Contour gauge conditions in non-Abelian gauge theories

S. V. Ivanov

Institute of Physics, Rostov State University
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The most important problems in noncovariant gauge conditions of axial type are analyzed. The entire exposition is based on the path-dependent formalism. The standard noncovariant gauges are regarded as a special limiting case of contour gauge conditions. A simple derivation of a translationally noninvariant free gluon propagator in $n^\alpha A_\alpha(x) = 0, A_0(x) = 0$ gauges is proposed. The multiplicative renormalizability of the propagators in the contour gauges is proved. A spectral representation of the gluon propagator in the Lorentz-noncovariant approach is given. A method for constructing the infrared asymptotic behavior of the Green's functions on the basis of an analysis of the Wilson loop with cusp singularities is explained.

INTRODUCTION

We must first of all explain what we understand by a nonstandard name. The class of contour gauges is rather large. In fact, under certain mathematical assumptions practically all noncovariant gauges except the Coulomb gauge $\partial_i A^i(x) = 0$ belong to it. Noncovariant gauges of axial type are used actively in theoretical investigations devoted to the diverse problems of quantum field theory. It is true to say that the quantum theory of gauge fields actually arose 60 years ago in the noncovariant time gauge $A_0(x) = 0$ (Ref. 1). Of the specialist literature on this question Leibbrandt's excellent review,[2] which contains practically all fundamental information on noncovariant gauge conditions for the period up to 1986, is outstanding. However, in recent years there has been important progress in our understanding of specific details of noncovariant gauges in non-Abelian gauge theories. Some mathematical imprecisions allowed in quantum electrodynamics and in theories with self-interacting massless gauge fields lead, as was shown, to contradictory results. We shall attempt to outline ways of solving the existing problems in the quantization of non-Abelian gauge fields in noncovariant gauges. It may seem surprising that so much attention is devoted to the study of individual gauges. At the first glance this question seems to be very special and purely technical. Nevertheless, it is well known that a felicitous choice of gauge condition greatly simplifies the solution of several problems in field theory. In addition, contour gauges may possess properties that put them in a distinguished position in the search for nonperturbative approaches to the problems of quantum chromodynamics. A positive property of noncovariant gauges is the absence of interaction of the gauge fields with "ghost" fields, but by itself this fact yields little; indeed, from the point of view of the simplicity of calculations in perturbation theory ghostless gauges do not lead to a simplification but, rather, introduce additional problems associated with unphysical gauge poles. However, it is precisely in noncovariant gauges that some fundamentally important theorems and general propositions have been proved. These include the proof of ultraviolet finiteness of the $N = 4$ supersymmetric Yang–Mills model.[3] The planar gauge $n^\alpha A^a_\alpha(x) = B^a(x)$, $n^2 < 1$, has been widely and effectively used by many authors together with the axial gauge in perturbative QCD in the study of hard processes.[4–8]

An important property of contour gauges is the absence of Gribov copies,[9–11] and this means that in principle they can be used in nonperturbative approaches. It is possible that active use of this property of noncovariant gauges is still ahead of us.

In recent years, important progress has been noted in the analysis of infrared divergences in QCD; here, use has been made of the method of re-expansion of the gluon and quark propagators, and also of the external lines of Feynman diagrams with respect to the corresponding operators in the axial gauges with different gauge vectors $\{n_\mu^{(i)}\}$.[12–14] It has been asserted that all information about the infrared structure of hard processes is contained in the averaging of a set of path-ordered exponentials with specially chosen paths of integration. The appearance of such nonlocal gauge-invariant structures is a consequence of the corresponding re-expansion.[12] In two-particle processes the entire burden of the infrared divergences can be transferred by a felicitous choice of the gauge vector into radiative corrections to the quark or gluon propagator. This means that by analyzing the properties of the quark and gluon propagators in the axial gauges we can extract important information about the part played by long-wave gluon exchanges in hard reactions, for example, the doubly logarithmic asymptotic behaviors and the K factor.[15–18] Thus, the region of application of noncovariant gauges in the quantum theory of gauge fields is fairly large. This stimulates their active study in non-Abelian theories.

1. WHAT ARE THE PROBLEMS?

The fundamental problems can be revealed by the example of the time gauge $A_0(x) = 0$. In this case, as, incidentally, in any gauge of axial type, the canonical Hilbert space is larger than the space of physical states. An additional state is generated by the invariance with respect to the remaining, time-independent, gauge transformations. Such transformations obviously do not affect the condition $A_0(x) = 0$. In Abelian theory without interaction the electric field is

$$E_i = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = F_{0i} = \partial_0 A_i.$$

and the magnetic field is

$$F_{ij} = \varepsilon_{ijk} B_k, \quad B_k = -(\nabla \times \mathbf{A})_k.$$

The canonical equal-time commutation relations have the form

$$[E_i(t, \mathbf{x}), A_j(t, \mathbf{y})] = -i\delta_{ij}\delta^3(\mathbf{x} - \mathbf{y}). \tag{1}$$

We can readily obtain the equations of motion

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$$\partial_0 A_i = \mathrm{i}\,[H,\ A_i] = E_i;$$
$$\partial_0 E_i = \mathrm{i}\,[H,\ E_i] = \partial_j F_{ji} = (\nabla \times \mathbf{B})_i. \qquad (2)$$

It is obvious that the Gauss law $\nabla\cdot\mathbf{E} = 0$ does not follow from the Hamiltonian equations (2). Nevertheless, since the Hamiltonian H and $\nabla\cdot\mathbf{E}$ commute, they can formally be diagonalized simultaneously. A physical state is defined as one that satisfies the condition

$$(\nabla\cdot \mathrm{E})\ |\ \Phi_{\mathrm{phys}}\rangle = 0. \qquad (3)$$

However, if we use the relation (1), then

$$\langle\Phi_{\mathrm{phys}}|[(\nabla\cdot\mathbf{E}(t,\ x)),\ A_i(t,\ y)]|\Phi_{\mathrm{phys}}\rangle = -\mathrm{i}\frac{\partial}{\partial x_i}\delta^3(\mathbf{x}-\mathbf{y})\ \langle\Phi_{\mathrm{phys}}|\Phi_{\mathrm{phys}}\rangle, \qquad (4)$$

from which it follows, when (3) is taken into account, that the physical states are not normalizable. In quantum electrodynamics the solution to this problem is well known. It is necessary to replace Φ_{phys} by normalizable states Φ_ε that satisfy (3) in the limit $\varepsilon\to 0$. This procedure has been fairly well justified for an Abelian field by many authors.[19,20] We shall consider the problems that occur in non-Abelian theories. In this case the residual symmetry has the form

$$\hat{A}_i^{\omega}(t,\ \mathrm{x}) = \omega^{-1}(\mathrm{x})\,\hat{A}_i\omega(\mathrm{x}) + \frac{\mathrm{i}}{g}\,\omega^{-1}(\mathrm{x})\,\partial_i\omega(\mathrm{x}), \qquad (5)$$

with the matrix $\omega(\mathbf{x}) = \exp((\mathrm{i}/2)g\alpha^a(\mathbf{x})\lambda^a)$. The canonical momentum $\pi_i^a(t,\mathbf{x}) = \partial_0 A_i^a - \partial_i A_0^a + gf^{abc}A_0^b A_i^c \equiv G_{0i}^a$ is not a gauge-invariant quantity:

$$G_{0i}^a(A^{\omega}) = \omega^{-1}(\mathrm{x})\,G_{0i}^a\omega(\mathrm{x}). \qquad (6)$$

Using the canonical equal-time commutator

$$[\pi_i^a(t,\ \mathbf{x}),\ A_j^b(t,\ \mathbf{y})] = -\mathrm{i}\delta^{ab}\delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}), \qquad (7)$$

we can determine the operator of the residual gauge transformation:

$$\hat{A}_i^{\omega}(t,\ \mathrm{x}) = U^{-1}(\omega)\,\hat{A}_i(t,\ \mathrm{x})\,U(\omega), \qquad (8)$$

where the matrix

$$U(\omega) = \exp\left\{\mathrm{i}\int d^3x\,T^a(\mathbf{x})\,\alpha^a(\mathbf{x})\right\} \qquad (9)$$

is a natural generalization of the corresponding operator in QED. Thus,

$$T^a(\mathbf{x}) = (\delta^{ab}\partial_i - gf^{abc}A_i^c(\mathbf{x}))\,\pi_i^b(\mathbf{x}) \qquad (10)$$

generates residual gauge transformations. The operator $T^a(\mathbf{x})$ commutes with the Hamiltonian of the system and satisfies the non-Abelian commutation relations

$$[T^a(\mathbf{x}),\ T^b(\mathbf{y})] = \mathrm{i}gf^{abc}T^c(\mathbf{x})\,\delta^3(\mathbf{x}-\mathbf{y}). \qquad (11)$$

Following Ref. 21, we investigate the possibility of going over to the interaction representation. Formally, this means that

$$S(t,\ 0)\,HS^{-1}(t,\ 0) = H_0 + V(t) \qquad (12)$$

and

$$V(t) = \int d^3x\,\frac{g}{2}\left\{f^{abc}(\partial_i A_j^a - \partial_j A_i^a)A_i^b A_j^c + \frac{1}{2}gf^{abc}f^{adl}A_i^b A_j^c A_i^d A_j^l\right\}. \qquad (13)$$

If we now assume the existence of the limit

$$\lim_{t\to\infty} V(t) = 0, \qquad (14)$$

then we can make the canonical transformation

$$S(\infty,\ 0)\,HS^{-1}(\infty,\ 0) = H_0. \qquad (15)$$

The action of the operator S on the operator T^a is defined as

$$T_{\infty}^a(\mathbf{x}) = \lim_{t_\Phi\to\infty} T^a(t_\Phi,\ \mathbf{x})$$
$$= \partial_i\pi_i^{a,L}(\mathbf{x}) - gf^{abc}A_i^{c,L}(\mathbf{x})\,\pi_i^{b,L}(\mathbf{x}),$$
$$A_i^{a,L}(\mathbf{x}) = \frac{\partial_i\partial_j}{\Delta}A_j^a(\mathbf{x}),\quad \pi_i^{a,L}(\mathbf{x}) = \frac{\partial_i\partial_j}{\Delta}\pi_j^a(\mathbf{x}). \qquad (16)$$

It is readily verified that despite the vanishing of the commutator $[H_0, T_{\infty}^a(\mathbf{x})]$ the relation (11) is not satisfied, i.e., the generators $T_{\infty}^a(\mathbf{x})$ do not form a Lie group. Thus, from the relations (7), (10), (11), and (16) it can be seen that in the limit $t\to\infty$ the limit of the product of operators at equal times is not equal to the product of the limits. From this it follows that an operator $S(\infty,0)$ of a canonical transformation that diagonalizes the total Hamiltonian H and the free Hamiltonian H_0 does not exist. This means that the interaction representation does not exist.

In fact, all that we have said above is a consequence of the erroneous assumption that $\lim_{t\to\infty}V(t) = 0$. A possible way of solving the problem is to eliminate the longitudinal gluons at the very beginning from the Hamiltonian of the system in non-Abelian theory. However, in the Coulomb gauge this leads to the appearance of Gribov copies.[9,11] In eliminating the residual symmetry one must not forget the possibility of taking into account field configurations with nontrivial topology.

A reflection of the general problem is the difficulties which arise in specific calculations of gauge-invariant quantities in noncovariant gauges. These problems appear in the nonleading orders of perturbation theory, i.e., where the non-Abelian structure of the theory shows up in the interaction of the longitudinal and transverse components of the gluon fields. It is paradoxical that hitherto investigations have been made with the standard propagator of the gluon field in the $n^\alpha A_\alpha(x) = 0$ gauge:

$$D_{\mu\nu}(x,\ x') = \int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{e}^{\mathrm{i}k(\mathbf{x}-\mathbf{x}')}}{k^2+\mathrm{i}\delta}\times\left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{(kn)} + \frac{n^2}{(kn)^2}k_\mu k_\nu\right), \qquad (17)$$

the expression (17) giving for $n = (1, 0, 0, 0)$

$$D_{ij}(x,\ x') = -\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{e}^{\mathrm{i}k(\mathbf{x}-\mathbf{x}')}}{k^2+\mathrm{i}\delta}\left(\delta_{ij} - \frac{k_i k_j}{k_0^2}\right). \qquad (18)$$

The fact is that the expression (17) is only a semifinished product and has no definite meaning until we have found a way to fix the rule for avoiding the gauge poles $1/(kn)$.[2] This can be done by solving the problems associated with the residual symmetry. We emphasize once more that the problem is important precisely in non-Abelian theories. The studies of Caracciolo, Curci, and Menotti[22,23] were crucial in the investigation of the ambiguity in the definition of the gluon propagator. In the fourth order in the coupling constant g they calculated in the $A_0(x) = 0$ gauge the Wilson loop

$$W_c = N^{-1}\langle|\ \mathrm{tr}\ P\exp(\mathrm{i}g\oint dz^\mu A_\mu(z))\ |\rangle. \qquad (19)$$

The contour lies in the (t,x) plane. In the limit $t_2 - t_1 = T \to \infty$,

$$W_c = \text{const} \exp\left(-\mathrm{i}TV(L)\right), \tag{20}$$

where $V(L)$ is the potential energy of the interaction of two static charges separated by distance L.

