Anomalous magnetic moments of charged leptons and problems of elementary-particle physics

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Problems in the theory of the anomalous magnetic moments (AMMs) of charged leptons and experiments in which they are measured are reviewed. The electrodynamic contributions to the AMMs, and also the contributions of the strong and weak interactions are analyzed. The dependence of the AMM on the particle energy and on the external conditions (the presence of electromagnetic fields, conducting surfaces) is discussed. The corrections to the AMMs that arise in various alternative theories that generalize the standard Glashow–Weinberg–Salam interaction model are calculated. The precession and resonance experiments made at CERN and at the University of Washington, in which the most accurate values for the muon and electron AMMs have been obtained, are described. The experiments being prepared at the Brookhaven National Laboratory in the United States with the aim of increasing the accuracy in the measurement of the muon AMM are also considered. The most recent theoretical and experimental values for the electron and muon AMMs are given in the review and compared. Because of the high accuracy of their determination, they impose very stringent conditions on the parameters of the various theories used to describe particle interactions. Bounds on the parameters of a number of alternative models are obtained. It is also shown how the currently best value for the fine-structure constant can be obtained from the results of the theoretical and experimental investigations of the lepton AMMs. In addition, the fundamental symmetry principle of matter and antimatter—the *CPT* theorem—can be verified to a high accuracy, and the scale of the possible deviation of the space–time dimension from 4 can be estimated.

INTRODUCTION

The development of elementary-particle physics during the last few decades has shown that further progress in this field is intimately related to the possibility of performing experiments at ever higher energies. However, despite the fact that new elementary-particle accelerators are being constructed[1] [for example, the UNK (Large Accelerator and Storage Facility) at IHEP at Serpukhov and LEP at CERN in Switzerland][1)] and projects for the creation of even more powerful accelerators in the middle of the nineties are being actively pursued [for example, the Superconducting Proton Supercollider SSC at Fermilab in the United States, and the colliding linear electron–positron beams (VLÉPP) of the Institute of Nuclear Physics of the Siberian Branch of the USSR Academy of Sciences], it is already clear that the traditional methods of accelerating particles will not make it possible to maintain the previous dynamics of the advance into the region of high energies. It has therefore become a matter of paramount importance to obtain in a fundamentally new way the information needed to develop ideas about the microscopic world. A basis for this is comparison of theoretical predictions and results of relatively low-energy experiments for various properties of elementary particles and processes in which they participate. Very important properties of this kind are the anomalous magnetic moments of the charged leptons (electron and muon).[2)]

The study of the anomalous magnetic moments (AMMs) of charged leptons combines different fields of modern physics: theoretical and experimental physics, solid-state and elementary-particle physics, high- and low-energy physics, and the physics of hyperfine resonance and precession measurements. Simultaneously, the result and cause of this unification is the fact that the electron and muon AMMs are currently calculated theoretically and measured with colossal accuracy.

Comparison of the experimental and theoretical values for the electron and muon AMMs plays an important part in quantum metrology in the determination of the best values for the fundamental physical constants. At the last adjustment of the constants,[5] the value of the electron g_e factor [$g_e = 2(1 + a_e)$, where a_e is the electron AMM] was included in the basic data from which the value of the fine-structure constant α can be obtained. It also participated in the adjustment as a fixed auxiliary constant. The value of the muon g_μ factor is known theoretically and experimentally with a larger error, making it impossible to determine α with satisfactory accuracy, and therefore this value was regarded as a fixed auxiliary constant in the adjustment.

At the last adjustments, the following values were recommended for the electron and muon AMMs:[5]

$$\left.\begin{aligned} a_e\,(g_e-2)/2 &= 0.001159652193\,(10),\\ a_\mu\,(g_\mu-2)/2 &= 0.0011659230\,(84),\end{aligned}\right\} \qquad (1)$$

the errors being given in the brackets.

The accuracy achieved in the measurement of the electron and muon AMMs is such that the contributions of the electromagnetic interactions proportional to α^4 are now experimentally detected; in addition, in the case of the muon AMM, contributions of the strong interactions are detected, and the increase in the accuracy of measurement of the muon AMM by approximately 20 times planned at the Brookhaven National Laboratory in the United States will make it

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possible to detect contributions of the weak interactions as well. The AMMs of charged leptons depend strongly on the energies of the particles, on the external conditions (in particular, on the strength of an external electromagnetic field), and on the vacuum structure, and as a result they are sensitive to the contributions that arise in the various theories which generalize the standard model of the Glashow–Weinberg–Salam interaction. Significant contributions to the AMMs may also be made by the supersymmetric particles and bosons predicted in grand-unification theories. In addition, comparison of the theoretical and experimental values for the lepton AMMs makes it possible to restrict the characteristic energy scales at which effects associated with the structure of leptons and bosons in theories with composite particles could be manifested. Equality of the electron and positron AMMs to a high accuracy is a good experimental verification of a consequence of the fundamental symmetry property of matter and antimatter—the *CPT* theorem.

Therefore, theoretical and experimental investigations of the AMMs of charged leptons play an important part in verifying the various theories of interactions and in the solution of fundamental problems of elementary-particle physics.

The remaining material of the review is organized as follows. Section 1 contains a general theoretical review of the anomalous magnetic moments of leptons (electron, muon, and τ lepton) and a discussion of the relationship between the contributions of the electromagnetic, weak, and strong interactions; attention is drawn to the existing disagreements in the theoretical calculations, and a comparison with the most recent experimental results is also made. Section 2 describes the principles and specific features of the experiments made to determine the electron and muon AMMs at CERN and the University of Washington, and the experiments being prepared at the Brookhaven National Laboratory with the aim of increasing the accuracy in the determination of the muon AMM are also considered. Section 3 is devoted to basic theoretical problems such as the dependence of the lepton AMMs on an external field and on the energy of the particles, and the contributions to the AMMs of the weak axial-vector Z and W bosons and scalar Higgs bosons. The contributions to the AMM of a lepton moving in an external field and of bosons of various types are calculated by a model-independent method, and the contributions that arise from supersymmetric particles and from allowance for the structure of leptons and bosons in theories with composite particles are also discussed. In conclusion, it is shown that comparison of the theoretical and experimental values for the electron AMM makes possible the currently most accurate determination of the fine-structure constant α. In addition, comparison of the electron and positron AMMs yields an experimental bound on the possible violation of the fundamental *CPT* theorem. In connection with the existence of a discrepancy between the theoretical and experimental values for the electron AMM, restrictions on the possible deviation of the space–time dimension from 4 are also discussed.

1. ANOMALOUS MAGNETIC MOMENTS OF CHARGED LEPTONS

The anomalous magnetic moment of a charged lepton characterizes the deviation of the so-called g_l factor of the particle, which establishes the connection between the magnetic dipole moment of the particle and its spin angular momentum, from the value $g_0 = 2$. In classical theory (see Ref. 6) a magnetic dipole moment arises in the presence of currents; for example, in the case of circular motion of a particle with charge e and mass m the resulting dipole magnetic moment is related to the orbital angular momentum by $\boldsymbol{\mu}_L = (e/2mc)\mathbf{L}$, and the g factor in this case is $g = g_{\rm cl} = 1$. In contrast to this classical expression, in the analogous relationship between the magnetic dipole moment and the spin for a point Dirac particle,

$$\boldsymbol{\mu}_m = g_l \frac{e_l}{2m_l c}\mathbf{S} \tag{2}$$

($\boldsymbol{\mu}_m$ is the operator of the magnetic dipole moment, $\mathbf{S} = \hbar\boldsymbol{\sigma}/2$ is the spin operator, e_l and m_l are the charge and mass of the lepton, and $\boldsymbol{\sigma}$ are the Pauli matrices), the g_l factor is

$$g_l = g_0 = 2. \tag{3}$$

This value of the g_l factor is obtained because the particle wave function satisfies the Dirac equation and the interaction with the external electromagnetic field is introduced as a minimal coupling—by extending the derivative: $\partial_\mu \to \partial_\mu + ieA_\mu$. This nontrivial result of Dirac's theory can also be obtained in the framework of a nonrelativistic approach on the basis of Pauli's equation.

We note that the g factor has the value $g_0 = 2$ as a reflection of the fundamental "purity" of the particle with respect to the electromagnetic interactions. For example, if the particle participates in several other interactions that lead to the occurrence of internal structure, this deviation from point structure is reflected in the g factor. Indeed, because of the presence of internal structure the neutron and proton g factors are, respectively, $g_n = -3.38$ and $g_p = 5.59$.

However, even in the absence of internal structure the quantum nature of the electromagnetic interaction leads to a deviation of the g factor from the Dirac value $g_0 = 2$, and therefore the gyromagnetic ratio for a charged lepton can be represented in the form

$$g_l = 2\,(1 + a_l), \tag{4}$$

where a_l is equal to the so-called anomalous magnetic moment $\Delta\mu_l$, divided by the Bohr magneton μ_l^0:

$$a_l = \frac{\Delta\mu_l}{\mu_l^0}, \quad \mu_l^0 = \frac{e_l\hbar}{2m_l c}. \tag{5}$$

It should be noted that if the interaction with the electromagnetic field is not introduced "minimally" and the equation with interaction is taken to be

$$\left[(i\hat{\partial} - e\hat{A}) - m + \frac{\Delta g}{2}\,\frac{e}{4m}\,\sigma^{\mu\nu}F_{\mu\nu}\right]\Psi = 0,$$

$$\sigma^{\mu\nu} = \frac{i}{2}\,(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

($F_{\mu\nu}$ is the tensor of the electromagnetic field), the g factor will also differ from 2: $g = 2 + \Delta g$.

In the framework of quantum electrodynamics it was first shown in Ref. 7 that the gyromagnetic ratio for the electron differs from $g_0 = 2$, the reason being the interaction of the electron with the vacuum of the photons. The calculations of the part of the energy of the vacuum interaction of the electron which arises in the presence of an external magnetic field, made in the framework of perturbation theory up

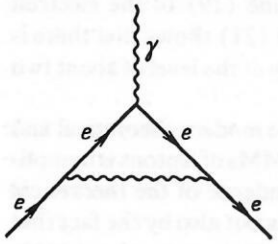

FIG. 1. Feynman diagram that describes the contribution to the electron AMM in the lowest order of perturbation theory in QED.

to the terms of first order in the fine-structure constant $\alpha = e^2/c\hbar$ (the corresponding Feynman diagram is shown in Fig. 1), lead to the Schwinger value for the electron AMM:

$$a_{\rm Sch} = \alpha/2\pi. \tag{6}$$

This result does not depend on the electron mass, and therefore remains valid for the case of any charged particle with spin $\frac{1}{2}$ and, in particular, for other charged leptons—the muon and τ lepton. However, beginning with the corrections of second order in α, the contributions to the AMMs (and g factors) of the different leptons will be different, since some of the corresponding Feynman diagrams (Fig. 2) contain a polarization operator, and this contains virtual pairs of leptons of all types. Thus, in QED one can obtain an expression in the form of a series in the fine-structure constant for the g_l factor of a charged lepton with allowance for the contributions of the higher orders of perturbation theory:

$$g_l = 2\left[1 + \sum_n \left(\frac{\alpha}{\pi}\right)^n A_n^{(l)} + \sum_n \left(\frac{\alpha}{\pi}\right)^n B_n^{(l)}\right],$$

and, accordingly, for the anomalous magnetic moment

$$a_l = \frac{g_l - 2}{2} = \sum_n \left(\frac{\alpha}{\pi}\right)^n A_n^{(l)} + \sum_n \left(\frac{\alpha}{\pi}\right)^n B_n^{(l)}. \tag{7}$$

The coefficients $A_n^{(l)}$ and $B_n^{(l)}$ differ in that $A_n^{(l)}$ does not depend on the lepton mass, while $B_n^{(l)}$ are functions of the ratio of the mass of the external lepton to the mass of the lepton in the vacuum polarization loops. Therefore, the first group of terms is the same for all charged leptons.

The contributions to the electron and muon AMMs are now calculated in the framework of QED with allowance for the terms proportional to α^4. Such calculations have been made necessary by the colossal accuracy of the experimental determination of the electron and muon AMMs. Below, we shall give the latest experimental values of $a_e^{\rm exp}$ and $a_\mu^{\rm exp}$, but we first discuss the theoretical values of the AMM for the electron ($a_e^{\rm th}$) and muon ($A_\mu^{\rm th}$).

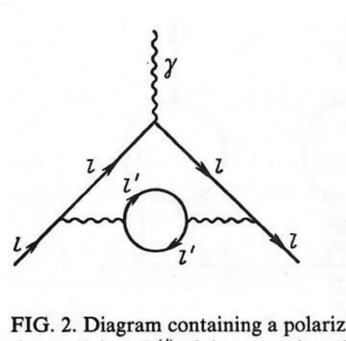

FIG. 2. Diagram containing a polarization operator and contributing to the coefficient $B_2^{(l)}$ of the expansion (7).

Electron anomalous magnetic moment

The volume of work that must be done can be gauged from the fact that in the fourth order in α there is a total of 891 Feynman diagrams for the electron AMM. Because of the extreme complexity, only a small fraction of these diagrams is amenable to exact analytic calculation. Therefore, from 1977 Kinoshita and collaborators[8] have considered the fourth-order contributions to the electron AMM on the basis of numerical computer calculations, and this work is only now approaching its end.[9]

By the middle of 1988, exact analytic calculations (Refs. 7, 10, and 11) had given for the coefficients C in the expansion

$$a^{\rm QED} = \frac{\alpha}{\pi} C_1 + \left(\frac{\alpha}{\pi}\right)^2 C_2 + \left(\frac{\alpha}{\pi}\right)^3 C_3 + \left(\frac{\alpha}{\pi}\right)^4 C_4 + \dots \tag{8}$$

the values

$$C_1 = 0.5, \quad C_2 = -0.328478965\dots, \tag{9}$$

and numerical calculations gave[12] (the exact analytic calculation of the coefficient C_3 has not yet been completed)[9]

$$C_3 = 1.1765\ (13), \quad C_4 = -0.8\ (1.4). \tag{10}$$

A detailed bibliography of studies devoted to the calculation and more accurate determination of the electromagnetic contributions to the electron AMM is given in the review of Ref. 13.

To determine the numerical value of the electron AMM with allowance for the fourth order in α, it is necessary to combine the results of theoretical calculations of the coefficients C_i and a value for the fine-structure constant α obtained on the basis of experiments that are not associated with measurement of the $(g-2)_e$ factor. Moreover, the result for the AMM depends strongly on which value of α is used. At the present time, the most accurate values of the fine-structure constant [without use of the $(g-2)_e$ factor] are obtained from measurement of the quantum Hall effect, on the basis of the Josephson effect, and from the hyperfine splitting of muonium (see Refs. 14 and 15 and the literature quoted there).

If for α we take the value that follows from the Josephson effect,

$$\alpha_J^{-1} = 137.035963\ (15)\ \text{(Ref. 16)}, \tag{11}$$

then the summation of the four terms of the expansion (8) leads to a contribution to the electron AMM with the value[12]

$$a_e^{\rm QED}(\alpha^4) = 1159652455\ (43)\ (128)\cdot 10^{-12}, \tag{12}$$

where the first number in brackets is the error associated with the calculations, and the second is due to the uncertainty in the value (11) of α.

When the theoretical and experimental values of the electron (and muon) AMM are compared, it is also necessary, because of the high accuracy of measurement that is achieved to take into account and estimate the contributions of various vacuum polarization effects (muon and τ-lepton loops), which are equal to[12]

$$a_\mu^{\rm QED} = 2.8\cdot 10^{-12}, \quad a_\tau^{\rm QED} = 0.01\cdot 10^{-12}. \tag{13}$$

In addition, it is also necessary to take into account contributions of nonelectromagnetic origin (hadron loops and weak nteractions). Bearing in mind that the corrections to the lepton AMM that arise from the strong (a_l^{had}) and weak (a_l^{w}) interactions play a more important part in the case of the muon AMM than in the case of the electron AMM,[3] we shall dwell in more detail on the calculations of a_l^{had} and a_l^{w} when we consider the muon AMM, and here we shall merely give numerical estimates:[12]

$$a_e^{had} = 1.6\,(2)\cdot 10^{-12}, \quad a_e^{w} = 0.05\cdot 10^{-12}. \tag{14}$$

Combining (12)–(14), we obtain for the theoretical value of the electron AMM

$$a_e^{th} = 1159652459\,(43)\,(128)\cdot 10^{-12}. \tag{15}$$

But if for the fine structure constant we use the value obtained from the measurements of the quantum Hall effect made up to 1986 (Ref. 5), $a_H^{-1} = 137.035\,994\,3(127)$, we arrive at a somewhat smaller value of the electron AMM:[9]

$$a_e^{th} = 1159652192\,(74)\,(108)\cdot 10^{-12}. \tag{16}$$

Recently, Kinoshita[9] gave new revised (though, as it turns out, not yet final) values of the coefficients C_3 and C_4,

$$C_3' = 1.17562\,(56), \quad C_4' = -1.472\,(152), \tag{17}$$

which correspond to a new still smaller value

$$a_e^{th} = 1159652164\,(108)\cdot 10^{-12}. \tag{18}$$

If in addition we use the latest and most accurate results for the fine-structure constant α that follow from the quantum Hall effect, $\alpha_H^{-1} = 137.035\,997\,9(33)$ (Ref. 17), then the theoretical value (using the coefficients C_3' and C_4') will be[9]

$$a_e^{th} = 1159652133\,(29)\cdot 10^{-12}, \tag{19}$$

in which the main part of the error is due to the uncertainty in the value of α.

The main stimulus to this truly titanic work to achieve a high accuracy in the theoretical value of the electron and muon AMMs is that the error in the result should be comparable with the errors in the experimental determination of a_l. Up to 1974, the theoretical value of the $(g-2)_e$ factor was determined with less accuracy than the experimental value, but in the period from 1974 (after the publication of Ref. 18) to 1981 the relation between the accuracies was reversed. The publication in 1981 of the result of the experimental determination of the $g-2$ factor of electrons and positrons confined in a Penning trap,[19]

$$a_{e^-}^{exp} = 1159652200\,(40)\cdot 10^{-12},$$
$$a_{e^+}^{exp} = 1159652222\,(50)\cdot 10^{-12},$$

again established the supremacy (in accuracy) of experiment over theory, and it still persists to this day. In Ref. 20, Dehmelt's group gives an even more accurate value (error reduced by about 10 times):

$$a_{e^-}^{exp} = 1159652193\,(4)\cdot 10^{-12}. \tag{20}$$

Finally, the most recent experimental results were published in Ref. 21:

$$a_{e^-}^{exp} = 1159652188.4(4.3)\cdot 10^{-12},$$
$$a_{e^+}^{exp} = 1159652187.9\cdot(4,3)\cdot 10^{-12}. \tag{21}$$

Comparison of the theoretical value (19) of the electron AMM with the experimental value (21) shows that there is still (see also Ref. 15) a discrepancy at the level of about two standard deviations.

