Nuclear bremsstrahlung in reactions involving protons

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The low-energy bremsstrahlung accompanying particle scattering is described using the Feshbach–Yennie approximation, that is, including only an external contribution to the amplitude of the process. A modified derivation of the Feshbach–Yennie approximation is given. Its range of applicability is analyzed. Expressions are given for the bremsstrahlung spectrum in the region of isolated and overlapping resonances for both monoenergetic and broadened beams of incident particles. The quantities needed to calculate the bremsstrahlung spectra, the current methods of calculating these spectra, and the information which can be extracted by analyzing the experimental data are discussed. The relation between the bremsstrahlung photon spectrum and the time behavior of a nuclear interaction is studied. The experimental studies on proton bremsstrahlung are reviewed.

INTRODUCTION

It has long been known that gamma transitions due to the interaction of nucleons with an electromagnetic field can be studied to obtain accurate information about the nuclear structure and the nuclear reaction mechanism. From the experimental point of view this is a consequence of the efficiency and simplicity of photon detection, the ease with which photons can be detected in the presence of background from other radiation, and the high accuracy with which excited-state energies can be determined. In addition, electromagnetic radiation has been studied more thoroughly than nuclear radiation. This is in large measure due to the possibility of using perturbation theory in electrodynamics, whereas great difficulties are encountered in the perturbation theory of the strong interaction.

There are two important problems in the quantum theory of radiation:

1. Electromagnetic transitions between bound states. 2. Bremsstrahlung or transitions between continuum states.

In the first case the excitation energy of the system is smaller than the particle separation energy, and the most probable mechanism for the system to make a transition from an excited state to the ground state is via interaction with the electromagnetic field. Since the bound-state wave functions have zero asymptote, the strength of the electromagnetic transitions is basically determined by the size of the system.

In this case we obtain information on the initial and final states of the radiating system. This problem for a nucleus is analogous to the problem of transitions between bound states of an atom.

In the second case the initial and final states of the radiating system belong to the continuum and the radiation is a consequence of the accelerated motion of one charged particle relative to the other. The theory of bremsstrahlung in the scattering of an electron in the Coulomb field of a stationary source can be found in Refs. 1–3. The approach used in these studies differs considerably from that in the case when both states are bound. It is assumed that the bremsstrahlung spectrum is formed at distances significantly larger than the size of the force center, so that the latter can be assumed to be a point source.

The situation is more complicated when the bremsstrahlung is related to the motion of a nucleon or another particle in the field of a nucleus. In this case it may not be possible to fully describe the particle–nucleus interaction by a potential. The particle might penetrate inside the nucleus and form long-lived states of the compound nucleus. As a result, the bremsstrahlung spectrum might give information about the characteristics of the compound system and might serve as a tool for studying the mechanism of the nuclear reaction accompanied by bremsstrahlung. The latter possibility was first demonstrated in the case of direct nuclear reactions by Dubovoĭ and Shapiro.[4] The possibility of using the bremsstrahlung spectrum for identifying the direct-reaction mechanism is based on the appearance of deviations of the spectrum of radiation with frequency ω from $d\omega/\omega$, which become important if the reaction amplitude has singularities near the physical region. A review of the work done on this problem up to 1971 can be found in the lectures of Kondratyuk.[5]

The bremsstrahlung spectrum is determined by the time dependence of the acceleration, that is, the mechanism by which the incident particle interacts with the nucleus. Neglecting radiation due to the motion of the recoil nucleus, there are two segments of the particle trajectory in which the particle is accelerated—when approaching the nucleus and when leaving it. The time separation of these two processes is equal to the delay time of the particle in the nucleus τ and, accordingly, the bremsstrahlung spectrum depends on τ. The use of bremsstrahlung to determine the time for a reaction was first suggested by Eisberg, Yennie, and Wilkinson in 1960.[6] The quantum-mechanical description of bremsstrahlung was developed by Feshbach and Yennie.[7] They proposed a method for describing bremsstrahlung based on the expansion of the reaction amplitude off the mass shell in the photon energies. In this method the bremsstrahlung cross section is expressed in terms of the experimentally measured elastic scattering cross section and the reaction time. Therefore, in those cases when the method is applica-

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ble it becomes possible to extract the reaction time by comparing the experimental results with the calculation. The appraisal of the applicability of the Feshbach–Yennie approximation is thus an important problem in bremsstrahlung studies.

The stationary quantum-mechanical approaches to describing bremsstrahlung which are currently used can be divided into two groups. The first contains the so-called model-independent theories. They are based on the expansion of the bremsstrahlung amplitude in a series in the photon energy E_γ:

$$T_{fi} = T_{-1}E_\gamma^{-1} + T_0 E_\gamma^0 + T_1 E_\gamma + \dots \qquad (1)$$

The coefficients T_{-1} and T_0 were first obtained by Low for nonresonance behavior of the elastic scattering amplitudes f.[8] The expressions for T_{-1} and T_0 were calculated by Feshbach and Yennie (Ref. 7; see also Ref. 9) in the case where f varies strongly in an interval of order E_γ. Strictly speaking, the derivation of the relation for T_0 in that study was rather qualitative; it was later done more accurately (using the potential approach) in Refs. 10–13. The first two terms of the expansion (1) are determined by the behavior of the elastic scattering amplitudes and do not contain any off-shell information. This makes it possible in the case of low-energy photons to calculate the bremsstrahlung cross section using elastic scattering data—it is sufficient to have some parametrization for f.

In the second group of approaches[14–22] the starting point is an explicit expression for the particle–target-nucleus interaction potential. In Refs. 15–17, 21, and 22 the bremsstrahlung amplitude was calculated after solving the Schrödinger equation for the continuum wave functions in a given potential. This method is quite time-consuming, owing to the slow convergence of the matrix elements corresponding to the electromagnetic interaction between continuum wave functions. At present its application is limited to either few-nucleon systems or the one-particle description of bremsstrahlung.

Dipole radiation was studied in Refs. 18–20. The Ehrenfest theorem was used to improve the convergence in coordinate space. By approximating the nuclear part of the potential by a square well, simple relations were obtained for the parametrization of T_{fi}. The validity of the Ehrenfest theorem in the description of bremsstrahlung was proved in Refs. 23 and 24.

Bremsstrahlung was studied in Refs. 25 and 26 in the nonstationary quantum-mechanical approach using wave packets. Eremin, Melikov, and Tulinov[26] first described the dependence of the shape of the bremsstrahlung energy spectrum on the energy spread of the incident particle beam and on the detector resolution. It was assumed that elastic scattering occurs on an isolated resonance, and only the first term of the expansion (1) was included in the amplitude T_{fi}. This approach made it possible to study the range of applicability of the stationary description for bremsstrahlung. If the level width is much larger than the experimental energy spread, the expression for the intensity of bremsstrahlung photon emission coincides with that obtained in the stationary quantum-mechanical approach. Otherwise, the expression for the radiation spectrum is obtained from that in the stationary approach after averaging over the experimental energy spread.

The expression for the amplitude T_{fi} in the form of a series expansion in E_γ can be used to relate the characteristics of the bremsstrahlung spectrum to the time delay of the particle in the field of the nucleus,[6,7,25–30] so let us study its derivation.

1. A MODIFIED DERIVATION AND DISCUSSION OF THE VALIDITY OF THE FESHBACH–YENNIE APPROXIMATION

According to Refs. 31 and 32, in first order in the electromagnetic interaction we have

$$T_{fi} = (\psi_{\mathbf{k}_f}^{(-)} V_{\text{em}} \psi_{\mathbf{k}_i}^{(+)}), \qquad (A)$$

where $\psi_{\mathbf{k}i}^{(+)}$ ($\psi_{\mathbf{k}f}^{(-)}$) is the wave function of the initial (final) nucleon with relative momentum $\hbar\mathbf{k}_i$ ($\hbar\mathbf{k}_f$), which asymptotically is a plane wave plus a spherically diverging (converging) wave. To simplify the notation we have omitted the spin indices on $\psi_{\mathbf{k}}^{(\pm)}$.

If we restrict ourselves to electric transitions, in the c.m.s. of the nuclear subsystem in the entrance channel the electromagnetic interaction potential V_{em} normalized to unit volume is

$$\left.\begin{aligned} V_{\text{em}} &= \mathrm{i}\frac{e\hbar^2}{m}\sqrt{\frac{2\pi}{E_\gamma}}(\hat{\boldsymbol{\varepsilon}}\mathbf{Q}), \\ \mathbf{Q} &= \left[\frac{Z_i}{A_i}\exp(-\mathrm{i}\boldsymbol{\varkappa}\mathbf{r}A_t/A) - \frac{Z_t}{A_t}\exp(\mathrm{i}\boldsymbol{\varkappa}\mathbf{r}A_i/A)\right]\nabla_{\mathbf{r}}. \end{aligned}\right\} \qquad (2)$$

Here, $\hat{\boldsymbol{\varepsilon}}$ and $\boldsymbol{\varkappa}$ are the photon polarization and momentum unit vectors, respectively, e and m are the proton charge and nucleon mass, Z_α and A_α are the charge and atomic number of the incident particle $\alpha = i$ and the target nucleus ($\alpha = t$), $A = A_i + A_t$, and $\mathbf{r}$ is the coordinate of the relative motion of the particle. Therefore,

$$T_{fi} = \mathrm{i}\frac{e\hbar^2}{m}\sqrt{\frac{2\pi}{E_\gamma}}(\hat{\boldsymbol{\varepsilon}}\mathbf{B}),\quad \mathbf{B} = (\psi_{\mathbf{k}_f}^{(-)}\mathbf{Q}\psi_{\mathbf{k}_i}^{(+)}). \qquad (3)$$

According to the general rules of quantum scattering theory,[33,34] in the c.m.s. the bremsstrahlung differential cross section [10^{-33} cm^2/(keV·sr)] is related to the vector amplitude $\mathbf{B}$ as

$$\frac{d^3\sigma}{dE_\gamma\, d\Omega_\gamma\, d\Omega_{\mathbf{k}_f}} = \text{const}\,\mu^2 E_\gamma \sqrt{\frac{E_{\mathbf{k}_f}}{E_{\mathbf{k}_i}}}\frac{1}{\hat{I}_i^2\hat{s}_i^2}\sum_{\substack{m_{s_i}m_{s_f}\\ m_{I_i}m_{I_f}}} M,$$

$$\text{const} = \frac{(2\pi)^2\cdot 10^4\, e^2}{(\hbar c)^3} \cong 0{,}073983,\quad \mu = A_iA_t/(A_i + A_t), \qquad (4)$$

where for an elliptically polarized photon[35]

$$M = \frac{1}{\pi}\int_0^{2\pi} d\chi |(\hat{\boldsymbol{\varepsilon}}\,(\chi)\mathbf{B})|^2,$$

χ is the angle between the polarization vector and the unit vector of the axis of abscissas in the plane perpendicular to the photon momentum, $\hat{I} = \sqrt{2I+1}$, I_α and s_α are the target-nucleus and particle spins, m_{I_α} and m_{s_α} are their projections, and $E_{k\alpha}$ is the energy of the relative motion of the particle in the entrance ($\alpha = i$) and exit ($\alpha = f$) channels. In writing down the expression (4) it was assumed that the functions $\psi_{\mathbf{k}}^{(\pm)}$ are normalized by $\delta(\mathbf{k}\text{-}\mathbf{k}')$.

In the c.m.s. of the entrance channel $E_\gamma = E_{\mathbf{k}_i} - E_{\mathbf{k}_f} - E_c$, where E_c is the energy of the motion of the center of mass of the nuclear subsystem. We shall approximately take $E_c \cong 0$ and

$$E_\gamma = E_{\mathbf{k}_i} - E_{\mathbf{k}_f}. \quad (5)$$

According to Ref. 36, even for proton bremsstrahlung on carbon the particle characteristics in the c.m.s. and the laboratory frame coincide with an accuracy of up to $\leqslant 17\%$. Therefore, in what follows we shall neglect the difference between these frames. The rigorous transformation to the laboratory frame can be done as in Ref. 37.

For simplicity we restrict ourselves to the bremsstrahlung of a spinless charged particle ($A_i = Z_i$) on a heavy target nucleus with $A_t \gg Z_t$, so that

$$Q = e^{-i\varkappa r}\nabla_r.$$

We rewrite the expression (3) for the **B**-vector amplitude of the process using the Lippmann–Schwinger equations.[38] As a result, we find[24]

$$\mathbf{B} = \sum_{i=1}^{3} \mathbf{B}_i;$$

$$\mathbf{B}_1 = \mathbf{k}_i t(\mathbf{k}_f, \mathbf{k}_i - \varkappa; E_{\mathbf{k}_f}) G(\mathbf{k}_f, \mathbf{k}_i - \varkappa) \cong$$

$$- \frac{\mathbf{k}_i t(\mathbf{k}_f, \mathbf{k}_i - \varkappa; E_{\mathbf{k}_f})}{E_\gamma [1 - (\beta_i \hat{\varkappa})]};$$

$$\mathbf{B}_2 = \mathbf{k}_f t(\mathbf{k}_f + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i}) G(\mathbf{k}_i, \mathbf{k}_f + \varkappa) \quad (6)$$

$$\cong \frac{\mathbf{k}_f t(\mathbf{k}_f + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i})}{E_\gamma [1 - (\beta_f \hat{\varkappa})]};$$

$$\mathbf{B}_3 = \int d\mathbf{q}\, G(\mathbf{k}_f, \mathbf{q})\, t(\mathbf{k}_f, \mathbf{q}; E_{\mathbf{k}_f})\, \mathbf{q} G(\mathbf{k}_i, \mathbf{q} + \varkappa)\, t$$

$$\times (\mathbf{q} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i});$$

$$(\beta_\alpha = v_\alpha/c, \quad E_\gamma = \hbar c \varkappa, \quad \hat{\varkappa} = \varkappa/\varkappa, \quad \mathbf{v} = \hbar\mathbf{k}/(m\mu), G(\mathbf{k}, \mathbf{q})$$

$$= 1/(E_{\mathbf{k}} - E_{\mathbf{q}} + i\eta),$$

$$E_{\mathbf{k}} = \hbar^2 \mathbf{k}^2/(2m\mu),$$

where $t(\mathbf{q}, \mathbf{k}, E) \equiv \langle \mathbf{q}|T(E)|\mathbf{k}\rangle$ is the T matrix of off-shell nuclear scattering. Here we have dropped the term $\langle \mathbf{k}_f|\mathbf{Q}|\mathbf{k}_i\rangle$ and the terms proportional to $\varkappa$, since they do not contribute to the amplitude T_{fi}.

$|\mathbf{Q}|\mathbf{k}_i\rangle$ and the terms proportional to $\varkappa$, since they do not contribute to the amplitude T_{fi}.

According to (6) the amplitude for the probability of the emission of a bremsstrahlung photon is the sum of three terms: (1) the amplitudes for emission before ($\mathbf{B}_1$) and after ($\mathbf{B}_2$) the nuclear interaction (external emission), which dominate at distances larger than the nuclear-force range; (2) the amplitudes $\mathbf{B}_3$ for emission when the particles are close to each other (internal emission or rescattering). Outside the range R of the interparticle potential V the term $\mathbf{B}_3$ is proportional to $\varkappa$ and does not contribute to the bremsstrahlung amplitude.

