The problem of l-forbidden magnetic dipole transitions in odd nuclei

N. A. Bonch-Osmolovskaya and V. A. Morozov

Joint Institute for Nuclear Research, Dubna

M. A. Dolgopolov and I. V. Kopymin

Voronezh State University

Fiz. Elem. Chastits At. Yadra **18**, 739–776 (July–August 1987)

The present status of the problem of l-forbidden magnetic diopole transitions in nuclei is reviewed. The available experimental data are systematized in three regions of mass numbers, 27–49, 105–151, and 193–211, and theoretical approaches to the solution of the problem are discussed. A method for calculating the reduced probabilities of l-forbidden M1 transitions in the framework of the theory of finite Fermi systems is proposed.

INTRODUCTION

It is known from experimental data in odd spherical nuclei that there is a group of electromagnetic transitions of multipolarity M1 for which $\Delta I = 1$, $\pi_i \pi_f = +1$ but the reduced probability $B(\mathrm{M1})$ is 1–3 orders of magnitude less than the corresponding probability of unhindered $\gamma(\mathrm{Ml})$ transitions (see the reviews of Refs. 1–4). Transitions of this type have become known as l-forbidden transitions (l is the orbital angular momentum). In the single-particle model of the nucleus (for them $\Delta l = 2$) they are strictly forbidden. This circumstance makes the l-forbidden $\gamma(\mathrm{M1})$ transitions a good object for testing new theoretical approaches. For these transitions, the matrix element is small (as already noted, it is zero in the purely single-particle model). Therefore, phenomena that are difficult to observe on the background of ordinary transitions may here be more clearly expressed. Therefore, by solving the problem of quantitative description of l-forbidden transitions, we can simultaneously use their probabilities to fix parameters of nuclear models to which these probabilities are sensitive.

It is natural to ask the following question: what mechanism (or mechanisms) is responsible for the lifting of the l forbiddenness? Many theoretical studies (which will be reviewed below) have been devoted to attempts to answer this question. It is now generally accepted that the lifting of the l forbiddenness is largely due to the residual interaction of the nucleons in the nucleus, although meson exchange currents may make a contribution. The interaction is manifested in various polarization phenomena, and also through coupling of the odd quasiparticle to the collective excitations of the core nucleus. The role of the residual interaction in lifting the l forbiddenness has been estimated quantitatively on the basis of both phenomenological and microscopic (or semimicroscopic) approaches. Choosing the parameters of the phenomenological (or semimicroscopic) model, the analysis of the properties of particular nuclei makes it possible in a number of cases to achieve a satisfactory description of the l-forbidden transitions as well. However, as a rule, it is found that these parameters must be changed on the transition from one nuclide to another. This obviously lowers the value of the physical information that is then obtained.

Among the microscopic approaches not based on perturbation theory the most promising is the theory of finite Fermi systems, which uses universal constants for the coupling of the quasiparticles. Calculations within this theory of the reduced probabilities $B(\mathrm{M1})$ of l-forbidden transitions for a large group of nuclei in which pairing is important reproduce the experimental results up to a coefficient of 2–3. However, in the same calculations for the nuclei nearest the doubly magic ^{208}Pb, where the states between which the $\gamma(\mathrm{M1})$ transition occurs can be regarded as single-quasiparticle states to a fairly high degree of accuracy, the discrepancy between the theoretical and experimental $B(\mathrm{M1})$ values reaches an order of magnitude and more, and this may indicate that here the transitions are more strictly l-forbidden.

The reliability of the theoretical conclusions will depend to a large degree on the extent to which a studied γ transition is correctly identified in an experiment with an l-forbidden transition. In fact, from the experimental point of view the identification of the l-forbidden transitions among all possible γ transitions of a particular nucleus is by no means simple. Its successful solution depends to a large degree on the extent to which the studied transition fits into a sequence of analogous l-forbidden transitions in neighboring isotopes or isotones. However, there are not always sufficient experimental data for reliable conclusions to be drawn. For these reasons the existing theoretical reviews are not always united in identifying individual $\gamma(\mathrm{Ml})$ transitions as l-forbidden transitions. This must be borne in mind in a theoretical analysis, and one should allow for a possible error of experimental classification. On the other hand, a theory that permits calculation of $B(\mathrm{M1})$ probabilities with a sufficiently high accuracy may be helpful in analysis of experimental data on l-forbidden transitions, making it possible to identify doubtful transitions or, at the least, cases that require a special theoretical investigation.

In the present paper, we have two aims. First, the significant experimental information on M1 transitions belonging to the l-forbidden category accumulated in recent years needs to be systematized, and this we do, using reviews of past years as well as new data.

Second, the use in recent years of the most varied theoretical approaches to the problem of the l-forbidden transitions calls for a review of the present status of the problem and the determination of the prospects. For our part, we also propose a theory that enables one in the single-quasiparticle approximation to calculate, without adjustable parameters, the probabilities of l-forbidden M1 transitions with allowance for the pairing, spin, spin-orbit, and one-pion exchange interactions of the quasiparticles. The theory is based on the theory of finite Fermi systems.

 0090-4759/87/040313-18$04.40

In Sec. 1 we systematize the available experimental data on l-forbidden γ(M1) transitions. We analyze M1 transitions in nuclei of three regions of mass numbers: $27 \leqslant A \leqslant 49$, $105 \leqslant A \leqslant 151$, and $193 \leqslant A \leqslant 211$.

In Sec. 2 we review theoretical studies in which there are calculations of the magnetic properties of nuclei, including the probabilities of M1 transitions that belong to the l-forbidden category. In this section, we formulate the basic propositions of our theory, the possibilities of which are illustrated by calculations of the B(M1) probabilities for a large number of nuclei. We end by giving the main conclusions drawn from the investigation.

1. SYSTEMATIZATION OF THE EXPERIMENTAL DATA

In this section, we present and analyze experimental data relating to l-forbidden magnetic dipole transitions in proton-odd and neutron-odd nuclei with mass numbers $27 \leqslant A \leqslant 49$, $105 \leqslant A \leqslant 151$, and $193 \leqslant A \leqslant 211$. In these regions there is, respectively, filling of subshells near the magic numbers $Z=20$, $N=20$, 28; $Z=50$, 64, $N=82$; and $Z=82$, $N=126$.

These regions of nuclei are characterized by the presence in them of both spherical and weakly deformed nuclei, the coexistence in a number of cases in one and the same nucleus of states due to both spherical and deformed shapes of the nucleus, and the presence in the nuclei of low-lying excitations of different collective natures. Therefore, the problem of identifying pure single-particle transitions is here fairly complicated, particularly in the cases when the γ transition takes place between excited states of the studied nucleus.

The most direct information about the single-particle nature of states can be obtained from single-nucleon transfer reactions. However, for many nuclei such data are not available. And even in the nucleí for which they are available and a fairly complete analysis of them is possible it is found in the majority of cases that the lowest states in the nucleus are single-particle states or carry the greater part of the single-particle component in the case of fragmentation of states due to interaction of the odd nucleon with the quadrupole excitations of the core.

Indirect methods can also be used to select single-particle transitions in nuclei:

1. Information from nuclear reactions in which collective states are excited.

2. The systematics of the variation of the energies of states in different subshells in isotopes.

3. Comparison of the acceleration of the admixture of the E2 component in the M1 transition in an odd nucleus with the acceleration of the E2 transition $2_1^+-0_1^+$ in the even–even core nucleus. The ratio of these two quantities in nuclei of medium and heavy masses can serve as a measure to estimate the degree of collectivization of the E2 transition in the odd nucleus, i.e., to estimate the admixture of collective components in the structure of the states of the odd nucleus. For light nuclei, a different picture is observed, proximity of the values of E_{odd} and E_{core}, suggesting a single-particle nature of the transition in the odd nucleus, since the 2_1^+ states in light nuclei are weakly collectivized, i.e., they are essentially two-quasiparticle states. In principle, this method is very productive, but in our case it must be used with care, since the admixtures of the E2 component in the l-forbidden M1 transitions are, as a rule, small, and in an experiment it is very difficult to obtain exact values of the small mixing parameters δ_γ(E2/M1), on which the reduced probabilities B(E2) depend strongly.

Thus, we have included magnetic dipole transitions in the l-forbidden category if for the initial and final states the

TABLE I. Neutron l-forbidden transitions $2s_{1/2} \rightleftarrows 1d_{3/2}$.

Nucleus	Initial state	E_{lev}/E_γ, keV[a]	Experiment		$B(M1)_{\text{exp}}$ [b]	$F_M(M1)$	$B(E2)_{\text{exp}}$ [b]	$E(E2)_{\text{odd}}$	$E(E2)_{\text{core}}$
			$T_{\frac{1}{2}\text{lev}}$, psec	δ_γ (E2/M1)					
$^{27}_{14}Si_{13}$	$d_{3/2}$	957/177	1.24(14)	—	35	3.3	—	—	—
$^{29}Si_{15}$	$d_{3/2}$	1273	0,28(4)	—0.197(9)	6,62	17	0,23	4,2	12,6
$^{31}Si_{17}$	$s_{1/2}$	753	0,54(12)	—	17,2	6,7	—	—	—
$^{31}_{16}S_{15}$	$d_{3/2}$	1249	0.50(12)	—	4,08	28	—	—	—
$^{33}S_{17}$	$s_{1/2}$	840	1.17(3)	0.151(4)	5.56	21	0.26	4,2	9.5
$^{35}S_{19}$	$s_{1/2}$	1572	2,3(3)	—	0,44	260	—	—	—
$^{37}_{18}Ar_{19}$	$s_{1/2}$	1410	0,72(10)	—	1,96	59	—	—	—
$^{39}_{20}Ca_{19}$	$s_{1/2}$	2470	0.21(4)	—	1.25	93	—	—	—
$^{41}Ca_{21}$	$s_{1/2}$	2670/660	2.1(5)	—	2.03	57	—	—	—
$^{43}Ca_{23}$	$s_{1/2}$	1957/967	1.07(31)	—	0.90	129	—	—	—

Note. The experimental values of $T_{\frac{1}{2}\text{lev}}$ and δ_γ are taken from the review in Nucl. Phys. **A301**, 1 (1978); the value of $T_{\frac{1}{2}}$ (^{43}Ca) is taken from J. Phys. G **5**, 1117 (1979).

[a] One value in this column means that the γ transition takes place to the ground state of the nucleus. In the case of transition to an excited state of the nucleus, the upper value corresponds to the energy of the intitial level, and the lower value to the energy of the γ transition.

[b] The values of B(M1) are given in units of $10^{-2}\mu_N^2$ and those of B(E2) in units of $10^{-50}e^2 \cdot cm^4$.

TABLE II. Proton l-forbidden transitions $2s_{1/2} \rightleftarrows 1d_{3/2}$.

Nucleus	Initial state	E_{lev}/E_γ, keV[a]	Experiment		$B(M1)_{exp}$ [b]	$F_M(M1)$	$B(E2)_{exp}$ [b]	$E(E2)_{odd}$	$E(E2)_{core}$
			$T_{\frac{1}{2}lev}$, psec	$\delta_\gamma(E2/M1)$					
$^{29}_{15}P_{14}$	$d_{3/2}$	1384	0.140(15)	0.17(2)	10.4	16	0.23	4.3	12.6
$^{31}P_{16}$	$d_{3/2}$	1266	0.52(3)	0.30(1)	3.44	48	0.27	5.2	7.0
$^{33}P_{18}$	$d_{3/2}$	1431	0.44(6)	0.60(9)	2.29	72	0.57	8.9	7.9
$^{33}_{17}Cl_{16}$	$s_{1/2}$	811	1.2(2)	—	6.19	28	—	—	—
$^{35}Cl_{18}$	$s_{1/2}$	1219	0.15(2)	0.106(8)	14.3	12	0.16	2.3	6.6
$^{37}Cl_{20}$	$s_{1/2}$	1727	0.128(21)	0.25(2)	5.63	29	0.17	2.3	2.6
$^{39}Cl_{22}$	$s_{1/2}$	396	>1.4	—	<45	>3.7	—	—	—
$^{39}_{19}K_{20}$	$s_{1/2}$	2523	0.061(3)	0.69(13)	2.73	61	0.30	3.7	3.6
$^{41}K_{22}$	$s_{1/2}$	980	0.28(3)	0.15(5)	14.8	11	0.49	5.8	9.6
$^{43}_{21}Sc_{22}$	$s_{1/2}$	855/704	22(3)	—	0.041	4070	—	—	—
$^{45}Sc_{24}$	$s_{1/2}$	939/927	$7.3^{+5.7}_{-3.2}$	—	0.65	255	—	—	—
$^{47}Sc_{26}$	$s_{1/2}$	1391/624	>4.2	—	<0.58	>285	—	—	—
$^{49}Sc_{28}$	$d_{3/2}$	2372/143	1400(90)	—	0.067	2490	—	—	—

Note. For references for the experimental values of $T_{\frac{1}{2}}$ and δ_γ, see the note to Table I; the value of $T_{\frac{1}{2}}$ (^{39}K) is taken from Yad. Fiz. **39**, 1069 (1984) [Sov. J. Nucl. Phys. **39**, 673 (1984)], and the values of $T_{\frac{1}{2}}$ and δ_γ (^{41}K) from Nucl. Phys. **A357**, 268 (1981).

[a,b] See the notes to Table I.

spins and parities have been observed to match the quantum numbers of the subshells and the multipolarity type and also if no contradiction has been found with the data of the nuclear reactions, the information on the probabilities of the E2 transitions, and the trend of the subshell energy systematics.

The experimental data that we have analyzed and calculated are given in Tables I–VI. These also include a number of nuclei in which the transitions do not correspond strictly to the selection criteria listed above but which nevertheless help to reveal the general tendency of the l-forbidden-

TABLE III. Neutron l-forbidden transitions $3s_{1/2} \rightleftarrows 2d_{3/2}$.

