction ^{35}Cl$(n,2\gamma)$ and comparing them with the values obtained from calculations using data in the literature. Comparison of the data obtained by us for six cascades with a total energy of 8579 keV, shown in Table I, with the results of Ref. 10 shows that there is practically complete agreement: the value of χ^2 per degree of freedom turned out to be 1.3.

Nevertheless, it is necessary to analyze the sources and possible magnitudes of systematic errors, since we cannot exclude the possibility of a mutual cancellation between them.

Among the sources of systematic error in the measurement of the cascade intensity we note the following:

a) detection of a three-step cascade as a two-step one;

b) energy dependence of the width of the resolving-time curve from the ratio of the cascade transition energies;

c) internal conversion of γ quanta;

d) angular correlations of cascade transitions;

e) self-absorption in the sample;

f) errors in determining the efficiency;

g) energy dependence of the width of the total-absorption peaks.

Analysis of these errors shows the following:

a. The probability of detecting a three-step cascade in the form of an isolated peak is fairly small. For example, in ^{146}Nd (Ref. 11) a cascade from the three successive transitions $6523 + 589 + 454$ is observed in the form of an experimentally resolved pair of peaks with an intensity of $(2.6 \pm 1.0) \times 10^{-4}$ per decay.

This is the worst case. Other cascades of three successive quanta for other energy ratios can, for all the nuclei that we have studied so far, make only a much smaller contribution to the observed cascade intensity distributions, owing to the lower probability for the simultaneous detection of a pair of transitions as a common peak. Such intense cascades are well known for all nuclei in the (n,γ) reaction, and it is not difficult to take their effect into account.

In an actual experiment cascades to a given excited state of the nucleus under study are distinguished. This state, in turn, decays with the emission of γ quanta of greater or lesser energy, or a cascade of γ transitions. When they fall on the detector, these γ transitions decrease the areas of the corresponding peaks in the SC spectra, since the detectors record the large γ-radiation energy. However, a distortion of the measured cascade intensity distributions in this case would have been possible only if the decay mode of the state depended on the mode by which the state was populated.

b. The width of the curve which can describe the time distribution of the coincidence pulses depends on the quantum energy.[12] If the time interval from which coincident γ quanta are selected is too narrow, this effect can significantly distort the shape of the cascade intensity distribution as a function of the cascade transition energy. However, this effect becomes negligible if the γ detection threshold is sufficiently high (for example, 520 keV), and the window from which coincidences are selected is wider than 3Δ, where Δ is the total width at half the maximum of the resolution curve for a pair of 511-keV quanta.

c. In practice, only $E1$, $M1$, and $E2$ transitions are observed in the $(n,2\gamma)$ reaction. The amplitude threshold of the detection scheme, which corresponds to a γ energy of 520 keV, makes it possible to neglect corrections to the measured cascade intensity distribution related to competition from internal γ conversion.

d. Angular correlations of cascade transitions can in principle distort the observed cascade intensity distribution by exciting intermediate levels of different spin (for fixed initial and final states of the cascade). However, this effect is small in the geometry that we used (Fig. 2). Modeling shows, for example, that the anisotropy-induced increase of the measured intensity of the cascade $1/2 \rightarrow 3/2 \rightarrow 1/2$

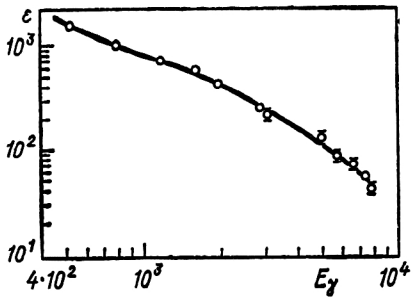

FIG. 8. Example of the behavior of the relative energy dependence $\varepsilon(E_\gamma)$ of the detector: points—experiment; line—approximation.

relative to the isotropic distribution is no more than 5%.

e. The role of photon self-absorption in the sample as a source of systematic error is weakened considerably by choosing the detection threshold to be 520 keV. Monte Carlo modeling shows that in our case the relative efficiency of detecting a 5.5 + 0.5 MeV cascade is 0.95 of the efficiency of detecting a 3 + 3 MeV cascade; for a 5.0 + 1.0 MeV cascade it is 0.985. This estimate was obtained for a target of powdered ytterbium oxide in the geometry shown in Fig. 2.

f. In the reconstruction of the true values of the cascade intensity from the experimental distributions it is necessary to specify the the energy dependence of the relative efficiency of γ detection by a semiconductor detector in the energy range $0.5 < E_\gamma < 9$ MeV.

To describe the functional dependence of the efficiency ε on the quantum energy E_γ we used the expression

$$\ln \varepsilon = a + b \ln E_\gamma + \alpha \ln(E_\gamma/1022) + \beta \ln^2(E_\gamma/1022), \quad (1)$$

where the parameters b, α, and β are obtained by fitting the values of the areas of the total-absorption peaks for γ quanta in the reaction $^{35}\mathrm{Cl}(n,\gamma)$ on thermal neutrons and their yields determined in Ref. 10.

An example of one of the relative dependences obtained for a detector with 5% efficiency is given in Fig. 8. To minimize the possible errors in the determination of the dependence $\varepsilon = f(E_\gamma)$ we used a NaCl sample with the same geometry as when studying specific nuclei, and the dependence itself was defined as the average from measurements of the $^{35}\mathrm{Cl}(n,\gamma)$ reaction made before and after the main experiment.

A remaining source of error in determining the efficiency is related to the different multiplicity of quanta for different decay modes of the compound state of the ^{36}Cl nucleus excited in neutron capture. Here the main factor giving rise to distortion is the decrease of the area of the total-absorption peak owing to the two cascade quanta hitting the same detector.

The experimental probability of detecting a quantum of energy $R_\gamma \sim 1$ MeV in the incomplete-energy-absorption continuum is about 20% for the geometry that we used.

The multiplicity of quanta in different cascades in the $^{35}\mathrm{Cl}(n,\gamma)$ reaction mainly differs by unity. We can therefore take 20% as an upper limit on the estimated systematic error in determining the energy dependence of the efficiency. The actual error in determining the cascade intensity in the nucleus under study must be even smaller, owing both to the mutual cancellation of the errors of the curves approximating the efficiencies of the two detectors and to the smoothing of the possible systematic errors for quanta from the decay of ^{36}Cl with different energies when ε is approximated by Eq. (1).

The width of the peak of the SC spectrum is independent of the energy of the γ quanta of the cascade, since the squared width of the peak in the spectrum of a single detector is linearly related to the γ energy. When the distance between the peaks in the SC spectrum is comparable with the width of the peak, which is known *a priori* from the location of low-lying states populated by two-quantum cascades, the background subtraction in obtaining the cascade intensity distributions can be made in such a way that the width of the "peak plus background" window is smaller than the width of the peak at its base, with the background windows to the left and right of the peak containing the "wings" of the peak. There is no reason to expect systematic distortions in the intensity distributions obtained in this case, since for any ratio of the energies of the cascade quanta E_1 and E_2 the square of the peak width depends only on the difference $E_\lambda - E_f$.

