The development of new methods for charged-particle acceleration at the Erevan Physics Institute (a review)

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Work on the development of new methods for charged-particle acceleration carried out during the last 15 years at the Erevan Physics Institute is reviewed. Results are given on the use of surface waves arising in a vacuum adjacent to a dielectric in total internal reflection and also the inverse Vavilov–Čerenkov effect for obtaining high acceleration rates inaccessible in traditional accelerators. The experimental setup and results of a demonstration experiment on laser acceleration in undulators (an inverse free-electron laser) are described. The acceleration of particles to high energies by wake fields in passive moderating structures is discussed. The results of theoretical studies of nonlinear effects arising in particle acceleration by wake fields in plasmas are given. The dependence of the wake-field strength and the transformation ratio on the γ factor of the bunch is found, and the possibility of self-acceleration of the electrons of a relativistic bunch in a plasma is considered. The influence of the ion motion and the transverse dimensions of the bunch on the self-focusing of a bunch in a plasma and on wake-field generation is taken into account. The status of the investigations in these areas at other laboratories and the future trends are discussed.

INTRODUCTION

The search for new methods of charged-particle acceleration different from the traditional ones used at present began long ago, in the early 1950s, when the principles of collective methods of acceleration in plasmas were formulated (see, for example, Ref. 1). After the discovery of lasers and the development of laser technology, in the early 1960s several new methods were proposed which are referred to in general as the laser acceleration of charged particles.[2] However, it was only at the beginning of the 1980s that these new acceleration methods came into their own and became an independent area of research in accelerator physics and technology. This was reflected, in particular, at the 12th and 13th International Conferences on High Energy Particle Accelerators,[3,4] and, in recent years, in the fairly regular scheduling of conferences devoted to new acceleration methods.[5–10] These new methods have been classified; this is described in Refs. 2, 11, and 12.

There are two factors which have motivated the search for new methods of charged-particle acceleration. The first is the rapid development of high-energy accelerators and storage rings during the late 1970s and early 1980s, which simultaneously made it clear that there is a natural limit to the possibilities of further development of accelerators and storage rings based on the traditional technology. Second, the revolutionary development of physics during the same period and later of high-energy astrophysics have made it necessary to reach ever higher energies, luminosities, and precision in charged-particle beams.

The original goal of studies on laser acceleration was to use existing technical achievements (lasers, microtechnology, controlled nuclear fusion, material synthesis, and so on) to improve the physical characteristics of accelerators: to increase the particle energy, the acceleration rate, and the beam luminosity. A variety of new methods were developed for designing accelerator components which, as a rule, were completely different from the traditional ones, but were used for the same purposes (acceleration, focusing, phase-space transformation, and so on).

During the course of investigations the emphasis shifted to the problem of developing powerful energy sources and energy exchange (the efficiency of transformation of the "line" energy into the beam energy), the efficiency of which must increase with the accelerator energy.

It also became clear that when lasers were used for acceleration, the lasers needed to have a number of unique characteristics related to requirements on the stability of the particle-acceleration regime. For example, for a width of the laser radiation spectrum of the order of 10% of the frequency, which ensures an acceleration rate of up to 80 GeV/m for Nd lasers in the ten-fold use of the structure to accelerate particles by surface waves, the phase and group velocities cannot differ by more than several parts in a thousand. This imposes strong constraints on the physical and technological characteristics of laser optics, forcing the tolerance to be less than a fraction of a percent of the wavelength.

As the understanding of the new, often quite stringent requirements on lasers improved, new laser schemes for charged-particle acceleration began to be realized, and it seemed almost natural to return to the traditional electromagnetic field sources—intense beams of relativistic electrons. The power which can be obtained in such beams is comparable with, or even sometimes larger than, that in present-day lasers. It is easier to focus electron beams or make them parallel, and it is easier to guide them and to ensure the required pulse repetition frequency. By using electron beams in various ways (free-electron lasers, or specially designed structures) it is also possible to obtain the high electromagnetic wave frequency needed to ensure a large acceleration rate for a relatively small peak output from the power supply. In addition, the coefficient describing the conversion of beam energy into electromagnetic wave energy can be made quite large, in some cases (resonance structures) approaching several tens of percent. These features are used in the two-beam accelerator, in

TABLE I. New acceleration schemes.

Number	Acceleration scheme	Center
1	Laser acceleration in plasmas	UCLA, RAL, NRC, LLNL, Paris
2	Plasma lenses	SLAC
3	Wake fields in passive and active structures	DESY, Osaka, ANL, SLAC, UCLA
4	Switched Linac	CERN, BNL; Rochester
5	Inverse free-electron laser	BNL, NRL
6	Scheme using inverse Čerenkov effect	UCSB
7	Two-beam systems	CERN, LBL/LLNL
8	Photocathodes	LANL, BNL
9	Investigation of the possibility of obtaining high gradients in accelerator structures	SLAC, LBL/LLNL, KEK, Novosibirsk
10	Collider physics	SLAC, CERN, KEK, Novosibirsk
11	Power sources:	
	a) klystrons, klystrinos, girokons	SLAC, Novosibirsk
	b) lasertrons	KEK, SLAC, Orsay
	c) lasertrons forming plane beams	Texas
	d) gyroklystrons	Maryland
	e) free-electron laser, CARM	LBL/LLNL, MIT, NRL, KEK
	f) relativisitic klystron	SLAC/LLNL/LBL

which an intense beam from an induction linear accelerator generates (in particular, in a free-electron laser) intense electromagnetic radiation with centimeter wavelength. This radiation is then used to accelerate the electrons of a low-current beam to high energies with an acceleration rate of hundreds of MeV per meter. Schemes for obtaining high acceleration rates have been proposed in which intense, low-energy electron beams are used to excite high-frequency electromagnetic wake fields in passive structures (for example, resonators and waveguides with diaphragms) and in a plasma.

In the case of passive structures, the wake-field acceleration schemes and the corresponding technology are very close to the conventional schemes, and therefore these schemes are classified as quasi-conventional or simply conventional.[1)] Schemes like the resonant wake-field transformer, the linear collider based on a relativistic klystron (TLC), and the CERN linear collider using a superconducting linear accelerator as the beam source (CLIC) (Refs. 13 and 14) are examples of schemes which will be realized only in the 1990s for electron–positron colliders operating in the TeV energy range. Also the technique based on nonlinear interactions of electron beams with a plasma appear to be very promising for future applications.

Because of these general trends and also the specific conditions for the future development of the Erevan Accelerator Center at the Erevan Physics Institute (EPI), in the early 1970s we began the active study and development of new schemes for charged-particle acceleration.

In Table I (Ref. 12) we list almost all of the new acceleration schemes and the principal centers where they are being developed. The Erevan Physics Institute should have been listed in this table as a center working on the development of several of these schemes (schemes 2, 3, 5, 6, and 11e). The present review was written to fill this gap and to summarize the work carried out during the last 15 years in these areas at the Erevan Physics Institute.

1. ACCELERATION BY SURFACE WAVES ARISING IN TOTAL INTERNAL REFLECTION

One of the features of surface waves propagating in a vacuum is the decrease of the propagation velocity, which is particularly attractive for charged-particle acceleration by means of lasers. This idea was first suggested by Lohmann (see, for example, Ref. 15), who proposed that the inverse Smith–Parcell effect[16] be used for particle acceleration (in 1953 Smith and Parcell discovered and studied the radiation emitted in the passage of a charged particle near the surface of a diffraction grating). Using the idea of a scaled-down waveguide with diaphragms for rf waves, first suggested in Ref. 18, Takeda and Matsui[17] proposed that radiation in the optical range be used for particle acceleration. Radiation from a CO_2 laser falls at normal incidence on a diffraction grating, and a beam of accelerated particles passes near the grating at distances on the order of the wavelength (see also Ref. 19). Lawson[20] later showed that in the geometry proposed in Ref. 17 the acceleration will fall off in inverse proportion to the particle Lorentz factor. After several years, Palmer[21,22] proposed that this defect be eliminated by using one or two laser beams, symmetric about the surface normal, and incident on the grating at an oblique angle.

Further developments and a review of the current status of the work on particle acceleration by surface waves on a diffraction grating are discussed in Refs. 23–26. An experiment on particle acceleration by surface waves is being prepared at Brookhaven National Laboratory[27] at the accelerator test facility (ATF), which consists of a 50-MeV linear accelerator which produces electron bunches synchronized with picosecond pulses from a CO_2 laser of power 100 GW.

A surface-wave acceleration scheme which is simpler, at least from the mathematical point of view, was considered in 1971 at the EPI by Kheifetz.[28] He proposed the use of surface waves appearing in a vacuum adjacent to the surface of a dielectric due to total internal reflection. Later it was pointed out[15] that this idea was first suggested by Lohmann in 1962. The same idea was proposed in Ref. 20. The problem is amenable to a rigorous analytic calculation, which is important at the present stage of development of the theory. One of the results stated in Ref. 28 is that in the stable state particles are not accelerated. Particle acceleration requires either an increase of $\beta = v/c = (1/\sqrt{\varepsilon})\sin\varphi$ (here ε is the dielectric constant of the dielectric material and φ is the angle of incidence of the laser beam on the interface) along the accelerator channel, or a change in the polarization of the incident elliptically polarized wave along the accelerator channel. Naturally, these methods give a relatively low acceleration rate. The analysis of the stability of the motion carried out in Ref. 28 shows that stability at each stage of the acceleration can be ensured if the particle energy increases by no more than a factor of 3.

Fairly recently, Nagorskiĭ, Amatuni, and Harutiunyan[29] returned to the analysis of the possible use of surface waves arising in total internal reflection for particle acceleration.

Total internal reflection in an optically less dense medium gives rise to a surface wave whose field components are sizable only right next to the interface, and fall off exponentially with increasing distance from the interface as

$$\exp\left\{-\frac{2\pi z}{\lambda}\sqrt{\varepsilon\sin^2\varphi-1}\right\}, \qquad (1)$$

where z is the distance along the normal to the flat interface, λ is the wavelength in the less dense medium (henceforth this will be the vacuum), and φ is the angle of incidence on the interface, which is larger than the limiting angle φ_0, $\sin\varphi_0 = 1/\sqrt{\varepsilon}$. Here if the wave vector $\mathbf{k}$ of the incident wave lies in the $x0z$ plane, the surface wave in the second medium propagates along the $0x$ axis with phase velocity $c(\sqrt{\varepsilon}\times\sin\varphi)^{-1}$, i.e., the effective "index of refraction" for it will be

$$1\leqslant n=\sqrt{\varepsilon}\sin\varphi,\quad \varphi_0<\varphi<\pi/2. \qquad (2)$$