In the case of proton–proton bremsstrahlung the term $\mathbf{B}_3$ is small in the c.m.s.,[37,39] and for incident-proton energies from 10 to 300 MeV its effect is $\lesssim 15\%$ (Ref. 15).

Let us obtain the expression for $\mathbf{B}_3$ accurate to terms linear in E_γ. We use the identity

$$t(\mathbf{k}_2, \mathbf{k}_1 - \varkappa, E_u) - t(\mathbf{k}_2 + \varkappa, \mathbf{k}_1, E_z)$$

$$= \int d\mathbf{q}\, t(\mathbf{k}_2, \mathbf{q}; E_u)[G(\mathbf{k}, \mathbf{q})$$

$$- G(\mathbf{z}, \mathbf{q} + \varkappa)\, t(\mathbf{q} + \varkappa, \mathbf{k}_1; E_z)],$$

which follows from the integral equation for the off-shell T matrix. This relation is satisfied for local interactions and has been obtained in Ref. 11. We have written it here using different notation. If we set $\mathbf{u} = \mathbf{k}_f$ and $\mathbf{z} = \mathbf{k}_i$ and take the gradient with respect to $\varkappa$, then for $E_{\mathbf{k}_i} - E_{\mathbf{k}_f} \equiv E_\gamma = \hbar c \varkappa$ we obtain

$$\int d\mathbf{q}\, t(\mathbf{k}_f, \mathbf{q}; E_{\mathbf{k}_f})\, G(\mathbf{k}_f, \mathbf{q})\, \mathbf{q} G(\mathbf{k}_i, \mathbf{q} + \varkappa)\, t$$

$$\times (\mathbf{q} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i})$$

$$= \frac{m\mu}{\hbar^2} \nabla_{\mathbf{k}_f + \varkappa} t(\mathbf{k}_f + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i}) + \frac{m\mu}{\hbar^2} \nabla_{\mathbf{k}_i - \varkappa} t$$

$$\times (\mathbf{k}_f, \mathbf{k}_i - \varkappa; E_{\mathbf{k}_f})$$

$$+ E_\gamma \int d\mathbf{q}\, t(\mathbf{k}_f, \mathbf{q}; E_{\mathbf{k}_f})\, G(\mathbf{q}, \mathbf{k}_f) \left\{ \frac{m\mu}{\hbar^2} \nabla_\varkappa t \right.$$

$$\times (\mathbf{q} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i})$$

$$\left. + \left[\mathbf{q} G(\mathbf{q}, \mathbf{k}_f) - \frac{1}{\hbar c} [(\mathbf{q}\varkappa) \nabla_\varkappa + \hat{\varkappa}] \right] G(\mathbf{q} + \varkappa, \mathbf{k}_i)\, t \right.$$

$$\left. \times (\mathbf{q} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i}) \right\}$$

$$- \frac{E_\gamma^2}{(\hbar c)^2} \int d\mathbf{q}\, t(\mathbf{k}_f, \mathbf{q}; E_{\mathbf{k}_f})\, G(\mathbf{q}, \mathbf{k}_i) \nabla_\varkappa G(\mathbf{q} + \varkappa, \mathbf{k}_i)\, t$$

$$\times (\mathbf{q} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i}). \quad (7)$$

Therefore, $\mathbf{B}_3$ does not contribute to the term T_0 of the expansion (1), and

$$\mathbf{B}_3 = \frac{m\mu}{\hbar^2} \nabla_{\mathbf{k}_f + \varkappa} t(\mathbf{k}_f + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i})$$

$$+ \frac{m\mu}{\hbar^2} \nabla_{\mathbf{k}_i - \varkappa} t(\mathbf{k}_f, \mathbf{k}_i - \varkappa, E_{\mathbf{k}_f}) + 0\left(\frac{E_\gamma}{\delta I}\right), \quad (8)$$

where δI is the energy range of the characteristic variation of the integral terms on the right-hand side of (7).

Let us isolate the dependence on the unit vectors $\hat{\mathbf{k}}$ in the matrices t. Taking into account (5), up to terms linear in E_γ we have

$$t(\mathbf{k}_f + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i}) = t(\hat{\mathbf{k}}_f \mathbf{k}_i (1 - E_\gamma/E_i)^{1/2} + \varkappa, \mathbf{k}_i; E_{\mathbf{k}_i})$$

$$\cong t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i, E_{\mathbf{k}_i}) + E_\gamma \left[\frac{\hat{\varkappa}}{\hbar c} - \frac{\hat{\mathbf{k}}_f \mathbf{k}_i}{2E_i} \right] \nabla_{\mathbf{p}_i} t(\mathbf{p}_i, \mathbf{k}_i; E_{\mathbf{k}_i});$$

$$t(\mathbf{k}_f, \mathbf{k}_i - \varkappa; E_{\mathbf{k}_f}) \cong t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i, E_{\mathbf{k}_f})$$

$$+ E_\gamma \left[\frac{\hat{\mathbf{k}}_i \mathbf{k}_f}{2E_f} - \frac{\hat{\varkappa}}{\hbar c} \right] \nabla_{\mathbf{p}_f} t(\mathbf{k}_f, \mathbf{p}_f, E_{\mathbf{k}_f}), \quad (9)$$

where $t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_i}) \equiv t(\hat{\mathbf{k}}_f \mathbf{k}_i, \mathbf{k}_i; E_{\mathbf{k}_i})$ and $t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i, E_{\mathbf{k}_f}) = t(\mathbf{k}_f \hat{\mathbf{k}}_i \mathbf{k}_f; E_{\mathbf{k}_f})$ are the on-shell T matrices, $\mathbf{p}_i = \hat{\mathbf{k}}_f \mathbf{k}_i, \mathbf{p}_f = \hat{\mathbf{k}}_i \mathbf{k}_f$, and terms of the form $\nabla_{\mathbf{p}} t$ are gradients of the T matrix half off-shell with subsequent passage to the energy shell.

Substitution of (9) into (6) and (8) gives

$$\mathbf{B} \equiv \Big(-\frac{\mathbf{k}_i}{[1-(\hat{\boldsymbol{\beta}}_i\hat{\boldsymbol{\varkappa}})]}\, t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_f}) + \frac{\mathbf{k}_f}{[1-(\hat{\boldsymbol{\beta}}_f\hat{\boldsymbol{\varkappa}})]}\, t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_i})\Big) E_\gamma^{-1} + \Big(\frac{m\mu}{\hbar^2}\frac{\boldsymbol{\beta}_i}{[1-(\hat{\boldsymbol{\beta}}_i\hat{\boldsymbol{\varkappa}})]}[\hat{\boldsymbol{\varkappa}} - (\hat{\boldsymbol{\beta}}_i\hat{\boldsymbol{\varkappa}})\hat{\boldsymbol{\beta}}_i]\nabla_{\mathbf{p}_f} t(\mathbf{k}_f, \mathbf{p}_f; E_{\mathbf{k}_f}) + \frac{m\mu}{\hbar^2}\frac{\boldsymbol{\beta}_f}{[1-(\hat{\boldsymbol{\beta}}_f\hat{\boldsymbol{\varkappa}})]}[\hat{\boldsymbol{\varkappa}} - (\hat{\boldsymbol{\beta}}_f\hat{\boldsymbol{\varkappa}})\hat{\boldsymbol{\beta}}_f]\nabla_{\mathbf{p}_i} t(\mathbf{p}_i, \mathbf{k}_i; E_{\mathbf{k}_i})\Big) E_\gamma^0. \quad (10)$$

The discarded terms are small, of order $O(E_\gamma/\min\{\delta I, \delta E_D\})$, where δE_D is the interval of the characteristic energy variation of the gradients from (10). Since δE_D is of order of the energy interval δE in which the elastic-scattering probability amplitude varies significantly, the accuracy of (10) can be estimated to be $O(E_\gamma \delta E/(E_i \delta I))$.

The approximation in which the first term in (10) is kept is usually referred to as the Feshbach–Yennie approximation (FYA). The coefficient of E_γ^0 in (10) is essentially the same as that obtained in Ref. 7 after the transformation using (5). An approximation of the form (10) in which the terms T_1 and T_0 of the expansion (1) are included is called the Feshbach–Yennie approximation with the correction term (FYCT). The result of Low[8] is obtained by further expansion of the elastic-scattering probability amplitudes in the average energy $(E_i + E_f)/2$.

Separating the angular ($\nabla_{\hat{\mathbf{p}}}$) and the modulus-dependent terms in the gradient, we find that in the case of particle bremsstrahlung on a heavy nucleus only the angular components of the gradient remain in **B**, and

$$\mathbf{B} = \Big(-\frac{\mathbf{k}_i}{(1-(\hat{\boldsymbol{\beta}}_i\hat{\boldsymbol{\varkappa}}))}\, t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_f}) + \frac{\mathbf{k}_f}{(1-(\hat{\boldsymbol{\beta}}_f\hat{\boldsymbol{\varkappa}}))}\, t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_i})\Big) E_\gamma^{-1} + \frac{\boldsymbol{\beta}_i}{\hbar v_f [1-(\hat{\boldsymbol{\beta}}_i\hat{\boldsymbol{\varkappa}})]}\hat{\boldsymbol{\varkappa}}\nabla_{\hat{\mathbf{k}}_i} t(\hat{\mathbf{k}}_f, \mathbf{k}_i; E_{\mathbf{k}_f}) + \frac{\boldsymbol{\beta}_f}{\hbar v_i [1-(\hat{\boldsymbol{\beta}}_f\hat{\boldsymbol{\varkappa}})]}\hat{\boldsymbol{\varkappa}}\nabla_{\hat{\mathbf{k}}_f} t(\hat{\mathbf{k}}_f, \hat{\mathbf{k}}_i; E_{\mathbf{k}_i}) + O\Big(\frac{E_\gamma \delta E}{E_i \delta I}\Big). \quad (11)$$

The expression for the bremsstrahlung amplitude was apparently first written in this form and is a consequence of the relation (5). In Ref. 40 the relation for **B** was obtained starting from the requirement of gauge invariance, viewing the T matrix as a function of the energy and the square of the momentum transfer. The terms with the gradients in (11) are the analogs of the derivatives with respect to the momentum transfer from Ref. 40 and, in accord with the picture of the time behavior of the scattering,[47] determine the displacement of the particle perpendicular to the initial direction owing to the interaction between the particle and the nucleus.

The recoil of the target nucleus can also easily be included by using Eq. (2) for **Q**. The amplitude **B** will contain eight terms of the same type as in (11).

The accuracy of approximating the bremsstrahlung amplitude by the first terms of the expansion (1) was studied in Refs. 9, 17, 24, 35, 36, 39, and 41–44. It was shown[39,41] that in the case of bremsstrahlung in the pp reaction at proton energies $10 \lesssim E_i \lesssim 200$ MeV the Low approximation, which in the absence of resonances is equivalent to the FYA, describes experiment with an accuracy of up to 25% even for $E_\gamma/E_i \sim 1$. In the case of bremsstrahlung in the $\alpha\alpha$ (Refs. 9 and 17), $p\alpha$, and dp (Ref. 35) scattering processes an important role is played by the kinematic constraints, and the FYA is valid for particle emission angles near 90° in the c.m.s. and small photon energies.

The validity of the FYA was studied in Refs. 36, 40, and 42–44 by comparing the calculations with the results of experiments on the bremsstrahlung of protons with resonance energies on carbon. The radiation for protons with energies 1.504–1.895 MeV (Refs. 36, 40, 42, and 43) and 0.461 MeV (Ref. 44) was studied. The R-matrix parametrization was used for the elastic scattering amplitudes. The data of the Maryland group[45] were used for the reduced widths and resonance energies in Refs. 40, 42, and 43. It was shown[43] that the calculated bremsstrahlung cross sections are quite sensitive to the resonance parameters. The cross sections for bremsstrahlung photon emission in the FYA and FYCT are similar far from the resonance in the final channel, while near the resonance they can differ by 40% (Refs. 40 and 42). The FYCT gives the best description of the experimental data near the resonance in the final channel. On the whole, the agreement with experiment obtained in these two approximations is about the same. Several results from these studies are shown in Fig. 1. For protons with energy near the resonance $E_r = 0.461$ MeV the FYA also gives a better de-

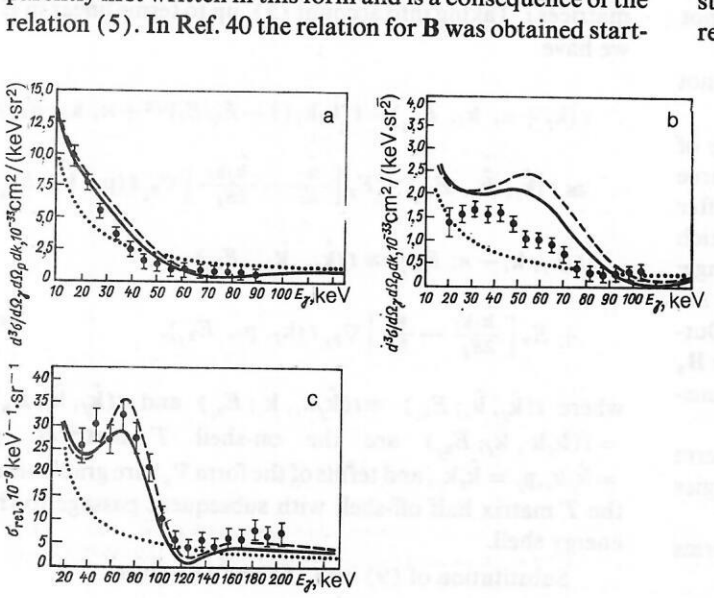

FIG. 1. Bremsstrahlung spectrum of protons on carbon: (a) at the incident-proton energy $E_i = 1.765$ MeV, $\theta_{k_f} \equiv \theta_f = 157°$; (b) $E_i = 1.795$ MeV, $\theta_f = 157°$; (c) yield of bremsstrahlung radiation G_{rel} relative to the elastic scattering cross section for $E_i = 1.81$ MeV and $\theta_f = 155°$. The solid lines are calculated using the FYCT, the dotted lines are calculated using the Low approximation, and the dashed lines are calculated using the FYA. The data in Fig. 1(c) are averaged over the photon solid angle.

scription of experiment than the FYCT (Ref. 44) (Fig. 2). Newer data on the reduced widths and resonance energies were used in Ref. 36. It was found that the shape of the bremsstrahlung photon spectrum is in agreement with that given by the FYA, although the absolute values differ at proton energies above the resonance in the entrance channel (see Sec. 4).

We have studied[24] the possibility of using the FYA to describe the interaction of a particle with a heavy nucleus as a function of the ratio of the photon and incident-particle energies and also the particle emission angle. We considered dipole radiation. We studied the differential cross section for bremsstrahlung in a model problem close to the case of bremsstrahlung of photons with energy below the Coulomb barrier on the carbon nucleus. We took the potential to be a square well with a step barrier (P1). Its parameters were selected such that there is a resonance of width $\Gamma \cong 85$ keV, in the s wave at the energy 0.216 MeV.