Therefore, the experimental values of the intensities of cascades between the compound state and a given final level for the experimental accuracy attained are primarily characterized by the statistical error in determining the area under the peaks in the measured distributions.

The best statistical accuracy in determining the intensity of the strongest cascades in the nuclei that we have studied so far, $144 \leqslant A \leqslant 187$, is usually 5–10%, that of the whole aggregate of experimentally resolved cascades is 20–30%, and at the sensitivity threshold it is 50%.

3. METHOD OF CONSTRUCTING THE γ-DECAY SCHEME AND RELIABILITY OF THE SPECTROSCOPIC INFORMATION OBTAINED

The Ritz combination principle is the commonly used method of constructing the γ-decay scheme from the energies and intensities of γ transitions determined in the (n,γ) reaction. It is believed that this method allows the decay scheme to be determined up to excitation energies of 1.5–2 MeV. When the probability for random coincidence of the transition energies and the energy-level differences is sufficiently large, it is very important to have an independent and effective method of finding the decay scheme. The use of the $(n,2\gamma)$ reaction offers this possibility.

The primary transition exciting a given intermediate level can give rise to the appearance of either one or several secondary transitions to different final nuclear levels. Accordingly, in the intensity distributions of cascades to different final levels the intense primary and secondary cascade γ transitions form pairs of peaks. The primary transition has the same value of the energy in the different spectra. The secondary transitions differ from each other in the difference of the energies of the final cascade levels.

Then the problem of constructing the decay scheme reduces to seeking the grouping of the peaks in several intensity distributions of cascade transitions to different final levels for which the mathematical expectation value of the difference between the positions of any pair of peaks i and j is equal to zero.

The maximum number of distributions obtained experimentally for the nucleus under study is limited only by the efficiency of the detectors used and varies from 3 to 10. In this situation (a limited statistical set) the problem of seeking the common primary transition must be solved in the most efficient and correct manner.

In practice, when the maximum-likelihood method is used, it is necessary to use the multidimensional normal distribution as the maximum-likelihood function for describing the combined distribution of the probability for a random deviation of the experimental positions of the peaks from their mathematical expectation value.

We define the difference of the positions of the peaks numbered k and l in the intensity distribution spectra of the cascades numbered i and j, respectively, as

$$R_{ijkl}=E_{ik}-E_{jl}, \tag{2}$$

where the peak position E can be specified both on the apparatus scale and on the energy scale. If σ^2 is the variance of the distribution of experimental values of E relative to their mathematical expectation value $\langle E\rangle$ and ξ is a random, normally distributed variable with zero mean and unit variance, then (2) can be written as

$$R_{ijkl}=\sigma_{ik}\xi_{ik}-\sigma_{jl}\xi_{jl}+q. \tag{3}$$

The parameter q in Eq. (3) is equal to the unknown difference of $\langle E_{ik}\rangle$ and $\langle E_{jl}\rangle$. For peaks corresponding to the same transition the mathematical expectation value of q is zero. Therefore, the problem of arranging the cascades in a decay scheme reduces to searching for those N peaks of M spectra of the $(n,2\gamma)$ reaction for which the mathematical expectation value of q is zero.

The likelihood function with the variables (3) is specified as follows:

$$L=\text{const}\cdot\exp\left[-\frac{1}{2}(\mathbf{R}^T\cdot B^{-1}\cdot\mathbf{R})\right](\det B)^{-1/2}. \tag{4}$$

Here $\mathbf{R}$ is a column vector, the $(N-1)$-th component of which is given by (3). The T denotes transposition. The values of the elements of the matrix B are determined by the mathematical expectation values of the elements of the matrix product $\mathbf{R}\cdot\mathbf{R}^T$ expressed in terms of the values of σ^2 with the indices corresponding to the spectrum number and the peak number.

The maximum of (4) in the parameter q is found by solving this equation numerically. Small modulus of the "best" value of q is a necessary but not sufficient condition for N peaks of M spectra to correspond to the same γ transition. Uniqueness of the choice is obtained by analyzing the expression $\mathbf{R}^TB^{-1}\cdot\mathbf{R}$. In the case where the analyzed peaks belong to the same γ transition and the values of σ^2 are not shifted, the distribution of values of this expression can be described by a χ^2 distribution with $N-1$ degrees of freedom.

In practice, the N peaks are considered to be coincident if the following conditions are satisfied for them:

$$\begin{aligned}&|q|\lesssim 2\langle\sigma\rangle;\\&\mathbf{R}^TB^{-1}\mathbf{R}<\chi^2_{N-1},\end{aligned} \tag{5}$$

where $\langle\sigma\rangle$ is the mean error in determining the transition energies in the $(n,2\gamma)$ reaction, and χ^2_{N-1} is equal to the value of the parameter of this distribution corresponding to 99% of the area.

The conditions (5) are checked independently for the first and second cascade transitions. As the primary transition we chose the transition with the minimum value of q. If the values for the two cascade quanta are comparable and satisfy the conditions (5), we choose the transition with the largest N as the primary one.

The likelihood function and the conditions (5) are insufficient for obtaining a unique arrangement of all the cascades in the γ-decay scheme. Additional possibilities in the assignment of transitions to the decay scheme arise when the level of reliability of the scheme is analyzed.

Such an analysis was carried out in Ref. 13 for the characteristic nuclei ^{178,179}Hf. The basic component of the analysis—the deviation from reality of the decay scheme obtained in the $(n,2\gamma)$ reaction via the above-described algorithm—can as a first approximation be considered to be the same as the discrepancy between the scheme from the $(n,2\gamma)$ reaction and its new value obtained for distortion of the transition energies by random errors and the subsequent reconstruction of the decay scheme.

The experimentally obtained "reference" decay schemes were distorted in the following manner: we added to the cascade transition energies E_1 and E_2 a random, normally distributed variable $\sigma\xi$ having variance σ^2 and zero mean:

$$\begin{aligned}\tilde{E}_{1i}&=\langle E_{1i}\rangle+\sigma\xi_i,\\\tilde{E}_{2i}&=\langle E_{2i}\rangle-\sigma\xi_i.\end{aligned} \tag{6}$$

This relation follows directly from the fact that when the resolution-improvement method[9] is used the errors in determining the cascade transition energies are completely anticorrelated.

Sets (6) were generated 10 times each for the values $\sigma=0.5$, 1, and 2 keV. The decay schemes obtained by the algorithm described above were compared with the reference scheme. In the latter we also included all cascades not placed in the decay scheme. (Such cascades arise primarily because their intermediate level decays with the appearance of only a single intense γ transition.) They are placed in the decay scheme assuming that their hard transition is the primary one, since on the average the γ emission probability is proportional to E_γ^3 ($E1$ and $M1$ transitions) or E_γ^5 ($E2$ transitions). This assertion can be based on the fact that most of the transitions placed in the decay scheme satisfy this condition.

