Traditional nuclear physics as a test of nuclear exotics

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The review considers the testing of some exotic hypotheses about the properties of the nucleon in a nuclear medium in phenomena of traditional nuclear physics. The hypothesis of nucleon swelling proposed to explain the EMC effects is considered in detail. The consequences of this hypothesis for the charge densities and cross sections for scattering of fast electrons and protons by nuclei are analyzed. Also considered are the Nolen–Schiffer anomaly, the Coulomb sum rule for inelastic electron scattering, y scaling, and some other nuclear processes. It is shown that one can estimate the possible scale of nuclear exotics by analyzing many of these phenomena. Thus, examination of high-precision data on the elastic scattering of electrons with energy 500–700 MeV using density distributions calculated on the basis of the self-consistent theory of finite Fermi systems yields a restriction on the amount of nucleon swelling: $\alpha = \delta r_N / r_N \lesssim 10\%$. A similar analysis for protons with energy 0.8–1.0 GeV using Glauber theory gives $\alpha \lesssim 6\%$. An even more stringent restriction, $\alpha \lesssim 3\%$, follows from data on y scaling in ^{56}Fe.

INTRODUCTION

At the present time it is widely accepted that the quantum chromodynamics of quarks and gluons is the exact theory of the strong interactions. In this sense, the quark and gluon degrees of freedom determine the structure of nuclei. However, in practice one can describe in the framework of QCD consistently only hard processes associated with the interaction of particles at short relative distances $r \lesssim 1/M$, where M is the characteristic hadronic mass. The description of phenomena at larger distances is intimately related to the unresolved problem of confinement and comes up against as yet insuperable technical difficulties. In particular, the problem of describing the properties of the nucleon in terms of quarks and gluons in the framework of QCD is still unsolved. In the light of this one cannot in the foreseeable future hope for the construction of a theory of the nucleus on the basis of the first principles of QCD. For this reason, it appears more promising for the description of nuclei to use simpler approximate approaches motivated by QCD in conjunction with microscopic theories of traditional type that operate with nucleon and meson degrees of freedom.

Extremely important for modern nuclear theory is the question of how well these degrees of freedom are chosen. To what extent are we justified in describing nuclei in terms of nucleons, mesons, and Δ isobars? Should the wave function of a nucleus contain multiquark configurations ($6q$, $9q$, etc.) with an appreciable weight? To what extent do the properties of nucleons in a nucleus differ from those of free nucleons?

The answers to these questions are severely restricted by the impossibility of finding exact solutions of many-body problems. Such solutions exist only in the case of the lightest nuclei. For medium and heavy nuclei the greatest success is achieved by microscopic approaches with phenomenological interactions. These include the Hartree–Fock method with effective forces and the self-consistent theory of finite Fermi systems, which here we shall also call the quasiparticle Lagrangian method. In these approaches one introduces, in place of nucleons, quasiparticles corresponding to excitations of the system near the Fermi surface. The interaction between the quasiparticles has a short range and is characterized by a few experimentally determined parameters.

There are many nuclear processes in which the nuclear structure is represented basically by a single-particle density distribution. In these cases the quasiparticle description makes it possible to draw conclusions about the properties of the particles themselves—the nucleons. This possibility is based on the well-known theorems of Hohenberg and Kohn[1] and Kohn and Sham[2] on the equality of the densities of the particles and quasiparticles. These theorems are derived in the framework of many-body theory and have a general nature.

If we now turn to the quark and gluon degrees of freedom in nuclei, one would expect a direct and explicit manifestation of them under certain special conditions, for example, hard interactions at high energies. There has been considerable interest in recent years in searching for "nuclear exotics" in such processes. A well-known example of this kind is the EMC effect discovered in 1983.[3] It consists of a pronounced difference between the structure functions of a free nucleon and a nucleon bound in a nucleus.

This effect generated numerous explanations, the majority of which appeal to nuclear exotics. Thus, it was suggested that there is an appreciable admixture of multiquark configurations, that the nucleon swells in the nuclear medium, and so forth. This hypothesis[4] arose naturally as a qualitative explanation of the softening observed in the EMC effect of the momentum distribution of the quarks within the nucleons of a nucleus. The swelling of the nucleon, i.e., the increase in the confinement radius that determines the region in which the quarks move, can be related to the probability of overlapping of the wave functions of the nucleons at short distances.[4] However, other mechanisms of the swelling effect are also possible.[5]

In what follows we shall see that there exists a possibil-

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ity of testing exotic models and hypotheses in low-energy nuclear physics by using high-precision experimental data and modern methods of nuclear theory.

The authors of Refs. 6–8 were apparently the first to recognize the importance of testing exotic models of the nucleon in different low-energy nuclear processes. Unfortunately, in many cases they used the results of somewhat imperfect nuclear-structure calculations. As will be shown below, this led to results in conflict with those that follow from a more accurate analysis.

In this review we shall consider various phenomena in traditional nuclear physics in which the effect of swelling of the nucleon in the nucleus could be manifested. These include the longitudinal and transverse response functions in inclusive quasielastic electron–nucleus scattering (e, e'); y scaling in the (e, e') reaction at high energies; details of the charge densities of nuclei (^{208}Pb, ^{206}Pb–^{205}Tl density difference); elastic scattering of electrons and protons of intermediate energies by nuclei; the total cross sections for interaction of high-energy hadrons with nuclei; and the Nolen–Schiffer anomaly in the mass difference of mirror nuclei. We shall show that the elastic scattering of electrons and protons by nuclei is a very sensitive and effective means of studying the swelling effect. It enables one to establish fairly stringent upper bounds on the possible increase in the radius of the nucleon in nuclear matter.

1. INCLUSIVE (e, e') SCATTERING

Response functions

The study of the response functions of nuclei in inclusive inelastic scattering of electrons has a long history.[9] It is well known that the inelastic scattering of unpolarized electrons by nuclei is characterized by two independent response structure functions, longitudinal $R_L(|\mathbf{q}|, \omega)$ and transverse $R_T(|\mathbf{q}|, \omega)$ ($\mathbf{q}$ is the momentum transfer to the nucleus, and ω is the energy transfer). The doubly differential cross section $d^2\sigma/d\Omega_2 dE_2$ can be expressed in terms of R_L and R_T as follows:

$$\frac{d^2\sigma}{d\Omega_2\, dE_2} = \sigma_M \left[\left(\frac{q^2}{|\mathbf{q}|^2} \right)^2 R_L(|\mathbf{q}|, \omega) + \left(\frac{1}{2} \frac{q^2}{|\mathbf{q}|^2} + \mathrm{tg}^2 \frac{\Theta}{2} \right) R_T(|\mathbf{q}|, \omega) \right], \quad (1)$$

where

$$\sigma_M = \frac{\tilde{\alpha}^2}{4E_1^2} \frac{\cos^2(\Theta/2)}{\sin^4(\Theta/2)}, \quad q^2 = \omega^2 - \mathbf{q}^2, \quad \omega = E_1 - E_2,$$

$\tilde{\alpha} = 1/137$, Θ is the scattering angle, and E_1 and E_2 are the initial and final energies of the electrons.

The functions R_L and R_T can be expressed in terms of the imaginary parts of the scalar (spin $S = 0$) and spin ($S = 1$) components of the photon polarization operator $\Pi^p(\mathbf{r}_1, \mathbf{r}_2, \omega)$:

$$R_{L,T}(|\mathbf{q}|, \omega) = -\frac{2}{\pi} \mathrm{Im} \int d^3r_1\, d^3r_2\, e^{i\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2)} \Pi^p_{S=0,1}(\mathbf{r}_1, \mathbf{r}_2, \omega). \quad (2)$$

The polarization operator is determined by the sum of the two diagrams shown in Fig. 1, in which the first diagram

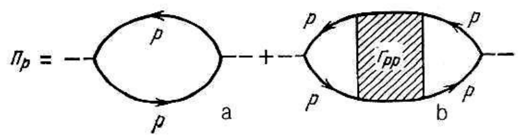

FIG. 1. Diagrams whose imaginary parts determine the response function.

describes the response of a system of noninteracting particles, while the second takes into account the nucleon correlations (Γ_{pp} is the total two-proton interaction amplitude).

Besides the response functions one studies the integrals

$$S_{L,T}(|\mathbf{q}|) = \int_0^\infty d\omega R_{L,T}(|\mathbf{q}|, \omega). \quad (3)$$

The integral S_L is particularly interesting, since for it there is an exact asymptotic Coulomb sum rule:

$$S_L(|\mathbf{q}|) \xrightarrow[|\mathbf{q}| \to \infty]{} Z(\tilde{G}^p_E(|\mathbf{q}|))^2. \quad (4)$$

Here, Z is the charge of the nucleus, and $\tilde{G}^p_E(|\mathbf{q}|)$ is the electric form factor of the proton, including the relativistic Darwin–Foldy correction:

$$\tilde{G}^p_E(|\mathbf{q}|) = G^p_E(q)/\left(\sqrt{1 + |\mathbf{q}|^2/4m^2}\right), \quad (5)$$

where $G^p_E(q)$ is the intrinsic proton form factor, and m is the proton mass.

An expression identical to the right-hand side of (4) arises from the imaginary part of the diagram in Fig. 1a under the condition $|\mathbf{q}| > 2p_F$ (p_F is the Fermi momentum). The correlation contribution, shown by the diagram in Fig. 1b, must decrease as $|\mathbf{q}| \to \infty$ much faster than the right-hand side of (4).[10]

Experimental data in which the longitudinal and transverse responses are separated exist only for a few nuclei at $|\mathbf{q}| \lesssim 500$–$600$ MeV/c.[9] The asymptotic sum rule (4) exceeds the corresponding experimental values by about 20–40%. It is known that the contribution, ignored here, of the exchange currents to S_L is appreciably less than this value.

In Refs. 6, 11, and 12 it was proposed that the resulting discrepancy should be interpreted as evidence of a modification of the proton form factor in a medium. The part played by correlations was not analyzed. To describe the data, an increase of the proton radius by about 20–30% in ^{12}C was required.

In Ref. 7 a modified nucleon form factor was calculated in the framework of one of the variants of the soliton model. In this model, the quarks in a nucleon are confined by a certain nonlinear scalar field χ. The influence of other nucleons on the quark wave function was taken into account in the approximation of the Hartree average field formed by the scalar meson σ (attraction) and vector meson ω (repulsion). The scalar field decreases the mass of a quark, and this leads to an increase of its kinetic energy. This is associated with an increase of the internal pressure within the soliton and an increase of its radius. This radius depends on the density $\rho(r)$ of the nuclear matter.

In Ref. 6 the functions R_L and R_T were calculated for the nuclei ^{12}C, ^{40}Ca, and ^{56}Fe, for which, in accordance with the conditions of the experiment of Ref. 9, $|\mathbf{q}| \simeq 400$–600 MeV/c. The diagram of Fig. 1a was calculated in the Fermi-

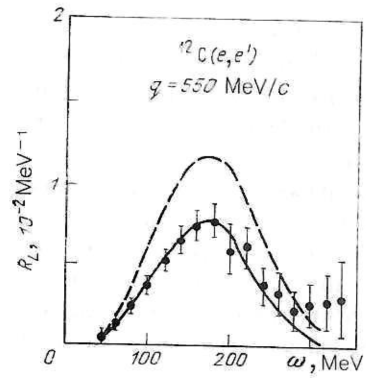

FIG. 2. Dependence of the longitudinal response function $R_L(q,\omega)$ for the $^{12}C(e,e')$ reaction on the energy transfer ω. The broken curve represents the calculation with the free nucleon form factor, and the continuous curve is for the calculation with the medium-modified form factor.[6]

gas model and in a relativistic single-particle model. The relativistic wave functions were plane waves for the continuum states and solutions of the Dirac equation in the potential field for the bound states. It was found that when the free nucleon form factor is used in the longitudinal response function R_L there is a significant excess over the experiment, on the average by a factor 2 (Fig. 2). In the transverse response function R_T a reasonable description of the data up to the quasielastic maximum is observed (Fig. 3).

If a nucleon form factor modified by the nuclear medium is used,[6] there is a significant suppression of the functions R_L and R_T. It markedly improves the agreement with the data in the longitudinal response, but at the same time makes it worse in the transverse response. This is a direct consequence of the more rapid decrease of the proton form factor with increasing q^2 in the case of an increase of the proton radius. It can be seen that the situation is rather uncertain, but nevertheless it was concluded in Ref. 6 that an effect of swelling of the nucleon in nuclei had been found from data on the (e, e') reaction.

