Nucleon and cluster knockout reactions induced in light nuclei by intermediate-energy protons

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Experimental and theoretical studies of nucleon and cluster knockout reactions induced in light nuclei by intermediate-energy (30–150 MeV) protons are reviewed. The methods of theoretical analysis of knockout reactions are considered. The main attention is devoted to the t-matrix approximation with distorted waves. It is shown for the example of cluster knockout reactions how quasielastic and two-step processes can be coherently included in the matrix element when the t-matrix approximation is used.

INTRODUCTION

Knockout reactions of the type $(p, 2p)$, (p, pn), (p, pd), $(p, p\alpha)$ and some others give two relatively independent forms of information: 1) about the reaction mechanism, i.e., about the two-particle forces and the interactions in the initial and final states; 2) about the properties of the states of the residual nuclear systems and about the wave functions of the particles that are knocked out. Of all the listed reactions, the most attention has been paid to the $(p, 2p)$ reaction, which is sometimes called quasielastic proton scattering. In the region of intermediate initial energies (150–1000 MeV), the experiments in which it has been studied have confirmed the validity of the model of direct (quasielastic) knockout of a particle from a nucleus and the shell structure of nuclei. From the angular correlations of the protons in $(p, 2p)$ experiments at sufficiently high particle energies ($\sim$400 MeV) one can directly obtain data about the intranuclear momentum distributions of the protons.

Because of the importance of such information, reviews of the various studies and their main results have often been published. As a rule, these cover only $(p, 2p)$ reactions. At various times, such reviews have been written by Jacob and Maris,[1,2] Riou,[3] Chant,[4] and McDonald.[5] These were all devoted to knockout reactions at initial proton energies greater than 150 MeV, and in each it was noted that, despite the qualitative progress in understanding quasielastic processes, problems already encountered in the sixties still remained unresolved. In the first place, there is no quantitative description of the experimental differential cross sections (angular correlation functions). The calculated cross sections must be multiplied by coefficients (reductions), which, taking into account the absorption, vary unpredictably when the initial proton energy is varied. For example, in the case of the $(p, 2p)$ reaction on ^{16}O, this coefficient is 1/3 at proton energy 450 MeV and 1/7 at 170 MeV.[3]

In the spectra of excitations of the residual nuclei, peaks corresponding to s, p, and d states are clearly observed. These peaks themselves have a complicated structure, but because of the poor energy resolution of the detectors the individual nuclear levels are not resolved (in the best case, the resolution reaches 1.5 MeV). The probability of their formation must be taken into account by introducing weight coefficients by means of model calculations with j–j, L–S, or intermediate couplings, and this lowers the accuracy of the calculations.

The knockout of particles from inner shells is accompanied by a broad (10–15 MeV) s peak in the excitation spectrum of the residual nuclei, this peak extending into the region of energies 30–35 MeV. The problem of identifying individual levels in this range of excitation energies is complicated and has not yet been solved. It is here necessary to take into account[2] the interaction of the nucleons of the outer shells with the inner shells, the interference between the direct knockout processes and rescattering, the two-particle and many-particle correlations of the nucleons within the nucleus, and other factors.

These problems had the consequence that interest in knockout reactions induced by protons of intermediate energies began to decline steadily in the seventies.

The energy resolution can be raised by decreasing the energy of the incident proton. For example, at proton energy 50 MeV the resolution for the incident protons reaches 300 keV, and it therefore becomes possible to observe directly the individual levels of the upper shells of nuclei. The first experiment that studied the $(p, 2p)$ reaction on ^{12}C at energy 50 MeV (Ref. 6) made it possible to identify at once five individual levels in the ^{11}B excitation spectrum. There then followed intensive investigations of knockout reactions induced by protons of intermediate energies (30–150 MeV). Accelerators that make it possible to vary the proton energy smoothly in the range 50–200 MeV have been constructed. In the Soviet Union, such an accelerator is operated at the Institute of Nuclear Research of the Ukrainian Academy of Sciences at Kiev. As before, the aim of the investigations is to study the reaction mechanism; however, in this range of energies not only quasielastic but also other processes are possible, for example, two-stage and exchange processes. If they are to be analyzed, one must take into account correctly: 1) the distortions of the wave functions (reflection, absorption, refraction, and focusing by the surface of the nucleus); 2) off-shell effects in the two-particle reaction amplitudes; 3) the influence of resonances of the intermediate nuclei.

Knockout reactions induced by intermediate-energy protons have been analyzed by means of various models; in particular, Faddeev's equations were used for nuclei with $A = 2$–4 in Ref. 7. In the majority of cases the solutions of these equations explained the nature of the experimental data. For $A \geqslant 6$, Faddeev's equations proved to be ineffective because of the absence of reliable data on the two-particle interactions.[8]

In the region of intermediate energies, the $(p, 2p)$ reac-

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tion has been the most fully studied. The largest contribution to the development of its theory has been made by McCarthy. The distorted wave t-matrix approximation (DWTA) that he developed[9] was made into the basis of the effective t-matrix method, DWETA,[10] which is used to analyze knockout from the nuclei of complex particles.

Despite definite successes in the description of the mechanism of nuclear reactions at intermediate proton energies, many questions still remain open. To some extent, this is due to unresolved problems in nuclear theory: the absence of reliable data on the intranuclear wave functions, two-particle potentials, etc.

If we are to advance further, it is now time to review what has been done during the last two decades, bearing in mind that neither in the Soviet nor the foreign literature have the data hitherto accumulated been systematized. In the present paper, we attempt an analysis of the experimental and theoretical studies of reactions in which the knockout of particles from nuclei with $A \geqslant 6$ at intermediate energies have been studied.

The review consists of two parts. In the first, we consider nucleon knockout reactions, while in the second the main attention is devoted to knockout of complex particles (clusters). For the example of such reactions we show how quasielastic, two-stage, and exchange processes can be included coherently in the matrix element when the DWETA is used.

1. THE ($p, 2p$) AND (p, pn) REACTIONS

The experiments

We shall consider only those methods of investigating knockout reactions in which the differential cross sections of the reactions are obtained, and we shall not consider the methods, mainly radiochemical, of obtaining integrated cross sections and reaction product yields.

For reactions with three particles in the final state, there are five degrees of freedom (independent parameters), not counting the polarization, and it is therefore important that the measurements be made in a way that is simultaneously convenient from the point of view of both theory and experiment.

One distinguishes coplanar, noncoplanar, and chamber experiments. The coplanar experiments are those in which the momenta of all the particles that participate in the reaction lie in one plane. Among the coplanar experiments, four types are distinguished:

1. The symmetric coplanar type, the scheme of which is shown in Fig. 1. The particles are detected at equal angles θ on different sides of the direction of motion of the incident proton (the Z axis). One selects events in which the momenta of the emitted particles are equal in magnitude ($P_1 = P_2$), i.e., depending on the value of the angle θ, the residual nucleus moves along (or in the opposite direction to) the direction of the incident proton.

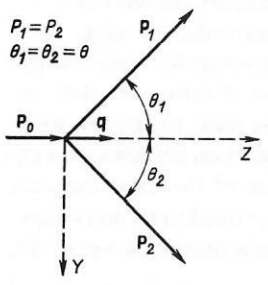

FIG. 1. Symmetric coplanar ($p, 2p$) reaction.

2. The coplanar asymmetric experiment, in which the angle of emission of one of the particles is fixed, $\theta_1 = \text{const}$, and the other angle θ_2 is varied. When ($p, 2p$) reactions are studied in such experiments, one usually selects events with equal momenta (energies); in the case of reactions with complex particles, one assumes equality of the projections of the momenta of the two emitted particles onto the X axis, i.e., the residual nucleus travels along the direction of the Z axis.

3. For equal angles, $\theta_1 = \theta_2 = \theta$, the particles are detected at a fixed value of the parameter $x = P_1/P_2 = \text{const}$ ($P_1 \geqslant P_2$), the value of the parameter x usually being near unity. In this arrangement of the experiment, the recoil nucleus moves in the (Z, Y) plane.

4. For fixed angles θ_1 and θ_2, usually $\theta_1 = \theta_2 = \text{const}$, and proton knockout one detects all events corresponding to the given reaction and investigates the energy spectra of the particles (energy-sharing spectrum). A distinctive (and attractive) feature of such experiments is the complete (100%) use of the measured material.

As follows from the name, in noncoplanar experiments the momenta of only two of the emitted particles lie in one plane. In principle, all the methods of measurement listed above (1–4) can also be used in noncoplanar geometry, but in fact this method began to be used only very recently, following the commissioning of high-current accelerators, since in such experiments one must usually contend with small cross sections.

In all the listed methods of measurement, the events are detected in coincidence by means of semiconductor ($\Delta E/\Delta x - \Delta E$) telescopes, total-absorption crystals, magnetic spectrometers, and, in the case of (p, pn) reactions, the time-of-flight technique. When one is considering events with equal energies of the emitted particles, this means that one selects events satisfying the condition $|E_1 - E_2| \leqslant C$. In other words, the cross sections are averaged within an energy interval of width $2C$, and it is assumed that the change of the cross section within the interval is slight. For ($p, 2p$) reactions, one usually has $C = 5$ MeV in the best experiments.

The angle spread $\Delta\theta$ is usually 2–3° and is determined by the size of the detectors. Their finite sizes (so-called instrumental effects) are the reason why all measurements in coplanar geometry contain an admixture of noncoplanar events. The influence of the instrumental effects has been investigated several times, by Jacob and Maris,[11] Sakamoto,[12] and Jackson[13]; recently, this question has been investigated by the Monte Carlo method[14] and the t-matrix approximation.[15] In all cases, it was noted that the spread $|E_1 - E_2| \leqslant C$ does not have a strong influence, but the finite size of the detectors may distort the results.

All the methods of measurement listed above are called correlation methods, or restricted-geometry experiments. The experimental cross sections measured in them are determined by means of the expression

$$D^3\sigma = \frac{d^3\sigma}{d\Omega_1\, d\Omega_2\, dE_1} = \frac{N_x}{N_0 N \varepsilon} \frac{1}{\Delta\Omega_1\, \Delta\Omega_2\, \Delta E_1}, \qquad (1)$$

where N_x is the number of events in the zone of the kinematic line of the biparametric spectrum corresponding to the chosen value of ΔE_1 (the biparametric spectrum is the dependence of the number of events on E_1 and E_2 for fixed angles θ_1 and θ_2), N_0 is the number of incident particles, N is the number of target nuclei per 1 cm^2, ε is the detection efficiency, and $\Delta\Omega$ is the solid angle of the detectors.

In contrast to the correlation experiments, the measurements by means of bubble, spark, and photoemulsion chambers are made in the entire accessible phase space of the reactions, which is limited only by the particle detection threshold. Such experiments are called chamber or complete-geometry experiments.

The chamber measurements are made with low energy resolution ($\sim$1–2 MeV), but the possibility of simultaneous detection of all events (with both small and large momentum transfers) is an advantage of these measurements compared with correlation experiments. It is usually assumed that the chamber experiments complement the correlation experiments. But if in the final state unstable nuclei are formed [the reactions ^{12}C($p, p3\alpha$), ^{16}O($p, p4\alpha$), and others], then only complete-geometry experiments give objective information about both the reaction mechanism and the states (levels) of the intermediate nuclei that are formed.

It may be noted that the theoretical analysis of knockout reactions measured in complete geometry is much more complicated than in restricted geometry. It is readily seen that the correlation measurements are made in the region of the phase space in which the expected reaction mechanism is kinematically distinguished (for example, symmetric geometry corresponds to direct knockout). When reactions are measured and analyzed in complete geometry, one must consider simultaneously the complete gamut of processes and phenomena that are possible at the selected initial energy of the particles. The direct processes in reactions that are studied by means of chambers can be distinguished in some cases[6] by means of the Treiman–Yang criterion.[17] As a rule, these are events with small momentum transfers.

Qualitative description of knockout reactions. Single-particle approximation

It is convenient to begin the qualitative description of knockout reactions with the idealized single-particle approximation, in which the inelastic interaction of the nucleons with the nuclei can be represented as a succession of independent intranuclear nucleon–nucleon collisions. In other words, it is assumed that only nucleon–nucleon and not three-particle interactions are possible in the nucleus. This is explained by the fact that the internucleon distances in nuclei are about 2 F, while the de Broglie wavelength corresponding to the mean momentum transfer to the nucleus is 1 F at an initial energy of 50 MeV. The Pauli principle precludes collisions of nucleons with small momentum transfers (large wavelengths).[2,18] In this scheme, quasielastic knockout is represented simply by a single interaction of the incident proton with an intranuclear proton. To explain the quasielastic knockout of clusters, it is necessary to assume the existence in the nucleus of short-range interactions of the nucleons (correlations), these giving rise to the association of the nucleons into clusters.[19]

For convenience in presenting the formalism for the knockout reactions, we introduce the following system of indices. We denote the incident proton, the intranuclear proton, and the two emitted protons by the indices 0′, 1′, 0, and 1; by the index A we denote the initial nucleus, and by $A-1$ the final nucleus, which may be in an excited state. In the nonrelativistic case

$$E_{0'} = E_0 + E_1 + E_{A-1} + |Q| + E_{\text{exc}}; \tag{2}$$

$$\hbar\mathbf{k}_{0'} = \hbar\mathbf{k}_0 + \hbar\mathbf{k}_1 + \hbar\mathbf{k}_{A-1}, \tag{3}$$

where E_i are the kinetic energies of the particles. We define the energy of separation of particles as $E_s = |Q| + E_{\text{exc}}$, where E_{exc} is the excitation energy of the residual nucleus, and Q is the reaction energy.

The separation energy E_s of the emitted proton and the momentum $\hbar\mathbf{k}_{A-1}$ of the recoil nucleus have a simple physical interpretation. The recoil nucleus has a hole in the shell from which the proton was emitted, and the separation energy is equal to the energy of this single-particle state. Since initially nucleus A was at rest, the momentum of the recoil nucleus is equal to the momentum of the intranuclear proton before the collision: $\hbar\mathbf{k}_{A-1} = -q$. By measuring the energies of the emitted particles and the angles of their emission, one can directly measure the energies and momentum distributions of the single-particle states in particular shells.

Plane-wave impulse approximation (PWIA)

This method of analyzing knockout reactions was developed at the beginning of the fifties by Chew and Low and since then has been used in practice with small modifications. Ignoring the spin and isospin variables, we write the differential cross section of the ($p, 2p$) reaction in the form

$$D^3\sigma = \frac{d^3\sigma}{d\Omega_1\, d\Omega_2\, dE_1} = (2\pi)^{-5}\, \frac{k_0 k_1}{k_{0'}}\, \frac{m_1^3}{(\hbar c)^6} \sum_{l,\, m} |T_l^m|^2, \tag{4}$$

where $d\Omega_i = 2\pi \sin\theta_i d\theta_i$ are the solid angles of emission of the protons, m_1 is the proton mass, and T_l^m is the matrix element, equal to

$$T_l^m = \int\int d\mathbf{r}_1 d\mathbf{r}_2 \exp(-i\mathbf{k}_1\mathbf{r}_1 - i\mathbf{k}_0\mathbf{r}_2) \cdot \times V(|\mathbf{r}_1 - \mathbf{r}_2|) \exp(i\mathbf{k}_0\mathbf{r}_1)\, \psi_l^m(\mathbf{r}_2). \tag{5}$$

Here, ψ_l^m is the single-particle coordinate wave function of the intranuclear proton 1′ with orbital angular momentum l and projection m of it, and $V(|\mathbf{r}_1 - \mathbf{r}_2|)$ is the two-particle local central potential of the pp interaction.

After the change of variables $\mathbf{r}_1 - \mathbf{r}_2 = \mathbf{r}$, $\mathbf{r}_2 = \mathbf{r}'$ (approximation of a zero-range interaction), the matrix element T_l^m can be represented as a product of two factors (it factorizes):

$$T_l^m = \left\{\int d\mathbf{r} \exp[i(\mathbf{k}_{0'} - \mathbf{k}_1)\mathbf{r}]\, V(r)\right\} \times \left\{\int d\mathbf{r}' \exp[i(\mathbf{k}_{0'} - \mathbf{k}_1 - \mathbf{k}_0)\mathbf{r}']\, \psi_l^m(\mathbf{r}')\right\}. \tag{6}$$

The first factor is the Fourier transform of the two-particle interaction at momentum equal to the momentum transfer by the initial proton. The best approximation for $V(r)$ is the pseudopotential for which the square of its Fourier transform gives the cross section of free pp scattering. This gives the impulse approximation, and in Eq. (4) the square of the Fourier transform of the two-particle interac-

tion potential is replaced by the experimental cross section $(d\sigma/d\Omega)_{p-p}$.