As an example, in Fig. 9 we give the levels of the experimentally observed decay scheme and three of the

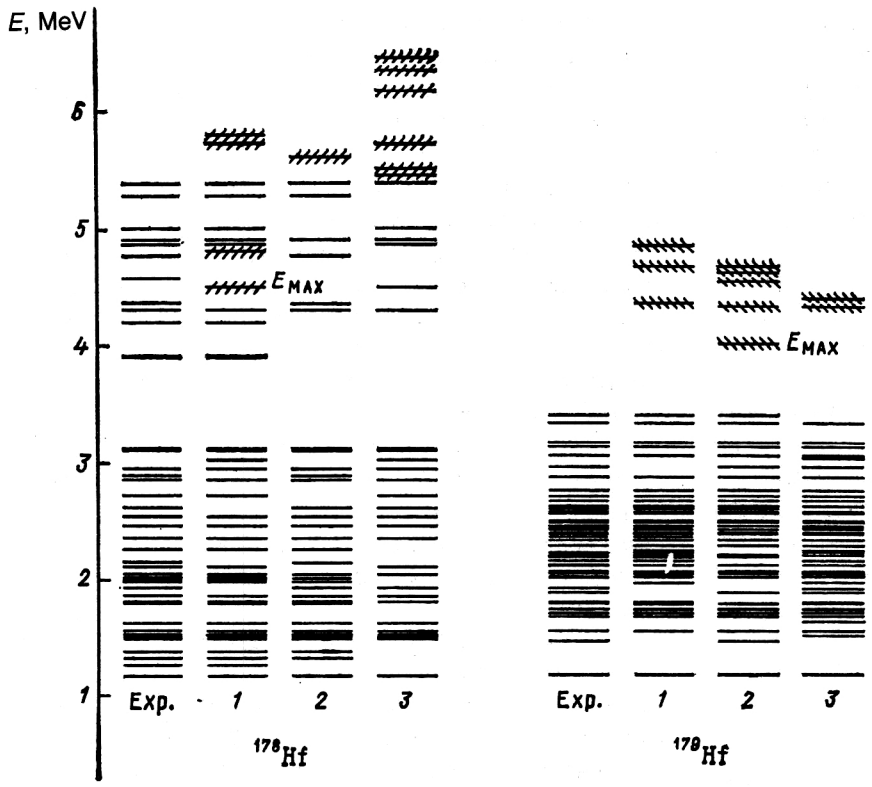

FIG. 9. Experimental (Exp.) and model schemes of excited states of ^{178,179}Hf. The false levels are hatched.

schemes that we obtained. These are labeled 1, 2, 3 and correspond to the value $\sigma = 1$ keV characteristic of most of the cascades that we found.

In Fig. 9, E_{max} is the position of the lowest of the "false" levels found in each of the ten sets of schemes for different values of σ. They can arise owing to the random coincidence of the secondary γ-transition energies in the intensity distributions of cascades to different final levels.

In Fig. 10 we show the values of the fraction of omitted and false levels, and also the values of E_{max} as a function of the parameter σ.

The modeling shows that for the experimental accuracy attained the false levels are concentrated in the upper part of the decay scheme. In practice, they are unlikely to appear at energies of less than half the neutron binding energy. Therefore, the levels established by the method described here should be viewed as reference levels when, for example, the Ritz combination principle is used.

4. THE POSSIBILITY OF CONSTRUCTING COMPLETE DECAY SCHEMES

The amount of experimental information which can be extracted from the spectroscopic data can be increased considerably by the joint study of the decay scheme which has been well established in the $(n,2\gamma)$ reaction, but which remains incomplete, in a wide range of excitation energies and the large amount of spectroscopic information obtained using a single detector in the (n,γ) reaction.

The effectiveness of combining the spectroscopic information from the (n,γ) and $(n,2\gamma)$ reactions is due to the fact that they supplement each other. Of course, the selection of total-absorption events in cascades of two, three, etc., transitions of a system from a fairly large number of highly efficient semiconductor detectors would make it possible to find all the γ transitions and their role in γ decay (beginning at a certain threshold intensity) independently of measurements made with a single detector. However, there do not yet exist any such setups based on Ge(Li) detectors.

a. The construction of a decay scheme by combining the spectroscopic information from two types of measure-

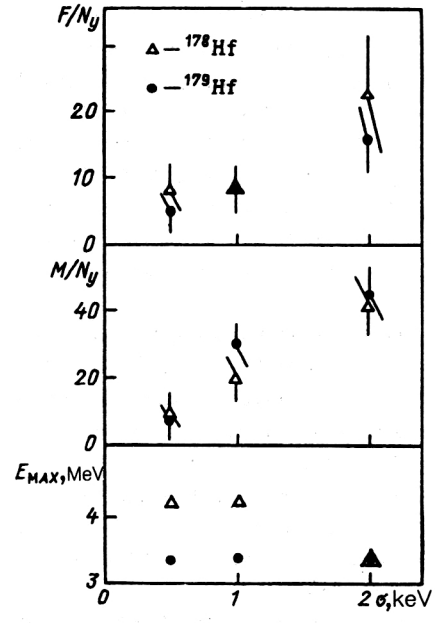

FIG. 10. Dependence of the fractions (%) M/N_y of omitted levels, F/N_y of false levels, and the energy E_{max} of the lowest of the false levels on the parameter σ.

ments makes it possible to effectively take into account such information as the uniqueness of the specification of the final level of the cascade. This is especially important in cases where the decay of a deformed nucleus with a high density of excited states is studied. Here the large number of multiplets of γ transitions of arbitrarily close energies are not only efficiently identified, but are also uniquely arranged in a decay scheme.

b. Joint analysis also makes it possible to reveal doublets of closely spaced levels ($\Delta E \sim 1$–3 keV) at arbitrary excitation energies, as long as they are populated by intense primary transitions.

c. The sum Σj_γ of secondary-transition intensities in the decay of states of any intermediate energy can be determined uniquely. If the spectra of a single detector are used to determine the intensities i_γ of γ transitions from the compound state to an intermediate state, and from the intermediate state to one of the final states (j_γ), for the two-quantum cascade this possibility follows from the expression for the ratio of the cascade intensity $i_{\gamma\gamma}$ and the intensity of the primary transition, normalized to the absolute number of decays:

$$i_{\gamma\gamma}/i_\gamma = \left(i_\gamma j_\gamma / \sum j_\gamma\right)/i_\gamma = j_\gamma / \sum j_\gamma. \qquad (7)$$

This also uniquely determines the sum Σj_γ of intensities, unobserved in the $(n,2\gamma)$ reaction, of transitions to final high-lying cascade levels. This quantity can be of interest for comparing various models of γ decay in the statistical approach to the description of this process. The sum Σj_γ is obviously equal to the sum i_γ of the intensities of all cascades of two, three,...γ quanta to the corresponding state excited by the transition.

