Description of deformed nuclei in the quasiparticle–phonon and interacting-boson models

V. G. Solov'ev

Joint Institute for Nuclear Research, Dubna

Fiz. Elem. Chastits At. Yadra **21**, 1360–1404 (November–December 1990)

The basic assumptions of the quasiparticle–phonon nuclear model (QPNM) and the interacting-boson model (IBM) with *sd*, *sdg*, and *sdf* bosons are presented. Comparison of these models has shown that there is a cardinal difference in the description of nonrotational states with $K^{\pi}_{\nu} = 0^{\pm}_2$, $0^{\pm}_3$, $0^{\pm}_4$, 1^{-}_2, 1^{-}_3, $2^{\pm}_2$, $2^{\pm}_3$, $2^{\pm}_4$, $2^{\pm}_5$, $3^{\pm}_2$, $3^{\pm}_3$, and 4^{+}_1 of even–even deformed nuclei. This difference is demonstrated by the example of five 2^+ states in ^{168}Er, the structure of which is correctly described in the QPNM but cannot be described in the IBM. The distribution of the $E\lambda$ strength between the low-lying states is investigated and is shown to differ in individual cases from the standard distribution—most of the $E\lambda$ strength is concentrated on higher states and not on the first state. The nonstandard distribution of the $E\lambda$ strength is correctly described in the QPNM but not in the IBM. It is shown that to treat states with positive and negative parity the *spdfg* IBM should be constructed. According to calculations in the QPNM, collective two-phonon states should not exist in well-deformed nuclei, but they certainly should in accordance with the IBM. Experiments are needed to look for two-phonon states in deformed nuclei and to study the structure of states with energy 2–3 MeV.

INTRODUCTION

Excited states of even–even deformed nuclei with energy up to 2.5 MeV are studied experimentally with success in β decays, in inelastic scattering of protons and α particles, in single- and two-nucleon transfer reactions, in (n, γ) reactions, etc. In even–even deformed nuclei there are low-lying rotational, vibrational, and two-quasiparticle states. The last can be described in the model of independent quasiparticles.[1] Collective γ-vibrational states with $K^{\pi}_{\nu} = 2^+_1$, β-vibrational states with $K^{\pi}_{\nu} = 0^+_1$, and the first octupole states with $K^{\pi}_{\nu} = 0^-_1$, 1^-_1, 2^-_1, and 3^-_1 can be fairly well described in the random-phase approximation (RPA) as single-phonon states.[2–4] In accordance with the widely adopted interpretation (see Ref. 5), single-, two-, and three-phonon states must exist in even–even nuclei.

The modern description of the vibrational states of deformed nuclei is based on the phenomenological interacting-boson model (IBM)[6–8] and the microscopic quasiparticle–phonon nuclear model (QPNM).[9–13] Calculations in other phenomenological models are restricted to giant resonances. Calculations in other microscopic models, such as the method of self-consistent collective coordinates[14] and the multiphonon method,[15] are mainly restricted to description of the anharmonicity of the two-phonon states.

To understand the structure of nuclear states, it is necessary to examine the basic assumptions of the QPNM and the IBM and to compare the results of calculations of excited states of even–even deformed nuclei in the two models, both with one another and with the experimental data. Such a comparison is helpful for establishing the effectiveness of these models and the limits of their applicability. In making such a comparison, we shall restrict ourselves to nonrotational states and electric transitions.

Before we compare the two models, we briefly review the basic assumptions of them.

1. DESCRIPTION OF NONROTATIONAL STATES OF DEFORMED NUCLEI IN THE QUASIPARTICLE–PHONON MODEL

Basic assumptions of the quasiparticle–phonon nuclear model

For the description of deformed nuclei, the Hamiltonian of the QPNM consists of the average field of the neutron and proton systems in the form of an axisymmetric Woods–Saxon potential, monopole and quadrupole pairing, and isoscalar and isovector particle–hole (p–h) and particle–particle (p–p) multipole and spin-multipole interactions between the quasiparticles.

Calculations in the QPNM are made in four stages. The *first stage* consists of finding the single-particle energy eigenvalues and wave functions of the Woods–Saxon potential. The parameters of this potential are chosen to obtain, with allowance for the quasiparticle–phonon interaction, a correct description of the low-lying states of odd-A nuclei. There is no doubt that one could take a different form of the potential of the average field or use the Hartree–Fock method to calculate the energies and wave functions of the single-particle states and use them in calculations in the framework of the QPNM. These differences are not fundamental. Use of the Hartree–Fock method means that one uses an earlier stage of parametrization, i.e., parametrization of an effective action, for example, in the form of Skyrme forces. Calculations in the QPNM are made with the parameters of the Woods–Saxon potential fixed during 1968–1973 and given in Refs. 3, 4, and 16. The single-particle states are labeled by the asymptotic quantum numbers introduced by Nilsson: $Nn_z\Lambda\uparrow$ for $K = \Lambda + \frac{1}{2}$ and $Nn_z\Lambda\downarrow$ for $K = \Lambda - \frac{1}{2}$, these being designated by $q\sigma$, $\sigma = \pm 1$.

The *second stage* is a Bogolyubov canonical transformation

$$a_{q\sigma} = u_q \alpha_{q\sigma} + \sigma v_q \alpha^{+}_{q-\sigma}, \tag{1}$$

 0090-4759/90/060579-20$04.00

which realizes a transition from the particle operators $a^+_{q\sigma}$, $a_{q\sigma}$ to the quasiparticle operators $\alpha^+_{q\sigma}, \alpha_{q\sigma}$ followed by calculations in the model of independent quasiparticles.[1] Simultaneous allowance for monopole pairing with constants G_τ and quadrupole pairing with G^{20} and use of the condition that 0^+ ghost states be eliminated led in Ref. 17 to the equations

$$1=\frac{G_\tau}{2}\sum_q^\tau \frac{C_\tau+f^{20}(qq)\,C_{2\tau}}{C_\tau\varepsilon_q}; \qquad (2)$$

$$1=2G^{20}\left\{\sum_q^\tau \frac{f^{20}(qq)\,C_\tau}{2C_{2\tau}\varepsilon_q}+\sum_{qq'}^\tau \frac{f^{20}(qq')\,(v^{(+)}_{qq'})^2}{\varepsilon_q}\right\}; \qquad (2')$$

$$N_\tau=\sum_q\left[1-\frac{\xi(q)}{\varepsilon_q}\right]. \qquad (2'')$$

If in Eq. (2') we ignore the nondiagonal matrix elements $f^{20}(qq')$, then we obtain the equations derived earlier in Ref. 18. Here

$$\left.\begin{aligned}
&\varepsilon_q=[\Delta_q^2+\xi^2(q)]^{1/2},\ \xi(q)=E(q)-\lambda_\tau;\\
&\Delta_q=C_\tau+f^{20}(qq)\,C_{2\tau},\quad C_\tau=G_\tau\sum_q^\tau u_qv_q;\\
&C_{2\tau}=2G^{20}\sum_q^\tau f^{20}(qq)\,u_qv_q;\\
&u^{(\pm)}_{qq'}=u_qv_{q'}\pm u_{q'}v_q,\ v^{(+)}_{qq'}=u_qu_{q'}\pm v_qv_{q'}.
\end{aligned}\right\} \qquad (3)$$

Further, $E(q)$ is the energy of the single-particle state, $\Sigma^\tau_{qq'}$ denotes a summation over the single-particle levels of the neutron system when $\tau=n$ or proton systems when $\tau=p$, λ_τ are the chemical potentials, G_τ are the constants of the monopole pairing, and $G^{\lambda\mu}$ are the constants of the p–p coupling of multipolarity λ with projection μ. The constants G_τ and G^{20} are determined[19] from the pairing energies and the energies of two-quasiparticle states with $K>4$. The energies of the two-quasiparticle states are calculated with allowance for the blocking effect.

The effective interactions between the quasiparticles are represented in the form of expansions with respect to multipoles and spin multipoles. The effective interactions play an important part in compensating the equations ignored in the Hartree–Fock–Bogolyubov approximation. They are also associated with nucleon–nucleon interactions in the nuclear medium, and some of their terms can be associated with the exchange of one or two mesons. To make calculations in the QPNM, it is important that the interaction between the quasiparticles be represented in a separable (factorized) form. Separable potentials are widely used to describe nucleon–nucleon interactions and to study three-body nuclear systems and light nuclei. The separable representations of rank $n_{\max}\leqslant 5$ of the Paris and Bonn potentials give a satisfactory approximation for these potentials. Separable potentials are used in cases when the results of the calculations are more sensitive to the form of the radial dependence of the forces as compared with QPNM calculations of the properties of complex nuclei. It should be borne in mind that the matrix elements of the effective interactions are used in the calculations. The single-particle wave functions cut out a small part of the interaction. One can construct separable interactions whose matrix elements are close to those of more complicated forces. One may suppose that felicitously chosen interactions between the quasiparticles in separable form do not limit the accuracy of the calculations. It is therefore justified to use separable interactions of finite rank to calculate the properties of complex nuclei.

As in Ref. 20 for spherical nuclei, we introduce a separable interaction of finite rank for deformed nuclei. We expand in multipoles the central, spin-independent interaction and write it in the second-quantized form

$$\sum_{\substack{q_1q_2q_2'q_1'\\ \sigma_1\sigma_2\sigma_2'\sigma_1'}}\langle q_1\sigma_1,\ q_2\sigma_2\Big|\sum_{\lambda\mu}[R^{\lambda\mu}(r_1r_2)+(\tau^{(1)}\tau^{(2)})\,\overline{R}^{\lambda\mu}(r_1r_2)]\sum_\sigma Y_{\lambda\sigma\mu}(\theta_1\varphi_1)\times Y_{\lambda-\sigma\mu}(\theta_2\varphi_2)|q_2'\sigma_2',\ q_1'\sigma_1'\rangle\, a^+_{q_1\sigma_1}a^+_{q_2\sigma_2}a_{q_2'\sigma_2'}a_{q_1'\sigma_1'}.$$

If we take the separable interaction of rank $n_{\max}>1$ for p–h and p–p interactions in the form

$$\left.\begin{aligned}
&R^{\lambda\mu}(r_1r_2)=\varkappa_0^{\lambda\mu}\sum_{n=1}^{n_{\max}}R_n^{\lambda\mu}(r_1)\,R_n^{\lambda\mu}(r_2),\\
&\overline{R}^{\lambda\mu}(r_1r_2)=\varkappa_1^{\lambda\mu}\sum_{n=1}^{n_{\max}}\overline{R}_n^{\lambda\mu}(r_1)\,\overline{R}_n^{\lambda\mu}(r_2),\\
&R^{\lambda\mu}(r_1r_2)=G_0^{\lambda\mu}\sum_{n=1}^{n_{\max}}\widetilde{R}_n^{\lambda\mu}(r_1)\,\widetilde{R}_n^{\lambda\mu}(r_2),\\
&\widetilde{\overline{R}}^{\lambda\mu}(r_1r_2)=G_1^{\lambda\mu}\sum_{n=1}^{n_{\max}}\widetilde{\overline{R}}_n^{\lambda\mu}(r_1)\,\widetilde{\overline{R}}_n^{\lambda\mu}(r_2),
\end{aligned}\right\} \qquad (4)$$

then the multipole expansion takes the form

$$\sum_{\lambda\mu}\sum_{n=1}^{n_{\max}}\Big\{\sum_{\tau\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})\sum_\sigma M_{\lambda\sigma\mu n}(\tau)\,M_{\lambda-\sigma\mu n}(\rho\tau)+\sum_{\tau\sigma}(G_0^{\lambda\mu}+G_1^{\lambda\mu})\,P^+_{\lambda\sigma\mu n}(\tau)\,P_{\lambda\sigma\mu n}(\tau)+\varkappa_1^{\lambda\mu}\sum_\sigma(M^{\mathrm{CH}}_{\lambda\sigma\mu n})^+M^{\mathrm{CH}}_{\lambda\sigma\mu n}+G_1^{\lambda\mu}\sum_\sigma(P^{\mathrm{CH}}_{\lambda\sigma\mu n})^+P^{\mathrm{CH}}_{\lambda\sigma\mu n}+(G_0^{\lambda\mu}-G_1^{\lambda\mu})\sum_\tau P^+_{\lambda\sigma\mu n}(\tau)\,P_{\lambda\sigma\mu n}(-\tau)\Big\}.$$

The first and second terms take into account the p–h and p–p interactions, respectively, the third and fourth terms take into account the charge-exchange p–h and p–p interactions, respectively, and the last takes into account the exchange of two nucleons. Here, $\varkappa_0^{\lambda\mu}$, $\varkappa_1^{\lambda\mu}$ and $G_0^{\lambda\mu}$, $G_1^{\lambda\mu}$ are the isoscalar and isovector constants of the p–h and p–p interactions; in what follows, we use $G^{\lambda\mu}=G_0^{\lambda\mu}+G_1^{\lambda\mu}$.

The introduction of a separable interaction of finite rank $n_{\max}>1$ as opposed to $n_{\max}=1$ leads to an additional summation over n. It is sensible to introduce a separable interaction of rank $n_{\max}$ if $n_{\max}$ is much less than the rank of the RPA secular-equation determinant for the nonseparable interaction. There is a certain arbitrariness in the choice of the radial dependence of the separable interactions. The existence of collective vibrational quadrupole and octupole states indicates that there must be a maximum in the radial dependence on the surface of the nucleus. We therefore take $R^{\lambda\mu}(r)=r^\lambda$ or $R^{\lambda\mu}(r)=\partial V(r)/\partial r$, where $V(r)$ is the central part of the Woods–Saxon potential. The calculations of Refs. 21 and 22 were made with $R^{\lambda\mu}(r)=\partial V(r)/\partial r$.

Further, one introduces RPA phonons $Q^+_{\lambda\mu i\sigma}$ and $Q_{\lambda\mu i\sigma}$,

$$Q^+_{\lambda\mu i\sigma}=\frac{1}{2}\sum_{qq'}[\psi^{\lambda\mu i}_{qq'}A^+(qq';\ \mu\sigma)-\varphi^{\lambda\mu i}_{qq'}A(qq';\ \mu-\sigma)], \qquad (5)$$

$$A^+(qq';\ \mu\sigma)=\begin{cases}\sum_{\sigma'}\delta_{\sigma'(K-K'),\,\sigma\mu}\sigma'\alpha^+_{q\sigma'}\alpha^+_{q'-\sigma'} & \text{or}\\ \sum_{\sigma'}\delta_{\sigma'(K+K'),\,\sigma\mu}\alpha^+_{q\sigma'}\alpha^+_{q'\sigma'},\end{cases} \qquad (6)$$

where $\sigma = \pm 1, i = 1, 2, 3, \ldots$, is the number of the root of the RPA secular equation. In the QPNM, single-phonon states are used as a basis. Therefore, the *third stage* is the calculation of the single-phonon basis. In this step, all the constants of the QPNM are fixed.

The QPNM Hamiltonian is transformed and, as a result, expressed as

$$H_{QPNM} = \sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma}\alpha_{q\sigma} + H_v + H_{vq}. \tag{7}$$

The first two terms in (7) describe the quasiparticles and phonons, while H_{vq} describes the interactions of the quasiparticles with the phonons. The *fourth stage* is to take into account these interactions. The wave functions of the excited states are represented in the form of series with respect to the number of phonon operators; in odd nuclei, each term is multiplied by a quasiparticle operator. The approximation is in the truncation of the series. Series truncation with respect to the number of phonons is an approximation of the same type as the truncation of the hierarchy of equations in the Hartree–Fock–Bogolyubov approximation. At the present time, our expansion is truncated at two phonons. It is as difficult to estimate the influence of the many-phonon terms of the wave functions on the calculated effects as it is to estimate the part played by the terms ignored in the Hartree–Fock–Bogolyubov approximation of the hierarchy equations of the nuclear many-body problem. In both cases, it is asserted that the approximate equations correctly describe the properties of the nuclear excitations, and the omitted terms are partly taken into account by the use of constants fixed on the basis of experimental data. When the calculations are made, the Pauli principle is taken into account by requiring fulfillment of the exact commutation relations between the phonon and quasiparticle operators.

