Present status of the quark bag model

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The present status of the bag model—the composite quark model of hadrons—is reviewed. The basic idea of the model is of quarks that move quasi-independently in a finite closed region of space. The Dubna formulation of the model is given together with versions of it (MIT, chiral model, and others). The most promising directions of development are identified. The parts played by symmetry and physical principles are emphasized, a critical review of mass formulas is given, and the relationship between the bag model and solitonlike models (in particular, the Skyrme model) is considered.

INTRODUCTION

Since 1964, composite quark models of hadrons have been developed at Dubna.[1-3] The first studies and those that followed them dealt basically with the formulation of the symmetry and dynamical principles of quark models. The introduction of quark color degrees of freedom,[1,4] the formulation of dynamical models of composite particles,[2,3] the proof of scaling of the asymptotic form factors in local field theory,[5] and the quark counting rules[6] are the essence of the most important results obtained during a decade of work by N. N. Bogolyubov and his pupils in the field of high-energy physics. This work had a direct influence on the creation in 1967 of the quark model of hadrons,[7,8] which subsequently became known as the bag model.

The basic postulate of the model is the dynamical assumption that quasi-independent relativistic constituent particles (quarks and gluons) move in a finite closed region of space. Any relativistic model that contains this proposition may be called a variant of the bag model.

The attractive side of the bag model is above all the fact that it gives a clear picture of hadrons and processes in which they participate. Over an almost 20-year development the model has not lost its significance. On the contrary, it can be readily refined and enriched by new experimental and theoretical data on the nature of strong interactions and still remains today an important tool for describing this domain of elementary-particle interactions. The model also owes its success to the simple practical calculations with it based on approximate solution of the model.

In this review, we wish to characterize the logical development of the model, its most important achievements, and also some problems that arise in the description of the properties of hadrons and the justification of the model and its structure. During the 20 years since its creation, a huge amount of material has been accumulated, and therefore we do not fear a considerable degree of overlap of our exposition with the well-known reviews on the bag model.[8-15] In particular, we shall devote more attention to the Dubna formulation of the model, to the discussion of the correspondence between the parameters of the model and QCD, to the description of the properties of nuclear matter, to questions that go beyond the framework of the static approximation, and to some other questions.

At the end of the forties and the beginning of the fifties the development of cosmic-ray physics and experimental techniques caused the number of detected elementary particles to begin to grow strongly. This work was responsible for the development of composite models of elementary particles, among which one of the first was the Fermi–Yang model.[16] A development of this model was the Markov–Sakata model.[17] This model was valuable in drawing the attention of physicists to the use of the group SU(3) in the theory of elementary particles, and it was thus a direct predecessor of quark models.

The prediction and subsequent discovery in the indicated mass range of the Ω^- particle made the symmetry approach the most attractive for hadron classification. The development and deepening of these ideas made it possible to abandon the assumption of true elementarity of the strongly interacting particles, the hadrons, and to introduce new structural objects—fractionally charged quarks.[18,19]

In this scheme, it is assumed that there exist several species (flavors) of quarks ($u, d, s,\dots$) carrying the following quantum numbers: electric charge ($+2/3$ or $-1/3$), spin projection ($\pm 1/2$) and isospin projection ($\pm 1/2$, 0), strangeness (± 1, 0), and baryon charge ($\pm 1/3$). Mesons are represented as quark–antiquark bound states ($q\bar{q}$), and baryons as bound states of three quarks (qqq). The corresponding combinations of quarks completely reproduce the dimensions of the lowest-lying observed mesons and baryons:

$$q\bar{q} \equiv 3 \times \bar{3} = 1 \oplus 8,$$

$$qqq \equiv 3 \times 3 \times 3 = 1 \oplus 8 \oplus 8 \oplus 10.$$

Here, the symbol $q \equiv 3$ means that the quark wave function transforms in accordance with the fundamental (three-dimensional) representation of the group $SU_f(3)$.

In the construction of the wave functions of baryons, a fundamental difficulty arose—three-quark states symmetric with respect to all variables (spatial, spin, unitary), $\Psi_{ijk} = \Psi_{P(ijk)}$, where $P(ijk)$ is any permutation, reproduced well the spectrum of ground states of the baryons but contradicted the Pauli principle, one of the fundamental principles of quantum theory.

To resolve this difficulty, Struminskiĭ suggested[4] in January 1965 that the quarks have an additional degree of freedom; this was subsequently called color. Antisymmetrization of the baryon wave function with respect to the color solved the problem:

$$\Psi_{ijk} = \frac{1}{\sqrt{6}}\,\varepsilon_{abc}\Psi_{ia,\,jb,\,kc}.$$

The final formulation of the new principle was completed in publications of Dubna physicists working under the guid-

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ance of N. N. Bogolyubov.[1,2] Independently and almost simultaneously, Han and Nambu[20] and Miyamoto[21] also concluded that quarks should be ascribed a new quantum number. Because the quarks have color degrees of freedom but there is the principle that only colorless states are observed, the formation of many-quark hadron states differs radically from the formation of nuclei from nucleons.

The Dubna studies[1-3] were among the first that posed the problem of developing the dynamical principles for the construction of quark models of hadrons. Relativistically invariant equations of motion for the constituent particles were taken as the basis of the approach.

Among the other ideas that preceded the appearance of the theory of quark bags, we should mention the hypothesis of heavy quarks. By means of this assumption, which simulates modern ideas about the complex vacuum structure of quantum chromodynamics, the nonobservation of quarks in the free state was explained. However, it was found to be difficult dynamically to combine the assumption of a sufficiently large quark mass with the values of the magnetic moments of the baryons composed from the Dirac magnetic moments $e/2M_q$ of the individual quarks. A mechanism for increasing the magnetic moment of a quark in a bound state was indicated in the pioneering work of Bogolyubov, Struminskiĭ, and Tavkhelidze.[1] They noted that the magnetic moment of a quark in a strong scalar field is determined by the energy of the bound state, which plays the part of an effective mass of the particle: $\mu_q \sim e/2\varepsilon_q \gg e/2M_q$.

This result does not depend on the chosen form of the scalar field. As was shown by Lipkin and Tavkhelidze,[22] an enhancement of the quark magnetic moment can be obtained only when a scalar potential is used and is completely absent for potentials with other transformation properties.[23]

The results mentioned above, obtained in the Dubna studies, provided the basis for the development of a new direction in elementary-particle physics that subsequently became known as the quark bag model. Essential here was the assumption of relativistic quasifree color $SU_f(3)$ quarks moving in a scalar confinement potential that prevents the escape of quarks to large distances. The "Dubna bag" model was finally formulated in Ref. 7 (see also Ref. 8), in which the far-reaching possibilities for applying the model to the calculation of the static properties of elementary particles were simultaneously demonstrated.

Before we pass from the prehistory to the main part of our exposition, we make two further introductory comments.

At the beginning of the seventies, the interest in quark models receded somewhat. This pessimistic stage was due to the absence of direct experimental proofs for the existence of quarks despite the intensive investigations made in this direction for almost ten years. A new stimulus to the development of quark models was obtained after the discovery in 1974 of the J/ψ particles, which were interpreted as bound states of quarks with a new quantum number C called charm.

To this period there belong the formulation of QCD and the proof of its asymptotically free behavior at short distances,[24,25] and also the formulation by the MIT group of their variant of the hadron model[26] (MIT bag).

At the first glance, it seems remarkable that for more than a decade there have coexisted a formulation of QCD, which lays claim to being the theory of the strong interactions, and a model of strong interactions. The explanation is that methods of calculation are restricted to perturbation theory and we have a poor understanding of the structure of QCD at large distances, at which the quark confinement forces are formed and the mechanism of spontaneous breaking of chiral symmetry works. A possibility to resolve these problems rigorously would lead to construction of the hadron spectrum in the framework of theory. The models are needed to fill the gap in our theoretical knowledge in the domain of interactions at large distances. Perturbation theory gives a description of strong-interaction physics at short distances in agreement with the experimental data.

There exists an important method of analyzing high-energy physics processes that does not rely directly on perturbation theory with respect to the coupling constant. This is the sum-rule method[27] developed in high-energy physics by A. A. Logunov and his collaborators. This method has been widely used in QCD,[28] and its field of application has also covered the static characteristics of the low-lying hadrons.[29] The most important thing in this approach is that by a comparison with experiment one can determine the values of the vacuum condensates that characterize the ground-state structure of QCD. This makes it possible to compare the results obtained by the QCD sum-rule method with the parameters employed in the bag model and must serve as a further stimulus in the refinement of the model.[30] (Here, we should also mention the existence of the lattice approach to the solution of the QCD equations[31]; this method, which does not rely on perturbation theory, is an alternative which has been actively developed in recent years.)

The second remark relates to the principles that determine the development of the bag model. In listing them, we characterize the structure of the present review.

The starting point of the evolution is the Dubna bag model. It contains two basic elements—the characteristic scale of confinement (the bag radius) of the relativistic quarks and the $SU_f(N) \otimes SU_s(2) \otimes SU_c(3)$ symmetry structure of the hadron systematics. The first direction is associated with the attempt to use the model to describe the complete set of experimental data in processes in which hadrons participate. The second direction is also the global problem of understanding by means of the bag model the microscopic nature of nuclear forces. The inclusion in the structure of the model of experimentally established symmetry principles is the essence of the third direction. These principles must include the gauge, relativistic, and chiral symmetries. The fourth direction is associated with technical problems that arise in the model itself (the finding of a solution of the model and calculation of its parameters). We have here questions concerning the justification of the model and the derivation of its relationship with QCD.

1. THE DUBNA BAG MODEL

In this part, we consider the symmetry and dynamical principles of the Dubna bag model. Among the former we include here the rules for constructing the wave functions of the hadrons with respect to their internal quantum numbers, and in the latter we include the choice of the equation of motion. In the review, we shall basically have in mind the case of the light u, d, and s quarks (unitary triplet).

The quarks are, in the first place, the carriers of the internal symmetry properties of the strong interactions. By hypothesis, their wave function transforms in accordance with the fundamental representation of the 3×3 group $SU_f(3)\times SU_c(3)$ and the spinor representation $SU_s(2)$ (the antiquarks q transform in accordance with the conjugate representations):

$$\left.\begin{array}{l} q_{Aia}=t_A\chi_i\lambda_a, \\ \bar{q}_{Aia}=q^{Aia}=t^A\chi^i\lambda^a, \end{array}\right\} \quad (1)$$

where χ_i is the spin function ($i=1,2$), t_A is the function in the unitary space ($A=1,2$ ($3=n_f$)), and λ_a is the color function ($a=1,2,3$). The quarks possess fractional electric ($e_u=+2/3$, $e_d=e_s=-1/3$, $e_{\bar{q}}=-e_q$) and baryon charges ($B_q=-B_{\bar{q}}=1/3$).

The hadron wave functions are formed as combinations of the quark wave functions (1) and are constructed in accordance with the laws for forming the elements of a representation of the symmetry group that characterizes the spin, unitary, and orbital degrees of freedom of the interacting quarks.[32] The resulting states have the values of the quantum numbers of the observed particles and, as noted in the Introduction, completely reproduce the dimensions of the ground states of the low-lying mesons and baryons: the nonets of pseudoscalar ($J^P=0^-$) and vector ($J^P=1^-$) mesons ($B=0$), and also the singlet ($J^P=1/2^-$), the octet ($J^P=1/2^+$), and the decuplet ($J^P=3/2^+$) of baryons ($B=1$).

