Spontaneous compactification of subspaces in Kaluza–Klein theories

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Fiz. Elem. Chastits At. Yadra **18**, 1035–1079 (September–October 1987)

The review considers mechanisms of spontaneous compactification of the additional dimensions and the vacuum configurations that result from them in multidimensional Einstein–Yang–Mills theories; $N=1, d=11$, and $N=2, d=10$ supergravities; and $N=1, d=10$ supergravities + SO(32) and $E_8 \times E_8$ Yang–Mills theories (these being the low-energy limits of superstring theories). Particular attention is devoted to the mechanism in which gauge and spin connections are identified as the most attractive from the geometrical and physical points of view, since in the listed Kaluza–Klein theories it ensures the possibility of formation of compact vacuum configurations having topological and geometrical properties capable of giving a realistic spectrum and dynamics of field interactions in the effective 4-dimensional theory.

INTRODUCTION

A fundamental tendency in physics throughout its entire development has been the striving of theoreticians to understand and describe all possible physical processes on the basis of fundamental principles that unify superficially different phenomena. After the discovery of the weak and strong interactions of the elementary particles, and also their gauge nature, the creation of a unified theory of all known interactions, including gravitation, became a fundamental task. The first successful steps in this direction have already been taken. A unified gauge theory of electromagnetic and weak interactions has been created, and, after the remarkable experiments at CERN which led to the discovery of the $W^{\pm}$ and Z^0 bosons (the massive partners of the photon in transmitting the electroweak interactions), hardly anyone doubts its correctness. There has been intensive study of numerous grand unification models, in which the strong and electroweak interactions arise through the spontaneous breakdown of the symmetry of a single fundamental interaction characterized by a single coupling constant and a gauge group that contains as a subgroup the symmetry group $SU(3)\times SU(2)\times U(1)$ of the standard model.

The development of modern elementary-particle physics (both theoretical and experimental) determined to a large degree the construction in the seventies of relativistic field-theory models invariant with respect to supersymmetry transformations, i.e., symmetry transformations between particles possessing different statistics: bosons and fermions. This made it possible to unify in a nontrivial manner the space–time and internal symmetries. Supersymmetry possesses a number of remarkable properties. It follows from many theoretical constructions that supersymmetry effects could already be manifested in the range of energies accessible to modern accelerators (around 100 GeV). It is believed that the existence of superpartners of the elementary particles, with masses $M\sim 100$–200 GeV, could solve the mass hierarchy problem in the electroweak scheme. Many experiments are now aimed at the search for superpartners; we may mention, in particular, the search already made using the proton–antiproton accelerator at CERN. The presence of supersymmetry can lead to a cancellation of divergences in quantum theories. An example of a completely finite theory is the $N=4$ supersymmetric Yang–Mills theory. It is therefore with supersymmetric theories, in the first place those with extended supergravity variants and, more recently, superstring theories, that the main hopes as paradigms of a future unified theory are associated.

The construction of extended supergravities with $N\geqslant 4$ in four-dimensional space–time by the methods used for the construction of $N=2$ and $N=3$ supergravities encountered serious difficulties.[1] This led to the construction of supergravities in spaces of $d=4+n$ dimensions, in particular, $d=11$ and $d=10$ supergravities,[2–7] after which supergravities with $N\geqslant 4$ were obtained from the multidimensional theories by dimensional reduction.[8–10] This approach revealed several interesting properties of extended supergravities, in particular, the presence of hidden internal symmetries. It could be that the multidimensional supergravity variants are not merely a convenient description of 4-dimensional theories but have a more fundamental significance—it may be necessary to construct a unified theory of the gravitational, strong, and electroweak interactions on the basis of multidimensional theories. In recent years there has therefore been a strong growth of interest in multidimensional unified theories of gravitational and gauge fields of Kaluza–Klein type.

In this review, we consider mechanisms of spontaneous compactification of the additional dimensions and the vacuum configurations that arise from them in multidimensional Einstein–Yang–Mills theories; $N=1$, $d=11$ and $N=2$, $d=10$ supergravities; and $N=1$, $d=10$ supergravity + SO(32) and $E_8\times E_8$ Yang–Mills theories (these being the low-energy limits of superstring theories). But particular attention is devoted to the mechanism of embedding the gauge connection in the spin connection and vice versa as the most attractive from the geometrical and physical points of view, since in the listed Kaluza–Klein theories it ensures the possibility of formation of compact vacuum configurations having topological and geometrical properties capable of giving a realistic spectrum and dynamics of field interactions in the effective 4-dimensional theory.

In Sec. 1, we give a brief history of the development of the Kaluza–Klein ideas, and we discuss the main problems that arise in the construction of a unified multidimensional theory.

In Sec. 2, we consider mechanisms of spontaneous compactification of multidimensional Einstein–Yang–Mills theories: the Cremmer–Scherk mechanism and the connection identification mechanism. The relation between these mech-

 0090-4759/87/050441-19$04.40

anisms and their differences are established.

In Sec. 3, the 10-dimensional Einstein–Maxwell theory is used to demonstrate the basic possibility of constructing a unified Kaluza–Klein theory.

Section 4 is devoted to mechanisms of spontaneous compactification and vacuum configurations of $N=1$, $d=11$ supergravity.

Compactification in $d=10$ supergravity and the connection between the compact vacuum configurations of $N=1, d=11$ supergravity and nonchiral $N=2, d=10$ supergravity are discussed in Sec. 5.

In Sec. 6, we consider the possibility of realistic spontaneous compactification of $N=1, d=10$ supergravity interacting with $N=1$ supersymmetric SO(32) and $E_8 \times E_8$ Yang–Mills fields.

The following notation is used in the review:

The signature of the $(4+n)$-dimensional metric is $(-,+,\ldots,+)$;

X^M $(M, N=1,\ldots,4+n)$ are the coordinates of the $(4+n)$-dimensional space–time M^{4+n};

$\hat{X}^M$ are the coordinates of the 11-dimensional space–time of $d=11$ supergravity;

x^μ $(\mu, \nu, \ldots = 1, 2, 3, 4)$ are the coordinates of the 4-dimensional space M^{1+3} (Minkowski space, anti-de Sitter space AdS^4);

Y^m $(l, m, n, p, \ldots = 1, \ldots, n)$ are the coordinates of the n-dimensional space K^n;

$\hat{Y}^m$ are the coordinates of the 7-dimensional compact subspace in 11-dimensional supergravity; indices in brackets (M), (n), (i), etc., are indices of the tangent space;

∇_m is the covariant derivative containing the Riemann connection;

Γ^L_{MN} are Christoffel symbols;

$\hat{D}_M$ is the covariant derivative containing the Riemannian, Γ^L_{MN}, and gauge, A^α_M, connections;

A^α_M are gauge fields of the group G, and $\alpha, \beta, \gamma, \ldots$ are indices that label the generators of G;

A, B, C, D are indices of a subgroup H of the group G;

$F^\alpha_{MN} = \partial_M A^\alpha_N - \partial_N A^\alpha_M + C_{\beta\gamma}{}^\alpha A^\beta_M A^\gamma_N$ is the intensity tensor of the gauge fields $A^\alpha_M(X)$;

$R_{MNL}{}^P$ is the Riemann curvature tensor;

$R_{MNL}{}^M = R_{NL}$ is the Ricci tensor, and $R = R^M{}_M$ is the scalar curvature of the space M^{4+n}.

The transition from the world basis (indices $M, N, \ldots$) to an orthogonal basis [indices (M), (N), ...] is made by means of vielbeins $e^{(M)}_N(X)$ (which determine the components of the moving frame on the space M^{4+n}) and the inverse vielbeins $e^N_{(M)}(X)$, for example, $R_{(M)(N)} = R_{LP} e^L_{(M)} e^P_{(N)}$; $e^{(M)}_N e^{(L)}_P \eta_{(M)(L)} = g_{NP}(X)$ is the metric of M^{4+n}, $\eta_{(M)(L)}$ is the $(4+n)$-dimensional Minkowski metric, and

$$E = \det e^{(M)}_N = \sqrt{-g} = \sqrt{-\det g_{MN}}.$$

1. KALUZA–KLEIN THEORY: BRIEF HISTORY AND PROBLEMS

In the twenties of this century, Kaluza and Klein[11,12] proposed a unified theory in which the gravitational and electromagnetic interactions in 4-dimensional space–time are different manifestations of a gravitational interaction in a 5-dimensional space–time for which one of the spatial dimensions is a circle of a small radius (S^1 sphere) and, thus, not directly observable. The S^1 sphere has the symmetry group U(1).

From the point of view of an observer living in 4-dimensional space–time, this group is an internal gauge group responsible for the existence of the electromagnetic interaction in $d=4$. The presence of the fifth dimension, forming the S^1 sphere, is also responsible for quantization of the masses and electric charges of the fields of the effective 4-dimensional theory.

The ideas advanced by Kaluza and Klein were developed by a number of well-known physicists, in particular, Einstein, Bergmann,[13] and Rumer.[14] After the creation by Yang and Mills of the theory of non-Abelian gauge fields, attempts were made at a unified description of the gravitational and gauge interactions using spaces of higher dimensions (see Refs. 15 and 16 and the references in them) in the spirit of Kaluza–Klein theory. However, the subspaces of additional dimensions in such theories played only an auxiliary role, being responsible for the existence of internal gauge symmetries in the 4-dimensional space–time; they could not be regarded as real physical dimensions, since the form of the metric of the multidimensional space was chosen "by hand" and did not satisfy multidimensional Einstein equations.

If one is to consider additional dimensions having the same physical significance as the spatial dimensions in 4-dimensional space–time, it is necessary to assume that these dimensions form (for certain reasons resulting from the interaction of the fields of the original theory) compact spaces K^n of small size, so that their existence does not contradict everyday experience. There exist models in which both spatial and time dimensions are additional dimensions,[17,30] and also models in which noncompact subspaces of small volume can be formed.[18] The inclusion of timelike and noncompact subspaces of additional dimensions leads to new possibilities for solving the problem of the smallness of the cosmological constant and the existence of chiral fermions, i.e., the asymmetry between the left and right fermions required by the electroweak scheme, in the effective 4-dimensional sector of Kaluza–Klein theory. However, it also generates a number of new difficulties: a) the existence of additional time dimensions leads to the appearance of ghosts and tachyons in $d=4$; these can be eliminated in the massless sector by a restriction on the possible topological and geometrical structure of the timelike subspaces[17]; b) if the additional dimensions are noncompact, it is difficult to obtain a discrete mass spectrum of the 4-dimensional fields; this difficulty can be overcome by the choice of suitable boundary conditions for solutions of such type.[19]

We shall limit ourselves to Kaluza–Klein theories in which the additional dimensions are spacelike and form small compact subspaces. Such spaces are manifested through the symmetry properties of the fields of the physical 4-dimensional sector of the theory (for reviews of the modern approach to Kaluza–Klein theories, see Refs. 20–22). The fields in the 4-dimensional space–time arise as excitations over a ground state with a compact subspace of additional dimensions (Kaluza–Klein vacuum) of the $(4+n)$-dimensional theory. Such excitations can be taken into account systematically by expanding the wave functions of the fields of the multidimensional theory in eigenfunctions of the mass operator on the space K^n corresponding (when K^n has a group of motions G) to the irreducible representations of the group of motions of the compact space. The expansion coefficients then represent the wave functions of the fields in the effective four-dimensional theory. After the substitution of such an expansion in the Lagrangian of the multidimensional theory and integration over the volume of the compact space, one can obtain a Lagrangian that describes a system of infinitely many interacting fields in the 4-dimensional space–time. Only some of these fields, for example, the gravitational and gauge fields of the group G, are massless. The symmetry of the Kaluza–Klein vacuum ensures that these fields are massless. (If the compact vacuum configuration permits the preservation of supersymmetry in the effective $d = 4$ theory, then the superpartners of the gravitational and gauge fields will also be massless.) However, the remaining fields have masses inversely proportional to the scale of the compact space ($M \sim 1/l$). If $l \sim 10^{-33}$ cm (the Planck length), then the massive fields have masses of the order of the Planck mass ($\sim 10^{19}$ GeV), and, generally speaking, cannot be observed at the energies currently attainable. When one is considering low-energy phenomena in theories for which the 4-dimensional space–time of the ground state is Minkowski space, one usually makes a restriction to the massless modes. (The situation is much more complicated when the 4-dimensional space–time is the anti-de Sitter (AdS^4) space; this happens, for example, in $d = 11$ and 10 supergravities. In this case, even the "massless" modes have anti-de Sitter masses, and it is difficult to take into account consistently the contributions of the different fields to the low-energy sector.[23])

It is not usually required that the massless fields satisfy the equations of motion of the original multidimensional theory, since it is assumed that these equations will be satisfied when allowance is made for the complete spectrum of massive fields, i.e., after the construction of the complete effective $d = 4$ theory. However, if one considers the low-energy sector (massless fields plus a restricted number of massive fields) as closed and not interacting with the complete massive spectrum, then the requirement that the wave functions of the fields of the low-energy sector satisfy both 4-dimensional equations of motion and the equations of the complete theory acquires particular importance and leads to a restriction on the possible form of the Kaluza–Klein ansatz. This was pointed out in Refs. 24–26 in connection with the difficulties that arise when $d = 4$ supergravity theories are derived from $d = 11$ supergravity with a compactified 7-dimensional subspace, in particular $N = 8$, $d = 4$ supergravity with compactification into S^7.[22] The problem of a self-consistent Kaluza–Klein ansatz is also intimately related to the problem of choosing the true ground state of the multidimensional theory.[22]

The existence of compact subspaces must not be inconsistent with the equations of motion of the original multidimensional theory. This is possible if in the multidimensional space–time not only the gravitational field but also matter fields are present, for example, gauge or scalar fields.[27–32] In this case, the compactification of the additional dimensions can be due to the interaction of the gravitational field with the nontrivial vacuum expectations of these fields, i.e., it can have a spontaneous nature. It should be noted here that although the formation of Ricci-flat compact spaces (for example, in SO(32) and $E_8 \times E_8$ superstring theories[33]) can also occur without the participation of matter fields, one can hardly hope to obtain a realistic spectrum of 4-dimensional fields without them. For example, the presence of a vacuum condensate of the original gauge fields with nontrivial topological structure (monopoles, instantons, etc.) is a most important condition for the existence in the effective 4-dimensional theory of chiral fermions that, depending on their chirality, transform in accordance with different representations of the gauge group; this is a necessary condition for Kaluza–Klein theory to be consistent with the standard electroweak scheme (the left fermions are doublets of the $SU(2) \times U(1)$ group, and the right fermions are singlets).[33–37]

As examples, we can give the 6- and 10-dimensional Einstein–Maxwell theories, in which compactification of additional dimensions occurs as a result of interaction of the gravitation with the original Abelian gauge field. It has been shown[35] that there exist solutions of the equations of motion of the $d = 6$ Einstein–Maxwell theory corresponding to spontaneous compactification of a two-dimensional subspace into the S^2 sphere [having symmetry group SU(2)] with the square of its radius proportional to the ratio of the gravitational and gauge coupling constants, $\varkappa_4^2/e_4^2$, of the effective 4-dimensional theory. Thus, if it is assumed that e_4^2 and $\varkappa_4^2$ take the experimentally observed values, then the compact subspace has a scale of the order of the Planck length $l_P \sim 10^{-33}$ cm. The effective $d = 4$ theory which arises as a result of such compactification describes the gravitational field and gauge fields of the group $SU(2) \times U(1)$. We note that the constants e_4^2 and $\varkappa_4^2$ are related to the original gauge, e^2, and gravitational, $\varkappa^2$, constants of the multidimensional theory by $e_4^2 \sim e^2/V$, $\varkappa_4^2 \sim \varkappa^2/V$, where V is the volume of the compact space. In the case of a 10-dimensional Einstein–Maxwell theory one can obtain after compactification an effective $d = 4$ theory describing gravitation and gauge fields of the $SU(3) \times SU(2) \times U(1)$ group.[38–41] It is interesting that in both theories the fermions can be transformed in accordance with chiral representations of the gauge groups.

