# Nonlinear optical waves in layered structures

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The current status of the problem of the propagation of nonlinear optical waves through dielectric and metallic layered structures is reviewed. We discuss the basic concepts and theoretical approaches used to analyze nonlinear surface waves (NSWs) and nonlinear guided-wave modes (NGWMs), the amplitudes of which are solutions of a particular class of nonlinear Schrödinger equation with coefficients depending on the transverse coordinate. The stability of NSWs and NGWMs is studied by numerical methods. We suggest some possible applications of NSWs and NGWMs to nonlinear integrated-optics devices.

## INTRODUCTION

The discovery of optical bistability in the semiconductors GaAs (Ref. 1) and InSb (Ref. 2) and their broad application in optical integrated processors for optical coupling and in optical computers has stimulated a considerable amount of theoretical and experimental activity during the last few years.[3,4] For the simple case of a plane wave, nonlinear optical devices such as bistable switches[1,2] and logic elements[5,6] have been built and put into operation. In general, planar optical waveguides ensure the optimal geometry for an effective nonlinear interaction and, in particular, for nonlinear optical processors.

The key concept on which all nonlinear guided-wave devices are based is that the local intensity of the guided waves is controlled by the propagation vector, i.e., the optical-field profile and the propagation constant can depend on the energy flux of the incident beam when one or several media in contact are characterized by an index of refraction which depends on the intensity of the incident beam. This optical phenomenon can be used as the basis for two types of integrated optical device.

The first type is devices in which the nonlinear variation in the index of refraction is insignificant in comparison with the difference of the indices of refraction of the media in contact. In this case the dependence of the wave propagation vector on the energy flux can be estimated using the coupled-mode theory,[7-9] while the distribution of the electromagnetic field of the coupled waves (the profile of the optical field) is approximated by linear guided-wave modes. Examples of devices which operate in this regime are nonlinear prism or grating coupling elements[10,11] and nonlinear coherent couplers.[12,13]

The second type corresponds to nonlinear optical devices in which the optically induced change of the index of refraction is comparable with or greater than the difference of the indices of refraction of the media in contact. In this case the wave propagation vector and the optical-field distribution depend on the energy flux of the incident beam. This dependence can be estimated using the more accurate approach of solving the nonlinear wave equation with the condition that the tangential electric and magnetic fields are continuous along all surfaces. Devices operating in this regime include nonlinear guided-wave optical limiters and also devices with low critical energy-flux threshold and optical switches.[14] Analytic solutions for optical fields propagating in media obeying the Kerr law were first found in Refs. 15–17. These studies laid the foundation for rapid progress in the study of nonlinear guided-wave modes (NGWMs) and nonlinear surface waves (NSWs).

The unique features of NGWMs and NSWs propagating in planar layered structures, namely, self-focusing and self-defocusing, have been investigated in detail in many studies.[18-39] A formalism was developed in Ref. 40 for arbitrary lossless optical nonlinearities (nonlinearities which are not of the Kerr type), and has been used in Refs. 41 and 42 to estimate numerically the dependence of the wave vector on the energy flux in various planar layered structures. The question of the stability of the propagation of various $TE_m$-nonlinear stationary wave solutions has been studied numerically in a number of articles.[43-50] In Ref. 51 the beam-propagation method was used to investigate the excitation of nonlinear $TE_0$-surface waves by means of a Gaussian light beam. It was shown that, in the case of a thin dielectric film sandwiched between two self-focusing media, it is possible to excite independently three different field distributions corresponding to the same level of energy flux of the wave using the corresponding Gaussian beams. The problem of multisoliton emission from a nonlinear waveguide was studied in Ref. 52. In that study it was shown numerically that, when the indices of refraction of the film and the substrate are linear and the index of refraction of the overlayer has nonlinear properties (the Kerr optical effect), the external excitation of NSWs can manifest a systematic threshold behavior, owing to the emission of multisolitons from the waveguide. This behavior is similar to that predicted for a nonlinear interface.[53,54] In Ref. 55 the beam-propagation method was used to study effects of nonlinear absorption of propagating $TE_0$ NSWs in an optical waveguide with a nonlinear overlayer obeying the Kerr law. Up to now there have been only a few experimental communications on NSWs.[56,57] The authors of those studies used a nonlinear self-focusing medium (the liquid crystal MBBA or $CS_2$) with a precipitated dielectric film. These experiments could be interpreted in


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terms of NSWs, the optical field distribution of which depends on the energy flux.

The aim of the present study is to illustrate as simply as possible, for several examples, the fundamental physical principles of NSW and NGWM phenomena in planar layered structures. The choice of material was dictated by the criterion of maximal simplicity, and all questions on a given topic not dealt with here are discussed in the recent review of Ref. 58. We carry out a detailed analysis of nonlinear TE-polarized waves propagating in planar layered structures, for which it is possible to obtain exact stationary solutions of the nonlinear wave equation.

This review is organized as follows. Section 1 is devoted to the study of surface electromagnetic waves guided by nonlinear interfaces. The basic concepts and the method of analyzing NSWs and NGWMs are discussed. In Sec. 2 we make a detailed study of nonlinear TE-polarized waves guided by thin dielectric films. In Sec. 3 we show that nonlinear TE-polarized waves can also be guided by very thin metal films (nonlinear surface plasmons). A brief summary of the results is given in the Conclusions.

## 1. NONLINEAR SURFACE ELECTROMAGNETIC WAVES GUIDED BY A SINGLE INTERFACE

### The dielectric tensor and the intensity-dependent index of refraction

The progress attained in recent years in the theory of nonlinear optics is related to the study of the interaction of an intense light field with a medium, which leads to the mixing of three optical fields. The nonlinear third-order term in the polarization vector in a nonlinear optical medium has the form

$$P_i^{NL}(\omega)=\varepsilon_0\chi^{(3)}_{ijkl}\,E_j(\omega)\,E_k^*(\omega)\,E_l(\omega), \qquad (1)$$

where $i = x, y, z$; $\chi^{(3)}$ is the third-order susceptibility, and $\mathbf{E}$ is the electric field vector. We note that one of the optical fields involved in the mixing must be a complex conjugate, so that the frequencies of the incoming and outgoing signals are the same, which is a fundamental requirement for all optical processors operating at a single frequency $\omega$.

If the optical field associated with the plane or guided waves is sufficiently large, it can change the index of refraction of the medium. For a plane wave propagating in an isotropic material, the Fourier component of a polarized field with frequency $\omega$ has the form

$$P_i(\omega)=\varepsilon_0[\chi^{(1)}_{ii}+3\chi^{(3)}_{\rm eff}\,|\,E_j(\omega)\,|^2]\,E_i(\omega), \qquad (2)$$

where $\chi^{(1)}_{ii}=n_0^2-1$ and $n_0$ is the linear part of the index of refraction. Expressing $|E_j(\omega)|^2$ in terms of the local intensity $I=\frac{1}{2}c\varepsilon_0 n_0|E_j(\omega)|^2$, the intensity-dependent index of refraction can be written as

$$n=n_0+n_{2I}I,\quad n_{2I}=\frac{3\chi^{(3)}_{\rm eff}}{c\varepsilon_0 n_0}, \qquad (3)$$

where the condition $n_{2I}>0$ corresponds to a self-focusing, and the condition $n_{2I}<0$ to a self-defocusing, nonlinearity of the Kerr type.

In the case of guided waves propagating along the $x$ axis with the normal to the surface in the $z$ direction, the electric field has the form

$$E_i=\frac{1}{2}E_i(z)\,\exp\,[{\rm i}(\beta k_0x-\omega t)]+{\rm c.c.} \qquad (4)$$

where $\beta$ is the effective index of refraction for guided waves and $k_0=\omega/c$ is the propagation constant in a vacuum. The nonlinear polarization vector in an isotropic medium is defined as[9,59]

$$P_i^{NL}(z)=c\varepsilon_0^2n_0^2n_{2I}\Big[\frac{2}{3}E_i(z)\sum_j E_j(z)E_j^*(z)+\frac{1}{3}E_i^*(z)\sum_j E_j(z)E_j(z)\Big]. \qquad (5)$$

For TE-polarized waves propagating along the $x$ axis the electric and magnetic field vectors have the form $\mathbf{E}=(0,E_y,0)$ and $\mathbf{H}=(H_x,0,H_z)$, so that the nonzero component of the polarization vector is

$$P_y^{NL}(z)=c\varepsilon_0^2n_0^2n_{2I}\,|E_y(z)|^2\,E_y(z). \qquad (6)$$

In this case the Maxwell equations involve only a single component of the nonlinear dielectric tensor $\varepsilon_{yy}$:

$$\varepsilon_{yy}=n_0^2+\alpha\,|\,E_y\,|^2,\quad \alpha=c\varepsilon_0n_0^2n_{2I}, \qquad (7)$$

and the Maxwell equations themselves have the form

$$\frac{dE_y}{dz}=-{\rm i}\omega\mu_0H_x,\quad \beta k_0E_y=\omega\mu_0H_z; \qquad (8)$$

$$\frac{dH_x}{dz}-{\rm i}\beta k_0H_z={\rm i}\omega\varepsilon_0\varepsilon_{yy}E_y. \qquad (9)$$

Equations (8) and (9) lead to the following nonlinear wave equation for the amplitude function $E_y(z)$:

$$\frac{d^2E_y}{dz^2}-k_0^2(\beta^2-n_0^2)\,E_y+\alpha k_0^2\,|\,E_y|^2\,E_y=0. \qquad (10)$$

In the case of real electric fields Eq. (10) has an analytic solution,[15-17] which will be discussed in detail below.

### Transverse-electric polarized NSWs

TE-polarized surface electromagnetic waves cannot exist at the interface between two dielectric media whose indices of refraction are independent of the intensity. However, if the index of refraction of one of the dielectric media depends on the energy of the incident beam, the existence of nonlinear surface waves becomes possible.[15,18,20,60] In this case, a self-focusing optical nonlinearity, which might not be small, generates optical waves of a new type which have no analog in the linear optics of surface modes (the critical energy level is reached before NSWs appear, so that the self-focusing channel is opened).

Let us consider the nonlinear surface between an optically linear semi-infinite dielectric medium (referred to as the substrate) characterized by the dielectric constant $\varepsilon_s$ in region I ($z<0$), and a semi-infinite nonlinear medium obeying the Kerr law (referred to as the overlayer) and characterized by the function $\varepsilon=\varepsilon_c+\alpha_c|\mathbf{E}|^2$ in region II ($z>0$). TE-polarized waves propagate along the $x$ axis with normal to the surface in the $z$ direction. The nonvanishing component of the electric field has the form

$$E_y(x,z,t)=\frac{1}{2}E_y(x,z)\,\exp\,[{\rm i}(\beta k_0x-\omega t)]+{\rm c.c.} \qquad (11)$$

and the nonlinear Maxwell equations for the guided-wave fields independent of $x$ (the stationary distribution of the field) can be written as

$$\frac{d^2E_y^{I}}{dz^2} - k_0^2q_s^2E_y^{I} = 0, \quad z < 0; \tag{12}$$

$$\frac{d^2E_y^{II}}{dz^2} - k_0^2q_c^2E_y^{II} + \alpha_c k_0^2 (E_y^{II})^3 = 0, \quad z > 0, \tag{13}$$

where $q_s^2 = \beta^2 - \varepsilon_s$, $q_c^2 = \beta^2 - \varepsilon_c$, and $\alpha_c = c\varepsilon_0 n_c^2 n_{2c}$.

For waves guided by a single interface characterized by the condition $E_y(z) \to 0$ for $|z| \to \infty$, i.e., for exponentially decreasing fields, the solution of Eqs. (12) and (13) is well known (Refs. 15–17,61)[1)]:

$$E_y^{I}(z) = E_0 \exp(k_0 q_s z), \; z < 0; \tag{14}$$

$$E_y^{II}(z) = \left(\frac{2}{\alpha_c}\right)^{1/2} q_c \{\mathrm{ch}[k_0 q_c (z - z_c)]\}^{-1}, \quad z > 0, \tag{15}$$

where $q_s = (\beta^2 - \varepsilon_s)^{1/2}$, $q_c = (\beta^2 - \varepsilon_c)^{1/2}$, and $\alpha_c > 0$ (a self-focusing nonlinearity).

For TE-polarized waves the field $E_y$ and its derivative $dE_y/dz$ are continuous functions along the interface $z = 0$ between the nonlinear and linear media. This leads directly to the eigenvalue equation

$$\varepsilon_s = \varepsilon_c + \frac{1}{2}\alpha_c E_0^2, \tag{16}$$

where $E_0$ is the value of the field at the surface. It follows from (16) that the amplitude of the field is fixed on the boundary, because $\varepsilon_s$ and $\varepsilon_c$ are constants, and in the limit $\alpha_c \to 0$ we must have $E_0 \to +\infty$, i.e., TE-polarized surface electromagnetic waves do not exist on the interface in the linear limit.

The energy flux of guided surface waves is defined in terms of the Poynting vector

$$P = \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*)_x \, dz = \frac{\beta}{2c\mu_0}\int_{-\infty}^{\infty} E_y^2(z)\, dz. \tag{17}$$

This expression can be estimated analytically for a medium obeying the Kerr law (see Refs. 15 and 18, for example):

$$P(\beta) = P_0\beta\left[\frac{(\varepsilon_s - \varepsilon_c)}{q_s} + 2(q_s + q_c)\right], \tag{18}$$

where $P_0 = (\varepsilon_0/\mu_0)^{1/2}(2\alpha_c k_0)^{-1}$. Expression (18) can be viewed as a nonlinear dispersion relation $\omega = \omega(k,P)$, i.e., the frequency $\omega$ and wave vector $k$ are related to each other at a given level of the energy flux.

The attenuation factor of NSWs and NGWMs can be estimated using the imaginary component of the dielectric constant, if it is assumed that the optical-field distribution corresponding to the lossless case is valid in the case of a small energy loss per unit wavelength.[60,62,63]

A formalism for dealing with an arbitrary lossless local nonlinearity has been developed in Ref. 40. For this technique it is not necessary to know the analytic solutions of the nonlinear wave equation in order to estimate the dependence of the wave vector on the energy flux.

It is well known that the form of the dielectric function is determined by the physical processes which give rise to the nonlinearity. The Kerr nonlinearity, which is a quadratic function of the local optical field, $\varepsilon^{NL}\alpha|\mathbf{E}|^2$, arises from nonlinearity of the electronics thermal effects, and so on. In semiconductors, where nonlinearity due to absorption leads to the formation of excitons, plasmons, and so on, the electric-field dependence of the dielectric function is not quadratic. In this case the optical nonlinearity has the form $\varepsilon^{NL} = \alpha_r|\mathbf{E}|^r$, where $1 < r < 2$ (Refs. 64–67). Moreover, in real materials it is impossible to vary optically the index of refraction up to infinity, owing to the saturation effect. The limits of the change of the index of refraction upon saturation $\Delta n_{sat}$ vary from $10^{-1}$ to $10^{-4}$. The saturation effect is important for a nonlinear interface, since the interesting, energy-flux-dependent properties of NSWs arise when the optically induced change of the index of refraction $\Delta n_{sat}$ is comparable with or larger than the difference of the indices of refraction of the substrate $n_s$ and the overlayer $n_c$ existing at small intensities of the energy flux.

We shall model the dielectric function of a nonlinear self-focusing ($\alpha_c > 0$) overlayer as follows[41,42,60,68]:

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_c + \varepsilon_{sat}\left[1 - \exp\left(-\frac{\alpha_c E_y^2}{\varepsilon_{sat}}\right)\right]; \tag{19}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_c + \frac{\alpha_c E_y^2}{\left(1 + \frac{\alpha_c E_y^2}{\varepsilon_{sat}}\right)}; \tag{20}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_c + \alpha_{c,r} E_y^r, \quad r \geqslant 1. \tag{21}$$

We note that for the two dielectric tensors in (19) and (20) the maximum change of the dielectric function is $\varepsilon_{sat}$, since for $|\mathbf{E}|^2 \to \infty$ we have $\varepsilon \to \varepsilon_c + \varepsilon_{sat}$. For small fields $\varepsilon \to \varepsilon_c + \alpha_c E_y^2$, i.e., we have the case of an ordinary medium obeying the Kerr law. The dielectric tensors (19)–(21) can be written in the general form

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_c + \varepsilon_c^{NL}(E_y^2). \tag{22}$$

The nonlinear wave equation for the real quantity (in the absence of losses) $E_y(z)$ has the form

$$\frac{d^2E_y}{dz^2} + k_0^2[\varepsilon_c + \varepsilon_c^{NL}(E_y^2) - \beta^2]E_y = 0. \tag{23}$$

For coupled surface waves, which are characterized by $E_y(z) \to 0$ for $|z| \to \infty$, the first integral of (23) can be written as

$$\left(\frac{dE_y}{dz}\right)^2 = \Phi_c(E_y, \beta) = k_0^2\left[q_c^2E_y^2 - \int_0^{E_y^2} \varepsilon_c^{NL}(E_y^2)\, d(E_y^2)\right]. \tag{24}$$

The continuity of $E_y$ and $dE_y/dz$ along a nonlinear boundary $z = 0$ between a linear substrate and a nonlinear overlayer leads to the dispersion relation

$$q_s + \bar{q}_c = 0. \tag{25}$$

Here

$$\bar{q}_c = (-1)^{M_c}(\beta^2 - \varepsilon_{cNL})^{1/2}; \tag{26}$$

$$\varepsilon_{cNL} = \varepsilon_c + \frac{1}{E_0^2}\int_0^{E_0^2} \varepsilon_c^{NL}(E_y^2)\, d(E_y^2), \tag{27}$$

where $E_0$ is the field at the surface; $\varepsilon_{cNL}$ is the averaged dielectric function of the nonlinear medium; $M_c = 1$ if the self-focusing peak occurs in the nonlinear overlayer and $M_c = 0$ if the field does not have a maximum in the nonlinear medium. In our case of a self-focusing nonlinear overlayer ($\alpha_c > 0$), the maximum of the self-focusing field occurs in the medium ($M_c = 1$), and from the dispersion relation

$$\varepsilon_s = \varepsilon_c + \frac{1}{E_0^2}\int_0^{E_y^2} \varepsilon_c^{NL}(E_y^2)\, d(E_y^2) \tag{28}$$

we can find the field at the surface, $E_0$. For an overlayer obeying the Kerr law, Eq. (28) leads to Eq. (16). We obtain an important result from (28): TE-polarized nonlinear surface waves can be supported by a nonlinear interface only when $\varepsilon_s > \varepsilon_c$.