For the proton, for example, representation theory determines the combination

$$P=\frac{1}{\sqrt{6}}\varepsilon_{abc}\frac{1}{\sqrt{2}}(\varphi_{M,S}\chi_{M,S}+\varphi_{M,A}\chi_{M,A})\Psi(r_1,r_2,r_3), \quad (2)$$

where Ψ is the symmetric spatial part of the wave function and χ and φ are spin and unitary wave functions of mixed symmetry[32]:

$$\varphi_{M,S}=\frac{1}{\sqrt{6}}[(ud+du)u-2uud];$$

$$\varphi_{M,A}=\frac{1}{\sqrt{2}}(ud-du)u, \text{ etc.}$$

The combination (2) has the quantum numbers $Q=+1$, $s=1/2$, $I=1/2$, $c=0$, which correspond to the quantum numbers of the proton. Applying the standard technique of lowering and raising operators for the spin and isospin, one can obtain from the proton wave function the remaining wave functions of the baryon octet ($J^P=1/2^+$).

It is important to emphasize[7] that the spin–isospin part of the hadron wave function is constructed in the same way as in the nonrelativistic model based on the group SU(6). It is merely necessary to bear in mind that in the bag model we are dealing with the total angular momenta of the quarks. As will be seen below, the replacement of the nonrelativistic Pauli spinors by Dirac spinors, proposed in Ref. 7, is important for studying the effects of the breaking of the SU(6) symmetry by the relativistic motion of the quarks.

The main dynamical postulate of the model of Ref. 7 was the assumption of a quasi-independent nature of the motion of relativistic quarks in a hadron. The single-particle method of describing the quarks makes it possible to treat in a unified manner not only systems consisting of two but also a larger number of quarks (baryons, many-quark states). This principle makes it possible to avoid the serious difficulties that arise when one attempts to treat many-particle systems on the basis of relativistically invariant equations. On the other hand, it subsequently became the fundamental principle in the construction of the spectroscopy of a huge number of hadronic states. The assumption of independence of the quarks was subsequently strengthened by the theoretically and experimentally discovered asymptotically free nature of the motion of the quarks at short distances. However, we do regard it as important to find the power-law corrections that give the deviations from this hypothesis.

As the equation of motion determining the single-particle quark wave function, $\Psi(\mathbf{x})\exp(-iEt)$, the relativistic Dirac equation with a scalar confinement potential was chosen:

$$\left.\begin{array}{l} [\gamma\mathbf{p}+M+V(r)]\Psi=\gamma_0E\Psi, \\ V(r)\xrightarrow[|r|\to\infty]{}\infty. \end{array}\right\} \quad (3)$$

In particular, for a confining potential in the form of a spherically symmetric rectangular well (bag of radius R)

$$V(r)=\begin{cases}\infty, & r>R;\\ 0, & r<R\end{cases} \quad (4)$$

we have as the solution for the nth mode of the quark wave function (s wave, massless quarks)

$$\Psi_n(\mathbf{x},t)=N\exp(-i\omega_nt)\begin{pmatrix} j_0(\varkappa_nr)\\ i(\sigma\hat{r})j_1(\varkappa_nr)\end{pmatrix}, \quad (5)$$

where the single-particle quark energies $\omega_n=\varkappa_n/R$ are determined from the boundary condition

$$j_0(\omega_nR)=j_1(\omega_nR). \quad (6)$$

In the Introduction, we have already noted how a scalar confinement potential, ensuring the mechanism of enhancement of the quark magnetic moment, is unique as regards its Lorentz structure. We give one further independent argument for this assertion.[33,34] If the confinement potential were to be transformed as the fourth component of a 4-vector, a Klein paradox would arise—the steeper the potential, the more free quark–antiquark pairs would be created in the exterior region of the hadron.

The Klein paradox does not occur for a scalar field. Physically, the following picture arises. A quark of fairly large mass M_q that is in a bound state acquires an effective mass $m=-V_0+M_q\ll M_q$, where V_0 is the depth of the well. The probability of finding the quark outside the well is proportional to $|\Psi|^2\xrightarrow[|r|\to\infty]{}\exp[-M(r-R)]$. In the limiting case $M\to\infty$, $m=\text{const}$, a regime of absolute confinement of the quark within the bag well occurs.

As already noted above, a fundamentally important feature of the model is the relativistic motion of the quarks. The use of the Dirac wave functions also makes it possible to clarify the nature of the relativistic corrections for the color, weak, and electromagnetic quark currents.

We consider gauge interactions introduced minimally:

$$i\partial_\mu\to i\partial_\mu+eA_\mu;$$
$$i\partial_\mu\to i\partial_\mu+G/2\gamma_5\tau^\pm l_\mu^\pm;$$
$$i\partial_\mu\to i\partial_\mu+gA_\mu^a\lambda^a.$$

The proton magnetic moment and the ratio g_A/g_V of the axial to the vector weak coupling constant take the form

$$\mu_p = \frac{e}{2\varepsilon_q}\langle\uparrow|\sigma_z + L_z|\uparrow\rangle;$$
$$g_A/g_V = -5/3\langle\uparrow|\,\sigma_z\,|\uparrow\rangle,$$

where ε_q is the binding energy of the quark in the proton, and $1/2\sigma_z$ and L_z denote the operators of the spin and orbital angular momenta, respectively. Taking the expectation value for the state in which the z component of the quark angular momentum is $+1/2$,

$$\langle\uparrow|J_z|\uparrow\rangle = \left\langle\uparrow\left|\frac{\sigma_z}{2} + L_z\right|\uparrow\right\rangle = \frac{1}{2},$$

we obtain

$$\mu_p = \frac{e}{2\varepsilon_q}(1-\delta);\quad g_A/g_V = -(5/3)(1-2\delta). \qquad (7)$$

In these expressions, the parameter δ [calculated with the wave functions (5)],

$$\delta = \langle\uparrow|L_z|\uparrow\rangle = \int d\mathbf{r}\Psi^*(r)L_z\Psi(r) = \frac{1}{6}\,\frac{2\varkappa_0-3}{\varkappa_0-1},$$

characterizes the magnitude of the relativistic corrections to the predictions of the naive relativistic $SU_{NR}(6)$ model. For massless quarks ($\varkappa_0 = 2.04$) $\delta = 0.17$, and for the ratio g_A/g_V this gives a correction of about 30%. The relativistic corrections qualitatively improve the results of the $SU_{NR}(6)$ model.

Subsequently, the idea of using Dirac wave functions that take into account the internal motions of the quarks in a hadron was employed[35] to study the breaking of the $SU_W(6)$ symmetry. These studies estimated, in particular, the decay characteristics of radially excited states of charmonium and other mesons with $L = 1$.

The only parameter of the model is the bag radius, which is fixed, for example, by the proton mass. An interesting prediction obtained in the Dubna model is the estimate for the mass of the Roper state ($M_R^{\exp} = 1470$ MeV). Assuming that this resonance is a bound state of a quark in a radially excited state and two quarks in the ground state, and that the radii are equal, $R_p = R_R$, we have

$$\frac{M_R}{M_p} = \frac{2\varkappa_0 + \varkappa_1}{3\varkappa_0} = 1.547;\quad \left(\frac{M_R}{M_p}\right)^{\exp} = 1.567,$$

where $\varkappa_1 = 5.396$ is the energy of the radial excitation of the quark. The agreement between the theoretical estimate of the mass and the experimental value was one of the most remarkable results of this model.

The discovery in the second half of the seventies of the J/ψ and Υ families of particles led to a mass of studies on quarkonium spectroscopy.[36] For the description of a two-particle system of heavy quarks, one mainly used a Schrödinger equation with a potential that confined the quarks at large distances and took into account exactly the QCD interaction at short distances.[37]

However, it can be shown that in composite particles in which the confinement forces are decisive the corrections in $(v/c)^2$ to the energy levels of the excited states may be very appreciable. In this respect, the results obtained on the basis of the Dubna bag model are more definite. The model was successfully applied to the description of the vector mesons ($\rho, J/\psi, D, \Upsilon$, and others).[33] The phenomenological potential used in the Dirac equation was

$$V(r) = -\alpha_s/r + br,$$

(where α_s is the running coupling constant), which is a combination of a linear scalar term responsible for the confinement of the quarks in the hadron and a Coulomb term corresponding to allowance for quark interaction by one-gluon exchange.

For the description of the quarkonium states, the hypothesis that the potential is independent of the quark flavor was advanced.[33,38] This resulted in satisfactory agreement between the calculations and the experimental data on the energy spectrum of the vector mesons ($\rho, J/\psi, D, \Upsilon$, and others) using only two potential parameters and the quark masses $m_u = m_d, m_s, m_c, m_b$.

To conclude this section, we must also mention that in the work of the Dubna group[1-3] the main principle underlying the construction of the composite models of elementary particles is the relativistically covariant approach (Dirac equation with a factorized potential,[2,3] the Logunov–Tavkhelidze relativistic quasipotential equation[39]). In this connection, the semirelativistic nature of the model of quasi-independent quarks was noted at that time, and the problem was posed of taking into account within this model the motion of the composite particle as a whole and the construction of relativistically covariant local currents of composite particles.[40]

Therefore, the logically next step in the development of the model, which began with the work of the MIT group,[26,41] was associated with the construction of relativistically invariant forms of the bag model. We shall be concerned with a class of such models in the following section.

The solution to the problem of constructing relativistically covariant local currents in such models proved to be a more subtle matter. It has been solved at the semiclassical level only recently.[42] A section in which we discuss corrections due to the recoil effect will be devoted to this question.

2. SYMMETRY PRINCIPLES IN THE BAG MODEL

A. Conservation laws

Symmetry principles play a fundamental role in the construction of physical theories. Fundamental in the physics of elementary particles is the assumption of relativistic invariance of the theory. A large body of experimental material indicates that in strong-interaction processes conservation laws hold with a high degree of accuracy for the baryon number and flavor, and chiral SU(3) symmetry and a number of other internal symmetries are manifested. It is therefore necessary to include these principles in the formulation of the bag model. This problem determined the further development of the model. From the phenomenological point of view, the establishment of each symmetry principle in the model leads to the introduction of either a new field or new parameters.

Symmetry principles can be most conveniently treated in the Lagrangian formulation of field theory. We consider a possible field-theoretical formulation of the model in which the quarks, as in the Dubna variant, can move freely in a finite region of space:

$$L = \int_{Bag} d^3x \overline{\Psi}(x)\,[i\hat{\partial} - m]\,\Psi(x). \qquad (8)$$

We verify the fulfillment of the conservation laws corresponding to global transformations of the space-time symmetry:

$$x \to x' = (\Lambda x + a); \quad \Psi \to \Psi' = S(\Lambda)\Psi(x') \qquad (9)$$

and the internal symmetries:

$$\left.\begin{array}{l} \Psi(x) \to \Psi'(x) = \exp(-i\theta I)\,\Psi(x); \\ \Psi_A(x) \to \Psi'_A(x) = \exp(-i\theta_A I)\,\Psi_A(x), \quad A = 1, \ldots, N_f; \\ \Psi_\alpha(x) \to \Psi'_\alpha(x) = \exp(-i\theta_a T^a)\,\Psi_\alpha(x), \quad a = 1, \ldots, N_f^2 - 1; \\ \Psi_\alpha(x) \to \Psi'_\alpha(x) = \exp(-i\theta_a T^a \gamma_5)\,\Psi_\alpha(x), \quad a = 1, \ldots, N_f^2 - 1. \end{array}\right\} \qquad (10)$$

Using the Lagrangian (8) and Noether's theorem, we write down the currents of the symmetries:

$$\left.\begin{array}{l} T^\mu_\nu = \frac{i}{2}\overline{\Psi}(x)\gamma^\mu\partial_\nu\Psi(x) - \frac{i}{2}\partial_\nu\overline{\Psi}(x)\gamma^\mu\Psi(x); \\ J^\mu_B = \frac{1}{N}\overline{\Psi}^A_\alpha(x)\gamma^\mu\Psi^A_\alpha(x); \\ (J_f)^\mu_A = \overline{\Psi}^A_\alpha(x)\gamma^\mu\Psi^A_\alpha(x), \quad A = 1, \ldots, N_f; \\ V^\mu_a = \overline{\Psi}^A_\alpha(x)\gamma^\mu(T^a)_{AB}\Psi^B_\alpha(x); \\ A^\mu_a = \overline{\Psi}^A_\alpha(x)\gamma^\mu\gamma_5(T^a)_{AB}\Psi^B_\alpha(x). \end{array}\right\} \qquad (11)$$

In the model with the Lagrangian (8), all currents have a discontinuity on the bag surface. Therefore, in order to restore the condition of continuity of the currents on the bag surface it is necessary to introduce compensating terms in some manner.