The second factor is the Fourier transform of the wave function of the bound proton (the overlap integral of the wave functions of the initial nucleus and core) $\psi_l^m(\mathbf{r})$ in the momentum representation: $g_l^m(\mathbf{q})$. Its substitution in (4) yields the proton form factor of shell l or the probability density of the momentum distribution of the protons in the nucleus (after summation over l and m). Thus, Eq. (4) can be written in the form

$$\frac{d^3\sigma}{d\Omega_1\, d\Omega_2\, dE} = \mathscr{K}' \left(\frac{d\sigma}{d\Omega}\right)_{p-p} \sum_{lm} |g_l^m(\mathbf{q})|^2, \qquad (7)$$

where $\mathscr{K}'$ is a kinematic factor by which, to eliminate its influence, the measured differential cross sections (spectra) in experiments of type 4 are divided; in experiments of types 1–3 it changes little.[20]

Information obtained by means of the PWIA

1. The analytic form of the single-particle wave functions $g_l^m(\mathbf{q})$ will be different for different types of potentials by which the $\psi_l^m(\mathbf{r})$ are determined. In particular, for all m a rectangular well gives spherical Bessel functions $j_l(q)$, and a harmonic oscillator gives $q^l \exp(-q^2/q_0^2)$. The explicit form of the functions $g_l^m(\mathbf{q})$ for the oscillator potential for some values of l and m can be found in Jackson's papers of Refs. 13 and 21. The properties of the wave functions $g_l^m(\mathbf{q})$ do not depend on the type of potentials used in practice (including the Woods–Saxon potential). For all $l \neq 0$, $g_l(q) = 0$ if $q = 0$ and, conversely, if $l = 0$ then the function $g_l(q)$ has a maximum at $q = 0$. This makes it possible, using the form of the experimentally measured differential cross sections, to distinguish the shell, say s or p, from which the proton has been knocked out. For example, in symmetric coplanar geometry $q = 0$ for $\theta \approx 45°$ ($\theta = 45°$ if $Q = 0$). When protons are knocked out of the s shell, the measured cross section $D^3\sigma$ will have a bell-shaped form with a maximum at angle 45°. If the protons are knocked out of other shells, the cross section must have a minimum (vanish) at 45°, and on both sides of this minimum there will be maxima at angles θ_1 and θ_2 ($\theta_1 < \theta < \theta_2$). The maximum at θ_1 must be higher than at θ_2, since it corresponds to a $(d\sigma/d\Omega)_{p-p}$ taken at a lower initial energy in the center-of-mass system (smaller momentum transfer); in other words, the maximum at θ_1 corresponds to a collision of protons moving in the same direction.

2. In symmetric coplanar geometry, the theoretical analysis of the experimental cross sections is greatly simplified, and for odd m we have matrix elements $T_l^m = 0$. This was first pointed out by Maris[22] and was later demonstrated by Jackson.[21] In the PWIA this leads to interesting consequences. In the case of interaction with p protons, one selects only the collisions that occurred on the periphery (more precisely, at the two poles) of the nucleus when the intranuclear proton moved collinearly with respect to the direction of the accelerator beam. Conversely, experiments of the types 2–4 enable one to investigate other orbitals with $m \neq 0$ for chosen values of l. At intermediate energies, such investigations were made at the beginning of the seventies for 1p-shell nuclei; their results are discussed in the review of Ref. 2, in which it was merely concluded that the PWIA is in qualitative agreement with the experimental results.

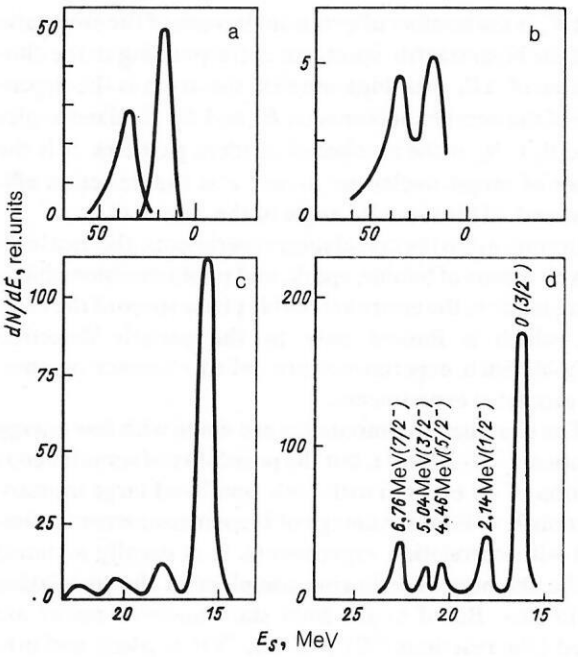

FIG. 2. Excitation spectra of residual nuclei in the $^{12}C(p, 2p)^{11}B$ reaction at energy 400 MeV (a), Ref. 23; 155 MeV (b), Ref. 24; 100 MeV (c), Ref. 20; and 50 MeV (d), Ref. 6.

3. For $l \neq 0$, the two maxima must be symmetrically placed with respect to the value $q = 0$ if as a variable along the abscissa one uses $q = q(\theta)$.

4. The angular correlation functions $D^3\sigma(\theta)$ alone do not enable one to establish from which of the outer shells the proton has been knocked out if $l \neq 0$. However, using additional information about the excitation spectra of the residual nuclei, one can also solve the problem of identifying the l values. Note that with increasing l the distance between the neighboring maxima (in the scale of q along the abscissa) increases.

Thus, the PWIA has a clear physical interpretation: The interaction mechanism is determined by the first factor in (6), and the second factor determines the probability for an intranuclear proton to have the momentum q (i.e., it determines the nuclear structure).

In Figs. 2 and 3 we give the excitation spectra of the residual nuclei and the angular correlation functions from various experiments in which the $^{12}C(p, 2p)^{11}B$ reaction was studied at 400 (Ref. 23), 155 (Ref. 24), 100 (Ref. 20), and 50 MeV (Ref. 6) in symmetric coplanar geometry. One can speak of agreement between the PWIA and the experimental data, and then only nominally, for the experiments at 400 MeV, at which events are selected simultaneously for all excitation levels of ^{11}B in the interval 0–16 MeV. In reality, not all the ^{11}B levels correspond to quasielastic knockout, but only the ones (see Fig. 2) that have spin $1/2^-$ or $3/2^-$, if it is assumed that in the ^{12}C nucleus there are no admixtures of f, d, or other nucleons.

5. For nucleon knockout reactions, the impulse approximation is equivalent to the assumption that the nucleons interact only in singlet $^1S_0$ (or triplet $^3S_1$) states, this being due to the isotropic nature of the differential cross sections of pp (or pn) scattering, $(d\sigma/d\Omega)_{cms}$, in Eq. (7). In fact, one

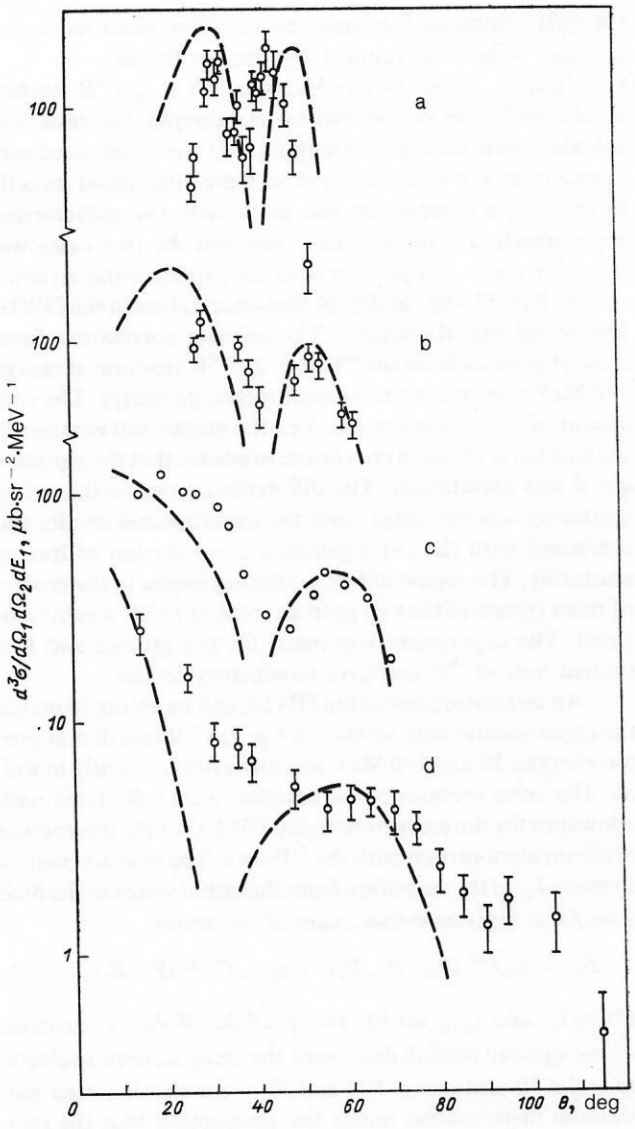

FIG. 3. Angular correlation functions of protons knocked out of the p shell in the $^{12}C(p, 2p)^{11}B$ reaction at energies 400 MeV (a), Ref. 23; 155 MeV (b), Ref. 24; 100 MeV (c), Ref. 20; and 50 MeV (d), Ref. 6. The curves were calculated in the PWIA.

here loses information about the phase of the reaction matrix element, and this, in its turn, prevents one from summing coherently the matrix elements corresponding to other processes (different from the quasielastic), for example, two-stage processes. It is particularly important to understand this when using the impulse approximation to describe reactions induced by intermediate-energy protons. For at such energies, the energy resolution being good, one can distinguish experimentally the different two-stage interaction mechanisms.

It can be seen from Fig. 3, which shows, for the example of the $^{12}C(p, 2p)^{11}B$ reaction, the angular correlation functions typical of the majority of nuclei when protons are knocked out of nuclear shells with $l \neq 0$, that at proton energies above 100 MeV the shape of the experimental distributions is close to that predicted by the impulse approximation. Such agreement was a stimulus for further improvement of the approximation, including a calculation of the effects of the distortions of the proton wave functions before and after the reaction; the result was the distorted-wave impulse approximation: DWIA. The form of the experimental distributions for initial proton energy 50 MeV (Fig. 3) differs from the form of the corresponding dependences at higher energies, and it was for this reason that other methods and theories were tested and improved on these distributions. In particular, in a series of papers McCarthy (Refs. 6, 9, and 25–28) proposed the pseudopotential model,[25,26] the local t-matrix model,[27] and finally created the nonlocal realistic t-matrix theory[9,28] for analysis of $(p, 2p)$ reactions; this was the distorted-wave t-matrix approximation: DWTA. At proton energies of order 150 MeV, his theory gives results analogous to the DWIA, but, in contrast to it, makes it possible to take into account the two-step processes coherently. As a result, in the region of intermediate proton energies we now have a situation in which the t-matrix approximation is used at proton energies below 100 MeV, while the DWIA is mainly used above 100 MeV. Before we discuss the results of the experimental studies, we briefly present the essence of the DWIA and dwell in more detail on the DWTA.

Distorted-wave impulse approximation: DWIA

Comparison of the experimental differential cross sections with PWIA calculations made for a large number of nuclei in the wide energy range 150–1000 MeV revealed the following features[1–5]:

a) Minima at $q = 0$ when $l \neq 0$ are observed, but they are usually nonzero, i.e., the minima are filled. With decreasing energy of the incident protons, their depth decreases. A measure of the filling of the minima is the ratio of the cross sections at the left or right maximum and at the minimum.

b) With decreasing initial proton energy, the position of a minimum is shifted with respect to the value $q = 0$ to larger momenta.

c) The ratio of the cross sections at the maxima is not equal to the ratio of the free scattering cross sections at the energies corresponding to the momentum transfers.

d) The positions of the maxima are not symmetric with respect to $q = 0$ or even with respect to the experimental minimum.

The reasons for these discrepancies are assumed to be the following:

1. Rescattering, i.e., interaction of the protons with intranuclear nucleons before and after the quasielastic interaction. There have been some discussions of the problem of three-particle interactions, i.e., the influence of short-range correlations of the nucleons during the interaction time.

2. Off-shell effects due to the presence of a third particle (core) at the time of the interaction. They are manifested in the fact that the differences between the kinetic energies of the relative motion of the protons in the entrance and exit channels lead to uncertainties in the energy at which it is necessary to determine the free scattering cross section in (7).

3. The insufficient accuracy in the determination of $\psi_l^m(\mathbf{r})$, i.e., the use of simple potentials, leads to solutions $\psi_l^m(\mathbf{r})$ that decrease rapidly outside the nucleus, and this part of the wave function (its tail) may have an important influence in the calculation of $g_l^m(\mathbf{q})$. Possible admixture states, for example, $2p$ in the ^{12}C nucleus, have a weak influence.

In the DWIA, rescattering is taken into account by dis-

tortion of the wave functions of the incident, 0′, and the emitted, 0 and 1, protons. For this, one uses optical potentials (most often the Woods–Saxon potential), ignoring the spin–orbit interaction. Two methods are widely used—the semiclassical and the eikonal approximations. The first was developed by Maris,[22] and the second by McCarthy.[29] In this section, we shall consider the semiclassical approximation, while in our consideration of the DWTA we shall use the eikonal approximation; the differences between them will then be evident.

The procedure for introducing the distortions can be represented formally as a change of the wave vectors $\mathbf{k}_i$ in the plane waves of Eq. (6), this being achieved by multiplying them by complex numbers of the form $D = 1 + Z$. As a result, Re Z changes the momentum of the particle in the field of the nucleus, while Im Z is responsible for absorption of the wave by the nucleus, i.e., one effectively introduces a complex refractive index D.

The overlap integral (6) in the DWIA with the semiclassical approximation has the form

$$\sum_{lm} |g_l^m|^2 = \sum_{lm} \left| \int \exp(i\mathbf{k}_{A-1}\mathbf{r}) \prod_{i=1}^{3} D_i(\mathbf{r})\, \psi_l^m(\mathbf{r})\, d\mathbf{r} \right|^2, \quad (8)$$

where

$$D_i(\mathbf{r}) = \exp\left[-i\,(E_i/\hbar^2 c^2 k_i) \int V_i(\mathbf{r}')\, dS_i\right],$$

$$i = 0',\ 0,\ 1; \quad (9)$$

V_i is the optical potential, whose parameters are chosen on the basis of elastic scattering of protons by the corresponding nuclei (target and core), i.e., independently of the studied (p, $2p$) reaction.

The integrals in (9) are taken along the classical paths S_i for the incident particle from $-\infty$ to r and from r to ∞ for the emitted particles. It will be understood that calculations in the DWIA with the semiclassical approximation require a considerable amount of computing time.

The properties of the overlap integrals (8) and the justification of such a method of calculating g_l^m can be found in Refs. 2 and 30. In the general case, the distortions $D_i(r)$ depend on the geometry of the experiment, are not spherically symmetric, and may deviate appreciably from unity.

How does the DWIA manifest itself relative to the PWIA? The real part of Z leads to the appearance of a phase factor of the form $\exp(i\Delta\mathbf{k}\cdot\mathbf{r})$, where $\Delta\mathbf{k}$ is parallel to $\mathbf{k}$ and can be expressed in terms of the real parameters of the distorting optical potential.

The imaginary parts of the optical potential give a real factor in the argument of the exponential. As a result, the calculated curves are shifted by $\Delta\mathbf{k}$ along the abscissa and there is a change in the absolute value of the cross section (its normalization), and this leads to a filling of the minimum. A detailed explanation of such effects can be found in Ref. 31 and the recently published study of Zelenskaya and Teplov.[32]

The region of intermediate energies was found to be convenient for testing the correctness of the DWIA, since here the distortions play an important part. By "correctness" we mean validity of the factorization in (7) if the distorted momentum distribution $g_l^m(q)$ from (8) is used. Such a division into two factors is valid only for plane waves, and the only argument for factorization when distorted waves are used is the short range of the nuclear forces.

In Refs. 33 and 34, in which the ^{12}C(p, $2p$)^{11}B reaction at 160 MeV was considered as an example, the results of calculations in accordance with Eq. (7) were compared with calculations for which distortions were introduced directly in (5), i.e., a comparison was made with the unfactorized approximation. The difference between the two cases was tens of percent, comparable with the experimental errors.

In Ref. 35, the validity of the factorization in the DWIA was tested experimentally. The angular correlation functions of protons from the ^{40}Ca(p, $2p$)^{39}K reaction at energy 148 MeV were measured in asymmetric geometry. The conditions of the measurements, i.e., the angles and energies of the emitted protons, were chosen to ensure that the equation $q = 0$ was maintained. The differential cross section of pp scattering was extracted from the experimental results and compared with the corresponding cross section of free pp scattering. The region of free scattering angles in the center-of-mass system of the two protons from 51 to 84° was investigated. The experiment was made for the ground and first excited state of ^{39}K and gave satisfactory results.

An interesting test of the DWIA and its ability to match the experimental data on the ^{12}C(p, $2p$)^{11}B reaction at proton energies 50 and 100 MeV was made quite recently in Ref. 36. The cross sections of the reaction were calculated with allowance for the correction to the DWIA for the interaction of the incident proton with the ^{11}B core. The reaction matrix element T_{if} of the transition from the initial state i to the final state f was represented as a sum of two terms:

$$T_{if} = t_{01} F^{01}(\mathbf{P}_{0'}, \mathbf{P}_0, \mathbf{P}_1) + t_{0^{11}\mathrm{B}} F^{0^{11}\mathrm{B}}(\mathbf{P}_0, \mathbf{P}_1), \quad (10)$$

where t_{01} and $t_{0^{11}\mathrm{B}}$ are the two-particle off-shell t matrices, whose squared moduli determine the cross sections of elastic pp and p^{11}B scattering; F^{01} and $F^{0^{11}\mathrm{B}}$ are the distorted momentum distributions under the assumption that the incident proton is scattered by a valence proton and the core.

In Fig. 4, we reproduce the results of calculations from Ref. 36 in a comparison with the experimental data. It can be seen that the correction for the interaction with the core improves the agreement of the DWIA with the experimental data at energy 50 MeV, but the question of the validity of the DWIA in the range of intermediate energies 50–100 MeV is said by the authors to be still open.