Nucleus	Initial state	E_{lev}/E_γ, keV[a]	Experiment		$B(M1)_{exp}$ [b]	$B(M1)_{th}$ [b]	$F_M(M1)$	$B(E2)_{exp}$ [b]	$E(E2)_{odd}$	$E(E2)_{core}$
			$T_{\frac{1}{2}lev}$, nsec	$\delta_\gamma(E2/M1)$ [c]						
$^{105}_{46}Pd_{59}$	$s_{1/2}$	344,5/63,98	0.88(5)	—0,025(30)	2.68	2.27 г	43	⩽2,78	⩽9.4	37
$^{111}_{48}Cd_{63}$	$d_{3/2}$	342.1	0.059(12)	0.39(2)	1.32	2.91	87	2.45	7.7	27
$^{113}Cd_{65}$	$d_{3/2}$	298.5	0,032(3)	0,29(1)	4.10	4.61	28	5.83	18	33
$^{119}Cd_{71}$	$d_{3/2}$	27.0	2.3(4)	0	3.74	4.53	31	—	—	—
$^{115}_{50}Sn_{65}$	$d_{3/2}$	497.4	0,011(2)	0.21(2)	2.78	2.52	41	0.70	2.1	15
$^{117}Sn_{67}$	$d_{3/2}$	158.6	0.279(9)	0.0133(15)	3,10	2.22	37	0.03	0,09	13
$^{119}Sn_{69}$	$d_{3/2}$	23.87	18.08(12)	<0,006	2.59	2.03	44	<0.23	<0.66	12
$^{121}_{52}Te_{69}$	$d_{3/2}$	212.2	0.062(16)	0.23(4)	5.82	4,17	20	9.80	27	30
$^{123}Te_{71}$	$d_{3/2}$	159.0	0.196(9)	0.111(4)	4.18	5,03	28	2.89	7.9	48
$^{125}Te_{73}$	$d_{3/2}$	35.50	1,48(1)	0,027(1)	3,98	3.08	29	3.27	8.7	31
$^{123}_{54}Xe_{69}$	$d_{3/2}$	97,35	0.380(30)	0	5.95	6.89	19	—	—	—
$^{125}Xe_{71}$	$d_{3/2}$	111.8	0.350(20)	0	5,00	6.77	23	—	—	—
$^{127}Xe_{73}$	$d_{3/2}$	124.8	0.28(1)	0.09(2)	4.97	6.77	23	3.68	9.6	41
$^{129}Xe_{75}$	$d_{3/2}$	39,58	1.01(4)	—0,027(5)	4.77	6.84	24	3,14	8.0	35
$^{131}Xe_{77}$	$s_{1/2}$	80.18	0,416(20)	0	7.17	7.56	16	—	—	—
$^{131}_{56}Ba_{75}$	$d_{3/2}$	108.4	0.35(5)	2%	4,86	3.14	24	11.8	30	64
$^{133}Ba_{77}$	$d_{3/2}$	12,33	7.0(3)	0	4.60	3.11	25	—	—	—
$^{135}Ba_{79}$	$s_{1/2}$	221.0	—	—	0.56 д	—	200	1.88 д	4.5	33
$^{135}_{58}Ce_{77}$	$d_{3/2}$	82,6	0,53(6)	0	4.38	2.97	26	—	—	—
$^{137}Ce_{79}$	$s_{1/2}$	160.3	0.79(14)	0	0,92	—	125	—	—	—
$^{139}Ce_{81}$	$s_{1/2}$	255.1	0.110(20)	⩽2.2%	1.59	—	73	⩽9.5	⩽22	—
$^{141}_{60}Nd_{81}$	$s_{1/2}$	193.7	1.17(15)	0.15(4) [e]	0.33	2,49	350	1.88	4.2	19
$^{145}_{64}Gd_{81}$	$d_{3/2}$	27.3	11.5(3)	0.090(22)	0,79	0.89	145	12.2	27	—

Note. References for the experimental values of $T_{\frac{1}{2}lev}$ and δ_γ are given in Ref. 4; the value of δ_γ (^{105}Pd) is taken from Nucl. Data Sheets **47**, 261 (1986).

[a,b] See the notes to Table. I.

[c] The values in the δ_γ column given in percentages correspond to a fraction of the E2 component in the form M1 + Δ%E2, where $\Delta = \delta^2/(1+\delta^2)$.

[d] The approximate formula (33) (see Sec. 2) was used in the calculations.

[e] Data obtained from Coulomb-excitation reactions.

[f] The value of δ^2_γ is given.

TABLE IV. Proton l-forbidden transitions $2d_{5/2} \rightleftarrows 1g_{7/2}$.

Nucleus	Initial state	E_{lev}/E_γ, keV [a]	Experiment							
			T_{1lev}, nsec	$\delta_\gamma\,(E2/M1)$ [c]	$B(M1)_{exp}$ [b]	$B(M1)_{th}$ [b]	$F_M\,(M1)$	$B(E2)_{th}$ [b]	$E(E2)_{odd}$	$E(E2)_{core}$
$^{119}_{51}\mathrm{Sb}_{68}$	$g_{7/2}$	270.5	0.035(10)	—0.13(4)	5.48	3.45	30	1.78	6.9	9.3
$^{121}\mathrm{Sb}_{70}$	$g_{7/2}$	37.14	3.46(3)	0	1.84	3.27	90	—	—	—
$^{123}\mathrm{Sb}_{72}$	$d_{5/2}$	160.3	0.61(4)	0.079(15)	1.33	2.89	125	0.46	1.25	11
$^{125}\mathrm{Sb}_{74}$	$d_{5/2}$	332.0	0.163(10)	—0.59	0.48	4.33	350	2.15	5.7	8.9
$^{121}_{53}\mathrm{I}_{68}$	$g_{7/2}$	132.8	0.35(2)	0.02(4)	3.56	3.47	47	0.11	0.3	30
$^{125}\mathrm{I}_{72}$	$g_{7/2}$	113.5	0.61(4)	0.12(2)	2.96	3.24	56	4.70	13	31
$^{127}\mathrm{I}_{74}$	$g_{7/2}$	57.61	1.96(1)	—0.084(6)	2.20	3.11	75	6.67	17	24
$^{129}\mathrm{I}_{76}$	$d_{5/2}$	27.79	16.8(2)	—0.045(14)	1.82	2.82	90	6.73	17	20
$^{131}\mathrm{I}_{78}$	$d_{5/2}$	149.7	0.94(3)	12(5)%	0.87	2.09	190	7.49	19	15
$^{129}_{55}\mathrm{Cs}_{74}$	$g_{7/2}$	188.9/182.3	2.26(6)	0.25(2)	0.23	0.59	725	0.62	1.6	82
$^{131}\mathrm{Cs}_{76}$	$g_{7/2}$	78.7	9.38(30)	0.5%	0.31	0.56	540	0.36	0.89	64
$^{133}\mathrm{Cs}_{78}$	$d_{5/2}$	81.00	6.36(3)	0.169	0.42	0.58	400	2.57	6.3	36
$^{135}\mathrm{Cs}_{80}$	$d_{5/2}$	249.7	0.28(8)	~ 15%	0.72	0.61	230	2.94	7.0	22
$^{131}_{57}\mathrm{La}_{74}$	$g_{7/2}$	195.5/169.4	0.20(5)	0.02 [d]	3.0	—	55	2.97	7.2	64
$^{133}\mathrm{La}_{76}$	$g_{7/2}$	130.8	1.12(18)	5.4(12)%	0.98	0.70	170	4.65	11	36
$^{135}\mathrm{La}_{78}$	$g_{7/2}$	119.4	4.1(1)	4%	0.33	0.52	500	1.41	3.3	33
$^{137}\mathrm{La}_{80}$	$d_{5/2}$	10.56	89(4)	0	0.31	0.62	530	—	—	—
$^{139}\mathrm{La}_{82}$	$d_{5/2}$	165.9	1.48(3)	0.034	0.47	0.62	350	0.028	0.06	9.7
$^{141}\mathrm{La}_{84}$	$d_{5/2}$	190.3	1.27^{+6}_{-10}	< 8%	0.35	—	470	< 1.2	< 2.7	—
$^{143}\mathrm{La}_{86}$	$d_{5/2}$	211.3	0.69(7)	0	0.53	—	320	—	—	—
$^{139}_{59}\mathrm{Pr}_{80}$	$g_{7/2}$	113.9	2.60(8)	2.6(3)%	0.52	1.17	320	1.51	3.4	—
$^{141}\mathrm{Pr}_{82}$	$g_{7/2}$	145.5	1.85(2)	0.069(2)	0.48	1.09	350	0.15	0.34	8.2
$^{143}\mathrm{Pr}_{84}$	$d_{5/2}$	57.37	4.16(4)	9(1) [d]	0.68	0.98	245	0.27	0.61	21
$^{145}\mathrm{Pr}_{86}$	$d_{5/2}$	62.6	4.0	0	0.67	1.03	245	—	—	—
$^{141}_{61}\mathrm{Pm}_{80}$	$g_{7/2}$	196.6	0.23(3)	~ 0.1	1.82	1.45	90	0.67	1.6	—
$^{143}\mathrm{Pm}_{82}$	$g_{7/2}$	271.8	1.06(8)	—	0.17	1.75	980	—	—	—
$^{145}\mathrm{Pm}_{84}$	$g_{7/2}$	61.25	2.69(8)	0.15(5)%	0.85	1.27	195	0.49	1.1	22
$^{147}\mathrm{Pm}_{86}$	$d_{5/2}$	91.11	2.50(5)	0.086(7)	0.68	1.44	245	0.86	1.8	32
$^{149}\mathrm{Pm}_{88}$	$d_{5/2}$	114.3	2.52(2)	0.180(24)	0.51	1.36	320	1.60	3.4	58
$^{147}_{63}\mathrm{Eu}_{84}$	$g_{7/2}$	229.3	0.18(2)	3%	1.49	1.34	110	1.25	2.7	—
$^{149}\mathrm{Eu}_{86}$	$g_{7/2}$	149.7	0.32(2)	0.15(4)	2.27	1.32	75	1.98	4.2	33
$^{151}\mathrm{Eu}_{88}$	$g_{7/2}$	21.54	9.4(4)	0.088%	1.43	1.35	115	3.83	8.0	—

Note. References for the experimental values of T_{1lev} and δ_γ are given in Ref. 4; the values of δ_γ (^{149}Pm and ^{149}Eu) are taken from Nucl. Data Sheets **46**, 1 (1985).
[a,b]See the notes to Table I.
[c]See the note in Table III.
[d]The value of $\delta_\gamma^2 \times 10^{-4}$ is given.

TABLE V. Proton l-forbidden transitions $3s_{1/2} \rightleftarrows 2d_{3/2}$ and $2f_{7/2} \rightleftarrows 1h_{9/2}$.

Nucleus	Initial state	E_{lev}, keV	Experiment: $T_{1/2 lev}$, nsec	Experiment: $\delta_\gamma(E2/M1)$	$B(M1)_{exp}$ [b]	$F_M(M1)$	$B(E2)_{exp}$ [t]	$E(E2)_{odd}$	$E(E2)_{core}$
				$(3s_{1/2} \rightleftarrows 2d_{3/2})$					
$^{193}_{79}Au_{114}$	$s_{1/2}$	38.22	3.81(18)	0.46(2)	0.15	1110	31	46	69
$^{195}Au_{116}$	$s_{1/2}$	61.43	3.0(2)	0.45(1)	0.35	477	27	39	58
$^{197}Au_{118}$	$s_{1/2}$	77.35	1.91(1)	−0.352(5)	0.76	220	22	32	46
$^{199}Au_{120}$	$s_{1/2}$	77.20	1.3(2)	0.22(6)	1.40	119	16	23	35
$^{201}_{81}Tl_{120}$	$d_{3/2}$	331.2	0.070(20)	1.33(6)	0.49	340	11.3	16	27
$^{203}Tl_{122}$	$d_{3/2}$	279.2	0.278(2)	1.17(5)	0.23	725	5.63	7.9	18
$^{205}Tl_{124}$	$d_{3/2}$	203.7	1.46(8)	1.56(15)	0.065	2570	5.40	7.5	12
$^{207}Tl_{126}$	$d_{3/2}$	351.1	0.030(7)	0.271(4)	2.29	73	0.93	1.3	—
				$(2f_{7/2} \rightarrow 1h_{9/2})$					
$^{209}_{83}Bi_{126}$	$f_{7/2}$	896.4	9.7(11) [c]	−0.70(5)	0.36	469	0.33	0.44	7.2
$^{211}Bi_{128}$	$f_{7/2}$	404.9	0.317(11)	−1.39(0)	0.058	2880	0.98	1.3	—

Note. The experimental values of $T_{1/2}$ and δ_γ are taken from the surveys of Nucl. Data Sheets **34**, 101 (1981) ($A = 197$); **46**, 287 (1985) ($A = 203$); **23**, 287 (1978) ($A = 205$); **25**, 397 (1978) ($A = 211$). For the remaining A values, see the note to Table VI. The value of δ_γ (^{209}Bi) is taken from Z. Phys. A **322**, 641 (1985).

[b] See the note to Table I.

[c] The value of $T_{1/2}$ is given in picoseconds.

ness phenomenon in the corresponding region of mass numbers.

To calculate the reduced probabilities B(M1) of the magnetic and B(E2) of the electric transitions we have used the experimental level lifetimes $T_{1/2}$ and the mixing parameters δ_γ (E2/M1), taken mainly from the reviews of *Nuclear Data Sheets, Table of Isotopes*, edited by Lederer and Shirley (1978), and for light nuclei ($27 \leqslant A \leqslant 43$) from the survey in Nucl. Phys. **A310**, 1 (1978). We have also introduced additions and corrections using the results of original studies in recent years.

To determine the hindrance factors $F_M = B(M1)_{th}/B(M1)_{exp}$ of the M1 transitions, we used the Moszkowski estimate of the radiative transition rate, which takes into account the difference between the proton and neutron magnetic moments. Since we are considering γ transitions that are l-forbidden in the strict single-particle model, we take the statistical factor equal to unity and $B(M1)_{th}$ in the form

$$B_M(M1)_{th} = \frac{1}{\pi} M_\mu \mu_N^2,$$

where $M_\mu = (\mu - g_l/2)^2$,) μ is the magnetic moment of the

TABLE VI. Neutron l-forbidden transitions $2f_{5/2} \rightleftarrows 3p_{3/2}$ and $1i_{11/2} \rightarrow 2g_{9/2}$.

Nucleus	Initial state	E_{lev}/E_γ, keV [a]	Experiment: $T_{1/2 lev}$, nsec	Experiment: $\delta_\gamma(E2/M1)$	$B(M1)_{exp}$ [b]	$F_M(M1)$	$B(E2)_{exp}$ [b]	$E(E2)_{odd}$	$E(E2)_{core}$
				$(2f_{5/2} \rightleftarrows 3p_{3/2})$					
$^{193}_{80}Hg_{113}$	$f_{5/2}$	39.49	0.63(3)	0.062(11)	4.02	29	14	21	—
$^{195}Hg_{115}$	$f_{5/2}$	53.30/16.20	0.72(3)	0.024(6)	3.70	31	11.6	17	—
$^{199}Hg_{119}$	$p_{3/2}$	208.2/49.83	0.069(5)	−0.045(3)	7.98	14	9.23	13	30
$^{201}Hg_{121}$	$f_{5/2}$	26.3	0.63(5)	—	5.67	20	—	—	—
$^{207}_{82}Pb_{125}$	$p_{3/2}$	897.7/328.1	0.13(3) [c]	—	4.98	23	—	—	—
				$(1i_{11/2} \rightarrow 2g_{9/2})$					
$^{209}_{82}Pb_{127}$	$i_{11/2}$	779.0	8.2(9) [c]	—	0.98	118	—	—	—

Note. The experimental values of $T_{1/2}$ and δ_γ are taken from the surveys of Nucl. Data Sheets **32**, 593 (1981) ($A = 193$); **23**, 607 (1978) ($A = 195$); **24**, 57 (1978) ($A = 199$); **25**, 193 (1978) ($A = 201$); **43**, 383 (1984) ($A = 207$); **22**, 545 (1977) ($A = 209$).