The guided-wave energy flux parallel to the surface is $P = P_s + P_c$, where $P_c$ is given by[40]

$$P_c = \frac{\beta}{2\mu_0 c}\left[\int_0^{\overline{E}_y} \frac{E_y^2\,dE_y}{\Phi_c^{1/2}} + (-1)^{M_c}\int_{\overline{E}_y}^{E_0} \frac{E_y^2\,dE_y}{\Phi_c^{1/2}}\right]. \qquad (29)$$

Here $M_c = 1$, $\Phi_c^{1/2} = dE_y/dz$, and $\overline{E}_y$ is the maximum of the field obtained from the condition $\Phi_c(\overline{E}_y) = 0$. Finally, for the total flux $P = P_s + P_c$ we find

$$P_s = \frac{1}{2}\,P_0\beta\,\frac{u}{q_s}; \qquad (30)$$

$$P_c = \frac{1}{2}\,P_0\beta\left[\int_0^{\bar{u}} \frac{dx}{[\varphi(x)]^{1/2}} + (-1)^{M_c}\int_{\bar{u}}^{u} \frac{dx}{[\varphi(x)]^{1/2}}\right]. \qquad (31)$$

Here

$$\varphi(x) = \beta^2 - \varepsilon_c - \varepsilon_{sat} + \frac{\varepsilon_{sat}^2}{x}\left[1 - \exp\left(-\frac{x}{\varepsilon_{sat}}\right)\right]; \qquad (32)$$

$$\varphi(x) = \beta^2 - \varepsilon_c - \varepsilon_{sat} + \frac{\varepsilon_{sat}^2}{x}\ln\left(1 - \frac{x}{\varepsilon_{sat}}\right); \qquad (33)$$

$$\varphi(x) = \beta^2 - \varepsilon_c - \frac{2}{(r+2)}\,x^{r/2} \qquad (34)$$

correspond to the dielectric tensors (19)–(21), $u = \alpha_c E_0^2$ is obtained from (28), and $\bar{u}$ is found from the condition $\varphi(\bar{u}) = 0$.

For a medium obeying the Kerr law we have $\varphi(x) = \beta^2 - \varepsilon_c - \frac{1}{2}x$ and the integrals (31) can be estimated analytically:

$$P_c = 2P_0\beta\left[(\beta^2 - \varepsilon_c)^{1/2} + \left(\beta^2 - \varepsilon_c - \frac{1}{2}\,u\right)^{1/2}\right], \qquad (35)$$

where $u = \alpha_c E_0^2 = 2(\varepsilon_s - \varepsilon_c)$ [see (16)].

In the case of an overlayer with nonlinearity reaching saturation and characterized by the dielectric functions (19) and (20), the maximum optically induced change of the dielectric constant $\Delta\varepsilon_c$ occurs, i.e., $\varepsilon \to \varepsilon_c + \varepsilon_{sat}$, for large fields $|\mathbf{E}| \to \infty$. Therefore, as the flux increases the effective index of refraction $\beta$ asymptotically approaches its maximum value $(\varepsilon_c + \varepsilon_{sat})^{1/2}$. A necessary condition for the existence of a solution of Eq. (28) for unknown $E_0$ is that the inequality $\varepsilon_s < \varepsilon_c + \varepsilon_{sat}$ be satisfied. From the condition $\Phi_c(\overline{E}_y) = 0$ we obtain $\beta^2 < \varepsilon_c + \varepsilon_{sat}$, so that in the case of NSWs the range of allowed values of $\beta$ is $\varepsilon_s^{1/2} < \beta < (\varepsilon_c + \varepsilon_{sat})^{1/2}$.

For an overlayer depending on the energy flux and characterized by the dielectric function (21) we have

$$u = \left[\frac{(r+2)\,(\varepsilon_s - \varepsilon_c)}{2}\right]^{2/r}, \qquad (36)$$

$$\bar{u} = \left[\frac{(r+2)\,(\beta^2 - \varepsilon_c)}{2}\right]^{2/r}, \qquad (37)$$

while the energy fluxes $P_s$ and $P_c$ are given by (30) and (31), with $P_0$ replaced by the quantity $P_{0,r} = (\varepsilon_0/\mu_0)^{1/2}(2k_0\alpha_{c,r}^{2/r})^{-1}$.

In Fig. 1 we show the results of numerical calculations for TE-polarized surface waves for the following types of overlayer:

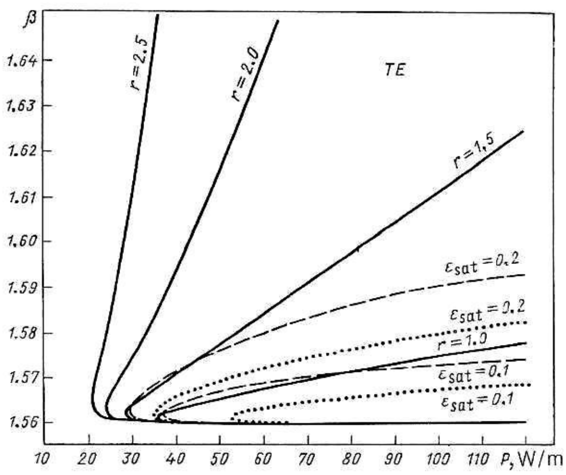


FIG. 1. Dependence of the effective index of refraction $\beta$ on the energy flux for TE waves guided by the interface between a self-focusing overlayer ($n_c = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W) and a linear substrate ($n_s = 1.56$). The dashed lines correspond to the dielectric function (19) and the dotted lines to the dielectric function (20) (Ref. 69).

a) obeying the Kerr law ($r = 2$);
b) not obeying the Kerr law ($r \neq 2$);
c) reaching saturation.

The dependence of the energy flux $P$ on the effective index of refraction $\beta$ was calculated for a nonlinear overlayer with $n_c = 1.55$ and $n_{2c} = 10^{-9}$ m$^2$/W (the liquid crystal MBBA), in contact with a linear substrate for which $n_s = 1.56$, at the light wavelength $\lambda = 0.515$ $\mu$m (an argon laser). The values of the nonlinear coefficients $\alpha_{c,r}$ were chosen so as to obtain similar minimum values of the energy flux: $\alpha_{c;1} = 4.7\times10^{-9}$ m/W, $\alpha_{c;1.5} = 1.75\times10^{-9}$ (m/W)$^{1.5}$, $\alpha_{c;2.5} = 2.3\times10^{-14}$ (m/W)$^{2.5}$ (see Ref. 69).

We see from Fig. 1 that the minimum power needed to excite TE-NSWs increases with decreasing $\varepsilon_{sat}$.

#### Stability of propagation of NSWs

The reflection of a plane wave from an interface between linear and nonlinear media was first studied explicitly in Refs. 17 and 70. After this pioneering work, theoretical[53,54,71] and experimental[72] studies of the interaction of Gaussian light beams with a nonlinear interface were carried out. The excitation of NSWs by means of Gaussian light beams was studied numerically in Ref. 43. The question of the stability of their propagation was crucial in the problem of NSW excitation by external sources. It was shown in Ref. 43 that both stable and unstable NSWs can be excited by means of Gaussian light beams from a linear medium incident on a nonlinear interface at grazing angles.

Let us consider a nonlinear interface between a linear substrate characterized by dielectric constant $\varepsilon_s$ in region I ($z < 0$) and a nonlinear overlayer obeying the Kerr law and characterized by dielectric function $\varepsilon = \varepsilon_c + \alpha_c|\mathbf{E}|^2$ in region II ($z > 0$). Let a polarized wave with nonzero electric field component $E_y$, uniform along the $y$ axis, propagate along the $x$ axis with frequency $\omega$. Then the weakly varying amplitude $A(x,z) = \alpha_c^{1/2}E_y(x,z)$ satisfies the parabolic equation

$$-2i\beta k_0\frac{\partial A}{\partial x} = \frac{\partial^2 A}{\partial z^2} - \gamma^2(z)\,k_0^2 A + \theta(z)\,k_0^2|A|^2 A. \qquad (38)$$

Here for $z<0$ we have $\theta(z)=0$ and $\gamma^2(z)=\beta^2-n_s^2$, while for $z>0$ we have $\theta(z)=1$ and $\gamma^2(z)=\beta^2-n_c^2$. We note that the usual stationary solution of Eq. (38), i.e., $A(0,z)=A_0(z)$, can be obtained analytically [see (14) and (15)].

Equation (38) has two integrals of motion

$$I(\beta)=k_0\int_{-\infty}^{\infty}|A|^2dz=(P_0\beta)^{-1}P(\beta); \quad (39)$$

$$H(\beta)=k_0\int_{-\infty}^{\infty}\left[\left|\frac{\partial A}{\partial z}\right|^2+k_0^2\gamma^2(z)|A|^2-\frac{1}{2}k_0^2\theta(z)|A|^4\right]dz \quad (40)$$

and for arbitrary solutions of Eq. (38) we then have $dI/dx=dH/dx=0$. Equation (38) is an equation of the mixed type, a linear/nonlinear Schrodinger equation with coefficients depending on the transverse coordinate $z$. The absence of translational symmetry along the $z$ axis means that we cannot use the elegant inverse-scattering method[73,74] to solve the problem analytically. To simplify the calculations we used the Crank–Nicolson difference scheme (see, for example, Refs. 75 and 76) with the following mesh sizes: $k_0\Delta x=k_0\Delta z=0.4$. The corresponding system of nonlinear equations was solved by the Newton–Picard method.[75] In this case two iterations proved to be sufficient for convergence. This scheme permitted the conservation of the integrals of motion (39) and (40) over the entire mesh. The conservation of the total energy flux was 99% in all cases. In the case of a nonlinear overlayer obeying the Kerr law and $\beta=1.5607$ the NSW is unstable in the vicinity of the branch with negative slope ($dI/d\beta<0$) (see Fig. 1). After further evolution it "extrudes" into the linear substrate (Fig. 2). The evolution of the NSWs for $\beta=1.574$ on the branch of positive slope ($dI/d\beta>0$) is shown in Fig. 3. For the chosen value of the propagation constant the nonlinear wave $A_0(z)$ is stable, at least at distances of order 300 wavelengths (Fig. 3).

In conclusion, we note that the authors of Ref. 77 have shown that self-focusing plane waves in an infinite medium are stable for $dI/d\beta>0$. Numerical results for TE-polarized

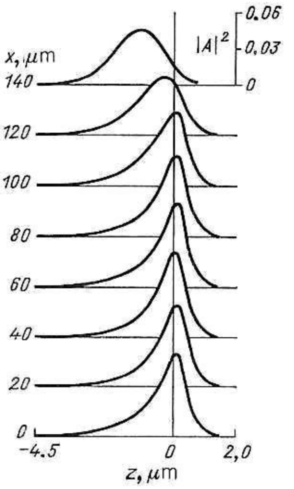


FIG. 2. Evolution of the field distribution as a function of the propagation distance $x$. Parameters of the calculation: $n_c=1.55$, $n_{2c}=10^{-9}$ m²/W, $n_s=1.56$, and the initial distribution of $A_0(z)$ corresponds to $\beta=1.5607$.

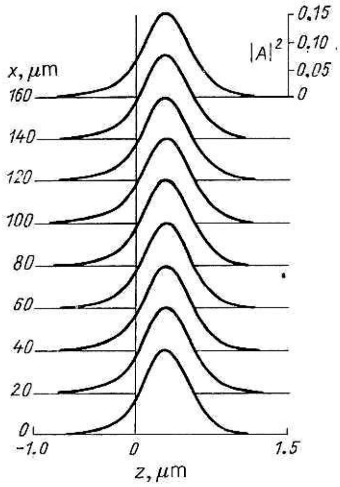


FIG. 3. The same as in Fig. 2 for $\beta=1.574$.

waves guided by a nonlinear interface were obtained for the same stability criterion, i.e., for $dI/d\beta>0$.

**Transverse-magnetic polarized NSWs**

Electromagnetic waves guided by the interface between two semi-infinite media or by a single or by several films surrounded by two semi-infinite media (see, for example, Refs. 78 and 79) are called surface polaritons. In all cases the falloff of the electromagnetic fields with increasing distance from the interface in a semi-infinite medium is exponential, so that the fields are localized near the surface. In the case of transverse-magnetic (TM) polarization the magnetic field vector is perpendicular to the plane of incidence, i.e., the plane defined by the direction of propagation and the normal to the surface.

Let us consider the simplest case of electromagnetic waves guided by the surface between two semi-infinite linear media. The dielectric constants $\varepsilon_c$ and $\varepsilon_s$ characterize the overlayer and the substrate, respectively. In the linear case the interface can support only TM-polarized surface polaritons, and this only for $\varepsilon_c>0$, $\varepsilon_s<0$, and $\varepsilon_c<|\varepsilon_s|$. The effective index of refraction $\beta=k/k_0$ is given by[79]

$$\beta^2=\frac{\varepsilon_c\varepsilon_s}{\varepsilon_s+\varepsilon_c}=\frac{|\varepsilon_s|\,\varepsilon_c}{(|\varepsilon_s|-\varepsilon_c)}. \quad (41)$$

Later we shall study effects caused by optical nonlinearities, which may be fairly large, for surface and guided electromagnetic waves. The presence of these nonlinearities leads to new types of waves which have no analog in the linear optics of surface and coupled waves. We note that the propagation of nonlinear TM-polarized surface waves in a plasma was first studied in Ref. 80. For TM polarization and media obeying the Kerr law there are two approximations commonly used in the literature:

a) the uniaxial $\varepsilon_{xx}(|E_x|^2)$ approximation, where the component of the dielectric tensor parallel to the surface, $\varepsilon_{xx}$, depends on the field component $E_x$ parallel to the surface[19];

b) the uniaxial $\varepsilon_{zz}(|E_z|^2)$ approximation, where the component of the dielectric tensor perpendicular to the surface, $\varepsilon_{zz}$, depends on the normal component of the field $E_z$ (Ref. 81).

The dispersion equation for TM-polarized nonlinear surface polaritons (NSPs) guided by the interface between a linear dielectric medium and a nonlinear dielectric medium obeying the Kerr law was first studied in detail in the uniaxial $\varepsilon_{xx}(|E_x|^2)$ approximation

$$\varepsilon_{xx} = \varepsilon_x + \alpha_{xx} \; |E_x|^2; \quad \varepsilon_{zz} = \varepsilon_z \tag{42}$$

in Ref. 19. Using this approximation, for the case $\varepsilon_x > 0$ and $\varepsilon_z > 0$ the authors of Ref. 60 obtained results for TM-polarized NSPs guided by the interface between quartz and the vacuum. The authors of Refs. 21 and 27 obtained exact dispersion relations for nonlinear TM-polarized waves propagating along the interface between two semi-infinite nonlinear uniaxial media characterized by diagonal dielectric tensors of the form (42). The effect of transition-layer oscillations on the spectrum of TM-polarized NSPs in the uniaxial $\varepsilon_{xx}(|E_x|^2)$ approximation was first discussed in Ref. 82 and in the uniaxial $\varepsilon_{zz}(|E_z|^2)$ approximation

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_x, \\ \varepsilon_{zz} &= \varepsilon_z + \alpha_{zz} \; |E_z|^2 \end{aligned} \tag{43}$$

in Ref. 83.

For TM waves propagating in a medium obeying the Kerr law, in the uniaxial $\varepsilon_{xx}(|E_x|^2)$ approximation the differential equation for the $E_x(z)$ component of the field has the form

$$\frac{d^2E_x}{dz^2} - k_0^2 q_\gamma^2 E_x - \frac{k_0^2 q_\gamma^2 \alpha_\gamma}{\varepsilon_\gamma} E_x^3 = 0, \tag{44}$$

where $\gamma = s, f, c$ denotes the substrate, film, and overlayer, respectively. This equation has an analytic solution. For example, if $\alpha_c < 0$ and $\alpha_s < 0$, we have

$$E_x(z) = \left(\frac{2\varepsilon_s}{|\alpha_s|}\right)^{1/2} \{\mathrm{ch}\,[k_0 q_s (z_s - z)]\}^{-1}, \quad z < 0; \tag{45}$$

$$E_x(z) = \left(\frac{2\varepsilon_c}{|\alpha_c|}\right)^{1/2} \{\mathrm{ch}\,[k_0 q_c (z_c + z)]\}^{-1}, \quad z > 0. \tag{46}$$

If $\alpha_c > 0$ and $\alpha_s > 0$, cosh is replaced by sinh. We note that in this case the sign of the last term in (44) is negative, while in the case of TE polarization it is positive. Because of this, the field distributions in the case of TE and TM polarizations for the $\varepsilon_{xx}(|E_x|^2)$ approximation are different.

An alternative method of solution is to eliminate the components $E_x(z)$ and $E_z(z)$ from the Maxwell equations and obtain the equation for the field $H_y(z)$. In the uniaxial $\varepsilon_{zz}(|E_z|^2)$ approximation we obtain

$$\frac{d^2H_y}{dz^2} - k_0^2 q_\gamma H_y + \frac{k_0^2\beta^4\alpha_\gamma}{c^2\varepsilon_0^2\varepsilon_{zz}^3} H_y^3 = 0. \tag{47}$$

This equation cannot be solved exactly, owing to the term $\varepsilon_{zz}^3$ in the denominator. We note that for many materials the value of $\Delta\varepsilon = |\alpha_\gamma E_z^2|$ is less than 0.01, while in exceptional cases, for example, InSb, it is of order 0.1. Use of the approximation $\varepsilon_{zz} \simeq \varepsilon_\gamma$ in the denominator of the third term of Eq. (47) leads to a small error when this term is small. In this limit the solutions for $H_y(z)$ have the same form as for the exactly solvable TE case with $\alpha_\gamma$ replaced by $\alpha_{\gamma'} = \beta^4(c_2 \varepsilon_0^2 \varepsilon_\gamma^3)^{-1}\alpha_\gamma$. Therefore, in the uniaxial $\varepsilon_{zz}(|E_z|^2)$ approximation the equivalence of the solutions leads to similar behavior of the flux for TE- and TM-polarized surface waves.

TM-polarized electromagnetic waves guided by an interface have been studied in detail in Ref. 63. Both approximations $\varepsilon_{xx}(|E_x|^2)$ and $\varepsilon_{zz}(|E_z|^2)$ were analyzed, and the propagation vector and attenuation factor were calculated as functions of the energy flux of the guided waves and the state of the materials. The NSW attenuation factor was calculated approximately using the imaginary components of the dielectric constants $\varepsilon_{sI}$ and $\varepsilon_{cI}$. Assuming that losses are small, it was shown in Ref. 63 that

$$\beta_I = \frac{1}{2\beta_R P}(\varepsilon_{sI} P_s + \varepsilon_{cI} P_c), \tag{48}$$

where $\beta_I$ and $\beta_R$ are, respectively, the imaginary and real parts of the effective index of refraction. A theory of nonlinear surface TM waves was developed in Ref. 84, but the analysis carried out there was limited to the case of an isotropic, nonlinear medium, whose dielectric constant contained two electric field components with equal weight. A numerical method of solving Maxwell's equations for TM waves propagating on a nonlinear interface was developed in Ref. 85. This method is applicable to arbitrary nonlinear dielectric tensors, which is achieved by transforming the infinite transverse plane into a finite interval and using asymptotic boundary conditions.