QPNM Hamiltonian and the phonon basis

For deformed nuclei, the original Hamiltonian is taken in the form

$$H = \sum_{\tau}\Big\{\sum_{q\sigma}{}^{\tau}(E'(q)-\lambda_\tau)\,a^+_{q\sigma}a_{q\sigma} - G_\tau \sum_{qq'}{}^{\tau} a^+_{q+}a^+_{q-}a_{q'-}a_{q'+} - \frac{1}{2}\sum_{n\lambda\mu\sigma}\Big[\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})\,M_{\lambda\sigma\mu n}(\tau)\,M_{\lambda-\sigma\mu n}(\rho\tau) + G^{\lambda\mu}P^+_{\lambda\sigma\mu n}(\tau)\,P_{\lambda\sigma\mu n}(\tau)\Big]\Big\}. \tag{8}$$

We restrict ourselves to multipole interactions. After the manipulations, the QPNM Hamiltonian takes the form (7), namely,

$$H_{QPNM} = \sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma}\alpha_{q\sigma} + H_v + H_{vq}.$$

Here

$$H_v = H_v^{00} + \sum_{\lambda} H_{Ev}^{\lambda 0} + \sum_{\substack{\lambda\mu\\ \mu\neq 0}} H_{Ev}^{\lambda\mu}; \tag{9}$$

$$H_v^{00} = -\sum_{ii'} W^{00}_{ii'}Q^+_{20i}Q_{20i'}; \tag{10}$$

$$W^{00}_{ii'} = \frac{1}{2}\sum_{\tau} G_\tau \sum_{qq'}{}^{\tau}[(u_q^2-v_q^2)(u_{q'}^2-v_{q'}^2)\,g^{20i}_{qq}g^{20i'}_{q'q'} + w^{20i}_{qq}w^{20i'}_{q'q'}]; \tag{10'}$$

$$H^{\lambda 0}_{Ev} = -\sum_{ii'} W^{\lambda 0}_{ii'}Q^+_{\lambda 0i}Q_{\lambda 0i'}; \tag{11}$$

$$W^{\lambda 0}_{ii'} = \sum_{n\tau}\{(\varkappa_0^{\lambda 0}+\rho\varkappa_1^{\lambda 0})\,D^{\lambda 0i}_{n\tau}D^{\lambda 0i'}_{n\rho\tau} + G^{\lambda 0}(D^{\lambda 0i}_{gn\tau}D^{\lambda 0i'}_{gn\tau} + D^{\lambda 0i}_{wn\tau}D^{\lambda 0i'}_{wn\tau})\}; \tag{11'}$$

$$H^{\lambda\mu}_{Ev} = -\sum_{ii'\sigma} W^{\lambda\mu}_{ii'}Q^+_{\lambda\mu i\sigma}Q_{\lambda\mu i'\sigma}; \tag{12}$$

$$W^{\lambda\mu}_{ii'} = \frac{1}{4}\sum_{n\tau}\Big\{\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})\,D^{\lambda\mu i}_{n\tau}D^{\lambda\mu i'}_{n\rho\tau} + G^{\lambda\mu}(D^{\lambda\mu i}_{gn\tau}D^{\lambda\mu i'}_{gn\tau} + D^{\lambda\mu i}_{wn\tau}D^{\lambda\mu i'}_{wn\tau})\Big\}; \tag{13}$$

$$H_{vq} = H^{00}_{vq} + \sum_{\lambda} H^{\lambda 0}_{vq} + \sum_{\substack{\lambda\mu\\ \mu\neq 0}} H^{\lambda\mu}_{vq}; \tag{14}$$

$$H^{00}_{vq} = -\sum_{\tau i} G_\tau \sum_{qq'}{}^{\tau}(u_q^2-v_q^2)\,u_{q'}v_{q'} \times\Big\{(\psi^{20i}_{qq}Q^+_{20i}+\varphi^{20i}_{qq}Q_{20i})\sum_{\sigma}\alpha^+_{q\sigma}\alpha_{q\sigma}+\text{h.c.}\Big\}; \tag{15}$$

$$H^{\lambda 0}_{vq} = -\sum_{\tau i}\sum_{qq'}{}^{\tau} V^{\lambda 0i}_{\tau}(qq')\{(Q^+_{\lambda 0i}+Q_{\lambda 0i})\,B(qq';\mu=0)+\text{h.c.}\}; \tag{16}$$

$$V^{\lambda 0i}_{\tau}(qq') = \frac{1}{2}\sum_{n}\Big\{f^{\lambda 0}_n(qq') \times\Big[\sum_{\rho=\pm1}(\varkappa_0^{\lambda 0}+\rho\varkappa_1^{\lambda 0})\,v^{(-)}_{qq'}D^{\lambda 0i}_{n\rho\tau} - G^{\lambda 0}u^{(+)}_{qq'}D^{\lambda 0i}_{gn\tau}\Big]\Big\}; \tag{17}$$

$$H^{\lambda\mu}_{vq} = -\frac{1}{2}\sum_{ni\tau\sigma}\sum_{qq'}{}^{\tau}\{V^{\lambda\mu i}_{n\tau}(qq')\,f^{\lambda\mu}_n(qq')\,Q^+_{\lambda\mu i\sigma}B \times(qq';\ \mu-\sigma)+\text{h.c.}\}; \tag{18}$$

$$V^{\lambda\mu i}_{n\tau}(qq') = \frac{1}{2}\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})\,v^{(-)}_{qq'}D^{\lambda\mu i}_{n\rho\tau} - \frac{1}{2}G^{\lambda\mu}u^{(+)}_{qq'}D^{\lambda\mu i}_{gn\tau}. \tag{19}$$

Here

$$D^{\lambda\mu i}_{n\tau} = \sum_{qq'}{}^{\tau} f^{\lambda\mu}_n(qq')\,u^{(+)}_{qq'}g^{\lambda\mu i}_{qq'};$$
$$D^{\lambda\mu i}_{gn\tau} = \sum_{qq'}{}^{\tau} f^{\lambda\mu}_n(qq')\,v^{(-)}_{qq'}g^{\lambda\mu i}_{qq'}; \tag{20}$$
$$D^{\lambda\mu i}_{wn\tau} = \sum_{qq'}{}^{\tau} f^{\lambda\mu}_n(qq')\,v^{(+)}_{qq'}w^{\lambda\mu i}_{qq'},$$

in which $f^{\lambda\mu}_n(qq')$ are the single-particle matrix elements of the multipole operators, whose explicit form is given in Ref. 12, and

$$g^{\lambda\mu i}_{qq'} = \psi^{\lambda\mu i}_{qq'} + \varphi^{\lambda\mu i}_{qq'},\quad w^{\lambda\mu i}_{qq'} = \psi^{\lambda\mu i}_{qq'} - \varphi^{\lambda\mu i}_{qq'}; \tag{21}$$

$$B(qq';\ \mu\sigma) = \sum_{\sigma'}\delta_{\sigma'(K-K'),\,\sigma\mu}\alpha^+_{q\sigma'}\alpha_{q'\sigma'} \quad \text{or}$$

$$\sum_{\sigma'}\delta_{\sigma'(K+K'),\,\sigma\mu\sigma'}\alpha^+_{q\sigma'}\alpha_{q'-\sigma'}. \tag{22}$$

Note that for RPA solutions with averaging over the phonon vacuum the following relation is satisfied:

$$\Big\langle\Big\{\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}+H_v\Big\}Q^+_{\lambda\mu i}Q^+_{\lambda\mu i'}\Big\rangle = 0.$$

Therefore, H_{QPNM} contains no terms proportional to $Q^+_{\lambda\mu i\sigma}Q^+_{\lambda\mu i'\sigma}$ and $Q_{\lambda\mu i'\sigma}Q_{\lambda\mu i\sigma}$.

We shall obtain RPA equations for the calculation of the energy $\omega_{\lambda\mu i}$ and wave functions of the single-phonon states of electric type:

$$Q^+_{\lambda\mu i\sigma}\Psi_0, \tag{23}$$

where Ψ_0 is the wave function of the ground state of the even–even nucleus, defined as the phonon vacuum. The nor-

malization of (23) has the form

$$\frac{1}{2}\sum_{qq'}[(\psi_{qq'}^{\lambda\mu i})^2-(\varphi_{qq'}^{\lambda\mu i})^2]=\frac{1}{2}\sum_{qq'}g_{qq'}^{\lambda\mu i}w_{qq'}^{\lambda\mu i}=1. \qquad (23')$$

To describe the single-phonon states, the part

$$\sum_{q\sigma}\varepsilon_q\alpha_{q\sigma}^{+}\alpha_{q\sigma}+\sum_{\lambda\mu}H_{Ev}^{\lambda\mu} \qquad (24)$$

of the Hamiltonian (7) is used. We find the expectation value of (24) with respect to the state (23) and, using the variational principle in the form

$$\delta\left\{\left\langle Q_{\lambda\mu i\sigma}\left\{\sum_{q\sigma}\varepsilon_q\alpha_{q\sigma}^{+}\alpha_{q\sigma}+H_{Ev}\right\}Q_{\lambda\mu i\sigma}^{+}\right\rangle\right.$$

$$\left.-\frac{\omega_{\lambda\mu i}}{2}\left[\sum_{qq'}g_{qq'}^{\lambda\mu i}w_{qq'}^{\lambda\mu i}-2\right]\right\}=0,$$

obtain the system of equations

$$D_{n\tau}^{\lambda\mu i}=\sum_{n'=1}^{n_{\max}}\left\{\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})X_{nn'}^{\lambda\mu i}(\tau)D_{n'\rho\tau}^{\lambda\mu i}\right.$$

$$\left.+G^{\lambda\mu}[X_{nn'}^{\lambda\mu i\varepsilon}(\tau)D_{gn'\tau}^{\lambda\mu i}+X_{nn'}^{\lambda\mu i\omega}(\tau)D_{wn'\tau}^{\lambda\mu i}]\right\}; \qquad (25)$$

$$D_{gn\tau}^{\lambda\mu i}=\sum_{n'=1}^{n_{\max}}\left\{\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})X_{nn'}^{\lambda\mu i\varepsilon}(\tau)D_{n'\rho\tau}^{\lambda\mu i}\right.$$

$$\left.+G^{\lambda\mu}[X_{nn'}^{\lambda\mu i\varepsilon-}(\tau)D_{gn'\tau}^{\lambda\mu i}+X_{nn'}^{\lambda\mu iv}(\tau)D_{wn'\tau}^{\lambda\mu i}]\right\}; \qquad (26)$$

$$D_{wn\tau}^{\lambda\mu i}=\sum_{n'=1}^{n_{\max}}\left\{\sum_{\rho=\pm1}(\varkappa_0^{\lambda\mu}+\rho\varkappa_1^{\lambda\mu})X_{nn'}^{\lambda\mu i\omega+}(\tau)D_{n'\rho\tau}^{\lambda\mu i}\right.$$

$$\left.+G^{\lambda\mu}[X_{nn'}^{\lambda\mu iv}(\tau)D_{gn'\tau}^{\lambda\mu i}+X_{nn'}^{\lambda\mu i\varepsilon+}(\tau)D_{wn'\tau}^{\lambda\mu i}]\right\}, \qquad (27)$$

where

$$X_{nn'}^{\lambda\mu i}(\tau)=\sum_{qq'}^{\tau}\frac{f_n^{\lambda\mu}(qq')f_{n'}^{\lambda\mu}(qq')(u_{qq'}^{(+)})^2\varepsilon_{qq'}}{\varepsilon_{qq'}^2-\omega_{\lambda\mu i}^2};$$

$$X_{nn'}^{\lambda\mu i\varepsilon}(\tau)=\sum_{qq'}^{\tau}\frac{f_n^{\lambda\mu}(qq')f_{n'}^{\lambda\mu}(qq')u_{qq'}^{(+)}v_{qq'}^{(-)}\varepsilon_{qq'}}{\varepsilon_{qq'}^2-\omega_{\lambda\mu i}^2};$$

$$X_{nn'}^{\lambda\mu i\omega}(\tau)=\sum_{qq'}^{\tau}\frac{f_n^{\lambda\mu}(qq')f_{n'}^{\lambda\mu}(qq')u_{qq'}^{(+)}v_{qq'}^{(+)}\omega_{\lambda\mu i}}{\varepsilon_{qq'}^2-\omega_{\lambda\mu i}^2};$$

$$X_{nn'}^{\lambda\mu i\varepsilon\pm}(\tau)=\sum_{qq'}^{\tau}\frac{f_n^{\lambda\mu}(qq')f_{n'}^{\lambda\mu}(qq')(v_{qq'}^{(\pm)})^2\varepsilon_{qq'}}{\varepsilon_{qq'}^2-\omega_{\lambda\mu i}^2};$$

$$X_{nn'}^{\lambda\mu iv}(\tau)=\sum_{qq'}^{\tau}\frac{f_n^{\lambda\mu}(qq')f_{n'}^{\lambda\mu}(qq')v_{qq'}^{(+)}v_{qq'}^{(-)}\omega_{\lambda\mu i}}{\varepsilon_{qq'}^2-\omega_{\lambda\mu i}^2};$$

$$\varepsilon_{qq'}=\varepsilon_q+\varepsilon_{q'}.$$

From Eqs. (25)–(27) for $\tau=p$ and $\tau=n$ we find the secular equation for the energies $\omega_{\lambda\mu i}$ of the single-phonon states in the form of the vanishing of a determinant of rank $6n_{\max}$. The use of separable interactions of rank $n_{\max}>1$ leads to an increase in the rank of the determinant by a factor $n_{\max}$ in comparison with simple separable interactions. If allowance is made for the spin-multipole interactions with $\lambda=L$, then the rank of the determinant is $12n_{\max}$. If only multipole p–h interactions are taken into account, the rank is $2n_{\max}$. In calculating the phonon amplitudes $\psi_{qq'}^{\lambda\mu i}$ and $\varphi_{qq'}^{\lambda\mu i}$, we use the condition (23').

The secular equation for the description of 0^+ states with a simple separable interaction ($n_{\max}=1$) was obtained in Ref. 17 in the form of the vanishing of a determinant of order 10. If separable interactions of rank $n_{\max}>1$ are taken into account, then the rank of the determinant for finding the energies ω_{20i} of the single-phonon 0^+ states is $4+6n_{\max}$.

The RPA equations for the charge-exchange states are obtained similarly. For charge-exchange multipole and spin-multipole states RPA equations are given in Ref. 23 for simple ($n_{\max}=1$) p–h interactions. For p–h and p–p Gamow–Teller interactions such equations were obtained in Ref. 24.

If the RPA secular equations have been solved and the energies $\omega_{\lambda\mu i}$ and phonon amplitudes $\psi_{qq'}^{\lambda\mu i}$ and $\varphi_{qq'}^{\lambda\mu i}$ have been found, then the Hamiltonian (7) is uniquely determined. It does not contain any free parameter or any unfixed constant.

The complete space of p–h and p–p two-quasiparticle states is replaced by the space of single-phonon states. For each value of K^π, the number of single-phonon states is equal to the number of two-quasiparticle states. In deformed nuclei, the phonon basis can be formed from phonons of electric type, and only the $K^\pi=1^+$ states are described by phonons of the new type.

QPNM equations for even–even deformed nuclei

We shall give expressions for describing nonrotational states with $K^\pi\neq0^+$, 1^+ for even–even deformed nuclei in the QPNM with allowance for p–h and p–p interactions, the wave functions of which consist of single- and two-phonon terms, namely,

$$\Psi_\nu(K_0^{\pi_0}\sigma_0)=\left\{\sum_{i_0}R_{i_0}^{\nu}Q_{g_0\sigma_0}^{+}\right.$$

$$+\sum_{g_1g_2\sigma_2\sigma_1}\frac{(1+\delta_{g_1,g_2})^{1/2}\sigma_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0}}{2[1+\delta_{K_0,0}(1-\delta_{\mu_1,0})]^{1/2}}$$

$$\left.\times P_{g_1g_2}^{\nu}Q_{g_1\sigma_1}^{+}Q_{g_2\sigma_2}^{+}\right\}\Psi_0, \qquad (28)$$

where $g=\lambda\mu i$, $\mu_0\equiv K_0$.

The normalization condition has the form

$$\sum_{i_0}(R_{i_0}^{\nu})^2+\sum_{g_1\geqslant g_2}(P_{g_1g_2}^{\nu})^2[1+\mathscr{K}^{K_0}(g_1g_2)]=1. \qquad (29)$$

To take into account the Pauli principle in the two-phonon terms of the wave function (28), we introduce the function

$$\mathscr{K}^{K_0}(g_2,\ \lambda_1\mu_1i'|g_1,\ g_2)=\frac{1}{1+\delta_{g_1g_2}}\sum_{\sigma_1\sigma_2}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0}$$

$$\times\langle Q_{g_2\sigma_2}[[Q_{\lambda_1\mu_1i'\sigma_1},\ Q_{g_1\sigma_1}^{+}],\ Q_{g_2\sigma_2}^{+}]\rangle, \qquad (30)$$

which for $i'=i$ is denoted by $\mathscr{K}^{K_0}(g_1g_2)$, its explicit form is given in Refs. 11–13.

By means of the variational principle, we obtain the following equations, from which, taking into account the condition (29), we can find the energies η_ν and the functions $R_{i_0}^{\nu}$ and $P_{g_1g_2}^{\nu}$:

$$(\omega_{g_0}-\eta_\nu)R_{i_0}^{\nu}-\sum_{g_1\geqslant g_2}(1+\delta_{g_1g_2})^{-1/2}$$

$$\times(1+\delta_{K_0,0}(1-\delta_{\mu_1,0}))^{-1/2}$$

$$\times P_{g_1g_2}^{\nu}U_{g_1g_2}^{g_0}[1+\mathscr{K}^{K_0}(g_1g_2)]=0; \qquad (31)$$

$$[\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)-\eta_\nu]P_{g_1g_2}^{\nu}$$

$$-\sum_{i_0}(1+\delta_{g_1g_2})^{-1/2}[1+\delta_{K_0,0}$$

$$\times(1-\delta_{\mu_1,0})]^{-1/2}R_{i_0}^{\nu}U_{g_1g_2}^{g_0}=0, \qquad (31')$$

where

$$\Delta\omega(g_1, g_2) = \sum_{i'} \{\mathscr{K}^{K_0}(g_2, \lambda_1\mu_1 i' | g_1, g_2) W^{\lambda_1\mu_1}_{i_1 i'^1} + \mathscr{K}^{K_0}(\lambda_2\mu_2 i', g_1 | g_1, g_2) W^{\lambda_2\mu_2}_{i_2 i'^2}\}; \quad (32)$$

$$U^{g_0}_{g_1g_2}[1 + \mathscr{K}^{K_0}(g_1g_2)] = -\frac{1}{2}\sum_{\sigma_1\sigma_2} \delta_{\sigma_1\mu_1+\sigma_2\mu_2,\, \sigma_0 K_0} \times \{\langle Q_{g_0\sigma_0} H_{vg} Q^+_{g_1\sigma_1} Q^+_{g_2\sigma_2}\rangle + \langle Q_{g_2\sigma_2} Q_{g_1\sigma_1} H_{vq} Q^+_{g_0\sigma_0}\rangle\}. \quad (33)$$

The function $W^{\lambda\mu}_{ii'}$ is given by (13), and the function $U^{g_0}_{g_1g_2}$ contains the functions $V^{\lambda 0i}_{\tau}(qq')$ and $V^{\lambda\mu i}_{n\tau}(qq')$, which are determined by (17) and (19). The explicit form of $U^{g_0}_{g_1g_2}$ in the case of a simple ($n_{\max} = 1$) interaction is given in Ref. 12.