In Figs. 3 and 4 we show the differential cross sections $\sigma(\theta_f, E_\gamma) = d^3\sigma/dE_\gamma d\Omega_\gamma d\Omega_{kf}$ calculated in the exact approach $[\sigma_T(\theta_f, E_\gamma)]$ and using the FYA $(\sigma_{\text{FYA}}(\theta_f, E_\gamma))$ at various incident-particle energies $E_i = E_{k_i}$ and particle

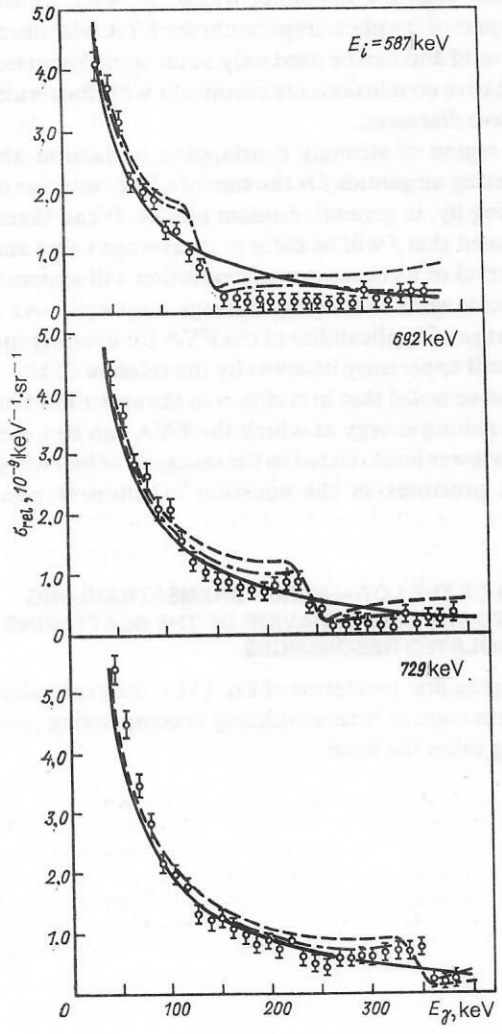

FIG. 2. Yield of bremsstrahlung radiation σ_{rel} compared to the elastic scattering cross section near the resonance $E_r = 461$ keV. The solid lines are calculated using the Low approximation, the dashed lines are the FYCT, and the dot–dash lines are the FYA.

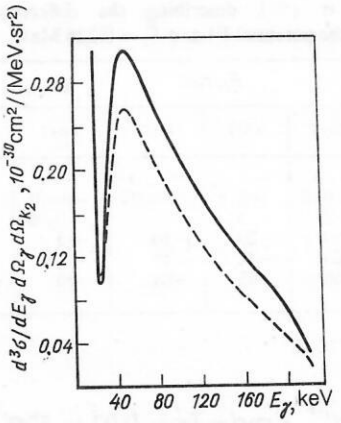

FIG. 3. Dependence of the bremsstrahlung differential cross section on the photon energy for $\theta_f = 3\pi/7$ and $E_i = 0.226$ MeV. The solid line is calculated using the FYA, and the dashed line is the exact calculation.

emission angles θ_f relative to the incident beam with $\theta_\gamma = \pi/2$ and $\varphi_\gamma = \varphi_f = 0$.

In Tables I and II we show the coefficients $\alpha[\sigma_T(\theta_f, E_\gamma) - \sigma_{\text{FYA}}(\theta_f, E_\gamma)]/\sigma_T(\theta_f, E_\gamma)$ describing the difference between the differential cross sections calculated in the FYA and the exact values as functions of $\Delta = E_\gamma/E_i$ and θ_f at $E_i = 0.226$ MeV. To calculate the cross sections in Table I we used the potential P1, and for those in Table II we used the potential P2, of the same form as P1 but with a barrier so narrow that there are no elastic-scattering resonances in any of the partial waves.

We see from Table I that the validity of the Feshbach–Yennie approximation depends not only on the energy ratio Δ, but also on the particle emission angle. As the incident-particle energy E_i increases for fixed Δ, the difference between $\sigma_T(\theta_f, E_\gamma)$ and $\sigma_{\text{FYA}}(\theta_f, E_\gamma)$ grows. For example, for the potential P1 with $E_i = 0.226$ MeV and $0.005 \leqslant \Delta \leqslant 0.97$ the modulus of α averaged over the eight

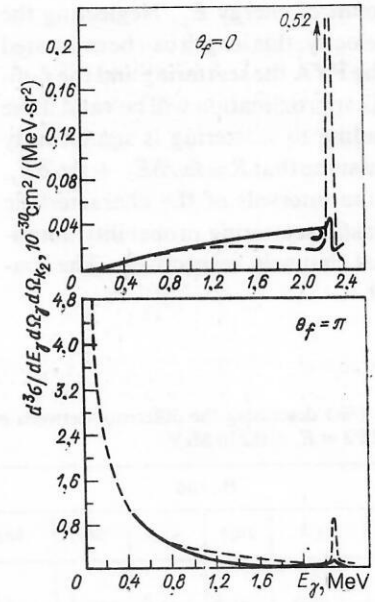

FIG. 4. Bremsstrahlung differential cross section for $E_i = 2.5$ MeV. The notation is the same as in Fig. 3. The arrow indicates the value of the cross section at the maximum.

TABLE I. The coefficients α (%) describing the difference between $\sigma_T(\theta_f, E_\gamma)$ and $\sigma_{FYA}(\theta_f, E_\gamma)$ for the potential P1 and $E_i = 0.226$ MeV.

Δ	θ_f, rad							
	0	$\pi/7$	$2\pi/7$	$3\pi/7$	$4\pi/7$	$5\pi/7$	$6\pi/7$	π
0.005	−0.01	−0.2	−0.2	−0.1	−0.05	−0.01	−0.01	0.00[illegible]
0.008	0.06	0.8	3	6	3	0.09	0.3	0.1
0.2	0.4	−0.8	−10	−20	−10	−3	−0,3	0.2
0.44	0.8	−5	−25	−32	−18	−7	−1	0.3
0.97	−38	−44	−56	−65	−64	−56	−48	−45

angles θ_f from Table I ($|\alpha|_{ave}$) varies from 0.07 to 52%, while for $E_i = 2.5$ MeV and $0.04 \leqslant \Delta \leqslant 0.92$ the coefficients $|\alpha|_{ave}$ vary from 6 to 173%, and for $\theta_f = 2\pi/7$ and $\Delta = 0.8$ we have $\alpha = -570\%$. The differences between the exact and approximate calculations are considerably smaller for the potential P2. For example, for $0.004 \leqslant \Delta \leqslant 0.92$ the coefficient $|\alpha|_{ave}$ varies only from 2 to 25%.

Comparison of the results of Tables I and II shows that the presence of a resonance in the elastic channel leads (for a given value of Δ) to deterioration of the validity of the FYA. In the resonance region the monotonic dependence of the coefficient α on Δ is spoiled.

On the whole, the studies indicate that in the absence of resonances the FYA gives a quite good description of experiment and that its accuracy is determined by Δ, the ratio of the bremsstrahlung-radiation energy and the incident-particle energy. The presence of a resonance at the same Δ causes the approximation to become less accurate. For sufficiently heavy nuclei, when the differences between the c.m.s. and the lab frame can be neglected even for small Δ, the shape of the bremsstrahlung photon spectrum given by the FYA is in agreement with the experimental spectrum.

The range of applicability of the FYA for describing the bremsstrahlung accompanying elastic scattering can be determined, following Low,[8] by introducing the bremsstrahlung formation length, namely, the distance R_b over which the particle loses an amount of energy E_γ. Neglecting the variation of the particle velocity, this length can be estimated as $R_b \cong \Delta t v \cong v\hbar/E_\gamma$. In the FYA the scattering and the radiation are separated, so this approximation will be valid if the range R of the forces leading to scattering is significantly smaller than R_b. We can assume that $R \cong \hbar v/\delta E_i + \hbar v/\delta E_f$, where δE_i and δE_f are the intervals of the characteristic energy variation of the elastic-scattering probability amplitudes in the initial and final channels, respectively. The condition $R \ll R_b$ implies that

$$E_\gamma \ll (\delta E_i \delta E_f)/(\delta E_i + \delta E_f).$$

In the absence of resonances for short-range interactions $\delta E_{i,f} = v_i p_i \sim 2E_i$ and the condition for the FYA to be applicable becomes

$$E_\gamma \ll E_i. \tag{12}$$

Near the resonances in the exit channel ($E_f \cong E_r$) the energy δE_f is of the order of the resonance width Γ, and the FYA will describe the resonance part of the bremsstrahlung spectrum when the corresponding energy $E_\gamma = E_i - E_r$ is significantly smaller than the resonance width ($E_\gamma \ll \Gamma$). Otherwise, in this part of the photon spectrum the FYA will not, in general, be valid and can be used only as an approximation. These qualitative conclusions are consistent with the results which we have discussed.

In the region of strongly overlapping resonances the elastic scattering amplitude f is the sum of a large number of terms differing by, in general, random phases. It can therefore be assumed that f will be close to its average value and that the interval of its characteristic variation will approximately coincide with that of the average amplitude. As a result, the range of applicability of the FYA for overlapping resonances will apparently be given by the relation (12).

It should be noted that in addition to the upper limit on the bremsstrahlung energy at which the FYA can be used, there is also a lower limit related to the necessity of including multiphoton processes in the emission of ultrasoft photons.[46]

2. RELATION OF THE LOW-ENERGY BREMSSTRAHLUNG SPECTRUM TO THE TIME BEHAVIOR OF THE SCATTERING PROCESS. ISOLATED RESONANCES

Keeping the first two terms of Eq. (11), the expression (4) for the spectrum of bremsstrahlung accompanying proton scattering takes the form

TABLE II. The coefficients α (%) describing the difference between $\sigma_T(\theta_f, E_\gamma)$ and $\sigma_{FYA}(\theta_f, E_\gamma)$ for the potential P2 at $E_i = 0.226$ MeV.

Δ	θ_f, rad							
	0	$\pi/7$	$2\pi/7$	$3\pi/7$	$4\pi/7$	$5\pi/7$	$6\pi/7$	π
0.045	0.02	−0.4	+0.5	−0.4	−0.2	−0.1	−0,02	−0.01
0.44	0,05	−1	−2	−2	−2	−1	−0.3	−0,1
0.97	−0.3	−0.5	−0.9	−1	−1	−1	−0.8	−0,7

$$E_\gamma \frac{d^3\sigma}{dE_\gamma\, d\Omega_\gamma\, d\Omega_f} = \mathbf{b}_f^2 \frac{d\sigma_{\rm el}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E_i)}{d\Omega_f} +$$

$$+ \mathbf{b}_i^2 \frac{d\sigma_{\rm el}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E_f)}{d\Omega_f} - 2(\mathbf{b}_i\mathbf{b}_f)\Phi_c(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E_i, E_f);$$

$$\mathbf{b}_p = \frac{\beta_p(\hat{\varkappa}\times\hat{\mathbf{k}}_p)}{1-(\beta_p\hat{\varkappa})}\left[\frac{\alpha}{(2\pi)^2}\frac{v_f}{v_i}\left(\frac{N\mu}{A}\right)^2\right]^{1/2}, \quad p = i, f; \ \alpha = e^2/\hbar c. \tag{13}$$

Here $d\sigma_{\rm el}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E)/d\Omega_f = \Sigma_{\delta,\gamma}|f_{\delta\gamma}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E)|^2/(\hat{s}_i^2\hat{I}_i^2)$ is the differential cross section for elastic scattering, $\Phi_c(\mathbf{k}_i, \mathbf{k}_f; E_i, E_f) = (1/\hat{s}_i^2\hat{I}_i^2)\ {\rm Re}\Sigma_{\gamma}^{\delta} f_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E_i) f_{\delta\gamma}^*(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E_f)$ is the real part of the correlation function of the elastic scattering amplitudes, $f_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E) = -(2\pi)^2 m\mu t(\hat{\mathbf{k}}_f\hat{\mathbf{k}}_i, E)/\hbar$ is the elastic-scattering probability amplitude, and the indices δ and γ denote the spin degrees of freedom of the particles in the entrance and exit channels. The recoil of the target nucleus has been taken into account by using for the protons the effective charge eN/A (N is the number of neutrons in the nucleus).

If we introduce $\varphi_{\delta\gamma}$ as the argument of the elastic-scattering probability amplitude, the function Φ_c can be rewritten as

$$\Phi_c(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f; E_i, E_f) =$$

$$= \frac{1}{\hat{s}_i^2\hat{I}_i^2}\sum_{\delta,\gamma}|f_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_i) f_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_f)|\cos\Delta\varphi_{\delta\gamma},$$

where

$$\Delta\varphi_{\delta\gamma} \equiv \varphi_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_i) - \varphi_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_f)$$

$$= \int_{E_f}^{E_i} \frac{d\varphi_{\delta\gamma}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E)}{dE}\, dE.$$

For monoenergetic beams $\Delta\varphi_{\delta\gamma}$ can be related to the time delay $\tau_{\delta\gamma} = \hbar d\varphi_{\delta\gamma}/dE$ in plane-wave scattering[34,47]:

$$\Delta\varphi_{\delta\gamma} = \frac{1}{\hbar}\int_{E_i-E_\gamma}^{E_i} \tau_{\delta\gamma}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f, E)\, dE = \Delta\tau_{\delta\gamma}E_\gamma/\hbar.$$

Here, according to the mean-value theorem for integrals, $\Delta\tau_{\delta\gamma}$ is some average time. For weakly varying $\tau_{\delta\gamma}$ on the interval E_γ, that is, far from resonances,

$$\Delta\tau_{\delta\gamma} \cong \tau_{\delta\gamma}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f, E_i) \cong \tau_{\delta\gamma}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f, E_f).$$

For an isolated resonance $\tau_{\delta\gamma}(E) = (\hbar\Gamma/2)/[(E-E_r)^2 + \Gamma^2/4]$ and

$$\Delta\tau_{\delta\gamma} = \frac{\hbar}{E_\gamma}\,{\rm arctg}\,\frac{2E_\gamma\Gamma}{\Gamma^2 + 4(E_f - E_r - E_\gamma)(E_f - E_r)}.$$

In this case, strictly speaking, the average time $\Delta\tau_{\delta\gamma}$ will coincide with $\tau_{\delta\gamma}(E_f)$ only for $E_\gamma/[\Gamma^2 + 4(E_f - E_r)^2]^{1/2} \ll 1$

For spinless particles the real part of the correlation function is obviously related to the elastic-scattering differential cross sections as

$$\Phi_c(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_i, E_f) = \sqrt{\frac{d\sigma_{\rm el}(E_i)}{d\Omega_f}\frac{d\sigma_{\rm el}(E_f)}{d\Omega_f}}\cos\Delta\varphi, \tag{14}$$

where

$$\Delta\varphi = \varphi(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E_i) - \varphi(\hat{\mathbf{k}}_i\mathbf{k}_f, E_f) = \Delta\tau E_\gamma/\hbar.$$

In the case of the interaction of spin-$\frac{1}{2}$ particles with a spinless nucleus we have $f = g + h(\bar{\mathbf{k}}_i\times\bar{\mathbf{k}}_f)\boldsymbol{\sigma}/|\mathbf{k}_i\times\mathbf{k}_f|$ (Ref. 33), where g and h are scalar amplitudes depending on the energy, the scattering angle, and the nature of the interaction, and $\boldsymbol{\sigma}$ is a vector formed from the Pauli matrices. As a result, the function Φ_c will have the form (14), but

$$\cos\Delta\varphi \equiv \cos\varphi_{if} = \frac{{\rm Re}\,[g_i g_f^* + h_i h_f^*]}{(|g_i|^2 + |h_i|^2)(|g_f|^2 + |h_f|^2)}, \tag{15}$$

where g_i, h_i and g_f, h_f are the scalar amplitudes in the entrance and exit channels, respectively.