However, when gauge fields are included in the multidimensional theory, it would seem that the main reason for using higher dimensions for a unified description of the different interactions is lost. This problem can be solved in the framework of multidimensional variants of supergravity, whose supermultiplets necessarily contain not only the gravitational field but also different types of gauge and scalar fields. Therefore, the main hope of constructing a realistic Kaluza–Klein theory has rested on multidimensional ($d = 10$ and 11) supergravity theories and, in recent years, mainly on those of them that arise as the low-energy limits of the anomaly-free SO(32) (type 1, Ref. 42) and SO(32) and $E_8 \times E_8$ heterotic (hybrid) superstring theories.

It is possible that it is precisely spontaneous compactification of the additional dimensions in these theories that gives rise to a definite group structure of the internal symmetries, the chirality of the fermions, the number of generations, etc.; it may also give the key to an understanding of the sequence of spontaneous breakdown of the internal symmetries and supersymmetry, determining thus the phenomenology of all known interactions.

In this connection, great attention is being devoted to the study of different mechanisms of spontaneous compactification in $(4+n)$-dimensional Einstein–Yang–Mills theories, $d=11$ and 10 supergravities, and superstrings, and also the properties of the resulting Kaluza–Klein vacua, which represent the first step in the construction of a realistic theory. The second step is the study of the spectra of the fields of the effective 4-dimensional theories, the dynamics of their interactions, and the possibility of associating these theories with the actual physics of the elementary particles.

We note that up to 1984 the main attention of theoreticians studying Kaluza–Klein theory was concentrated on $N=1$, $d=11$ (Ref. 22) and $N=2$, $d=10$ supergravities, whose chiral[7] and nonchiral[5,6] variants appeared to be the most general of the supergravity theories that could be constructed. The supermultiplets of these theories contain only Abelian gauge fields. Therefore, the main problem in the study of the spontaneous compactification of the $N=2$, $d=10$ and $N=1$, $d=11$ supergravities was to find mechanisms of spontaneous compactification for which the interaction of the gravitational field with a *minimal* number of gauge fields would ensure the formation of compact 6- and 7-dimensional spaces with the maximal possible symmetry groups; thus, the complete symmetry group of the Kaluza–Klein vacuum would contain the group $SU(3)\times SU(2)\times U(1)$ corresponding to the symmetry group of the strong and electroweak interactions.[34,38,41]

In 1984 a remarkable property was discovered—the cancellation of the gauge and gravitational anomalies in the $SO(32)$ and $E_8\times E_8$ superstring theories[42–45]; serious arguments were also advanced for their being finite. The low-energy limit of these theories is $N=1$, $d=10$ supergravity interacting with $N=1$, $d=10$ Yang–Mills superfields ($N=1$, $d=10$ supergravity + Yang–Mills) with gauge groups $SO(32)$ and $E_8\times E_8$. The action of $N=1$, $d=10$ supergravity + Yang–Mills with an arbitrary gauge group was constructed by Chapline and Manton[4] and generalized by Green and Schwarz[44] by the addition of terms with higher derivatives (Lorentz Chern–Simons terms) that arise in the low-energy limit of the superstring. The distinguished position of the $SO(32)$ and $E_8\times E_8$ superstrings stimulated intensive investigations of these theories (especially the heterotic string[43] with the group $E_8\times E_8$) as the most likely contenders for a realistic Kaluza–Klein theory providing a framework for the solution of important problems in $N=2$, $d=10$ and $N=1$, $d=11$ supergravities as, for example, the fermion chirality. Since the original gauge groups of the anomaly-free superstring versions are very high, in theories of such type it is not necessary for spontaneous-compactification to lead to the formation of compact spaces with high symmetry. In fact, the spontaneous-compactification mechanisms must act in such a way as to *break* suitably the original gauge group $SO(32)$ or $E_8\times E_8$ [for example, one of the E_8 down to E_6 or $SU(5)$] and lead to a realistic effective grand unification theory.[33]

Despite the difference between the requirements imposed on the outcomes of the spontaneous-compactification mechanisms in $N=2$, $d=10$ and $N=1$, $d=11$ supergravities and in $SO(32)$ and $E_8\times E_8$ superstring theories, they are united by the fact that in both approaches a fundamental role is played by the mechanisms of identifying gauge and spin connections on the compactified subspace.[29–31]

We recall briefly the ten-year history of the study of spontaneous-compactification mechanisms.

The first spontaneous-compactification mechanism in a $(4+n)$-dimensional Einstein–Yang–Mills theory was proposed by Cremmer and Scherk [27] for compactification into the S^n sphere and was later generalized by Luciani[28] to the case of symmetric spaces G/H (G is the symmetry group, and H is the holonomy group of these spaces). In this mechanism, the gauge fields that participate in the compactification are associated with Killing vectors of the symmetric space and, thus, transform in accordance with the group G.

A more universal spontaneous-compactification mechanism was proposed in Refs. 29 and 30 and, later, in Ref. 31. It is based on associating the vacuum expectation values of the gauge fields that participate in the compactification with the components of the spin connection of the compact space.[1)] Thus, for example, the formation of the compact symmetric space G/H is ensured by gauge fields of the group H. In the case when H is a direct product of groups, $H=H_0\times H_1\times\ldots\times H_8$, the formation of the space G/H can be ensured by interaction of the gravitational field with gauge fields that transform with respect to one of the invariant subgroups of the group H.[46] For example, if H contains an Abelian subgroup, then for the formation of the space G/H it is sufficient to have the participation of one Abelian field, whose components are equated with the components of the spin-connection form of G/H, this form transforming in accordance with the Abelian invariant subgroup of H. As investigations showed, the mechanism of equating (or embedding) gauge and spin connections plays a decisive role in the formation of physically interesting vacuum configurations in Kaluza–Klein theories whose multiplets contain different types (vector and tensor) of gauge fields, in particular $d=10$ and 11 supergravities[38,41,47–51] and anomaly-free superstring versions.[33]

We note that the monopole and instanton mechanisms of spontaneous compactification considered in Refs. 35 and 36 are special cases of this mechanism of identification of connections.

Other mechanisms of spontaneous compactification of subspaces of multidimensional theories are also known. One of them is based on allowing for the Casimir energy of the matter fields that arises after formation of the compact space as a manifestation of quantum effects and can ensure its stability.[52,53] Also possible is a compactification mechanism that leads to the formation of parallelizable compact spaces (i.e., spaces that possess zero curvature but, in the general case, nonvanishing torsion), such as group manifolds and the 7-dimensional sphere. In this case, the spontaneous compactification is the result of the interaction of gravitational fields with tensor gauge fields A_{MN} and A_{MNL}, which play the role of the torsion of the parallelizable manifolds.[54–56] Spontaneous compactification can also occur when the gravitational field interacts with scalar σ-model fields.[32]

To eliminate a large (in absolute magnitude) cosmological constant Λ, which arises in $d=4$ after compactification of the additional dimensions in many variants of Kaluza–Klein theory, mechanisms have been proposed in which smallness of Λ is ensured by the vacuum condensate of fermions.[57,58]

The most popular mechanism of spontaneous compactification in $d = 11$ supergravity is the Freund–Rubin mechanism,[59] in which an 11-dimensional space–time is compactified into the direct product of a 4-dimensional anti-de Sitter space (AdS^4) and a 7-dimensional compact space K^7. However, here too the mechanism of associating the connections has its influence, ensuring in a number of cases a successive compactification of the 11-dimensional space–time.[48-50] The Fruend–Rubin mechanism has yielded a number of interesting variants of Kaluza–Klein theory based on 11-dimensional supergravity (Refs. 22, 47–50, and 60). In a number of cases, the simultaneous action of several mechanisms of spontaneous compactification can lead to the formation of new vacuum configurations not obtained when only one of them is operating. For example, the simultaneous action in $N = 1$, $d = 11$ supergravity of the Freund–Rubin,[59] Englert,[54] and connection-identification[69,70] mechanisms leads to the occurrence of a vacuum configuration called a "stretched" S^7 sphere with symmetry group SU(4).[71]

The class of possible Kaluza–Klein vacua and effective $d = 4$ theories is significantly extended if one includes for consideration vacuum configurations with the metric

$$g^{4+n}_{MN} = \begin{pmatrix} \Delta^{-1}(Y)\, g^{M^{1+3}}_{\mu\nu}(x) & 0 \\ 0 & g^{K^n}_{mn}(Y) \end{pmatrix},$$

which corresponds to a space–time of the ground state that is not a direct product $M^{1+3} \times K^n$ (Refs. 72 and 73).

The classical stability of the Kaluza–Klein vacua was investigated in Refs. 74–78.

The composition of the (super)multiplets of fields that arise in the effective $d = 4$ theories from $d = 11$ supergravity as a result of Freund–Rubin compactification was studied in Refs. 23 and 79–84.

Of particular interest from the physical point of view is the spectrum of massless 4-dimensional fields that arise as excitations over the compactified ground state of the $E_8 \times E_8$ superstring including gauge fields of the E_6 group and four generations of chiral fermions.[32]

It must, however, be said that the realization of the beautiful and attractive Kaluza–Klein ideas contains many important questions which must be resolved before a unified theory based on the idea of the existence of additional dimensions can be called realistic. We emphasize the basic problems already mentioned:

1. In a realistic effective $d = 4$ theory (as follows from experiment) the cosmological constant Λ must be zero or very small in absolute magnitude. When the $N = 2$, $d = 10$ and $N = 1$, $d = 11$ supergravities are compactified in physically interesting cases, a large cosmological constant of anti-de Sitter type arises as a rule. At the same time, the spontaneous breakdown of supersymmetry and allowance for quantum corrections[85] can lead to the appearance of a cosmological constant of equal magnitude but opposite sign.

On the other hand, in $N = 2$, $d = 6$ supergravity + Yang–Mills[86] and anomalously free $N = 1$, $d = 10$ SO(32) and $E_8 \times E_8$ supergravity + Yang–Mills the phenomenological requirement of the retention of $N = 1$ supersymmetry in the 4-dimensional space leads to a choice of vacuum configurations in which the 4-dimensional space-time is Minkowski space without the fitting of any parameters.[33] The main problem in this case is to find a mechanism of spontaneous breakdown of $N = 1$, $d = 4$ supersymmetry that does not lead to a large positive vacuum energy density, i.e., to a large Λ of de Sitter type.

Thus, the general solution to the problem of the cosmological term is intimately related to the solution of the problem of spontaneous breakdown of supersymmetry.

2. The chirality problem. As was noted by Witten,[34] in $d = 11$ supergravity there are no classical solutions with $M^{11} = M^{1+3} \times K^7$ that could ensure the appearance of chiral fermions in $d = 4$. However, chiral fermions can evidently arise if the spontaneous compactification of M^{11} is accompanied by the production in M^{1+3} of Kaluza–Klein monopoles.[87] The difficulties in obtaining chiral fermions also exist in $N = 2$, $d = 10$ supergravity, in which there is no minimal interaction of the spinor fields with the Abelian gauge field.

Of the currently known variants of solution of this problem (noncompactness of the additional dimensions,[18] Kaluza–Klein monopoles,[87] quasi-Riemannian geometry in many dimensions,[88] etc.) the one that is currently most attractive is the connection-identification mechanism, which ensures the possibility of appearance of chiral fermions in the Kaluza–Klein theories that admit minimal interaction of the spinor fields with the gauge fields (multidimensional Einstein–Yang–Mills theories, $N = 2$, $d = 6$ supergravity + Yang–Mills, anomaly-free $N = 1$, $d = 10$ supergravity + Yang–Mills).

3. An important problem is the selection of the true Kaluza–Klein vacua on the basis of verification of the classical (absence of ghost and tachyons) and quantum (tunnel transitions) stability of the possible vacua configurations. The complexity of allowing for quantum gravitational effects makes it difficult to study the quantum stability of the vacua in many physically interesting cases.

4. The problem of infinities in (super)gravity and (super)string theories. There are grounds for hoping that the SO(32) and $E_8 \times E_8$ superstrings are free of divergences and that a consistent quantum theory of gravitation can be constructed on their basis.