If $P^{\nu}_{g_1g_2}$ is found from Eq. (31′) and substituted in (31), the following secular equation is obtained:

$$\det\left\| (\omega_{g_0} - \eta_\nu)\,\delta_{i_0 i'_0} - \sum_{g_1 \geqslant g_2} \frac{1 + \mathscr{K}^{K_0}(g_1g_2)}{(1+\delta_{g_1g_2})\,[1+\delta_{K_0,\,0}\,(1-\delta_{\mu_1,\,0})]} \times \frac{U^{\lambda_0\mu_0 i_0}_{g_1g_2} U^{\lambda_0\mu_0 i'_0}_{g_1g_2}}{\omega_{g_1} + \omega_{g_2} + \Delta\omega(g_1g_2) - \eta_\nu} \right\| = 0. \quad (34)$$

The rank of this determinant is equal to the number of single-phonon terms in the wave function (28). Allowance for the Pauli principle in the two-phonon terms (28) leads to the appearance in (34) of the factor $1 + \mathscr{K}^{K_0}(g_1g_2)$ and generates a shift $\Delta\omega(g_1g_2)$ of the two-phonon pole. The equations for the 0^+ states have the same form.

The form of Eqs. (31), (31′), and (34) is the same for $n_{\max} > 1$ and $n_{\max} = 1$ in the cases when both *p–h* and *p–p* interactions or only *p–h* interactions are taken into account. It is very important that the transition from simple separable interactions with $n_{\max} = 1$ to separable interactions of finite rank with $n_{\max} > 1$ does not raise the rank of the determinant (34). It leads to more complicated expressions for $U^{g_0}_{g_1g_2}$ and $\Delta\omega(g_1g_2)$. This complication of the functions is not significant for numerical calculations on a computer.

It should be emphasized that allowance for the finite rank of the separable interactions does not lead to any significant complication of the equations for calculating the fragmentation of the quasiparticle and collective motions. This means that the QPNM may in the future provide a basis for calculation of many properties of deformed nuclei.

2. APPLICATION OF THE INTERACTING-BOSON MODEL TO DEFORMED NUCLEI

The *sd* IBM

The phenomenological IBM was formulated by Arima and Iachello,[6] who used the methods of group theory. They introduced bosons of two types: *s* bosons with $J = 0$ and *d* bosons with $J = 2$ (d_μ, $\mu = 0, \pm 1, \pm 2$). These interact with one another and with the others through single- and two-particle interactions. Arima and Iachello assumed that the 0^+ and 2^+ nucleon pairs play the dominant part in the description of the collective quadrupole states, i.e., that the *s*- and *d*-boson degrees of freedom are associated with excitations with $J^\pi = 0^+$ and $J^\pi = 2^+$ fermion pairs. Therefore, in the IBM the total number of bosons,

$$N = N_s + N_d = s^+ s + \sum_\mu d^+_\mu d_\mu, \quad (35)$$

is conserved and equal to half the number of valence nucleons. In the original and most popular version of the model, the IBM-1, there is a single species of bosons, and there are no separate proton and neutron bosons, as in the IBM-2.

Together, the *s* and *d* bosons have six components and therefore determine a six-dimensional space; this leads to a description in terms of the group $SU(6)$. To construct a complete basis, invariant subgroups of $SU(6)$ are found. The following chains of subgroups are used:

$$SU(6) \supset SU(5) \supset O(5) \supset O(3);$$
$$SU(6) \supset SU(3) \supset O(3);$$
$$SU(6) \supset O(6) \supset O(5) \supset O(3).$$

We restrict ourselves to consideration of deformed nuclei, and for the *sd* IBM-1 we write the Hamiltonian in the form

$$H_{sd} = a_0 P^+ \cdot P + a_1 L_d \cdot L_d + a_2 Q_d \cdot Q_d, \quad (36)$$

where

$$P = \frac{1}{2}[(\tilde{d}\cdot\tilde{d}) - s\cdot s], \quad L_d = \sqrt{10}\,(d^+ \times \tilde{d})^{(1)},$$
$$Q_d = (s^+ \times \tilde{d} + d^+ \times s)^{(2)} + \chi\,(d^+ \times \tilde{d})^{(2)}, \quad \tilde{d}_\mu = (-1)^\mu d_{-\mu}, \quad (37)$$

The tensor product is denoted by $(d^+ \times \tilde{d})^{(J)}$, and the scalar product by $(\tilde{d}\cdot\tilde{d})$. Here, a_0, a_1, a_2, and χ are phenomenological parameters. The $E2$ transition operator has the form

$$\mathfrak{M}(E2, \mu) = \tilde{e}\{[(d^+ \times s) + (s^+ \times \tilde{d})]^{2\mu} + \chi'\,(d^+ \times \tilde{d})^{2\mu}\}. \quad (38)$$

where $\tilde{e}$ is the effective charge, and the parameter χ' must be taken to be different from χ.

For an axisymmetric even–even deformed nucleus, the intrinsic wave function of the unperturbed ground state has the form

$$|g) = (N!)^{-1/2}\,(b^+)^N\,|0), \quad b^+ = (1 + \beta^2)^{-1/2}\,(s^+ + \beta d^+_0), \quad (39)$$

where $|0)$ is the boson vacuum. In the harmonic approximation, there are two types of excitation, which have the symmetry of β and γ vibrations and are determined by the quantum numbers n_β and n_γ. Their wave functions [see (25)] have the following form: for a state with $K^\pi_\nu = 0^+_1$ and $n_\beta = 1$,

$$|n_\beta = 1) = [(N-1)!]^{-1/2} b^+_\beta\,(b^+)^{N-1}\,|0); \quad (40)$$

for a state with $K^\pi_\nu = 2^+_1$ and $n_\gamma = 1$,

$$|n_\gamma = 1) = [(N-1)!]^{-1/2}\,d^+_2\,(b^+)^{N-1}\,|0), \quad (40')$$

where

$$b^+_\beta = (1 + \beta^2)^{-1/2}\,(-\beta s^+ + d^+_0),$$

and one usually takes $\beta = 1.24$. The other excited states are characterized as follows: $K^\pi_\nu = 4^+_1$ as $n_\gamma = 2$; $K^\pi_\nu = 2^+_2$ as $n_\gamma = 1, n_\beta = 1$; $K^\pi_\nu = 0^+_2$ and 0^+_3 as $n_\beta = 2$ and $n_\gamma = 2$, etc.

In the general case, when the interactions between the bosons are taken into account, the wave function has the form

$$|K^\pi \nu) = \sum_{N_d N_\beta} C^{K\nu}_{N_d N_\beta}\,(d^+_2)^{N_d}_K\,(b^+_\beta)^{N_\beta}\,(b^+)^{N-N_d-N_\beta}|0), \quad (41)$$

where $N \geqslant N_d + N_\beta$. Since in well-deformed nuclei the anharmonicity of the vibrations is not very great, the wave function (41) has one dominant component—a single-boson or two-boson component.

In the *sd* IBM a good description has been obtained (see, for example, Ref. 7) for rotational bands constructed on ground and on β- and γ-vibrational states and for $E2$ transitions between them. This is an undoubted success of the IBM.

The *sdg* IBM

Disagreements with the experimental data for states lying above the β- and γ-vibrational states, and also the need to describe $K^\pi = 3^+$ states, led to a refinement of the *sd* IBM. Besides the *s* and *d* bosons, a *g* boson with $J = 4$ was introduced. The introduction of the *g* boson was effected in two directions: 1) renormalization of the boson Hamiltonian without explicit introduction of the *g* boson;[26] 2) explicit introduction of the *g* boson.[27-29] In the latter case, i.e., in the *sdg* IBM, there appear in deformed nuclei states not present in the *sd* IBM, namely, states with dominant single-boson components with K^π equal to 1^+, 3^+, and 4^+ and additional states with $K^\pi = 0^+$ and 2^+. There is a possibility of calculating $E4$ transitions. In the version of the *sdg* IBM proposed in Ref. 27, calculation of the ^{168}Er levels led to prediction of a level with $K^\pi_\nu = 1_1^+$ and a low energy that was not observed experimentally. A systematic treatment of the *sdg* IBM was given in Refs. 28 and 29.

The Hamiltonian of the *sdg* IBM has the form

$$H_{sdg} = a_0 P^+ \cdot P + \varepsilon_g g^+ \cdot \tilde{g} + \varkappa Q^{(2)} \cdot Q^{(2)} + \varkappa' L^{(1)} \cdot L^{(1)}, \quad (42)$$

where

$$Q^{(2)} = Q_d + \frac{9}{7}(d^+ \times \tilde{g} + g^+ \times \tilde{d})^{(2)} - \frac{3}{14}\sqrt{55}\,(g^+ \times \tilde{g})^{(2)};$$
$$L^{(1)} = L_d + \sqrt{60}\,(g^+ \times \tilde{g})^{(1)}.$$

The operators of $E2$ and $E4$ transitions are

$$\mathfrak{M}(E2,\ \mu) = e_1[(d^+ \times s) + (s^+ \times \tilde{d})]^{2\mu} + e_2(d^+ \times \tilde{d})^{2\mu} + e_3[(d^+ \times \tilde{g}) + (g^+ \times \tilde{d})]^{2\mu} + e_4(g^+ \times \tilde{g})^{2\mu}; \quad (43)$$

$$\mathfrak{M}(E4,\ \mu) = e_{04}[(s^+ \times \tilde{g}) + (g^+ \times s)]^{4\mu} + e_{22}(d^+ \times \tilde{d})^{4\mu} + e_{24}[(d^+ \times \tilde{g}) + (g^+ \times \tilde{d})]^{4\mu} + e_{44}(g^+ \times \tilde{g})^{4\mu};$$
$$N = N_s + N_d + N_g. \quad (44)$$

In the harmonic approximation, the wave functions have the following form:[29] for the ground state,

$$|g) = \frac{1}{\sqrt{N!}}(\Sigma^+)^N |0); \quad (45)$$

for the single-boson states,

$$\left.\begin{aligned} |\gamma) &\approx \Pi_2^+ (\Sigma^+)^{N-1}|0), \\ |\beta) &\approx \Pi_0^+ (\Sigma^+)^{N-1}|0), \\ |2_2^+) &\approx \Gamma_2^+ (\Sigma^+)^{N-1}|0), \\ |0_2^+) &\approx \Gamma_0^+ (\Sigma^+)^{N-1}|0), \\ |4_1^+) &\approx \Gamma_4^+ (\Sigma^+)^{N-1}|0); \end{aligned}\right\} \quad (46)$$

for the two-boson states,

$$\left.\begin{aligned} |2^+) &\approx \Pi_2^+ \Pi_0^+ (\Sigma^+)^{N-2}|0), \\ |0^+) &\approx (\Pi_0^+)^2 (\Sigma^+)^{N-2}|0), \\ |4^+) &\approx (\Pi_2^+)^2 (\Sigma^+)^{N-2}|0), \\ |0^+) &\approx (\Pi_2^+)^2 (\Sigma^+)^{N-2}|0), \end{aligned}\right\} \quad (47)$$

where

$$\left.\begin{aligned} \Sigma^+ &= 5^{-1/2} s^+ + 2 \cdot 7^{-1/2} d_0^+ + 2\sqrt{2}\,35^{-1/2} g_0^+; \\ \Pi_0^+ &= 2 \cdot 15^{-1/2} s^+ + 21^{-1/2} d_0^+ - 2\sqrt{6}\,35^{-1/2} g_0^+; \\ \Pi_2^+ &= 7^{-1/2} d_2^+ + \sqrt{6}\,7^{-1/2} g_2^+; \\ \Gamma_0^+ &= 2\sqrt{\frac{2}{15}}\, s^+ - 2\sqrt{\frac{2}{21}}\, d_0^+ + \sqrt{\frac{3}{35}}\, g_0^+; \\ \Gamma_2^+ &= \sqrt{\frac{6}{7}}\, d_2^+ - \sqrt{\frac{1}{7}}\, g_2^+; \\ \Gamma_4^+ &= g_4^+. \end{aligned}\right\} \quad (47')$$

To them one should add single-boson states with $K^\pi = 1^+$ and $K^\pi = 3^+$. A wave function can be constructed in accordance with the type (41) with allowance for interactions between the bosons.

In the framework of the *sdg* IBM, some contradictions in the description of ^{168}Er have been eliminated,[28,30] the energies of collective levels in ^{178}Hf and ^{234}U have been described,[31] the distribution of the $E4$ strength in ^{150}Nd and ^{156}Gd has been calculated,[32] and some other calculations have been made.

The *sdf* IBM

The *sd* IBM and *sdg* IBM cannot describe negative-parity states. To describe collective octupole states, an *f* boson with $J = 3$ was introduced additionally in Ref. 33. A detailed exposition of the *sdf* IBM was given in Ref. 34.

The Hamiltonian of the *sdf* IBM has the form

$$H_{sdf} = H_{sd} + \varepsilon_f (f^+ \times \tilde{f}) + a_3 5 (\tilde{d}^+ \times \tilde{f})^{(3)} (f^+ \times \tilde{d})^{(3)} + a_4 L_d \cdot L_f + a_5 Q_d \cdot Q_f, \quad (48)$$

where

$$L_f = 2\sqrt{7}\,(f^+ \times \tilde{f})^{(1)}; \quad Q_f = -2\sqrt{7}\,(f^+ \times \tilde{f})^{(2)}.$$

The $E3$ transition operator is

$$\mathfrak{M}(E3,\ \mu) = e_3\{[(s^+ \times \tilde{f}) + (f^+ \times s)]^{3\mu} + \chi_3[(d^+ \times \tilde{f}) + (f^+ \times \tilde{d})]^{3\mu}\}. \quad (49)$$

Here, $N = N_s + N_d + N_f$, and $N_f = 0$ or 1. In the harmonic approximation, the wave functions of the single-boson states with $K^\pi = 0^-$, 1^-, 2^-, and 3^- have the form ($K \equiv \mu$)

$$|K^\pi) = [(N-1)!]^{-1/2} f_\mu^+ (b^+)^{N-1} |0). \quad (50)$$

The dependence of the energies of the excited negative-parity states and rotational bands based on them on the parameters of the Hamiltonian (48) was investigated in Ref. 34. Also studied was the dependence of these bands on the calculated energies of the 0_1^+ and 2_1^+ states of the *sd* core. In Ref. 34, calculations of octupole states of nine nuclei from ^{154}Sm to ^{182}W were made. It was asserted that three or four parameters are needed to describe the energies of the negative-parity states. Two parameters are needed to determine the $B(E3)$ values, and one parameter was taken to be constant. It was asserted in Ref. 34 that because the parameters vary from nucleus to nucleus, and the tendencies of these

variations are not simple, it is impossible to make predictions for nuclei for which appropriate experimental data are not available. Different versions of the *sdf* IBM were used to describe rotational bands with negative parity in, for example, ^{156}Gd (Ref. 35) and ^{168}Er (Ref. 36).

We note that to achieve a more consistent description of negative-parity states a p boson with $J = 1$ was introduced in Refs. 37 and 38, and investigations were made in the *spdf* IBM. In the *spdf* IBM a description of rotational bands on $K^\pi = 0^+$ and 0^- states in even–even Th isotopes was obtained in Ref. 39.

Other modifications of the IBM

The IBM-2 version is widely used to describe deformed nuclei. In Ref. 40, F spin was introduced, and states of mixed symmetry were studied. A great success of the IBM-2 is associated with the description of $K^\pi = 1^+$ states with energy around 3 MeV and large $B(M1)$ values. We note that these 1^+ states and $M1$ transitions are correctly described in Ref. 41 in the RPA with an axisymmetric deformed Woods–Saxon potential. In what follows, we shall not discuss these states. Great interest attaches to low-lying isovector collective states. There have been attempts to interpret a $K^\pi_\nu = 2^+_2$ state in ^{154}Sm (Ref. 42) and $K^\pi_\nu = 0^+_2$ state in ^{156}Gd (Ref. 43) in the framework of the IBM-2 as mixed-symmetry states. However, the experimental data on such states are too sparse to make a comparison of the two models on their basis. Most calculations in the IBM-2 (see Refs. 44 and 45) are restricted to description of the rotational bands based on the ground, β-, and γ-vibrational states and $E2$ and $M1$ transitions between them. Therefore, there is no possibility of making a comparison with calculations in the QPNM, since the differences in the descriptions occur for the higher excited states. Calculations[27] of ^{168}Er in the IBM-2 yielded eight rotational bands with bases below 2 MeV. They are not observed experimentally.