Below we obtain the exact dispersion relation for TM-polarized surface waves on the interface of a linear dielectric or metal and a nonlinear dielectric obeying the Kerr law.[39] This dispersion relation is an equation with many terms, including the values of the electric field components on the interface, the parameters of the medium, and the effective index of refraction $\beta$ for surface waves. We note that surface electromagnetic waves guided by the boundary between a nonlinear dielectric and a metal are of particular interest, since they correspond to NSWs with no energy threshold.

It is well known that TM-polarized waves have two electric field components, one of which ($E_x$) is parallel to the wave vector, while the other ($E_z$) is perpendicular to the surface. In order to determine the effects arising from an intensity-dependent index of refraction, we must first study a nonlinear polarization field. The electric field vector is given by

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2}[E_x(z)\, x + E_z(z)\, z] \; \exp\,[\mathrm{i}\,(\beta k_0 x - \omega t)] + \mathrm{c.c.} \tag{49}$$

where the phases of the components $E_x(z)$ and $E_z(z)$ differ by $\pi/2$, i.e., $|E_z|^2 = E_z^2$ and $|E_x|^2 = -E_x^2$. For the nonzero components of the nonlinear polarization vector we have[9]

$$P_x^{NL}(z) = \varepsilon_0 (\alpha_{xx} |E_x(z)|^2 + \alpha_{xz} |E_z(z)|^2) E_x(z); \tag{50}$$

$$P_z^{NL}(z) = \varepsilon_0 (\alpha_{zx} |E_x(z)|^2 + \alpha_{zz} |E_z|(z)|^2) E_z(z). \tag{51}$$

Then for the components of the dielectric tensor characterizing a medium obeying the Kerr law we obtain

$$\varepsilon_{xx} = \varepsilon_x + \alpha_{xx} \; |E_x|^2 + \alpha_{xz} \; |E_z|^2; \tag{52}$$

$$\varepsilon_{zz} = \varepsilon_z + \alpha_{zx} \; |E_x|^2 + \alpha_{zz} \; |E_z|^2, \tag{53}$$

where the quantities $\alpha_{ij}$ (Kerr optical nonlinearities) depend on the nonlinear mechanism studied below. For nonlinearities in the electronics, which can be obtained by series expansion of the polarization in terms of the field variables, we obtain $\alpha_{xx} = \alpha_{zz} = 3\alpha_{xz} = 3\alpha_{zx} = c\varepsilon_0 n_0^2 n_2 I$, while for nonlinearities related to electrostriction we find $\alpha_{xx} = \alpha_{zz} = \alpha_{xz} = \alpha_{zx} = c\varepsilon_0 n_0^2 n_{2I} I$, where $n = n_0 + n_{2I}$, where $n_0$ is the linear part of the index of refraction and $n_{2I}$ is the intensity-dependent coefficient of the index of refraction.

The electric components $E_x(z)$ and $E_z(z)$ for TM-polarized surface waves satisfy the equations

$$\frac{dE_x}{dz} = -i\frac{k_0}{\beta}(\varepsilon_{zz} - \beta^2)E_z; \quad (54)$$

$$\frac{d(E_z\varepsilon_{zz})}{dz} = -i\beta k_0 \varepsilon_{xx} E_x; \quad (55)$$

$$H_y = -\frac{c\varepsilon_0}{\beta}\varepsilon_{zz}E_z. \quad (56)$$

The key point in the analysis is the condition that for surface TM waves i.e., for the condition $E \to 0$ and $dE/dz \to 0$ when $z \to \pm\infty$, Eqs. (54) and (55) have the first integral

$$\frac{1}{2}\left(\frac{dE_x}{dz}\right)^2 + U(E_x, E_z) = 0, \quad (57)$$

where

$$U(E_x, E_z) = \frac{1}{2}k_0^2\varepsilon_x E_x^2 + \frac{1}{2}k_0^2(\beta^2 - \varepsilon_z)E_z^2 + \frac{1}{2}k_0^2\alpha_{xz}E_x^2E_z^2 - \frac{1}{4}k_0^2\alpha_{xx}(E_x^4 + E_z^4), \quad (58)$$

as was first shown in Ref. 86.

The solution of Maxwell's equations (54) and (55) in a semi-infinite linear substrate characterized by dielectric constant $\varepsilon_s$ (for a dielectric $\varepsilon_s > 0$); for a metal $\varepsilon_s < 0$) and filling the lower half-plane $z < 0$, can be written as

$$E_x(z) = E_{0x}\exp(k_0 q_s z), \ z < 0, \quad (59)$$

where $q_s = (\beta^2 - \varepsilon_s)^{1/2}$, $E_{0x} = E_x(0)$, and $\beta^2 > \varepsilon_s$, for a dielectric. For a linear medium Eq. (54) can be transformed to

$$E_z = \frac{i\beta}{k_0(\varepsilon_{zz} - \beta^2)}\frac{dE_x}{dz}, \quad (60)$$

which gives

$$D_z = \frac{i\beta\varepsilon_{zz}}{k_0(\varepsilon_{zz} - \beta^2)}\frac{dE_x}{dz}, \quad (61)$$

where the quantity $D_z$ is the $z$-component of the electric induction vector $\mathbf{D}$. Equation (61) also holds in a linear medium with $\varepsilon_{zz}$ replaced by $\varepsilon_s$. From the viewpoint of standard electrodynamics, the quantities $D_z$ and $E_x$ must be continuous functions along the interface $z = 0$. We define $E_{0z} = E_z(0)$ and $\varepsilon_{nl}$, the $z$ component of the dielectric tensor on the interface $z = 0$, depending on the value of the field at the interface:

$$\varepsilon_{nl} = \varepsilon_z - \alpha_{zx}E_{0x}^2 + \alpha_{zz}E_{0z}^2. \quad (62)$$

From the continuity of $D_z$ on the interface $z = 0$ we obtain the following relation between the boundary values of the field:

$$E_{0x} = \frac{iq_s}{\beta\varepsilon_s}(\varepsilon_z - \alpha_{zx}E_{0x}^2 + \alpha_{zz}E_{0z}^2)E_{0z}. \quad (63)$$

In the weak-field limit Eq. (63) reproduces the usual relation between the boundary values of the field. Using the first integral (57), we obtain the following eigenvalue equation for $\beta$ (Ref. 39):

$$\beta^4\left[2\varepsilon_s^4\varepsilon_{nl} - \varepsilon_x\varepsilon_s^2\varepsilon_{nl}^2 - \varepsilon_z\varepsilon_s^4 - \frac{1}{2}\alpha_{zz}E_{0z}(\varepsilon_s^4 + \varepsilon_{nl}^4) - \alpha_{xz}E_{0z}^2\varepsilon_s^2\varepsilon_{nl}^2\right] + \beta^2[\varepsilon_x\varepsilon_s^3\varepsilon_{nl}^2 + \alpha_{zz}E_{0z}^2\varepsilon_s\varepsilon_{nl}^4 + \alpha_{xz}E_{0z}^2\varepsilon_s^3\varepsilon_{nl}^2 - \varepsilon_s^4\varepsilon_{nl}^2] - \frac{1}{2}\alpha_{zz}E_{0z}^2\varepsilon_s^2\varepsilon_{nl}^4 = 0. \quad (64)$$

We note that the special case of an isotropic nonlinear overlayer, i.e., $\varepsilon_x = \varepsilon_z = \varepsilon_c$ and $\alpha_{xx} = \alpha_{zz} = \alpha_{xz} = \alpha_{zx} = \alpha_c$, has been studied in detail in Ref. 84, and the solution for this case has the simple form

$$\beta^2 = \frac{\varepsilon_s\varepsilon_{nl}^2(2\varepsilon_s - \varepsilon_{nl} - \varepsilon_c)}{\varepsilon_s^2(3\varepsilon_{nl} - \varepsilon_c) - \varepsilon_{nl}^2(\varepsilon_{nl} + \varepsilon_c)}, \quad (65)$$

where $\varepsilon_{nl} = \varepsilon_c + \alpha_c(E_{0z}^2 - E_{0x}^2)$ is the dielectric constant of the nonlinear overlayer obeying the Kerr law at the interface $z = 0$.

Introducing the parameters characterizing the material, from Eqs. (63) and (64) we can determine the boundary values of the electric field components inside the nonlinear medium as functions of the effective index of refraction. Then, using them and integrating Eqs. (54) and (55), we obtain the field distribution. In a linear medium this distribution has a simple exponential form [see Eq. (59)]. Using the field distribution and integrating the time-averaged Poynting vector over the variable $z$, we obtain the surface-wave energy flux. Finally, we have $P = P_s + P_c$, where

$$P_s = \frac{\varepsilon_{nl}^2 E_{0z}^2}{4\mu_0\omega\beta q_s\varepsilon_s}; \quad (66)$$

$$P_c = \frac{k_0}{2\mu_0\omega\beta}\int_0^\infty \varepsilon_{zz}(z)E_z^2(z)\,dz. \quad (67)$$

The types of field and the range of allowed values of the effective index of refraction were found by analyzing the NSW "phase trajectories."[40,84] Let us consider some special cases for an isotropic nonlinear substrate with the set of parameters $\varepsilon_x = \varepsilon_z = \varepsilon_c$ and $\alpha_{xx} = \alpha_{zz} = \alpha_{xz} = \alpha_{zx} = \alpha_c$.

Case (a): $\varepsilon_c > 0$, $\alpha_c > 0$, and $\varepsilon_s < 0$. In Fig. 4a we show the dependence of the dimensionless energy flux $P/P_0$ on the effective index of refraction $\beta$ for $\varepsilon_c = 2.25$ and several values of $\varepsilon_s$. As the effective index of refraction $\beta$ increases, the surface-wave energy flux increases up to some critical value, and then falls to zero. This is related to the fact that in a medium characterized by negative dielectric constant the energy flux and wave vector have opposite directions and for a certain range of $\beta$ the energy flux decreases with increasing

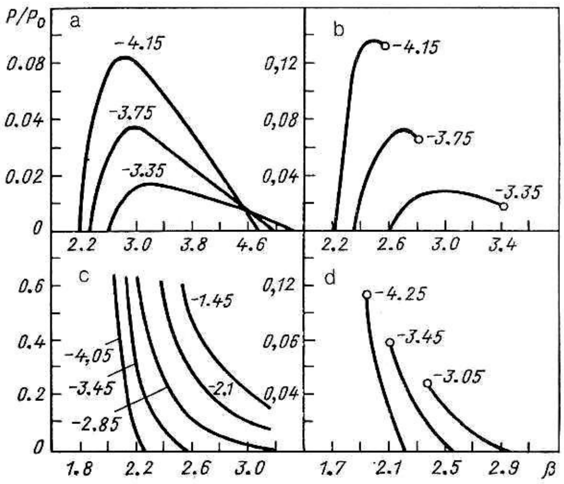


FIG. 4. Dependence of the dimensionless energy flux $P/P_0$ on the effective index of refraction $\beta$. The value $\varepsilon = 2.25$ was chosen for the medium with positive dielectric constant ($\varepsilon_c$ or $\varepsilon_s$). The numbers on the curves are the values of the dielectric constants of the bounding media.[84]

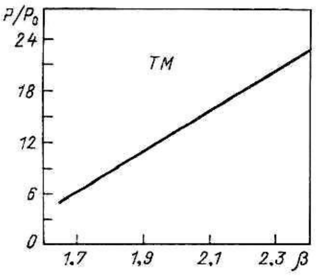


FIG. 5. The same as in Fig. 4 for $\varepsilon_c = 2.25$, $\varepsilon_s = 2.5$, and $\alpha_c > 0$ (Ref. 84).

$\beta$. We note that the magnetic field for these values reaches a maximum at the interface of the two media, and the effective index of refraction $\beta$ for NSWs is larger than the value $\beta_l = [\varepsilon_c|\varepsilon_s|(|\varepsilon_s| - \varepsilon_c)^{-1}]^{1/2}$, corresponding to TM-polarized linear surface polaritons.

Case (b): $\varepsilon_c < 0$, $\alpha_c > 0$, and $\varepsilon_s > 0$. In Fig. 4b we show the dependence of the dimensionless energy flux $P/P_0$ on the effective index of refraction $\beta$ for $\varepsilon_s = 2.25$ and several values of $\varepsilon_c$. In this case surface waves exist in a limited range of values larger than $\beta_l = [|\varepsilon_c|\varepsilon_s(|\varepsilon_c| - \varepsilon_s)^{-1}]^{1/2}$. We note that, as in case (a), the transmitted power has a maximum.

Case (c): $\varepsilon_c < 0$, $\alpha_c < 0$, and $\varepsilon_s > 0$. We see from Fig. 4c that for $\beta = \beta_l$ the energy flux is equal to zero and increases to infinity as soon as $\beta$ reaches the value $n_s = \varepsilon_s^{1/2}$. We recall that for $\varepsilon_s > |\varepsilon_c|$ TM-polarized linear surface polaritons do not exist. However, TM-polarized NSPs can exist for $\varepsilon_s > |\varepsilon_c|$, when the energy flux exceeds some threshold value (see the curves for $\varepsilon_c = -2.1$ and $\varepsilon_c = -1.45$ in Fig. 4c).

Case (d): $\varepsilon_c > 0$, $\alpha_c < 0$, and $\varepsilon_s < 0$. Here (see Fig. 4d), when the nonlinearity is negative, as in the preceding case, as the energy flux increases the effective index of refraction $\beta$ decreases, beginning at the value $\beta = \beta_l$, corresponding to TM-polarized linear surface polaritons.

Case (e): $\varepsilon > 0$, $\alpha > 0$, and $\varepsilon > 0$. In this case the phase diagram and field profile show that the magnetic field reaches its maximum not at the interface $z = 0$, but in the self-focusing nonlinear overlayer ($\alpha_c > 0$). A given nonlinear wave can be guided by the interface between a self-focusing overlayer and a linear dielectric substrate when the energy-flux threshold is exceeded, which is similar to the case of TE-polarized surface polaritons on a nonlinear interface.[15,18,20,60] In Fig. 5 we show the $\beta$ dependence of the dimensionless energy flux $P/P_0$ for the values $\varepsilon_c = 2.25$ and $\varepsilon_s = 2.5$. The corresponding TM-polarized waves have no analog in the linear optics of surface waves.[84]

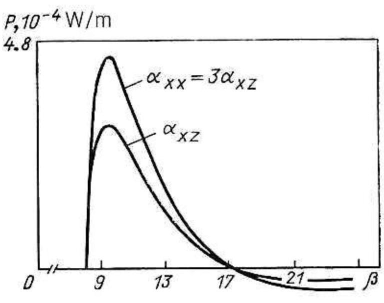


FIG. 6. Dependence of the energy flux on the effective index of refraction $\beta$ for the following parameters of the calculation: $\omega = 3.66\times 10^{15}$ rad·sec$^{-1}$, $\varepsilon_c = \varepsilon_x = \varepsilon_z = 2.405$, $\varepsilon_s = -2.5$, and $\alpha_{xx} = \alpha_{zz} = 6.4\times 10^{-12}$ m$^2$/W.

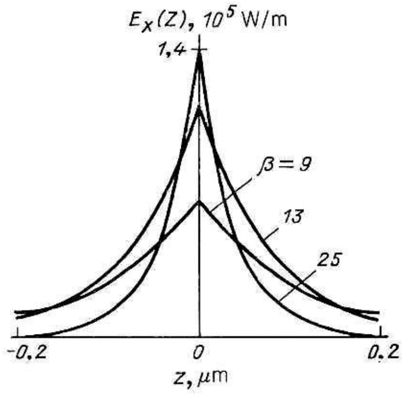


FIG. 7. Dependence of $E_x$ on the transverse coordinate $z$ for various values of $\beta$.

In Fig. 6 we show the $\beta$ dependence of the energy flux for a nonlinear, self-focusing dielectric–metal interface for both the case of nonlinearity in the electronics, $\alpha_{xx} = 3\alpha_{xz}$, and the case of nonlinearity owing to electrostriction, $\alpha_{xx} = \alpha_{xz}$. In Fig. 7 we show the transverse distribution of the electric field component $E_x$ for nonlinearity in the electronics and several values of the propagation constant $\beta$. We note that the maximum transmitted power occurs for self-focusing nonlinearities in the electronics and nonlinearities due to electrostriction, and that the effective index of refraction $\beta$ is larger than the value $\beta_l$ corresponding to TM-polarized linear surface waves.

## 2. TRANSVERSE-ELECTRIC POLARIZED NONLINEAR OPTICAL WAVES GUIDED BY THIN DIELECTRIC FILMS

### Nonlinear guided-wave modes propagating in three-layered structures

A guided-wave mode is an electromagnetic field guided by a medium with large index of refraction. A dielectric plate is the simplest example of an optical waveguide used as a light pipe in integrated-optics schemes (see, for example, Refs. 87–89). A film waveguide is a thin dielectric film of thickness $d$ and index of refraction $n_f$ surrounded by a medium with lower index of refraction: a substrate and overlayer with indices of refraction $n_s$ and $n_c$, respectively. For thin-film waveguides and TE waves (polarized along the $y$ axis) the nonzero components of the electric field have the form

$$E_y^{\mathrm{I}}(z) = E_s \exp(k_0 q_s z),\ z < 0; \tag{68}$$

$$E_y^{\mathrm{II}}(z) = E_f \cos(k_0 q_f z - \Phi_{sf}),\ 0 < z < d; \tag{69}$$

$$E_y^{\mathrm{III}}(z) = E_c \exp[-k_0 q_c (z - d)],\ z > d, \tag{70}$$

where $q_s = (\beta^2 - n_s^2)^{1/2}$, $q_f = (n_f^2 - \beta^2)^{1/2}$, and $q_c = (\beta^2 - n_c^2)^{1/2}$.