The transverse part of the free gluon propagator is

$$\langle | T A^a_{\perp i}(x) A^b_{\perp j}(y) | \rangle = \mathrm{i}\delta^{ab}\left(\delta_{ij} - \frac{\partial_i \partial_j}{\Delta}\right) \Delta_F(x-y), \tag{21}$$

where $\Delta_F(x-y)$ is the well-known Feynman propagator. The general expression for the longitudinal part of the gluon propagator that satisfies the equation of motion has the form

$$D^L_{ij}(x, y) = \mathrm{i}D(t, t') \int \frac{d^3k}{(2\pi)^3} \mathrm{e}^{\mathrm{i}\bar{\mathbf{k}}(\mathbf{x}-\mathbf{y})} \frac{k_i k_j}{\mathbf{k}^2}, \tag{22}$$

where

$$D(t, t') = -\frac{1}{2}(t-t') + \frac{1}{2}\alpha(t+t') + \gamma, \tag{23}$$

in which α and γ are arbitrary constants. Obviously, $D(t, t') \to \infty$ as $t, t' \to \infty$. The prescription for avoiding the pole $1/k_0^2$ in the expression (18) in the sense of the principal value corresponds to the choice $\alpha = 0$, and then from (23) translational invariance of the gluon propagator is recovered.[1)] The expansion of the expression (19) in the g^2 approximation leads to the diagrams of Fig. 1. The broken lines represent the longitudinal part of the gluon propagator. The contributions from the diagrams with the transverse part of the gluon propagator are suppressed by inverse powers of $t_2 - t_1 = T$ as $T \to \infty$. The three diagrams of Fig. 1 give

$$\begin{aligned} W_c^{(2)} &= -g^2 C_F \int_0^L dx \int_0^L dy \left[\theta(x-y) D^L_{\parallel}(t_2, t_2; x, y) \right. \\ &\quad \left. - D^L_{\parallel}(t_2, t_1; x, y) + \theta(y-x) D^L_{\parallel}(t_1, t_1; x, y)\right] \\ &= \frac{\mathrm{i}g^2}{4\pi^{\omega}} C_F\, \Gamma(\omega - 1) L^{2(1-\omega)} |T| \mu^{2\omega-3}; \quad \omega = \frac{1}{2}(n-1), \end{aligned} \tag{24}$$

in which μ is the parameter of the dimensional regularization. As can be seen from (24), the parameters α and γ have canceled. Therefore, the g^2 approximation does not depend on the ambiguity in the definition of $D(t, t')$. In quantum electrodynamics one can show that in any order of perturbation theory there is no dependence of $D(t, t')$ on the parameters α and γ, and the result exponentiates. In QCD, the g^4 approximation leads to the diagrams of Fig. 2, which give non-Abelian contributions $\sim C_A C_F$. We consider the coefficients of T and T^2 separately. The T term is determined by the diagram of Fig. 2d. A calculation in the MS scheme gives

$$\begin{aligned} V(L) &= \frac{g^2 C_F}{(2\pi)^3} \int \frac{d\bar{p}}{\bar{p}^2} \exp(\mathrm{i}\bar{p}\bar{L}) \\ &\quad \times \left\{1 + \frac{g^2 C_A}{(4\pi)^2}\left[\frac{11}{6}\left(\ln\frac{4\pi\mu^2}{\bar{p}^2} - \gamma_E\right) + \frac{31}{9}\right]\right\}, \end{aligned} \tag{25}$$

which agrees with the calculation by Fischler[27] of the sum of the diagrams in the Feynman gauge. The T^2 terms from the diagrams of Figs. 2a and 2b, which are related by a Ward identity, and the diagram of Fig. 2c give together

$$\begin{aligned} &-\frac{1}{4} g^4 C_A C_F L^{4(1-\omega)} \left(\frac{\Gamma(\omega-1)}{4\pi^{\omega}}\right)^2 \mu^{4\omega-6} \\ &\quad \times \left[D^2(t_2, t_1) - \frac{1}{2} D^2(t_2, t_2) - \frac{1}{2} D^2(t_1, t_1)\right]. \end{aligned} \tag{26}$$

The diagram of Fig. 2d leads to the expression

$$\begin{aligned} &\frac{1}{4} g^4 C_A C_F L^{4(1-\omega)} \left(\frac{\Gamma(\omega-1)}{4\pi^{\omega}}\right)^2 \mu^{4\omega-6} \left\{\left[D^2(t_2, t_1)\right.\right. \\ &\left.\left. - \frac{1}{2} D^2(t_2, t_2) - \frac{1}{2} D^2(t_1, t_1)\right] - \frac{3}{4}(1-\alpha^2)(t_2 - t_1)^2\right\}. \end{aligned} \tag{27}$$

The T^2 terms in the non-Abelian structures cancel only if $\alpha^2 = 1$. This means that the result agrees with the calculations in the covariant gauges, i.e., the gauge invariance is not broken only for a special choice of the constant $\alpha \neq 0$, and this is not realized in any of the trivial prescriptions. This result can be readily generalized to the case of an arbitrary axial gauge with $n^2 \neq 0$. In addition, it is not obvious that in the higher orders of perturbation theory (g^6, etc.) gauge invariance can be recovered by the choice $\alpha = 1$.

A possible way of solving this problem was proposed by Slavnov and Frolov.[28] They formulated most clearly the proposition that in the $A_0 = 0$ gauge the Gauss law does not linearize for the asymptotic states:

$$\lim_{\dots} \mathrm{e}^{\mathrm{i}H_0 t} G^a \mathrm{e}^{-\mathrm{i}H_0 t} \neq G^a_0, \tag{28}$$

where $G^a = \partial_k \mathscr{E}^a_k + g f^{abc} \mathscr{E}^b_k A^c_k + j^a_0 = 0$; $\mathscr{E}^a_k = F^a_{0k}$ and $G^a_0 = \partial_k \mathscr{E}^a_k$.

The relation (28) is a consequence of the fact that the solution of the free equation for $\widehat{A}^L_k$ has rising asymptotics:

$$\left.\begin{aligned} A^{a,L}_k(t, \mathbf{x}) &= \frac{1}{2}\int_{t_1}^{t_2} |t - S|\, J^{a,L}_k(s, \mathbf{x})\, dS + C^a_k(\mathbf{x})\, t + D^a_k(\mathbf{x}); \\ C^a_k(\mathbf{x}) &= \frac{1}{(t_2 - t_1)}\left\{A^{a,L}_{k,i} - A^{a,L}_{k,j}\right. \\ &\quad \left. + \int_{t_1}^{t_2} S J^{a,L}_k(s, \mathbf{x})\, dS - \frac{1}{2}(t_2 + t_1)\int_{t_1}^{t_2} J^{a,L}_k(s, \mathbf{x})\, dS\right\}; \\ D^a_k(\mathbf{x}) &= A^{a,L}_{k,j} - \frac{1}{2}\int_{t_1}^{t_2} (s - t_1) J^{a,L}_k(s, \mathbf{x})\, dS - C^a_k(\mathbf{x})\, t_1. \end{aligned}\right\} \tag{29}$$

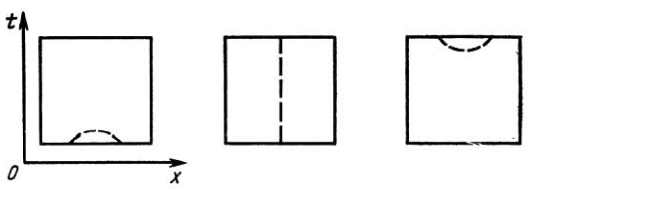

FIG. 1. Expansion of the expression (19) in the g^2 approximation. The square in the (t, x) plane represents the path of integration in the argument of the exponential.

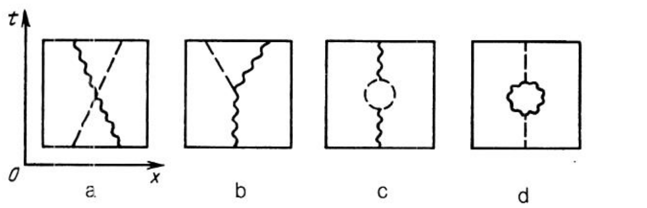

FIG. 2. Diagrams of the g^4 approximation. The broken lines denote the longitudinal propagator, and the wavy lines the total gluon propagator.

However, in perturbation theory

$$G_0 |\Phi\rangle = 0 \text{ and } (\partial_k \mathscr{E}_k^{L,a} + g f^{abc} \mathscr{E}_k^{L,b} \quad A_k^{L,c}) \,|\,\Phi\rangle \tag{30}$$

determine the same set of physical states

$$|\,\Phi\rangle = |\,\psi^T\rangle \otimes |\,0\rangle = |\,\psi^T,\ 0\rangle.$$

Therefore, the scattering matrix S in the space of physical states is unitary:

$$\langle E^L,\ \psi^L \,|\, S \,|\, \psi,\ 0\rangle \sim \delta\,(E^L). \tag{31}$$

In the expression $\langle E^L,\psi^T|S|\widetilde{E}^L,\psi^T\rangle$ the dependence on the transverse and longitudinal components of the field factorizes, and it is therefore sufficient to calculate $\langle\psi^T|S(J^T)|\psi^T\rangle$ and $\langle E^L|S(J^L)|E^L\rangle$. We emphasize that from the very beginning the procedure of the calculations is constructed in such a way that at all stages the residual gauge invariance is preserved:

$$\hat{A}_i^{\omega}(t,\ \mathbf{x}) = \omega^{-1}(\mathbf{x})\,\hat{A}_i\,(t,\ \mathbf{x})\,\omega\,(\mathbf{x}) + \frac{\mathrm{i}}{g}\,\omega^{-1}(\mathbf{x})\,\partial_i\omega(\mathbf{x}).$$

The expression for

$$S^T = \exp\left\{\mathrm{i}\int J_i^{a,\,T} A_{i(0)}^{a,\,T}(x)\,dx + \frac{\mathrm{i}}{2}\int J_i^a(x)\,D_{ij}^{ab,\,T}(x-y)\,J_j^b(y)\,dx\,dy\right\} \tag{32}$$

gives the three-dimensionally transverse part of the gluon propagator $A_{i(0)}^{a,T}$, the solution of the free equation. The expression for $\langle E^L|S(J^L)|E^L\rangle$ can be calculated by using the solutions (29). A key point is the unitarity of the total S matrix in the space of the physical states (31), which enables us to write

$$\langle 0,\ \psi^T\,|S|\,\psi^T,\ 0\rangle = \int \prod_x dE^L \exp\left\{\frac{\mathrm{i}}{2}\,\gamma\int d^3x\,E_k^L(\mathbf{x})\,E_k^L(\mathbf{x})\right\} \times \langle E^L,\ \psi^T\,|S|\,\psi^T,\ 0\rangle, \tag{33}$$

where γ is an arbitrary constant. The relation (33) leads to the expression

$$\langle 0\,|S\,(J^L)|\,0\rangle = \int dE^L \exp\left\{\frac{\mathrm{i}}{2}\,\gamma\int (E_k^L(\mathbf{x}))^2\,d^3x\right\} \times\langle E^L\,|S\,(J^L)|\,0\rangle \times\exp\left\{\frac{\mathrm{i}}{2}\int J_k^{a,\,L}(\mathbf{x})\,D_{kn}^{ab,\,L}(\mathbf{x},\ \mathbf{y})\,J_n^{b,\,L}(y)\,d\mathbf{x}\,d\mathbf{y}\right\}. \tag{34}$$

The expression (34) determines the translationally noninvariant longitudinal part of the gluon propagator in the $A_0 = 0$ gauge:[28]

$$D_{ij}^{ab,\,L}(x,\ x') = \frac{1}{2}\,(|x_0 - y_0| \pm (x_0 + y_0) + 2\gamma)\,\frac{\delta^{ab}}{(2\pi)^3}\int\frac{d^3k}{\mathbf{k}^2}\,k_i k_j e^{\mathrm{i}\bar{k}(\bar{\mathbf{x}}-\bar{\mathbf{y}})}. \tag{35}$$

Thus, we have obtained a result that explains the calculation of Ref. 22, but it is now necessary to show that translational invariance can be recovered in the gauge-invariant quantities in all orders of perturbation theory. In addition, the problems posed by Saradzhev and Faĭnberg[21] remain open.

2. FORMULATION OF THE PATH-DEPENDENT APPROACH

The basic idea of the traditional approach to the quantization of non-Abelian fields[29–31] is to separate the "volume" of each fiber $\mathscr{P}_x$ from the functional integral and go over to an integral over a surface in the manifold of all fields that intersects once each fiber of the bundle $\mathscr{P}(R^4,\pi)$. The problem of quantizing non-Abelian gauge fields is solved in the framework of perturbation theory. The main motive for the choice of some particular local gauge condition $\Phi(A, w) = 0$ was in practice always the desire to eliminate in the simplest possible manner the unphysical degrees of freedom; in QED uniqueness of the choice of the element of the group orbit was automatically ensured by the absence of photon self-interaction, but in non-Abelian theories the local gauge conditions usually lead to a system of nonlinear partial differential equations for the matrices $\hat{\omega}(x)$ of the adjoint representation of the corresponding group. These equations can be uniquely solved in the framework of perturbation theory under certain boundary conditions. Outside perturbation theory uniqueness of the solution is not realized.[9]

However, one can proceed in a different way, namely, one can propose an equation that must be satisfied by the matrices of the adjoint representation of the gauge group, this being such that it definitely has a unique solution and permits the possibility of transition to an integral over a surface in the field manifold that intersects once the orbits of the gauge group, and one can then obtain on the basis of the existing solution a condition on the gauge field. Naturally, in this case simplicity of the elimination of the unphysical degrees of freedom is not guaranteed. For implementation of such a procedure a geometrical interpretation of the gauge fields is convenient. We restrict ourselves to the introduction of only those definitions needed for the subsequent exposition. Detailed information about this question can be found in the monographs of Refs. 32 and 33.

In the general case, the fiber space $\mathscr{P}(x,\pi)$ consists of the set $\mathscr{X}$ of the bundle base, whose elements can be specified by means of coordinates $x = (x_1,...,x_n)$, the set $\mathscr{P}$, which is called the bundle over the base $\mathscr{X}$, and a projection of the bundle onto its base, i.e., a mapping π. To each point $x\in\mathscr{X}$ there corresponds a set of points $\mathscr{P}_x$, each of which goes over into x under the projection π. This entire set is called the fiber over the point x. The further development of the geometrical interpretation of gauge fields is associated with the introduction of definite structures on the fiber bundle. In particular, one can identify each fiber $\mathscr{P}_x$ with the group G. Then on the manifold $\mathscr{P}_x$ there will be defined the action of a group that carries each fiber into itself. Strictly, this in fact is the definition of the principal fiber bundle $\mathscr{P}(\mathscr{X},G,\pi)$. In the simplest case, the bundle is a direct product of the base and the group, $\mathscr{P} = \mathscr{X}\times G$, and this permits the introduction of coordinates of each point of the bundle $P = (x,g)x\subset\mathscr{X}$, $g\in G$ with projection $\pi(x, g) = x$. The action of the group G on the trivial fiber bundle shifts each point (x, g) along its fiber $\mathscr{P}_x$, i.e., the points P and P' belong to the same fiber if and only if $P' = Pg$ and g is an element of the group G. In the general case, any sufficiently small region in $\mathscr{P}$ can be represented as a direct product, i.e., it can be trivialized. If for every point $x\in\mathscr{X}$ one can choose in accordance with some rule a unique point $\sigma(x)\in\mathscr{P}_x$, then one says that a section of the bundle has been chosen. The existence of such a section permits one to construct a trivialization by associating with each point $P=\sigma(x)g\in\mathscr{P}_x$ the coordinates (x,g). In nontrivial cases, smooth sections exist only over sufficiently small regions $U\in\mathscr{X}$. In addition, it is by no means necessary that the complete set $\mathscr{X}$ be one-to-one mapable to a region of Euclidean space. In the general case, the coordinates are introduced on individual regions $\mathscr{X}$. Then on the intersection of the coordinate neighborhoods one can establish a rule for passing between the different coordinates: $x'_\mu=f_\mu(x_1,...,x_n)$. In the bundle it is natural to introduce a direction of displacements: vertical along the fibers and horizontal, which are invariant with respect to the action of the structure group. The specification of the horizontal directions is in fact specification of a connection in the principal fiber bundle. The fields of such directions are determined by vector fields

$$H_\mu=\frac{\partial}{\partial x_\mu}+\mathrm{i}\hat{A}_\mu g\frac{\partial}{\partial g},$$

where $\hat{A}_\mu(x)=A^a_\mu(x)\lambda^a$, and $\lambda^a=\{\lambda^a_{\alpha\beta}\}$ are the generators of the group. The coefficients $A^a_\mu(x)$ are arbitrary functions of x that do not depend on the vertical coordinates g. The last requirement is necessary, since otherwise the commutation relation

$$\left[\lambda^a g\frac{\partial}{\partial g},\ H_\mu\right]=0$$

does not hold. The fields A^a_μ determine the gauge field. The set of horizontal fields H_μ forms the horizontal subspace of the space tangent to the bundle $\mathscr{P}$ at a certain point. This permits the introduction of a procedure that is the inverse of the projection operation. The requirement that a certain vector at the point P be horizontal permits unique determination of it from its projection. A similar procedure can be carried out for a certain smooth curve $x(\tau)$ defined in the base R^4. Obviously, different curves of the manifold $\mathscr{P}$ can have the curve $x(\tau)$ as their projection. However, the requirement that a curve be horizontal at each of its points uniquely determines it from its projection $x(\tau)$. The curve in the bundle is characterized by the coordinates $P(\tau)=(x(\tau),g(\tau))$ and is called the lift of the curve $x(\tau)$. At the same time, the matrix element $g\equiv E$ of the group must satisfy the equation

$$\frac{\partial}{\partial\tau}X^\mu(\tau)H_\mu(A)E=\frac{\partial X^\mu}{\partial\tau}\left(\frac{\partial}{\partial X_\mu}E(\tau)+\mathrm{i}gA^a_\mu\lambda^a E(\tau)\right)=0. \quad (36)$$