Within the framework of these possibilities we constructed the complete decay scheme of the compound state of ^{187}W excited in thermal-neutron capture. A decay scheme containing the positions of about 130 excited states was constructed from the data on cascade transitions of Ref. 14 (235 cascades with a threshold intensity of 3×10^{-4} per decay, exciting 102 intermediate levels of the ^{187}W nucleus and ending at five final states with energies $0 \leqslant E_f \leqslant 303$ keV) and the data on 520 γ transitions,[15] measured with an average energy error of $\langle\sigma\rangle = 0.196$ keV. These 130 excited states decay by more than 500 secondary γ transitions.

Modeling of the conditions determining the accuracy with which the decay scheme is constructed and its level of reliability shows that the scheme gives a stable (reliable) reproduction of the positions of about 420 secondary γ transitions. Here it should be noted that about 270 secondary transitions in this decay scheme were fixed either using the data on the $(n,2\gamma)$ reaction, or on the basis of the decay scheme established earlier (the decay of levels with excitation energy below 780 keV).

It should also be noted that here the presence of excited states populated by intense primary transitions has been established fairly reliably. In the energy range $780 \leqslant E^* \leqslant 3380$ keV the positions of about 60 excited states have been determined from the data on the $(n,2\gamma)$ reaction with practically 100% reliability.

The other cascades with a single secondary transition, left out of the scheme in Ref. 13, were placed in the decay scheme in such a way that practically everywhere their primary transition is harder. This conclusion was drawn after comparing two alternative schemes in which either one or the other cascade quantum was assumed to be the primary one; the position of the excited state was thereby fixed, and by comparing the possible secondary transitions (the list of energies from Ref. 14) we determined the more likely variant of arrangement of cascade transitions in the γ-decay scheme in this case.

Let us demonstrate the statements listed above for the example of the ^{179}Hf nucleus. This nucleus has been studied as thoroughly as is presently possible[16] by the traditional techniques of the (n,γ) reaction. It has also been studied simultaneously and independently by the $(n,2\gamma)$ reaction.[17]

In Table II we compare the γ-transition energies measured in the GAMS spectrometer at the ILL (Grenoble, France) and by us in the $(n,2\gamma)$ reaction. Owing to the extremely high resolution of the GAMS at energies $E_\gamma \lesssim 1$ MeV and the relatively small number of transitions of this energy found under the actual cascade detection conditions, unresolved multiplets appear in the data of Ref. 16 only for transitions of energy $E_\gamma > 1$ MeV.

It should be noted that because of this the authors of Ref. 16 in most cases either placed such multiplets in the ^{179}Hf decay scheme incorrectly or completely left them out.

Here we should note that the presence of a quantum of energy $E_\gamma \pm \Delta E_\gamma$ in a cascade ending at the level E_f is indisputable. Although the algorithm described above for constructing the decay scheme does not permit the sequence of the quanta in a cascade to be determined in some cases, while in others it gives a false sequence, the difference between the values of E_f in Table II implies that the placement of the transition in the scheme of Ref. 16 is false.

Equally important is the fact that several levels form a multiplet. This follows from the data of Table III. Here we give the decay of the five intermediate levels of ^{178}Hf excited by the strongest cascades. We give the energy of the final level E_f determined from the precise data of Ref. 16, and the most probable secondary-transition energies E_γ. Some of these transitions are shown in the decay scheme of this nucleus,[16] and are incorporated on the basis of the results from the $(n,2\gamma)$ reaction. The average error in the cascade transition energies $\langle\sigma\rangle = 1.1$ keV in Ref. 17 is an order of magnitude higher than the error in determining the quantum energies in Ref. 16. Therefore, from the list of Ref. 16 we chose those quantum energies for which the sums $E_f + E_\gamma$ would differ least from those included in the scheme of Ref. 16.

It follows from this table that the levels given in it are most likely excited-state multiplets, since the variations of the sums $E_\gamma + E_f$ lie beyond the errors with which the energies E_γ and E_f are determined.

This example demonstrates the following:

TABLE II. Multiplets of γ transitions of the reaction $^{178}Hf(n,\gamma)$.

Quantum energy (Ref. 16), keV	Energy of the cascade transition (Ref. 17), keV	Cascade intensity $i_{\gamma\gamma}$ (per 10^4 decays)	Energy E_i of an intermediate level of the cascade, keV	Energy E_f of the final level of the cascade, keV
1605,1 (4)	1606,2 1605,8	8,9 25,7	2394,8 2082,9	788 476
1588,4 (5)	1589,5 1588,9	9,5 6,3	2309,7 (2268,9)	720 680
1462,24 (9)	1464,3 1462,9	6,9 11,5	2183,7 2250,3	720 788
1446,16 (7)	1446,0 1445,6	5,6 19,5	2149,0 1821,1	701 375
—	1438,5 1436,9	7,3 8,7	1813,2 (2116,9)	375 680
1391,09 (17)	1392,3 1391,6	5,0 8,8	(1606,3) 2093,6	214 701
1387,84 (6)	1388,5 1387,2	63,9 11,9	1764,0 (1808,1)	375 * 421
—	1383,70 1383,70	11,4 6,0	(1997,8) 2082,9	614 701
1350,75 (9)	1352,9 1351,5 1350,7	6,3 31,2 11,0	2053,9 1727,0 2071,2	701 375 * 720
1334,23 (8)	1335,3 1334,4 1333,1	82,7 9,2 15,6	1757,3 (2035,4) (1947,2)	420 * 701 614
1330,95 (20)	1332,7 1332,6 1332,5	9,3 13,6 43,1	2053,3 (2120,8) 1707,1	720 788 375 *
1293,49 (18)	1295,6 1293,7	5,4 14,4	1813,2 1669,1	518 * 375
1192,94 (4)	1193,1 1192,8	21,5 5,3	1912,6 1669,1	720 * 476

Remarks. a. The average statistical error in determining the cascade transition energy is 1.1 keV.

b. The statistical error in determining the cascade intensities $i_{\gamma\gamma}$, in units of 10^{-4} quanta per decay is:
about 30–50% for $5 < i_{\gamma\gamma} < 10$;
about 15–30% for $10 < i_{\gamma\gamma} < 30$;
less than 6% for $i_{\gamma\gamma} > 60$.

c. The stars denote correspondences on the energies of levels excited by the secondary transition of the cascade to the scheme of Ref. 16.

a. The degree to which the data on the decay schemes of complex nuclei are approximate.

b. The ever growing experimental possibilities arising when study of the $(n,2\gamma)$ reaction is included with the traditional methods.

5. γ-DECAY CHANNELS IN *4S*-SHELL NUCLEI

The most important conclusion following from study of the cascade γ decay of compound states of nuclei near the 4*S* maximum of the neutron strength function is that roughly half the total radiative width of the resonance can be attributed to the contribution of several dozen very intense primary transitions. Their intensity clearly exceeds the values expected when:

a) the matrix elements of *E*1, *M*1, and *E*2 transitions are independent of the structure of the levels they connect;

b) the fluctuations of the partial widths about their mean values are purely random, and the law describing them is the Porter–Thomas distribution.[18]

From the first statement it is easy to see that the intensity $i_{\gamma\gamma}$ of a single cascade considered separately must decrease very rapidly with increasing excitation energy of its intermediate level.