[a,b] See the notes to Table I.

[c] The value of $T_{1/2}$ is given in picoseconds.

nucleon, $\mu = 2.79$ and $g_l = 1$ (for the proton), $\mu = -1.91$ and $g_l = 0$ (for the neutron), and $\mu_N = e\hbar/2mc$ is the nuclear magneton.

For the Moszkowski estimates of the rates of the proton and neutron transitions we obtain the ratio

$$\frac{B(M1)^p_{\rm th}}{B(M1)^n_{\rm th}} = 1.44.$$

As is well known, the Moszkowski and Weisskopf estimates for the acceleration of the E2 transitions, $E = B(\mathrm{E2})_{\exp}/B(\mathrm{E2})_{\rm th}$, agree.

We consider in more detail the l-forbidden M1 transitions in the different regions of nuclei.

1. Neutron and proton transitions $2s_{1/2} \rightleftarrows 1d_{3/2}$ in $27 \leqslant A \leqslant 49$ nuclei

In this region of nuclei, the first excited states have a high energy in the region 1–2.5 MeV. We analyzed neutron and proton l-forbidden M1 transitions taking place between levels of the subshells $2s_{1/2} \leftrightarrows 1d_{3/2}$, which here, as a rule, are the lowest (Tables I and II). The data given in these tables differ from the data of Ref. 3—additional B(M1) values have been calculated for transitions in the nuclei ^{39}Cl, ^{43}Ca, $^{43-47}$Sc and have been eliminated in the nuclei ^{25}Na, ^{25,27}Al, ^{25,27}Mg, which are too close to the region of stable deformation at $A = 25$.

It can be seen from Tables I and II that no significant difference is observed in the values of the reduced probabilities B(M1) of the neutron and proton transitions. For the overwhelming majority of transitions, the hindrance factors $F_{\rm M}$ are in the range $F_{\rm M} = 10$–100. The only exceptions are transitions in the nuclei $^{43-49}$Sc and ^{35}S, where the factors $F_{\rm M}$ are very high. If one follows the trend of the variation of the reduced probabilities of the M1 transitions within the groups of isotopes, one may note that, as a rule, the probabilities decrease (the hindrance factors increase) on the approach of N to the magic numbers 20 and 28. We note that large hindrance factors are observed for transitions in $^{43-49}$ Sc in nuclei in a certain island between the magic numbers $N = 20$ and $N = 28$. This could possibly explain why their properties differ from those of the other nuclei which we consider.

In the systematics of the energy position of the $2s_{\frac{1}{2}}$ levels (Fig. 1) jumps are observed in the positions of the levels as N approaches $N = 20$ ($Z =$ const). For the Sc isotopes, a jump occurs at $N = 28$.

It would be interesting to analyze whether or not there is a dependence of the hindrance factors $F_{\rm M}$ on the values of the spectroscopic factors obtained in single-nucleon transfer reactions, i.e., on the extent to which the states are single-particle states. However, because of the incompleteness of the data and the absence of a common method of analyzing the experimental results of nuclear reactions in studies in different years, it is very difficult to do this for a large number of nuclei. Nevertheless, from analysis of special cases one may conclude that it is difficult to establish a single-valued dependence. For example, in (d, ^{3}He) reactions for the ^{35,37}Cl isotopes[6] similar values of the spectroscopic factors are obtained, $C^2S(1/2^+) = 1.34$, $C^2S(3/2^+) = 2.2$ (^{35}Cl) and $C^2S(1/2^+) = 1.19$, $C^2S(3/2^+) = 2.32$ (^{37}Cl), but the values of $F_{\rm M}$ for the transitions in them differ by 2.5 times (Table II). For the ^{45,49}Sc isotopes in (d, ^{3}He) reactions[7] similar values of the spectroscopic factors are also obtained, $C^2S(1/2^+) = 1.50$, $C^2S(3/2^+) = 3.43$ (^{45}Sc) and $C^2S(1/2^+) = 1.40$, $C^2S(3/2^+) = 3.62$ (^{49}Sc), but the values of $F_{\rm M}$ for transitions in them differ by an order of magnitude (Table II). It should be noted that in not only the ^{35,37}Cl nuclei but also the ^{45,49} Sc nuclei the fragmentation of the $1d_{3/2}$ state is very weak ($2s_{1/2}$ is hardly fragmented), and the lowest 3/2 levels contain not less than 80–90% of the $1d_{3/2}$ component.

We now consider the case when the fragmentation of the $2s_{1/2}$ or $1d_{3/2}$ states is very great. In the ^{39,41}K isotopes [(d, ^{3}He) reactions[8]] the $1d_{3/2}$ and $2s_{1/2}$ states in ^{39}K are not fragmented, but the lowest $1/2^+$ level in ^{41}K contains only 50% of the $2s_{1/2}$ component. At the same time, the hindrance factor $F_{\rm M}$ of the transition in ^{41}K is less than in ^{39}K. However, the opposite phenomenon is also observed; the hindrance factor of the transition in ^{39}Ca is somewhat less than in ^{43}Ca, though according to the data of (d, t) reactions[8] there is no fragmentation of the $2s_{1/2}$, $1d_{3/2}$ states in the first nucleus but there is appreciable fragmentation in the second—the lowest $1/2^+$, $3/2^+$ levels carry 70–60% of the $2s_{1/2}$, $1d_{3/2}$ components.

Obviously, there are many different factors that influence the hindrance of the magnetic dipole transitions, but this is most clearly seen when the number of neutrons in the

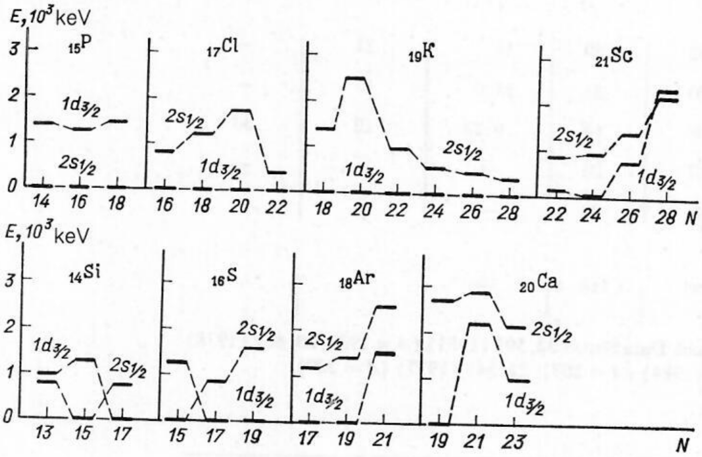

FIG. 1. Dependence on N and Z of the energy positions of the single-particle levels of the $2s_{1/2}$ and $1d_{3/2}$ subshells.

considered nuclei approaches the magic values of N equal to 20 and 28.

If we turn to the analysis of the accelerations of the E2 transitions, then, as can be seen from Tables I and II, the values of E_{odd} and E_{core} are, as a rule, closer together, the closer N is to the magic number $N = 20$.

Attention was drawn in Ref. 3 to the properties of the l-forbidden transitions in mirror nuclei ($^{29}_{15}P_{14}$, $^{29}_{14}Si_{15}$ and $^{33}_{17}Cl_{16}$, $^{33}_{16}S_{17}$), since each pair of nuclei has the same even–even core. It was noted that the reduced probabilities B(M1) of the proton transitions are always somewhat higher than for the neutron transitions. However, this difference disappears if one considers, not the B(M1) values, but the hindrance factors F_M in the Moszkowski estimates which take into account the difference between the proton and neutron magnetic moments. Indeed the factors F_M for each pair of mirror nuclei are found to be very nearly the same: F_M is equal to 16 and 17 for ^{29}P and ^{29}Si and equal to 28 and 21 for ^{33}Cl and ^{33}S. For transitions in mirror nuclei for which the even–even cores $(A-1)$ are different, as, for example, for the pair of nuclei $^{39}_{19}K_{20}$ and $^{39}_{20}Ca_{19}$, we find a significant difference in both the reduced probabilities B(M1) and the values of F_M: $B(\mathrm{M1}) = 3.20\times10^{-2}\mu_N^2$, $F_M = 52$ (^{39}K) and $B(\mathrm{M1}) = 1.25\times10^{-2}\mu_N^2$, $F_M = 92$ (^{39}Ca).

2. Transitions in the region of nuclei $105 \leqslant A \leqslant 151$

In this region of nuclei neutron and proton l-forbidden M1 transitions taking place, respectively, between the levels of the subshells $3s_{1/2} \leftrightarrows 2d_{3/2}$ and $2d_{5/2} \leftrightarrows 1g_{7/2}$ have been considered. These states are here the lowest-lying states, and the majority of transitions takes place to the ground state of the nucleus (Tables III and IV).

Neutron transitions $3s_{1/2} \rightleftarrows 2d_{3/2}$

The most extensive data in this region of nuclei are for neutron transitions of the type $3s_{1/2} \leftrightarrows 2d_{3/2}$ (Table III). In Fig. 2 we show the systematics of the energy positions of the states of the $3s_{1/2}$, $2d_{3/2}$ subshells as functions of N for $Z = \mathrm{const}$. As a rule, we observe a small decrease (or increase) in the energy of the levels on the transition from isotope to isotope. At a certain N, there is an inversion of the position of the $3s_{1/2}$ and $2d_{3/2}$ subshells.

The reduced probabilities B(M1) of the l-forbidden transitions $3s_{1/2} \leftrightarrows 2d_{3/2}$ can be followed in the range of neutron numbers $N = 59$–81 and proton numbers $Z = 46$–64 (Table III). It may be noted that in the region $N = 59$–77 the hindrance factors F_M of the transitions are comparatively small, constant in magnitude, and depend neither on Z nor on N. They also do not change for transitions in the Sn nuclei at the magic value $Z = 50$. The hindrance factors F_M increase only at $N = 79$, 81, i.e., in the immediate proximity of $N = 82$ ($^{135}Ba_{79}$, $^{137}Ce_{79}$, $^{141}Nd_{81}$, $^{145}Gd_{81}$). The only exception to this rule is the transition in $^{111}_{48}Gd_{63}$ ($F_M = 87$). This increase of F_M is correlated with the energy systematics of the $3s_{1/2}$ subshell trend in the corresponding isotopes (for the Nd isotopes also with the trend for the $2d_{3/2}$ subshell). At the values of N corresponding to the jump in the values of F_M there is also a jump in the state energy of the subshell (or, at least, in the immediate vicinity of the given N, as in the case of $^{141}Nd_{81}$ or $^{111}Cd_{63}$; Fig. 2).

For the nuclei given in Table III the literature contains rather varied and incomplete information on single-nucleon transfer reactions. In addition, it is not always possible to determine accurately the degree of fragmentation of the $2d_{3/2}$ state, since one often observes highly excited states with $l = 2$ whose spin is determined either as $3/2^+$ or as $5/2^+$. However, a comparison of data where it is possible shows that, as in the region of light nuclei, there is no definite dependence of the F_M values on the spectroscopic factor or on the degree of fragmentation of a particular state. Thus, in Ref. 9, on the basis of data from (^{3}He, α) and (d, t) reactions in the ^{121,123,125}Te isotopes it is assumed that only the $2d_{3/2}$ state in ^{123}Te is fragmented. However, comparison of the spectroscopic factors presented in Ref. 9 shows that their values in the case of ^{121}Te are somewhat less $[C^2S(1/2^+) = 0.67$ and $C^2S(3/2^+) = 1.24]$ than in the case of ^{123,125}Te $[C^2S(1/2^+) = 1.22$–1.00 and $C^2S(3/2^2) = 1.70$–$1.80]$. It could be that this is due to a certain fragmentation of the states in ^{121}Te as well, since in it there are some highly excited states with $l = 2$ whose spin is determined as ($3/2^+$, $5/2^+$), and highly excited states with $l = 0$ ($1/2^+$) may have energy outside the limits of the experimentally observed levels. Nevertheless, the hindrance factors F_M for the M1 transitions in all three Te isotopes differ little (Table III). For the transitions in the $^{139}Ce_{81}$ and $^{141}Nd_{81}$ nuclei, the hindrance factors F_M differ by five times, although the spectroscopic factors in the (d, t) and (^{3}He, α) reactions[10]

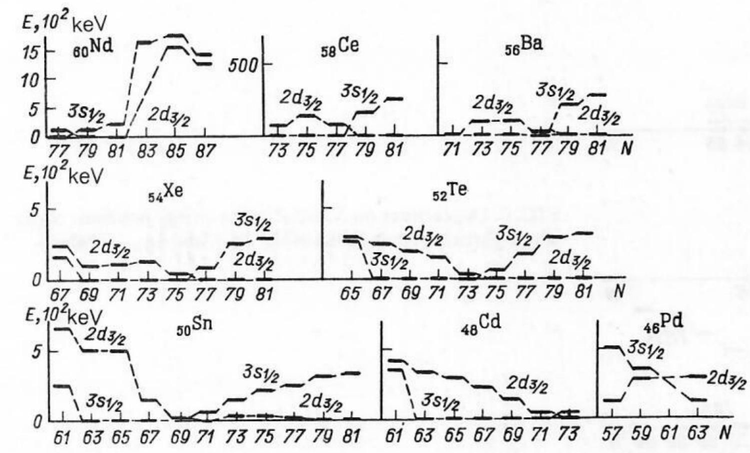

FIG. 2. Dependence on N and Z of the energy positions of the single-particle neutron levels of the $3s_{1/2}$ and $2d_{3/2}$ subshells.

for the lowest $1/2^+$, $3/2^+$ states are practically the same, $C^2S(3/2^+) = 4$ and $C^2S(1/2^+) = 1.7$–1.8, and the possible fractions of fragmentation of the $3s_{1/2}$ and $2d_{3/2}$ states are nearly the same in the two nuclei.