Equation (36) is solved by the P-ordered exponential

$$E(A)=\left(P\exp\int\limits_{\gamma(x,y)}dz^\mu A_\mu(z)\right)E_{\gamma(y,y)}. \quad (37)$$

The P (path) ordering is a natural generalization of T (time) ordering. Matrices $\hat{E}$ of the structure group G which satisfy Eq. (36) are functions on the set of continuous curves and form a representation of the groupoid of paths P. (An element of the set P is a class of curves that differ in "appendices."[32]) In accordance with its definition, a horizontal curve in a bundle has a unique intersection with each fiber $\mathscr{P}_x$. This makes it possible, by fixing the initial point of a path in the bundle, $P(0)=(Y,\mathbb{1})$, and specifying the first coordinate by the choice of the path $\gamma(Y,x)$ in the base R^4, to sample all points $x\in R^4$, joining them by a given curve $\gamma(Y,x)$ to the initial point Y. Then over each x the intersection of the curve with a fiber distinguishes a unique element $E(\gamma(Y,x);A)$. This element of the group enables us to distinguish uniquely the elements of an orbit of the gauge field (Fig. 3):

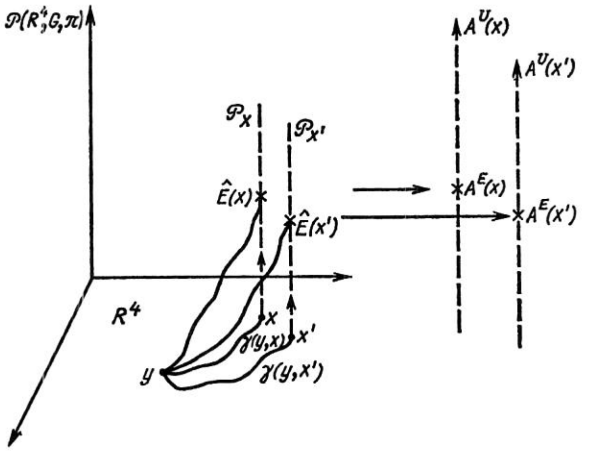

FIG. 3. The fixing of the path $\gamma(Y,x)$ in the base R^4 fixes its shape and initial point Y.

$$\hat{A}^E_\mu=\hat{E}^{-1}\hat{A}_\mu\hat{E}+\frac{\mathrm{i}}{g}\hat{E}\partial_\mu\hat{E}. \quad (38)$$

In accordance with the requirement of the definition of a horizontal field H, the field $\hat{A}^E_\mu$ does not depend on the vertical coordinates of the bundle. In other words,

$$\begin{aligned}\hat{A}^E_\mu&=E^{-1}(A)\,\hat{A}_\mu E(A)+\frac{\mathrm{i}}{g}\hat{E}^{-1}(A)\,\partial_\mu\hat{E}(A)\\&=E^{-1}(A^\omega)\,\hat{A}^\omega_\mu E(A^\omega)+\frac{\mathrm{i}}{g}E^{-1}(A^\omega)\,\partial_\mu E(A^\omega),\end{aligned} \quad (39)$$

where

$$\hat{A}^\omega_\mu=\omega^{-1}A_\mu\omega+\frac{\mathrm{i}}{g}\omega^{-1}\partial_\mu\omega.$$

The invariance of (39) holds up to global transformations. Equation (38) is equivalent to the expression (Refs. 12 and 34)[2)]

$$\begin{aligned}\hat{A}^E_\mu(x)=\hat{A}_\beta(y)\frac{\partial z^\beta}{\partial x_\mu}\Big|_{z=y}+\int\limits_{\gamma(y,x)}dz_\alpha\frac{\partial z^\beta}{\partial x_\mu}\hat{E}^{-1}(A)\,\hat{G}_{\alpha\beta}(A)\,\hat{E}(A)\\-\frac{\mathrm{i}}{g}\int\limits_{\gamma(y,x)}dz_\alpha\frac{\partial z^\beta}{\partial x_\mu}\hat{E}^{-1}(A)\left[\frac{\partial}{\partial z_\alpha},\ \frac{\partial}{\partial z_\beta}\right]\hat{E}(A),\end{aligned} \quad (40)$$

where

$$\hat{G}_{\alpha\beta}(A):=\partial_\alpha\hat{A}_\beta-\partial_\beta A_\alpha-\mathrm{i}g\,[A_\alpha,\ A_\beta].$$

The possibility of trivializing a bundle makes the choice of the "level" of the base nominal. Choosing vertical variables E over each point $x\in R^4$ and setting

$$E_\gamma(A)=P\exp\left\{\mathrm{i}g\int\limits_{\gamma(y,x)}dz^\alpha A_\alpha(z)\right\}=\mathbb{1} \quad (41)$$

at each point x, we effectively identify the base with the surface of the horizontal intersection. The field (40) obviously satisfies the gauge condition (41). Under the gauge transformations, the P-ordered exponential transforms as follows:

$$\begin{aligned}&P\exp\left\{\mathrm{i}g\int\limits_{\gamma(y,x)}dz^\mu A^U_\mu(z)\right\}\\&=U^{-1}(Y)\left(P\exp\mathrm{i}g\int\limits_{\gamma(y,x)}dz^\mu A_\mu(z)\right)U(X).\end{aligned} \quad (42)$$

It follows from the relation (42) that under the gauge condition

$$\Phi(A,\ U)=P\exp\Big\{ig\int\limits_{\gamma(y,\ x)} dz_\mu A^\mu(z)\Big\}-\mathbb{1}=0 \qquad (43)$$

there is no interaction of the gauge field with the ghost fields, since

$$\det\left|\frac{\partial\Phi}{\partial\hat{\omega}}\right|=\det\left|\frac{\partial}{\partial\hat{\omega}}U^{-1}(Y)\,U(x)\right|. \qquad (44)$$

Since (43) is realized for any point x, we can use the relation

$$\frac{\partial z^\alpha(\tau)}{\partial\tau}A_\alpha(z(\tau))\Big|_{z=x}=0.$$

However, the condition (43) includes topologically nontrivial situations.

We consider somewhat more complicated configurations:

$$\left.\begin{aligned}
A_\mu^{(1)}&=\hat{E}_1^{-1}(A)\,A_\mu\hat{E}_1(A)+\frac{i}{g}\hat{E}_1^{-1}(A)\,\partial_\mu\hat{E}_1(A);\\
A_\mu^{(2)}&=E_2^{-1}(A^{(1)})\,A_\mu^{(1)}E_2(A^{(1)})+\frac{i}{g}E_2^{-1}(A^{(1)})\,\partial_\mu E_2(A^{(1)});\\
&\ \ \vdots\\
A_\mu^{(n)}&=E_n^{-1}(A^{n-1})\,A_\mu^{(n-1)}E(A^{n-1})+\frac{i}{g}E_n^{-1}(A^{n-1})\,\partial_\mu E_n(A^{n-1}).
\end{aligned}\right\} \qquad (45)$$

The set of exponentials $\{E_i\}$ is determined by the paths $\{\gamma_i(Y,x)\}$. It is obvious that

$$A_\mu^{(n)}(x)=(E_1(A),\ \dots,\ E_n(A^{n-1}))^{-1}A_\mu(E_1(A),\ \dots,\ E_n(A^{n-1}))+\frac{i}{g}(E_1(A),\ \dots,$$
$$E_n(A^{n-1}))^{-1}\partial_\mu(E_1(A),\ \dots,\ E_n(A^{n-1})), \qquad (46)$$

where E_i contains information about fixed curves in the bundle. This means that the vector A_μ^{i+1} depends in the general case on a set of $i+1$ paths in the space R^4. If the path γ_{n+1} coincides with γ_n, then the function

$$\varphi_{n+1}=\int\limits_{\gamma_{n+1}} dz_\mu\left\{E_n^{-1}A_\mu^nE_n+\frac{i}{g}E_n^{-1}\partial_\mu E_n\right\}, \qquad (47)$$

which occurs in the argument of the exponential E_{n+1}, vanishes. Thus, we obtain $\varphi_{n+1}=\varphi_n=\ldots=0$ for all φ_i with coincident paths of integration. If $\gamma_{n+1}\neq\gamma_n$, then the function $\tilde{Z}_\mu\in[0,z]$ defines a path $\gamma_n[0,\ Z(t,x)]$, and the function $Z_\mu\ (t,\ x)$ defines the path $\gamma_{n+1}(0,x)$. In fact, $\gamma_n[0,Z(t,x)]$ is a set of paths parametrized by the variable t. The derivative $\partial/\partial z_\mu$ and the increment $t+dt$ lead to transition from one path $\gamma_n[0,Z(t,x)]$ to another $\gamma_n[0,Z(t+dt,x)]$ in the same series γ_n (Fig. 4). This case leads to a nonvanishing function φ_i. Transferring the derivative with respect to x to the right-hand side of the relation (40), we obtain

$$\frac{i}{g}\hat{E}_{n+1}^{-1}(x)\frac{\partial}{\partial x_\mu}\hat{E}_{n+1}(x)$$
$$=-\frac{\partial}{\partial x_\mu}\Big(\int\limits_{\gamma_{n+1}(y,\ x)} dz_\alpha E_n^{-1}(z)\,A_\alpha^n(z)\,E_n(z)\Big)$$
$$+ig\int\limits_{\gamma_{n+1}(y,\ x)} dz^\alpha\Big[E_n^{-1}(z)\,A_\alpha^nE_n(z);$$
$$\frac{\partial}{\partial x_\mu}\int\limits_{\gamma_{n+1}(y,\ z)} dz_1^\beta E_n^{-1}(z_1)\,A_\beta^nE_n(z_1)\Big]$$
$$-(ig)^2\int\limits_{\gamma_{n+1}(y,\ x)} dz^\alpha\Big[E_n^{-1}A_\alpha^nE_n,\ \int\limits_{\gamma_{n+1}(y,\ z)} dz_1^\beta$$
$$\times\Big[E_n^{-1}A_\beta^nE_n,\ \frac{\partial}{\partial x_\mu}\int\limits_{\gamma_{n+1}(y,\ z_1)} dz_2^\delta E_n^{-1}A_\delta^nE_n\Big]\Big]+\ldots$$
$$+(-1)^{m+1}(ig)^m\int\limits_{\gamma_{n+1}(y,\ x)} dz^\alpha$$
$$\times\Big[\ldots,\int\limits_{\gamma_{n+1}(y,\ z)} dz_1^\beta\Big[\ldots,\int\limits_{\gamma_{n+1}(y,\ z_1)} dz_2^\delta$$
$$\times\Big[\ldots,\int\limits_{\gamma_{n+1}(y,\ z_2)} dz_3^\rho\Big[\ldots,\int\limits_{\gamma_{n+1}(y,\ z_{m-1})}$$
$$\times\ dz_m^\eta E_n^{-1}A_\eta^n(z_m)\,E_n\Big]\ldots\Big]\Big]\Big]. \qquad (48)$$

From the last term the procedure for separating the commutators can be continued to infinity. It can be seen from the expression (48) that the transition to the explicit dependence of the function φ_{n+1} on the field A_μ is not a simple problem. If we restrict ourselves to just the terms linear in A_μ in φ_{n+1}, then

$$E_1\ldots E_n=P\exp ig\Big\{\int\limits_{\gamma_n(y,\ x)} dz_\alpha A^\alpha(z)$$
$$+\oint\limits_{\gamma_{n/(n-1)}} dz_\alpha A^\alpha(z)+\ldots+\oint\limits_{\gamma_{n/1}} dz_\alpha A^\alpha(z)\Big\}. \qquad (49)$$

If $n\to\infty$, then the corresponding field A_μ^∞ will take into account all possible loops $\gamma_{i/j}$ formed by drawing the paths j to a single fixed path i (see, for example, Fig. 4 in the limit $t\to 0$). Formally, we represent this fact in the form of the path integral

$$\prod_{i=1}^{\infty}E_i=P\exp ig$$
$$\times\Big\{\int\limits_{\gamma^*} A_\mu dz^\mu+\sum_{j=1}^{N}\int d\gamma^{(j)}\oint\limits_{C(\gamma^*\gamma^j)} A^\mu(z)\,dz_\mu\Big\}, \qquad (50)$$

in which N is the number of homotopy classes; $\oint A_\mu dz_\mu$ is a functional on which a path integral is defined. If in the gluon field there are solutions that lead to a space that is not simply connected, then loops from different homotopy classes cannot be carried into each other by a continuous deformation. However, in this review we shall be interested in the possibilities of a path-dependent approach in the solution of a number of problems of noncovariant gauges in standard perturbation theory.

3. FREE GLUON PROPAGATOR

We now establish the mathematical assumptions under which gauges of axial type belong to the class of contour gauges defined above. We show that in non-Abelian gauge

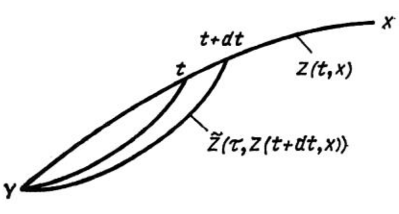

FIG. 4. The function Z_μ parametrizes the path γ_{n+1}, while the function $\tilde{Z}_\mu$ determines the path γ_n.

theory direct use of the gauges $A_0 = 0$, $n^\alpha A_\alpha = 0$ leads to a need to regularize the propagator of the gauge fields in each order of perturbation theory.

