Two-particle nuclear molecular states

B. V. Danilin and M. V. Zhukov

I. V. Kurchatov Institute of Atomic Energy, Moscow

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The present state of the theory and experiments on reactions with the formation of quasimolecular states is reviewed. The main attention is devoted to the $^{12}C + ^{12}C$ system, which is the one that has been most investigated from the theoretical and experimental points of view. The characteristic features of near-Coulomb resonances and resonances above the Coulomb barrier are considered. The existing theoretical models are analyzed and compared. There are, on the one hand, spectroscopic models, including the collective model, group-theoretical approaches, and calculations of low-lying spectra in a microscopic basis; on the other hand, there are the dynamical models, beginning with the phenomenological ones and ending with methods that use simplified variants of microscopic theories. The review ends with a summary of the conclusions that can be reached on the basis of an analysis of the present state of the experimental and theoretical investigations of quasimolecular resonances.

INTRODUCTION

Nuclear molecular states, or nuclear quasimolecules, or resonances of intermediate structure, were introduced about 25 years ago to explain very interesting results obtained in the first studies of heavy-ion interactions ($^{12}C + ^{12}C$). The resonances found in these experiments were well correlated in different channels and had a width much less than the single-particle width and much greater than the compound-nucleus width for the given collision energy.

Since then, such phenomena have been intensively investigated, both theoretically and experimentally (particularly in recent years). However, the problem of the occurrence of unexpectedly narrow and well-separated resonances in the interaction of heavy ions (with excitation energy of the compound nuclear system around 15–50 MeV) is still not completely clarified.

It should be noted that from both the experimental and the theoretical point of view the system hitherto most fully investigated is the $(^{12}C + ^{12}C)^{24}Mg^*$ system, although analogous phenomena have been observed in the $^{12}C + ^{16}O$, $^{14}C + ^{14}C$, $^{16}O + ^{16}O$, $^{12}C + ^{14}N$, $^{24}Mg + ^{24}Mg$, $^{28}Si + ^{28}Si$ systems and some others. For this reason, we shall in Secs. 1 and 2 consider only the experimental data on the $^{12}C + ^{12}C$ system and the theoretical models, which were largely stimulated by the experiments with this system.

In Sec. 1, we discuss the below-barrier resonances, the Imanishi model, resonances at energies above the Coulomb barrier in transfer reactions and inelastic scattering, the fusion reaction, and unusual entrance and exit channels leading to $^{24}Mg^*$.

In Sec. 2, we give the main theoretical models and methods—the classical estimates, spectroscopic approaches (collective model, the approach based on group theory, calculations in a collective cluster basis), and dynamical calculations of the reaction and fusion cross sections, beginning with the phenomenological ones (diffraction model and Regge-pole model) and continuing with semimicroscopic ones (coupled-channel method, DWBA) and fully microscopic ones (resonating-group method, projection-operator method, interpolation approach, Hartree–Fock method).

Section 3 is devoted to a review of complementary methods of studying nuclear molecular states through γ transitions in molecular bands and measurement of the populations of magnetic substates (alignments).

The main conclusions are drawn at the end of each section, while in Sec. 4 we summarize the study of nuclear molecular states and consider certain prospects.

1. EXPERIMENTAL DATA ON THE $(^{12}C + ^{12}C)^{24}Mg^*$ SYSTEM

The below-barrier resonances

Experimental facts

The history begins with the discovery in 1960 of three resonances by Bromley, Kuehner, and Almqvist[1] in the $^{12}C + ^{12}C$ system; these resonances are shown in Fig. 1, which reveals their presence at energies below the Coulomb barrier. These structures are not statistical fluctuations, since they are correlated at all angles and in all exit channels. Their widths are of order 100 keV, and their reduced widths of decay to the $^{12}C + ^{12}C$ channel are also large. The spins of these resonances are 2^+ (5.6 MeV), 4^+ (6.0 MeV), and 2^+ (6.3 MeV). The presence of such clearly defined states at ^{24}Mg excitation energies of about 20 MeV and the large

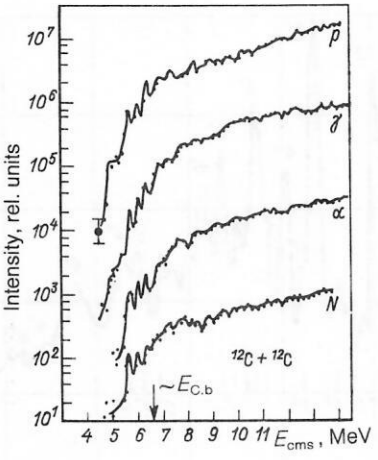

FIG. 1. Inclusive yield of p, N, γ, and α from the $^{12}C + ^{12}C$ reaction. The arrow shows the position of the Coulomb barrier.

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width (compared with statistical estimates) of decay to the $^{12}C + {}^{12}C$ channel immediately served as the ground for proposing for them a quasimolecular nature (i.e., a state in which an appreciable fraction of the wave function is a configuration for which the two ^{12}C nuclei move around each other with weak overlapping of their wave functions).

Subsequent more exact measurements near the Coulomb barrier were made already in the seventies, when the $^{12}C(^{12}C, \alpha)^{20}Ne^*$ and $^{12}C(^{12}C, p)^{23}Na^*$ reactions were studied.[2,3] Study of the reaction with α-particle emission and formation of ^{20}Ne in the ground state (in the cases when such a process is observed) makes it possible to determine from the angular distribution the angular momentum J of the original state [see, for example, Fig. 2 (from Ref. 4), in which the angular distribution $|P_L(\cos\theta)|^2$ for $L = 4$ is fitted for the state $E_{cms} = 7.71$ MeV]. Figure 3 shows the nuclear structure factor $\tilde{S} = E\sigma(E)/\Sigma_{L=0}^{10}[(2L+1)T_L]$ (T is the barrier penetrability)[5] deduced from the count of γ rays with energies corresponding to decay of the low excited states of the ^{20}Ne, ^{23}Na, and ^{23}Mg nuclei. It was assumed that such transitions make the main contribution to the total reaction cross section. In the figure, one can clearly see the peaks corresponding to the initially open resonances 2^+, 4^+, 2^+, and also 4^+ (7.71 MeV), mentioned above. The same figure shows some of the more low-lying states and the results of a calculation of the spectrum in Ref. 6 made by means of Imanishi's model.[7]

The absence of such structure for the neighboring α-particle systems $^{12}C + {}^{16}O$ and $^{16}O + {}^{16}O$ was remarkable. However, recent experiments[8] with better resolution have shown that such spectra are found not only for these systems but also in the $^{14}C + {}^{14}C$ reaction,[9] in which the presence of pairs of "extra" neutrons should, according to expectations, have completely smeared the resonance picture.

Before we discuss Imanishi's model, we show that these same states can also be clearly seen in $^{12}C + {}^{12}C$ elastic scattering (Fig. 4).[10] Although the data given in Fig. 3 correspond well to calculations in accordance with Imanishi's model, investigations of the $^{12}C + {}^{12}C$ system in the region of energies near the Coulomb barrier are continuing. In particular, accurate measurements have been made (with step < 50 keV) of the elastic scattering in the region $E_{cms} = 6.5$–7.05 MeV (i.e., in the region of resonances shown in Fig. 4).

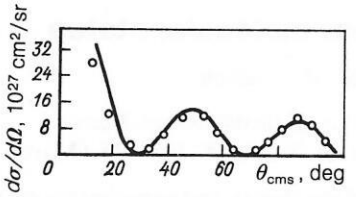

FIG. 2. Angular distribution of α particles from the $^{12}C(^{12}C, \alpha)^{20}Ne$ reaction at $E_{cms} = 7.71$ MeV. The continuous curve is $|P_4(\cos\theta)|^2$.

The phase-shift analysis of Ref. 11 and the inclusion of the total reaction cross sections confirmed once more the spins of these states, 2^+ and 4^+, and also showed that the elastic width is an appreciable fraction of the total width (from 15% to 30%). This confirms once more the hypothesis made long ago of a quasimolecular ($^{12}C + {}^{12}C$) nature of these resonances. It should be noted that the density of compound states ($^{24}Mg^*$) in this region of energies is of order $100\ MeV^{-1}$ and that four further channels of decay to many levels are opened. Estimates show that if the reaction were to proceed through the formation of ordinary compound states, then the mean elastic width would have to be 20–40 times smaller than the experimental width, and this again is an indication of an important part played by a quasimolecular configuration in the wave function of the resonance state.

Investigations made during the last six years in the region of energies near the Coulomb barrier, mainly with the detection of γ rays for the $^{12}C + {}^{12}C$ system, reveal a very large number of resonances in the region $E_{cms} = 3$–11 MeV.[12,13] The total number of resonances currently known at these energies with spins $J = 0, 2, 4, 6$ reaches 28–30 (see Table I). It is rather difficult to explain such an abundance of resonances in simple ways. Nevertheless, there now exist models which permit one to understand the essence of the problem qualitatively. We discuss these models in more detail below.