We note that a universal description of collectivity in heavy nuclei was obtained in Refs. 46 and 47 and in other studies in terms of the parameter $P = N_p N_n/(N_p + N_n)$, which characterizes the number of valence protons, N_p, and valence neutrons, N_n, that interact with one another. The important role of n–p interactions is manifested above all in global nuclear properties, for example, in the moments of inertia. They play the same part in the ground state and in many excited states.

There have been several further modifications of the IBM. For example, in Ref. 48 the *sd* IBM was augmented by the introduction of s', d', and g bosons, and the energies and $B(E2)$ values in even–even Gd isotopes were described. For the description of backbending, one of the bosons in the IBM-2 was replaced in Ref. 49 by a nucleon pair, and in Ref. 50 the IBM-1 plus a two-quasiparticle pair was used in calculations of even–even Dy isotopes.

3. CARDINAL DIFFERENCES BETWEEN THE IBM AND MICROSCOPIC MODELS

Essence of the cardinal differences

It is possible to compare the descriptions of the nonrotational states of even–even deformed nuclei in the phenomenological IBM and in microscopic models because the ideal bosons in the IBM can be represented in the form of series with respect to fermion pairs, i.e., there is a connection between the bosons and the microscopic fermion structure. An important part in the IBM is played by the number N of valence nucleons, as a consequence of which the results of calculations are tied to a specific nucleus, and comparison of the spectra of different nuclei becomes natural. In the IBM one can describe two-nucleon transfer reactions of the (t, p) or (p, t) type, i.e., the coupling of one nucleus to another. Another factor of no small importance is that in the phenomenological IBM and in the microscopic models the same nonrotational excited states are described.

There are two cardinal differences between the IBM and many microscopic models such as the RPA and its modifications, the theory of finite Fermi systems,[51] nuclear field theory,[52] the QPNM, etc.

First difference. The IBM distinguishes a subspace of collective states that forms a very small fraction of the total space of two-quasiparticle states. Namely, it contains only the part present in the s, d, and f bosons, or in the s_p, d_p and s_n, d_n bosons. Introduction of the g bosons extends the space of two-quasiparticle states with $K^\pi = 0^+$ and $K^\pi = 2^+$, and there is additional allowance for the subspace with $K^\pi = 1^+$, 3^+, and 4^+. The introduction of the s' and d' bosons also extends the space of the two-quasiparticle states.

In the microscopic models, a large, fairly complete space of two-quasiparticle states of particle–hole type is used. Some microscopic models also take into account a space of states of particle–particle and hole–hole type. The completeness of the employed space of two-quasiparticle states needed to describe giant resonances can be tested by the fulfillment of model-independent and model-dependent sum rules.

A strong aspect of the IBM is the partial allowance for many-quasiparticle components of the wave functions, since the boson operator is represented in the form of a series in fermion pairs. In the microscopic models, allowance for the $2p$–$2h$ and, *a fortiori*, $3p$–$3h$ configurations is associated with great difficulties.

Second difference. The separation of the subspace of collective states in the IBM breaks the coupling to the other collective states, for example, the states that form the giant resonances, and also to a large number of weakly collective and two-quasiparticle states. It should be borne in mind that the separation of the collective states is not unique because there is no clear boundary between the strongly collective and somewhat less collective states. The inclusion in the IBM of a number of weakly collective states when the g, s', and d' bosons are introduced reduces the attractiveness of the model, since there are many other weakly collective states not included.

As the excitation energy is increased, the density of states and the complexity of their structure increase. There is a fragmentation (distribution of the strength) of the simple single-quasiparticle or single-phonon states over many nuclear levels. The coupling of the collective and noncollective motions or the interaction of the quasiparticles with the phonons is responsible for the fragmentation.[53,54] This coupling is taken into account to some degree in microscopic models and, as a rule, is absent in the IBM. As a consequence the IBM is powerless to describe one of the most important properties of nuclei—the more complicated structure of the states at higher excitation energies. In addition, the weakly

collective and two-quasiparticle states are taken into account in the IBM in an artificial manner, and this leads to the appearance of a large number of new parameters.

Specific main differences between the QPNM and the IBM

The cardinal differences between the QPNM and the IBM in the description of a number of nonrotational states of even–even deformed nuclei can be represented quite concretely, and therefore ways to test the differences experimentally can be formulated. The concrete differences can be formulated because the wave functions of the nonrotational states with excitation energies up to 2.5 MeV each have a single dominant component in both models. And the cardinal differences are the different dominant components in the QPNM and the IBM, which were first noted in Ref. 55 and discussed in detail in Refs. 56–58.

The dominant components of the wave functions in the QPNM, in the *sd* IBM, and in the *sdg* IBM for states with $K_\nu^\pi = 0_1^+$, 0_2^+, 0_3^+, 0_4^+, 2_1^+, 2_2^+, 2_3^+, 2_4^+, 2_5^+, 4_1^+, and 4_2^+ are shown schematically in Fig. 1. In the QPNM, the wave function (18) has one dominant single-phonon component, $\lambda\mu i$. In the *sd* IBM, there are dominant single-phonon states with $n_\gamma = 1$ and $n_\beta = 1$, two-boson states with $\{n_\gamma = 1, n_\beta = 1\}$, $n_\gamma = 2$, and $n_\beta = 2$, and three-boson states with $\{n_\gamma = 2, n_\beta = 1\}$, $n_\gamma = 3$, $n_\beta = 3$, etc. In the *sdg* IBM, the dominant components are represented in terms of the operators of the single-boson states (46) and two-boson states (47); the order of these states may be different from the order given in Fig. 1.

The wave functions of the γ-vibrational state with $K_\nu^\pi = 2_1^+$ and the β-vibrational state with $K_\nu^\pi = 0_1^+$ have a dominant single-boson or single-phonon component. Therefore, there is no significant difference between the descriptions of these states in the QPNM, the *sd* IBM, and the *sdg* IBM.

The wave functions of states with $K_\nu^\pi = 2_2^+$, 0_2^+, 4_1^+, and 0_3^+ in the QPNM each have one dominant single-phonon component, and in the *sd* IBM and the *sdg* IBM they each have one dominant two-boson component. Here, the QPNM and the IBM give a cardinal difference in the structure of these states. There is the same difference between the QPNM and the *sd* IBM for states with $K_\nu^\pi = 2_3^+$, 0_4^+, and 4_2^+, since their wave functions have dominant single-phonon components in the QPNM and three-boson components in the *sd* IBM.

In the *sdg* IBM, the wave function of the $K_\nu^\pi = 4_2^+$ state has a dominant single-boson component, while the wave functions of the $K_\nu^\pi = 2_3^+$ and 0_4^+ states consist of a combination of single-boson operators. There is no cardinal difference with the QPNM. An important difference is that the wave function of a single-phonon state, for example, $\lambda_{\mu i} = 223$ can differ strongly as regards the contributions of the different two-quasiparticle components to its normalization from the wave functions of the single-phonon states 221 and 222. In the *sdg* IBM, the operators Γ_2^+ and Π_2^+ are expressed in terms of the operators d_2^+ and g_2^+ with fixed weights. This also applies to the state $K_\nu^\pi = 0_4^+$.

In the wave functions of the states included in Fig. 1 there are, in addition to the dominant components, other components. According to the calculations of Ref. 2 in the QPNM, the admixtures of the two-phonon components to the normalization of the wave functions of these states are 1–3% and never exceed 10%. In individual cases of nearly equal energies of states with a fixed value of K^π there is a mixture of the components $\lambda\mu_{i_K}$ and $\lambda\mu_{i_{K+1}}$.

$\lambda\mu i$ QPNM	K_ν^π	*sd* IBM	*sdg* IBM
225	2_5^+		$\Pi_2^+ (\Pi_0^+)^2$
224	2_4^+		$(\Pi_2^+)^3$
442	4_2^+	$n_\gamma = 2$, $n_\beta = 1$	Γ_4^+
204	0_4^+	$n_\beta = 3$	Γ_0^+
223	2_3^+	$n_\gamma = 3$	Γ_2^+
203	0_3^+	$n_\beta = 2$	$(\Pi_0^+)^2$
441	4_1^+	$n_\gamma = 2$	$(\Pi_2^+)^2$
202	0_2^+	$n_\gamma = 2$	$(\Pi_2^+)^2$
222	2_2^+	$n_\gamma = 1$, $n_\beta = 1$	$\Pi_2^+ \Pi_0^+$
201	0_1^+	$n_\beta = 1$	Π_0^+
221	2_1^+	$n_\gamma = 1$	Π_2^+

FIG. 1. Scheme of dominant components of wave functions of nonrotational states with $K^\pi = 0^+$, 2^+, and 4^+ in the QPNM, the *sd* IBM, and the *sdg* IBM.

In the *sd* IBM, the interaction between the bosons leads to a wave function of the form (41). Because of the interaction, some of the single-boson strength is displaced from the 2_1^+ state to the 2_2^+ state and from 0_1^+ to 0_2^+. In the *sdg* IBM, the interaction between the bosons has the consequence that some of the single-boson strength is transferred from the 2_1^+ and 2_3^+ states to the 2_2^+ state, from 0_1^+ and 0_4^+ to 0_2^+ and 0_3^+, and from the 4_2^+ state to the 4_1^+ state.

There is a cardinal difference between the QPNM and the IBM with regard to the collective two-phonon states in deformed nuclei. Allowance for the Pauli principle in the two-phonon components of the wave function (28) has the consequence that the energy centroids of the two-phonon states are shifted to higher energies. In accordance with the calculations of Refs. 13 and 22, in the QPNM the energy centroids are above 2.5 MeV. At excitation energies above 2.5 MeV, the two-phonon states must be fragmented over many levels. It was therefore concluded in Ref. 13 that there are no collective two-phonon states in deformed nuclei.

In the *sd* and *sdg* IBM there must be states with dominant two-boson components in their wave functions. The parameters of the IBM can be chosen in such a way as to describe correctly the energy of the 2_1^+ state, the $B(E2)$ and $B(E4)$ values, and the appreciable deviation of the energy of the $K_\nu^\pi = 4_1^+$ state from twice the energy of the 2_1^+ state. Nevertheless, the two-phonon component in the wave function of the 4_1^+ state remains dominant. If it is demonstrated experimentally that there are no two-phonon collective states in deformed nuclei, this will necessitate a radical modification of the IBM.

Experimental manifestation of cardinal differences in the structure of nonrotational states

We must explain how states whose wave functions have dominant single-phonon components are to be distinguished from states with dominant two-boson components. We shall consider inelastic scattering of electrons, protons, α particles, and heavy ions, single-nucleon transfer reactions, and allowed unhindered au β decay.

If states with $K_\nu^\pi = 2_2^+$, 0_2^+, and 4_1^+ are collective and have large $B(E2)$ values, they can be distinguished from two-boson states. Two-boson states are excited in two stages, and their excitation cross sections are small. In many cases, states with $K_\gamma^\pi = 2_2^+$, 0_2^+, and 4_1^+ are not collective, and for them $B(E2)$ and $B(E4)$ are small. Therefore, by inelastic scattering one can distinguish only sufficiently collective single-phonon states from states with dominant two-boson components.

In single-nucleon transfer reactions, quite definite large two-quasiparticle components of the single-phonon wave functions are manifested. They, in their turn, uniquely determine the existence of a large single-phonon component of the wave function (28). The cross sections for single-nucleon transfer reactions with excitation of two-boson states are small. Namely, single-nucleon transfer reactions in cases in which there are corresponding large two-quasiparticle components make it possible to distinguish clearly states with dominant single-phonon components from states with dominant two-boson components.

In deformed nuclei in the region $150 < A < 190$ there are two au β transitions between states: $p523\uparrow \rightleftarrows n523\downarrow$ and $p514\uparrow \rightleftarrows n514\downarrow$. In the case of an au β decay, one can unambiguously distinguish a transition to single-phonon states from one to a two-phonon state. If one of the single-particle states of the matrix element of the au β transition corresponds to an odd–odd nucleus, then the au β decay takes place to the component of the single-phonon part of the wave function (18) that contains the single-quasiparticle state in the matrix element of the au β decay, while the second single-quasiparticle state corresponds to the state of the odd–odd nucleus that does not participate in the β decay. If the two-quasiparticle state of the odd–odd nucleus does not contain a single-particle state that occurs in the matrix element of the au β decay, then the au β decay takes place to a four-quasiparticle component belonging to the two-phonon state. This is the case, for example, for the au β^- decay with log ft = 5.0 of the $K^\pi = 5^+$ state $p411\uparrow + n633\downarrow$ of ^{164}Tb to the $K_\gamma^\pi = 4_2^+$ state of ^{164}Dy with energy 2.313 MeV and structure $nn(633\uparrow + 523\downarrow)\cdot 1.0 - pp(413\uparrow - 411\uparrow)\cdot 0.73$, which was calculated in Ref. 59.

Because of the interaction between the bosons, the wave functions of the states with $K_\gamma^\pi = 2_2^+$, 0_2^+, 4_1^+, and 0_3^+ in Fig. 1 contain admixtures of single-boson components. In the *sdg* IBM, the states with $K_\gamma^\pi = 2_3^+$, 0_4^+, and 4_2^+ have dominant single-boson components. The question arises of whether it is possible to demonstrate experimentally which of the two models, the QPNM or the *sdg* IBM, describes the structure of these states more correctly. The results of measurements of single-nucleon transfer reactions, au β decays, and inelastic scattering make it possible to answer this question in the affirmative in individual cases.

The wave functions of the single-phonon states $\lambda\mu_i$ consist of different two-quasiparticle components. Some states with fixed $\lambda\mu$ or K^π can, for example, be excited in the $(t\alpha)$ reaction, others in the (dp) reaction, still more in both reactions, yet more in neither but in au β decay, etc. As a result of calculations in the QPNM, one can find the contribution of the single-phonon component to the normalization (19) of the wave function (28), and also the contribution of definite two-quasiparticle components to the normalization (23′) of the wave function of the single-phonon state (23). As a result of calculations, one can find the spectroscopic factors of the single-nucleon transfer reactions and the intensities of the au β decays.

In the *sdg* IBM, the operators Π_0^+, Π_2^+, Γ_0^+, Γ_2^+, and Γ_4^+ consist of combinations of the s^+, d^+, and g^+ operators with quite definite weights. These weights determine rigorous selection rules for single-nucleon transfer reactions and au β decays. If, for example, the 0_1^+ and 2_1^+ states are strongly excited in the (dp) reaction but not in the $(t\alpha)$ reaction, then the 0_4^+ and 2_3^+ states with the dominant components given in Fig. 1 must also be excited in the (dp) reaction and not in the $(t\alpha)$ reaction. The states 2_2^+, 0_2^+, and 0_3^+ must not be excited in the $(t\alpha)$ reaction, but they can be excited in the (dp) reaction, since the interaction between the bosons can transfer some of the single-boson strength to these states.

We illustrate the possibilities of excitation of states in single-nucleon transfer reactions and in au β decays in the *sdg* IBM and in the QPNM for the example of the five $K^\pi = 2^+$ states shown in Fig. 2. The dominant components of the wave functions of these five 2^+ states are given in Fig. 1. The possible excitations of these states in the *sdg* IBM and in the QPNM are shown schematically in Fig. 2. Let us consider the $(t\alpha)$ reaction. If the 2_1^+ state is strongly excited, then in accordance with the *sdg* IBM the 2_3^+ state must be strongly excited, and the 2_2^+ state may be weakly excited. In accordance with the QPNM, the 2_1^+, 2_4^+, and 2_5^+ states can be strongly excited, and the 2_2^+ and 2_3^+ states are not excited. For example, in the QPNM the 2_5^+ state can be excited in the (dp) reaction, and the first four 2^+ states are not excited, while in (dt) reactions all five 2^+ states are excited. If, for example, the 2_1^+ state is not excited in the (dt) reaction, then in the *sdg* IBM the remaining 2^+ states should not be excited. In the (dp) reaction the 2_5^+ state cannot be excited if

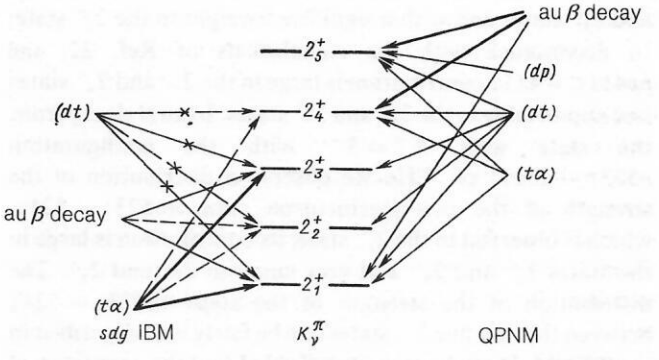

FIG. 2. Excitation scheme of $K^\pi = 2^+$ states in single-nucleon transfer reactions and in au β decay in the QPNM and the *sdg* IBM. The heavy arrows represent strong transitions, and the thin and broken arrows represent weak and very weak transitions; the arrows with crosses represent forbidden transitions.