The basis of the path-dependent approach is determined by the possibility of parametrizing each element of the bundle $g(x)$ in the fiber space $\mathscr{P}(x,G,\pi)$ by a P-ordered exponential:

$$g(x) \equiv \hat{E}(x) = P \exp\left\{ig \int_{\gamma(y,\,x)} dz^\alpha \hat{A}_\alpha(x)\right\},$$

where $\gamma(y, x)$ is a path defined in the base of the fiber space which connects the fixed point Y to any point x over which the fiber is defined. Varying the path in the base, we fill the complete fiber over the point x with group matrices, identifying the group with the fiber. The separation of a unique element of each orbit of the gauge field, which is needed for quantization, presupposes the existence of a unique intersection of each fiber in the fiber space. This implies the possibility of specifying a rule for unique determination of the vertical variable $g(x) = \hat{E}(x, A)$ over each point of the base and lifting the base to the surface of the horizontal intersection. A nonperturbative gauge is specified at each point x by the condition (41). The initial point Y of the path $\gamma(y, x)$ is deleted, since the limit $x \to Y$ is not defined in accordance with the construction, and this, in its turn, leads to the presence of a residual gauge invariance. This can be readily seen by going over to curvilinear coordinates and identifying the path $\gamma(Y, x)$ with the curve of the radius vector, after which the invariance with respect to transformations by matrices that depend on three angles is obvious. This assertion can be formulated in the reverse order: If after imposition of the gauge condition there is a residual gauge invariance, then there exists at least one point of the space at which the gauge is not fixed at all. Extension of the definition of the gauge at the point Y in the condition (41) eliminates the residual gauge arbitrariness. From the condition (41) we obtain

$$\hat{A}_\mu(x) = \int_\gamma dz^\alpha \frac{\partial z^\beta}{\partial x_\mu} \hat{G}_{\alpha\beta}(A). \tag{51}$$

In (51) the condition $A_\mu(y) = 0$ has been extended at the beginning of the path $\gamma(y, x)$. In a sufficiently small neighborhood of the point y, the expression (51) can be represented in the form

$$\hat{A}_\mu(x) = (x-y)_\beta \int_0^1 d\tau\, \tau \hat{G}_{\beta\mu}(A). \tag{52}$$

In a spherical coordinate system with center at the point y, we obtain

$$\hat{A}_\mu(R,\ \varphi_1,\ \varphi_2,\ \theta) = \int_0^R dr\, \hat{G}_{1\mu}(A), \tag{53}$$

where the index μ takes values from 1 to 4, corresponding to the spherical coordinates $r, \varphi_1, \varphi_2, \theta$. Obviously, $A_1 = 0$. In the limit $R \to 0$, we have $A_\mu(R,\varphi_1,\varphi_2,\theta) \to 0$. If we were to extend the definition of the gauge at the origin, then we should have the relation

$$\hat{A}_\mu(R,\ \varphi_1,\ \varphi_2,\ \theta)$$
$$= \frac{i}{g} U^{-1}(\varphi_1,\ \varphi_2,\ \theta)\, \partial_\mu U(\varphi_1,\ \varphi_2,\ \theta) + \int_0^R dr\, U^{-1} G_{1\mu}(A)\, U. \tag{54}$$

It can be seen from the relation (54) that the extension of the definition of the field $\hat{A}_\mu(0,\varphi_1,\varphi_2,\theta)$ at the origin amounts to fixing the topology, since the residual invariance holds with respect to transformations by singular matrices. There is a fundamental difference between gauges (41) and the conditions $n^\alpha A_\alpha(x) = 0$, $A_0(x) = 0$, which determine the axial and time gauges. These conditions are obtained from (41) by fixing the paths at every point x that passes along the vector n_α or along the time axis to infinity. This is tantamount to replacement of the unique point y, at which an extension of the condition is required, by an infinitely distant three-dimensional surface. At the same time, spherical symmetry is replaced by cylindrical symmetry, and there exists a residual invariance in the class of both singular and nonsingular transformations.

Having in mind the results of Refs. 21, 22, and 28, we follow the transition from a gauge condition that is not fixed at a single point to a condition that is not fixed on an infinitely distant three-dimensional surface. We choose a path that leads to an analog of the time gauge ($A_0 = 0$) with parametrization

$$Z_\mu = \left(x_\mu + \frac{1}{\varepsilon} n_0 \delta_{0\mu}\right) e^{-\varepsilon\tau} - \frac{1}{\varepsilon} n_0 \delta_{0\mu}, \tag{55}$$

where ε is a small parameter that we set equal to zero in the final result. The vector $n_\mu = (n_0, 0, 0, 0)$ specifies a direction (we can choose $n_0 = 1$ or -1):

$$\left.\begin{aligned} Z_\mu &\xrightarrow[\tau\to 0]{} x_\mu - \tau n_0 \delta_{0\mu} + O(\varepsilon); \\ Z_\mu &\xrightarrow[\tau\to\infty]{} -\frac{1}{\varepsilon} n_0 \delta_{0\mu} \equiv Y_\mu. \end{aligned}\right\} \tag{56}$$

We use the expression (40), which connects the potential in the gauge

$$P \exp\left\{ig \int_{\gamma(y,\,x)} dz^\alpha A^E_\alpha(z)\right\} = \mathbb{1} \tag{57}$$

to the potential in an arbitrary gauge. In the general case, the transformation matrices can be singular; then the third term in (40) is nonzero. The first term on the right in (40) is zero, since $\partial z^\beta/\partial x_\mu|_{z=y} = g_{\beta\mu} e^{-\varepsilon\tau}|_{\tau\to\infty} = 0$. For the simple path which we have chosen we obtain the expression

$$\hat{A}^E_\mu = -(n_0\delta_{0\beta} + \varepsilon x_\beta) \int_0^\infty d\tau e^{-2\varepsilon\tau} E^{-1}(A)\, \hat{G}_{\beta\mu}(A)\, E(A), \tag{58}$$

which can be conveniently used to find the propagator

$$\langle 0|\, A^E_\mu(x)\, A^E_\nu(x')\, |0\rangle$$
$$= \int_0^\infty d\tau \int_0^\infty d\tau'\, e^{-2\varepsilon(\tau+\tau')} \langle 0|\, (n_0 + \varepsilon x_0)(n_0 + \varepsilon x'_0)$$
$$\times (E^{-1}G_{0\mu}E)(E^{-1}G_{0\nu}E) + \varepsilon x_i (n_0 + \varepsilon x'_0)(E^{-1}G_{i\mu}E)(E^{-1}G_{0\nu}E)$$
$$+ \varepsilon(n_0 + \varepsilon x_0) x'_j (E^{-1}G_{0\mu}E)(E^{-1}G_{j\nu}E)$$
$$+ \varepsilon^2 x_i x_j (E^{-1}G_{i\mu}E)(E^{-1}G_{j\nu}E)\, |0\rangle \tag{59}$$

of the free gluon field. For this, we ignore all the terms on the right that contain the coupling constant and use any gauge that is convenient for calculation, in particular, the Feynman gauge. We represent the first term in (59) as a sum of the transverse and longitudinal propagators, $D^{(1)}_{ij} = D^T_{ij} + D^L_{ij}$:

$$D_{ij}^{(1)}=\frac{1}{n_0^2}(n_0+\varepsilon x_0)(n_0+\varepsilon x_0')$$
$$\times\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{k^2+i\delta}\left(\delta_{ij}-\frac{k_ik_j}{(k_0-2i\varepsilon)(k_0+2i\varepsilon)}\right);$$
$$D_{ij}^T=-\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{k^2+i\delta}\left(\delta_{ij}-\frac{k_ik_j}{\bar{\mathbf{k}}^2}\right)$$
$$\times\left(1+\frac{\varepsilon}{n_0}x_0\right)\left(1+\frac{\varepsilon}{n_0}x_0'\right) \qquad (60)$$

$$D_{ij}^L(x,\ x')=-\frac{1}{2\varepsilon}\left(1+\frac{\varepsilon}{n_0}x_0\right)\left(1+\frac{\varepsilon}{n_0}x_0'\right)$$
$$\times e^{-\varepsilon|x_0-x_0'|}\int\frac{d^3k}{(2\pi)^3}\frac{k_ik_j}{\bar{\mathbf{k}}^2}e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}. \qquad (61)$$

In the limit $\varepsilon\to0$ the Slavnov–Frolov result follows from (61). The other components of the propagator have the form

$$\left.\begin{aligned}D_{l0}(x,\ x')&=e^{-\varepsilon|x_0-x_0'|}\frac{(x_i'\partial^i)\partial^l}{2n_0\nabla}\delta^3(x-x');\\ D_{0l}(x,\ x')&=e^{-\varepsilon|x_0-x_0'|}\frac{(x_i\partial^i)\partial^l}{2n_0\nabla}\delta^3(x-x').\end{aligned}\right\} \qquad (62)$$

We have not written out the components of the propagator proportional to ε. The parameter ε that we have introduced uniquely fixes the rule for avoiding the additional poles of the propagator. We emphasize that in the limit $\varepsilon\to0$ we have $A_0(x)\to0$, but the components D_{0l} and D_{l0} of the propagator are nonzero. This is a consequence of the pinch structure $[(k_0-i\varepsilon)(k_0+i\varepsilon)]^{-1}$, the integration of which leads to a factor $1/\varepsilon$ that cancels the ε in the numerators of the expressions for D_{0i} and D_{i0}.

We consider the analog of the axial gauge. For this, we choose a path from the fixed point $Y_\mu=-\varepsilon^{-1}n_\mu$ to every point x in the form $Z_\mu=(x_\mu+(1/\varepsilon)n_\mu)e^{-\varepsilon\tau}-(1/\varepsilon)n_\mu$. As a result,

$$\hat{A}_\mu^E(x)=-(n_\beta+\varepsilon x_\beta)\int_0^\infty d\tau\, e^{-2\varepsilon\tau}\hat{G}_{\beta\mu}(A). \qquad (63)$$

It is obvious that in the limit $\varepsilon\to0$ the condition $n^\alpha A_\alpha(x)=0$ follows from (63). For the gluon propagator we obtain in this case

$$\langle0|\,\hat{A}_\mu^E(x)\hat{A}_\gamma^E(x')\,|0\rangle=r_\alpha(x)r_\beta(x')\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{(k^2+i\delta)}$$
$$\times\frac{(k_\alpha k_\beta g_{\mu\nu}-k_\mu k_\beta g_{\alpha\nu}-k_\nu k_\alpha g_{\mu\beta}+k_\mu k_\nu g_{\alpha\beta})}{((kn)+2i\varepsilon)((kn)-2i\varepsilon)}, \qquad (64)$$

where $r_\alpha=\varphi(x)n_\alpha+\varepsilon x_\alpha^T$, $\varphi(x)=1+\varepsilon x^L$. The coordinates and momenta can be conveniently represented as components orthogonal to the vector n and longitudinal with respect to it:

$$k_\mu^T=k_\mu-\frac{(kn)}{n^2}n_\mu,\quad x_\mu^T=x_\mu-\frac{(xn)}{n^2}n_\mu;$$
$$k_\mu^L=k^Ln_\mu,\quad x_\mu^L=x^Ln_\mu.$$

We transform (64) to the form

$$D_{\mu\nu}(x,\ x')=\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{(k^2+i\delta)}\Big\{\varphi(x)\varphi(x')P_{\mu\nu}(k,\ n)$$
$$-\frac{\varepsilon}{n^4}\frac{((x_\alpha^Tk_\alpha^T)k_\mu^Tn_\nu+(x_\alpha'^Tk_\alpha^T)k_\nu^Tn_\mu)}{(k_L-2i\tilde{\varepsilon})(k_L+2i\tilde{\varepsilon})}+O(\varepsilon)\Big\},$$
$$P_{\mu\nu}(k,\ n)=\Big[g_{\mu\nu}-\frac{k_\mu n_\nu}{(kn)+2i\varepsilon}-\frac{k_\nu n_\mu}{(kn)-2i\varepsilon}$$
$$+\frac{n^2k^\mu k^\nu}{((kn)-2i\varepsilon)((kn)+2i\varepsilon)}\Big]=P_{\mu\nu}^T+P_{\mu\nu}^L;$$

where

$$P_{\mu\nu}^T=g_{\mu\nu}-\frac{k_\mu^Tk_\nu^T}{(k^T)^2}-\frac{n_\mu n_\nu}{n^2}$$
$$=g_{\mu\nu}-\frac{(nk)^2}{(nk)^2-n^2k^2}\Big[\frac{k_\mu n_\nu+k_\nu n_\mu}{(kn)}-\frac{1}{(kn)^2}(n^2k_\mu k_\nu+k^2n_\mu n_\nu)\Big];$$
$$P_{\mu\nu}^L=\frac{k^2}{(k^L-2i\varepsilon)(k^L+2i\varepsilon)}\frac{1}{n^2}\frac{k_\mu^Tk_\nu^T}{(k^T)^2}. \qquad (65)$$

Obviously, $P_{\mu\alpha}^TP_{\alpha\nu}^T=P_{\mu\nu}^T$, $n_\mu P_{\mu\nu}^T=k_\mu P_{\mu\nu}^T=0$. In the expression (65) we have written out explicitly only the terms that are proportional to ε, which lead on integration over k^L to a finite contribution in the limit $\varepsilon\to0$. As a result, we obtain

$$D_{\mu\nu}(x,\ x')=D_{\mu\nu}^T+D_{\mu\nu}^L+D_{\mu\nu}^{\text{anom}}. \qquad (66)$$

In (66) we have introduced the notation

$$D_{\mu\nu}^T(x,\ x')=\varphi(x)\varphi(x')\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-x')}}{k^2+i\delta}P_{\mu\nu}^T(k,\ n); \qquad (67)$$

$$D_{\mu\nu}^L(x,\ x')=\frac{1}{2\varepsilon}(1+\varepsilon x^L)(1+\varepsilon x'^L)e^{-\varepsilon|x_L-x_L'|}$$
$$\times\int\frac{d^3k_T}{(2\pi)^3}\frac{k_\mu^Tk_\nu^T}{(k^T)^2}e^{ik^T(x^T-x'^T)}; \qquad (68)$$

$$D_{\mu\nu}^{\text{anom}}=\frac{1}{2}e^{-\varepsilon|x^L-x'^L|}\int\frac{d^3k^T}{(2\pi)^3}\frac{e^{ik^T(x_T-x_T')}}{(k_T)^2}$$
$$\times[k_\mu^Tn_\nu(x^Tk_T)+k_\nu^Tn_\mu(x_T'k_T)]. \qquad (69)$$

In the limit $\varepsilon\to0$, the anomalous term (69) has the consequence that $n^\mu D_\mu(x,x')\neq0$, despite the fact that $(nA)=0$ follows from Eq. (63). As in the preceding case, the manner in which the poles are avoided is fixed, and this guarantees, by construction, gauge invariance of the physical quantities. However, the possibility of going to the limit $\varepsilon\to0$ at the level of the Green's functions is questionable. We emphasize that in the limit $\varepsilon\to0$ the symmetry of the residual gauge transformations changes abruptly, and it is in this that the mathematical incorrectness of using the incompletely defined gauges $A_0=0$ and $(nA)=0$ consists. In the higher orders of perturbation theory powers of the pole $1/\varepsilon$ will accumulate from the longitudinal terms of the propagators, and, multiplied by the terms $D_{\mu\nu}^T$ and $D_{\mu\nu}^L$, which are proportional to ε^n, may make a finite contribution. If we let ε tend to zero prematurely, we are forced in each subsequent order of perturbation theory to redefine the longitudinal and transverse parts of the gluon propagator. Of course, the possibility of mutual cancellation of the finite contributions of the poles $1/\varepsilon^n$ in the higher orders of perturbation theory cannot be ruled out, and in such a case one can go to the limit $\varepsilon\to0$ in the Green's function. However, such compensation appears unlikely. A direct calculation of the Wilson loop in the order α_s^3 could cast light on this problem.