From the condition of continuity of $E_y$ and $dE_y/dz$ at the interfaces $z = 0$ and $z = d$, we obtain the dispersion relation

$$\mathrm{tg}(k_0 q_f d) = \frac{q_f(q_s + q_c)}{(q_f^2 - q_s q_c)}. \tag{71}$$

Equation (71) can be rewritten as (the condition for constructive interference)

$$k_0 q_f d = \Phi_{sf} + \Phi_{cf} + m\pi,\quad m = 0, 1, 2 \ldots, \tag{72}$$

where $\tan \Phi_{sf} = q_s/q_f$ and $\tan \Phi_{cf} = q_c/q_f$. Solutions of (72) exist for discrete sets of values of $m$ and are numbered with the notation $TE_m$ ($m = 0, 1, 2, \ldots$). In addition, we have the following relations for the field amplitudes:

$$E_s^2 (n_f^2 - n_s^2) = E_f^2 (n_f^2 - \beta^2) = E_c^2 (n_f^2 - n_c^2). \quad (73)$$

We still must relate the amplitudes $E_f$ of the electromagnetic field to the power carried by the mode. The guided-wave energy flux is obtained by integration of the $x$ component of the Poynting vector:

$$P = \frac{1}{4} \beta \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} E_f^2 d_{\text{eff}}, \quad (74)$$

where $d_{\text{eff}} = d + (k_0 q_c)^{-1} + (k_0 q_s)^{-1}$ is the effective thickness of the thin-film waveguide. Therefore, from Eq. (74) we can find the amplitude $E_f$ as a function of the energy flux $P$ and the effective index of refraction $\beta$.

There also exist transverse-magnetic (TM) polarized modes, the magnetic field of which is polarized along the $y$ axis, i.e., $H_y \neq 0$ and $E_x \neq 0$, $E_z \neq 0$ (Ref. 87). Therefore, TM-polarized waves have two electric field components, one of which ($E_x$) is parallel to the wave vector, while the other ($E_z$) is perpendicular to the surface. We note that the dispersion relation for the TM modes of a linear asymmetric waveguide is given by Eq. (71), with $q_\gamma$ replaced by $q_\gamma/\varepsilon_\gamma$, where $\gamma = s, f, c$. A variant of the guided-wave method for weakly varying phase and an amplitude approximation have been developed for guided-wave modes and are known as the coupled-mode theory.[7,8] This method is useful for analyzing the appearance of new waves, and also in the case of an intensity-dependent index of refraction. If the optical nonlinearity does not lead to any significant changes in the distribution of the guided-wave field, the coupled-mode theory can be used to calculate the intensity-dependent wave vector or phase shift.[9] When the optically induced change of the index of refraction is comparable with or larger than the differences of the indices of refraction $n_f - n_c$ and $n_f - n_s$ existing for low values of the energy flux between the dielectric film and the media with which it is in contact, the field profiles and the distribution constants become dependent on the energy flux. In this case, the coupled-mode theory, which is essentially first-order perturbation theory, becomes inadequate even for a qualitative description. For a medium obeying the Kerr law and TE-surface guided waves, the exact theory based on analysis of the nonlinear wave equation permits analytic solutions to be obtained.

Let us consider an asymmetric dielectric layered structure composed of an optically linear medium (the substrate) with index of refraction $n_s$ in the lower half-plane $z < 0$ (region I), a dielectric film of thickness $d$ with index of refraction $n_f$ in region II ($0 < z < d$), and a nonlinear self-focusing overlayer obeying the Kerr law and having dielectric function $\varepsilon = \varepsilon_c + \alpha_c |\mathbf{E}|^2$ with $\alpha_c > 0$ in region III ($z > d$). Maxwell's equations for the guided-wave fields, which are independent of the $x$ components, have the form

$$\frac{d^2 E_y^{\text{I}}}{dz^2} - k_0^2 (\beta^2 - \varepsilon_s) E_y^{\text{I}} = 0, \quad z < 0; \quad (75)$$

$$\frac{d^2 E_y^{\text{II}}}{dz^2} - k_0^2 (\beta^2 - \varepsilon_f) E_y^{\text{II}} = 0, \quad 0 < z < d; \quad (76)$$

$$\frac{d^2 E_y^{\text{III}}}{dz^2} - k_0^2 (\beta^2 - \varepsilon_c) E_y^{\text{III}} + \alpha_c k_0^2 (E_y^{\text{III}})^3 = 0, \quad z > d. \quad (77)$$

The exact solutions of Eqs. (75)–(77) for $\alpha_c > 0$ (self-focusing optical nonlinearities and $\beta < n_f$) can be written in the form

$$E_y^{\text{I}} (z) = \frac{1}{\alpha_c^{1/2}} \tilde{A} \exp (k_0 q_s z), \quad z < 0; \quad (78)$$

$$E_y^{\text{II}} (z) = \left( \frac{1}{\alpha_c} \right)^{1/2} \tilde{A} \left[ \cos (k_0 q_f z) + \frac{q_s}{q_f} \sin (k_0 q_f z) \right], \quad 0 < z < d; \quad (79)$$

$$E_y^{\text{III}} (z) = \left( \frac{2}{\alpha_c} \right)^{1/2} q_c \{ \text{ch} [k_0 q_c (z - z_c)] \}^{-1}, \quad z > d, \quad (80)$$

where

$$\tilde{A} = [2 (1 - \nu^2)]^{1/2} q_c \left[ \cos (k_0 q_f d) + \frac{q_s}{q_f} \sin (k_0 q_f d) \right]^{-1}; \quad (81)$$

$$\nu = \text{th} [k_0 q_c (z_c - d)]. \quad (82)$$

For the corresponding tangential electric and magnetic fields at the interface, we obtain the dispersion relation

$$\text{tg} (k_0 q_f d) = \frac{q_f (q_s - \nu q_c)}{(q_f^2 + \nu q_s q_c)}. \quad (83)$$

This result is very similar to that obtained for the linear case, when $q_c$ is replaced by $-\nu q_c$. If $\alpha_c \to 0$, then $z_c \to -\infty$ and $\nu = -1$, and we obtain the dispersion relation for the TE-polarized modes of a linear asymmetric thin-film waveguide [see (71)].

For $\beta > n_f$ the exact solutions of Maxwell's equations (75)–(77) have the form

$$E_y^{\text{I}} (z) = \alpha_c^{-1/2} \tilde{B} \exp (k_0 q_s z), \quad z < 0;$$

$$E_y^{\text{II}} (z) = \alpha_c^{-1/2} \tilde{B} \left[ \frac{q_s + \tilde{q}_f}{2\tilde{q}_f} \exp (k_0 \tilde{q}_f z) + \frac{(\tilde{q}_f - q_s)}{2\tilde{q}_f} \exp (-k_0 \tilde{q}_f z) \right], \quad (84)$$

$$0 < z < d \quad (85)$$

and $E_y^{\text{III}}(z)$ is given by Eq. (80). Here $\tilde{q}_f = (\beta^2 - n_f^2)^{1/2}$ and

$$\tilde{B} = [2 (1 - \nu^2)]^{1/2} q_c \left[ \text{ch} (k_0 \tilde{q}_f d) + \frac{q_s}{\tilde{q}_f} \text{sh} (k_0 \tilde{q}_f d) \right]^{-1}. \quad (86)$$

When the quantities $E$ and $dE_y/dz$ are continuous along the interfaces $z = 0$ and $z = d$, the dispersion relation is given by

$$\text{th} (k_0 \tilde{q}_f d) = \frac{\tilde{q}_f (\nu q_c - q_s)}{(\tilde{q}_f^2 - \nu q_s q_c)}, \quad (87)$$

where for $\nu$ we have the expression (82).

The energy flux of the guided-wave modes per unit length along the $y$ axis is calculated by integrating the Poynting vector over the variable $z$:

$$P = \frac{\beta}{2c\mu_0} \int_{-\infty}^{\infty} E_y^2 (z) \, dz = P_s + P_f + P_c. \quad (88)$$

For $\beta < n_f$ we have (see Refs. 91 and 92)

$$P_s = \frac{1}{2} P_0 \beta \frac{\tilde{A}^2}{q_s}; \quad (89)$$

$$P_f = \frac{1}{2} P_0 \beta \tilde{A}^2 \left\{ k_0 d \left( 1 + \frac{q_s^2}{q_f^2} \right) + \frac{1}{q_f} \sin (k_0 q_f d) \times \left[ \left( 1 - \frac{q_s^2}{q_f^2} \right) \cos (k_0 q_f d) + 2 \frac{q_s}{q_f} \sin (k_0 q_f d) \right] \right\}; \quad (90)$$

$$P_c = 2 P_0 \beta q_c (1 + \nu). \quad (91)$$

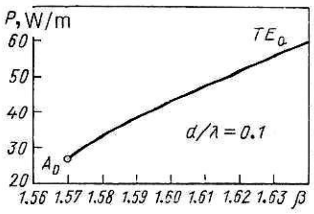


FIG. 8. Energy flux $P$ of $TE_0$ waves as a function of $\beta$. Parameters: $n_c = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W, $n_s = 1.52$, $n_f = 1.61$, and $\lambda = 0.515$ $\mu$m.

For $\beta > n_f$ we obtain

$$P_s = \frac{1}{2} P_0\beta \frac{\tilde{B}^2}{q_s}; \tag{92}$$

$$P_f = \frac{1}{2} P_0\beta\tilde{B}^2 \left\{k_0 d \left(1 - \frac{q_s^2}{\tilde{q}_f^2}\right) + \frac{1}{\tilde{q}_f} \operatorname{sh}(k_0\tilde{q}_f d)\left[\left(1 + \frac{q_s^2}{\tilde{q}_f^2}\right) \operatorname{ch}(k_0\tilde{q}_f d) + 2\frac{q_s}{\tilde{q}_f} \operatorname{sh}(k_0\tilde{q}_f d)\right]\right\}, \tag{93}$$

and $P_c$ is given by the expression (91).

Let us take the nonlinear overlayer to be the liquid crystal MBBA ($n_c = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W), deposited on a glass waveguide ($n_f = 1.61$, $n_c = 1.52$). The presence of a nonlinear overlayer affects the transmission conditions of an asymmetric film waveguide. It is well known that a linear asymmetric optical waveguide ($n_c \neq n_s$) cannot support guided-wave modes below the critical thickness $d_{cr}$ (Refs. 87 and 88).

In the case of an asymmetric nonlinear optical waveguide there is an energy threshold for the propagation of $TE_0$ waves in a film of thickness $d < d_{cr}$ (Fig. 8 for $d/\lambda = 0.1$). This effect can be used for designing an apparatus which lowers the threshold energy, i.e., it begins to transmit an energy flux above a certain minimum value. Such a device can be constructed using a self-focusing overlayer reaching saturation when the value of $n_{sat}$ at which saturation occurs is not too large. In Fig. 9 we show the dependence of the guided-wave energy flux on the effective index of refraction for the following values of the parameters: $d = 2$ $\mu$m, $n_c = n_s = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W, and $n_f = 1.57$. The unique features of the solutions for $TE_0$ waves are that they can propagate when $\beta > n_f$ and that there exists a local maximum of the energy of the guided-wave modes. For $TE_1$ waves (Fig. 9) the value of $\beta$ never exceeds $n_f$, and the guided-wave energy flux has an absolute maximum. Moreover, the $TE_1$ waves cease to branch at some value $\beta < n_s$ (Fig. 9).

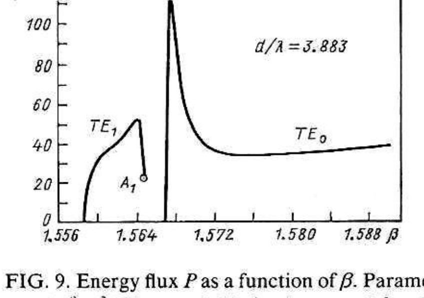


FIG. 9. Energy flux $P$ as a function of $\beta$. Parameters: $n_c = n_s = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W, $n_f = 1.57$, $d = 2$ $\mu$m, and $\lambda = 0.515$ $\mu$m (Ref. 93).

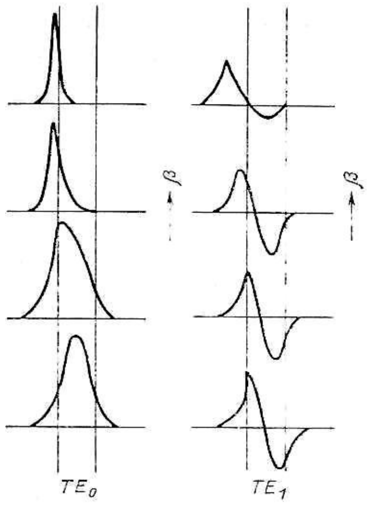


FIG. 10. Distribution of the fields for $TE_0$ and $TE_1$ NGWMs as a function of $\beta$ (Ref. 14).

The evolution of the field distribution for $TE_0$ and $TE_1$ waves with increasing $\beta$ is shown in Fig. 10, which illustrates one of the characteristic features of NGWMs, namely, the energy dependence of the field distributions. As $\beta$ increases, the maximum of the $TE_0$-wave field narrows and moves into the nonlinear self-focusing overlayer, while the maximum of the $TE_1$ waves, which is close to the nonlinear overlayer, is shifted into this medium.

The variation of the guided-wave flux with varying propagation constant $\beta$ for $d/\lambda = 6$ (Fig. 11) demonstrates that all of the higher-order $TE_m$ ($m \geqslant 1$) branches terminate at certain values $\beta < n_f$. For the self-focusing overlayer the energy flux has an absolute maximum, which can propagate in any $TE_m$ ($m \geqslant 1$) mode. For all film thicknesses at high energies the lowest branch (the $TE_0$ wave) degenerates into a self-focusing surface wave guided by the nonlinear interface between the film and the overlayer. These NGWMs can obviously be used in the design of optical power limiters for various applications. The limiting action of a self-focusing overlayer has been demonstrated experimentally for the case of $TE_1$ waves in Ref. 56.

Let us consider a symmetric thin-film waveguide consisting of a thin dielectric film of thickness $d$ in contact on both sides with a self-focusing medium obeying the Kerr law

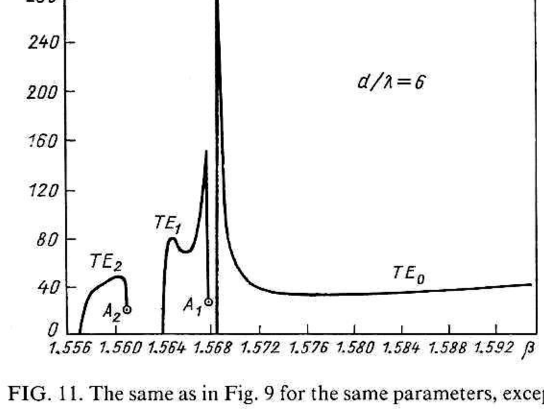


FIG. 11. The same as in Fig. 9 for the same parameters, except $d/\lambda = 6$.

($n_s = n_c$ and $\alpha_s = \alpha_c > 0$). The dielectric film occupies region II ($-d/2 \leqslant z \leqslant d/2$), and the two nonlinear dielectrics occupy regions I ($z < -d/2$) and III ($z > d/2$). In this case Maxwell's equations take the form

$$\frac{d^2E_y^{\mathrm{I}}}{dz^2} - k_0^2(\beta^2 - \varepsilon_c)E_y^{\mathrm{I}} + k_0^2\alpha_c(E_y^{\mathrm{I}})^3 = 0, \quad z < -d/2; \quad (94)$$

$$\frac{d^2E_y^{\mathrm{II}}}{dz^2} - k_0^2(\beta^2 - \varepsilon_f)E_y^{\mathrm{II}} = 0, \quad -d/2 < z < d/2; \quad (95)$$

$$\frac{d^2E_y^{\mathrm{III}}}{dz^2} - k_0^2(\beta^2 - \varepsilon_c)E_y^{\mathrm{III}} + k_0^2\alpha_c(E_y^{\mathrm{III}})^3 = 0, \quad z > d/2, \quad (96)$$

and the exact solutions of these equations (94)–(96) in the case of a self-focusing optical nonlinearity ($\alpha_c > 0$) are given by

$$E_y^{\mathrm{I}}(z) = \left(\frac{2}{\alpha_c}\right)^{1/2} q_c\{\mathrm{ch}\,k_0q_c(z - z_s)\}^{-1}, \quad z < -d/2; \quad (97)$$

$$E_y^{\mathrm{II}}(z) = \begin{cases} A_1\cos[k_0q_f(z - z_f)], & \beta < n_f \\ A_2\,\mathrm{ch}[k_0\tilde{q}_f(z - z_f)], & \beta > n_f \end{cases} \quad -d/2 < z < d/2; \quad (98)$$

$$E_y^{\mathrm{III}}(z) = \left(\frac{2}{\alpha_c}\right)^{1/2} q_c\{\mathrm{ch}[k_0q_c(z - z_c)]\}^{-1}, \quad z > d/2. \quad (99)$$

The boundary conditions lead to an equation for the unknown quantity $z_f$:

$$\left\{1 - b_1^2\,\mathrm{th}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} - z_f\right)\right]\right\}\mathrm{ch}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} + z_f\right)\right] - \left\{1 - b_1^2\,\mathrm{th}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} + z_f\right)\right]\right\}\mathrm{ch}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} - z_f\right)\right] = 0, \quad (100)$$

where $b_1 = \tilde{q}_f/q_c$. Equation (100) has the unique solution $z_f = 0$ for all $\beta > n_f$. The solution (98) for $z_f = 0$ corresponds to a symmetric wave (S) propagating in a symmetric three-layered planar structure. In this case the field distribution is symmetric about the center of the thin-film waveguide and $z_c = -z_s$. The eigenvalue equations for the symmetric branch have the form

$$\mathrm{th}\left[k_0q_c\left(\frac{d}{2} + z_c\right)\right] = b_2\,\mathrm{tg}\left(k_0q_f\frac{d}{2}\right), \quad \beta < n_f; \quad (101)$$

$$\mathrm{th}\left[k_0q_c\left(\frac{d}{2} + z_c\right)\right] = -b_1\,\mathrm{th}\left(k_0q_f\frac{d}{2}\right), \quad \beta > n_f, \quad (102)$$

where $b_2 = q_f/q_c$. For $\alpha_c \to 0$ we have $z_c \to +\infty$ and Eq. (101) leads to the familiar dispersion relation for the symmetric (even) modes of a symmetric dielectric waveguide:

$$\mathrm{tg}\left(k_0q_f\frac{d}{2}\right) = \frac{q_c}{q_f}. \quad (103)$$

For the symmetric solution (S) the electric field amplitudes inside the thin-film dielectric are given by

$$A_1^2 = \frac{2}{\alpha_c}q_c^2\left[\cos\left(k_0q_f\frac{d}{2}\right)\right]^{-2}\left[1 - b_2^2\,\mathrm{tg}^2\left(k_0q_f\frac{d}{2}\right)\right], \quad \beta < n_f; \quad (104)$$

$$A_2^2 = \frac{2}{\alpha_c}q_c^2\left[\mathrm{ch}\left(k_0\tilde{q}_f\frac{d}{2}\right)\right]^{-2}\left[1 - b_1^2\,\mathrm{th}^2\left(k_0\tilde{q}_f\frac{d}{2}\right)\right], \quad \beta > n_f. \quad (105)$$