B. Relativistically invariant formulations of the bag model

The most popular formulation in which the problem of constructing a Poincaré-invariant model of hadrons is solved is the variant proposed by the MIT group,[26]

$$L = \int_{Bag} d^3x \{\overline{\Psi}(x)\,[i\gamma^\mu\partial_\mu - m]\,\Psi(x) - B\}, \qquad (12)$$

$$j^\mu|_{s-} \equiv \frac{i}{2}(\overline{\Psi}\gamma^\mu\partial_0\Psi - \partial_0\overline{\Psi}\gamma^\mu\Psi) = j^\mu|_{s+} \equiv Bn^\mu.$$

Here, n^μ is the 4-vector of the outer normal to the surface. Thus, it is here assumed that the potential energy of the field that ensures the quark confinement is proportional to the volume of the bag, i.e., the volume of the region of motion of the quarks. Using geometrical arguments, one can also assume that this energy is partly or fully due to surface-tension forces of the bag.[43,44]

In the model (12), one can choose relativistic equations of motion and boundary conditions for which the additional physical requirement of quark confinement is satisfied:

$$i\gamma^\mu n_\mu \Psi|_s = \Psi|_s. \qquad (13)$$

We have already described one way of deriving the equations. It reduces to considering the limit in which the mass of a quark outside the bag becomes infinitely large.[7] It was in this manner that the correct confinement boundary conditions were obtained in the work of the MIT group.

We present a different derivation,[45] which is more traditional for the derivation of equations of motion by means of variational principles. In Ref. 45, it was suggested that the equations of motion of the bag model should be sought by varying the action subject to the subsidiary relativistically invariant condition

$$G(\overline{\Psi}\Psi,\; n_\mu\overline{\Psi}\gamma^\mu\Psi) = 0, \qquad (14)$$

which is determined by the quark confinement requirement

$$n_\mu T^{\mu\nu}|_s \equiv -\frac{i}{2}[n_\mu\overline{\Psi}\gamma^\mu\partial^\nu\Psi - (\partial^\nu\overline{\Psi})\,n_\mu\gamma^\mu\Psi]|_s - Bn^\nu = 0. \qquad (15)$$

The variational problem (14)–(15) with a moving boundary arises[46]; it is assumed in this problem that the variation of the field depends on the variation of the boundary. As a result, we obtain the equations of the bag model:

$$\left.\begin{array}{ll} G = \overline{\Psi}\Psi; & \\ (i\hat{\partial} - m)\,\Psi = 0; & \text{in the bag} \\ \left\{\begin{array}{l} in^\mu\gamma_\mu\Psi = \pm\Psi; \\ \pm n_\mu\partial^\mu(\overline{\Psi}\Psi) = 2B. \end{array}\right. & \text{on the boundary of the bag} \end{array}\right\} \qquad (16)$$

As was shown in Ref. 47, to obtain the correct equations the usual variational principle can be applied to the action function

$$S = \int d^4x\,\{\theta_{bag}(x)\,\{\Psi(x)\,[i\gamma\partial - m]\Psi(x) - B\} + \Delta_{bag}(\overline{\Psi}\Psi)\}, \qquad (17)$$

where

$$\theta_{bag}(x) = \begin{cases} 1 & \text{in the bag,} \\ 0 & \text{outside the bag,} \end{cases}$$

and Δ_{bag} is the surface δ function of the bag.

Thus, in the MIT model the quarks move within the hadron freely. On the bag, boundary conditions are satisfied that ensure continuity of the quark density $\overline{\Psi}\Psi$ and absence of a quark energy flux through the bag surface. These conditions guarantee invariance of the classical bag theory[26] with respect to transformations of the Poincaré group.

The sharp boundary separating the region of free motion of the quarks from the outer region in which their motion is forbidden is a three-dimensional surface in a four-dimensional space. This surface is parametrized by infinitely many dynamical degrees of freedom, which, by virtue of the boundary conditions (17), are not independent but are functionals of the quark fields.

In the adiabatic approximation (to be discussed below), the MIT model with a sharp boundary reduces to a potential model with an infinitely deep spherically symmetric well. This approximation is a special case of the Dubna bag model.[17]

For numerous reasons, a sharp boundary of the bag is a physically unsatisfactory approximation. For example, the nucleon form factors become negative[48,49] at momentum transfers $q^2 \gtrsim 1/R^2$, and a too large Casimir effect arises in the bag.[50] Taken together, these facts indicate that the MIT model is invalid in processes in which the influence of the boundary on the results is not small.

To overcome these difficulties, we proposed[51] a relativistic model that generalizes the Dubna bag model.[7] The action proposed for the model was the functional

$$S_1 = \int_{t_1}^{t_2} dt \left\{ \int dr \left[\frac{i}{2} \overline{\Psi} \overleftrightarrow{\partial} \Psi - U(\{g\}, x) \overline{\Psi}\Psi \right] - \int_V d\mu(\mathbf{r}) B \right\}. \tag{18}$$

Here, $U(\{g\}, x)$ is a scalar function characterized by a set of parameters $\{g\}$ and possessing the property

$$U(\{g\}, x) \xrightarrow[t \text{ fixed}]{|\mathbf{r}| \to \infty} \infty. \tag{19}$$

The term $\int d\mu(\mathbf{r}) B$ is the potential energy needed to ensure energy stability of the system. The measure $\mu(\mathbf{r})$ is a given functional of $U(\{g\}, x)$ with the asymptotic behavior

$$\mu(U, x) \xrightarrow[t \text{ fixed}]{|\mathbf{r}| \to \infty} O\left(\frac{1}{r^{3+}}\right). \tag{20}$$

The parameters $\{g\}$ determine the geometry of the bag surface. The MIT variant of the model is obtained with the choice

$$\left.\begin{aligned} U(\{g\}, x) &= \begin{cases} m_q & \text{in the bag,} \\ \infty & \text{outside the bag,} \end{cases} \\ \mu(U, g) &= \begin{cases} 0 & U = \infty, \\ 1 & U = m < \infty. \end{cases} \end{aligned}\right\} \tag{21}$$

The dynamical parameters of the model determined by the action S_1 are the quark field Ψ and the set of parameters $\{g\}$, which determine the dynamics of the geometrical degrees of freedom of the bag. Assuming that the variations of the field Ψ are independent of the variation of the parameters $\{g\}$, we obtain the equations of motion

$$\left.\begin{aligned} &[i\hat{\partial} - U]\, \Psi(x) = 0, \\ &\overline{\Psi}\Psi \xrightarrow[t \text{ fixed}]{|\mathbf{r}| \to \infty} \infty \\ &\int dr \frac{\partial U}{\partial g_\alpha} \overline{\Psi}\Psi = -B \int dr \frac{\delta \mu}{\delta U} \frac{\partial U}{\partial g_\alpha}. \end{aligned}\right\} \tag{22}$$

In Ref. 51, a specific realization of the model (18)–(19) was considered, a solution of it was found in the adiabatic approximation,

$$\begin{aligned} U &= gx, \\ d\mu(x) &= \theta(gx - E)\, dx, \end{aligned} \tag{23}$$

and it was shown that "softening" of the confinement in this manner leads to a finite contribution of the sea of quarks to the nucleon structure functions. Investigation of the scattering characteristics at $q^2 \sim 1/R^2$ must give more detailed information about the form of the function U, i.e., about the dependence of the size of the confinement region on the internal state of the quarks within a hadron.

Some other relativistically invariant formulations of the bag model are known,[52–55] for example, a hadron model with surface tension[44] and the Salam–Strathdee model.[52,53] The topological (spatial) aspects of models of extended objects of bag type can be found in Ref. 54.

Thus, relativistically invariant bag models can be constructed in different ways. In particular, there is considerable uncertainty relating to the specification of the geometrical properties of the bag itself (the confinement properties). Moreover, the choice of a preferred model by comparison with experiment is limited by the serious computational difficulties that arise. Therefore in each model one can obtain only a number of estimates. In this sense, the most advanced variant of the bag model is the MIT bag.

In the standard formulations of the model, the electromagnetic current in the configuration space, for example, is represented by a sum of single-particle operators of the individual quarks. As was noted in the Introduction, this independence is approximate. Using the ideas of the quasipotential approach,[39] Gerasimov proposed a formulation of the model[56] in which the behavior of the quarks has an essential dependence on the quantum numbers and behavior of the neighboring quarks. In this theory, the quark velocities are not assumed to be small but the energy functional has a quasinonrelativistic form:

$$\begin{aligned} W = \Big\langle \Psi_{p=0} \Big| \sum_{i=1}^{N} \frac{1}{2\varepsilon_i} \Big\{ \varepsilon_i^2 + \mathbf{p}_i^2 + \Big[m_i + \frac{1}{2(N-1)} \sum_{j \neq i} V_s(r_{ij}) \Big]^2 \\ - \Big[\frac{1}{2(N-1)} \sum_{j \neq i} V_v(r_{ij}) \Big]^2 \Big\} + \frac{1}{N-1} \sum_{i<j} V_v(r_{ij}) \Big| \Psi_{p=0} \Big\rangle. \end{aligned} \tag{24}$$

The quasipotentials V_s and V_v of the universal $q\bar{q}$ interaction are determined in the framework of the bag model. Thus, an attempt is made here to use the achievements of the nonrelativistic models and the advantages of the relativistic treatment.

C. Color symmetry

Quarks are objects with color [$SU_c(3)$ triplets]. Their interaction with the Yang–Mills gauge field is introduced minimally: $i\partial_\mu \to i\partial_\mu + gA^a_\mu \lambda^a$. From the variational principle, we obtain the boundary condition

$$n_\mu F_{\mu\nu}|_s = 0, \tag{25}$$

which guarantees the absence of a color flux into the exterior region in accordance with the principle of color neutrality of the hadrons. A color singlet in the bag can be formed from purely gluonic states. Such particles are usually called glueballs.[57]

The color degrees of freedom play a key role in many-quark systems.[58] The wave function of such systems is represented in the form of an expansion with respect to a complete system of more elementary states, among which there exist components with "hidden color":

$$q^6 = (3q) \otimes (3q) = \begin{cases} 1 \oplus 1 \\ 8 \oplus 8' \end{cases}$$

i.e., in such a state the color of any of its parts is compensated ("covered") by the color of the remaining part. In recent years, such states have attracted interest in connection with the problem of taking into account the quark degrees of freedom in the description of the structure of nuclei and their interactions at short distances.[59]

Thus, the color symmetry is taken into account rigorously in the bag model, the boundary conditions ensure dynamical fulfillment of confinement, and the color degrees of freedom must be taken into account in constructing the hadron spectroscopy.