2. MECHANISMS OF SPONTANEOUS COMPACTIFICATION IN (4+n)-DIMENSIONAL EINSTEIN–YANG–MILLS THEORY

In Secs. 2–5, we shall adhere to the classical philosophy of Kaluza and Klein (which we shall dub the "principle of economy"), which was dominant up to 1985 (until the discovery of the anomaly-free SU(32) and $E_8 \times E_8$ superstring versions) and proceeded from the assumption that the multidimensional theory must contain a minimal number of nongravitational fields ($d = 11$ and $N = 2$, $d = 10$ supergravities) and that the majority (or all) gauge fields observed in 4-dimensional space–time arise as a result of spontaneous compactification of additional dimensions into spaces with a fairly high group of symmetries. It was on the basis of these assumptions that the mechanisms of spontaneous compactification in Einstein–Yang–Mills theories were discovered.[27-31]

Gauge fields in mechanisms of spontaneous compactification

We consider the system of interacting Einstein and gauge fields defined by the following Lagrangian in $(4 + n)$-

dimensional space–time M^{4+n}:

$$L=-E\left(\frac{1}{4\varkappa^2}R+\frac{1}{4e^2}F^{\alpha'}_{MN}F^{MN\alpha'}+\lambda\right), \qquad (1)$$

where λ is a cosmological constant introduced into the Lagrangian of the multidimensional theory to ensure that after compactification the 4-dimensional space–time remains flat (in the multidimensional variants of supergravity, in particular in $N=2$, $d=6$ supergravity + Yang–Mills,[86] the vacuum value of the interaction potential of scalar fields can play the role of λ), and α' is the index of the gauge group G'.

The equations of motion that arise from the variation of the Lagrangian (1) have the form

$$R_{MN}-\frac{1}{2}g_{MN}R=-2\varkappa^2\left[\frac{1}{e^2}F^{\alpha'}_{ML}F^{L\alpha'}_{N}-\left(\frac{1}{4e^2}F^{\alpha'}_{LP}F^{LP\alpha'}+\lambda\right)g_{MN}\right]; \qquad (2a)$$

$$(\hat{D}_M F^{MN})^{\alpha'}=\frac{1}{E}D_M(EF^{ML})^{\alpha'}=0. \qquad (2b)$$

When one considers the spontaneous compactification of the additional dimensions of $(4+n)$-dimensional Einstein–Yang–Mills theory, one seeks, as a rule, solutions of (2) for which the space M^{4+n} is the direct product of 4-dimensional Minkowski space M^{1+3} and an n-dimensional compact space K^n. 2) The metric of such a space has the block diagonal form

$$g_{MN}(X)=\begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(Y)\end{pmatrix}. \qquad (3)$$

It is assumed that the compactification occurs as a result of interaction of the gravitational field with the vacuum (in the classical sense) expectation values of the gauge fields:

$$\langle A^{\alpha'}_{\mu}\rangle=0; \qquad (4a)$$

$$\langle A^{\alpha'}_{m}\rangle=A^{\alpha'}_{m}(Y). \qquad (4b)$$

The condition (4a) ensures that Lorentz invariance is preserved in M^{1+3}. The main task is to establish the minimal number of Yang–Mills fields and find the form of the vacuum expectation values of the gauge potentials $\langle A^{\alpha}_{m}\rangle$ that ensure spontaneous compactification of the space M^{4+n} and the formation of the compact subspace K^n with a sufficiently high symmetry group G that includes as a subgroup, for example, SU(3)×SU(2)×U(1), which corresponds to the gauge group of the strong and electroweak interactions. Such spaces are the homogeneous spaces $K^n=G/H$, where G is a group of motions and H is the group of isotropy (stationary subgroup of G), whose action leaves the origin of the space G/H fixed. For simplicity, we restrict our consideration to n-dimensional *symmetric* spaces G/H, which possess the maximally possible symmetry groups in the class of homogeneous spaces K^n. The generalization of the results given here to the complete class of homogeneous spaces K^n does not contain fundamental difficulties and is presented in detail in Ref. 90.

The isotropy group H of a symmetric space is identical to its holonomy group, and this ensures simplicity and clarity of the mechanisms of spontaneous compactification that lead to the formation of spaces of such type.

The curvature tensor of a symmetric space satisfies the condition of covariant constancy

$$\nabla_m R_{lnpq}=0, \qquad (5)$$

and in an orthogonal basis has the form

$$R_{(m)(n)(l)}{}^{(p)}=kC_{(m)(n)}{}^{A}C_{A(l)}{}^{(p)}, \qquad (6)$$

where k is the mean curvature, which characterizes the size of the compact space, and $C_{(m)(n)}{}^{A}$ are the structure constants of the group G. The transition to a natural basis (with local coordinates Y^m) is made by means of vielbeins $e^{(m)}_n(Y)$ of the space G/H. Depending on which group of transformations, G or H, is associated with the group of local transformations of the vacuum expectation values (4b) of the gauge fields, there are two mechanisms of spontaneous compactification of the subspaces of the additional dimensions in the case of interaction of Einstein fields with gauge fields.

In the mechanism proposed by Cremmer and Scherk,[27] and subsequently generalized by Luciani,[28] the vacuum values (4) of the gauge fields that transform in accordance with the group G are associated with Killing vectors $K^{\alpha}_{m}(Y)$ of the symmetric space. It was shown that when this is done it is possible to choose the gauge and cosmological constants in such a way that the equations of motion are satisfied not only for the gauge fields (4) but also for the Einstein fields determined by the metric (3). The intensity tensor of the gauge field $\langle A^{\alpha}_{m}\rangle=K^{\alpha}_{m}(Y)$ corresponding to the compactified solution has the following nonvanishing components in an orthogonal basis:

$$\langle F^{\alpha}_{(m)(n)}\rangle=-kC_{(m)(n)}{}^{A}D^{\alpha}{}_{A}(Y), \qquad (7)$$

where $D^{\alpha}_{\beta}(Y)=D^{\alpha}_{\beta}(K_Y)$ are the components of the matrix of the adjoint representation of the group G, and K_Y is a transformation of the group G that belongs to the space of G/H cosets.

A more economic mechanism of spontaneous compactification was proposed in Refs. 29–31. In this mechanism, only the gauge fields of the group H participate in the formation of the symmetric space G/H. To find nontrivial solutions (4) of Eq. (2) possessing a sufficiently high symmetry (so that they can be regarded as vacuum solutions), it was proposed in Ref. 29 to impose on the vacuum values of the gauge fields (4) $(G'=H)$ the condition of parallelizability (covariant constancy), analogous to the condition (5):

$$(\hat{D}_m\langle F_{nl}\rangle)^{A}=0, \qquad (8)$$

the Yang–Mills equations being satisfied identically when this holds.3) To find solutions of (8) on a given symmetric space G/H, we consider the condition of integrability of these equations:

$$[\hat{D}_l,\hat{D}_m]\langle F^{A}_{np}\rangle=0. \qquad (9)$$

Because of the Maurer–Cartan structure equations of the group G, a solution of (9),

$$\langle F^{A}_{(m)(n)}\rangle=-kC_{(m)(n)}{}^{A}, \qquad (10)$$

is also a solution of (8) and corresponds to a spontaneous transition from a flat vacuum configuration to a compact configuration with metric (3). The constants λ and k, as in the first mechanism, take the following values:

$$\lambda=\frac{1}{4e^2}\langle F^{A}_{mn}\rangle^2,\quad k=\frac{e^2}{2\varkappa^2}.$$

Using the structure equations of the group G, one can show[30,91] that the vacuum values $\langle A^{B}_{m}\rangle$, characterized by the intensity tensor (10), have the same form as the components of the Cartan forms $\theta^B_m(Y)$ of the group,

$$\langle A^B_m \rangle = \theta^B_m(Y), \tag{11}$$

these components specifying the Riemannian (spin) connection on the symmetric space $G/H(\omega^{(l)}_{m(n)} = -\theta^B_m C^{(l)}_{B(n)})$. Thus, this mechanism of spontaneous compactification "works" by associating the gauge and spin connections.

We note that on compactification into homogeneous asymmetric spaces G/H the conditions (5) and (8) are not satisfied, and the validity of choosing the vacuum values of the gauge fields in the form of the spin-connection components of the space K^n must be confirmed by direct solution of the equations of motion (2).

The symmetry group of the vacuum configurations that arise as a result of the action of both mechanisms is the group $SO(3,1)\times G$ if the original gauge group G' of the multidimensional theory does not exceed G or H, respectively ("economy" principle).[4] Therefore, when allowance is made for excitations above Kaluza–Klein vacua (Refs. 28, 35, 39, and 40), gauge fields of the group G arise in the effective 4-dimensional theory. We emphasize that the original gauge group G or H (in the general case) is broken when nonvanishing vacuum expectation values (4) arise. (From the point of view of a 4-dimensional observer, $\langle A^{\alpha'}_m \rangle$ corresponds to the vacuum expectation values of scalar fields in the space M^{1+3}.) However, when the spontaneous compactification takes place through the connection-identification mechanism and the group H contains an Abelian invariant subgroup H_0 or is identical to it, the Kaluza–Klein vacuum has an additional symmetry H_0, this corresponding to the appearance in the physical sector of the theory of gauge fields of the Abelian subgroup H_0.[92]

It should be noted that the symmetry group G of the irreducible symmetric space is a semisimple group, and therefore in the case of Cremmer–Scherk–Luciani spontaneous compactification the original gauge symmetry G is always completely broken.

Because of the similarity considered above between the vacuum configurations that arise as a result of the action of the two possible mechanisms of spontaneous compactification, it is natural to consider whether they are connected. Such a connection does indeed exist, i.e., in the Cremmer–Scherk–Luciani mechanism not all the gauge fields of the group G participate effectively in the spontaneous compactification, but only those of its subgroup H. At the same time, the additional components of the gauge fields of the group G can be eliminated by a gauge transformation that belongs to the group G and is specified by the matrix $(D^{\beta}_{\alpha})^{-1}$ in (7). After the elimination of these purely gauge degrees of freedom, the Cremmer–Scherk–Luciani spontaneous-compactification mechanism reduces to the connection-identification mechanism.[91]

Despite this similarity of the spontaneous-compactification mechanisms and the vacuum configurations to which they lead, there is an important difference between them, which is as follows. An important criterion in the selection of the real Kaluza–Klein vacua is their stability with respect to small fluctuations, i.e., the absence of tachyon modes. As was shown in Ref. 74, tachyon modes do not arise after compactification into the homogeneous space G/H if the original gauge group of the multidimensional Einstein–Yang–Mills theory is isomorphic to the holonomy group or one of its subgroups. Thus, the vacuum configurations that arise as a result of the action of the connection-identification mechanism are classically stable. But if the original gauge group is larger than H, for example, G (as in the case of Cremmer–Scherk–Luciani compactification), the corresponding vacuum configurations may be unstable. Therefore, the connection-identification mechanism is preferable to the first mechanism of spontaneous compactification not only because it is more economic but also because it always leads to the formation of classically stable Kaluza–Klein vacua of $(4+n)$-dimensional Einstein–Yang–Mills theory. A detailed investigation of the stability of vacuum configurations in multidimensional Einstein–Yang–Mills theories was made in Ref. 75. We also note that precisely the connection-identification mechanism is the most natural in the case of compactification into spaces that do not possess a group of motions (there are no Killing vectors), for example, into Calabi–Yau spaces (see Sec. 6).

Spontaneous compactification into spaces with holonomy group $H=\prod_{i=1}^{s}\times H_i$

The economy principle is realized to the maximal degree by the connection-identification mechanism in the case of the formation of compact spaces whose holonomy group is the direct product of simple and Abelian groups: $H=\prod_{i=1}^{s}\times H_i$ (s is their number). Of this type there is a large class of symmetric spaces, including ones with orthogonal, unitary, and symplectic groups of motion ($SO(p_1+p_2)/SO(p_1)\times SO(p_2)$, $SU(p_1+p_2)/SU(p_1)\times SU(p_2)\times U(1)$, $Sp(p_1+p_2)/Sp(p_1)\times Sp(p_2)$). Spontaneous compactification into such spaces can be realized by gauge fields of the group $H=\prod_{i=1}^{s}\times H_i$ with independent coupling constants e_i corresponding to each subgroup H_i and, in particular, only the gauge fields of one of the subgroups H_i, this leading to an additional reduction in the number of gauge fields that participate in the spontaneous compactification.[92]

Since in this case the values of the Yang–Mills fields that ensure the compactification are equated to the spin-connection components of the space G/H that transform only with respect to the subgroup H_i, the connection-identification mechanism consists of embedding a gauge connection in the spin connection. For example, the vacuum expectation values of the gauge fields whose interaction with gravitation leads to the formation of symmetric spaces with $H=\prod_{i=1}^{s}\times H_i$ have the form (10) and (11), where the index A can take values corresponding to one of the groups (for more details, see Ref. 92).

The economy principle is realized to the maximal extent when the spontaneous compactification is produced by a single Abelian field. This occurs when the holonomy group of the compact space contains invariant Abelian subgroups. This last circumstance is very important for the possibility of the formation of physically interesting vacuum configurations in $d=11$ and $N=2$, $d=10$ supergravities, since the supermultiplets of these theories contain only Abelian (vector and tensor) gauge fields (see Secs. 4 and 5).

Note that in the considered case of compactification into symmetric spaces with holonomy group $H=\prod_{i=1}^{s}\times H_i$ the Cremmer–Scherk–Luciani mechanism does not reduce to the connection-identification mechanism; for although the gauge group G of the first mechanism can be reduced to the gauge group H of the second by local transformations of the gauge fields, the further reduction of H to one of its invariant subgroups is not possible.

3. TEN-DIMENSIONAL MODEL OF KALUZA–KLEIN THEORY

As a simple and clear example, we consider the connection-identification mechanism in 10-dimensional Einstein–Maxwell theory. It is worth studying this theory, since, on the one hand, it possesses elements of a realistic Kaluza–Klein theory (there is the possibility of compactification with the formation of gauge fields of the group $SU(3)\times SU(2)\times U(1)$ and realistic values of the coupling constants, and in $d=4$ chiral fermions arise), and this demonstrates the basic possibility of realizing the Kaluza–Klein ideas; on the other hand, it is a constituent in $N=1$, $d=10$ supergravity + Yang–Mills and nonchiral $N=2$, $d=10$ supergravity (which, in their turn, are low-energy limits of superstrings) and, therefore, indicates the possibility of realistic spontaneous compactification in these theories.