The problem of reconciling the modern theoretical and experimental predictions for the AMMs of leptons is complicated not only by the strong dependence of the theoretical value a_l^{th} on the employed value of α but also by the fact that in electrodynamics itself the contributions to the AMMs proportional to α^3 and α^4 cannot be calculated exactly in analytic form. Because of the complexity of the theoretical expressions, a large proportion of the calculations must be made on a computer using different methods of approximate calculation of multidimensional integrals, which sometimes give seriously differing results. Thus, in 1968 Samuel[22] calculated the contribution to the electron AMM proportional to α^3 from diagrams of "light–light scattering" (Fig. 3). He used an approximate method that had made it possible earlier,[23] on the basis of a numerical calculation, to predict the corresponding exact analytic result for the contribution to the muon AMM from diagrams of light–light scattering. The value of the electron AMM that follows from the calculations of Ref. 22, with allowance for all the electromagnetic, hadronic, and weak contributions [for α the value (11) is used],

$$\tilde{a}_e^{th} = 1159652797\,(76)\,(128)\cdot 10^{-12}, \tag{22}$$

is appreciably larger than the theoretical values (15), (16), (18), and (19). Comparing (22) with the most recent experimental value (21), we find[24] a discrepancy at the level of four standard deviations:

$$\tilde{a}_e^{th} - a_e^{exp} = 609\,(149)\cdot 10^{-12}. \tag{23}$$

The existence of such an appreciable difference between the theoretical and experimental values of the electron AMM opens up a large space for the discussion of the contributions that arise outside the framework of electrodynamics and the standard interaction model.

Samuel's paper[22] stimulated commentaries[25] that questioned the theoretical result, but in answer[26] Samuel justified his confidence in the correctness of the employed approximate method of calculation.

The discussion about the theoretical value of the electron AMM shows that it is desirable to make exact analytic calculations (despite all their complexity and cumbersome nature) of the various contributions to the lepton AMMs; in all probability, it will be only such calculations that can resolve the existing contradiction.

Completing our discussion of the value a_e^{th} of the elec-

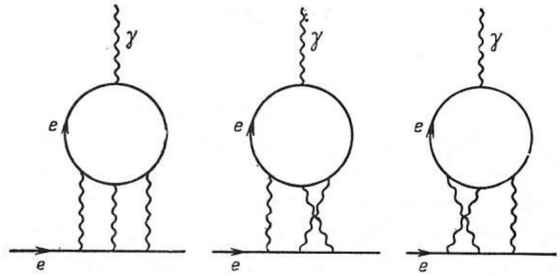

FIG. 3. Diagrams of light–light scattering that contribute to the electron AMM in the third order in α.

tron AMM, we draw attention to Ref. 27, in which a numerical method of accelerated convergence of the electrodynamic series of perturbation theory is used, on the basis of the known coefficients C_1, C_2, and C_3 in the expansion (8), to determine the values of the following coefficients C_4 and C_5. Although the calculated value $C_4 = -5.65(0.64)$ differs from the results of (10) and (17), a final conclusion about the adequacy of this approach can be drawn only after completion of the work being done by Kinoshita and collaborators to determine the coefficient C_4 more accurately. In addition, so far as we know, a numerical value was obtained for the first time in Ref. 27 for the following ($\sim\alpha^5$) term in the expansion (8): $C_5 = 31(6)$. We note that it may become necessary to take into account the contribution $C_5(\alpha/\pi)^5$ in the electron AMM if the accuracy of the experimental determination becomes of order 10^{-13}, and that the application to the calculation of C_5 of the standard methods used to find C_3 and C_4 requires a great deal of computing time.

Muon anomalous magnetic moment

We have already mentioned that the value of the muon AMM is more sensitive to various vacuum polarization effects than the electron AMM. At the accuracy achieved in the experimental determination of a_e [see Eq. (21)], the contributions proportional to $B_n^{(e)}$ (7) are not important, but in the case of a_μ they play a significant part. The basic possibility of obtaining an exact analytic expression for the muon AMM in the framework of QED for any order in α cannot, as in the case of the electron, be realized because of the rapid growth in the number of Feynman diagrams that contribute in the higher orders of perturbation theory. The presence of the small parameter $m_e/m_\mu \ll 1$ (m_e and m_μ are the electron and muon masses) makes it possible in the calculations of the coefficients B_n to reduce the number of diagrams, but even in the limit $m_e/m_\mu \to 0$ there remain 469 terms in $B_4^{(\mu)}$. A detailed classification of the QED diagrams and a calculation of their contributions to a_μ can be found in Ref. 28.

In Ref. 29, Kinoshita and collaborators obtained

$$a_\mu^{\text{QED}} = 0.5\left(\frac{\alpha}{\pi}\right) + 0.76585810(10)\left(\frac{\alpha}{\pi}\right)^2 + 24.073(11)\left(\frac{\alpha}{\pi}\right)^3 + 140(6)\left(\frac{\alpha}{\pi}\right)^4, \qquad (24)$$

and, if the value (11) for α is used, this gives

$$a_\mu^{\text{QED}} = 11658480\,(3)\cdot 10^{-10}. \qquad (25)$$

Work to improve the accuracy in the calculation of the coefficients $C_i^{(\mu)}$ is continuing, and the most recent result known to us[30] (see also Ref. 31) is $C_2^{(\mu)} = 0.765\,857\,577$, $C_3^{(\mu)} = 24.0725(123)$, $C_4^{(\mu)} = 137.96(250)$ and leads to a decrease of a_μ^{QED} by about $0.67\cdot 10^{-10}$.

The contradiction between the results of Refs. 22 and 9 that we noted above in the estimate of the contributions to the electron AMM made by the light–light scattering diagrams also appears in the case of the muon AMM. Although as yet the difference between the contributions given in Refs. 22, 23, and 29 is not amenable to experimental observation, the discrepancy between the two results could become important with further increase in the accuracy of measurement of the muon AMM.[4)]

Before we compare the theoretical and experimental values of the muon AMM, we must take into account the contributions of the strong and weak interactions (a_μ^{had} and a_μ^{w}). The calculations of a_μ^{had} take into account the circumstance that the lepton is coupled to hadrons through a virtual photon (see, for example, the Feynman diagram in Fig. 4, which describes the contribution a_l^{had} in the lowest order of perturbation theory in α), and therefore it is possible to establish a correspondence between this virtual process and the cross section $\sigma(e^+e^-\to\text{hadrons})$ for annihilation of an electron–positron pair into hadronic states. Using the dispersion technique for calculation[33,34] and the assumption of dominance and annihilation of the single-photon process, we can determine the hadronic contribution as[35,36]

$$a_\mu^{\text{had}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\,\sigma(e^+e^-\to\text{hadrons})\,(s)K(s),$$

where s is the square of the total energy of the e^+e^- pair in the rest frame, and the function $K(s)$, which is a combination of two lepton propagators and the propagator of a virtual photon with mass $\sqrt{s}$, can be calculated in the framework of electrodynamics:

$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)(s/m_\mu^2)}. \qquad (26)$$

In the region of integration of the expression (26), $K(s)$ is positive and has a limiting value as $s/m_\mu^2 \to \infty$ equal to $\frac{1}{3}(m_\mu^2/s)$. It follows from this in particular that the hadronic contribution to the electron AMM will be about 10^5 times smaller than the corresponding contribution to the muon AMM.

Since the cross section $\sigma(e^+e^-\to\text{hadrons})$ cannot be obtained by an exact analytic calculation in quantum chromodynamics, a_μ^{had} is determined by means of the cross section $\sigma(e^+e^-\to\text{hadrons})$ measured in experiments with e^+e^- colliders. Thus, the accuracy in the determination of a_μ^{had} depends not only on the quality of the calculations that are made, but also on the accuracy of the measurement of $\sigma(e^+e^-\to\text{hadrons})$. The tempo at which the accuracy of a_μ^{had} is increased can be deduced from Refs. 28 and 29. We note that the main contribution to a_μ^{had} is made by the region of energies $\sqrt{s} \lesssim 1$ GeV, which is associated with the ρ and ω resonances.

The following value for a_μ^{had} was obtained in Ref. 29 on the basis of a detailed study:

$$a_\mu^{\text{had}} = 702\,(19)\cdot 10^{-10}. \qquad (27)$$

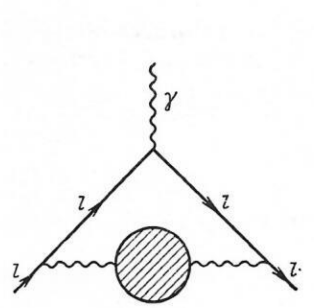

FIG. 4. Hadronic contribution to the lepton AMM (lowest order of perturbation theory in α).

In calculating the hadronic contribution, the Novosibirsk group[37] used information on the pion form factors and obtained the more accurate value $a_\mu^{\rm had} = 684(11)\cdot 10^{-10}$, in which the error is basically due to the errors in the experimental data used in the region $\sqrt{s} > 2$ GeV. There are indications (Refs. 32 and 38) that in the near future it will be possible to reduce still further the error in the determination of $a_\mu^{\rm had}$. This should be achieved using the detector KMD-2, which will make it possible to measure all cross sections in the region of energies $\sqrt{s} \sim 1.5$–2 GeV with an error $\lesssim 1\%$ and, in its turn, to lower by 2–3 times the error in the determination of $a_\mu^{\rm had}$.[5)]

The creation of the renormalizable gauge theories with spontaneous symmetry breaking for the description of the weak interactions made it possible in principle to calculate the corresponding contributions to the lepton AMMs with any accuracy. The actual idea of determining the contributions of the weak processes to the AMM of a charged lepton was first realized in the studies of Refs. 39 and 40, but the absence of a consistent method for eliminating the ultraviolet divergences in the higher orders of perturbation theory made it impossible to obtain reliable results. In some studies[41–46] calculations were made, in the second order of perturbation theory, of the weak contributions $a_l^{\rm w}$ to the lepton AMM, and results that agreed with each other were obtained.

The diagram shown in Fig. 5 contributes to the AMM of a charged lepton in the second order of perturbation theory in an arbitrary gauge theory.[47] It corresponds to the following expression for the current:

$$\bar{u}(p')\, e\Gamma_\mu(p',\ p)\, u(p)$$

$$= \bar{u}(p')\left[e\gamma_\mu F_1(q^2) + \frac{ie\sigma_{\mu\nu}q^\nu}{2m_l} F_2(q^2)\right] u(p).$$

The magnetic form factor, taken at zero value of q^2, determines the contribution to the lepton AMM: $a_l = F_2(0)$. Moreover, because of the renormalizability of the gauge theory with spontaneous symmetry breaking, a_l will be finite. We note that the contributions that arise in different models depend strongly on the parameters of the theory, and it is only in the case of the standard Weinberg–Salam model of the electroweak interactions, for which the parameters are well determined experimentally, that a sufficiently accurate numerical estimate of $a_l^{\rm w}$ can be obtained.

In the Weinberg–Salam theory, processes in which virtual axial-vector Z and W bosons and scalar Higgs bosons participate contribute to the muon AMM. For the Z and W contributions, one can obtain[41]

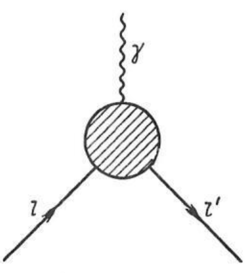

FIG. 5. Contribution to the lepton AMM in the second order of perturbation theory for an arbitrary gauge model.

$$a_\mu^Z = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\,\frac{1}{3}\,[(3-4\cos^2\theta_W)^2-5],\quad a_\mu^W = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\,\frac{10}{3}, \tag{28}$$

where G_F is the Fermi constant, and θ_W is the Weinberg angle. The contribution of the Higgs boson H will, if its mass is much greater than the muon mass ($M_H \gg m_\mu$), be appreciably smaller than a_μ^Z and a_μ^W (Refs. 41, 48, and 49). The modern numerical estimate of the contribution of the weak interactions on the basis of (28) gives

$$a_\mu^{\rm w} = 19.5\ (0.1)\cdot 10^{-10}. \tag{29}$$

Thus, summing (25), (27), and (29), and taking into account the contribution of the τ-lepton polarization loop, $a_\mu(\tau) = 4.2\cdot 10^{-10}$, we arrive at the theoretical value for the muon AMM:[29]

$$a_\mu^{\rm th} = 11659202\ (20)\cdot 10^{-10}. \tag{30}$$

But if for the hadronic contribution we use the result of Ref. 37, then

$$a_\mu^{\rm th} = 11659184\ (12)\cdot 10^{-10}. \tag{31}$$

The theoretical values (30) and (31) agree with the data of the experiments to measure the $g-2$ factors of μ^+ and μ^- carried out at CERN:[50]

$$a_{\mu^-}^{\rm exp} = 11659370\ (120)\cdot 10^{-10},$$
$$a_{\mu^+}^{\rm exp} = 11659110\ (110)\cdot 10^{-10}. \tag{32}$$

τ-lepton anomalous magnetic moment

Compared with the electron and muon, the problem of the anomalous magnetic moment of the charged lepton of the third generation, the τ lepton, has been much less studied. In part this is the case because the τ lepton was itself discovered comparatively recently.[51] Moreover, the experimental study of the properties of the τ lepton is hindered by its short lifetime: $T \sim 3\cdot 10^{-13}$ sec.[52] However, there has been in the literature a fairly systematic theoretical investigation[53] of the AMM of the heavy lepton. The electrodynamic contribution to the AMM of the heavy lepton l was determined in accordance with the formula

$$a_l = \frac{1}{N}\sum_{l'}[(a_l - a_{l'}) + a_{l'}], \tag{33}$$

where l', denotes a lepton lighter than the lepton l, and N is the total number of such leptons. The value of $a_l - a_{l'}$ in the case $l = \tau$ can be calculated approximately on the basis of the expression for a_μ: $a_\tau = a_\mu(m_\mu \to m_\tau)$. Such an approach gives a good result if $m_{l'} \ll m_l$, and a_μ in its turn, can be calculated analytically from the value of a_e. We note that the contribution to the expression (33) from a lepton l', with mass greater than m_l is negligibly small, since it does not contain the factor $\ln(m_l/m_{l'})$.[28]

We give the numerical values of the coefficients $C_i^{(\tau)}$ in the expansion of $a_\tau^{\rm th}$ in a perturbation series:[54]

$$a_\tau^{\rm th} = C_1^{(\tau)}\frac{\alpha}{\pi} + [C_2^{(\tau)}({\rm QED}) + C_2^{(\tau)}({\rm had})]\left(\frac{\alpha}{\pi}\right)^2$$
$$+ C_2^{(\tau)}({\rm w}) + O\left(\frac{\alpha}{\pi}\right)^3 + \dots$$

The first coefficient determines the Schwinger value of the

AMM and is equal to $C_1^{(\tau)} = 0.5$; for the following contribution, it is possible to obtain $C_2^{(\tau)}(\text{QED})(\alpha/\pi)^2 = 1.11243\cdot 10^{-5}$.

The method for determining the hadronic contribution to the τ-lepton AMM does not differ from that used in the calculations of a_μ^{had}, but there is no accurate result and a rough estimate gives[54]

$$a_\tau^{\text{had}} = C_2^{(\tau)}(\text{had})\cdot\left(\frac{\alpha}{\pi}\right)^2 = 0{,}35\cdot 10^{-5}. \quad (34)$$

This contribution is approximately 50 times larger than a_μ^{had}. The weak contribution a_τ^{w} can be estimated from the value $a_\mu^{\text{w}}(m_\mu \to m_\tau)$:

$$a_\tau^{\text{w}} = 6\cdot 10^{-7}, \quad (35)$$

and this value is larger than a_μ^{w} by a factor 285.

Figure 6 (from Ref. 53) shows the dependence of the hadronic and weak contributions to the AMM of the lepton on its mass m_l; to estimate the contribution of the Higgs boson, the mass value $M_H = 4.5$ GeV was taken. It can be seen that with increasing mass of the lepton the relative importance of the weak contributions increases; in addition, with increasing m_l there is also an increase in the value of the mass M_H at which the contribution of the Higgs boson begins to dominate over the other part $a_l^Z + a_l^W$ of the contribution of the weak interactions.[49]

Experimentally, the value of a_τ has been studied still less than it has theoretically. A direct measurement of the τ-lepton AMM like the experiments to determine $(g-2)_\mu$ at CERN[50] or $(g-2)_e$ at the University of Washington[21] is not planned, this being due to the short lifetime of the particle. Precession experiments, of the CERN type, would, even at the energies that it is planned to achieve at the LEP and SLC accelerators, require magnetic fields of strengths several orders of magnitude greater than the highest yet achieved. However, one can still speak of the existence of certain experimental bounds on a_τ. The value of a_τ can influence the total and differential cross sections of the process of production of a $\tau^+\tau^-$ pair in a collision of e^+e^- beams:

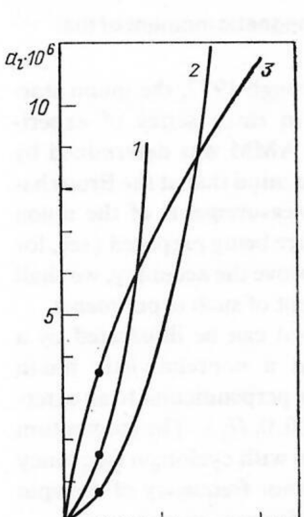

FIG. 6. Dependence of the hadronic, a_l^{had}, and weak, a^{w}, contributions on the lepton mass. Curve 1 corresponds to $a_l^Z + a_l^W + a_l^H$, curve 2 to $a_l^Z + a_l^W$, and curve 3 to a_l^{had}.

$$e^+e^- \to \tau^+\tau^-. \quad (36)$$

Interest in this process arose in connection with composite theories,[55–57] which predict a larger value for a_τ than theories of point particles, and the difference should be manifested in the cross section of the process (36). This circumstance made it possible,[56] on the basis of experimental data from PEP and PETRA, to obtain the estimate $a_\tau^{\exp} < 0.02$, this bound exceeding the theoretical value of a_τ^{w} by about an order of magnitude. A bound on a_τ can also be obtained from the experimental data on the angular distribution of the particles in the reaction (36). It is hoped that such measurements with the accelerators of the next generation will permit a first determination of a_τ.[6)]

2. EXPERIMENTAL DETERMINATION OF THE ANOMALOUS MAGNETIC MOMENTS OF LEPTONS (e AND μ)

The experiments to measure the AMMs of charged leptons (e^-, e^+, μ^-, μ^+) can be divided into two groups: resonance and precession experiments. A common feature of both methods is that the lepton moves in a constant magnetic field; the difference is that when the resonance method is used an alternating electromagnetic field also acts on the lepton, and the AMM is deduced from the results of observation of the induced resonant transitions of the lepton to excited levels. The use of the precession method makes it possible to observe directly the precession of the lepton spin in the magnetic field and, by comparing its frequency with the cyclotron frequency of the lepton in the field, to determine the AMMs.