In Ref. 13 a more detailed analysis of the longitudinal response in ^{12}C, ^{40}Ca, and ^{56}Fe was made with allowance for the nucleon correlations in the random-phase approximation (RPA). The effect of the finiteness of the nucleus was taken into account in the approximation of a local density. In this case the expression (2) takes the form

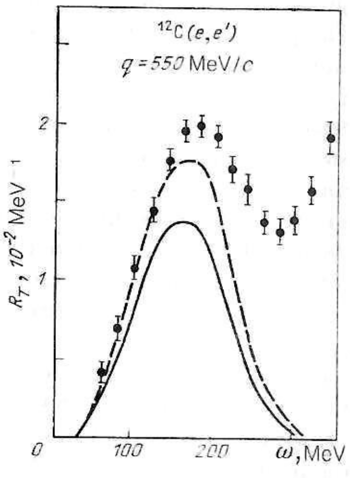

FIG. 3. The same as in Fig. 2 for the transverse response function.

$$R_L(q,\omega) = -\frac{2}{\pi}\int d^3r\, d^3R\, e^{i\mathbf{qr}}\, \mathrm{Im}\, \Pi^p(\mathbf{r}, \mathbf{R}, \omega); \quad (6)$$

$$R_L(q,\omega) = -\frac{2}{\pi}\int d^3R\, \mathrm{Im}\, \Pi^p(q, p_F(R), \omega), \quad (7)$$

where $p_F(R)$ is the local Fermi momentum. In the framework of the RPS for nuclei with $N = Z$ the function $R_L(q,\omega)$ has the form[13]

$$R_L(q,\omega) = -4\, \mathrm{Im}\int_0^{R_0} R^2 dR\, \Pi^0 \left[\frac{1}{1-F^+(q,\omega)\Pi^0} + \frac{1}{1-F^-(q,\omega)\Pi^0}\right], \quad (8)$$

where $\Pi^0 = \Pi^0[q, p_F(R), \omega]$ is the polarization operator of the noninteracting Fermi gas, R_0 is the classical turning point, and $F^{\pm} = F^{nn} \pm F^{np}$ are the amplitudes of the effective interaction of the nucleons in the particle–hole channel. The dependence of $F^{\pm}(q,\omega)$ on ω was not taken into account, while the dependence on q was taken in the form

$$F^{\pm}(q^2) = \Lambda^{\pm}(q^2)\frac{2\pi^2\hbar^2}{p_F(R)\, m^*(R)} f^{\pm}(R); \quad (9)$$

$$\Lambda^{\pm}(q^2) = \left(\frac{\mu_{\pm}^2}{q^2+\mu_{\pm}^2}\right)^{p_{\pm}}, \quad (10)$$

where $m^*(R)$ is the coordinate-dependent effective mass of the nucleon, and $\mu_{\pm}$ and $p_{\pm}$ are parameters chosen for each nucleus separately ($\mu_{\pm} \simeq 400$–500 MeV/c, $p_+ \simeq 2$–3, $p_- \simeq 1$).[13]

The function $f^+(R)$ was found from the well-known relation of Ref. 14, which connects this quantity to the compressibility modulus K of nuclear matter with allowance for the density dependence of the amplitude. To find $f^-(R)$, the connection between this quantity and the symmetry-energy coefficient[14] was used.

The calculations made in the framework of this scheme lead to a comparatively small contribution of the correlations on the scale of 10% (Fig. 4). Therefore, to achieve agreement with experiment it was necessary to increase the proton charge radius by 13% in ^{12}C, 23% in ^{40}Ca, and 21% in ^{56}Fe (Ref. 13). These numbers were chosen to give the best agreement with the Coulomb sum rule (Fig. 5).

It should be noted that the choice of the q^2 dependence of the amplitude F^+ in the form (10) is very different from the form of this dependence in the quasiparticle Lagrangian method[15] and in the Hartree–Fock method with Skyrme

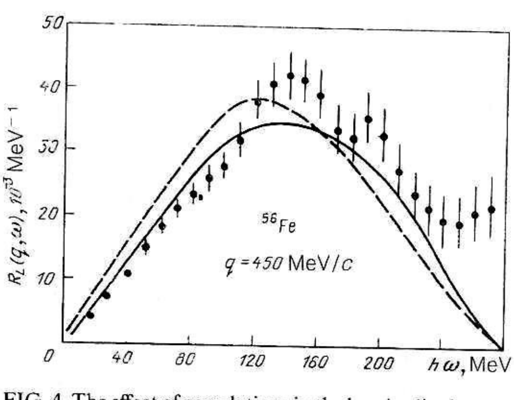

FIG. 4. The effect of correlations in the longitudinal response function for the (e,e') reaction on ^{56}Fe (Ref. 13). The continuous and broken curves are with and without allowance for the correlations, respectively.

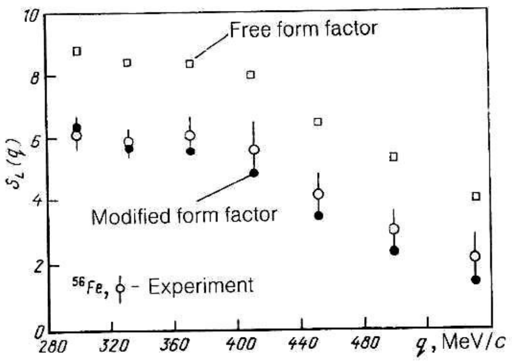

FIG. 5. The longitudinal response function, integrated over the energy, compared with the Coulomb sum rule.[13]

forces.[16] In these approaches $F^+(q^2)$ has the form $F^+(q^2) = a + bq^2$. The coefficient b is uniquely related to the surface energy of the nucleus.[15] Since $b > 0$, $F^+(q^2)$ decreases rapidly with increasing q^2. In contrast to this, $F^+(q^2)$ in the form (9) decreases rapidly, and this leads to a lowering of the correlation contribution at large q^2. There is also some danger if one uses the approximation of a local density in the case of light and medium nuclei.

A more realistic calculation of the longitudinal response function R_L in ^{12}C using the wave functions of finite nuclei was made in the first of the studies of Ref. 17. As an effective nucleon interaction the G matrix constructed for one of the variants of the Bonn NN potential was used. Besides the ordinary $1p$-$1h$ configurations included in the RPA, states of type $2p$–$2h$ were included. The analysis made showed that the employed basis is fairly complete for the description of the nuclear response at momentum transfers $q \lesssim 300$ MeV/c. In this region it was possible to obtain very good agreement with the sum rule using the free proton form factor. At larger q the theoretical sum-rule value remains appreciably greater than the experimental value. In the opinion of the authors, this result is due to insufficient completeness of the employed basis.

There have recently appeared several studies in which the influence of short-range correlations in nuclear matter on the Coulomb sum rule has been investigated.[17] It has been shown that when a realistic nucleon–nucleon interaction is used one can almost completely reproduce the experimentally observed suppression of the sum rule. We note also the last of the studies of Ref. 7, in which the relativistic response function was calculated in the framework of Walecka's model. A degree of suppression close to that of experiment (somewhat greater than in ^{12}C, and somewhat less than in ^{40}Ca) was also obtained.

It should be noted that there also exists purely experimental evidence against the introduction of a modified proton form factor to describe the Coulomb sum rule. If a change of the proton radius were responsible for the discrepancy with this sum rule, then in accordance with (4) the values of $S_L(q)$ for nuclei with the same Z should be very nearly equal. But the experiment of Ref. 9, made on the ^{48}Ca and ^{40}Ca isotopes, gave $S_L(^{48}\mathrm{Ca})/S_L(^{40}\mathrm{Ca}) = 1.20 \pm 0.04$ at $q \simeq 500$ MeV/c. The deviation of this ratio from unity is rather close to the value of the discrepancy between the theory and experiment in the case of one nucleus (see Fig. 5).

We note also that for the lightest nuclei ^{3}He and ^{4}He there exist rigorous relations that connect $S_L(q)$ to the elastic form factor of the nucleus.[18] In the case of ^{3}He comparison with experiment shows that the corresponding relation is satisfied to within the experimental errors ($\sim 3\%$) when the free nucleon form factors are used.

Evidence against nucleon swelling also comes from the situation that has developed in the description of the transverse response function R_T. Good agreement with experiment for R_T is achieved if the free nucleon form factor is used.[6] But the use of the form factor corresponding to an enlarged radius necessarily leads to a worsening of the description.

Overall, it can be said that analysis of nuclear response functions in quasielastic electron scattering does not provide sufficiently weighty evidence for a modification of the proton form factor in the nuclear medium. The problem of more correct allowance for the correlations still awaits its solution. The present level of nuclear theory makes it possible to carry out the corresponding calculations, though technically they are rather complicated.

y scaling

We have seen that in the description of the nuclear response functions in (e, e') scattering at $q < 600$ MeV/c and $\omega \lesssim 200$ MeV there is a difficulty associated with the calculation of the correlation corrections. However, this difficulty can be avoided by going over to consideration of the (e, e') reaction at much larger q and ω. In this region of the kinematic variables the cross section has an interesting property known as y scaling. It was predicted theoretically[19] and confirmed experimentally.[20] The variable y arises naturally from the energy conservation law at large ω and q when correlations and exchange currents are ignored. It has the significance of the longitudinal (along $\mathbf{q}$) component of the momentum of a nucleon in a nucleus, $y = k_\parallel = m\omega/q - q/2$ (the initial energy of the target nucleon and the transverse component of its momentum are negligibly small in the given case). The property of y scaling takes the form that the cross section of inclusive (e, e') scattering, divided by the elementary cross section of the eN interaction and the known function $dy/d\omega$, depends solely on $y = y(q,\omega)$, but not on q and ω separately:

$$\frac{d\sigma}{d\Omega}(q,\ \omega)/\sigma_{eN}(q)\ \frac{dy}{d\omega} = F(y). \tag{11}$$

The function $F(y)$ is the probability of finding a nucleon with momentum $y = k_\parallel$. The point $y = 0$ corresponds to elastic scattering on a nucleon at rest, and the region $y < 0$ corresponds to the low-energy wing of the quasielastic peak. The features of the scaling behavior make it possible to study both the reaction mechanism and the properties of the nucleon on which the scattering occurs (its form factor and mass). Thus, if a bound nucleon is increased in size, this leads to a change of the cross section $\sigma_{eN}(q)$ and, therefore, to a q-dependent change of $F(y)$. This signifies a breakdown of y scaling. It will be particularly pronounced if for fixed y the data cover a wide range in q.

Figure 6 shows the realization of the scaling dependence for ^{56}Fe according to SLAC data ($E_e = 2$–36 GeV, $q = 3$–12 F^{-1}) in the case when the free cross section σ_{eN} was used in (11).[21] The experimental points, taken for different ω and q but for the same y, fit on the single line $F(y)$

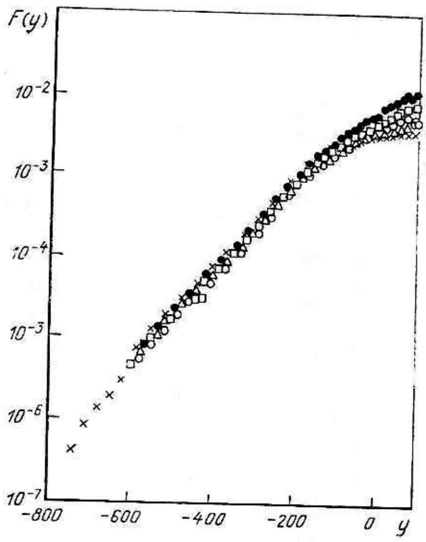

FIG. 6. Scaling function $F(y)$ for the $^{56}Fe(e,e')$ reaction in the case of free nucleon form factors.[21]

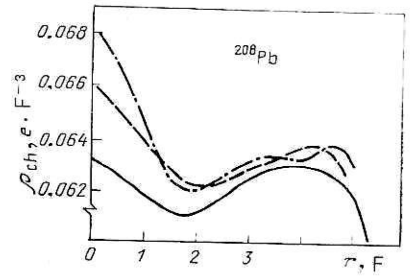

FIG. 8. Charge density $\rho_{ch}(r)$ in the central region of ^{208}Pb, calculated in the relativistic mean-field theory with the free proton form factor (chain curve) and the form factor modified by the nuclear medium (broken curve)[17]; The continuous curve is the model-independent density $\rho_{ch}(r)$.

(realization of y scaling). If the radius of the bound nucleon is increased, the quality of the y scaling becomes worse. This can be seen in Fig. 7, which gives the dependence of χ^2 (which characterizes the degree of smearing of the line) on the relative increase of the nucleon radius.[21] Besides a change of the radius, a change of the nucleon mass in accordance with the relation $\Delta m/m = -\Delta r/r$, which holds in the bag model, was introduced. It can be seen that the minimum of χ^2 is attained at values of the radius close to the free-nucleon radius. With allowance for possible uncertainties, it was concluded[21] that $\Delta r/r \lesssim 3\%$, i.e., significant swelling of the nucleon in the nucleus is ruled out.