Ground states with a compact 6-dimensional subspace

As was shown in the previous section, interaction of the gravitational field with an Abelian gauge field can lead to the compactification of subspaces and the formation of a nontrivial Kaluza–Klein vacuum in which the $(4+n)$-dimensional space–time is the direct product of 4-dimensional space-time M^{1+3} and an n-dimensional homogeneous compact space $K^n=G/(H\times H_0)$ whose isotropy group (for symmetric spaces, this is identical to the holonomy group) contains an Abelian invariant subgroup H_0. Thus, in $d=10$ Einstein–Maxwell theory the subspace of the additional dimensions can be compactified by virtue of the connection-identification mechanism into the following manifolds[38–41]:

$$\left.\begin{array}{l} K^6=CP^3,\ S^4\times S^2,\ CP^2\times S^2,\ S^2\times S^2\times S^2,\\ \dfrac{SO(5)}{SO(3)\times SO(2)},\ \dfrac{\mathrm{Sp}(4)}{\mathrm{Sp}(2)\times U(1)},\ \dfrac{SU(3)}{U(1)\times U(1)};\\ (\mathrm{Sp}(4)\sim SO(5),\ \mathrm{Sp}(2)\sim SU(2)).\end{array}\right\}\qquad(12)$$

The symmetry group of the Kaluza–Klein vacuum with M^6 of the form (12) is $G^{1+3}\times G\times H_0$, where G^{1+3} is the group of motions of M^{1+3}, G is the group of motions of M^6, and H_0 is the gauge group of the original Abelian field that is not broken after compactification (see Sec. 2).

The equations of motion of the system of interacting Einstein–Maxwell fields in $d=10$ have the form (2), where $n=6$, and F_{MN} is the intensity tensor of the Abelian field.

From the point of view of physical interest, spontaneous compactification of the 6-dimensional space into the direct product of the projective space $CP^2=SU(3)/SU(2)\times U(1)$ and the two-dimensional sphere $S^2=SU(2)/U(1)$ is most worth studying. The ground state that results from such compactification is invariant with respect to transformations of the group $G^{1+3}\times SU(3)\times SU(2)\times S(1)$. Thus, in the effective $d=4$ theory gauge fields of the group $SU(3)\times SU(2)\times U(1)$ appear.[38–41]

It is interesting to note that the formation, as a result of K^6 compactification, of the space $SU(3)/[U(1)\times U(1)]$ [see (12)] can be regarded as a topological deformation of the space $CP^2\times S^2$ due to the appearance of instanton excitations above CP^2, which break the SU(2) group.[39,48] From the geometrical point of view, the occurrence of instantons over CP^2 results in transformation of the space $CP^2\times S^2$ into a nontrivial associated fiber bundle $E(CP^2, S^2, SU(2))\sim SU(3)/U(1)\times U(1)$ with base CP^2, standard fiber S^2, structure group SU(2), and connection given by the field of the SU(2) instanton on CP^2 (Refs. 39 and 48).

The metric of the ground-state space $M^{1+3}\times CP^2\times S^2$ has the following nonvanishing components:

$$g_{MN}=(g_{\mu\nu}(x),\ g_{ab}(y),\ g_{ik}(z)),\qquad(13)$$

where $g_{\mu\nu}(x)$, $g_{ab}(y)$, $g_{ik}(z)$ are the metrics and x^μ, y^a, z^i are the coordinates of the spaces M^{1+3}, CP^2, and S^2, respectively.

The curvature tensors of the symmetric spaces M^{1+3}, CP^2, and S^2 satisfy the conditions of covariant constancy and have the form

$$\left.\begin{array}{l} R^{M^{d'}}_{(\mu)(\nu)(\lambda)(\rho)}=\dfrac{2\varkappa^2}{d'-1}\Lambda\left(g_{(\mu)(\nu)}g_{(\nu)(\rho)}-g_{(\nu)(\lambda)}g_{(\mu)(\rho)}\right);\\ R^{CP^2}_{(a)(b)(c)(d)}=k_1\left(f_{(a)(b)}{}^{A}f_{A(c)(d)}+f_{(a)(b)}{}^{8}f_{8(c)(d)}\right);\\ R^{S^2}_{(i)(j)(k)(l)}=k_2C_{(i)(j)}{}^{3}C_{3(k)(l)},\end{array}\right\}\qquad(14)$$

where Λ is the cosmological constant in the effective 4-dimensional theory (for $\Lambda>0$, M^{1+3} is de Sitter space with the group of motions SO(4,1); for $\Lambda<0$, M^{1+3} is the anti-de Sitter space with the group of motions SO(3,2); and for $\Lambda=0$, M^{1+3} is Minkowski space); $d'=4$; $k_1>0$ and $k_2>0$ are the mean curvatures, which characterize the size of the spaces CP^2 and S^2; $f^{\gamma}_{\alpha\beta}$ are the structure constants of the group SU(3) in the Gell-Mann basis ($A=1, 2, 3$ are the indices of the generators of the subgroup SU(2); 8 is the index of the generator of the subgroup U(1) of SU(3); and (a), (b), (c), (d) are the indices of the generators belonging to the space of $SU(3)/SU(2)\times U(1)$ cosets, these being identical to the indices of the space tangent to CP^2 in an orthogonal basis); C^{C}_{AB} are the structure constants of the group SU(2) (3 is the index of the generator of the subgroup U(1) of the group SU(2); (i), (j), (k), (l) are the indices of the generators belonging to the space of the SU(2)/U(1) cosets, these being identical to the indices of the space tangent to S^2 in an orthogonal basis).

The generators of the groups SU(3) and SU(2) are normalized in such a way that the Killing–Cartan metrics of these groups have the form

$$g_{\alpha\beta}=f_{\alpha\gamma}{}^{\delta}f_{\delta\beta}{}^{\gamma}=\frac{3}{2}\delta_{\alpha\beta};\quad g_{AB}=C_{AC}{}^{D}C_{DB}{}^{C}=\delta_{AB}.\qquad(15)$$

For this normalization of the generators, the structure constants f^{C}_{AB} and C^{C}_{AB} are equal to each other, and this is convenient for what follows.

The transition from the world basis to the orthogonal basis is made by means of the vielbeins $e^{(\mu)}_{\nu}(x)$, $e^{(a)}_{b}(y)$, and $e^{(i)}_{k}(z)$ of the spaces M^{1+3}, CP^2, and S^2, respectively.

The metric (13) is form-invariant, i.e., has the same functional form in different coordinate systems, with respect to transformations of the group of motions $G^{1+3}\times SU(3)\times SU(2)$ of the spaces $M^{1+3}\times CP^2\times S^2$. The infinitesimal transformations of the coordinates under the action of the elements of this group have the form

$$\delta x^\mu=\varepsilon^r K_r^\mu(x),\ \ \delta y^a=\varepsilon^\alpha K_\alpha^a(y),\ \ \delta z^i=\varepsilon^A \widetilde{K}_A^i(z), \qquad (16)$$

where $K_r^\mu(x)$, $K_\alpha^a(y)$, and $\widetilde{K}_A^i(z)$ are the Killing vectors of the spaces M^{1+3}, CP^2, and S^2, respectively; ε^r, ε^α, ε^A are the parameters of infinitesimal transformations of the group of motions $G^{1+3}\times SU(3)\times SU(2)$ (r is the index of G^{1+3}).

Since we study the spontaneous compactification of K^6 into $CP^2\times S^2$ through the connection-identification mechanism, the nonvanishing vacuum expectation values of the gauge field have the form

$$\langle A_a\rangle=A_a(y)=\sqrt{\frac{3}{2}}\frac{\rho_1}{k_1}\frac{e^2}{2\varkappa^2}\theta_a^8(y); \qquad (17a)$$

$$\langle A_i\rangle=A_i(z)=\frac{\rho_2}{k_2}\frac{e^2}{2\varkappa^2}\theta_i^3(z), \qquad (17b)$$

where ρ_1 and ρ_2 are numerical parameters, and $\theta_a^8(y)$ and $\theta_i^3(z)$ are the components of the Cartan θ forms on the spaces CP^2 and S^2 that relate to the U(1) subgroups of the groups SU(3) and SU(2), respectively. The form of the coefficients in (17) is chosen for convenience in the subsequent calculations.

The corresponding values of the components $F_{ab}(y)$ and $F_{ik}(z)$ of the Maxwell field tensor are found by means of the Maurer–Cartan structure equations for the groups SU(3) and SU(2) and have the form

$$\left.\begin{aligned}F_{ab}(y)&=-\sqrt{\frac{3}{2}}\rho_1\frac{e^2}{2\varkappa^2}f_{(c)(d)}{}^8 e_a^{(c)}(y)\,e_b^{(d)}(y);\\ F_{ik}(z)&=-\rho_2 C_{(j)(l)}{}^3 e_i^{(j)}(z)\,e_k^{(l)}(z).\end{aligned}\right\} \qquad (18)$$

By virtue of the transformation properties of the Cartan θ forms,[20] the solutions (17) are invariant (up to Abelian gauge transformations) with respect to the transformations (16). Note that in contrast to non-Abelian gauge fields, for which the values of the coefficients in the expression (10) are uniquely fixed by the term in the equations of motion that describes the self-interaction of the non-Abelian gauge fields, in the Abelian case there exists a freedom in the choice of these coefficients. The domain of values of the parameters $\rho(\rho_1,\rho_2)$ can be restricted only by the features of the topology of the compact manifolds. The field (17b) is none other than the field of a monopole on the S^2 sphere, and therefore $\rho_2 e^2/2\varkappa^2 k_2$ satisfies the Dirac quantization condition; moreover, since $k_2=\rho_2 e^2/2\varkappa^2$ [see (20)], ρ_2 is inversely proportional to an integer n ($\rho_2=2\sqrt{2}/n$).[35] A restriction on the parameter ρ_1 arises from the difficulties of introducing a spin structure on the space CP^2 that are associated with the global properties of this space. As was shown in Refs. 41 and 93, fermions can exist on CP^2 if they interact with the Abelian field (17a), whose parameter ρ_1 satisfies the quantization condition $\rho_1=6/(m+\frac{1}{2})$ (m is an integer).

Substituting in (2a) the expressions (14) and (18) and using the equation $f_{(a)(c)}{}^8 f_{(b)}^{(c)\,8}=\frac{1}{4}\delta_{(a)(b)}$, we obtain for Λ, k_1, and k_2 the values

$$\left.\begin{aligned}\Lambda&=\frac{1}{4}\frac{e^2}{2\varkappa^2}(\rho_1^2+\rho_2^2)-\frac{1}{4}\lambda;\\ k_1&=\frac{e^2}{16\varkappa^2}(3\rho_1^2-\rho_2^2)+\varkappa^2\lambda;\\ k_2&=\frac{e^2}{16\varkappa^2}(7\rho_2^2-\rho_1^2)+\varkappa^2\lambda.\end{aligned}\right\} \qquad (19)$$

If $\lambda<[e^2/(4\varkappa^2)^2](\rho_1^2+\rho_2^2)$, then $\Lambda>0$ and the space M^{1+3} is de Sitter space; if $\lambda=[e^2/(4\varkappa^2)^2](\rho_1^2+\rho_2^2)$, then $\Lambda=0$ and M^{1+3} is Minkowski space. For $\Lambda=0$, the relations (19) become

$$\lambda=\frac{e^2}{(4\varkappa^2)^2}(\rho_1^2+\rho_2^2),\ \ k_1=\frac{e^2}{4\varkappa^2}\rho_1^2,\ \ k_2=\frac{e^2}{2\varkappa^2}\rho_2^2. \qquad (20)$$

Thus, the Kaluza–Klein vacuum state with the metric (13) and nonvanishing vacuum values of the gauge-field components (17) is invariant up to a U(1) transformation with respect to the group of motions $G^{1+3}\times SU(3)\times SU(2)$ of the space $M^{1+3}\times CP^2\times S^2$.

All the possibilities of compactification of the 10-dimensional space listed in (12) are realized in the nonchiral variant of $N=2$, $d=10$ supergravity (see Sec. 5).

Excitations above the Kaluza–Klein vacuum

We consider the contribution to the action of excitations above the possible ground-state configuration of 10-dimensional Einstein–Maxwell theory with $K^6=CP^2\times S^2$.[40]

To take into account systematically excitations above a ground state with a compact subspace of additional dimensions (Kaluza–Klein vacuum), it is necessary to expand the wave functions of the fields of the multidimensional theory in a series in the harmonics corresponding to the irreducible representations of the group of motions G of the compact space.[20,94] For example, the expansion of the vector wave function has the form $A_{\mu p}(x,Y)=\sum_n\sum_{q,l}D^n_{pl,q}(Y)A^n_{\mu ql}(x)$, where $D^n_{pl,q}$ are the matrices of the irreducible representations of the group G, n is the number of the representation of dimension d_n, p is the matrix index corresponding to the restriction of the representation $\mathbf{D}^n$ to the isotropy group H, a subgroup of G, and l tells us how many times the representation $\mathbf{D}(h)$ ($h\in H$) occurs in the representation $\mathbf{D}^n(g)$ ($g\in G$).[20,94] After the substitution of such an expansion into the action of the multidimensional theory

$$S=-\int E\left(\frac{1}{4\varkappa^2}R+\frac{1}{4e^2}F_{MN}F^{MN}+\lambda\right)d^{10}X \qquad (21)$$

and integration over the volume of the compact space, we can obtain a Lagrangian that describes a system of an infinite number of interacting fields with spins 2, 1, 0 in 4-dimensional space–time. Only some of these fields (corresponding, for example, to the terms of the expansion relating to the trivial and adjoint representations of the group of motions[94]) are massless; the remainder have masses whose values depend on the size of the compact region.

We restrict ourselves to considering excitations that correspond in the 4-dimensional space–time to massless fields with spin 2 (gravitational field) and 1 (vector gauge fields). In seeking such massless excitations, our point of departure will be the requirement of invariance of an excited state with respect to local transformations, i.e., ones whose parameters depend on the coordinates of the 4-dimensional space–time, of the groups SU(3), SU(2), and U(1).