Thus, gauge invariance of the S matrix is equivalent to independence of the choice of the path in the path-dependent formalism. The breaking of the gauge invariance of the Green's function in the standard formalism is transformed into breaking of the translational invariance in the path-dependent approach. The asymptotic behavior of the longitudinal part of the propagator depends in principle on the order in which the limit $\varepsilon\to0$ is taken. The parameter ε, which keeps the point of the beginning of the path in a finite region of space, actually specifies the rule for avoiding the additional poles of the propagator.

4. RENORMALIZABILITY OF PROPAGATORS IN CONTOUR GAUGES

Formally, the absence of interaction of the "ghost" fields with the gauge fields in the contour gauges automatically leads to a simple connection between the divergent parts of the renormalization constants:

$$Z_A = Z_{3A} = Z_{4A}, \tag{70}$$

where Z_A is the renormalization constant of the gluon wave function, and Z_{3A} and Z_{4A} are the renormalization constants of the three- and four-gluon vertices, respectively. The Ward identities (70) were verified by Kummer[35] by a direct calculation of the single-loop QCD corrections in the gauge $(nA) = 0$. Problems of the renormalizability of noncovariant gauges have been discussed by many authors,[36-39] but as yet there is no proof of the multiplicativity of the renormalization in all orders. In addition, the calculation of the polarization operator has always used a prescription for avoiding the additional poles $1/(kn)^\beta$ (Refs. 24, 25, 40, and 41), and, naturally, this does not permit us to generalize with confidence the result to the higher orders of perturbation theory, since it is necessary to prove in each order that the prescription does not break the gauge invariance of observable quantities or the unitarity of the elements of the S matrix. The results of Caracciolo, Curci, and Menotti[22] shook confidence in the assumption that the noncovariant gauges in their traditional form have an equal status with the well-known and well-studied covariant gauges. At the least, in non-Abelian massless theories the situation is seriously complicated by the fact that the longitudinal, translationally noninvariant part of the propagator, interacting with the transverse gluons, seriously deforms the result. At the same time, as we saw in the previous section, none of the prescriptions is sufficient in the general case.

One of the possible ways of proving multiplicative renormalizability of the propagator of the gauge field in gauges of axial type is to develop the method that enables us in each order of perturbation theory to calculate the propagator of the gluon field in a noncovariant gauge by using, in an appropriate approximation, the propagator of the gluon field in a covariant gauge.

In this section we shall show that in any order of perturbation theory there is a simple connection between the propagators in the noncovariant and covariant gauges, the connection being equivalent to a special choice of the gauge parameter α in the covariant α gauge. In this way, the problem of renormalizability and unitarity can be reduced to a problem that has been solved.

We use Eq. (40), setting $E^{-1}[\partial_\alpha \partial_\beta]E = 0$, and then

$$D^{E,c}_{\mu\nu}(x, x') = -i\langle 0|T\hat{A}^E_\mu(x)\hat{A}^E_\nu(x')|0\rangle$$
$$= -i\langle 0|T\left\{\int_{\gamma(x,y)} dz_\alpha \int_{\gamma(x',y)} dz'_\beta \frac{\partial z_\eta}{\partial x_\mu}\frac{\partial z'_\rho}{\partial x'_\nu}\right.$$
$$\left.\times \widetilde{E}^{-1}_{ad}(A(z))\, G^d_{\alpha\eta}(z)\, G^p_{\beta\rho}(z')\, \widetilde{E}_{pb}(A(z'))\right\}|0\rangle. \tag{71}$$

The relation (71) connects the propagators in different gauges: $D^{E,c}_{\mu\nu}(x, x')$ is the causal propagator in the noncovariant gauge fixed by the choice of the path $\gamma(x, y)$, where y is the initial point, and $\widetilde{E}_{ab}(A) = P\exp\{ig\int_{\gamma(y,x)} dz^\alpha A^c_\alpha(z) f^{abc}\}$ is the P-ordered exponential in the adjoint representation; f^{abc} is a structure constant of the group $SU(3)$. Note the nonuniqueness in the definition of the time ordering on the right-hand side of Eq. (71) if a transition to Minkowski space is made. It is known (see, for example, Ref. 42) that different types of time ordering (Wick or Dyson) lead to a difference by a quasilocal operator, which is unimportant in the calculation of observables. In the case when the vector tangent to the contour $\gamma(z, Y)$ is timelike over a certain section of the path, time ordering does not commute with integration. The difference between the different definitions leads to different forms of the longitudinal part of the propagator. In the case of non-Abelian gauge fields, this is important. To preserve the gauge invariance of the observables, it is necessary to introduce the operation of time ordering both under the integral sign and under the differential operator. The integrand of Eq. (71) is shown in Fig. 5. On the right-hand side of Eq. (71) we can use the gluon propagator in the covariant α gauge. In the sum of the diagrams the α dependence cancels in each order of perturbation theory. This is clear if one bears in mind that under gauge transformations of the potential on the right-hand side of Eq. (71)

$$\hat{E}(A^U) = U^{-1}(x)\,\hat{E}(A)\,U(y),$$

$$\text{a}\ \hat{G}_{\mu\nu}(A^U) = U^{-1}(x)\,\hat{G}_{\mu\nu}(A)\,U(x),$$

from which gauge invariance of the right-hand side of Eq. (71) follows. In other words, Eq. (71) projects the complete set $\{A^U_\mu\}$ of physically equivalent fields to a single element of this set.

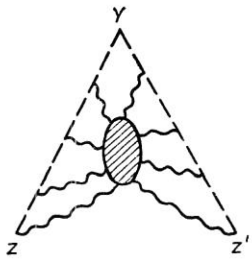

FIG. 5. Graphical representation of the integrand of Eq. (71). The broken line denotes the P-ordered exponential with an integral in the argument from the point Y to the point z; the wavy line is the gluon propagator in the covariant gauge; and the hatched block denotes an arbitrary order of perturbation theory.

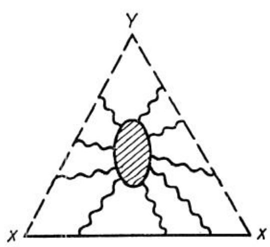

FIG. 6. Graphical representation of Eq. (72). The straight line corresponds to the quark propagator.

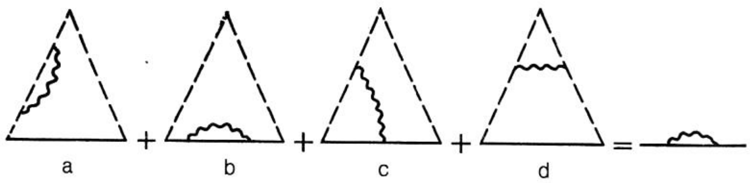

FIG. 7. Single-loop approximation of Eq. (72). The continuous line on the left-hand side of the equation denotes the fermion propagator in the α gauge.

For the quark propagator, the situation is completely analogous. The connection of the quark propagator in the noncovariant gauge to the propagator in the α gauge has the form

$$\langle 0 \mid T\bar{q}(x)\, q(x') \mid 0\rangle^E = \langle 0 \mid T\{E^{-1}(\gamma(y, x); A)\, \bar{q}(x)\, q(x')\, E(\gamma(x', y); A)\} \mid 0\rangle. \quad (72)$$

Graphically, this equation is shown in Fig. 6. In the single-loop approximation, the expression (72) leads to the diagrams of Fig. 7. Each of these diagrams contains a dependence on the gauge parameter α. At the same time, the diagram of Fig. 7d gives information about the infrared structure of the propagator. In what follows, we shall show that the infrared and ultraviolet properties of the propagators in noncovariant gauges are essentially related. We write down the divergent constants for each of the diagrams of Fig. 7:

$$Z_a = 1 + \frac{\alpha_s}{4\pi} C_F (3-\alpha)\frac{1}{\varepsilon}, \quad Z_d = Z_a^{-1} Z_{\text{cusp}};$$
$$Z_b = 1 - \frac{\alpha_s}{4\pi} C_F \alpha \frac{1}{\varepsilon};$$
$$Z_c = 1 + \frac{\alpha_s}{4\pi} C_F \alpha \frac{1}{\varepsilon}, \quad (73)$$

where Z_a is the renormalization constant of the P-ordered exponential, and Z_b is the renormalization constant of the quark propagator in the α gauge. The constant Z_c is a consequence of the divergence in the integral that characterizes the interaction of the P-ordered exponential with the quark propagator, and, finally, the constant Z_d, which corresponds to the diagram of Fig. 7d, is represented as a product of two constants. The significance of the constant $Z_{\text{IR}}^{\text{cusp}}$, which does not depend on the parameter α, will be explained below. Thus, the renormalization constant of the quark propagator in the noncovariant gauge is

$$Z_q^E = Z_a^2(\alpha) Z_b(\alpha) Z_c^2(\alpha) Z_a^{-1}(\alpha) = 1 + 3\frac{\alpha_s}{4\pi} C_F \frac{1}{\varepsilon}. \quad (74)$$

As one would expect, the gauge invariance of the expression (72) has led to cancellation of the parameter α, but this fact can be used in what follows for significant simplification of the calculations and for the analysis of some general properties of the scattering amplitudes in hard processes. It is important in the given case that the equation

$$Z_a Z_c^2 = 1 \quad (75)$$

has a solution. In the chosen approximation, it is $\alpha = -3$, i.e., for $\alpha_0 = -3$ there is compensation of the ultraviolet singularities of the diagrams of Figs. 7a, 7c, and 7d. This means that the ultraviolet singularities of the quark propagator in the noncovariant gauge are equivalent to the ultraviolet singularities of the propagator in the covariant α gauge for $\alpha = \alpha_0$:

$$Z_q^E = Z_q(\alpha = -3). \quad (76)$$

We rewrite Eq. (71) in the form

$$Z_A^E \langle 0|\, T A_\mu^E(x)\, A_\nu^E(x')\, |0\rangle_R^{ab} = \int dz_\alpha \frac{\partial z^\eta}{\partial x_\mu} \int dz'_\beta \frac{\partial z'_\rho}{\partial x'_\nu} \{ Z_A(\alpha)\, K_{\eta\rho}^{\alpha\beta\gamma\delta}(\partial\partial)\, \langle 0|\, T \times (A_\gamma^a(z)\, A_\delta^b(z'))\, |0\rangle_R$$
$$+ \langle 0|\, [(\widetilde{E}^{-1} - 1)_{ad}\, \hat{G}_{\alpha\eta}^d G_{\beta\rho}^p \widetilde{E}_{pb} + \widetilde{E}_{ad}^{-1} G_{\alpha\eta}^d G_{\beta\rho}^p (\widetilde{E} - 1)_{pb} + (\widetilde{E}^{-1} - 1)_{ad}\, G_{\alpha\eta}^d G_{\beta\rho}^p (\widetilde{E} - 1)_{pb}]\, |0\rangle\}, \quad (77)$$

where $A_\mu^E = Z_{A,E}^{1/2} A_{\mu,R}^E$, $Z_{A,E}$ is the renormalization constant of the gluon wave function in the noncovariant gauge,

$$K_{\eta\rho}^{\alpha\beta\gamma\delta}(\partial\partial) = g_{\gamma\eta} g_{\rho\delta} \partial_\alpha \partial'_\beta - g_{\alpha\gamma} g_{\rho\delta} \partial_\eta \partial'_\beta - g_{\eta\gamma} g_{\delta\beta} \partial_\alpha \partial'_\rho + g_{\alpha\gamma} g_{\beta\delta} \partial_\eta \partial'_\rho, \quad (78)$$

$A_\mu = Z_A(\alpha) A_{\mu,R}$, and $Z_A(\alpha)$ is the renormalization constant of the gluon wave function in the covariant α gauge. It is obvious that in the general case $Z_A^E \neq Z_A(\alpha)$. In the order α_s the diagrams of Fig. 8 follow from the expression (77). In complete analogy with the case of the quark propagator (74), the gauge invariance of the right-hand side of Eq. (77) leads to cancellation of the α parameter in the sum of the diagrams of Figs. 8b–8n. In this case

$$Z_A^E = Z_A(\alpha = -3). \quad (79)$$

But because there is no interaction of the ghosts with the gauge fields in the noncovariant gauges, the constant Z_A^E determines the β function:

$$g_B^2 \frac{\partial \ln Z_A^E}{\partial \ln \mu^2}\bigg|_{g_B^2=\text{const}} = \beta(g_R^2(\mu^2)). \quad (80)$$

We now verify the hypothesis that it is possible to make a choice of the gauge parameter α,

$$\alpha_R^* = \alpha_0 \sum_{n=0}^{\infty} C_n \left(\frac{\alpha_s}{4\pi}\right)^n, \quad (81)$$

for which in any order of perturbation theory and for definite values of the coefficients C_n one can achieve the equality

$$Z_A^E = Z_A(\alpha = \alpha_R^*) \quad (82)$$

in each order of perturbation theory. The constants C_n are determined from the equation

$$Z_\eta(\alpha_R^*) = Z_{\eta A\eta}(\alpha_R^*), \quad (83)$$

where Z_η is the renormalization constant of the ghost wave function, and $Z_{\eta A \eta}$ is the renormalization constant of the gluon–ghost vertex. The condition (83) leads to a simplification of the connection between the "bare" and renormalized coupling constants:

$$g^2 = Z_\eta^{-2} Z_{\eta A\eta}^2 Z_A^{-1} g_R^2 = Z_A^{-1}(\alpha_R^*)\, g_R^2. \quad (84)$$

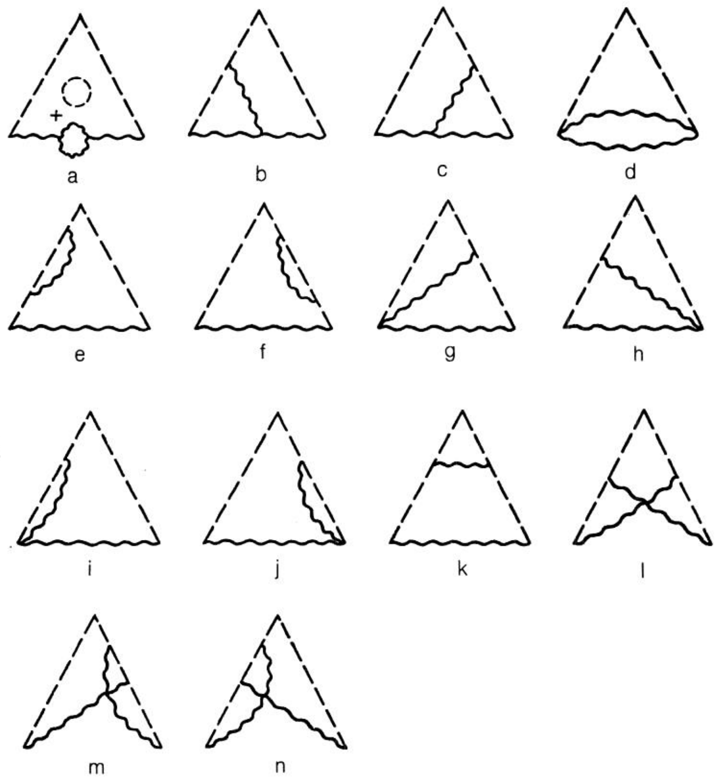

FIG. 8. The α_s approximation of the expression (77). In diagram (a), the broken circle denotes the contribution of the ghost fields to the gluon polarization operator.