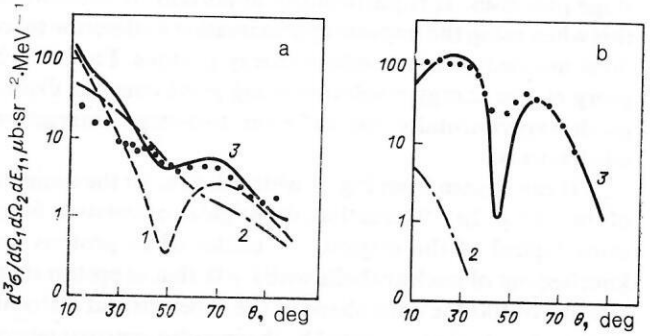

FIG. 4. Calculated angular correlation functions of protons from the ^{12}C(p, $2p$)^{11}B reaction at energy 50 MeV (a) and 100 MeV (b).[36] 1) DWIA; 2) correction for interaction with the core; 3) resultant curve. The points are the experimental data of Refs. 6 and 20.

TABLE I. Experimental investigations of (p, $2p$) reactions at intermediate proton energies ($\geqslant$100 MeV) (references are given in square brackets).

Target nucleus	Energy, MeV	References	Geometry of experiment	Location of experiment	Resolution, ΔE, MeV	$\lvert E_1-E_2\rvert$, MeV
^{6,7}Li	185	[37]	Symmetric	Uppsala	3	—
^{12}C	158	[38]	»	Harvard	2	8
^{6,7}Li, ^{9}Be	156	[39]	»	Orsay	2	10
^{40}Ca, ^{45}Sc, ^{48}Ti, ^{52}Cr, ^{58}Mn	156	[40]	»	»	2	10
^{6,7}Li, ^{10,11}B, ^{12}C	155	[24]	»	»	1.5	10
^{40}Ca	148	[35]	Symmetric + asymmetric	Maryland	—	—
^{6}Li	100	[41]	Symmetric	Montreal	1	10
^{6}Li	100	[42]	»	Maryland	1	10
^{7}Li, ^{12}C	100	[29]	»	»	1	10
^{12}C	100	[43]	Symmetric + asymmetric	Indiana	3	—

Experimental data and problems of their analysis in the range of intermediate proton energies near 100 MeV

In Table I we list experiments in which the (p, $2p$) reaction has been studied at energies 100–200 MeV. In the majority of cases, the experiments were made with good energy resolution (especially at proton energies near 100 MeV), and this made it possible to obtain the differential cross sections for individual excitation levels of the residual nuclei. It can be assumed that at proton energies above the nominal value of 100 MeV (the precise value of this number is different for each particular nucleus and is determined by the Q value of the reaction) the angular correlation functions of the protons from (p, $2p$) reactions are similar to those measured earlier at higher energies and in their form agree with the predictions of the DWIA. Figure 5 shows, for this interval of intermediate energies, a typical excitation spectrum of the residual nucleus and the angular correlation function of the protons from the (p, $2p$) reaction on the ^{7}Li nucleus at energy 100 MeV together with the DWIA calculation. An exception is the ^{6}Li nucleus, for which the angular correlation functions have a shallow minimum, and the angular distance between the two maxima is less than for the other nuclei (Fig. 6). This behavior can be explained by the high degree of clustering of ^{6}Li. In Ref. 44, the ^{6}Li(p, $2p$)^{5}He reaction was analyzed by means of the α–d cluster model of ^{6}Li. The minimum at $q=0$ was filled by the contribution from the s state that arises on the decay of ^{5}He into a neutron and ^{4}He. At proton energy 70 MeV, the intranuclear wave function of a proton in ^{6}Li from the three-particle p–^{4}He–n model[45] was used in Ref. 10. The agreement obtained was good, though admittedly the t-matrix approximation was used in the calculation.

We consider the proton energy 100 MeV in more detail. The results obtained here are instructive and interesting because of the good energy resolution in the experiments. Figure 6 shows the angular correlation functions of the protons from the ^{6}Li(p, $2p$)^{5}He reaction at 100 MeV from Refs. 41 and 42. It can be seen that the results differ both in absolute value, by nearly two times, and in form, although the experiments were made under the same conditions. In Ref. 42 it was asserted that a minimum is not observed at the angle 42°, in contrast to the results of Ref. 41. It is clear that in such a situation it is difficult to draw conclusions about the possibilities of the DWIA, though considered together the data of Fig. 6 give another interesting conclusion—at proton energies from 47 to 185 MeV the angular correlation functions of the ^{6}Li(p, $2p$)^{5}He reactions retain approximately the same form: two maxima and a minimum at $q=0$. For no other nucleus hitherto investigated has such a feature been observed.

In symmetric coplanar geometry at proton energy 100 MeV, the (p, $2p$) reaction was studied on the ^{7}Li and ^{12}C nuclei.[20] For these nuclei, a clear minimum is observed in the angular correlation functions. Calculations made in the DWIA showed that the results (the shape of the curve) were

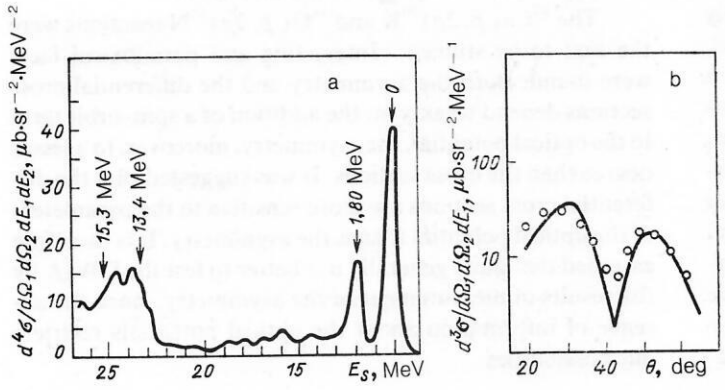

FIG. 5. Energy spectrum of excitation of residual nuclei (a) and angular correlation function of protons (b) from the ^{7}Li(p, $2p$)^{6}He reaction at energy 100 MeV.[20]

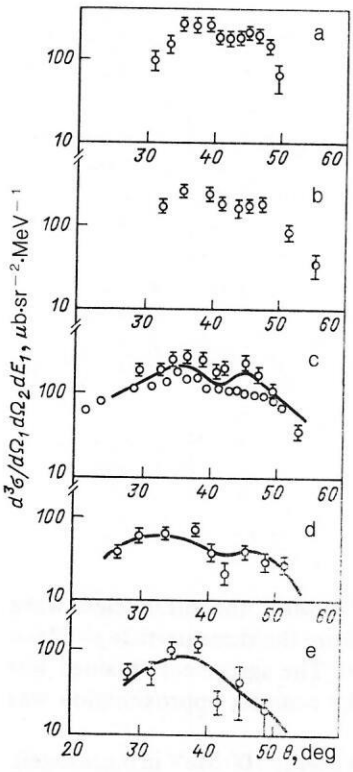

FIG. 6. Angular correlation functions of protons from the ^{6}Li(p, $2p$) ^{5}He reaction at the following energies: a) 185 MeV (Ref. 37); b) 156 MeV (Ref. 39); c) black circles for 100 MeV (Ref. 41), open circles for 100 MeV (Ref. 42); d) 70 MeV (Ref. 45); e) 47 MeV (Ref. 46). The curves were calculated using the DWTA.[76]

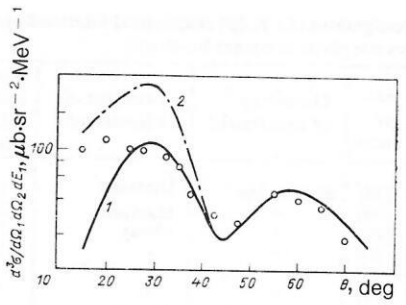

FIG. 7. Angular correlation functions of protons from the ^{12}C(p, $2p$) ^{11}B reaction at proton energy 100 MeV. The curves were calculated in the DWIA with different approximations for the relative energy of the pp interaction: 1) approximation by the initial relative energy (e_i); 2) approximation by the final relative energy (e_f).

stable with respect to the parameters of the optical potential used to describe the distorted waves, while the spectroscopic amplitude differed appreciably from the values usually used to calculate the ^{7}Li(p, $2p$) ^{6}He reaction at intermediate energies.

Figures 2c and 3c give experimental data on the (p, $2p$) reaction on ^{12}C at 100 MeV. In the spectrum of excitation energies of the ^{11}B nuclei excitation levels are clearly revealed at 2.14 ($3/2^-$), 4.4 ($5/2^-$), 5.02 ($3/2^-$), and 6.76 MeV ($7/2^-$). In the model of quasielastic knockout of protons from the p shell the levels at 4.4 and 6.76 MeV cannot be excited, and their occurrence indicates the presence of two-stage processes. The admixtures of the $1f_{7/2}$ and $1d_{5/2}$ states are 1.3 and 0.12% and cannot be the reason for the occurrence of such peaks.[6]

At intermediate proton energies, the differential cross sections of (p, $2p$) reactions measured in symmetric coplanar geometry usually disagree with DWIA calculations at small angles θ. The calculated curves fall more steeply with decreasing angle than the experimental data. In Ref. 20, a study was made of the behavior of the calculated curves in this region of angles as a function of the method of determining the relative energy e of the interaction of the protons, (i.e., as a function of the off-shell effects). The energy e was specified in two ways: $e_i = (P_{0'} - P_1 \cos\theta)^2/m$, the relative energy in the initial channel, and $e_f = (P_1 \sin\theta)^2/m$, in the final channel. Figure 7 shows the results of these calculations together with the experimental data. We note that the off-shell effects observed here are manifested in the region of angles $\theta = 0$–$40°$ and take the form that for $e = e_f$ the calculated curve falls more steeply than for $e = e_i$, and the curve itself for $e = e_f$ is higher than for $e = e_i$. This behavior of the cross section corresponds completely to the results of Ref. 48, in which there is a thorough analysis of these effects for different methods of calculating (p, $2p$) reactions.

In Ref. 43, the (p, $2p$) reaction on ^{12}C was studied simultaneously in symmetric and asymmetric coplanar geometries. Cases in which the ^{11}B nucleus was formed in different excited states were considered. It was concluded that at large q the calculated curves drop more rapidly than the experimental cross sections, while the spectroscopic factors for all the observed ^{11}B levels were found to be 1.5–2 times smaller than in the case of symmetric coplanar geometry.

Before we consider the experiments at energies below 100 MeV, where the DWIA is not used, let us attempt to draw some first conclusions about the use of the DWIA to study (p, $2p$) reactions at intermediate energies.

The confidence in this method, in the expression of Maris, is due to the correct prediction of the shape of the differential cross sections. Its main shortcomings are the absence of reliable information on the optical potentials for calculating the distorted waves and the separation of the two-nucleon interaction with the overlap integral of the distorted waves. Because of this, the spectroscopic amplitudes at different initial energies of the protons are different for the same reaction or for measurements in different geometries.

To some degree, the elimination of these shortcomings is the aim of the studies made at the beginning of the eighties with the new high-current TRIUMF accelerator, where the (p, $2p$) reaction is studied with polarized proton beams. By means of the asymmetry, a possibility was presented for more reliable separation of the excitation levels of the residual nuclei at proton energy around 200 MeV.

The ^{40}Ca($\vec{p}$, $2p$) ^{39}K and ^{16}O($\vec{p}$, $2p$) ^{15}N reactions were the first to be studied.[5] Interesting and paradoxical facts were found: Both the asymmetry and the differential cross sections depend weakly on the addition of a spin–orbit term to the optical potential, the asymmetry, moreover, to a lesser degree than the cross sections. It was suggested that the differential cross sections are more sensitive to the parameters of the optical potential than is the asymmetry. It is therefore assumed that quite generally it is better to test the DWIA by the results of measurement of the asymmetry, since the absence of information about the optical potentials restricts our possibilities.

The *t*-matrix approximation (DWTA) with a local *t* matrix (pseudopotential)

The best energy resolution (better than 300 keV) is achieved for measurement of the proton energy in the (p, $2p$) reactions when the initial energy is near 50 MeV. This is due to the properties of the semiconductor (based on Li–Si and other materials) ($\Delta E/\Delta x$-ΔE) telescopes, basically their size. In Ref. 6, an experiment with such accuracy was made by Pugh's group on ^{12}C in symmetric and asymmetric coplanar geometries. In contrast to the earlier study of Ref. 49, in which the (p, $2p$) reaction on ^{12}C was also studied at 50 MeV, the experiment of Ref. 6 measured the differential cross sections of the reaction for formation of the residual ^{11}B nucleus in the ground state and the first excited states at 2.14, 4.44, 5.04, and 6.76 MeV. As already noted, the levels at 4.44 and 6.76 MeV appear because of two-stage processes, while the levels at 2.14 and 5.04 MeV are associated with quasielastic processes. From the ratio of the heights of the peaks of the levels corresponding to the different processes one can draw conclusions about the parts played by these processes. If the excitation spectra of ^{11}B at initial energies 50 and 100 MeV are compared (see Fig. 2), an increasing importance of the two-stage processes when the initial energy is reduced to 50 MeV can be noted.

In the differential cross sections measured in Refs. 6 and 49 oscillations were observed, and these differed from the predictions of the impulse approximation by the absence of the minimum at $q = 0$ ($\theta = 31°$ in symmetric coplanar geometry at initial energy 50 MeV). According to the authors of Ref. 6 the cross section has a diffraction structure with two maxima at 0 and 65° (coinciding with the position of the maxima of the spherical Bessel function). The maximum at 40° was explained by distortions. The studies of Refs. 6 and 49 demonstrated the need to develop a new theory capable, in contrast to the DWIA, of taking into account correctly all possible processes: direct, two-stage, exchange, etc. Such a theory was created by McCarthy (Refs. 6, 9, and 25–27).

In his studies, he does not divide the matrix element (5) into two factors but represents it in the form

$$T_l^m = \int d\mathbf{r}_1 d\mathbf{r}_2 \chi^{-*}(\mathbf{k}_1, \mathbf{r}_1)\, \chi^{-*}(\mathbf{k}_0, \mathbf{r}_2) \times V(|\mathbf{r}_1 - \mathbf{r}_2|)\, \chi^{+}(\mathbf{k}_{0'}, \mathbf{r}_1)\, \psi_l^m(\mathbf{r}_2), \quad (11)$$

where $\chi^{\pm}$ are distorted waves determined from the Schrödinger equation by means of optical potentials, and $V(|\mathbf{r}_1 - \mathbf{r}_2|)$ is a two-particle pseudopotential of Yukawa type with three one-pion components. Its parameters were fitted to reproduce the cross section of free pp scattering in the interval of energies 0–300 MeV by means of only singlet (even L) states. The contribution of the triplet (odd L) channel was determined as a correction to the singlet contribution by requiring that the calculated curves agree with the experimental data. In other words, in the first studies the parameters of the potential V for the triplet channel of pp scattering were the same as the singlet ones, but the potentials themselves differed only in the normalization. The calculations of the differential cross sections of the (p, $2p$) reactions on ^{12}C gave an estimate of the fraction of the triplet channel of pp scattering to the total cross section of the (p, $2p$) reaction for the case when ^{11}B is formed in the ground state. At proton energy 155 MeV, the contribution of the triplet component was small (about 1%), while for 50 MeV it was 20%.[27] After these estimates, interest in the DWIA at proton energies below 100 MeV decreased considerably, and all subsequent (p, $2p$) experiments were analyzed by means of the DWTA.

Experimental data at proton energies below 100 MeV and problems of their analysis

A list of experiments in which the (p, $2p$) reaction has been studied at proton energies below 100 MeV is given in Table II. The nuclei ^{6,7}Li, ^{12}C, ^{14}N, ^{16}O, ^{19}F, ^{40}Ca, and ^{89}Y were studied in correlation and chamber experiments with different methods of event selection and different resolutions. In Fig. 8, we give the differential cross sections of the (p, $2p$) reactions measured in symmetric coplanar geometry together with DWTA calculations made in different studies using local potentials $V(r)$. In some studies,[50,51]

TABLE II. Experimental investigations of (p, $2p$) reactions at intermediate proton energies (below 100 MeV) (references are given in square brackets).