In the considered case, when the 6-dimensional subspace of the ground state is the space $CP^2\times S^2$, the massless excitations over such a vacuum correspond to a system of interacting gravitational and gauge fields of the group $SU(3)\times SU(2)\times U(1)$ in 4-dimensional space–time.

Indeed, the requirement of local $SU(3)\times SU(2)$ invariance can be satisfied by extending the Cartan forms of the groups SU(3) and SU(2) by means of quantities that transform in the same way as gauge fields of the SU(3) and SU(2) groups in 4-dimensional space–time (as occurs when a gauge interaction is taken into Yang–Mills theory by extending the derivatives ($\partial_M \to \partial_M + A_M$)). Thus, we shall consider a state of the 10-dimensional Einstein–Maxwell theory in which the vielbeins of the space M^{10} have, in the notation of differential forms, the form (Kaluza–Klein ansatz)

$$e^{(\mu)} = dx^{\nu}\, e_{\nu}^{(\mu)}(x);$$

$$\left.\begin{aligned} e^{(a)} &= dy^b\, e_b^{(a)}(y) + \frac{1}{\sqrt{k_1}}\, G_\mu^{(a)}(x)\, D_\alpha^a(y)\, dx^\mu; \\ e^{(i)} &= dz^k\, e_k^{(i)}(z) + \frac{1}{\sqrt{k_2}}\, H_\mu^A(x)\, D_A^{(i)}(z) dx^\mu, \end{aligned}\right\} \tag{22}$$

and the field A_M has the form

$$A(x, y, z) = dx^\mu A_\mu(x) + \sqrt{\frac{3}{2}}\,\frac{\rho_1 e^2}{2\varkappa^2 k_1}\,(dy^a \theta_a^8(y) + dx^\mu G_\mu^\alpha(x)\, D_\alpha^8(y)) + \frac{\rho_2 e^2}{2\varkappa^2 k_2}\,(dz^i \theta_i^3(z) + dx^\mu H_\mu^A(x)\, D_A^3(z)), \tag{23}$$

where $G_\mu^\alpha(x)$ and $H_\mu^A(x)$ are gauge fields, and $D_\alpha^\beta(y)$ and $D_A^B(z)$ are elements of matrices of the adjoint representations of the groups SU(3) and SU(2), respectively.

The contracted curvature tensor R corresponding to (21) and the components of the field intensity tensor (23) in the orthogonal basis have the form

$$R = R^{M^{1+3}} + R^{CP^2} + R^{S^2} + \frac{1}{4k_1}\, K_{\alpha a}(y)\, K_\beta^a(y)\, G_{\mu\nu}^\alpha(x)\, G^{\mu\nu\beta}(x) + \frac{1}{4k_2}\, \tilde{K}_{Ai}(z)\, \tilde{K}_B^i(z)\, H_{\mu\nu}^A(x)\, H^{\mu\nu B}(x); \tag{24}$$

$$F_{(M)(N)} = \Big(F_{(\mu)(\nu)} + \sqrt{\frac{3}{2}}\,\frac{\rho_1 e^2}{2\varkappa^2 k_1}\, G_{(\mu)(\nu)}^\alpha D_\alpha{}^8(y) + \frac{\rho_2 e^2}{2\varkappa^2 k_2}\, H_{(\mu)(\nu)}^A D_A{}^3(z), - \sqrt{\frac{3}{2}}\,\frac{\rho_1 e^2}{2\varkappa^2} f_{(a)(b)}{}^8, - \frac{\rho_2 e^2}{2\varkappa^2}\, C_{(i)(k)}{}^3 \Big), \tag{25}$$

where

$$G_{\mu\nu}^\alpha(x) = \partial_\mu G_\nu^\alpha - \partial_\nu G_\mu^\alpha + f_{\beta\gamma}^{\alpha} G_\mu^\beta G_\nu^\gamma;\ H_{\mu\nu}^A(x) = \partial_\mu H_\nu^A - \partial_\nu H_\mu^A + C_{BC}^A H_\mu^B H_\nu^C;\ F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x).$$

Substituting (24) and (25) in the action (21) and integrating over the volume of the compact space using the integral properties of the matrices $D_\alpha^\beta(y)$ and $D_A^B(z)$ (see Refs. 20 and 28), we can obtain a Lagrangian that describes a system of interacting Einstein fields and gauge fields of the group SU(3)×SU(2)×U(1) in 4-dimensional space–time:

$$L_4 = -(\det e_\nu^{(\mu)}(x)) \Big(\frac{1}{4\varkappa_4^2} R^{M^{1+3}} + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + \frac{1}{4g_2^2} (H_{\mu\nu}^A)^2 + \frac{1}{4g_3^2}\, g_{\alpha\beta} G_{\mu\nu}^\alpha G^{\mu\nu\beta} \Big), \tag{26}$$

where $\varkappa_4^2 = \varkappa^2/V$, $g_1^2 = e^2/V$, $g_2^2 = (3/2)\rho_2^2 g_1^2$, $g_3^2 = \rho_1^2 g_1^2$ ($V = V^{CP^2} \cdot V^{S^2}$ is the volume of $CP^2 \times S^2$) are the effective gravitational and gauge coupling constants of the groups U(1), SU(2), and SU(3), respectively. [The form of (26) corresponds to the case when $\Lambda = 0$ and k_1 and k_2 have the form (19).] It is interesting to note that the coupling constants in such a model can have values close to the coupling constants of the standard model of the strong and electroweak interactions. For example, $\tan\theta_W = \sqrt{2} g_1/g_2 = n/\sqrt{6}$ (θ_W is the Weinberg angle, and $\sqrt{2}$ arises because of the chosen normalization of the SU(2) generators) for $n = 2$ has the value $\sqrt{2/3}$, which is close to the asymptotic value $\sqrt{3/5}$ of the SU(5) grand unification model. After normalization of the coupling constants to the energies ~100 GeV currently accessible to experiment, $\sin^2\theta_W$ takes a value close to the experimental one (0.22–0.23). This fact, first noted in Ref. 20 in the consideration of $d = 6$ Einstein–Maxwell theory, shows that it is in principle possible to obtain realistic parameters of the standard SU(3)×SU(2)×U(1) model as a consequence of spontaneous compactification in Kaluza–Klein theory.

If the considered SU(3)×SU(2)×U(1) vacuum configuration of $d = 10$ Einstein–Maxwell theory is to be a contender for the role of the true Kaluza–Klein vacuum, it must be stable. A vacuum configuration is classically unstable if the excitation spectrum contains tachyon modes. The results of Refs. 74 and 75 indicate that in the considered model tachyons are absent.

Chirality of the fermions

As already noted, one of the most important problems that arise in the construction of a realistic Kaluza–Klein theory is that of obtaining in the effective $d = 4$ theory a spectrum of massless Weyl fermions transforming in accordance with different complex representations of the gauge group, i.e., one must obtain chiral fermions. In the case of compactification of M^{4+n} into $M^{1+3} \times K^n$, the number of massless fermions in M^{1+3} is equal to the number of zero modes of the Dirac operator that acts on the compact subspace. The condition that determines the occurrence of chiral fermions on the compact subspace is a nonvanishing index of the Dirac operator. This indicates (by virtue of the Atiyah–Singer index theorem; see Ref. 95) that the vacuum field configurations must have a nontrivial topological structure. As was shown by Witten,[96] chiral fermions can arise in M^{1+3} ($M^{4+n} = M^{1+3} \times K^n$) only if the original multidimensional theory itself contains chiral fermions. Therefore, the dimension d of the multidimensional space–time must be even.

Let us consider, for example, the 10-dimensional Lagrangian for a massless Weyl fermion field interacting minimally with the gravitational and gauge fields of $d = 10$ Einstein–Maxwell theory:

$$L_F = \mathrm{i} E \overline{\Psi} \Gamma^M \hat{D}_M \Psi, \tag{27}$$

$$\Gamma^M \hat{D}_M = \Gamma^M (\partial_M + \omega_M + \mathrm{i} A_M),\quad \Psi = \Gamma_{11} \Psi.$$

The Dirac equation which results from the variation of this Lagrangian has the form

$$\Gamma^M \hat{D}_M \Psi = 0. \tag{28}$$

In the case when contributions are made to (28) only by fields corresponding to a ground state with $M^{10} = M^{1+3} \times K^6$ [for example, (14) and (17)], the solution of Eq. (28) can be sought in the form

$$\Psi = \psi(x) \otimes \eta(Y), \tag{29}$$

where $\psi(x)$ is an anticommuting spinor in M^{1+3}, and $\eta(Y)$

is a commuting spinor in K^6. The Γ matrices of the 10-dimensional space can be conveniently chosen in the form

$$\Gamma_\mu=\gamma_\mu\otimes I,\ \Gamma_m=\gamma_5\otimes\bar{\gamma}_m,\ \Gamma_{11}=\gamma_5\otimes\bar{\gamma}_7 \qquad (30)$$

(γ_μ, γ_5 and $\bar{\gamma}_m$, $\bar{\gamma}_7$ are Dirac matrices on M^{1+3} and K^6, respectively).

The Dirac equation (28) takes the form

$$(\gamma^\mu\partial_\mu\psi)\otimes\eta(Y)+\gamma_5\psi\otimes\bar{\gamma}^m\hat{D}_m\eta(Y)=0, \qquad (31)$$

from which it follows that the field $\psi(x)$ will be massless in $d=4$ if and only if

$$\bar{\gamma}^m\hat{D}_m\eta(Y)=0. \qquad (32)$$

Note that by virtue of (30) the Weyl spinor Ψ can be represented in two different ways in the form (29):

$$\Psi=\psi_L(x)\otimes\eta_L(Y);\quad \Psi=\psi_R(x)\otimes\eta_R(Y), \qquad (33)$$

where $\psi_{L,R}$ and $\eta_{L,R}$ are eigenfunctions of the operators γ_5 and $\bar{\gamma}_7$.

Thus, if chiral fermions are to arise in $d=4$ there must be different numbers ($N_L-N_R\neq 0$) of left (N_L) and right (N_R) zero modes of the operator (32) that transform in accordance with equivalent representations of the symmetry group of the ground state; for all paired zero modes form a vector representation of the gauge group. The difference N_L-N_R is called the index of the Dirac operator $\bar{\gamma}^m\hat{D}_m$ and can be determined by means of the Atiyah–Singer theorem.[95]

If only the spin connection $\omega^{(1)}_{m(n)}$ contributes to the operator (32), then N_L-N_R vanishes for compact spaces with any topology,[96] and, therefore, chiral fermions cannot arise. However, if in the ground state there are nonvanishing vacuum expectation values of gauge fields whose values are equated with the values of the corresponding components of the spin connection, N_L-N_R becomes nonzero and chiral fermions can arise in M^{1+3}.

For example, in the model which we consider of 10-dimensional Kaluza–Klein theory with an SU(3)×SU(2)×U(1)-symmetric vacuum the presence of monopoles on S^2 and CP^2 "suppresses," as it were, some of the zero modes of one of the chiralities and ensures the formation of chiral fermions in the effective $d=4$ theory.[11,45,50]

The chirality problem was studied in detail in Kaluza–Klein theories in Refs. 34–37 and 96.

4. SPONTANEOUS COMPACTIFICATION OF SUBSPACES IN $d=11$ SUPERGRAVITY

Mechanisms of spontaneous compactification in $d=11$ supergravity began to be studied particularly extensively after Witten[34] had shown qualitatively the possibility of formation of a 7-dimensional compact subspace with the symmetry group SU(3)×SU(2)×U(1), which corresponds to the group of the strong, weak, and electromagnetic interactions of the physical sector of the theory.

In this section, we consider the structure of the compact vacuum configurations of $d=11$ supergravity that arise as a result of the Freund–Rubin spontaneous-compactification mechanism[51] and also the connection-identification mechanism generalized to the case of antisymmetric gauge fields (Refs. 47–49, 69, and 70).

The supermultiplet of $N=1$, $d=11$ supergravity consists of the gravitational field $e_N^{(\hat{M})}(\hat{X})$, a Rarita–Schwinger Majorana field $\Psi_{\hat{M}}$, and an antisymmetric Abelian field $A_{\hat{M}\hat{N}\hat{L}}$ ($\hat{M},\hat{N},\hat{L}=1,\ldots,11$), which determines the torsion of the 11-dimensional space.[97,98]

We shall assume (as is customary when compactification is being considered at the classical level) that the fermion fields do not participate in the formation of the compact subspaces ($\langle\Psi_{\hat{M}}\rangle=\langle\bar{\Psi}\Psi\rangle=0$), and we shall consider only the boson sector of the theory. Imposition of this condition leads to the study of vacuum configurations that have the highest symmetries. The Lagrangian of the boson sector of 11-dimensional supergravity has the form[2]

$$L=-E\left(\frac{1}{4\varkappa^2}R+\frac{1}{48}(F_{\hat{M}\hat{N}\hat{L}\hat{P}})^2\right.$$
$$\left.-\frac{2\varkappa}{(6\cdot 4!)^2E}\varepsilon^{\hat{M}_1\ldots\hat{M}_{11}}F_{\hat{M}_1\ldots\hat{M}_4}F_{\hat{M}_5\ldots\hat{M}_8}A_{\hat{M}_9\hat{M}_{10}\hat{M}_{11}}\right), \qquad (34)$$

where $F_{\hat{M}\hat{N}\hat{L}\hat{P}}=4\partial_{[\hat{M}}A_{\hat{M}\hat{L}\hat{P}]}$ is the intensity tensor of the antisymmetric gauge field $A_{\hat{N}\hat{L}\hat{P}}$; [ ] is the operation of antisymmetrization with division by the number of permutations of the indices.

In the Lagrangian (34), all interactions are determined by a single coupling constant—the gravitational constant $\varkappa$, whose dimension is $l^{(d-2)/2}$, where l has the dimensions of a length. In the following formulas, we shall take $\varkappa$ equal to unity.