2. MANIFESTATION OF A POSSIBLE EFFECT OF NUCLEON SWELLING IN THE SINGLE-PARTICLE NUCLEAR DENSITIES AND THEIR DESCRIPTION IN THE QUASIPARTICLE LAGRANGIAN METHOD

In this section we discuss the studies of Refs. 7 and 8, in which an investigation was made of the manifestation of the proton swelling effect in the charge density of ^{208}Pb and the ^{206}Pb–^{205}Tl density difference. The problem of calculating the single-particle nucleon densities in self-consistent approaches of nuclear theory will be considered in detail. We briefly describe a method for calculating these densities in the quasiparticle Lagrangian method. We give the results of calculations of the charge densities of various nuclei on the basis of this method. It will be shown that they agree well with experiment without the introduction of any nucleon swelling.

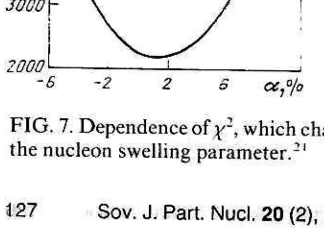

FIG. 7. Dependence of χ^2, which characterizes the accuracy of scaling, on the nucleon swelling parameter.[21]

Influence of proton swelling on the ^{208}Pb charge density and the ^{206}Pb–^{205}Tl density difference

In Ref. 7 these densities were calculated by convolving the theoretical distribution densities of point protons with the proton charge form factor modified by the nuclear medium. The results of Hartree–Fock theory with effective density-dependent forces[22] and of relativistic "mean-field" theory[23] were used. In the case of mean-field theory an increase of the proton radius when the convolution is calculated leads to a better description of the ^{208}Pb charge density (Fig. 8). However, in the case of the Hartree–Fock density this procedure leads to worse agreement with experiment.[7] It is known that the accuracy of the employed variant of Hartree–Fock theory[22] is basically much higher than the accuracy of mean-field theory. There are also other calculations of the ^{208}Pb charge density[24,25] that agree very well with experiment without any modification of the proton form factor. Despite this, it was concluded in Ref. 7 that analysis of the ^{208}Pb charge density indicates an increase of the nucleon radius in the medium ($\sim 25\%$ at the center of ^{208}Pb).

We now discuss the difference of the charge densities, $\delta\rho(^{206}Pb$–$^{206}Tl)$, extracted from high-precision data on the elastic scattering of electrons by the neighboring nuclei.[26] From this one can obtain information about the wave function of the proton added to ^{205}Tl and its spectroscopic factor. The difference is also sensitive to the form factor of the added proton and its possible modification in the medium.

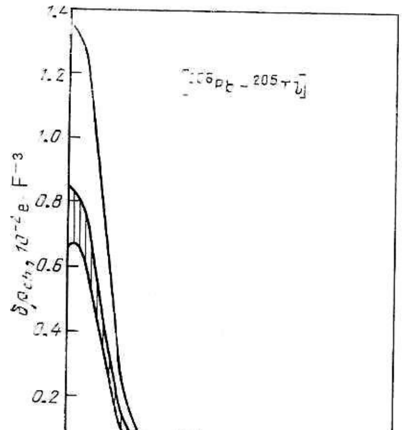

FIG. 9. Difference of the ^{206}Pb and ^{205}Tl charge densities calculated in the Hartree–Fock framework with effective forces.[26] The hatched region represents the experimental values.

In Fig. 9 the difference $\delta\rho(r)$, determined from so-called model-independent analysis of the cross sections of inelastic electron scattering, is compared with the result of the Hartree–Fock calculation.[8] In this calculation a unit spectroscopic factor for the $3s_{1/2}$ state and the free proton form factor were used. It can be seen that the form of $\delta\rho(r)$ is well reproduced, but there is a difference in the absolute value.

In Ref. 8 this difference was eliminated by introducing a medium-modified proton form factor, while the spectroscopic factor of the $3s_{1/2}$ state was taken equal to unity. However, it should be noted that this last assumption contradicts recent experiments on transfer reactions[27] and on the $(e,e'p)$ reaction.[28] These data indicate that there is a fragmentation of the $3s_{1/2}$ state on the scale 20–30% in the region of low excitation energies. This fragmentation is largely due to coupling to the low-lying collective states (phonons). Thus, the state $|3s_{1/2}^{-1}\rangle$ is strongly mixed with the state $|2d_{3/2}^{-1}\otimes 2^+\rangle_{J=1/2}$, where 2^+ is a phonon $(J^\pi = 2^+)$ in ^{206}Pb. With allowance for the experimental spectroscopic factor $[\sim(0.7–0.8)]$ the difference $\delta\rho(^{206}\mathrm{Pb}–^{205}\mathrm{Tl})$ can be well described without introducing a nuclear modification of the proton form factor.[21] It will be shown below that a similar result is also obtained in the quasiparticle Lagrangian method.

Density distribution of the particles in the ground state of a many-body system

In what follows we shall show that very stringent restrictions on the possible change of the nucleon radius in a medium can be obtained by analyzing the cross sections of elastic scattering of electrons and protons of intermediate energies. In these processes the nuclear structure is basically represented by single-particle nucleon densities. Therefore, the problem of calculating them as accurately as possible is very important. As we noted in the Introduction, in quasiparticle approaches the calculation of these densities is based on the Kohn–Sham theorem[2] on the equality of the densities of the particles and quasiparticles. This theorem, in its turn, is based on the Hohenberg–Kohn theorem,[1] according to which the ground-state energy of a many-body system is a functional of the particle density $\rho(r): E = E[\rho]$. The particle density of the ground state corresponds to the minimum of this functional under the condition $\int d^3\mathbf{r}\rho = N$, where N is the total number of particles.

We now suppose that we know the true energy functional $E[\rho]$. We write it in the form

$$E[\rho] = \int d^3r \left[\frac{p_F^2}{2m}\tau(\mathbf{r}) + W(\rho(\mathbf{r}))\right], \qquad (12)$$

where τ is the kinetic-energy density. The factor $p_F^2/2m$ is introduced in order to normalize the distribution $\tau(\mathbf{r})$ like the ordinary density ρ. In the ground state, the function τ can be expressed in terms of the function ρ and its derivatives.[29] In this sense the representation (12) does not violate the Hohenberg–Kohn theorem.

Following the logic of Ref. 2, we shall seek $\rho(\mathbf{r})$ and $\tau(\mathbf{r})$ in the form

$$\rho(\mathbf{r}) = \sum_\lambda n_\lambda|\varphi_\lambda(\mathbf{r})|^2; \qquad (13)$$

$$\tau(\mathbf{r}) = \frac{1}{p_F^2}\sum_\lambda n_\lambda|\nabla\varphi_\lambda(\mathbf{r})|^2, \qquad (14)$$

where n_λ are the quasiparticle population numbers, equal to 1 and 0 for occupied and unoccupied states, respectively. The condition of a minimum of the functional $E[\rho]$ on variation with respect to $\varphi_\lambda^*(\mathbf{r})$ leads to a Schrödinger equation for $\varphi_\lambda(\mathbf{r})$ with the potential $U = \delta W/\delta\rho$. Of course, the representation of $E[\rho]$ in the form (12) is not unique. For example, in the Vautherin–Brink functional[16] the kinetic term is expressed in the form $\tau f(\rho)$. As a result, an equation for $\varphi_\lambda(\mathbf{r})$ with a coordinate-dependent effective mass is obtained.

By definition, the density (13) is the quasiparticle density. But since it realizes a minimum of the functional $E[\rho]$, it is equal to the particle density of the system. This fact is the fundamental basis for the application of the quasiparticle approach to the description of nuclear reactions involving nuclear densities.

In real calculations every employed energy functional (we shall call it the quasiparticle functional $E_q[\rho]$) differs to some extent from the true functional. Accordingly, the density ρ_q that minimizes it also differs from the true density ρ. Their connection can be expressed in the form

$$\rho(\mathbf{r}) = \int d^3r' f_q(\mathbf{r}, \mathbf{r}')\rho_q(\mathbf{r}'), \qquad (15)$$

where $f_q(\mathbf{r}, \mathbf{r}')$ is the quasiparticle form factor. More precisely, the form factor is the coordinate-dependent Fourier transform of the function $f_q(\mathbf{r}, \mathbf{r}')$ with respect to the variable $\mathbf{r} - \mathbf{r}'$: $f_q((\mathbf{r}+\mathbf{r}')/2, q)$. We write its expansion in powers of q^2 in the form

$$f_q\left(\frac{\mathbf{r}+\mathbf{r}'}{2}, q\right) = 1 + \alpha_2\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)q^2 + \alpha_4\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)q^4 + \dots \qquad (16)$$

The first term on the right-hand side of (16) is equal to 1 by virtue of the normalization of ρ_q to the number of particles. Under certain conditions the coefficient α_2 can be made very small. Essentially, the difference between f_q and 1 is a measure of the difference of the quasiparticle energy functional from the true one. The good agreement between the densities calculated in the quasiparticle Lagrangian method and the experimental densities[24] makes it possible to assume that to high accuracy $f_q \simeq 1$, i.e., $\rho_q(\mathbf{r}) \simeq \rho(\mathbf{r})$. If appreciable discrepancies for the densities were to arise, this should be regarded as a signal for improving the energy functional. It is this principle that we shall follow.

We note that in the Hohenberg–Kohn–Sham theorems it is the density of point, i.e., structureless, particles that is considered. The charge density $\rho_{ch}(r)$, which determines the electron scattering cross sections, is obtained by convolving the proton and neutron point densities $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ with the corresponding charge distributions $f_p(\mathbf{r})$ and $f_n(\mathbf{r})$ within the proton and neutron:

$$\rho_{ch}(r) = \int d^3r' [f_p(\mathbf{r}-\mathbf{r}')\rho_p(\mathbf{r}') + f_n(\mathbf{r}-\mathbf{r}')\rho_n(\mathbf{r}')]. \qquad (17)$$

As in Ref. 30, the functions $f_p(r)$ and $f_n(r)$ are chosen in the form

$$f_p(r) = (8\pi a_0^3)^{-1}\exp(-r/a_0), \quad a_0 = 0.25\ \mathrm{F};$$

$$j_n(r) = (8\pi a_1^3)^{-1}\exp(-r/a_1) - (8\pi a_1^3)^{-1}\exp(-r/a_2),$$
$$a_1 = 0.22\ \text{F},\ a_2 = 0.242\ \text{F}.$$

The value of a_0 is increased by 10% compared with the corresponding value in Ref. 30 through allowance for the relativistic Darwin–Foldy correction $(a_0^2 \to a_0^2 + 1/8m^2)$. In what follows, when calculating the electron and proton scattering cross sections, we shall use the point nucleon densities calculated in the framework of the quasiparticle Lagrangian method.