It follows from (15) that in general the phase φ_{if} cannot be identified with the phase shift of the elastic scattering amplitudes in the entrance and exit channels and therefore have no simple relation to the time delay of the particle in the nucleus. However, the latter can be done near an isolated resonance. In this case[33]

$$g_\alpha \cong i\hat{g}P_l(\cos\theta)\exp(2i\delta(E_\alpha)),$$

$$h_\alpha \cong \hat{h}P_l'(\cos\theta)\sin\theta\exp(2i\delta(E_\alpha)),$$

where $\hat{g}$ and $\hat{h}$ are smooth, real functions of the energy which can be assumed to be constant, and the expression for Φ_c takes the form

$$\Phi_c(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_i, E_f) \cong \frac{d\sigma_{\rm el}(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f; E_i)}{d\Omega_f}\cos(2\Delta\delta),$$

$$\Delta\delta = \delta(E_i) - \delta(E_f) = \Delta\tau E_\gamma/(2\hbar),$$

which is similar to that for the case of spinless particles.

The spectrum of bremsstrahlung of resonance protons on carbon was measured experimentally in Refs. 27 and 36, and the quantities $\cos\Delta\varphi$ were determined using (14). Then the relation $\Delta\varphi = \Delta\tau E_\gamma/\hbar$ was used to extract the quantities $\Delta\tau$, which were interpreted as the delay time of the protons in the field of the nucleus. This interpretation is limited in two respects—the phase difference $\Delta\varphi$ from (15) can, in general, be expressed in terms of the average time only near a resonance, and $\Delta\tau$ cannot be identified with $\tau(E_f)$ for all energies E_i.

The concept of a delay time in stationary scattering is rather artificial. It can be introduced rigorously in a temporal description of the collision process using wave packets.[29,34,47-50] In this case the instant that the collision occurs is determined with accuracy no better than $\hbar/\Delta E$, where ΔE is the energy spread in the wave packet, and for sufficiently large ΔE it can be considered fixed. One can therefore speak of time intervals τ during which secondary particles appear and also of their probability distribution $P(\tau)$.

The time distribution function has been determined formally by Ericson and Mayer-Kuckuk.[51] A physically motivated expression for $P(\tau)$ was first obtained by Kopylov, Lyuboshits, and Podgoretskiĭ[29,50]:

$$P(\tau) = \frac{1}{2\pi\hbar}\int_{-2\Delta E}^{2\Delta E} W(\varepsilon, \theta)\, e^{-i\varepsilon\tau/\hbar}\, d\varepsilon, \tag{16}$$

where

$$W(\varepsilon, \theta) = \int \rho(E', E'+\varepsilon)\, dE'$$

$$\times \frac{\int f(E, \theta) f^*(E-\varepsilon, \theta)\rho(E, E-\varepsilon)\, dE}{\int |f(E, \theta)|^2 \rho(E, E)\, dE}.$$

Here $\rho(E,E') = c(E-E_0)c^*(E-E_0)$, $c(E-E_0)$ is the weight function describing the energy spread in the wave packet near the average energy E_0, $\int\rho(E,E)dE = 1$, and $f(E,\theta) \equiv f(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E)$.

In the case where the normalized correlation function between the amplitudes

$$F_c(\varepsilon) \equiv \frac{\langle f(E,\theta) f^*(E-\varepsilon,\theta)\rangle_{E_0}}{\langle |f(E,\theta)|^2\rangle_{E_0}}, \tag{17}$$

where the symbol $\langle ...\rangle_{E_0}$ stands for averaging over $\rho(E,E)$, satisfies the conditions (A) that (i) it is concentrated near $\varepsilon = 0$ and (ii) the characteristic energy range of its variation δE_F, close to the range of the characteristic variation of the scattering amplitude, is much smaller than the averaging interval

$$\delta E_F \ll \Delta E,$$

it can be identified with $W(\varepsilon,\theta)$ and the definition (16) takes the form

$$P(\tau) = \frac{1}{2\pi\hbar}\int\limits_{-\delta E_F}^{\delta E_F} F_c(\varepsilon)\, e^{-i\varepsilon\tau/\hbar}d\varepsilon$$

$$\cong \frac{1}{2\pi\hbar}\int\limits_{-\infty}^{\infty} F_c(\varepsilon)\, e^{-i\varepsilon\tau/\hbar}\, d\varepsilon. \tag{18}$$

Up to terms of order $\varepsilon/\Delta E$,

$$F_c(\varepsilon) = F_c^*(-\varepsilon).$$

Because of this the function $P(\tau)$ defined by (18) will be real, which is consistent with its physical interpretation.

It follows from Eq. (18) that

$$F_c(\varepsilon) = \int\limits_{-\infty}^{+\infty} d\tau\, P(\tau)\, e^{i\varepsilon\tau/\hbar}. \tag{19}$$

When the conditions A are satisfied, the contribution of negative τ can be neglected, that is, it can be assumed that the integration over τ runs from 0 to ∞.[29,50]

Averaging, at fixed energy E_γ, the expression (13) over the incident-beam energy with weight function $\rho(E,E)$, when the conditions A are satisfied we have[29,50]

$$E_\gamma\left\langle\frac{d^3\sigma}{dE_\gamma\, d\Omega_\gamma\, d\Omega_f}\right\rangle_{E_0} = \left\langle\frac{d\sigma_{el}(\hat{\mathbf{k}}_i,\hat{\mathbf{k}}_f,E)}{d\Omega_f}\right\rangle_{E_0}$$

$$\times\left[\mathbf{b}_i^2 + \mathbf{b}_f^2 - 2(\mathbf{b}_i\mathbf{b}_f)\int\limits_0^\infty d\tau\, P(\tau)\cos\left(\frac{\tau E_\gamma}{\hbar}\right)\right],$$

$$E_\gamma \ll \delta E_F \ll \Delta E. \tag{20}$$

When the spread ΔE is sufficiently large, the average values $\langle d\sigma_{el}/d\Omega_f\rangle$ and F_c are not sensitive to the form of the weight function $\rho(E,E)$ (Ref. 52). For example, this occurs for $\Delta E \gg D, \Gamma$, where D is the average distance between compound resonances and Γ is the average width of these resonances. Therefore, the average values in (20) can be understood as the corresponding quantities for monoenergetic beams averaged over the experimental energy spread $\Delta E_{exp} \cong \Delta E$ of the incident particle.

The expression (20) for the bremsstrahlung spectrum coincides with that obtained by Eisberg, Yennie, and Wilkinson[6,53] using the classical approach. As was first shown by Kopylov, Lyuboshits, and Podgoretskiĭ,[29] this agreement between the classical and quantum results occurs only for spectra averaged in the interval $\Delta E \gg E$; the classical approach becomes meaningless in the scattering of monoenergetic particles. In fact, it is not possible to go to the case of monoenergetic beams in the expression (20), since the conditions A were used in its derivation.

It follows from (20) that the averaged spectrum of the bremsstrahlung of low-energy photons gives information on the real part of the correlation function F_c:

$$\mathrm{Re}\, F_c(E_\gamma) = \int\limits_0^\infty d\tau\, P(\tau)\cos(\tau E_\gamma/\hbar), \tag{21}$$

that is, on the Fourier component of the probability distribution of the delay time of the particle in the field of the nucleus.

In general, the distribution $P(\tau)$ has a complicated dependence on the particle emission angle. This dependence is simplified in the case of an isolated resonance or for strongly overlapping resonances, when they all have the same orbital angular momenta. In the former case[54]

$$\left.\begin{aligned} &F_c(\varepsilon) = (1 - i\varepsilon/\Gamma)^{-1}, \\ &P(\tau) = \theta(\tau)\exp(-\Gamma\tau/\hbar)\,\Gamma/\hbar. \end{aligned}\right\} \tag{22}$$

Therefore, the resonance width can be determined by studying the averaged bremsstrahlung spectra near an isolated resonance.

In the second case, when the level widths are identical and the level separations obey a Poisson distribution, $P(\tau)$ consists of two components, an instantaneous component and a component with a normal distribution.[29,55]

In reality, the case where all the resonances have the same orbital angular momentum is of low probability. In the general case it is necessary to explicitly isolate the dependence on the angle θ_f between the directions of the particle motion in the entrance and exit channels.

3. LOW-ENERGY BREMSSTRAHLUNG IN THE REGION OF OVERLAPPING RESONANCES

The averaged differential cross section for the emission of bremsstrahlung photons taking into account the spin structure of the channels

We represent the scattering amplitude $f_{\delta\gamma}$ as an expansion in Legendre polynomials $P_l(\cos\theta_f)$ and use the j representation for the S-matrix elements.[56] As a result, the elastic scattering cross sections and the correlation function will be determined by

$$g(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f;\ E_1,\ E_2) =$$

$$= \sum A_{ij}(\theta_f)\,(S^{J\pi}_{p_jp_i}(E_1) - \delta_{p_ip_j})\,(S^{J'\pi'}_{p'_jp'_i}(E_2) - \delta_{p'_ip'_j}),$$

where $S^{J\pi}_{p_fp_i} \equiv S^{J\pi}_{l_fj_f\alpha_f,l_ij_i\alpha_i}$ are the S-matrix elements for scattering with total spin J and parity π, the α_i are the quantum numbers characterizing channel i which are different from l_i and j_i, the orbital and total angular momenta of the relative motion of the particles in channel i, and from J^π:

$$A_{if}(\theta_f) = \frac{1}{4k_i k_f \hat{s}_i^2 \hat{I}_i^2} \sum_n P_n(\cos\theta_f)$$
$$\times B(l_i j_i l_i' j_i';\ J,\ J';\ n)\, B(l_f j_f l_f' j_f';\ J,\ J';\ n),$$
$$B(ljl'j';\ JJ'n)$$
$$= (-)^{s_i+j+j'+I_i+J} \hat{l}\hat{l}'\hat{j}\hat{j}'\hat{J}\hat{J}' C^{n0}_{l0l'0} \begin{Bmatrix} jj'n \\ l'ls_i \end{Bmatrix} \begin{Bmatrix} JJ'n \\ j'jI_i \end{Bmatrix},$$

$C^{ab}_{cd,ef}$ is the Clebsch–Gordan coefficient, and $\{^{abc}_{def}\}$ are the $6j$ symbols; the summation runs over the quantum angular momenta $l_i, j_i, l_i', j_i', l_f, l_f', j_f', J$ and J'. Here for the elastic scattering cross sections $d\sigma_{el}(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_f, E)/d\Omega_f = g(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E, E)$, and in the case of the correlation function $\Phi_c(\varepsilon) = \mathrm{Re}\, g(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_f, E_i, E_f = E_i - \varepsilon)$.

In the resonance interaction of a particle with a nucleus it is possible to isolate the instantaneous stage related to the average S matrix, which is usually identified with direct processes.[57] It is therefore convenient to split the scattering matrix into two components, the average component $\langle S\rangle$ and the so-called fluctuation component $S^{(\mathrm{fl})} = S - \langle S\rangle$, and to use this representation for studying the average bremsstrahlung spectrum. If we now average the expression (13) in the interval ΔE near the energy E_0 satisfying the conditions A, we obtain[1)]

$$E_\gamma \left\langle \frac{d^3\sigma}{dE_\gamma\, a\Omega_\gamma\, d\Omega_f} \right\rangle = \frac{d\sigma_{el}^{(d)}}{d\Omega_f} (\mathbf{b}_i - \mathbf{b}_f)^2$$
$$+ \frac{d\sigma_{el}^{(\mathrm{fl})}}{d\Omega_f} (\mathbf{b}_i^2 + \mathbf{b}_f^2) - 2(\mathbf{b}_i\mathbf{b}_f)\, \mathrm{Re}\, \Phi^{(\mathrm{fl})}(E_\gamma),\ E_\gamma \ll \Delta E, \quad (23)$$

where

$$\frac{d\sigma_{el}^{(d)}}{d\Omega_f}$$
$$= \Sigma A_{if}(\theta_f) [\langle S^{J\pi}_{p_f p_i}(E_i)\rangle_{E_0} - \delta_{p_f p_i}] [\langle S^{J\pi *}_{p_f' p_i'}(E_i)\rangle_{E_0} - \delta_{p_f' p_i'}]$$

is the direct ("instantaneous") contribution of elastic scattering to the cross section,

$$\frac{d\sigma_{el}^{(\mathrm{fl})}}{d\Omega_f} = \Sigma A_{if}(\theta_f) \langle S^{J\pi(\mathrm{fl})}_{p_f p_i}(E_i)\, S^{J\pi'(\mathrm{fl})*}_{p_f' p_i'}(E_i)\rangle_{E_0} \equiv \Sigma \frac{d\sigma_{el}^{J\pi(\mathrm{fl})}(\theta_f)}{d\Omega_f}$$

is the fluctuation contribution, which is related to the production of long-lived states of the compound system, and

$$\Phi^{(\mathrm{fl})}(\varepsilon) = \Sigma A_{if}(\theta_f) \langle S^{J\pi(\mathrm{fl})}_{p_f p_i}(E_i)\, S^{J\pi'(\mathrm{fl})*}_{p_f' p_i'}(E_i - \varepsilon)\rangle_{E_0} \quad (24)$$

is the fluctuation component of the amplitude correlation function.

Therefore, in the general case the averaged spectrum of low-energy photon bremsstrahlung is determined by the direct and fluctuation contributions to the elastic scattering cross section and by the fluctuation component of the correlation function. The latter quantities are bilinear combinations of the fluctuation components of the S matrices. The calculation of these quantities is a complicated problem which has been studied intensively in connection with the statistical theory of binary nuclear reactions.[58,59]

Methods of calculating the averaged products of elements of the scattering matrix

The techniques for calculating the averaged bilinear combinations of the S matrices consist of parametrizing them in terms of the averaged elements of the scattering matrix. The latter are calculated using either the usual optical model or the strongly-coupled-channel method. It is assumed that the energy is automatically averaged over when a complex potential is used.

In the derivation of the expressions for the fluctuation contributions the scattering matrix is replaced by some random function and the energy averages are replaced by averages over the ensemble. Let us discuss the validity of this approach.

In the general case the scattering matrix S with fixed total spin J and parity π is a function of the energy and of the matrix elements H_{lm} of the Hamiltonian H of the system between states of the discrete ($\{l, m\} = d$) and continuous ($\{l, m\} = c$) spectra: $S_{ab} = S_{ab}(E, \{H_{lm}\})$. In principle, the elements S_{ab} can be calculated from the known $\{H_{lm}\}$. However, for systems with a large number of states this method of calculating the scattering matrix cannot be used in practice. Numerical studies[60–63] have shown that at a fixed energy for a sufficiently complicated system the matrix elements H_{dd} and H_{dc} are a sample from the statistical ensemble specified by some distribution $f(E, \{H_{dd}, H_{cd}\})$. Since the quantities H_{cd} vary with energy, the distribution also depends on the energy. In the interval δE, where the function f can be assumed to be uniform in E, the elements of the scattering matrix S_{ab} will be determined by different samples H_{cd} from the ensemble f. If the elements $H_{cc'}$ vary just as weakly in the interval δE, quantities averaged over the energy in the interval $\Delta E \lesssim \delta E$ can be identified with ensemble averages:

$$\overline{S}_{ab} = \int S_{ab}(E, \{H_{cd}, H_{dd}\})\, f(\{H_{cd}, H_{dd}\})\, d\{H_{cd}, H_{dd}\}.$$

The replacement of the averages is valid if the ensemble of random S matrices is ergodic. This property has been verified numerically for a number of situations[67] and has been proved analytically.[59,64,65]

Let us consider the case where the energy averages and the ensemble averages coincide and find the limits on the averaging interval ΔE for which this replacement will be valid. As in Ref. 66, we shall use the criterion of the rms deviation.