Let us find the dispersion relations for the antisymmetric wave (AS) of a symmetric thin-film waveguide. The solution of Maxwell's equations (95) inside the film has the form

$$E_y^{\mathrm{II}}(z) = \begin{cases} B_1\sin[k_0q_f(z - z_f)], & \beta < n_f; \\ B_2\,\mathrm{sh}[k_0\tilde{q}_f(z - z_f)], & \beta > n_f. \end{cases} \quad (106)$$

In this case the equation for the unknown $z_f$ is written as

$$\left\{1 - b_2^2\,\mathrm{ctg}^2\left[k_0q_f\left(\frac{d}{2} - z_f\right)\right]\right\}\sin^2\left[k_0q_f\left(\frac{d}{2} + z_f\right)\right] - \left\{1 - b_2^2\,\mathrm{ctg}^2\left[k_0q_f\left(\frac{d}{2} + z_f\right)\right]\right\}\sin^2\left[k_0q_f\left(\frac{d}{2} - z_f\right)\right] = 0 \quad (107)$$

for $\beta < n_f$ and

$$\left\{1 - b_1^2\,\mathrm{cth}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} - z_f\right)\right]\right\}\mathrm{sh}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} + z_f\right)\right] - \left\{1 - b_1^2\,\mathrm{cth}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} + z_f\right)\right]\right\}\mathrm{sh}^2\left[k_0\tilde{q}_f\left(\frac{d}{2} - z_f\right)\right] = 0 \quad (108)$$

for $\beta > n_f$. It is easily verified that Eqs. (107) and (108) have the solution $z_f = 0$, which corresponds to an antisymmetric wave (AS) in a symmetric layered structure. In addition, there is a solution $z_f \neq 0$, i.e., an asymmetric wave (A) propagating in a nonlinear symmetric layered structure. Here the asymmetric wave exists only above a certain energy-flux threshold.[23] From the boundary conditions we obtain the following dispersion relations for the antisymmetric wave (AS):

$$\mathrm{th}\left[k_0q_c\left(\frac{d}{2} + z_c\right)\right] = -b_2\,\mathrm{ctg}\left(k_0q_f\frac{d}{2}\right), \quad \beta < n_f, \quad (109)$$

$$\mathrm{th}\left[k_0q_c\left(\frac{d}{2} + z_c\right)\right] = -b_1\,\mathrm{cth}\left(k_0\tilde{q}_f\frac{d}{2}\right), \quad \beta > n_f. \quad (110)$$

For $\alpha_c \to 0$ we have $z_c \to +\infty$, and from Eq. (109) we obtain the dispersion relation for the antisymmetric (odd) modes of a linear symmetric waveguide:

$$\mathrm{ctg}\left(k_0q_f\frac{d}{2}\right) = -q_c/q_f. \quad (111)$$

The amplitudes $B_1$ and $B_2$ of the electric field inside a linear medium in the case of the antisymmetric solution are given by

$$B_1^2 = \frac{2}{\alpha_c}q_c^2\left[\sin\left(k_0q_f\frac{d}{2}\right)\right]^{-2}\left[1 - b_2^2\,\mathrm{ctg}^2\left(k_0q_f\frac{d}{2}\right)\right], \quad \beta < n_f; \quad (112)$$

$$B_2^2 = \frac{2}{\alpha_c}q_c^2\left[\mathrm{sh}\left(k_0\tilde{q}_f\frac{d}{2}\right)\right]^{-2}\left[1 - b_1^2\,\mathrm{cth}^2\left(k_0\tilde{q}_f\frac{d}{2}\right)\right], \quad \beta > n_f. \quad (113)$$

The time-averaged energy flux (in W/m) along the $y$ axis of the symmetric wave (S) has the form

$$P = 4P_0\beta q_c(1 - r_2)\left\{1 + \frac{q_c}{4}(1 + r_2) \times \left[\cos\left(k_0q_f\frac{d}{2}\right)\right]^{-2}\left[k_0d + \frac{\sin(k_0q_fd)}{q_f}\right]\right\} \quad (114)$$

for $\beta < n_f$ and

$$P = 4P_0\beta q_c(1 - r_1)\left\{1 + \frac{q_c}{4}(1 + r_1) \times \left[\mathrm{ch}\left(k_0\tilde{q}_f\frac{d}{2}\right)\right]^{-2}\left[k_0d + \frac{\mathrm{sh}(k_0\tilde{q}_fd)}{\tilde{q}_f}\right]\right\} \quad (115)$$

for $\beta > n_f$, where

$$r_1 = -b_1\,\mathrm{th}\left(k_0\tilde{q}_f\frac{d}{2}\right); \quad r_2 = b_2\,\mathrm{tg}\left(k_0q_f\frac{d}{2}\right). \quad (116)$$

Equations (114) and (115) give a dependence of the form $\omega = \omega(\beta, P)$, i.e., they determine the dispersion relation for a nonlinear symmetric wave (S). We note that for $P = 0$ Eq.

(114) gives $1 - r_2 = 0$, i.e., the dispersion relation for the TE-polarized symmetric (even) modes of a symmetric linear waveguide. It is also easy to verify that for $d \to \infty$ Eq. (115) reproduces the expression for the energy flux of surface waves on a nonlinear interface (see Sec. 1). Similarly, using Eqs. (112) and (113), we obtain an expression for the energy flux in a nonlinear antisymmetric wave (AS):

$$P = 4P_0\beta q_c(1-t_2)\left\{1+\frac{q_c}{4}(1+t_2)\times\left[\sin\left(k_0 q_f\frac{d}{2}\right)\right]^{-2}\left[k_0 d-\frac{\sin(k_0 q_f d)}{q_f}\right]\right\} \qquad (117)$$

for $\beta < n_f$ and

$$P = 4P_0\beta q_c(1-t_1)\left\{1+\frac{q_c}{4}(1+t_1)\times\left[\mathrm{sh}\left(k_0 \tilde{q}_f\frac{d}{2}\right)\right]^{-2}\left[\frac{\mathrm{sh}(k_0 \tilde{q}_f d)}{\tilde{q}_f}-k_0 d\right]\right\}, \qquad (118)$$

for $\beta > n_f$, where

$$t_1 = -b_1\,\mathrm{cth}\left(k_0\tilde{q}_f\frac{d}{2}\right);\quad t_2 = -b_2\,\mathrm{ctg}\left(k_0 q_f\frac{d}{2}\right). \qquad (119)$$

In this case also for $P = 0$ expression (117) leads to the relation $1 - t_2 = 0$, i.e., to the dispersion relation for the antisymmetric (odd) modes of a symmetric linear waveguide. For $d \to \infty$ expression (118) reproduces the expression for the NSW energy flux.

In the systems with full symmetry, i.e., when $n_c = n_s$ and $n_{2c} = n_{2s} > 0$ (self-focusing nonlinearities), it can be expected that self-focusing fields may occur in one or both of the media in contact for large energy fluxes. In Fig. 12 we show the dependence of the dimensionless energy flux $P/P_0$ on the propagation constant $\beta$ for a symmetric layered structure.[23] The field distribution of branch S is symmetric with respect to the center of the film (the symmetric $TE_0$ branch). As the energy flux increases in the center of the film, the field minimum grows, and the two symmetric maxima of the field move into the overlayer and the substrate. Branch A exists only above the threshold value of the energy flux, and the fields related to it are self-focusing either in the overlayer or in the substrate (the asymmetric $TE_0$ branch). For the curve AS the field distribution preserves the symmetry with respect to the center of the thin film (the symmetric

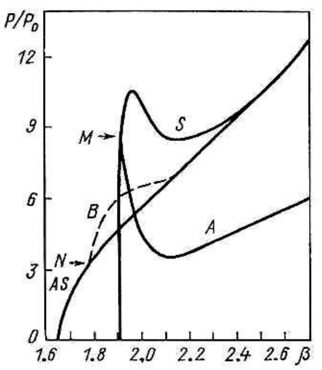


FIG. 12. Normalized dimensionless energy flux $P/P_0$ as a function of $\beta$ for a symmetric layered structure. Parameters: $n_f = 2.0$, $n_c = n_s = 1.5$, and $d/\lambda = 0.6$. The curves are labeled as follows: S—symmetric $TE_0$ branch; A—asymmetric $TE_0$ branch; AS—symmetric $TE_1$ branch; B—asymmetric $TE_1$ branch.[23]

$TE_1$ branch). This branch changes from an ordinary, low-power $TE_1$ mode with field extrema inside the film to a $TE_1$ branch of high power with symmetric field maxima localized in the two nonlinear media in contact with the film. Curve B has an energy-flux threshold and is similar to curve A, since the field distribution is asymmetric with respect to the center of the film (the asymmetric $TE_1$ branch).

Let us find the exact dispersion relations for TE-polarized guided-wave modes in a planar structure consisting of an optically linear dielectric film embedded in optically heterogenous nonlinear semi-infinite media. The three layers of the waveguide structure are as follows: a nonlinear substrate with Kerr dielectric function $\varepsilon = \varepsilon_s + \alpha_s|E|^2$ in region I ($z<0$), a thin dielectric film of thickness $d$ and dielectric constant $\varepsilon_f$ in region II ($0<z<d$), and a nonlinear overlayer with Kerr dielectric function $\varepsilon = \varepsilon_c + \alpha_c|E|^2$ in region III ($z>d$). Maxwell's equations for TE-polarized waves propagating along the $x$ axis have the form

$$\frac{d^2E_y^{\mathrm{I}}}{dz^2} - k_0^2(\beta^2-\varepsilon_s)E_y^{\mathrm{I}} + k_0^2\alpha_s(E_y^{\mathrm{I}})^3 = 0,\quad z<0; \qquad (120)$$

$$\frac{d^2E_y^{\mathrm{II}}}{dz^2} - k_0^2(\beta^2-\varepsilon_f)E_y^{\mathrm{II}} = 0,\quad 0<z<d; \qquad (121)$$

$$\frac{d^2E_y^{\mathrm{III}}}{dz^2} - k_0^2(\beta^2-\varepsilon_c)E_y^{\mathrm{III}} + k_0^2\alpha_c(E_y^{\mathrm{III}})^3 = 0,\quad z>d. \qquad (122)$$

The field solutions for the nonlinear substrate have the form

$$E_y^{\mathrm{I}}(z) = \left(\frac{2}{\alpha_s}\right)^{1/2} q_s\{\mathrm{ch}[k_0 q_s(z_s - z)]\}^{-1},\quad z<0, \qquad (123)$$

in the case $\alpha_s > 0$ (self-focusing nonlinearity) and

$$E_y^{\mathrm{I}}(z) = \left(\frac{2}{|\alpha_s|}\right)^{1/2} q_s\{\mathrm{sh}[k_0 q_s(z_s - z)]\}^{-1},\quad z<0, \qquad (124)$$

for $\alpha_s < 0$ (self-defocusing nonlinearity), where $q_s = (\beta^2 - \varepsilon_s)^{1/2}$. The fields inside the optically linear dielectric film can be written as

$$E_y^{\mathrm{II}}(z) = E_y^{\mathrm{II}}(0)\left\{\cos(k_0 q_f z) + \frac{q_s}{q_f}[\mathrm{th}(k_0 q_s z_s)]^{k_\alpha}\sin(k_0 q_f z)\right\}, \qquad (125)$$

where $0<z<d$, $q_f = (\varepsilon_f - \beta^2)^{1/2}$ and $k_\alpha = +1$ for $\alpha_s > 0$ and $k_\alpha = -1$ for $\alpha_s < 0$. The nonlinear solution in the case of a nonlinear substrate is given by

$$E_y^{\mathrm{III}}(z) = \left(\frac{2}{\alpha_c}\right)^{1/2} q_c\{\mathrm{ch}[k_0 q_c(z_c - z)]\}^{-1},\quad z>d, \qquad (126)$$

for $\alpha_c > 0$ and

$$E_y^{\mathrm{III}}(z) = \left(\frac{2}{|\alpha_c|}\right)^{1/2} q_c\{\mathrm{sh}[k_0 q_c(z_c - z)]\}^{-1},\quad z>d, \qquad (127)$$

for $\alpha_c < 0$, where $q_c = (\beta^2 - \varepsilon_c)^{1/2}$.

The dispersion relation for the corresponding tangential electric and magnetic fields continuous at the film–substrate interface ($z = d$) has the form

$$\mathrm{tg}(k_0 q_f d) = \frac{q_f(\nu_s^{k_\alpha} q_s + \nu_c^{k_\alpha} q_c)}{(q_f^2 - \nu_s^{k_\alpha}\nu_c^{k_\alpha} q_s q_c)}, \qquad (128)$$

where $\nu_c = \tanh[k_0 q_c(d - z_c)]$ and $\nu_s = \tanh(k_0 q_s z_s)$. This result is very similar to that of the linear case, except that $q_s$ and $q_c$ are replaced by $\nu_s^{k_\alpha} q_s$ and $\nu_c^{k_\alpha} q_c$, respectively. For $\alpha_s \to 0$ and $\alpha_c \to 0$ we have $z_s \to +\infty$, $z_c \to -\infty$, $\nu_s \to +1$, and $\nu_c \to +1$, and from (128) we obtain the dispersion relation for the TE-polarized modes of a linear asymmetric waveguide [see (71)].

For $\beta > n_f$ the analytic solution of Maxwell's equations is

$$E_y^{II}(z) = E_y^{II}(0)\left\{\operatorname{ch}(k_0\tilde{q}_f z) + \frac{q_s}{\tilde{q}_f}[\operatorname{th}(k_0 q_s z_s)]^{k_\alpha}\operatorname{sh}(k_0 q_f z)\right\}, \quad (129)$$

where $0<z<d$, $\tilde{q}_f = (\beta^2 - \varepsilon_f)^{1/2}$ and $k_\alpha = +1$ correspond to a self-focusing medium ($\alpha_s > 0$), and $k_\alpha = -1$ corresponds to a self-defocusing medium ($\alpha_s < 0$). The continuity of the magnetic fields leads to the dispersion relation for $\beta > n_f$:

$$\operatorname{th}(k_0\tilde{q}_f d) = -\frac{\tilde{q}_f(v_s^{k_\alpha}q_s + v_c^{k_\alpha}q_c)}{(\tilde{q}_f^2 + v_s^{k_\alpha}v_c^{k_\alpha}q_s q_c)}. \quad (130)$$

The energy flux of the guided-wave modes, scaled to unit length, propagating along the $y$ axis is given by the expression $P = P_s + P_f + P_c$, where

$$P_s = \frac{\beta q_s}{k_0 n_s^2 n_{2s}}(1 - v_s^{k_\alpha}); \quad (131)$$

$$P_c = \frac{\beta q_c}{k_0 n_c^2 n_{2c}}(1 - v_c^{k_\alpha});$$

(132)
$$P_f = \frac{\beta q_s^2}{2k_0 n_s^2 n_{2s}}(1 - v_s^{2k_\alpha})\left\{k_0 d\left(1 + \frac{q_s^2 v_s^{2k_\alpha}}{q_f^2}\right) + \frac{\sin(k_0 q_f d)}{q_f}\left[\left(1 - \frac{q_s^2 v_s^{2k_\alpha}}{q_f^2}\right)\cos(k_0 q_f d) + 2\frac{q_s v_s^{k_\alpha}}{q_f}\sin(k_0 q_f d)\right]\right\} \quad (133)$$

for $\beta < n_f$ and

$$P_f = \frac{\beta q_s^2}{2k_0 n_s^2 n_{2s}}(1 - v_s^{2k_\alpha})\left\{k_0 d\left(1 - \frac{q_s^2 v_s^{2k_\alpha}}{\tilde{q}_f^2}\right) + \frac{\operatorname{sh}(k_0\tilde{q}_f d)}{\tilde{q}_f}\left[\left(1 + \frac{q_s^2 v_s^{2k_\alpha}}{\tilde{q}_f^2}\right)\operatorname{ch}(k_0\tilde{q}_f d) + 2\frac{q_s v_s^{k_\alpha}}{\tilde{q}_f}\operatorname{sh}(k_0\tilde{q}_f d)\right]\right\} \quad (134)$$

for $\beta > n_f$ (Ref. 93).

Since the integration constants $z_s$ and $z_c$ in expressions (123), (124), (126), and (127) are related to the boundary conditions and depend on the energy flux transmitted by the NGWM, the propagation constant $\beta$ obtained by solving the dispersion relations (128) and (130) also depends on the energy flux.