D. Chiral-invariant bag models

A new stimulus to the development of the bag model came after studies were made of the chiral symmetry in the model. The chiral symmetry of the strong interactions is one of the symmetries that has been established experimentally most accurately. It is assumed that in the limit of massless pions the axial current is conserved and there exists the algebra of vector and axial currents corresponding to the symmetry SU(2)×SU(2). The effects of the partial conservation of the axial current (PCAC) are attributed to the nonzero (but small) pion mass. The PCAC hypothesis has been confirmed by numerous experiments, and there are no facts which contradict it.[60]

All this shows the importance of considering the principle of chiral symmetry in the bag model. In Refs. 61 and 62 it was pointed out for the first time that the axial current is not conserved in the formulation of the MIT bag model. The reason for this is that the quark confinement in the model is realized by a scalar field (the bag is an object with a scalar nature). This can be interpreted as an effective quark mass that depends on the distance to the center of the bag. It is well known that mass terms explicitly break the symmetry between the left and right components of the spinors:

$$\Psi_L \to \exp(i\tau\gamma_5\alpha)\,\Psi_L;\quad \Psi_R \to \exp(i\tau\gamma_5\alpha)\,\Psi_R. \qquad (26)$$

With this transformation there is associated in the model (16) the axial current

$$A^\mu(x) = \overline{\Psi}(x)\,\gamma^\mu\gamma_5\frac{\tau}{2}\,\Psi(x)\,\theta_V,$$

whose divergence is proportional to the surface δ function:

$$\partial_\mu A^\mu(x) = -i\overline{\Psi}(x)\,\frac{\tau}{2}\,\gamma_5\Psi(x)\,\Delta_s.$$

To restore the chiral symmetry, it was proposed in Ref. 61 that on the bag surface the quark degrees of freedom should be coupled to the exterior pion field:

$$L_{\rm int} = -\frac{\eta}{2}\,\overline{\Psi}(x)\,(\sigma(x) + i\tau\pi\gamma_5)\,\Psi(x)\,\Delta_s,\quad \eta = (\sigma^2+\pi^2)^{-1/2}. \qquad (27)$$

Subsequently, interest in such hybrid models was revived in connection with attempts to explain internucleon forces on the basis of the bag model. In such models, the pion is treated as a Goldstone boson, and in the considered approximation its internal structure is completely ignored. Such a situation leaves one somewhat dissatisfied, since in the bag model the pion is regarded as a bound state of quarks moving in a cavity on an equal footing with other particles. It must be pointed out that up to the present day we still do not have an unambiguous dynamical treatment of the pion in the bag model, and nor do we have a complete understanding of the part played by the pion in nucleon structure.

Chiral symmetry can be realized in the bag model in infinitely many ways. The problem consists of finding solutions of such models and establishing the correspondence between these solutions and the description of the hadron world. A detailed exposition of different approaches based on the hybrid models can be found in the review of Ref. 14. Here, we can only dwell briefly on some aspects.

In 1979, Brown and Rho published a paper[63] with conclusions that revived interest in the problem. They considered a two-phase system in which the pion field exists only outside the bag. Chiral symmetry is realized nonlinearly:

$$\pi = \xi\,(1+\xi^2/f^2)^{-1/2};\quad \sigma = f\,(1+\xi^2/f^2)^{-1/2}.$$

Among the qualitative results obtained in Ref. 63, one of the most impressive was the reduction of the bag radius to 0.3 F. This generated the hope that the nucleon core might be interpreted as a quark bag.

However, the problems that arise in such a treatment are more serious. The interaction of the bag with the pions was found to be too strong, and the bag itself was unstable with respect to collapse.[64] Attempts were made to stabilize the bag by taking into account as well its coupling to the ω mesons.[65] In addition, the significant reduction of the bag radius was due to the application of nonperturbative calculations. This strongly distorted the quantitative successes in the description of hadrons obtained in the MIT model.

Later, Jaffe[66] and Musakhanov[67] concluded that the pion field must be described in the long-wave approximation if the MIT results are not to be strongly changed. To this end, a nonlinear realization was chosen in the form

$$\left.\begin{array}{l}\pi = f\hat{\Phi}\sin(|\Phi|/f),\ \text{where}\ \sigma^2+\pi^2=f^2,\\ \sigma = f\cos(|\Phi|/f),\quad |\Phi| = \sqrt{\Phi^2},\ \hat{\Phi}=\Phi/|\Phi|.\end{array}\right\}$$

The expansion parameter in this model, which determines the degree of interaction of the pion field with the bag,

$$\varepsilon = g_A/(8\pi f_\pi^2 R^2)$$

takes a value $\varepsilon \sim 0.2$ at $R \sim 1$ F and $\varepsilon \sim 2$ at $R \sim 0.3$ F. This model not only preserves the main MIT results; the pion contribution also makes a perturbation-theory contribution to the N–Δ mass splitting and, in addition, the numerical values of the magnetic moments of the proton, neutron, and Λ particle are improved.[68]

In the model of Ref. 66, the pions are coupled to the bag on its surface and do not penetrate into the interior. The contribution of these surface pions to the axial current increased by 1.5 times the value of the constant g_A compared with the satisfactory result of the bag model.[41] The calculations of the higher corrections in the pion interaction made this situation still worse.

To overcome this obstacle, it was proposed in Refs. 69–71 that pions should be considered in the complete region of space, including the bag interior. Then the chiral-invariant Lagrangian of the bag model takes the form

$$L_{CBM} = (i\overline{\Psi}\,\hat{\partial}\Psi - B)\,\theta_V - \frac{1}{2}\,\overline{\Psi}\Psi\Delta_s + \frac{1}{2}\,(\partial\mu\Phi)^2 - \frac{1}{2}\,m_\pi^2\Phi^2 - \frac{i}{2f}\,\overline{\Psi}\gamma_5\tau\Psi\Phi\Delta_s. \qquad (28)$$

Allowance is here already made for the fact that the treatment is given in the long-wave approximation, i.e., only the terms linear in Φ are retained in the interaction, and the breaking of the chiral symmetry is due to the nonzero mass of the pion. From the point of view of QCD, the penetration of the pion field into the bag can be treated as a condition of correlation within the bag of the quark–antiquark pairs with the pion quantum numbers.

The quark and pion fields are treated quantum mechanically. The bag surface is assumed to be a static classical sphere of radius R. The main technical difficulty in the model is to prove rapid convergence of the perturbation series in the quark–pion interaction and that, accordingly, the lowest orders of perturbation theory give the main contributions. One can, in fact, obtain on the one hand results for the static properties of the hadrons that do not differ too strongly from the standard results,[70] and, on the other, describe the πN and NN scattering processes.[71] Moreover, one can construct the total chiral-invariant Lagrangian in such a way that it enables one to obtain the basic results of current algebra for low-energy pion scattering and generalize the well-known Weinberg Lagrangian.[72]

In 1983, after the publication of Witten's paper,[73] interest in the Skyrme model[74] as a model of baryons was reanimated. The Skyrme Lagrangian corresponding to the chiral $SU(2)\times SU(2)$ group has the form

$$L=-\frac{1}{4}f_\pi^2 Tr\,(L_\mu L^\mu)-\frac{1}{4}\varepsilon^2 Tr\,\{[L_\mu,\ L_\nu]\}^2, \tag{29}$$

where $L_\mu = U^+\partial_\mu U$,

$$U=\exp\,\{f_\pi^{-1}\,[\sigma\,(x)+i\tau\pi\,(x)]\};\quad U^+U=1; \tag{30}$$

f_π is the pion decay constant, ε is a parameter of the model, σ is the scalar meson field, and π is the triplet Goldstone boson (pion) field. Witten showed, in particular, that the effective Lagrangian (29) arises in the leading order of the $1/N_c$ expansion of QCD. The Lagrangian (29) contains only effective Bose (meson) fields. The model has a soliton solution, the skyrmion

$$U\,(r)=\exp\,[i\tau\hat{r}\theta\,(r)], \tag{31}$$

whose topological charge is identified with the baryon charge. In Ref. 75, it was argued that in the limit of large N_c the mass of the Skyrme baryon is proportional to N_c, and in the tree approximation for the Lagrangian (29) the validity of the low-energy theorems of current algebra was established. Thus, two opposite approaches to the description of baryons began to coexist: the quark bag containing three (almost massless) quarks and the skyrmion, determined solely by effective boson fields and making no explicit appeal to quark degrees of freedom. Moreover, both approaches described the region of low-energy physics.

Very soon after this a phenomenon was discovered[76] that made it possible to combine the two approaches and raise the question of the reasons for this unexpected agreement. Under fairly general assumptions, it was found that in the bag model with boundary conditions on a sphere

$$n_\mu\gamma^\mu\Psi=\exp\,(i\hat{n}\tau\gamma_5\theta\,(R))\,\Psi, \tag{32}$$

which guarantee chiral invariance of the model, the baryon charge for $\theta\neq 0$ is not completely concentrated within the bag:

$$B_{\rm in}=1-\frac{1}{\pi}\left[\theta\,(R)-\frac{1}{2}\sin 2\theta\,(R)\right]. \tag{33}$$

There is a leakage of baryon charge from the bag. Polarization of the negative Dirac sea within the bag is considered as a cause of this. For $\theta\neq 0,\ \pi/2,\ \pi$ the boundary conditions violate the CP symmetry between the positive and negative energy spectra of the quarks. Because of this, the negative sea disappears on the transition $\theta\colon 0\to\pi$. This phenomenon is due to the anomaly in the baryon current associated with the boundary conditions for the time-dependent chiral angle θ. At the same time, the baryon current ceases to be conserved.[77–79]

It was then noted[80] that the lack of baryon charge can be identified with a skyrmion emerging through the bag. Indeed, the solution (31) of the model (29) carries the baryon charge

$$B_{\rm out}=\frac{1}{\pi}\left[\theta\,(R)-\frac{1}{2}\sin 2\theta\,(R)\right] \tag{34}$$

and, thus, the total charge is $B_{\rm tot}=B_{\rm in}+B_{\rm out}=1$.

On the other hand, the leakage of the baryon charge must obviously be accompanied by leakage of other charges (electric charge, magnetic moment, axial charge). The skyrmion must compensate the leakage, and in the limit of zero bag radius it must determine the total charges. If one assumes in addition that on variation of R, which separates the region of the bag and the Skyrme solution, the charge g_A, for example, remains fixed, then it can be shown that the other physical quantities (energy, magnetic moment, charge radius) vary weakly with R. Thus, R is not fixed in this picture.

Rho[81] assumed that R can be determined solely by a process in which a baryon participates. This could explain qualitatively why the pion degrees of freedom behave quite differently in different processes. They dominate in the description of the electrodecay of the deuteron, an axial transition, the magnetic form factor of ^{3}He, and elsewhere. However, in processes in which the asymptotically free behavior of the quarks is decisive, the pion degrees of freedom can be hardly noticed. In other words, in some processes the bag is small, in others large. In such a picture, the skyrmion and the bag represent two limiting QCD regimes, and the radius R is an arbitrary scale parameter that determines the physics of the phenomenon in an optimal manner. However, the nature of the regulation of R remains obscure.

3. MASS FORMULAS OF THE BAG MODEL

In the previous section, methods were developed for constructing different formulations of the bag model invariant with respect to the relativistic and internal symmetries. Neither at the quantum nor the classical levels is it possible to solve the corresponding equations of motion exactly. Therefore, the success of the model depends on the choice of the zeroth approximation of the theory.

It is generally assumed[7,26,82] that the static properties of the hadrons are basically determined by a system of noninteracting quarks moving in a bag, and that their interaction at short distances can be regarded as a weak "residual" interaction that can be taken into account by perturbative methods.