Target nucleus	Energy, MeV	Reference	Geometry of experiment	Location of experiment	Resolution, ΔE,	$\lvert E_1 - E_2 \rvert$, MeV
^{12}C	50	[6]	Symmetric + asymmetric	Berkeley	0.3	5
^{12}C	50	[49]	Symmetric	»	—	5
^{12}C, ^{16}O	49.5	[50]	»	Harwell	0.3	12
^{12}C	45.5	[51]	Symmetric + asymmetric	Los Angeles	0.3	4—6
^{89}Y	46	[52]	Symmetric	»	0.5	14
^{14}N	46	[53]	»	»	—	5
^{16}O	45	[54]	»	»	—	4
^{40}Ca	45	[55]	»	Winnipeg	0.6	3.5
^{19}F	42.7	[56]	»	Washington	0.6	10
^{7}Li	47	[47]	»	Kiev	—	—
^{6}Li	70	[46]	»	»	—	—
^{6}Li	70	[57]	»	»	—	—
^{12}C	50—110	[58]	Complete	Orsay	1	—
^{12}C	50	[59]	»	Leningrad	1	—
^{14}N	50	[60]	»	»	1	—
^{16}O	50	[61]	»	»	1	—

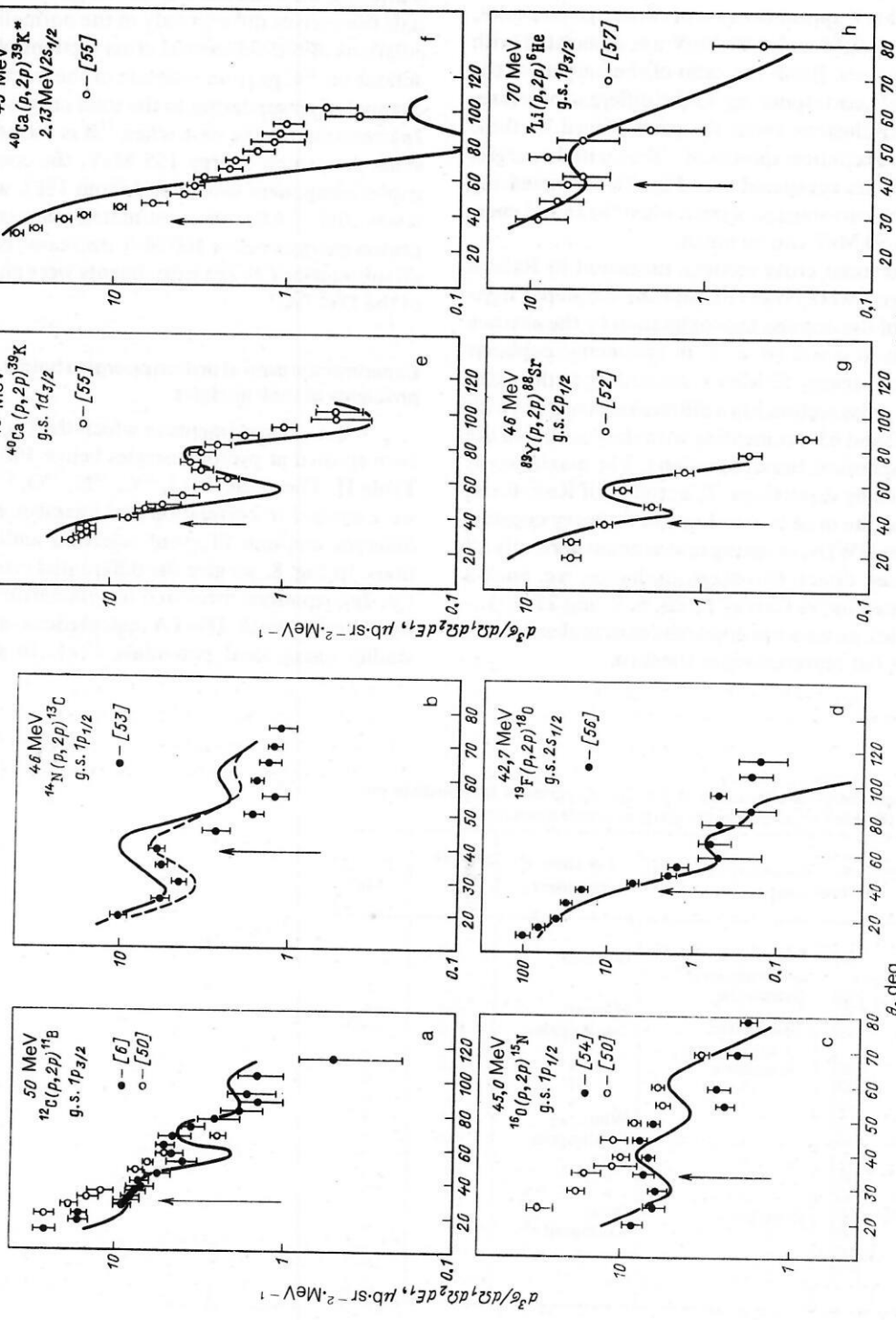

FIG. 8. Angular correlation functions of protons from (p, $2p$) reactions on different nuclei at proton energies around 50 MeV. The calculated curves were obtained in various studies using the DWTA: with local (continuous curves in Figs. 8a–8g), nonlocal (broken curve in Fig. 8b), and realistic (the curve in Fig. 8h) t matrices. The arrows indicate the angles for which $q = 0$ (references are given in square brackets).

Pugh's results were tested. In Ref. 50, the cross section of the (p, $2p$) reaction on ^{12}C and ^{16}O at initial proton energy 50 MeV was averaged in the interval $|E_1 - E_2| < 12$ MeV; the accuracy was low, and the cross sections were too high and not as found by Pugh. However, we give them (Figs. 8a and 8c) together with the data of Refs. 6 and 54 to show how the form of the angular correlation functions changes with the accuracy of the measurements. In Ref. 51, the ^{12}C(p, $2p$) ^{11}B reaction was studied in asymmetric coplanar geometry at proton energy 46 MeV; the results confirmed the data of Ref. 6.

All the angular correlation functions in Fig. 8 have a complicated structure; oscillations are observed, and there are no minima at the angles corresponding to $q = 0$. It is interesting that the differential cross sections for the nuclei ^{12}C, ^{14}N, and ^{16}O are similar in form and close in absolute magnitude. At the present time, they are regarded as the most reliable in the sense of the accuracy of the measurements, and they are usually used to investigate different methods of calculation.

Figures 8d–8h give the angular correlation functions of the protons knocked out of different shells of the nuclei ^{19}F, ^{40}Ca, ^{89}Y, and ^{7}Li. It can be seen that here too there are no maxima for proton knockout from the 2s shell and no minima for knockout from the 1p, 1d, and 2p shells at $q = 0$.

We shall also discuss the question of the diffraction nature of the experimental cross sections. This term first appeared in Ref. 6 from the analogy of the form of the angular correlation functions of protons of the (p, $2p$) reaction on ^{12}C and the diffraction pattern of light on an opaque screen (bright spot at the center of the shadow). The reason for such behavior of the cross sections has frequently been discussed and disputed[50] in the literature. It seems to us that an answer to the question of why all the measured differential cross sections (see Fig. 8) lack the first maximum can be obtained by comparing Figs. 3 and 5. It is only for the ^{6}Li nucleus that the angular correlation functions do not change their shape, i.e., retain both maxima in the complete investigated interval of proton energies 47–155 MeV. This can be explained by the small Q value of the (p, $2p$) reaction on ^{6}Li, which is -4.59 MeV, and the features of the ^{6}Li momentum distribution $g_l^m(q)$, the first maximum of which is situated at $q = -40$ MeV/c. For the ^{12}C nucleus, $Q = -16$ MeV, and the value of q corresponding to the first maximum is -100 MeV/c. When the initial proton energy is 50 MeV, the largest value of q ($\theta \simeq 10°$) in the region of the first maximum of the ^{12}C(p, $2p$) ^{11}B reaction does not exceed -45 MeV/c, i.e., at energy 50 MeV the kinematics simply prevents one from observing a picture with two maxima. With regard to ^{6}Li, for it at proton energy 47 MeV the situation in the case of (p, $2p$) reactions is the same as for ^{12}C at initial proton energy 100 MeV, i.e., q values exceeding -40 MeV/c are allowed by the kinematics. Similar conclusions can be drawn with regard to all the other nuclei represented in Fig. 8. Of course, the reaction cross sections in the DWTA (Fig. 8) must not be taken as distorted momentum distributions of the protons in the nuclei like g_l^m. The agreement between the calculated and experimental cross sections merely indicates that the wave functions of the intranuclear nucleon, the two-particle potential $V(r)$, and the distortions of the proton wave functions have been correctly determined. That is why in the study of knockout reactions induced by intermediate-energy protons one poses only the problem of studying the reaction mechanism—the problem of estimating correctly the distorting and two-particle potentials.

Two-step processes

In (p, $2p$) reactions, two-step processes were considered in Ref. 62 as a correction to the quasielastic processes in the framework of perturbation theory. A two-step process was understood in this paper as scattering of the proton by the core of the nucleus with excitation of the core to the region of the giant resonance with subsequent de-excitation of it by a valence proton. These processes were taken into account by means of the theory of Satchler and Love,[63] which is called core polarization theory, and in which the particle–core interaction is described by a generalized deformation optical potential. In Ref. 62, the effective two-nucleon transition operator was represented in the form of a coherent sum:

$$\bar{t}(0,\ 1) = V(0,\ 1) + \sum_{\lambda\mu} y_\lambda^\tau(\bar{Q}) \left[\delta_{\tau,\,0} + \left(\frac{j_{01}}{j_{00}}\right)^2 \times \delta_{\tau,\,1}\tau_\eta(0)\,\tau_{-\eta}(1)\,(-1)^\eta \right] \times V_{\lambda\mu}(0)\,V_{\lambda-\mu}(1)\,(-1)^\mu. \qquad (12)$$

Here, the notation of Ref. 62 is used, and $V(0, 1)$ is a potential of McCarthy type; $y_\lambda^\tau(\bar{Q})$ is the coupling constant corresponding to excitation of the core to the level $\bar{Q}$ of multipolarity $2\lambda + 1$; j and τ are the spin and isospin transferred through the excitation of the core; μ is the projection of λ; and $V_{\lambda\mu}$ are single-particle potentials, explicit forms of which are given in Refs. 9, 62, and 63.

In Ref. 62, the angular correlation functions of protons of the (p, $2p$) reaction on ^{16}O were calculated for initial proton energy 45 MeV. The potentials of McCarthy, Hamada and Johnston, and of Gaussian form were used. Excitation of the levels corresponding to $\lambda = 1, 2, 3$ were taken into account.

For the first time in study of the (p, $2p$) reaction it was demonstrated that one could achieve agreement between the calculated and measured cross sections through the constructive interference of the quasielastic and two-step processes. In Fig. 9, we give the results of Ref. 62 together with the corresponding experimental data obtained for the ^{15}N

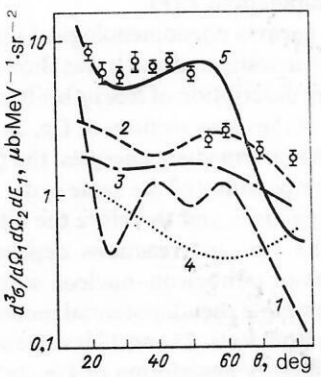

FIG. 9. Angular correlation function of protons from the ^{16}O(p, $2p$) ^{15}N reaction at energy 45 MeV. The calculated curves are from Ref. 62: 1) quasielastic approximation; 2)–4) excitation of core resonances of different multipolarities; 5) the sum.

ground state in the (p, $2p$) reaction on ^{16}O. It is here appropriate to point out that at certain angles the part played by the polarization of the core is clearly overestimated. Instead of being a correction it actually determines the behavior of the cross section. Since in the experiment of Ref. 54 events were selected that corresponded to direct interactions (from the region of the proton energy spectrum $|E_1 - E_2| < 4$ MeV, where there are no oscillations indicating two-step processes), the need for a more accurate description of quasielastic proton knockout at intermediate energies arose.

Another method of describing two-step processes was proposed in Ref. 64, in which the ^{16}O(p, $2p$)^{15}N reaction was analyzed by a microscopic description in the Born approximation with allowance for the second term of the Born series. The total amplitude of the (p, $2p$) reaction was represented as a sum of one- and two-step amplitudes. The one-step transitions were described by a local pp potential parametrized by means of the free scattering cross sections. The use of plane waves in the calculation made it impossible to obtain satisfactory agreement with experiment at energy 45 MeV. However, some important conclusions were drawn: 1) the two-step processes correspond to excitation of the ^{16}O nucleus to the 1d and 1f levels, and the cross section of these processes falls rapidly when the initial energy of the protons increases in the interval of energies 45–100 MeV; 2) the cross section of the two-step processes is about an order of magnitude smaller than that of the quasielastic processes. The importance of a more accurate description of the quasielastic processes was also emphasized in the paper.

The need to use nonlocal potentials to describe quasielastic interactions

Besides the local three-term potential $V(r)$ of McCarthy, other approaches have been used to describe quasielastic scattering of protons at an energy around 50 MeV. Jackson introduced a diproton model to take into account the corrections to the impulse approximation,[65] Strobel proposed a modification of the Born approximation with distorted waves,[8] Meboniya and Kvarackheliya developed a three-body unitarized impulse approximation,[66] and Deutchman and Old proposed a method to take into account the nonsphericity of nuclei.[67] The DWTA still takes into account most fully the specific features of the direct processes at intermediate energies, but it has not been possible to improve the results solely by a modification of the parameters of the three-term pseudopotential $V(r)$.

In Refs. 25, 27, and 48, various phenomenological nucleon–nucleon potentials were investigated and it was shown that while giving a satisfactory description of free pp scattering they give different values of the cross sections of (p, $2p$) reactions off the mass shell. At intermediate energies, the Q values of the reactions are comparable (of the same order) with the initial energy of the particle, and therefore the differential cross sections of the (p, $2p$) reactions depend strongly on the off-shell behavior of nucleon–nucleon scattering. It is clear that with the simple pseudopotential model of the t matrix of pp scattering from Refs. 25 and 27 it will not be possible to achieve a satisfactory description of (p, $2p$) reactions. In (p, $2p$) reactions, the off-shell t matrix must depend on three variables, namely, the relative coordinates of the two protons before ($\mathbf{r}' = \mathbf{r}_0' - \mathbf{r}_1'$) and after ($\mathbf{r} = \mathbf{r}_0 - \mathbf{r}_1$) the scattering, and also on their relative energy e, i.e., the matrix must be nonlocal.

There have also been other proposals to improve the agreement between the calculated and experimental cross sections; in particular, it was shown in Ref. 86 that to describe the (p, $2p$) reaction on ^{12}C it is expedient to use a complex pp scattering t matrix.

Nonlocal potentials had also been used earlier in calculations, but they were used in the factorized DWTA, in which the reaction matrix element is equal to the first term of (10). For the example of the ^{12}C(p, $2p$)^{11}B reaction at proton energy 155 MeV, such an approximation was used to investigate the behavior of the differential cross sections as a function of the type of nonlocal pp potential that determines the t matrix. It was found that different nonlocal potentials give approximately the same angular correlation functions.

Various nonlocal potentials that have proved themselves well in calculations of nucleon–nucleon interactions are now known and often used. Examples are the potentials of Mongan, Strobel, Chrepinshek, and others. For the development of the theory of a nonlocal t matrix for analysis of (p, $2p$) reactions, these potentials were found to be unsuitable, since with them it is difficult to calculate the multidimensional integrals of the type (11); in addition, the parameters of the potentials are determined only for a limited number of channels of free pp scattering and this is clearly insufficient for calculating cross sections of (p, $2p$) reactions. In Ref. 9, McCarthy proposed a nonlocal separable potential with Gaussian form factors whose parameters were determined from the phase shifts for the five channels $^1S_0$, $^1D_2$, $^3P_0$, $^3P_1$, $^3P_2$ of elastic pp scattering in the interval of proton energies 0–350 MeV.

In the notation of Ref. 9, $V_L(r, r')$ can be written in separable form as

$$V_L(r, r') = (g_L(r)\, h_L(r)) \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \begin{pmatrix} g_L(r') \\ h_L(r') \end{pmatrix}, \qquad (13)$$

where $g_L(r)$ and $h_L(r)$ are form factors of the form $r^L \exp(-a_i^2 r^2)$, and L is the relative orbital angularmomentum of the two protons. The potential (13) makes it possible to calculate analytically the integrals that occur in the matrix element.

A nonlocal separable energy-dependent realistic t matrix of free pp scattering can be found by solving the Lippmann–Schwinger equation with the potential (13):

$$\left.\begin{aligned} t_L(r, r'; e) &= \tilde{V}_L^{\dagger}(r)\, M_L(e)\, \tilde{V}_L(r'), \\ \tilde{V}_L^{i} &= r^L \exp(-a_i^2 r^2), \end{aligned}\right\} \qquad (14)$$

where the matrix $M_L(e)$ has rank 2.[9]

In the potential (13), the components g_L and h_L are responsible for attraction and repulsion, and C_i are their strengths. The parameters of the potential $V_L(r, r')$ of elastic nucleon–nucleon scattering for the isospin values $T = 0$ and 1 are given in Table III.

Distorted waves

For convenience in calculations of multidimensional integrals of the type (11), McCarthy proposed the use of an analytic form of the representation of the distorted waves of

TABLE III. Parameters of separable potentials of *pp*, *pn*, *pα*, and *pd* scattering.* (References are given in square brackets).