Let us show that the determination of the dispersion relations requires knowledge of the shape of the field.[29,94,95] Integrating Maxwell's equations (120)–(122) using the fact that for $z \to \pm\infty$ we have $E_y(z) \to 0$ and $dE_y/dz \to 0$, we obtain

$$\left(\frac{dE_y^{I}}{dz}\right)^2 - k_0^2(\beta^2 - \varepsilon_s)(E_y^{I})^2 + \frac{k_0^2\alpha_s}{2}(E_y^{I})^4 = 0; \quad (135)$$

$$\left(\frac{dE_y^{II}}{dz}\right)^2 - k_0^2(\beta^2 - \varepsilon_f)(E_y^{II})^2 = c_f; \quad (136)$$

$$\left(\frac{dE_y^{III}}{dz}\right)^2 - k_0^2(\beta^2 - \varepsilon_c)(E_y^{III})^2 + \frac{k_0^2\alpha_c}{2}(E_y^{III})^4 = 0. \quad (137)$$

Here $c_f$ is an integration constant. We assume that on the waveguide boundaries $z = 0$ and $z = d$ the electric field takes the values $E_0$ and $E_d$, respectively. It is useful to define the quantities $\gamma_s = (\beta^2 - \varepsilon_s - \frac{1}{2}\alpha_s E_0^2)^{1/2}$ and $\gamma_c = (\beta^2 - \varepsilon_c - \frac{1}{2}\alpha_c E_d^2)^{1/2}$, after which the field gradients at $z = 0$ and $z = d$ can be written as

$$\lim_{z\to 0}\frac{dE_y^{I}}{dz} = \pm k_0\gamma_s E_0, \quad \lim_{z\to d}\frac{dE_y^{III}}{dz} = \pm k_0\gamma_c E_d. \quad (138)$$

Using (138), we can express the integration constant as

$$c_f = k_0^2(q_f^2 + \gamma_s^2)E_0^2 = k_0^2(q_f^2 + \gamma_c^2)E_d^2. \quad (139)$$

After some algebra, it can be shown that the electric fields $E_0^2$ and $E_d^2$ on the boundaries of the waveguide structure are related to each other through the following equation for a conic section (a hyperbola or an ellipse):

$$\frac{\alpha_s^2\alpha_c}{(\alpha_c\eta_s^2 - \alpha_s\eta_c^2)}\left(E_0^2 - \frac{\eta_s}{\alpha_s}\right)^2 - \frac{\alpha_s\alpha_c^2}{(\alpha_c\eta_s^2 - \alpha_s\eta_c^2)}\left(E_d^2 - \frac{\eta_c}{\alpha_c}\right)^2 = 1, \quad (140)$$

where $\eta_s = \varepsilon_f - \varepsilon_s$ and $\eta_c = \varepsilon_f - \varepsilon_c$. Depending on the specific parameters of the materials used in the construction of the three-layered planar structure, for a given value of $E_0^2$ there may exist two, one, or no values of $E_d^2$. We note that for a completely symmetric waveguide, i.e., if $\varepsilon_c = \varepsilon_s$ and $\alpha_c = \alpha_s$, the conic section is transformed into straight lines which are perpendicular to each other and have the equation

$$(E_0^2 - E_d^2)\left[(\varepsilon_f - \varepsilon_s) - \frac{\alpha_s}{2}(E_0^2 + E_d^2)\right] = 0, \quad (141)$$

which implies the existence of symmetric ($E_0 = E_d$) and antisymmetric ($E_0 = -E_d$) waves in a symmetric planar structure. The third possibility is an asymmetric wave, for which $E_0^2 = E_d^2$ and

$$E_d^2 = \frac{2(\varepsilon_f - \varepsilon_s)}{\alpha_s} - E_0^2. \quad (142)$$

This type of wave does not exist in the linear limit.[23,29] For NGWMs in an asymmetric layered structure we have the following eigenvalue equations ($\beta < n_f$):

$$\cos(k_0 q_f d) = \pm\frac{q_f^2 \pm \gamma_s\gamma_c}{[(q_f^2 + \gamma_s^2)(q_f^2 + \gamma_c^2)]^{1/2}}. \quad (143)$$

The plus sign in front of the right-hand side is used for even solutions, when $E_0$ and $E_d$ have the same sign ($E_0 > 0$, $E_d > 0$ or $E_0 < 0$, $E_d < 0$), and the minus sign is used for odd solutions, when $E_0$ and $E_d$ have opposite signs ($E_0 > 0$, $E_d < 0$ or $E_0 < 0$, $E_d > 0$). Here $q_f = (\varepsilon_f - \beta^2)^{1/2}$.

For NSWs ($\beta > n_f$) the eigenvalue equations are obtained from (143) and have the following form:

For even solutions

$$\operatorname{ch}(k_0\tilde{q}_f d) = \frac{-\tilde{q}_f^2 \pm \gamma_s\gamma_c}{[(\gamma_s^2 - \tilde{q}_f^2)(\gamma_c^2 - \tilde{q}_f^2)]^{1/2}}; \quad (144)$$

For odd solutions

$$\operatorname{ch}(k_0\tilde{q}_f d) = -\frac{\tilde{q}_f^2 \pm \gamma_s\gamma_c}{[(\gamma_s^2 - \tilde{q}_f^2)(\gamma_c^2 - \tilde{q}_f^2)]^{1/2}}. \quad (145)$$

We shall use the method of Ref. 29 to calculate the guided-wave energy flux without using any information about the optical fields outside the nonlinear medium. We again start from the integrals of Maxwell's equations [Eqs. (135)–(137)].

Differentiating Eqs. (135) and (136) with respect to $z$, we obtain

$$\frac{d}{dz}\left(\frac{1}{E_y^{\rm I}}\frac{dE_y^{\rm I}}{dz}\right)=-\frac{k_0^2\alpha_s}{2}(E_y^{\rm I})^2; \qquad (146)$$

$$\frac{d}{dz}\left(\frac{1}{E_y^{\rm III}}\frac{dE_y^{\rm III}}{dz}\right)=-\frac{k_0^2\alpha_c}{2}(E_y^{\rm III})^2. \qquad (147)$$

Using Eqs. (146) and (147) and the fact that

$$\lim_{z\to-\infty}\frac{1}{E_y^{\rm I}}\frac{dE_y^{\rm I}}{dz}=k_0 q_s; \qquad (148)$$

$$\lim_{z\to+\infty}\frac{1}{E_y^{\rm III}}\frac{dE_y^{\rm III}}{dz}=-k_0 q_c, \qquad (149)$$

after some algebra we obtain the expressions for the energy fluxes $P_s$ and $P_c$ in the nonlinear bounding media:

$$P_s=2\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}(2\alpha_s k_0)^{-1}\beta(q_s\pm\gamma_s); \qquad (150)$$

$$P_c=2\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}(2\alpha_c k_0)^{-1}\beta(q_c\pm\gamma_c). \qquad (151)$$

In a linear waveguide film we have

$$[E_y^{\rm II}(z)]^2=\frac{1}{2k_0^2q_f^2}\left[c_f-\frac{d}{dz}\left(E_y^{\rm II}\frac{dE_y^{\rm II}}{dz}\right)\right]. \qquad (152)$$

Integrating over the variable $z$, we obtain the following contribution to the total energy flux:

$$P_f=\frac{c\varepsilon_0\beta}{2}\int_0^d[E_y^{\rm II}(z)]^2dz$$

$$=\frac{c\varepsilon_0\beta}{2}\frac{1}{2k_0^2q_f^2}\left[c_fd-\left(E_y^{\rm II}\frac{dE_y^{\rm II}}{dz}\right)_d+\left(E_y^{\rm II}\frac{dE_y^{\rm II}}{dz}\right)_0\right]. \qquad (153)$$

Using (138), the energy flux in the linear film can be written as

$$P_f=\frac{c\varepsilon_0\beta}{2}\frac{1}{2k_0^2q_f^2}[k_0^2(q_f^2+\gamma_s^2)E_0^2d\mp k_0\gamma_cE_d^2\pm k_0\gamma_sE_0^2]. \qquad (154)$$

Then the total energy flux in the planar structure is $P=P_s+P_f+P_c$, where $P_s$, $P_c$, and $P_f$ are given by Eqs. (150), (151), and (154), respectively. Since $E_d^2$ is expressed in terms of $E_0^2$, and $E_0^2$ is a function of the effective index of refraction $\beta$ via the eigenvalue equation, the energy flux $P$ can vary with varying $\beta$ for a particular frequency $\omega$.

As a special example, let us consider the case $n_{2s}>0$ and $n_{2c}>0$, i.e., the two media surrounding the film have a self-focusing nonlinearity. This case is a rich source of new phenomena.[93] We see from Fig. 13 that for $n_c=n_s$ and $n_{2c}\neq n_{2s}$, i.e., different optical nonlinearities, the nonlinear-wave solutions split up into two decoupled branches A and B. If $n_c\neq n_s$, the curves shift and bend relative to the energy-flux axis. Branch B exists only above a certain energy-flux level. Branch A, which develops from the linear limit, leads to localization of the field in the medium with largest nonlinearity (the overlayer), i.e., for large energy fluxes it degenerates into the corresponding surface wave propagating on the interface. Branch B appears when the field extremum is in the medium with the smallest nonlinearity (the substrate) and terminates with the field maximum in the two media

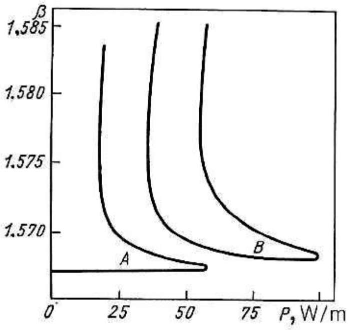


FIG. 13. Behavior of the effective index of refraction as a function of the energy flux of $TE_0$ waves. Parameters: $d=2$ μm, $n_c=n_s=1.55$, $n_f=1.57$, $n_{2c}=2\times10^{-9}$ m$^2$/W, $n_{2s}=10^{-9}$ m$^2$/W (Ref. 93).

surrounding the film. A nonlinear guided-wave mode in branch A is excited until the maximum of the field is reached. Further increase of the energy flux of the guided wave is possible only upon switching to branch B, in which the field distribution and, consequently, the attenuation differ from the analogous quantities in branch A. Therefore, the switching from branch to branch will be accompanied by a change of the transmitted intensity. Subsequent decrease of the energy flux of the guided wave in branch B leads to reverse switching to branch A at a significantly lower value of the energy flux than that when the switching to branch B occurred. This behavior can lead to a hysteresis look or to bistability.[94]

It has been shown in a number of studies[16,24,28,95–98] that the solutions of Maxwell's equations for a waveguide film obeying the Kerr law can be expressed in terms of Jacobi elliptic functions.

Let us consider an asymmetric planar layered structure composed of an optically linear medium (the substrate) with dielectric constant $\varepsilon_s$ in region I ($z<0$), a dielectric film of thickness $d$ obeying the Kerr law and having dielectric function $\varepsilon=\varepsilon_f+\alpha_f|E|^2$ in region II ($0<z<d$), and a linear medium (the overlayer) with dielectric constant $\varepsilon_s$ in region III ($z>d$). Maxwell's equations for TE-polarized waves propagating along the $x$ axis have the form

$$\frac{d^2E_y^{\rm I}}{dz^2}-k_0^2(\beta^2-\varepsilon_s)E_y^{\rm I}=0, \quad z<0; \qquad (155)$$

$$\frac{d^2E_y^{\rm II}}{dz^2}-k_0^2(\beta^2-\varepsilon_f)E_y^{\rm II}+k_0^2\alpha_f(E_y^{\rm II})^3=0, \quad 0<z<d; \qquad (156)$$

$$\frac{d^2E_y^{\rm III}}{dz^2}-k_0^2(\beta^2-\varepsilon_c)E_y^{\rm III}=0, \quad z>d. \qquad (157)$$

Let us trace the solutions, localized near the surface of the thin film, for which the field tends to zero for $|z|\to\infty$. Then from Eqs. (155) and (157) we find

$$E_y^{\rm I}(z)=E_0\exp(k_0q_sz), \quad z<0; \qquad (158)$$

$$E_y^{\rm III}(z)=E_d\exp[-k_0q_c(z-d)], \quad z>d, \qquad (159)$$

where $q_s=(\beta^2-\varepsilon_s)^{1/2}$ and $q_c=(\beta^2-\varepsilon_c)^{1/2}$. Let us consider the case of a self-defocusing nonlinearity of the Kerr type ($\alpha_f=-|\alpha_f|<0$). In this case $\beta<n_f$, where $n_f=\varepsilon_f^{1/2}$, and the exact solution of Eq. (156) can be expressed in terms of Jacobi elliptic functions[99] in the form

$$E_y^{\rm II}(z)=\left(\frac{2}{|\alpha_f|}\right)^{1/2}m^{1/2}t\,\mathrm{sn}(k_0tz+\theta/m) \qquad (160)$$

for $0<z<d$, where $t=(1+m)^{-1/2}q_f$, $q_f=(n_f^2-\beta^2)^{1/2}$, and $\theta$ is the integration constant. Here sn is the sin-type Jacobi elliptic function, and $m$ is the parameter of this function $(0<m<1)$. From the boundary conditions we obtain the eigenvalue equations

$$\frac{\mathrm{cn}(k_0td+\theta/m)\,\mathrm{dn}(k_0td+\theta/m)}{\mathrm{sn}(k_0td+\theta/m)}=-\frac{q_c}{t}; \qquad (161)$$

$$\frac{\mathrm{cn}(\theta/m)\,\mathrm{dn}(\theta/m)}{\mathrm{sn}(\theta/m)}=\frac{q_s}{t}, \qquad (162)$$

where $\mathrm{cn}^2(\theta/m)=1-\mathrm{sn}^2(\theta/m)$, $\mathrm{dn}^2(\theta/m)=1-m\,\mathrm{sn}^2(\theta/m)$, and cn is the cos-type Jacobi elliptic function.

We shall show that for $m=0$ Eqs. (161) and (162) can be used to find the dispersion relation for the TE-polarized mode of a linear asymmetric waveguide. Using Eqs. (155)–(157), we find the total energy flux $P=P_s+P_f+P_c$ transmitted by the NGWM (Refs. 24, 96, and 97):

$$P_s=P_0\beta mt^2\frac{\mathrm{sn}^2(\theta/m)}{q_s}; \qquad (163)$$

$$P_f=2P_0\beta t\,[k_0td-E(k_0td+\theta/m)+E(\theta/m)]; \qquad (164)$$

$$P_c=P_0\beta mt^2\frac{\mathrm{sn}^2(k_0td+\theta/m)}{q_c}, \qquad (165)$$

where $P_0=(2|\alpha_f|k_0)^{-1}(\varepsilon_0/\mu_0)^{1/2}$ and $E(\theta/m)$ is the elliptic integral of the second kind.[99]

In the approach proposed in Refs. 96 and 97, the propagation constant $\beta$ is treated as a function of the Jacobi elliptic-function parameter. Solving Eqs. (161) and (162), we obtain the values of $\beta$ for each $m$ $(0<m<1)$. Then, using (163)–(165) we determine the quantities $P_s$, $P_f$, and $P_c$, thereby obtaining the dependence $P=P(\beta)$. A different approach developed in Ref. 95 makes use of the boundary field amplitudes $E_0$ and $E_d$ and can be used to calculate the dependence of the propagation constant on the energy flux.

In the case of a self-focusing dielectric film $(\alpha_f>0)$, the exact solution of the nonlinear wave equation (156) has the form

$$E_y^{\mathrm{II}}(z)=\left(\frac{2}{\alpha_f}\right)^{1/2}m^{1/2}t\,\mathrm{cn}(k_0tz+\theta/m), \qquad (166)$$

where $t=(1-2m)^{-1/2}q_f$ for $\beta<n_f$ and $t=(2m-1)^{-1/2}\tilde{q}_f$ for $\beta>n_f$ and $\tilde{q}_f=(\beta^2-n_f^2)^{1/2}$.

The dispersion relations for these NGWMs are given by

$$\frac{\mathrm{sn}(k_0td+\theta/m)\,\mathrm{dn}(k_0td+\theta/m)}{\mathrm{cn}(k_0td+\theta/m)}=\frac{q_c}{t}; \qquad (167)$$

$$\frac{\mathrm{sn}(\theta/m)\,\mathrm{dn}(\theta/m)}{\mathrm{cn}(\theta/m)}=-\frac{q_s}{t}. \qquad (168)$$

Using (158), (159), and (166), we obtain the expressions for the energy flux in the linear substrate $P_s$, in the nonlinear waveguide film $P_f$, and in the overlayer $P_c$:

$$P_s=P_0\beta mt^2\frac{\mathrm{cn}^2(\theta/m)}{q_s}; \qquad (169)$$

$$P_f=2P_0\beta t\,[E(k_0td+\theta/m)-E(\theta/m)-(1-m)k_0td]; \qquad (170)$$

$$P_c=P_0\beta mt^2\frac{\mathrm{cn}^2(k_0td+\theta/m)}{q_c}. \qquad (171)$$

In order to determine the energy flux $P$ for each value of the propagation constant $\beta$, it is necessary to calculate numerically the dependence of $\beta$ on the parameter $m$ using the dispersion relations (167) and (168). In Ref. 95 it was shown

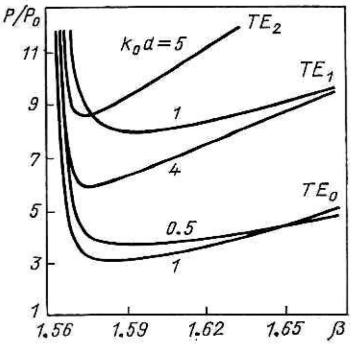


FIG. 14. Beta dependence of the dimensionless energy flux for nonlinear $TE_0$, $TE_1$, and $TE_2$ waves guided by a self-focusing dielectric film. Parameters: $\varepsilon_s=2.45$, $\varepsilon_f=2.3$, and $\varepsilon_c=1$ (Ref. 95).

for a particular choice of the parameters $(n_c<n_f<n_s)$ that all $TE_m$ $(m\geqslant 0)$ branches induced by a self-focusing nonlinearity begin to propagate at some energy-flux threshold (Fig. 14). This threshold behavior was first demonstrated in Refs. 96 and 97 for the $TE_m$ $(m\geqslant 1)$ branches. The authors of that study used different sets of parameters, allowing them to obtain a linear limit for the TE branch. The existence of two different values of the propagation constant for a single value of the energy flux, and also the existence of a threshold value of the energy flux for a nonlinear layered structure are certainly of interest, for example, for designing optical switches.

### Nonlinear guided-wave modes in a medium with saturation characteristics

Let us consider a dielectric planar structure consisting of a linear substrate with dielectric constant $\varepsilon_s$ in region I $(z<0)$, a linear dielectric film of thickness $d$ and dielectric constant $\varepsilon_f$ in region II $(0<z<d)$, and a nonlinear self-focusing overlayer in region III $(z>d)$ characterized by the dielectric tensor (19).

For TE-polarized waves the electric field vector has the form $\mathbf{E}=(0,E_y,0)$ and Maxwell's equations are given by the relations

$$\frac{d^2E_y^{\mathrm{I}}}{dz^2}-k_0^2(\beta^2-\varepsilon_s)E_y^{\mathrm{I}}=0,\quad z<0; \qquad (172)$$

$$\frac{d^2E_y^{\mathrm{II}}}{dz^2}-k_0^2(\beta^2-\varepsilon_f)E_y^{\mathrm{II}}=0,\quad 0<z<d; \qquad (173)$$

$$\frac{d^2E_y^{\mathrm{III}}}{dz^2}-k_0^2[\beta^2-\varepsilon_c-\varepsilon_c^{NL}(E_y^{\mathrm{III}})]E_y^{\mathrm{III}}=0,\quad z>d. \qquad (174)$$

Integrating Eqs. (172)–(174) once, we obtain

$$\Phi_s=\left(\frac{dE_y}{dz}\right)^2=k_0^2(\beta^2-\varepsilon_s)E_y^2,\quad z<0; \qquad (175)$$

$$\Phi_f=\left(\frac{dE_y}{dz}\right)^2=k_0^2(\beta^2-\varepsilon_f)E_y^2+c_f,\quad 0<z<d; \qquad (176)$$

$$\Phi_c=\left(\frac{dE_y}{dz}\right)^2=k_0^2\left[(\beta^2-\varepsilon_c)E_y^2-\int_0^{E_y^2}\varepsilon_c^{NL}(E_y^2)\,d(E_y^2)\right],\quad z>d. \qquad (177)$$

Here $c_f$ is the integration constant.