Formalism of the quasiparticle Lagrangian method

The point of departure for the construction of the quasiparticle Lagrangian[15] is the form of the quasiparticle mass operator $\Sigma_q(\mathbf{r}, \mathbf{p}, \varepsilon)$ in the theory of a Fermi liquid. For it we use the following form linear in p^2 and ε:

$$\Sigma_q(\mathbf{r}, \mathbf{p}, \varepsilon) = \Sigma_0(\mathbf{r}) + \frac{1}{2m\varepsilon_F^0}\mathbf{p}\Sigma_1\mathbf{p} + \frac{\varepsilon}{\varepsilon_F^0}\Sigma_2(\mathbf{r}) \equiv \xi_i\Sigma_i(\mathbf{r}), \tag{18}$$

where $\xi_0 = 1$, $\xi_1 = \hat{p}_2^2/(2m\varepsilon_F^2)$, $\xi_2 = \hat{\varepsilon}/\varepsilon_F^0, \varepsilon_F^0 = p_F^2/(2m)$, and p_F is the Fermi momentum of the nuclear matter. The mass operator Σ_q has the meaning of the average field that acts on a quasiparticle. In Hartree–Fock theory only the first two terms of the expansion (18) are present, and the term $\sim\Sigma_2$ is absent. This term determines the weight of the quasiparticle component in the exact single-particle wave function through the factor

$$Z(\mathbf{r}) = (1 - \Sigma_2(\mathbf{r})\,\varepsilon_F^0)^{-1}. \tag{19}$$

The basic quantity in the quasiparticle Lagrangian method is the quasiparticle Green's function G_q, which is determined by the Dyson equation

$$[\varepsilon - \varepsilon_F^0 - \Sigma_q(\mathbf{r}, \mathbf{p}, \varepsilon)]\,G_q(\mathbf{r}, \mathbf{r}', \varepsilon) = \delta(\mathbf{r} - \mathbf{r}'). \tag{20}$$

It can be seen from (20) that the operator G_q is diagonal on functions ψ_λ satisfying the equation

$$\left[\varepsilon_p^0 + \Sigma_0(\mathbf{r}) + \frac{1}{2m\varepsilon_F^0}\mathbf{p}\Sigma_1\mathbf{p}\right]\psi_\lambda(\mathbf{r}) = \varepsilon_\lambda(1 - \Sigma_2(\mathbf{r})/\varepsilon_F^0)\,\psi_\lambda(\mathbf{r}). \tag{21}$$

This equation is the equation of motion of the quasiparticles. In Ref. 15 it was shown that there exists a quasiparticle Lagrangian L_q such that the Lagrange equation of motion corresponding to it is identical to Eq. (21). The variational definition of L_q is

$$\delta L_q(t) = \int d^3r \left[i\frac{\partial}{\partial\tau} - \varepsilon_p^0 - \Sigma_q\left(\mathbf{r}, \mathbf{p}; t, i\frac{\partial}{\partial\tau}\right)\right] \delta G_q(\mathbf{r}, \mathbf{r}'; t, \tau)\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau\to-0}}. \tag{22}$$

The Lagrangian L_q can be expressed as a sum $L_q = L_q^0 = L_q'$, where L_q^0 is the Lagrangian of the noninteracting quasiparticles. It can be seen from (22) that the density of the Lagrange function $\mathscr{L}_q^0$ corresponding to L_q^0 is

$$\mathscr{L}_q^0(\mathbf{r}, t) = \left(i\frac{\partial}{\partial\tau} - \varepsilon_p^0\right) G_q(\mathbf{r}, \mathbf{r}'; t, \tau)\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau\to-0}}. \tag{23}$$

It is convenient to introduce the functions

$$\nu_0(\mathbf{r}, t) = G_q(\mathbf{r}, \mathbf{r}; t, \tau = -0) = \sum_\lambda n_\lambda|\psi_\lambda(\mathbf{r}, t)|^2; \tag{24}$$

$$\nu_1(\mathbf{r}, t) = \frac{1}{2m\varepsilon_F^0}\nabla\nabla' G_q(\mathbf{r}, \mathbf{r}'; t, \tau = -0)\Big|_{\mathbf{r}'\to\mathbf{r}} = \frac{1}{2m\varepsilon_F^0}\sum_\lambda n_\lambda|\nabla\psi_\lambda(\mathbf{r}, t)|^2; \tag{25}$$

$$\nu_2(\mathbf{r}, t) = \frac{i}{\varepsilon_F^0}\frac{\partial}{\partial\tau}G_q(\mathbf{r}, \mathbf{r}; t, \tau)\Big|_{\tau\to-0} = \frac{1}{2i\varepsilon_F^0}\sum_\lambda n_\lambda[\dot{\psi}_\lambda^*(\mathbf{r}, t)\,\psi_\lambda(\mathbf{r}, t) - \psi_\lambda^*(\mathbf{r}, t)\,\dot{\psi}_\lambda(\mathbf{r}, t)]. \tag{26}$$

They are the quasiparticle density, the kinetic-energy density, and the total energy density of the quasiparticles, respectively. In abbreviated form, these densities can be written as

$$\nu_i(\mathbf{r}, t) = \xi_i G_q(\mathbf{r}, \mathbf{r}; t, \tau = -0) = \int \frac{d\varepsilon}{2\pi i}\xi_i G_q(\mathbf{r}, \mathbf{r}; t, \varepsilon), \tag{27}$$

where ξ_i are defined in (18).

It follows from (23), (25), and (26) that

$$\mathscr{L}_q^0(\mathbf{r}, t) = \varepsilon_F^0[\nu_2(\mathbf{r}, t) - \nu_1(\mathbf{r}, t)]. \tag{28}$$

The interaction Lagrangian L_q' is determined by the variational condition

$$\delta L_q'(t) = -\int d^3r\,\Sigma_q\left(\mathbf{r}, \mathbf{p}; t, i\frac{\partial}{\partial\tau}\right)\delta G_q(\mathbf{r}, \mathbf{r}'; t, \tau)\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau\to-0}}. \tag{29}$$

Taking Σ_q in the form (18), we obtain

$$\delta L_q'(t) = -\int d^3r\,\Sigma_i(\mathbf{r}, t)\,\delta\nu_i(\mathbf{r}, t), \tag{30}$$

where, as before, the sum over $i = 0, 1, 2$ is understood.

Thus,

$$\Sigma_i(\mathbf{r}) = -\frac{\delta L_q'(\mathbf{r})}{\delta\nu_i(\mathbf{r})}. \tag{31}$$

It follows from the definition of the densities ν_i that

$$\left.\begin{aligned} &\frac{\partial\nu_0}{\partial\psi_\lambda^*} = n_\lambda\psi_\lambda; \quad \frac{\partial\nu_1}{\partial(\nabla\psi_\lambda^*)} = n_\lambda\nabla\psi_\lambda/(2m\varepsilon_F^0);\\ &\frac{\partial\nu_2}{\partial\dot{\psi}_\lambda^*} = in_\lambda\psi_\lambda/(2\varepsilon_F^0); \quad \frac{\partial\nu_2}{\partial\dot{\psi}_\lambda^*} = -in_\lambda\psi_\lambda/(2\varepsilon_F^0). \end{aligned}\right\} \tag{32}$$

By means of these relations we can readily show that the Lagrange equation

$$\frac{\delta L_q}{\delta\psi_\lambda^*} - \frac{\partial}{\partial t}\frac{\delta L_q}{\delta\dot{\psi}_\lambda^*} - \nabla_\alpha\frac{\delta L_q}{\delta(\nabla_\alpha\psi_\lambda^*)} = 0 \tag{33}$$

with Lagrangian determined by Eqs. (28) and (30) is identical to the quasiparticle equation of motion in the form (21).

For the Lagrangian L_q dependent on the densities ν_i we can readily calculate the 4-current. In accordance with the canonical rules its spatial component is

$$j_\alpha(\mathbf{r}) = i\sum_\lambda\left[\psi_\lambda^*(\mathbf{r})\frac{\delta L_q}{\delta(\nabla_\alpha\psi_\lambda^*(\mathbf{r}))} - \psi_\lambda(\mathbf{r})\frac{\delta L_q}{\delta(\nabla_\alpha\psi_\lambda(\mathbf{r}))}\right]. \tag{34}$$

Using (32), we find

$$j_\alpha(\mathbf{r}) = \frac{\delta L_q}{\delta\nu_1(\mathbf{r})}\frac{1}{\varepsilon_F^0}j_\alpha^0(\mathbf{r}), \tag{35}$$

where

$$j_\alpha^0 = \frac{i}{2m} \sum_\lambda n_\lambda (\psi_\lambda^* \nabla_\alpha \psi_\alpha - \psi_\alpha \nabla_\alpha \psi_\lambda^*). \quad (36)$$

The time component of the 4-current, which is the baryon-charge density, can be found similarly:

$$j_0(\mathbf{r}) = i \sum_\lambda \left[\psi_\lambda^*(\mathbf{r}) \frac{\delta L_q}{\delta \dot{\psi}_\lambda^*(\mathbf{r})} - \psi_\lambda(\mathbf{r}) \frac{\delta L_q}{\delta \dot{\psi}_\lambda(\mathbf{r})} \right] = \frac{\nu_0}{\varepsilon_F^0} \frac{L_q}{\nu_2(\mathbf{r})}. \quad (37)$$

Hence, using (19), (28), and (30), we find

$$\nu_0(\mathbf{r}) = Z(\mathbf{r}) j_0(\mathbf{r}) \equiv Z(\mathbf{r}) \rho_q(\mathbf{r}), \quad (38)$$

where $\rho_q(\mathbf{r})$ is the ordinary quasiparticle density normalized to the total number N of particles. It is equal to the particle density $\rho(\mathbf{r})$ by virtue of the Kohn–Sham theorem. The density $\nu_0(\mathbf{r})$ is normalized to N with weight $Z^{-1}(\mathbf{r})$ because of the explicit dependence of L_q on ν_2.

The total quasiparticle energy E_q corresponding to the Lagrangian L_q is given by the integral

$$E_q = \int d^3\mathbf{r}\, \mathscr{H}_q(\mathbf{r}), \quad (39)$$

where the density of the quasiparticle Hamiltonian $\mathscr{H}_q$ has the form

$$\mathscr{H}_q(\mathbf{r}) = \sum_\lambda \left[\dot{\psi}_\lambda^*(\mathbf{r}) \frac{\delta L_q}{\delta \dot{\psi}_\lambda^*(\mathbf{r})} + \dot{\psi}_\lambda(\mathbf{r}) \frac{\delta L_q}{\delta \dot{\psi}_\lambda(\mathbf{r})} \right] - \mathscr{L}_q(\mathbf{r}). \quad (40)$$

Since our Lagrangian L_q depends on $\dot{\psi}_\lambda$ and $\dot{\psi}_\lambda^*$ only through the density $\nu_2(\mathbf{r})$, the expression (40) can be written in the form

$$\mathscr{H}_q(\mathbf{r}) = \nu_2(\mathbf{r}) \frac{\delta L_q}{\delta \nu_2(\mathbf{r})} - \mathscr{L}_q(\mathbf{r}). \quad (41)$$

As was shown in Ref. 15, the energy E_q is equal to the total binding energy E_0 of the system. This assertion is analogous to the well-known Landau–Luttinger theorem on the equality of the particle number to the quasiparticle number.

The densities ν_0 and ν_1 are analogous to the Hartree–Fock densities $\rho(\mathbf{r})$ and $\tau(\mathbf{r})$ from Ref. 16. The density ν_2 does not have a Hartree–Fock analog; it arose because of the allowance for the explicit dependence of Σ_q on the energy.

To construct the quasiparticle Lagrangian in the quasiparticle Lagrangian method, a polynomial form L'_q in ν_i and $\nabla \nu_0$ is used. The minimal Lagrangian that takes into account the effects of the velocity and energy dependences contains the densities ν_1 and ν_2 to the first power. This is due to the fact that both the effective mass $m^*(\mathbf{r})$ and the renormalization factor $Z(\mathbf{r})$ can be assumed to depend only on the quasiparticle density. In this case, as follows from (41), $\mathscr{H}_q(\mathbf{r})$ does not depend explicitly on $\nu_2(\mathbf{r})$. In terms of the functions, ψ_λ, $\mathscr{H}_q(\mathbf{r})$ has exactly the same form as the corresponding Hartree–Fock functional with effective Skyrme forces.[16] The entire difference between the Hartree–Fock approach and the quasiparticle Lagrangian method is concentrated in the right-hand side of Eq. (21) for ψ_λ.

The Hartree–Fock equations are usually obtained on the basis of an effective Hamiltonian by means of a variational principle.[16] Equation (21) can also be obtained on the basis of $\mathscr{H}_q$; it is merely necessary to require, in place of the usual normalization condition for the single-particle functions, fulfillment of the weighted normalization condition

$$\int d^3r \psi_\lambda^*(\mathbf{r}) (1 - \Sigma_2(\mathbf{r})/\varepsilon_F^0) \psi_\lambda(\mathbf{r}) = 1. \quad (42)$$

It is obvious that one can go over to the functions

$$\varphi_\lambda(\mathbf{r}) = Z^{-1/2}(\mathbf{r}) \psi_\lambda(\mathbf{r}), \quad (43)$$

which have the standard normalization to 1 and satisfy the equation

$$h\varphi_\lambda = \varepsilon_\lambda \varphi_\lambda, \quad (44)$$

where

$$h = Z^{1/2}(\mathbf{r}) \left[\frac{p^2}{2m} + \Sigma_0(\mathbf{r}) + \frac{1}{2m\varepsilon_F^0} \mathbf{p} \Sigma_1(\mathbf{r}) \mathbf{p} \right] Z^{1/2}(\mathbf{r}). \quad (45)$$

In terms of the functions $\varphi_\lambda(\mathbf{r})$ the density $\rho(\mathbf{r})$ has the standard Hartree–Fock form (13).

To obtain Eq. (44) from the variational principle for the energy, it is necessary to express the Hamiltonian density (41) in terms of the functions ρ and τ. Then to the simple expression for L_q in terms of the densities ν_i there corresponds a very complicated expression for $\mathscr{H}_q$ in terms of the densities ρ and τ.