Channel	a_1, F^{-1}	a_2, F^{-1}	C_1, MeV·F^{-2L-3}	C_2, MeV·F^{-2L-3}
		(p, p) [9]		
$^1S_0$	7.8438	7.8118(−1)	8.9826(6)	−7.911(1)
$^1D_2$	4.1252(−1)	9.6861(−1)	−7.0461(−3)	−9.1961(1)
$^3P_0$	3.0428	5.6896(−1)	2.227(7)	−3.4283
$^3P_1$	3.4924(−1)	1.0147	2.3237(−2)	4.1748(2)
$^3P_2$	7.2076(−1)	1.3132	−5.225	−1.187(3)
		(p, n), $T=1$ [78]		
$^1S_0$	7.13	0.774	1.01(7)	−82.5
$^1D_2$	3.08	0.561	2.247(7)	3.46
$^3P_0$	1.01	0.349	405	0.0281
$^3P_1$	1.31	0.721	−1187	−5.23
$^3P_2$	0.969	0.413	−91.96	−0.00705
		(p, n), $T=0$ [78]		
$^3S_1$	1.3(1)	0.74	1.16(7)	−9.5(1)
$^1P_1$	1.37	0.25	1.14(4)	0.002
$^3D_1$	0.848	0.388	6.8(1)	0.017
$^3D_2$	0.95	0.51	−1.9(2)	−0.33
$^3D_3$	1.11	0.4	−2.18(2)	−0.001
		(p, α), [113]		
$S_{1/2}$	0.88	8.42	−2.3(2)	−3.1(6)
$P_{3/2}$	0.549	1.12	−1.16(1)	−1.89(2)
$P_{1/2}$	0.645	1.15	−2.16(1)	−3.74(2)
$D_{5/2}$	0.548	1.31	−0.76	−2.74(2)
$D_{3/2}$	0.63	1.18	−2.32	−1.60(2)
		(p, d) [115]		
$^2S_0$	1.809	0.4223	−4.285(3)	−1.942(1)
$^4S_0$	1.892	0.5423	−3.786(3)	−1.492(1)
$^2P_1$	0.164	0.707	0.384(−3)	−3.578(1)
$^4P_1$	0.3113	0.6994	−0.99(−1)	−1.807(1)
$^2D_2$	0.43	0.795	−0.988(−1)	−2.763(1)
$^4D_2$	0.268	0.705	0.120(−2)	−1.0

*The numbers in the brackets give the exponent of the factor 10^X by which the values given in the table must be multiplied.

the protons:

$$\chi^{\pm}(\mathbf{k}, \mathbf{r}) = \exp(-\gamma k R_N) \exp[\pm(1+\beta \mp i\gamma)\mathbf{kr}], \quad (15)$$

which is called the eikonal approximation. In (15), the parameter β determines the effective wave number in the region of interaction of the particles, and the parameter γ determines the decrease in the amplitude of the distorted wave due to absorption by the nucleus. The parameter R_N normalizes the wave; its value is close to the radius R_0 of the nucleus. In essence, the eikonal approximation is close to the semiclassical approximation, and the condition for its applicability is identical to the usual one for that approximation: $\lambda = 1/k \ll R_0$. On the boundary of the nucleus, the following estimates hold for the parameters β and γ:

$$\left.\begin{aligned} \beta &\approx U_1(R_0)/2[E+U_1(R_0)]; \\ \gamma &\approx U_2(R_0)/2[E+U_2(R_0)], \end{aligned}\right\} \quad (16)$$

where $U(r) = U_1(r) + iU_2(r)$ is the distorting optical potential, and E is the kinetic energy of the particle in the field of the nuclear forces. The properties of the eikonal approximation were considered in Ref. 32, where it was shown that within the nucleus the parameter β is positive ($\beta \approx 0.5$), while on its periphery an estimate of β in accordance with (16) gives small negative values. In accordance with its meaning, the parameter γ is always positive. From the relations between the real and imaginary parts of the potential $U(r)$, one can also obtain an estimate of the parameters β and γ: $\gamma \approx 0.1\beta$. In actual calculations, these estimates of the parameters must be made more precise, or one must regard β and γ as adjustable parameters. The next step in improving the eikonal approximation was to take into account the focusing of the incident and outgoing proton waves by the surface of the nucleus[9]:

$$\chi_{\mathbf{k}}^{\pm}(\mathbf{r}) = \exp(\pm iDkr)\{1 + F\exp[-(R\hat{\mathbf{k}} \mp \mathbf{r})^2/S^2]\}, \quad (17)$$

where $D = 1 + \beta + i\gamma$ is a generalized distorting potential that must be understood as a complex refractive index averaged over the surface of the nucleus; F, R, and S are the focusing parameters that determine the focusing capacity of

the nucleus (its focus F), the position of the surface peak of the distorted wave resulting from the focusing (R), and the width of this peak (S).

DWTA with the nonlocal realistic *t* matrix of McCarthy

Having at his disposal a nonlocal t matrix of the type (14), McCarthy gave up the zero-range interaction approximation (11) and included in the matrix element the coordinates of all particles:

$$T_{IF} = \sum_{j,\,m} (jmJ_FM_F|J_IM_I) \times \int \hat{A}\,[\chi^-_{\mathbf{k}_1}(\mathbf{r}_0)\,S_{\sigma_1}(0)\,\chi^-_{\mathbf{k}_2}(\mathbf{r}_1)\,S_{\sigma_2}(1)]^* \times \bar{\tau}(0,\,1;\,0',\,1';\,e)\,\chi^+_{\mathbf{k}_0}(\mathbf{r}'_0)\,S_{\sigma_0}(0')\,\Psi_{jm}(1')\,d0'\,d1'\,d0\,d1. \tag{18}$$

Here, $\hat{A}$ is the antisymmetrization operator, S_σ are the spin wave functions of the particles, σ are the projections of the spins of the particles; j, m, J_F, M_F, J_I, M_I are their angular momenta and their projections, respectively, for the knocked-out proton, the core, and the initial nucleus; and **0**, **1**, **0′**, **1′** are generalized (spatial and spin) coordinates of the protons.

The effective quasi-two-particle $\bar{\tau}$ matrix is the coherent sum of the quasielastic τ matrix and the core polarization operator:

$$\bar{\tau}(0,\,1;\,0',\,1';\,e) = \tau(0,\,1;\,0',\,1';\,e) + \sum_{\lambda\mu} y^T_\lambda(\bar{Q})\,V_{\lambda\mu}(0)\,V_{\lambda-\mu}(1)\,(-1)^{\mu+1}\,\delta_{0,\,0'}\delta_{1,\,1'}. \tag{19}$$

The transition from the matrix $\tau(0,\,1;\,0',\,1';\,e)$ to the quasielastic matrix $t(r,\,r';\,e)$ is made by expanding the matrix τ with respect to partial waves and separating the relative coordinates $\mathbf{r}$, $\mathbf{r}'$ and the center of mass $\mathbf{R}$, $\mathbf{R}'$. After this, the 12-fold integral (18) reduces to a 9-fold integral, and it can be calculated analytically by virtue of the exponential representation (expansion with respect to Gaussian functions) of the wave function $\Psi_{jm}(\mathbf{r})$ of the intranuclear proton and the use of generating functions in the exponential form for calculating the angular-momentum eigenfunctions. After all these manipulations, all the integrands in (18) become exponentials, the argument of the product of the exponentials being a quadratic form that is transformed to a sum of squares.

Analysis of experimental data by means of the DWTA and a nonlocal realistic *t* matrix

At the present time, the term "t-matrix approximation" is used in two cases—when Eq. (10) or Eq. (18) is used. To distinguish them, the approximation in the form (10) is usually called the factorized DWTA, and the one in the form (18) is called the DWTA.

The first calculations of (p, $2p$) reactions on the ^{12}C and ^{16}O nuclei at proton energies 50 and 45 MeV, respectively, made to test the possibilities of the DWTA,[9] did not give a significant improvement in the agreement with experiment compared with the models used earlier (Fig. 10). In the subsequent studies of Refs. 10 and 59–61, the reasons for such discrepancies were carefully analyzed and it was shown that, first, it is necessary to parametrize the distorted waves $\chi^\pm$ in the interval of proton energies 10–70 MeV using the data of

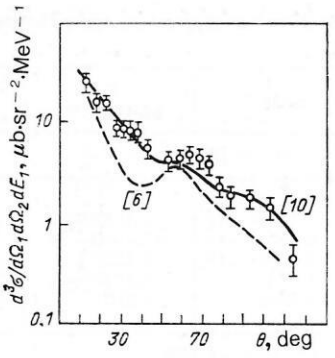

FIG. 10. Angular correlation function of protons from the ^{12}C(p, $2p$)^{11}B reaction at 50 MeV (references are given in square brackets).

elastic scattering and ensuring continuity (and smoothness) of the energy dependence of the parameters. Second, the study of Ref. 9 took into account virtual excitations of only quadrupole ($\lambda = 2$) giant resonances, and it was shown in Ref. 62 that this was clearly insufficient. It is also necessary to include in the calculation the dipole ($\lambda = 1$) resonances. Third, the calculations of Ref. 9 used random values of the coupling constants $y^T_\lambda(\bar{Q})$, and this evidently explained the absence in the calculations of interference between the direct processes and the core polarization.

The study of Ref. 69 was devoted to the parametrization of the distorted waves; the optimal values of their parameters for the nuclei ^{12}C, ^{14}N, and ^{16}O were given in Refs. 59–61, in which simple analytical dependences of the values of some of the parameters on the proton energy were also given. The inclusion in the calculations of the dipole resonances and the optimal values of the coupling constants y_λ in Ref. 61 made it possible to improve the agreement between the calculations and the experimental data.

The results of the calculations in Refs. 10 and 59–61 are given in Figs. 8h, 10, and 11. The interference between the processes of direct knockout and core polarization can be clearly seen in Fig. 11.

A serious shortcoming of these studies is the actual method of parametrizing the distorted waves $\chi^\pm$ by means of the optical potentials $U(r)$. The (p, $2p$) reaction is, in

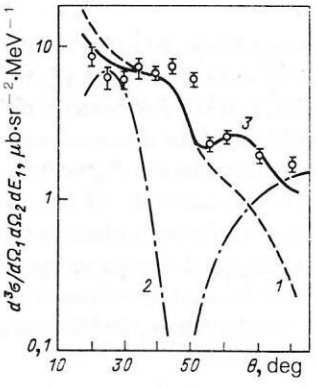

FIG. 11. Angular correlation function of protons from the ^{16}O(p, $2p$)^{15}N reaction at energy 45 MeV. The calculated curves are from Ref. 61: 1) quasielastic knockout; 2) core polarization; 3) the sum of the two processes.

TABLE IV. Parameters of proton distorted waves for inelastic scattering and (p, pN) reactions on 1p-shell nuclei.

Nucleus	E, MeV	$\|F\|$	arg F, deg	β	γ	S, F	R, F
^{16}O	50	2	30	0,035	0,008	1,2	2.46
	20	6	120	0,55	0,3	1.2	1,51
^{14}N	50	2	30	0,035	0,008	1,2	2.4
	20	6	120	0,5	0.2	1,2	1,56
^{12}C	50	2	30	0,035	0,008	1,2	2.28
	20	6	120	0,4	0.1	1,3	1,7
^{7}Li	50	2	30	0,035	0,008	1,5	1.53
	20	4	150	0,2	0,06	1,5	0.956
^{6}Li	50	2	30	0,035	0,008	1,6	1.45
	20	3	150	0,04	0,03	1,6	0.91

essence, a special case of inelastic proton scattering, and it is therefore natural to ask whether the inelastic scattering of protons by nuclei might not be described by the same proton –proton t matrix. The corresponding problem was subsequently solved in Ref. 70.

This then provided a real possibility to use the data of inelastic scattering to parametrize the distorted waves $\chi^{\pm}$ and verify the result independently by an analysis of knock-out reactions. To a degree, this makes it possible to solve the problem of the ambiguity in the parametrization of the distorted waves $\chi^{\pm}$.

Table IV gives the parameters of the distorted waves $\chi^{\pm}$ of the protons for (p, 2p) reactions and inelastic scattering by 1p-shell nuclei. Comparing their values, one can see a growth of the distorting parameters β and γ with both decrease of the proton energy and increase in the number of nucleons in the 1p shell.

Table V gives the values of the coupling constants $y_\lambda^{T=1}$ of the dipole and quadrupole giant resonances obtained by analyzing (p, 2p) reactions on these nuclei. As follows from (19), the accuracy in the determination of $|y_\lambda|$ is directly related to the accuracy in the determination of the depth of the optical potentials $V_{\lambda\mu}$. In all the cases which we investigated, complex Woods–Saxon potentials were used for the incident particle and the emitted particles. Their depths were determined from the data of Ref. 71. For the potential of the particle–core coupling a real potential was used, and its depth was found using a fragment of the OLYMP program.[72]

The values of the coupling constants for $\lambda = 1, 2, 3$ had been determined earlier in Ref. 62 in an analysis of proton inelastic scattering by ^{16}O and ^{12}C. For comparison, Fig. 12 gives the results of Ref. 62, which agree with our data to within the indicated errors (see Table V).

We note that the coupling constants given in Table V and in Fig. 12 in reality correspond to virtual excitation of the core, for example, in the case of the ^{16}O nucleus to excitation of ^{15}N. This should be borne in mind when comparing the values of the coupling constants if they are obtained from an investigation of different reaction channels. In other words, the value of y_λ from Table V for the ^{12}C nucleus could be compared with the analogous value from the ^{14}N(p, p^3He)^{11}B reaction, etc. At the present time, there is practically no information about the coupling constants y_λ.

TABLE V. Coupling constants $y_\lambda^{T=1}$ obtained from study of (p, 2p) reactions.

Nucleus	$\|y_1\|$, MeV^{-1}	arg y_1, deg	$\|y_2\|$, MeV^{-1}	arg y_2, deg
^{16}O	0,208	70	0,01	15
^{14}N	0,01	60	0,005	50
^{12}C	0,12	50	0,02	58
^{7}Li	0,047	48	0,067	81
^{6}Li	0,029	17	0,037	44

Analysis of chamber experiments

The (p, 2p) reaction was studied for the first time in complete geometry on the ^{12}C nucleus by means of a bubble chamber and in the interval of proton energies 50–110 MeV.[58] The problem was posed of tracing the change in the dynamics of the processes, i.e., seeing how the mechanism of the (p, 2p) reaction changes within the chosen interval of initial energies. A theoretical analysis of the experimental results was made by Sakamoto,[73] using the impulse approximation. Figure 13 gives the results of his calculations together with the experimental data. Unfortunately, the poor sta-

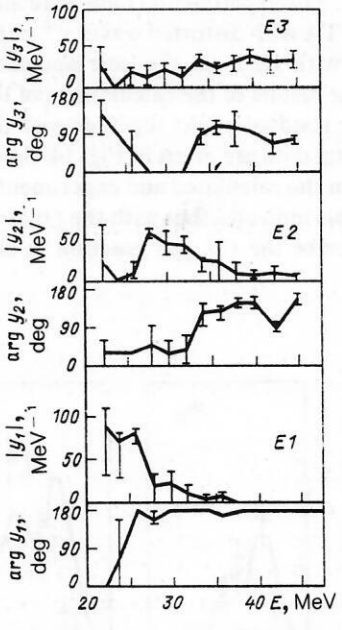

FIG. 12. Coupling constants $y_\lambda = |y_\lambda|\, e^{i\Phi} \cdot 10^3$ for excitation of giant dipole (El), quadrupole (E2), and octupole (E3) resonances deduced from inelastic scattering of protons by ^{12}C (Ref. 62).

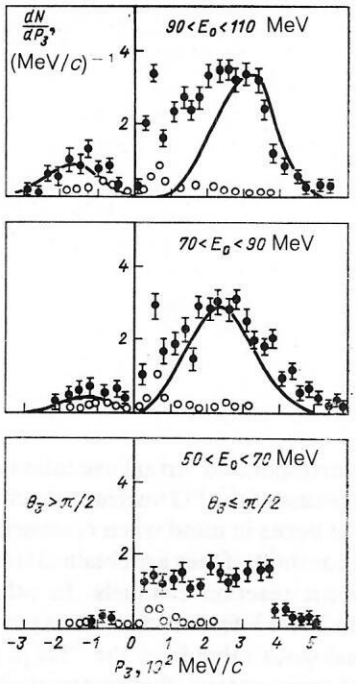

FIG. 13. Momentum spectrum of residual nuclei from the $^{12}C(p, 2p)^{11}B$ reaction in complete geometry for different initial energies of the protons. The open circles are for coplanar events, and the black circles give the complete statistics. The calculated curves are by Sakamoto.[71]

tistics inherent in all chamber experiments and the experimental errors did not allow an unambiguous solution to the problem that had been posed. The enhanced importance of the nondirect processes on the transition to the energy 50 MeV can be seen even without calculation. In the momentum spectrum corresponding to the direct processes there must be a maximum in the region of small momentum transfers, at $q \approx 100$ MeV/c.

The photoemulsion method was used to study the $(p, 2p)$ reactions on the nuclei ^{12}C, ^{14}N, and ^{16}O at proton energy 50 MeV in Refs. 59–61. The experimental data were analyzed by means of the DWTA with distorted waves $\chi^{\pm}$ parametrized in accordance with data on the corresponding coplanar experiments. The results of the calculations of the momentum spectra of the residual nuclei together with the corresponding experimental data are given in Fig. 14. Satisfactory agreement between the calculated and experimental spectra is observed, and this indicates that with the t matrix (12) it is possible to describe the $(p, 2p)$ reaction in any geometry.

In Refs. 59–61 there was noted an increasing importance of the triplet channels $^3P_0$, $^3P_1$, and $^3P_2$ in the total cross section of the $(p, 2p)$ reaction measured in experiments with complete geometry as compared with coplanar geometry for all three nuclei investigated in these studies.

Off-shell effects

One of the fundamental questions that arise in the description of knockout reactions by means of an effective two-particle amplitude is that of the off-shell effects. They arise because, in contrast to free pp scattering, by means of which all the two-particle potentials are parametrized, the incident proton in knockout reactions interacts with a bound intranuclear proton, whose separation energy is $E_s \neq 0$. As a result, the relative momenta of the interacting particles in the entrance, $P_i = (P_0 + q)/2$, and exit, $P_f = (P_0 - P_1)/2$, channels are different, and this leads to an ambiguity in the calculation of the effective proton–proton matrix element T_{pp}.

Off-shell phenomena are inherent in all three-particle (or many-particle) processes when the scattering in the two-particle system is distorted by a third particle. Only when $E_s = 0$ and one can ignore the kinetic energy of the recoil nucleus are the relative momenta in the initial and final channels equal and the two-particle amplitude $|T_{pp}|^2$ will be determined on the mass shell.