For TE-polarized waves the quantities $E_y$ and $dE_y/dz$ are continuous along the interfaces $z=0$ and $z=d$. Using this fact, we obtain a relation between the integration constant $c_f$ and the values of the field $E_0$ and $E_d$ at the corresponding boundaries $z = 0$ and $z = d$:

$$\frac{c_f}{k_0^2} = (\varepsilon_f - \varepsilon_s) E_0^2 = (\varepsilon_f - \varepsilon_c) E_d^2 - \int_0^{E_d^2} \varepsilon_c^{NL}(E_y^2)\, d(E_y^2). \quad (178)$$

Since $dE_y^{II}/dz = \Phi_f^{1/2}$, and the result of the integration over region II can be written as

$$\int_{E_0}^{E_d} \frac{dE_y}{\Phi_f^{1/2}} = d, \quad (179)$$

we arrive at the following dispersion relation for $TE_m$ waves[40]:

$$\mathrm{tg}\,(k_0 q_f d) = q_f (q_s + \bar{q}_c)(q_f^2 - q_s \bar{q}_c)^{-1}, \quad (180)$$

where $q_s = (\beta^2 - \varepsilon_s)^{1/2}$, $q_f = (\varepsilon_f - \beta^2)^{1/2}$, $\bar{q}_c = (-1)^{M_c}(\beta^2 - \varepsilon_{cNL})^{1/2}$, and

$$\varepsilon_{cNL} = \varepsilon_c + \frac{1}{E_d^2} \int_0^{E_d^2} \varepsilon_c^{NL}(E_y^2)\, d(E_y^2), \quad (181)$$

with $M_c = 1$ if the self-focusing peak (field maximum) occurs in the nonlinear overlayer, and $M_c = 0$ otherwise (no maximum). If $M_c = 1$, the amplitude of the field $\overline{E}_y$ can be computed from the condition $\Phi_c(\overline{E}_y, \beta) = 0$.

In the case $\beta > n$ the dispersion relation is given by

$$\mathrm{th}\,(k_0 \tilde{q}_f d) = -\frac{\tilde{q}_f (q_s + \bar{q}_c)}{(\tilde{q}_f^2 + q_s \bar{q}_c)}, \quad (182)$$

which has a solution only for $M_c = 1$ and $\bar{q}_c = -(\beta^2 - \varepsilon_{cNL})^{1/2} < 0$. In the absence of a self-focusing peak in the overlayer ($M_c = 0$) we have $\bar{q}_c = q_c = [\varphi(u)]^{1/2}$, where $\varphi(u)$ is given by (32) or (33). Using Eqs. (178) and (179), we obtain the following expressions for the energy fluxes:

$$P_s = \frac{1}{2} P_0 \beta \frac{u}{q_s} \frac{(q_c^2 + q_f^2)}{(q_s^2 + q_f^2)}; \quad (183)$$

$$P_f = \frac{1}{2} P_0 \beta u \left[ k_0 d \frac{(q_c^2 + q_f^2)}{q_f^2} + \frac{q_c}{q_f^2} + \frac{q_s}{q_f^2} \frac{(q_c^2 + q_f^2)}{(q_s^2 + q_f^2)} \right]; \quad (184)$$

$$P_c = \frac{1}{2} P_0 \beta \int_0^u \frac{dx}{[\varphi(x)]^{1/2}}, \quad (185)$$

where $u = \alpha_c E_d^2$. In the case of an overlayer satisfying the Kerr law, the integral (185) can be computed analytically, giving

$$P_c = 2P_0\beta \left[ \left(\beta^2 - \varepsilon_c + \frac{1}{2} u\right)^{1/2} - (\beta^2 - \varepsilon_c)^{1/2} \right]. \quad (186)$$

For a self-focusing overlayer ($\alpha_c > 0$) the field maximum is reached at a fairly large value of the energy flux, i.e., $M_c = 1$ and $\bar{q}_c = -q_c$, where $q_c = [\varphi(u)]^{1/2}$ and $\varphi(u)$ are given by Eqs. (32) and (33).

We can summarize the result as follows:

$$P_f = \frac{1}{2} P_0 \beta u \left[ k_0 d \frac{(q_c^2 + q_f^2)}{q_f^2} - \frac{q_c}{q_f^2} + \frac{q_s}{q_f^2} \frac{(q_c^2 + q_f^2)}{(q_s^2 + q_f^2)} \right]; \quad (187)$$

$$P_c = \frac{1}{2} P_0 \beta \left[ \int_0^u \frac{dx}{[\varphi(x)]^{1/2}} - \int_{\bar{u}}^{u} \frac{dx}{[\varphi(x)]^{1/2}} \right]. \quad (188)$$

with $P_s$ given by Eq. (183).

For a self-focusing overlayer, the integrals in (188) can be evaluated analytically, giving

$$P_c = 2P_0\beta \left[ \left(\beta^2 - \varepsilon_c + \frac{1}{2} u\right)^{1/2} + (\beta^2 - \varepsilon_c)^{1/2} \right]. \quad (189)$$

For $\beta > n_f$ we finally obtain

$$P_s = \frac{1}{2} P_0 \beta \frac{u}{q_s} \frac{(q_c^2 - \tilde{q}_f^2)}{(q_s^2 - \tilde{q}_f^2)}; \quad (190)$$

$$P_f = \frac{1}{2} P_0 \beta u \left[ \frac{q_c}{\tilde{q}_f^2} - k_0 d \frac{q_c^2 - \tilde{q}_f^2}{\tilde{q}_f^2} - \frac{q_s}{\tilde{q}_f^2} \frac{(q_c^2 - \tilde{q}_f^2)}{(q_s^2 - \tilde{q}_f^2)} \right], \quad (191)$$

with $P_c$ given by Eq. (188).

Let us consider the liquid crystal MBBA ($n_c = 1.55$, $n_{2c} = 10^{-9}\, m^2/W$) deposited on a glass waveguide ($n_f = 1.57$, $d = 2\ \mu$m) with substrate chosen such that $n_s = n_c = 1.55$. In Fig. 15 we show the results of numerical calculations of the energy fluxes of TE-polarized waves guided by a planar layered structure with nonlinear overlayer, characterized by the dielectric function (19), for several values of the dimensionless parameter $\varepsilon_{sat}$. Comparing the behavior of the solutions for $TE_0$ and $TE_1$ waves with the case in which the Kerr law is satisfied ($\varepsilon_{sat} = \infty$), we see that the characteristics of these waves are preserved if the value of $n_{sat} = (\varepsilon_c + \varepsilon_{sat})^{1/2} - n_c$ is not small. The absolute maximum of the energy flux of the $TE_1$ wave depends strongly on the value of $n_{sat}$, and for small values it grows sharply (for example, for $\varepsilon_{sat} = 0.0155$ or $n_{sat} = 0.005$). In the case of the $TE_0$ branch, as the energy flux increases the propagation constant $\beta$ reaches its asymptotic value $(\varepsilon_c + \varepsilon_{sat})^{1/2}$ (the curve corresponding to $\varepsilon_{sat} = 0.1256$ in Fig. 15). Therefore, the characteristic features of the $TE_m$ branches are preserved when $n_{sat}$ is much smaller than the difference of the indices of refraction $n_f - n_c$ or $n_f - n_s$ for small values of the energy flux. An important conclusion which follows from these calculations is that if the saturation effects are sufficiently large, they might change, and in some cases even eliminate, a number of the interesting energy flux-dependent features of NGWMs.

In Ref. 100 it was shown that saturation effects considerably influence the two uncoupled TE branches in the case

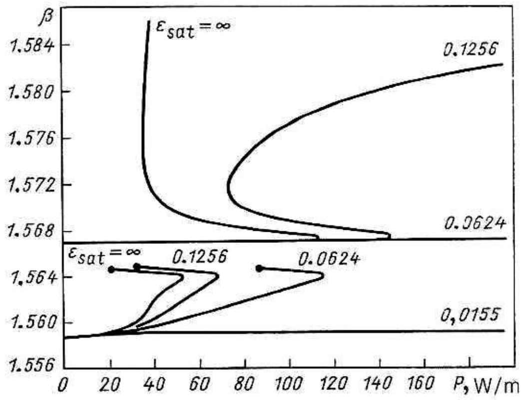


FIG. 15. Behavior of the propagation constant $\beta$ as a function of the energy flux $P$ for a self-focusing overlayer. The parameters are the same as in Fig. 9. Each curve corresponds to the given value of the saturation $\varepsilon_{sat}$ (Ref. 100).

of self-focusing bounding media ($n_{2c} \neq n_{2s}$) capable of saturation. When saturation effects occur in the substrate and in the overlayer, the properties of the NGWM switching change markedly, which might have applications in integrated optical devices.

Let us consider a three-layered asymmetric structure consisting of a linear substrate with dielectric constant $\varepsilon_s$, a thin dielectric film of thickness $d$ with dielectric constant $\varepsilon_f$, and a nonlinear self-defocusing ($\alpha_c < 0$) overlayer characterized by one of two types of nonlinearity capable of saturation:

$$\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_c-\varepsilon_{sat}\left[1-\exp\left(-\frac{|\alpha_c|E_y^2}{\varepsilon_{sat}}\right)\right]; \quad (192)$$

$$\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_c-\frac{|\alpha_c|E_y^2}{\left(1+\frac{|\alpha_c|}{\varepsilon_{sat}}E_y^2\right)}. \quad (193)$$

We note that both types of dielectric tensor are tensors of the Kerr type, i.e., $\varepsilon \to \varepsilon_c - |\alpha_c|E_y^2$ for small field strengths, and they have the same saturation level ($\varepsilon_c - \varepsilon_{sat}$). In this case the field has a virtual maximum in the nonlinear overlayer ($M_c = 0$), and the dispersion relation is of the form

$$\mathrm{tg}\,(k_0 q_f d)=\frac{q_f(q_s+q_c)}{(q_f^2-q_s q_c)}, \quad (194)$$

where $q_c = [\varphi(u)]^{1/2}$, $u = |\alpha_c|E_d^2$, and

$$\varphi(u)=\beta^2-\varepsilon_c+\varepsilon_{sat}-\frac{\varepsilon_{sat}^2}{u}\left[1-\exp\left(-\frac{u}{\varepsilon_{sat}}\right)\right]; \quad (195)$$

$$\varphi(u)=\beta^2-\varepsilon_c+\varepsilon_{sat}-\frac{\varepsilon_{sat}^2}{u}\ln\left(1+\frac{u}{\varepsilon_{sat}}\right), \quad (196)$$

which correspond to the dielectric tensors (21) and (22).

The total energy flux is $P = P_s + P_f + P_c$, where $P_s$, $P_f$, and $P_c$ are given by expressions (183)–(185). We have carried out numerical calculations of the effective index of refraction $\beta$ as a function of the total energy flux for several values of the dimensionless parameter $\varepsilon_{sat}$. In Fig. 16 we show the results for TE waves guided by a $GaAl_xAs_{1-x}$ structure for the dielectric functions (21) and (22). In the case of a self-defocusing overlayer obeying the Kerr law and $n_c > n_s$, there is a maximum energy flux which can be transmitted. For example, for a realistic overlayer with the condition that $n_{sat} = n_c - (\varepsilon_c - \varepsilon_{sat})^{1/2}$ is sufficiently large (in Fig. 16, the curves with $\varepsilon_{sat} = 0.0676$, i.e., $n_{sat} = 0.01$) the transmitted energy flux has a limiting value. This phenomenon can be used in the design of devices which limit the maximum value of the transmitted energy flux. In such devices the energy-flux cutoff might be achieved, for example, by using a thin film of $GaAl_xAs_{1-x}$ with variable index of refraction $n_f(x)$ (see Ref. 58).

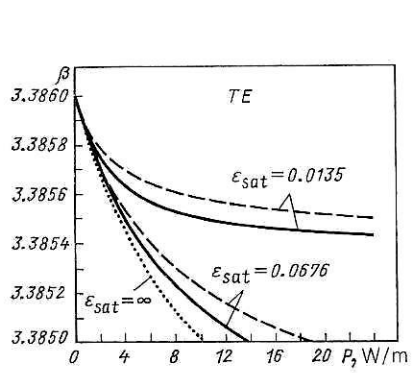


FIG. 16. Behavior of the effective index of refraction as a function of the energy flux $P$ in the case of a self-defocusing overlayer. Parameters: $n_f = 3.39$, $n_c = 3.385$, $n_{2c} = -2\times10^{-9}$ m²/W, $n_s = 3.38$, $d = 1.07$ μm, and $\lambda = 0.82$ μm. The solid lines correspond to the dielectric function (19) and the dashed lines to (20).

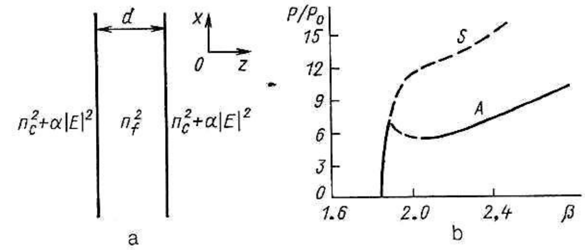


FIG. 17. Dimensionless energy flux $P/P_0$ as a function of $\beta$ for a symmetric planar waveguide. The curves are labeled S for the symmetric wave and A for the asymmetric wave. Above the bifurcation point $\beta = \beta_{cr} \simeq 1.89$ both branches are unstable (broken line). The doubly degenerate A-wave is stabilized in the region where the slope of the curve is positive.

#### Stability of propagation of NGWMs

The question of the stability of the various $TE_m$ solutions for NGWMs has been studied, for the most part numerically, in Refs. 44–50. The analytical analysis is complicated by the fact that the eigenvalues of the linearized system lie on the imaginary axis, making it impossible to use the usual techniques for analyzing dissipative systems (as long as losses are not deliberately introduced into the system).

Let us consider the case of TE-polarized guided-wave modes in a symmetric nonlinear planar waveguide (Fig. 17a) consisting of a linear waveguide film with index of refraction $n_f$, bounded on both sides by identical overlayer and substrate obeying the Kerr law, i.e., $n_c = n_s$ and $n_{2c} = n_{2s}$. In Fig. 17b we show the dependence of the dimensionless energy flux on $\beta$ for a symmetric layered structure with the following parameters: $n_c = n_s = 1.5$, $n_f = 2$, and $d/\lambda = 0.4$ (see Refs. 23 and 44). In Fig. 18 we show the evolution of the field distribution as a function of the propagation distance. As the initial data we used the instantaneous distribution of the electric field before and after the bifurcation point of the symmetric $TE_0$ branch (S).

For $\beta = 1.89 < \beta_{cr}$ the symmetric wave is stable to propagation for distances of order 180 wavelengths (Fig. 18a), whereas for $\beta = 1.90 > \beta_{cr}$ the S wave loses its stability after traveling only a distance of 18 wavelengths (Fig. 18b). In this case the wave goes either into the overlayer or into the substrate. Therefore, the symmetric $TE_0$ branch (S) loses its stability in the region of positive slope of the branch of the dispersion curve (Fig. 18b) at the bifurcation point (the critical value is $\beta = \beta_{cr} \simeq 1.89$), where a doubly degenerate asymmetric wave A appears.

Let us consider an asymmetric three-layered planar structure consisting of a linear substrate with dielectric constant $\varepsilon_s = nn_s^2$ in region I ($z < 0$), a thin dielectric film of thickness $d$ with dielectric constant $\varepsilon_f = n_f^2$ in region II ($0 < z < d$), and a nonlinear self-focusing overlayer characterized by a diagonal dielectric tensor of the type (19) or (20) in region III ($z > d$). A TE-polarized wave of frequency $\omega$ propagates along the $x$ axis, and the electric field is uniform in the $y$ direction ($z$ is the transverse coordinate).

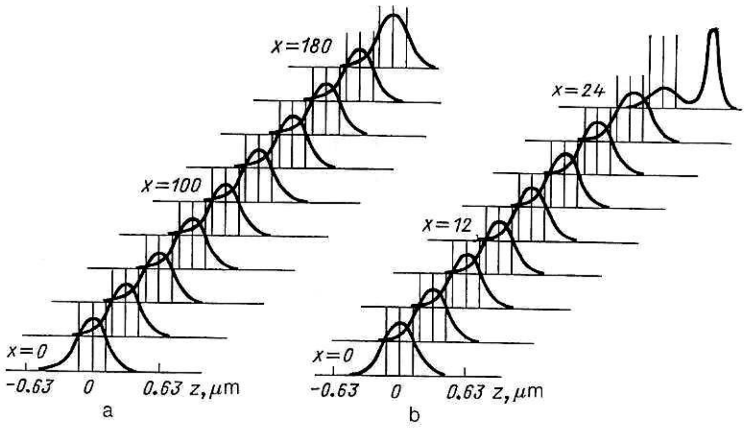


FIG. 18. Evolution of the field distribution as a function of the propagation distance for the symmetric wave (S) below, $\beta = 1.89$ (a) and above, $\beta = 1.90$ (b), the bifurcation point $\beta = \beta_{cr}$ (Ref. 44).

The nonzero component of the electric field, $E_y(\mathbf{r}, t)$, is given by (11). Then in the usual approximation of a slowly varying envelope, we obtain the following equation for the amplitude $A(x, z) = \alpha_c^{1/2} E_y(x, z)$:

$$-2i\beta k_0 \frac{\partial A}{\partial x} = \frac{\partial^2 A}{\partial z^2} - \gamma^2(z) k_0^2 A + \theta(z) k_0^2 f(|A|^2) A. \quad (197)$$

Here $\theta(z) = 0$ for $-\infty < z < d$ and $\theta(z) = 1$ for $z > d$; $\gamma^2(z) = \beta^2 - n_s^2$ for $z < 0$; $\gamma^2(z) = \beta^2 - n_f^2$ for $0 < z < d$; $\gamma^2(z) = \beta^2 - n_c^2$ for $z > d$; and

$$f(|A|^2) = \varepsilon_{sat}\left[1 - \exp\left(-\frac{|A|^2}{\varepsilon_{sat}}\right)\right]; \quad (198)$$

$$f(|A|^2) = \frac{|A|^2}{1 + |A|^2/\varepsilon_{sat}}, \quad (199)$$

which correspond to the dielectric functions (19) and (20). We note that for a medium obeying the Kerr law, $f(|A|^2) = |A|^2$. The solution of Eq. (197) independent of the $x$ components, $A(0,z) = A_0(z)$, corresponds to stationary NGWMs, whose effective index of refraction $\beta$ is determined by a dispersion relation of the form $\beta = \beta(P)$. Equation (197) has two integrals of the motion: $I(\beta)$ is given by (39), and

$$H(\beta) = k_0 \int_{-\infty}^{\infty} \left\{\left|\frac{\partial A}{\partial z}\right|^2 + k_0^2\gamma^2(z)|A|^2 - k_0^2\theta(z) g(|A|^2)\right\} dz. \quad (200)$$

Here

$$g(|A|^2) = \int_0^{|A|^2} f(|A|^2)\, d(|A|^2), \quad (201)$$

where $F(|A|^2)$ is given Eqs. (198) and (199). For a medium obeying the Kerr law we obtain $g(|A|^2) = 1/2|A|^4$. Then for a medium with saturation properties we find

$$g(|A|^2) = \frac{1}{2}\varepsilon_{sat}\left[1 - \exp\left(-\frac{|A|^2}{\varepsilon_{sat}}\right)\right]|A|^2; \quad (202)$$

$$g(|A|^2) = \frac{1}{2}\varepsilon_{sat}\left[\ln\left(1 + \frac{|A|^2}{\varepsilon_{sat}}\right)\right]|A|^2, \quad (203)$$

corresponding to the dielectric functions (19) and (20). We note that for arbitrary solutions of Eq. (197) we have $dI/dx = dH/dx = 0$.