If the interval δE on which the distribution function f is independent of the energy is larger than or equal to ΔE, then in the averaging interval $\overline{S}_{ab}$ it is independent of the energy—it is not changed by repeated averaging over the ensemble and $\langle \overline{S}_{ab}\rangle = \overline{S}_{ab}$. Without loss of generality we can choose the weight function for the averaging to be a square step of width ΔE, and then we find

$$\overline{|\Delta S_{ab}|^2} = \overline{|\langle S_{ab}\rangle - \overline{S}_{ab}|^2} = \frac{1}{(\Delta E)^2} \int_0^{\Delta E} dt \int_0^{\Delta E} dt'\, K_{ab}(t, t'), \quad (25)$$

where

$$K_{ab}(t, t') = \overline{S_{ab}^{(\mathrm{fl})}(E_i + t)\, S_{ab}^{(\mathrm{fl})*}(E_i + t')},\ E_i = E_0 - \frac{\Delta E}{2} \quad (26)$$

is the complex correlation function of the fluctuation components of the S matrix.

A random process specified by an ensemble of random functions S_{ab} is stationary on the interval δE, so that[68]

$$K_{ab}(t,\ t') \equiv K_{ab}(t-t') = K_{ab}^{*}(t'-t).$$

As a result, we obtain

$$\overline{|\langle S_{ab}\rangle - \overline{S}_{ab}|^2} \leqslant \frac{E_c}{\Delta E}\,\overline{|S_{ab}^{(\mathrm{fl})}(E_i)|^2},$$

where the quantity

$$E_c \equiv \int_0^{\Delta E} d\tau\, |\operatorname{Re} K_{ab}(\tau)|/K_{ab}(0) \qquad (27)$$

will be referred to as the energy length of the correlations.

Therefore, if $\Delta E \lesssim \delta E$ is significantly larger than the correlation length E_c, the modulus of the squared deviation of the average of the S matrix over energy from the ensemble average is small and the ergodicity condition will be satisfied by the matrix elements S_{ab}. Similarly, for bilinear combinations we find

$$\overline{|\langle S_{ab}S_{cd}^{*}\rangle - \overline{S_{ab}}\,\overline{S_{cd}^{*}}|^2} \leqslant \frac{E_c^{*}}{\Delta E} K_{abcd}(0),$$

where

$$E_c^{*} = 2\int_0^{\Delta E} d\tau\, |\operatorname{Re} K_{abcd}(\tau)|/K_{abcd}(0);$$

$$K_{abcd}(t-t') = \overline{S_{ab}(E_i+t)\,S_{cd}^{*}(E_i+t)\,S_{ab}^{*}(E_i+t')\,S_{cd}(E_i+t')} - \overline{|S_{ab}(E_i)\,S_{cd}^{*}(E_i)|^2}. \qquad (28)$$

Therefore, the ergodicity condition applied to binary products of the S matrices will be satisfied for $\Delta E \lesssim \delta E$ significantly greater than E_c^{*}.

The correlation length E_c and E^{*} can be found after computing the corresponding ensemble averages and are discussed later in this section.

As a consequence of the invariance under rotations and time reversal there is no general condition like the unitarity relation which would lead to a relation between scattering matrix elements with different J^{π}. It is usually assumed that the wave functions of compound states are so complicated that their phases are random functions of J, so that the ensembles $S^{J^{\pi}(\mathrm{fl})}$ are independent and uncorrelated, that is,

$$\overline{S_{ab}^{J^{\pi}(\mathrm{fl})}(E_1)\,S_{cd}^{J^{\pi'}(\mathrm{fl})*}(E_2)} = \delta_{JJ'}\delta_{\pi\pi'}\,\overline{S_{ab}^{(\mathrm{fl})}(E_1)\,S_{cd}^{(\mathrm{fl})*}}(E_2). \qquad (29)$$

We have dropped the indices J^{π} on the right-hand side of (29) and in the formulas below.

In the studies of Weidenmüller *et al.*[67,69] and Moldauer[71] a method was proposed for calculating $\overline{S_{ab}^{(\mathrm{fl})}S_{cd}^{(\mathrm{fl})*}}$ for $\overline{S}_{ab} \neq 0$ $(a=b)$, that is, when there are direct inelastic transitions between channels, which is consistent with the unitarity of the scattering matrix. This method is based on the Engelbrecht–Weidenmüller unitary transformation,[69] the matrix of which, U, diagonalizes the averaged scattering matrix,

$$(U\overline{S}U^{T}) = \hat{S}_{ab} = \delta_{ab}s_a, \qquad (30)$$

where U^{T} is the transpose of U.

The Satchler transmission matrix is diagonalized simultaneously:

$$\left.\begin{aligned} &P_{ab} \equiv \delta_{ab} - \sum_c \overline{S}_{ac}\overline{S}_{bc}^{*}, \\ &(UPU^{-1})_{ab} = p_a\delta_{ab},\ \ p_a = 1 - s_a^2. \end{aligned}\right\} \qquad (31)$$

A method of calculating U and s_a using the relation (30) was described in Ref. 72 and realized in Refs. 73 and 74.

The transformation U effects a transition to new effective channels between which there are no direct inelastic transitions. The average values of products of the fluctuation components of the S matrix satisfy the relation

$$\overline{S_{ab}^{(\mathrm{fl})}S_{cd}^{(\mathrm{fl})*}} = \sum_{efgh} U_{ea}^{*}U_{fb}^{*}U_{gc}U_{hd}\,\overline{\hat{S}_{ef}^{(\mathrm{fl})}\hat{S}_{gh}^{(\mathrm{fl})*}}, \qquad (32)$$

where $\overline{\hat{S}_{ef}^{(\mathrm{fl})}\hat{S}_{gh}^{(\mathrm{fl})*}}$ are the corresponding averages when inelastic channels are not directly coupled. As a result, for averaging intervals which are significantly smaller than the intervals of the characteristic variation of $\overline{S}_{ab}$, it is possible to explicitly isolate the correlations related to direct inelastic transitions.[2)] Here

$$\begin{aligned}&\overline{\hat{S}_{ef}^{(\mathrm{fl})}(E_1)\,\hat{S}_{gh}^{(\mathrm{fl})*}(E_2)} \\ &= (\delta_{eg}\delta_{fh} + \delta_{eh}\delta_{fg})(1-\delta_{ef})\,\overline{\hat{S}_{ef}^{(\mathrm{fl})}(E_1)\,\hat{S}_{ef}^{(\mathrm{fl})*}}(E_2) \\ &\quad + \delta_{ef}\delta_{gh}\hat{S}_{ee}^{(\mathrm{fl})}(E_1)\,\hat{S}_{gg}^{(\mathrm{fl})*}(E_2). \end{aligned} \qquad (33)$$

General expressions for the averaged binary products $\overline{\hat{S}_{ef}^{(\mathrm{fl})}(E_1)\hat{S}_{gh}^{(\mathrm{fl})*}(E_2)}$ which are convenient for applications have been obtained in Refs. 67, 70, 71, and 73–75. Weidenmüller *et al.*[67,70] have used the method of numerical modeling of the random S matrix on the basis of the K-matrix representation, which automatically preserves the unitarity of the scattering matrix independently of the distribution of the resonance parameters of the K matrix. Approximation formulas were proposed which accurately reproduce the numerical calculations.

Moldauer[71,75] used the pole expansion of the S matrix and the so-called M-compensation hypothesis, according to which terms related to correlations between the amplitudes of the reduced widths are cancelled by terms containing correlations between the amplitudes of the reduced widths and the resonance energies. This leads to formulas of the same form as those in the case of weak absorption ($p_a \ll 1$) in all channels. In the general case the validity of the M-compensation hypothesis is confirmed indirectly—by the agreement between the fluctuation cross sections calculated by different methods.

In Refs. 73 and 74 numerical modeling was used to formulate and study a modified computational technique. In contrast to Refs. 67 and 70, an expression[76] valid for a large number of closed channels was used as the initial approximation formula.

An integral representation for $\overline{\hat{S}_{ef}^{(\mathrm{fl})}\hat{S}_{gh}^{(\mathrm{fl})*}}$ has recently been obtained[77–79] in the limit of a large number of resonance levels ($\Lambda \to \infty$). A unitary expression[80] close to that of the shell model[81] was used as the initial scattering matrix. The matrix elements of the residual interaction V_{dd} and V_{cd} were assumed to obey a normal distribution.

When the resonances overlap weakly or strongly ($\Gamma \ll D$ and $\Gamma \gg D$, respectively) a large number of different, fairly simple expressions is again obtained for the averaged binary products of the scattering matrix. However, the ratio Γ/D of the average resonance width to the average separation for fixed J^{π} is determined by the averaged scattering matrix[75,82] (the Moldauer–Simonius theorem),

$$|\det \overline{S}| = \exp(-\pi\Gamma/D),$$

that is, using (30),

$$\Gamma/D = -(\ln|\det \overline{S}|)/\pi = -(\ln \prod_g s_g)/\pi$$
$$= (\sum_g \ln(1/(1-p_g)))/(2\pi), \quad (34)$$

and, in general, before calculating and diagonalizing $\hat{S}$ it is impossible to say which situation actually occurs in a specific case.

Calculational methods and expressions for the correlation function. Relation to Ericson fluctuations

In the calculation of the two-point ($E_1 \neq E_2$) averaged binary products of the scattering matrix these products are expressed in terms of the one-point ($E_1 = E_2$) products, defining $\varphi_{ab}(\varepsilon)$ as the autocorrelation function of the fluctuation components of the scattering matrix ($|E_1 - E_2| \leqslant \delta E$):

$$\overline{S_{ab}^{(\mathrm{fl})}(E_1)\, S_{ab}^{(\mathrm{fl})*}(E_2)} \equiv \varphi_{ab}(E_1 - E_2)\, \overline{|S_{ab}^{(\mathrm{fl})}(E_1)|^2}.$$

According to Eq. (26), $\varphi_{ab}(\varepsilon) = K_{ab}(\varepsilon)/K_{ab}(0)$ and its real part characterizes the ergodicity of the scattering matrix.

It follows from Eq. (33) that in the absence of direct inelastic transitions [$\varphi_{ab}(\varepsilon) \equiv \hat{\varphi}_{ab}(\varepsilon)$] the binary two-point products of the fluctuation components of the scattering matrix (with the possible exception of the case $e = f$, $g = h$, $e \neq g$) are expressed in terms of the normalized autocorrelation function $\bar{\varphi}_{ab}(\varepsilon)$.

The function $\hat{\varphi}_{ab}(\varepsilon)$ is also related to the normalized autocorrelation function of the cross sections $c_{ab}(\varepsilon)$ in the theory of Ericson fluctuations[51,83]:

$$c_{ab}(\varepsilon) = \frac{\overline{\sigma_{ab}(E+\varepsilon)\,\sigma_{ab}(E)} - (\bar{\sigma}_{ab})^2}{(\bar{\sigma}_{ab})^2},$$

where $\sigma_{ab} = (\pi/k_a^2) g |\delta_{ab} - S_{ab}|^2$ is the partial cross section for the transition between channels a and b with fixed J^π, and k_a and g are the wave number in the channel a and the statistical weight.

For $\overline{S}_{ab} = \delta_{ab}\overline{S}_{aa}$ and when the matrix elements $S_{ab}^{(\mathrm{fl})}$ obey a normal distribution, which holds with good accuracy when the number of open channels is $\gtrsim 10$ (Refs. 84 and 85), we have

$$c_{ab}(\varepsilon) \equiv \hat{c}_{ab}(\varepsilon) = \frac{\delta_{ab}|1-\overline{S}_{aa}|^2 \operatorname{Re} \overline{S_{aa}^{(\mathrm{fl})}(E+\varepsilon)\, S_{aa}^{(\mathrm{fl})*}(E)} + \overline{|S_{ab}^{(\mathrm{fl})}(E+\varepsilon)\, S_{ab}^{(\mathrm{fl})*}(E)|^2}}{(k_a^2 \bar{\sigma}_{ab}/\pi)^2},$$

where it is assumed that $\varepsilon < \Delta E \lesssim \delta E$, so that the averages are identified at E and $E + \varepsilon$.

Therefore,

$$\hat{c}_{ab}(\varepsilon) = |\hat{\varphi}_{ab}(\varepsilon)|^2 \quad (a \neq b).$$

In accordance with the relations (28), in the case $\overline{S}_{ab} = \delta_{ab}\overline{S}_{aa}$ with $a \neq b$ the autocorrelation function $\hat{c}_{ab}(\varepsilon) = K_{abab}(\varepsilon)/K_{abab}(0)$ determines the ergodicity of binary products of the scattering matrix.

The autocorrelation functions of the amplitudes and cross sections were first calculated in the Ericson fluctuation theory.[51,83] In this theory the scattering matrix is not unitary. Its real and imaginary parts are represented as sums of a very large number of Breit–Wigner terms with constant width, and it is assumed that the residues at the poles are uncorrelated and that they have random phases, zero mean values, and identical dispersions. The autocorrelation functions in this case are identical for all channels and, as in the case of isolated resonances (22), are Lorentzian[51,59,83]:

$$\hat{\varphi}_{ab}(\varepsilon) \equiv \varphi_L(\varepsilon) = i\gamma/(\varepsilon + i\gamma), \quad (35)$$

where

$$\gamma \equiv \Gamma. \quad (36)$$

In this case the correlation length is $E_c \cong \pi\gamma$ [see Eq. (27)], and the ergodicity condition for the elements of the scattering matrix will be satisfied for energy averaging intervals $\Delta E \gg \pi\gamma$.

It was originally thought that the assumptions about the behavior of the elements of the scattering matrix are satisfied in the region of strongly overlapping resonances, and the condition for the validity of (35), (36) was determined in Refs. 51 and 83 as $\Gamma \gg D$. Subsequent studies (Refs. 54, 69, 75, 79, and 86–89) showed that this is not correct.

Agassi, Weidenmüller, and Mantzouranis[87] gave an analytic derivation of the expression for the autocorrelation functions in the limit of strongly overlapping resonances ($\Gamma \gg D$). It was assumed that the matrix elements of the residual interaction V_{dd} and V_{cd} obeyed a normal distribution with zero averages and known second moments. It was shown that, independently of the distribution of the resonance energies and the presence of direct processes, the autocorrelation function for the elements of the scattering matrix is Lorentzian, although the correlation width does not coincide with the average width Γ, but is equal to

$$\gamma \equiv D\,\mathrm{Tr}P/(2\pi). \quad (37)$$

According to (34), the width γ will coincide with the average width in the range

$$D \ll \Gamma \ll D/n,$$

where n is the number of open channels.

The same results for the autocorrelation functions of the cross sections, that is, $\hat{c}(\varepsilon) \equiv |\varphi_L(\varepsilon)^2|$, in the absence of direct inelastic processes was obtained by Moldauer[75] by numerical modeling. The K-matrix representation was used for the scattering matrix. The amplitudes of the reduced widths were assumed to obey a normal distribution, and the resonance spacings were assumed to obey the Wigner distribution. The model of statistically equivalent channels, that is, $p_a \equiv p \neq f(a)$, was used. The values of the ratio $2\pi\Gamma/(Dn)$ were found to lie in the range from 1.1 to 2.7.