At the same time the simple Ward identity (70) is realized. If we can prove the proposition that has been advanced, then we shall have justified the renormalization equivalence of the noncovariant gauges to the covariant gauge with the special parameter choice $\alpha_R = \alpha_R^*$, this being a sufficient condition of multiplicative renormalizability and unitarity.

The approximation that follows the leading approximation $\alpha = \alpha_0 = -3$ for the parameter α can be obtained by using the results of calculation of the renormalization constants:

$$Z_\eta = 1 + \frac{1}{4\varepsilon} A(3-\alpha) + \frac{A^2}{4\varepsilon}\left[\frac{1}{8}\left(\alpha + \frac{95}{3}\right) - \frac{5N_f t}{3N_c}\right] + \frac{A^2}{32\varepsilon^2}\left[3\alpha^2 - 35 + 16\frac{T^2}{N_c}\right]; \quad (85)$$

$$Z_{\eta A\eta} = 1 - \frac{A}{2\varepsilon}\alpha - \frac{A^2}{16\varepsilon}(5\alpha+1)\alpha + \frac{A^2}{8\varepsilon^2}(2\alpha+3)\alpha; \quad (86)$$

$$Z_A = 1 - \frac{A}{6\varepsilon}\left[(3\alpha - 13) + \frac{8tN_f}{N_c}\right] - \frac{A^2}{16\varepsilon}\left[2\alpha^2 + 11\alpha - 59 + \frac{2N_f t T^2}{N_c^2} + \frac{5}{2}\frac{N_f t}{N_c}\right] + \frac{A^2}{24\varepsilon^2}\left[(6\alpha^2 - 17\alpha - 39) + \frac{24N_f t}{N_c}\left(1 + \frac{2}{3}\alpha\right)\right]; \quad (87)$$

$$Z_q = 1 - \frac{A}{\varepsilon}\frac{\alpha}{N_c}T^2 - \frac{A^2}{8\varepsilon}T^2 \times\left[(25 + 8\alpha + \alpha^2)\frac{1}{N_c} - \frac{8N_f t}{N_c^2} - \frac{6T^2}{N_c^2}\right] + \frac{A^2}{4\varepsilon^2}T^2\left[\alpha(\alpha+3) + \frac{2\alpha^2 T^2}{N_c^2}\right], \quad (88)$$

where N_f is the number of quarks, $A = \alpha_s N_c/4\pi$, $T^2 = (n^2-1)/2n$, $t = \frac{1}{2}$, and n is the rank of the group. Since the renormalization constants can be expanded in inverse powers of ε (in 't Hooft's scheme[59]),

$$Z_\eta^{-1} Z_{\eta A\eta} = 1 + \sum_{n=1}^{\infty} \frac{d_n(g_R^2, \alpha_R)}{\varepsilon^n}. \quad (89)$$

By an appropriate choice of the coefficients C_n in the expression (81) it is possible to achieve in all orders of perturbation theory the equalities

$$d_1(g_R^2, \alpha_R^*) = 0. \quad (90)$$

This means that the β function and, therefore, the nature of the interaction are determined by the renormalization constant Z_A of the gluon wave function, since

$$\beta(g^2) \equiv g^4 \frac{da_1(g^2)}{dg^2}, \quad (91)$$

where a_1 is the first coefficient of the renormalization series for the coupling constant:

$$g_B^2 = \mu^{2\varepsilon} g_R^2 \left(1 + \sum_{n=1}^{\infty} \frac{a_n(g_R)}{\varepsilon^n}\right). \quad (92)$$

Thus, the relation (83) can be satisfied in the weak sense, i.e., to accuracy d_n/ε^n with $n > 1$. In this case the coefficients C_n are finite in the limit $\varepsilon \to 0$. In the case when we require fulfillment of (83) for all poles $1/\varepsilon^n$,

$$C_n = C_n^{(1)} + \frac{1}{\varepsilon^n} C_n^{(2)} \quad (93)$$

will contain a term that is singular with respect to ε. This means that when the regularization is lifted $C_n \to \infty$. However, the part played by the terms d_n/ε^n for $n > 1$ is such that, contributing to the coefficients a_n of the series (92) for $n > 1$, they ensure fulfillment of the renormalization-group restriction

$$\frac{da_{n+1}}{dg^2} = \frac{da_1}{dg^2}\left(a_n + g^2\frac{da_n}{dg^2}\right) \quad (94)$$

and at the same time in no way influence the nature of the interaction. It follows from the relations (85) and (86) that

$$C_0=1,\ C_1^{(1)}=\left(\frac{5}{6}\right)^2 N_c-\frac{5}{9}N_f t,\ N_c=3 \tag{95}$$

in the expression (81). The renormalization-group equation for the parameter α has the form

$$\frac{d\alpha}{dl}=b(\alpha_s,\ \alpha)\,\alpha;\ l=\ln \mu^2/\mu'^2, \tag{96}$$

where $b(\alpha_s,\alpha)=b_1(\alpha_s/4\pi)+b_2(\alpha_s/4\pi)^2+\cdots$.

The two-loop calculation[43] gives in the $\overline{\text{MS}}$ scheme

$$\left.\begin{aligned} b_1&=\frac{13-3\alpha}{2}-\frac{2}{3}N_f;\\ b_2&=\frac{531}{8}-\frac{99}{8}\alpha-\frac{9}{4}\alpha^2-\frac{61}{6}N_f.\end{aligned}\right\} \tag{97}$$

Using the relations (95) and (81), we can readily show that $b_1=\beta_1$ and $b_2=\beta_2$. These equations are a consequence of the fact that for $\alpha=\alpha_R^*$

$$\alpha_B=\frac{g_R^2}{g_B^2}\alpha_R^*\mu^{2\varepsilon}. \tag{98}$$

Here, $\varepsilon=(4-n)/2$, n is the dimension of space, and μ is a parameter that has the dimensions of mass. Thus,

$$\alpha_s b(\alpha_s,\ \alpha^*)=-\beta(\alpha_s). \tag{99}$$

In the MOM scheme the relation (99) is violated, and the situation is completely unstudied.[3)] Thus, the special choice of the gauge parameter α effectively regroups the singular terms, leaving them only in the first term on the right-hand side of Eq. (77). This leads to the simple expression

$$D_{\mu\nu}^{c,E}(x,\ x')=Z_A(\alpha_R^*)\,\hat{L}_{\mu\nu}^{\gamma\eta}D_{\gamma\eta(R)}^{c\ \mathrm{cov}}(z,\ z'), \tag{100}$$

which relates the propagator in the noncovariant gauge to the propagator in the special covariant gauge; $\hat{L}_{\mu\nu}^{\gamma n}$ is the integro-differential operator in Eq. (77). In contrast to the covariant gauges $(1/2\alpha)(\partial^\mu \hat{A}_\mu)^2$, in which only the transverse part of the propagator is renormalized, in our case the complete propagator is renormalized. This is a consequence of the property

$$\hat{L}_{\mu\nu}^{\gamma\eta}\partial_\gamma=\hat{L}_{\mu\nu}^{\gamma\eta}\partial_\eta=0. \tag{101}$$

Equation (100) proves the multiplicative renormalizability of the gluon propagator in the contour gauge.

5. SPECTRAL REPRESENTATION OF THE GLUON PROPAGATOR

Interest in the spectral representation of the gluon propagator was largely stimulated by the work of Oehme and Zimmermann,[60] who pointed out a possible contradiction between positive definiteness of the spectral function and the property of asymptotic freedom. The arguments were based on results of Källén.[61] It was shown that the value of the physical charge e_R in electrodynamics is always less than that of the bare charge e_B. This fact is a consequence of the Lehmann sum rule for the photon renormalization constant:

$$\int_0^\infty \rho(q^2)\,dq^2=1-Z_A, \tag{102}$$

where $Z_A=(e_R/e_B)^2$. Positive definiteness of the spectral function $\rho(q^2)$ proves the assertion

$$Z_A<1 \tag{103}$$

from which there follow a positive value of the β function and, therefore, a zero-charge behavior of the coupling constant.

In QCD, the situation is much more complicated, since, first, in the general case $Z_A\neq(g_R/g_B)^2$ and Z_A depends on the gauge (Z_A is the gluon renormalization constant), and second, the corresponding spectral function is not positive definite. This last assertion follows from the fact that in covariant gauges the Hilbert space contains "ghost" fields. However, in the axial gauge, as in all contour gauges, ghost states are absent, and this makes it possible to avoid the need to project onto a positive-definite space of states, and, thus, it would appear that we arrive at a clear contradiction with the property of asymptotic freedom, since in this case

$$Z_A=g_R^2/g_B^2 \tag{104}$$

is a gauge-invariant quantity. A way of resolving this paradox was outlined by Frenkel and Taylor,[26] and later analyses were made by many authors.[62,63] In achieving the absence of ghosts, we have sacrificed Lorentz invariance, this being reflected in the appearance of additional unphysical gauge singularities. The treatment of these singularities plays a decisive role in the resolution of the problem. The formally positive quantity ρ contains a singularity in the presence of poles, $k^\alpha n_\alpha=0$. It was shown that the principal-value prescription leads to a change in the sign of the spectral function ρ, and, therefore, we have the inequality

$$Z_A>1, \tag{105}$$

which is necessary in an asymptotically free theory. However, as we now know, the principal-value prescription does not solve all the problems in determining the propagator of a non-Abelian gauge field. In addition, the very definition of the gluon propagator in noncovariant gauges,

$$D_{\mu\nu}^{ab}(x-x')\equiv-\frac{\delta^2 W[0]}{\delta J_\mu^a(x)\,\delta J_\nu^b(x')}, \tag{106}$$

where

$$W[J]=\frac{\int DA_\mu^a\exp\left\{i\int d^4x\,(\mathcal{L}+J_\mu^a A_\mu^a)\right\}\delta(nA)}{\int DA_\mu^a\exp\left(i\int \mathcal{L}\,d^4x\right)\delta(nA)} \tag{107}$$

is the standard generating functional, is not correct, but must be regarded merely as the limit of a more general expression. As a result, the propagator is not a function of the coordinate difference $(x-x')_\alpha$, and therefore the standard definition

$$\rho_{\mu\nu}^{ab}(q)\equiv\int d^4x\,e^{iqx}\,\langle 0|\,[A_\mu^a(x),\ A_\nu^b(0)]\,|0\rangle \tag{108}$$

of the spectral function is also inconsistent for fields in noncovariant gauges. It is obvious that the traditional form of the spectral function,

$$\begin{aligned}\rho_{\mu\nu}^{ab}(q)=\int d^4x\,e^{iqx}\Big\{&-\rho_1(q,\ n)\\ &\times\Big[g_{\mu\nu}-\frac{k_\mu n_\nu+k_\nu n_\mu}{(kn)}+\frac{n^2}{(kn)^2}k_\mu k_\nu\Big]\\ &+\rho_2(q,\ n)\Big(g_{\mu\nu}-\frac{n_\mu n_\nu}{n^2}\Big)\Big\}\end{aligned} \tag{109}$$

which follows from (108), does not contain essential translationally noninvariant terms.

Using the relation (100), we attempt to calculate the gluon propagator, expressing it in terms of the spectral function $\rho(k)$ in the special α_R^* gauge. In doing so, we shall attempt to represent the result in a form analogous to (109) and find a connection between the functions ρ_1 and ρ_2 and the spectral function $\rho_{\alpha_R^*}$. We choose the path of integration in (100) in the form $Z_\mu = (x_\mu + (1/\varepsilon)n_\mu)e^{-\varepsilon\tau} - (1/\varepsilon)n_\mu$. On the right-hand side of Eq. (100) we shall use the general form of the spectral representation of the gluon propagator in the covariant α gauge for $\alpha = \alpha_R^*$. As a result,

$$D_{\mu\nu}^{c,E}(x, x') = -\int_0^\infty d\mu\rho(\mu^2) \int_{\gamma(x,\frac{n}{\varepsilon})} dz_\alpha \int_{\gamma(x',\frac{n}{\varepsilon})} dz'_\beta \mathscr{K}_{\mu\nu}^{\alpha\beta}(\partial\partial) D^c(z-z', \mu^2), \tag{110}$$

with the operator $\mathscr{K}_{\mu\nu}^{\alpha\beta} = g_{\mu\nu}\partial_\alpha\partial'_\beta - g_{\alpha\nu}\partial_\mu\partial'_\beta - g_{\mu\beta}\partial_\alpha\partial'_\nu + g_{\alpha\beta}\partial_\mu\partial'_\nu$. In deriving the expression (110), we used the property (101), as a result of which (110) contains only one of the two spectral functions of the gluon propagator in the α gauge:

$$D^c(z-z', \mu^2) = -\int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(z-z')}}{k^2-\mu^2+i\delta} \tag{111}$$

Integrating (110) over the paths, we obtain the expression

$$D_{\mu\nu}^{c,E}(x, x') = \varphi_\alpha(x)\varphi_\beta(x') \int_0^\infty d\mu^2\rho(\mu^2) \times \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-x')}}{k^2-\mu^2+i\delta} \frac{\mathscr{K}_{\mu\nu}^{\alpha\beta}(k)}{((kn)-2i\varepsilon)((kn)+2i\varepsilon)}, \tag{112}$$

where $\varphi_\alpha(x) = n_\alpha + \varepsilon x_\alpha$; $\varphi_\alpha(x') = n_\alpha + \varepsilon x'_\alpha$.