The Crank–Nicolson scheme[75,76] was used for a numerical investigation of the stability of thin-film NGWMs. The solution of the system of nonlinear equations for successive steps in $x$ was found by the Newton method in conjunction with a matrix representation in $z$. The chosen difference scheme (the mesh size was $k_0\Delta x = k_0\Delta z = 0.4$) ensured that the integrals $I(\beta)$ and $H(\beta)$ were conserved on the mesh. The conservation of the total energy flux $P(\beta)$ was always at least 99%. Unstable waves were defined as waves whose field distribution along the $z$ coordinate varied in proportion to the propagation along the $x$ coordinate.

For a nonlinear overlayer obeying the Kerr law, stationary waves are unstable where the nonlinear dispersion curve $\beta = \beta(P)$ has negative slope and they enter the nonlinear medium. In this case, a single soliton is emitted, i.e., a self-focusing channel arises in which the NGWM decays (Fig. 19).[47,49,52]

In the case of a nonlinear overlayer with saturation properties described by the dielectric tensor (19) and $\varepsilon_{sat} = 0.1256$, in the region where the nonlinear dispersion curve has negative slope (see Fig. 15) the $TE_0$ wave is unstable. In Fig. 20 we show the result of a numerical calculation of the propagation of a $TE_0$ wave over a distance corresponding to the first 400 wavelengths for $\beta = 1.5685$. The wave remains fairly stable and its field is bounded by the waveguide film, whose maximum oscillates aperiodically between the film boundaries. In Fig. 21 we show the evolution of the field distribution of a $TE_0$ NGWM during the propa-

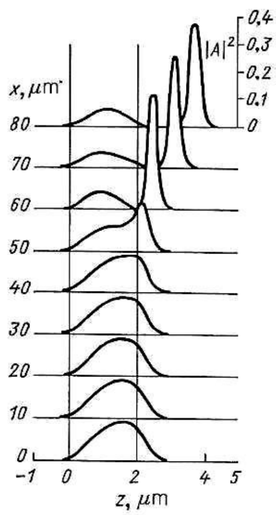


FIG. 19. Evolution of the field distribution of TE NGWMs as a function of the propagation distance for an overlayer obeying the Kerr law. The initial field distribution $A_0(z)$ corresponds to $\beta = 1.5685$.

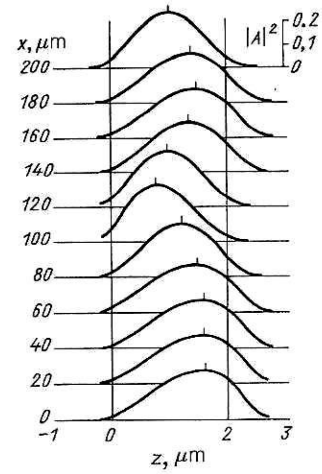


FIG. 20. Evolution of the field distribution of $TE_0$ NGWMs as a function of the propagation distance for an overlayer with the dielectric function (19) and $\varepsilon_{sat} = 0.1256$.

gation for the case of an overlayer with saturation properties characterized by the dielectric tensor (19) and $\varepsilon_{sat} = 0.1256$ for $\beta = 1.58$ in the region where the slope of the dispersion curve $\beta = \beta(P)$ is positive. In this case the nonlinear stationary wave $A_0(z)$ is stable. The results of numerical analysis of stationary-wave propagation confirm the fact that $TE_0$ NGWMs are stable in asymmetric layered structures in which self-focusing occurs only in one of the bounding media in the region where the nonlinear dispersion curve has positive slope.[52]

When dissipation is present the Kramers–Kronig relation states that nonlinear refraction must at least be accompanied by linear absorption. Therefore, Eq. (197) should include an absorption term

$$-2i\beta k_0 \frac{\partial A}{\partial x} = \frac{\partial^2 A}{\partial z^2} - \gamma^2(z) k_0^2 A + \theta(z) k_0^2 f(|A|^2) A + i\beta k_0^2 \Gamma(z) A, \qquad (204)$$

where $\Gamma(z)$ is the absorption profile. The effects of linear absorption of $TE_0$ NGWMs in asymmetric waveguides with overlayer obeying the Kerr law have been studied in Ref. 55 using the beam-propagation method.[101,102]

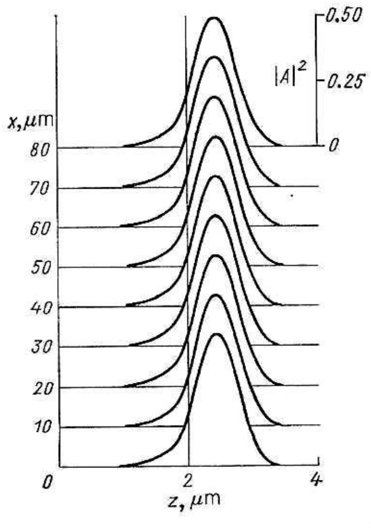


FIG. 21. The same as in Fig. 20, but for $\beta = 1.58$.

## 3. TRANSVERSE-ELECTRIC POLARIZED NONLINEAR SURFACE PLASMONS (NSPs)

### Bounding media obeying the Kerr law

There has recently been a surge of interest in the theoretical and experimental study of NSPs.[78,79,103] It is well known that the boundary between two linear dielectric media cannot support TE-polarized surface polaritons. However, if the index of refraction of one of the two media depends on the flux intensity, TE-polarized surface polaritons can exist for energy fluxes above the threshold.

A number of studies[105-108] have predicted that, in addition to the well studied NGWMs which can be supported by linear dielectric films surrounded by at least one medium (the overlayer or the substrate) with intensity-dependent index of refraction, there exists a new type of wave for metallic films bounded on one side by a nonlinear medium. Nonlinear TE-polarized waves guided by very thin metal films surrounded on both sides with a nonlinear medium exist only at energies above the threshold characteristic of the given material.[106] In Ref. 107 it was shown that TE NSP also exist in planar configurations with a nonlinear substrate or a nonlinear overlayer. A threshold value of the energy flux and a limited range of possible values of the propagation constant exist in the case of such asymmetric structures for nonlinear dispersion curves.

Let us study the characteristics of nonlinear TE-polarized waves guided by an asymmetric three-layered configuration. The configuration contains the following: a linear substrate with dielectric constant $\varepsilon_s$ in region I ($z<0$), a very thin metal film with dielectric constant $\varepsilon_f = -|\varepsilon_f| < 0$ in region II ($0<z<d$), and a nonlinear self-focusing Kerr-type overlayer in region III ($z>d$). The field distribution in the substrate, the film, and the overlayer is given by the following expressions:

$$E_y^{I}(z) = E_0 \exp(k_0 q_s z), \quad z < 0; \qquad (205)$$

$$E_y^{II}(z) = E_0 \left[ \frac{q_s + \tilde{q}_f}{2\tilde{q}_f} \exp(k_0 \tilde{q}_f z) + \frac{(\tilde{q}_f - q_s)}{2\tilde{q}_f} \exp(-k_0 \tilde{q}_f z) \right], \quad 0 < z < d; \qquad (206)$$

$$E_y^{III}(z) = \left( \frac{2}{\alpha_c} \right)^{1/2} q_c \{\mathrm{ch}[k_0 q_c (z - z_c)]\}^{-1}, \quad z > d, \qquad (207)$$

where $q_s = (\beta^2 - \varepsilon_s)^{1/2}$, $\tilde{q}_f = (\beta^2 + |\varepsilon_f|)^{1/2}$, and $q_c = (\beta^2 - \varepsilon_c)^{1/2}$. Here the surface field $E_0$ has the form

$$\alpha_c E_0^2 = 2\{1 - \mathrm{th}^2[k_0 q_c (z_c - d)]\} \times \left[ \mathrm{ch}(k_0 \tilde{q}_f d) + \frac{q_s}{\tilde{q}_f} \mathrm{sh}(k_0 \tilde{q}_f d) \right]^{-2}. \qquad (208)$$

Owing to the condition that $E_y$ and $dE_y/dz$ be continuous at the interfaces $z = 0$ and $z = d$, the effective index of refraction $\beta$ obeys the dispersion relation

$$\mathrm{th}(k_0 \tilde{q}_f d) = \frac{\tilde{q}_f \{q_c \,\mathrm{th}[k_0 q_c (z_c - d)] - q_s\}}{\{\tilde{q}_f^2 - q_s q_c \,\mathrm{th}[k_0 q_c (z_c - d)]\}}, \qquad (209)$$

which has a solution only when $n_s > n_c$ and $z_c > d$, i.e., the field maximum must always be located in the overlayer. The energy flux per unit length along the $y$ axis carried by the wave is given by the equations

$$P_s = \frac{1}{2} P_0 \beta \frac{(\alpha_c E_0^2)}{q_s}; \qquad (210)$$

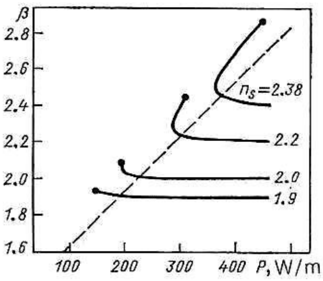


FIG. 22. Dependence of the propagation constant $\beta$ on the guided-wave energy flux. Parameters: $n_c = n_s = 1.55$, $n_{2c} = n_{2s} = 10^{-9}$ m$^2$/W, $\varepsilon_f = -10$, $\lambda = 0.515\,\mu$m, and $d = 0.005\,\mu$m. For the solid lines $n_{2s} = 0$.

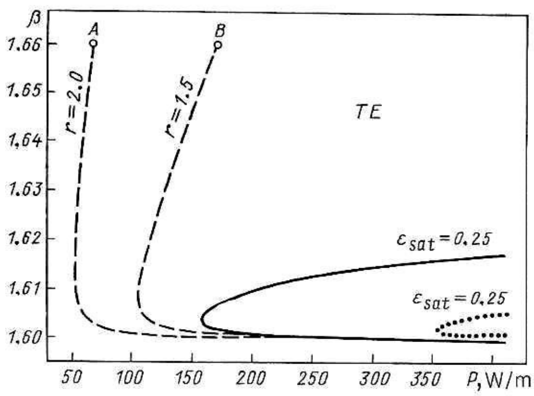


FIG. 23. The same as in Fig. 22. Parameters: $n_c = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W, $n_s = 1.6$, $\varepsilon_f = -10$, $d = 10^{-3}\,\mu$m, $\lambda = 0.515\,\mu$m, $\alpha_c = 1.75\times 10^{-9}$ (m/W)$^{1.5}$ ($r = 1.5$), and $\alpha_c = 6.4\times 10^{-12}$ (m/W)$^2$ ($r = 2$). The solid and dotted lines correspond to the dielectric functions (19) and (20). The broken lines correspond to an energy-dependent overlayer for different parameters $r$.

$$P_f = \frac{1}{\alpha} P_0 \beta \alpha_c E_0^2 \left\{ k_0 d \left(1 - \frac{q_s^2}{\tilde{q}_f^2}\right) + \frac{1}{\tilde{q}_f} \operatorname{sh}(k_0 \tilde{q}_f d) \left[ \left(1 + \frac{q_s^2}{\tilde{q}_f^2}\right) \operatorname{ch}(k_0 \tilde{q}_f d) + 2 \frac{q_s}{\tilde{q}_f} \operatorname{sh}(k_0 \tilde{q}_f d) \right] \right\}; \quad (211)$$

$$P_c = 2P_0 \beta q_c \{1 + \operatorname{th}[k_0 q_c (z_c - d)]\}, \quad (212)$$

where $P_0 = (2\alpha_c k_0)^{-1}(\varepsilon_0/\mu_0)^{1/2}$. Numerical calculations have been carried out for the following values of the parameters: $n_c = 1.55$, $n_{2c} = 10^{-9}$ m$^2$/W, $\varepsilon_f = -10$, and $\lambda = 0.515\,\mu$m (an argon laser). In Fig. 22 we show the dependence of the propagation constant $\beta$ on the surface-wave energy flux $P$ for various values of the substrate index of refraction. As in the symmetric case,[106] a definite energy threshold appears, beginning at which TE-polarized NSPs arise. The broken line corresponds to a symmetric planar structure with the following parameters: $n_s = n_c = 1.55$ and $n_{2s} = n_{2c} = 10^{-9}$ m$^2$/W. It follows from Fig. 22 that for an asymmetric configuration with $n_s > n_c$ and $n_{2s} = 0$, a single value of the parameter $P$ corresponds to two different values of the propagation constant. In addition, there exists an upper limit on $\beta$ determined by the value of $n_s$. The field distribution for $\beta$ close to the lower limit shows that the electric field penetrates deeply into the substrate, and when $\beta$ reaches its upper limit the field energy is concentrated in the nonlinear overlayer. This is preferable, since it decreases the losses related to absorption in real metal films. Therefore, a possible switching between the upper and lower branches of the dispersion curve is related to transitions from the state with high energy-flux transmission capability to the state with low capability.

### Bounding media which do not obey the Kerr law

Let us consider an asymmetric configuration consisting of a linear dielectric substrate with dielectric constant $\varepsilon_s$, a thin metal film of thickness $d$ with dielectric constant $\varepsilon_f = -|\varepsilon_f| < 0$, and a nonlinear self-focusing overlayer characterized by one of the dielectric tensors of the type (19)–(21). We shall use the formalism of Ref. 40 to study the dependence of the effective index of refraction on the energy flux in the case of a dielectric tensor displaying saturation properties and having a power-law energy dependence.

The dispersion relation for TE-polarized NSPs has the form

$$\operatorname{th}(k_0 \tilde{q}_f d) = \frac{\tilde{q}_f (q_c - q_s)}{\tilde{q}_f^2 - q_s q_c}, \quad (213)$$

where $\tilde{q}_f = (\beta^2 + |\varepsilon_f|)^{1/2}$, $q_s = (\beta^2 - \varepsilon_s)^{1/2}$, $q_c = [\varphi(u)]^{1/2}$, $u = \alpha_c E_d^2$, $E_d$ is the electric field at the interface $z = d$, and $\varphi(u)$ is given by (32)–(34). The dispersion relation (213) has a solution $\beta > n_s$ only for $n_s > n_c$. In this case the field maximum (self-focusing peak) must be located inside the self-focusing overlayer. The total energy flux per unit length is $P = P_s + P_f + P_c$, where $P_c$, $P_s$, and $P_f$ are given by Eqs. (188), (190), and (191), respectively. In Fig. 23 we show the results of numerical calculations of the dependence of $\beta$ on the energy flux $P$ in the case of a self-focusing overlayer characterized by the dielectric tensors (19)–(21). As in the case of a symmetric configuration,[106] TE-polarized NSPs appear at a certain value of the energy flux. The upper limit on $\beta$ is determined from the condition $u = \alpha_c E_d^2 = 0$, i.e., $P_s = P_f = 0$ and $P_c \neq 0$. When $\beta$ reaches its upper limit, the optical field begins to be concentrated in the nonlinear self-focusing overlayer. It follows from Fig. 23 that the minimum energy flux required to excite TE-polarized NSPs increases with decreasing $\varepsilon_{sat}$. As the energy flux increases the effective index of refraction $\beta$ asymptotically reaches its limiting value $(\varepsilon_c + \varepsilon_{sat})^{1/2}$. Saturation effects can change the specific energy flux-dependent properties of TE-polarized NSPs in configurations composed of media obeying the Kerr law. Apparently, the experimental observation of NSPs is difficult at present, owing to the particular conditions under which they arise (the extreme thinness of the metal film, the strong variation of the index of refraction, and so on).

### CONCLUSIONS

In summarizing our discussion, we note that the use of materials with nonlinear index of refraction significantly enriches the phenomenon of NSW and NGWM propagation in planar structures. If one or more media in contact with a dielectric or metal film is characterized by an intensity-dependent index of refraction, then the number of NGWM solutions, the propagation constant, the field distribution, the attenuation coefficient, and the threshold conditions all begin to depend on the energy flux. In this article we have paid special attention to planar structures and their possible applications in integrated optics.

The models we have discussed can be used as the theoretical foundation for the design of various types of optical switches in waveguides, and the study of the behavior of the energy flux-dependent wave vector is interesting from the viewpoint of the design of optical power limiters. For example, devices which lower the energy-flux threshold and can transmit optical pulses only above a certain value of the energy flux will probably be based on the use of a self-focusing nonlinearity. Planar optical waveguides with nonlinear self-focusing overlayer and linear waveguide film, whose thickness is smaller than that typical for the phenomenon of TE-wave cutoff at a certain value of the energy flux, might be used to raise the level of the transmitted energy flux. Nonlinear guided-wave power limiters can also be designed using one or several self-defocusing media in contact with a linear waveguide film. Optical switching might be achieved by using an overlayer and substrate which simultaneously have self-focusing nonlinearities. In addition, a combination whose threshold and limiting actions can be controlled might be used to select the most favorable regimes for the transmission of energy flux by NGWMs and NSWs.

At present, the experimental realization of NSW and NGWM phenomena is difficult, owing to the limited choice of available materials. The fact that the difference of the indices of refraction $n_f - n_c$ and $n_f - n_s$, existing between the film and the bounding medium at small values of the energy flux, must be smaller than the change of the index of refraction related to saturation, $\Delta n_{sat}$, limits the combination of materials which can be used in the construction of nonlinear planar optical waveguides. It is necessary to create new materials with optical nonlinearity $n_{2f}$ larger than $10^{-13}$ m$^2$/W and attenuation factor smaller than 1 cm$^{-1}$ in the waveguide.

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[1)] *Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the article in the displayed equations.


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