Even such a problem is unsolvable, owing to the essentially nonlinear coupling that determines the dynamics of the bag surface through the quark fields. (The classical equations of motion of the model have been solved only in two-dimensional space–time.[26]) It is necessary to introduce further simplifying but not readily controllable assumptions.[83,84] The most widely used approximation is that of a static cavity.[82] In the static approximation, a spherically symmetric cavity of radius R is the stable form of the bag surface.[82]

In this case, the equations of the bag model (16) take the form

$$(i\gamma\partial - m)\,\Psi = 0 \qquad (35)$$

within the sphere, while on its surface

$$-i\gamma n\Psi = \Psi; \qquad (36)$$

$$-n\partial\,(\overline{\Psi}\Psi) = 2B. \qquad (37)$$

Then, as is well known,[85] the general solution of the Dirac equation in a spherically symmetric field has the form

$$\Psi(\mathbf{r},\, t) = \sum_{nJlm} a_{nJlm}\Psi_{nJlm}(\mathbf{r})\exp(-i\varepsilon_{nl}t), \qquad (38)$$

where

$$\Psi_{nJlm}(\mathbf{r}) = \begin{pmatrix} f_l(\varepsilon_{nl}r)\,\Omega_{Jlm}(\mathbf{n}) \\ ig_{l'}(\varepsilon_{nl}r)\,\Omega_{Jl'm}(\mathbf{n}) \end{pmatrix};$$

$$l + l' = 2J; \quad \sigma n\Omega_{nlm} = -\Omega_{nl'm}.$$

The explicit form of the functions $f(r)$ and $g(r)$ is found by solving the radial Dirac equation with the boundary condition (36). If the left-hand side of the boundary condition (37) is to be independent of the time and angles, the quark field must be represented by one fixed mode $a_{\overline{nJlm}}$ with $J = \frac{1}{2}$.

We give the form of the quark wave function with total angular momentum $J = \frac{1}{2}$:

$$\Psi_s(\mathbf{r}) = \frac{N_s}{\sqrt{4\pi}}\begin{pmatrix} \sqrt{\dfrac{\omega_n + m}{\omega_n}}\, ij_0\left(\dfrac{\varkappa_n^s r}{R}\right)U_m \\ -\sqrt{\dfrac{\omega_n - m}{\omega_n}}\, j_1\left(\dfrac{\varkappa_n^s r}{R}\right)U_m(\sigma\hat{\mathbf{r}}) \end{pmatrix}, \qquad (39)$$

where $N_s^{-2} = R^3 j_0^2(\varkappa_n^s)\,[2\omega_n(\omega_n - 1/R) + m/R]/[\omega_n(\omega_n - m)]$, and

$$\Psi_p(\mathbf{r}) = \frac{N_p}{\sqrt{4\pi}}\begin{pmatrix} i\sqrt{\dfrac{\omega_n + m}{\omega_n}}\, j_1\left(\dfrac{\varkappa_n^p r}{R}\right)\sigma\hat{\mathbf{r}}U_m \\ \sqrt{\dfrac{\omega_n - m}{\omega_n}}\, j_0\left(\dfrac{\varkappa_n^p r}{R}\right)U_m \end{pmatrix}, \qquad (40)$$

where $N_p^{-2} = R^3 j_0^2(\varkappa_n^p)\,[2\omega_n(\omega_n + 1/R) + m/R\,]/[\omega_n(\omega_n - m)]$.

Here, U_m is the two-component spinor corresponding to the projection of the angular momentum onto the quantization axis z, and ω_n is the single-particle energy of a quark with mass m:

$$\omega_n = (m^2 + \varkappa_n^2/R^2)^{1/2}. \qquad (41)$$

The solutions Ψ_s and Ψ_p correspond to states with different parities (s: $k = -1$; p: $k = 1$). The boundary condition (36) determines the rule for quantization of the energy values:

$$j_1(\varkappa_n^s) = \sqrt{\frac{\omega_n^s + m}{\omega_n^s - m}}\, j_0(\varkappa_n^s); \qquad (42)$$

$$j_1(\varkappa_n^p) = -\sqrt{\frac{\omega_n^p - m}{\omega_n^p + m}}\, j_0(\varkappa_n^p). \qquad (43)$$

The numerical values of the solutions of these equations for $m = 0$ are $\varkappa_1^s = 2.04$, $\varkappa_1^p = 3{,}84$, $\varkappa_2^s = 5.40$, etc.

We assume that in the hadron ground state all the quarks are in the lowest s state. Then in the approximation of a static cavity, the energy of such a state takes the form

$$E_{\text{bag}} = \frac{N\varkappa_s}{R} + B\,\frac{4\pi}{3}\,R^3 + E_{\text{int}}(R). \qquad (44)$$

The condition of pressure balance (37) on the bag surface reduces to minimization of the total energy of the hadron with respect to the bag radius R (for fixed value of the "external" pressure B):

$$\partial E_{\text{bag}}/\partial R = 0. \qquad (45)$$

The relations (38)–(44) represent the solution of the equations of the MIT bag model in the approximation of a static cavity. The solution (38)–(40) and the conditions of quantization (42) and (43) of the Dirac modes are identical to the solutions (5) and (6) of the Dubna bag model.[7,8]

In the original model, without inclusion of additional types of interaction between the quarks, one could only estimate the centroid characteristics of the energy spectrum of the mesons and baryons and estimate the static properties of the hadrons. As was noted earlier,[7] such estimates give satisfactory results if one takes the bag radius to be $R \sim 1$ F($B^{1/4} \sim 140$ MeV). To describe the energy spectrum of all the hadrons and their static properties, it is necessary to introduce an interaction between the quarks, which generates a structure of the spectrum.

It is usually assumed that the splitting between the hadron multiplets is due to processes of one-gluon exchange between the quarks within the bag.

The quarks within the bag, as color-charged particles, are sources of color electric and magnetic fields. Within the bag, these fields satisfy the Maxwell equations

$$\begin{aligned} &\text{curl}\ \mathbf{B}_i^a = \mathbf{j}_i^a; \quad \text{div}\ \mathbf{B}_i^a = 0; \\ &\text{div}\ \mathbf{E}_i^a = j_i^a; \quad \text{curl}\ \mathbf{E}_i^a = 0, \end{aligned} \qquad (46)$$

where $j_\mu^{ai} = g\bar{\Psi}_i\gamma^\mu\lambda^a\Psi_i$ is the color current of the quarks, and on the bag surface the fields satisfy the boundary conditions

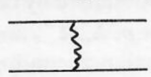
$$\hat{\mathbf{r}}\left(\sum_i \mathbf{E}_i^a\right) = 0, \quad \hat{\mathbf{r}}\times\left(\sum_i \mathbf{B}_i^a\right) = 0. \qquad (47)$$

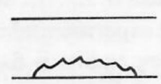
The boundary conditions guarantee that there is no escape of the color-electric field from the bag. The form of the boundary conditions enables us to regard the vacuum within the bag as a perfect color insulator (see below).

The contribution to the hadron energy due to the quark–quark one-gluon interaction is determined by the expression

$$\Delta E_g = \alpha_s\sum_{a=1}^{8}\frac{1}{2}\int dx\,[\mathbf{E}^a\mathbf{E}^a - \mathbf{B}^a\mathbf{B}^a] = \Delta E_g^E + \Delta E_g^M. \qquad (48)$$

In the considered order of perturbation theory, diagrams of two types contribute; they are an exchange diagram (a) and a self-energy diagram (b):


a


b

While the exchange diagrams can be calculated exactly, some assumptions are made about the self-energy diagrams. It is usually assumed that the diagrams (b) in some manner renormalize the quark masses, which, generally speaking, are parameters of the model. However, some of the self-energy contributions must be kept in order to guarantee fulfillment of the boundary conditions. Indeed, for a color-electric field having radial direction the boundary condition can be satisfied only for color-singlet states: $\Sigma_i \lambda_i^a = 0$. For states that are a combination of identical quarks, the contribution to the energy of the color-electric field is zero, $\Delta E^{E} = 0$, and for different quarks it does not exceed 5 MeV.

The color-magnetic interaction of the quarks has a more important influence on the formation of the spectrum of ground states. The magnetic field $\mathbf{B}_i^a$ of each quark satisfies the bag-model boundary condition (47), and therefore, to describe its contribution to the energy, one uses the formula

$$\Delta E_g^M = -\alpha_s \sum_{a=1}^{8} \sum_{i<j} \frac{1}{2} \int d\mathbf{x} \mathbf{B}_i^a \mathbf{B}_j^a$$
$$= -\frac{3\alpha_s}{R} \sum_{a=1}^{8} \sum_{i<j} (\sigma_i \lambda_i^a)(\sigma_j \lambda_j^a)$$
$$\times \overline{M}(m_i, m_j, R) = \frac{\eta \alpha_s}{R} \sum_{i<j} \overline{M}(m_i, m_j, R)(\sigma_i \sigma_j). \tag{49}$$

On the transition to the final row, we have taken here the expectation value for the color-singlet state ($\eta = 1$ for baryons and $\eta = 2$ for mesons); $\overline{M}(m_i, m_j, R)$ is the integral over the wave functions of the quarks.

The non-Abelian structure of the theory is manifested in the identity of the signs of the forces for both baryons and mesons. The expression (49) determines the contribution of the color-magnetic forces to the hadron mass.

The most important aspect of the application of the bag model in the static approximation to the description of the hadron spectrum is the choice of the mass formula (44). In the MIT version of the model, the mass formula was given by the expression

$$E(R) = \sum_{\text{flav}} \frac{n_i \omega(m_i, R)}{R} + \frac{4}{3}\pi B R^3 - \frac{Z_0}{R} + \Delta E_g. \tag{50}$$

The first term in (50) is the kinetic energy of the $N = \Sigma n_i$ quarks confined in the bag of radius R, and the ω are the eigenvalues of the solutions of the Dirac equation in the cavity. By hypothesis, the second term determines the (volume) energy needed to create the confining potential. The constant B characterizes the degree of disruption of the physical vacuum within the bag. In the papers of the MIT group, it was assumed that the quarks within the bag interact in accordance with perturbation theory with the strong coupling constant α_s and that the physical vacuum within the bag is completely destroyed. The term $-Z_0/R$ contains contributions identified with the energy of "zero-point vibrations," a correction due to the center-of-mass motion, and other possible contributions.

The quantities B, Z_0, α_s, and m_q were obtained by comparison with the experimental masses of the p, Δ, Ω^-, and ω particles. The bag radius is fixed by the balance condition (45). The constant B determines the energy scale of the hadron masses, α_s is responsible for obtaining the correct splitting between the hadron multiplets (the N–Δ splitting), the parameters N_q (the number of quarks) and Z_0 characterize the choice of scale of the meson and baryon masses, and, finally, the quark masses m_u, m_d, m_s determine the electromagnetic mass splitting in the isospin multiplets and the mass difference of the strange and nonstrange hadrons. The values obtained in Ref. 41 were

$$B^{1/4} = 145 \text{ MeV}, \quad Z_0 = 1.84, \quad \alpha_s = 2.2, \quad m_s = 279 \text{ MeV} \quad \text{for} \quad m_u = m_d = 0 \tag{51}$$

or

$$B^{1/4} = 125 \text{ MeV}, \quad Z_0 = 1.95, \quad \alpha_s = 3.0, \quad m_s = 353 \text{ MeV} \quad \text{for} \quad m_u = m_d = 108 \text{ MeV}.$$

Except for the pion, the predictions for the masses of the ground states of the other hadrons agree reasonably well with experiment. An analogous procedure, containing new model assumptions that made it possible to reduce the number of independent parameters of the model, was carried out in Ref. 86, which used the mass formula

$$M_{\text{Hadr}}^2 = E_{\text{bag}}^2 - \sum_i n_i \left(\frac{\varkappa_i}{R}\right)^2, \quad \frac{dM}{dR} = 0, \tag{52}$$

which makes it possible to take into account explicitly the contribution of the center-of-mass motion of the quarks to the bag energy.[87,88] The following values of the parameters were obtained by comparison with experiment:

$$B^{1/4} = 228 \text{ MeV}, \quad Z_0 = 2.33, \quad \alpha_s = 0.96, \quad m_s = 288 \text{ MeV}, \quad m_u = m_d = 0. \tag{53}$$

In Ref. 86, Z_0 behaved like a color-electric energy, α_s was assumed to be the running coupling constant, and B was calculated in accordance with the QCD-vacuum model of Ref. 89.