Included in the data of Table III there is information on the $_{50}$Sn isotopes, which are spherical nuclei, and the transitions in them are single-particle transitions. This is manifested in a slight acceleration of the admixture of the E2 transition in these isotopes relative to the acceleration of the E2 transitions in the even core nuclei. It can be seen from Table III that the majority of the E2 transitions in the odd nuclei are characterized by accelerations several times less than those of the E2 transitions in the corresponding even nuclei, and this is some indication that they have a single-particle nature. An exception is the E2 transition in ^{121}Te, whose acceleration is very close to that of the E2 transition in the even core. In the neighboring ^{123,125}Te isotopes, this is not observed. As noted above, fragmentation of the ^{121}Te states cannot be ruled out. However, one also cannot rule out an error in the experimental value of δ_γ for the transition in ^{121}Te.

Proton transitions $2d_{5/2} \rightleftarrows 1g_{7/2}$

For proton *l*-forbidden M1 transitions of the type $2d_{5/2} \leftrightarrows 1g_{7/2}$ there is a fairly extensive systematics (Table IV). The positions of the levels of the $2d_{5/2}$ and $1g_{7/2}$ subshells vary smoothly with N (Fig. 3), and, as in the case of the neutron $3s_{1/2}$ and $2d_{3/2}$ subshells, an inversion of them is observed at a certain N ($Z = \text{const}$). As a rule, one of the $2d_{5/2}$ or $1g_{7/2}$ states is the ground state and the other is the lowest-lying state.

The dependence of the reduced probabilities $B(\text{M1})$ on N can be analyzed using a fairly large number of isotopes in the transitional region of nuclei from the near-magic $Z = 51$ to $Z = 63$, which is close to the semimagic $Z = 64$, at values $N = 68$–88. We note first of all that the reduced probabilities $B(\text{M1})$ of the proton transitions $2d_{5/2} \leftrightarrows 1g_{7/2}$ are much less (the hindrance factors F_M are much greater) than the $B(\text{M1})$ values for the neutron transitions $3s_{1/2} \rightleftarrows 2d_{3/2}$. A second difference from the neutron transitions is that for the proton transitions the hindrance factors increase fairly monotonically as $N = 82$ is approached and are not constant, increasing only abruptly in the neighborhood of $N = 82$. Further, for $N > 82$ the opposite picture is observed—a decrease of the F_M values, but now in an irregular fashion. An exception to the general rule is provided by the M1 transition in the Cs isotopes, in which we observe on the approach of N to $N = 82$, not a decrease, but an increase of $B(\text{M1})$ (a decrease of F_M).

In the majority of cases (Table IV) in the odd nuclei we observe smaller values of the acceleration of the E2 transitions than the accelerations of the collective $2_1^+ \to 0_1^+$ transitions in the corresponding even core nuclei. This can be regarded as evidence for a single-particle nature of the transitions in the odd nuclei. However, there are exceptions. Thus, in the ^{119,125}Sb isotopes the E2 transitions have accelerations only 1.5 times less than the E_{core} accelerations, and in the ^{127,129,131}I isotopes we observe accelerations of the E2 transitions practically equal to those of the collective E2 transitions in the even core nuclei (Table IV).

In the (^{3}He, d) reactions the structure of the states in the $^{119-125}$Sb isotopes[11] and $^{121-131}$I isotopes[12] has been investigated. The experiments show that the $1g_{7/2}$ state in the Sb isotopes is not fragmented, but the $2d_{5/2}$ state is. It could also be possible that in the ^{119,125}Sb isotopes the fraction of the $2d_{5/2}$ component in the first $5/2^+$ state is somewhat less (60–70%) than in the ^{121,123}Sb isotopes (80%). In the I isotopes, both the $1g_{7/2}$ and $2d_{5/2}$ states are fragmented, but the first only slightly and the second very appreciably and much more than in the Sb isotopes. The fraction of the $2d_{5/2}$ component in the first $5/2^+$ levels in the I nuclei may be from 40 to 60%. The reduced numbers of the fractions of the $2d_{5/2}$ component for Sb and I must be taken as estimates, since in single-nucleon transfer reactions for highly excited states with $l = 2$ the spin is, as a rule, found to be either $5/2^+$ or $3/2^+$. Nevertheless, these estimates do not contradict the data on the E2 transitions, and it is probable that in the ^{119,125}Sb and ^{127,129,131}I isotopes there is a significant coupling of the odd nucleon to the phonon excitations of the core, particularly in the case of I, i.e., we have here a structure of the states more complicated than the single-particle case. However, for both series of isotopes, Sb and I, the same tendency for growth of the hindrance factors of the M1 transitions with increasing number of neutrons in the nucleus is observed. It appears that the proximity of N to the magic number $N = 82$ is the main factor that influences the *l*-forbiddenness of the proton transitions in this region, and this is

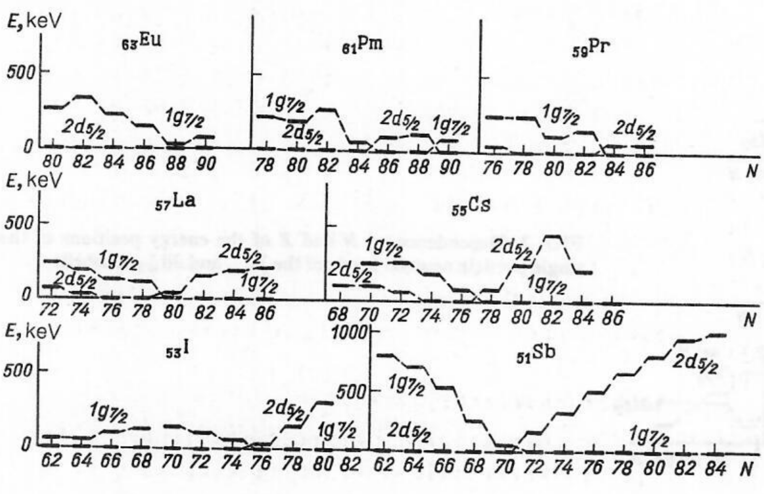

FIG. 3. Dependence on N and Z of the energy positions of the single-particle proton levels of the $2d_{5/2}$ and $1g_{7/2}$ subshells.

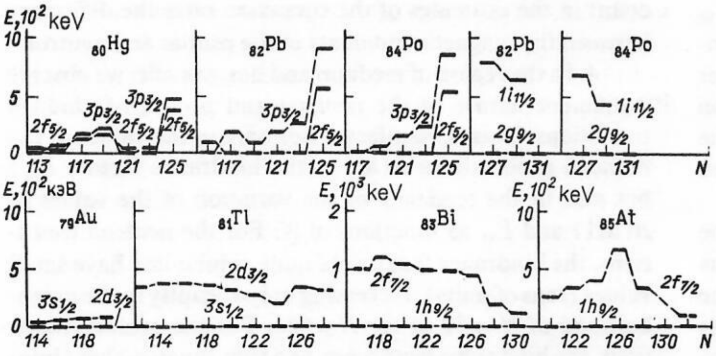

FIG. 4. Dependence on N and Z of the energy positions of the lowest single-particle levels of different subshells in the region of heavy nuclei.

manifested most clearly even though in reality there may in a number of cases be a superposition of other factors influencing the hindrance of the M1 transitions—for example, coupling of the odd nucleon to the phonon excitations of the core or a certain degree of deformation of the nucleus.

3. Transitions in the region of nuclei 193⩽A⩽211

The structure of the states of this transitional region of nuclei is the most complicated. On the one hand, this group of isotopes adjoins the region of stable deformation ($A < 191$) while on the other it is close to the magic numbers $Z = 82$, $N = 126$.

Information on the possible neutron and proton l-forbidden transitions from the lowest levels is here very sparse, and we have included in the analysis transitions from different subshells of the nuclei (Tables V and VI). In systematizing positions of the levels of these subshells we have used information on the Po and At isotopes (Fig. 4), although there are no experimental data for calculations of the reduced probabilities of the l-forbidden transitions for them.

Proton transitions $3s_{1/2} \rightleftarrows 2d_{3/2}$

The proton transitions between the lowest states, which may belong to the $3s_{1/2}$ and $2d_{3/2}$ subshells, are considered in the Tl and Au isotopes (Table V). It can be seen that the hindrance factors F_M for the proton transitions are very high. We observe a monotonic variation of the F_M values as N approaches the magic number $N = 126$. This recalls the situation with the proton transitions in the region of nuclei with $105 \leqslant A \leqslant 151$ when N there approaches $N = 82$. However, there is also a difference; for at $N = 126$ there is a sharp drop in the value of F_M ($^{207}_{81}Tl_{126}$), whereas at $N = 82$ an increase of F_M occurred as a rule (Table IV).

It is interesting to note that despite the very significant change of F_M as a function of N ($Z =$ const) the systematics of the levels of the $3s_{1/2}$ and $2d_{3/2}$ subshells in the Tl and Au isotopes exhibits a very smooth trend undergoing a small jump (Tl isotopes) only at $N = 126$ (Fig. 4).

As N approaches $N = 126$, the values of F_M move in different directions in the Au and Tl nuclei, decreasing in the first case but increasing in the second. A possible reason for this is that the Au nuclei are more deformed than the Tl nuclei. The Au isotopes are near the region of stable deformation $A < 191$ and much further from the doubly magic nucleus $^{208}_{82}Pb_{126}$. The acceleration of the E2 transition admixture in the Au nuclei is greater than in the Tl nuclei, and their importance gradually decreases with increasing N. It is natural to attribute this acceleration of the odd Au isotopes basically to the deformation of the nucleus, since in the even $(A-1)$ core nuclei the acceleration of the E2 transitions also decreases with increasing N in the same proportion. The ratio E_{odd}/E_{core} is practically constant and equal to 0.66–0.69 for the $^{193-199}$Au isotopes (Table V).

The lack of data on single-nucleon transfer reactions does not permit a more detailed analysis of the structure of the $3s_{1/2}$ and $2d_{3/3}$ states in the $^{193-199}$Au nuclei. Information about the structure of the $^{203-207}$Tl isotopes is available from the $(\vec{t}, \alpha)$ reactions[13] and others. In the speherical nucleus $^{207}_{81}Ti_{126}$ the $3s_{1/2}$ and $2d_{3/2}$ states are not fragmented. In th ^{203,205}Tl nuclei, the $3s_{1/2}$ ground state is also hardly fragmented. However, the lowest $3/2^+$ levels carry 60–70% of the $2d_{3/2}$ component, and this indicates a significant coupling of the odd nucleon to the phonon excitations of the core. Nevertheless, despite the fairly complicated structure of the states in the Tl and Au nuclei, the hindrance factors for the M1 transitions in them change as N approaches the magic number $N = 126$ in a manner similar to the analogous factors for the proton l-forbidden transitions in the other regions of nuclei.

Proton transitions $2f_{7/2} \to 1h_{9/2}$

Transitions of this type between the lowest states are observed only in the isotopes $^{209}_{81}Bi_{126}$ and $^{211}_{81}Bi_{128}$ (Table V). However, they are of undoubted interest, since they occur in nuclei for which Z and N are in the immediate vicinity of the magic values $Z = 82$, $N = 126$. Both of the Bi isotopes are spherical nuclei, and in them the transitions are single-particle transitions. The acceleration of the E2-component admixture in them is very small.

As in the case of the Tl isotopes, a larger value of B(M1) (smaller F_M) is observed for the transition at $N = 126$ (^{209}Bi). However, the jump in the values of F_M on the change of N from $N = 126$ to $N = 128$ (^{211}Bi) is not so great as on the change of N from $N = 124$ to $N = 126$ in the transitions in the ^{205,207} Tl nuclei.

The behavior of the states of the $2f_{7/2}$ subshell reflects the change in the values of B(M1) and F_M in the neighborhood of $N = 126$ (Fig. 4), namely, on the transition from $N = 126$ to $N = 128$ the energy of the $2f_{7/2}$ level drops sharply. A jump in the energy of the $2f_{7/2}$ state at $N = 126$ is also observed in the neighboring odd $_{85}$At isotopes (Fig. 4).

Neutron transitions $2f_{5/2} \rightleftarrows 3p_{3/2}$

The characteristics of the neutron transitions of this type are given in Table VI for the Hg isotopes and the nucleus $^{205}_{82}Pb_{125}$. It can be seen that the hindrance factors F_M for the neutron l-forbidden transitions are practically constant, do not change as N approaches the magic number $N = 126$, and are significantly less than F_M for the proton transitions in the nuclei of the given region. This recalls the situation for the neutron l-forbidden transitions in the region $105 \leqslant A \leqslant 151$ (see Sec. 1.2).

For a fuller picture of the phenomenon, it would be interesting to investigate the neutron l-forbidden transitions in the nuclei $^{203,205}Hg$ and $^{205}_{82}Pb_{123}$. As can be seen from the systematics of the energies of the $2f_{5/2}$ and $3p_{3/2}$ subshells (Fig. 4), the positions of the levels in the Hg, Pb, and Po isotopes change abruptly on the transition from $N = 123$ to $N = 125$. It could be that at $N = 123$ there is also a jump in the value of F_M, as was found, for example, in the case of $^{41}_{60}Nd_{81}$ (see Sec. 1.2).

For the nuclei $^{199,201}Hg$ and ^{207}Pb data from single-nucleon transfer reactions are known. It follows, for example, from the results of (d, t) reactions that the $3p_{3/2}$ and $2f_{5/2}$ states in the ^{207}Pb[14] nucleus are hardly fragmented, whereas in the $^{199,201}Hg$ isotopes[15] their fragmentation is very appreciable. The lowest $3/2^-$ levels in ^{199}Hg and ^{201}Hg contain 40% and 60% of the $3p_{3/2}$ component, respectively. Nevertheless, this appreciable difference between the structures of these states in the $^{199,201}Hg$ and ^{207}Pb nuclei does not significantly influence in the hindrance factors of the M1 transitions (Table VI).

Neutron transitions $1i_{11/2} \to 2g_{9/2}$

An l-forbidden M1 transition of this type is observed only in the nucleus $^{209}_{82}Pb_{127}$ (Table VI). However, it is interesting in that it occurs in a nucleus whose even core $^{208}_{82}Pb_{126}$ is a doubly magic spherical nucleus. Accordingly, the nucleus ^{209}Pb can also be assumed to be spherical and the transition in it to be a single-particle transition. As also follows from the (d, p), $(\alpha, {}^3He)$, and (d, n) reactions,[14] the $2g_{9/2}$ and $1i_{11/2}$ states are not fragmented in the ^{209}Pb nucleus. The hindrance factor of the M1 transition $1i_{11} \to 2g_{9/2}$ is appreciably higher ($F_M = 118$) than for the neutron transitions $2f_{5/2} \rightleftarrows 3p_{3/2}$ in the Hg isotopes and the ^{207}Pb nucleus ($F_M = 14$–31).

The systematics of the levels of the $1i_{11/2}$ subshell for $N > 126$ in the Pb and Po isotopes reveals a smooth variation of the energy as a function of N.