Bauer, Mello, and McVoy[86] have shown that for the Lorentzian form of the autocorrelation function $\hat{\varphi}(\varepsilon)$ the width γ will always be given by Eq. (37).

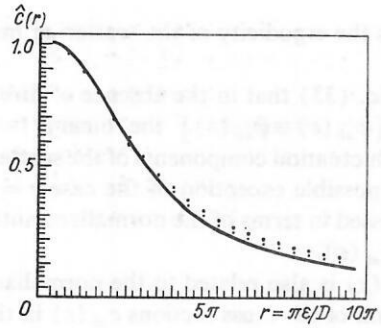

FIG. 5. Autocorrelation function of the cross sections $\bar{c}$; $n = 20$, the upper points correspond to $p = 0.01$, the lower points to $p = 1.0$, and the solid line is calculated using (35) and (37).

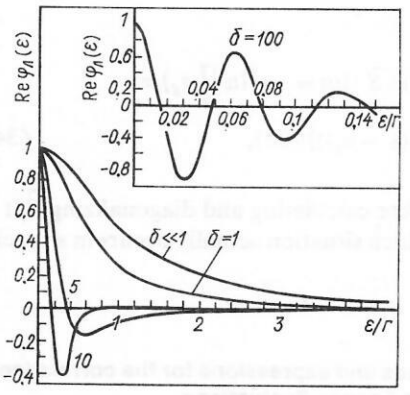

FIG. 6. Real part of the autocorrelation function $\varphi_\Lambda(\varepsilon)$.

Verbaarschot[79] has studied the behavior of the function $\hat{c}(\varepsilon)$ numerically, starting from the integral representation for the averaged bilinear combinations of the scattering matrix (see above). He has considered the case[20] of statistically equivalent channels with the quantity $2\pi\Gamma/(nD)$ equal to 0.01, 0.05, 0.105, and ∞. The form of the function $\hat{c}(\varepsilon)$ is nearly Lorentzian with γ given by (37) (Fig. 5). The calculated autocorrelation functions fall off somewhat more slowly than $\hat{c}_L(\varepsilon)$. Lyuboshits[50,54,88,89] has studied the autocorrelation functions $\hat{\varphi}(\varepsilon)$ in connection with the probability distribution of the particle delay times in a compound nucleus. The unitary representation of Simonius[90] was used as the initial S matrix. The model of equivalent channels was formulated and used in the case of a large number of open channels. In this model one has

$$\overline{\det S} = \det \bar{S},$$
$$\overline{\det S(E)\, S^+(E-\varepsilon)} = (\det \overline{S(E)\, S^+(E-\varepsilon)}),$$
$$(\bar{S})_{ab} = s\delta_{ab},$$
$$(\overline{S(E)\, S^+(E-\varepsilon)})_{ab} = r\delta_{ab},$$
$$\varphi_{ab}(\varepsilon) = \varphi(\varepsilon).$$

In addition, it was assumed that all the resonances have the same width and that their energies obey a Poisson distribution.[55] As a result,

$$\hat{\varphi}(\varepsilon) \equiv \varphi_\Lambda(\varepsilon) = (\exp[i\delta/(i+y)] - 1)/(\exp(\delta) - 1),$$
$$\delta \equiv 2\pi\Gamma/(nD),\quad y \equiv \varepsilon/\Gamma. \tag{38}$$

In general, autocorrelation functions of this form oscillate and do not have the Lorentzian shape. For example, for the overlap parameter $\delta > 2.2$, $\mathrm{Re}\,\varphi_\Lambda(\varepsilon)$ passes through zero and goes into the region of negative values. For $\delta \gg 1$ it oscillates, taking positive and negative values, and only for $\delta \ll 1$ does it go into the Ericson expression (35), (36). This is illustrated in Fig. 6 of Ref. 50.

The autocorrelation function determines the probability distribution $P_c(\tau)$ of the particle delay times in a compound nucleus by formulas similar to (16)–(19), that is,[50]

$$P_c(\tau) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \varphi(E)\, e^{-\frac{i}{\hbar}\varepsilon\tau}\, d\varepsilon. \tag{39}$$

Therefore the difference of $\varphi_\Lambda(\varepsilon)$ from the Lorentzian shape is related to the divergence of the time-delay probability distribution from the damped exponential (22). According to (38) and (39), for $\delta \gg 1$ the time-delay probabilities obey a normal distribution.[50,91]

We note that a nonexponential distribution $P_c(\tau)$ has also been obtained in Ref. 92 for the one-channel case. Kadmenskiĭ and Furman[93,94] have studied the problem of possible deviations of the compound-nucleus decay law from the exponential form in a physically realistic situation. They used Eqs. (34) and (37) to calculate the ratio of the widths γ/Γ for states with $J^\pi = 11^+/2$ of the nucleus ^{189}Os in the excitation energy range $6 \lesssim E^* \lesssim 100$ MeV. The transmission coefficients were calculated using the optical model, and decay channels with nucleon, deuteron, and α-particle emission were included. The contribution of direct inelastic processes was neglected. The ratio γ/Γ grows up to excitation energies $\cong$45 MeV, and then it saturates, never exceeding 3 (Ref. 94). In the equivalent-channel model

$$\gamma/\Gamma = \delta/(1 - \exp(-\delta)),$$

so that in this case the "total-absorption" regime, that is, $\delta \gg 1$, is not realized. It was stated in Ref. 93 that the relation $\delta \gg 1$ does not hold in general and that the time-delay distribution therefore differs little from the exponential law.

Let us calculate the ratios γ/Γ and δ in the model with identical transmission coefficients $t_a = 1 - |\bar{S}_{aa}|^2 \equiv t$. We consider the case when

$$t = 1 - \Delta, \tag{40}$$

where Δ is the "transmission window" of the nucleus and the relative contribution (β_a^d) of direct inelastic processes to the average cross section $\bar{\sigma}_a^r$ is identical for all channels a, that is,

$$\beta_a^d = \sigma_a^d/\bar{\sigma}_a^r \equiv \beta^d. \tag{41}$$

According to Ref. 73, there are at least three cases in which the eigenvalues of the Satchler matrix p_a (31) can be calculated analytically and are $p_a = P_{aa} = 1 - \Sigma_b|\bar{S}_{ab}|^2 = t_a - \Sigma_{b\neq a}|\bar{S}_{ab}|^2$: (1) when there is a large number $n \gg 1$ of open, identically coupled channels; (2) when the contribution of direct inelastic processes is small; (3) when there is strong absorption in all channels. Using the unitarity condition for the scattering matrix and neglecting the compound contribution in the elastic channel, we obtain $p_a = (1 - \beta_a^d)t_a$. Returning to Eq. (34), we see that the presence of direct inelastic processes causes the ratio Γ to de-

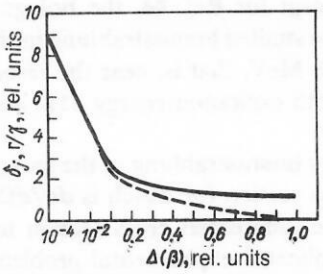

FIG. 7. Dependence of the overlap parameter $\delta = 2\pi\Gamma/(nD)$ (dashed line) and Γ/γ (solid line) on $\Delta(\beta)$.

crease to D. For the model defined by (40) and (41) we have

$$\delta = -\ln(1-p) = -\ln(\Delta+\beta-\beta\Delta),$$
$$\Gamma/\gamma = -\frac{\ln(1-p)}{p} = -\frac{\ln(\Delta+\beta-\beta\Delta)}{[1-\beta][1-\Delta]}.$$

Since $\Delta \leqslant 1$ and $\beta \leqslant 1$, the presence of either a direct inelastic process or a transmission window Δ leads to the same effect—the decrease of the overlap parameter δ and the approach of the correlation width γ to the average width Γ. The calculated values of δ and Γ/γ are shown in Fig. 7. For $\Delta = 0$ the value of q coincides with β, while for $\beta = 0$ it is equal to Δ. For $q < 0.1$ the scale on the horizontal axis is logarithmic. We see that in the case of a completely black nucleus ($t = 1$) a 10% contribution from direct processes already leads to an overlap parameter smaller than 3. When direct processes are absent, this occurs for $\Delta \gtrsim 0.1$. This is consistent with the results of Kadmenskiĭ and Furman.[93,94]

As a rule, either $\Delta \neq 0$ or $\beta \neq 0$, so that large values of δ and the differences between the widths Γ and γ should not be expected. For example, when the dominant contribution comes from neutron decay channels, assuming that $\Delta = 0$ and taking, as usual, $\beta = 0.1$ (Ref. 95), we have $\delta < 2.5$ according to the result in Fig. 7.

On the whole, at present there is no single view about the behavior of the autocorrelation functions $\hat{\varphi}(\varepsilon)$ or, equivalently, about the time-delay distribution $P_c(\tau)$ in a compound nucleus. The overlap parameter has been calculated only in the cases studied above. This problem is interesting from the viewpoint of investigating the dynamics of the behavior of compound systems and deserves further study. Important information about the behavior of the autocorrelation functions might be obtained from experiments on low-energy bremsstrahlung in the region of overlapping resonances.

The relation between the averaged bremsstrahlung spectrum and the autocorrelation function

Let us express the bremsstrahlung differential cross section (23) explicitly in terms of the autocorrelation function $\hat{\varphi}(\varepsilon)$. According to (33), the averaged bilinear products of the fluctuation components of the S matrix ($\overline{S}_{ab} = \delta_{ab}\overline{S}_{aa}$) are expressed in terms of $\overline{\hat{S}^{(\mathrm{fl})}_{ef} S^{(\mathrm{fl})}_{ef}}$ and $\overline{\hat{S}^{(\mathrm{fl})}_{ee}\hat{S}^{(\mathrm{fl})}_{gg}}$.

It follows from the foregoing discussion that the first product can be rewritten in terms of the autocorrelation function $\hat{\varphi}$ which does not depend on the channel indices:

$$\overline{\hat{S}^{(\mathrm{fl})}_{ef}(E)\,\hat{S}^{(\mathrm{fl})*}_{ef}(E-\varepsilon)} = \hat{\varphi}(\varepsilon)\,\overline{|\hat{S}^{(\mathrm{fl})}_{ef}(E)|^2}. \qquad (42)$$

The second product with $e \neq g$ can be treated in either of two ways; one can assume that it satisfies a relation of the type (42), or one can drop this relation. The latter alternative will be a good approximation in the case of a large number of open channels, since the number of such products is roughly n^2 times smaller than that of the others. In either case, using Eqs. (32) and (33) we obtain the following expression for $\Phi^{(\mathrm{fl})}(\varepsilon)$, the fluctuation component of the amplitude correlation function (24):

$$\Phi^{(\mathrm{fl})}(\varepsilon) = \sum_{J^\pi} \hat{\varphi}^{J^\pi}(\varepsilon)\,\frac{d\sigma_{\mathrm{el}}^{J^\pi(\mathrm{fl})}}{d\Omega_f}. \qquad (A)$$

Here $d\sigma_{\mathrm{el}}^{J^\pi(\mathrm{fl})}/d\Omega_f$ is the fluctuation component of the cross section for elastic scattering with given total spin J and parity π of the compound system, and the dependence of the function $\hat{\varphi}(\varepsilon)$ on J^π has been noted explicitly.

The expression (23) for the low-energy bremsstrahlung spectrum takes the form

$$E_\gamma \left\langle \frac{d^3\sigma}{dE_\gamma\, d\Omega_\gamma d\Omega_f} \right\rangle = \frac{d\sigma_{\mathrm{el}}^{(d)}}{d\Omega_f}(\mathbf{b}_i-\mathbf{b}_f)^2 + \frac{d\sigma_{\mathrm{el}}^{(\mathrm{fl})}}{d\Omega_f}(\mathbf{b}_i^2+\mathbf{b}_f^2) - 2(\mathbf{b}_i\mathbf{b}_f)\sum_{J^\pi}\frac{d\sigma_{\mathrm{el}}^{J^\pi(\mathrm{fl})}}{d\Omega_f}\,\mathrm{Re}\,\hat{\varphi}^{J^\pi}(E_\gamma). \qquad (43)$$

Independently of whether or not direct processes are present, the spectrum (43) depends on the autocorrelation function $\hat{\varphi}(\varepsilon)$ for the $\hat{S}$ matrix with no coupled channels.

The autocorrelation function varies weakly with J for heavy nuclei and when the elastic scattering receives contributions from compound-nucleus states with spins which are not too large. If in addition the incident-particle energy is sufficiently high, the appearance of states with $\pi = \pm 1$ is equally probable and it is possible to use the approximation

$$\hat{\varphi}^{J^\pi}(\varepsilon) = \hat{\varphi}(\varepsilon).$$

We note that an approximation of this type is frequently used in fluctuation analyses of the cross sections based on the Ericson method.[96,97]

As a result, according to (43)

$$E_\gamma \left\langle \frac{d^3\sigma}{dE_\gamma\, d\Omega_\gamma\, d\Omega_f} \right\rangle = \frac{d\sigma_{\mathrm{el}}^{(d)}}{d\Omega_f}(\mathbf{b}_i-\mathbf{b}_f)^2 + \frac{d\sigma_{\mathrm{el}}^{(\mathrm{fl})}}{d\Omega_f}[\mathbf{b}_i^2+\mathbf{b}_f^2 - 2(\mathbf{b}_i\mathbf{b}_f)\,\mathrm{Re}\,\hat{\varphi}(E_\gamma)].$$

Therefore, from the averaged spectrum of bremsstrahlung of low-energy photons with $\Delta E \gg \gamma \gtrsim E_\gamma$ and $E_i \gg E_\gamma$ it is possible, in principle, to extract the direct and the fluctuation contributions to the elastic cross section and also to obtain information on the autocorrelation fluctuation, that is, on the Fourier component of the time-delay probability distribution in a compound nucleus: $\mathrm{Re}\hat{\varphi}(E_\gamma) = \int_0^\infty d\tau\, P_c(\tau)\cos(\tau E_\gamma/\hbar)$.

We note that in the usual methods of studying correlation functions based on the study of the Ericson fluctuations of the cross sections, one determines not $\mathrm{Re}\,\hat{\varphi}$, but rather the modulus squared of the autocorrelation function. In addition, in the study of the correlation properties of strongly overlapping resonances using the bremsstrahlung it is possible also to employ nonmonoenergetic incident-particle beams of width[3)] $\Delta E \gg \gamma$, whereas in the case of Ericson fluc-

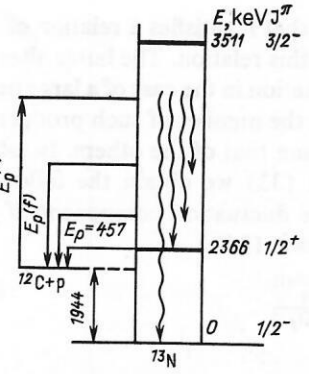

FIG. 8. Scheme of the reaction $^{12}C(p, \gamma p')^{12}C$.

tuations a sufficiently accurate determination of $\hat{c}(\varepsilon)$ can be obtained only for $\Delta E \ll \gamma$.