Imanishi's model

For the theoretical description of these resonances in the $^{12}C + {}^{12}C$ system, the currently most widely used model is Imanishi's model,[7] in which it is assumed that in the collision process one of the nuclei is excited to a 2^+ ($E^* = 4.43$ MeV) state, after which the system falls into a quasibound

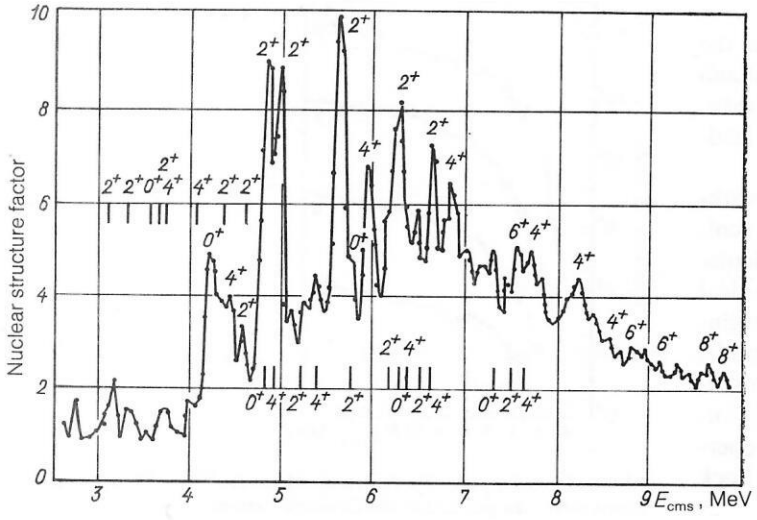

FIG. 3. Nuclear structure factor of the $^{12}C + {}^{12}C$ reaction from measurement of γ rays of the final products $^{20}*Ne$, $^{23}*Na$, and $^{23}*Mg$. The vertical bars indicate the results of calculations of the spectra of quasimolecular states in Imanishi's model.

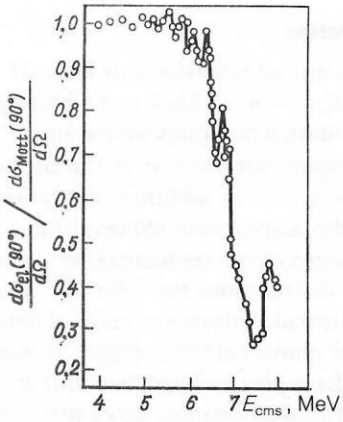

FIG. 4. The ratio $[d\sigma_{el}(90°)/d\Omega]/[d\sigma_{Mott}(90°)/d\Omega]$ for $^{12}C + ^{12}C$ elastic scattering.

state of the $^{12}C + ^{12}C^*$ potential, which is taken to be the same as the $^{12}C + ^{12}C$ interaction potential, remaining in the quasibound state for a long time. Decay of such a state into the elastic channel is possible only if the excitation is removed, while decay into the inelastic channel is suppressed by the wide barrier (the system is in a potential lower in energy by 4.43 MeV; see Fig. 5); fusion of the nuclei into a compound nucleus is hindered by the repulsive part of the interaction potential of the complex nuclei, which is due to the Pauli principle (see, for example, Ref. 14) and leads to an effective nucleus–nucleus potential of surface or quasimolecular type (Fig. 5). This model had significant success in describing the positions and widths of the low-lying quasimolecular resonances (and to a considerable degree was used in subsequent theoretical models). However, even with allowance for the mutual excitations of the two ^{12}C nuclei in the 2^+ state, this model does not give the number of near-Coulomb resonances that has so far been obtained in experiments.

Conclusions

Investigations of the $^{12}C + ^{12}C$ system at near-Coulomb energies are still continuing. For example, in Ref. 15 γ-ray detection was used to measure the total reaction cross section in the range $E_{cms} = 5.5$–10 MeV with step $\Delta E = 50$ keV. Analysis of these data led to a determination of the ratios of the widths to the elastic channel, Γ_C, to the width Γ_{tot} for the resonances in this region of energies. In Ref. 16 the total reaction cross section was recovered from measurements of the angular distributions of elastic scattering, while in Ref. 17 the total fusion cross sections were measured by means of γ rays. It may be noted that all the recently obtained results agree rather well with the results of the earlier studies and make it possible to draw quite definite conclusions about the intermediate structure of the quasimolecular resonances at near-Coulomb energies:

(1) In the region of energies $E_{cms} = 3$–10 MeV there exist a large number (28–30) of resonances with widths 100–150 keV and spins 0^+, 2^+, 4^+, 6^+ of a nonstatistical nature; they are clearly seen not only in the total cross sections and individual reaction channels but also in elastic scattering.

(2) Investigations have confirmed the hypothesis of a quasimolecular ($^{12}C + ^{12}C$) nature of these states. All the investigations give $\Gamma_C/\Gamma_{tot} \sim 10\%$–$40\%$ and show that the widths of the decays of these resonances to the elastic channel are 10–40 times greater than the mean widths obtained from a statistical nuclear model.

(3) In this region of energies, the $^{12}C + ^{12}C(2^+)$ decay channel is strongly suppressed by the Coulomb barrier and therefore has not been directly studied, but it still makes a large contribution to the formation of the quasimolecular states, i.e., the system remains for an appreciable fraction of the time in the quasibound $^{12}C + ^{12}C(2+)$ state, a situation that leads in accordance with Imanishi's model to total widths of the resonances of 100–150 keV.

(4) Some of the resonances predicted by Imanishi's model have been discovered experimentally, but there is a large number of resonances that do not fit into this model.

Resonances at energies above the Coulomb barrier

Study of the $^{12}C + ^{12}C$ reaction at higher energies ($E_{cms} > 10$ MeV) immediately leads to serious difficulties. In the first place, these are due to the fact that the elastic scattering cross section decreases rapidly with increasing energy and the reaction cross sections increase accordingly. The elastic-scattering excitation function (for $\theta_{cms} = 90°$) is characterized by a gross structure with widths 3–4 MeV and a fine structure (not particularly clearly expressed) superimposed on it. This excitation function does not permit one to speak of definite resonances (and, because of the large number of participating partial waves and the large inelasticity, does not permit a phase-shift analysis to be made). For this reason, intensive experimental investigations of differ-

TABLE I. Dependence of the energy E_{cms} (MeV) on the spin J^π ot quasimolecular resonances found for $^{12}C + ^{12}C$ collisions near the Coulomb barrier. The energies of resonances with insufficiently well-determined spins are given in brackets.

$J^\pi = 0^+$	$J^\pi = 2^+$	$J^\pi = 4^+$	$J^\pi = 6^+$
(3.17)	(3.75)	(4.46)	(6.49)
(3.35)	4.62	5.25	7.55
4.25	4.88	5.96	8.86
5.80	5.0	6.85	9.05
	(5.37)	7.3	9.33
	5.64	(7.45)	9.98
	6.25	7.71	10.45
	6.41	8.10	(11.20)
	6.63	8.26	
		8.45	
		8.58	

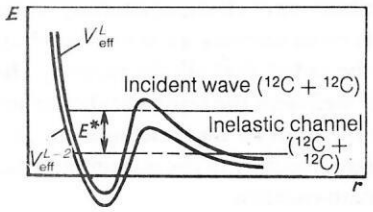

FIG. 5. Scheme of formation of quasimolecular states in Imanishi's model.

ent reaction channels with $^{12}C + ^{12}C$ entrance channel have been made since about 1975.

However, it was soon found that the reaction cross sections fluctuate strongly, an appreciable fraction of the fluctuations being purely statistical in nature. To separate the true peaks from the statistical fluctuations, several criteria have been proposed:

(1) agreement of the spin and parity of the resonances when determined by different methods;

(2) the partial width of the resonance must be greater than 2% of the Wigner limit in at least one channel;

(3) the appearance of resonances in the cross sections integrated over the angle for at least two exit channels at correlated energies;

(4) the resonance remains clearly visible in the fixed-angle cross section summed over many final states of the nuclei.

The combined use of all these criteria makes it possible, in principle, to eliminate random fluctuations. However, as a rule, not all the criteria are used simultaneously. In addition, there are some complicating factors related to the large orbital angular momenta introduced into the entrance channel at high energies. As a result, transitions to some channels are suppressed, since in these channels it is difficult to introduce a large angular momentum and accordingly correlations are not manifested. In channels with light particles it is possible to have manifestation of resonance effects associated with the penetrabilities and yrast band in the residual nuclei, since only such states can carry away the large angular momentum of the doorway state. In addition, at such energies there is in the exit channels a large nonresonance background that can interfere with the resonance amplitude being sought and lead to a shift of the peaks and dips with respect to the energy and change their nature. Thus, the problem of determining nonstatistical peaks (or resonances) that are correlated in different channels is, as before, fairly complicated. We now list briefly the experimental results obtained in recent years in the $^{12}C + ^{12}C$ system.

The $^{12}C + ^{12}C \rightarrow ^{8}Be + ^{16}O^*$ reaction

This channel (especially with a transition to the ground state of the ^{16}O nucleus) has been widely investigated in the range $E_{cms} = 9$–20 MeV and with step $\Delta E \sim 100$ keV in Refs. 18–20 and is mainly used to determine the spins of the resonances. The analysis of the $^{8}Be + ^{16}O$ (in the ground state[20]) excitation functions integrated over the angles shows that the structure of the excitation functions becomes less well expressed and less suitable for analysis with increasing energy.