Therefore, the surface wave in the vacuum arising in total internal reflection is slowed down. The energy flux in the surface wave, averaged over a period, is nonzero and is directed along the $0x$ axis. Therefore, if, above the interface in the region where the field strength of the surface wave is still large, we release a charged particle with velocity $v = c\beta$ at an angle to the $0x$ axis given by the Čerenkov condition

$$\cos\theta = 1/n\beta < 1, \qquad (3)$$

this particle will be located in the region with the surface wave. From (2) and (3) it follows that the particle Lorentz factor will be bounded below:

$$\gamma > (1-n^{-2})^{-1/2} \equiv \gamma_n. \qquad (4)$$

The conditions for particle acceleration by the field of a surface wave and for stability of the particle motion can be derived from the solutions of Maxwell's equations for a surface wave in the vacuum with wave vector

$$\mathbf{k}=\frac{\omega}{c}\,(n\mathbf{e}_1+i\sqrt{n^2-1}\,\mathbf{e}_3),\quad k^2=\omega^2/c^2, \qquad (5)$$

and arbitrary polarization specified by the complex parameters α_1 and α_2 in the coordinate system (x', y', z', t') moving with the wave along the direction of the Ox axis with velocity c/n (the frame in which the wave is static, SWF):

$$\mathbf{E}'=\mathbf{e}_1(1-\gamma_n)(\mathbf{e}_1\mathbf{E})+\gamma_n\left(\mathbf{E}+\frac{1}{n}\mathbf{e}_1\times\mathbf{H}\right)$$

$$=E_0e^{-\xi}\,\mathrm{Re}\,\alpha_1(\mathbf{e}_1\mp i\mathbf{e}_3)\,e^{i\Phi}; \qquad (6)$$

$$\mathbf{H}'=\mathbf{e}_1(1-\gamma_n)(\mathbf{e}_1\mathbf{H})+\gamma_n\left(\mathbf{H}-\frac{1}{n}\mathbf{e}_1\times\mathbf{E}\right)$$

$$=-\frac{n}{\gamma_n}E_0e^{-\xi}\,\mathrm{Re}\,i\alpha_2(\mathbf{e}_1+i\mathbf{e}_2)\,e^{i\Phi}, \qquad (7)$$

where

$$\xi=\frac{\omega'}{c}z;\quad \Phi=\frac{\omega'}{c}x;\quad \omega'=\omega\sqrt{n^2-1}. \qquad (8)$$

The condition for vertical equilibrium of the particle requires that ξ = const, and the condition for a Čerenkov resonance requires that the phase Φ be constant, so according to (6)–(8) a particle in equilibrium in the SWF can move only in the direction $\mathbf{e}_2$ with velocity

$$\boldsymbol{\beta}'=\gamma_n\mathbf{e}_2\sqrt{\beta^2-n^{-2}}, \qquad (9)$$

which also follows from Lorentz transformations for the particle velocity $\mathbf{v}$. As can be seen from (6), owing to the absence of an electric field component along $\mathbf{e}_2$, particles moving with velocity (9) in the SWF will not acquire a longitudinal acceleration from the surface field. This also follows from the results of Ref. 28. The electric field component directed along $\mathbf{e}_2$ in the SWF needed for particle acceleration can, however, be obtained by switching on, in the laboratory frame (LF), a constant vertical magnetic field b_3 pointing antiparallel to the $0z$ axis (this is the essential difference between this acceleration mechanism and the mechanism proposed in Ref. 28). Then an electric field arises in the SWF. According to (6), it is given by

$$\mathbf{E}_2'=\frac{\gamma_n}{n}\,\mathbf{e}_2b_3 \qquad (10)$$

and ensures longitudinal acceleration. At the same time the field b_3 in the LF is needed to maintain the phase equilibrium of the Čerenkov resonance in the accelerating phase of the wave. This condition is what defines the field b_3.

Vertical equilibrium obviously requires a field perpendicular to the particle trajectory and parallel to the surface of the dielectric in the LF. However, for this equilibrium to be stable (neglecting the field produced by the beam charge), its sign must change in going from one segment of the accelerator channel to another. It follows from the equations of motion that this fact, in turn, requires that the phase of the wave undergo jumps, as shown in Ref. 30 (see also Ref. 18). The inclusion of the effect of the beam-charge field on the stability of the motion (the current-loading effect), due to G. A. Nagorskiĭ, simplifies the conditions for vertical equilibrium.

Later, more detailed studies were carried out of the dynamics of particles in a system consisting of a surface electromagnetic wave (SEW) plus a magnetic field. The stability of the particle motion near the Čerenkov threshold was studied by computer,[31] and conditions on the external-wave polarization were found for which the focusing parameters (the magnet period and the SEW phase shifts) are independent of

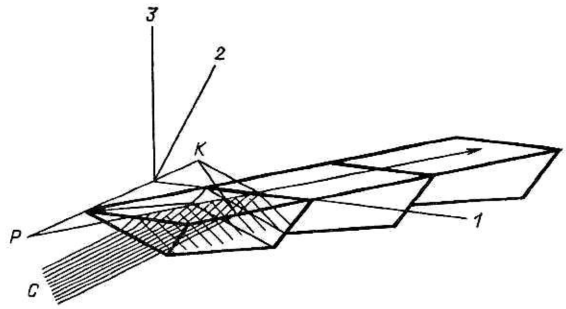

FIG. 1. General form of the generator for the SEW laser accelerator. Three crystals of the accelerator channel and the passage of a light ray in one of them are shown. The ultrarelativistic particle *P* and wave front in the equlibrium phase *C* start simultaneously. The geometry of the crystals, which are cut from the wedge with edge *PK*, is selected to achieve the maximum acceleration rate and minimum reflection of the elliptically polarized wave at the entrance (the lower surface of the wedge) to the crystal. The surface electromagnetic wave propagates in the direction *1*, and its wave front is parallel to the axis *2*. The axis *3* is perpendicular to the upper surface of the crystals.

the particle velocity, thus allowing a significant increase of the acceptance.

An algorithm was developed for analyzing more complicated focusing systems (of the type FODO) allowing a sizable increase in the acceleration rate, owing to more rational use of the laser power.[32] A rigorous solution of Maxwell's equations for the SEW with phase jumps was obtained, which revealed the possibility of the propagation of a complex wave front along the surface of a dielectric without refractive and dispersive distortions.

At the present time a demonstration experiment on surface-wave acceleration is under preparation; the schematics of the setup are shown in Fig. 1.

2. THE INVERSE VAVILOV–ČERENKOV EFFECT

In 1962 Shimoda[33] suggested that the inverse Vavilov–Čerenkov effect be used for particle acceleration. Here an electromagnetic wave is decelerated by the presence of a medium with index of refraction $n(\omega)$, and the resonance nature of the interaction of a particle with the incident wave is ensured by making the angle between the directions of the incident wave and the particle velocity equal to the Čerenkov angle:

$$\cos\theta = 1/n(\omega)\,\beta.$$

In 1972–1980 the Erevan group made detailed studies[34–41] of charged-particle acceleration using the inverse Vavilov–Čerenkov effect in various media, including gaseous media. The studies of Nagorskiĭ, Orlov, and Petrosyan,[35–37] which were essentially the first on the theory of laser acceleration, gave special attention to problems concerning the stability of the motion of particles accelerated by the inverse effect. In particular, they drew attention to the role of the magnetic field of the laser wave, the strength of which in the medium is larger than that of the electric field. When the particle velocities are relativistic, the effect of this magnetic field is important and disrupts the stability of the wave–particle Čerenkov resonance. It was suggested that the stability be ensured by the inclusion of a constant magnetic field directed along the magnetic vector of the wave. A system of three Čerenkov waves in which it is also possible to have a regime of stable particle acceleration with large phase space of the beam was proposed and analyzed.

In Ref. 37 a method was developed for including, in both these acceleration variants, the effect of multiple scattering on the stability of the acceleration regime. It was shown that long-term stability occurs only when the electric field strength of the wave is sufficiently large. In particular, in the field of picosecond laser pulses with flux 10^{14} W/cm^2, when breakdown of the medium does not yet occur, multiple scattering cannot seriously hinder long-term particle acceleration.

The inverse Vavilov–Čerenkov effect on free electrons passing through various media was first observed experimentally in Ref. 42. The preliminary data of Ref. 42 were increased and confirmed in Ref. 43. In the latter study gaseous hydrogen and methane were used, in addition to the helium medium used in Ref. 42. In addition, the influence of a laser pulse on the electron distribution was studied when the Čerenkov condition was violated as the index of refraction of the media was varied (see also Ref. 44).

The authors of Refs. 45–47 discovered the interesting and, apparently, promising possibility of creating large fields by means of a second inversion of the Vavilov–Čerenkov effect. Here particle beams accelerated and modulated using the inverse effect are allowed to fall on a different, specially chosen medium, where they produce radiation with large field strength. In this method the inverse Vavilov–Čerenkov effect is used twice. First, particle beams which have acquired the necessary amount of energy in a traditional accelerator are modulated with the use of a laser, then this energy is used to excite a Čerenkov wave in a natural or artificial medium, and finally the inverse effect is used to accelerate the particles.

A review by Fontana[48] gives the results of studying the effects of breakdown of the medium and the influence of electron bremsstrahlung in the medium during electron acceleration via the inverse Vavilov–Čerenkov effect.

To conclude this section, we should note that there has been some slackening of interest in recent years in the development of particle acceleration by means of the inverse Vavilov–Čerenkov effect.[1–8] This may be related to the appearance of new acceleration schemes, which are being discussed intensively in the present literature. However, it should always be remembered that a "subtle form" of the Vavilov–Čerenkov effect is silently used in practically all acceleration schemes (for example, in the various types of crystal accelerator).

3. THE INVERSE FREE-ELECTRON LASER

As is well known, ideas for using lasers to accelerate charged particles appeared right after the successful operation of the first lasers in the early 1960s. In 1962 Kolomenskiĭ and Lebedev[49–51] proposed the autoresonance acceleration of particles moving simultaneously in the field of an electromagnetic wave and in a magnetic field of various configurations. Similar ideas were proposed at nearly the same time by Davydovskiĭ.[52]

Ten years after the appearance of Refs. 49 and 52, in 1972 Palmer[53] made a detailed study of the interaction of a particle moving in a magnetic undulator or a helical magnetic field with a laser wave and carried out calculations for a setup for particle acceleration which later was called the inverse free-electron laser (IFEL). Here it turned out that the acceleration rate is inversely proportional to the particle energy and can reach 100 MeV/m for particle energies of up to several MeV.

In 1982 Kondratenko and Saldin[54] carried out a detailed calculation of an inverse free-electron laser with a constant transverse magnetic field of alternating sign.

The reviews of Pellegrini[5,6] and Ranieri[7] discuss specific schemes for accelerators to energies of up to several hundred GeV based on an IFEL. Here, even at energies of order 10 GeV the electron acceleration rate can be maintained at a level ~100 MeV/m, with no important restrictions imposed on the current. Acceleration by the IFEL method is also attractive because nearly all of the huge amount of theoretical and experimental results on free-electron lasers can be used (see, for example, the reviews of Refs. 55–58). It can therefore be hoped that the first prototypes of accelerators in the GeV range based on the IFEL principle will be created in the next few years. Here the fundamental difficulty is to increase the average laser power while observing certain requirements on the beam quality.

However, for the experimental verification of laser acceleration in undulators and the refinement of the technique, it is sufficient to use lasers of relatively low power and low-energy electrons. This has been done at the EPI by the group working under Petrosyan.[59,60] This was a continuation of the work of Ref. 61 on the development of a free-electron laser operating at infrared wavelengths using a microtron. The results of these studies are logically connected to and re-echo the results obtained at other laboratories around the world in recent years (see the reviews of Refs. 54–58).

Ref. 60 gives the results of experimental study of the spectral distribution of the beam electrons after their interaction with an external electromagnetic wave in an undulator. The experimental setup is shown schematically in Fig. 2. It is similar to the setup of Ref. 59, but the fundamental parameters are closer to optimal. In particular, the electron energy was increased to 12 MeV, which allowed a significant expansion of the range of variation of the parameters determining the interaction between the electrons and the wave. An electron beam of energy up to 12 MeV extracted from the microtron is shaped by two quadrupole lenses and four correcting magnets in such a way that the diameter of the transverse cross section of the beam at the center of the undulator magnet is 4 mm. Radiation from a CO_2 laser enters the undulator and is focused on the center of the undulator magnet by means of two copper mirrors: a plane mirror and a spherical mirror with radius of curvature 5 m. An iron-free flat undulator magnet was used. The fundamental parameters are given in Table II.

TABLE II. Fundamental parameters of the setup.