The intensity

TABLE III. Decay of some levels of ^{179}Hf.

Level E_i, keV	Final level E_f (Ref. 16), keV	Transition energy E_γ (Ref. 16), keV	Cascade intensity $i_{\gamma\gamma}$, 10^4	Transition intensity i_γ, 10^4	$E_\gamma + E_f$, keV
1572,34	375,032(2)	1197,46(8)	11(4)	7,65	1572,49(8)
	476,331(1)	1095,77(8)	9(3)	2,45	1572,10(8)
	518,332(2)	1054,25(3)	12(2)	14,81	1572,57(3)
	679,512(3)	892,00(10)	3(1)	0,51	1571,51(10)
	788,179(3)	782,78(7)	9(2)	0,45	1570,96(7)
1706,12	375,032(2)	1330,95(20)	43(4)	8,79	1705,98(20)
	420,890(1)	1285,97(8)	7(4)	3,24	1706,86(8)
	518,322(2)	1187,83(13)	38(3)	6,16	1706,15(13)
	614,124(2)	1092,00(13)	15(4)	1,49	1706,12(13)
	679,512(3)	1024,71(8)	9(2)	2,99	1704,23(8)
	720,611(3)	985,46(3)	31(4)	3,96	1706,07(3)
1725,85	375,032(2)	1350,75(9)	31(5)	5,04	1725,78(9)
	420,890(1)	1305,45(13)	12(4)	2,14	1726,34(13)
	518,332(2)	(1210,6)	18(3)	—	—
	614,120(2)	1111,55(7)	66(4)	4,78	1725,67(7)
	679,512(3)	1046,16(6)	17(2)	3,89	1725,67(6)
	701,052(3)	1024,71(8)	21(3)	2,99	1725,76(8)
	720,607(3)	1005,24(3)	28(3)	4,70	1725,85(3)
	788,179(3)	937,55(3)	17(3)	1,34	1725,73(3)
1756,68	214,336(2)	1543,75(30)	14(3)	1,14	1758,08(30)
	375,032(2)	1381,24(30)	44(4)	3,50	1756,27(30)
	420,890(1)	1334,23(8)	83(9)	10,74	1755,11(8)
	476,331(1)	1279,45(11)	54(3)	3,59	1755,78(11)
	518,322(2)	1239,18(23)	34(3)	2,20	1757,50(23)
	614,120(2)	1141,16(3)	99(16)	11,31	1755,38(3)
	616,751(2)	1138,03(16)	50(17)	1,15	1754,78(16)
	679,512(3)	1078,37(8)	21(2)	1,76	1757,88(8)
	701,052(3)	1055,06(5)	80(3)	9,88	1756,11(5)
	720,607(3)	1038,11(4)	15(4)	2,91	1758,71(4)
	788,179(3)	969,49(11)	19(3)	0,94	1757,67(11)
1763,12	375,032(2)	1387,84(6)	64(4)	9,0	1762,87(6)
	420,890(1)	1342,60(30)	43(9)	4,4	1763,52(30)
	476,331(1)	1285,97(8)	11(3)	3,2	1762,30(8)
	518,322(2)	1244,73(6)	24(3)	5,9	1763,05(6)
	701,052(3)	1061,63(5)	25(3)	9,2	1762,68(5)
	720,607(3)	1042,28(7)	24(4)	1,4	1762,89(7)
	788,179(3)	975,72(6)	18(3)	4,1	1763,90(6)

$$i_{\gamma\gamma} = \Gamma_{\lambda g}\Gamma_{gf}/\Gamma_\lambda\Gamma_g \qquad (8)$$

is determined by the partial widths of the cascade transitions between states $\lambda \to g \to f$ and the total radiative widths Γ of the decaying states λ and g. The ratio $\Gamma_{\lambda g}/\Gamma_\lambda$ has the same energy dependence as the partial widths of the primary transitions, $\Gamma_{\lambda g}$, since for the compound state $\Gamma_\lambda \simeq$ const. $\Gamma_{\lambda g}$ is proportional to some power of the γ-transition energy; in particular, in the description of the $E1$-transition widths by the giant electric dipole resonance (GEDR) model $\Gamma_{\lambda g} \sim E_{\lambda g}^5$. The ratio Γ_{gf}/Γ_g falls rapidly with increasing excitation energy, since the number of possible γ transitions of its decay grows very rapidly, owing to the exponential increase of the density of excited states.

The intensities of observed individual cascades as a function of the energy of their primary transition are compared with the calculated value (8) for the ^{165}Dy, ^{168}Er, and ^{179}Hf nuclei in Figs. 11–13. Here the cascades which are not placed in the decay scheme are included in the set, assuming that their primary transition is the hardest one.

We see that the experimentally observed energy dependence of the intensity does not correspond to the model calculation.

It should be noted that there is a general trend, characteristic of all the nuclei that we have studied, for the energy dependence of the experimentally observed cascades to differ most from the theoretical prediction in the

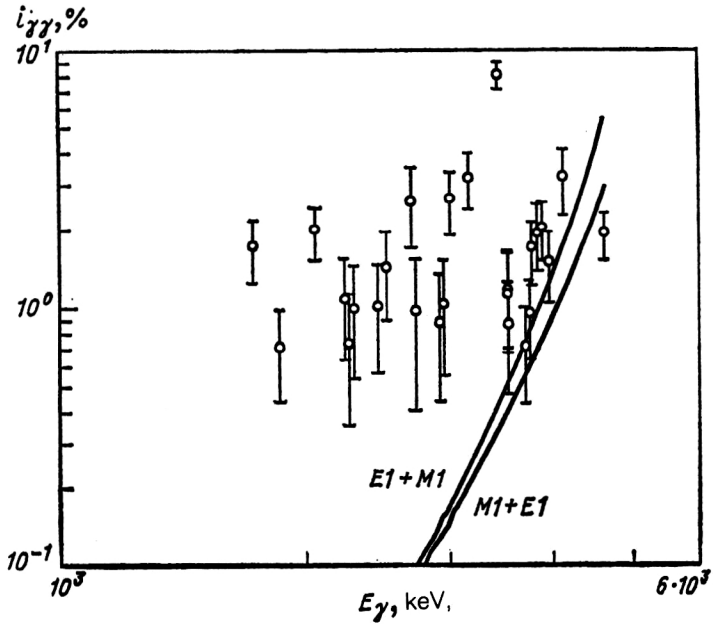

FIG. 11. Intensity $i_{\gamma\gamma}$ of the strongest cascades with total energy 5556 keV in ^{165}Dy for various energies of their primary transitions E_γ: points—experiment; lines—calculation for various possible sequencings of transitions of different multipole order in the cascade.

region $A \approx 165$ and to decrease quite rapidly with increasing A. This dependence can be qualitatively understood if we assume that the observed enhancement of the cascade intensity is related to the existence of one-particle transitions between the $4S$ and $3P$ shells. The large value of the neutron strength function of the nuclei that we have studied from the region $144 \leqslant A \leqslant 187$ is due to the fragmentation of the strength of the $4S$ state in neutron resonances. The $3P$ shell is fragmented at lower excitation energies. In particular, in the region $A \approx 165$ its component $K^\pi[Nn_z\Lambda] = \frac{1}{2}^-[510]$ is located[19] in the vicinity of half the neutron binding energy. As A increases this component moves downward in excitation energy. For all the nuclei that we have studied the strongest cascades have an intensity of roughly several percent of the sum of the intensities of all the possible cascades between the compound state and a given low-lying level. Similar cascade intensities are also expected with the primary transitions of maximum energy (Figs. 11–13).