Measurement of the anomalous magnetic moment of the electron

We cannot discuss here the early experiments on the electron AMM (a review of these studies can be found in Ref. 13), and we merely mention that the first results for the electron g factor were obtained from magnetic resonance measurements with atomic beams in 1947 (Ref. 58).

We shall examine the main ideas and methods of their realization in the experiments that yield the most accurate value for the electron AMM.[59,60] The most recent measurements made by Dehmelt's group[19,21] at the University of Washington used a single electron that was kept in a Penning trap for many days. Such a system had become known as a "geonium," since the electron is confined by a device that is directly coupled to the earth.[59]

In the experiment of Ref. 19, the electron was in a trap with a homogeneous magnetic field of strength $H = 18.3$ kOe, applied along the direction of the OZ axis (Fig. 7).[60]

In the most recent measurements[21] of g_{e^-} and g_{e^+} the strength of the magnetic field was $H = 50$ kOe. Besides the magnetic field, which localizes the motion of the electron (or positron) in the plane perpendicular to the OZ axis, a cylindrically symmetric electric quadrupole field with potential $V(r, z) = (V_0/b^2)(r^2 - 2z^2)$ also acts on the electron. This field is produced by two end electrodes [their form is described by the equation $z^2 = (r^2 + b^2)/2$] and one ring electrode ($r^2 = b^2 + 2z^2$, $b = 0.473$ cm). The nonrelativistic motion of an electron in such a field is determined by the Hamiltonian[13]

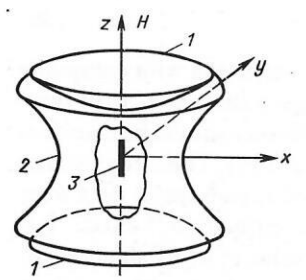

FIG. 7. Arrangement of electrodes in the Penning trap [$(g-2)_e$ experiment of the University of Washington]. The characteristic scale of the trap is 1 cm: 1) negatively charged ring electrodes; 2) positively charged ring electrode; 3) trajectory of an electron (or positron) confined in the trap.

$$H=\frac{\mathbf{p}^2}{2m_e}-eV+\mu_e^0\frac{g}{2}(\sigma\mathbf{H}),\quad \mathbf{P}=-i\hbar\nabla+\frac{e}{c}\mathbf{A},$$
$$\mathbf{A}=\left(-g\frac{H_z}{2},\ x\frac{H_z}{2},\ 0\right).$$

The corresponding energy levels[61]

$$E=\hbar\left[\left(n+\frac{1}{2}\right)\omega_c'+\left(n_z+\frac{1}{2}\right)\omega_z' -\left(n_m+\frac{1}{2}\right)\omega_m+m_s\omega_L\right] \qquad (37)$$

depend on the four quantum numbers n, n_z, $n_m=0,1,2,...$, and $m_s=\pm\frac{1}{2}$. The number n characterizes the cyclotron orbital motion with modified frequency $\omega_c'=eH/(m_ec)-\omega_m$, $\omega_c'/(2\pi)\approx 51$ GHz, the number n_z characterizes the quantum axial oscillations with frequency

$$\omega_z=2\sqrt{eV_0/(mb^2)},\quad \omega_z/2\pi\approx 59\ \text{MHz},$$

and the number n_m corresponds to the still slower magnetron gyrations of the center of the cyclotron orbit with frequency

$$\omega_m=(\omega_0/2)\left(1-\sqrt{1-2\omega_z^2/\omega_0^2}\right),$$
$$\omega_m/2\pi\approx 34\ \text{kHz},\ \omega_0=eH/m_ec.$$

In the limit of a weak confining potential ($V_0\to 0$) we obtain from (37)

$$E=\hbar\omega_0\left(n+\frac{1}{2}+\frac{g_e}{2}m_s\right).$$

The experiments of Refs. 19 and 21 determined not $g_e=2\omega_s/\omega_0$ [$\omega_s=(1+a)eH/m_ec$] but, directly, the much smaller quantity $a_e=(g_{e^-}-2)/2$, which is related to the characteristic frequencies:

$$a_e=\frac{\omega_a}{\omega_0}=\frac{\omega_a'-\omega_m}{\omega_c'+\omega_m},\quad \omega_a'=\omega_L-\omega_c'\ (\omega_L=\omega_s). \qquad (38)$$

The three frequencies ω_c', ω_a', and ω_z were measured, and, in accordance with (38) and with allowance for the relation $2\omega_c'\omega_m=\omega_z^2$, the value of the electron AMM a_e was calculated. The anomalous frequency ω_a' was determined by irradiating the electron confined in the Penning trap by the occurrence of resonance transitions from the lowest ($n=0$) level with spin flip of the electron (see Refs. 13 and 60). The latest most accurate results for the electron and positron AMMs[21] were given above in (21). The main part of the error in (21), $\pm 4\cdot 10^{-12}$, is due to the systematic error which arises from the use of the cavity formed by the electrodes of the Penning trap.

In this case we encounter a clear example of the way in which the increase in the accuracy of experiments poses for the theoreticians new problems associated with allowance for effects that are more and more delicate. In Refs. 62–64, calculations were made of the change of the AMM of an electron between two conducting plates, and it was shown that the modification of the photon propagator due to the presence of the plates (the introduction of which can simulate the effect of the Penning trap) leads to the appearance of appreciable corrections to the electron AMM: $\Delta a_e=(\alpha/2m_ed)[2-\ln(4md)]$, where d is the distance between the plates. And if we set $d=1$ cm (the characteristic scale of the Penning trap in the latest experiments), then $\Delta a_e=-3.3\cdot 10^{-12}$, a value comparable with the contribution of the strong interactions. However, it was pointed out in Refs. 65–67 that the results obtained in Refs. 62–64 should not be used to estimate the shift of the value of the electron AMM measured in the Penning trap. The authors of the latest papers of this series[68,69] reanalyze the assumptions made in Refs. 62–67 and discuss the region of applicability of the results. A general conclusion that may be drawn (see also Refs. 59, 68, and 69) is that what were actually calculated in Refs. 62–64 were the corrections to the AMM of an almost free electron between parallel conducting plates, while an estimate of the correction to the AMM of a bound electron, corresponding to the condition under which the electron was confined by the strong magnetic field of the Penning trap, can be obtained from the results of Refs. 66 and 67: $\Delta a_e\sim 10^{-20}$. On the other hand, the presence of the trap leads, not to a change in the spin-precession frequency ω_s (something that would directly indicate a change of the anomalous magnetic moment itself), but to a shift of the electron cyclotron frequency ω_c, and precisely this may be important in the realized scheme of the experiments.

A detailed discussion of the motion of an electron near conducting surfaces can be found in Ref. 70.

Measurement of the anomalous magnetic moment of the muon

In the period from 1959 through 1977, the muon storage ring at CERN was used in three series of experiments[71,72,50] in which the muon AMM was determined by the precession method. Bearing in mind that at the Brookhaven National Laboratory new measurements of the muon AMM using the CERN method are being prepared (see, for example, Ref. 26) in order to improve the accuracy, we shall consider in more detail the concept of such experiments.

The basic idea of the method can be illustrated by a simple example.[73] Suppose that a nonrelativistic muon moves in a circular orbit in a plane perpendicular to an external constant magnetic field $\mathbf{H}=(0,0,H_z)$. The momentum vector of the electron then rotates with cyclotron frequency $\omega_c=eH/m_\mu c$, whereas the Larmor frequency of the spin precession of the particle has the form

$$\omega_L=g_\mu\frac{e\mathbf{H}}{2m_\mu c}=(1+a_\mu)\frac{e\mathbf{H}}{m_\mu c}.$$

If the g_μ factor is exactly equal to 2, the frequencies are equal, and the muon retains the original polarization of the spin relative to the momentum. But if $g_\mu > 2$, then $\omega_c < \omega_L$ and the spin rotates faster than the momentum vector, and the frequency difference

$$\omega_a = \omega_L - \omega_c = a_\mu \frac{e}{m_\mu c} \mathbf{H} \qquad (39)$$

is proportional to the muon AMM. Therefore, by observing the change in time of the angle between the spin and the direction of the motion, one can determine the muon AMM.

It follows from the relation between the frequencies ω_L and ω_c that during the time of rotation of the spin through an angle 360° the muon can execute about a thousand revolutions in its orbit. Therefore, to obtain a sufficiently accurate result, it is necessary to keep the muon in the storage ring for a time much greater than the lifetime of the muon at rest, i.e., it is necessary to use relativistic muons. It is a remarkable fact that the basic relation (39) nevertheless remains unchanged. Indeed, the cyclotron frequency in the relativistic case has the form

$$\omega_c = \frac{e}{\gamma m_\mu c} \mathbf{H}, \ \gamma = \sqrt{1-\beta^2}, \ \beta = v/c.$$

In addition it is necessary to take into account the relativistic Thomas precession, namely, for circular motion of the particle its rest frame, viewed by an observer at rest, rotates with frequency

$$\omega_T = (1-\gamma^{-1}) \frac{e}{m_\mu c} \mathbf{H}.$$

Then the resulting frequency of the rotation of the spin in the laboratory system will be

$$\omega_s = \omega_L - \omega_T = (a_\mu + \gamma^{-1}) \frac{e}{m_\mu c} \mathbf{H}.$$

Therefore, for the relative frequency of the change of the muon polarization we obtain

$$\omega_a = \omega_s - \omega_c = a_\mu \frac{e}{m_\mu c} \mathbf{H}.$$

Thus, for relativistic motion too the relative spin-precession frequency ω_a is proportional to a_μ.

In accordance with the general idea of the method for determining the muon AMM, it is necessary to measure $\mathbf{s}\cdot\boldsymbol{\beta}$ as a function of the time ($\mathbf{s}$ is the polarization vector, and $\boldsymbol{\beta}$ is the muon velocity). This quantity oscillates with the frequency ω_a. At each cycle of the CERN proton synchrotron, a pion beam was directed into a storage ring with a magnetic field. The pions, revolving in the ring, decayed in flight, and the resulting muons were accumulated in stable orbits during their lifetime. An essential part in the entire scheme is played by parity nonconservation in weak interactions, as a consequence of which the muons have 100% longitudinal polarization in the pion rest frame. The determination of the spin direction of the muon is based on the fact that in the decay $\mu \to e + \bar{\nu} + \nu$ the angular distribution of the electrons has a maximum in the direction of the muon spin. By observing a large number of decays of muons in the ensemble, it is possible to measure the spin direction with any accuracy. Since the variation in time of $\mathbf{s}\cdot\boldsymbol{\beta}$ is similar to pendulum motion, the larger the number of oscillations determined, the greater the accuracy that will be achieved in the determination of the frequency ω_a. The increase in the duration of the observation time is achieved by the relativistic retardation of the flow of time. This explains the tendency to use muons of ever higher energies in the three series of CERN experiments.

We shall discuss the last series of experiments, in which the highest accuracy in the determination of the muon AMM was achieved.[73] One of the main difficulties in realization of this type of experiment is the use of the method of weak focusing—the magnetic field in the ring has a radial gradient, by means of which one can ensure vertical focusing of the muon beam. The strength of the magnetic field varies in the range $\pm 0.1\%$, and this also gives rise to a dependence of the frequency ω_a on the radius. The conditions under which the experiment is performed can be improved if one can eliminate the dependence of ω_a on the radius without loss of the vertical focusing. This can be achieved by introducing in the laboratory system an electromagnetic field that appears as an electric field in the muon rest frame. At the same time, as follows from (39), the relative spin-precession frequency ω_a is determined solely by the magnetic field strength. These two fields can be specified independently by means of an appropriate choice of the magnetic and electric fields in the laboratory system.

The equations of motion of a muon, with allowance for its spin, in the combination of the transverse magnetic and electric fields ($\boldsymbol{\beta}\cdot\mathbf{H} = \boldsymbol{\beta}\cdot\mathbf{E} = 0$) have the form

$$\frac{d\boldsymbol{\beta}}{dt} = \boldsymbol{\omega}'_c \times \boldsymbol{\beta}, \quad \frac{d\boldsymbol{\sigma}}{dt} = \boldsymbol{\omega}'_s \times \boldsymbol{\sigma},$$

and the frequencies $\boldsymbol{\omega}'_c$ and $\boldsymbol{\omega}'_s$ differ from the frequencies $\boldsymbol{\omega}_c$ and $\boldsymbol{\omega}_s$ because of the influence of the electric field:[74]

$$\boldsymbol{\omega}'_c = \frac{e}{m_\mu c} [\mathbf{H}\gamma^{-1} - \gamma(\gamma^2-1)^{-1} \boldsymbol{\beta} \times \mathbf{E}],$$

$$\boldsymbol{\omega}'_s = \frac{e}{m_\mu c} [(a_\mu + \gamma^{-1}) \mathbf{H} - ((\gamma^2-1)^{-1} - a_\mu - \gamma(\gamma^2-1)^{-1}) \boldsymbol{\beta} \times \mathbf{E}].$$

It follows from the last expressions that the relative spin-precession frequency ω_a will depend not only on the strength of the magnetic field but also on that of the electric field. Thus, the confining electric field shifts the frequency by $\Delta\omega_a = \omega'_a - \omega_a$. This shift can be made to vanish by a unique special choice of the muon energy, which is determined by the condition

$$\gamma = \sqrt{1 + a_\mu^{-1}} \approx 29.3, \qquad (40)$$

and it is equivalent to muon energy $p_0 = 3.098$ GeV. In the last series of experiments at CERN, the muons did have this energy, so that the frequency ω_a was entirely determined by the strength of the magnetic field and did not depend on the strength of the electric quadrupole field that ensured vertical focusing of the muon beam.

Figure 8 shows the basic arrangement of the muon storage ring at CERN for the experiments to determine $(g-2)_\mu$. The pions injected into the storage ring have momentum slightly greater than 3.098 GeV/c, for which the ring was designed (the spread in the pion momentum is about $\pm 0.75\%$, the strength of the magnetic field in the last series of experiments was $H = 1.47$ kOe, and the ring radius is $R = 7$ m). Only about a tenth of the pions succeed in decaying into a muon and a neutrino, and the remaining pions leave the ring. The mean "efficiency" of muon production is

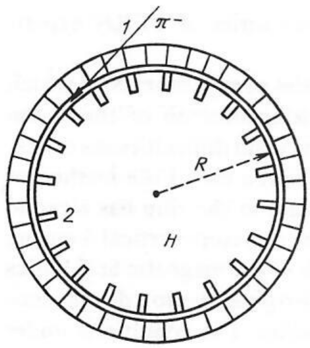

FIG. 8. Schematic representation of the CERN storage ring [$(g-2)_\mu$ experiment]: 1) direction of the beam of injected pions; 2) shower counters of the electrons.

about 10^{-4} per pion. Retaining in the ring muons with momentum in a narrow interval that makes up only about 1.5% of the complete admissible range, it is possible to exploit with high efficiency the property of complete longitudinal polarization of the muons in the pion rest frame. In this way it is possible to obtain muon beams with polarization exceeding 90%.

Along the circumference of the storage ring 22 electron counters detect the direction of maximal emission of the electrons that arise from the decay $\mu \to e + \nu + \bar{\nu}$; this direction coincides with the direction of the mean momentum of the muons. It follows from (39) that to obtain the frequency ω_a from the information extracted from the counters it is also necessary to measure the magnetic field strength H. This was done by the method of proton magnetic resonance. Direct measurements gave R, which is equal to the ratio of the mean frequency ω_a and the effective Larmor frequency of a proton. The value obtained in the experiment was

$$R = \frac{\omega_a}{\omega_L(p)} = 3.707213\,(27)\cdot 10^{-3}.$$

The muon AMM was then calculated in accordance with the formula $a_\mu = R(\lambda - R)^{-1}$ using the known[75,76] value of the ratio of the Larmor frequencies of the muon and proton:

$$\lambda = 3.1833417\,(39).$$

After analysis of the results of the experiments with positive (μ^+) and negative (μ^-) particles, the values of $a_{\mu^+}^{\exp}$ and $a_{\mu^-}^{\exp}$ (32) were obtained, and from them the mean value of the muon AMM was determined as

$$a_\mu^{\exp} = 11659240\,(85)\cdot 10^{-10}. \qquad (41)$$

The relative error is $7.3\cdot 10^{-6}$. A large part of it ($7.0\cdot 10^{-6}$) is associated with the statistical error in the determination of the frequency ω_a. The main part of the systematic error ($1.2\cdot 10^{-6}$) is due to the uncertainty in the determination of the average value of the magnetic field strength over space, time, and the muon distribution. The additional systematic error ($0.3\cdot 10^{-6}$) is associated with the deviation of the muon orbits from the standard orbit.

Planned increase in the accuracy of measurement of the muon AMM

Because improved accuracy in the measurement of the AMMs of charged leptons is fundamentally important, it is planned to carry out a new series of experiments to determine the $g-2$ factor of the muon using the AGS synchroton at the Brookhaven National Laboratory.[26,77] The aim of the new experiment is to reduce the relative error in the determination of a_μ to about $0.3\cdot 10^{-6}$, corresponding to a 20-fold increase in the accuracy over that of the latest experiments at CERN. The method of determining $(g-2)_\mu$ is unchanged—muons will again be confined in a storage ring by means of a magnetic field H and a quadrupole electric field, and the frequency ω_a will be measured by means of the asymmetry of the electron emission. It is planned to reduce the main part of the statistical error in the value of the muon AMM by about an order of magnitude by raising the intensity of the primary proton beam to about 100 times that of the CERN experiment. On the basis of the latest advances in technology, it is also planned to use superconducting magnets, giving a confining magnetic field of strength $H = 5$ kOe. This value is approximately three times higher than the field strength in the CERN experiments. The use of this stronger magnetic field will make it possible to reduce by three times the statistical error in the determination of the frequency ω_a for the same number of detected electrons.