Electron energy, MeV	12
CO_2-laser power, MW	20
Length of helical undulator, cm	20
Length of undulator cycle, mm	9.5
Strength of undulator field, Oe	0–10 000
Transverse cross section of beam, cm^2	0.25

The dependence of the location of the maximum in the electron energy distribution on the magnetic field strength of the undulator was measured (Fig. 3). It is well known[54] that the average change of the electron energy in a single passage through the undulator has the form

$$\left\langle \frac{\Delta\gamma}{\gamma} \right\rangle = 4\pi^3 \left(\frac{\Omega}{\omega} \right)^4 N^3 F(x), \tag{11}$$

where $\Omega^2 = 4\pi E_l k/(m\gamma_p^2 \Lambda)$ is the frequency of the phase oscillations, $\omega = 2\pi c/\Lambda$ is the frequency of the undulator oscillations, $k = 10^{-4} H\Lambda$ is the undulation coefficient, $F(x) = (1/x)(\cos x - 1 + (x/2)\sin x)$ is the standard interaction function, Λ is the period of the undulator in cm, H is the strength of the undulator magnetic field in Oe, N is the number of periods of the undulator, $\Lambda N = L$ is the undulator length, E_l is the field strength of the laser wave, $x = 2\pi N(1 - \gamma_p/\gamma)$, $\gamma = E/(m_e c^2)$, and γ_p is determined from the synchronism condition

$$\lambda = \frac{\Lambda}{2\gamma_p}(1 + k^2),$$

where λ is the wavelength of the laser.

The condition for a small signal and linear operating regime

$$\Omega/\omega \ll 1 \tag{12}$$

is satisfied for the chosen setup parameters. The standard

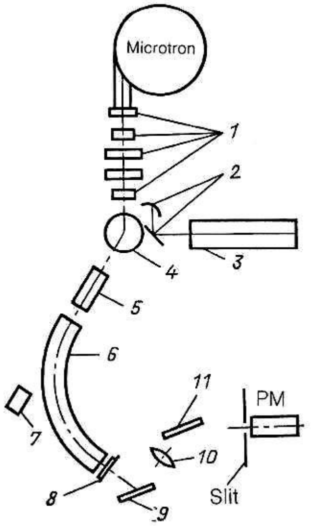

FIG. 2. Scheme for an experimental setup: *1*—correcting magnets and lenses; *2*—copper mirrors; *3*—CO_2 laser; *4*—rotating magnet; *5*—undulator; *6*—spectrometer magnet; *7*—detector; *8*—scintillator; *9*—mirror; *10*—lens; *11*—scanning mirror.

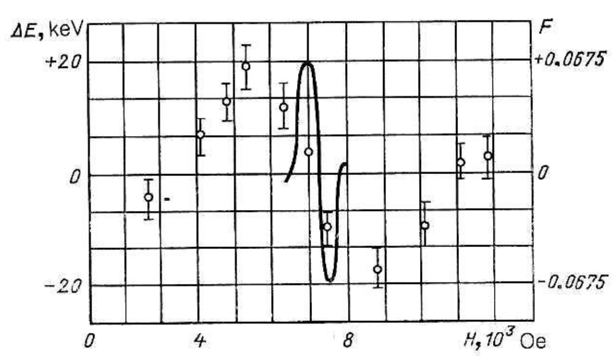

FIG. 3. Dependence of the shift of the electron energy distribution ΔE and F on the undulator field strength.

TABLE III. Parameters of the setup.

Electron energy, MeV	12
CO_2-laser power, GW	5
Laser field strength in interaction region, W/cm^2	10^{12}
Undulator length, cm	20
Length of initial undulator cycle, cm	0.95
Strength of undulator field, Oe	6000
Expected increase of electron energy, MeV	6
Expected acceleration rate, MeV/m	30

interaction function $F(H)$ is shown in Fig. 3 as a function of H (with x expressed in terms of H). We see from the experimental results that the range of H in which the wave and electron beam interact is much larger than predicted by the standard curve $F(H)$. Since the condition for a small signal and linear regime (12) is satisfied, this discrepancy is probably due to the nonmonochromaticity of the laser radiation and the nonuniformity of the undulator magnetic field over the transverse cross section. Expression (11) was derived for a monochromatic beam and uniform field. The nonuniformity of the undulator magnetic field over the transverse cross section amounts to several tens of percent, and the CO_2 laser beam is a multi-mode beam. If we substitute into (11) the maximum value of $F(H)$ and the values of the fundamental parameters of this experiment (see Table II), we find the maximum value $\Delta\gamma/\gamma \approx 10^{-3}$, which is in good agreement with the results of the measurements. The maximum change of the electron energy can be estimated from the expression

$$\Delta\varepsilon_{\max} = E_l \frac{k}{\gamma} L, \tag{13}$$

which was derived for the case of laser acceleration.[59] For the undulator length $L = 20$ cm, we have $\Delta\varepsilon_{\max} = 60$ keV, which is of the same order of magnitude as the measured results, taking into consideration the fact that it is the shift of the maximum of the spectral distribution that is measured, not the maximum change of the electron energy.

At the present time, scientists and engineers at the EPI, the Kurchatov Institute of Atomic Energy, and the Efremov Institute of Electrophysical Apparatus are working on the design of a new experimental setup for electron acceleration by the IFEL method. The main goal is to experimentally verify the possibility of obtaining a higher acceleration rate, which is basically determined by the power of the CO_2 laser used in the experiment, which was designed at the Institute of Atomic Energy and the Efremov Institute. The parameters of the setup are given in Table III.

The next stage of the study should be the design of a prototype accelerator based on an IFEL at an energy of about 5 GeV. Examples of calculations for such systems can be found in the reviews of Pellegrini[5] and Joshi.[62] In Table IV we give an example of a single-stage accelerator with an undulator simultaneously possessing a variable field and a period (see Fig. 4, which was taken from Ref. 62).

In conclusion, we note that free-electron lasers are used in various laser acceleration schemes directly as electromagnetic field sources.[63,64]

When lasers are used directly as sources of superhigh-frequency power, the restriction on the maximum energy of the accelerated particles is lifted. However, the accelerated currents cannot be as large as in the case of acceleration by the IFEL method. But, as is well known, in the IFEL method the energy of the accelerated particles is limited to about 100 GeV, owing to the decrease of the acceleration rate with increasing energy and, more importantly, the loss to magnetic bremsstrahlung of the electrons in the undulator (undulator radiation), which increases with energy.

TABLE IV. Laser parameters.

Power, W	$2\cdot10^{13}$
Pulse duration, nsec	1
Spot diameter, mm	3.5
Wavelength, mm	1
Laser field, V/m	$2\cdot10^{10}$
Accelerator length, m	39
Undulator parameters	
Field amplitude, kG	2–50
Length of cycle, cm	2–10
Parameters of the electron beam	
Initial energy, MeV	50
Final energy, GeV	5.3
Current, kA	2
Beam radius, mm	2
Average rate, MeV/m	120
Energy spread, %	0.05

4. WAKE-FIELD ACCELERATION IN PASSIVE MODERATING STRUCTURES

In wake-field acceleration in moderating structures, a particle beam of relatively low particle energy but large current is injected into a passive (for example, a waveguide with a diaphragm) or active (for example, a plasma) moderating structure and creates an electromagnetic field, which is then used to accelerate a second beam of particles of lower density to higher energies.

In some sense all the existing accelerators are wake-field accelerators. For example, in ordinary linear rf accelerators an electron beam of low energy and high current generates an rf electromagnetic field in the resonator of a klystron, which in this case is a "wake field." This field is transferred via a waveguide to an accelerating structure,

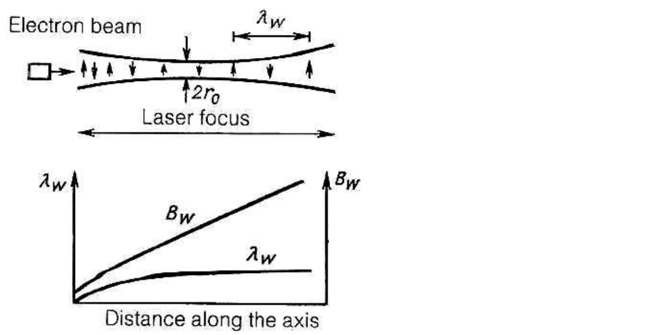

FIG. 4. Scheme for a single-stage accelerator with variable field B_w and period λ_w.

where the particle beam is accelerated to the required energy. The new feature which makes wake-field acceleration in passive structures a new acceleration method is the important quantitative changes in the characteristics of the corresponding accelerator setups. These changes are so large that they sometimes even require qualitative modifications, particularly in rf power-supply systems. First of all, the acceleration rate can be increased by at least an order of magnitude in comparison with the traditional methods, reaching 150–200 MeV/m. This in turn requires that the field frequency be increased in order to avoid breakdown or disruption in the passive structures. Increase of the frequency to tens of GHz is also required to preserve the possibly lower level of pulse power input into the accelerating structure, and also the decrease of the current loading. Nevertheless, the pulse power, for example, in a given frequency range reaches several gigawatts per meter of the accelerating structure for a pulse duration of tens of nanoseconds. All this requires the development of new rf (actually, ultra-rf) power sources, and the matching (or near-matching) of the electromagnetic wave-generating structures to the accelerating structures.[13,65–67]

Here we shall discuss the studies devoted to passive structures, although some of the general results are also valid for wake-field acceleration in a plasma (see Sec. 5).

The first proposals for wake-field acceleration in passive structures were made by Voss and Weiland.[68,69] The effect has been studied experimentally using a device designed at DESY, which the authors call a wake-field transformer[70] (Fig. 5). In this setup a ring-shaped electron beam passes through the accelerating structure (a waveguide fitted with diaphragms) through a slot in the diaphragms located near the outer wall of the waveguide. The wake fields which are generated are reflected from the outer wall and move along the radii toward the center of the waveguide. This causes the volume occupied by the field to decrease.

The net field formed on the axis of the waveguide increases in inverse proportion to the square root of the radius of the volume containing the wake field. For this configuration of the moderating structure, the usual constraint on the value of the accelerating field E_+ following from energy conservation for successive, longitudinally symmetric bunches, $E_+/E_- \equiv R \leqslant 2$ (where E_- is the field which decelerates the initial beam and R is the transformation ratio), is no longer applicable. For reasonable dimensions of the structure a transformation ratio $R = 10$–20 can be obtained.

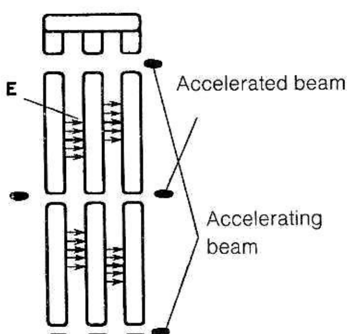

FIG. 5. Wake-field acceleration.

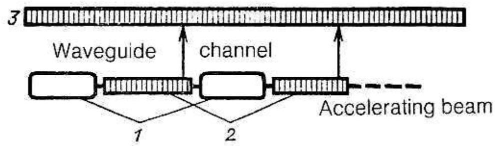

FIG. 6. The relativistic klystron. A high-current beam of intermediate energy is produced by the ILA module *1*. The bunched beam from the ILA excites a superhigh-frequency field in the resonator system *2* with small impedance. The energy losses of the accelerating (driving) beam to radiation are compensated by preacceleration in the ILA modules; *3* is a linear accelerator with high acceleration rate at frequency 36 GHz.

The main difficulty in realizing the scheme proposed in Refs. 68–70 (and, to a significant degree, any other scheme) is the generation of an accelerating beam of sufficiently high density, as has been pointed out by the scientists working on the original experiment.[13] They succeeded in obtaining only 10% of the density of the accelerating beam, and therefore the observed acceleration rate was only 8 MeV/m instead of the expected 100 MeV/m.