CONCLUSIONS

1. Most clearly revealed is the dependence of the reduced probabilities B(M1) of the l-forbidden transitions on the proximity of the number of neutrons in the nucleus to the magic values $N = 20, 28$; $N = 82$, $N = 126$.

2. For the l-forbidden transitions in light nuclei ($27 \leqslant A \leqslant 49$) we do not, as a rule, observe significant differences between the rates of the neutron and proton transitions. The transitions in the Sc isotopes, for which the hindrance factors are much higher than for the remaining isotopes, are an exception. Both the neutron and the proton transitions are characterized by an increase of the hindrance factors (for $Z =$ const) as N approaches $N = 20$ and 28.

3. For l-forbidden transitions in pairs of mirror nuclei that have the same even–even $(A - 1)$ cores, we obtain hindrance factors F_M that are close in value if we take into account in the estimates of the transition rates the difference between the magnetic moments of the proton and neutron.

4. In the region of medium and heavy nuclei we observe a different nature of the neutron and proton l-forbidden transitions. This is manifested not only in the values of the reduced probabilities B(M1) (the hindrance factors F_M) but also in the tendency of the variation of the values of B(M1) and F_M as functions of N. For the neutron transitions, the hindrance factors are quite similar and have small values (tens of units), increasing only abruptly in the neighborhood of $N = 82$ or for $N > 126$. For the proton transitions, the hindrance factors are, as a rule, much higher (hundreds and even thousands of units), and within the groups of isotopes they increase monotonically (in the Cs and Au isotopes they decrease) as N approaches $N = 82$ or $N = 126$.

2. THEORETICAL APPROACHES TO THE DESCRIPTION OF *l*-FORBIDDEN M1 TRANSITIONS

As was first noted in Ref. 16, the residual interaction of the nucleons in the nucleus is responsible for lifting the l-forbiddenness. In accordance with the procedure used to take into account the residual interaction of the nucleons, the theoretical studies devoted to this problem can be nominally divided into two basic groups. In one of them we have the studies in which the choice of some particular form of the residual interaction is used to determine more accurately the wave-function structure of the initial and final states of the nucleus for an unchanged operator of the M1 transition. As a rule, these calculations are made perturbatively and the interaction parameters are not universal.

The studies of the second group are based on the theory of finite Fermi systems.[17] In this theory, the principal effect from taking into account the interaction of the quasiparticles is mainly reflected in a change in the form of the M1-transition operator, specifically, in the appearance of an additional term that lifts the l-forbiddenness. The calculations on the basis of the theory of finite Fermi systems are made with universal coupling constants of the quasiparticles and do not use perturbation theory. Below, we shall discuss in detail the results of the studies of each group on this problem.

1. Calculations based on perturbation theory. Cluster-vibration model

In the framework of the shell model, calculations of the reduced probabilities B(M1) for l-forbidden transitions were made in Refs. 16 and 18. The l-forbiddenness is lifted by taking into account admixtures of configurations with nonvanishing seniority in the wave functions of the states between which the γ transition takes place. The admixtures were found in the first order of perturbation theory in the residual interaction between the nucleons, which had the form

$$V_{12} = (V_0 + V_1 \sigma_1 \cdot \sigma_2)\, \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{1}$$

where V_0 and V_1 are parameters. The results described satisfactorily the experimental data only for nearly magic nuclei, but even in this case the validity of perturbation theory in the system of strongly interacting particles is doubtful. As was shown in Ref. 19 for the example of a calculation in the same scheme of the magnetic moments of nearly magic nuclei, allowance for the second order of perturbation theory already destroys the agreement between theory and experiment obtained in Ref. 16.

Another possibility for lifting the l-forbiddenness is associated with taking into account the interaction of a particle or group of particles (cluster) with low-lying collective excitations of the even–even core. In this approach, the Hamiltonian of the odd nucleus is written in the form

$$H = H_p + H_{col} + H_{int}, \quad (2)$$

where H_p is the Hamiltonian of the particle (cluster), H_{col} is the Hamiltonian of the collective motion in the core nucleus and H_{int} is the interaction Hamiltonian. The various studies that have calculated the magnetic properties of nuclei, including the probabilities $B(M1)$ of l-forbidden γ transitions, differ in the choice of the particular form of these Hamiltonians.[20–31]

In Refs. 20 and 21, in the framework of a generalized model, the influence of the quasiparticle–phonon interaction was estimated for nuclei in the region of ^{208}Pb. The Hamiltonian H_{int} was taken in the form

$$H_{int} = -kr\frac{\partial U}{\partial r}\sum_{\mu}\alpha_{\mu\lambda}Y^*_{\lambda\mu}(\theta, \varphi), \quad (3)$$

where k is a phenomenological parameter, U is the single-particle potential, and λ is the phonon angular momentum. The calculations of the probabilities $B(M1)$ in Refs. 20 and 21 lead to an unsatisfactory description of the experimental data for the l-forbidden transitions $[B(M1)_{exp}/B(M1)_{th} \sim 10]$, the authors seeing the reason for this in the neglect of the spin polarizability of the core.

A computational scheme that takes into account the interaction of the single-particle and collective degrees of freedom of the nucleus was also used in Ref. 22 to calculate the spectra and probabilities of transitions between the low-lying states of the ^{113,115}In isotopes. In this study, the Hamiltonian of the nucleus had the form

$$H = E_0 + \sum_{\lambda} b^+_{\lambda}b_{\lambda}[\hbar\omega_{\lambda} + (2\lambda+1)/2] + \sum_{\alpha}\varepsilon_{\alpha}a^+_{\alpha}a_{\alpha} + \sum_{\alpha,\alpha'}\sum_{\lambda,\mu}\langle\alpha|Y^*_{\lambda\mu}|\alpha'\rangle[b^+_{\lambda\mu} + (-1)^{\mu}b_{\lambda-\mu}]a^+_{\alpha}a_{\alpha'} + \frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta}V_{\alpha\beta\gamma\delta}a^+_{\alpha}a^+_{\beta}a_{\gamma}a_{\delta}. \quad (4)$$

Here, E_0 is the total energy of the ground (0^+) state of the even–even core nucleus ^{114}Sn, b^+_{λ} (b_{λ}) is the operator of creation (respectively, annihilation) of a phonon of multipolarity λ (the 2^+ and 3^- vibrational states were taken into account), and a^+_{α} (a_{α}) is the operator of creation (annihilation) of a proton in the state α. The first stage was to solve the problem of diagonalizing the Hamiltonian in order to describe the low-lying states of the even–even ^{114}Cd nucleus, from which the parameters of the model were chosen. The second step took into account the interaction of the odd proton with the low-lying core excitations and calculated the spectra of the ^{113,115}In isotopes, the spectroscopic factors, the magnetic and quadrupole moments, and the reduced probabilities $B(M1)$ and $B(E2)$. For the E2 transitions the theoretical values $B(E2)$ agreed satisfactorily with experiment, but for $B(M1)$ the agreement was not nearly so good. In particular, for the l-forbidden transitions the $B(M1)_{th}$ values were found to be an order of magnitude less than the experimental probabilities.

The approach based on the general Hamiltonian (2) was developed further in the framework of the so-called cluster–vibration model.[23–31]

The fundamental concept in this model is a dynamical cluster, which consists of three protons (or neutrons) of a 1g–2d–3s shell interacting with quadrupole core vibrations. Accordingly, the Hamiltonian H_{int} is taken in the form

$$H_{int} = k\sum_{i=1}^{3}\sum_{\mu}\alpha_{\mu}Y^*_{2\mu}(\theta_i, \varphi_i), \quad (5)$$

where k is a strength parameter and the summation over i is over the nucleons of the cluster. In addition, a residual interaction between the particles of the cluster is included in H_p [see (2)]. The interaction of the cluster with the collective vibrations of the core nucleus leads to the appearance of states of vibrational and rotational types, and also states associated with excitations of the cluster itself. In the model the M1-transition operator is taken in the form

$$\Gamma(M1) = g_R\mathbf{I} + (g_l - g_R)\mathbf{J} + (g_s - g_l)\mathbf{S} + \left(\frac{3}{4\pi}\right)^{1/2}g_p(Y_2\times\mathbf{S})_1. \quad (6)$$

Here, I is the total angular momentum of the nucleus, and J and S are the total angular momentum and spin of the cluster. For the gyromagnetic ratio g_R the hydrodynamical value $g_R = Z/A$ is used, and the residual interaction of the nucleons is taken into account by a renormalization of the g_s factor of the proton ($g_s \approx 0.5g_s^{(0)}$, where $g_s^{(0)}$ is the spin gyromagnetic ratio for the free nucleon). The last term in (6) takes into account phenomenologically the polarization of the core and the corrections associated with meson exchange currents.

In Ref. 24, this model was used to calculate the spectra and reduced probabilities of M1 and E2 γ transitions between the lowest states of iodine and silver isotopes. A satisfactory description of these characteristics was obtained, including the probabilities of the M1 transitions that are classified in the single-particle model as l-forbidden transitions. The use of the cluster–vibration model to describe the properties of xenon, gold, thallium, and astatine was also fairly successful.

In Ref. 32, the magnetic properties of isotones with neutron number $N = 126$ were investigated. The main attention was devoted to the change in the form of the M1-transition operator due to the meson corrections. This operator was expressed in the form

$$\Gamma(M1) = g_l\mathbf{l} + g_s\mathbf{S} + g_p(\mathbf{Y}_2\times\mathbf{S})_1, \quad (7)$$

the following effective values being used for the proton g factors: $g_l = 1.155$, $g_s = 5.699$, $g_p = -0.063$. The wave functions of the nuclear states were constructed with allowance for the spin polarization of the core in three different schemes: in the first order of perturbation theory, in the Tamm–Dancoff approximation, and in the random-phase approximation, the form of the residual interaction being varied as well. Harmonic-oscillator wave functions were used as basis single-particle functions. Satisfactory agreement with the experimental values of $B(M1)$ of l-forbidden transitions was obtained.

In the recent paper of Ref. 33 calculations were made of reduced probabilities of M1 transitions, including l-forbidden ones, for a number of Cs and Xe isotopes in the framework of the model of interacting fermions and bosons. There are various versions of models of this type, which are currently being strongly developed (a review of these models can be found, for example, in Ref. 34). The mathematical formalism of these models is rather complicated, and therefore, omitting the details of the actual calculation,[33] we note merely that the theory gives values of $B(M1)$ that are too large compared with the experimental values.

A general shortcoming of studies of this group is the use, at some stage, of perturbation theory with respect to the internucleon interaction, which is not small in nuclei, and the absence of universal parameters of this interaction as well as of the particle (cluster)–phonon interaction. As a consequence, it is necessary to vary the parameters that are introduced, sometimes even for neighboring isotopes. However, it should be noted that, except for Refs. 16 and 18, the authors not only aim at a calculation of the probabilities of l-forbidden transitions but also attempt to describe theoretically a large number of experimental data on particular isotopes, including their magnetic properties.

2. Approaches based on the theory of finite Fermi systems

According to the theory of finite Fermi systems,[17] the effect of an external field, given by an operator V_0, in a nucleus is to give rise to an effective field (operator V) whose matrix elements determine the probability of a γ transition. To find V, it is necessary to solve an integral equation having the operator form

$$V = e_q V_0 + \mathcal{F} A V. \tag{8}$$

Here, $\mathcal{F}$ is the quasiparticle scattering amplitude, irreducible in the particle–hole channel; A is the particle–hole propagator; and e_q is the effective charge of the quasiparticle with respect to the given type of field. In the case of magnetic-dipole transitions, the operator V_0 acts on the spin coordinates of the nucleons (we shall consider the structure of V_0 below). Accordingly, in the scattering amplitude $\mathcal{F}$ we can retain only the terms that include the spin operators. In the momentum representation,

$$\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_\pi + \mathcal{F}_{1s}. \tag{9}$$

Here

$$\mathcal{F}_0 = C_0\,(g_0 \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + g_0' \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2), \tag{10}$$

$\mathcal{F}_\pi$ is the irreducible amplitude of the one-pion exchange interaction in the annihilation channel[35] (here and below we use a system of units with $\hbar = m_e = c = 1$):

$$\mathcal{F}_\pi = -1.38 C_0 (1-2\zeta_s)^2\, \boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}\gamma_\pi(k^2)\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2; \tag{11}$$

$$\gamma_\pi(k^2) = \left[m_\pi^2 + k^2 - \frac{0.9\,(1-\alpha)\,k^2}{1+0.23k^2/m_\pi^2} \right]^{-1}; \tag{12}$$

$\mathcal{F}_{1s}$ is the amplitude of the two-particle spin–orbit interaction:

$$\mathcal{F}_{1s} = \frac{1}{2} C_1 (\varkappa + \varkappa' \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,[\mathbf{p}_{12}, \mathbf{k}]\cdot(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2). \tag{13}$$

In Eqs. (10)–(13), $\boldsymbol{\sigma}_1$ and $\boldsymbol{\tau}$ are the spin and isospin operators, $\mathbf{k}$ is the momentum transferred in the process of interaction of two quasiparticles, m_π is the pion mass, $\mathbf{p}_{12} = \mathbf{p}_1 - \mathbf{p}_2$, where $\mathbf{p}_i$ is the nucleon momentum operator, g_0, g_0', ζ_s, α, $\varkappa$, $\varkappa'$ are the universal constants of the theory of finite Fermi systems, and C_0 and C_1 are factors introduced to make the quantities dimensionless.

The studies of the various authors based on the theory of finite Fermi systems and devoted to the problem under discussion differ in the allowance made for the various terms in the scattering amplitude $\mathcal{F}$ and the method of solving the equation for finding the effective field V. Equation (8) was used for the first time to investigate l-forbidden M1 transitions by Khodel'.[36] He showed that the solution of this equation has in the general case the structure

$$V = \alpha(r)\,\boldsymbol{\sigma} + \beta(r)\,\mathbf{n}\,(\boldsymbol{\sigma}\cdot\mathbf{n}),$$

where $\alpha(r)$ and $\beta(r)$ are certain functions [their form depends on the choice of the particular system (nucleus)], and $\mathbf{n} = \mathbf{r}/r$. Because of the presence of the term $\mathbf{n}(\boldsymbol{\sigma}\cdot\mathbf{n})$, due to the spin–spin interaction of the quasiparticles in the nucleus, the l-forbiddenness is lifted. Although the reduced probabilities of specific transitions were not calculated in Ref. 36, a general estimate gives $B(M1) \sim 10^{-2}$ in units of the nuclear magneton, and this corresponds in order of magnitude to the experimental data.