The $^{12}C + ^{12}C \rightarrow \alpha + ^{20}Ne^*$ reaction

This channel was investigated intensively in Refs. 21–25 in the range of energies $E_{cms} = 4$–15 MeV and with step $\Delta E \sim 50$ keV. Individual excitation functions were studied[21] and also the excitation functions integrated over the angles for the six lowest ^{20}Ne states.[20,23] In addition, excitation functions integrated over the angles were obtained for 22 excited ^{20}Ne states.[24] The spins of the resonances were determined from the angular distributions for ^{20}Ne (0^+ and 2^-) states. In Ref. 25, a statistical analysis was made of data in the $\alpha + ^{20}Ne$ channel for above-barrier energies. It was shown that nonstatistical effects play an important part and that the discovered nonstatistical resonances agree with the results of other authors.

Figure 6, taken from Ref. 24, shows the $^{12}C + ^{12}C \rightarrow \alpha + ^{20}Ne^*$ reaction cross sections for different ^{20}Ne states and their sum.

In Fig. 6, particular attention should be paid to the cross section for excitation of the $E^* = 7.83$ MeV (2^+) level of ^{20}Ne, which on the average is $4 \cdot 10^{-26}$ cm^2, while the sum of the cross sections for 22 levels is about 26.10^{-26} cm^2. This indicates that the given level is excited with an anomalously large cross section. The structure of this state is well known ($8p4h$), i.e., its excitation can be related to transfer of two α particles to the ^{12}C nucleus [for example, from the excited 0_2^+ (7.65 MeV) level of one of the ^{12}C nuclei], and this, in its turn, indicates that the α-cluster mechanism plays an important part in the formation of the quasimolecular resonances (see also Sec. 2).

Inelastic $^{12}C + ^{12}C$ scattering

The $^{12}C + ^{12}C$ inelastic-scattering reaction with excitation of one or both nuclei to low collective states (2_1^+, 4.43 MeV; 0_2^+, 7.65 MeV; 3_1^-, 9.64 MeV; 4_1^+, 14.08 MeV) has long been a subject of great interest. In recent years, such

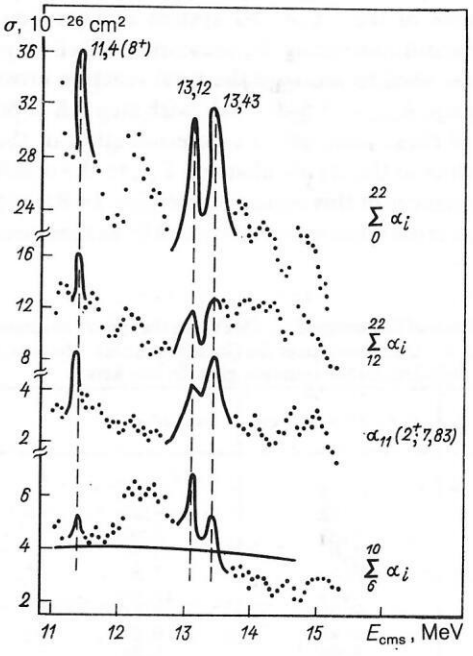

FIG. 6. Cross sections of the $^{12}C(^{12}C, \alpha)^{20}Ne^*$ reaction for different ^{20}Ne states and their sums. The continuous curve at the bottom is the result of a calculation by the Hauser–Feshbach method.

inelastic-scattering channels have been intensively studied in the region of energies $E_{cms} = 15$–40 MeV.[26–29] For example, the cross sections of inelastic scattering with the excitation of one or both of the ^{12}C nuclei to a 2^+ state were measured in Refs. 26 and 27. Both gross structure and fine structure were found, and on the basis of linear interpolation spins up to 18^+ ($E_{cms} \sim 37$ MeV) were proposed. Analysis of these cross sections, the fusion cross sections, and the excitation function for elastic scattering showed that the ratio of the elastic width to the total width for the 10^+, 12^+, 14^+ resonances is 20%–30%, and this is again an indication of a quasimolecular nature of these states. It is interesting to note that the reduced widths for decay to the 2^+ (^{12}C) state are close to the reduced widths for decay to the elastic channel. In Ref. 28, inelastic scattering to 0_2^+, 3_3^-, 4_1^+ states in the region of energies $E_{cms} = 22$–40 MeV was studied. It was noted that all the inelastic channels ($2^+, 2^+2^+, 0^+, 3^-, 4^+$) at $E_{cms} \approx 30$ MeV give a cross section close to the unitarity limit (and one cross section to 2^+ exhausts almost 40%). In Ref. 29, the angular correlations between the γ ray and the ^{12}C nucleus were used to study inelastic scattering to the 2^+ (^{12}C) state in the region of energies $E_{cms} = 15$–30 MeV. The most probable values of the spins agree with the estimates from Ref. 26.

Figure 7 shows the cross sections of inelastic scattering (from Refs. 27 and 28) to excited states of the ^{12}C nucleus ($2^+, 3^-, 0_2^+, 4^+$) and the cross section of excitation of both nuclei to the 2^+ states. In these figures, one can see a strong resonance structure in the region of energies $E_{cms} = 12$–20 MeV, and also a well-defined but less copious structure at energies E_{cms} equal to 25 and 30 MeV.

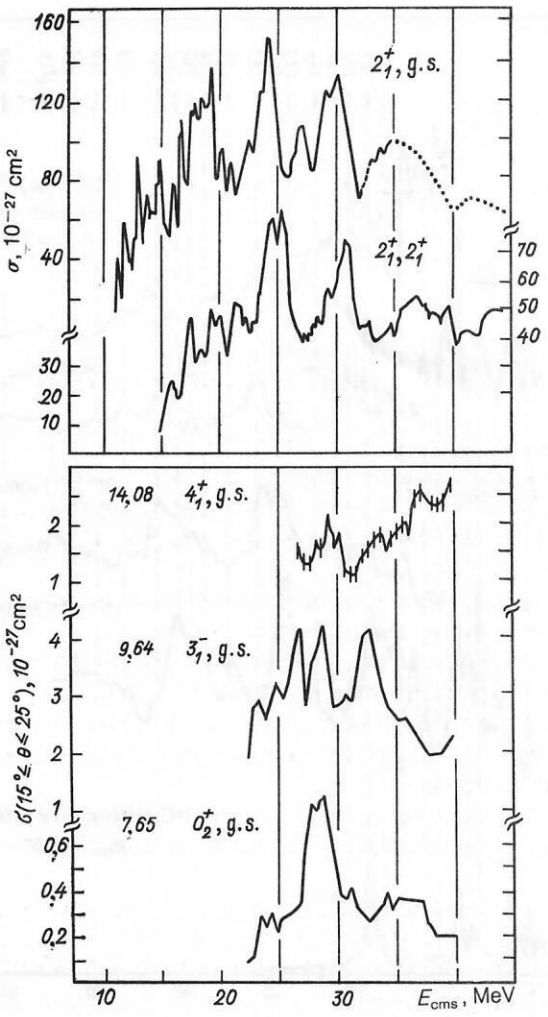

FIG. 7. Cross sections of inelastic $^{12}C + ^{12}C$ scattering with excitation of one or both nuclei.

Fusion reaction

Besides the channels listed above, investigations were made in Ref. 17 of the total fusion cross sections in the range of energies $E_{cms} = 5$–20 MeV with step $\Delta E \approx 125$ keV, for which measurements were made of the excitation functions of ten residual nuclei by means of γ-ray detection. The statistical analysis indicates a large number of nonstatistical anomalies coincident with resonances in other channels.

Figure 8 shows the total fusion cross section from Ref. 17; one can again see appreciable structure in the cross section well correlated with many other channels.

Elastic scattering

All the analysis discussed above related exclusively to investigation of the inelastic channels at E_{cms} above the Coulomb barrier and hardly touched elastic scattering. It must be borne in mind that the elastic scattering cross sections are small and that structure is not seen in them as clearly as in the inelastic channels. It is therefore only in recent years that data on elastic scattering at high energies have begun to be used in the analysis of resonances of intermediate structure.

For example, the angular distributions of $^{12}C + ^{12}C$ elastic scattering in the energy range $E_{cms} = 15$–23 MeV were investigated in Ref. 30. The results can be described by using an optical potential with the addition of resonance terms.

For the investigation of the elastic channel, the excitation function at 90° (c.m.s.) is the most convenient, since the ratio of the resonance effects to the background is maximal at this angle.

A correlation analysis in Ref. 31 compared the excitation function of elastic scattering with inelastic ^{12}C scattering and with the α-particle and proton channels in the region of energies $E_{cms} = 10$–35 MeV. It was shown that in all cases the maxima are correlated (Fig. 9), though sometimes not in all channels. The elastic scattering was compared with calculations in which resonance parameters were chosen. The results indicate that the ratio of the widths of decay to the

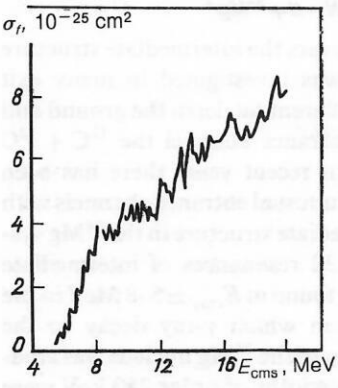

FIG. 8. Total fusion cross section in the $^{12}C + ^{12}C$ reaction.