The equations of motion for the fields $e_{\hat{N}}^{(\hat{M})}$ and $A_{\hat{M}\hat{N}\hat{L}}$, that follow from the Lagrangian (34) have the form

$$R_{\hat{M}\hat{N}}-\frac{1}{2}g_{\hat{M}\hat{N}}R=-2T_{\hat{M}\hat{N}}; \qquad (35)$$

$$\nabla_{\hat{M}}F^{\hat{M}\hat{L}\hat{N}\hat{P}}=-\frac{1}{(4!)^2E}\varepsilon^{\hat{M}_1\ldots\hat{M}_8\hat{L}\hat{N}\hat{P}}F_{\hat{M}_1\ldots\hat{M}_4}F_{\hat{M}_5\ldots\hat{M}_8}, \qquad (36)$$

where $T_{\hat{M}\hat{N}}$ is the energy–momentum tensor of the gauge fields $A_{\hat{M}\hat{N}\hat{P}}$:

$$T_{\hat{M}\hat{N}}=\frac{1}{6}\left(F_{\hat{M}\hat{L}\hat{K}\hat{P}}F_{\hat{N}}^{\hat{L}\hat{K}\hat{P}}-\frac{1}{8}\left((F_{\hat{L}\hat{K}\hat{P}\hat{S}})^2g_{\hat{M}\hat{N}}\right)\right). \qquad (37)$$

Freund–Rubin spontaneous compactification

The simplest and most natural and, therefore, most popular mechanism of spontaneous compactification of $d=11$ supergravity is the Freund–Rubin mechanism,[59] in which compactification of the 11-dimensional space into the direct product of space–time M^{1+3} and a 7-dimensional compact space is ensured by the "vacuum condensate" of the field $A_{\hat{M}\hat{N}\hat{L}}$, whose intensity tensor has the following nonvanishing components:

$$\langle F_{\mu\nu\lambda\rho}\rangle=(\det e_\delta^{(\sigma)}(x))\,3m\varepsilon_{\mu\nu\lambda\rho}, \qquad (38)$$

where m is a parameter which characterizes the size of the compactified subspace. Note that the tensor (38) satisfies the condition of parallelizability ($\Delta_\mu F_{\nu\lambda\rho\sigma}=0$), analogous to (8). At the same time, the solutions of the equations of motion of the fields of the boson sector of $d=11$ supergravity, Eqs. (35) and (36), determine the vacuum configuration of the space–time as the direct product of 4- and 7-dimensional Einstein spaces with Ricci tensors given by

$$R_{\mu\nu}=12m^2g_{\mu\nu}(x), \qquad (39)$$

$$R_{\hat{m}\hat{n}}=-6m^2g_{\hat{m}\hat{n}}(\hat{Y}), \qquad (40)$$

where $g_{\mu\nu}(x)$ is the metric of the 4-dimensional space–time, which we shall throughout assume is the anti-de Sitter space; $g_{\hat{m}\hat{n}}(\hat{Y})$ is the metric of the 7-dimensional space with coordinates $\hat{Y}$.

We note that there are no solutions of (39) corresponding to flat 4-dimensional space–time except for the cases when the curvature of the compact subspace is equal to zero ($m = 0$). On the other hand, if we impose the requirement of a small size of the compact subspace, this leads to the appearance of a large cosmological term in the effective 4-dimensional theory. This is an important but as yet unsolved problem of Kaluza–Klein theories based on $d = 10$ and 11 supergravitites and is due to the impossibility of including in these supergravity variants an original cosmological term capable of compensating the effective 4-dimensional cosmological term, in contrast to what happens in multidimensional Einstein–Yang–Mills theory (see Secs. 2 and 3). One of the possible ways of solving this problem was proposed by Duff and Orzalesi,[57] Wu,[58] and Candelas and Raine[85] and involves allowance for nonvanishing vacuum expectation values of bilinear combinations of fermion fields that contribute to the torsion of the compactified subspace and lead to flattening of the 4-dimensional space–time.

There exists an infinite set of Freund–Rubin solutions characterized by the relations (39) and (40). The main task is to identify in this set candidates for the most content-rich vacuum configurations of the theory. The vacuum configurations are selected under the assumption that the ground states must be stable and have a sufficiently high symmetry. We note, however, that the true quantum-mechanical vacuum can be determined by a superposition of different vacuum configurations with allowance for possible tunneling transitions from one configuration to another.

It was shown that solutions in which the 7-dimensional space is a direct product of spaces, for example, $CP^2 \times S^3$ and $S^4 \times S^3$, are unstable.[76] Compactification into such spaces completely breaks the supersymmetry in the effective 4-dimensional theory. In contrast, compact vacuum configurations that preserve supersymmetry in $d = 4$, and also spatial configurations obtained from supersymmetry configurations by changing the orientation (i.e., by the vielbein substitution $e^{(\hat{m})}_{\hat{n}} \to -e^{(\hat{m})}_{\hat{n}}$) and break supersymmetry, are classically stable.[76]

The number of unbroken supersymmetries in the Freund–Rubin solutions is characterized by the number of Majorana spinors η (Killing spinors) of the 7-dimensional compact subspace satisfying the equation[60–63]

$$\left(\partial_{\hat{m}} - \frac{1}{4}\omega_{\hat{m}}{}^{(\hat{n})(\hat{l})}\Gamma_{(\hat{n})(\hat{l})} - \frac{1}{2} m e^{(\hat{n})}_{\hat{m}}\Gamma_{(\hat{n})}\right)\eta = 0, \qquad (41)$$

where $\Gamma_{(\hat{n})}$ are 8×8 Dirac matrices. Equation (41) follows from the requirement of invariance of the ground state with respect to supertransformations. In particular, the supervariation of the Rarita–Schwinger field must vanish:

$$\delta\langle\Psi_{\hat{M}}\rangle = \Big(\nabla_{\hat{M}} + \frac{i}{144}(\Gamma_{\hat{M}}^{\hat{N}\hat{L}\hat{P}\hat{Q}} - 8\delta_{\hat{M}}^{[\hat{N}}\Gamma^{\hat{L}\hat{P}\hat{Q})})F_{\hat{N}\hat{L}\hat{P}\hat{Q}}\Big)(\varepsilon \otimes \eta) = 0 \qquad (42)$$

($\Gamma_{(\hat{M})}$ are 11-dimensional Dirac matrices, and $\varepsilon(x)$ is the parameter of the supertransformations in $d = 4$). It can be seen from (41) that a change in the orientation of the space K^7 is equivalent to a change in sign of the parameter m. The condition of integrability of (41) is the equation

$$C_{(\hat{m})(\hat{n})}{}^{(\hat{l})(\hat{p})}\Gamma_{(\hat{l})}\Gamma_{(\hat{p})}\eta = (R_{(\hat{m})(\hat{n})}{}^{(\hat{l})(\hat{p})} - m^2\delta^{(\hat{l})}_{[(\hat{m})}\delta^{(\hat{p})}_{(\hat{n})]})\Gamma_{(\hat{l})}\Gamma_{(\hat{p})}\eta = 0. \qquad (43)$$

The matrices $C_{(\hat{m})(\hat{n})} = C^{(l)(\hat{p})}_{(\hat{m})(\hat{n})}\Gamma_{(\hat{l})}\Gamma_{(\hat{p})}$ and all their commutators generate the holonomy group of the tensor $C_{(\hat{m})(\hat{n})}$, this being a subgroup of the group Spin(7). Therefore, the number of solutions of Eq. (43) is equal to the number of singlet representations of this subgroup.[99]

As examples of supersymmetric Freund–Rubin vacuum configurations, we can take the 7-dimensional sphere with the symmetry group SO(8); this preserves $N = 8$ supersymmetry in the $d = 4$ theory (in accordance with the assumption of Ref. 22), equivalent to the $N = 8$ supergravity of De Wit and Nicolai[10]; the squashed S^7 sphere (i.e., a manifold topologically equivalent to the standard S^7 sphere but with a different Einstein metric) with symmetry $SO(5) \times SU(2) \sim Sp(4) \times Sp(2)$ and $N = 1$ supersymmetry (a survey of the Freund–Rubin and Englert solutions with the topology of the 7-dimensional sphere is given in Refs. 4 and 62); $SU(3) \times SU(2) \times U(1)/SU(2) \times U'(1) \times U''(1)\gamma$ spaces, a subclass of which admits preservation of $N = 2$ supersymmetry[63]; $SU(3) \times U(1)/U'(1) \times U''(1)$ spaces, with $N = 3$ and 1 supersymmetries (Refs. 47, 48, 64, 67, and 68); and a number of other examples (Refs. 47, 48, 65, 66, and 100).

A list of the 7-dimensional compact spaces that are Freund–Rubin solutions of $d = 11$ supergravity is given in Table I.

$AdS^5 \times G/(H \times H_0)$ vacuum configurations in $d = 11$ supergravity

Apart from the Freund–Rubin mechanism, other mechanisms of spontaneous compactification of subspaces in $d = 11$ supergravity have been found. In particular, Englert[54] considered the possibility that the formation of the compact subspace M^7 might involve not only a Freund–Rubin field but also vacuum expectation values of a field $\langle A_{mnl}\rangle$ with indices of the space K^7, these playing the role of the torsion of these spaces. Details of the investigation of 11-dimensional Kaluza–Klein supergravity can be found in the review of Ref. 22.

We shall consider a different possible mechanism of spontaneous compactification of $d = 11$ supergravity leading to vacuum configurations of 11-dimensional space–time that are a direct product of a 5-dimensional space AdS^5 and a 6-dimensional symmetric space $K^6 = G/H \times H_0$ (Refs. 69 and 70). The compactification in this case is through the connection-identification mechanism, generalized to the case of antisymmetric gauge fields.

The solutions of Eqs. (35) and (36) for $A_{\hat{M}\hat{N}\hat{L}}$ or, equivalently, for $F_{\hat{M}\hat{N}\hat{L}\hat{P}}$ that ensure spontaneous compactification have nonzero values only for components with indices of the compact space, and in an orthogonal basis can be expressed in terms of the structure constants of the group G, one of whose indices (0) corresponds to the Abelian subgroup H_0 and the other to the space $G/H \times H_0$:

$$F_{(m)(n)(l)(p)} = -\rho C_{[(m)(n)}{}^0 C_{(l)(p)]}{}^0; \qquad (44)$$

examples of possible spaces M^6 are CP^3, $CP^2 \times S^2$, $S^4 \times S^2$, and $SO(5)/SO(3) \times SO(2)$.

TABLE I. Freund–Rubin solutions of $d = 11$ supergravity.

Compact 7-dimensional spaces	Holonomy group of C_{mn}	No. of supersymm. in $d = 4$	Vacuum invariance group
$T^7 = T^6 \times S^1$	I	$N=8$	Super-Poincaré $\times U(1)^7$
$S^7 = P(CP^3, S^1) \sim U(4)/U(3) \sim P(E(S^4, S^2, SU(2)), S^1) \sim E(S^4, S^3, SU(2)) \sim P(S^4, SU(2))$	I	$N=8$	$OSp(4, 8)$
$K3 \times T^3 = (K3 \times T^2) \times S^1$	$SU(2)$	$N=4$	Super-Poincaré $\times U(1)^3$
$\frac{SU(3)}{U(1)} \sim P(E(CP^2, S^2, SU(2)), S^1) \sim E(CP^2, S^3, SU(2)) \sim P(CP^2. SU(2)) \sim SU(3) \times SU(2)/SO(2) \times U(1)$	$SU(2)$	$N=3$	$OSp(4, 3) \times SU(3)$
$\frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U'(1) \times U''(1)} \sim P(CP^2 \times S^2, S^1)$	$SU(3)$	$N=2$	$OSp(4, 2) \times SU(3) \times SU(2)$
	$SO(7)$	$N=0$	$SO(3, 2) \times SU(3) \times SU(2) \times U(1)$
$\frac{SO(5) \times SO(2)}{SO(3) \times SO(2)} \sim P\left(\frac{SO(5)}{SO(3) \times SO(2)}, S^1\right)$	$SU(3)$	$N=2$	$OSp(4, 2) \times SO(5)$
$(SU(2)^3 \times U(1))/U(1)^3 \sim P(S^2 \times S^2 \times S^2, S^1)$	$SU(3)$	$N=2$	$OSp(4, 2) \times SU(2)^3$
	$SO(7)$	$N=0$	$SO(3, 2) \times SU(2)^3 \times U(1)$
$CY \times S^1$*	$SU(3)$	$N=2$	Super-Poincaré $\times U(1)$
Squashed $S^7 \sim P(E(S^4, S^2, SU(2)), S^1) \sim E(S^4, S^3, SU(2)) \sim P(S^4, SU(2)) \sim (SO(5) \times SU(2))/SU'(2) \times SU''(2)$	G_2	$N=1$	$OSp(4, 1) \times SO(5) \times SU(2)$
Squashed $\frac{SU(3)}{U(1)} \sim P(CP^2, SU(2)) \sim P(E(CP^2, S^2, SU(2)), S^1) \sim E(CP^2, S^3, SU(2)) \sim \frac{SU(3) \times SU(2)}{SU'(2) \times U(1)}$	G_2	$N=1$	$OSp(4, 1) \times SU(3) \times SU(2)$
$\frac{SU(3) \times U(1)}{U'(1) \times U''(1)} \sim P\left(\frac{SU(3)}{U(1) \times U'(1)}, S^1\right)$	G_2	$N=1$	$OSp(4, 1) \times SU(3) \times U(1)$
$S^4 \times S^3 = P(S^4 \times S^2, S^1)$	$SO(7)$	$N=0$	$SO(3, 2) \times SO(5) \times SO(4)$
$\frac{SU(3)}{SO(3)} \times S^2$	$SO(7)$	$N=0$	$SO(3, 2) \times SU(3) \times SO(3)$
$SO(5)/SO(3)$ max	G_2	$N=1$	$OSp(4, 1) \times SO(5)$

*Calabi–Yau spaces are discussed in Sec. 6.

Note that this compactification mechanism can result in the formation of only symmetric spaces K^6, since the intensity tensor (44) must satisfy the equations of motion (36), and this is possible when $\nabla_l C^0_{(m)(n)} = 0$, a condition that is satisfied only if the space $K^6 = G/H \times H_0$ is symmetric (see Sec. 2).

The possibility of subsequent compactification of five dimensions into the direct product of 4-dimensional Minkowski space and the S^1 sphere is associated with the solution to the problem of the cosmological term and cannot be solved in the framework of the considered compactification mechanism.