However, the ideas of Refs. 68–70 had great heuristic value, since they led to a number of new ideas about wake-field acceleration in various types of moderating waveguide structures. An example of one of these ideas is the linear accelerator with switched power.[71] Here it is proposed that, instead of the ring-shaped beam of Refs. 68 and 69 traveling along the periphery of the structure, electrons emitted by photodiodes under the influence of suitably phased laser radiation be used. However, estimates have shown that to generate a photoelectron current of the required density the laser must be very large. The idea of the relativistic klystron was suggested in Ref. 13 and later developed in Refs. 66, 67, and 72. Here electron beams of high luminosity (~1–2 kA), large peak power (~GW), and low energy (~1 MeV), created by existing induction accelerators, are used to generate an rf field of wavelength 1–3 cm (see, for example, Fig. 6 of Ref. 67).

The relativistic klystron represents the natural unification of two well developed technologies—klystron tubes and the induction linear accelerator. A collaboration consisting of scientists from the Stanford linear accelerator (SLAC) and induction linear accelerator (ILA), the Lawrence Laboratory at Berkeley (LBL), and the Lawrence Livermore National Laboratory (LLNL) is actively working on this idea, with the ultimate goal of designing an electron–positron collider operating at energies in the TeV region and compact accelerators at lower energies (see, for example, Refs. 66 and 67, where the difficulties in this program are also discussed). The first experimental data on the design of relativistic klystrons of various types were given in Ref. 67. Typical results are given in Table V.

Another variant of the relativistic klystron is the two-stage accelerator proposed by Schnell at CERN.[13,73,74] Here, in contrast to the relativistic klystron, the ILA is replaced by the superconducting resonators operating at 350 MHz de-

TABLE V.

Exit frequency, GHz	8.57—11.4
Peak power, MW	75—200
Efficiency, %	50
Field strength in accelerating sector at 11.4 GHz, MV/m	140

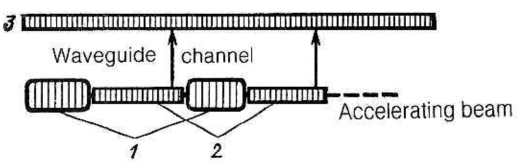

FIG. 7. A two-stage linear accelerator. Bunches of energy 3 GeV generate superhigh-frequency fields in resonators *2* at frequency 30 GHz. Energy losses are compensated by preacceleration in superconducting linear-accelerator modules *1*; *3* is a linear accelerator with high acceleration rate at frequency 30 GHz.

signed at CERN for the upcoming upgrade of LEP (Fig. 7).

Further development of the original idea of the wake-field transformer[68–70] taking into account the parameters of the two-stage accelerator[74] led to the idea of the resonant wake-field transformer[13] (Fig. 8). This uses a resonance regime generated by a train of many (for example, 50) ring-shaped bunches, with a temporal separation of 1 ns, and resonance-exciting wake fields in the accelerating structure. This scheme leads to a significant increase (by several orders of magnitude) of the transformation ratio over that of the single-pulse regime. The obvious difficulty is the creation of the required sequence of intense and short ring-shaped bunches, which, moreover, must have different energies, since the leading bunches are decelerated less than the following ones.

All the schemes proposed and described here have the obvious advantage that the associated technology is nearly the same as that for existing linear accelerators (except for the use of ultrahigh frequencies). Therefore, they are sometimes (see, for example, Ref. 9) referred to as quasiconventional linear accelerators at ultrahigh frequencies. As noted above, this area is developing rapidly in connection with the design of linear colliders operating in the TeV region by the CERN, SLAC–LBL–LLNL, and DESY groups, and significant progress should be made in the next few years (see, for example, Ref. 14).

In connection with this, beginning in 1985 Laziev and Tsakanov at the EPI carried out a series of theoretical studies on the calculation of wake fields in various structures and possible ways of increasing the transformation ratio in modulating structures.[75–80]

In an accelerator scheme using a cylindrical resonator of elliptical cross section, the accelerating and accelerated bunches move with a certain delay along the axes passing through the foci of the ellipse. Here the electromagnetic waves excited at one of the foci by the accelerating bunch are reflected from the walls and accumulate at the other focus, along which the accelerated beam can be injected. Calculations based on the technique of expansion in the eigenmodes of the resonator show that this splitting of the trajectories gives the optimal energy exchange between the bunches.[75] In Fig. 9 we show graphs of the energy-loss function $V(s)$ along the lines passing through the two foci of the ellipse, taking into account the first 1000 excited modes. The dimensions of the resonator in cm are: length, 2.5; large semiaxis, 4; small semiaxis, 2.4. Wake fields are excited along the line passing through the first focus by a cylindrical ultrarelativistic bunch of length 0.5 cm, radius 2 mm, and number of particles equal to 10^{13}.

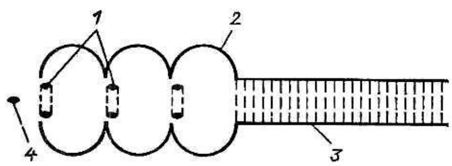

FIG. 8. Resonant excitation of a wake field. The passage of 50–100 ring-shaped bunches, each of charge 500–100 kC, causes resonant excitation of a mode with longitudinal component E of the field: *1*—driving electron rings; *2*—electron-ring accelerator; *3*—system for resonant excitation of a wake field and acceleration; *4*—a high-energy bunch.

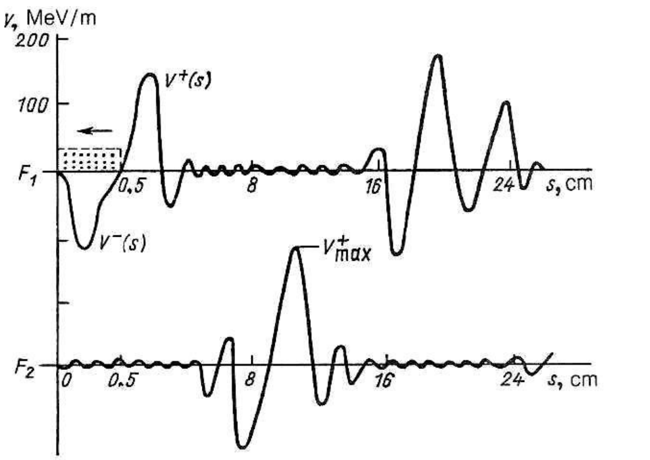

FIG. 9. Graphs of the loss function $V(s)$ along lines passing through the foci F_1 and F_2, including the first 1000 excited modes; s is the distance from the leading bunch.

We see that electromagnetic field bursts of alternating polarity appear along the axes passing through the two foci. Here for the given geometry the maxima are shifted by 8 cm along the direction of motion. For the given dimensions of the resonator this approximately corresponds to the time needed for the fields excited at the first focus to be reflected from the walls and reach the axis passing through the second focus. A difference of ~1 cm arises, owing to the longitudinal dimension of the resonator.

The fields reach the line passing through the second focus at a distance of 6.4 cm. An electron bunch to be accelerated can be injected at the maximum of the function $V(s)$ along the line passing through the second focus. Here the maximum acceleration rate reaches ~190 MeV/m. The width of the peak is 0.5–0.8 cm. Particles of the opposite sign (positrons) can be accelerated at points where the loss function is negative. However, in this scheme the transformation ratio is low. Calculations show[75] that for acceleration schemes using driving bunches which are symmetric in the longitudinal direction, the transformation ratio does not exceed 1.5.

In Fig. 10 we give dispersion curves for a given mode and various types of waveguide.[79] The straight lines emanating from the origin correspond to the phase velocity equal to the speed of light.

We see that in a periodic structure (a waveguide with diaphragms) each mode can propagate with arbitrary phase velocity. A charge interacts with those modes whose phase velocity coincides with the particle velocity. The exact calculation of the wake fields in such structures by expansion in eigenmodes is rather difficult.[76] However, it is possible to

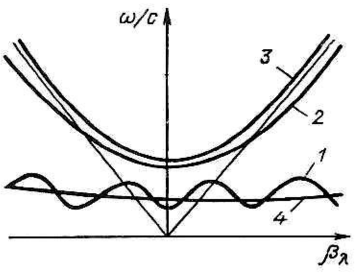

FIG. 10. Dispersion curves for a given mode and various types of waveguide. *1*—a waveguide with diaphragms; *2*—a dielectric-filled waveguide; *3*—a hollow waveguide; *4*—a set of resonators.

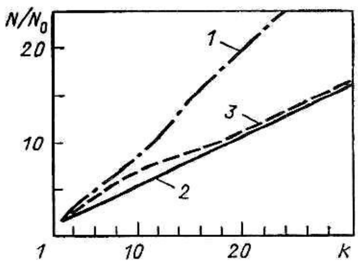

FIG. 12. Dependence of the number of particles generated by wake fields on the transformation ratio R for several acceleration schemes at a given acceleration rate: *1*— the leading bunch with linearly increasing particle density; *2*—a sequence of bunches with the number of particles increasing with bunch number; *3*—a piecewise-linear leading bunch.

make a rough estimate of the magnitude of the wake fields in structures with weak coupling between neighboring cells. Here the error, in particular, for a cylindrical waveguide, is of order a^3/b^2D, where D is the length of the cell, a is the internal radius of the diaphragm, and b is the waveguide radius. This approximation amounts to replacing the eigenmodes for a cell by the corresponding modes for a closed resonator, the analytic representation of which is well known.

In Fig. 11 we show the distributions of the wake-field loss function in the passage of a Gaussian bunch along the axis of a linear accelerator—the EPI injector. The number of particles per bunch is 10^7, and the bunch width is 2 mm. We see from these figures that when the rms radius of the bunch is $\sigma = 10$ mm, the trailing part of the bunch is accelerated, a phenomenon which by itself is worthy of close attention. In connection with this, we mention Refs. 81 and 82, reporting work carried out in the early 1970s at the EPI, in which the acceleration of the tail of an electron bunch passing through a resonator (the "head–tail" effect) was studied in more detail (Ref. 82 contains references to earlier studies). Although the maximum acceleration rate in a bunch does not exceed 1 keV/m, in the case of an intense driving bunch containing 10^{13} particles this value can reach 100 MeV/m. We note that for such densities of the accelerating bunch the effect of transverse forces on the dynamics of both the accelerating and the accelerated bunches can become important.

The problem of ensuring a high transformation ratio in a wake-field acceleration scheme is related to the possibility of obtaining intense bunches with nonstandard longitudinal particle density distributions.[83] In Refs. 83 and 84 it was noted that for symmetric driving bunches the transformation ratio cannot exceed 2. It can be shown[77] that electromagnetic waves with the maximum transformation ratio are generated by a driving bunch for which all the particles lose the same amount of energy. Bunches of this type can have linear, piecewise-linear, and piecewise-exponential longitudinal distributions.[75,78–80]

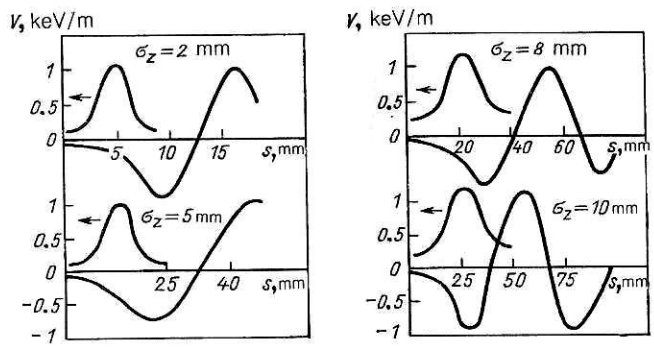

FIG. 11. Distribution of the wake-field loss function in the passage of a Gaussian bunch along the axis of a linear accelerator (the EPI injector) for various values of the rms size of the bunch σ_z.