TABLE I. $K^{\pi} = 2^{+}$ states in ^{168}Er.

K_{ν}^{π}	Experiment		Calculations in the QPNM	
	$\mathscr{E}$, MeV	Structure	$\mathscr{E}$, MeV	Structure
2_1^+	0,821	$(\vec{t}\alpha)$: $pp413\downarrow - 411\downarrow$ 50% $pp411\uparrow + 411\downarrow$ 37% lg $ft = 5{,}2$: $nn523\downarrow - 521\downarrow$ appreciable	0,8	221 : 96% 221 : $pp413\downarrow - 411\downarrow$ 36% $pp411\uparrow + 411\downarrow$ 30% $nn523\downarrow - 521\downarrow$ 18% $nn521\uparrow + 522\downarrow$ 18%
2_2^+	1,848	lg $ft = 6{,}1$: $nn523\downarrow - 521\downarrow$ small	1,8	222 : 98% 222 : $nn512\uparrow - 521\downarrow$ 97% $pp411\uparrow + 411\downarrow$ 2%
2_3^+	1,930	lg $ft = 6{,}2$: $nn523\downarrow - 521\downarrow$ small	1,9	223 : 94% 223 : $nn521\uparrow + 521\downarrow$ 60% $pp411\uparrow + 411\downarrow$ 13% $nn523\downarrow - 521\downarrow$ 3%
2_4^+	2,193	$(\vec{t}\alpha)$: $pp411\uparrow + 411\downarrow$ 20÷30% lg $ft = 4{,}8$: $nn523\downarrow - 521\downarrow$ large	2,2	224 : 98% 224 : $nn523\downarrow - 521\downarrow$ 60% $pp411\uparrow + 411\downarrow$ 28% $nn521\uparrow + 521\downarrow$ 4%
2_5^+	2,425	$(\vec{t}\alpha)$: $pp411\uparrow + 411\downarrow$ appreciable lg $ft = 4{,}6$: $nn523\downarrow - 521\downarrow$ large	2,5	225 : 97% 225 : $nn633\uparrow - 651\uparrow$ 36% $pp411\uparrow + 411\downarrow$ 15% $nn523\downarrow - 521\downarrow$ 15% $nn521\uparrow + 521\downarrow$ 6%

in the (dp) reaction the 2_1^+ and 2_3^+ states are not excited. Similar selection rules hold for the au β transitions. For example, if log ft is small for the transition to the 2_1^+ state, then in accordance with the *sdg* IBM it must be small for the transition to the 2_3^+ state. In the QPNM, small values of log ft can occur for au β transitions, for example, to the 2_1^+, 2_4^+, and 2_5^+ states.

The possibility of describing in the QPNM such excitation of the five 2^+ states illustrated in Fig. 2 occurs in the case of ^{168}Er. The results of calculations of the energy and structure of the five 2^+ states in ^{168}Er made in Ref. 22 and experimental data from Refs. 60 and 61 are given in Table I. The table gives the calculated contributions (as percentages) of the single-phonon components $22i$, $i = 1, 2, 3, 4, 5$, to the normalization (29) of the wave functions (28) for the first five 2^+ states. For each phonon $22i$ we give the contribution (as a percentage) of the largest two-quasiparticle components to the normalization of their wave functions. It follows from the experimental data of Ref. 60 on the $(\vec{t}\alpha)$ reaction that the two-quasiproton state $pp411\uparrow + 411\downarrow$ occurs with a large weight in the wave functions of the 2_1^+ and 2_4^+ states and with a significant weight in the 2_5^+ state. In accordance with the calculations of Ref. 22, the $pp411\uparrow + 411\downarrow$ contribution is large in the 2_1^+ and 2_4^+ states and appreciable in the 2_3^+ and 2_5^+ states. In au β decay from the state with $K^{\pi} = 3^+$ with the configuration $p523\uparrow - n521\downarrow$ of ^{168}Ho we observe a distribution of the strength of the two-quasineutron state $nn523\downarrow - 521\downarrow$, which is observed in the 2_1^+ state; its contribution is large in the states 2_4^+ and 2_5^+ and very small in 2_2^+ and 2_5^+. The distribution of the strength of the state $nn523\downarrow - 521\downarrow$ between the first five 2^+ states can be fairly well described in the QPNM. It can be seen from Table I that the properties of the excitation of the 2^+ states in the $(\vec{t}\alpha)$ reaction and in au β decay illustrated in Fig. 2 are given a qualitatively correct description in the QPNM. According to the calculations, all five 2^+ states can be excited in the ^{169}Er$(dt)^{168}$Er reaction, but it is very difficult to realize experimentally because of the insufficient lifetime of ^{169}Er. The state 2_5^+ can be observed in the (dp) reaction.

In the framework of the *sdg* IBM it is not possible to give a qualitatively correct description of the experimental data on the excitation of the first five 2^+ states in ^{168}Er in the $(\vec{t}\alpha)$ reaction and in au β decay. For the description of the structure of these 2^+ states in ^{168}Er the *sdg* IBM reveals its weakness. It is not clear how the model should be modified in order to eliminate the contradiction with the experimental data on the 2^+ states in ^{168}Er.

In accordance with the calculations of Ref. 62, the energies of the third and fourth states, 2_3^+ and 2_4^+, are 3.6 and 4.4 MeV in the *sdg* IBM-2. This means that even in the framework of the *sdg* IBM-2, in which there is an additional freedom not present in the *sdg* IBM-1, it is not possible to describe all five 2^+ states in ^{168}Er.

In individual cases of the inelastic scattering reaction one can assess the accuracy of the description of certain states in the QPNM and in the *sdg* IBM. As is well known, one of the reasons for the introduction of the *g* boson in Ref. 28 was for description in ^{168}Er of the $K_{\nu}^{\pi} = 3^+$ state and the large anharmonicity of the 4_1^+ state, which was treated as a two-boson state. In Ref. 28, the parameters of the model were chosen in such a way as to describe correctly in ^{168}Er the probabilities of $E2$ transitions between the bands constructed on the ground, 0_1^+, and 2_1^+ states and the $B(E4)$ values for excitation of the 4^+0, 4^+2_1, and 4^+4_1 states, for which experimental data were given in Ref. 63. As a result, the value predicted in the *sdg* IBM for $B(E4)$ for excitation of the 4^+3_1 state in ^{168}Er was 50.8 s.p.u.[28] According to the calculations of Ref. 22, $B(E4) = 0.4$ s.p.u. in the QPNM for excitation of the 4^+3 state of ^{168}Er. The predictions for excitation of the 4^+3_1 state in ^{168}Er are shown in Fig. 3. The predictions of the QPNM and the *sdg* IBM differ by a factor of 100, and such a large difference could be observed experimentally.

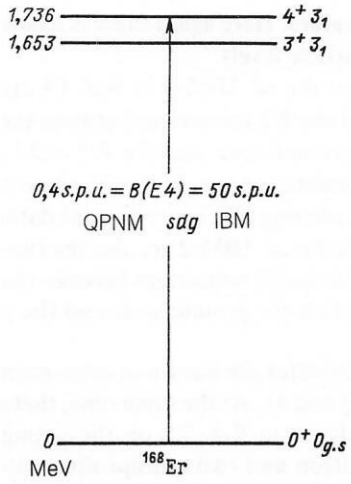

FIG. 3. Predictions of $B(E4)$ values for excitation of the $I^{\pi}K_{\nu} = 4^{+}3_{1}$ state of ^{168}Er in the QPNM (Ref. 22) and in the *sdg* IBM.[28]

4. COMPARISON OF THE DESCRIPTIONS OF LOW-LYING NONROTATIONAL STATES OF DEFORMED NUCLEI IN THE QPNM AND IN THE IBM

Structure of nonrotational states

A description of excited nonrotational states of nuclei by means of quasiparticle and phonon operators is effective. It takes into account the most important degrees of freedom of the nuclei responsible for the nuclear excitations. The collective degrees of freedom are manifested in the form of vibrational states. In all nuclei, there are collective quadrupole and octupole low-lying states and high-lying collective states—giant resonances of different types. The general picture of the vibrational states is determined by the symmetry of the equilibrium ground state. In deformed nuclei, the equilibrium shape possesses axial symmetry. The excited states of deformed nuclei differ from those of spherical nuclei in the richness and diversity of their properties.

In the QPNM strongly and weakly collective vibrational states and two- and many-quasiparticle states are described in a unified manner. The calculations of Refs. 22, 64, and 65 in the QPNM with the wave function (28) and Eqs. (29), (31), and (31′) showed that the energies and structure of the nonrotational states of even–even deformed nuclei are mainly determined by the single-particle energies and wave functions of the Woods–Saxon potential, monopole pairing, and *p–h* isoscalar multipole interaction. The multipole *p–h* isovector interaction, quadrupole pairing, and the multipole *p–p* interaction play an auxiliary part. Allowance for the *p–p* interaction somewhat improves the description of the vibrational states. In addition, it makes the validity of the RPA for the description of states with energy less than 1 MeV more convincing.

The investigations of Refs. 22, 64, and 65 showed that the nonrotational states with energies up to 2.5 MeV have dominant single-phonon or two-quasiparticle components. For the states with $K^{\pi} = 0^{\pm}, 1^{-}, 2^{\pm}, 3^{\pm}$, and 4^{+} and energies up to 2 MeV the dominant single-phonon component provides more than 90% in the normalization of the wave

TABLE II. Nonrotational states in ^{178}Hf.

K^{π}_{ν}	Experiment: η_{ν}, MeV [$B(E\lambda)$ s.p.u]	Experiment: Structure	Calculations in the QPNM: η_{ν}, MeV [$B(E\lambda)$ s.p.u]	Calculations in the QPNM: Structure
8^{-}_{1}	1,147	lg *ft*=4,7 : *pp*404↓+514↑ 34% *nn*514↓+624↑ 66%	1,11	981 : 100% 981 : *pp*404↓+514↑ 25% *nn*514↓+624↑ 75%
2^{+}_{1}	1,174 [3,9]	(*dp*) : *nn*514↓—512↓	1,12 [4,1]	221 : 94% {221, 441} : 1% 221 : *nn*514↓—512↓ 29%; *nn*512↑—510↑32%
0^{+}_{1}	1,199			
2^{-}_{1}	1,260 [4,0]	(*dt*) : *nn*624↑—512↑	1,2 [2,0]	321 : 98% 321 : *nn*624↑—512↑ 86% *pp*514↑—402↑ 8%
1^{-}_{1}	1,310		1,4 [0,5]	311 : 89% 311 : *nn*514↓—624↑ 96%
0^{+}_{2}	1,434			
0^{+}_{3}	1,444			
8^{-}_{2}	1,479	lg *ft*=4,7 : {*pp*404↓+513↑66% *nn*514↓+624↑34%	1,42	982 : 100% 982 : *pp*404↓+514↑ 75% *nn*514↓+624↑ 25%
4^{+}_{1}	1,514	(*dp*) : *nn*514↓+510↑	1,5 [2,0]	441 : 94% {221, 221} 3% 441 : *nn* 514↓+510↑ 60%: *nn* 512↑+512↓ 23%
6^{+}_{1}	1,554		1,7	*nn* 512↑+514↓100%
2^{-}_{2}	1,568		1,9 [0,8]	322 : 96% 322 : *pp*514↑—402↑ 88% *nn*624↑—512↑ 10%
5^{-}_{1}	1,637		1,8	551 : 100% 551 : *pp*514↑+411↓ 73%; *nn*624↑+521↓ 22%
3^{+}_{1}	1,758	(*dp*) : *nn*514↓—510↑	1,8 [1,7]	431 : 99% 431 : *pp*404↓—411↓ 37% *nn*514↓—510↑ 21%
0^{+}_{4}	1,772			
3^{-}_{1}	1,803	(*dp*) : *nn*514↓—512↓	1,9 [4,0]	331 : 98% 331 : *nn*615↑—512↑ 45% *pp*514↑—411↑ 24%
2^{+}_{2}	1,808		2,0 [0,01]	222 : 93% {202, 221} 3% 222 : *nn* 514↓—512↓ 56%; *nn*512↑—510↑ 40%
2^{-}_{2}	1,857		2,6 [0,02]	323 : 98% 323 : *nn*615↑—514↓ 98%
4^{+}_{2}	(2,007)		2,0 [0.0004]	442 : 98% 442 : *pp*404↓+411↓ 98%

function; the contribution of an individual two-phonon component does not exceed 5%. Weak anharmonicity is a characteristic feature of the low-lying vibrational states of deformed nuclei.

We shall demonstrate the effectiveness of the description of nonrotational states in the QPNM for the example of ^{178}Hf. The results of calculations[22,66] and experimental data[67-69] for ^{178}Hf are given in Table II. We give $\log ft$ and the experimental values of the two-quasiparticle components for the states $K^{\pi}_{\nu} = 8^{-}_{1}$ and 8^{-}_{2}, obtained in Ref. 70 from au β decay and the two-quasineutron components through which the (dp) and (dt) reactions proceed. The $B(E\lambda)$ values are given in single-particle units. The calculated structure is given (in percentages) in the form of the contribution of the single-phonon, $\lambda\mu_i$, and two-phonon, $\{\lambda_1\mu_1 i_1,\ \lambda_2\mu_2 i_2\}$, components to the normalization of the wave function (28). For the single-phonon states we give the contribution of the two-quasineutron, nn, and two-quasiproton, pp, components.

Let us discuss the data of Table II. The first 2^{+}_{1} state is a collective state, and the two-quasineutron configuration $nn514\downarrow - 512\downarrow$ is contained in the wave functions of the 2^{+}_{1} and 2^{+}_{2} states. Therefore, they are well excited in (dp) reactions. These properties of the 2^{+}_{1} and 2^{+}_{2} states are well reproduced by the calculations. However, the calculated energy of the 2^{+}_{2} state is 2 MeV, this being due to the absence of two-quasiparticle poles with energy less than 2.2 MeV. If the $I^{\pi}2^{+}$ state with energy 1.561 MeV has $K = 2$, this contradicts the results of the calculations. This contradiction may be due to the hexadecupole deformation $\beta_4 = -0.16$ given in Ref. 71, our calculations having been made for $\beta_4 = -0.03$.

According to the calculations, the 3^{+}_{1} state in ^{178}Hf is collective and must be well excited in the (dp) reaction; the second state, 3^{+}_{2}, is weakly collective. The states 3^{+}_{3}, 3^{+}_{4}, and 3^{+}_{5} have energies 2.2, 2.3, and 2.4 MeV. The wave functions of these states each contain one dominant single-phonon component. The state 4^{+}_{1} in ^{178}Hf is collective with dominant component 441 in the wave function; the contribution of the two-phonon component $\{221, 221\}$ is 3%. The experimental data of Ref. 69 do not provide evidence for collectivization of the $E2$ transition from the 4^{+}_{1} to the 2^{+}_{1} state. The contribution of the two-phonon component $\{221, 221\}$ to the 4^{+}_{2}, 4^{+}_{3}, and 4^{+}_{4} states with energies 2.0, 2.3, and 2.4 MeV does not exceed 2%. The Coriolis interaction has an effect on the energy and structure of the $K^{\pi} = 2^{+}$, 3^{+}, and 4^{+} states in ^{178}Hf.

Experimental data on octupole states in ^{178}Hf are sparse. The $K^{\pi}_{\nu} = 0^{-}_{1}$ has not been observed. Among the 1^{-}_{1}, 2^{-}_{1}, and 3^{-}_{1} states, only the $B(E3)$ value for excitation of the $3^{-}2_1$ state has been measured. According to our calculations, the energy of the 0^{-}_{1} state is 2 MeV, and $B(E3)$ is 4 s.p.u. The calculated values of $B(E3)$ for the 1^{-}_{1}, 2^{-}_{2}, and 2^{-}_{3} states are small, and it is therefore not surprising that they have not been measured experimentally. The calculated energies of the 2^{-}_{2} and 2^{-}_{3} states lie above the experimental values.

The energies of the two-quasiparticle states calculated with monopole and quadrupole pairing are close to the values given in Ref. 3. The energies of the two states with $K^{\pi}_{\nu} = 6^{+}_{1}$ and 5^{-}_{1} are somewhat overestimated in comparison with the experimental values. Here again there is a lack of a scheme of the single-particle levels.

Calculations of ^{178}Hf in the *sd* IBM-1 in Ref. 69 are limited to the description of the $E2$ transitions between the bands constructed on the ground state and the $K^{\pi}_{\nu} = 2^{+}_{1}$, 0^{+}_{1}, and 0^{+}_{2} states. The calculated energy of the 0^{+}_{2} state is greater than 2 MeV, contradicting the experimental data. The calculations in Ref. 44 in the *sd* IBM-2 are also restricted to the relative intensities of the $E2$ transitions between the rotational levels constructed on the ground state and the γ and β states.

Many two-quasiparticle states are known in even–even deformed nuclei (see Refs. 3 and 4). At the same time, there are experimental data, analyzed in Ref. 72, on the strong mixing of the two-quasineutron and two-quasiproton configurations in states with large values of K. It was shown in Ref. 66 in the framework of the QPNM that in the cases when the energies of the two-quasineutron and two-quasiproton states with the same values of K^{π} are nearly equal and the corresponding matrix elements are not small, high multipole interactions with $\lambda = 5$–9 play an important part in the mixing of such states.