It is clear that the observed discrepancy between the experimentally determined cascade intensities and the values predicted theoretically is most clearly manifested at small values of the calculated intensities even when the nature and scale of the enhancement are identical in different nuclei.

One-particle transitions between the $3S$ and $2P$, $3P$ and $3S$ shells have been observed experimentally earlier.[20] It is difficult to observe one-particle transitions between the $4S$

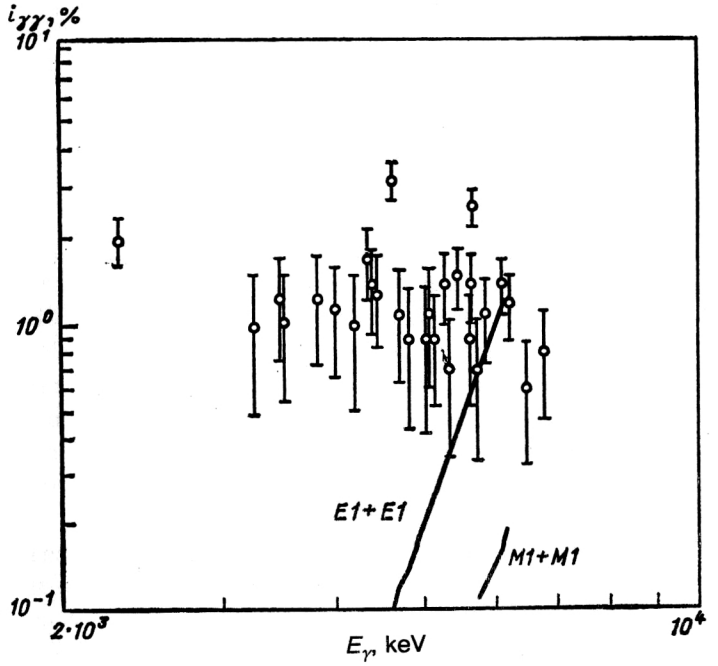

FIG. 12. Energy dependence of the intensity of cascades with total energy 7507 keV in ^{168}Er. See Fig. 11 for the notation.

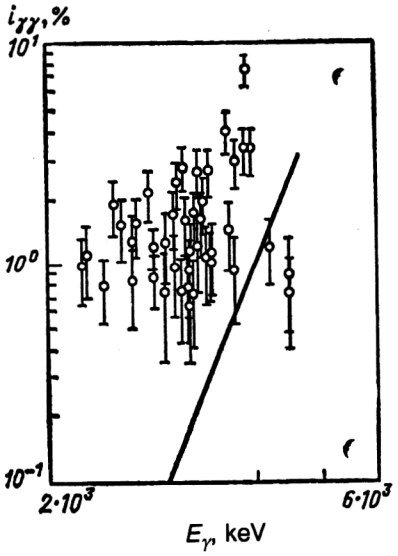

FIG. 13. Cascades with total energy 5262 keV in ^{175}Yb. See Fig. 11 for the notation.

and $3P$ states experimentally in the (n,γ) reaction—the strength of the one-particle state is smeared over several levels, and the expected effects of correlations of the widths of various processes in deformed nuclei are weakened considerably in relation to the case of spherical nuclei.[20]

The experimentally observed (see Figs. 11–13) enhancement of the cascade intensities can also be explained purely formally as random fluctuations of the widths of primary and secondary transitions. However, analysis of the cascade intensity distribution about the expected mean value shows that this is not so—the full sum of the intensity of the strongest cascades exceeds the analogous value expected in the case where the cascade intensity fluctuates as the convolution of two Porter–Thomas distributions.

The proof is given in Ref. 21. It is carried out using several assumptions, chosen so that the estimate obtained for the probability of the random appearance of a given or larger deviation of the intensities from the mean would be overestimated. For this:

a) The cascades resolved experimentally which are not placed in the level scheme are included in the analysis, assuming that their hard transition is the primary one. In this case the deviation of the experimental intensity from the calculated one (8) is a minimum.

b) Cascades which are not resolved by the detectors are included in the analysis, assuming that the γ transition of the lowest energy is the primary one.

c) The intensity of any cascade not resolved experimentally does not exceed that calculated using the model by more than a factor of K (henceforth we take $K = 30$). This means that at the primary transition energy $E_\gamma \gtrsim 2$ MeV the intensity of any unresolved individual cascade does not exceed 0.5% of the sum of the intensities of all the two-quantum cascades to a given level. This is the usual practical threshold for observing an individual cascade in an experiment.

d) The comparison is made for the relative experimental and calculated cascade intensities. Since the experimental absolute cascade intensities in practice exceed the calculated ones in many cases, this condition very strongly raises the upper estimate obtained below for the probability of the appearance of a group of cascades of a given or greater intensity than is observed experimentally.

Using $Z = i^E_{\gamma\gamma}/\langle i^C_{\gamma\gamma}\rangle$ to denote the ratio of the experimental cascade intensity $i^E_{\gamma\gamma}$ to the corresponding calculated value $\langle i^C_{\gamma\gamma}\rangle$, for n experimentally resolved cascades we define the growing sum as a function of the parameter Z describing the intensity excess:

$$D(Z) = \sum^{n} Z. \tag{9}$$

For cascades which are not resolved experimentally a similar expression can be defined for an arbitrary, sufficiently small range of transition energies in which the change of $\langle i^C_{\gamma\gamma}\rangle$ can be neglected:

$$C(K) = \sum_j \frac{1}{\langle i^C_{\gamma\gamma}\rangle} \int_0^K Z\langle i^C_{\gamma\gamma}\rangle F(Z)\,dZ. \tag{10}$$

The value of the integral in (10) is simply equal to the sum of the intensities of the experimentally unresolved cascades, and the parameter K from condition (c) determines the level above which the tail of the cascade intensity distribution can be analyzed.

The function $F(Z)$ is a distribution whose dispersion cannot exceed that of the convolution of two Porter–Thomas distributions.