Great efforts were then made to calculate the parameters of the model and compare their values with the analogous parameters in other approaches. On the basis of these data, it was found that the use of the MIT mass formula (50) is not self-consistent.

In Ref. 90, which used the standard MIT assumptions, the formula (50) was generalized to the case of hadrons containing one heavy quark Q and light quarks:

$$E(R) = m_Q + E_{\text{kin}}^q + \frac{4\pi}{3} B R^3 + \frac{Z_0}{R} + \Delta E_g, \tag{54}$$

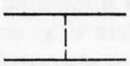
where ΔE_g is the color-magnetic correction to the energy. This case is important in that the theoretical uncertainty in the estimate of the parameters is much less than for light quarks alone. It was shown later in Ref. 91 to be impossible to describe by means of (50) the observed D^*–D and Σ_c–Λ_c mass splittings, and the need to introduce additional, spin-dependent terms was noted.

In addition, in all the calculations (for light quarks) the Δ–N splitting can be described only if a fairly large constant $\alpha_s \approx 1$–2 within the bag is chosen. Recent calculations of the quark self-energy have indicated the invalidity in this case of perturbation theory, since the first correction to the quark self-energy was found to be almost 50%.[92] Hopes of obtaining a large negative value of the parameter Z_0 by taking into account the geometrical Casimir effect were also not fulfilled.[93]

Another source indicating inconsistency of the mass formula (50) is comparison with the results obtained by the QCD sum-rule method.[28] The most important conclusion from applications of the sum-rule method is that the static characteristics are determined primarily by effects associated with the presence in the QCD vacuum of different condensates. It was found that the bag constant B, which characterizes the degree of disruption of the vacuum, is an order of magnitude smaller than the "depth" of the vacuum determined by the quantity $\langle (\alpha_s/\pi) G^a_{\mu\nu} G^a_{\mu\nu} \rangle$, which was estimated by the sum-rule method of Ref. 28. Thus, one of the basic hypotheses of the MIT version of the model—that the vacuum within the bag has a trivial structure—is not confirmed. Allowance for the interaction of the valence constituents with the condensate fields has the consequence that the parameters of the bag model acquire an essential dependence on the considered state. In all probability, such an interaction may have a dominant effect on the formation of the hadron spectroscopy.[30,94]

Among the other factors that influence the description of the static properties of the hadrons, we must mention the pion interactions[67,69] and the interaction with high-frequency vacuum fields, an example of which is instantons.[30,95]

The contributions of a chiral and instanton quark–quark interaction to the energy of the hadron states are analogous to the color-magnetic contribution. For example, the diagrams corresponding to one-pion exchange have the form


a

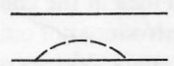

b

Diagram (a) leads to a spin–isospin structure of the interaction of the form

$$(\tau^a_i \sigma_i)\,(\tau^a_j \sigma_j).$$

For correct allowance of the pion field outside the baryon, one must also take into account the pion self-energy diagram (b). But summation over all possible intermediate quark states leads to divergent expressions.[96] Such a situation is analogous to the bag-model calculations of the contributions of the quark sea to the structure functions of deep inelastic scattering. It is a consequence of the inclusion in the sum of intermediate states with very high momentum, for which semiclassical ideas about confinement are unreliable. Therefore, one usually retains only the principal term in the sum, or, alternatively, introduces an effective cutoff of the contributions of the high-momentum intermediate states.[51,97]

In its totality, this procedure is very successful in describing the static properties of the ground states of the baryons and the decay widths of the excited baryon states N^* and Δ^*.[98] However, for many years there has existed the problem of describing the energy spectrum of excited baryon states. In Refs. 99 and 100, the first attempt was made in the MIT model to calculate the masses of negative-parity baryons in which two quarks are in the ground state and one quark is excited to the $P_{1/2^-}$ or $P_{3/2^-}$ state.[99] A fuller technique for calculating the spectrum of excited states with allowance for interactions due to one-gluon and one-pion exchanges was developed in Ref. 101. This yielded a significant improvement in the results of the bag model for the calculation of the spectrum of excited negative-parity N^* and Δ^* states. However, problems remain. For example, to obtain the required splitting of the $N^*(3/2^-)$ states unrealistically large values of the constant α_s are needed.

The calculation of the excited N^* and Δ^* states is also important from the point of view of the attempt to reproduce in the framework of the bag model the long-range nucleon–nucleon forces. We assume that in this situation the effects of the interaction of the quarks with the QCD vacuum must also play an important part.

Everything that we have considered above has been related to the problem of taking into account the residual interaction between the quarks. In Sec. 2 it was already noted that one of the basic assumptions of the bag model[7,26] is that of quasi-independent behavior of the quarks in the bag, and the need to go beyond this approximation was pointed out. We also mentioned there one of the attractive ways of taking into account the many-particle forces proposed by Gerasimov.[56]

This approach is based on the mass formula for a state $|\Psi\rangle$ of the quarks with momenta $\mathbf{p}_i$, energies ε_i, and masses m_i at the points r_i:

$$E = \Big\langle \Psi_{\mathbf{p}=0} \Big| \sum_{i=1}^{N} \frac{1}{2\varepsilon_i} \Big\{ \varepsilon_i^2 + \mathbf{p}_i^2 + \Big[m_i + \frac{1}{2(N-1)} \sum_{j\neq i} V_s(r_{ij}) \Big]^2 - \Big[\frac{1}{2(N-1)} \sum_{j\neq i} V_v(r_{ij}) \Big]^2 \Big\} + \frac{1}{N-1} \sum_{i<j} V_v(r_{ij}) \Big| \Psi_{\mathbf{p}=0} \Big\rangle \tag{55}$$

and on the variational principle

$$\partial E/\partial \alpha_l = 0; \quad \partial E/\partial x_i = 0 \quad (x_i = \varepsilon_i/E), \tag{56}$$

where the varied parameters are the single-particle quark energies ε_i and the set of potential parameters $\{\alpha_l\}$, which determine the trial wave function of the hadron state with total momentum $\mathbf{p} = 0$:

$$\Psi_{\mathbf{p}=0}\ (\boldsymbol{\rho}_1, \ldots, \boldsymbol{\rho}_{N-1};\ \{\alpha\}), \tag{57}$$

where $\boldsymbol{\rho}_N = [1/\sqrt{n(n+1)}]\,(\Sigma_{i=1}^n \mathbf{r}_i - n\mathbf{r}_{N+1})$ are Jacobi coordinates ($n = 1,2,\ldots,N-1$).

In the derivation of (55), the following assumptions are made. There exists a universal interaction $V_0(r)$ (r is the relative interquark distance) of the quark–antiquark system ($q\bar{q}$) described by the wave function

$$\Psi_{q\bar{q}} = N \exp\left(-\tfrac{1}{2}\gamma r\right). \tag{58}$$

The interaction potential of the N quarks is found in terms of V_0:

$$V(1,\ 2,\ \ldots,\ N) = \frac{1}{N-1} \sum_{i<j} V_0(r_{ij}), \tag{59}$$

where r_{ij} is the distance between quarks i and j. Such a choice involves regarding the interaction of quark i with its neighbors as a universal $q\bar{q}$ interaction of the ith quark q with the "antiquark" q^{N-1}. It is assumed that the trial wave function of the state has the form

$$\Psi_{q^N;\ N\geqslant 3} = N \exp\left[-\tfrac{1}{2}\alpha^2(\rho_1^2 + \ldots + \rho_{N-1}^2)\right]. \tag{60}$$

In the approach of Ref. 56, the spin interactions are taken into account by perturbation theory in such a way that the results of minimizing E with the wave functions (58) and (60) are compared with linear combinations of the masses of the physical particles, from which the contributions of the spin–spin interaction are eliminated approximately.

The variational principle $\partial E/\partial x_i = 0$ postulates a method for taking the particles off the "mass shell" when the interaction is introduced. As can be seen from (55), the interaction is introduced as a (quasi)potential that depends parametrically on the energy of the particles.

To determine the potential of the universal $q\bar{q}$ interaction, one uses the expression for the energy of two color charges $q\bar{q}$ at rest obtained in the framework of the bag model[102]:

$$V_s = ar; \quad V_v = ar - \varkappa/r, \tag{61}$$

where V_s and V_v are the scalar and vector parts of the confinement potential that arises in the model.

With the parameters $a = 0.055$ GeV, $m_{u,d} = 0$, $m_s = 0.33$ GeV, $m_c = 1.65$ GeV, $m_b = 5.1$ GeV the masses of the ground states of the hadrons and the radiative decay widths of the vector mesons are well reproduced. The magnetic moments of the baryons have the same form as in the nonrelativistic SU(6) model—the expressions for the quark moments $e_i/2\varepsilon_i$ contain the energy of the quarks, which depends on the state of the hadron and the state in which the quark neighbors are. We have specially devoted attention to the approach of Ref. 56 in order to emphasize the promise of the ideas developed in this paper.

4. SOLITONLIKE BAG MODELS

The problem of justifying the bag model can be approached from several points of view. We may include here the problem of the formal consistency of the model, the problem of reconciling it with other well-known approaches to the description of elementary particles (nonrelativistic models, quasipotential approach, Reggeistics, behavior of the structure functions), and attempts to calculate the parameters of the model from QCD (QCD sum rules and, in particular, the instanton approach). In this section, we shall dwell on connections between the bag model and local quantum field theory and consider the principle of dynamical breaking of chiral invariance.

In the MIT formulation,[26] the bag model is a theory of nonlocal objects (bags). The dimension of these objects is equal to the dimension of the space of the considered theory. In a two-dimensional theory (x,t), the bag is a relativistic string. This circumstance makes possible the furthest advance in the formal solution of the bag model in the two-dimensional model (Refs. 26, 42, and 103–105), since the theory of a relativistic string has been fairly well developed.[106] But even in this case, the problem of ordering the operators that arises on quantization presents difficult obstacles if one attempts to extract complete information about the theory from the resulting solutions. Only in the finite-mode (effectively, single-mode) approximation[104] can one overcome the resulting problems. The main result that can be extracted in this way is the construction of a translationally invariant approximate solution of the bag model. We shall return to this question in an appropriate place.

In theories with higher dimension, in particular, in a four-dimensional theory, one cannot even construct the classical bag theory. Therefore, in practice one uses two assumptions that drastically impoverish the dynamics—that the bag is static and that it be described classically.

Since the middle of the seventies, there have been intensive investigations of field-theory models with nontrivial particlelike solutions. The methods developed for quantization in the neighborhood of such solutions[107] have made it possible to give a real meaning to classical extended objects. Usually, the energy spectrum of the model's excitations consists of several discrete levels and a band of the continuous spectrum. The first excitations correspond to bound states, and the continuum excitations correspond to processes in which excited quanta are scattered by the soliton. As a rule, the soliton is regarded as a baryon and the excitation quanta as mesons.[108] Many-soliton solutions make it possible to consider baryon scattering processes.