The off-shell properties of the two-particle amplitude, which could, in principle, be obtained from knockout reactions, could play a part in the solution of the inverse problem (recovery of the two-particle amplitude from data on nucleon–nucleon scattering). Even with a complete set of data on the nucleon–nucleon scattering (there are no data on nn scattering), one can construct an infinite number of phase-equivalent potentials that give the same two-particle amplitudes on the mass shell but differ off it. Information about the off-shell behavior of the two-particle amplitude in knockout reactions would make it possible to choose among the complete manifold of potentials the true potential, i.e., to solve the inverse scattering problem.

It was found that in reality it is not simple to obtain such information, since in all the experiments so far made the off-shell effects have been manifested simultaneously with the distortions, and it is in practice impossible to separate them.

Maris and Jacob[2] noted in connection with the applicability of the impulse approximation to $(p, 2p)$ reactions that at energies 150–500 MeV, where the cross section of free pp scattering depends weakly on the energy of the incident proton and the scattering angle, the influence of the off-shell

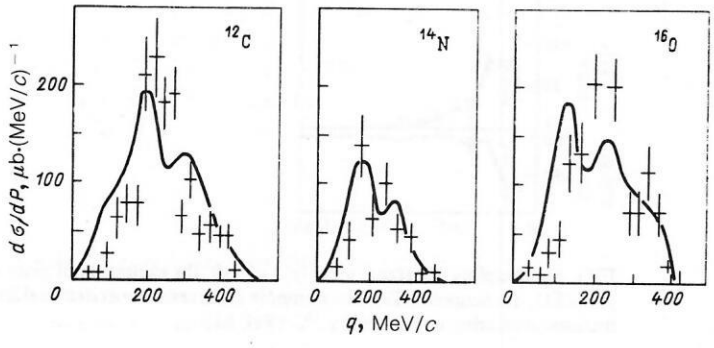

FIG. 14. Momentum spectra of residual nuclei from $(p, 2p)$ reactions on different nuclei at energy 50 MeV.[59-61] The calculation is in the DWTA.

effects can be ignored. However, they recommended that the impulse approximation with distorted waves should not be used if off-shell effects are important.

In the plane-wave impulse approximation, Redish[48] investigated the sensitivity of the two-particle amplitude $|T_{pp}|^2$, as determined by means of the Reid potential with a soft core, to the method of determining this amplitude and the relative energy of the protons. He proposed the use of the matrix in the half-off-shell form:

$$T_{pp} = \langle \mathbf{P}_{\text{on}} \mid t(e) \mid \mathbf{P}_{\text{off}} \rangle, \tag{20}$$

where $\mathbf{P}_{\text{on}} \equiv \mathbf{P}_f$ and $\mathbf{P}_{\text{off}} = \mathbf{P}_i$ are the relative momenta of the two protons on the mass shell and off it; e is the relative energy of the two protons on the mass shell, $e = e_f$, off it, $e = e_i$, or their mean value $e = (e_i + e_f)/2$. The name "half-off-shell" follows from the form of the expression (20); if $\mathbf{P}_{\text{on}}$ in it is replaced by $\mathbf{P}_{\text{off}}$, the matrix element will be completely off-shell.

According to the data of Redish, the $(p, 2p)$ reactions are not sensitive to the off-shell behavior of the two-particle amplitude at energies above 300 MeV, and therefore precisely this region of energies is recommended for the direct (directly from experiment) study of the intranuclear wave functions. The region of energies below 200 MeV is sensitive to the off-shell behavior of the pp interaction and is therefore the most convenient for the study of off-shell effects. For the energy e in Ref. 48 it was proposed to use e_i or $(e_i + e_f)/2$.

Would it be possible to establish off-shell properties of the two-particle potentials on the basis of the behavior of the calculated cross sections of $(p, 2p)$ reactions in the DWTA? To examine this question, we took the example of the ^{6}Li$(p, 2p)^5$He reaction at proton energy 100 MeV in symmetric coplanar geometry and compared the results of DWTA calculations using some well-known nonlocal potentials, which we first expanded in series in Gaussian functions. We chose the potentials of Mongan (Mo)[74] and Strobel (S).[75] The results of these calculations were normalized by the experimental cross section at angle 30°.

In Table VI, we give the ratios of the cross sections of the $(p, 2p)$ reactions calculated with these potentials to the cross sections obtained using McCarthy's potential (M). It can be seen that all three potentials give values of the cross sections that do not deviate beyond the experimental errors. For comparison, we also give in Table VI the results of similar calculations from Ref. 28, in which the ^{12}C$(p, 2p)^{11}$B reaction was studied at energy 155 MeV. The calculations there were made by the factorized DWTA with the potentials HM (Ref. 76) and GTGW.[77] All the calculated values in Table VI were obtained for the $^1S_0$ channel of pp scattering; as in Ref. 28, it is not possible to give preference to any one of the three considered potentials.

TABLE VI. Comparison of calculated angular correlation functions of protons from $(p, 2p)$ reactions for different pp potentials.

θ, deg	S/M	Mo/M	GTGW/HM (Ref. 28)
10	0.91	0.90	—
20	0.96	1.00	1.11
30	1.00	1.00	1.00
40	1.03	1.06	1.00
50	1.02	1.27	1.41
60	1.02	1.03	1.22

In Refs. 8, 70, and 78 a study was made of the behavior of the angular correlation function of the protons of $(p, 2p)$ reactions in symmetric coplanar geometry at initial energy 50 MeV as a function of the method of specifying the relative interaction energy e of the protons: in the initial, $e = e_i$, and in the final, $e = e_f$, channels. It was found that the cross sections differ only in the region of small angles, $\theta \leqslant 40°$, where for $e = e_f$ the calculated cross sections are always greater than for $e = e_i$. The nature of such a dependence is determined by the behavior of $|t(k, k'; e)|$ (the t matrix in the momentum representation, which for $e < 10$ MeV rapidly increases with decreasing e). With regard to the application of these conclusions to specific $(p, 2p)$ reactions at initial proton energy 50 MeV, it was established that at large Q (the nuclei C, N, and O), when the importance of the off-shell effects is large, only the choice $e = e_f$ gives agreement with the experiments, while conversely,[78] when Q is small (^{6}Li), only $e = e_i$ leads to the correct result. At proton energies of 100 MeV and above for all the investigated nuclei (C, N, O, and Li) agreement with experiment in the region of small angles can be obtained only for $e = e_i$.

These results confirm the validity of the assertion that the simultaneous variation of the distortions and of the off-shell properties makes it impossible to draw unambiguous conclusions about the off-shell effects in knockout reactions.

In a systematic study of the off-shell effects, it is evidently necessary to separate the distortions and the off-shell effects already at the experiment stage. For this purpose, Jackson proposed the establishment of a kinematic control of the measured variables.[79] He suggested that the variables measured (and controlled) in the experiment should be divided into two groups: a) those responsible for the off-shell effects (P_{on}, P_{off}, and the angle between these vectors); b) the variables responsible for the distortion (q, E_1, E_2). Making the assumption that the cross section of the $(p, 2p)$ reaction does not change significantly with variation of the angles θ_1 and θ_2, one can choose the kinematic conditions of measurement to ensure that the momentum q of the recoil nucleus and the sum of the energies $E_1 + E_2$ are constant. Then there remain only the four variables θ_1, θ_2, $\lambda = E_1/(E_1 + E_2)$, and θ_3 (the angle of emission of the recoil nucleus with respect to the direction of the incident beam of protons) that will vary in the experiment.

By a variation of θ_3 for fixed λ all the variables in group (b) can be fixed, and, depending on θ_3, the ratio $P_{\text{off}}/P_{\text{on}}$ can be made arbitrarily large. Varying λ for fixed θ_3, we can fix P_{off} and P_{on} in the region of small θ_3 (less than 45°), and the variables of group (b) can be changed.

Such a program of investigations can be realized experimentally with the high-current accelerators of the meson-factory or TRIUMF type. It would be very desirable to make such experiments at the Institute of Nuclear Research of the USSR Academy of Sciences.

Study of (p, pn) reactions

Despite the large cross sections of free pn scattering, study of (p, pn) reactions began only recently, this being due to the difficulties of neutron detection. For this purpose,

the time-of-flight method is usually employed. The (p, pn) reactions are regarded as complementing the (p, $2p$) reactions, and therefore they are usually studied together. A generalization of the results on neutron knockout reactions at intermediate energies is contained in the review of McDonald.[5] In this region of energies, the ratio of the (p, $2p$) and (p, pn) cross sections, measured under identical kinematic conditions, is usually equal to the ratio of the corresponding cross sections of free pp and pn scattering, and this is an additional criterion for the validity of the impulse approximation.

At intermediate proton energies, (p, pn) reactions have been studied only in Refs. 80 and 81; at energy 46 MeV, Miller[80] obtained spectra of protons from (p, pn) reactions on the nuclei ^{6}Li, ^{9}Be, and ^{13}C, while the group of Chant[81] obtained data at 47 MeV for the nuclei ^{6,7}Li and ^{9}Be. There are preliminary communications on a study of the ^{6}Li(p, pn)^{5}Li reaction at 70 MeV at Kiev.[82]

In the energy spectra of the protons from the (p, pn) reactions for all the studied nuclei except ^{6}Li a minimum is observed at energies corresponding to $q = 0$, this agreeing with the impulse approximation. The calculations using the DWIA made in Refs. 80 and 81 gave good agreement with experiment. An analogous form of the proton spectra from (p, pn) reactions is also characteristic of higher initial energies of the protons, i.e., on the transition to intermediate energies the shape of these spectra does not change. However, this fact does not contradict the results of investigations of the (p, $2p$) reactions at intermediate energies, since for these nuclei the neutron binding energy is low (between 1.7 MeV for ^{9}Be and 7.3 MeV for ^{6}Li).

The energy spectrum of the protons from the (p, pn) reaction on ^{6}Li does not have a dip at $q = 0$; as in the case of the (p, $2p$) reactions, this can be explained by the cluster structure of this nucleus. The use of a d–α cluster model for the ^{6}Li nucleus[44] made it possible in the calculations of Ref. 81 to obtain agreement between the calculated and experimental spectra.

For (p, pn) reactions on nuclei for which the neutron separation energy is greater, one must expect changes in the structure of the differential cross sections at intermediate proton energies. In this case, the impulse approximation, for which the transition from the (p, $2p$) to the (p, pn) reactions reduces in essence to merely an increase in the cross section of free scattering, is incorrect.

In Ref. 78, the t-matrix approximation was extended to (p, pn) reactions by inclusion in the nucleon–nucleon t matrix of the pn scattering channel with $T = 0$. This approximation gives a more accurate description of the dynamics of quasifree pn scattering at intermediate energies. The results of the calculations given in Ref. 78 for the known experimental data using the DWTA agree well with experiment.

Intranuclear wave functions of nucleons

The experimental data that are obtained in investigations of knockout reactions undoubtedly contain information about nuclear structure, but the difficulty of interpreting the reaction mechanism makes it difficult to extract this information. The wave function $\Psi_{jm}(r)$ which occurs in the expression (18) is calculated on the basis of the shell or cluster nuclear models. It is usually represented in the form of an overlap integral:

$$\Psi_{jm}(\mathbf{r}) = \int d^3\xi \Psi_F^*(\xi)\, \Psi_I(\xi, \mathbf{r})$$

$$= \sum_{j,m} (J_F M_F jm | J_I M_I)\, C_{J_I J_F} \Psi_j^m(\mathbf{r}), \qquad (21)$$

where Ψ_F and Ψ_I are complete sets of states of the final F, and initial, I, nuclei, ξ are the internal coordinates of nucleus F, $C_{J_I J_F}$ is the coefficient of fractional parentage for separation of a nucleon (or cluster) in the original nucleus, and Ψ_j^m ($\mathbf{r}$) is the normalized single-particle wave function.

The total angular momentum of particle j is the sum of the orbital angular momentum l and the spin s. If in a nucleus there are N_{lj} identical particles (usually, this is the number of particles in a distinguished subshell), the concept of a spectroscopic factor is introduced:

$$S_{J_I J_F}(l, j) = N_{lj}\{C_{J_I J_F}\}. \qquad (22)$$

The asymptotic behavior of the wave function $\Psi_{jm}(\mathbf{r})$ can be determined from the Schrödinger equation

$$(H_F + T_F + V)\, \Psi_I = E_I \Psi_I. \qquad (23)$$

Multiplying this expression from the left by Ψ_F^* and integrating over ξ, we obtain

$$(T_F + V)\, \Psi(\mathbf{r}) = (E_I - E_F)\, \Psi(\mathbf{r}), \qquad (24)$$

i.e., the behavior of the radial part $R_{jl}(\mathbf{r})$ outside the range of the effective potential V ($V = 0$) has the form

$$R_{jl}(r) \to \exp(-kr)/r, \qquad (25)$$

where

$$k^2 = \frac{2\mu_F (E_I - E_F)}{\hbar^2} = \frac{2\mu_F E_s}{\hbar^2}. \qquad (26)$$

The usual notation is employed in the equations: H_F and T_F are the Hamiltonian and the kinetic-energy operator of the final state, E_I and E_F are the energy eigenvalues of the Hamiltonians of the initial and final states, $E_S = E_I - E_F$ is the separation energy of the particle, and μ_F is the reduced mass of the particle that is knocked out (proton) and the residual nucleus.

For the potential V, one usually takes the spherically symmetric Woods–Saxon potential, the parameters of which are chosen to make the nucleon binding energy equal to the known value E_S of the separation energy. The values of the mean separation energy of protons obtained from (p, $2p$) and (e, ep) experiments are given in Ref. 83. Sometimes, V is taken to be a harmonic oscillator potential; the asymptotic behavior of the functions that are then obtained is different from (25) [at large values of r, the functions $\Psi(r)$ decrease more rapidly than (25)], although they correctly reproduce the physically measurable quantity (the rms charge radius of the nucleus).

The separation-energy method makes it possible to describe correctly the asymptotic radial dependence of the intranuclear wave functions but not their behavior within the nucleus nor the absolute value in the asymptotic region. To obtain the wave functions more accurately, it would be necessary to take into account the residual interaction, but this leads to a system of coupled equations. Methods that permit

practical use of these equations are reviewed in Ref. 84.

When the matrix element (18) is calculated, it is in practice convenient to use oscillator wave functions, which can be obtained by means of the program OLYMP.[72] The incorrect asymptotic behavior of these functions at large r does not seriously affect the accuracy of the calculation, since $|\bar{\tau}(r, r'; e)|$ decreases rapidly with increasing r.

Information about the nuclear structure is contained in the spectroscopic factor $S_{J_F J_F}$, which is determined experimentally or theoretically. In the first case, the spectroscopic factor is a normalization coefficient by means of which the calculated differential cross section of the (p, $2p$) reaction is normalized by means of the experimental cross section. The theoretical spectroscopic factor is considered in the framework of some model, and therefore it depends on the type of potential or coupling scheme, j–j or intermediate l–s, used in the calculation. This opens up the possibility of testing the various models for particular nuclear states.

In the region of high energies (several hundred megaelectron-volts) the analysis of cluster knockout reactions must take into account the rearrangement of the intranuclear wave functions of the clusters, i.e., it must take into account excitation of the clusters. Calculations of such processes were undertaken for the first time at the Institute of Nuclear Physics at Moscow State University by a group led by V. G. Neudachin.[85] Allowance for excitation of the clusters permits a number of experimentally verifiable predictions to be made, in particular, a dependence of the cross section on the orientation of the recoil-nucleus momentum, anisotropy with respect to the Treiman–Yang angle, etc. In Ref. 85, a method was proposed for calculating the spectroscopic factors for 1p-shell nuclei in a translationally invariant model. It was found that allowance for cluster excitation increases appreciably the effective number of nucleon associations (for example, for ^{16}O the effective number of α particles increases from 13 to 35 if cluster excitation is taken into account).

We mention one further property of the nuclear knockout reactions—the possibility of studying the populations of the states of the residual nucleus. This arises when the (p, $2p$) reaction is studied in symmetric geometry and even components m contribute to the cross section. If the proton is knocked out of the p shell of the ground state of the O^+ nucleus, then in symmetric coplanar geometry the possible states of the residual nucleus are $M = \pm 1/2$, in contrast to the asymmetric case, in which all states, $-3/2 \leqslant M \leqslant 3/2$, are populated. Changing the geometry of the reaction, it is possible to study the relative population of the states.[13]

In conclusion, we consider for the case of (p, $2p$) reactions on ^{6}Li the sensitivity of the differential cross section to the form of the intranuclear proton wave function. It is well known that the ^{6}Li nucleus is strongly clustered, though many of its properties (structure of the levels, charge form factor, etc.) can be independently obtained (and explained) in α, d, three-body, or shell models. Then the (p, $2p$) reaction can be used as a test of the correctness of the determination of the intranuclear wave function, i.e., of the nuclear model.

Figure 15 gives the results of our calculation of the (p, $2p$) reaction on ^{6}Li at energy 47 MeV with the wave function of the three-body n–α–p model of Ref. 86. Compared with the analogous calculation (see Fig. 6e) in which the shell model is used, one can see that there is a significant improvement of the agreement with experiment. This was made possible by the contribution (20%) of the $(S_{1/2})^2$ states of the valence nucleons due to the $S_{1/2}$ interaction of the α–N system. In the shell model, the possible states of the ^{6}Li valence nucleons are only $p_{3/2}$, and it is not possible to reproduce the experimental cross section shown in Fig. 6e with any set of parameters in the DWTA.[78] A similar conclusion had already been drawn in an analysis of experimental data for the (p, $2p$) reaction on ^{6}Li at energy 70 MeV (Ref. 10).