A scheme using a sequence of driving bunches is more useful from the practical point of view. In particular, in the single-mode approximation the particles of N pointlike driving bunches lose the same amount of energy if the distance between bunches is equal to half the wavelength of the excited wave and the number of particles in the nth bunch is $N_n = N_1(2n-1)$. Here the transformation ratio will be $2N$. In Fig. 12 we show the dependences of the number of particles involved in generating wake fields on the transformation ratio for several acceleration schemes.

We see that the sequence of pointlike bunches discussed above is optimal. Another advantage of this variant is that the total number of particles participating in the energy exchange can be increased significantly, while keeping the number of particles in each bunch within reasonable limits.

In conclusion, we note that our discussion is also valid for wake-field acceleration in a plasma, where one mode at the plasma frequency is excited.

There are plans to use the new linear accelerator at the EPI to experimentally verify the features of various schemes of wake-field generation. At the present time the installation of the three-section experimental bench have begun (see Table VI for the parameters) and the construction of the experimental room has been completed. The accelerator will have a beam monochromatization system, a system for varying the distances between bunches, and a system for regulating the delay of one pulse relative to another. The three-

TABLE VI. Parameters of the three-section bunch.

Energy, MeV	30 (50 MeV at half current)
Current per pulse, A	1.0–1.5
Pulse duration, sec	$8\cdot10^{-6}$
Bunch duration, sec	$3.6\cdot10^{-11}$
Repetition frequency, Hz	50–100
Average current, μA	1000–1500
Particle number density in the bunch, cm^{-3}	$(2.2–3.3)\cdot10^9$
Bunch length, cm	0.5–1
Bunch diameter, cm	0.5–0.7
Average power, kW	120–150

section bench is expected to start operation at the end of 1990.

5. NONLINEAR EFFECTS IN WAKE-FIELD ACCELERATION IN A PLASMA

The possibility of using longitudinal wake fields excited by electrons or electron bunches moving in a plasma to accelerate charged particles was first suggested by Bolotovskiĭ[85] and Faĭnberg[86,87] in the early 1950s. In the mid-1980s the interest in plasma wake-field acceleration (PWFA) revived in a series of studies, carried out almost simultaneously at SLAC and UCLA, on an alternative acceleration method using laser beat waves in a plasma (see the reviews of Refs. 1 and 88–92). In particular, Chen and Ruth[93] made a comparative analysis of the two acceleration schemes and showed that the PFWA scheme has higher efficiency and requires less beam energy than the scheme based on beats of two laser waves for the same driving gradient. Detailed studies have been carried out on the problem of increasing the transformation ratio R, the ratio of the driving field of the wake wave to the field inside it which decelerates the beam or, equivalently, the ratio of the energy gained by the accelerated particles to the energy of the beam particles. As noted above, in the case of passive structures with certain assumptions, this ratio should not exceed 2. This statement remains true in the case of a plasma under the same conditions (the Dawson theorem). Beams with linearly increasing (to the tail) density and noncollinear driven and driving beams have been considered in attempts to significantly increase the transformation ratio.[84,94–96]

In Ref. 97 it was suggested that the transformation ratio can be increased by using the nonuniformity of the field over the cross section of the moderating structure. It has also been suggested that more efficient acceleration can be obtained by placing the flux exciting the field and the accelerated particles in different resonances relative to the wave.

The influence of the boundary of a semi-infinite plasma on the formation of the wake field of a charged particle was studied in Ref. 98. It was shown that transition effects related to the presence of the vacuum–plasma interface are important at distances of the order of several plasma wavelengths from the boundary. This conclusion also follows from the results of Ref. 99, where problems of particle emission on the plasma boundary were studied.

The authors of Ref. 100 studied the effect of current loading of the accelerated beam on the acceleration process and estimated the maximum values of the density of the accelerated beams as a function of the electron density in the plasma and the accelerating beam. In addition to the accelerating wake field, focusing magnetic fields (circling the beam) and electric fields (directed radially) also arise when the finiteness of the transverse dimensions of the beam is taken into account. An ever increasing number of studies are being devoted to the analysis of the focusing of the initial beam in the plasma.[100–106] The interest in this problem is particularly strong, owing to the necessity of obtaining, in order to increase the luminosity, micron and submicron beams after the final focusing (before the collision) in colliders operating at superhigh energies. Numerical estimates show, for example, that the luminosity of the planned electron–positron collider at CERN operating in the TeV range (CLIC) can be increased by an order of magnitude when this focusing technique is used instead of the traditional methods.[104]

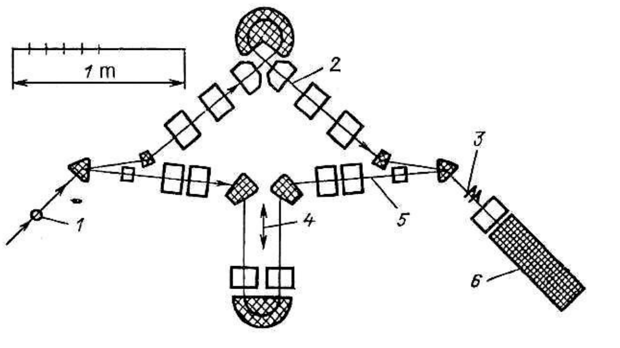

FIG. 13. Scheme for the setup at Argonne National Laboratory for testing wake-field acceleration: *1*—target; *2*—trajectory of the high-energy beam; *3*—testing section; *4*—trombone; *5*—trajectory of the low-energy beam; *6*—spectrometer.

The Argonne–Wisconsin group has carried out experimental studies of plasma wake-field acceleration.[107,108] In their experiment they used intense electron beams (10^{10} e/pulse) from the 21.4-MeV accelerator of the Chemistry Department at Argonne National Laboratory. After passing through a short graphite target, the beam in a special spectrometer was split into two beams. One of them, the accelerating beam, had energy 21.4 MeV, while the other, the witness beam, was of lower intensity and had energy 15 MeV. The spectrometer has a "trombone" allowing a time delay of the witness beam relative to the driving beam in the range from −50 to + 200 psec. The two beams passed through a uniform column of plasma of length 33 cm and electron density $2.3\cdot10^{13}$ cm^{-3}. After passing through the plasma column, the energies of the two beams were measured by a spectrometer located at the end of the beam channel (Fig. 13). Depending on the value of the time delay, the witness beam encountered either the accelerating field or the decelerating field of the longitudinal wake wave excited by the driving beam. Periodic acceleration and deceleration of the witness beam depending on the time delay was observed experimentally (Fig. 14). The maximum observed acceleration gradient was 1.6 MeV/m; in future experiments it will be increased, owing to the introduction of a new beam-pulse compression system.

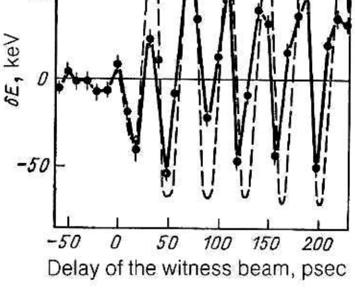

FIG. 14. Acceleration of the witness beam by a wake field in a plasma at the Argonne setup: the dashed line shows the theoretically predicted energy variation under the following experimental conditions: driving-beam duration, 2.1 nsec; length of the plasma column, 33 cm; plasma electron density, $2.3\cdot10^{13}$ cm^{-3}. The corresponding acceleration rate is roughly 1 MeV/m. The solid line is the experimental result.

All the theoretical studies mentioned above (except for that of Ref. 101) use the linear approximation for which the perturbation of the electron density of the plasma is $\Delta n \ll n_0$ (or $n_{b0}/n_0 \ll 1$), where n_0 is the equilibrium electron density of the plasma and n_{b0} is the electron density of the beam, which is taken to be constant and fixed.

It is also assumed that the motion of the plasma electrons is nonrelativistic. The inclusion of nonlinear effects related to the increasing degree of nonequilibrium of the system and relativity, which under certain conditions can be qualitatively and quantitatively important, has been studied by the Erevan group since 1977 (Refs. 109–116). A number of recent papers[101,117,118] have also been devoted to study of the generation of nonlinear wake fields by relativistic electron beams in a plasma.

Here we should draw attention to the fundamental studies carried out by Akhiezer and Polovin in 1955–1956 (Refs. 119 and 120), in which the equations for free longitudinal waves in an unbounded relativistic plasma were first formulated and solved. In particular, in Ref. 119 (see also Ref. 121) they obtained the maximum value of the electric field strength of a free longitudinal wave in a relativistic cold plasma:

$$\frac{eE_{\max}}{m\omega_p c}=\sqrt{2}\gamma_{\max}^{1/2},$$

where

$$\gamma_{\max}=\left(1-\frac{u_{\max}^2}{c^2}\right)^{-1/2},$$

and $u_{\max}$ is the maximum velocity of the plasma electrons.

The value of $E_{\max}$ differs considerably from the maximum value for a nonrelativistic cold plasma obtained later by Dawson,[122,123] $eE_{\max}/(m\omega_p c)\simeq 1$, which is usually cited as the maximum value of the field strength, above which the wave begins to break down. The authors of Refs. 110–113, in generalizing Ref. 119, studied longitudinal waves in a plasma generated by bunches of relativistic electrons moving with constant velocity $v_0 \lesssim c$ and Lorentz factor $\gamma_0 \simeq (1-v_0^2/c^2)^{-1/2}$. They showed that for $n_{b0}\approx n_0/2$, where n_{b0} and n_0 are the bunch and plasma electron densities, respectively, and $(n_{b0}^2/n_0^2)/(1-2n_{b0}/n_0)\gg\gamma_0^2=1$, the maximum value of the field strength is given by $eE_{\max}/m\omega_p c=\sqrt{2}\gamma_0^{1/2}$. We see that the field dependence proportional to $\gamma_0^{1/2}$ (rather than the $\gamma_{\max}^{1/2}$ of the plasma electrons, as in Ref. 119) is obtained only when certain conditions are imposed on the Lorentz factor. In those cases in which $\gamma_0^2 \gg (n_{b0}^2/n_0^2)/(1-2n_{b0}/n_0)\gg 1$, the field strengths are also large, but now owing to the factor $(1-2n_{b0}/n_0)^{-1}$. This fact is not always clearly stated in the literature.[117,118,122] We note that the equations of Refs. 119 and 120 were also used in Ref. 124 to study the generation of longitudinal waves in a plasma by beat waves from two lasers. More details about the various types of dependence of the electric field of the longitudinal wave on the γ_0 factor of the bunch electrons and the applicability conditions for the corresponding equations will be given below, where we discuss the main results of theoretical studies of intense longitudinal-wave excitation in a plasma by a relativistic electron bunch.[109–115]

The back effect of the field of the beam on its longitudinal-momentum distribution was taken into account in a self-consistent manner in a study by the Erevan group.[116] The same group has recently carried out studies taking into account the ion motion and transverse dimensions of the bunch. In the latter case approximate expressions were found for the focusing (defocusing) forces which generalize the results of earlier studies.[100–106]

The entire process by which a relativistic bunch interacts with a plasma can be split into stages. The bunch is injected into the plasma over a time $\tau=d/v_0$, where d is the length of the bunch and v_0 is the initial velocity of the bunch electrons. The injection of the bunch into the plasma is accompanied by the appearance of transient fields which fall off with time. For $t_{cr}\geqslant\tau+nT$ (see also Ref. 98), where T is the period of the stationary wake field and $n\geqslant 1$ is an integer, in the system of the plasma and the bunch a stationary regime of plasma oscillations is established. In this regime all the quantities characterizing the beam and the plasma, namely, the electron densities of the beam n_b and of the plasma n_e, the longitudinal momenta of the beam and plasma electrons $\rho_b=P_{bz}/mc$ and $\rho_e=P_{ez}/mc$, and the fields E^b and E inside and outside the bunch, cease to depend explicitly on the time and are functions of the variable $\tilde z=z-v_f t$, $v_f\approx v_0$, where v_f is the phase velocity of the excited wave. After a time t_{cr} a wake field E is excited behind the bunch. The field inside the bunch is completely ($E^b=0$) or partially ($E^b\neq 0$) canceled, and a certain distribution in the momenta ρ_e and ρ_b of the plasma and bunch electrons is established in the region occupied by the bunch and in the region outside the bunch. Then the tail electrons in the bunch, which acquire momenta $\rho_p\gg\rho_0$, where ρ_0 is the momentum of the electrons at the front of the bunch and is equal to the initial electron momentum, overtake the electrons at the front of the bunch and the bunch breaks up after a time $t_H=2d\gamma_0^2/c\gg t_{cr}$.