In the quasiparticle Lagrangian method an interaction Lagrangian density $\mathscr{L}'_q$ of the following form is used:

$$\mathscr{L}'_q = -C_0 \left\{ \frac{\lambda_{00}}{2} [(\nu_0^+)^2 - r_p^2 (\nabla \nu_0^+)^2] + \frac{\lambda'_{00}}{2} (\nu_0^-)^2 + \lambda_{01} \nu_0^+ \nu_1^+ + \lambda'_{01} \nu_0^- \nu_1^- + \lambda_{02} \nu_0^+ \nu_2^+ + \frac{2}{3} \frac{\gamma}{\rho_0} \nu_0^+ \nu_0^n \nu_0^p \right\} + \mathscr{L}_c + \mathscr{L}_{sl}. \quad (46)$$

Here $\nu_i^\pm = \nu_i^n \pm \nu_i^p$, $C_0 = 300$ MeV·F^3; λ_{00}, λ'_{00}, λ_{01}, λ'_{01}, λ_{02}, γ, r_p^2 are free parameters; $\mathscr{L}_c$ and $\mathscr{L}_{sl}$ are the Coulomb and spin–orbit terms of $\mathscr{L}'_q$. Their explicit form is given in Ref. 15.

In terms of the densities ν_i the Hamiltonian density $\mathscr{H}_q$ also has a simple form:

$$\mathscr{H}_q(\mathbf{r}) = \varepsilon_F^0 \nu_1^+ + C_0 \left\{ \frac{\lambda_{00}}{2} \left[(\nu_0^+)^2 - r_p^2 (\nabla \nu_0^+)^2 \right] + \frac{\lambda'_{00}}{2} (\nu_0^-)^2 + \lambda_{01} \nu_0^+ \nu_1^+ + \lambda'_{01} \nu_0^- \nu_1^- + \frac{2}{3} \frac{\gamma}{\rho_0} \nu_0^+ \psi \nu_0^n \nu_0^p \right\} + \mathscr{H}_c + \mathscr{H}_{sl}, \quad (47)$$

where $\mathscr{H}_c = -\mathscr{L}_c$ and $\mathscr{H}_{sl} = -\mathscr{L}_{sl}$.

We recall that $\nu_0^{n,p}(\mathbf{r}) = Z(\mathbf{r}) \rho^{n,p}(\mathbf{r})$, where the factor $Z(\mathbf{r})$ itself depends on the density. For the chosen $\mathscr{L}'_q$ in the form (46) we can show that

$$Z(\mathbf{r}) = [1 - C_0 \lambda_{02} \nu_0^+(\mathbf{r})/\varepsilon_F^0]^{-1} = \frac{2}{1 + \sqrt{1 - 4C_0 \lambda_{02} \rho^+(\mathbf{r})/\varepsilon_F^0}}. \quad (48)$$

Therefore, $\nu_0(\mathbf{r})$ depends on $\rho(\mathbf{r})$ nonlinearly. The connection between $\nu_1(\mathbf{r})$ and $\tau(\mathbf{r})$ is more complicated:

$$\nu_1^{n,p}(\mathbf{r}) = Z(\mathbf{r}) \tau^{n,p} + \frac{1}{4m\varepsilon_F^0} \frac{dZ}{d\rho^+} \nabla_\alpha \rho^+ \left[\nabla_\alpha \rho^{n,p} + \frac{\rho^{n,p}}{2Z} \frac{dZ}{d\rho^+} \nabla_\alpha \rho^+ \right]. \quad (49)$$

The use of these expressions for ν_0, ν_1, and Z in (47) makes it possible to express $\mathscr{H}_q$ in terms of τ and ρ and the derivatives of ρ. However, the corresponding expression will be very complicated. It is simply impossible to devise such a construction as an ansatz. Of course, one may also have the opposite situation, in which a very complicated Lagrangian corresponds to a simple Hamiltonian (for example, a Skyrme Hamiltonian). We emphasize that more information is contained in the Lagrangian than in the Hamiltonian. Thus, the term $\sim\lambda_{02}\nu_0\nu_2$, which determines $Z(\mathbf{r})$, is present in $\mathscr{L}'_q$. But if $\mathscr{H}_q$ is specified, information about the factor $Z(\mathbf{r})$ is absent.

Results of calculations of the charge densities in the framework of the quasiparticle Lagrangian method

The parameters of the quasiparticle Lagrangian density (46) were determined from energy characteristics, namely, the total binding energies and the single-particle spectra of magic nuclei. A fairly good description of the charge densities was obtained. As examples, Figs. 10 and 11 show the ^{40}Ca, ^{208}Pb, and $^{116,124}Sn$ densities. It can be seen that the degree of agreement with experiment does not require the introduction of any modification of the proton form factor. There is agreement of approximately the same quality for the other nuclei calculated in the quasiparticle Lagrangian method.[31]

In this approach one can also describe fairly well the charge-density difference $\delta\rho[^{206}Pb-^{205}Tl]$ discussed above (Fig. 12).[32] The main reason for this is that, in contrast to the Hartree–Fock method, the spectroscopic factor of the single-particle state in the quasiparticle Lagrangian method differs from $1 (Z\simeq 0.8)$. This arises from the allowance for the dependence of the effective interaction of the quasiparticles on the energy.

The good description of the densities in the quasiparticle Lagrangian method is a manifestation of the Hohenberg–Kohn–Sham theorems. Indeed, the parameters of the energy functional were chosen to give the best description of the nuclear binding energies.

The "experimental" charge densities discussed in this section were obtained from the cross sections of electron elastic scattering on nuclei by means of the so-called model-independent analysis. However, this procedure unavoidably introduces errors in addition to those of the original data. For this reason, the properties of the proton form factor in nuclear matter are better studied directly in the cross sections themselves. In addition, in this case it is easy to separate the region of large momentum transfers, where the properties of the nucleon form factor are most strongly manifested. This is the subject of the following section.

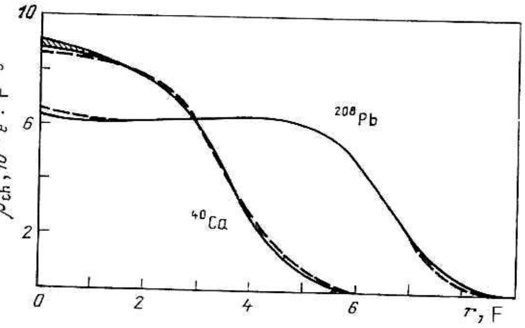

FIG. 10. Charge densities of ^{40}Ca and ^{208}Pb calculated in the framework of the quasiparticle Lagrangian method[15] (broken curve) and the corresponding model-independent distributions (continuous curves).

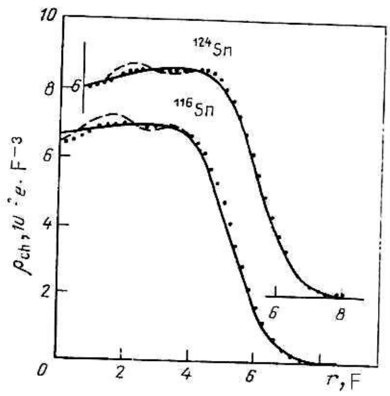

FIG. 11. Charge densities of the isotopes ^{116}Sn and ^{124}Sn. The continuous curve represents the experimental values, the dotted curve the calculation in the quasiparticle Lagrangian method,[32] and the broken curve the Hartree–Fock calculation with the forces of Ref. 25.

3. ELECTRON ELASTIC SCATTERING

In Ref. 33 different self-consistent approaches to nuclear theory were compared from the point of view of the quality of the description of the cross sections for elastic scattering of high-energy electrons by nuclei.

The cross sections were calculated in the framework of a phase-shift analysis for the Dirac equation in the central Coulomb field of the nucleus. The corresponding expressions are given in Ref. 34. It is known that at $q \lesssim 3$ F^{-1} the corrections to this scheme due to exchange currents and also the dispersion corrections are small.

In this process the nuclear structure is basically represented by the charge density $\rho_{ch}(r)$. This density was calculated by the convolution (17) of the theoretical point distributions of the protons, $\rho_p(r)$, and neutrons, $\rho_n(r)$, with the corresponding charge distributions within the proton, $f_p(r)$, and neutron, $f_n(r)$. In contrast to the quasiparticle Lagrangian method, the Hartree–Fock calculation also took into

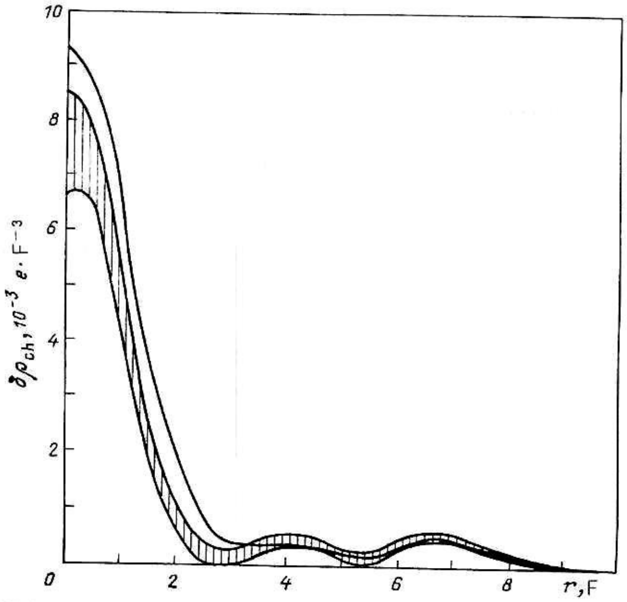

FIG. 12. Difference of the ^{206}Pb and ^{205}Tl charge densities calculated in the quasiparticle Lagrangian method[32] (continuous curve). The hatched region represents the experimental values.

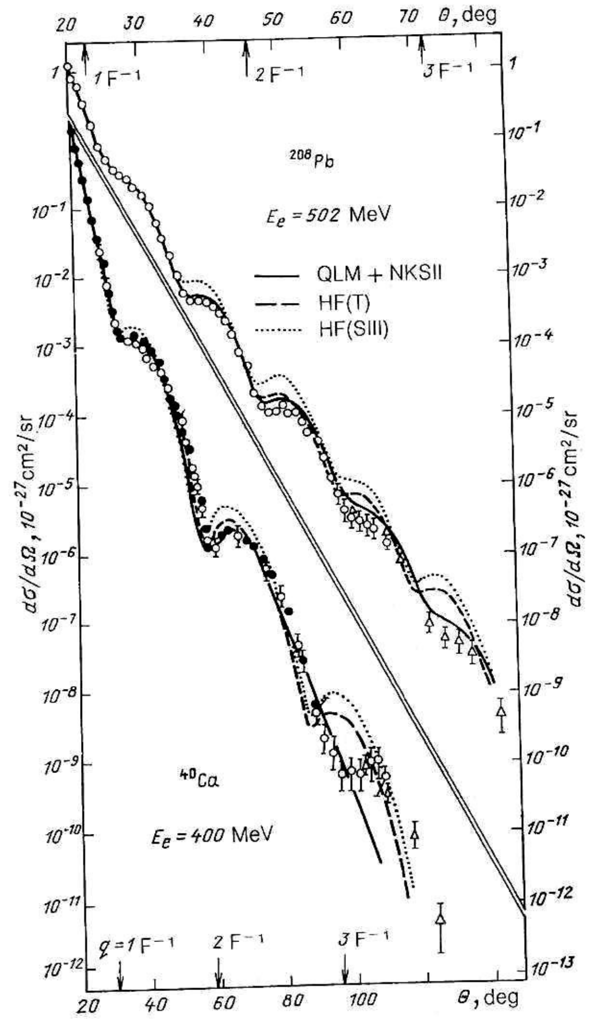

FIG. 13. Cross sections for elastic scattering of electrons by ^{208}Pb (E_e = 502 MeV) and ^{40}Ca(E_e = 400 MeV), calculated with different densities. The figure is taken from Ref. 33.

account the contribution to $\rho_{ch}(r)$ from the spin–orbit density:

$$\rho_{sl}(\mathbf{r}) = \sum_{\lambda} n_{\lambda} (\sigma \mathbf{l})_{\lambda\lambda} \varphi_{\lambda}^{*}(\mathbf{r}) \varphi_{\lambda}(\mathbf{r}). \tag{50}$$

It was found that the Tondeur functional,[35] which corresponds to modified Skyrme forces, leads to the best description of the cross sections among the Hartree–Fock functionals. However, the best result is obtained by using the densities $\rho_{ch}(r)$ calculated in the quasiparticle Lagrangian method. This can be seen in Fig. 13, which shows the cross sections for elastic scattering of electrons corresponding to the charge densities in the various self-consistent methods.