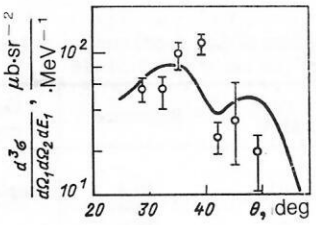

FIG. 15. The same as in Fig. 6e. The curve is calculated in the DWTA with the ^{6}Li wave function of the three-body n–α–p model.[86]

2. CLUSTER KNOCKOUT REACTIONS

Types of experiment

In this section, we shall consider reactions in which deuterons and α particles are knocked out of nuclei with $9 \leqslant A \leqslant 16$. As in the case of the (p, $2p$) reactions, one here distinguishes coplanar, noncoplanar, and chamber experiments. A list of experimental studies made at initial proton energy below 100 MeV is given in Table VII. As a rule, the coplanar experiments are made by one of the following methods:

1. The proton emission angle is fixed and the cluster emission angle is varied. Events that satisfy the condition $P_p \sin\theta_p = P_x \sin\theta_x$ are selected.

2. The emission angles of both the proton and the cluster are varied, and $\theta_p = \theta_x$. Events that satisfy the condition $|P_p - P_x| < C$ are selected.

In these two cases, the angular correlation functions are measured. The restrictions imposed on the momenta have the consequence that one considers only events in which the direction of motion of the residual nucleus coincides with the direction of the momentum of the incident proton.

3. The proton and cluster emission angles are fixed and the energy spectrum of the proton is measured.

In the majority of the experiments, the proton energy spectra are determined; moreover, as a rule, one selects a quasielastic pair of angles, i.e., the angles of the emitted proton and cluster, for which zero momentum of the residual nucleus is kinematically allowed.

Analysis of coplanar experiments makes it possible to obtain information about the mechanism of the nuclear reactions and about the nuclear structure. Thus, a bell-shaped angular correlation function with a maximum corresponding to minimal momentum transfer indicates that the interaction is quasielastic and that the wave function of the relative motion of the cluster and core has zero angular momentum; the presence of narrow (with width $\lesssim 0.25\ F^{-1}$ in the scale of the wave functions of the residual nucleus) peaks in the proton energy spectra indicates that the reaction takes place in two steps: inelastic scattering of the proton

TABLE VII. Experimental investigations of the (p, pd) and ($p, p\alpha$) reactions at intermediate proton energies ($\lesssim 100$ MeV) on nuclei with $9 \leqslant A \leqslant 16$.

Reaction	Target nucleus	Energy, MeV	Reference	Geometry of experiment
(p, pd)	^{14}N	44	[87]	Coplanar
	^{14}N	46	[88]	»
	^{12}C, ^{16}O	75	[89]	»
	^{9}Be, ^{12}C, ^{14}N	58	[90, 91]	»
	^{12}C, ^{14}N, ^{16}O	50	[92]	Complete
(p, $p\alpha$)	^{12}C	57	[93]	Coplanar
	^{9}Be	57	[94]	»
	^{9}Be	55	[95]	»
	^{9}Be	26, 35, 46, 8	[96]	»
	^{16}O	46,8	[97]	»
	^{9}Be, ^{12}C	100	[98]	»
	^{16}O	101,5	[99, 100]	»
	^{9}Be	101,5	[101]	Noncoplanar
	^{12}C	85	[102]	Complete
	^{16}O	50	[103]	»

with formation of excited states of the intermediate nuclei and their subsequent decay with the emission of particle x, etc. However, the coplanar experiments are made in a restricted region of the phase space. As a result, it is not possible to analyze the contribution of all mechanisms to the reaction cross section. Therefore, in a theoretical analysis of cluster knockout reactions, calculations in the coplanar case must be used to test the correctness of the choice of the wave functions and the potentials, while a final conclusion about the reaction mechanisms must be obtained by analyzing experimental data obtained in complete geometry.

Analysis of (p, px) reactions in the PWIA

As in the case of (p, $2p$) reactions, the methods traditionally employed to analyze cluster knockout reactions are the PWIA and DWIA. The expression for the cross section of the (p, px) reaction in the PWIA is obtained in the same way as in the (p, $2p$) case and has the form

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_x\, dE_p} = KFS_x |g(q)|^2 \frac{d\sigma}{d\Omega}\Big|_{p-x}. \qquad (27)$$

Here, KF is a kinematic factor, S_x is the spectroscopic factor (the effective number of x associations in the target nucleus), $(d\sigma/d\Omega)|_{p-x}$ is the cross section of px scattering, and $g(q)$ is the momentum distribution of the x cluster in the target nucleus (the Fourier transform of the wave function of the relative motion of the cluster and core).

The use of the PWIA makes it possible in the case when the calculated curve describes the shape of the experimental distribution to obtain the values of the spectroscopic factors by normalization. The PWIA curves describe the shape of the experimental dependence of reactions with initial energy above 150 MeV reasonably well. For example, a PWIA analysis of the ^{12}C($p, p\alpha$)^{8}Be reaction made at initial energy 150 MeV (Ref. 104) and 160 MeV (Ref. 105) gave nearly equal values of the spectroscopic factors: $0.30^{+0.23}_{-0.11}$ and 0.13 ± 0.07.

At energy below 100 MeV, the PWIA permits description of only the ($p, p\alpha$) reaction on ^{9}Be. Figure 16 shows the experimental angular correlation functions of the ^{9}Be(p, $p\alpha$)^{5}He reaction at $E_0 = 55$ MeV.[95] They are curves with a single broad maximum in the region corresponding to the minimal momentum transfer, and this indicates the following:

1) that there is an appreciable contribution of quasielastic knockout to the reaction mechanism;

2) that there is a dominant contribution from the s component of the wave function of the relative motion of the α cluster in the target nucleus.

The large contribution of quasielastic knockout in the ^{9}Be($p, p\alpha$)^{5}He reaction can be explained by the low binding energy of the α particles in ^{9}Be ($Q = 2.53$ MeV, a value that is appreciably less than for the other light nuclei: $Q = 7.36$ MeV for ^{12}C and $Q = 7.16$ MeV for ^{16}O). The shape of these angular distributions can be described reasonably well in the PWIA if for the momentum distribution of the α particles in the nucleus one uses the oscillator distribution $\exp[-(q/\sigma)^2]$ with the parameter $\sigma = 0.304$ F^{-1} taken from analysis of the ^{9}Be(α, 2α) reaction.[106] However, the probability of finding α clusters in the nucleus obtained in Ref. 95 (0.87) is appreciably greater than the corresponding value obtained from analysis of the ^{9}Be(α, 2α) reaction,[106] which is 0.12. This indicates the need to take into account the distortions of the wave functions of the incident particle and the emitted particles.

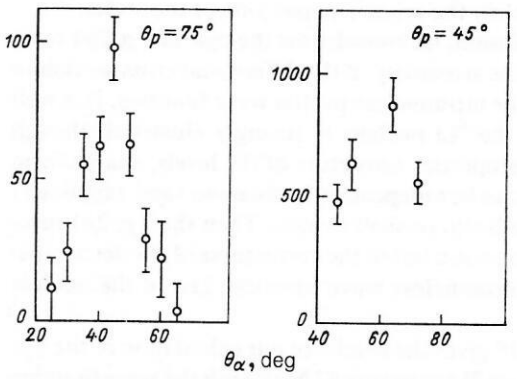

FIG. 16. Experimental angular correlation functions of the ^{9}Be(p, $p\alpha$) ^{5}He reaction at energy 55 MeV.[95]

To model the distortions of the wave functions of the incident and emitted particles (to take into account the effects of absorption), a radial cutoff is used, i.e., the wave functions are set equal to zero in the region $r \leqslant R_{\text{cut}}$, where R_{cut} is the cutoff radius (only particles outside the region bounded by the cutoff radius participate in the reaction). In Ref. 107, the PWIA with a radial cutoff was used to analyze the ^{9}Be$(p, p\alpha)$ reaction at energies below 160 MeV. The wave functions of a harmonic-oscillator, rectangular-well, and Woods–Saxon potential were used for the wave function of the relative motion of the α cluster and the core. Analysis of the reaction at 160 MeV yielded the spectroscopic factor $S_\alpha = 0.208$ (for cutoff radius equal to zero). To obtain the same value of S_α at lower energies, it is necessary to change the cutoff radius (it is equal to 1.2 F at 46.8 MeV and 2.8 F at 35 MeV). In fact, the cutoff radius is a free parameter, and the reliability of the nuclear-structure information obtained from PWIA analysis with a radial cutoff is very questionable. For analysis of cluster knockout from other nuclei the PWIA was found to be unsuitable.

Use of the DWIA to analyze (p, px) reactions

Transition to the distorted-wave impulse approximation leads to replacement of the plane waves in the expression for $g(q)$ in (27) by distorted waves. As in the case of $(p, 2p)$ reactions, the simplest way of introducing distortions is through the semiclassical approximation. The form of the distorting factors for the α particles and deuterons is the same as for protons. Such wave functions were used by Sakamoto,[108,109] who also attempted to calculate the absolute values for $(p, p\alpha)$ reactions. The wave function of the cluster–core relative motion was represented by a harmonic-oscillator wave function. The application of this method to the analysis of the angular correlations of the ^{12}C$(p, p\alpha)$ reaction at 150 MeV (experimental data of Ref. 104) did not lead to agreement with the experiment (the results of the calculation were about five times smaller). In other cases, the semiclassical approximation for the description of (p, px) reactions was not used.

Calculation of the cross section of the (p, px) reaction in the DWIA was considered rigorously in Ref. 110. The expression for the cross section (without allowance for the spins) has the form

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_x\, dE_p} = PSF \cdot C^2 \sum_J \left| \sum_{\alpha L\Lambda} S^{1/2}_{\alpha LJ} \pi^{\alpha L\Lambda}_{BA} \right|^2, \qquad (28)$$

where PSF is a kinematic factor, C is an isospin Clebsch–Gordan coefficient, $S_{\alpha LJ}$ is a spectroscopic amplitude, L, Λ, and J are the orbital angular momentum, its projection, and the total angular momentum of the cluster, α is the set of residual quantum numbers needed to describe the reaction, A and B are quantum numbers of the nuclei, and π is the three-particle transition amplitude. In the impulse approximation, π is represented as a product of a two-particle transition amplitude and an overlap integral, and (28) can be written as

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_x\, dE_p} = KFC^2 \frac{d\sigma}{d\Omega}\Big|^{\text{off}}_{p-x} \sum_{\Lambda LJ} \left| \sum_{\alpha} S^{1/2}_{\alpha LJ} T^{\alpha L\Lambda}_{BA} \right|^2, \qquad (29)$$

where $(d\sigma/d\Omega)|^{\text{off}}_{p-x}$ is the off-shell px scattering cross section, T is the overlap integral of the wave functions of the incident and emitted particles (distorted waves) and the wave function of the cluster–core relative motion, and KF is a kinematic factor. The expression (29) is valid when $L = 0$ or when the cluster spin is 0 or $\frac{1}{2}$. Thus, for (p, pd) reactions the expression (29) is valid for transitions with $L = 0$ and for transitions with $L = 0$ and 2 with a weak D component. When the S component is small or a pure D component is present, $(d\sigma/d\Omega)|_{p-x}$ cannot be separated, and the two-particle px interaction amplitude interferes with $T^{\alpha L\Lambda}_{BA}$.

If $T^{\alpha L\Lambda}_{BA}$ does not depend on α, then the spectroscopic factor $S_{LJ} = |\Sigma_\alpha S^{1/2}_{\alpha LJ}|^2$ is determined, and for each set of L and J the corresponding partial-wave cross section is proportional to S_{LJ}. The function $\varphi(q) = \Sigma_\Lambda |T^{L\Lambda}_{BA}|^2$ is called the distorted momentum distribution of the cluster in the target nucleus. To calculate $T^{L\Lambda}_{BA}$, the distorted waves, expanded with respect to partial waves, are calculated using the Woods–Saxon optical potential. In the calculation of the bound-state wave function a real Woods–Saxon potential with depth chosen to fit the description of the cluster separation energy is used.

The px scattering cross section $(d\sigma/d\Omega)|^{\text{off}}_{p-x}$ is usually replaced by the on-shell cross section, for which different choices of the energy E_{cms} and the scattering angle θ_{cms} are possible. As a rule, for E_{cms} the approximation of the initial energy E_i or the final energy E_f is chosen; for θ_{cms}, the effective angle approximation, θ_{eff}, or the momentum-transfer approximation, $\mathbf{P}_t$. In the first, θ_{cms} is taken to be equal to θ_{eff}, the angle between the incident and the emitted protons; in the momentum-transfer approximation, the two-particle cross section is taken for angle θ_{cms} corresponding to the same value of the momentum transfer $P_t = |P_0 - P_p|$ as in the three-particle reaction. Thus, one generally uses one of the following four representations: $(E_i, \theta_{\text{eff}})$, $(E_f, \theta_{\text{eff}})$, $(E_i, \mathbf{P}_t)$, $(E_f, \mathbf{P}_t)$. In the DWIA analysis of experimental data the two-particle cross section is chosen in order to give the best description of the data. The choice is simpler if the two-particle cross section depends strongly on the energy and angle, as, for example, in the case of $(\alpha, 2\alpha)$ reactions. For (p, pd) and $(p, p\alpha)$ reactions, the two-particle cross section varies weakly, and therefore the corresponding three-particle cross sections have the same shape and differ only in the absolute values, and this makes it difficult to choose the best representation. Since, strictly, one cannot choose either E_i or E_f, analysis of experiments sometimes yields two sets of spectroscopic factors, S_i and S_f, that differ appreciably. For example, in Ref. 88 analysis of the reactions ^{12}C$(p, pd)^{10}$B*(0.72 MeV) and ^{16}O(p, pd) ^{14}N*(3.95 MeV) with initial energy 75 MeV gave for the first reaction $S_f = 0.14$, $S_i = 0.71$ and for the second $S_f \leqslant 0.19$, $S_i \leqslant 0.95$. The impossibility of making a proper choice of the representation for the two-particle cross section is a serious shortcoming of the DWIA and reduces the reliability of the spectroscopic information obtained from DWIA analysis of experimental data.

Comparison of the results of calculations in the DWIA and PWIA shows that the effects of the distortions reduce the cross section [for $(p, p\alpha)$ reactions on $1p$-shell nuclei, the ratio of the cross section calculated at $E_0 = 100$ MeV near the point corresponding to zero momentum transfer in the PWIA to the cross section calculated in the DWIA varies from approximately 2 to 10 for different nuclei]. This

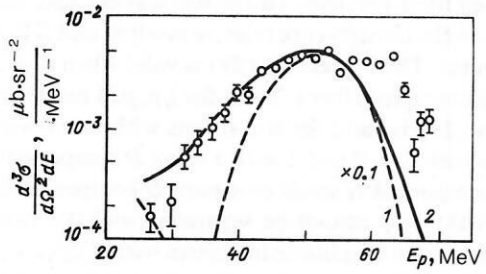

FIG. 17. Energy spectrum of protons from the $^{12}C(p, p\alpha)^{8}Be$ reaction at energy 100 MeV.[92] The calculated curves are as follows: 1) the PWIA cross section; 2) the DWIA cross section.

indicates appreciable absorption. Figure 17 gives the results of calculation of the energy spectrum of protons of the $^{12}C(p, p\alpha)^{8}Be$ reaction induced by 100-MeV protons as obtained in the DWIA and PWIA and compared with the experimental data.[98] In the PWIA curve, we observe a deep minimum that is not present in the experimental data [analogous minima in the PWIA curves are also present in the energy spectra of protons in $(p, p\alpha)$ reactions on other nuclei]. The use of a radial cutoff does not eliminate this minimum. At the same time, the DWIA makes it possible to obtain good agreement with the experiment.

At proton energies above 100 MeV, the impulse approximation gives a good description of the experimental data. At lower energies, the DWIA was used to analyze $(p, p\alpha)$ reactions with initial energy 100 MeV (Refs. 98–101 and 110) and (p, pd) reactions with initial energy 75 MeV (Ref. 89) and 58 MeV (Ref. 91). The DWIA gives a good description of the experimental data on the $(p, p\alpha)$ reactions at 100 MeV (see, for example, Fig. 17). Normalization of the DWIA calculations to the experimental data made it possible to obtain the spectroscopic factors S_α. Table VIII gives the spectroscopic factors S_α for ^{9}Be, ^{16}C, and ^{16}O obtained from DWIA analysis of the $(p, p\alpha)$ reactions at 100 MeV and $(\alpha, 2\alpha)$ reactions at 140 MeV (Ref. 111) and a comparison with theoretical values calculated in the shell model. The spectroscopic factors obtained in different ways agree well for the nuclei ^{9}Be and ^{12}C. The large differences in the values of S_α for ^{16}O can be explained by the high sensitivity of the results of the DWIA calculations to the parameters of the cluster–core relative-motion wave function. However, already at 100 MeV the effects of the distortion of the wave functions of the incident particle and the emitted particles

TABLE VIII. Spectroscopic factors obtained from DWIA analysis of $(p, p\alpha)$ and $(\alpha, 2\alpha)$ reactions.