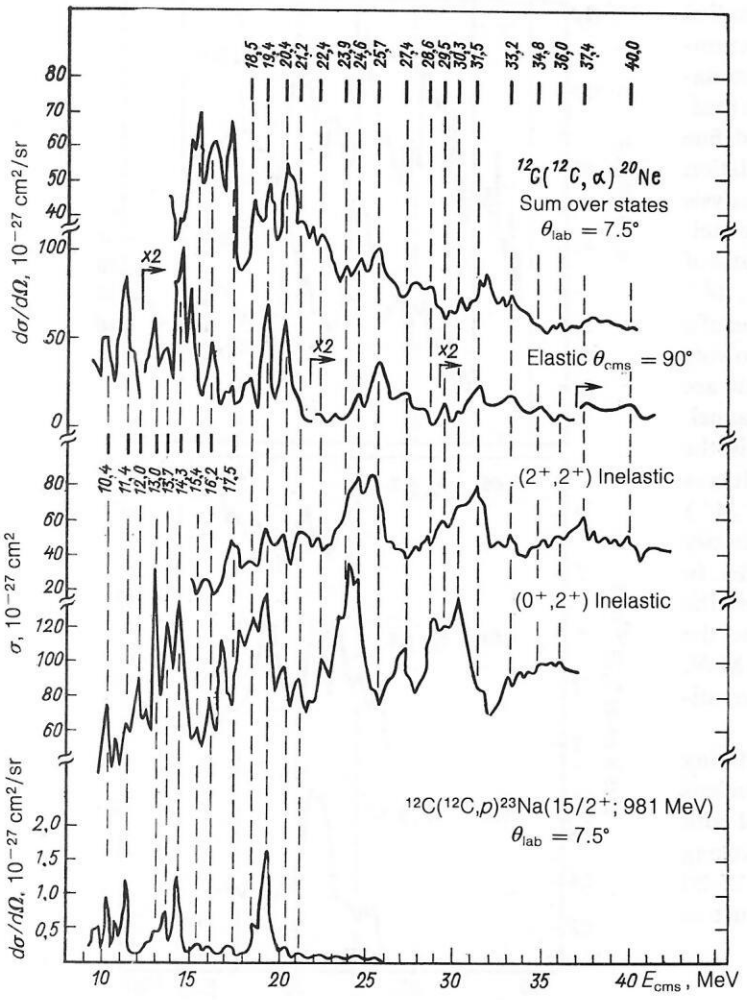

FIG. 9. Correlation analysis of the excitation functions of different channels in the $^{12}C + {}^{12}C$ reaction. The vertical bars show the positions of the resonances found in some channels.

elastic channel, Γ_{el}, to the total widths Γ_{tot} of the resonances is about 10%–50%, in good agreement with the strength of the resonances in the total reaction cross section. The investigation directly demonstrates the appearance of quasimolecular states in elastic scattering, giving, in principle, a possibility of measuring the spin and Γ_{el} in the elastic channel. The authors again draw attention to the appreciable admixture of channels with ^{12}C excitation in the wave function of these states, and also to the admixture of the $\alpha + {}^{20}Ne$ channel.

Unusual entrance and exit channels of reactions leading to the system $^{24}Mg^*$: $^{12}C + {}^{12}C \to \gamma + {}^{24}Mg^*$, $^{20}Ne + \alpha \to {}^{12}C + {}^{12}C$, $^{12}C + {}^{16}O \to \alpha + {}^{24}Mg^*$, $^{12}C + {}^{14}N \to d + {}^{24}Mg^*$

Whereas for almost 20 years the intermediate structure in the $^{12}C + {}^{12}C$ reaction was investigated in many exit channels with emission of different nuclei in the ground and excited states and as the entrance channel the $^{12}C + {}^{12}C$ channel was always used, in recent years there has been much interest in the study of unusual entrance channels with excitation of states of intermediate structure in the ^{24}Mg system. For example, in Ref. 32 resonances of intermediate structure with spins 2^+ were found at $E_{cms} \approx 5$–8 MeV in the $^{12}C(^{12}C, \gamma)^{24}Mg$ reaction, in which γ-ray decay to the ground and low excited states of the ^{24}Mg nucleus was studied. Several resonances with widths of order 250 keV were found. Although some of the resonances did not agree with the known results of other studies, a more careful analysis is required, since the conclusions in Ref. 32 were drawn on the basis of excitation functions measured at one angle θ_γ.

In Ref. 33, the angular distributions were measured with step 10 keV in the region of energies $E_\alpha = 13.4$–20.8 MeV $[E^*(^{24}Mg) \approx 20.5$–$26.6$ MeV] for the $^{20}Ne(\alpha, {}^{12}C){}^{12}C$ reaction. Altogether about 10 000 cross sections were measured, and this made it possible to carry out a detailed analysis by means of Legendre polynomials and to determine the spins and widths of the resonances by fitting (by the χ^2 method) of the cross sections as sums of a resonance and a nonresonance amplitude. By means of this method it was possible to determine about 55 resonances (of which only 10 were known from other experiments). It should be borne in mind that for the majority of resonances the $(\Gamma_\alpha \Gamma_C)^{1/2}\Gamma_{tot}$ values are very small and can therefore hardly be seen in the total cross sections; in addition, they overlap strongly, and this also makes the analysis ambiguous (generally speaking, in such a situation a statistical analysis would be very helpful).

From 1979 the $^{12}C(^{16}O, \alpha)^{24}Mg$ reaction began to be used to study quasimolecular states in the $^{12}C + {}^{12}C$ system. The spectrum of α particles emitted at angles 7° and 15° was measured, for example, in Ref. 34 at energy 145 MeV of the incident ^{16}O nucleus. After subtraction of the smooth background, characteristic structures in the α spectra that fitted into the known quasimolecular band for ^{24}Mg were found.

From this it was concluded that in this reaction there is a direct transfer of the ^{12}C nucleus and that high-lying $J = 16^+$, 18^+ states of ^{24}Mg are observed. We note that the effect was not observed at the angle 40°. Although a justification for direct transfer was given in Ref. 35, it is nevertheless evident that the hopes of observation of high-lying quasimolecular states in this reaction were not justified. For example, the $^{12}C(^{16}O, \alpha)^{24}Mg$ reaction was studied in Ref. 36 in the range $E_{^{16}O} = 60$–100 MeV. Measurements were made of the energy, width, and decay branch for ^{24}Mg excited states at E^* up to 27.5 MeV. From the resulting data the authors concluded that the excited states in this region of energies are not associated with quasimolecular resonances in ^{24}Mg excited in the $^{12}C(^{12}C, ^{8}Be)^{16}O$ and $^{12}C(^{12}C, \alpha)^{20}Ne$ reactions. With regard to the region of higher excitations, the authors noted the possibility of explaining the corresponding α groups by inelastic ^{16}O scattering (to highly excited α-cluster states) with subsequent emission of α particles. This explains, in particular, the result of Ref. 34, in which an effect was not observed at $\theta_{lab} = 40°$. This conclusion is confirmed by Ref. 37, from which it follows that in the ^{16}O reaction on the ^{12}C and ^{13}C nuclei at $E = 140$ MeV the dominant process in the α–^{12}C coincidence spectra is that of excitation and decay of ^{16}O nuclei and not the process of interaction of the nuclei $^{12}C + ^{12}C$ in the final state. The same conclusion was drawn in Ref. 38, in which measurements were made of two ^{12}C nuclei in coincidence from the $^{12}C(^{16}O, ^{12}C, \alpha)^{12}C$ reaction at energies $E_{lab}(^{16}O) \approx 70$–$80$ MeV. Since in these reactions there is a large background of α particles, it was suggested that the α–^{12}C coincidences are not sensitive to the interaction of the ^{12}C nuclei (which give weak peaks at the level of the large background).

With a view to further study of this question and to establish whether the $^{12}C(^{16}O, \alpha)^{24}Mg$ reaction could be a new tool for investigating quasimolecular resonances in the $^{12}C + ^{12}C$ system, a study was made in Ref. 39 of the energy dependence of the inclusive α spectra in the $^{16}O + ^{12}C$ reaction at $E_{lab}(^{16}O) \approx 94$–$150$ MeV. The results obtained in this study show that the clearest peaks in the α spectrum are not $^{24}Mg^*$ states but derive from the successive decay of an excited ^{16}O nucleus. The structures observed at $E^*(^{24}Mg)$ equal to 25 and 28 MeV do have the kinematics of the ^{24}Mg final state but consist of a large number of narrow states, and the cross sections of these states fluctuate with the beam energy (this indicating importance of the compound-nucleus process). Only two states, at $E^*(^{24}Mg)$ equal to 20.3 and 20.9 MeV, are definitely ^{24}Mg states, but these states are excited very weakly. For this reason, the $^{12}C(^{16}O, \alpha)^{24}Mg$ reaction does not give a clear proof of direct transfer of the ^{12}C nucleus and is not yet a good tool for investigating quasimolecular resonances in the $^{12}C + ^{12}C$ system. This reaction does make it possible to investigate possible three-particle (^{12}C–α–^{12}C) (see Ref. 40) quasimolecular states of the ^{28}Si nucleus.