The accuracy of the description of the cross sections was characterized quantitatively in Ref. 33 calculating

$$\chi_0^2 = \frac{1}{N} \sum_{i=1}^{N} (\sigma_i^{th} - \sigma_i^{exp})^2 / (\sigma_i^{exp})^2. \tag{51}$$

This quantity determines the mean relative theoretical error. Calculations were made for five nuclei (^{40}Ca, ^{58}Ni, ^{116,124}Sn, ^{208}Pb) for which there are high-precision experimental data. The analysis of χ_0^2 covered the data in the region $q \lesssim 2.5$ F^{-1}, where the experimental errors are smaller and scattering theory is more reliable. The results of this analysis are given in Table I. It can be seen that in the case of the Hartree–Fock calculation with SIII Skyrme forces the error is very great (about 60%). In the case of the Tondeur forces (T) the error is appreciably smaller (on the average about 7%). The calculation based on the quasiparticle Lagrangian method "beats" this variant too—the mean error is about 5%. Such accuracy in the description of the cross sections with the densities of the quasiparticle Lagrangian method is sufficient to investigate the question of a possible change of the proton charge radius in the nucleus. This question was analyzed in Ref. 36.

To estimate how a change in the proton charge radius influences the value of the cross section, one can use the Born approximation, in which

$$\frac{d\sigma}{d\Omega} \sim |\rho_{ch}(q)|^2 \simeq |f_p(q)\, \rho_p(q)|^2, \tag{52}$$

and the proton form factor $f_p(q)$, corresponding to the exponential distribution (17), is

$$f_p(q) = (1 + q^2 a_0^2)^{-2}. \tag{53}$$

In the expression (52), which is used only for an estimate, the neutron contribution is omitted. In the exact calculation it is, of course, taken into account.

If it is assumed that the nucleons swell in the nucleus, it is natural to make the substitution $a_0 \to a_0(1+\alpha)$. The parameter α characterizes the relative increase of the radius. Using (52) and (53) for the relative change of the cross section, we obtain

$$\delta\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right) = -\frac{8q^2a_0^2}{1+q^2a_0^2}\,\alpha. \tag{54}$$

The coefficient in front of $-\alpha$ is 0.4 for $q = 1$ F^{-1}, 1.4 for $q = 2$ F^{-1}, and 2.6 for $q = 3$ F^{-1}. In the region $q \lesssim 3$ F^{-1} there are experimental data on the cross sections with errors

TABLE I. Values of χ_0^2 (electron scattering), %, for different forms of self-consistent calculation.

Nucleus	HF–SIII	HF–T	QLM
^{40}Ca	49.6	9.5	13
^{58}Ni	—	7.0	2.3
^{116}Sn	—	5.3	4.2
^{124}Sn	—	8.7	2.7
^{208}Pb	67.1	5.9	5.5
Average of χ_0^2 over the nuclei	—	7.3	5.5

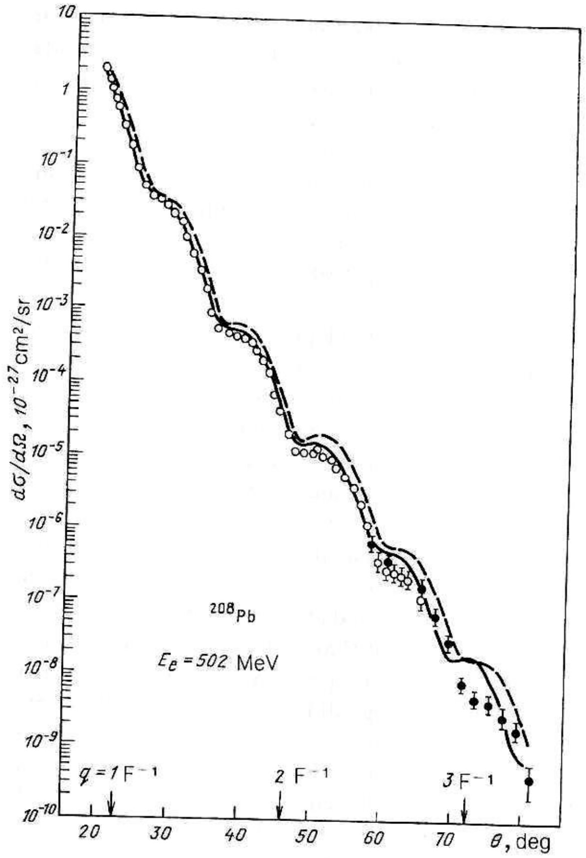

FIG. 14. Sensitivity of the cross section for elastic scattering of electrons by ^{208}Pb ($E_e = 502$ MeV) to a change in the proton charge radius. The continuous and broken curves correspond to $\alpha = 0.2$ and 0, respectively.

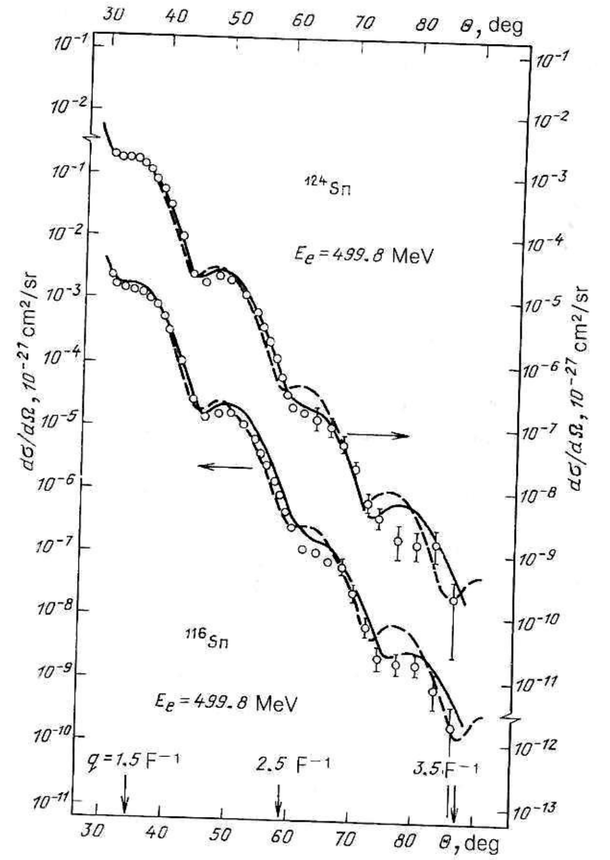

FIG. 15. The same as in Fig. 14 for ^{116}Sn and ^{124}Sn at $E_e = 500$ MeV.

less than 5% for the nuclei listed above. In the ideal case of validity of the Born approximation and an exact nuclear theory for the nucleon densities one could rely on determination of the value of α from (54) with an error not worse than 2–3%. As we shall see below, this estimate is basically confirmed by an exact calculation in the framework of the phase-shift analysis for the Dirac equation. The results of such a calculation[36] for ^{116}Sn and ^{208}Pb for different values of the swelling parameter α are given in Figs. 14 and 15. The effect of a change of the proton charge radius is evident. Quantitatively, it can be characterized by the quantity χ_0^2 defined in (51). The results for each of the five considered nuclei are given in Table II, from which it can be seen that the value of χ_0^2 has a minimum at $\alpha = -10\%$ for Ca and Ni, at $\alpha = 10\%$ for the two Sn isotopes, and at $\alpha = 0$ in the case of Pb. Averaging over all these nuclei gives $\alpha = (-3 \pm 12)\%$.[36] Thus, the analysis of the cross sections for elastic scattering of electrons gives evidence against significant swelling of the proton in the nuclear medium. An increase of the proton radius less than 10% is not ruled out. If one makes the natural assumption that neutrons and protons placed in the nuclear medium behave similarly, the same restriction can be applied to the neutron. A direct verification of the swelling hypothesis simultaneously for protons and neutrons can be made by analyzing the cross sections for elastic scattering of protons of the intermediate energies by nuclei. This problem is discussed in the following section.

TABLE II. Values of χ_0^2 (electron scattering) for different values of the swelling parameter α.

Nucleus	α, %			
	−10	0	10	20
^{40}Ca	0.103	0.130	0.168	0.214
^{58}Ni	0.010	0.023	0.055	0.100
^{116}Sn	0.135	0.042	0.007	0.015
^{124}Sn	0.103	0.027	0.003	0.017
^{208}Pb	0.024	0.021	0.038	0.070

4. ELASTIC SCATTERING OF PROTONS OF INTERMEDIATE ENERGIES

If the radius of a bound nucleon were to increase in a medium, this should lead not only to softening of its form factor but also to an increase of its cross section for interaction with an incident hadron. Among the various hadron–nucleus reactions that include such a cross section, the elastic scattering of protons with energy around 1 GeV by nuclei occupies a special position.[37] This region has been actively studied by theoreticians and experimentalists. There are here numerous measurements with high accuracy of the differential cross sections for elastic scattering in a fairly wide range of momentum transfers, $q \lesssim 3.5\ \mathrm{F}^{-1}$. On the other hand, this is the region of applicability of Glauber's diffraction theory of multiple scattering.[38] In this theory the nuclear structure is represented by single-, two-, three-, etc., many-particle densities of the ground state, of which the single-particle densities make the main contribution. As we have already said, these can be calculated with high accuracy in the framework of the quasiparticle Lagrangian method. This is indicated, in particular, by the good description of the inelastic scattering of high-energy electrons by nuclei achieved using the results of the quasiparticle Lagrangian method. As was demonstrated in the previous section, the characteristic discrepancies between the theory and experiment do not exceed 10–20%.

Of course, the electron scattering is determined practically exclusively by the proton distribution. However, it is natural to assume that the neutron distribution can also be calculated at approximately the same level of accuracy.

The accuracy of the Glauber approximation

There have been numerous attempts to justify the Glauber approximation on the basis of Watson's theory of multiple scattering. The fullest study was made in Ref. 39, in which corrections of three sorts were taken into account: deviation from eikonal propagation between two successive scatterings of the incident particle in the nucleus, the Fermi motion of the target nucleons, and the kinematic corrections associated with the transition from the many-particle scattering operator to the physical two-particle amplitudes. For the example of ^{4}He it was shown in Ref. 39 that there is an appreciable cancellation between the contributions of these corrections. In this sense it is better to use the standard Glauber approximation than to use it with one of those three corrections but without the others.

We note also that, as was shown in Refs. 40 and 41, the total effect of the correlation corrections is also small, since there is a mutual compensation of the contributions from the short-range correlations (repulsion of nucleons at short distances) and long-range correlations (virtual excitation of collective states).

Despite all this work, the program of complete and systematic allowance for the various corrections to the Glauber approximation is still not yet complete. In this situation, one must appeal to the experience of practical calculations. At the end of the seventies several calculations of the cross sections for elastic proton–nucleus scattering were made using microscopic nuclear densities.[42–44] The accuracy of the description of the data was at the level 20–30% at small momentum transfers q; however, with increasing q the discrepancy between the theory and experiment increased, reaching a factor 2–3. It is obvious that such a level of accuracy is not sufficient to analyze delicate effects such as swelling. However, it is noteworthy that all the features of the description of the proton–nucleus cross sections (including the nature of the discrepancies) repeated those in electron scattering, for which the theory is fairly reliable. This fact suggested that the main source of error in the description of the proton cross sections came from the defects of the densities that were used, and not from Glauber theory.