4. EXPERIMENTAL METHODS OF STUDYING THE BREMSSTRAHLUNG ACCOMPANYING PROTON SCATTERING

According to the expression (13) for the experimental determination of the correlation function and, consequently, the temporal characteristics of the interaction with a nucleus, it is necessary to measure the differential cross section $d^3\sigma/d\Omega_\gamma d\Omega_f dE_\gamma$.

The first measurements of this differential cross section for proton bremsstrahlung were carried out at the University of Bologna,[27] the JINR Neutron Physics Laboratory, and Kiev State University.[99,101] Later the studies of the Brooklyn Group,[40,42-44] the Tokyo Group,[36] and the Group from the Scientific Research Institute of Nuclear Physics, Moscow State University[102] appeared. The reaction $^{12}C(p, \gamma p')^{12}C$ at proton energies 0.5–3.0 MeV was studied. The reaction scheme is shown in Fig. 8.

For proton energies $E_i \leqslant 4.5$ MeV there are three possibilities: proton elastic scattering on the nucleus ^{12}C, radiative capture, and bremsstrahlung. There are resonances in the ^{13}N nucleus at the excitation energies 3547, 3511, and 2365 keV.[98] The widths of these resonances (in the c.m.s.) are 47, 62, and 33.7 keV, respectively, and the proton resonance energies are 1734, 1699, and 457 keV.[98] The cross section for the elastic scattering of protons with energies 1.5–2.0 MeV is shown in Fig. 9.[42]

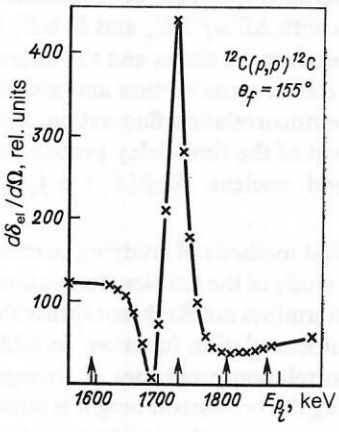

FIG. 9. The cross section for proton elastic scattering on the nucleus ^{12}C.

In all their studies except for Ref. 44, the Bologna, Brooklyn, and Tokyo groups studied bremsstrahlung in the proton energy range 1.6–1.8 MeV, that is, near the resonances of the ^{13}N nucleus with excitation energy 3511 and 3547 keV.

The study of low-energy bremsstrahlung in the energy range 10–200 keV, the cross section for which is $d\sigma/d\Omega_\gamma \sim 10^{-8}$ keV$^{-1}\cdot$sr^{-1} of the differential cross section for elastic scattering, is a complicated experimental problem. The main difficulty is the high level of background in this energy range. The background is due to x rays, rescattered γ rays, and bremsstrahlung from δ electrons. Since the intensity of this background is several orders of magnitude greater than the bremsstrahlung intensity, the only possible procedure here is to measure the γ spectrum in coincidence with protons. The Bologna group used the traditional technique of fast-slow coincidences. In this technique it is difficult to correctly take into account the contribution of background and random coincidences and also the distortions of the spectrum related to pulse pileup in the spectrometer channel.

The more refined technique of two-dimensional analysis was used in Ref. 36. An electrostatic accelerator served as the proton source, and the beam current was $\sim 0.5\ \mu$A. A self-supporting carbon target of thickness 30 mg/cm^2 was used, and the current was measured by means of a Faraday cup. The gamma radiation was detected by a scintillation detector with a thin (~ 6 mm) NaI(Tl) crystal placed at an angle $\theta_\gamma = 90°$. The protons were registered by solid-state surface-barrier detectors placed at angles of 85 and 157.5° in the lab frame (90 and 158.9° in the c.m.s.). Two sets of detectors were used in order to increase the detection efficiency. The experimental setup is shown in Fig. 10. In Fig. 11 we show the two-dimensional spectrum of p–γ coincidences at the incident-proton energy $E_{i\,\mathrm{lab}} = 1795$ keV and $\theta_{p\,\mathrm{lab}} = 157.5°$.

The diagonal line of Fig. 11 marks the events for which $E_{f\,\mathrm{lab}} - E_\gamma = \mathrm{const}$ and corresponds to coincidences of photons detected at the total-absorption peak with the corresponding protons. The two-dimensional analysis makes it possible to determine the momenta corresponding to the photons detected through Compton scattering and to correctly take into account random coincidences. The charged-particle detector operates under high-charge conditions, owing to the pulses from elastically scattered protons. In order to take into account the dead time and pulse pileup,

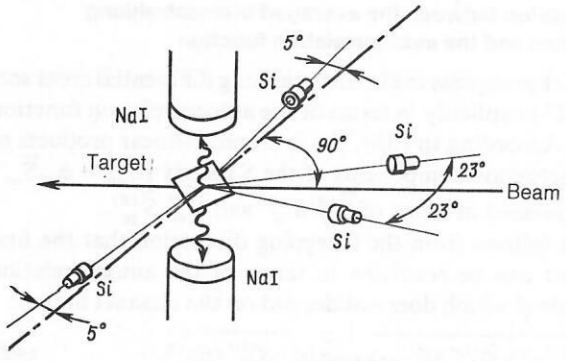

FIG. 10. Experimental setup for studying bremsstrahlung.

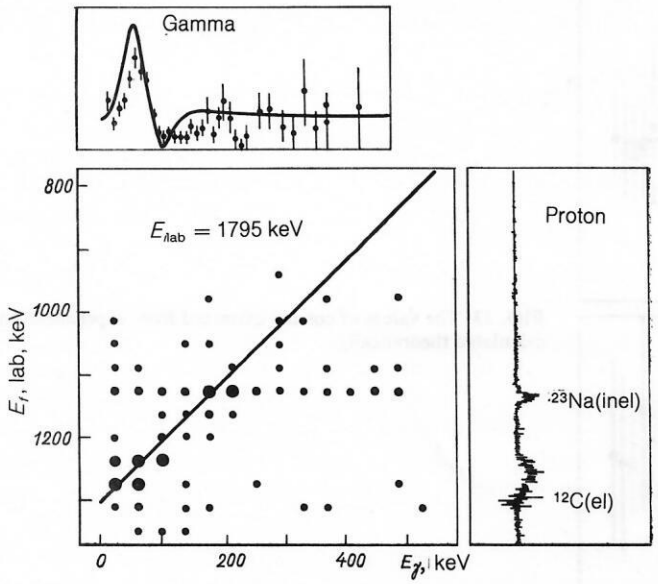

FIG. 11. The two-dimensional spectrum of p–γ coincidences.

pulses from a generator of stable frequency are input to the detectors. The background pulses from proton inelastic scattering on ^{23}Na are visible in the two-dimensional spectrum.

In Fig. 12 we show the bremsstrahlung spectrum $E_\gamma d^3\sigma/(d\Omega_\gamma d\Omega_p dE_\gamma)$ for the proton energies 1700, 1765, 1795, and 1835 keV.

The results were analyzed using Eqs. (13) and (14). The values of cos $\Delta\varphi$ and then the averaged times $\Delta\tau$ were extracted. Whereas in the other studies cos $\Delta\varphi$ was found by simultaneous counting of elastic protons and proton-photon coincidences,[27] the authors of Ref. 36 used the relative rates of proton-photon coincidences at c.m.s. angles of 90° and 158.9°. The values of cos $\Delta\varphi$ and $\Delta\tau$ extracted from experiment are shown in Figs. 13 and 14 together with the values of these quantities calculated using the FYA. The difference between the experimental and calculated values characterizes the degree of accuracy of the FYA.

A hybrid approach was proposed in Ref. 30 for experimentally determining the average delay time. This approach is based on the description of the bremsstrahlung spectrum using the FYCT approximation (see Sec. 1). The values of $\Delta\tau$ extracted differ by roughly a factor of two from those obtained using the FYA.

As was shown in Sec. 2, the times $\Delta\tau$ and $\tau(E_f)$ do not always agree. The extent of agreement is demonstrated in Fig. 15 for the case of proton elastic scattering on carbon near the resonance with $E_r \cong 1735$ keV and $\Gamma \cong 60$ keV. The solid line corresponds to the times $\tau(E_f)$ given in Fig. 8 of Ref. 26, the dashed line to $\Delta\tau$ at $E_i = 1765$ keV, and the dot–dash line to $\Delta\tau$ at $E_i = 1795$ keV. The values of $\Delta\tau$ were found using the theoretical values of cos $\Delta\varphi$ from Ref. 26. We see that near the resonance ($E_f = E_r$) the discrepancy between the values of $\Delta\tau$ and τ increases with increasing energy $E_\gamma = E_i - E_f = E_i - E_r$; for example, for $(E_i - E_r)/\Gamma \sim 0.5$ we have $(\tau - \Delta\tau)/\tau \sim 20\%$, while for $(E_i - E_r)/\Gamma \sim 1$ we have $(\tau - \Delta\tau)/\tau \sim 60\%$. The experimental results on bremsstrahlung in the region $E_\gamma \ll E_i$, where the FYA can be used, can be viewed as direct experimental proof of the presence of interference between the two extertributions to the bremsstrahlung amplitude.

The bremsstrahlung of protons on carbon was studied at the JINR Neutron Physics Laboratory with the proton in the exit channel near the $s_{\frac{1}{2}}$ resonance $E_r = 2365$ keV of the nucleus ^{13}N. The ÈG-5 electrostatic generator was used to supply a 5–8 μA current of protons of energy 1200–1450 keV. The self-supporting carbon target of approximate thickness 20 μg/cm^2 was obtained by spraying reactor graphite in a vacuum. In order to decrease the background the reaction chamber contained a tube of length 1.5 m for withdrawing the beam after its passage through the target. The measurements were monitored by means of a current integrator and an Si solid-state detector which registered protons elastically scattered at an angle of 135°. The γ radiation was detected by a Ge(Li) detector of volume 40 cm^3 and resolution 3 keV along the 1.17-MeV line of the ^{60}Co source. A 4000-channel amplitude analyzer was used to analyze the spectrum.

The energy calibration of the spectrometer channel was carried out using the lines 186.1, 241.9, 351.9, 609.3, 768.4, 934.1, 1120.3, 1377.3, 1764.6, 1847.4, 2204.1, and 2447.6 keV of a ^{226}Ra source.

The points obtained in the N_k–E_γ plane (N_k is the

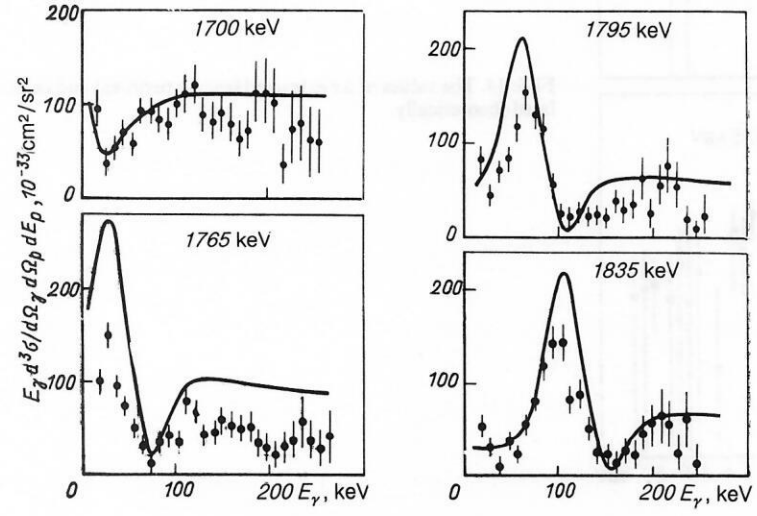

FIG. 12. The triple bremsstrahlung differential cross section $E_\gamma d^3\sigma/d\Omega_\gamma d\Omega_p dE_p$ for the reaction ^{12}C(p, $\gamma p'$)^{12}C at $\theta_p = 157.5°$. The solid line was calculated using the Feshbach–Yennie formula.

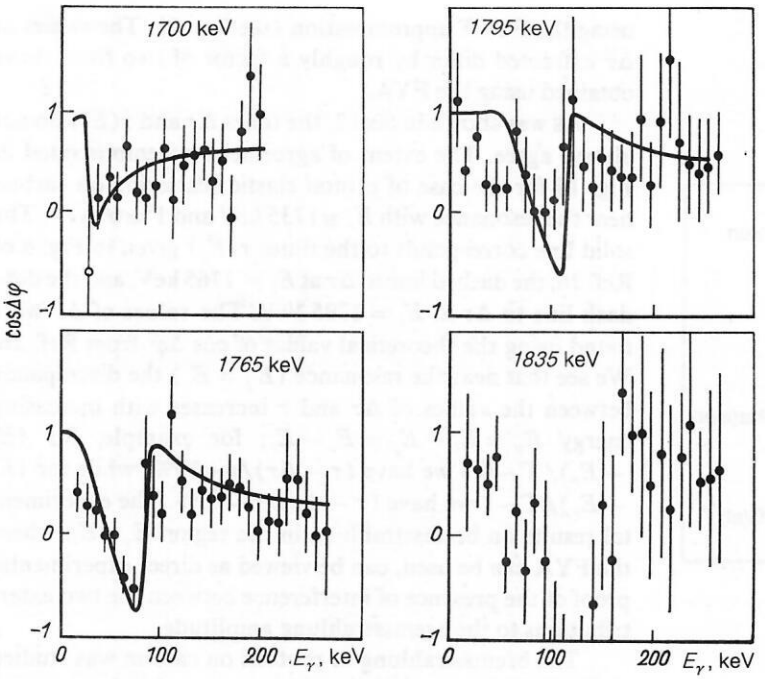

FIG. 13. The values of cos $\Delta\varphi$ extracted from experiment and calculated theoretically.

channel number corresponding to the maximum of the photopeak) were approximated by a parabola at the minimum of χ^2:

$$E_\gamma = a_1 + a_2 N_k + a_3 N_k^2.$$

Here $a_1 = 8.5$ keV, $a_2 = 1.956$ keV/channel, and $a_3 = -6.4\times10^{-6}$ keV/channel^2. With this calibration the photon energy was determined throughout the entire energy range with an accuracy of better than ± 1 keV. Data on the relative intensity of the γ lines were used to obtain the curve describing the relative efficiency of the Ge(Li) detector.

The spectra of photons from the reaction $^{12}C(p,\gamma p')^{12}C$ ($E_i = 1230, 1350$ keV) in the energy range 500–900 keV are shown in Fig. 16.