Nonlinear wake fields and the particle acceleration rate for a given bunch

Here we find the exact solution of the following problem: a relativistic electron beam moving with constant velocity v_0 passes through a cold, collisionless plasma of density n_0 with stationary ions. The electron beam has a given density n_{b0}, its longitudinal dimensions are d, and its transverse dimensions are infinite. We are interested in the stationary regime which is established, where all the variables cease to depend explicitly on the time and are functions of only the single variable $\tilde z=z-v_f t$, where $v_f\approx v_0\approx c$ is the phase velocity of the longitudinal wake field in the plasma excited by the bunch. The equations of relativistic magnetic hydrodynamics for the plasma electrons and Maxwell's equations for the wave fields inside the bunch and outside the bunch in this case reduce to the following system of equations:

$$\left.\begin{aligned}&\frac{d^2}{d\tilde z^2}\left(\beta\rho_e-\sqrt{1+\rho_e^2}\right)+\frac{\omega_p^2\rho_e}{c^2\left(\beta\sqrt{1+\rho_e^2}-\rho_e\right)}\\&\qquad+\frac{\omega_b^2}{c^2}\left[\theta(\tilde z)-\theta(\tilde z-d)\right]=0;\\&\left(\beta-\frac{\rho_e}{\sqrt{1+\rho_e^2}}\right)\frac{d\rho}{d\tilde z}=\frac{eE}{mc^2};\\&\frac{\partial}{\partial\tilde z}\left(n_e v_e-n_e v_\Phi\right)=0,\end{aligned}\right\}\qquad(14)$$

where

$$\beta = v_r/c,\ \omega^2_{p,b} = 4\pi e^2 n_{0,b0}/m,$$

with the boundary conditions $n_e(d) = n_0$, $\rho_e(d) = 0$, and $E^b(d) = 0$.

At time t_{cr} after the bunch enters the plasma, a stationary wake field is excited behind the bunch. This wake field has length

$$\tilde{z}_\lambda = 4\sqrt{2}\frac{v_0}{\omega_p}\gamma_0^{1/2} \tag{15}$$

and maximum field strength

$$E_{\max} \simeq \sqrt{2}\frac{mc\omega_p}{e}\gamma_0^{1/2} \tag{16}$$

under the conditions

$$\frac{n_{b0}^2}{n_0^2}\Big/\left(1-\frac{2n_{b0}}{n_0}\right) \gg \gamma_0^2 \gg 1,\ n_{b0} \approx \frac{n_0}{2}, \tag{17}$$

and

$$E_{\max} = \sqrt{2}\frac{mc\omega_p}{e}\frac{(n_{b0}/n_0)^{1/2}}{\left(1-\frac{2n_{b0}}{n_0}\right)^{1/4}},\quad \tilde{z}_\lambda = 4\sqrt{2}\frac{v_0}{\omega_p}\frac{(n_{b0}/n_0)^{1/2}}{\left(1-\frac{2n_{b0}}{n_0}\right)^{1/4}}, \tag{18}$$

under the opposite condition

$$\gamma_0^2 \gg \frac{n_{b0}^2/n_0^2}{1-2n_{b0}/n_0} \gg 1,\ n_{b0} \approx \frac{n_0}{2}. \tag{19}$$

The condition for complete cancellation of the bunch charge (the field inside the bunch is $E^b = 0$) leads to a limit on the bunch length $d \leqslant n_{b0}\tilde{z}_\lambda/(n_0 - n_{b0})$. If the field inside the bunch is not completely canceled ($E^b \neq 0$), the expressions (15), (16), and (18) for the maximum values of the field strength and the wake-field wavelength are obtained for the bunch length

$$d = \frac{2v_0}{\omega_p}\gamma_0 \quad\text{or}\quad d = \frac{2v_0}{\omega_p}\frac{n_{b0}/n_0}{(1-2n_{b0}/n_0)^{1/2}} \tag{20}$$

under the conditions (17) or (19), respectively. For lengths which are larger or smaller than these, the maximum values of the field strength and wavelength are, respectively, larger or smaller than the values given by Eqs. (15), (16), and (18). For example, when the bunch length is given by $d_0 = (8v_0/\omega_p)\gamma_0^2$, the field strength and wake wavelength are given by

$$E_{\max} \simeq 2\frac{mc\omega_p}{e}\gamma_0.\ \tilde{z}_\lambda = \frac{8v_0}{\omega_p}\gamma_0 \tag{21}$$

for the condition (17). The general dependence of the wake field strength on the bunch length can be found only numerically. General expressions for the implicit dependence $d = d(E)$ are given in Refs. 111–113.

Here we give some numerical estimates of the maximum values of the strength of the longitudinal wake field and the particle acceleration rate in it for various values of the parameters n_0, n_{b0}, and γ_0 of the plasma and the bunch.

1. Let the equilibrium density of the plasma electrons be $n_0 = 10^{13}$ cm^{-3}, $\omega_p = \sqrt{3\cdot 10^9 n_0} = 1.7\cdot 10^{11}$ sec^{-1}, the bunch density be $n_{b0} \simeq n_0/2$ [see the condition (17)], and the relativistic factor of the bunch be $\gamma_0 = 10^2$ (50 MeV). (These parameters are close to the values of the parameters at the ARC ILA at Livermore.) Then $E_{\max} = 4.1\cdot 10^9$ W/m, and the maximum acceleration rate is $eE_{\max} = 4.1$ GeV/m. Here the bunch length for complete cancellation of the field inside the bunch is $d \leqslant 10$ cm, while for the case of partial cancellation ($E^b \neq 0$) these values of the field strength and acceleration rate are obtained for the bunch length $d \lesssim 35$ cm. The distance traveled by the bunch in the plasma before the stationary regime is established is $L = v_0 t = d + n\tilde{z}_\lambda = 54$ cm for $n = 2$ (Figs. 15 and 16).

2. Let $n_0 = 2\cdot 10^{11}$ cm^{-3}, $\omega_p = 2.4\cdot 10^{10}$ sec^{-1}, $n_{b0} = n_0/2 = 10^{11}$ cm^{-3}, $\gamma_0 = 60$ (30 MeV). In this case $E_{\max} = 4.6\cdot 10^8$ W/m, $eE_{\max} = 460$ MeV/m, and $d \leqslant 53$ cm for complete cancellation of the field in the bunch and $d \simeq 1.5$ m for incomplete cancellation; $L = 256$ cm.

When the electron density in the bunch is small, the expression for the wake field strength is independent of the

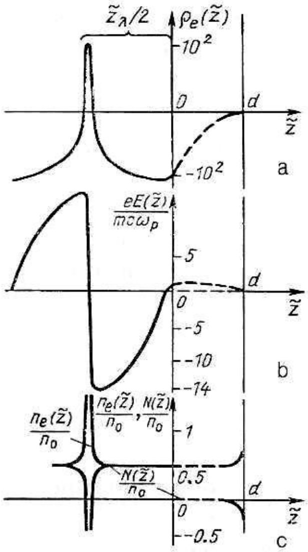

FIG. 15. Schematic graphs of the quantities $\rho_e(\tilde{z})$ (a), $eE(\tilde{z})/m\omega_p c$ (b), $n_e(\tilde{z})/n_0$ and $N(\tilde{z})/n_0$ (c) [where $N(\tilde{z}) = n_0 - n_e(\tilde{z}) - n_{b0}$ inside the bunch and $N(\tilde{z}) = n_0 - n_e(\tilde{z})$ outside the bunch] for a given, fixed bunch with $d = 8V_0\gamma_0^2/\omega_p$, $n_0 = 10^{12}$ cm^{-3}, $n_b \simeq n_0/2$, and $\gamma_0 = 10^2$.

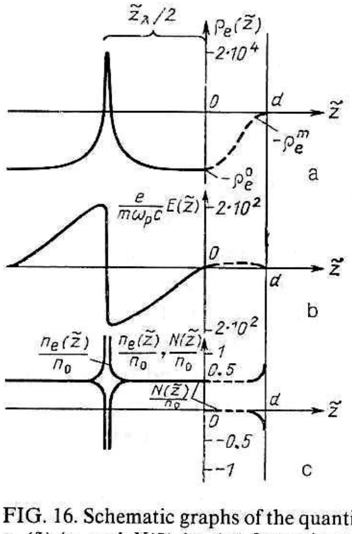

FIG. 16. Schematic graphs of the quantities $\rho_e(\tilde{z})$ (a), $eE(\tilde{z})/m\omega_p c$ (b), $n_e(\tilde{z})/n_0$ and $N(\tilde{z})/n_0$ (c) for a given, fixed bunch with $d = 2v_0\gamma_0/\omega_p$, $n_0 = 10^{12}$cm^{-3}, $n_b \simeq n_0/2$, and $\gamma_0 = 10^2$.

γ_0 factor and its maximum value is

$$E_{\max}=\frac{m\omega_p c}{e}\cdot\frac{n_{b0}}{n_0}\beta=0.96\cdot 10^2 n_{b0}^{1/2}\left(\frac{n_{b0}}{n_0}\right)^{1/2} \tag{22}$$

for bunch length $d=\pi v_0/2\omega_p$. The optimal choice of n_{b0} and n_0 taking into account the parameters n_{b0} and γ_0 of existing accelerators gives for this case the accelerating wake field $E_{\max}=10^8$ W/m and the corresponding acceleration rate $eE_{\max}=100$ MeV/m.

Nonlinear self-acceleration of the electrons of a relativistic bunch in a plasma

The results quoted above were obtained in the approximation of a given, "fixed" bunch, where its parameters appear in Maxwell's equations as given functions and the back effect on the bunch from the fields excited by it is neglected. The inclusion of this back effect can significantly change the bunch parameters and the conditions for wake-field excitation, and this is one of the reasons why the bunch itself becomes unstable. The dynamics of a one-dimensional infinitely long bunch with initially uniform charge distribution has been studied using perturbation theory in Refs. 125–127. In particular, the authors of Ref. 127 noted that fields are excited only in that region of the plasma from which a point source has already escaped. Thus, on the basis of physical considerations we can assume that the leading front of the electron bunch always moves with the injection velocity v_0. In Ref. 128 it was shown that the wave excited by the beam in a plasma is basically a single-mode wave, and the electric field in the plasma is established practically instantaneously (the first stage of the development of the two-beam instability). In the next (the second) stage the field of the wave is stabilized by the capture of some of the bunch electrons and the amplitude of the wave reaches its maximum (saturated) value. Then the energy of the wave decreases somewhat, owing to the increase of the bunch-electron energy. In the third, most drawn-out stage, the process of decrease of the wave energy is replaced by an almost equilibrium, "quasistationary" state of the bunch and plasma electrons with a practically constant ratio of the wave energy and the bunch-electron energy. Here there are electrons in the tail of the bunch which are not entrained by the wave with heightened (twice the initial) energy.