We represent the coordinates and momenta as sums of transverse and longitudinal components

$$\left.\begin{aligned} &k_\mu = k_\mu^T + k_\mu^L; \quad x_\mu = x_\mu^T + x_\mu^L; \\ &k_\mu^T = k_\mu - k^L n_\mu; \quad x_\mu^T = x_\mu - x^L n_\mu; \\ &k^L = (nk)/n^2; \quad x^L = (nx)/n^2 \end{aligned}\right\} \tag{113}$$

and introduce the function $\varphi_L = (1 + \varepsilon x_L)$; then $\varphi_\alpha = n_\alpha\varphi_L + \varepsilon x_\alpha^T$. As in Sec. 2, we divide the propagator $D_{\mu\nu}^{c,E}(x,x')$ into two terms:

$$D_{\mu\nu}^{c,E}(x, x') = D_{\mu\nu}^{(1)} + D_{\mu\nu}^{(2)}.$$

In $D_{\mu\nu}^{(1)}(x,x')$ the integral is multiplied by φ_L, φ'_L, and in the definition of the function $D_{\mu\nu}^{(2)}(x,x')$ the factors $\varepsilon\varphi_L x'^T_\alpha$ and $\varepsilon x_\alpha^T\varphi'_L$ occur in front of the integral. It can be seen from (112) that the rule for avoiding the poles is determined by the parameter ε and, therefore, is defined until we take the limit $\varepsilon \to 0$. Taking into account the pinch structure of the poles in the relation (112), we rewrite it in the equivalent form

$$D_{\mu\nu}^{(1)}(x, x') = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} \Big\{-\sigma_1(k, n; x_L, x'_L) \times \Big[g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{(kn)} + \frac{n^2}{(kn)^2} k_\mu k_\nu\Big] + \sigma_2(k, n; x_L, x'_L)\Big(g_{\mu\nu} - \frac{n_\mu n_\nu}{n^2}\Big)\Big\} \tag{114}$$

Here

$$\sigma_1(k, n, x_L, x'_L) = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{k^2-\mu^2+i\delta} D_1(k, n, x_L, x'_L);$$

$$\sigma_2(k, n, x_L, x'_L) = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{k^2-\mu^2+i\delta} (D_1(k, n, x_L, x'_L) - 1). \tag{115}$$

In (115) we have introduced the notation

$$D_1(k, n; x_L, x'_L) = (nk)^2 \Big[-\frac{1}{(k^2-\mu^2)n^2-(nk)^2} + \frac{\pi}{\varepsilon}\varphi_L(x)\varphi_L(x') e^{-\varepsilon|x_L-x'_L|}\delta(kn)\Big].$$

Formally, the expression (114) has the same structure as the traditional expression (108). However, the difference is great. First, σ_1 and σ_2 contain translationally noninvariant terms, which determine the longitudinal part of the gluon propagator. Both functions σ_1 and σ_2 can be expressed in terms of the spectral function $\rho(\mu^2)$ of the gluon propagator in the covariant α gauge for $\alpha = \alpha_R^*$. From (115) we obtain a simple translationally invariant relation:

$$\sigma_1(k, n, x_L, x'_L) - \sigma_2(k, n, x_L, x'_L) = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{k^2-\mu^2+i\delta}. \tag{116}$$

We now analyze the structure of the term $D_{\mu\nu}^{(2)}(x,x')$. As was shown in Refs. 34 and 39, this term leads to the anomalous result $\lim_{\varepsilon\to 0}(n^\alpha A_\alpha(x)) = 0$ but $\lim_{\varepsilon\to 0} n^\mu D_{\mu\nu}^{c,E} \neq 0$. A simple calculation gives

$$\left.\begin{aligned} &D_{\mu_L\nu}^{(2)}(x, x') = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')}\sigma^{\text{anom}}(k, n, x, x') n_\mu k_\nu; \\ &D_{\mu\nu_L}^{(2)}(x, x') = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')}\sigma^{\text{anom}}(k, n, x, x') n_\nu k_\mu, \end{aligned}\right\} \tag{117}$$

where

$$\sigma^{\text{anom}}(k, n, x, x') = \int_0^\infty d\mu^2\rho(\mu^2) \frac{D^{\text{anom}}(k, n, x, x')}{k^2-\mu^2+i\delta};$$

$$D^{\text{anom}}(k, n, x, x') = \pi n^2(kx)\delta(kn) e^{-\varepsilon|x_L-x'_L|};$$

$$D^{\text{anom}}(k, n, x', x) = \pi n^2(kx')\delta(kn) e^{-\varepsilon|x_L-x'_L|}.$$

The anomalous propagator, like the longitudinal part of the propagator $D_{\mu\nu}^{(1)}(x,x')$, is a consequence of the pinch structure of the poles (112). The limit $\varepsilon \to 0$ changes the residual symmetry of the system. It is obvious that for $\varepsilon = 0$ there is invariance with respect to continuous gauge transformations by matrices that depend on the coordinates orthogonal to the gauge vector n_μ:

$$\hat{A}_\mu^E(x) = U^{-1}(x_T)\hat{A}_\mu^E(x)U(x_T) + \frac{i}{g}U^{-1}(x_T)\partial_\mu U(x_T). \tag{118}$$

The presence of the residual symmetry leads to the problem of determining the operator of the canonical transformation to the interaction representation.[21] Strictly speaking, the passage to the limit $\varepsilon \to 0$ in the Green's function may lead to the loss of a number of terms in the higher orders of perturbation theory when observable quantities are calculated. The part played by the anomalous components of the propagator can be elucidated by direct calculation of the equal-time canonical commutation relation. We choose the timelike vector $n = (n_0,0,0,0)$. In this case the limit $\varepsilon \to 0$ leads to the gauge $A_0(x) = 0$. The commutator can be calculated in two ways:

a) by using the connection to any other gauge, in particular a covariant gauge,

$$\langle 0 \mid [\hat{G}^E_{0i}(x),\ \hat{A}^E_j(x')] \mid 0\rangle_{t=t'}$$

$$= \langle 0 \mid [E^{-1}\hat{G}_{0i}(A)\,E,\ \int dz_\alpha E^{-1}\hat{G}_{\alpha j}E] \mid 0\rangle, \qquad (119)$$

where $E(A) = P\exp\{ig(n_0\delta_{0\nu} + \varepsilon x_\nu)\int_0^\infty d\tau e^{-\varepsilon\tau}A_\nu(z)\}$;

b) by direct use of the relations (114) and (117) we represent the spectral function $\rho(\mu^2)$ in the form

$$\rho_B(\mu^2) = Z_A(\alpha_R^*)(\delta(\mu^2) + \tilde{\rho}(\mu^2))_R. \qquad (120)$$

We then obtain

$$\langle 0 \mid [G^a_{0i}(x),\ A^b_j(x')] \mid 0\rangle_{t=t'}$$

$$= i\delta_{ij}\delta^{ab}Z_A(\alpha_R^*)\left(1 + \int_0^\infty d\mu^2\,\tilde{\rho}(\mu^2)\right)_R,$$

and thus

$$1 - Z_A(\alpha_R^*) = \int_0^\infty d\mu^2\tilde{\rho}_B(\mu^2).$$

The results of the two approaches will be the same only if in the calculation (b) we take into account the anomalous terms $\langle 0|\partial_i A^a_0(x),\ A_j(x')]|0\rangle_{t=t'}$ of the commutator, which are nonzero despite the fact that $\lim_{\varepsilon\to 0}A_0(x) = 0$ by virtue of the relation (117).

Since the formal limit of the Faddeev–Popov determinant is

$$\lim_{\varepsilon\to 0}(\det M_F) = \int D\bar{\eta}D\eta\exp\left\{i\int d^4x\bar{\eta}^a(x)(n^\alpha\partial_\alpha\delta^{ab} + gf^{abc}n^\alpha A^c_\alpha)\eta^b(x)\right\}, \qquad (121)$$

one gets the impression that the interaction of the anomalous propagator with the ghost fields makes a contribution to the elements of the S matrix, but this is not so. Since the chosen path leads to the condition $(n^\mu + \varepsilon x^\mu)A(x) = 0$, and for the propagator

$$(n^\mu + \varepsilon x^\mu)D^E_{\mu\nu}(x, x') = n_\mu D^{(2)}_{\mu\nu}(x, x') + \varepsilon x_\mu D^{(1)}_{\mu\nu}(x, x') = 0 \qquad (122)$$

the singular ($\sim 1/\varepsilon$) longitudinal part of the propagator $D^{(1)}_{\mu\nu}(x,x')$ plays a part in the limit $\varepsilon\to 0$ in the second term of (122). Therefore, formal passage to the limit $\varepsilon\to 0$ in the expression

$$\det M_F = \det\{(n_\mu + \varepsilon x_\mu)[\delta^{ab}\partial_\mu + gf^{abc}A^c_\mu(x)]\} \qquad (123)$$

without allowance for the singular longitudinal part of the gluon propagator gives an incorrect result.

6. INFRARED ASYMPTOTICS OF THE GREEN'S FUNCTIONS

It may be that the greatest satisfaction comes from the analysis of the infrared structure of the Green's functions of the fermion and gluon fields in the contour gauges. This is due to both the purely esthetic properties of the approach and the fact that precisely the infrared asymptotics of the propagators have a direct applied significance in the calculation of long-wave gluon exchanges in hard reactions with the production of hadron jets or a real photon and a hadron jet propagating in the opposite direction. Re-expansion of the propagators with respect to the corresponding functions in the axial gauges is used fruitfully in the investigation of the infrared structure of hard processes (Refs. 12, 13, 18, 44, and 45).

We analyze the infrared properties of the quark propagator in the Fock–Schwinger (FS) gauge:[46,47]

$$(x-y)^\mu\hat{A}_\mu(x) = 0. \qquad (124)$$

This is sometimes called the coordinate gauge; in theoretical studies it has been used by many authors.[4,48–50] Thus, the quark propagator is

$$S^{FS}(x, x') = \langle 0 \mid E_1^{-1}(A)\,S(x, x')\,E_1(A) \mid 0\rangle, \qquad (125)$$

where $E_1(A) = P\exp\{ig(x-y)^\alpha\int_0^1 d\tau A_\alpha(z)\}$ and $z_\mu = y_\mu + \tau(x-y)_\mu$. We take the field A_ν in the argument of the exponential and in the function $S(x,x',A)$ on the right in the α gauge. Suppose that the point Y does not lie on the straight line that connects the points x and x' in Fig. 6. From Eq. (125) we can pass to the expression

$$S^{FS}(x, x') = \langle 0|\,E_1^{-1}(A)\,E_2(A)\,\tilde{S}^{FS}(x, x';\,A_{FS})\,E_2^{-1}(A)\,E_1(A) \mid 0\rangle. \qquad (126)$$

In Eq. (126), $E_2(A) = P\exp\{ig(x-\eta)^\alpha\int_0^1 d\tau A_\alpha(\tilde{z})\}$, and we take the point η_μ on the line $(x - x')$. The field A^μ_{FS} on the right-hand side of (126) satisfies the condition $(x-\eta)^\beta A^{FS}_\beta(x) = 0$, i.e., with a fixed point different from that in (124). The right-hand side of (126) is shown in Fig. 9. We see that the propagators in the Fock–Schwinger gauges with different fixed points are connected by a transformation that forms a Wilson loop. The propagator $\tilde{S}^{FS}(x, x')$ does not contain infrared singularities if the fixed point lies on the line connecting the points x and x'. As was shown earlier, the renormalization constants of the ultraviolet divergences do not depend on the choice of the fixed point, and, therefore, for an arbitrary point Y the infrared asymptotic behavior is determined by the calculation of the Wilson loop,

$$W_c = \langle|\,P\exp\left[ig\oint_c dz^\alpha\hat{A}_\alpha(z)\right]|\rangle. \qquad (127)$$

We calculate the loop W_c with the contour shown in Fig. 9 in the order α_s:

$$W_c = 1 - g^2\int_0^1 d\tau\int_0^1 d\tau'\{\theta(\tau-\tau')[\Delta_1^\alpha\Delta_1^\beta D_{\alpha\beta}((\tau-\tau')\Delta_1) + \Delta_2^\alpha\Delta_2^\beta D_{\alpha\beta}((\tau-\tau')\Delta_2) + \Delta_3^\alpha\Delta_3^\beta D_{\alpha\beta}((\tau-\tau')\Delta_3)] - \Delta_1^\alpha\Delta_2^\beta D_{\alpha\beta}(\tau\Delta_1 - \tau'\Delta_2) - \Delta_1^\alpha\Delta_3^\beta D_{\alpha\beta}(\tau\Delta_1 - \tau'\Delta_3) + \Delta_2^\alpha\Delta_3^\beta D_{\alpha\beta}(\tau\Delta_2 - \tau'\Delta_3)\}. \qquad (128)$$

In the relation (128) we have introduced the notation $\Delta_1^\alpha = (x-y)^\alpha$, $\Delta_2^\alpha = (x'-y)^\alpha$, $\Delta_3^\alpha = (x-x')^\alpha$:

$$\left.\begin{aligned} D_{\alpha\beta}(z-z') &= D^{(1)}_{\alpha\beta}(z, z') + D^{(2)}_{\alpha\beta}(z, z'); \\ D^{(1)}_{\alpha\beta}(z, z') &= \frac{1}{(z-z')^2}; \\ D^{(2)}_{\alpha\beta}(z, z') &= \frac{(1-\alpha)}{(4\pi)^2}\frac{\partial^2}{\partial z_\alpha\,\partial z'_\beta}\ln(z-z')^2. \end{aligned}\right\} \qquad (129)$$

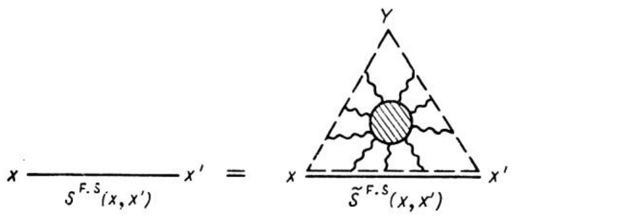

FIG. 9. Representation of Eq. (126). The broken triangle determines the path of integration in the Wilson loop.

The representation (129) of the gluon propagator in the covariant α gauge greatly simplifies the calculation, since the term $D^{(2)}_{\alpha\beta}$ enables us to carry out the integration over the paths. In the α_s approximation the regularization by the straight-line contour gives

$$z_E = 1 + \frac{\alpha_s}{2\pi}(3-\alpha) C_F \ln \delta^{-1}, \tag{130}$$

where δ is the regularizing dimensionless parameter.