The existence today of much more accurate microscopic nuclear densities makes the problem of testing Glauber theory at the new level of accuracy topical. We have made such tests for the example of elastic scattering of protons by the nuclei ^{40,42,44,48}Ca, ^{48}Ti, and ^{208}Pb (Ref. 45) (Figs. 16 and 17) at energy 1 GeV and by the nuclei ^{58}Ni, ^{90}Zr, and ^{114,216}Sn at 0.8 GeV (Fig. 18). The details of the calculations are given in Refs. 43 and 44. As can be seen from the figures,

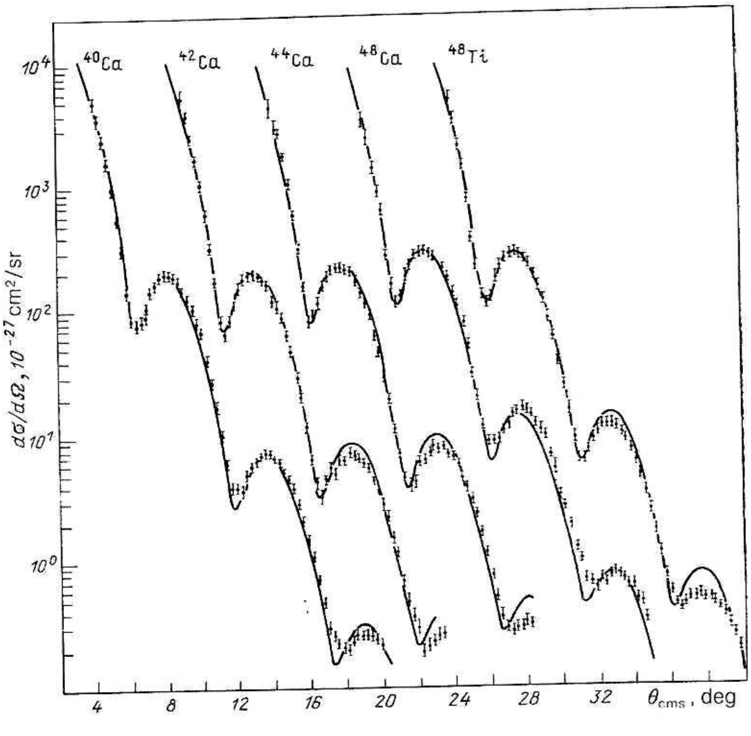

FIG. 16. Cross sections for elastic scattering of protons (E_p = 1.04 GeV)[46] by ^{40,42,44,48}Ca and ^{48}Ti (shifted relative to each other by 5°), calculated with the densities of the quasiparticle Lagrangian method.[15,31]

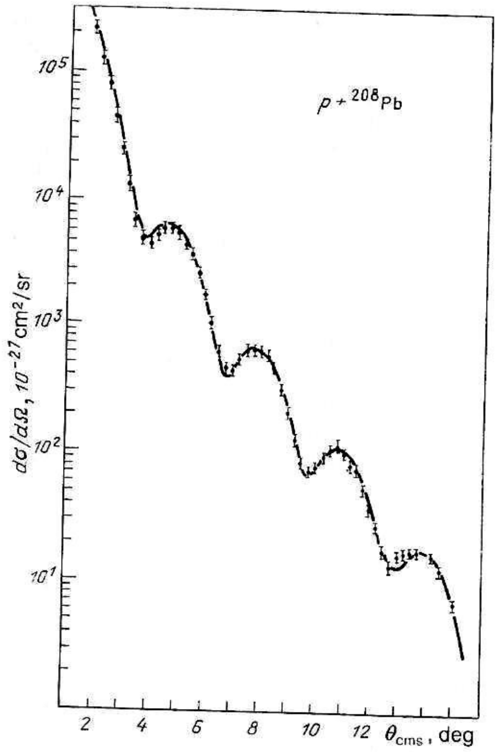

FIG. 17. Cross section for elastic scattering of protons (E_p = 1.0 GeV)[47] by ^{208}Pb, calculated with the densities of the quasiparticle Lagrangian method.[15,31]

the deviations from experiment are very small; they are of the same kind and on the same scale as in electron scattering. Such high accuracy of the Glauber approximation is somewhat remarkable and requires new theoretical examination. But if it is accepted an an "experimental fact," then there is hope of obtaining new information about the scale of the putative nucleon swelling in the nucleus from an analysis of the differential cross sections for elastic proton–nucleus scattering at an energy near 1 GeV.

Analysis of the manifestation of swelling in proton scattering

In the Glauber method the amplitude of proton–nucleus scattering can be expressed in terms of the amplitude of the free *NN* interaction, which is usually parametrized in the form[43,44]

$$f_{n,p}(q) = \frac{ik\sigma_{p,n}}{4\pi}(1 - i\gamma_{p,n})\exp\left(-\frac{\beta^2 q^2}{2}\right), \quad (55)$$

where at energy 1 GeV

$$\sigma_p = 4.75\ \text{F}^2, \quad \gamma_p = -0.05, \quad \beta^2 = 0.21\ \text{F}^2;$$
$$\sigma_n = 4.04\ \text{F}^2, \quad \gamma_n = -0.5;$$

and at energy 0.8 GeV (Ref. 49)

$$\sigma_p = 4.73\ \text{F}^2, \quad \gamma_p = 0.056, \quad \beta^2 = 0.20\ \text{F}^2;$$
$$\sigma_n = 3.79\ \text{F}^2, \quad \gamma_n = -0.48.$$

In (55), k is the momentum of the incident proton, q is the momentum transfer, σ_p and σ_n are the total cross sections for *pp* and *pn* scattering, and β^2 is the slope of the diffraction peak.

If it is assumed that a nucleon bound in a nucleus swells, i.e., increases its radius, then it is natural to assume that the parameters σ and β change:

$$\sigma \to (1 + \alpha)\,\sigma; \ \beta \to (1 + \alpha/2)\,\beta. \quad (56)$$

It is obvious that the change of the nucleon radius in the medium must depend on the density, and therefore on the coordinate r, i.e., $\alpha = \alpha(r)$. With allowance for the small expected role of this dependence, we shall assume that

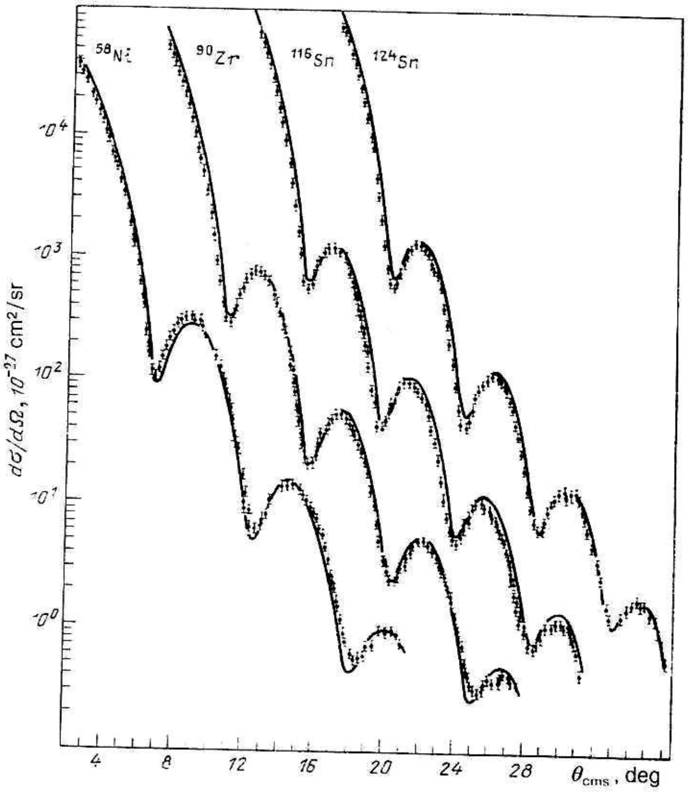

FIG. 18. Cross sections for elastic scattering of protons (E_p = 0.8 GeV)[48] by ^{58}Ni, ^{90}Zr, ^{116,124}Sn (shifted relative to each other by 5°), calculated with the densities of the quasiparticle Lagrangian model.[15,31]

$\alpha(r) = (\alpha_0/\rho_0)\rho(r)$, where ρ_0 is the density of nuclear matter. This assumption does not depend on the specific mechanism of the swelling effect, provided it is not associated with some phase transition of the QCD vacuum at normal nuclear density. Such a phase transition could lead to a nonanalytic dependence of α on ρ. To estimate the effect, we temporarily ignore the coordinate dependence, using the parameter $\alpha_{\rm eff} \simeq \alpha_0/2$, since it is basically the effective density $\rho_{\rm eff} \simeq \rho_0/2$ that works in Glauber scattering.

Note that the connection between the relative change of the cross section, α, and the relative change of the nucleon radius, α_N $[r_N \to (1+\alpha_N)r_N]$, is not completely unambiguous. If it is assumed that a nucleon that enters a nucleus swells under the influence of the nucleon environment in the same way as a target nucleon, then one must assume that $\alpha \simeq 2\alpha_N$. But if it does not change its properties, then $\alpha \simeq \alpha_N$. Which of these possibilities occurs depends on the ratio of the characteristic times of swelling and of passage of the proton through the nucleus. The attempt to obtain an estimate of the swelling time must necessarily involve a particular model of the phenomenon. As yet, such models do not have a sufficient degree of reliability. We shall not attempt to clarify this question, but assume that $\alpha_N \simeq \alpha$. From the point of view of obtaining a stringent restriction on α_N this is the least favorable case.

For the example of ^{40}Ca, Fig. 19 demonstrates the sensitivity of the cross section for proton–nucleus scattering to changes in σ and β for three values of the parameter $\alpha_{\rm eff}$: $\alpha_{\rm eff} = 0, \alpha_{\rm eff} = 0.1$, and $\alpha_{\rm eff} = 0.2$. Variations of σ and β separately and in combination were considered. The realistic situation corresponds to a simultaneous change of these quantities. As was to be expected, the sensitivity to variations of σ is appreciably greater than to variations of β. Why this is so can be readily seen in the example of the impulse approximation. In this case the amplitude for proton–nucleus scattering is proportional to the NN amplitude and, therefore, to σ. The variations of σ are proportional to the variations of the proton–nucleus amplitude, whereas the variation of the slope β^2 influences only the correction term $[\exp(-\beta^2q^2/2) \simeq 1 - \beta^2q^2/2]$, which is important only at large q^2. If we go beyond the impulse approximation, the situation is not changed qualitatively.

The high sensitivity of the results to variations of σ makes proton–nucleus scattering in some respect, a more suitable tool for studying the swelling effect than electron scattering. In the latter case the cross section is determined by the proton form factor, which has the same structure as the q^2-dependent factor in the NN amplitude. Therefore, the sensitivity of the cross sections for electron scattering to a change of the proton radius in the nucleus is approximately the same as the sensitivity of the cross sections for proton scattering to variations of β.

It can be seen from Fig. 19 that swelling of the nucleon leads to a significantly poorer description of the cross section. For a quantitative characterization of the degree of agreement with experiment, we shall use

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}(\sigma_i^{\rm th} - \sigma_i^{\rm exp})^2/(\Delta\sigma_i)^2, \quad (57)$$

where $\Delta\sigma_i$ is the error of the cross section $\sigma_i^{\rm exp}$ at the point i. In contrast to χ_0^2 (51), each point here enters with weight $1/(\Delta\sigma_i)^2$ determined by the error $\Delta\sigma_i$. In the case when the error at all points is the same, χ^2 and χ_0^2 are essentially equal.

Cases with energy of the incident protons equal to 800

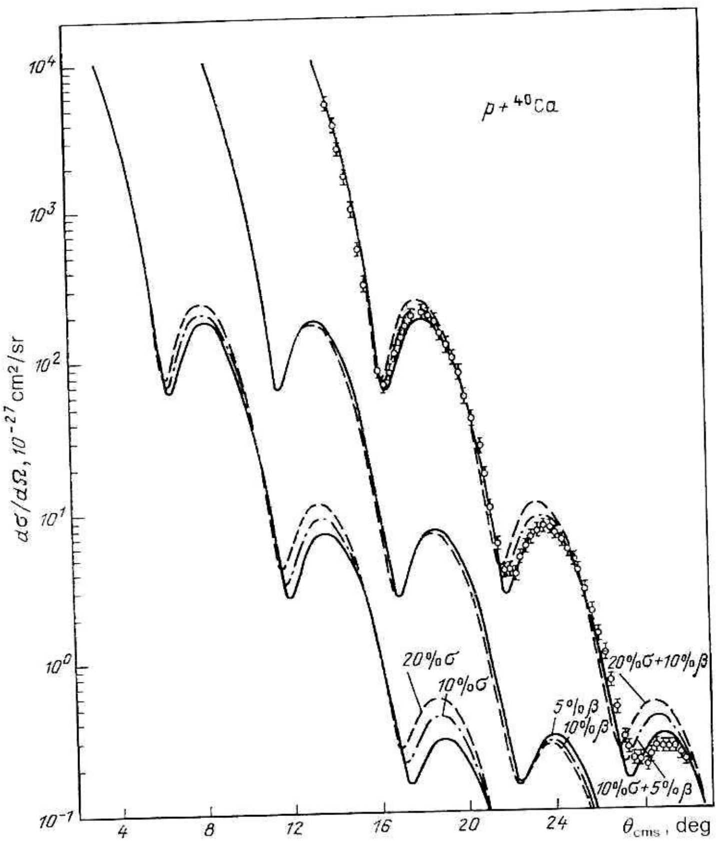

FIG. 19. Sensitivity of the cross section for elastic scattering of protons by ^{40}Ca(E_p = 1.04 GeV)[46] to the value of the effective swelling parameter $\alpha_{\rm eff}$. The continuous curves are for $\alpha_{\rm eff} = 0$, the broken curves for $\alpha_{\rm eff} = 0.1$, and the chain curves for $\alpha_{\rm eff} = 0.2$. On the left-hand curves only σ is varied, in the central ones only β^2, and on the right-hand ones both σ and β^2.