From the point of view of studying quasimolecular resonances, the $^{12}C(^{14}N, d)$ reaction offers a different perspective. Until recently, it was taken to be well known that this reaction proceeds through the stage of formation of a compound nucleus with subsequent evaporation of deuterons. However, in Ref. 41 measurement of the d–α angular correlation function with the α emitted from a ^{24}Mg excited state [it is true, at $E^*(^{24}Mg) < 14$ MeV] showed that direct transfer of the ^{12}C nucleus makes a large contribution to the process. If it will be shown that in this process one can also have direct transfer of the nucleus $^{12}C(2^+)$ in the (2^+) excited state, then such a reaction will be exceptionally important for the study of quasimolecular states in the $^{12}C + ^{12}C$ system.

Conclusions

The experimental data discussed above give convincing evidence for the existence in the $^{12}C + ^{12}C$ system of specific quasimolecular states.

In Fig. 10 we have collected together all the currently known and exactly established resonances of intermediate structure (from the studies quoted above) in the $^{12}C + ^{12}C$ system up to energies $E_{cms} \sim 25$ MeV. It can be seen from this figure that the picture is very rich (and, generally speaking, resembles the rotational–vibrational spectra of diatomic molecules or deformed nuclei), the centroids of the resonances with different spins (J) lying well on a rotational band with moment of inertia corresponding approximately to the

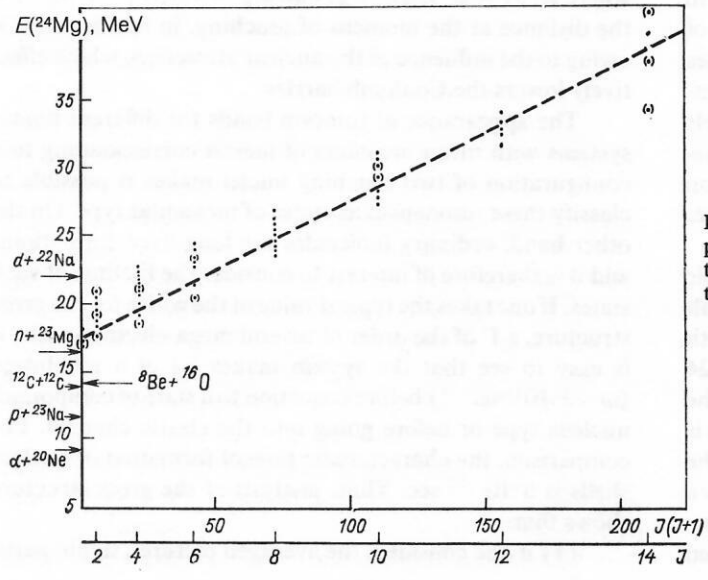

FIG. 10. Rotational quasimolecular band of the $^{12}C + ^{12}C$ system. The points show the positions of the resonances of intermediate structure, with the resonances having insufficiently well-established J given in brackets; the arrows indicate the positions of the thresholds of the various channels.

moment of inertia of two ^{12}C nuclei separated by ~ 5.3 F (we recall that the radius of the ^{12}C nucleus is ~ 2.7 F).

It should be noted that all the theoretical estimates in accordance with different models are based to a large degree on knowledge of the spins of the resonances in the region of energies in which the calculations are made. For this reason, the determination of the spins of the states is of paramount importance. However, in the region of energies $E_{cms} \sim 17$–20 MeV the situation is not entirely clear in this sense, since investigations of the $^{8}Be + {}^{16}O$ channel[20] give $J = 12^+$ for the spin values in this region, whereas study of the $^{20}Ne(g.s.) + \alpha$ channel[42] in this region of energies gives preference to spins 10^+, and study of the angular distributions of elastic scattering in Ref. 30 gives preference to spins 14^+. All this indicates the need to make a more careful study of the spins of resonances at higher excitation energies.

The investigations of the $^{12}C + {}^{12}C$ system at energies above the Coulomb barrier made during the last ten years permit the drawing of a number of conclusions:

(1) in the region of energies $E_{cms} \approx 10$–30 MeV there exists a large number of resonances with widths 100–400 keV and spins 6^+–14^+ of a nonstatistical nature that are well correlated in different channels;

(2) the investigations indicate a quasimolecular nature of these resonances, and the elastic widths of these resonances vary from 10 to 50% of the total widths;

(3) on the average, these resonances fit onto a rotational band with moment of inertia close to the moment of inertia of two weakly overlapping ^{12}C nuclei;

(4) channels with excitation of one of the ^{12}C nuclei to 2^+ (4.43 MeV) and α-particle channels are important in the formation of the wave function of these resonances;

(5) the spins of the high-lying states are at the present time determined with insufficient definiteness, and this can lead to certain difficulties in attempts at theoretical description of these states;

(6) the $^{12}C(^{16}O, \alpha)$ reaction excites weakly or not at all high-lying quasimolecular states in ^{24}Mg.

2. THEORETICAL MODELS

From the middle of the sixties, when the first quasimolecular resonances (see Fig. 1) were discovered in the $^{12}C + {}^{12}C$ system, numerous attempts were already made to create a theoretical model suitable for describing this class of phenomena. However, a unified theory or model that makes it possible to explain (and calculate) the existing experimental data does not yet exist. As a rule, the existing models make it possible to explain only one aspect of the phenomenon (for example, the inelastic cross section, or the fusion cross section, or the elastic cross section, or a large number of states with one J, etc.).

The solution of this problem by a purely microscopic approach is at the present time an exceptionally difficult task—even the groups of theoreticians best equipped with computers are not in a position to solve the problem of 24 nucleons in the continuum with an excitation energy of the compound system of order 15–50 MeV. This problem is further complicated by the diversity of manifestation of the different degrees of freedom of a complex system such as a strongly excited nucleus. On the one hand, there is the amorphous compound state, and on the other the well-defined cluster phase; there is a variety of collective phenomena plus a certain indeterminacy in the nature of the interaction of the nucleons in the medium.

Therefore, the only realistic way of clarifying the nature of the resonance phenomena in reactions with light nuclei (up to ^{40}Ca) is to make a broad analysis of the consequences that follow from different theoretical assumptions and compare them with experimental data.

In this section, we review the main theoretical models and methods used to study resonances of intermediate structure. Feshbach's scheme of doorway states, proposed in Ref. 43, makes it possible to understand the general picture of the phenomenon. In accordance with this scheme, a resonance state in the incident wave with a width of the order of the single-particle width interacts with so-called doorway states. These states of relatively simple structure are long-lived and can decay into the entrance channel, or into the reaction channel, or through a long chain of transitions into very complicated states (of compound-nucleus type). The coupling of the doorway (long-lived) states to the incident wave can lead to a set of fairly narrow resonances. If one adopts this point of view, then the various specific models differ in the way in which the doorway state is described.

The majority of the theoretical models employ Imanishi's model,[7] which we have already mentioned (see Sec. 1), and in which the doorway states are taken to be quasibound states of the $^{12}C + {}^{12}C$ system with excitation of one or both nuclei to low-lying collective states: 2^+, 4.43 MeV; 0_2^+, 7.65 MeV; 3^-, 9.64 MeV.

Classical estimates

We consider the typical features of the resonances from the classical point of view. First of all, one can see from Fig. 10 that there is a large number of resonances grouped into a band with gross-structure width of the order of several mega-electron-volts. All the bands of quasimolecular states (not only $^{12}C + {}^{12}C$) observed in heavy-ion reactions satisfy the rotation rule[44,45]

$$E_{rot} = E(J) - E(0) = J(J+1)\frac{\hbar^2}{2\mu R_B^2}, \qquad (1)$$

where $E(0) = ce^2 Z_1 Z_2 / R_B$, and $R_B = 1.4(A_1^{1/3} + A_2^{1/3})$ is the distance at the moment of touching, in fermis; $c \approx 0.9$, owing to the influence of the nuclear attraction, which effectively lowers the Coulomb barrier.

The appearance of rotation bands for different binary systems with mean moments of inertia corresponding to a configuration of two touching nuclei makes it possible to classify these resonances as states of molecular type. On the other hand, ordinary molecules are long-lived formations, and it is therefore of interest to consider the lifetime of such states. If one takes the typical value of the width for the gross structure, a Γ of the order of several mega-electron-volts, it is easy to see that the system makes $\geqslant \frac{1}{2}$ of a revolution ($\omega \sim 2 \cdot 10^{21}$ sec^{-1}) before transition to a state of compound-nucleus type or before going into the elastic channel. For comparison, the characteristic time of formation of nuclear shells is $5 \cdot 10^{-22}$ sec. Thus, analysis of the gross structure shows that:

(1) if one considers the averaged picture (single-parti-

cle resonances), then the picture that is realized is not one of molecular type (number of revolutions of the system $n \gg 1$) but is rather of quasimolecular type ($n \gtrsim 1$);

(2) the nuclei spend an appreciable time before going into the elastic channel without losing their individuality.