The sum of (9) and (10) for values $Z > K$ can be associated with the theoretical distribution

$$T(Z) = \int_0^Z ZF(Z)\,dZ. \tag{11}$$

Graphs of the distributions $T(Z)$ and $Q(Z) = D(Z) + C(K)$ for the ^{165}Dy nucleus are shown in Fig. 14. The probability for the distribution $Q(Z)$ to deviate from the expected one $T(Z)$ can be found by direct calculation of $Q(Z)$ for $Z = K$ for a set of random quantities from the distribution $F(Z)$. The number m of such quantities is uniquely determined by the number of possible cascades in the nucleus in question between the compound state and a given low-lying level. The model estimate is $m \sim 10^3$ for an even–odd compound nucleus.

In Table IV we give the values of $q = 1 - Q(K)$, the fractional area of the distribution, and the estimated $P_{\max}$, the probability for the random observation of a given or larger deviation of q from the theoretical value $q = 6.2\%$ for $K = 30$.

It should be noted that besides the conditions (a)–(d) raising the value of $P_{\max}$, the latter is also very much exaggerated by the condition that the function $F(Z)$ be a convolution of two Porter–Thomas distributions. If the real fluctuations of the widths $\Gamma_{\lambda g}$ and Γ_{gf} are described by that or a narrower distribution, then, owing to the correlation of the random quantities Γ_{gf} and Γ_g, the distribution

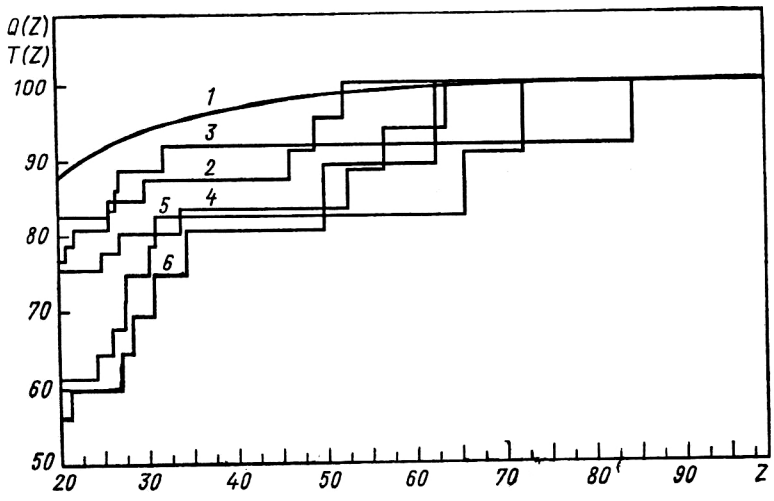

FIG. 14. Fractions of the area of the theoretical (curve 1) and experimental (curves 2–6) distributions for various total energies of cascades in ^{165}Dy. The numbers denote cascades with the total energy: 2—5607; 3—5556; 4—5534; 5—5176; 6—5142 keV.

$F(Z)$ will be narrower, and the probability P_{max} will be smaller. In other words, the γ-decay "channels" are seen experimentally.

The general conclusion that decay "channels" appear in the γ decay of compound states of 4S-shell nuclei was drawn by us in an earlier study,[22] in analyzing the fluctuations of the population of low-lying states by cascades of γ transitions.

The possibility of estimating the "effective" number of primary transitions (i.e., the presence of a small number of intense cascades/decay channels) in the γ decay of a compound state is determined[23] by the following considerations:

a. A primary γ transition with mean relative intensity $\langle i_m \rangle$ populates a low-lying level f with probability $\langle i_m \rangle b_{mf}$ via the decay of the level m excited by it.

b. Then, assuming that the fluctuations of the reduced primary-transition widths obey the Porter–Thomas distribution, the dispersion of the population $P_{\lambda f}$ of the level f in various resonances is equal to $2\langle i_m \rangle^2 b_{mf}^2$, summed over all possible primary transitions.

c. The probability b_{mf} of population of the level f in the decay of a state m populated by a primary transition is determined by the mean radiative widths and the density of states below the neutron binding energy.

The relative fluctuations of the population of the level f are larger, the lower the effective number of primary transitions in the sum:

$$P_{\lambda f} = \sum_m \langle i_m \rangle b_{mf}. \qquad (12)$$

TABLE IV. Experimental fraction q of the contribution to the total area of the distribution of the intensity fluctuations $Q(Z)$ of cascades with intensity exceeding the average calculated value by a factor of 30 or more; upper estimate of the probability for the random appearance of these or larger values P_{max} for cascades with total energy ΣE_γ

Nucleus	ΣE_γ, keV	q	P_{max}, %
^{165}Dy	5607	0,13	7
	5556	0,11	13
	5534	0,20	0,4
	5175	0,31	$<0,1$
	5142	0,25	$<0,1$
^{168}Er	7691	0,17	$<0,1$
	7507	0,10	$<0,1$
	7222	0,53	$<0,1$
	6950	0,65	$<0,1$
	7875	0,43	$<0,1$
	6775	0,65	$<0,1$
^{175}Yb	5307	0,12	9
	5267	0,18	2
	5220	0,15	5
	5183	0,25	0,3
	5011	0,18	2
	4950	0,23	0,3
	4902	0,07	30
	4830	0,57	$<0,1$