Whereas in the study of the two-dimensional bag model knowledge of solutions of the relativistic string model was helpful, for the four-dimensional model, for which there is no such information, it would be natural to use the methods of local field theory. This approach generated an entire class of so-called solitonlike models,[109–114] whose (classical) solutions reproduce qualitatively the structure of the solutions of the bag models. Indeed, in the approximation in which the dynamics of the bag surface is given and is classical (in the approximation of a static cavity, for example), the bag can be regarded as an extended classical object, quantization around which leads to an infinite discrete spectrum of excited states—Dirac quanta in the cavity.

The general idea in the construction of solitonlike Lagrangians is to introduce not only a spinor field $\Psi(x)$ corresponding to the quark degrees of freedom but also an effective (scalar) field $\sigma(x)$ which determines the effective quark mass and whose self-interaction forms an extended object—a bag. The bag is a spatially inhomogeneous vacuum state of the theory possessing its own momentum P and energy E. The quarks are determined as single-particle fermion excitations above such a vacuum. Note that in contrast to the MIT model the field σ is an independent variable.

We consider the Lagrangian of the Friedberg–Lee model,[112] which is the most advanced model of this type:

$$L = \frac{i}{2}\bar{\Psi}(\hat{\nabla} - m)\Psi - \frac{i}{2}g\bar{\Psi}\sigma\Psi - U(\sigma) + \frac{1}{2}(\nabla_\mu\sigma)^2 - \frac{1}{4}\left(1 - \frac{\sigma}{\sigma_0}\right)(G_{\mu\nu})^2, \tag{62}$$

where

$$U(\sigma) = \frac{a}{4!}\sigma^4 + \frac{b}{3!}\sigma^3 + \frac{c}{2}\sigma^2 + B.$$

It is assumed that the quarks effectively interact with the gluon field $G_{\mu\nu}$ in accordance with perturbation theory. The Lagrangian (62) may, in general, also include effective pseudoscalar Higgs fields as well as other fields; in addition, to construct a renormalized perturbation theory it is necessary to introduce counterterms. (The requirement of renormalizability of the theory leads to a choice of the self-interaction $U(\sigma)$ in the form of a polynomial of fourth degree.) But to describe the qualitative picture in the semiclassical approximation, these modifications are unimportant.

From the point of view of modern ideas about QCD, any model of hadrons is determined by the information about the vacuum structure contained in it. The nontriviality of the QCD vacuum structure is a manifestation of the self-interaction of the gluon field and is important at scales characteristic of the strong interactions.

In the Friedberg–Lee model, these QCD features are, by hypothesis, approximated by the effective scalar field σ

with self-interaction $U(\sigma)$ chosen in the form of a function of σ having one local minimum at $\sigma = 0$, $U(0) = B$ and an absolute minimum at $\sigma(x,t) = \sigma_v$, $U(\sigma_v) = 0$. Such behavior corresponds to a picture of two phases separated by a sharp closed boundary (a bag); in the exterior region we have the vacuum σ_v, which is transparent for the quarks and gluons, while in the bag we have the hadronic phase with $\sigma = 0$. Quarks exist in the vacuum "well." The constant B is determined from the condition $U(\sigma_v) = 0$, and σ_v is determined through the parameters of the self-interaction $U(\sigma)$. Confinement in the model is ensured by fulfillment of the boundary conditions

$$\sigma_0 = \begin{cases} -m/g, & r \ll R; \\ \sigma_v \to \infty, & r \gg R, \end{cases} \qquad (63)$$

for which the quark mass tends to infinity, $M_q \to \infty$, in the exterior region. The limit of the theory in which the energy and thickness of the transition zone between the vacua tend to zero and all the masses except those of the quarks within the bag are assumed to tend to infinity[110] corresponds to the bag model (Fig. 1).

It is not possible to construct the quantum theory of the model (62), and therefore one uses semiclassical perturbation theory, which is based on consideration of the classical equations of motion, the solution being a nontopological soliton.[112] The topological charge of such a field is zero; in other words, $\sigma(x) \to_{|x| \to \infty} \sigma_v$. Stability of the solution is ensured by the conservation of some Noether charges (electric charge in a charged scalar field, fermion number in a spinor theory) and is directly related to the existence of valence (quark) degrees of freedom. This is a consequence of Derrick's well-known theorem.[140] In the classical equations, the spinor fields are c-number Grassmann functions.

It was shown in Ref. 112 that for a definite choice of the model's parameters the soliton solution is energetically more advantageous than the plane-wave solutions and, therefore, is stable with respect to decay.

Functional-integration methods and equations that take into account the vacuum-polarization contributions are considered in Ref. 115.

In the classical limit, the self-consistent solution of the coupled system of equations for the Dirac and scalar fields is a bound state of strongly interacting fermions ($\lambda \gg 1$).

The wave functions $\Psi(x)$ of the quark states and the field σ in the mean-field approximation satisfy the equations

$$[-i\alpha\nabla + \beta m + \beta g\sigma(\mathbf{r})]\,\Psi_k(\mathbf{r}) = \varepsilon_k \Psi_k(\mathbf{r});$$

$$-\nabla^2\sigma(\mathbf{r}) + \frac{\partial U(\sigma)}{\partial\sigma} = -g\sum \overline{\Psi}_k(\mathbf{r})\,\Psi_k(\mathbf{r}) \qquad (64)$$

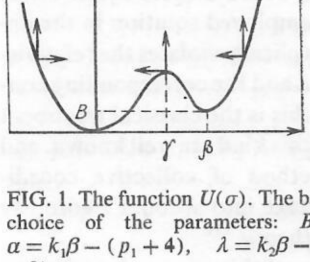


FIG. 1. The function $U(\sigma)$. The bag model corresponds to the following choice of the parameters: B fixed, $\beta \to 0$, $\frac{1}{2} < \gamma/\beta < 1$ fixed, $\alpha = k_1\beta - (p_1 + 4)$, $\lambda = k_2\beta - (p_2 + 2)$, $(k_1 > 0, k_2 > 0, 0 < p_2 < p_1 < 2)$.

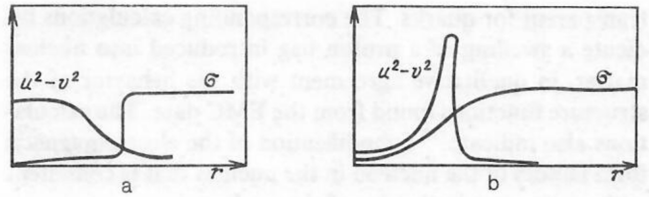


FIG. 2. Behavior of the functions $\sigma(r)$ and $(u^2 - v^2)(r)$: a) MIT variant ($a \gg 1, g \sim 1$); b) SLAC variant ($a \sim 1, g \gg 1$).

with the normalization condition

$$\int \Psi_k^{+}(\mathbf{r})\,\Psi_k(\mathbf{r})\,d\mathbf{r} = 1,$$

where α and β are the Dirac matrices, and g is the constant of the coupling of the quarks to the field σ. The sum in (64) is over all the populated states minus the contribution of the Dirac sea of levels.

Compared with the static-cavity approximation in the bag model, the solitonlike models have a number of particular features.

First of all, the "degree" of confinement is determined in such a model by a certain limit of the self-interaction function $U(\sigma)$ for a specially chosen limiting behavior of the four parameters of this function. In fact, depending on the choice of the parameters, one can realize variants of the bag model: the MIT bag (quark in the volume of the bag) or SLAC model (a quark that "lives" on the surface of the bag) (Fig. 2).

In Ref. 116, in which great attention was devoted to the development of numerical methods, solutions were obtained to the classical equations of the Friedberg–Lee model for a wide range of variation of the parameters of the function $U(\sigma)$. (In the last paper, solutions for excited states of the model were studied.)

One can, however, show that the qualitative behavior of these solutions can be obtained from solution of the Dirac equation with scalar confining potentials which depend on parameters that have a meaning analogous to the parameters in the Friedberg–Lee model.

In the studies cited above, it was shown that one cannot reconcile quantitatively the static characteristics of the nucleon with the parameters of the model, and it was concluded that it is necessary to introduce new effective fields and take into account some further effects.

In addition, the Friedberg–Lee model admits in principle many-soliton solutions. This fact makes it possible to study in the framework of this model processes in which hadrons participate, namely, the collision dynamics of nonstatic bags, the normal modes of bag oscillations leading to decay, and a meson field $q\bar{q}$ surrounding the hadrons as a representation of the oscillations of the effective fields (σ,π).

From this (physical) point of view it was interesting to study in the model the change in the properties of the nucleon in a nucleus.[117] If in a nucleus one distinguishes a certain arbitrary nucleon and for simplicity assumes that all the nucleons surrounding it form homogeneous nuclear matter, then in the model language such a situation reduces to replacement of the external physical vacuum by the external nuclear medium. Moreover, the latter is, in general, semitransparent for quarks. The corresponding calculations indicate a swelling of a proton bag introduced into nuclear matter, in qualitative agreement with the behavior of the structure functions found from the EMC data. The calculations also indicate[118] a modification of the electromagnetic form factors of the nucleon in the nucleus that is consistent with an increase in the size of the nucleon.

For the description of the EMC effect, it is not only the changes in the properties of the nucleon in the nuclear medium that are important but also the change in the properties of the medium itself with increasing mass number.[59,119] In particular, nuclear matter need not be simply "pure" nucleon matter but may be a homogeneous mixture (heterophase) of different states of quark matter characterized by different numbers of quarks in one bag: $n = 0, 3, 6, 9, \ldots$ (the phase with $n = 0$ takes into account the nontrivial nature of the QCD vacuum). In Ref. 120, the possible existence of such phase states of bags was estimated approximately. It would be interesting now to combine the dynamical approach of the solitonlike bag model with study of the static properties of nuclear matter.

The following clear ideas of a color-insulating vacuum provide an explanation of the confinement of the color gluon fields in the Friedberg–Lee model. The color fields $A_\mu^a(x)$ are introduced as in QCD, except that they interact with the soliton field through a permittivity $k(\sigma)$ chosen such that $k(0) = 1$ and $k(\sigma_v) = 0$. For example, $k(\sigma) = |1 - \sigma/\sigma_v|$.

Color confinement arises from these general requirements on k. In accordance with Gauss's law,

$$\operatorname{div} \mathbf{D}^a = J_0^a.$$

If the total color charge does not vanish within a certain finite cavity, then the D field behaves in accordance with the law $r^{-(2-\varepsilon)}$ as $r \to \infty$ and the color-electric energy in the medium behaves as

$$\frac{1}{2} \int dr \frac{[D(r)]^2}{k(r)} \to \infty, \quad \text{since} \quad k(r) \underset{r \to \infty}{\longrightarrow} 0.$$

Within the bag, where $k \simeq 1$, the gluon field is practically free. In such a picture, the use in the calculations of a series expansion in α_s acquires a qualitative explanation. Estimation of the higher corrections is a technical problem. However, it is known from the calculations of Ref. 121 that the corrections arising from allowance for the quantum fluctuations of the quark fields due to their interaction with the gluons are small.