This gives a physical justification for regarding the resonance phenomena in heavy-ion collisions as a new class of phenomena or a new phase of the state of nuclear matter.

The general conclusions from the estimates of the gross structure leave open, nevertheless, the question of the explanation of the fine structure of the resonances that is observed in the reactions or the large number of resonances with the same J^π (see Fig. 10). A consistent treatment requires the inclusion of new degrees of freedom and a quantum-mechanical approach. Before we consider the dynamical aspects of the problem, we dwell on the existing structure calculations.

Spectroscopy of high-lying states

Overall, the arrangement of the states in Fig. 10 recalls the picture of the well-known low-lying rotational–vibrational spectra of deformed nuclei or diatomic molecules. For this reason, it is tempting to use the methods developed to describe analogous phenomena in other branches of physics, although the quantitative aspect can be determined only by comparison with experiment. The methods most widely employed are: (a) study of collective degrees of freedom; (b) application of group theory to the classification of the states; (c) calculations in a microscopic cluster basis.

Collective model

The collective model uses a scheme with coupling of the rotations of a rigid rotator to surface β and γ vibrations (analogous to the case of a deformed nucleus). In the first order in the coupling, the following expression was obtained in Ref. 44 for the spectrum of the $^{12}C + {}^{12}C$ system:

$$E_{LKn_2n_0} = (L(L+1) - K^2)\,\varepsilon - \left\{\frac{1}{2}\,|K| + 1 + 2n_2\right) E_\gamma + \left(n_0 + \frac{1}{2}\right) E_\beta, \qquad (2)$$

$$\varepsilon = \hbar^2/2I \approx 0{,}1\ \text{MeV}.$$

In this study, it was noted that the values of n_0 and n_2 can only be guessed. From the position of the 0^+ states, taking $n_2 = 0$, $n_0 = 0$ for the ground state and $n_2 = 0$, $n_0 = 1$ (β-vibration band), it follows that $E_\gamma = 3.40$ MeV, $E_\beta = 1.65$ MeV. Calculations for the complete spectrum with fixed E_γ and E_β give a spectrum similar to the experimental one but with much larger distances between the levels. But if one takes $n_2 = 1$, $n_0 = 1$ and $n_2 = 1$, $n_0 = 2$ for the 0^+ ($E_\gamma = 0.64$ MeV, $E_\beta = 1.55$ MeV) doublet, much better agreement with experiment is obtained, but in this case there must be one additional 0^+ level at $E_{\text{cms}} = 1.50$ MeV, which has not yet been observed.

Another attempt to introduce explicitly vibrational degrees of freedom was made in Refs. 45 and 46, in which the possibility of quadrupole deformations of the ^{12}C nuclei forming the molecule was considered. Thus, the complete system was described by quadrupole–octupole symmetric and antisymmetric vibrations with allowance for rotations of the system as a whole (collective two-center model). The model leads to coupled-channel equations; the parameters of the potentials are taken from calculations by the folding method, while the parameters of the vibrational part of the Hamiltonian are taken from calculations using the two-center shell model, and also from fitting with respect to the 2^+ (4.43 MeV) level in ^{12}C. It should be noted that, in contrast to Ref. 44, allowance is made here for the nonadiabatic effects of the coupling of the moment of inertia and the vibration parameters. The results show that in this approach it is easy to obtain a large number of levels with given J, but again it is not possible to obtain the correct splitting (the correct level density). For example, the distance between the bands is 5 MeV, in contrast to the experimental values, which are 250–300 keV for large J and 1.4 MeV for the 0^+ states.

Group-theory methods (dynamical symmetries)

Group-theory methods, which have proved themselves well in molecular spectroscopy and the spectroscopy of low-lying nuclear states, give a picture of the global dynamical symmetries of the interaction between heavy ions. In Ref. 47 it is assumed that the spectra are determined by dipole degrees of freedom (like a diatomic molecule) with $\mathbf{R}$, the radius vector joining the atoms, as the fundamental quantity. Taking into account also the vibrational degree of freedom with molecular potential $V(R) = \Sigma_n \alpha_n [(R-c)/b]^n$ and using the Lie algebra of the group $U(4)$ (three rotational degrees of freedom and one vibrational degree), we obtain for the spectrum the expression

$$E(v,\ L) = -D + a(v + 1/2) - A(v + 1/2)^2 + BL(L+1), \qquad (3)$$

where ν is the oscillator frequency, and D, a, A, and B are constants.

In Ref. 12, this expression [for a potential $V(R)$ with $n = 4$] was used to investigate the $^{12}C + {}^{12}C$ system at $E_{\text{cms}} \simeq 3.1$–13 MeV. To determine the free parameters D, a, A, and B, 28 resonances correlated in different channels were used, and very good agreement ($\sigma = \pm 44$ keV) was obtained in the positions of the levels (Fig. 11). Moreover, even the resonances with spin insufficiently well determined

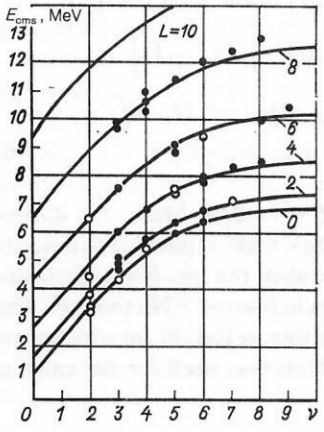

FIG. 11. Comparison of experimental quasimolecular spectrum (see Fig. 10 and Table I) with the spectrum calculated in accordance with Eq. (3). The black circles represent the experimental results with well-established spins, and the open circles are the results with insufficiently well-established spins.

lay on the curves describing the different bands. However, it is here necessary to bear in mind the following facts, which are not readily explained in the simple physical scheme just considered:

(1) The reduced moment of inertia obtained from the fit, $B = 0.076$ MeV, corresponds to an average distance $r \simeq 6.75$ F between the ^{12}C nuclei, and this is appreciably greater than the total radius of two spherical nuclei. Note that for the formation of a double nuclear system there must be at least partial overlapping of the wave functions of the ^{12}C nuclei to enable the attractive nuclear forces to come into play. To explain such a large relative distance r, one must either assume a very large polarization deformability of the colliding ions or take into account explicitly the α-cluster structure of the excited states of the ^{12}C nucleus [in particular, the 0_{12}^+ (7.6 MeV) state]. Such a possibility was pointed out in Refs. 12, 48, and 49.

(2) The parameter $a = 1.44$ MeV corresponds to a very "broad" oscillator potential, which cannot be obtained with short-range NN forces.

(3) As was noted in Ref. 12, it is necessary to include in the treatment other important channels as well, and this must lead to a large number of additional states, since in such a case the levels will be determined by a representation of the direct product of groups $U(N) \otimes U(4)$, where N is the number of channels.

In Ref. 50, allowance for the quadrupole–quadrupole interaction between the ^{12}C nuclei and use of a cluster representation of the wave function in the $SU(3)$ basis,

$$\Psi_{\text{clust}} = \hat{A}\{[\Phi_{I_1}(^{12}\text{C})\,\Phi_{I_2}(^{12}\text{C})]_{I_c}\,\varphi_L(\mathbf{R})\}_{JM_T} \qquad (4)$$

[where $\hat{A}$ is the antisymmetrization operator, Φ_{I_1} and Φ_{I_2} are the wave functions of fragments with spins I_1 and I_2, $\varphi_L(\mathbf{R})$ is the wave function of the relative motion with orbital angular momentum L, and J and M_J are the total angular momentum and its projection], lead to the interaction Hamiltonian

$$\hat{H}_{\text{int}} = \hat{Q}_{i_1}\hat{Q}_{i_2} + \hat{Q}_{\text{rel}}(\hat{Q}_{i_1} + \hat{Q}_{i_2}), \qquad (5)$$

where $\hat{Q}_{i_1}$, $\hat{Q}_{i_2}$, and $\hat{Q}_{\text{rel}}$ are the operators of the quadrupole moments of the fragments and of the relative motion.

The schematic Hamiltonian can be expressed in terms of the Casimir operators $C(\lambda\mu)$ of the group $SU(3)$:

$$H = E_{i_1} + E_{i_2} + \alpha\left[c\,(\lambda\mu) - \tfrac{3}{4}J\,(J+1)\right] + \tfrac{3}{4}\beta[I_1(I_1+1) + I_2(I_2+1)] + \gamma I_c\,(I_c+1) + \delta L\,(L+1) + \nu c\,(\lambda_c\mu_c). \qquad (6)$$

The parameter values $\alpha = \beta = -0.04$ MeV, $\gamma = 0$, $\delta = 0.15$ MeV, and $\nu = -0.10$ MeV make it possible to describe satisfactorily the distance between the low-lying levels and the positions of the resonances in the $\alpha + {}^{20}Ne$ channel. This philosophy was developed further in Ref. 51, in which a microscopic oscillator cluster basis was used for the calculations.