To achieve the aim, it will, of course, also be necessary to reduce the systematic error to about the new value of the statistical error, i.e., it will be necessary to reduce the systematic error to one fifth of the error in the CERN experiments. This requirement can be met by using the existing method (see Ref. 77), so that the average of the magnetic field strength over the muon trajectories can be determined with an error down to $(0.1\text{–}0.2)\cdot 10^{-6}$. The other part of the systematic error, associated with the deviation of the muons from the ideal standard trajectory, will be reduced by decreasing the size of the muon beam. The basic arrangement of the new experiment at Brookhaven is shown in Fig. 9. Note that, because of the use of the stronger magnetic field (at unchanged energy of the primary muons), the radius of the muon orbit is reduced to $R_0 = 2.1$ m.

3. BASIC THEORETICAL QUESTIONS

The entire philosophy in the planning of the experiments to measure the electron and muon $g-2$ factors and the theoretical constructions corresponding to the experiments are based on the assumption that the object of investigation is a free particle at rest. Similarly, the theoretical calculations of, for example, Refs. 7, 9, and 29 were also made for a free particle at rest.[7)] However, the concept of investigation of a free particle at rest was always associated with a certain idealization, and it would be more correct to assume that in a measurement of the $g-2$ factor of the electron and muon we obtain a property, not of a free particle at rest, but

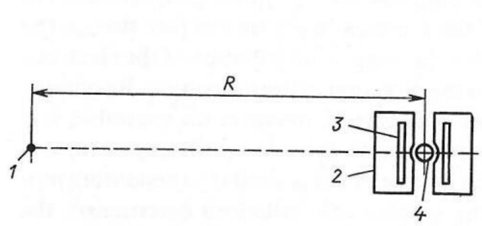

FIG. 9. Arrangement of the $(g-2)_\mu$ experiment under preparation at the Brookhaven National Laboratory: 1) center of storage ring; 2) cryostat; 3) windings of magnets; 4) active region measuring 5×4 cm. The radius of the ring is $R = 2.1$ m, and the strength of the magnetic field is $H = 5$ kOe.

of a particle that is moving and bound, i.e., is in external electromagnetic fields. At the existing accuracy of the measurements, the effects due to the motion of the particle and to the presence of the external confining fields and leading to a change of the $(g-2)_l$ factor itself cannot be detected experimentally. However, they may become more important with further increase in the accuracy of the measurements. The effects of the dependence of the lepton AMMs on the energy and field strength may be manifested under the conditions that exist in the neighborhood of astrophysical objects, when particles with ultrarelativistic energies move in electromagnetic fields of colossal strengths.

In addition, study of the dynamical and field effects will make it possible to understand more deeply the nature of the anomalous magnetic moments of the charged leptons, and it therefore has fundamental importance.

Electrodynamic contributions to the anomalous magnetic moment of a lepton moving in an external field

The dependence of the electron AMM on the strength of the external magnetic field was actually first pointed out in Ref. 78, in which the method employed had already been used[79] to calculate the QED contribution to the electron AMM in the lowest order of perturbation theory, i.e., the order proportional to α. The essence of the approach of Ref. 79 is that one considers the state of an electron in a given magnetic field of strength H with energy equal to $m_e c^2$ (the energy of the orbital magnetic moment is canceled by the energy of the spin magnetic moment) and calculates the correction to the electron energy due to interaction with radiation. The term in the expansion of this correction in powers of H that does not depend on the field strength—it is the electron self-energy in the absence of the field H—is infinite, while the term proportional to the first power of H leads to the Schwinger value for the electron AMM. In Ref. 78, the following terms in the expansion of the energy in powers of H were retained, thus making it possible to study the dependence of the electron AMM on the strength of the external magnetic field, and it was concluded that the field (or H-dependent) corrections to the AMM are finite for all orders of the expansion in powers of H. A more systematic investigation of the dependence of the electron AMM on the strength of the external magnetic field was made on the basis of a calculation of the mass operator in Ref. 80.

An important restriction in the application of the method and of the results of Refs. 78–80 is the fact that the entire treatment was given for an electron at rest—in the nonrelativistic approximation. However, in the precession scheme of the experiments to measure the AMMs of charged leptons realized at CERN and planned at BNL, the leptons have relativistic energies, and it is therefore interesting to study the dependence of the AMM of a lepton on its energy. In Refs. 81 and 82 the energy dependence of the AMM of an electron moving in an external magnetic field was demonstrated and investigated for the first time. In these papers, a method was developed for calculation in the case of relativistic motion of an electron in a magnetic field with strength $H<H_0=m_e^2c^3/|e|\hbar=4.41\cdot10^{13}$. The expressions for the AMM of an electron moving with relativistic energy in an arbitrary constant field and crossed electromagnetic field were obtained on the basis of a dispersion method in Ref. 83 and were confirmed directly in terms of a calculation of the contribution to the mass operator of an electron in an external constant field in Ref. 84 (see also Ref. 85). In addition, the AMM of an electron in a crossed electromagnetic field was considered in Ref. 86, and in a magnetic field of arbitrary strength in Ref. 87.

Bearing in mind that under the conditions of the experiments to determine the $g-2$ factor the charged leptons move in a combination of electromagnetic fields, and in the case of resonance $g-2$ experiments the electron is also subject to a high-frequency electromagnetic field, it is of interest to consider investigations like those made in Refs. 88–90, in which the corrections to the electron AMM due to the effect of a superposition of a constant electromagnetic field and the field of a plane electromagnetic wave were considered.

Summarizing the results of the investigations of the influence of external electromagnetic fields and motion of the lepton on the anomalous magnetic moment, considered in the leading order in the framework of QED, we can draw the following conclusions. First, the AMM depends on the strength of the external field, and the dependence is nonlinear. Further, the behavior of the lepton AMM is different in weak and superstrong fields. Whereas in a weak magnetic field ($H<H_0$) there appear only four field corrections to the vacuum (or fieldless) value of the AMM, equal to $a_{\rm Sch}=\alpha/2\pi$ in the lowest order in α, in a strong field the value of the AMM is completely determined by the field strength H, decreases with increasing H, and at a definite field strength even changes sign.

Second, the anomalous magnetic moment is an essentially dynamical quantity, since it changes with the lepton energy, and, moreover, the AMM of a lepton in motion in the field of an electromagnetic wave depends on its polarization,[90] an effect that also belongs in the considered category. We note also that in the case of a relatively weak field the AMM of a lepton at ultrarelativistic energies decreases rapidly with increasing energy.

Investigations like those mentioned above are of interest, since the field-dependent and energy-dependent corrections to the AMM in the leading order in α may be greater than the contributions of the following orders of the expansion in α and the nonelectrodynamic contributions. For example, under the conditions of experiments made with the linear collider at Stanford (SLAC), with the energy of the electron beam raised to 46 GeV and effective strength of the magnetic field reaching 10^5 Oe (Ref. 181), the dynamical field correction in the leading order in α will be $\Delta a=0.6\cdot10^{-8}$ (Ref. 184).

The importance of the study of the dynamical and field dependences of the lepton AMMs can also be illustrated by the fact that this subject regularly attracts the interest of theoreticians. For example, in Refs. 91 and 92 many of the conclusions contained in the studies listed above were confirmed.

We also mention the study of Ref. 93, which considered the influence of an external magnetic field on the electron AMM (leading to a shift of the g_e factor) and in which, in addition, allowance was made for the specific conditions under which experiments using a Penning trap are performed. The main result of the study was the discovery of pronounced resonance peaks in the correction to the electron AMM at integer values of the ratio of the electron cyclotron frequency and the frequency that characterizes the stationary states of a photon between the two ideal conducting plates that model the Penning trap. This assertion agrees with the results of Refs. 66 and 67. We note that this correction, associated with the presence of a magnetic field and a Penning trap, is as yet significantly smaller than the discrepancy between the theoretical and experimental values of the electron AMM.

Contributions of the axial-vector Z and W bosons and Higgs H bosons to the AMM of a lepton moving in an external electromagnetic field

The dependence of the contributions of the weak interactions to the AMM of a charged lepton on the lepton energy and on the strength of the external field was first considered in Refs. 94 and 95 (see also Ref. 96) in the framework of the Weinberg–Salam model in 1982, i.e., before the direct discovery, with the proton collider at CERN,[97,98] of the carriers of the weak interactions—the axial-vector neutral Z and charged W bosons. In particular, the contribution of the neutral weak current (Z-boson contribution) to the AMM of a lepton moving in a constant field[8)] was obtained in Ref. 94 on the basis of a calculation of the mass operator of an electron in an external field, and it was concluded for the first time that in the case of relativistic energies of particles in a sufficiently strong field the contribution of the weak interactions can become equal in order of magnitude to the photon contribution, i.e., the purely electrodynamic contribution. The contribution of the charged weak currents (W-boson contribution) was first discussed in Ref. 100 on the basis of the method of analytic continuation[83] in a magnetic field. Later,[101] a calculation was made of the W-boson contribution to the mass operator of a charged lepton in a constant field, and this made it possible to obtain the corresponding contribution to the lepton AMM more rigorously. The dependence of the contribution to the lepton AMM from the third boson predicted in the Weinberg–Salam model but not yet experimentally discovered—the Higgs H boson—on the lepton energy and on the strength of a constant electromagnetic field was studied in Ref. 102 (see also Refs. 49 and 103–105). The values of the photon (γ) and Z-, W-, and H-boson contributions to the AMM of a charged lepton for arbitrary lepton masses and with allowance for motion of the lepton in a constant external field were compared in Ref. 106. The particular specific case of the influence of a superstrong magnetic field on the Z-, W-, and H-boson contributions to the AMM can be considered on the basis of the results of Refs. 107–109.

In this section we shall dwell on the theoretical treatment, in the framework of the Weinberg–Salam model of the electroweak interactions, of the contributions to the AMM of a charged lepton with allowance for the dependence on the energy and on the strength of an external magnetic field. The most consistent way of finding the corresponding contributions to the AMM is based on use of the expression for the lepton mass operator $M(x', x)$, which occurs in the Schwinger equation and describes the motion of the lepton in the presence of the field with allowance for radiative corrections:

$$(\hat{p}-m_l)\,\Psi(x)=\int M(x',\,x)\,\Psi(x')\,dx',$$

$$\hat{p}=i\hat{\partial}-eA^{\text{ext}},\quad \hat{\partial}=\gamma_\nu\frac{\partial}{\partial x_\nu},$$

where A^{ext} is the 4-potential of the external electromagnetic field. As the initial field one can choose an external constant crossed field ($\mathbf{E}\perp\mathbf{H}, E=H$), which in the case of relativistic particles is a good approximation in wide limits for any constant electromagnetic field (for more details, see Ref. 85). The results then obtained will depend on the characteristic parameter

$$\chi=\sqrt{-(eF^{\mu\nu}p_\nu)^2}\,m_l^{-3},\tag{42}$$

where $F^{\mu\nu}$ is the tensor of the electromagnetic field, and e, m_l, and p_ν are the charge, mass, and momentum of the lepton.

The expectation value of the mass operator calculated in the second order of perturbation theory, $M^{(2)}$, and taken on the mass shell (in what follows, we use a system of units in which $c=\hbar=1$) between constant spinors

$$U=\frac{1}{\sqrt{4npp_0}}\begin{pmatrix} m_l+np+(\sigma\mathbf{n})(\sigma\mathbf{p}_\perp)\\ (m_l-np)(\sigma\mathbf{n})+(\sigma\mathbf{p}_\perp)\end{pmatrix}V,\quad VV^+=1,$$

$$np=n_\mu p^\mu,$$

where $p_\perp$ is the component of the lepton momentum transverse to the direction of the magnetic field H, determines the contribution to the amplitude of elastic scattering of the lepton in the external field: $T_{pp'}=-\bar{U}M^{(2)}(p,p')\,U$. The constant two-component spinor V describes the orientation of the lepton spin and satisfies the equation

$$(\sigma\mathbf{l})\,V=\xi V,$$

where $\xi=\pm 1$ characterizes the two possible values of the spin projection of the particle onto the direction of the unit vector $\mathbf{l}$. The expectation value of the Pauli matrices σ is calculated in accordance with the formula

$$\bar{\sigma}=V^+\sigma V=\xi\frac{1+\xi\xi'}{2}\,\mathbf{l}+\frac{1-\xi\xi'}{2}\left(\frac{i\xi\mathbf{l}\times\mathbf{n}+\mathbf{l}\times\mathbf{l}\times\mathbf{n}}{1-(ln)^2}\right).$$

The ξ-dependent part of the amplitude is related to the correction to the lepton AMM in the rest frame by the equation[85,110]

$$\Delta\mu=(V\cdot T\cdot\xi\mathbf{H})^{-1}\ \mathrm{Re}\ T_{pp'}.$$

In contrast to electrodynamics, in the Weinberg–Salam theory of the electroweak interactions not only photon processes contribute to the lepton mass operator, but also processes involving neutral Z bosons, charged W bosons, and neutral Higgs H bosons (the unitary gauge, which does not require the consideration of processes with the participation of unphysical particles—Faddeev-Popov ghosts—is used).[111] If a restriction is made to processes of second order in the coupling constant in the Weinberg–Salam theory, then the photon, Z-, W-, and H-boson contributions to the mass operator do not interfere with one another, and for $M^{(2)}$ we have the decomposition

$$M^{(2)}(x',\,x)$$
$$=M^{(2)}_\gamma(x',\,x)+M^{(2)}_Z(x',\,x)+M^{(2)}_W(x',\,x)+M^{(2)}_H(x',\,x).$$

These four contributions to the mass operator of the lepton in the external field are determined by the Feynman diagrams shown in Fig. 10, and the corresponding analytic expressions have the form

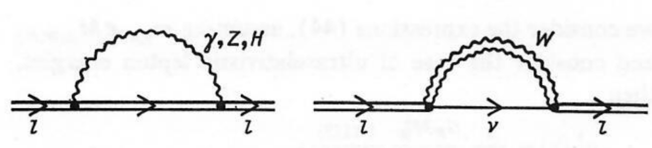

FIG. 10. Feynman diagrams that describe the contributions of the γ, Z, W, and H bosons to the mass operator of a charged lepton moving in an external field.

$$M_{\gamma}^{(2)}(x',\ x)=-\mathrm{i}e^2\gamma^{\mu}G^l(x',\ x)\,\gamma^{\nu}D_{\mu\nu}^{\gamma}(x',\ x),$$

$$e=\frac{gg'}{(g^2+g'^2)^{1/2}};$$

$$M_Z^{(2)}(x',\ x)=-\mathrm{i}G_Z^2\tilde{\gamma}^{\mu}G^l(x',\ x)\ \tilde{\gamma}^{\nu}D_{\mu\nu}^{Z}(x',\ x),$$

$$\tilde{\gamma}^{\mu}=\gamma^{\mu}(1+\bar{\alpha}\gamma^5),\ G_Z=\frac{g^2-3g'^2}{4\,(g^2+g'^2)^{1/2}},$$

$$\bar{\alpha}=\frac{g^2+g'^2}{g^2-3g'^2};$$

$$M_W^{(2)}(x',\ x)=-\mathrm{i}G_W^2\tilde{\gamma}'^{\mu}G^{\nu}(x',\ x)\,\tilde{\gamma}'^{\nu}D_{\mu\nu}^{W}(x',\ x),$$

$$\tilde{\gamma}'^{\mu}=\gamma^{\mu}(1+\gamma^5),\ G_W=\frac{g}{2\sqrt{2}},$$

$$M_H^{(2)}(x',\ x)=-\mathrm{i}G_H^2G^l(x',\ x)\,D^H(x',\ x),\ G_H=\frac{g}{2}\frac{m_l}{M_W},$$

where g and g' are the gauge coupling constants of the Weinberg–Salam theory, $G^{l,\nu}(x',x)$ are the lepton and neutrino propagators, and $D^{\gamma,Z,W,H}(x',x)$ are the propagators of the photon and of the Z, W, and H bosons, and they take into account exactly the effect of the external crossed electromagnetic field on the charged particles.[85,101,112]

Note that comparison of the constants e, G_Z, G_W, and G_H, which describe the vertices of the corresponding Feynman diagrams, shows that the H-boson contribution is relatively small, since $G_H\sim m_l/M_W$, $m_l/M_W\ll 1$. However, the contribution of the Higgs boson depends strongly on the boson mass M_H, and in the presence of a strong electromagnetic field a final conclusion about the relative importance of the H-boson contribution can be drawn after the calculations have been made.[49]

Calculations in accordance with the above scheme[185] give (see the references cited earlier in this section)

$$a_l^{\gamma}(\chi)=\frac{e^2}{2\pi^2}\int_0^{\infty}\frac{du}{(1+u)^3}\left(\frac{u}{\chi}\right)^{2/3}\Upsilon(z_{\gamma}),\ z_{\gamma}=\left(\frac{u}{\chi}\right)^{2/3};$$

$$a_l^{Z}(\chi)=-\frac{G_Z^2}{2\pi^2}\lambda_Z^{-1}\int_0^{\infty}\frac{du}{(1+u)^3}\left(\frac{u}{\chi}\right)^{2/3}\Upsilon(z_Z)$$

$$\times\left\{\frac{1+\bar{\alpha}^2}{2}\left(u+2\lambda_Z\frac{1+u}{u}-\lambda_Z\right)\right.$$

$$\left.-\frac{1-\bar{\alpha}^2}{2}\left(u+2\lambda_Z\frac{1+u}{u}\right)\right\},$$

$$z_Z=\left(\frac{u}{\chi}\right)^{2/3}\left(1+\lambda_Z\frac{1+u}{u^2}\right),\ \lambda_Z=\frac{M_Z^2}{m_l^2};$$

$$a_l^{W}(\chi)=\frac{G_W^2}{2\pi^2}\lambda_W^{-1}\int_0^{\infty}\frac{du}{(1+u)^3}\left(\frac{u}{\chi}\right)^{2/3}$$

$$\Upsilon(z_W)\left\{\lambda_W\frac{1+2u}{u}-\frac{1}{2u}\right\};$$

$$z_W=\left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{1}{u}+\lambda_W\frac{1+u}{u}\right),\ \lambda_W=M_W^2/m_l^2;$$

$$a_l^{H}(\chi)=\frac{G_H^2}{2\pi^2}\int_0^{\infty}\frac{du}{(1+u)^3}\left(\frac{u}{\chi}\right)^{2/3}\Upsilon(z_H)\frac{1+u/2}{2};$$

$$z_H=\left(\frac{u}{\chi}\right)^{2/3}\left(1+\lambda_H\frac{1+u}{u^2}\right),\ \lambda_H=M_H^2/m_l^2. \qquad (43)$$

Here, the dynamical field parameter has the form $\chi=|e|(p_0-p_3)H/m_l^3$, and p_0 and p_3 are the energy and projection of the lepton momentum onto the vector **H**.