Numerical calculations of the magnetic properties of nuclei including the probabilities $B(M1)$ of l-forbidden transitions, were made in Refs. 37–49. The most extensive calculations together with a systematization of the experimental data are given in Ref. 37. In this paper, Eq. (8) was solved in the λ representation ($\lambda = n, l, j, m$) using an oscillator basis with allowance for pairing of the quasiparticles. In the amplitude $\mathcal{F}$ the terms $\mathcal{F}_\pi$ and $\mathcal{F}_{1s}$ were not included ($\mathcal{F} = \mathcal{F}_0$), and the constants g_0 and g_0' were used as free parameters. Their values were found by requiring the best agreement between $B(M1)_{\text{th}}$ and the experimental values for all the considered groups of nuclei (transitions in nuclei in the range of mass numbers $61 \leqslant A \leqslant 205$ were studied). The operator of the external field V_0 had the form (in nuclear-magneton units)

$$e_q V_0 = \tilde{g}_s \mathbf{s} + \tilde{g}_l \mathbf{l}; \tag{14}$$

$$\tilde{g}_s^{(n)} \equiv e_q g_s^{(n)} = g_s^{(n)}(1-\zeta_s) + g_s^{(p)}\zeta_s;\ \tilde{g}_s^{(p)} \equiv e_q g_s^{(p)} = g_s^{(p)}(1-\zeta_s) + g_s^{(n)}\zeta_s; \tag{15}$$

$$\tilde{g}_l^{(n)} = \zeta_l g_l^{(p)};\ \tilde{g}_l^{(p)} = (1-\zeta_l)\, g_l^{(p)};$$

here, ζ_l is the constant of the theory of finite Fermi systems, and g_s and g_l are the g factors of free nucleons.

It was found that for all the considered groups of nuclei (2p–1f, 1g–2d–3s shells and the region of ^{208}Pb) universal values of the constants g_0 and g_0' can be obtained, but the theoretical and experimental values of $B(M1)$ in the 2p–1f and 1g–2d–3s shells differ on the average by a factor of 3, and in the region of ^{208}Pb by 20 times.

In Refs. 38–41, a study was made of the magnetic properties, including the l-forbidden M1 transitions, of the nearly magic nuclei ^{207}Tl, ^{207,209}Pb, and ^{209}Bi. Equation (8) was also solved in the λ representation, and the terms $\mathcal{F}_\pi$ and $\mathcal{F}_{1s}$ were not included in the anplitude $\mathcal{F}$; however, the single-particle basis functions were calculated in the Woods–Saxon potential. The most important point in Refs. 38–41 was the introduction into the external-field operator V_0 [see (14)] of the tensor term $(3/4\pi)^{1/2} kr^2 (Y_2 \times \boldsymbol{\sigma})_1$, which lifts the l-forbiddenness, the factor k being regarded as free. Because of the presence in the operator V_0 of this term, it was possible to obtain a satisfactory description of the l-forbidden transitions for the four known transitions in these nuclei $B(\mathrm{M1})_{\mathrm{exp}}/B(\mathrm{M1})_{\mathrm{th}} \sim 3$ on the average, i.e., this ratio reached the same level as for the l-forbidden M1 transitions in the other groups of nuclei].

In Ref. 40, which used the same scheme of calculation, terms associated with exchange of π and ρ mesons were added to the scattering amplitude $\mathscr{F}_0$. As a result, a much better description of the l-forbidden transitions in the ^{207}Tl, ^{207,209}Pb, and ^{209}Bi isotopes was obtained than in Refs. 38 and 39.

The calculation of the magnetic properties of the ^{205}Tl isotope in Ref. 41 went beyond the framework of the purely single-quasiparticle approximation, although the scheme for calculating the operator of the effective field was the same as in Ref. 38. The wave functions of the states of the ^{205}Tl nucleus were constructed as three-quasiparticle states, the diagonalization problem being solved as a preliminary in order to obtain the states of the core nucleus ^{206}Pb. For the construction of the wave functions of the odd nucleus ^{205}Tl, an interaction of the odd proton with the neutron hole was introduced and the problem of diagonalizing the interaction Hamiltonian was again solved. The interaction of the proton and the neutron hole was studied in three forms: surface, contact, and dependent on the density. The best results were obtained with the density-dependent forces. In particular, for the l-forbidden $3/2^+–1/2^+$ transition $B(\mathrm{M1})_{\mathrm{exp}}/B(\mathrm{M1})_{\mathrm{th}} \approx 5$.

From the theoretical point of view, the scheme for calculating the magnetic properties of nuclei used in Refs. 38–41 can be questioned,[17] since the most important role for the l-forbidden transitions in it is played by the empirical term $r^2(Y_2\times\sigma)_1$, which lifts the l-forbiddenness. Its introduction is equivalent to specifying an effective quasiparticle charge e_q of the same form. But this structure of the operator of the local charge e_q contradicts translational invariance and has the wrong limit as $A\to\infty$ (the local charge increases as $A^{2/3}$ instead of decreasing or at least tending to a constant). At the same time, a good description of the l-forbidden M1 transitions in the region of lead could be obtained mainly only by the introduction of this term.

The unsatisfactory nature of the approach proposed in Refs. 38–41 and the large discrepancy between the theoretical and experimental values of $B(\mathrm{M1})$ for the l-forbidden transitions obtained in Ref. 37 stimulated further investigations of the magnetic properties of the nuclei in the region of lead based on the theory of finite Fermi systems. In Ref. 42, the influence of the one-pion exchange interaction of the quasiparticles on the $B(\mathrm{M1})$ of l-forbidden transitions was studied. The quasiparticle scattering amplitude was taken in the form $\mathscr{F} = \mathscr{F}_0 + \mathscr{F}_\pi$, and the external-field operator V_0 in the form (14). Equation (8) for the effective field V was solved in the coordinate representation, and this made it possible to overcome the shortcomings of the λ representation due to the use of a restricted single-particle basis. As a result of the calculations that were made, it was concluded that the probabilities of the l-forbidden M1 transition are sensitive to $\mathscr{F}_\pi$ and to the value of g_0'. However, the enhancement of $B(\mathrm{M1})$ sufficient to explain the experimentally observed values was obtained only in the region of g_0' values for which the nuclear matter becomes unstable with respect to formation of a π condensate. Therefore, the problem remained open, and as a possible explanation for the enhancement of the l-forbidden transitions the authors of Ref. 42 suggested one-pion exchange in the scattering channel, though estimates were not made.

The importance of the spin–orbit interaction $\mathscr{F}_{1s}$ of the quasiparticles in lifting the l-forbiddenness was estimated in Refs. 43–46. When Eq. (8) is solved in the case of electromagnetic fields, the amplitude $\mathscr{F}_{1s}$ is manifested in two ways. First, being included in the polarization term $\mathscr{F}AV$ in Eq. (8), it leads, like the amplitude $\mathscr{F}_0$, to a renormalization of the external field and a lifting of the l-forbiddenness. Second, when an external electromagnetic field is applied to the system (the nucleus), the amplitude $\mathscr{F}_{1s}$ itself is changed, since it depends on the momenta of the interacting nucleons, and these are replaced when the electromagnetic field is applied by the generalized momenta, i.e., $\mathbf{p}$ is replaced by $\mathbf{p}_i - e\mathbf{A}(\mathbf{r}_i)$, where $\mathbf{A}(\mathbf{r}_i)$ is the vector potential. As a result, the two-particle charge-exchange current between the nucleons is changed, and, as a result, there are additional corrections to the magnetic moments and probabilities of M1 transitions in the nuclei. Corrections of such kind to the magnetic moments were obtained in Refs. 50 and 51, and to the M1-transition operator in Ref. 43. With allowance for this change, the external-field operator V_0 can be represented in the form[44-46]

$$e_qV_0 \to e_qV_0^{(1s)} = e_qV_0 + \Delta V^{(1s)}$$
$$= \sqrt{4\pi}\,\{V_0^{(0)}(r)\,T_{10}^{\mu}(\sigma) + V_0^{(2)}(r)\,T_{12}^{\mu}(\sigma) + g_l T_{10}^{\mu}(\mathbf{j})\}. \quad (16)$$

Here, in the case of an M1 transition of an odd neutron

$$V_0^{(0)}(r) = \frac{1}{2}\,\tilde{g}_s^{(n)} - 2C_1 m(\varkappa-\varkappa')\,\frac{Z}{A}\Big[\rho(r) + \frac{2}{3}\,r\,\frac{d\rho}{dr}\Big]; \quad (17)$$

$$V_0^{(2)}(r) = -\frac{\sqrt{2}}{3}\,C_1 m(\varkappa-\varkappa')\,\frac{Z}{A}\,r\,\frac{d\rho}{dr}, \quad (18)$$

and in the case of an M1 transition of an odd proton

$$V_0^{(0)}(r) = \frac{1}{2}\,(g_s^{(p)}-1) - 2C_1 m\Big[(\varkappa-\varkappa')\,\frac{Z}{A}\,\rho(r) - \frac{2}{3}(\varkappa-\varkappa')\,\frac{N}{A}\,r\,\frac{d\rho}{dr}\Big]; \quad (19)$$

$$V_0^{(2)}(r) = \frac{\sqrt{2}}{3}\,C_1 m(\varkappa-\varkappa')\,\frac{N}{A}\,r\,\frac{d\rho}{dr}. \quad (20)$$

For convenience, we have gone over to spherical notation, introducing the tensor operators

$$T_{KL}^{\mu}(\mathbf{a}) = \sum_v C_{1vL\mu-v}^{K\mu}\,a_v Y_{L\mu-v}, \quad (21)$$

where a_v are the spherical components of the vector $\mathbf{a}$ and in the expressions (17)–(20) $\rho(r)$ is the nuclear density. As follows from (16) the action of the spin–orbit forces has the consequence that in the operator that produces the M1 transition a term containing the tensor $T_{12}(\sigma)$, which lifts the l-forbiddenness, has appeared.

Equation (8) with the operator V_0 in the form (16) and with the scattering amplitude $\mathscr{F} = \mathscr{F}_0 + \mathscr{F}_{1s}$ was solved in the coordinate representation in Ref. 44 in the calculation of the magnetic properties of nuclei in the region of lead. It was found that allowance for the spin–orbit interaction of the quasiparticles also does not solve the problem of obtaining a satisfactory description of the l-forbidden M1 transitions in this region.

The l-forbidden M1 transitions in the region of the 1g–2d–3s shell ($A = 111$–141) were investigated in Refs. 45–47. In this region, in contrast to the region of Pb, pairing of the nucleons is important. The influence of the quasiparticle–phonon interaction on the probabilities $B(\mathrm{M1})$ was estimated in Ref. 47. In contrast to Ref. 37, single-particle wave functions obtained with the Woods–Saxon potential were used to solve the equation for the effective field in the λ representation. The characteristics of the quadrupole phonon (the 2_1^+ states of the even–even core were considered) and of the quasiparticle–phonon interaction amplitudes were also calculated in the framework of the theory of finite Fermi systems. It was shown that although an interaction of this type also lifts the l-forbiddenness, it does not enable one to reduce significantly the discrepancy between the theoretical and experimental $B(\mathrm{M1})$ values, which, in individual cases, was found to be about an order of magnitude [allowance for the quasiparticle–phonon interaction changed the value of $B(\mathrm{M1})$ by not more than 10–15%].

In Refs. 45 and 46 the part played by the spin–orbit interaction for nuclei in the same region was investigated. Equation (8) was also solved in the λ representation, the operator V_0 was taken in the form (16), and the scattering amplitude in the form $\mathcal{F} = \mathcal{F}_0$. As a result of the calculations it was concluded that allowance for the spin–orbit interaction in this region of mass numbers ($111 \leqslant A \leqslant 141$) permits a significant improvement of the situation with regard to the l-forbidden transitions, an important role being played here, in contrast to the region of lead, precisely by the replacement of the operator V_0 in the form (14) by the operator $V_0^{(\mathrm{ls})}$ (16). However, since in the numerical calculations Eq. (8) was solved in the λ representation and one-pion exchange was not taken into account, the fact that the role of this exchange was not elucidated and also the unknown contribution of virtual transitions with the participation of the continuum meant that the problem of quantitative explanation of the $B(\mathrm{M1})$ values remained open for this group of nuclei too.

3. Equations for calculating the effective field of M1 transitions with allowance for the continuum and quasiparticle pairing

In Refs. 42–44, Eq. (8) for the effective field of the M1 transitions was solved in the coordinate representation. This method makes it possible when one is solving the equation for the effective field to avoid the difficulties associated in practice with the limitation of the single-particle basis when the λ representation is used. However, the method of solving Eq. (8) in the coordinate representation developed in Ref. 52 and used in Refs. 42–44 could be used only for nearly magic nuclei, for which the pairing interaction of the nucleons is unimportant. But the overwhelming majority of the known l-forbidden M1 transitions take place to shells in which the pairing effect cannot be ignored (see Sec. 1). In addition, the inclusion of one-pion exchange (the amplitude $\mathcal{F}_\pi$) in the scattering amplitude $\mathcal{F}$ makes it quite impossible to solve Eq. (8) in the λ representation. Therefore, to make calculations of $B(\mathrm{M1})$ in nuclei with developed pairing and with inclusion in $\mathcal{F}$ of the amplitude $\mathcal{F}_\pi$, it is necessary to transform the computational scheme in such a way that, on the one hand, the advantages of the solution in the coordinate representation are retained, while, on the other, the pairing interaction of the quasiparticles can be taken into account. This problem was solved in Refs. 48 and 49.

The calculation of the particle–hole propagator A creates the greatest difficulty in the solution of Eq. (8). For nearly magic nuclei, the problem of constructing A was solved in Ref. 52, in accordance with which $A(\mathbf{r}_1, \mathbf{r}_2; \omega)$ has the following form in the coordinate representation:

$$A(\mathbf{r}_1, \mathbf{r}_2; \omega) = \sum_\lambda n_\lambda \varphi_\lambda(\mathbf{r}_1) \varphi_\lambda^*(\mathbf{r}_2) \times [G(\mathbf{r}_1, \mathbf{r}_2; \varepsilon_\lambda + \omega) + G(\mathbf{r}_1, \mathbf{r}_2; \varepsilon_\lambda - \omega)]. \quad (22)$$

Here, ε_λ and $\varphi_\lambda(\mathbf{r})$ are the single-particle energies and corresponding wave functions (we recall that $\lambda = n, l, j, m_j \equiv \nu, m_j$); n_λ are the population numbers of the corresponding states λ; $G(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is the Green's function of the single-particle Schrodinger equation, and it can be expressed in the standard manner in terms of the solutions of this equation that are regular and irregular at the origin; finally, ω is the transition energy.