Many-particle cluster basis

In Ref. 51, a microscopic calculation was made of low-lying below-barrier states with a central NN potential of Gaussian type and with allowance for 12 to 20 excitation quanta with respect to the coordinate of the relative motion. Diagonalization was carried out in the framework of a simplified variant of the resonating-group method, the orthogonal-condition model,[52] only the quadrupole moments and the main channels, $^{12}C + {}^{12}C$, $\alpha + {}^{20}Ne$, $^{8}Be + {}^{16}O$, being considered. The partial widths of decay to all channels were calculated. It can be seen from the results of the calculations that, despite the incorrect positions of the thresholds, the number and approximate distance between the quasimolecular states, 0^+, 2^+, 4^+, are in agreement with experiment, this confirming their cluster nature; it is found that the partial widths are too small for wave functions having the nature of truly surface molecular states, and they are therefore, rather, to be regarded as quasimolecular states. It must here also be noted that allowance for an admixture of higher-lying states leads to an increase in the weight of the wave function in the surface region.

Conclusions

Summarizing what has been said, we may conclude that the spectroscopic calculations for the $^{12}C + {}^{12}C$ system with allowance for the collective and cluster degrees of freedom give overall a reasonable picture of the low-lying (below-barrier) states and the approximate distance between the levels of intermediate structure for the above-barrier resonances. However, each of the models considered above describes well only one particular feature of the spectrum; a complete description is not obtained (or the description leads to unphysical parameters of the model). It should be noted here that the extension of the spectroscopic models to heavier systems must lead to similar spectra of the quasimolecular states, which are not seen at the present resolution in experiments with nuclei heavier than ^{28}Si. This may mean: (a) the presence of an additional dynamical selection rule preventing transitions to such configurations; (b) enhancement of the coupling to such states in heavier systems, leading to overlapping of the quasimolecular resonances; (c) a much higher density of the levels of cluster nature.

Dynamics. Microscopic approaches

A mathematical formalism suitable for representation of a nucleus as a system that spends some time in a completely thermalized state, called the compound nucleus, and some of the time in other more ordered states, called at the present time composite systems, is the projection approach to nuclear theory developed in Refs. 43 and 53–55. To give a definite meaning to the reaction mechanism, it is necessary to separate the parts of the total wave function that make the dominant contribution to the observed quantities—the cross sections, positions of the resonances, and their widths. In the projection approach, one can identify three directions that have been most strongly developed in recent years: the resonating group method,[54] the projection-operator method,[43,53] and the interpolation approach,[55] which uses aspects of a hyperspherical basis (K harmonics).

Resonating-group method

The resonating-group method is based on a cluster representation of the wave function (4). The original many-particle Hamiltonian with two-body NN interactions $V(r_{ij})$

is projected onto the intrinsic states Φ_{Ii} of the fragments, and as a result one obtains for $\varphi_L(R)$ a well-known system of integro-differential equations[54] (the integral kernel is associated with systematic allowance for the Pauli principle). The difficulties associated with allowance for antisymmetry can be partly avoided in the orthogonal-condition model,[52] in which the results of numerous calculations show that the elimination of the Pauli-principle-forbidden states in the wave function of the relative motion (and, thus, in the total wave function) is more important than allowance for the residue of the nonlocal exchange kernel. Unfortunately, because of the great computational difficulties, it has hitherto been possible in this method to describe only the low-lying (below-barrier) resonances for complicated systems of the $^{12}C + {}^{12}C$ type (see Sec. 2).

Feshbach's projection-operator method

In Feshbach's projection-operator method,[43,53] one uses a decomposition of the complete Hilbert space into two parts—the shape-elastic subspace $\hat{P}$ and the residue $\hat{Q}(\hat{P} + \hat{Q} = 1)$. In its turn, $\hat{P}$ is decomposed into a resonance part, $\hat{R}$, and a smooth background, $\hat{p}$, while $\hat{Q}$ is decomposed into the subspace of doorway states, $\hat{d}$, which are complementary to the elastic resonances, and more complicated (compound) states, $\hat{q}$. Thus, the wave function consists of four orthogonal parts:

$$\Psi^{(+)} = \hat{p}\Psi^{(+)} + \hat{R}\Psi^{(+)} + \hat{d}\Psi^{(+)} + q\Psi^{(+)} \qquad (7)$$

and, in accordance with this, four terms can be separated in the scattering amplitude $\hat{T}$:

$$\hat{T} = \langle \chi_f^{(-)} | \hat{H}_{\text{int}} | \Psi^{+} \rangle = T_1 + T_2 + T_3 + T_4, \qquad (8)$$

corresponding to resonance and nonresonance elastic scattering, transition to doorway states, and transition to compound states.

Because of the ambiguity in the decomposition of the space and the as yet insuperable difficulties of many-particle calculations with *NN* potentials, the equations of the method merely provide a scheme for the construction of models that make it possible, on the basis of physical considerations, to parametrize and calculate effective potentials, using them subsequently in various model problems.

It should be noted that the developed method is widely used to interpret qualitative effects observed in heavy-ion physics.

Interpolation approach

In the interpolation approach,[55] the wave function is sought by the variational method in the form of a sum $\Psi_{\text{int}}(\rho) + \Psi_{\text{ext}}(\rho)$ using hyperspherical functions; as in the case of three-dimensional spherical functions, this makes it possible to reduce the problem, after projection of the many-particle Hamiltonian onto the multidimensional angular functions, to motion with respect to the single collective hyperradial variable $\rho(\rho^2 = 1/A\,\Sigma_{i>j}^{A}(\mathbf{r}_i - \mathbf{r}_j)^2$. The functions Ψ_{int} and Ψ_{ext} are orthogonal and are required to describe adequately the behavior of the system in the interior region (compact states of A nucleons) and in the exterior region, where asymptotically the wave function represents the relative motion of the clusters in the different binary channels. For the radial parts of such a wave function, one obtains a system of coupled differential equations (in contrast to the system of integro-differential equations in the resonating-group method), the system having the same form as in the coupled-channel scheme. We give a somewhat simplified version of the system[55]:

$$(\hat{h}_0 - E)\,\Psi_{\text{int}}(\rho) = \sum_{\varkappa} D_{\varkappa}(\rho)\,\Psi_{\text{ext}}^{\varkappa}(\rho);$$

$$(\hat{h}_{\varkappa} - \varepsilon_{\varkappa})\,\Psi_{\text{ext}}^{\varkappa}(\rho) = \sum_{\varkappa' \neq \varkappa} V_{\varkappa\varkappa'}(\rho)\Psi_{\text{ext}}^{\varkappa'}(\rho) + D_{\varkappa}^{+}(\rho)\,\Psi_{\text{int}}(\rho), \qquad (9)$$

which preserves the qualitative features of the original system. Here, h_0 and $h_{\varkappa}$ are the effective Hamiltonians in the interior and exterior regions; E and $\varepsilon_{\varkappa}$ are the total energy and the energy of the relative motion in channel $\varkappa$; $D_{\varkappa}$ describes the coupling of channel $\varkappa$ to states of compound-nucleus type; and $V_{\varkappa\varkappa'}$ describes the coupling of the channels.

One can here repeat almost literally the remarks made on the formalisms of the resonating-group method and Feshbach's method.

Note that these approaches yield under suitable simplifications (basically, approximate allowance for antisymmetrization and replacement of transitions to unimportant reaction channels by an imaginary part in the potential) the widely used methods—DWBA, weak and strong channel coupling, and the independent-cluster model. Concrete applications of the methods described above to the investigation of quasimolecular states will be considered below.

Hartree–Fock method

For calculations of binary reactions, one usually employs a modified Hartree–Fock method, in which a two-center basis is used to obtain the correct asymptotic behavior of the wave function in the form of a product of wave functions of individual clusters.

In Ref. 56, elastic scattering at low (below-barrier) energies was studied by means of the Hartree–Fock method with a two-center deformed oscillator basis and with allowance for pairing correlations, this being usually done to describe soft modes of collective nuclear vibrations. This yielded the dependence of the mass parameter (the reduced mass) on the distance between the nuclei, an adiabatic potential close to the potential obtained in the contact approximation or from the folding procedure, and the density of the system at certain resonance energies. The calculations show that: (1) in the framework of this method it is not possible to reproduce a large number of below-barrier resonances, the ^{24}Mg binding energy, and the distance to the first quasimolecular 0^+ level; (2) as in calculations in the framework of the collective model, the effective reduced mass depends on the distance between the ions, increasing with their approach toward each other (this is equivalent to allowance for a nonlocal interaction); (3) there is a large admixture of an α–^{16}O–α configuration in the case of fission into two fragments.

In Ref. 57, the same method, with a number of simplifying assumptions, was used to obtain equations of coupled-channel type for the description of $^{12}C + {}^{12}C$ scattering and reactions with excitation of one of the ^{12}C nuclei to a 2^+ state; the potentials were calculated as both diagonal and

nondiagonal in two limiting cases: collisions with the ^{12}C deformation axes along the line joining the centers and perpendicular to it.

It appears to be an important circumstance that the averaged diagonal potential obtained for the interaction of the two ^{12}C nuclei has a quasimolecular nature (i.e., surface attraction with a repulsive core), irrespective of the mutual disposition of the deformation axes of the ^{12}C nuclei; the nondiagonal potentials have variable sign. As in the majority of other models, the fusion cross sections were fitted by introducing an imaginary part into the diagonal potentials.