The formulas that we have given in the second order of perturbation theory take into account exactly the influence of the external field on the behavior of the contributions a_l^{γ}, a_l^{Z}, a_l^{W}, and a_l^{H} the lepton AMM. The further investigation is associated with approximate calculations that use the properties of the functions

$$\Upsilon(z)=\int_0^{\infty}\sin(zx+x^3/3)\,dx$$

in two characteristic ranges of variation of the parameter χ: 1) $\chi\ll\lambda$, corresponding to the case of weak fields and not very high energies, 2) $\chi\gg\lambda^{3/2}$, for strong fields but when $H<H_0$ and at ultrarelativistic energies:

$$a_l^{\gamma}(\chi)=\frac{e^2}{4\pi}\begin{cases}\frac{1}{2\pi}\left[1-12\chi^2\left(\ln\chi^{-1}+c+\frac{1}{2}\ln 3-\frac{37}{12}\right)\right], & \chi\ll 1;\\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}, & \chi\gg 1;\ c=0{,}577\end{cases}$$

$$a_l^{Z}(\chi)=\frac{G_Z^2}{2\pi}\begin{cases}\frac{1}{\pi\lambda_Z}\left[\Delta_Z-\frac{1+\bar{\alpha}^2}{2}\frac{\chi^2}{\lambda_Z^3}\left(2\ln\lambda_Z-\frac{217}{30}\right)\right.\\ \left.+\frac{1-\bar{\alpha}^2}{2}\frac{\chi^2}{\lambda_Z^3}\left(4\ln\lambda_Z-\frac{79}{5}\right)\right], & \chi\ll\lambda_Z;\\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}\left[-\frac{1+\bar{\alpha}^2}{2}5+\frac{1-\bar{\alpha}^2}{2}6\right], & \chi\gg\lambda_Z^{3/2};\end{cases}$$

$$a_l^{W}(\chi)=\frac{G_W^2}{2\pi}\begin{cases}\frac{1}{\pi\lambda_W}\left(\Delta_W+\frac{1}{10}\frac{\chi^2}{\lambda_W^3}\right), & \chi\ll\lambda_W;\\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}11, & \chi\gg\lambda_W^{3/2};\end{cases}$$

$$a_l^{H}(\chi)=\frac{G_H^2}{2\pi}\begin{cases}\frac{1}{\pi\lambda_H}\frac{1}{2}\left[\Delta_H+\frac{1}{3}\frac{\chi^2}{\lambda_H^2}\right.\\ \left.+\frac{\chi^2}{\lambda_H^3}\left(\ln\lambda_H-\frac{257}{60}\right)\right], & \chi\ll\lambda_H;\\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}}\frac{7}{4}, & \chi\gg\lambda_H^{3/2}.\end{cases} \qquad (44)$$

For the parts that do not depend on the external field we have introduced here the notation

$$\Delta_Z=\frac{1+\bar{\alpha}^2}{2}\left[\left(\frac{\lambda_Z^4}{2}-\frac{5}{2}\lambda_Z^3+\frac{5}{2}\lambda_Z^2\right)\varepsilon_Z^{-1}\ln K_Z\right.$$

$$\left.+\left(\frac{\lambda_Z^3}{2}-\frac{3}{2}\lambda_Z^2+\frac{1}{2}\lambda_Z\right)\ln\lambda_Z-\lambda_Z^2+\frac{3}{2}\lambda_Z-\frac{1}{2}\right]$$

$$+\frac{1-\bar{\alpha}^2}{2}\left[\left(\frac{1}{2}\lambda_Z^3-\frac{3}{2}\lambda_Z^2\right)\varepsilon_Z^{-1}\ln K_Z\right.$$

$$\left.+\left(\frac{1}{2}\lambda_Z^2-\frac{1}{2}\lambda_Z\right)\ln\lambda_Z-\lambda_Z+\frac{1}{2}\right],$$

$$\varepsilon=[|\lambda(\lambda-4)|]^{1/2},\ K=|(\lambda-\varepsilon)/(\lambda+\varepsilon)|;$$

$$\Delta_W=\left(\lambda_W^3-\frac{5}{2}\lambda_W^2+\frac{3}{2}\lambda_W\right)\ln|\lambda_W(\lambda_W-1)^{-1}|$$

$$-\lambda_W^2+2\lambda_W+1/4;$$

$$\Delta_H/\lambda_H=\left(\frac{\lambda_H^3}{4}-\frac{5}{4}\lambda_H^2+\lambda_H\right)\varepsilon_H^{-1}\ln K_H$$

$$+\left(\frac{\lambda_H^2}{4}-\frac{3}{4}\lambda_H\right)\ln\lambda_H-\frac{\lambda_H}{2}+\frac{3}{4}. \qquad (45)$$

We note that the expressions (45) take into account exactly the dependence of the vacuum (not containing the parameter χ) contributions to the lepton AMM on the square of the ratio of the boson mass to the lepton mass ($\lambda_B = M_B^2/m_l^2$), and therefore the expressions (45) are valid for all values of λ_B.

We now give the leading terms in the expansion of the vacuum parts Δ_Z, Δ_W, and Δ_H under the assumption that $\lambda^{-1}_{Z,W,H} \ll 1$, i.e., for the case of the electron and the muon (it is assumed here that the Higgs boson is fairly heavy,[113,114] $M_H \gg m_l$):

$$\left.\begin{aligned} \Delta_Z &= -\frac{1+\bar{\alpha}^2}{2}\frac{2}{3}+\frac{1-\bar{\alpha}^2}{2}(1-\lambda_Z^{-1}\ln\lambda_Z)+O(\lambda_Z^{-2});\\ \Delta_W &= 5/6+\lambda_W^{-1}1/3+O(\lambda_W^{-2});\\ \Delta_H &= \frac{1}{2}\ln\lambda_H-7/12+O(\lambda_H^{-1}). \end{aligned}\right\} \tag{46}$$

It follows from (44) and (46) that the vacuum contributions of the Z and W bosons are dominant, and that their total value depends on the value used for the Weinberg angle θ_W. Thus, if $\sin^2\theta_W = 0.22$, then for the muon we have $a_\mu^Z(0) = -1.82\cdot 10^{-9}$, $a_\mu^W(0) = 3.89\cdot 10^{-9}$, $a_\mu^Z(0) + a_\mu^W(0) = 2\cdot 10^{-9}$, but if, for example, $\sin^2\theta_W = -0.217$, then $a_\mu^Z(0) + a_\mu^W(0) = 19.5\cdot 10^{-10}$.

Bearing in mind that in the Weinberg–Salam theory the mass of the Higgs boson is not determined, and at the present time there are no unambiguous experimental data on its value,[115,116] it is of interest to compare the contributions a_l^Z, a_l^W, and a_l^H for different ratios of the lepton, m_l, and boson, M_B, masses. Since the masses of the Z and W bosons are approximately equal, $M_Z \sim M_W \sim 90$ GeV $= M_B$, there are six possible characteristic relationships between the masses m_l, M_B, and M_H, and for each of them we obtained the following estimates for the ratios of the contributions $A = |a_l^H(0)/a_l^B(0)|$ and $a_l^B(0) = a_l^Z(0) + a_l^W(0)$:

1) $m_l < M_B < M_H$, $A \sim m_l^2/M_H^2 < 1$;
2) $m_l < M_H < M_B$, $A \sim m_l^2/M_H^2 < 1$;
3) $M_H < m_l < M_B$, $A \sim 1$;
4) $M_B < m_l < M_H$, $A \sim \lambda^{-1}\ln\lambda < 1$;
5) $M_B < M_H < m_l$, $A \sim 1$;
6) $M_H < M_B < m_l$, $A \sim 1$.

Thus, if the lepton rest mass m_l is close to the mass of the H boson or exceeds it, then for all ratios of m_l to the masses of the Z and W bosons the Higgs vacuum correction to the lepton AMM will be of the same order as, $|a_l^H(0)| \sim |a_l^B(0)|$, or greater than, $|a_l^H(0)| > |a_l^B(0)|$, the total Z- and W-boson contribution.[49] This fact, in the absence of unambiguous experimental data on the mass of the H bosons and with allowance for the ever increasing accuracy in the measurement of the AMMs of the leptons, permits us to draw the conclusion that in a comparison of the experimental and theoretical values of the AMMs it is necessary to take into account the possible existence of significant contributions of the Higgs bosons. We note also that such a comparison can in principle provide information about the mass of the Higgs boson.[9)]

To conclude this subsection, we compare the weak contributions to the electron and muon AMMs with the electrodynamic contribution at different energies of the particles. If we consider the expressions (44), assuming $m_{e,\mu} \ll M_{Z,W,H}$, and consider the case of ultrarelativistic lepton energies, then

$$a_l^\gamma(\chi) = \frac{1}{2\pi}\frac{G_F M_W^2}{9\sqrt{6}}\frac{\Gamma(1/3)}{(3\chi)^{2/3}};$$

$$a_l^Z(\chi) = -\frac{1}{2\pi}\frac{G_F M_W^2}{9\sqrt{6}}\frac{\Gamma(1/3)}{(3\chi)^{2/3}}\frac{11}{3};$$

$$a_l^W(\chi) = \frac{1}{2\pi}\frac{G_F M_W^2}{9\sqrt{6}}\frac{\Gamma(1/3)}{(3\chi)^{2/3}}\,11$$

(where $\theta_W = 30°$). These estimates for the AMMs $a_l(\chi)$ show that in this range of energies ($\chi \gg \lambda^{3/2}_{Z,W}$) the weak ($Z$- and W-boson) contributions exceed by almost an order of magnitude the electrodynamic (photon) contribution; the W-boson contribution is the largest, and, as at energies $\chi \ll 1$, the total contribution to the AMMs is positive: $a_l^\gamma(0) + a_l^Z(0) + a_l^W > 0$. The contribution of the Higgs boson is suppressed in relation to $a_l^Z(0)$ and $a_l^W(0)$ by the ratio $m_l^2/M_W^2 \ll 1$.

We consider in more detail the comparison of $a_l^\gamma(\chi)$ and $a_l^B(\chi)$ in the presence of an external constant field. For small values of the parameter ($\chi \ll 1$), the value of a_l^γ is much larger than a_l^B, since the photon contribution is close to the Schwinger value of the AMM,

$$a_l^\gamma(\chi) \sim \frac{\alpha}{2\pi} \sim 10^{-3},$$

and the boson contribution has the order $a_l^B(\chi) \sim 0.5\cdot 10^{-13}$.

An appreciable difference between the contributions $a_l^\gamma(\chi)$ and $a_l^B(\chi)$ appears if we consider how a_l^γ and a_l^B vary with increasing value of the parameter χ. Whereas a_l^γ immediately begins to decrease with increasing χ and for $\chi \gg 1$ tends to zero in accordance with the law $\chi^{-2/3}$, the value of a_l^B increases with increasing χ for $\chi \ll \lambda_{Z,W}$. In addition, estimates show that the ratio a_l^B/a_l^γ will also increase with increasing χ in the intermediate range of values of the parameter χ ($\lambda_{Z,W} \ll \chi \ll \lambda^{2/3}_{Z,W}$). Moreover, in the intermediate region the terms that determine the leading contribution to $a_l^B(\chi)$ in the region $\chi \ll \lambda_{Z,W}$ will, as before, be the leading terms. Making a comparison of a_l^B and a_l^γ in the intermediate region and using at the same time the expressions (44) and (46) to estimate $a_l^B(\chi)$,

$$a_l^B \approx \frac{G_F m_l^2}{\pi^2\sqrt{2}}\left[\frac{5}{24}+\frac{2}{3}\left(\sin^2\theta_W-\frac{1}{4}\right)^2\right],$$

we conclude that for $\chi \ll \lambda^{3/2}_{Z,W}$ the photon contribution to the AMMs will exceed the boson contribution. Since, as we have already noted, the main contribution to the lepton AMM in the region $\chi \gg \lambda^{3/2}_{Z,W}$ is made by the weak processes, we conclude that for $\chi \sim \lambda^{3/2}_{Z,W}$ the contributions to $a_l^B(\chi)$ and $a_l^\gamma(\chi)$ become equal in order of magnitude.

Anomalous magnetic moments of charged leptons in alternative theories

At the present time, the standard model of the Glashow–Weinberg–Salam interaction adequately describes all the available experimental data. However, the standard model is not free of shortcomings (such as the large number of parameters, the unresolved problem of the number of lepton and quark generations, the absence of gravitation, etc.), and this has stimulated active study of alternative models that go beyond the framework of the standard model but predict the entire low-energy picture that corresponds to it. Comparison of quantum electrodynamics with the Weinberg–Salam theory shows that an increase in the types of interaction of a charged lepton can significantly change the value and nature of the dependence of its anomalous magnetic moment on the energy and on the external field.

Contributions to lepton anomalous magnetic moments from bosons of various types

In the various generalizations that have been considered for the standard SU(3)×SU(2)×U(1) model (such as the theory of technicolor,[117,118] grand unification,[119] superstrings,[120] etc.) there are predictions of a rich spectrum of new bosons (see also Ref. 121), and the virtual processes in which they participate lead to new contributions to the AMM of a charged lepton. In this connection, it is of interest to study the contributions made by bosons of different types (B_i) and arbitrary mass M_i (Ref. 122): 1) scalar and neutral; 2) pseudoscalar and neutral; 3) vector and neutral; 4) axial-vector and neutral; 5) vector and charged; 6) axial-vector and charged (see also Ref. 185).

As follows from the descriptions of the experiments at the University of Washington to measure the electron $g-2$ factor, the CERN experiments to measure the muon $g-2$ factor, and the new experiments planned at BNL, the nature of the motion of the lepton is largely determined by the effect of the external constant field. In addition, in the case of the $(g-2)_\mu$ experiments the muons have a relativistic energy, and this, as is shown by the example of the electrodynamic interaction and the contribution of the weak interactions, can, like the presence of an external field, lead to a significant change of the lepton AMM. Therefore, when we consider below the contributions to the lepton AMM from the various bosons B_i we shall take into account the effect of the magnetic field, and also the possible influence of the motion of the lepton itself.

Assuming once more that the lepton is relativistic, we shall use as a model of the magnetic field a constant crossed field. To determine the contributions to the AMM of the charged lepton, we calculate, initially in the single-loop approximation, the part $M^{(2)}_{B_i}$ of the mass operator of the lepton moving in the crossed field. The characteristic Feynman diagram is shown in Fig. 11. The double lines signify that the

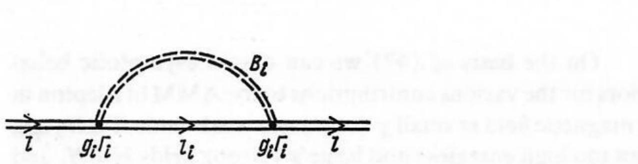

FIG. 11. Contribution of bosons of various species to the mass operator of a lepton in an external field.

wave functions and propagators of the charged particles are exact solutions of the corresponding equations with allowance for the effect of the external field. We then find the contributions $T_i^{(2)}$ to the elastic scattering amplitude, of which the real parts of the terms that depend on the spin variable are proportional to the contribution $a_l^i(\chi)$ to the anomalous magnetic moment.

Making the calculations, we obtain, for the contributions to the AMM of a charged lepton of mass m_l moving with relativistic energy in the magnetic field,

$$a_l^{B_i}(\chi) = \frac{g_i^2}{(2\pi)^2} \int_0^\infty \frac{du}{(1+u)^3} \left(\frac{u}{\chi}\right)^{2/3} \Omega_i \Upsilon(z_i), \qquad (47)$$

where $\Upsilon(z_i) = \int_0^\infty \sin(z_i x + x^3/3)dx$; and g_i is the constant of the coupling of the lepton to the boson B_i. In Table I we give the values of Γ_i, which characterize the structure of the coupling of the lepton and the bosons B_i, the type of the virtual fermion l_i, the functions Ω_i, which determine the integrand in (47), and the arguments z_i of the upsilon functions.

The contributions (47) depend on the single dynamical field parameter $\chi = \sqrt{-(eF_{\mu\nu}p^\nu)^2} m_l^3$. Note that Eqs. (47) give exact expressions for the contributions of the bosons B_i to the AMM of a lepton moving in a crossed electromagnetic field. In this case, the parameter takes the form $\chi = (H/H_0)(p_0 - p_3)/m_l$ where p_0 and p_3 are the energy and projection of the lepton momentum onto the vector **E**×**H**. Because of the "universality" of the crossed field, the expressions (47) are also valid for the motion of a relativistic lepton in an arbitrary constant field. For a purely magnetic field, the dynamical parameter has the form $\chi_H(H/H_0)/(p_\perp/m_l)$, where $p_\perp = \sqrt{2\gamma n}$ is the projection of the lepton momentum onto the plane perpendicular to the vector **H** (n is the ordinal number of the Landau levels in the magnetic field, and $\gamma = eH$).

TABLE I. Values of Γ_i, l_i, Ω_i, z_i for processes with allowance for bosons of different species.