We use the circumstance that in the calculation of the nuclear polarizability [the second term on the right in Eq. (8)] the pairing effect introduces the most important correction only to the virtual particle–hole excitations near the Fermi surface (as a rule, within a shell with incompletely filled states). One can then use a mixed representation, representing the propagator A in the form

$$A \to A_s(\mathbf{r}_1, \mathbf{r}_2; \omega) = A(\mathbf{r}_1, \mathbf{r}_2; \omega) - A'(\mathbf{r}_1, \mathbf{r}_2; \omega) + L'(\mathbf{r}_1, \mathbf{r}_2; \omega). \quad (23)$$

Here, $A(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is, as before, the propagator (22) without allowance for pairing; we define $A'(\mathbf{r}_1, \mathbf{r}_2; \omega)$ as the part of the total propagator A that includes a summation only over the states of the incompletely filled shell:

$$A'(\mathbf{r}_1, \mathbf{r}_2; \omega) = {\sum_{\lambda\lambda'}}' \frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'} - \omega} \varphi_\lambda(\mathbf{r}_1) \varphi_\lambda^*(\mathbf{r}_2) \varphi_{\lambda'}^*(\mathbf{r}_1) \varphi_{\lambda'}(\mathbf{r}_2), \quad (24)$$

and, finally, $L'(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is the propagator constructed by means of the Green's function of the system with pairing and also including a summation over the states of the incompletely filled shell [this fact is reflected, as in (24), by the prime on the summation sign]:

$$L'(r_1, r_2; \omega) = -{\sum_{\lambda\lambda'}}' \eta_{\lambda\lambda'}^{(-)} (E_{\lambda\lambda'} \eta_{\lambda\lambda'}^{(-)} + \omega \eta_{\lambda\lambda'}^{(+)}) (E_{\lambda\lambda'}^2 - \omega^2)^{-1} \varphi_\lambda(\mathbf{r}_1) \varphi_\lambda^*(\mathbf{r}_2) \times \varphi_{\lambda'}^*(\mathbf{r}_1) \varphi_{\lambda'}(\mathbf{r}_2). \quad (25)$$

Here,

$$E_{\lambda\lambda'} \equiv E_\lambda + E_{\lambda'}; \quad E_\lambda = (\tilde{\varepsilon}_\lambda^2 + \Delta_\lambda^2)^{1/2};$$

$$\eta_{\lambda\lambda'}^{(\pm)} = u_\lambda v_{\lambda'} \pm u_{\lambda'} v_\lambda;$$

$$u_\lambda = \left[\frac{1}{2}\left(1 + \frac{\tilde{\varepsilon}_\lambda}{E_\lambda}\right)\right]^{1/2}; \quad v_\lambda = \left[\frac{1}{2}\left(1 - \frac{\tilde{\varepsilon}_\lambda}{E_\lambda}\right)\right]^{1/2}; \quad (26)$$

$\tilde{\varepsilon}_\gamma$ are the single-particle energies measured from the corresponding chemical potential, and Δ_λ is the energy gap.

The proposed method of calculating the propagator $A_s(\mathbf{r}_1, \mathbf{r}_2; \omega)$ preserves the scheme for calculating the propagator $A(\mathbf{r}_1, \mathbf{r}_2; \omega)$, and this enables one in a problem with nucleon pairing to take into account fully the contribution of the continuum when calculating the nuclear polarizability. Therefore we obtain the possibility of calculating magnetic and also β-decay (see Ref. 53) properties of nuclei that are far from magic, preserving all the advantages of the coordinate representation. Although the calculation of A' and L' involves summation over the quasiparticle states λ and λ', the stringent selection rules for the spin operators mean that the number of terms is small, and it is not particularly difficult to take them into account in Eq. (8). For actual examples, we estimated the accuracy of the proposed method of extending the region of summation to shells neighboring the incompletely filled shell. We found that this extension of the region of summation hardly changes the transition probabilities (by not more than 2–3%).

The computational scheme of Refs. 48 and 49 uses an external-field operator V_0 in the form (16), which takes into account the contribution due to the change of the spin–orbit interaction $\mathscr{F}_{ls}$ in the electromagnetic field. As the numerical calculations made in Refs. 43 and 44, and ours also, showed, once the spin–orbit corrections have been introduced into the transition operator V_0 the inclusion in the quasiparticle scattering amplitude $\mathscr{F}$ of the spin–orbit term $\mathscr{F}_{ls}$ has little influence on $B(\mathrm{M1})$. Therefore, in what follows we keep the amplitude $\mathscr{F}$ in the form $\mathscr{F}=\mathscr{F}_0+\mathscr{F}_\pi$ [see (10) and (11)].

If we expand the amplitude $\mathscr{F}$ and the effective field V with respect to the spherical tensor operators $T^{\mu}_{KL}(\sigma)$,

$$\mathscr{F}(\mathbf{r}_1,\ \mathbf{r}_2)=\sum_{KL_1L_2\mu}\mathscr{F}^{(K)}_{L_1L_2}(r_1,\ r_2)\,[T^{\mu}_{K,L}(\sigma)]^{+}\,T^{\mu}_{KL_2}(\sigma);\quad (27)$$

$$V(r)=\sqrt{4\pi}\left\{\sum_{L=0,2}v^{(L)}(r)\,T^{\mu}_{1L}(\sigma)+g_l T^{\mu}_{10}(\mathbf{j})\right\},\quad (28)$$

then after integration in Eq. (8) over the angular variables and summation over the magnetic quantum numbers we obtain the following system of radial equations for determining the functions $v^{(L)}(r)$:

$$v^{(L)}(r)=e_q V_0^{(L)}+\int_0^{\infty} r_1^2\,dr_1\int_0^{\infty} r_2^2\,dr_2\sum_{L_1L_2}\mathscr{F}^{(1)}_{LL_1}(r,\ r_1)\times A_{sL_1L_2}(r_1,\ r_2;\ \omega)\,v^{(L_2)}(r_2).\quad (29)$$

Here, each of the indices L, L_1, L_2 takes the values 0 and 2. The functions $V_0^{(0)}(r)$ and $V_0^{(2)}(r)$ are determined by Eqs. (17) and (18) for the neutron γ transitions and by Eqs. (19) and (20) for the proton γ transitions.

The form of the functions $\mathscr{F}^{(1)}_{L_1L_2}(r_1, r_2)$ and $A_{S,L_1L_2}(r_1, r_2)$ is given in Ref. 35.

The reduced probability $B(\mathrm{M1})$ of the l-forbidden transition $j_i\to j_f$ is determined as follows:

$$B(M1)=(2j_i+1)^{-1}\,|M_{j_i\to j_f}(M1)|^2,\quad (30)$$

where the nuclear matrix element in the single-quasiparticle approximation has the form

$$M_{j_i\to j_f}(M1)=\sqrt{4\pi}\,\langle\nu_f\,||v^{(2)}(r)\,T_{12}(\sigma)||\,\nu_i\rangle\,\xi^{(-)}_{\nu_i\nu_f}\quad (31)$$

and

$$\xi^{(-)}_{\nu_1\nu_2}=u_{\nu_1}u_{\nu_2}+v_{\nu_1}v_{\nu_2}.\quad (32)$$

Thus, to obtain the reduced probability $B(\mathrm{M1})$ of the l-forbidden transition we must solve Eq. (29), which takes into account the spin, spin–orbit, and one-pion exchange interactions of the quasiparticles, and, in addition, pairing of the nucleons in the nucleus is also taken into account.

4. Results of calculations and their discussion

The single-particle energies, wave functions, and Green's functions needed to solve Eq. (29) were calculated for the Woods–Saxon potential with inclusion of spin–orbit and Coulomb (for the protons) terms with the parameters taken from Ref. 54. The nuclear density $\rho(r)$ [see (17)–(20)] was taken in the form $\rho(r)=\rho_0[1+\exp(r-R)/a]^{-1}$, $\rho_0=9.6\times10^6$ $(=0.17\ \mathrm{F}^{-3})$, $R=3.1\times10^{-3}A^{1/3}$, $a=1.51\times10^{-3}$. For the constants that parametrize the quasiparticle scattering amplitude $\mathscr{F}$ we used the values from the monograph of Ref. 17 and from Ref. 43. In the calculation of the chemical potentials and the values of $\eta^{(+)}_{\lambda\lambda'}$, $\xi^{(-)}_{\lambda\lambda'}$, the dependence of the energy gap Δ_λ on the state λ was, as a rule, ignored and the values of Δ_λ were taken from Ref. 55. An exception was made only for the calculation of the characteristics of the neutron γ transitions in nuclides with $67\leqslant N\leqslant 81$, for which there is an inversion in the population of the neutron subshells $3s_{1/2}$ and $1i_{11/2}$. The introduction into the calculation of a dependence of Δ_λ on the state λ in these nuclides improves the description of the neutron Fermi surface (for more details, see Ref. 48).

Analysis of the contribution of the individual components of the quasiparticle scattering amplitude to the renormalization of the M1-transition operator shows that to explain the experiments it is not sufficient to take into account only the spin interaction of the quasiparticles. The spin–orbit correction to the M1-transition operator has an important influence on the agreement between the theoretical and experimental values of $B(\mathrm{M1})$. This makes it possible to obtain an approximate estimate of the probability of an l-forbidden M1 transition in the single-particle approximation. Ignoring the residual interaction of the quasiparticles but using as the transition operator V_0^{ls} [see (16)], we obtain

$$B(M1)=4\pi\,(2j_i+1)^{-1}|\,\langle\nu_f||V_0^{(2)}(r)\,T_{12}(\sigma)||\nu_i\rangle\,|^2\,(\xi^{(-)}_{\nu_i\nu_f})^2.\quad (33)$$

This expression was used in a number of cases to estimate the $B(\mathrm{M1})$ of γ transitions between excited states. If one goes further and takes into account the fact that to terms of order $(a/R)^2$ one can set $\partial\rho/\partial r\approx-\rho_0\delta(r-R)$, ignore the pairing ($\xi^{(-)}_{v_iv_f}\approx1$), and use as the functions $R_\nu(r)$ harmonic-oscillator wave functions, then, independently of the values of v and the mass number A, we obtain the approximate single-particle estimate (in nuclear-magneton units) $B(\mathrm{M1})=7\times10^{-2}$. This value can be used as a unit of measurement for estimating the lifetime of nuclear states with respect to l-forbidden transitions.

In our theory, we limited ourselves to the single-quasiparticle scheme based on a spherical basis, but the possibilities of the theory of finite Fermi systems enable one, in principle, to take into account not only the deformation of the nucleus[17] but also, for example, interaction of a quasiparticle with phonons.[56,57] The realization of both possibilities is associated with a significant complexification of the computational scheme, although the necessity for this need not always arise. First, if the deformation is small and the states between which the γ transition takes place are correctly identified, then in the wave functions the main components will still be the ones that remain on the transition to the spherical limit, and our scheme constructed on the spherical basis will be a reasonably good approximation.

Second, the addition to the single-quasiparticle states of a collective component through inclusion of a quasiparticle–phonon interaction also cannot always significantly change the probability $B(\mathrm{M1})$ calculated in the single-quasiparticle approximation.[47] A role is played here by the rather stringent selection rules for the M1 transition and also by the mutual cancellation in the matrix element of individual terms that arise as a result of the allowance made for the collective excitations of the core. Of course, exceptions are also possible.

Our single-quasiparticle scheme takes into account to the maximum extent possible the spin polarization of the core, which, as is well known, is most important precisely for M1 transitions. Therefore, if a significant deviation of the results of a calculation from the experiments is found, this will, for the reasons listed above, indicate a need to establish the particular reasons for the failure of the computational scheme, one of which may be incorrect identification of the nuclear states.

We discuss separately for each of the considered groups of nuclei the results of the calculations of the probabilities $B(\mathrm{M1})$ (see Sec. 1).

a) The region $27 \leqslant A \leqslant 49$. The nuclei of this region are rather complicated for calculations, since, on the one hand, the pairing constants Δ_λ are not always known and, on the other, the small number of particles in the shell means that few-particle configurations, for example, three-quasiparticle ones, play a relatively important part. We considered only neutron l-forbidden transitions of the type $2s_{1/2} \leftrightarrows 1d_{3/2}$ in nuclei with $N = 19$. We obtained the following values of $B(\mathrm{M1})$: 1.17 for ^{35}S, 1.94 for ^{37}Ar, and 1.27 for ^{39}Ca (all values are given in units of $10^{-2}\mu_N^2$). Comparison of these values with the experimental $B(\mathrm{M1})$ reveals good agreement for the isotopes Ar and Ca and somewhat less good agreement for ^{35}S (see Table I). In Sec. 1.1, we noted that the $B(\mathrm{M1})$ values are nearly equal for the proton and neutron transitions. This is evidently due to the similar structure of the neutron and proton states between which the transitions in the light nuclei take place—in the two cases the neutrons and protons fill the same subshells.

b) The region $105 \leqslant A \leqslant 151$. In this region of mass numbers, calculations of $B(\mathrm{M1})$ were made for practically all M1 transitions to the ground state, and also for some M1 transitions between excited states (39 transitions in all). The results are given in Tables III and IV. It can be seen from these tables that for the overwhelming majority of γ transitions our calculated $B(\mathrm{M1})$ values differ from the experimental values by less than two times. This justifies the assumption that such γ transitions are basically single-quasiparticle transitions. However, there are three l-forbidden γ transitions for which the discrepancy between $B(\mathrm{M1})_{\mathrm{th}}$ and $B(\mathrm{M1})_{\mathrm{exp}}$ is appreciable (about an order of magnitude): for ^{125}Sb, ^{141}Nd, and ^{143}Pm. This must be regarded as an indication of a need to look for the reasons for such a discrepancy. A role could be played here by the quasiparticle–phonon interaction, by the presence of three-quasiparticle configurations, and other factors. There could also be as yet unknown factors that single out, by virtue of their hindrance factors, the M1 transitions in nuclei with numbers of neutrons or protons equal to 81–83 [see Sec. 1 and also (c) below].

As we noted in Sec. 1.2, the probabilities $B(\mathrm{M1})$ of the proton transitions $2d_{5/2} \leftrightarrows 1g_{7/2}$ are much less than the probabilities of the neutron transitions $3s_{1/2} \leftrightarrows 2d_{3/2}$. Our computational scheme reflects this fact. The analysis shows that the main reasons are both the difference of the states between which the M1 transition takes place (with increasing orbital number l of the states the radial integrals decrease) and the different structure of the neutron and proton Fermi surfaces, which influences the population numbers of the corresponding single-particle states.

c) The region $193 \leqslant A \leqslant 211$. Calculations of $B(\mathrm{M1})$ for l-forbidden transitions in the nuclei ^{207}Ti, ^{207,209}Pb, and ^{209}Bi were made in Ref. 44. Our scheme for calculating the nearly magic nuclei differs from that of Ref. 44 only by the introduction in the scattering amplitude $\mathscr{F}$ of the term $\mathscr{F}_\pi$ [see (11)]. Our calculations of $B(\mathrm{M1})$ for the ^{207,209}Pb isotopes gave values practically equal to those of Ref. 44, i.e., values that differed from the experimental values by 1–2 orders of magnitude (we recall that a similar situation for the nuclei of this region was also noted in Ref. 37). Our result is not unexpected, since the allowance for the amplitude $\mathscr{F}_\pi$ made earlier in Ref. 42 also failed to improve the agreement between theory and experiment (it is true that in Ref. 42 the spin–orbit interaction was not taken into account).