TABLE III. Values of χ^2 (proton scattering) as functions of the effective swelling parameter α_{eff}.

Nucleus	α_{eff}, %						α_{eff}^{min}
	−10	−5	0	5	10	20	
^{40}Ca	15.1	9.0	7.3	9.8	17.3	49.2	−0.6
^{42}Ca	5.4	4.7	7.8	14.5	25.5	62.9	−6.6
^{44}Ca	5.8	5.3	11.3	23.9	44.1	111	−7.1
^{48}Ca	19.4	13.4	10.2	9.8	12.8	31.0	1.9
^{48}Ti	6.1	7.5	15.7	30.7	53.2	132	−8.5
^{208}Pb	29.0	26.4	31.5	44.3	64.6	126	−5.8

MeV are not included in the χ^2 analysis. The reason for this is that at this energy the parameters of the amplitude (55) are not so well known as at energy 1 GeV (Ref. 48). For all the remaining cases the values of χ^2 are given in Table III for variations of the parameter α_{eff} from − 10 to 20%. In the final column we give the values of α_{eff}^{min} corresponding to the minimum of χ^2. For ^{40}Ca, $\alpha_{eff}^{min} \simeq 0$; for ^{48}Ca, $\alpha_{eff}^{min} \simeq 2\%$; and in the remaining cases $\alpha_{eff}^{min} < 0$. Thus, on the average our analysis suggests a decrease of the nucleon radius in the nucleus rather than an increase. We note that such a possibility was discussed in Refs. 50 and 51.

We now turn to the explicit allowance for the density dependence of α. Using the fact that the sensitivity of the cross sections to variations of σ is much stronger than to variations of β, we shall take into account the density dependence only in σ in the form $\delta\sigma/\sigma = \alpha_0 \rho(r)/\rho_0$. For β we substitute the effective value $\beta_{eff} = \beta(\rho_0/2)$, taken at half density.

The results of the calculations with variable $\alpha(r)$ are given in Table IV. Comparison with Table III shows that the influence of swelling on the cross section for elastic scattering of protons is indeed weakened by the density dependence of the parameter α. It can also be seen from the comparison that in the majority of cases the effective density is close to $\rho_{eff} = \rho_0/2$. Except for ^{48}Ca and ^{40}Ca, the values of α_0^{min} corresponding to the minimal value of χ^2 are negative. Averaging over all the considered nuclei gives $\alpha_0^{min} = (-3.6 \pm 9.8)\%$ (the rms error is given).

Of course, it must be borne in mind that the Glauber approximation and the nuclear densities used in the calculations have their own uncertainties, and it is rather difficult to estimate them. In view of this, the value given for α_0^{min} should not be taken too literally. Rather, it should be interpreted simply as evidence against nucleon swelling in the nucleus. Strictly speaking, we have here a restriction on the change in the nucleus of the total cross section for NN interaction compared with the free cross section. But if we adopt the swelling hypothesis, then we can speak of a restriction $\alpha_N = \delta r_N / r_N \lesssim 6\%$ on the possible increase of the effective nucleon radius in the nucleus.

5. TOTAL HADRON–NUCLEUS CROSS SECTIONS AT HIGH ENERGIES

In Ref. 52 an analysis was made of numerous data on the absorption cross sections and total cross sections of high-energy hadrons (p, $\bar{p}$, π, K; $E \simeq 10$–300 GeV) on the nuclei C, Al, Cu, and Pb with a view to finding effects due to a possible swelling of the nucleon. These cross sections were described by Glauber theory with corrections for inelastic screening, which are important at high energies.

The total cross section for scattering of an incident hadron by a nuclear nucleon, σ_{hN}, which occurs in the analysis was chosen to give the best description of the experiments. If there were swelling of the nucleon in nuclear matter, this would make the cross section σ_{hN} larger than the free cross section. As a result of the analysis in Ref. 52 it was concluded that there is an increase of σ_{hN} on the average by 5–15%.

6. NOLEN–SCHIFFER ANOMALY IN THE MASS DIFFERENCE OF MIRROR NUCLEI

In Ref. 53 an attempt was made to explain the so-called Nolen–Schiffer anomaly[54] by the hypothesis of nucleon swelling in the nucleus. The anomaly is associated with calculation of the mass difference ΔM of mirror nuclei, i.e., pairs of odd nuclei with the same A that differ by replacement of a neutron by a proton or of a proton by a neutron.

TABLE IV. Values of χ^2 (proton scattering) as functions of the swelling parameter α.

Nucleus	α, %								α_0^{min}
	−15	−10	−5	0	5	10	15	20	
^{40}Ca	14.3	11.1	8.8	7.3	6.7	7.3	9.0	11.2	5
^{42}Ca	4.4	4.6	5.7	7.8	10.8	14.8	19.9	26.2	−13.6
^{44}Ca	8.8	8.1	8.9	11.3	15.4	21.3	29.2	39.2	−10.2
^{48}Ca	22.5	17.7	13.6	10.2	7.6	6.0	5.4	6.1	14.0
^{48}Ti	10.3	9.7	11.5	15.7	22.6	32.6	46.0	63.1	−11.2
^{208}Pb	34.5	31.0	30 9	31.5	35.4	41.8	50.6	61.9	−5.5

The mass difference ΔM can be expressed as a sum of two terms,

$$\Delta M = \Delta E_{em} + \Delta m_{np}, \tag{58}$$

where ΔE_{em} is the difference of the electromagnetic energies of the nuclei, and $\Delta m_{np} = m_n - m_p$ is the mass difference of the neutron and proton within the nucleus. Usually, the problem reduces to calculation of ΔE_{em}, while the difference Δm_{np} is set equal to the mass difference of the free nucleons. The calculated contributions of the direct and exchange Coulomb potentials of the nucleus to ΔE_{em} lead, as a rule, to values of ΔM that are low compared with the experiments. It is this fact that is called the Nolen–Schiffer anomaly.

The results of numerous attempts to describe ΔE_{em}, including allowance for relativistic corrections and the effects of charge-exchange forces, are collected together in the review of Ref. 55. In Ref. 55 the direct Coulomb potential was calculated using the experimental charge densities, which are known from high-precision data on electron elastic scattering, so that the term that makes the main contribution to ΔE_{em} is calculated very reliably. The uncertainties are concentrated in the exchange Coulomb term, where there is a difficulty in taking into account consistently the nucleon correlations.

The measure of the anomaly is the difference

$$\Delta = \Delta M_{exp} - (\Delta E_{em} + \Delta m_{np})_{th}. \tag{59}$$

According to the calculations of Ref. 55, $\Delta = 0.21$ MeV at $A = 13$, and $\Delta = 0.62$ MeV at $A = 41$; the value of Δ increases rapidly with increasing A.

In Ref. 56 the first attempt was made to explain the anomaly at the quark level through a difference between Δm_{np} in the nucleus and the value for free nucleons. The estimate of this difference was based on the assumption that $6q$ bags are present in the nucleus.

Using this idea, the authors of Ref. 53 related the change of Δm_{np} in the nucleus to nucleon swelling. Estimates of Δm_{np} within the nucleus were made in various quark models of the nucleon. In particular, in the MIT model the following expression is obtained[53]:

$$\Delta m_{np}(R) = 0.42 \Delta m_{ud} f(R) + \frac{0.34}{R} \frac{e^2}{4\pi} g(R), \tag{60}$$

where R is the radius of the bag, $\Delta m_{ud} = -4.2$ MeV is the difference of the current masses of the u and d quarks, e is the electron charge, and $f(R)$ and $g(R)$ are known functions. The second term in (60) is due to the change of the electromagnetic interaction of the quarks of the nucleon when the u quark is replaced by the d quark. Under the assumption of an increase of the nucleon radius in the nucleus, the expression (60) leads to a contribution to the Nolen–Schiffer anomaly with the necessary sign. However, the contribution is too small. Thus, for $\delta R/R = 10\%$ we have $\delta(\Delta m_{np}) = 50$ keV, i.e., an order of magnitude less than what is required.

In models of the nucleon with constituent quarks in an oscillator potential a change of the radius results in a much more sensitive reponse. Thus, for the same $\delta R/R = 10\%$ we have $\delta(\Delta m_{np}) = 0.67$ MeV in the model of Ref. 57 and $\delta(\Delta m_{np}) = 0.3$ MeV in the model of Ref. 58.

If the expressions of Ref. 53 for Δm_{np} are used for the values of $\delta R/R$ that follow from the EMC effect, then for Δm_{np} (MeV) the following A dependence is obtained[59]:

$$\Delta m_{np} = (1.43 - 0.16\, A^{-1/3}). \tag{61}$$

However, such an A dependence does not in any way agree with the one that follows from the experimental data on the Nolen–Schiffer effect:

$$\Delta m_{np} = (1.3 + 0.0145\, A). \tag{62}$$

It should be noted that in the calculation of ΔE_{em} significant progress has been achieved in recent years through more correct allowance for the correlation corrections. For example, in Ref. 60 some correlation effects not hitherto considered were taken into account. Allowance for them significantly reduced the Nolen–Schiffer anomaly. Finally, in Ref. 61 a new method was used to calculate the correlation contribution, and it appears to have a high accuracy. As a result, the anomaly was practically eliminated without any assumptions about a change in the nucleus of the difference Δm_{np}.

In the light of this the additional contribution calculated in Ref. 53 to Δ associated with the swelling effect can be regarded as an argument against significant swelling of the nucleon in the nucleus.

7. CONCLUSIONS

In this review we have considered various phenomena of traditional nuclear physics in which there could be a manifestation of the swelling of a nucleon in the nucleus proposed as one of the possible explanations of the EMC effect. We have shown that the claim made in a number of studies of the detection of significant ($\sim 20\%$) swelling of the nucleons should be attributed rather to an inadequate description of the structural characteristics of nuclei, in the first place the distributions of the nuclear density. We have shown that a more accurate nuclear calculation makes it possible as a rule to describe the experimental data without introducing any swelling.

A situation that is not entirely clear has developed in the description of the longitudinal response in inclusive (e,e') scattering at intermediate energies, where the problem of taking into account the correlation corrections is as yet unresolved. The available estimates offer hope that a correct calculation of these contributions will lead to agreement with experiment without the need to invoke the swelling hypothesis in this process too. We note also that the attempt to reconcile the theory with experiment in the description of the longitudinal response by means of a medium-modified nucleon form factor leads to difficulties in the description of the transverse response function. At the same time, one cannot explain the 20% deviation from unity of the ratio of the longitudinal responses, integrated over the energy, for the isotopes ^{48}Ca and ^{40}Ca.

As we have shown, a very sensitive and effective tool for studying the swelling effect is the elastic scattering of fast electrons and protons by nuclei. Our analysis has made it possible to establish a rather stringent upper bound on the swelling. It follows from electron scattering that this bound is $\simeq 10\%$, and from proton scattering $\simeq 6\%$. An even more stringent restriction, $\delta r_N / r_N \lesssim 3\%$, was obtained from an analysis of y scaling in the (e,e') reaction at high energies for ^{56}Fe (Refs. 21). These bounds on the possible swelling agree with the estimates in Refs. 62 and 63. The restrictions obtained are important not only for nuclear physics but also for elementary-particle physics. They cast doubt on models of the nucleon that lead to a large swelling. Such models do not pass the test of nuclear physics.

Our study shows that traditional nuclear physics can, under conditions when there is a high-precision experiment and a sufficiently good theoretical "support," give important information for particle physics. This is particularly important in the present situation, in which the confinement problem in QCD has not been solved and various model ideas about nucleon structure have been developed.

Of course, the restrictions obtained cannot be mechanically transferred to all other forms of nuclear exotics. For example, the results of attempts to extract from the cross sections for elastic scattering of electrons by nuclei the admixture of the $6q$ configuration depends strongly on the form factor of this configuration. If it is the same as the nucleon form factor, the determination of the admixture in the given process is difficult. Another example is the impossibility of determining from electron elastic scattering the degree of softening of the pion mode in nuclei. This softening is usually interpreted as an increase of the effective number of pions in the nucleus. Since the numbers of π^+ and π^- mesons are increased by about the same amount, the charge distribution in the nucleus is hardly changed. It is therefore difficult to observe the softening of the pion degrees of freedom in electron elastic scattering. These examples show that in each particular case a special analysis is required to verify any particular exotic hypothesis.

We hope that this review demonstrates the importance of testing exotic hypotheses in different phenomena of traditional nuclear physics.

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