Species (and charge) of boson	Γ_i	l_i	Ω_i	z_i
Scalar (0)	1	l	$1+u/2$	$z_i = \left(\frac{u}{\chi}\right)^{2/3}\left(1+\lambda_i \frac{1+u}{u^2}\right)$
Pseudoscalar (0)	γ_5	l	$-u/2$	
Vector (0)	γ_μ	l	1	
Axial-vector (0)	$\gamma_\mu\gamma_5$	l	$-3-\frac{4}{u}-\frac{2u}{\lambda_4}$	
Vector (−1)	γ_μ	ν	$2+\frac{1}{u}-\frac{1}{u\lambda_i}$	$z_i = \left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{1}{u}+\lambda_i \frac{1+u}{u}\right)$
Axial-vector (−1)	$\gamma_\mu\gamma_5$	ν		

On the basis of (47) we can obtain asymptotic behaviors for the various contributions to the AMM of a lepton in a magnetic field at small χ (relatively weak fields $H \ll H_0$ and not too high energies) and large χ (strong fields $H < H_0$ and ultrarelativistic energies):[10)]

$$a_l^{B_1}(\chi) = \frac{g_1^2}{2\pi} \begin{cases} \frac{1}{\pi\lambda_1} \frac{1}{2} \left[\Delta_1 + \frac{1}{3} \frac{\chi^2}{\lambda_1^2} + \frac{\chi^2}{\lambda_1^3} \left(\ln \lambda_1 - \frac{257}{60} \right) \right], \\ \chi \ll \lambda_1; \\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{7}{4}, \quad \chi \gg \lambda_1^{3/2}; \end{cases}$$

$$a_l^{B_2}(\chi) = \frac{g_1^2}{2\pi} \begin{cases} -\frac{1}{\pi\lambda_2} \frac{1}{2} \left[\Delta_2 + \frac{2}{3} \frac{\chi^2}{\lambda_2^2} \right], \quad \chi \ll \lambda_2; \\ -\frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{5}{4}, \quad \chi \gg \lambda_2^{3/2}; \end{cases}$$

$$a_l^{B_3}(\chi) = \frac{g_3^2}{2\pi} \begin{cases} \frac{1}{\pi\lambda_3} \frac{1}{2} \left[\Delta_3 + 2 \frac{\chi^2}{\lambda_3^3} \left(\ln \lambda_3 - \frac{257}{60} \right) \right], \\ \chi \ll \lambda_3; \\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{1}{2}, \quad \chi \gg \lambda_3^{3/2}; \end{cases}$$

$$a_l^{B_4}(\chi) = \frac{g_4^2}{2\pi} \begin{cases} \frac{1}{\pi\lambda_4} \frac{1}{2} \left[\Delta_4 + \frac{\chi^2}{\lambda_4^3} \left(-6 \ln \lambda_4 + \frac{691}{30} \right) \right], \\ \chi \ll \lambda_4; \\ -\frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{11}{2}, \quad \chi \gg \lambda_4^{3/2}; \end{cases}$$

$$a_l^{B_{5,6}}(\chi) = \frac{g_{5,6}^2}{2\pi} \begin{cases} \frac{1}{\pi\lambda_{5,6}} \frac{1}{2} \left[\Delta_{5,6} + \frac{1}{10} \frac{\chi^2}{\lambda_{5,6}^3} \right], \quad \chi \ll \lambda_{5,6}; \\ \frac{\Gamma(1/3)}{9\sqrt{3}(3\chi)^{2/3}} \frac{11}{2}, \quad \chi \gg \lambda_{5,6}. \end{cases} \tag{48}$$

It follows from the expressions (48) that for any boson species ($B = S^0, P^0, V^0, A^0, V^-, A^-$) the dependence of the contributions on the parameter χ is the same, namely, at small χ ($\chi \ll \lambda_i$) there are small corrections quadratic in χ to the vacuum contributions $a_l^{B_i}(0)$ determined by the quantities Δ_i. At large χ ($\chi \gg \lambda_i^{3/2}$), all the contributions clearly demonstrate the dynamical nature of the lepton AMM, varying with increasing χ as $\chi^{-2/3}$. For the magnetic field strengths and lepton energies in the experiments to measure the $g-2$ factors of the electron and muon, the parameter χ is small and, therefore, the dynamical field effects are also small. However, for the motion of relativistic leptons in the neighborhood of astrophysical objects, where, according to modern estimates,[123] the magnetic fields can reach values $H = 10^{-1} H_0 = 4.41 \cdot 10^{12}$ Oe, it is necessary to use the expressions (48) for the contributions of the various bosons to the lepton AMM. Boson field effects can also be manifested in the AMM of charged leptons when they move under the conditions of modern accelerators (see the end of the first subsection of Sec. 3).

We now discuss the vacuum contributions to the AMM of a charged lepton, for which we obtain[185]

$$\left. \begin{aligned} & a_l^{B_i}(0) = \frac{g_i^2}{4\pi^2} \frac{\Delta_i}{\lambda_i}, \quad i = 1, \ldots, 6; \\ & \frac{\Delta_1}{\lambda_1} = \left(\frac{\lambda_1^3}{4} - \frac{5}{4} \lambda_1^2 + \lambda_1 \right) \varepsilon_1^{-1} \ln K_1 \\ & \qquad + \left(\frac{\lambda_1^2}{4} - \frac{3}{4} \lambda_1 \right) \ln \lambda_1 - \frac{\lambda_1}{2} + \frac{3}{4}; \\ & \Delta_2 = \frac{1}{2} \left[\left(\frac{\lambda_2^3}{2} - \frac{3}{2} \lambda_2 \right) \varepsilon_2^{-1} \ln K_2 \right. \\ & \qquad \left. + \frac{1}{2} (\lambda_2^3 - \lambda_2^2) \ln \lambda_2 - \lambda_2^2 - \frac{\lambda_2}{2} \right]; \\ & \Delta_3 = \left(\frac{\lambda_3^4}{2} - 2\lambda_3^3 + \lambda_3^2 \right) \varepsilon_3^{-1} \ln K_3 \\ & \qquad + \left(\frac{\lambda_3^3}{2} - \lambda_3^2 \right) \ln \lambda_3 - \lambda_3^2 + \frac{\lambda_3}{2}; \\ & \Delta_4 = \left(\frac{\lambda_4^4}{2} - 3\lambda_4^3 + 4\lambda_4^2 \right) \varepsilon_4^{-1} \ln K_4 \\ & \qquad + \left(\frac{\lambda_4^3}{2} - 2\lambda_4^2 + \lambda_4 \right) \ln \lambda_4 - \lambda_4^2 + \frac{5}{2} \lambda_4 - 1; \\ & \Delta_5(\lambda_5 = \lambda) = \Delta_6(\lambda_6 = \lambda) = \left(\lambda^3 - \frac{5}{2} \lambda^2 + \frac{3}{2} \lambda \right) \ln \left| \frac{\lambda}{\lambda - 1} \right| \\ & \qquad - \lambda^2 + 2\lambda + \frac{1}{4}; \\ & K = |(\lambda - \varepsilon)/(\lambda + \varepsilon)|, \quad \varepsilon = [|\lambda(\lambda - 4)|]^{1/2}. \end{aligned} \right\} \tag{49}$$

Note that in deriving these expressions we have not imposed any restrictions on the boson masses M_i, i.e., these expressions describe exactly the dependence of the vacuum contributions to the lepton AMM on the masses of the corresponding bosons. When the specific coupling constants g_i and boson masses M_i are substituted in them, the expressions (49) enable us to find the contributions to the AMM of a lepton of mass m_l that arise in the various theories. Thus, using these expressions, we can find the vacuum contribution of the Z, W, and H bosons (45) to the AMM of a lepton in the standard Weinberg–Salam model of the electroweak interactions.

The vacuum contributions of neutral massive scalar, pseudoscalar, vector, and axial-vector bosons for the electron and muon AMMs were recently considered in Refs. 124 and 125 by a method different from ours. Analytic expressions were obtained in Ref. 125 for the corresponding vacuum contributions to the AMMs for two cases (we use here the notation of the present paper): 1) $\lambda_i > 4$; 2) $\lambda_i < 4$; these cases can be combined and must agree with the result (49) ($\lambda \neq 4$). However, in Ref. 125 the contributions of the scalar and pseudoscalar bosons are given with the incorrect sign, and this is also reflected in the sign of the corresponding limiting values for $\lambda \gg 1$ and $\lambda \ll 1$.[11)] This error was corrected in Ref. 126 by one of the authors of Refs. 124 and 125. In addition, the contribution of the neutral axial-vector boson obtained in Refs. 124–126 differs from (49) and the identical expression that can be obtained from the results of Ref. 127.

For the case of light ($\lambda_i \ll 1$) and heavy ($\lambda_i \gg 1$) bosons, we can obtain from (49) the limiting values for the contributions to the AMM of a negatively charged lepton (see also Refs. 99, 102, 106, and 185):

$$a_l^{B_i}(0)=\frac{g_i^2}{8\pi^2}k_i,\quad B_i=S^0,\ P^0,\ V^0,\ A^0,\ V^-,\ A^-;$$

1) $\lambda_i \ll 1$:

$$k_{S^0}=\frac{3}{2},\ k_{P^0}=-\frac{1}{2},\ k_{V^0}=1,\ k_{A^0}=-\frac{2}{\lambda_4}\to-\infty,$$

$$k_{V^-}=k_{A^-}=\frac{2}{\lambda_{5,6}}\to\infty;$$

2) $\lambda_i \gg 1$,

$$k_{S^0}=\lambda_1^{-1}\ln\lambda_1-\lambda_1^{-1}\frac{7}{6},\quad k_{P^0}=-\lambda_2^{-1}\ln\lambda_2+\lambda_2^{-1}\frac{11}{6},$$

$$k_{V^0}=\lambda_3^{-1}\frac{2}{3}-\lambda_3^{-2}2\ln\lambda_3,$$

$$k_{A^0}=-\frac{10}{3}\lambda_4^{-1}+\lambda_4^{-2}2\ln\lambda_4,\quad k_{V^-}=k_{A^-}=\frac{10}{3}\lambda_{5,6}^{-1}+\frac{2}{3}\lambda_{5,6}^{-2}.$$

We note that the contributions of the scalar and pseudoscalar bosons are distinguished in a definite manner in the region $\lambda_i \gg 1$ by virtue of the presence, in the leading terms of the expansion, of the factor $\ln\lambda_i$, which can be large.

Figure 12 shows the dependence of the functions k_i on the ratio of the mass of the boson to the mass of the lepton, $\sqrt{\lambda_i}$, for bosons of the considered types.[185]

Thus, we may draw the general conclusion that the contributions of the bosons B_i to the lepton AMMs depend not only on the constant of the coupling to the lepton but also on the properties of the boson itself—its species and mass.

Contribution to lepton AMMs of "horizontal" bosons

Although the contributions to the AMMs considered above can, by virtue of the diversity of properties of the bosons, be used to analyze the lepton $g-2$ factor in the various theories that generalize the standard interaction model, they do not exhaust the complete gamut of processes that lead to corrections to the AMMs. Recently, attempts have been made to resolve one of the open problems of the standard Glashow–Weinberg–Salam model—the existence of generations of quarks and leptons—by the introduction of an additional "horizontal" symmetry between the generations of fermions (Refs. 114 and 128–133), and these have become quite popular.

As was first shown in Refs. 128–130, spontaneous breaking of the generation symmetry, which can lead to the observed mass hierarchy of the fermions, leads to the appearance in the theory of a neutral Goldstone boson—a familon,[128] which has an interaction with the fermions that is nondiagonal with respect to the flavors. Various processes with the participation of the familon in the presence of an external magnetic field were considered in Refs. 134–136 and 183, in which, in particular, the familon contribution to the muon AMM was discussed.

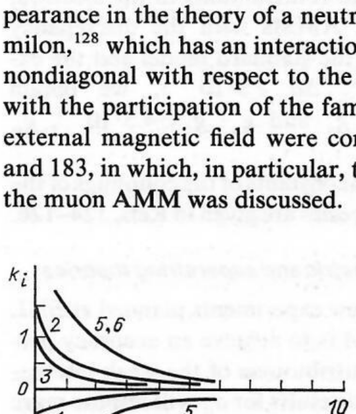

FIG. 12. Dependence of the functions k_i, which determine the vacuum contributions of the bosons of species i to the lepton AMM, on the mass ratio $\sqrt{\lambda_i}=M_i/m_l$ [1) P^0; 2) S^0; 3) V^0; 4) A^0; 5) V^-; 6) A^-].

The flavor-nondiagonal familon–fermion coupling can be either scalar or pseudoscalar,[129,130] and the corresponding Lagrangian in the case of two fermion flavors (electron and muon) can be represented in the form

$$\mathcal{L}=g_f\left[\overline{\Psi}_\mu(1-\alpha'\gamma^5)\Psi_e+\overline{\Psi}_e(1+\alpha'\gamma^5)\Psi_\mu\right]f(x),$$

where $\Psi_l(x)$ and $f(x)$ are the wave functions of the lepton ($l=e,\mu$) and the familon. The constants g_f and α are determined by the specific model, but in any case g_f is inversely proportional to V, which represents the energy scale associated with the spontaneous breaking of the global "horizontal" symmetry:

$$g_f=V^{-1}\sqrt{2m_\mu m_e}.$$

It follows from astrophysical estimates[137] that $V \gtrsim 7\cdot 10^9$ GeV.

We shall obtain the familon contribution to the muon AMM with allowance for the effect of a constant magnetic field, using the method described above. Considering the contribution to the mass operator of a muon moving in a magnetic field—it is given by the Feynman diagram in Fig. 13—we arrive at the expression[134–136]

$$a_\mu^f=\frac{g_f^2}{4\pi^2}\int_0^\infty\frac{du}{(1+u)^2}\left(\frac{u}{\chi}\right)^{2/3}$$

$$\times\left[\frac{1+\alpha'^2}{2}\frac{m_\mu^2}{m_e^2}\frac{1}{1+u}+\frac{1-\alpha'^2}{2}\frac{m_\mu}{m_e}\right]\Upsilon(z_f),$$

$$z_f=\left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{m_\mu^2}{m_e^2}\frac{1}{u}+\frac{1+u}{u}+\frac{M_f^2}{m_e^2}\frac{1+u}{u^2}\right),$$

$$\chi=\sqrt{-(eF_{\mu\nu}p^\nu)^2/m_e^3}.$$

Considering a strictly massless familon ($M_f=0$) in the two ranges of values of the dynamical field parameter, we obtain

$$a_\mu^f(\chi)=\frac{g_f^2}{16\pi^2}(1+\alpha'^2)\left(1+8\chi^2\left(\ln\frac{\chi}{\sqrt{3}}-c+3\right)\right),$$

$$c=0{,}577\ldots,\ \frac{m_e}{m_\mu}\ll\chi\ll 1,$$

$$a_\mu^f(\chi)=\frac{g_f^2}{72\sqrt{3}\pi}(1+\alpha'^2)\frac{\Gamma(1/3)}{(3\chi)^{2/3}}$$

$$\times\left[1-\frac{3\Gamma(2/3)}{\Gamma(1/3)(3\chi)^{2/3}}+O(\chi^{-4/3})\right]\chi\gg 1.$$

As in the cases discussed above with "diagonal" bosons of different species, at large χ the familon contribution a_μ^f decreases with increasing χ in proportion to $\chi^{-2/3}$. However, in the case of small χ the correction to the vacuum value of the anomalous magnetic moment is proportional, not to χ^2, as in (48), but to $\chi^2\ln\chi$. This singularity in the

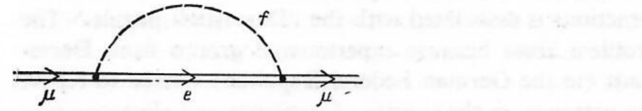

FIG. 13. Familon contribution to the mass operator of a muon in an external field.

asymptotic behavior may be related to the circumstance that, in contrast to the considered contributions to the mass operator of diagonal bosons (see Figs. 10 and 11), which are characterized by the fact that the total mass of the virtual particles is at least not less than the lepton mass, in the case of the familon contribution the total mass of the virtual particles is significantly less than the lepton mass ($m_e \ll m_\mu$), and therefore the external field has a stronger influence on the AMM.

We note that an attempt to explain the discrepancy between the theoretical and experimental values of the muon AMM ($\Delta a_\mu < 9\cdot 10^{-9}$) by a familon contribution a_μ^f does not lead to significant bounds on the constants g_f and α'.

In another version of the theory with "horizontal" symmetry, the model with a so-called "inverse hierarchy" (discussed in Ref. 138), the muon is most strongly coupled to the heavy τ lepton, and for the coupling constant we have the estimate

$$g_f' = \frac{\sqrt{m_\mu m_\tau}}{V'}, \quad V' \gg 10^6 \text{ GeV}.$$

For the vacuum familon contribution to the muon AMM that arises in this model, we obtained[184]

$$a_\mu^{f'} = \frac{g_f'^2}{4\pi^2}\left\{\frac{1+\alpha'^2}{2}\left[\left(-\frac{1}{\varepsilon}+\frac{1}{\varepsilon^2}\right)\ln|1-\varepsilon| - \frac{1}{2}+\frac{1}{\varepsilon}\right]\right.$$
$$\left. + \frac{1-\alpha'^2}{2}\sqrt{\varepsilon}\left[\frac{1}{\varepsilon}+\left(-\frac{1}{\varepsilon}+\frac{1}{\varepsilon^2}\right)\ln|1-\varepsilon|\right]\right\},$$
$$\varepsilon = \frac{m_\mu^2}{m_\tau^2} \approx 3.5\cdot 10^{-3},$$

or, if we make an expansion with respect to $\varepsilon \ll 1$,

$$a_\mu^{f'} = \frac{g_f'^2}{16\pi^2}\left\{\frac{m_\mu}{m_\tau}+\frac{1}{3}\left(\frac{m_\mu}{m_\tau}\right)^2 + \alpha'^2\left[-\frac{m_\mu}{m_\tau}+\frac{1}{3}\left(\frac{m_\mu}{m_\tau}\right)^2\right]\right.$$
$$\left. + O\left(\frac{m_\mu^3}{m_\tau^3}\right)\right\}.$$

If we require that $a_\mu^{f'}$ not exceed the discrepancy between the theoretical value calculated in the framework of the standard model and the experimental value of the muon AMM, i.e., if we require $a_\mu^{f'} < \Delta a_\mu < 9\cdot 10^{-9}$, then for the constant α' we obtain a stronger bound than in the case of the theory with an ordinary hierarchy: $\alpha' \leqslant 1\cdot 10^4$. If the 20-fold increase in the accuracy of the experimental determination of the muon $g-2$ factor planned at BNL results in a decrease by the same amount of the discrepancy between the experimental and theoretical values of the AMM in the standard model, this will impose an even more stringent bound on the parameter α': $\alpha' \leqslant 3\cdot 10^3$.

Contribution of a light pseudoscalar boson to the lepton AMMs

One further case in which study of the lepton AMMs can lead to definite conclusions about the properties of elementary particles and the parameters that describe their interactions is associated with the "Darmstadt puzzle." The problem arose because experimental groups from Darmstadt (in the German Federal Republic) started to report the presence, in the spectra of positrons and electrons produced in collisions of heavy ions, of narrow peaks with an energy of order $E \approx 300$ keV and width $\Delta E \approx 70$ keV.[139–142] Meanwhile, various groups have observed whole series of peaks in the range of energies from 220 to 430 keV (Ref. 126). One of the possible explanations of the nature of these peaks is associated with the assumption[124,143] that the positrons and electrons whose spectra exhibit the peaks are produced by the decay of an unknown particle X, whose mass can be estimated from the experimental data at $M_X = 1$–2 MeV.[126,144]

Suppose that the unknown particle is a pseudoscalar boson.[145–147] If, as was noted above (see also Refs. 49 and 106), the mass of a scalar or pseudoscalar particle is comparable with the lepton mass, the corresponding contribution to the lepton AMM will be comparable with (or will exceed) the total contribution of the Z and W bosons which it is intended to determine in the new experiments at BNL. This suggests that study of the AMM in the given case ($M_X \leqslant m_l$) will make it possible to obtain significant bounds on the constant of the coupling of the leptons to the new X boson.