Experimental data[60,70,73-77] and the results of calculations in Ref. 66 in the RPA with the wave functions of the Woods–Saxon potential and constants as in Ref. 22 are given in Table III. The mixing of the two-quasineutron state $nn514\downarrow + 624\uparrow$ and the two-quasiproton state $pp404\downarrow + 514\uparrow$ in ^{178}Hf can be described by taking into account the $\lambda\mu = 98$ multipole interaction. It can be seen from the table that a good description is obtained for the mixing with a constant $\varkappa^{98}_{0}$, close to the constants $\varkappa^{22}_{0}$, $\varkappa^{33}_{0}$, and $\varkappa^{44}_{0}$ with which a good description was obtained in Ref. 22 for the quadrupole, octupole, and hexadecupole states in ^{178}Hf. An important role of the interaction with such high multipolarity as $\lambda = 9$ with projection $\mu = 8$ was unexpected. We note that although the 8^{-}_{1} and 8^{-}_{2} states are not pure two-quasiparticle states, the isomer state with $K^{\pi} = 16^{+}$ and energy 2.477 MeV in ^{178}Hf is a pure four-quasiparticle state: $p514\uparrow + p404\downarrow + n514\downarrow + n624\uparrow$. This is due to the fact that the total strength of the states $p514\uparrow + p404\downarrow$ and $n514\downarrow + n624\uparrow$ is contained in the two levels 8^{-}_{1} and 8^{-}_{2}.

In accordance with the experimental data of Ref. 73, the two $K^{\pi} = 6^{+}$ configurations $nn514\downarrow + 512\uparrow$ and $pp404\downarrow + 402\uparrow$ in ^{176}Hf are mixed. The calculated energies of the 6^{+}_{1} and 6^{+}_{2} states and the amounts of mixing are close to the experimental values. However, the calculated structure of the first state is close to the experimentally observed second state and vice versa. This discrepancy is due to the scheme of the single-particle states. The proton scheme does not take account of the fact that, in accordance with the experimental data, the state $p402\uparrow$ in ^{175}Lu has a lower energy than the state $p514\uparrow$. Therefore, the energy of the two-quasiproton state $pp404\downarrow + 402\uparrow$ is higher than the energy of the two-quasineutron state $nn514\downarrow + 512\uparrow$. This may be due to the fact that the calculations were made with a hexadecupole deformation parameter $\beta_4 = -0.03$, while in accordance with the experimental data (Ref. 71) $\beta_4 = -0.16$.

According to the calculations, the $K^{\pi} = 8^{-}_{1}$ and 8^{-}_{2} states in ^{176}Hf have energies 1.52 and 1.84 MeV. There is somewhat less mixing of the configurations $pp404\downarrow + 514\uparrow$ and $nn514\downarrow + 624\uparrow$ than in ^{178}Hf. The description of the

TABLE III. Mixing of two-quasiproton and two-quasineutron configurations in deformed nuclei.

Nucleus	$\lambda\mu$ κ_ν^π	$\mathcal{E}$, MeV		First config.		Second config.	
		Experiment	Calculation	Experiment	Calculation	Experiment	Calculation
^{178}Hf	98			$pp404\downarrow + 514\uparrow$		$nn514\downarrow + 624\uparrow$	
	8_1^-	1,147	1,11	(34±4)%	25%	(66%)	75%
	8_2^-	1,479	1,42	(66%)	75%	(34±4)%	24%
^{176}Hf	66			$pp404\downarrow + 402\uparrow$		$nn514\downarrow + 512\uparrow$	
	6_1^+	1,333	1,35	62%	26%	38%	73%
	6_2^+	1,762	1,75	38%	71%	62%	27%
	98			$pp404\downarrow + 514\downarrow$		$nn514\downarrow + 624\uparrow$	
	8_1^-	1,559	1,52	95%	86%		14%
				(86±6)%			
	8_2^-	—	1,84		14%		86%
	77			$nn514\downarrow + 633\uparrow$		$nn512\uparrow + 624\uparrow$	
	7_1^-	1,860	1,76	~ 100%	99,7%		0,04%
	7_2^-	—	1,81		0,2%		99,8%
^{174}Yb	55			$pp411\downarrow + 514\uparrow$		$nn521\downarrow + 624\uparrow$	
	5_1^-	1,885	1,9	(46±2)%	78%		21%
	5_2^-	2,379	2,2	(54±2)%	21%		77%
^{168}Er	54			$nn633\uparrow + 521\downarrow$		$pp411\downarrow + 523\uparrow$	
	4_1^-	1,094	1,0	70%	81%	25	18%
	4_2^-	1,905	1,6	30%	18%	60	80%
^{158}Gd	44			$pp413\downarrow + 411\uparrow$		$nn523\downarrow + 521\uparrow$	
	4_1^+	1,380	1,32	large	94%	admixture	4%
	4_2^+	1,920	1,9		4%	75%	95%
	54			$nn521\uparrow + 642\uparrow$		$pp532\uparrow + 411\uparrow$	
	4_1^-	1,636	1,66	72%	92%	—	6%
	4_2^-	—	1,86	—	7%	—	87%

structure of the first 8_1^- state agrees with the experimental data,[73,74] in accordance with which the contribution of the configuration $pp404\downarrow + 514\uparrow$ is $86 \pm 6\%$. It would be interesting to detect experimentally the second 8_2^- state. According to the calculations, in ^{176}Hf there are 7_1^- and 7_2^- states with energies 1.76 and 1.81 MeV. Although these states have nearly equal energies, the two-quasineutron configurations $nn514\downarrow + 633\uparrow$ and $nn512\uparrow + 624\uparrow$ are hardly mixed. Both of the corresponding neutron matrix elements are small.

Mixing of the two-quasiproton configuration $pp411\downarrow + 514\uparrow$ with the two-quasineutron configuration $nn521\downarrow + 624\uparrow$ was found in the states 5_1^- and 5_2^- with energies 1.885 and 2.379 MeV in ^{174}Yb by study of the β decay of ^{174}Tm. The calculations of Ref. 66 with an interaction $\lambda\mu = 55$ give a mixing smaller than is found experimentally. If $\varkappa^{55}$ is increased by 10%, the mixing of these configurations is increased to 72 and 27%. The mixing of the configurations $nn633\uparrow + 521\downarrow$ and $pp411\downarrow + 523\uparrow$ in the states $K_\nu^\pi = 4_1^-$ and 4_2^- in ^{168}Er was originally calculated in Ref. 19.

The analysis in Ref. 77 of (dp) reactions and γ transitions associated with the states $K_\nu^\pi = 4_1^+$ (energy 1.380 MeV), 4_2^+ (1.920 MeV), and $K_\nu^\pi = 4_1^-$ (1.636 MeV) in ^{158}Gd showed that the states given above are not pure two-quasiparticle states. The wave function of the state 4_1^+ has a dominant component $pp413\downarrow + 411\uparrow$ and a small admixture of $nn521\uparrow + 523\downarrow$. It follows from the (dp) reaction that the contribution to the normalization of the wave function of the state 4_1^- by the configuration $nn521\downarrow + 642\uparrow$ is 72%, and that of the state 4_2^+ by the configuration $nn521\downarrow + 523\downarrow$ is $\approx$75%. In this case, one can readily explain the strong $E1$ transition between the states 4_2^+ and 4_1^- as $n523\downarrow \rightarrow n642\uparrow$. It follows from the $E1$ transition $4_1^- \rightarrow 4_1^+$ that the 4_1^- state has admixtures of the $pp532\uparrow + 411\uparrow$ configuration. It can be seen from Table III that in accordance with the calculations there is in the states 4_1^+, 4_2^+, 4_1^-, and 4_2^- a weak mixing of the two-quasineutron and two-quasiproton configurations, and this agrees with the available experimental data.

We give one further example of the effectiveness of the description of nonrotational states in the QPNM from Ref. 65. From β^+ decay of ^{170}Lu there was found[78,79] to be a large number of states with $K^\pi = 0^-$ and 1^- of ^{170}Yb in the energy interval 2.0–3.4 MeV. The number of $K^\pi = 0^-$ and 1^- states of ^{170}Yb obtained from the experimental data of Refs. 78 and 79 and the results of calculations are given in Table IV. Of course, there is some uncertainty in the experimental data due to the errors in the ascription of I^π values to the levels and in the results of the calculations. Nevertheless, the results of the calculations agree with the experimental

TABLE IV. Number of nonrotational $K^\pi = 0^-$ and 1^- states.

Energy interval, MeV	^{170}Yb		^{174}Yb
	Experiment	Calculation	Calculation
Up to 2	2	2	3
2,0—3,0	17	13	7
3,0—3,4	16	14	9
Total up to 3.4	35	29	19

TABLE V. Distribution of $E2$ strength.

Nucleus	K_ν^π	$\mathscr{E}$, MeV $\Delta\mathscr{E}$, MeV	$B(E2)_{\text{s.p.u}}$		
			Experiment	Calculation in QPNM (Ref. 22)	Calculation in IBM (Ref. 36)
^{168}Er	2_1^+	0,821	4,7	4,6	4,7
	0^+, 2^+	1,2—2,5	—	1,5	0,02
^{172}Yb	2_1^+	1,47	1,4	1,5	1,0
	2_2^+	1,61	0,42	0,7	0,007
	0^+	1,0—2,0	0,005	0,9	0,2
			0,66		
	0^+, 2^+	2,0—3,0	2,4	1,0	0,2
^{178}Hf	2_1^+	1,174	3,9	4,1	—
	2^+	3,0—3,2	—	0,6	

data—in ^{170}Yb in the interval 2.0–3.4 MeV there is an anomalously large number of states with $K^\pi = 0^-$ and 1^-. For comparison, Table IV gives the calculated number of $K^\pi = 0^-$ and 1^- states in ^{174}Yb; it was found to be 1.5 times smaller than in ^{170}Yb.

From these examples it can be seen that the possibilities of the QPNM for the description of the structure of nonrotational states are much greater than those of the IBM.

Distribution of the $E\lambda$ strength between low-lying states

It is assumed that the first quadrupole β- and γ-vibrational states are collective, and that the other collective states form giant isoscalar and isovector quadrupole resonances. In the case of octupoles, the collective states are, besides the first octupole states, those that form the low- and high-lying isoscalar and isovector giant octupole resonances. In some nuclei, low-lying collective hexadecupole states are observed. Phenomenological models, including the IBM, are based on the collectivization of the first quadrupole and octupole states and on its absence in the higher-lying states up to the giant resonances.

We shall study the distribution of the $E\lambda$ strength between the low-lying states with energy up to 3 MeV. Such a study was made possible after the publication of experimental results on inelastic scattering of α particles.[36] In the calculations of the $E\lambda$ strength made in Refs. 22 and 65, the coupling between the vibrational motion and the rotational motion was ignored.

Let us consider the distribution of the $E2$ strength. The experimental data and the results of calculations in the QPNM (Ref. 22) and in the *sd* IBM (Ref. 36) are given in Table V. In ^{168}Er and ^{178}Hf we find the standard case—the main fraction of the $E2$ strength is concentrated on the γ-vibrational state. According to the calculations, the remaining states in ^{168}Er receive about 30% of the $E2$ strength of the γ-vibrational state, and in ^{178}Hf they receive 15%.

The distribution of the $E2$ strength in ^{172}Yb differs qualitatively from that in ^{168}Er, ^{178}Hf, and a number of other nuclei. A feature of the distribution of the $E2$ strength in ^{172}Yb is, first, that not only the first state 2_1^+ but also the state 2_2^+ is collective. Second, an appreciable fraction of the $E2$ strength is concentrated in the energy interval 2–3 MeV. Cases when the first, 2_1^+, and second, 2_2^+, states are both collective present great difficulties for the phenomenological description. For example, according to the calculations of Ref. 36 in the *sd* IBM, the $E2$ strength ascribed to the 2_2^+ state in ^{172}Yb is 200 times less than for the 2_1^+ state, i.e., there is a pronounced disagreement with experiment. The calculations of Ref. 22 give a qualitatively correct description. According to the experimental data of Ref. 36, the energy interval 2–3 MeV in ^{172}Yb contains 1.3 times more $E2$ strength than in the two first collective states, and 1.7 times more than in the 2_1^+ state. This is a new and very important experimental result, indicating a limitation of the widely accepted picture of the distribution of the $E2$ strength. Such a distribution of the $E2$ strength cannot be described in the *sd* IBM. According to calculations in the QPNM, the interval 2–3 MeV contains an appreciable fraction of the $E2$ strength, about 2/3 of the $E2$ strength on the 2_1^+ state.

We now consider the distribution of the $E3$ strength in ^{168}Er. According to Ref. 36, there are first collective states with $K_\nu^\pi = 0_1^-$, 1_1^-, and 2_1^-. Six collective $K^\pi = 3^-$ states have been observed. The first three states, 3_1^-, 3_2^-, and 3_3^-, take 1.3 of the single-particle strengths, while the fourth state, 3_4^-, takes 4.68, i.e., almost three times more than the first three states. In the interval 2.25–2.50 MeV, 7.9 s.p.u. is concentrated. Such a distribution of the $E3$ strength differs strongly from the standard distribution. Table VI gives the results of calculations in the QPNM in Ref. 22, and in the *sdf* IBM in Refs. 36 and 34. In the calculations of Ref. 36, the normalization of the $B(E3)$ values is based on the state 3^-1_1. As a result, for the first state, 3_1^-, the calculated $B(E3)$ value was found to be about 500 times greater than the experimental value. It is difficult to explain why a large proportion of the $E\lambda$ strength is not concentrated on the first K_1^π state. This conclusion is confirmed by calculations in Ref. 34 in the *sdf* IBM, in which the first three $K^\pi = 3^-$ levels were omitted. The main part of the $E3$ strength is concentrated on the state 3_4^-, which was regarded as the first collective $K^\pi = 3^-$ state. The first three $K^\pi = 3^-$ states were omitted as two-quasiparticle states. The states 3_1^-, 3_2^-, and 3_3^- in ^{168}Er cannot be assumed to be two-quasiparticle states. According to the experimental data of Refs. 60 and 76 on the (dp) and $(\vec{t}\alpha)$ reactions, their wave functions contain a superposition of two-quasiproton and two-quasineutron terms. According to the experimental data of Ref. 36 on the $(\alpha\alpha')$ reaction, the $B(E3)$ values for the $K_\nu^\pi = 3_1^-$, 3_2^-, and 3_3^- states are 30–60 times greater than for states that are nearly two-quasiparticle states.

According to the calculations of Ref. 22, the $B(E3)$ values in the QPNM for the 0_1^-, 1_1^-, 2_1^-, and 3_1^- states in ^{168}Er agree with the experimental values. Among the $K^\pi = 3^-$ states, the largest $B(E3)$ value occurs for the fourth state, 3_4^-, in agreement with experiment. The total $E3$

TABLE VI. Distribution of $E3$ strength in ^{168}Er.

$I=3$, K^{π}_{ν}	Experiment (Ref. 36)		$B(E3)_{s.p.u}$, calculations		
	$\mathscr{E}$, MeV	$B(E3)_{s.p.u.}$	QPNM (Ref. 22)	*sdf* IBM (Ref. 36)	*sdf* IBM (Ref. 34)
1^-	1,431	3,92	4,6	3,92	5,5
3^-_1	1,541	0,25	0,14	134,6	—
2^-_1	1,633	4,94	4,6	4,94	8,0
3^-_2	1,828	0,60	0,6	2,30	—
0^-_1	1,913	1,96	3,0	1,53	4,6
3^-_3	1,999	0,42	0,3	0,085	—
1^-_2	2,022	—	0,3	—	1,1
3^-_4	2,269	4,68	2,0	8,5	5,8
2^-_2	2,302	—	0,2	—	0,3
(3^-_5)	2,324	1,53	—	0,034	—
1^-_3	—	—	4,9	—	2,6
(3^-_6)	2,486	1,70	—	0,017	—

strength concentrated on the ^{168}Er states with energy up to 2.5 MeV is 20 s.p.u. according to the experimental data of Ref. 36, and 20.3 s.p.u. according to the calculations of Ref. 22.

The distribution of the E3 strength calculated[22,65] in the QPNM is given in Table VII. In ^{170}Yb, the states $K^{\pi}_{\nu}=0^-_1$, 0^-_2, 1^-_1, 1^-_2, 2^-_1, and 2^-_2 are collective, and among the $K^{\pi}=3^-$ states an appreciable fraction of the $E3$ strength is concentrated on the third, 3^-_3, and fourth 3^-_4, states. In this nucleus, the distribution of the $E3$ strength is nonstandard.

In Ref. 36, all the first octupole states that were collective were found experimentally in ^{172}Yb. Their total $E3$ strength is 12 s.p.u. According to the calculations of Ref. 22, all the first octupole states are collective, and their total $E3$ strength is 8 s.p.u. The total $E3$ strength in the interval 2–3 MeV is 11 s.p.u. According to the calculations of Ref. 34, in the *sdf* IBM the $E3$ strength is concentrated on the first octupole states, and only about 3% is on the 3^-1_2 state. It would be interesting to confirm experimentally this difference in the distributions of the $E3$ strength in the energy interval 2–3 MeV.