Treating the gluon field by perturbation theory, one can develop a technique of Green's functions in the spherical cavity and solve the linearized equations, which have the same form as Maxwell's equations for a field in a medium. An estimate of the N–Δ splitting enables one to fix the constant α_s. As in the MIT model, it is approximately equal to 2, which rules out the use of perturbation theory and forces one to assume that there are more important contributions to the N–Δ splitting.[30]

The Friedberg–Lee model has proved to be convenient for developing methods to take into account QCD perturbation theory in a cavity,[121,122] for study of the problem of the center-of-mass motion,[42,123,124] and for the study of chiral invariance.[114,116] In Ref. 114, the ideas developed in Refs. 125–128 (pion–Goldstone mode, etc.) were used to recover chiral invariance of the theory.

5. RELATIVISTIC EFFECTS IN THE BAG MODEL

Throughout the preceding exposition, we have frequently emphasized the importance of relativistic effects in the predictions of the bag model. In this section, we examine especially ways of taking into account this most important principle of the model. All the relativistic aspects are due to the fact that quarks are spin-$\frac{1}{2}$ particles whose dynamics is described by the relativistic Dirac equation. We list these aspects.

The transition from the Pauli spinors used in nonrelativistic models to Dirac spinors[7] made it possible to take into account the relativistic spin–orbit coupling. The corrections due to the contribution of the lower component of the wave function to the static characteristics of the hadrons are not small and improve qualitatively the results of the nonrelativistic $SU_{NR}(6)$ theory[7,35] (see Sec. 1). Another important point here is the use of the scalar field to produce the quark confinement forces.

In addition, it is well known that under relativistic transformations the components of the Dirac field are transformed. It is essential to have the possibility of describing the motions of the hadron bag when one is calculating the form factors and the structure functions of scattering processes. It is important to emphasize that the static quantities too, for example, the magnetic moments, characterize the reaction of the particle to its motion (this point frequently gives rise to confusion). Therefore, the calculations should be made not in the rest system with $\mathbf{p} = 0$ but as the limit when $\mathbf{p} \to 0$. In this section, we shall examine in more detail the calculation of the effects due to the transformation properties of the quarks.

Another problem is associated with the composite nature of the hadron. This problem arises because the quarks move in a closed space bounded by the bag surface. The center of mass of the quarks is in motion. It makes a fictitious contribution to the energy and to the other parameters and is particularly important for light quarks.[87,88] We have already considered an alternative approach to the solution of this problem.[56]

Finally, the effects due to the influence of the Dirac sea of quarks are an important manifestation of relativity in the model. We have earlier noted the contribution of the sea when calculating the baryon number in the bag model.[76] Even more important is the part played by the Dirac sea in the recently developed approach to the low-energy physics.[129]

Progress in the solution of each of these problems resulted in a significant improvement in the predictive power of the bag model. Below, we consider in detail a method for taking into account the effects of recoil and the center-of-mass motion of the quarks.[42]

The problem of taking into account these effects arises from the fact that the usually employed solution in the approximation of a static cavity explicitly violates the relativistic and translational symmetries and the corresponding conservation laws. The reason for this is the classical treatment of the bag surface. Problems of this kind are well known, and there exists Bogolyubov's method of collective coordinates,[130] which enables one to take into account rigorously the original symmetries of the theory.[131]

For a systematic exposition of this approach to the bag model, it is convenient to use the formulation of the model in the Friedberg–Lee variant (62), whose Lagrangian

$$L=g^{-1}\int d^4x\left\{\frac{1}{2}\dot{\sigma}^2-\frac{1}{2}(\nabla\sigma)^2-U(\sigma)\right.$$
$$\left.-\bar{\Psi}(-i\gamma^{\mu}\partial_{\mu}+h\sigma)\Psi\right\}$$

was determined in Sec. 4. An advantage of the local formulation is, in particular, that the fields Ψ and σ are determined in the complete region of space–time and satisfy the (usual) canonical commutation relations

$$\left.\begin{array}{l}[\sigma(\mathbf{x},t)\dot{\sigma}(\mathbf{y},t)]=ig\delta(\mathbf{x}-\mathbf{y});\\ \{\Psi(\mathbf{x},t),\ \Psi^{+}(\mathbf{y},t)\}=g\delta(\mathbf{x}-\mathbf{y}),\end{array}\right\} \quad (65)$$

where g is the parameter of the semiclassical expansion of the theory.

Applied to the considered case of the relativistic symmetries, the idea of Bogolyubov's method is to separate from the dynamical variables of the theory those that describe the motion of the extended object (bag) as a whole and relate them to the symmetry parameters (collective coordinates). Collective coordinates are introduced only for those symmetry transformations that are broken by the classical solution.

We shall assume for simplicity that the static approximation of the bag model violates only the translational invariance and that the space of the collective coordinates is spatially homogeneous. The problem can be examined more consistently by using the method developed in Ref. 42.

As a result, in the semiclassical approximation one can construct the Heisenberg operator corresponding to a quark moving in the bag[42]:

$$\left.\begin{array}{l}\Psi(\mathbf{x},t)=\int d\mathbf{f}\,d\omega\exp[i(\omega\hat{\tau}(\mathbf{x},t)\\ -\mathbf{f}\hat{\xi}(\mathbf{x},t))]\,\Phi^{j}(\omega,\mathbf{f})\,D_{ij}(\theta(\mathbf{p}));\\ \Phi(\omega,\mathbf{f})=\int\frac{d\beta\,d\alpha}{(2\pi)^4}\exp[-i(\beta\omega-\mathbf{f}\alpha)]\,Q(\alpha,\beta);\\ Q(\alpha,\beta)=\sum\limits_{n=-\infty}^{\infty}b_m\Psi_m(\alpha)\exp(-ik_m\beta);\\ \{b_m,\ b_n^{+}\}=\delta_{mn},\end{array}\right\} \quad (66)$$

where Ψ_m and k_m are the wave functions (38)–(40) and energies (42) and (43) of the quarks in the static bag, $D(\theta(\mathbf{p}))$ is the matrix that is the irreducible spinor component of the representation of the Poincaré group, and the form of the operators $\hat{\xi}_{\alpha}(x)$,

$$\left.\begin{array}{l}\hat{\tau}(\mathbf{x},t)=Ht-P_z z-D;\\ \hat{\xi}(\mathbf{x},t)=Hz-P_z t-L_z;\\ \hat{\eta}_{\perp}=\mathbf{x}_{\perp}-\mathbf{q}_{\perp},\end{array}\right\} \quad (67)$$

which commute with each other and depend on the generators of the Poincaré group [L is a boost in the direction z, and D is the operator of longitudinal (t,z) scale transformations], guarantees the fulfillment for the spinor field $\Psi(x)$ of the correct transformation properties with respect to the relativistic transformations.

The bag state $|\Omega_p\rangle$ is a vector with respect to relativistic transformations,

$$P_{\mu}|\Omega_p\rangle=p_{\mu}|\Omega_p\rangle,\quad P_{\mu}^2=M^2, \quad (68)$$

and is the vacuum with respect to the second-quantized field Ψ:

$$b_n\mid\Omega_p\rangle=0. \quad (69)$$

We make two remarks concerning the choice of the variables. First, they are sufficient for representation of a spherically symmetric bag in the Breit system when the particle moves along a straight line during the entire process. For arbitrary motion of such a bag (or for a deformed bag), it is necessary to use the variables constructed in Ref. 132.

Second, the composite nature of the hadron means that each constituent particle possesses independent energy–momentum eigenvalues. Allowance for this fact leads to a further generalization of the coordinates and to a consistent solution of the problem of the center-of-mass motion.[132] In both of these cases, in contrast to (66), problems arise with regard to the commutativity of the operators ξ_{α}. As in the case of the canonical transformation (66), one can satisfy the canonical relations (65) by systematically expanding the fields in a series in $g(\hbar)$,[133] and in this case too the problem of ordering the noncommuting operators ξ_a can be solved by additionally expanding the fields in series in v/c.[134]

The procedure outlined here makes it possible to construct local currents of the composite particle and calculate the various matrix elements of them. Relativistic invariance is guaranteed.

We give the results of calculations, by means of this method, of the mean-square electromagnetic radius and the magnetic moment of the proton[49]:

$$\langle r^2\rangle=\langle r^2\rangle^{(st)};\quad \mu_p=\frac{M}{M_{st}}\mu_p^{(st)}+\Delta\mu_p, \quad (70)$$

where the correction to the static approximation has the form

$$\Delta\mu_p=\frac{e}{2M}\int\limits_0^{\infty}drr^2\left[u^2(r)-\frac{1}{3}l^2(r)\right],\text{where}\,\Psi(r)=\begin{pmatrix}u(r)\\ l(r)\end{pmatrix}\chi.$$

If the model parameters are fixed by means of the experimental proton radius, $\langle r^2\rangle^{1/2}=0.83$ F, then the correction to the magnetic moment calculated in the approximation of a static cavity is positive and has the value $\Delta\mu_p=0.65e/2M_p$.

The paper of Ref. 49 also gives the form of the proton electric, magnetic, and axial-vector form factors. At $q^2\simeq0.5$ GeV^2, their form becomes physically unsatisfactory. This is due to the use of the quark wave functions in a cavity, which, as noted in Sec. 1, is unjustified in this region of momentum transfers.

We end with some comments. Recoil effects in the bag model have been studied in many papers.[135–137] Inconsistencies in the approaches led to a diversity of results, some of which were found to be incorrect. The method considered above decisively eliminates the uncertainty in the calculations of the static properties of hadrons made in the approximation of a static cavity.

For the solution of the problems considered in this section, wide use is made of the Peierls–Yoccoz projection method,[138] which is widely used in nonrelativistic nuclear physics. Being in essence a particular realization of Bogolyubov's method, the projection method comes up against certain difficulties if it is directly generalized to the relativistic case. This is due to a dilemma that then arises[139]—the use of a basis of localized functions to describe fluctuations in the neighborhood of the classical solution violates the translational invariance of the vacuum and leads to a problem that cannot be solved in such an approach, namely, the calculation of the overlapping of bags situated at different space-time points. On the other hand, use of a plane-wave basis makes the solutions manifestly translationally invariant, but one then completely loses the property of localization of the solution. As we have shown above, use of a Bogolyubov transformation in the "double projection" form (66) eliminates this contradiction.

Finally, as was shown in Ref. 42, the solution of the two-dimensional model obtained by the method of collective coordinates is identical to the well-known solution of the translationally invariant L_0 approximation[104,105] obtained as an approximate solution to the exact solution. In the four-dimensional bag model, the exact solution has not been found and, therefore, it is impossible to construct the L_0 approximation. By analogy with the two-dimensional model, one can probably regard the solutions (66) as solutions corresponding to the L_0 approximation. Finally, a topical problem in the relativistic bag model is the need to go beyond the semiclassical approximation.

CONCLUSIONS

The bag model is a composite quark model of hadrons. The fundamental principles of the model are relativistic covariance, gauge symmetries, and the idea that quasi-independent quarks move in a finite closed region of space. The bag model is a theory with "built-in" confinement, and its parameters characterize the vacuum structure of the theory of the strong interactions.

The model has the greatest success in describing the static properties of hadrons. By going beyond the static approximation, one can also consider scattering processes. For interpretation of the complicated situation that arises in hadron spectroscopy, bag-model investigations of the excited states of hadrons and multiquarks are important. These investigations, and also the study of dynamical problems (deformed bags), help us to achieve a better understanding of the microscopic nature of internucleon forces.

In this review, we have identified the main directions of development of the bag model. We should emphasize that besides the successes of the model several problems await their resolution. Among these, the most important are, in our view, the construction of a field-theoretical solution of the model, and also the inclusion in the principles of the model of information about the structure of strong interactions that follows from the experimental data and the other independent theoretical approaches (QCD sum-rule method, lattice models, etc.). These directions will probably determine the positive part that the model plays in the future ahead of it.

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