Using the expressions for the contribution of the pseudoscalar boson to the lepton AMMs [see the expression (47) and Table I], we can obtain the contribution to the electron and muon AMMs[146,147] in the case of small values of the parameter χ:

$$a_e^X = \frac{g_e^2}{8\pi^2}\frac{1}{\lambda_e}\left[\frac{11}{6} - \ln\lambda_e - \frac{2}{3}\frac{\chi_e^2}{\lambda_e^2}\right]$$
$$= \frac{g_e^2}{8\pi^2}[-0.17 - 1.35\cdot 10^{-4}\chi_e^2], \quad \chi_e \ll \lambda_e \approx 10,$$
$$M_X = 1.6 \text{ MeV},$$
$$a_\mu^X = -\frac{g_\mu^2}{8\pi^2}\left[\frac{1}{2}+\lambda_\mu+\frac{\lambda_\mu}{2}\ln\lambda_\mu - \chi_\mu^2\left(\frac{29}{6}+\ln\lambda_\mu\right)\right]$$
$$= -\frac{g_\mu^2}{8\pi^2}[0.50 - 3.45\chi_\mu^2], \quad \chi_\mu \ll \lambda_\mu^{1/2} \sim 1.5\cdot 10^{-2},$$
$$\chi_l = \sqrt{-(eF_{\mu\nu}p^\nu)^2}/m_l^3, \quad \lambda_l = M_X^2/m_l^2,$$

and in the case of large χ ($\chi_e \gg \lambda_e^{3/2}$, $\chi_\mu \gg 1$):

$$a_l^X = -\frac{g_l^2}{72\sqrt{3}\pi}\frac{5\Gamma(1/3)}{(3\chi_l)^{2/3}} = -\frac{g_l^2}{8\pi^2}0.21\chi_l^{-2/3}.$$

Comparing the vacuum contributions to the electron, $a_e^X(0)$, and muon, $a_\mu^X(0)$, AMMs with the discrepancy between the predictions of the standard model and the experiments ($\Delta a_e < 3\cdot 10^{-10}$, $\Delta a_\mu < 9\cdot 10^{-9}$), we obtain bounds on the constants g_e and g_μ: $g_e < 4.5\cdot 10^{-4}$, $g_\mu < 1.1\cdot 10^{-3}$.

Similar estimates of the constants of the couplings of the leptons to bosons of other species are given in Refs. 124–126.

Lepton AMMs in supersymmetric and superstring theories

The main task of the new experiments planned at BNL to measure the muon AMM is to achieve an accuracy permitting detection of the contributions of the weak interactions. The new experimental results for a_μ will impose more stringent restrictions on the parameters of alternative theories than those that follow from the CERN value of a_μ^{exp}. The widely discussed supersymmetric theories[148] predict a large number of new particles with masses of order 100 GeV, and virtual processes in which these participate can contribute to the muon AMM. It is in fact the value of the muon AMM

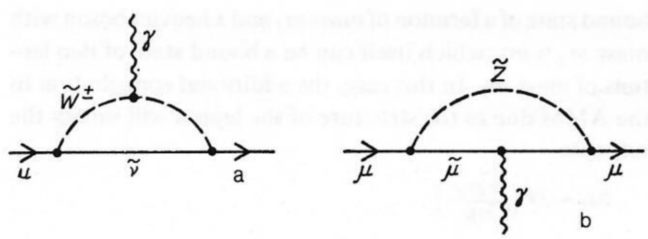

FIG. 14. Contributions of supersymmetric particles to the muon AMM.

that is of interest; for although it is calculated and measured with lower accuracy than the electron AMM, it is, because of the greater mass of the muon ($m_u \gg m_e$), more sensitive, as we noted above, to processes at short distances.

A model-independent analysis of the contributions $a_\mu^{\rm SUSY}$ to the muon AMM in supersymmetric unified theories based on $N=1$ supergravity was made in Refs. 149 and 184. The additional contributions $a_\mu^{\rm SUSY}$ which arise in the lowest order of perturbation theory through processes with the participation of the wino ($\widetilde{W}$) and sneutrino ($\tilde{\nu}$) (Fig. 14a), and the zino ($\widetilde{Z}$) and smuon ($\tilde{\mu}$) (Fig. 14b), are determined by the set of parameter values of the theory, but in most models this dependence can be reduced to the dependence on the wino mass $M_{\widetilde{W}}$ and the sneutrino mass $M_{\tilde{\nu}}$. We note that the dynamical field dependence of the contributions $a_\mu^{\rm SUSY}(\chi)$ is analogous to the χ dependence of the contributions $a_\mu^B(\chi)$ (see Ref. 184).

Figure 15 shows the dependence of the supersymmetric contributions to the muon AMM on the wino mass for different values of the sneutrino mass characteristic of two definite models. A general conclusion that follows from the analysis (see Ref. 149) is that the contribution of supersymmetric particles to the AMM can be appreciable and, in particular, can appreciably exceed the contributions of the weak interactions in the standard Weinberg–Salam model. For a certain choice of the parameters in a number of supersymmetric theories, it will be necessary to increase the accuracy of the muon AMM by about 2–3 times in order to detect $a_\mu^{\rm SUSY}$ experimentally. It is also noteworthy that if the BNL experiments are successful and the accuracy in the measurement of a_μ is increased by 20 times, it will be possible to detect not only the contributions of the standard theory of the electroweak interactions but also the contributions predicted by most of the supersymmetric theories for a wide spectrum of parameter values.

The contributions to the muon AMM in supersymmetric theories (in particular, in ones based on the symmetry group E_6) have been considered in a number of studies.[150–153] The corrections to the muon AMM obtained in these theories can also become comparable with the contributions of the weak interactions in the standard model, and therefore the new experimental values of a_μ will play an important part in the choice of the parameters of these theories.

Lepton AMMs in theories of composite particles

Another of the popular ideas that lead to alternatives to the standard model is that all the low-energy interactions are due to the propagation of massless gauge bosons. In this case, the massive Z and W bosons are not gauge particles and have structure. The definite model proposed in Ref. 154 on the basis of SU(2)×U(1) symmetry can, under certain assumptions, reproduce at low energies all the results of the standard model but at high energies it gives very different predictions. In this model, the gauge group of the $SU(2)_L$ symmetry is not broken, and the observed weak interactions arise as residual interactions between $SU(2)_L$ singlet composite states, which are interpreted as the leptons, quarks, Z, W, and Higgs bosons. This theory predicts the existence of a new neutral boson with spin 2 and mass $M \sim 100$ GeV.[155] Such a boson not only changes the structure of the weak interactions at high energies ($\gtrsim 100$ GeV), but also has an effect at the low-energy level through an additional contribution to the muon AMM. The calculation of the contribution made by the new boson to the muon AMM in the framework of a strongly coupled theory of this kind, in which there are ultraviolet divergences that can be eliminated by a cutoff at energies $\Lambda \sim G_F^{-1/2} \approx 300$ GeV, leads[155] to the result $|\Delta a_\mu| \approx 1.5\cdot 10^{-8}$, which is approximately an order of magnitude greater than the contribution of the weak interactions in the standard model.

Although we have shown by a number of examples that one can effectively use the values of the lepton AMMs to test the validity of various interaction theories, a serious problem in obtaining definite bounds on the parameters of a theory is

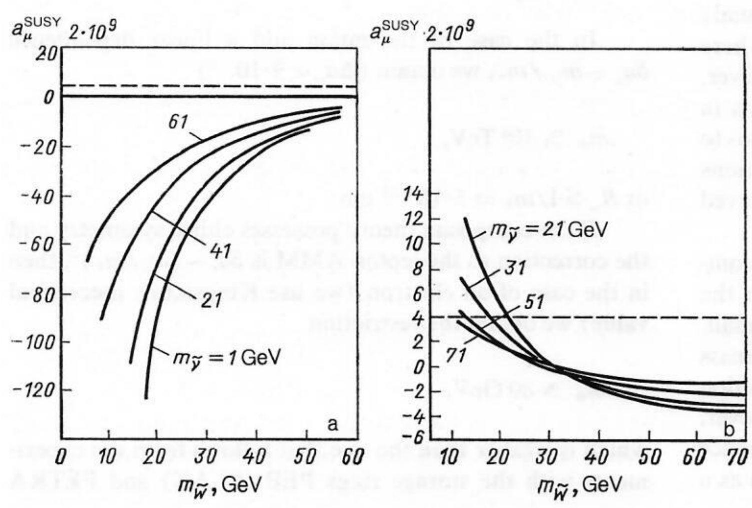

FIG. 15. Dependence of the supersymmetric contributions $a_\mu^{\rm SUSY}$ to the muon AMM on the wino mass $m_{\widetilde{W}}$ for photino mass $m_{\tilde{\gamma}} = 7$ GeV for different sneutrino masses in two models (Ref. 149): a) the curves correspond to $m_{\tilde{\nu}} = 1$, 21, 41, and 61 GeV; b) the curves correspond to $m_{\tilde{\nu}} = 21$, 31, 51, and 71 GeV. The broken lines are the contributions of the weak interactions in the standard Weinberg–Salam model.

that, as a rule, the value of an AMM depends simultaneously on several parameters (for example, on the coupling constants and masses of the particles). Therefore, in a certain sense it may be more effective to make a test based on values of the lepton AMMs, not of specific models, but deduced from more general concepts on which the models are based and which are determined by one or two parameters.

In this connection, let us analyze the concept of composite particles without going into the details of particular models that realize it. In accordance with the main idea of the composite theories, the well-known difficulties of the standard $SU(3)\times SU(2)\times U(1)$ model can be overcome if the leptons, quarks, and bosons are unified on the basis of a small number of common constituents.[156-160] We note that the direct tests of quantum electrodynamics at high energies in collisions of electron and positron beams impose restrictions on the scale at which the composite nature of the particles is manifested and indicate that leptons behave as point particles right down to distances of order 10^{-16} cm. Let us consider the question of the value of the AMM of a composite lepton.[55,161] It in no way follows *a priori* that in composite theories the g factor of a lepton must be equal to 2. If we regard a lepton as a truly composite particle, two circumstances must be taken into account—the lepton must be simultaneously light and small. These two requirements impose very stringent requirements on the theory of composite leptons.

In the case of an ordinary bound nonrelativistic system (atom, nucleus) its size is determined by the relation $R\sim(ME_B)^{-1/2}>1/M$, where M is the mass, and $E_B<M$ is the binding energy. However, in the case of leptons, for example, for an electron, the opposite relation holds, namely, the radius of the electron and, therefore, of its possible constituent parts satisfies $R_e\sim(ME_B)^{-1/2}>10^{-16}$ cm, and this value is appreciably less than the Compton wavelength of the electron: $\lambda_e^C\sim 1/m_e\sim 4\cdot 10^{-11}$ cm. Thus, any theory of a composite electron must explain why its mass is small compared with its radius: $R_e^{-1}\gtrsim 100$ GeV$\gg m_e$.

The simplest model is to assume that the electron consists of strongly bound particles—preons with very large mass $m^*\gg m_e$. In this case, it can be shown that the correction δa to the AMM will satisfy the estimate

$$\delta a\sim O\,(m_e/m^*). \tag{50}$$

This result reflects the fact that the natural (usual) scale of the magnetic moment has the order $\mu\sim eR$, where $R\sim m_*^{-1}$ is the characteristic radius of the system. However, it must be noted that any model of composite leptons in which (50) is valid immediately leads to large corrections to the lepton self-energy, and thus there are large corrections $\delta m_l\sim O(m_*)$ to the lepton masses, whereas the observed lepton masses are negligibly small compared with m_*.

An important requirement imposed on models of composite leptons is that the additional contributions to the AMM and to the lepton mass must be simultaneously small.

The problem of obtaining a small correction to the mass can be lifted if, for example, it is assumed that the correction is compensated by a large bare mass of the lepton. However, as was noted in Ref. 161, a more natural way is to introduce one further large mass scale, i.e., to regard the lepton as a bound state of a fermion of mass m_f and a heavier boson with mass $m_B\gg m_f$, which itself can be a bound state of two leptons of mass m_f. In this case, the additional contribution to the AMM due to the structure of the lepton will satisfy the estimate

$$\delta a_l\sim O\left(\frac{m_l m_f}{m_B^2}\right).$$

Another way of avoiding the appearance of large corrections to the lepton mass foresees a construction of the interaction which is such that the right- and left-helicity fermions that are the composite parts of the lepton (and have large mass $m_*\gg m_l$) arise with equal weight in the wave functions of the composite lepton. Such theories possess chiral symmetry, and the amplitudes of processes must be symmetric with respect to the substitution $m_*\to -m_*$, and therefore the correction to the AMM cannot contain terms $\sim m_l/m_*$, so that we can expect

$$\delta a_l\sim O\left(\frac{m_l^2}{m_*^2}\right). \tag{51}$$

Let us estimate the restrictions on the mass scales of the preons that follow from the existing discrepancies between theoretical and experimental values of the electron and muon AMMs. Suppose that the estimate (50) is valid; then, if Samuel's calculations for the electrodynamic contribution to the electron AMM (see Sec. 1) are correct, $a_e^{\rm th}-a_e^{\rm exp}=6.09(1.49)\cdot 10^{-10}$, we obtain for the preon mass

$$m_*\gtrsim 10^3\ \text{TeV}. \tag{52}$$

If for the theoretical value of the electron AMM we use the result obtained by Kinoshita, the discrepancy between the theory and experiment will be reduced by approximately a factor of 2, and

$$m_*\gtrsim 2\cdot 10^3\ \text{TeV}. \tag{53}$$

These restrictions on the energy scale of the electron structure are much more stringent than those that follow from the accelerator experiments in high-energy physics. In particular, it follows from (53) that the electron must be regarded as a point particle right down to distances of order

$$R_e\lesssim\frac{1}{m_*}\approx 10^{-20}\ \text{cm}.$$

In the case of the muon and a linear dependence $\delta a_\mu\sim m_\mu/m_*$, we obtain ($\Delta a_\mu<9\cdot 10^{-9}$)

$$m_*\gtrsim 10^4\ \text{TeV},$$

or $R_\mu\lesssim 1/m_*=5\cdot 10^{-21}$ cm.

If the composite theory possesses chiral symmetry and the correction to the lepton AMM is $\delta a_l\sim(m_l/m_*)^2$, then in the case of an electron (we use Kinoshita's theoretical value) we obtain the restriction

$$m_*\gtrsim 30\ \text{GeV},$$

which is weaker than the one that follows from the experiments with the storage rings PEP (SLAC) and PETRA

(DESY).[162] In the case of the muon and the quadratic dependence (51), we obtain a restriction on the preon mass at the level $m_* \gtrsim 1$ TeV, and the muon must be regarded as a point particle down to distances

$R_\mu \leqslant 0.5 \cdot 10^{-17}$ cm.

We now consider some consequences of the existence of structure of the bosons of the standard model of the electroweak interactions. To determine whether the axial-vector bosons really are gauge particles or whether they are composite, it is necessary to make a detailed study both of their interaction with leptons and of the interaction between the bosons themselves.

If the Z and W bosons are composite particles, then we can expect (as in the case of hadrons, which consist of quarks) that the first manifestations of structure will be the form-factor corrections to the propagators and vertex functions. However, it should be noted that there are other possibilities for modification of the predictions of the standard model by taking into account structure of the bosons (for example, by considering excited bosons or new nonstandard interactions between the bosons).[127]

It is natural that the propagators and vertex functions for composite bosons will depend on a characteristic parameter Λ_B, the scale at which the composite nature is manifested. If we compare the restrictions on the possible scale at which the structure of fermions is manifested, $\Lambda_f \gtrsim 1$ TeV (Refs. 162 and 163), which follow from accelerator experiments, with the scale Λ_B for the gauge bosons, we can draw the conclusion that at the present time such stringent bounds do not exist for Λ_B. This is explained by the fact that hitherto the properties of the bosons have been little studied, and there is particularly little information about the interactions of the virtual bosons at energies appreciable compared with their mass. Therefore, bearing in mind the absence of stringent bounds on the structure of the bosons, and also the level of the energies attained in the experiments, we can assume that the scale at which the composite nature of the fermions is manifested satisfies $\Lambda_f \gg \Lambda_B$. If this is not the case, it would be necessary to take into account as well the form factors associated with the structure of the fermions. Below, following Refs. 127 and 162–164, we shall consider possible values of Λ_B in the range 0.1 TeV$\leqslant \Lambda_B \leqslant 1$ TeV.

In the most general case, the presence of structure of the bosons can be described[162,163] by a modification of the propagators through the introduction of a certain function F:

$$\frac{1}{p^2-M^2} \to \frac{1}{p^2-M^2} F(p^2, \Lambda_B),$$

this function depending on the boson momentum and the scale Λ_B. It is natural to assume that $F(p^2, \Lambda_B) \to 1$ as $p^2/\Lambda_B^2 \to 0$, i.e., we arrive at the standard model in the limit $\Lambda_B \to \infty$. In addition, if we do not wish to enhance the divergences of the gauge theory, F need not be an increasing function of p^2, and in what follows we shall assume that $F(p^2, \Lambda_B) \to 0$ as $p^2 \to \infty$. Finally, a further condition that determines the form of the function F is that we do not want to change the pole properties of the propagator, and we will, as before, assume that the mass of a particle is determined by the pole of its propagator. Although these restrictions do not uniquely fix the form of the function F, they are quite stringent and enable us to choose it in the form

$$F(p^2, \Lambda_B) = \frac{1}{1+\rho p^2/\Lambda_B^2}, \quad \rho p^2 > 0, \ \rho = \pm 1.$$

Generally speaking, it must also be borne in mind that the vertex functions that describe the γWW interaction on the transition to composite bosons acquire an analogous additional factor: $\Gamma_{\mu\nu\lambda} \to \Gamma_{\mu\nu\lambda} F_1(s/\Lambda_B)$. The vertex function $\Gamma_{\mu\nu\lambda}$ contains charge, magnetic dipole (μ_W), and electric quadrupole (Q_W) interactions,[165] and each of these can be characterized by a corresponding form factor with different value and different behavior as $s \to 0$. A general picture of this kind is very complicated,[162] and therefore we shall simplify it by taking into account only the form-factor corrections in the boson propagators, and we choose the static moments of the W boson to be the same as in the standard model (see below). The modifications that arise because the bosons are composite will be much more complicated, but this simplified approach gives a picture of the order of the possible effects.