Nucleus	S_α^{exp}		S_α^{theor}
	Reaction $(p, p\alpha)$	Reaction $(\alpha, 2\alpha)$	
^{9}Be	0.45 $L=0$ 0.55 $L=2$	0.633	0.56 $L=0$ 0.55 $L=2$
^{12}C	0.59	0.558	0.56
^{16}O	1.15	0.244	0.23

TABLE IX. Deuteron spectroscopic factors obtained from analysis of the $^{12}C(p, pd)^{10}B^*(0.72\ MeV)$ reaction.

Energy, MeV	S_d^{exp} Representation		S_d^{theor}
	(E_f, θ_{eff})	(E_i, θ_{eff})	
75	0.14	0.71	1.19
58	0.22	1.86	

are appreciable, and it was noted in Ref. 110 that the DWIA is not valid for analysis of $(p, p\alpha)$ reactions at lower energies.

Overall, the use of the DWIA in Refs. 89 and 91 permitted a satisfactory description of the shape of the energy spectra of the protons in the (p, pd) reactions on the nuclei ^{9}Be, ^{12}C, ^{14}N, and ^{16}O at energies 75 and 58 MeV. The best agreement between calculation and experiment was obtained for the reaction $^{12}C(p, pd)^{10}B^*(0.72\ MeV)$. Normalization made it possible to determine the spectroscopic factors S_d, which are given in Table IX together with the theoretical values. We observe an appreciable discrepancy between the experimental spectroscopic factors obtained at different energies—between one another and the theoretical value. In Ref. 91 this difference is attributed in the first place to the difficulty of choosing a particular representation for $(d\sigma/d\Omega)|_{p-x}^{off}$ and to the high sensitivity to the optical potentials. The energy spectra of the reaction $^{16}O(p, pd)^{14}N^*(3.95\ MeV)$ at initial energy 75 MeV obtained for different sets of angles (θ_p, θ_d) could not be described using just one value of S_d. Therefore, Ref. 89 gives only an upper bound for S_d: $S_f \leqslant 0.19$, $S_i \leqslant 0.95$, this being appreciably less than the theoretical value 1.75. The DWIA analysis of the $^{9}Be(p, pd)^{7}Li$ reaction at 58 MeV made in Ref. 91 showed that to describe the experimental data it is sufficient to take into account the value $L = 0$. At the same time, it follows from the predictions of the shell model that the contribution of the D component must be dominant in the cross section of this reaction. The possibility of describing this reaction with allowance for only $L = 0$ indicates that at an energy of about 50 MeV the effects of distortion are very large and the use of the DWIA is not justified.

Several experimental dependences [angular correla-

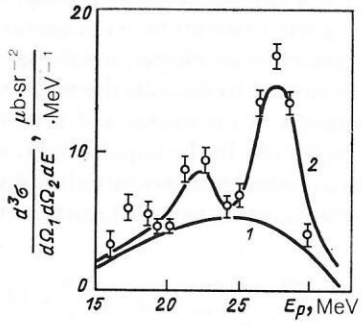

FIG. 18. Energy spectrum of protons from the $^{12}C(p, p\alpha)^{8}Be$ reaction at energy 57 MeV (Ref. 87); $\theta_p = 101°$, $\theta_\alpha = 30°$. The calculated curves are from Ref. 70: 1) quasielastic knockout; 2) coherent sum of quasielastic knockout and two-step processes (calculation in the DWETA).

tions of the reactions $^{16}O(p, p\alpha)^{12}C$ at 46 MeV (Ref. 96) and $^{14}N(p, pd)^{12}C$ at 46 MeV (Refs. 87 and 88), the energy spectrum of the protons of $^{12}C(p, p\alpha)^{8}Be$ reactions at 57 MeV (Ref. 92), and others] cannot be described in the framework of the DWIA. Figure 18 shows the energy spectrum of the protons of the $^{12}C(p, p\alpha)^{8}Be$ reaction at 57 MeV.[93] The peaks observed in it—they are also observed in the proton spectra of other knockout reactions—correspond to excited states of intermediate nuclei, i.e., the reaction takes place in two steps through inelastic scattering of the proton by the target nucleus. It was shown in Ref. 87 that in the $^{14}N(p, pd)^{12}C$ reaction at 46 MeV an important part is played by not only direct knockout but also two-step processes (in the first stage, the proton replaces in the nucleus a valence deuteron, and then the excited intermediate nucleus emits a proton). This indicates that the computational scheme must take into account not only quasielastic scattering of the proton by the valence cluster but also other, more complicated processes. This cannot be done correctly by means of the impulse approximation, since the factorization of the quasielastic interaction amplitude and the separation of the free scattering cross section result in a loss of information about the phase of the process. It is then impossible to take into account the interference effects that result from coherent addition of the amplitudes corresponding to the different mechanisms.

Thus, the impulse approximation does not enable one to take into account correctly the off-shell effects and the distortion effects at energies around 50 MeV or to obtain a coherent sum of the amplitudes of the different mechanisms that contribute to the reaction cross section.

The DWETA for analysis of (p, px) reactions

The need to take into account more correctly than in the DWIA the off-shell effects makes it necessary to give up factorization of the three-particle amplitude. In Refs. 112 and 113, we extended the distorted-wave t-matrix approximation to cluster knockout reactions. The quasi-three-particle transition operator is represented as the sum of a nonlocal t matrix of quasifree px scattering and a correction for core polarization. This transition operator is called the effective t matrix, since:

1) the t matrix of the quasifree px scattering is constructed under the assumption of just one (elastic) open channel of free px scattering;

2) the transition operator takes into account the core polarization (virtual excitation of the core by the incident proton with subsequent de-excitation and transfer of excitation to the valence cluster).

The construction of the t matrix of quasifree px scattering is considered in the Appendix; the core polarization is taken into account in the same way as in the case of $(p, 2p)$ reactions.

In the calculations, it is convenient to use wave functions of the initial and final nuclei in the shell model with intermediate coupling.[114] The cluster wave function in the initial nucleus is separated by means of the coefficients of fractional parentage of the translationally invariant shell model in accordance with the method of Ref. 19. For the wave function of the center-of-mass motion of the cluster in the initial nucleus an oscillator function is used.[85] The wave

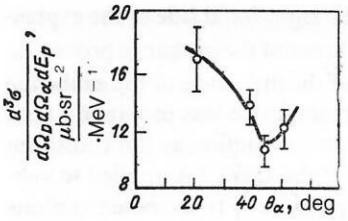

FIG. 19. Angular correlation function of the $^{16}O(p, p\alpha)^{12}C$ reaction at energy 46.8 MeV, $\theta_p = 81°$ (Ref. 91). The curve was calculated in the DWETA (Ref. 107).

functions of the incident particle and the emitted particles (distorted waves) are represented in the form (17). For deuterons and α particles, focusing effects are not observed, and therefore for them the focusing parameters in (17) are set equal to zero. A method analogous to the one explained for $(p, 2p)$ reactions makes it possible to calculate analytically the integrals that occur in the matrix element.

The distorted-wave t-matrix approximation makes it possible to reproduce the experimental data on (p, px) reactions at energy below 100 MeV, and the use of the coefficients of fractional parentage makes it possible to describe the absolute values of the cross sections. For example, Fig. 19 gives the results of calculation of the angular correlations of the $^{16}O(p, p\alpha)^{12}C$ reaction induced by 46-MeV protons; these show that the DWETA makes it possible to describe correctly the quasielastic knockout of clusters from light nuclei.

To take into account two-step processes (inelastic scattering of protons with subsequent breakup of the excited intermediate nucleus with emission of particle x), the matrix element was represented in Ref. 70 in the form

$$T_{IF} = T_0 + \sum_g \delta (E_p - E_g)\, T_{Ig} \Phi_{gF}, \qquad (30)$$

where the first term corresponds to quasielastic knockout (calculated in the DWETA[111,112]) and the second to the two-step processes. Here, E_g is the kinetic energy of the proton scattered inelastically with formation of state g of the intermediate nucleus, T_{Ig} is the amplitude of inelastic scattering of protons with formation of the excited state g of the intermediate nucleus, and Φ_{gF} is the overlap integral associated with the emission of particle x by the excited nucleus. To calculate the matrix element of inelastic scattering, it was proposed to use the DWTA with the effective t matrix of the pN interaction. Calculation of the inelastic scattering cross sections using the DWTA made it possible to reproduce the experimental data. Allowance for the two-step processes in Ref. 30 improved the agreement between the results of calculation of (p, px) reactions and the experimental data, in the first place, the description of the proton energy spectra. As an example, Fig. 18 gives the results of calculations of the energy spectrum of the $^{12}C(p, p\alpha)$ reaction in the DWETA with and without allowance for the two-step processes.

In Ref. 115, it was proposed to use the t-matrix approximation for exchange processes as well [in the first stage, a (p, x) substitution reaction; in the second, the excited intermediate nucleus emits a proton]. Finally, the matrix element of the (p, px) reaction is written in the form

$$T_{IF} = T_0 + T_1 + T_2, \qquad (31)$$

where $T_0 + T_1$ is equal to the right-hand side of the expression (30), and T_2 takes into account the exchange processes. To calculate the amplitude of the first stage of the exchange processes (the substitution reaction) it was proposed to use the effective t matrix of the px interaction as the transition operator. This modification of the DWETA applied to substitution reactions makes it possible to treat these reactions in the same way as quasielastic knockout. Allowance for the exchange processes made it possible to describe the experimental angular correlations of the $^{14}N(p, pd)^{12}C$ reaction given in Ref. 87.

The use of the t-matrix approximation to analyze (p, px) reactions made it possible to treat various processes that contribute to the reaction cross section from a unified point of view. The coherent summation of the amplitudes of direct knockout and of the two-step processes calculated in the DWETA made it possible to obtain good agreement with the experimental data. At the same time, the t-matrix approximation can also be used more successfully than the impulse approximation for the description of quasielastic knockout, since it enables one to take into account the off-shell effects more correctly. All this indicates that at the present time the DWTA is the most promising method for analyzing (p, px) reactions.

The cluster knockout reaction in complete geometry

As in the case of $(p, 2p)$ reactions, chamber experiments for the study of cluster knockout reactions provide a possibility for testing theoretical models created on the basis of an analysis of coplanar experiments in the complete range of variation of energies and angles of the secondary particles. However, at the present time such experimental data are very sparse.

In Ref. 102, the $^{12}C(p, p\alpha)$ reaction with initial energy 85 MeV was studied by means of a bubble chamber. The measured differential cross sections showed that this reaction possesses signatures of quasielastic processes. However, the high cutoff with respect to the α-particle energies in this study may have significantly distorted the shape and value of the differential cross sections. The total cross section of this reaction given in Ref. 102 (5.5 mb) is appreciably lower than the values obtained in experiments without an energy cutoff (50 mb at initial energy 50 MeV).[116] An investigation of (p, pd) and $(p, p\alpha)$ reactions in complete geometry was made in Refs. 92 and 103 at 50 MeV by the photoemulsion method. The measured differential cross sections indicate that an important part is played by rescattering effects in the final state and that processes different from direct knockout are present.

The use, for theoretical analysis, of the DWETA with allowance for two-step processes made it possible to obtain good agreement with the experimental data of Ref. 117. Because the calculations yielded not only qualitative agreement with the experiment, in the shape of the curves, but also quantitative agreement, for the partial-wave cross sections for the production of the residual nuclei in different states, the possibility is opened up of going over to analysis of reactions more complicated than three-particle reactions. The photoemulsion experimental data on the four- and five-particle reactions $^{12}C(p, p3\alpha)$ (Ref. 116) and $^{16}O(p, p4\alpha)$ (Ref. 118) induced by 50-MeV protons indicate that in them two-step processes taking place through the formation of different intermediate nuclei are dominant [similar results are obtained from analysis of the reactions $^{12}C(n,n3\alpha)$ (Ref. 119) and $^{16}O(n, n4\alpha)$ (Ref. 120)]. The results of these experiments give a unique possibility for testing the correctness of the allowance for the complete manifold of processes in the corresponding reactions. In Ref. 118 there is a theoretical analysis of α-particle breakup of ^{16}O induced by protons, made in the DWETA with allowance for two-step processes; the experimental differential cross sections were reproduced with fairly high accuracy. The success of the DWETA in the analysis of many-particle reactions indicates the validity of the description of cluster knockout reactions on the basis of an effective t matrix.

CONCLUSIONS

The analysis of experiments in which knockout reactions have been studied at intermediate proton energies permits the following conclusions to be drawn.

At proton energies above 100 MeV, the factorized DWTA permits description of quasielastic knockout of nucleons and clusters. As the energy is decreased, one observes not only quasielastic but also two-step processes, which can be correctly described by means of the DWTA or the distorted-wave effective t-matrix approximation: DWETA.

The use of the DWETA makes it possible to estimate the part played by virtual resonances (in particular, to obtain the coupling constants) and to verify the correctness of the determination of the intranuclear wave functions and the distorted waves. For this, however, accurate experiments in a wide kinematic range are needed.

A serious problem is the parametrization of the distorted waves, which is done, as a rule, by means of optical potentials. In the analysis of knockout reactions in the DWTA, it would also be correct to use the t-matrix approximation to parametrize the distorted waves using inelastic scattering data. The description of the inelastic scattering of protons with a nonlocal realistic t matrix was considered in Refs. 70 and 78. Such an analysis of knockout and inelastic scattering reactions, which can be made from a unified point of view in the DWTA, makes it possible to eliminate the problem of the nonuniqueness of the approximation of the wave functions and potentials.

We now consider problems of experimental and theoretical study of knockout reactions.

The region of intermediate energies has proved to be the most suitable for the study of off-shell effects, and therefore, with the commissioning of new high-current accelerators, it is necessary to concentrate efforts on their study. This applies particularly to the region of small angles θ, which is the most informative for the analysis of two-particle potentials. From the point of view of the analysis of knockout reactions, the region of large momentum transfers to the residual nucleus may be interesting. Experiments to study such reactions should be made in not only coplanar but also noncoplanar geometry.

It is necessary to develop more strongly the investigation of reactions of the DWETA type by the correlation method; for precisely in these reactions one observes a variety of two-step processes (for example, inelastic scattering or pickup in the first stage). Analysis of these reactions

makes it possible to determine both the partial and the total contributions of these processes to the total reaction cross section.

The theoretical analysis of knockout reactions requires a more rigorous study of the intranuclear wave functions, in particular allowance for cluster excitation; for the analysis of reactions on heavier nuclei, more complete information about the wave functions of these nuclei is needed. The DWETA can be extended to analysis of knockout reactions induced by composite particles (deuterons and α particles) and pions. For this, it is necessary to construct the corresponding free scattering t matrices. Consideration of these reactions does not require significant changes in the algorithm for calculating the three-particle cross sections in the DWETA. At the same time, analyzing the reactions induced by different particles in the framework of a unified method, one can extract more reliable information about the nuclear structure and verify the correctness of the treatment of the knockout reactions on the basis of an effective two-particle interaction.

APPENDIX: PARAMETRIZATION OF NONLOCAL SEPARABLE TWO-PARTICLE POTENTIALS

We briefly consider questions of the construction of the t matrices that describe elastic pn, pd, and $p\alpha$ scattering.

The potential of the form (13) proposed by McCarthy[9] for pp scattering is essentially the $T = 1$ part of the pn scattering potential. By analogy with Ref. 9, one can also readily construct the second part of this potential, corresponding to $T = 0$. The technique for solving the Lippmann–Schwinger equation with a separable potential having form factors of Gaussian form is described in detail in Ref. 9. This potential can be parametrized on the mass shell using the phase shifts of pn scattering. The parameters of the potential of Ref. 13 corresponding to these phase shifts for pn scattering at relative energies of the two nucleons from 0 to 350 MeV are given in Table III. Since the phase shifts of pn scattering for $T = 1$ differ somewhat from the corresponding values for pp scattering, the table gives the parameters that describe the corresponding phase shifts best, although the use of pp scattering potentials to describe pn scattering in the $T = 1$ channel does not lead to a significant change in the results of the calculations in nucleon knockout reactions.

To describe cluster knockout reactions, it was proposed in Ref. 113 to use the same technique of solution of the Lippmann–Schwinger equation for two-particle elastic p^4He scattering as in Ref. 9. It is assumed that the elastic channel of $p\alpha$ scattering is the only open channel in the complete range of relative energies of the protons and α particles in which the potential is parametrized. Neglect of the inelastic $p\alpha$ scattering channels in the phase-shift analysis has the consequence that the corresponding phase shifts are real at any energy. In Ref. 113, such phase shifts are called elastic. At small values of the relative energy of the proton and the α particle, the elastic phase shifts are equal to the real part of the complex phase shifts that describe the open inelastic channels of the two-particle scattering along with the elastic channel.

A complete phase-shift analysis of elastic pd and $p\alpha$ scattering has been made so far only for energies from 0 to 50 MeV. This energy range is not sufficient for analysis of (p, pd) and (p, $p\alpha$) reactions at initial energies from 0 to 150 MeV. Therefore, initially in Ref. 113 for $p\alpha$ scattering in the energy range from 0 to 300 MeV and later in Ref. 115 for pd scattering at energies from 0 to 150 MeV, a phase-shift analysis of the known experimental data for these processes was made with a view to finding the corresponding elastic phase shifts.

In the process of the phase-shift analysis it was found that for satisfactory description of the experimental data on pd and $p\alpha$ scattering it is sufficient to take into account the two-particle channels of S, P, and D scattering; at the same time, the calculations of the corresponding knockout reactions also revealed convergence with increasing L. From the obtained phase shifts, the parameters of the corresponding potentials of pd and $p\alpha$ scattering given in Table III were found.

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