CONCLUSIONS

We formulate some general conclusions of our study of magnetic dipole transitions that belong to the l-forbidden category.

1. Transitions in nuclei in which pairing of the nucleons is important can be very satisfactorily described both in the semiphenomenological models of the cluster–vibration type and in the microscopic theories based on the theory of finite Fermi systems. At the least, for the overwhelming majority of transitions the single-quasiparticle computational scheme of the theory of finite Fermi systems, which has no adjustable parameters and takes into account the pairing of the nucleons, the spin polarization of the core, and the spin–orbit corrections to the M1-transition operator, makes it possible to reflect the experimental behavior of the $B(\mathrm{M1})$ values and reproduce their values to within a coefficient 2 for a large group of transitions.

Microscopic calculations for this group of nuclei showed that the spin–orbit interaction of the quasiparticles plays the main role in lifting the l-forbiddenness. With regard to the quasiparticle–phonon interaction, which may be important for forming the quasiparticle states, its influence on the $B(\mathrm{M1})$ values of l-forbidden transitions is not apparently decisive apart from individual exceptions.

In the quantitative calculations it is still not clear to what extent the deformation of the nucleus affects the $B(\mathrm{M1})$ values of transitions that belong to the l-forbidden category. This has become a topical problem in connection with the discovery in nuclei regarded as spherical of rotational bands [for example, in $^{117-127}$I (Refs. 58 and 59), $^{119-133}$Cs (Ref. 60), $^{133-135}$La (Ref. 61), and others]. However, as follows from comparison of the experimental $B(\mathrm{M1})$ values with those calculated by our theory, the presence of a small deformation ($\beta \lesssim 0.15$) cannot apparently affect the agreement between the theory and experiment (see Table IV).

2. In the framework of the most consistent approaches (for example, those based on the theory of finite Fermi systems) it is not possible to obtain a satisfactory description of the experimental data for isotopes in the region of ^{208}Pb (nuclei containing a doubly magic core $\pm$ one nucleon). Theories that take into account the spin polarization of the core and spin–orbit and one-pion exchange interactions of the quasiparticles in the annihilation channel give B(M1) values that differ by 1–2 orders of magnitude from the experimental values. This fact seems remarkable, since in these nuclei the states between which the γ transition takes place are classified to a high accuracy as single-particle states. We recall that an unsatisfactory agreement between the calculated and experimental values of B(M1) was also obtained in nuclei of the region $105 \leqslant A \leqslant 151$ when the number N of neutrons approached the magic number $N = 82$ ($^{141}Nd_{81}$, $^{143}Pm_{82}$). In connection with these results, it would be interesting to investigate the consequence of taking into account one-pion exchange in the quasiparticle scattering channel alongside the terms listed above in the amplitude $\mathcal{F}$.

As noted in Sec. 2.2, it was possible in Refs. 38–40 to achieve a good description of the experimental data in the Tl, Pb, and Bi nuclides by introducing into the local charge of a quasiparticle a term of the form $(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}$, though this contradicts translational invariance. It would be interesting for this problem to estimate the part played by a translationally invariant term of the type $(\boldsymbol{\sigma}\cdot\mathbf{p})\mathbf{p}$. The investigation of the l-forbidden β transition in the ^{39}Ca nucleus made in Ref. 62 showed that in this transition at least its contribution is important.

Finally, it could be that the unsatisfactory nature of the traditional theoretical approaches to the problem of the lifting of the l-forbiddenness in the region of ^{208}Pb is a sign of inadequacy of our ideas of the nucleus as a system of particles. In this case, new possibilities for solving this problem are opened up by the relativistic nuclear theory developed by Birbrair *et al.* (see, for example, Ref. 63). Application of this theory to our problem could be helpful as regards the particularization of the parameters introduced in it as well as for detailed description of the experiments, in the first place, in nuclei that neighbor doubly magic nuclei.

3. In all the regions of nuclei we observe shell effects that influence the values of the reduced probabilities $B(\mathrm{M1})_{\mathrm{exp}}$ on the approach to the magic numbers $N = 20$, 28; $N = 82$; $N = 126$.

As was noted in the systematics of the l-forbidden M1 transitions (see Sec. 1), the reduced probabilities of the neutron transitions are on the average larger than those of the proton transitions in the transitional regions of the nuclei ($105 \leqslant A \leqslant 151$ and $193 \leqslant A \leqslant 211$). In these nuclei the protons and neutrons occupy different subshells and one must compare hindrance factors of different types (for example, $3s_{1/2} \leftrightarrows 2d_{3/2}$ and $2d_{5/2} \rightleftarrows 1g_{7/2}$). At the present time, there are very few experimental data on single-particle same-type transitions, and the statistics is not adequate for an unambiguous conclusion. Therefore, for a final conclusion about the relationship between the probabilities of the proton and neutron transitions it would be expedient to extend the experimental information on proton $3_{1/2} \leftrightarrows 2d_{3/2}$ and neutron $2d_{5/2} \leftrightarrows 1g_{7/2}$ transitions in nuclei in which one of these states is the ground state.

[1] E. E. Berlovich, preprint No. 110 [in Russian], Physicotechnical Institute, Leningrad (1968).
[2] N. Z. Marupov, V. A. Morozov, and T. M. Muminov, Preprint R6-9005 [in Russian], JINR, Dubna (1975).
[3] W. Andreitcheff, L. Zamick, N. Z. Marupov *et al.*, Nucl. Phys. **A351**, 54 (1981).
[4] N. A. Bonch-Osmolovskaya, M. A. Dolgopolov, I. V. Kopytin, and V. A. Morozov, Preprint R6-85-868 [in Russian], JINR, Dubna (1985).
[5] M. E. Voĭkhanskiĭ, in: Gamma-luchi (Gamma Rays), USSR Academy of Sciences, Moscow–Leningrad (1961), p. 10.
[6] P. Doll, H. Mackh, G. Mairle, and G. J. Wagner, Nucl. Phys. **A230**, 329 (1974).
[7] P. Doll, G. Mairle, and H. Breuer, J. Phys. G **5**, 1421 (1979).
[8] P. Doll, G. J. Wagner, K. T. Knöpfle, and G. Mairle, Nucl. Phys. **A263**, 210 (1976).
[9] M. A. G. Fernandes and M. N. Rao, J. Phys. G **3**, 1397 (1977).
[10] G. Berrier, M. Vergnes, G. Rotlard, and I. Kalifa, J. Phys. (Paris) **37**, 311 (1976).
[11] M. Conjeaud, S. Harar, and Y. Cassagnou, Nucl. Phys. **A117**, 449 (1968).
[12] A. Szanto de Toledo, H. Hafner, and H. V. Klapdor, Nucl. Phys. **A320**, 309 (1979).
[13] E. R. Flynn, R. A. Hardekopf, J. D. Sherman *et al.*, Nucl. Phys. **A279**, 394 (1977).
[14] R. A. Moyer, B. L. Cohen, and R. C. Diehlt, Phys. Rev. C **2**, 1898 (1970).
[15] R. A. Moyer, Phys. Rev. C **5**, 1678 (1972).
[16] A. Arima, H. Horie, and M. Sano, Prog. Theor. Phys. **17**, 567 (1957).
[17] A. B. Migdal, Teoriya konechnykh fermi-sistem i svoĭstva atomnykh yader, 2nd Ed., Nauka, Moscow (1983); English translation of 1st Ed.: Theory of Finite Fermi Systems and Application to Atomic Nuclei, Interscience, New York (1967).
[18] É. E. Berlovich and G. M. Bukat, Izv. Akad. Nauk SSSR Ser. Fiz. **28**, 214 (1964).
[19] H. A. Mavromatis, L. Zamik, and G. E. Brown, Nucl. Phys. **A80**, 545 (1966).
[20] I. Hamamoto, Phys. Lett. **B61**, 343 (1976).
[21] O. Häusser, F. C. Khanna, and D. Ward, Nucl. Phys. **A194**, 113 (1972).
[22] K. Heyde, M. Waroquier, and R. A. Meyer, Phys. Rev. C **17**, 1219 (1978).
[23] G. Alaga and G. Ialongo, Nucl. Phys. **A97**, 600 (1967).
[24] V. Paar, Nucl. Phys. **A211**, 29 (1973).
[25] V. Paar, Phys. Lett. **B39**, 587 (1972).
[26] V. Paar and S. Brant, Nucl. Phys. **A303**, 96 (1978).
[27] V. Paar, Phys. Rev. C **11**, 1432 (1975).
[28] V. Paar, Ch. Vieu, and J. S. Dionisio, Nucl. Phys. **A284**, 199 (1977).
[29] V. Paar and B. K. S. Koene, Z. Phys. A **279**, 203 (1976).
[30] A. Szanto de Toledo, M. N. Rao, O. Sala, and F. Kempotic, Phys. Rev. C **16**, 438 (1977).
[31] A. Giannatiempo and A. Perego, Z. Phys. A **317**, 79 (1984).
[32] I. S. Towner, F. C. Khanna, and O. Hausser, Nucl. Phys. **A277**, 285 (1977).
[33] J. M. Arias, C. E. Alonso, and R. Bijker, Nucl. Phys. **A445**, 333 (1985).
[34] R. V. Jolos, I. Kh. Lemberg, and V. M. Mikhaĭlov, Fiz. Elem. Chastits At. Yadra **16**, 280 (1985) [Sov. J. Part. Nucl. **16**, 121 (1985)].
[35] I. N. Borzov, É. E. Sapershteĭn, S. V. Tolokonnikov, and S. A. Fayans, Fiz. Elem. Chastits At. Yadra **12**, 848 (1981) [Sov. J. Part. Nucl. **12**, 338 (1981)].
[36] V. A. Khodel', Yad. Fiz. **2**, 24 (1965) [Sov. J. Nucl. Phys. **2**, 17 (1966)].
[37] B. L. Birbrair, K. I. Erokhina, V. I. Isakov, and I. Kh. Lemberg, Izv. Akad. Nauk SSSR Ser. Fiz. **32**, 1618 (1968).
[38] B. Bauer, J. Speth, V. Klemt *et al.*, Nucl. Phys. **A209**, 535 (1973).
[39] J. Speth, R. Werner, and W. Wild, Phys. Rep. **33C**, 127 (1977).
[40] J. Speth, V. Klemt, J. Wambach, and G. E. Brown, Nucl. Phys. **A343**, 382 (1980).
[41] L. Zamick, V. Klemt, and J. Speth, Nucl. Phys. **A245**, 365 (1975).
[42] É. E. Sapershteĭn, S. V. Tolokonnikov, and S. A. Fayans, Izv. Akad. Nauk SSSR Ser. Fiz. **41**, 2061 (1977).
[43] V. A. Sadovnikova, Yad. Fiz. **32**, 1527 (1980) [Sov. J. Nucl. Phys. **32**, 791 (1980)].
[44] V. F. Dmitriev and V. B. Telitsin, Nucl. Phys. **A402**, 581 (1983).
[45] M. A. Dolgopolov and I. V. Kopytin, Izv. Akad. Nauk SSSR Ser. Fiz. **48**, 102 (1984).
[46] M. A. Dolgopolov and I. V. Kopytin, Izv. Akad. Nauk SSSR Ser. Fiz. **49**, 85 (1985).
[47] M. A. Dolgopolov, I. V. Kopytin, and L. F. Kulapin, Izv. Akad. Nauk SSSR Ser. Fiz. **44**, 2397 (1980).

[48]N. A. Bonch-Osmolovskaya, M. A. Dolgopolov, I. V. Kopytin, and V. A. Morozov, Soobshchenie (Communication) R4-85-759 [in Russian], JINR, Dubna (1985).
[49]M. A. Dolgopolov and I. V. Kopytin, Izv. Akad. Nauk SSSR Ser. Fiz. **50**, 1023 (1986).
[50]G. A. Pik-Pichak, Yad. Fiz. **6**, 265 (1967) [Sov. J. Nucl. Phys. **6**, 192 (1967)].
[51]E. Lipparini, S. Stringari, and M. Traini, Nucl. Phys. **A293**, 29 (1977).
[52]É. E. Sapershteĭn, S. A. Fayans, and V. A. Khodel', Fiz. Elem. Chastits At. Yadra **9**, 221 (1978) [Sov. J. Part. Nucl. **9**, 91 (1978)].
[53]M. A. Dolgopolov and I. V. Kopytin, Yad. Fiz. **43**, 579 (1986) [Sov. J. Nucl. Phys. **43**, 368 (1986)].
[54]S. A. Fayans, Preprint No. 1593 [in Russian], I. V. Kurchatov Institute of Atomic Energy, Moscow (1968).
[55]L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963).
[56]B. L. Birbrair and K. E. Kir'yanov, preprint No. 140 [in Russian], Leningrad Institute of Nuclear physics (1975).
[57]B. L. Birbrair, Yad. Fiz. **28**, 1223 (1978) [Sov. J. Nucl. Phys. **28**, 631 (1978)].
[58]D. M. Gordon, M. Gai, A. K. Gaigalas *et al.*, Phys. Lett. **B67**, 161 (1977).
[59]U. Hagemann, H. J. Keller, and H.-F. Brinckmann, Nucl. Phys. **A289**, 292 (1977).
[60]U. Gard, T. P. Sjoreen, and D. B. Fossan, Phys. Rev. C **19**, 207 (1979).
[61]J. Chiba, R. S. Hayano, M. Sekimoto *et al.*, J. Phys. Soc. Jpn. **43**, 1109 (1977).
[62]R. U. Khafizov, in: Yadernaya spektroskopiya i struktura atomnogo yadra. Tezisy dokl. XXXVI soveshchaniya (Nuclear Spectroscopy and Nuclear Structure. Abstracts of papers at the 36th Symposium), Nauka, Leningrad (1986), p. 255.
[63]B. L. Birbrair, L. N. Savushkin, and V. N. Fomenko, Yad. Fiz. **38**, 44 (1983) [Sov. J. Nucl. Phys. **38**, 25 (1983)].

Translated by Julian B. Barbour