According to the calculations of Ref. 65, a standard distribution of the $E3$ strength in ^{174}Yb is obtained in the QPNM only between the $K^{\pi}=3^-$ states. For the $K^{\pi}=0^-$ and 1^- states the largest fraction of the $E3$ strength is concentrated in the interval 2.4–3.2 MeV. The two states with $K^{\pi}_{\nu}=2^-_1$ and 2^-_2 are both collective. The distribution of the $E3$ strength in ^{174}Yb calculated in the QPNM cannot be reproduced in the phenomenological models and in the IBM.

Experimental data on the distribution of the $E3$ strength in ^{178}Hf are sparse; they are restricted to excitation of the 3^-2_1 state with $B(E3)=4$ s.p.u. According to the calculations in Ref. 22, for $K^{\pi}=2^-$ and 3^- the $E3$ strength is concentrated on the first states. The states 0^-_1 and 0^-_2 have energies 2.0 and 2.4 MeV and $B(E3)$ values 2 and 4 s.p.u., and, in addition, in the interval 1.5–3.0 MeV there is a further 0.8 s.p.u. The distribution of the $E3$ strength over the $K^{\pi}=1^-$ states is as follows. On the first two states there is 0.8 s.p.u., and in the interval 2–3 MeV there is 9.5 s.p.u., i.e., the main fraction of the $E3$ strength is situated above the first two states, 1^-_1 and 1^-_2. In this case there is also a strong difference from the results of the calculations[34] in the *sdf* IBM.

On the basis of the calculations of Refs. 22 and 65, it can be asserted that there are cases of distribution of the $E\lambda$ strength different from the standard case and having a large proportion of the $E\lambda$ strength concentrated, not on the first

TABLE VII. Distribution of $E3$ strength calculated in the QPNM.

Nucleus	$I=3$, K^{π}_{ν}	$\mathscr{E}$ or $\Delta\mathscr{E}$, MeV	$B(E3)_{s.p.u}$	Nucleus	$I=3$, K^{π}_{ν}	$\mathscr{E}$ or $\Delta\mathscr{E}$, MeV	$B(E3)$s.p.u
^{170}Yb	0^-_1	1,6	2,8	^{174}Yb	0^-_1	1,7	1,0
	0^-_2	2,1	1,4		0^-_2	2,2	0,5
	0^-_3	2,4	0,9		0^-_3	2,5	1,1
	1^-_1	1,5	0,7		0^-_4	3,0	3,6
	1^-_2	2,2	1,0		1^-_1	1,8	0,1
	2^-_1	1,6	2,4		1^-_2	1,9	0,8
	2^-_2	1,9	2,0		1^-	2,4—3,2	3,6
	3^-_1	1,6	0,1		2^-_1	1,2	1,6
	3^-_2	2,0	0,2		2^-_2	2,7	1,7
	3^-_3	2,4	1,8		3^-_1	2,0	4,0
	3^-_4	2,5	1,2		3^-_2	2,2	0,2
^{172}Yb	0^-_1	1,7	1,1	^{178}Hf	0^-_1	2,0	2,0
	0^-_2	2,0	0,5		0^-_2	2,4	4,0
	1^-_1	1,2	1,8		1^-_1	1,4	0,5
	1^-_2	2,2	2,2		1^-_2	1,5	0,3
	2^-_1	1,6	2,5		1^-	2,0—3,0	9,5
	2^-_2	2,2	0,2		2^-_1	1,2	2,0
	2^-_3	2,6	1,4		2^-_2	1,9	0,8
	3^-_1	2,0	2,3		2^-_3	2,6	0,2
	3^-_2	2,2	2,0		3^-_1	1,9	4,0
					3^-_2	2,4	0,01

state, but on higher states. A nonstandard distribution of this kind has been discovered experimentally for the $E3$ strength among the $K^{\pi}=3^{-}$ states in ^{168}Er and the $E2$ strength among the $K^{\pi}=2^{+}$ states in ^{172}Yb. It would be interesting to study experimentally the distribution of the $E\lambda$ strength in many deformed nuclei. A distribution of the $E\lambda$ strength different from the standard case cannot be described in the existing versions of the IBM.

Need for the *spdfg* IBM and the situation with regard to collective two-phonon states

In the framework of the *sdf* IBM one can describe quadrupole and octupole collective states, i.e., states with positive and negative parity. The scheme of excited states is as follows. The states with the dominant single-boson component are the β- and γ-vibrational states and the first octupole states and the states with the dominant two-boson components. The scheme of the excited states is shown in Fig. 4. There are two-boson states of positive parity constructed from two quadrupole or two octupole bosons. The number of octupole phonons with a given value of K^{-} is denoted by n_f^k. In addition, there are negative-parity two-boson states constructed from quadrupole and octupole bosons. The interactions between the bosons have the consequence that the wave functions of states with dominant single-boson components will contain admixtures of two-boson components. The wave functions of states with dominant two-boson components must contain admixtures of single- and three-boson components.

It is asserted in Refs. 37 and 38 that for a consistent description of the negative-parity states it is necessary to introduce a p boson. In Refs. 28 and 29 the need to introduce a g boson to describe excited positive-parity states lying above the β- and γ-vibrational states is convincingly demonstrated. To describe 4_2^+ states in spherical nuclei, the g boson is also introduced. To describe in the framework of a single model the collective positive- and negative-parity states it is necessary to have a model with s, p, d, f, and g bosons. It is in the framework of the *spdfg* IBM that one should describe the collective states in even–even deformed nuclei. Such a version of the IBM has not yet been consistently formulated.

In the *spdfg* IBM, the scheme of the states is more complicated than the scheme in Fig. 4. There are additional states with dominant single-phonon and two-phonon components. The interaction between the bosons may lead to a small deviation of the energies of the two-boson states from the sum of the energies of the two corresponding single-boson states. The presence of a large number of states with dominant two-boson components is a fundamental characteristic of the *spdfg* IBM.

On the basis of the calculations in the QPNM it was concluded in Ref. 13 that there are no collective two-phonon states in deformed nuclei. A state is regarded as a two-phonon state if the contribution of the two-phonon component to the normalization of the wave function exceeds 50%. The question of the existence of two-phonon states is still being discussed (see, for example, Refs. 14, 15, 22, 28, and 80). So far, there are no firmly established experimental data on collective two-phonon states in deformed nuclei.

A raising of the energy centroids of the two-phonon states $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ relative to the sum of the energies of vibrational states with dominant single-phonon components of their wave functions is due to two factors. The first is anharmonicity of the vibrations, since the energies of the two-phonon states are higher than the sum of the energies of the two single-phonon states. The second is the shift $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ of the two-phonon pole (32) due to allowance for the Pauli principle in the two-phonon terms of

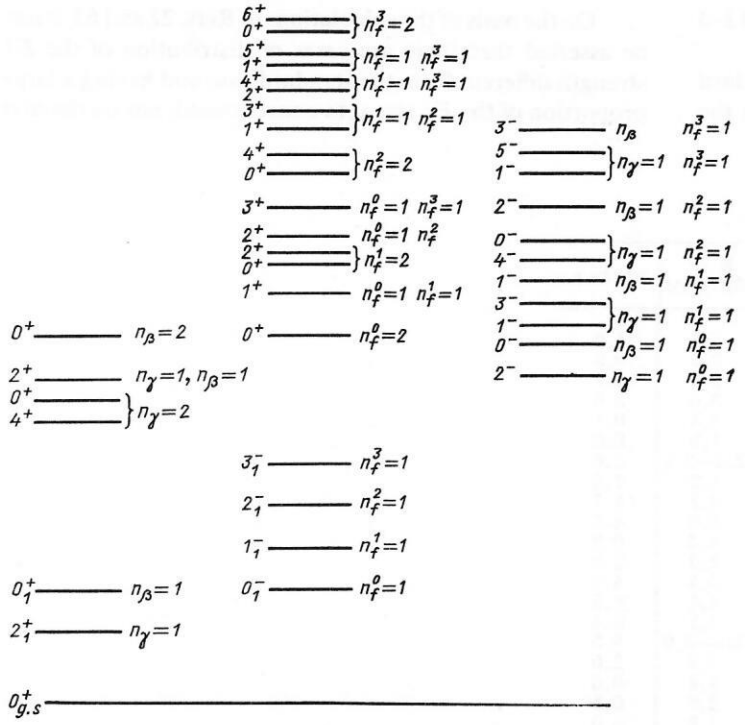

FIG. 4. Scheme of excited states with dominant quadrupole and octupole single- and two-boson components (energies in relative units).

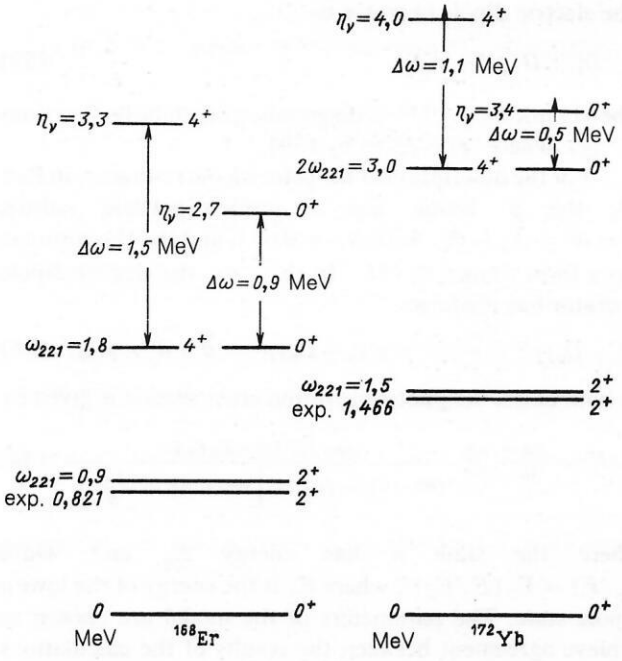

FIG. 5. Energy centroids of two-phonon γ-vibration states in ^{168}Er and ^{172}Yb.

the wave function (28). These two factors leading to raising of the energy centroids are demonstrated in Fig. 5 for the γ-vibrational two-phonon states with $K^\pi = 4^+$ and 0^+ in ^{168}Er and ^{172}Yb, calculated in the QPNM.[22] The calculations of Ref. 22 with allowance for p–h and p–p interactions yielded energies and $B(E\lambda)$ values for the 2_1^+ and the first octupole states in agreement with the experimental data. Allowance for p–p interactions reduced the collectivity of the 2_1^+ state and the first octupole states and thus gave smaller shifts $\Delta\omega$ than those calculated in Ref. 13, in which the p–p interactions were not taken into account. In ^{168}Er there is a lowest-lying and collective γ-vibrational state, and therefore the shift $\Delta\omega$ for 4^+ {221, 221} is maximal and equal to 1.5 MeV. In ^{172}Yb, the γ-vibrational state is less collective, and the shift $\Delta\omega(221, 221)$ is 1.1 MeV for the $K^\pi = 4^+$ state and 0.5 MeV for the $K^\pi = 0^+$ state.

We note that the shift $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ of the two-phonon pole was also calculated in Ref. 81. In Ref. 81, the method of boson expansions was used to construct new phonons, in terms of which the Hamiltonian was expressed. Up to the principal terms, the shift calculated by this method agrees with the result of Eq. (32).

The results of calculations[22,65] of the shifts $\Delta\omega$ $(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$, the values of $1 + \mathscr{K}^K(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ and the energy centroids of two-phonon states are given in Table VIII. The deviation of $1 + \mathscr{K}^K(\mu_1\lambda_1 i_1, \lambda_2\mu_2 i_2)$, which occurs in the normalization (29), from unity determines the influence of the Pauli principle. It can be seen from the table that the shifts $\Delta\omega$ take values from 0.1 to 1.5 MeV. As a rule, a decrease in $1 + \mathscr{K}^K$ leads to an increase in $\Delta\omega$. It is this that is responsible for the large shift $\Delta\omega(221, 221)$ for the $K^\pi = 4^+$ state from the $K^\pi = 0^+$ state. The largest shifts are obtained for the $K^\pi = 4$ {221, 221} states; for the other ^{168}Er and ^{178}Hf states, they do not exceed 1 MeV.

The energy centroid of the 0^+ {221, 221} state in ^{168}Er is 2.7 MeV according to Ref. 22; according to the calculations in Ref. 14 it is 2.9 MeV, and in Ref. 15 it is 2.8 MeV. The energy centroids of the 0^+ {221, 221} states calculated in the QPNM are approximately the same as those calculated in Ref. 15 by the multiphonon method. There is a discrepancy for the 4^+ {221, 221} states. It is not clear from this discrepancy which is more important—the large number of

TABLE VIII. Energy centroids of two-phonon states.

Nucleus	K^π	$g_1 = \lambda_1\mu_1 i_1$	$g_2 = \gamma_2\mu_2 i_2$	$\Delta\omega\,(g_1, g_2)$, MeV	$1+\mathscr{K}^K(g_1, g_2)$	Energy centroid, MeV
^{168}Er	0^+	221	221	0,9	0,90	2,7
	4^+	221	221	1,5	0,75	3,3
	2^-	221	301	0,4	0,95	3,2
	3^-	221	311	0,4	0,95	2,7
^{170}Yb	0^+	221	221	0,8	0,89	3,5
	4^+	221	221	1,1	0,78	3,7
	3^-	221	311	0,3	0,92	3,2
^{172}Yb	0^+	221	221	0,5	0,94	3,4
	0^+	221	222	0,2	0,97	3,3
	0^+	321	321	0,1	0,98	3,3
	2^+	221	441	0,6	0,42	4,1
	4^+	221	222	0,6	0,47	3,6
	4^+	221	221	1,1	0,60	4,0
	4^+	222	222	0,4	0,62	3,7
	1^-	221	311	1,0	0,45	3,7
	2^-	221	301	0,3	0,95	3,4
	3^-	221	311	0,2	0,96	2,7
^{174}Yb	0^+	221	221	0,5	0,92	3,8
	0^+	321	321	0,1	0,99	2,5
	2^+	301	321	0,1	0,99	3,0
	4^+	221	221	1,3	0,79	4,5
	2^-	221	301	0,4	0,92	3,7
^{178}Hf	0^+	221	221	1,0	0,90	3,5
	0^+	321	321	0,1	0,99	2,5
	2^+	301	321	0,1	0,97	3,1
	4^+	221	221	1,5	0,78	4,0
	4^+	221	222	0,4	0,88	3,9
	0^-	221	321	1,3	0,65	3,9
	2^-	221	301	0,6	0,88	3,8
	3^-	221	311	0,2	0,95	2,8
	3^-	431	301	0,2	0,93	4,0

degrees of freedom in the form of a large number of single-phonon and two-phonon states in the QPNM or the single degree of freedom, the γ-vibrational phonon, and wave function with multiphonon configurations in the multiphonon method. The multiphonon terms of the wave function do not usually lead to a strong departure of the root from the corresponding pole. But if the root η_ν does sink strongly relative to the pole $\omega_{g_1}+\omega_{g_2}+\Delta\omega(g_1g_2)$, then this two-phonon state will be strongly fragmented. It is therefore hard to expect large displacements of the energy centroids of two-phonon states without strong fragmentation when many-phonon terms are added to the wave function (28).

If a state is very collective and its energy is low, then the shift $\Delta\omega$ is large. If the energy of a state with a dominant single-phonon component is not low and the collectivization is not strong, then the shift $\Delta\omega$ is small. In all cases, the energy centroids of the lowest collective two-phonon states are 2.5–40 MeV. At these energies, the strength of the two-phonon states must be distributed over many levels. Therefore, the assertion made earlier in Ref. 13 that collective two-phonon states cannot exist in well-deformed nuclei is true.

According to the phenomenological models, different modifications of the IBM, and the methods of Refs. 14 and 15, collective two-phonon states must exist in deformed nuclei. Recently, attention has concentrated on the $K^\pi=4^+$ level with energy 2.055 MeV in ^{168}Er. According to calculations in the QPNM, this level has a dominant hexadecupole single-phonon component, but according to Refs. 7, 14, 15, and 25 it is a two-phonon state characterized by a high anharmonicity. High anharmonicity of the two-phonon 4^+ state is explained in Refs. 15 and 28 from diametrically opposed positions. In Ref. 28, the γ-vibrational state has a dominant single-boson component, and the anharmonicity is due to interaction between the ideal bosons. In Ref. 15, Tamm–Dancoff phonons are used, and the anharmonicity is due to the difference between them and ideal bosons.

The absence, in deformed nuclei, of two-phonon collective states necessitates a radical modification of the IBM or restricts the region of applicability of the phenomenological models to the first quadrupole and octupole states. Therefore, the question of the existence or nonexistence of collective two-phonon states in deformed nuclei is of fundamental scientific interest and must be clarified by experiment.

CONCLUSIONS

We have compared the descriptions in the quasiparticle–phonon nuclear model and the interacting-boson model of the low-lying vibrational states of even–even deformed nuclei. In the IBM, a description has been given of the isovector giant dipole resonance[82,83] and the isoscalar giant monopole and quadrupole resonances.[84] We shall give the general scheme of such a description and discuss the shortcomings and limitations of the description of giant resonances in the IBM.