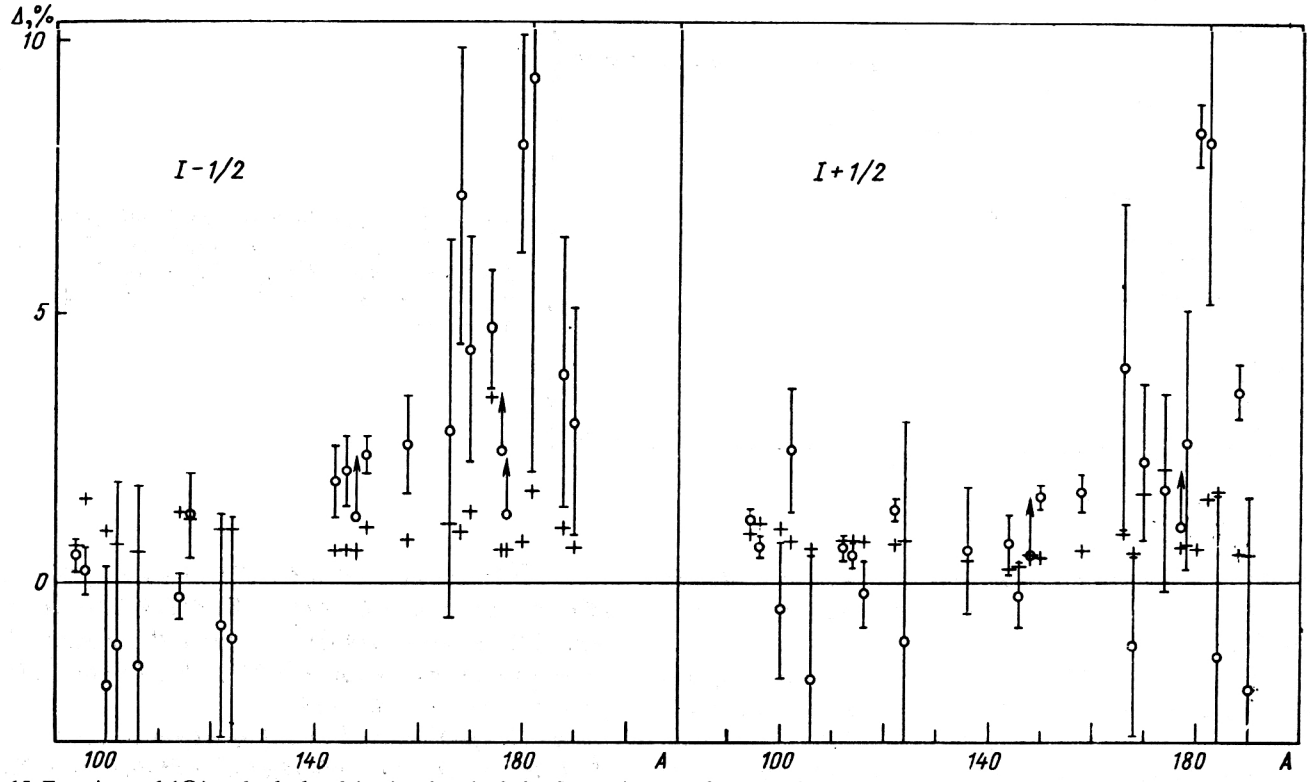

FIG. 15. Experimental (○) and calculated (+) reduced relative fluctuations as a function of the atomic mass A of the compound nucleus. The arrows indicate the lower estimates.

The population $P_{\lambda f}$ and its dispersion can be calculated simply within the framework of the model description of the transition widths and level density, and in practice in experiment one can measure the ratio of the intensities R_{gf} of γ transitions arising in the decay of low-lying excited states f and g for different resonances of the same spin.

The dispersion of R_{gf} is the sum of two dispersions reflecting different sources of its fluctuations: the S^2 dispersion determined by the counting statistics in the measured spectra, and $\sigma^2_{PT} \sim \langle \nu \rangle^{-1}$, the dispersion determined by random fluctuations of the widths of the primary transitions in different resonances.

According to Ref. 23, the mathematical expectation value of the difference of the values of R at the resonances i and $i+1$ is

$$M[(R_{i+1}-R_i)^2]=S_i^2+S_{i+1}^2+2\sigma^2_{PT}. \qquad (13)$$

Using the multidimensional normal distribution (4) as the likelihood function, we have developed[23] an algorithm for determining σ^2_{PT}.

The calculated and experimental values of σ^2_{PT} were compared both for special measurements of the ratios of secondary-transition intensities in neutron resonances of the target nuclei ^{111,113}Cd, ^{149}Sm (Ref. 23), ^{95}Mo, and ^{157}Cd (Ref. 24), and on the basis of the corresponding data in the literature for a wide range of nuclei from the region $93 \leqslant A \leqslant 189$ (Ref. 22).

The results of this comparison for two possible spins of compound states of s resonances are shown in Fig. 15.

The variations of the experimental values of σ^2_{PT} in different nuclei depend strongly on the resonance density D. The reduction of this dependence is ensured by the fact that we are comparing the calculated and experimental values of the new variable $\Delta = \sigma_{PT}/(\langle R \rangle \sqrt{D})$, which depend weakly on the atomic mass of the isotope studied.

We see from this figure that in the vicinity of the 4S maximum of the neutron strength function the reduced relative fluctuations of the population and the ratios of the populations of low-lying states of such nuclei determined by them are clearly higher than predicted by the calculation. In the vicinity of the minimum of the s-neutron strength function ($A \lesssim 130$) there is no such excess.

Therefore, the presence of enhanced cascades of the decay of compound states of nuclei of the 4S shell (examples are given in Figs. 11–13) is typical. In other words, the effective number $\langle \nu \rangle$ of primary transitions in the γ decay of nuclei in the vicinity of the 4S maximum of the neutron strength function is smaller, and the value of $\sigma^2_{PT} \sim \langle \nu \rangle^{-1}$ is higher than the values given by the model calculation assuming that the γ emission probability is independent of the structure of the states connected by this transition. This, the main premise of the statistical theory of γ decay, is therefore refuted by the two different types of experiment discussed above. It is very important that the analysis of the fluctuations of ratios of the intensities of secondary γ transitions of resonance neutron capture did not reveal this effect near the minimum of the strength function. This is simultaneously established by:

a. The absence of noticeable systematic errors in the technique of Refs. 22–24.

b. The existing features of γ decay are related to relatively large one- (two-) quasiparticle states in the wave functions of the compound state.

CONCLUSION

1. We have developed and put into practice a technique which allows the systematic study of the γ-decay process in a range of excitations which has not been studied before.

2. We have shown that the decay modes of levels of complex nuclei excited by intense primary transitions can be determined with a high degree of reliability up to excitation energies of 4 MeV and higher in any even–even or even–odd deformed nucleus.

3. Groups of high-intensity cascades exciting several dozen levels of the nucleus under study are observed directly in experiment. The enhancement of these cascades cannot be explained by random fluctuations of the widths of the primary and secondary γ transitions of the decay of the compound state. This conclusion is valid only assuming that random deviations from the average ratio of the partial widths of the transitions to the total radiative width of the decaying state are described by the Porter–Thomas distribution (or any other distribution with smaller dispersion).

4. γ-transition cascades of high intensity are observed in nuclei of the $4S$ maximum of the neutron strength function.

The simplest explanation of this phenomenon is that in experiment one observes transitions between $4S$ and $3P$ neutron shells fragmented into a series of states of deformed nuclei.

5. We have made essential improvements and extensions of the data on the decay schemes of the following nuclei: ^{144}Nd (Ref. 25), ^{146}Nd (Ref. 11), ^{163}Dy (Ref. 26), ^{165}Dy (Refs. 27 and 19), ^{168}Er (Ref. 28), ^{174}Yb (Ref. 29), ^{175}Yb (Ref. 30), ^{178}Hf (Ref. 31), ^{179}Hf (Ref. 17), ^{183}W (Ref. 32), ^{187}W (Ref. 14).

We have obtained new spectroscopic data on the decay schemes and on a number of other nuclei.

The authors would like to take this opportunity to thank A. A. Bogdzel', V. A. Bondarenko, O. I. Elizarov, G. P. Zhukov, I. L. Kuvaga, V. D. Kulik, B. P. Osipenko, P. T. Prokof'ev, G. L. Rezvaya, L. I. Simonova, V. G. Tishin, V. I. Khorozova, A. S. Khrykin, and Ya. Yurkovski for their contribution to the realization of the method described here and its use to obtain the results.

We gratefully remember the late Yu. S. Yazvitskĭ, who began the experimental studies with us.

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Translated by Patricia Millard