There are clearly expressed maxima of approximate width 40 keV at the energies 821 and 712.5 keV corresponding to the photons from proton radiative capture on the resonance state. The energy (in keV) of these photons at the maximum can be determined from the formula

$$E_{\gamma_0} = E_{i,\,\text{c.m.}} - 2366 + Q. \qquad (44)$$

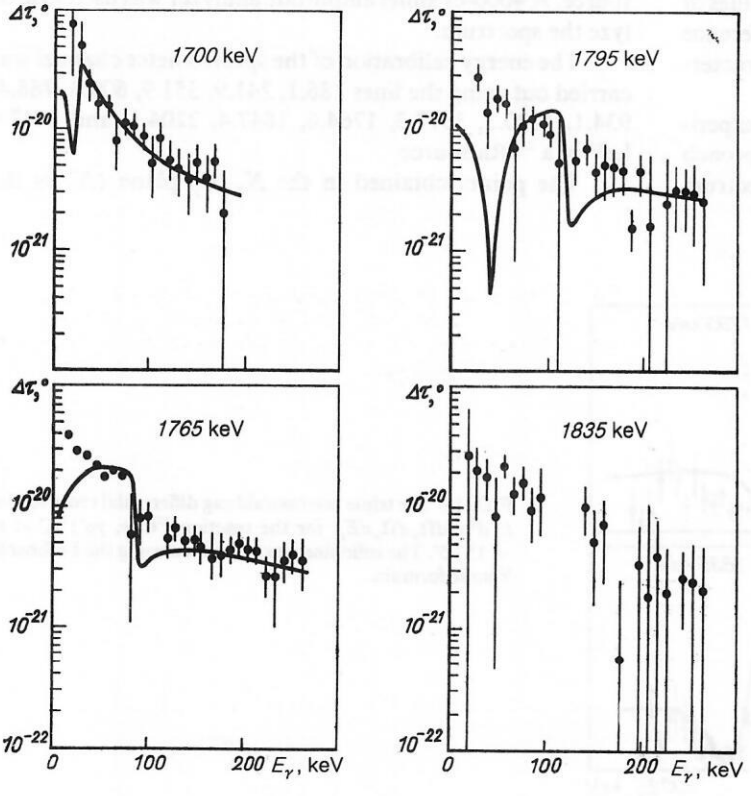

FIG. 14. The values of $\Delta\tau$ extracted from experiment and calculated theoretically.

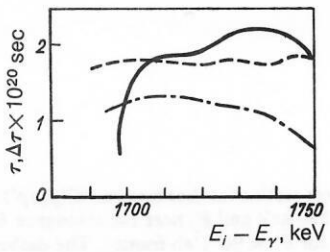

FIG. 15. Dependence of $\Delta\tau$ and τ on the photon energy.

The positions of the maxima in the measured spectra are in rough agreement with those given by (44).

The cross section for the capture of protons of energy 1230 keV on the 2366- keV resonance in the nucleus ^{13}N was measured in Ref. 104: $d\sigma/d\Omega_\gamma \sim 1.5\times 10^{-31}$ cm$^2\cdot$sr^{-1} (detection of γ radiation at 90°). Photons from capture on the ground state of ^{13}N were observed simultaneously with the photons accompanying the capture on the resonance. The photopeak from these photons is shown in Fig. 16. The width of the photopeak is determined by the target thickness, the instability and energy spread of the incident proton beam, the resolution of the γ spectrometer, and the stability of the electronics. The continuous observation of the photopeak made it possible to continuously monitor these parameters during the measurement process. It was possible to neglect Doppler broadening for the measurement geometry that was chosen. The width of the photopeak at half-max was 8.4 keV. The measured cross section for the capture of protons of energy 1230 keV on the ground state of the ^{13}N nucleus was $d\sigma/d\Omega_\gamma = 4.5\times 10^{-32}$ cm$^2\cdot$sr^{-1}.

A large part of the background in the measurements was due to the natural activity of the surrounding materials. In order to decrease this component of the background the Ge(Li) detector was surrounded by lead. The background measured during a period of 32 h for the same geometry as for the principal measurements was normalized to the principal measurements according to the area of the photopeak from the most intense background line (the 1461-keV line of ^{40}K) and was then subtracted. This normalization of the spectra made it possible to take into account random pulse pileups.

In Fig. 17 we show the photon spectrum for the reaction ^{12}C$(p, \gamma p')^{12}$C (after subtraction of the background) at the incident-proton energy 1230 keV. We see that the background related to the proton beam is negligible (at least in the energy range considered, $E_\gamma = 500$–900 keV).

The photon spectrum consists of two components—a smooth component (dashed line) and a resonance component with maxima near the energy given by (44). After subtraction of the background and inclusion of the efficiency of the γ spectrometer, the number of counts corresponding to p–γ coincidences was approximated by the function (Fig. 17)

$$N_\gamma = a + \frac{b}{N_k} + \frac{c}{N_k^2} + d\,\frac{(\Delta N)^2}{(N_k - N_0)^2 + (\Delta N)^2} \qquad (A)$$

which is the sum of a polynomial describing the first component and a Lorentzian term for the second component; N_0 is

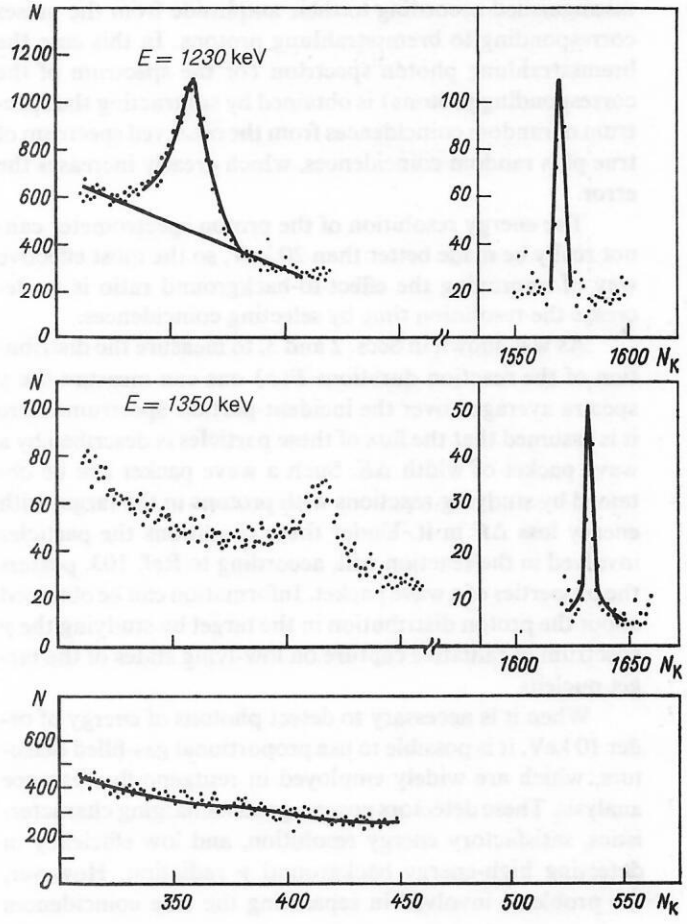

FIG. 16. Spectrograms of the γ radiation from the reaction ^{12}C$(p, \gamma p')^{12}$C at $E_i = 1230$, 1350 keV and E_f in the vicinity of the resonance with $E^* = 2365$ keV for θ_γ 90° (lab frame). The lower figure shows the background in this energy range.

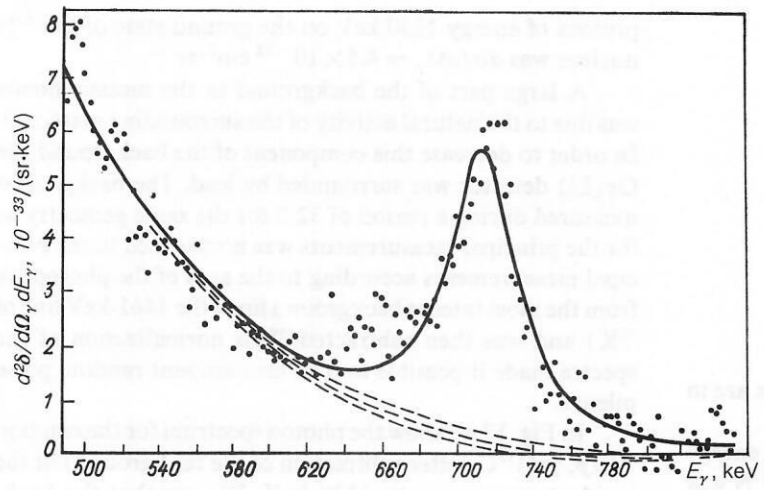

FIG. 17. Cross section for the reaction $^{12}C(p, \gamma p')^{12}C$ at $E_i = 1230$ keV and E_f near the resonance $E_r = 461$ keV for $\theta_\gamma = 90°$ (lab frame). The dashed lines show the smooth component of the bremsstrahlung spectrum (Ref. 101).

the channel number at the maximum, and ΔN is the half-width in units of the channel rate. The parameters of the curve were found by the method of least squares. The presence of the term a in the expression for N_γ can apparently be viewed as the deviation of the shape of the spectrum from that given by the FYA for $E_\gamma / E_i \cong 0.5$.

Interference between the amplitudes of the various contributions to the bremsstrahlung spectrum was sought at the energies $E_i \cong 1230$ keV and $E_\gamma \cong 715$ keV in Ref. 101. The authors studied the variation of the shape of the spectrum for small changes of the incident-proton energy without specifying the analytic form of the curve describing the spectrum near the maximum. The criterion for such a variation was taken to be a shift of the position of the center of mass (E^0_{cc}) of the experimental spectrum relative to the position of the maximum given by Eq. (44). The energy difference $E_{\gamma 0} - E^0_{cc}$ as a function of the incident-proton energy is shown in Fig. 18. The dependence of $E_{\gamma 0} - E^0_{cc}$ on the incident-proton energy is related to the change of the shape of the resonance component of the bremsstrahlung spectrum and can be viewed as an indication of the presence of interference in the vicinity of the maximum of the bremsstrahlung spectrum, that is, for $E_\gamma / E_i \sim 0.5$, where the FYA is apparently a poor approximation.

The p–γ coincidence technique is preferable from the viewpoint of studies of bremsstrahlung on other nuclei and in a wide range of E_γ. The experimental setup is complicated, but does not require any special components or devices. The technique of multidimensional analysis makes it possible to reliably separate the true events from the background. The greatest difficulty arises from the need to use spectrometer channels which enable spectra of high intensity to be measured without significant distortions. This is particularly important for the spectrometer channel used to detect the protons. It is loaded mainly by pulses from elastically scattered protons which are $\sim 10^6$ times larger than the bremsstrahlung pulses. This also makes it difficult to use solid-state detectors with good energy resolution for the spectrometry of low-energy bremsstrahlung. The use of scintillation detectors can limit the amount of information gained from the experiment.

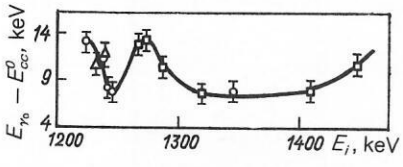

FIG. 18. Dependence of the parameter characterizing the shape of the maximum $E_{\gamma 0} - E^0_{cc}$ on the incident-proton energy.

The high charges and rather poor time-resolution capability (greater than 7 nsec) make it necessary to carry out special measurements of the spectra of randomly coincident pulses, which significantly complicates the processing of the results for E_γ below the proton-detector resolution, that is, when the pulses from elastically scattered protons cannot be distinguished according to their amplitude from the pulses corresponding to bremsstrahlung protons. In this case the bremsstrahlung photon spectrum (or the spectrum of the corresponding protons) is obtained by subtracting the spectrum of random coincidences from the observed spectrum of true plus random coincidences, which greatly increases the error.

The energy resolution of the proton spectrometer cannot really be made better than 20 keV, so the most effective way of improving the effect-to-background ratio is to decrease the resolution time by selecting coincidences.

As was shown in Secs. 2 and 3, to measure the distribution of the reaction durations $P(\tau)$ one can measure the γ spectra averaged over the incident-particle spectrum. Here it is assumed that the flux of these particles is described by a wave packet of width ΔE. Such a wave packet can be obtained by studying reactions with protons in the target with energy loss ΔE in it. Under these conditions the particles involved in the reaction will, according to Ref. 103, possess the properties of a wave packet. Information can be obtained about the proton distribution in the target by studying the γ spectrum of radiative capture on low-lying states of the target nucleus.

When it is necessary to detect photons of energy of order 10 keV, it is possible to use proportional gas-filled detectors, which are widely employed in rentgeno-fluorescence analysis. These detectors possess good recharging characteristics, satisfactory energy resolution, and low efficiency in detecting high-energy background γ radiation. However, the problems involved in separating the true coincidences

from the intense background of random ones require the design of experimental setups possessing a high degree of time resolution (better than 1 nsec).

CONCLUSIONS

Analysis of the theoretical and experimental studies of the γ radiation accompanying the bremsstrahlung of light particles on nuclei shows that in the absence of kinematic constraints the low-energy part of the γ spectrum can be approximated by the FYA, which can be used to obtain new data on the general features of particle–nucleus interactions, in particular, to study the interference between the elastic scattering amplitudes.

In the case of bremsstrahlung of monoenergetic protons on even–even nuclei the interference term is expressed in terms of $\cos \varphi_{if}$ [see Eq. (15)]. Strictly speaking, near an isolated resonance it is possible to determine the average time $\Delta\tau$ from this. The question of the relation between $\Delta\tau$ and the delay time τ of a plane wave in the field of a nucleus must be studied separately for each individual case. The estimates presented in Secs. 2 and 4 indicate that these quantities can differ by a factor of two. In this case an additional uncertainty arises, owing to the use of the FYA near the resonance, and this also increases the error in the determination of $\Delta\tau$ by about a factor of two.[30]

As in the classical case, the spectrum of low-energy bremsstrahlung photons averaged over the incident-particle energy depends on the Fourier component of the probability distribution of the delay times in the nucleus and, in particular, contains information about the dynamics of the nuclear system in the region of overlapping resonances. We note that when the formulas of Sec. 3 are used to analyze the results of experiments involving broad incident beams the conditions $\Delta E \gg \gamma \gtrsim E_\gamma$ and $E_i \gg E_\gamma$ must be satisfied. In addition, for the identification of only the bremsstrahlung accompanying elastic scattering it is necessary that $\Delta E + \Delta E_d < \varepsilon_1$, where ΔE_d is the energy resolution of the detecting apparatus and ε_1 is the energy of the first excited level of the target nucleus. Otherwise, the spectrum will contain photons from inelastic scattering, whose spectrum can also have a continuous component.

The agreement between the theoretical description of the γ spectrum near the resonance and at large photon energies can be improved by including terms of order E_γ^0 in the bremsstrahlung amplitude T_{fi}, that is, by using Eq. (11) for the vector amplitude **B**. In this case it is possible to extract from experiments with monoenergetic beams information on the phase shift of a plane wave in the field of a nucleus (see Sec. 2) or, from the viewpoint of classical physics, the impact parameter of the scattered particles.

By using Eq. (2) for the potential of the interaction with an electromagnetic field, it is possible also to obtain an expression for the spectrum of nuclear bremsstrahlung accompanying heavy-ion scattering.[4)] According to Refs. 106 and 107, the measurement of the bremsstrahlung in such reactions will give insight into the nature of the observed quasimolecular structures.

From the experimental point of view the study of low-energy bremsstrahlung on intermediate and heavy nuclei in coincidence with particles is a complicated problem requiring fast detectors and electronics and the use of multidimensional analysis. However, the present level of experimental technique apparently allows such investigations to be carried out.

The authors are grateful to V. M. Strutinskiĭ, Yu. P. Popov, I. V. Sizov, S. N. Ezhov, and S. Yu. Kun for useful discussions.

[1)] As usual, the range of the characteristic variation of the elements of the average scattering matrix is assumed to be much larger than ΔE.
[2)] We note that the situation $\overline{S}_{ab} \neq \delta_{ab} s_a$ can arise, owing to the presence of states of the intermediate structure. The information on such states is contained in s_a and U.
[3)] The upper limit on ΔE is discussed in the concluding section.
[4)] The theory of bremsstrahlung in reactions involving heavy ions based on classical electrodynamics is given in Ref. 105.

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