The authors of Ref. 116 obtained an exact solution to the stationary, nonlinear, self-consistent problem of the interaction of a one-dimensional monoenergetic relativistic electron bunch with a cold plasma, when both the bunch and the plasma are described by a system of nonlinear hydrodynamic equations and Maxwell's equations. Those authors took the model of the third stage of the two-beam instability, in which all quantities are functions of the variable $\tilde{z}=z-v_f t$, as the stationary case, although the leading edge of the beam continues to move with the initial velocity v_0. We note that the difference between the phase velocity and the initial velocity was pointed out in Refs. 125–129, where the electron beam was treated as an unknown quantity. In Ref. 116 it was shown that the inclusion of the interaction between the bunch electrons and the longitudinal field excited by them can, under certain conditions, lead to the appearance of an unstable stationary state, in which some of the electrons in the tail of the bunch have momenta significantly larger than the initial momenta.

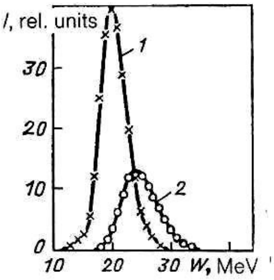

FIG. 17. Energy spectra of a beam before (1) and after (2) passing through a plasma (Ref. 131).

We note that the fact that a beam of electrons which have passed through a plasma contains a significant fraction of particles with energies considerably higher than the energy of the electrons upon entering the plasma was pointed out by Langmuir.

The monograph by Nezlin[130] and the review by Faĭnberg[88] give the results of some experimental studies of the energy spectrum of relativistic electrons which have passed through a plasma column.

The Khar'kov group has shown[131] that when a beam of electrons of energy 20 MeV passes through a plasma with electron density $n_e\simeq(2\text{–}3)\cdot 10^{16}$ cm^{-3}, the passage of about 15–20% of the electrons of the beam is observed and among these are some which have been accelerated to 24 MeV (Fig. 17).

The behavior of an electron bunch with density $n_b(\tilde{z})$ whose front moves with constant velocity v_0 ($\beta_0=v_0/c$) through a plasma with electron density $n_e(z)$ and ions at rest is described by a system of the equations of motion for the z components of the dimensionless momenta $\rho_e=P_{ez}/mc$ and $\rho_b=P_{bz}/mc$ of the plasma and beam electrons, respectively,

$$\frac{d}{d\tilde{z}}\left(\beta\rho_e-\sqrt{1+\rho_e^2}\right)=\frac{eE^b}{mc^2}, \tag{23}$$

$$\frac{d}{d\tilde{z}}\left(\beta\rho_b-\sqrt{1+\rho_b^2}\right)=\frac{eE^b}{mc^2}, \tag{24}$$

the continuity equations

$$\frac{d}{d\tilde{z}}[n_e(v_e-v_f)]=0;\quad \frac{d}{d\tilde{z}}[n_b(v_b-v_f)]=0, \tag{25}$$

and the Poisson equation for the longitudinal component of the field inside the bunch,

$$\frac{dE^b}{d\tilde{z}}=4\pi e(n_0-n_e-n_b), \tag{26}$$

where n_0 is the equilibrium density of the plasma electrons, equal to the density of stationary ions, and $\beta=v_f/c$. We shall consider the case $n_b<n_0$ and assume that the field E^b and the plasma electron momentum ρ_e and density n_e at the leading edge of the beam $\tilde{z}=d$, where d is the boundary of the bunch (or its length), are continuous. Since there are no plasma perturbations in front of the bunch, the boundary conditions have the form $E^b(d)=0$, $\rho_e(d)=0$, and $n_e(d)=n_0$. We also assume that at the front of the beam $\rho_b(d)=\rho_0$, where $\rho_0=\beta_0/(1-\beta_0^2)^{1/2}$ and $n_b(d)=n_{b0}$, with n_{b0} being the constant initial density of the bunch electrons.

This problem has an exact analytic solution, and some of the results will be given below.

After the stationary regime of the plasma oscillations has been established (after a time $\tau + nT$ or after the bunch has traveled a distance $L = d + n\tilde{z}_\lambda$), the momenta of the plasma and beam electrons in the region occupied by the bunch, which has length d_0 (see Refs. 113–116), have the following distributions for $v_f \lesssim v_0 \simeq c$:

$$-\rho_e^0 \leqslant \rho_e \leqslant 0, \quad \rho_b^0 \geqslant \rho_b \geqslant \rho_0; \tag{27}$$

$$\left.\begin{aligned} \rho_e^0 &= \frac{2\beta \frac{n_{b0}}{n_0} \alpha \left(1 - \frac{n_{b0}}{n_0} \alpha\right)}{1 - 2\frac{n_{b0}}{n_0} \alpha + \frac{n_{b0}^2}{n_0^2} \alpha^2 (1-\beta^2)}, \\ \rho_b^0 &= \rho_0 + \frac{2\beta^2 \frac{n_{b0}}{n_0} \alpha}{(1-\beta)\left[1 - 2\frac{n_{b0}}{n_0} \alpha + \frac{n_{b0}^2}{n_0^2} \alpha^2 (1-\beta^2)\right]}, \end{aligned}\right\} \tag{28}$$

where $\alpha = (\beta_0 - \beta)/(1-\beta)$.

Near the tail of the bunch ρ_e and ρ_b take the values given on the left-hand sides of the inequalities (27), from which it follows that the plasma electrons are accelerated backward, while the electrons near the tail of the bunch (up to 10% of the total number of particles in the bunch; see Ref. 116) acquire an additional forward momentum proportional [when the condition (17) holds] to γ_0^4 for the bunch length $d_0 = (8v_0/\omega_p)\gamma_0^2$ and γ_0^2 for $d_0 = \pi v_0/\omega_p$ and $n_{b0}/n_0 \ll 1$:

$$\rho_b^0 \approx \rho_0 + 8\gamma_0^4, \quad \frac{n_{b0}}{n_0} \approx \frac{1}{2}, \quad d_0 \approx \frac{8v_0}{\omega_p}\gamma_0^2, \tag{29}$$

$$\rho_b^0 \approx \rho_0 + \frac{4n_{b0}}{n_0}\gamma_0^2, \quad \frac{n_{b0}}{n_0} \ll 1, \quad d_0 \approx \frac{\pi v_0}{\omega_p}. \tag{30}$$

However, the lengths d_0 for $n_{b0}/n_0 \simeq \frac{1}{2}$ with the values of γ_0 and n_{b0} of presently attainable bunches are very large ($\sim 10^2$ m), and therefore we also give the expressions for some intermediate values of ρ_e and ρ_b for bunch lengths $d_1 = (2v_0/\omega_p)\gamma_0$ (for $n_{b0} \simeq n_0/2$). The range of variation of the bunch and plasma electron momenta in the region occupied by the bunch is given by the following inequalities in this case:

$$\left.\begin{aligned} &-\rho_e^m \leqslant \rho_e \leqslant 0, \quad \rho_b^m \geqslant \rho_b \geqslant \rho_0, \\ &\rho_e^m = \frac{\beta \frac{n_{b0}}{n_0} \alpha}{\left[1 - \frac{2n_{b0}}{n_0} \alpha + \frac{n_{b0}^2}{n_0^2} \alpha^2 (1-\beta^2)\right]^{1/2}}, \\ &\rho_b^m = \rho_0 + \frac{1 - \frac{n_{b0}}{n_0} \alpha (1-\beta^2)}{(1-\beta)\left[1 - \frac{2n_{b0}}{n_0} \alpha + \frac{n_{b0}^2}{n_0^2} \alpha^2 (1-\beta^2)\right]^{1/2}}. \end{aligned}\right\} \tag{31}$$

We see from these equations that at a distance d_1 from the front of the bunch the electrons near the tail of the bunch acquire, when the condition (17) is satisfied, an additional momentum over the initial momentum:

$$\rho_b^m \simeq \rho_0 + 4\gamma_0^3, \quad \frac{n_{b0}}{n_0} \simeq \frac{1}{2}, \quad d_1 \simeq \frac{2v_0}{\omega_p}\gamma_0; \tag{32}$$

$$\rho_b^m \simeq \rho_0 + \frac{2n_{b0}}{n_0}\gamma_0^2, \quad \frac{n_{b0}}{n_0} \ll 1, \quad d_1 \simeq \frac{\pi v_0}{2\omega_p}. \tag{33}$$

The distances over which Eqs. (29)–(33) for the maximum momenta change insignificantly correspond to roughly 10% of the total length of the bunch: $\Delta d_{1,0} = 10^{-1} d_{1,0}$. The particles located in the segment Δd_1 of the tail in the case (32), (33) acquire, for example, for $n_0/2 = n_{b0} = 10^{13}$ cm^{-3} and $\gamma_0 = 10^2$ (50 MeV) a maximum energy

$$\varepsilon_{\max} = mc^2 [(\rho_b^m)^2 + 1]^{1/2} \approx 2 \text{ TeV}.$$

The number of particles with this energy is $N^b = n_{b0}\Delta d_1 A$, where A is the cross section of the bunch; $N^b \simeq 3.5 \cdot 10^{13}$ if A is 1 cm^2. Here $d_1 = 35$ cm and $\Delta d_1 = 3.5$ cm. For small bunch-electron densities $n_{b0}/n_0 = 10^{-3}$, $n_0 = 10^{13}$ cm^{-3}, and $\gamma_0 = 10^4$ we have $d_1 = \pi v_0/2\omega_p = 2.7$ mm, $\Delta d_1 = 0.27$ mm, $N^b = 2.7 \cdot 10^{11}$, and $\varepsilon_{\max} = 10^2$ GeV. These energies are acquired by electrons in the tail of the bunch after the bunch travels a distance L in the plasma and before the bunch breaks up (before the tail overtakes the leading edge). For $n_0 = 2n_{b0} \simeq 10^{13}$ cm^{-3} and $\gamma_0 = 10^2$, we have $L = 54$ cm; for $n_{b0}/n_0 = 10^{-3}$ and $n_0 \simeq 10^{13}$ cm^{-3}, we have $L = d_1 + 2\pi v_0/\omega_p = 2$ cm. When the condition (19) is satisfied, the expressions (29) and (32) for the lengths $d_0 = (2v_0/\omega_p)/(1 - 2n_{b0}/n_0)$ and $d_1 = (v_0/\omega_p)/(1 - 2n_{b0}/n_0)^{1/2}$ are replaced by the expressions

$$\rho_b^0 = \rho_0 + 2\gamma_0^2 \Big/ \left(1 - \frac{2n_{b0}}{n_0}\right), \quad \rho_b^m = \rho_0 + 2\gamma_0^2 \Big/ \left(1 - \frac{2n_{b0}}{n_0}\right)^{1/2}, \tag{34}$$

while the expressions (30) and (33) for ρ_b^0 and ρ_b^m with $n_{b0}/n_0 \ll 1$ are proportional to γ_0^2 for any γ_0. However, it should be noted that for $n_{b0}/n_0 \simeq \frac{1}{2}$ the inclusion of the facts that the ions are stationary and that the transverse dimensions of the bunch are finite introduces certain restrictions on the value of γ_0 for which the expressions obtained above are valid (see below).

The expressions given in this section, like some of the other results of Refs. 113–116, can also have astrophysical applications in connection with searches for mechanisms of cosmic-ray acceleration in the relatively dense plasmas of certain astrophysical objects. Some of these nonlinear effects in charged-particle acceleration might help to explain the origin of high- and superhigh-energy cosmic particles.[114,118,132]

Inclusion of the effect of ion motion and the transverse dimensions of the bunch

In practically all the studies cited above on longitudinal-wave generation in a plasma, the ions are assumed to be stationary. By generalizing the results of Ref. 116 to the case of "three liquids"—the electrons of the beam, the plasma electrons, and the plasma ions (with infinite extent in the directions perpendicular to the beam velocity), we can study the self-consistent problem of the interaction of a relativistic electron bunch and a cold plasma with moving ions. This problem has an exact solution, which in the limit of the ion mass $M_i \to \infty$ reduces to the results of Ref. 116. The inclusion of the ion motion in the case $n_{b0}/n_0 \ll 1$ is not important; however, for $n_{b0}/n_0 \simeq \frac{1}{2}$ the results of Ref. 116 can remain valid only if the beam energy is not too large: the γ factor of the beam must be bounded above by a value proportional to the ratio M_i/zm_e ($\gamma_0^4 \ll M_i/16zm_e$), where z is the ion charge. When the inverse condition $\gamma_0^4 \gg M_i/16zm_e$ is satisfied, the momenta ρ_b of the bunch electrons turn out to depend on the square of the γ_0 factor of the bunch. We also note that in the case of moving ions the ion motion becomes directional, but the ion acceleration is small.