We consider in more detail the case when the compact space K^6 is the direct product of the space CP^2 and the S^2 sphere. The symmetry group of such a Kaluza–Klein vacuum is $SO(4,2) \times SU(3) \times SU(2)$ (Refs. 69 and 70).

To find solutions of Eqs. (35) and (36) for which the metric has the form

$$g_{\hat{M}\hat{N}}=\begin{pmatrix} g_{\mu\nu}(x) & 0 & 0 \\ 0 & g_{ab}(y) & 0 \\ 0 & 0 & g_{ik}(z) \end{pmatrix}, \tag{45}$$

[where $g_{\mu\nu}(x)$ is the metric of AdS^5, and the 6-dimensional subspace of additional dimensions is the $CP^2\times S^2$ symmetric space with structure determined by (14)], we assume that the nonvanishing values of the tensor $F_{\hat{M}\hat{N}\hat{L}\hat{P}}$ of the gauge fields $A_{\hat{M}\hat{N}\hat{L}}$ are the components

$$\langle F_{abcd}\rangle = F_{abcd}(y); \quad \langle F_{ikab}\rangle = F_{ikab}(y, z) \tag{46}$$

and that these components satisfy the parallelizability condition that generalizes the conditions (8) to the case of tensor gauge fields:

$$\nabla_j F_{abcd} = \nabla_d F_{ikab} = \nabla_j F_{ikcd} = 0. \tag{47}$$

The fulfillment of the condition (47) has the consequence that the equations of motion (36) are satisfied identically.

From (47), we find the following values for the components (46) of the intensity tensor in the orthogonal basis:

$$F_{(a)(b)(c)(d)}=-3\rho_1 f_{[(a)(b)}{}^{8} f_{(c)(d)]}{}^{8}; \quad F_{(i)(k)(a)(b)}=-\rho C_{(i)(k)}{}^{3} f_{(a)(b)}{}^{8}, \tag{48}$$

where ρ_1 and ρ_2 are arbitrary parameters.

The method employed here is based on finding manifestly covariant entities, i.e., intensity tensors. As in Refs. 29–31 and 91, the transition to the gauge fields is made in the standard manner using Cartan forms. For the components of the antisymmetric gauge field A_{abc}, A_{abi}, and A_{ika} we obtain the following expressions in terms of the $\theta^8_a(y)$ and $\theta^3_i(z)$ components of the Cartan forms, corresponding to the U(1) subgroup of the groups SU(3) and SU(2):

$$\left.\begin{aligned} A_{abc} &= \frac{3}{2}\frac{\rho_1}{k_1} f_{(a)(b)}{}^{8} e^{(a)}_{[a} e^{(b)}_{b}\theta^{8}_{c]}; \\ A_{ika} &= \frac{\rho_2}{k_1} C_{(j)(l)}{}^{3} e^{(j)}_{i} e^{(l)}_{k}\theta^{8}_{a}(y); \\ A_{abi} &= \frac{\rho_3}{k_2} f_{(a)(b)}{}^{8} e^{(a)}_{a} e^{(b)}_{b}\theta^{3}_{i}(z), \end{aligned}\right\} \tag{49}$$

where $\rho_2+\rho_3=\rho$ is the parameter in the definition of F_{ikab}. From Eq. (35), taking into account (14) and the contribution to the energy–momentum tensor (37) of the vacuum solutions, we obtain for the curvatures k_1 and k_2 the values

$$k_1=\frac{1}{6}(\rho_1^2+\rho^2); \quad k_2=\frac{1}{6}\left(4\rho^2-\frac{1}{2}\rho_1^2\right). \tag{50}$$

If k_1 and k_2 are to be positive, ρ and ρ_1 must satisfy the inequality $\rho^2>(1/8)\rho_1^2$. It follows from Eq. (35) that the 5-dimensional space determined by the curvature tensor (14a) ($d'=5$) is an anti-de Sitter space with Λ that depends on the quantities which characterize the vacuum solutions (48):

$$\Lambda=-\frac{1}{12}\left(\frac{1}{4}\rho_1^2+\rho^2\right). \tag{51}$$

Thus, the possibility of spontaneous compactification of the 11-dimensional space into $AdS^5\times CP^2\times S^2$ due to interaction of the gravitational field $e^{(\hat{M})}_{\hat{N}}$ and the gauge field $A_{\hat{M}\hat{N}\hat{L}}$ (as in Einstein–Yang–Mills theories; see Secs. 2 and 3) is essentially related to the structure of the holonomy group of the space CP^2, which makes it possible to associate definite components of the gauge field $A_{\hat{M}\hat{N}\hat{L}}$ [see Eq. (49)] with only the part of the connection on the space CP^2 determined by the U(1) subgroup of the group SU(3). We emphasize once more that the subsequent separation in the 5-dimensional space–time M^{1+4} of the $M^{1+3}\times S^1$ ground-state configuration cannot have a consistent solution without a solution to the problem of the cosmological term in this supergravity variant. However, in the framework of the proposed spontaneous-compactification mechanism it is possible to study cosmological models in which the expansion of the 5-dimensional universe occurs much more rapidly in three spatial directions than in the fourth.[70]

As already noted, the Kaluza–Klein theories based on $d=11$ supergravity encounter a serious difficulty—the absence in the 4-dimensional theory of chiral fermions. One of the ways of solving this problem is to consider preon models on the basis of hidden symmetry groups of the type SU(8) in $N=8$ supergravity.[19,22] However, at the present time such an approach does not appear promising in view of the great complexities associated with the description of the dynamics of particles of composite models.

Another possibility is to study the influence of the fermion spectrum of Kaluza–Klein monopoles.[101,102] The occurrence of such monopoles leads to a more complicated (fiber) structure of the ground-state space (different from a direct-product structure of spaces) and, as is suggested in Ref. 87, may ensure the formation of chiral fermions.

5. SPONTANEOUS COMPACTIFICATION IN $d=10$ SUPERGRAVITIES

Three variants of supergravity in 10-dimensional space–time are currently known. They can all be obtained as low-energy limits of 10-dimensional superstring theories[42]:

a) $N=1$, $d=10$ supergravity interacting with $N=1$ Yang–Mills fields arises as the low-energy limit of 10-dimensional theories of interacting open and closed superstrings[4,44,45] or the heterotic string[43];

b) the chiral [c) nonchiral] variants of $N=2$, $d=10$ supergravity can be obtained from the $d=10$ theory of a closed superstring; in addition, nonchiral $N=2$, $d=10$ supergravity can be obtained from $d=11$ supergravity by dimensional reduction.[5–7]

In chiral $N=2$, $d=10$ supergravity Kaluza–Klein vacua sufficiently interesting from the physical point of view have not been found. A list of the known solutions can be found in the papers of Ref. 103. We note that the structure of the vacuum configurations with $M^{10}=M^{1+4}\times K^5$ can be investigated in the same way as in $d=11$ supergravity by means of the connection-identification mechanism.[48,49]

The possibilities of realistic spontaneous compactification of $N=1$, $d=10$ supergravity interacting with $N=1$ Yang–Mills fields are considered in the following section.

In this section, we consider variants of spontaneous compactification of nonchiral $N=2$, $d=10$ supergravity and the connection between the 10-dimensional compact vacuum configurations and the Freund–Rubin solutions of the 11-dimensional theory.

Spontaneous compactification in nonchiral $N=2$, $d=10$ supergravity

We obtain $N=2$, $d=10$ supergravity by compactification of one dimension of $d=11$ supergravity into the S^1 sphere and by requiring all fields to be independent of the S^1 coordinate. The supermultiplets of $N=2$, $d=10$ supergravity contain the gravitational field $e_N^{(M)}$, antisymmetric fields A_{MNL} and B_{MN}, a "Maxwell" field A_M, a scalar $\phi = \exp \sigma$, a Rarita–Schwinger Majorana field ψ_M, and a spinor field λ, which can be decomposed into Weyl components.[5,6]

The fields A_M and ϕ appear in $d = 10$ through the choice of the metric of the $d = 11$ space in the form

$$g_{\hat{M}\hat{N}} = \begin{pmatrix} \phi^{-\frac{1}{4}} g_{MN}(X) + R_0^2\phi^2 A_M(X) A_N(X) & R_0\phi^2 A_M(X) \\ R_0\phi^2 A_N(X) & \phi^2(X) \end{pmatrix}, \quad (52)$$

where $g_{MN}(X)$ is the 10-dimensional metric, and R_0 is the radius of the S^1 sphere.

The form of the metric (52) is the same as in the classical 5-dimensional Kaluza–Klein theory.

As is customary in finding classical vacuum configurations, the values of the fermion fields are taken to be zero and one considers the equations of the bosonic sector of $N = 2$, $d = 10$ supergravity; these have the form[5,6]

$$R_{MN} = \frac{9}{8}\partial_M\sigma\partial_N\sigma + \frac{R_0^2}{2}\exp\left(\frac{9}{4}\sigma\right) \times\left[F_{MP}F_N^{\ P} - \frac{1}{16}g_{MN}(F_{PQ})^2\right] + \exp\left(-\frac{3}{2}\sigma\right)\left[H_{MPQ}H_N^{\ PQ} - \frac{1}{12}g_{MN}(H_{PQR})^2\right] + \frac{1}{3}\exp\left(\frac{3}{4}\sigma\right)\left[F'_{MPQR}F'^{PQR}_N - \frac{3}{32}g_{MN}(F'_{PQRS})^2\right]; \quad (53)$$

$$-36\,\Box\,\sigma + 9\exp\left(\frac{9}{4}\sigma\right)(F_{MN})^2 R_0^2 - 8\exp\left(-\frac{3}{2}\sigma\right)(H_{MNP})^2 + \exp\left(\frac{3}{4}\sigma\right)(F'_{MNPQ})^2 = 0, \quad (54)$$

where

$$F_{MNPQ} = F_{MNPQ} + 4R_0A_{[M}H_{NPQ]};\ \ H_{NPQ} = \partial_{[N}B_{PQ]}.$$

In addition, there are Maxwell equations for the fields A_{MNL}, B_{MN}, and A_M analogous to Eq. (36) of $d = 11$ supergravity. The explicit form of these equations is not given here, since they are satisfied identically for the considered mechanism of spontaneous compactification.

It can be seen from Eqs. (53) and (54) that for the compactification of the six additional coordinates of $N = 2$, $d = 10$ supergravity it is no longer sufficient to have just one Freund–Rubin vacuum field (38), in contrast to the 11-dimensional theory. It is also necessary for the field A_M to have nontrivial vacuum values on the compact space. This is due to the fact that the equations of motion (54) of the scalar field σ require fulfillment of the following relation for the vacuum values of A_M and A_{MNL} ($\langle\sigma\rangle = \langle B_{MN}\rangle = 0$):

$$\langle F_{MNLP}\rangle^2 + 9R_0^2\langle F_{MN}\rangle^2 = 0. \quad (55)$$

In Sec. 3, we pointed out the possibilities of spontaneous compactification of 10-dimensional Einstein–Maxwell theory, which is a constituent part of nonchiral $N = 2$, $d = 10$ supergravity. It can be seen from Eqs. (53)–(55) that all these possibilities are also realized in the complete $N = 2$, $d = 10$ theory when F_{MNLP} is chosen in the form (38) and F_{MN} in the form

$$\langle F_{(\mu)(\nu)}\rangle = 0;\ \langle F_{(m)(n)}\rangle = -\frac{\rho}{R_0^2}C_{(m)(n)}{}^0, \quad (56)$$

where $C_{(m)(n)}{}^0$ are the structure constants of the symmetry group $K^6 = G/H \times H_0$ (0 is the index of H_0), and ρ is a numerical parameter whose range of values may be restricted by the topology of K. At the same time, the relation (55) determines the connection between the Freund–Rubin parameter m^2 and the parameter ρ^2, which characterizes the scale of compactification of the subspaces by the Abelian field (56). Thus, the compact spaces K^6 listed in (12) are solutions of nonchiral $N = 2$, $d = 10$ supergravity. However, in contrast to $d = 10$ Einstein–Maxwell theory (see Sec. 3), the considered variant of supergravity contains difficulties associated with the formation of chiral fermions in the 4-dimensional space, since there is no minimal interaction of the field A_M with the original spinor fields.

The properties of the known vacuum configurations of $N = 2$, $d = 10$ supergravity are given in Table II.

Connection between the $d = 11$ and $d = 10$ vacuum configurations

Since $N = 2$, $d = 10$ supergravity is obtained as a result of reduction of $d = 11$ supergravity, and the solutions of the $d = 10$ theory mentioned above arise as a result of a combined compactification mechanism, including a Freund–Rubin field and the Abelian gauge field A_M (which appears in $d = 10$ supergravity as a consequence of compactification of one dimension into S^1 in the spirit of Kaluza and Klein), it is natural to assume that all $d = 10$ vacuum solutions of such type must be related to corresponding Freund–Rubin solutions of the $d = 11$ theory.

A relationship was established in Refs. 5 and 47–50 and takes the form that the vacuum configurations of $d = 11$ supergravity corresponding to the $d = 10$ solutions can be represented in the form of fiber bundles $K^7 = P(K^6, S^1)$ with base K^6, structure group $U(1) \sim S^1$, and a definite choice of the connection in the fiber bundle corresponding to the vacuum solutions for the field A_M in the $d = 10$ theory (see Table I). From the physical point of view, this means that spontaneous compactification in $d = 11$ supergravity can occur in a sequence. In the first stage, an S^1 cycle is formed together with the field A_M (as in classical Kaluza–Klein theory), and in the second stage the field A_M together with the field A_{MNL} ensures compactification of the six additional dimensions. Moreover, in the case of S^7 and $SU(3)/U(1)$, compactification can occur in three stages. The first is as above; in the second, A_M ensures compactification of a two-dimensional subspace into S^2 and the formation of gauge fields of the $SU(2)$ group of instanton type, which in the third stage ensure the compactification of a 4-dimensional subspace. Geometrically, this means that K^6, in its turn, can be an associated fiber bundle $E(K^4, S^3, SU(2))$ with base K^4, fiber S^2, and structure group $SU(2)$.