We shall obtain expressions for the additional contributions to the AMM of a lepton moving in a magnetic field which arise because the bosons are composite (we shall consider not only Z and W bosons but also the scalar Higgs bosons) [122,166–168]

Calculating the corresponding contributions to the lepton mass operator using the modified boson propagators for the additional contributions to the lepton AMM that arise because the bosons are composite, we obtain

$$\Delta_l^B = -\frac{1}{M_B^2+\Lambda_B^2}\left[M_B^2 a_l(M_B^2, \chi) + \Lambda_B^2 a_l(-\Lambda_B^2, \chi)\right], \quad (54)$$

where $B = Z, W, H$, and Λ_B is the scale corresponding to the composite nature of the boson of species B. The functions $a_l(M_B^2, \chi)$ depend on the boson mass M_B and on the invariant dynamical field parameter χ. We note that the functions $a_l(M_B^2, \chi)$ describe the corresponding contributions of point bosons to the AMM of a lepton in an external field [see (43)]. The functions $a_l(-\Lambda_B^2, \chi)$ differ from $a_l(M_B^2, \chi)$ only by the substitution $M_B \to -\Lambda_B^2$. Further, using (54) in conjunction with (44) and (56), we obtain asymptotic expressions for the contributions to the AMM which arise because the bosons are composite:

$$\left.\begin{aligned}
\Delta_l^Z = &-\frac{G_Z^2}{2\pi^2}(\lambda_Z+\lambda_\Lambda)^{-1}\Big\{-\frac{1+\bar{\alpha}^2}{2}\Big[\frac{4}{3}+O(\lambda_Z^{-1}, \lambda_\Lambda^{-1}) \\
&+\chi^2\Big(\lambda_Z^{-3}\Big(2\ln\lambda_Z-\frac{217}{30}\Big)-\lambda_\Lambda^{-3}\Big(2\ln\lambda_\Lambda-\frac{217}{30}\Big)\Big)\Big] \\
&+\frac{1-\bar{\alpha}^2}{2}\Big[2+O(\lambda_Z^{-1}, \lambda_\Lambda^{-1})+\chi^2\Big(\lambda_Z^{-3}\Big(4\ln\lambda_Z-\frac{79}{5}\Big) \\
&\qquad -\lambda_\Lambda^{-3}\Big(4\ln\lambda_\Lambda-\frac{79}{5}\Big)\Big)\Big]\Big\}, \quad \lambda_\Lambda = \Lambda_B^2/m_l^2; \\
\Delta_l^W = &-\frac{G_W^2}{2\pi^2}(\lambda_W+\lambda_\Lambda)^{-1}\Big[\frac{5}{3}+O(\lambda_W^{-1}, \lambda_\Lambda^{-1}) \\
&\qquad +\frac{1}{10}\chi^2(\lambda_W^{-3}+\lambda_\Lambda^{-3})\Big]; \\
\Delta_l^H = &-\frac{G_H^2}{4\pi^2}(\lambda_H+\lambda_\Lambda)^{-1}\Big[\ln\lambda_H+\ln\lambda_\Lambda-\frac{7}{3} \\
&+O(\lambda_H^{-1}, \lambda_\Lambda^{-1})+\frac{2}{3}\chi^2(\lambda_H^{-2}+\lambda_\Lambda^{-2})\Big].
\end{aligned}\right\} \quad (55)$$

For the experiments at CERN and BNL, the dynamical parameter satisfies $\chi \ll 1$, and we therefore ignore the terms proportional to χ^2. Therefore, with allowance for the fact that in the case of the muon $\lambda_B = M_B^2/m_\mu^2 \gg 1$ and $\lambda_\Lambda = \Lambda_B^2/m_\mu^2 \gg 1$, we obtain from (55) estimates of the discrepancy between the theoretical predictions for the muon AMM in the standard model and in the model with composite bosons:

$$\Delta_\mu^{Z,W} = \frac{-2}{1+\Lambda_B^2/M_{Z,W}^2} a_\mu^{Z,W}, \quad \Delta_\mu^H = \frac{-2}{1+\Lambda_B^2/M_H^2} \times \left[1 + \frac{\ln \lambda_\Lambda - \ln \lambda_H}{2\left(\ln \lambda_H - \frac{7}{6}\right)}\right] a_\mu^H. \quad (56)$$

We can now obtain bounds on the scale for composite bosons by comparing the experimental and theoretical (obtained in the standard model) values for the muon AMM by requiring that the corrections (56) to the AMM due to the composite nature of the bosons be less than the difference: $|\Delta a_\mu| = |a_\mu^{\exp} - a_\mu^{\rm th}| < 9\cdot 10^{-9}$. Because of the smallness of the constant $G_H \sim m_\mu/M_W$, we ignore the contribution of the Higgs boson; this simplification can be made if its mass is greater than the muon mass (see above). If $\Lambda_B \lesssim 100$ GeV, the additional contribution $|\Delta_\mu^Z + \Delta_\mu^W|$ is less than the discrepancy $|\Delta a_\mu|$, and we arrive at a weaker restriction (than the requirement $\Lambda_B \gtrsim 100$ GeV) at the existing accuracy with which the muon AMM is measured. But if the planned increase by 20 times in the accuracy with which the muon AMM is determined at BNL is accompanied by a similar decrease in the discrepancy $|\Delta a_\mu|$ between the theoretical and experimental values, it will be possible to test the hypothesis of composite Z and W bosons at the energy level $\Lambda_B \sim 250$ GeV.

The connection between the muon AMM and the energy scale at which the nonpointlike structure of the vector bosons would be manifested can also be established in another way.[169-171] The point is that the magnetic moment of the charged vector W boson has a significant influence on the muon AMM and could play a decisive role in establishing whether the boson does have structure, since the magnetic moment of a composite W boson will differ from the value given by the standard gauge model.[172]

In the general case, the local γWW interaction is, with allowance for the requirement of C, P, and T invariance and conservation of the electromagnetic current, characterized by two independent parameters k_0 and λ_0, which are related to the magnetic dipole moment μ_W and the electric quadrupole moment Q_W (Ref. 165): $\mu_W = (e/M_W)(1 + k_0 + \lambda_0)$, $Q_W = -(e/M_W^2)(k_0 - \lambda_0)$. In the standard SU(2) $\times$U(1) gauge theory, these parameters have the definite values $k_0 = 1$ and $\lambda_0 = 0$ [apart from radiative corrections $\sim O(e^2/4\pi)$]. However, if the W boson is a composite particle, this will be reflected in the magnetic dipole and (or) electric quadrupole moments of the boson. In the case of a nongauge theory, the anomalous values of k_0 and λ_0 could lead to significant deviations in the value of the muon AMM.

Let us suppose, as is done in Refs. 169 and 170, that $\lambda_0 = 0$, $k_0 \neq 1$; then we shall have two free parameters: k_0 and the scale Λ_B. If we assume that the interaction of the fermions with photons and with the W and Z bosons is not changed, then the Lagrangian of such a theory will have the form

$$\mathscr{L} = \mathscr{L}_{\rm SM} - {\rm i}\, e(1-k_0) W_\mu^+ W_\nu F^{\mu\nu},$$

where $\mathscr{L}_{\rm SM}$ is the Lagrangian of the standard model.

As a result of calculations,[169] making a cutoff of the divergent integrals at Λ_B and using the Feynman parametrization, we can obtain

$$\Delta a_\mu = \frac{e^2(1-k_0)}{64\pi^2 \sin^2\theta_W}\left(\frac{m_\mu}{M_W}\right)^2 \left\{\ln\eta - \frac{2-\eta}{\eta(1-\eta)} - \frac{2+\eta^2}{\eta^2}\ln(1-\eta) + O(\eta)\right\}, \quad \eta = \frac{M_W^2}{\Lambda_B^2}. \quad (57)$$

We are interested in the dependence of this expression on the scale Λ_B at which the composite nature of the W boson is manifested for different values of the boson AMM k_0.

We again compare the calculated correction with the experimental restrictions on the muon AMM, making the assumption[150] that $-42\cdot 10^{-10} < \Delta a_\mu < 96\cdot 10^{-10}$. Figure 16a shows the correction Δa_μ to the muon AMM as a function of the parameter Λ_B for different values of the W-boson AMM. The hatched regions are precluded by the experimental bounds. Figure 16b shows the region of allowed and forbidden values of k_0 as a function of the scale Λ_B. In the region of small Λ_B, the graphs are shown by broken lines, since the expression obtained for the correction is valid to accuracy $\sim O(\eta)$.

The study permits the following conclusions to be drawn:

1. A large deviation of k_0 from $k_0 = 1$ (large $|k_0 - 1|$) is possible if the parameter Λ_B is not very far from the value M_W.

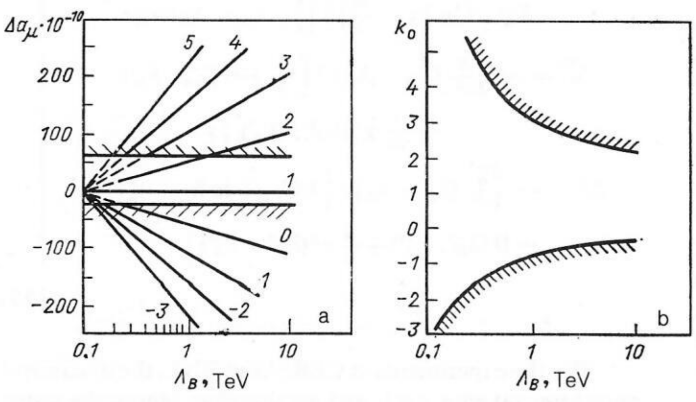

FIG. 16. Dependence of the correction Δa_μ to the muon AMM for different values of the W-boson AMM k_0 (a) and the regions of allowed and forbidden values of k_0 (b) as functions of Λ_B. The forbidden regions are hatched.

2. On the other hand, if $\Lambda_B \gg M_W$, then k_0 cannot differ significantly from its value in the standard theory.

3. If $\Lambda \sim 1$ TeV, then the W-boson AMM can take values in the range $-1 \lesssim k_0 \lesssim 3$.

4. If the scale at which a composite nature of the W boson is manifested is $\Lambda_B \sim 1$ TeV, then even a small deviation $|\Delta k_0| \sim 0.2$ from the standard value $k_0 = 1$ leads to a change in the muon AMM by $|\Delta a_\mu| \sim 1.1 \cdot 10^{-9}$.

Note that all the restrictions obtained in this section on the structure of leptons and bosons deduced from the values of the lepton AMMs are fairly general and do not depend on the particular model in which the principle of composite particles is realized.

CONCLUSIONS

The theoretical and experimental study of the AMMs of charged leptons plays at the present time an important part in the development of our ideas about the interactions of elementary particles and the properties of the theories that describe them.

The anomalous magnetic moments of charged leptons, being dynamical quantities in nature, depend essentially on the energies of the particles, and also on external factors such as external electromagentic fields or the presence of conducting surfaces. In addition, the lepton AMMs are sensitive to the vacuum structure, and therefore the existing estimates of the discrepancy between the theoretical and experimental results for the electron and muon AMMs impose very stringent restrictions on all alternative theories and exotic interactions, and each new model that is proposed for the interaction of the elementary particles must be tested to see whether its predictions agree with the experimental values for the lepton AMMs.

Comparison of the theoretical and experimental values of the electron AMM is a criterion for the validity of quantum electrodynamics, while comparison of the corresponding quantities for the muon AMM must also take into account the significant contribution of the strong interactions. After realization of the 20-fold increase in the accuracy of the experimental determination of the muon AMM planned at the Brookhaven National Laboratory and a certain increase in the accuracy with which the contributions of the strong interactions are calculated, the comparison of theory and experiment will also require allowance to be made for the contributions of the weak interactions, and this will therefore present a further possibility for testing the validity of the standard Glashow–Weinberg–Salam model.

In particular, if the mass M_H is small, comparison of a_μ^{th} and a_μ^{exp} will make it possible to measure the contribution of the standard Higgs boson.

The results of the theoretical and experimental investigations of the AMMs of the charged leptons are important not only for the physics of elementary particles but also have a larger significance. The tremendously high accuracy of the calculations and measurements make it possible to obtain on the basis of the electron AMM the currently most accurate value of a fundamental physical constant—the fine-structure constant.[173] Calculations using the theoretical (19) and experimental (21) values for the electron AMM lead to the value[9]

$$\alpha^{-1}_{(g-2)} = 137.0359914\ (11), \qquad (58)$$

where the relative error is $0.0081 \cdot 10^{-6}$, in which the main part ($0.0072 \cdot 10^{-6}$) is associated with the error in the theoretical calculations, while the experimental error is approximately $0.0037 \cdot 10^{-6}$. This value is more accurate than the new values for α that are obtained from measurements of the quantum Hall effect,[17]

$$\alpha^{-1} = 137.0359979\ (33) \quad (0.024 \cdot 10^{-6}),$$

or from the Josephson effect,[175–177]

$$\alpha^{-1} = 137.0359769\ (77) \quad (0.056 \cdot 10^{-6}),$$

or from the hyperfine splitting of muonium,[175–177]

$$\alpha^{-1} = 137.0359925\ (224) \quad (0.17 \cdot 10^{-6}).$$

Note that if we use the theoretical result of Samuel,[22] then for the fine-structure constant we obtain[21]

$$\alpha^{-1}_{(g-2)} = 137.0360302\ (76).$$

The errors in the theoretical and experimental values of the muon AMM lead to a much less accurate value for α.

There is another problem that can be solved on the basis of the values of the anomalous magnetic moments of charged leptons and has a general theoretical significance. It is the fundamental principle of matter–antimatter symmetry, which is a consequence of the so-called CPT theorem, which was formulated by Lüders[178] and in accordance with which a quantum field theory with a local Lorentz-invariant Hermitian Hamiltonian must be invariant with respect to the action of the product of the operators of charge conjugation C, parity P, and time reversal T.[13] One of the consequences of the CPT theorem is equality of the magnetic moments of a particle and its antiparticle. Therefore, comparison of the experimental values for the AMMs of a lepton and its antilepton makes possible a test of the CPT theorem. In the case of exact invariance, the ratio of the g factors of the particle and the antiparticle must be equal to unity: $g_{e^-}/g_{e^+} = 1$. Such a comparison can be made on the basis of the results of two experimental groups: first, using the independently measured values of the AMMs for nonrelativistic electrons and positrons confined in a Penning trap,[21]

$$g_{e^-}/g_{e^+} = [1 + (0.5 \pm 2.1)] \cdot 10^{-12},$$

and second, by direct measurement of the difference between the AMMs of electrons and positrons moving under identical conditions, as made in the storage ring VEPP-2M at Novosibirsk: (Ref. 179) $|a_{e^+} - a_{e^-}|/a_e < 1 \cdot 10^{-8}$ (95% confidence level), the difference from unity of the ratio g_{e^-}/g_{e^+} in this case being $\sim 10^{-12}$. We should point out that in the second case a difference between the spin-precession frequencies (leading to a deviation of the ratio g_{e^-}/g_{e^+} from unity) can arise not only because of a difference between the electron and positron AMMs but also because of nonequality of the masses, charges, or normal magnetic moments of the particle and antiparticle. However, an advantage of the second approach over the first is that the measurements are made with electrons and positrons under identical conditions, whereas in the experiments of the University of Washington the electron and positron AMMs are measured separately.[21,179]

One can also attempt to explain the difference between the theoretical [see Eqs. (19) and (22)] and experimental values for the electron AMM at a level between two and four standard deviations (under the assumption that the estimates of the errors in the calculations and in the measurements are not too low) not only as a consequence of certain new interactions but also in the framework of existing electrodynamics through a deviation of the space–time dimension ρ from 4 (Ref. 180).

Using the estimates of the difference between a_e^{th} and a_e^{exp} and the Hausdorff definition of the dimension of space–time,[180] one can obtain for the deviation $\Delta = 4 - \rho$ the value

$$\Delta \sim (0.5 - 1.6)\cdot 10^{-6}.$$

Qualitatively, this result can be explained by the circumstance that a decrease of the dimension leads to a decrease of the vacuum fluctuations around an electron, and, as a consequence, the radiative corrections (including the AMM) becomes less than in standard quantum electrodynamics.

In conclusion, we note that all the achievements in the investigation of the anomalous magnetic moments of charged leptons, which are of great interest for the solution of fundamental problems, are due to decisive collaboration and rivalry of theoreticians and experimentalists, and studies of the anomalous magnetic moments of leptons can be identified as a rapidly developing direction of modern physics.

I thank I. M. Ternov and V. N. Rodionov for joint investigation of problems associated with the anomalous magnetic moments of leptons, and also V. M. Mostepanenko and A. A. Tyapkin for their interest in the work and for valuable discussions.

[1)] The first stage of the LEP accelerator was commissioned in August 1989.

[2)] A further promising direction of the "nonaccelerator" low-energy approach to the investigation of particle properties is comparison of calculations and measurements of Casimir forces.[2-4]

[3)] This is explained by the fact that the muon mass is appreciably greater than the electron mass, and therefore $(g-2)_\mu$ is more sensitive than $(g-2)_e$ to the physics at short distances.

[4)] A general expression was obtained analytically in Ref. 32 for the tensor that takes into account the contributions of light–light scattering to a lepton AMM; in the case of the muon AMM a result which agrees with that of Refs. 22 and 31 follows from it.

[5)] I am grateful to É. A. Kuraev for the information provided.

[6)] Analysis of the experimental results for the differential cross section of the reaction (36) obtained by various groups (CELLO, JADE, TASSO, PLUTO, and others) up to 1986 led to the estimate (Ref. 182) $a_\tau = 0.0023 \pm 0.0052$.

[7)] Among the studies listed above, exceptions in this sense are those of Refs. 24, 49, and 62–69.

[8)] The Z-boson contribution to the lepton AMM was calculated by the method of analytic continuation[83] in Ref. 99.

[9)] If with the 20-fold increase in the accuracy with which the muon AMM is measured at BNL the discrepancy $|\Delta a_\mu| = |a_\mu^{\text{exp}} - a_\mu^{\text{th}}|$ can be reduced as much, i.e., so that the estimate $|\Delta a_\mu| \lesssim 5\cdot 10^{-10}$ holds, then from the muon AMM there will effectively arise (with allowance for the results of Ref. 116) a restriction on the mass M_H: $M_H \gtrsim 700$ MeV (Ref. 185).

[10)] In the determination of the χ-dependent terms it was assumed that the bosons are fairly heavy ($\lambda_i = M_i^2/m_l^2 \gg 1$).

[11)] Note also that an estimate of the discrepancy $a_e^{\text{th}} - a_e^{\text{exp}}$ reduced by 10 times was used in Ref. 124, and this led the authors to bounds on the squares of the electron–boson coupling constants that are too small.

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