The inclusion of the thermal motion of the plasma electrons was accomplished by Katsouleas and Mori[122,133] and refined in the study by Rosenzweig.[118]

The results quoted above were obtained for a beam which is infinitely large in the transverse directions. Owing to the nonlinearity of the problem, it is reasonable to assume that the inclusion of the finite transverse dimensions can lead to nontrivial results. It is therefore necessary to find conditions on the transverse dimensions of the beam for which the results quoted above are valid. This problem has been solved in the linear approximation in a number of studies[100–106] assuming a given, fixed electron beam. This assumption also has to be abandoned if one wishes to find the influence of the transverse dimensions on the self-acceleration of the electrons in a relativistic bunch.

The problem is formulated as follows. An ultrarelativistic cylindrical electron beam of length d, transverse dimensions a, and velocity of the leading edge of the beam $v_0 \sim c$ travels along the z axis through a plasma which is of infinite extent in both the z and r directions. The entire system is placed in a fairly strong external longitudinal magnetic field $B_0(\omega_{B0} \gg \omega_p)$. As before, we consider the stationary (more precisely, quasistationary) state of the system to be the state of the relativistic plasma and beam in which all the characteristics of the system can be expressed in terms of the variables $\tilde{z} = z - v_f t$ and r. The plasma ions are stationary, the plasma is neutral in its unperturbed state, and the velocities of the plasma electrons are ordered only in the single component v_{ez}. The electric fields in the system which we wish to find have two nonzero components E_z and E_r, and the magnetic field has one, B_θ. The equations describing this plasma–beam system consist of Maxwell's equations with sources, the equations of motion for the plasma electrons and the beam electrons, and the continuity equations for the plasma electrons and beam electrons separately (the two-liquid model). The initial density of the beam electrons is taken to be

$$n_{bd}(r) = \begin{cases} n_{b0}(1 - r^2/a^2), & r \leqslant a, \\ 0 & r > a, \end{cases} \quad 0 \leqslant \tilde{z} \leqslant d. \qquad (35)$$

The boundary conditions are specified at the leading edge of the beam: $\mathbf{E}(d) = 0$, $\mathbf{B}(d) = 0$, and $\mathbf{v}_e(d) = 0$. Introducing the potentials $\varphi(\tilde{z},r)$ and $A(\tilde{z}, r)$ in the usual manner with the Lorentz condition, for the scalar potential we obtain the equation

$$\gamma_f^2 \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r}\right) + \frac{\partial^2 \varphi}{\partial \tilde{z}^2} = -4\pi e \gamma_f^2 (n_0 - n_e - n_b), \quad (36)$$

where

$$\gamma_f = (1 - v_f^2/c^2)^{-1/2}; \quad v_f = (n_b v_{bz} - n_{bd} v_{bd})/(n_b - n_{bd}).$$

In the ultrarelativistic case considered here, $\rho_e = P_e / mc \gg 1$, $v_{bd} \simeq v_{bz} \simeq c$, $v_f \simeq c$, $n_e \simeq n_0(1 - \beta_e)^{-1}$, and $n_b \simeq n_{bd}$. Further, from the equations of motion for the plasma electrons we find $\varphi' = \sqrt{\rho_e^2 + 1} - \rho_e$, where $\varphi' = \varphi / \gamma_f^2 mc^2$; $\varphi'(d) = 1$, and $n_e = (n_0/2)(1 + 1/\varphi'^2)$. Substituting the expressions for n_e and n_b into (36) and introducing the dimensionless quantities φ', $r' = r/\lambda_p$, and $\tilde{z}' = \tilde{z}/\lambda_p$, we obtain the following equation for $\varphi'(\tilde{x}', r')$:

$$\gamma_f^2 \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r}\right) + \frac{\partial^2 \varphi}{\partial \tilde{z}^2} - \frac{2\pi^2}{\varphi^2} = -2\pi^2 \left(1 - \frac{2n_b}{n_0}\right) \qquad (37)$$

(the primes have been dropped).

Equation (37) can be solved approximately. We seek the zeroth-order approximation as the solution of the equation

$$\frac{d^2 \varphi_0(\tilde{z})}{d\tilde{z}^2} - \frac{2\pi^2}{\varphi_0^2(\tilde{z})} = -2\pi^2\left(1 - \frac{2n_{b0}}{n_0}\right), \qquad (38)$$

which has an exact solution coinciding with the solutions obtained earlier for a beam of infinite extent in the transverse directions in the approximation $v_b \to c$.

An approximate solution of Eq. (37) is sought in the form of the expansions

$$\left.\begin{aligned} &\varphi(\tilde{z}, r) = \varphi_1(\tilde{z}) + \varphi_2(\tilde{z}) \frac{r^2}{a^2} + \ldots, \\ &\varphi_1(\tilde{z}) = \varphi_0(\tilde{z}) + \varphi_{01}(\tilde{z}) + \ldots \end{aligned}\right\} \qquad (39)$$

Imposing the restrictions

$$|\varphi_{01}| \ll |\varphi_0|, \quad |\varphi_2| \ll |\varphi_0|, \quad 0 \leqslant \tilde{z} \leqslant d \qquad (40)$$

and keeping only terms of first order in the small quantity, from Eqs. (37) and (38) we find the following system:

$$\left.\begin{aligned} &\frac{d^2\varphi_2}{d\tilde{z}^2} + \frac{4\pi^2}{\varphi_0^3} \varphi_2(\tilde{z}) = -4\pi^2 \frac{n_{b0}}{n_0}, \\ &\frac{d^2\varphi_{01}}{d\tilde{z}^2} + \frac{4\pi^2}{\varphi_0^3} \varphi_{01}(\tilde{z}) = -\frac{4\gamma_f^2}{a^2} \varphi_2(\tilde{z}). \end{aligned}\right\} \qquad (41)$$

It follows from Eq. (38) that

$$1 \leqslant \varphi_0(\tilde{z}) \leqslant 1/b, \quad 0 \leqslant b \equiv 1 - 2\frac{n_{b0}}{n_0} \leqslant 1. \qquad (42)$$

In the case $n_{b0}/n_0 \ll 1$, which is most often encountered in practice, $\varphi_0 \simeq 1$ for $0 \leqslant \tilde{z} \leqslant 1$, and the system (41) is solved with zero boundary conditions on the leading edge of the beam $\tilde{z} = d$. The approximate expression for φ close to unity, $\varphi = 1 + 2\pi(n_{b0}/n_0)((d - \tilde{z})/\lambda_p)^2$, shows that the condition for $\varphi \simeq 1$ is $(n_{b0}/n_0)(d/\lambda_p)^2 \ll 1$, i.e., for short beams $d/\lambda_p \ll 1$ the ratio n_{b0}/n_0 can be arbitrary in the range $0 \leqslant n_{b0}/n_0 \leqslant \frac{1}{2}$. The system is also relatively easy to solve for very large φ_0, when the term with φ_0^{-3} in (41) can be neglected. Physically, this corresponds to the condition $b \to 0$, i.e., $n_{b0}/n_0 \to \frac{1}{2}$. The condition (40) for the results to be applicable for a beam of infinite extent in the transverse directions is

$$\left.\begin{aligned} &\frac{n_{b0}}{n_0} \frac{d^2}{\lambda_p^2} \ll 1, \quad \frac{n_{b0}}{n_0} \frac{r^2}{a^2} \ll 1, \\ &\lambda_p^2 \frac{\gamma_f^2}{a^2} \frac{n_{b0}}{n_0} \ll 1, \quad 0 \leqslant r \leqslant a. \end{aligned}\right\} \qquad (43)$$

We also note that in our approximation the self-acceleration effect follows directly from the equations of motion for the beam electrons:

$$(v_b - v_f) \frac{dp_{bz}}{d\tilde{z}} = -eE_z, \quad E_z = -\frac{1}{\gamma_f^2} \frac{\partial \varphi}{\partial \tilde{z}}.$$

Integrating these, taking into account the boundary conditions for $\tilde{z} = d$, $0 \leqslant r \leqslant a$,

$$\rho_b(d) = \rho_{b0} = p_{b0}/mc, \quad \varphi'(d) = 1 = [e\varphi(d)]/[mc^2\gamma_f^2],$$

we have

$$\rho_b = \rho_{b0} + 2\gamma_f^2 (\varphi' - 1). \qquad (44)$$

Equation (44) is true in general, independently of the specific form of $\varphi(\tilde{z},r)$. Therefore, for $\varphi > 1$, i.e., in the tail of a long ($d \gg \lambda_p$) beam, accelerated electrons always arise. Their appearance is the reaction of the beam to the field created by them and to the reverse current of the plasma electrons. We recall that, as discussed above, acceleration of electrons in the tail of the beam was also predicted in the case of wake fields in passive structures (see Sec. 4).

Knowing φ, we can find $E_r = -\partial\varphi/\partial r$ and $E_z = -(1/\gamma_f^2)(\partial\varphi/\partial\tilde{z})$, and then also B_θ from Maxwell's equations (this is more convenient than solving the nonlinear equation again for A_z). Knowing E_r and B_θ, we can calculate the force $f_r = -|e|E_r + |e|(v_b/c)B_\theta$; the sign and value of f_r determine the "self-focusing" ($f_r < 0$) and "self-defocusing" ($f_r > 0$) of the beam. We note that the field B_θ is determined both by the beam current, which is (partially) canceled by the plasma electron current, and by the displacement current, which is proportional to $\partial E_z/\partial\tilde{z}$.

The general expression for the force f_r is quite complicated. For short beams $d \ll \lambda_p$ it simplifies considerably and corresponds to rather strong focusing. For long beams $d \gg \lambda_p$ the focusing strength varies with the period λ_p, i.e., the beam appears to pass through an equivalent cylindrical symmetric undulator, which leads to magnetic bremsstrahlung radiation from the beam electrons and, in addition, can serve as a source of a constriction type of instability.

The results discussed in this section are certainly interesting, especially those concerning the self-acceleration and self-focusing of the bunch electrons. It is possible that with a suitable choice of parameters (even at high beam and plasma electron densities) it will be possible to obtain large values of the accelerating and focusing fields. These features may prove decisive in choosing the optimal conditions for the application of wake fields in plasmas and in passive structures (see Sec. 4).[13,14]

Owing to the stringent constraints on the spread in the transverse momenta in linear colliders operating in the TeV energy range,[134] schemes of plasma wake-field acceleration and also the nonlinear self-acceleration effect appear to be promising for the design of fixed-target accelerators, where the constraints on the spread in the transverse momenta are not as stringent.

Our discussion points to a clear need for systematic experimental studies of wake-field generation in plasmas and of the self-acceleration and self-focusing of electron bunches passing through a plasma.

[1)] We note that the construction of linear accelerators with colliding electron–positron beams at superhigh energies was proposed by V. E. Balakin, G. I. Budker, and A. N. Skrinskiĭ at the Sixth All-Union Conference on Charged Particle Accelerators in 1978 [V. E. Balakin, G. I. Budker, and A. N. Skrinskiĭ, in *Proceedings of the Sixth All-Union Conference on Charged Particle Accelerators* (in Russian), Dubna, 1978 (Dubna, 1979), p. 27].

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