It must be emphasized that despite this correspondence between the vacua of the two theories, the effective 4-dimensional theories that result from compactification of the $d = 10$ and 11 supergravities are different, since a consequence of the reduction is that $N = 2$, $d = 10$ supergravity does not contain the complete spectrum of fields of the $d = 11$ theory. This difference is already manifested when one studies the supersymmetries of the corresponding 4-dimensional theories.[49,50]

All the vacuum configurations of $N = 2$, $d = 10$ supergravity corresponding to $d = 11$ vacua with $N > 1$ supersym-

TABLE II. Vacuum configurations of nonchiral $N=2$, $d=10$ supergravity.

Compact subspaces K^6	No. of super-symm. in $d=4$	Vacuum symmetry group
T^6 $(m^2=0)$	$N=8$	Super-Poincaré $\times U(1)^7$
$CP^3=SU(4)/SU(3)\times U(1)$	$N=6$	$OSp(4,6)\times SU(4)\times U(1)$
$K3\times T^2$ $(m^2=0)$	$N=4$	Super-Poincaré $\times U(1)^3$
Squashed $CP^3=\frac{Sp(4)}{Sp(2)\times U(1)}=\frac{SO(5)}{SU(2)\times U(1)}$	$N=1$	$OSp(4,1)\times SO(5)\times U(1)$
$SU(3)/U(1)\times U'(1)$	$N=0$	$SO(3,2)\times SU(3)\times SU(2)\times U(1)$
$S^4\times S^2$	$N=0$	$SO(3,2)\times SU(5)\times SU(2)\times U(1)$
$S^2\times S^2\times S^2$	$N=0$	$SO(3,2)\times SU(2)^3\times U(1)$
$O(5)/SO(3)\times SO(2)$	$N=0$	$SO(3,2)\times SO(5)\times U(1)$
$CP^2\times S^2$	$N=0$	$SO(3,2)\times SU(3)\times SU(2)\times U(1)$
CY $(m^2=0)$	$N=1$	Super-Poincaré

metry and nonvanishing Freund–Rubin parameter $(m^2\neq 0)$ contain two fewer supersymmetries (see Tables I and II). For example, the vacuum configuration with $K^6=CP^3$ corresponding to the "round" sphere S^7 in $d=11$ with $N=8$ supersymmetries possesses $N=6$ supersymmetry, and the ground state with $K^6=CP^2\times S^2$ (associated with $K^7=SU(3)\times SU(2)\times U(1)/SU'(2)\times U'(1)\times U''(1)$ and $N=2$ supersymmetries) does not possess a supersymmetry at all.

The number of supersymmetries preserved in $d=4$ is the same for the connected $d=11$ and $d=10$ vacua if the $d=11$ vacuum configurations have $N=1$ supersymmetry or if the compact subspace of the ground state is Ricci-flat $(m^2=0)$ (Tables I and II).

6. TOWARDS A REALISTIC KALUZA–KLEIN THEORY

In the previous sections, we have considered mechanisms of spontaneous compactification in Kaluza–Klein theories (for example, $N=1$, $d=11$ and $N=2$, $d=10$ supergravities) based on the "economy" principle, namely, that the original theory should contain a minimum of nongravitational fields (in particular, gauge fields), so that it is necessary to ensure the occurrence of a larger group of internal symmetries by a minimal number of gauge fields participating in the compactification. However, the discovery at the end of 1984 of the cancellation of all anomalies in the "modernized" $N=1$, $d=10$ supergravity interacting with fields of $N=1$ supersymmetry and Yang–Mills fields of the SO(32) and $E_8\times E_8$ groups, and also in the theory of a type-I superstring with gauge group SO(32) (Refs. 44 and 45) (the low-energy limit of which is $N=1$, $d=10$ supergravity + SO(32) Yang–Mills), stimulated intensive study of these theories, which contain a large set of gauge fields, in the framework of the Kaluza–Klein approach.

Since the group $E_8\times E_8$ cannot be a gauge group of a type-I superstring,[42] and $N=1$, $d=10$ supergravity + $E_8\times E_8$ Yang–Mills is an anomaly-free quantum theory, it was suggested that this theory is the low-energy limit of a new string theory based on a bosonic string in $d=26$.[104] Such a theory was constructed in Ref. 43 and is called the heterotic (hybrid) string (mixture of a bosonic 26-dimensional string and a fermionic 10-dimensional string).

The gauge fields that arise in such a theory as massless vibrations of the string can transform only in accordance with the groups SO(32) or $E_8\times E_8$.

The main advantages of the SO(32) and $E_8\times E_8$ theories compared with the ones considered earlier are the following:

1. There are good grounds for believing[89] that the type-I superstring and the new heterotic strings with gauge groups SO(32) and $E_8\times E_8$ may be theories completely free of divergences, i.e., mathematically consistent quantum theories of gravitation.

2. The low-energy limits of these theories ($N=1$, $d=10$ supergravity + SO(32) or $E_8\times E_8$ Yang–Mills) contain a minimal interaction of Majorana–Weyl fermions with gauge fields, and this, as noted above, may lead through the connection-identification mechanism to the appearance in $d=4$ of chiral fermions; this, in its turn, indicates a possibility of constructing a theory realistic from the phenomenological point of view.

Following Ref. 33, let us consider how in $N=1$, $d=10$ supergravity interacting with $E_8\times E_8$ supersymmetric Yang–Mills theory [there are grounds for believing that the gauge group $E_8 \times E_8$ is better suited to the phenomenology than SO(32)] spontaneous compactification leading to a realistic effective $d=4$ theory can take place. Note that in considering such a theory we can abandon the "economy" principle, since the original gauge group is large. Moreover, one of the tasks is now the breaking of this group in such a way that a realistic 4-dimensional grand unification theory arises.

The supermultiplet composition of $N=1$, $d=10$ supergravity + Yang–Mills is as follows: ($e_N^{(M)}$, ψ_M, B_{MN}, λ, ϕ), a gravitational supermultiplet (see Sec. 5); (A_M^α, χ^α), a gauge supermultiplet (χ^α are Majorana–Weyl fields, and α is the index of the adjoint representation of the gauge group, identical in the case of the group $E_8 \times E_8$ to the fundamental representation).

As before, we shall assume that the compactification of M^{10} is realized by fields of the bosonic sector, whose Lagrangian is chosen in the form

$$L = -\frac{E}{4\varkappa^2} R - \frac{E}{2\varkappa^2} \phi^{-2}\partial_M\phi\partial^M\phi - \frac{E}{4e^2}\phi^{-1}(\mathrm{tr}\,F_{MN}F^{MN} - (R_{MNLP})^2) - \frac{3\varkappa^2E}{2e^4}\phi^{-2}(H_{MNP})^2 + \frac{E\phi^{-1}}{4e^2}(R^2 - 4R_{MN}R^{MN}), \tag{57}$$

where $H = H_{MNP}dX^M \wedge dX^N \wedge dX^P = dB - \omega_{3Y} + \omega_{3L}$, and $\omega_{3Y} = \mathrm{tr}(A\Lambda F - \frac{1}{3} A\Lambda A\Lambda A)$, $\omega_{3L} = \mathrm{tr}(\omega\Lambda R - \frac{1}{3}\omega\Lambda\omega\Lambda\omega)$ are Chern–Simons corrections to the intensity field B_{MN} in the notation of differential forms.[44]

It is assumed that the Lagrangian (57) arises from consideration of the low-energy limit of the heterotic string.[43] In contrast to the Chapline–Manton Lagrangian,[4] it contains terms with higher derivatives of the gravitational field (of the form R^2), and in the general case this leads to the appearance of fields of ghost states in the spectrum. However, in the theory with the Lagrangian (57) this does not occur, since the terms with higher derivatives occur in (57) in a definite combination ($L_{R^2} = R^2 - 4R_{MN}R^{MN} + R_{MNLP}R^{MNLP}$; see Ref. 105). The presence of such terms in the Lagrangian makes it possible to avoid the no-go theorem of Ref. 106 (which prevents realization of realistic spontaneous compactification in the Chapline–Manton theory[4]) and to satisfy all the equations of motion that follow from the Lagrangian (57).[33]

The main requirements imposed on the compactified vacuum configurations of the theory are the following:

1. Preservation of Lorentz invariance in the effective $d=4$ theory. As a consequence of this, M^{10} must compactify into $M^{1+3} \times K^6$, where M^{1+3} is a space–time with a maximal symmetry group (Minkowski space or de Sitter or anti-de Sitter space).

2. The presence of $N=1$ supersymmetry in $d=4$ (since $N=1$ supersymmetry can solve the problem of the gauge hierarchy of masses).

3. The spectrum of fields in the $d=4$ theory must be realistic.

It is found that these requirements strongly restrict the choice of possible vacuum configurations of the theory, since the quantum numbers of the fields of the effective 4-dimensional theory are determined by the topological characteristics of the space K^6 and its symmetry group, and also by the vacuum values of the gauge fields of the group $E_8 \times E_8$ defined on K^6.

The requirement of preservation of $N=1$ supersymmetry ($\langle\psi\rangle = \delta_\varepsilon\langle\psi\rangle = 0$) has the following consequences:

a) M^{1+3} is Minkowski space;

b) K^6 is a Ricci-flat Kählerian manifold with holonomy group SU(3) (Calabi–Yau space). For details of the Calabi–Yau spaces, see Refs. 33 and 107. However, if requirement 2 is given up, the class of compact spaces K^6 is greatly extended. For example, it is possible to have compactification into the spaces listed in Table II.

Requirement 3 (and, in particular, the existence of chiral fermions) has the consequence that:

c) the vacuum values of the gauge fields must be equal to K^6 spin-connection coefficients that transform in accordance with the group SU(3) (i.e., the mechanism of embedding of the spin connection in the gauge connection must operate):

$$\langle A_m^\alpha \rangle = \omega_{m(l)}{}^{(p)} \ (\alpha = (l, p)),$$

and then one of the groups E_8 is broken down to E_6 ($E_6 \times SU(3) \subset E_8$), the chiral fermions form a 27-plet of this group, and the number of generations of fermions in such a theory is equal to half the Euler characteristic of the space K^6 ($N = |\chi(K^6)|/2$).[33]

At the present time, a considerable number of Calabi–Yau spaces are known; they include spaces with $N = \chi/2 = 1, 2, 3, 4$.[33,107] The fields that transform in accordance with representations of the other (unbroken) group E_8 interact with the fields E_6 only through the gravitation and form a "hidden" sector of the theory. This sector may play an important part in the mechanism of spontaneous breakdown of $N=1$ supersymmetry.[33,116]

A very important problem in the considered theory is to find a mechanism of spontaneous breakdown of supersymmetry that does not lead to the formation in the spontaneously broken phase of a large positive vacuum energy density, i.e., to the transformation of the space M^{1+3} into de Sitter space (a consequence that would contradict experimental data).

Thus, the problem of the cosmological term still awaits its solution in the Kaluza–Klein theory based on string theory.

Searches for a realistic variant of supersymmetric Kaluza–Klein theory are continuing (see, for example, Ref. 108).

It should be emphasized that the compact vacuum configurations considered in the present section are possible ground states of the $N=1$, $d=10$ supergravity + Yang–Mills theory that arises from superstring theory in the limit when the string "size" tends to zero ($\alpha' \to 0$). However, the question of what is the ground state of the complete string theory and why and how spontaneous compactification of the additional dimensions occurs in it remains open. The most promising approach to its solution appears to be the effective string-action method proposed in Ref. 109.

CONCLUSIONS

We note in conclusion that in string theories too attempts are being made to return to the traditional philosophy of Kaluza and Klein, i.e., to take as a basis a multidimensional theory in which there are no original gauge fields. The 26-dimensional bosonic string is considered, for example, as such a theory (Refs. 104, 110, and 111). One of the possible variants is the theory of the heterotic string formulated in 26-dimensional space–time.[43] In such a theory, 16 dimensions are compactified into a 16-dimensional torus T^{16} and, despite the fact that the symmetry group of T^{16} is Abelian, gauge fields of the non-Abelian group SO(32) or $E_8 \times E_8$ arise in $d = 10$; this is a specific feature of the string theory of Ref. 43 and is an example of a new mechanism of generation of non-Abelian gauge symmetries radically different from the Kaluza–Klein mechanism.

Duff, Nilsson, and Pope[112] (see also Ref. 104) have suggested that one should not stick to 26 dimensions but take as a basis the theory of a closed bosonic string in a 506-dimensional space–time. Such a theory will be consistent from the quantum point of view if 496 dimensions are compactified and form the group manifold of the SO(32) or $E_8 \times E_8$ group.[113] As a result of such compactification, the bosonic sector of the heterotic string arises in $d = 10$ dimensions.[112]

It could also perhaps be the case that the additional dimensions of the space–time in which the strings "live" have a nature analogous to superspace harmonic coordinates.[114,115]

Which variant of the unified theory are we to choose? We hope that the intensive investigations being made in the field of Kaluza–Klein theories will provide the answer to this question.

Our understanding of the problems touched upon in this review is to a large degree due to collaboration with D. V. Volkov, to whom we are deeply grateful. We are also grateful to R. É. Kallosh, V. I. Ogievetskiĭ, and V. N. Pervushin for discussion and helpful remarks.

[1)]This mechanism was used earlier in Ref. 51 to study the influence of gravitation on the topology of Yang–Mills fields.

[2)]As we have already noted, more general possibilities, when the space M^{4+n} is not a direct product of subspaces[72,73] or K^n is noncompact, are also studied.

[3)]Note that the condition of parallelizability is a generalization of the condition of constancy of the vacuum values of the scalar fields in theories with spontaneously broken symmetry to the case of curved space–time and tensor fields.

[4)]But if $G' \supset G$, then the complete symmetry group of the Kaluza–Klein vacuum that arises as a result of the first mechanism will be SO(3, 1) $\times G \times N(G)$ and, as a result of the second, SO(3, 1) $\times G \times N(H)$ (where $N(G)$ and $N(H)$ are subgroups of G' that commute with G or H, respectively). Such a situation is realized, in particular, in the SO(32) and $E_8 \times E_8$ superstring versions.

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Translated by Julian B. Barbour