Let us consider the description of a giant dipole resonance in the IBM. In Ref. 82, a p' boson of particle–hole type is introduced, and the Hamiltonian is expressed in the form

$$H=H_{sd}+\varepsilon_{p'}p'^{+}\cdot\tilde{p}'+H_{\rm int}; \tag{51}$$

$$H_{\rm int}=a_6(p'^{+}\times\tilde{p}')^2\cdot(s^{+}\times\tilde{d}+d^{+}\times\tilde{s})^2 + \sum_{l=0,\,1,\,2}t_l(p'^{+}\times\tilde{p}')^l\cdot(d^{+}\times\tilde{d})^l. \tag{51'}$$

The electric dipole operator is

$$D^1_\mu=D\,(p'^{+}_\mu+p_\mu). \tag{52}$$

The Hamiltonian (51) is diagonalized with basis functions $p'^{+}|g)$, where $|g)$ is given by (39).

For the description of the giant dipole resonance in Ref. 83, the p' boson has a particle–particle nature, $N=N_s+N_p+N_{p'}$ with $N_{p'}=0$ or 1, and the Hamiltonian has a form similar to (51). In this case, the electric dipole operator has the form

$$D^1_\mu=D_0(p'^{+}\times\tilde{s}+s^{+}\times\tilde{p}')^1_\mu+D_2(p'^{+}\times\tilde{d}+d^{+}\times\tilde{p}')^1_\mu. \tag{53}$$

In both cases, the photoabsorption cross section is given by

$$\sigma_t(E)=\frac{8\pi e^2}{3\hbar c}\sum_n\frac{|(gp'|D^1|g)|^2E_n\Gamma_nE^2}{(E^2-E_n^2)+(\Gamma_n/2)^2\left[E^2+E_n^2+\frac{\Gamma_n^2}{8}\right]}, \tag{54}$$

where the state n has energy E_n and width $\Gamma_n(E)=\Gamma_0(E/E_0)^\gamma$, where E_0 is the energy of the lowest dipole state. The parameters of the model are chosen to achieve agreement between the results of the calculations and the experimental data. In this way one obtains a description of the splitting of the giant dipole resonance in deformed nuclei in accordance with $\mu\equiv K=0$ and 1. If the Hamiltonian H_{sd} has dynamical $SU(3)$ symmetry and $t_2=-(1/\sqrt{2})\sqrt{7}a_6$, $t_0=0$, then the Hamiltonian (51) preserves the $SU(3)$ symmetry. In this case, the dipole strength is split for $\mu=0$ and $\mu=1$ in the ratio 1:2. In the case of weak breaking of the $SU(3)$ symmetry, the fragmentation of the dipole strength is greater, but the separation of the two dominant components remains fairly clear.

To describe isoscalar giant monopole and quadrupole resonances in the IBM, s' and d' bosons of particle–hole type are also introduced. The Hamiltonian is expressed in the form

$$H=H_{sd}+\varepsilon_{s'}s'^{+}\cdot\tilde{s}'+\varepsilon_{d'}d'^{+}\cdot\tilde{d}'+H_{\rm int},$$

where $H_{\rm int}$ is constructed from s, d and s', d' bosons. The operators of $E0$ and $E2$ transitions are expressed in the form

$$D^0=\bar{D}^0(s'^{+}+s'),\quad D^2_\mu=D^2(d'^{+}_\mu+d'_\mu).$$

As a result of calculations, one finds the distributions of $B(E2)$ and $B(E0)$ corresponding to the isoscalar quadrupole and monopole resonances.

We note that the coupling between the low-lying and high-lying (of the type of giant resonances) collective states was studied many years ago in Greiner's dynamical collective model and in other phenomenological models. This resulted in a rough description of the widths of the giant resonances. In essence, the same coupling was taken into account in Refs. 82–84 in the IBM, and in the general features a description of the photoabsorption cross sections demonstrating the giant resonances was obtained. It remains an open question whether it is possible to calculate in the IBM other properties of giant resonances, for example, the fine structure and the partial decay widths.

In the description of giant resonances in deformed nuclei in the IBM in Refs. 82–84, the entire set of collective states that form the giant resonance for each value of the quantum number K is described by means of a single boson. The following question should be answered: Is it possible to

describe by means of a single boson the entire richness of the large number of states that form the giant resonance?

Calculations in the QPNM show that giant resonances are formed through fragmentation of a large number of single-phonon states. For example, according to Ref. 10, 150 single-phonon states exhaust 80% of the energy-weighted sum of the isovector dipole resonance in ^{238}U. In the calculation of giant quadrupole resonances in the region of the rare earths and actinides, 2000–3000 single-phonon states are taken into account.

It follows from the microscopic calculations that a large set of two-quasiparticle states is needed for formation of a giant resonance in deformed nuclei. Therefore, it is difficult to expect that the large number of shell configurations that participate in the formation of a giant resonance can be replaced by a single boson for each value of K.

From the formation of giant resonances by fragmentation of a single boson there must follow definite predictions for the probabilities of its decay with emission of a neutron or proton or by γ transitions to the ground and excited states as one passes from the low-energy part of the resonance to its upper boundary. Direct indications could be given by single-nucleon transfer reactions, provided that the valence-particle–particle (or hole) configurations make a large contribution to the wave functions of the states that form the giant resonance. The operators of the γ transitions from giant resonances couple states that differ by a particle–hole boson. Thus, γ transitions take place to γ-vibrational states from components s'^{+}, or p'^{+}, or d'^{+}, multiplied by a d^{+} boson of wave functions of the states that form the giant resonance. In the IBM, such probabilities of γ transitions to the β, γ, and first octupole states should be calculated for a number of deformed nuclei. By comparing these calculations with experimental data one could probably establish whether a giant resonance could or could not be formed by a single boson.

It can be asserted that in the framework of the QPNM a basis has been created for description of the structure of the low-lying states of spherical and deformed nuclei. The mathematical formalism of the QPNM was generalized in Refs. 20 and 85 to separable interactions of finite rank $n_{\max} > 1$ capable of reproducing complex effective interactions between quasiparticles.

For understanding the structure of deformed nuclei, great interest attaches to comprehensive experimental study of excited states with energy 2–3 MeV. It would be desirable to start by making experimental measurements as detailed as those for ^{168}Er in many other deformed nuclei. It may be hoped that an experimental study of states with energy 2–3 MeV will be made by multidetector systems with the new generation of accelerators with high energy resolution.

I thank V. O. Nesterenko, A. V. Sushkov, and N. Yu. Shirikov, with whom I participated in investigations that yielded many of the results presented here.

[1] V. G. Solov'ev, *Theory of Complex Nuclei* (Pergamon Press, Oxford, 1976) [Russ. original, Nauka, Moscow, 1971].
[2] V. G. Soloviev, At. Energy Rev. **3**, 117 (1965).
[3] E. P. Grigor'ev and V. G. Solov'ev, *Structure of Odd Deformed Nuclei* [in Russian] (Nauka, Moscow, 1974).
[4] S. P. Ivanova, A. L. Komov, L. A. Malov, and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **7**, 450 (1976) [Sov. J. Part. Nucl. **7**, 175 (1976)].
[5] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975) [Russ. transl., Mir, Moscow, 1977].
[6] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975); Ann. Phys. (N.Y.) **99**, 253 (1976).
[7] D. D. Warner, R. F. Casten, and W. F. Davidson, Phys. Rev. Lett. **45**, 1761 (1980); Phys. Rev. C **24**, 1713 (1981).
[8] R. F. Casten and D. D. Warner, Rev. Mod. Phys. **60**, 389 (1988).
[9] V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **9**, 860 (1978) [Sov. J. Part. Nucl. **9**, 343 (1978)].
[10] L. A. Malov and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **11**, 301 (1980) [Sov. J. Part. Nucl. **11**, 111 (1980)].
[11] V. G. Soloviev, Prog. Part. Nucl. Phys. **19**, 107 (1987).
[12] V. G. Solov'ev, *Nuclear Theory. Quasiparticles and Phonons* [in Russian] (Énergoatomizdat, Moscow, 1989).
[13] V. G. Soloviev and N. Yu. Shirikova, Z. Phys. A **301**, 263 (1981); Yad. Fiz. **36**, 1376 (1982) [Sov. J. Nucl. Phys. **36**, 799 (1982)].
[14] M. Matsuto and K. Matsyuanagi, Prog. Theor. Phys. **74**, 1227 (1985); **76**, 93 (1986).
[15] R. Pupenbring and M. K. Jammari, Nucl. Phys. **A481**, 81 (1988).
[16] V. G. Solov'ev, *Nuclear Theory. Nuclear Models* [in Russian] [Énergoizdat, Moscow, 1981].
[17] V. G. Soloviev, Z. Phys. A **34**, 143 (1989).
[18] I. Rignarsson and B. A. Broglia, Nucl. Phys. **A263**, 315 (1976).
[19] L. Karadzhev, V. G. Solov'ev, and A. V. Sushkov, Izv. Akad. Nauk SSSR, Ser. Fiz. **53**, 2150 (1989).
[20] V. G. Solov'ev, Yad. Fiz. **50**, 40 (1989) [Sov. J. Nucl. Phys. **50**, 25 (1989)].
[21] V. G. Solov'ev and N. Yu. Shirikova, Izv. Akad. Nauk SSSR, Ser. Fiz. **52**, 2095 (1988).
[22] V. G. Soloviev and N. Yu. Shirikova, Z. Phys. A **334**, 149 (1989).
[23] V. G. Soloviev, A. V. Sushkov, and N. Yu. Shirikova, Z. Phys. A **316**, 65 (1984).
[24] V. G. Soloviev and A. V. Sushkov, Phys. Lett. **216B**, 259 (1989).
[25] A. Bohr and B. R. Mottelson, Phys. Scr. **25**, 28 (1982).
[26] T. Otsuka and S. Ginoechio, Phys. Rev. Lett. **59**, 276 (1985).
[27] Wu Hua-Chan and Zhou Xiao-Oian, Nucl. Phys. **A417**, 67 (1984).
[28] N. Yoshinaga, Y. Akiyama, and A. Arima, Phys. Rev. Lett. **56**, 1116 (1986); Phys. Rev. C **38**, 419 (1988).
[29] N. Yoshinaga, Nucl. Phys. **A456**, 21 (1986); **A493**, 323 (1989).
[30] Y. Akiyama, K. Heyde, A. Arima, and N. Yoshinaga, Phys. Lett. **173B**, 1 (1986).
[31] Y. Akiyama, P. von Brentano, and A. Gelberg, Z. Phys. A **326**, 517 (1987).
[32] H. C. Wu, A. E. Dieperink, O. Scholten *et al.*, Phys. Rev. C **38**, 1638 (1988).
[33] O. Scholten, F. Iachello, and A. Arima, Ann. Phys. (N.Y.) **115**, 325 (1978).
[34] A. F. Barfield, B. R. Barrett, J. L. Wood, and O. Scholten, Ann. Phys. (N.Y.) **182**, 344 (1988).
[35] J. Konijn, F. W. de Boer, A. van Poelgest *et al.*, Nucl. Phys. **A352**, 191 (1981).
[36] I. M. Govil, H. W. Fulbright, D. Cline *et al.*, Phys. Rev. C **33**, 793 (1986); **36**, 1442 (1987).
[37] J. Engel and F. Iachello, Phys. Rev. Lett. **54**, 1126 (1985).
[38] I. N. Mikhailov, E. G. Nadjakov, M. Aiche *et al.*, J. Phys. G **15**, L19 (1989).
[39] T. Otsuka and M. Sigita, Phys. Lett. **209B**, 140 (1988).
[40] A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. **66B**, 205 (1977); **76B**, 139 (1978).
[41] O. Civitarese, A. Faessler, and R. Nojarov, Phys. Rev. C **35**, 2310 (1987).
[42] T. Otsuka and J. N. Ginocchio, Phys. Rev. Lett. **54**, 777 (1985).
[43] O. Scholten, K. Heyde, P. Van Isacker, and T. Otsuka, Phys. Rev. C **32**, 1729 (1985).
[44] A. Novoselsky, Nucl. Phys. **A483**, 282 (1988).
[45] P. Sala, P. von Brentano, H. Harter *et al.*, Nucl. Phys. **A456**, 269 (1986).
[46] R. F. Casten, P. von Brentano, A. Gelberg, and H. Harter, J. Phys. G **12**, 711 (1986).
[47] R. F. Casten, D. S. Brenner, and P. E. Haustein, Phys. Rev. Lett. **58**, 658 (1987).
[48] P. Van Isacker, K. Heyde, M. Waroquier, and G. Wenes, Nucl. Phys. **A380**, 383 (1982).
[49] C. E. Alonso, J. M. Arias, and M. Lozano, Phys. Lett. **177B**, 130 (1986).
[50] D. S. Chuu and S. T. Hsieh, Phys. Rev. C **3**, 960 (1988).
[51] A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic Nuclei*, transl. of 1st Russ. ed. (Interscience, New York, 1967) [Russ. original, 2nd ed., Nauka, Moscow, 1983].
[52] P. F. Bortignon, R. A. Broglia, D. R. Bes, and R. Liotta, Phys. Rep. **30C**, 306 (1977).
[53] V. G. Soloviev, *Nuclear Structure Dubna Symposium* (IAEA, Vienna, 1968), p. 101; *Neutron Capture Gamma-Ray Spectroscopy* (Reactor

Centrum Nederland, Petten, 1977), p. 99.
[54] V. G. Solov'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. **35**, 666 (1971).
[55] V. G. Solov'ev, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 398 (1984) [JETP Lett. **40**, 1216 (1984)].
[56] V. G. Soloviev, Z. Phys. A **324**, 393 (1986).
[57] V. G. Solov'ev, Yad. Fiz. **47**, 332 (1988) [Sov. J. Nucl. Phys. **47**, 209 (1988)].
[58] V. G. Soloviev, Inst. Phys. Conf. Ser. N88/J. Phys. G Suppl. **14**, S39 (1989); *Modern Developments in Nuclear Physics* (World Scientific, Singapore, 1988), p. 315.
[59] V. O. Nesterenko, V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Yad. Fiz. **44**, 1443 (1986) [Sov. J. Nucl. Phys. **44**, 938 (1986)].
[60] D. G. Burke, W. F. Davidson, W. F. Cizewski *et al.*, Nucl. Phys. **A445**, 70 (1985).
[61] V. S. Shirley, Nucl. Data Sheets **53**, 223 (1988).
[62] S. Kuyucak and I. Morrison, in: *Cocoyc '88* (World Scientific, Singapore, 1988), p. 169.
[63] I. M. Govil, H. W. Fulbright, D. Cline *et al.*, Phys. Rev. C **33**, 793 (1986).
[64] V. G. Solov'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. **53**, 2066 (1989).
[65] V. G. Solov'ev and N. Yu. Shirikova, Izv. Akad. Nauk SSSR, Ser. Fiz. **54**, 818 (1990).
[66] V. G. Soloviev and A. V. Sushkov, J. Phys. G **16**, L57 (1990).
[67] E. Browne, Nucl. Data Sheets **54**, 199 (1988).
[68] R. M. Ronningen, J. H. Hamilton, and A. V. Ramayya, Phys. Rev. C **15**, 1671 (1977).
[69] A. M. Hague, R. F. Casten, I. Forster *et al.*, Nucl. Phys. **A455**, 231 (1986).
[70] C. J. Gallagher and H. L. Nielsen, Phys. Rev. **126**, 1520 (1962).
[71] W. G. Nettles, A. V. Ramayya, J. H. Hamilton *et al.*, J. Phys. G **14**, L223 (1988).
[72] P. G. Sood and R. K. Sheline, Mod. Phys. Lett. **A4**, 1711 (1989).
[73] T. L. Khoo, J. C. Waddington, R. A. O'Neil *et al.*, Phys. Rev. Lett. **28**, 1717 (1972).
[74] T. Hammer, H. Ejiri, and G. B. Hagemann, Nucl. Phys. **A202**, 321 (1973).
[75] N. Kaffrell and W. Kurcewicz, Nucl. Phys. **A255**, 339 (1975).
[76] D. G. Burke, B. L. Maddock, and W. F. Davidson, Nucl. Phys. **A442**, 424 (1985).
[77] R. C. Greenwood, C. K. Reich, H. A. Baader *et al.*, Nucl. Phys. **A304**, 327 (1978).
[78] B. S. Dzhelepov, V. E. Ter Nersesyants, and S. A. Shestopalova, *Decay Schemes of the Radioactive Nuclei A-169, 170* [in Russian] (Nauka, Leningrad, 1988).
[79] Zhou Chunmei, Nucl. Data Sheets **50**, 351 (1987).
[80] V. G. Soloviev, Rev. Roum. Phys. **33**, 347 (1988).
[81] R. V. Jolos, G. P. Ivanova, R. Pedrosa, and V. G. Solov'ev, Teor. Mat. Fiz. **70**, 154 (1987).
[82] F. J. Scholtz, Phys. Lett. **151B**, 87 (1985); F. G. Scholtz and F. T. Hahne, Nucl. Phys. **A471**, 545 (1987).
[83] G. Maino, A. Ventura, L. Zuffi, and F. Iachello, Phys. Rev. C **30**, 2101 (1984); Phys. Lett. **152B**, 17 (1985).
[84] G. Maino, A. Ventura, P. Van Isacker, and L. Zuffi, Europhys. Lett. **2**, 345 (1986).
[85] V. G. Soloviev, Fizika **22**, 78 (1990).

Translated by Julian B. Barbour