The term $D^{(2)}_{\alpha\beta}$ in the calculation of the vertex leads to the result

$$\Gamma^{(2)}_W = \frac{\alpha_s}{2\pi} C_F (1-\alpha) \ln \delta. \tag{131}$$

Let us consider the calculation of the vertex with the propagator $D^{(1)}_{\alpha\beta}$ in more detail:

$$\Gamma^{(1)}_W = -\frac{\alpha_s}{\pi} C_F (\Delta_1\Delta_2) \int_0^1 d\tau \int_0^1 d\tau' \frac{1}{[\Delta_1^2\tau^2 - 2(\Delta_1\Delta_2)\tau\tau' + \Delta_2^2\tau'^2]}. \tag{132}$$

The expression (132) is invariant with respect to the scale transformations $\tau \to a\tau$, $\tau' \to a\tau'$ and $\Delta_1 \to C\Delta_1$, $\Delta_2 \to C\Delta_2$. The regularization violates this invariance. As a consequence, we have an arbitrariness in the definition of the finite part of the integral (132). The calculation of the expression (132) is fairly simple. If we restrict ourselves to the singular part, then

$$\Gamma^{(1)}_W = \frac{\alpha_s}{\pi} C_F \gamma_1 \operatorname{cth} \gamma_1 \ln \delta. \tag{133}$$

We have introduced the notation $\cosh \gamma_1 = (\Delta_1\Delta_2)/\sqrt{\Delta_1^2\Delta_2^2}$. In what follows we shall write $\cosh \gamma_2 = (\Delta_1\Delta_3)/\sqrt{\Delta_1^2\Delta_3^2}$ and $\cosh \gamma_3 = (\Delta_2\Delta_3)/\sqrt{\Delta_2^2\Delta_3^2}$. Now, taking into account the results (130), (131), and (133), we obtain

$$W_c = Z^{(1)}_{\text{cusp}} Z^{(2)}_{\text{cusp}} Z^{(3)}_{\text{cusp}} W^c_R; \tag{134}$$

$$Z^{(i)}_{\text{cusp}} = 1 - \frac{\alpha_s}{2\pi} C_F (\gamma_i \operatorname{cth} \gamma_i - 1) \ln \frac{\mu^2}{\mu'^2}. \tag{135}$$

As one would expect, the gauge invariance has the consequence that W_c does not contain the gauge parameter α. In (133) we have assumed that $|(\Delta_i\Delta_j)|/\sqrt{\Delta_i^2\Delta_j^2} > 1$. Depending on the properties of the integrals Δ_i, one or other of the angles γ may vanish. Thus, for the propagator (125) we obtain

$$S^{\text{FS}}(x, x') = Z_q \prod_{i=1}^{3} Z^{(i)}_{\text{cusp}} S^{\text{FS}}_R(x, x'). \tag{136}$$

Note that our method also enables us to obtain at once the infrared asymptotic behavior of the fermion propagator in an arbitrary covariant gauge. It follows from the relation

$$S(x, x') = \langle 0 \mid E(A) \mid 0 \rangle S^{\text{FS}}(x, x'), \tag{137}$$

where $E(A) = P\exp\{ig(x-x')^\alpha \int_0^1 d\tau A_\alpha(x + \tau(x'-x))\}$.

Since for such a choice of the path of integration the propagator $S^{\text{FS}}(x,x')$ does not contain infrared divergences, the infrared renormalization is equivalent to renormalization of the P-ordered exponential $E(A)$, i.e.,

$$Z_{IR} = 1 + \frac{\alpha_s}{4\pi} C_F (3-\alpha) \ln(\mu^2/\mu'^2), \tag{138}$$

in agreement with the well-known result of Abrikosov (1955) (see, for example, Ref. 51). The structure $(\gamma_j \coth \gamma_j - 1)(\alpha_s/2\pi)C_F$ is called the cusp anomalous dimension. It was introduced by Polyakov[52] and was later actively used in Refs. 13, 18, 44, 45, and 53. In observable quantities the infrared divergences lead to the appearance of the dimensional parameter λ_k, which characterizes the greatest wavelength of the field that can be detected by the experimental apparatus. In its turn, the renormalization of the ultraviolet divergences introduces into the theory the dimensional parameter Λ_{QCD}, which characterizes the interaction strength. The ratio λ_k^{-2}/μ^2 (μ is the parameter of the ultraviolet regularization) is unavoidably present in all regularized amplitudes of reactions with the production of hadron strings, together with the ratios of the parameter μ^2 to the Mandelstam variables. Naturally, the ultraviolet properties of the theory influence the infrared asymptotics of the reactions. It appears that the renormalization-group method was first used to analyze the infrared asymptotics of QCD in the studies of Ref. 15. The renormalization-group equation for the propagator (136) has the form

$$\left(\frac{\partial}{\partial \ln t} + \beta\frac{\partial}{\partial \alpha_s} - \sum_{i=1}^{3} \Gamma^{(i)}_{\text{cusp}} - \Gamma_q\right) S^{\text{FS}}(x, x'; Y) = 0. \tag{139}$$

Here

$$\frac{4\pi d\alpha_s^R}{d\ln t} = \beta, \quad \frac{d \ln Z^{(i)}_{\text{cusp}}}{d\ln t} = \Gamma^{(i)}_{\text{cusp}}, \quad \frac{d\ln Z_q}{d\ln t} = \Gamma_q, \quad t = \mu/\mu'^2.$$

Equation (139) is solved by

$$S^{\text{FS}}(x, x', Y) = S(x, x') \times \exp\left\{4\pi^2 \int_{\alpha_s(\mu^2)}^{\alpha_s(\tilde{\mu}^2)} \frac{d\alpha}{\beta}\left(\sum_i \Gamma^i_{\text{cusp}} + \Gamma_q\right)\right\}. \tag{140}$$

We use the single-loop calculations

$$\beta = -\alpha_s^2\beta_1, \quad \beta_1 = \frac{1}{3}(11N_c - 2N_f)$$

and write down the result of the integration for

$$W_{\text{cusp}} = \exp\left\{4\pi \int_{\alpha}^{\tilde{\alpha}} \frac{d\alpha}{\beta} \sum_i \Gamma^i_{\text{cusp}}\right\} = \exp\left\{\frac{2C_F}{\beta_1} \sum_i (\gamma_i \operatorname{cth} \gamma_i - 1) \ln\left(\frac{\alpha_s(\tilde{\mu}^2)}{\alpha_s(\mu^2)}\right)\right\}. \tag{141}$$

For the validity of perturbation theory in QCD we require $\alpha_s(\mu^2) \ll 1$, and if at the same time $\beta_1\alpha_s \ln(\tilde{\mu}^2/\mu^2) \ll 1$, then

$$W_{\text{cusp}} = \exp\left\{\frac{C_F\alpha_s(\mu^2)}{2\pi} \sum_i (\gamma_i \operatorname{cth} \gamma_i - 1) \ln \frac{\tilde{\mu}^2}{\mu^2}\right\}. \tag{142}$$

If $|(\Delta_i\Delta_j)|/\sqrt{\Delta_i^2\Delta_j^2} \gg 1$, then $\gamma_i \simeq \ln[(\Delta_i\Delta_j)/\sqrt{\Delta_i\Delta_j}]$. Then

$$W_{\text{cusp}} = \exp\left\{\frac{C_F\alpha_s(\mu^2)}{2\pi} \sum_{\substack{i,j=1 \\ i>j}} \ln\left(\frac{(\Delta_i\Delta_j)}{\sqrt{\Delta_i^2\Delta_j^2}}\right) \ln \frac{\tilde{\mu}^2}{\mu^2}\right\}. \quad 43)$$

For a felicitous choice of the contours that define the Wilson loop one can achieve correspondence between the doubly logarithmic asymptotic behavior (143) and the asymptotic behavior of a real jet process.[12,13,44] Note that, using the expression (138) for the infrared asymptotic behavior of the fermion Green's function in quantum electrodynamics in the covariant α gauge, we can readily obtain the well-known expression

$$S_F^{IR}(x, x')=\int\limits_{(k^2\sim m^2)} \frac{d^4k}{(2\pi)^4}\, e^{ik(x-x')}\, \frac{\hat{k}+m}{k^2-m^2} \exp\left\{\frac{\alpha_{\rm QED}}{2\pi}(3-\alpha) \times \ln\left(\frac{m^2}{k^2-m^2}\right)\right\}. \qquad (144)$$

The analysis of the infrared structure of the gluon propagator does not contain fundamental difficulties, but it is more awkward. As the initial expression for the investigation one should take the relation (71). The reader can learn about the results of calculations and the renormalization-group approach in the axial gauge in Ref. 54. The cusp anomalous dimension was calculated in the order α_s^2 in Ref. 57.

CONCLUSIONS

Thus, proceeding from the geometrical interpretation of a gauge field, we have attempted to understand from the point of view of unified principles the collection of problems that occur in a large class of noncovariant gauge conditions. The contour gauge condition that we have introduced (path-dependent gauge)[12] leads to field configurations which can be used to construct gauge theories in terms of gauge-invariant quantities. The price of such an approach is the explicit breaking of the Lorentz invariance of the Green's functions. The contour gauges contain as a limiting case gauge conditions of axial type. At the same time, the rules for avoiding the unphysical gauge poles are uniquely fixed. The infinitesimal parameter ε, which occurs in the definition of the path and characterizes the distance of the initial point, is naturally absent in the gauge-invariant structures. However, the traditional method of calculation in perturbation theory requires knowledge of the gauge-dependent Green's functions. This leads to a fundamental question: Is it possible to use gauges of axial type in the framework of standard perturbation theory in non-Abelian theories? In the higher orders of perturbation theory the poles $1/\varepsilon^n$ require inclusion of the following terms of the expansion with respect to the parameter ε, since otherwise finite contributions to observable quantities $(1/\varepsilon^n)m(s,u,t)\varepsilon^n$ would be lost, and this would entail violation of the gauge invariance. Theoretically, all information for the separation of such terms is available, but in computational practice this obviously leads to a huge complication of the calculations. This fact was first discussed in Refs. 34 and 39, and Andraši and Taylor[55] recently arrived at a similar conclusion. It is possible that that this problem has deeper roots directly related to the residual symmetry.[21] At the same time, in problems that require summations of infinite sequences of diagrams, going beyond the framework of the perturbative approach, the advantages of contour gauges are quite strong. The proof of multiplicative renormalizability has opened up the possibility of obtaining equations that relate the propagator spectral functions, which are actively used in the nonperturbative analysis of the Dyson equations. The only rigorously solvable nonperturbative problem of quantum gauge theory—the summation of the infrared divergences—acquires particular elegance in contour gauges. Should the conclusion be drawn that the main successes of the path-dependent formalism are possible with the development of nonperturbative methods? At the least, such a possibility should be borne in mind.

Note that our review has not covered all problems. We have not made a comparative analysis of conditions with timelike and spacelike contours. The case of an isotropic gauge, $(nA)=0$, $n^2=0$, has not been sufficiently studied in any of the approaches. The most systematic investigation of the isotropic gauge was made in Ref. 56.

APPENDIX

Several derivations of Eq. (40) are known; we give one of them, which was used in Ref. 12. We differentiate Eq. (36) with respect to x, obtaining

$$\frac{\partial}{\partial x_\nu}\left(\frac{\partial z^\mu}{\partial \tau}\frac{\partial E(A)}{\partial z_\mu}-ig\frac{\partial z^\mu}{\partial \tau}\hat{A}_\mu(z)E(A)\right)=0, \qquad (A1)$$

where $E(A)=P\exp(ig\int_0^\tau d\eta(\partial_z{}^\mu/\partial\eta)\hat{A}_\mu(z))$; $z_\mu(\eta)$ is some parametrization of the path from y to x. The expression (A1) can be transformed to

$$\frac{\partial z^\mu}{\partial \tau}\left\{\left(\frac{\partial}{\partial z_\mu}-ig\hat{A}_\mu(z)\right)\left(\frac{\partial}{\partial x_\nu}E(A)\right)-\left[\frac{\partial}{\partial z_\mu},\frac{\partial}{\partial x_\nu}\right]E\right\} =ig\frac{\partial}{\partial x_\nu}\left(\frac{\partial z^\mu}{\partial \tau}\hat{A}_\mu(z)\right)E. \qquad (A2)$$

Equation (A2) can be readily solved by the method of variation of arbitrary constants. Let

$$\frac{\partial E}{\partial x_\nu}=\Phi B_\nu(x, y), \qquad (A3)$$

and suppose that the function Φ satisfies the equation

$$\frac{\partial}{\partial \tau}\Phi-ig\frac{\partial z^\mu}{\partial \tau}A_\mu(z)\Phi=0; \qquad (A4)$$

then, since Eqs. (A4) and (36) are identical,

$$B_\nu(x, y)=ig\int_0^\tau d\eta E^{-1}\frac{\partial}{\partial x_\nu}\left(\frac{\partial z^\mu}{\partial \eta}A_\mu(z)\right)E + \int_0^\tau d\eta\frac{\partial z^\alpha}{\partial \eta}\frac{\partial z^\beta}{\partial x_\nu}E^{-1}\left[\frac{\partial}{\partial z_\alpha},\frac{\partial}{\partial z_\beta}\right]E, \qquad (A5)$$

i.e.,

$$-\frac{i}{g}E^{-1}\frac{\partial}{\partial x_\nu}E=\int_0^\tau d\lambda E^{-1}(\lambda)\frac{\partial}{\partial x_\nu}\left(\frac{\partial z^\alpha}{\partial \lambda}A_\alpha(z)\right)E(\lambda) +\frac{i}{g}\int_0^\tau d\lambda\frac{\partial z_\alpha}{\partial \lambda}\frac{\partial z_\beta}{\partial x_\nu}E^{-1}\left[\frac{\partial}{\partial z_\alpha},\frac{\partial}{\partial z_\beta}\right]E; \qquad (A6)$$

$$E^{-1}\partial_\nu E|_{\tau=0}=0. \qquad (A7)$$

Integrating the first term on the right by parts and using Eq. (36), we obtain

$$\frac{i}{g}E^{-1}\partial_\mu E=-E^{-1}\hat{A}_\mu(x)E+\frac{\partial z^\nu}{\partial x_\mu}E^{-1}A_\nu E\Big|_{\substack{\tau=0\\ z=Y}} +\int_0^\tau ds\frac{\partial z^\nu}{\partial x_\mu}\frac{\partial z^\eta}{\partial s}E^{-1}\{\partial_\eta\hat{A}_\nu-\partial_\nu\hat{A}_\eta+ig[\hat{A}_\nu,\hat{A}_\eta]\}E -\frac{i}{g}\int dz_\alpha E^{-1}\left[\frac{\partial}{\partial z_\alpha},\frac{\partial}{\partial z_\beta}\right]E\frac{\partial z_\beta}{\partial x_\mu}. \qquad (A8)$$

Since

$$A_\mu^E=E^{-1}A_\mu E+\frac{i}{g}E^{-1}\partial_\mu E,$$

we obtain Eq. (40) from (A8). All the important properties of the P-ordered exponential in fact follow from Eq. (36). Thus, for the transformed $A_\mu^U = U^{-1}A_\mu U + (i/g)U^{-1}\partial_\mu U$ we obtain

$$\frac{\partial E(A^U)}{\partial\tau} = igU\frac{\partial z^\mu}{\partial\tau}A_\mu(z)U^{-1}E(A^U) + \frac{\partial U}{\partial\tau}U^{-1}E(A^U),$$

from which it follows that

$$\frac{\partial}{\partial\tau}(U^{-1}E(A^U)) = ig\frac{\partial z^\mu}{\partial\tau}A_\mu(z)U^{-1}E,$$

i.e.,

$$U^{-1}(x)\,E(A^U)\,U(y) = E(A).$$

The generalization of Stokes's theorem to non-Abelian fields can also be readily obtained by using Eq. (36) for two P-ordered exponentials with contours that connect x and y but noncoincident γ_1 and γ_2: $\gamma_1 - \gamma_2 = \delta\gamma$.

[1)]The principal-value prescription was introduced in Refs. 24–26.

[2)]Derivations of Eq. (40), and also of the properties of P-ordered exponentials are given in the Appendix.

[3)]Two-loop calculations in the MOM scheme in the α gauge, and also an investigation of the dependence of the effective charge α_s on the gauge parameter α, can be found in Ref. 58.

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