The shortcomings of the calculations in Refs. 56 and 57 such as the approximate fulfillment of the asymptotic symmetries, the absence of good quantum numbers of the ^{12}C nuclei, and the neglect of translational invariance were partly overcome in Ref. 58. The J dependence of the molecular resonances and the possible existence of a simple equivalent local potential were investigated. The study used an NN potential with not only a central part but also a spin–orbit part. The results of the calculations reveal a number of serious shortcomings: (1) in the $E^J(R)$ curves there is a "gap" between J equal to 0–8 and 10–12 due to the limitation of the employed oscillator basis; (2) for $J > 8$, the moment of inertia is ~ 140 keV, a value appreciably greater than the one observed experimentally; (3) even the band based on the ground state is poorly reproduced, and the number of below-barrier states obtained is much smaller than is seen in the experiments; (4) there is an extra below-barrier state with $J = 0$; (5) the local equivalent potential depends on J and has range $a \simeq 3.31$ F, which is much smaller than the value obtained in other approaches; (6) the intensity of the spin–orbit part of the interaction obtained by fitting was found to be 80–100 MeV, which is much greater than its actual value.

Despite the fact that the Hartree–Fock method considered above is a modern and powerful means of investigating the interaction of heavy ions, in actual calculations it is necessary to make a large number of poorly controlled approximations, and as a consequence the results must be regarded as qualitative and preliminary.

Very interesting results have been obtained recently for central $^{12}C + {}^{12}C$ collisions in a microscopic variant of the time-dependent Hartree–Fock method.[59] In these calculations, the initial configuration was specified in the form of the ^{12}C ground state and the excited 0^+ (7.6 MeV) state, which has a linear 3α structure. This enabled the authors to model the large range of the interaction of the two ^{12}C nuclei needed to describe the spectrum of the quasimolecular states,[48] as discussed above. The picture obtained for the density distribution indicates a large amplitude of the surface vibrations with energy around 1.79 MeV and a dominant quadrupole mode.

Calculations of noncentral collisions in the systems $^{16}O + {}^{16}O$ ($E_{cms} = 52.5$ MeV, $L = 13\,\hbar$) and $^{40}Ca + {}^{40}Ca$ ($E_{cms} = 139$ MeV, $L = 70\,\hbar$) made in Ref. 60 again reveal a molecular nature of the interaction, the motion having a vibrational–rotational nature.

Dynamics. Phenomenological schemes

Many features of the elastic and inelastic excitation functions and of the fusion cross section at energies in the region of the potential barrier and above can be reproduced in the framework of phenomenological models in which allowance is made for the coupling of the intrinsic (collective) degrees of freedom and the relative motion, processes of transition to a thermalized compound state, and fusion. The concept of an internuclear potential is very attractive, making it possible to describe in a compact and clear form many of the basic features of the reactions and scattering, and also to obtain a physically perspicuous picture of the mechanism of formation of the resonance structure. Therefore, during the last fifteen years physically motivated schemes have been proposed. They can be divided nominally into three classes: (1) diffraction schemes; (2) parametrization of the amplitude in the spirit of the DWBA and Regge poles; (3) scattering and fusion models which use molecular and geometrical notions (the pioneering study in this direction was Imanishi's model[7]; see Sec. 1). Strictly speaking, the amplitude (8) can be regarded as the result of scattering by an optically inhomogeneous dispersive medium. Leaving on one side the traditional optical model, which is in contradiction with the experiments due to the fact that in the case of a large reaction cross section (and, therefore, large imaginary part of the optical potential) it is difficult to reproduce even the gross structure (the widths of the resonances are found to be too large), we turn to the actual calculations.

Diffraction model

The diffraction model has been widely used and is still used in nuclear physics, particularly to describe elastic scattering. In principle, diffraction by an absorbing medium with a diffuse edge distinguishes several partial waves (grazing angular momentum), giving correct angular distributions at forward angles. It was therefore natural to extend this picture to the $^{12}C + {}^{12}C$ system. For this, it was necessary to take into account not only elastic but also inelastic scattering, and also to find a felicitous parametrization of the reflection coefficients.

In Ref. 61, the adiabatic approximation was used to derive simple relationships between the inelastic $\hat{S}$ matrix in the angular-momentum space and the elastic reflection coefficient η_l calculated in the optical model. To study the inelastic amplitude, one usually employs the DWBA with the interaction of the collective model:

$$V_T = R\,\frac{\partial U(R)}{\partial R}\sum_{\lambda,\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}(\mathbf{n}) + O(\alpha^2),$$

where $U(R)$ is the interaction potential, which is most often chosen in the Woods–Saxon parametrization. The quantity η_l occurs in the elastic scattering amplitude in the form

$$f(\theta) = f_c(\theta) + \frac{i\sqrt{\pi}}{k}\sum_l \hat{l}\,(1-\eta_l)\,e^{2i\sigma_l}Y_{l0}(\theta,\,0), \quad (10)$$

where $f_c(\theta)$ and σ_l are the Coulomb amplitude and the phase shift, and for η_l the proposed parametrization is

$$\eta_l = g(t) + \gamma\frac{dg(t)}{dt} + i\left[\mu_1\frac{dg(t)}{dt} + \mu_2\frac{d^2g(t)}{dt^2}\right], \quad (11)$$

$$\text{with } g(t) = \left[1 + \exp\frac{(T-t)}{\Delta}\right]^{-1},\ t = l + 1/2.$$

The quantities $T(E)$ and Δ have a meaning close to that of the grazing angular momentum and the diffuseness in the model with "smeared" absorption. The reaction amplitude is expressed[61] in terms of the radial matrix element

$$\beta_{ll'} = \int u_l(k_i r)\frac{\partial U(r)}{\partial r} u_{l'}(k_f r)\, dr \qquad (12)$$

with optical wave functions $u_l(kr)$. For a strongly absorbing medium, the matrix element (12) can be approximately expressed[62] as the geometric mean

$$\beta_{ll'}(k_i k_f) = [\beta_{ll}(k_i k_i)\,\beta_{l'l'}(k_f k_f)]^{1/2}, \qquad (13)$$

and $\beta_{ll}(k_i k_i)$ itself is related to η_l by

$$\beta_{ll}(k_i k_i) = -\frac{1}{2} i E \frac{d\eta_l}{dl}. \qquad (14)$$

Note that the representation of the grazing angular momenta in the form $T_{f,i} = k_{f,i}R\,[1 - 2n_{f,i}/(k_{f,i}R)]^{\frac{1}{2}}$, where $n_{f,i}$ are the Coulomb Sommerfeld parameters, makes it possible to find the reaction amplitude in closed form. Such a program was realized for $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$ in Ref. 63 with the parametrization

$$\eta_L(E) = \left\{1 + \exp\left(\frac{E - E_G}{\Delta}\right)\right\}^{-1};\quad E_G = E_0 + \frac{\hbar^2}{2I}L(L+1). \qquad (15)$$

This representation does not lead to resonances in the elastic channel and is satisfactory for diffraction by a sphere with a diffuse edge (smooth cutoff with respect to l) and also for strong absorption for small l and transparency for large (grazing) angular momenta.

However, the model has a number of serious shortcomings, which put in doubt the nonresonance diffraction reaction mechanism, namely: (1) it gives incorrect peak/dip ratios in the $^{12}C^*(2^+)$ inelastic channel (Fig. 12a) and an incorrect value of the absolute cross section; (2) it describes poorly the angular distribution in the region of angles $70 \leqslant \theta_{cms} \leqslant 110°$ (this distribution is sensitive to the reaction mechanism); (3) to describe the resonance structure, one must use a too small value of Δ (~ 1 MeV) (15), in disagreement with the diffraction picture.

Regge-pole model

Another parametrization of the amplitude in the DWBA and the optical model is the model of moving Regge poles.[64,65] Instead of calculation of the partial-wave amplitudes by means of an optical potential or form factors in the DWBA, one introduces from the very beginning a physically motivated universal parametrization of these amplitudes. Allowance is made for the fact that direct reactions and transfer reactions are peripheral with $l \simeq k(R_1 + R_2)$ and a "window" $\Delta l \simeq 1/5$.

The partial-wave amplitude is expressed as

$$f(l) = \text{const}\ e^{i[\sigma_i^l + \sigma_f^l]}\,(l + L_0 + iy)e^{-\frac{(l-L_0)^2}{\Delta^2}}\,\frac{l - z}{(l - p_1)(l - p_2)} \qquad (16)$$

with nine parameters: z, p_1, p_2 (complex), L_0, y, Δ. The factor $(1 + L_0 + iy)$ reproduces the specific beats in multistep processes described by strong channel coupling.

In Ref. 64 it is noted that such a parametrization makes it possible to reproduce very well the DWBA results and the experiments.

In Ref. 66, such considerations led the authors to a parametrization of the DWBA radial integral in the form of two moving Regge poles,

$$I(l, l') = \text{const}/(l - \mathscr{L})\,(l' - \mathscr{L}'), \qquad (17)$$

where $\mathscr{L} = \mathscr{L}_r + i\Gamma_l/2$, with $\mathscr{L}_r(E)$ taken from the optical potential $V_{ll} = -(220 + i25)\exp(-r^2/9.46)$, which is obtained from the convolution procedure, and with $\Gamma_l \sim 40\,\mu^{-1}$ (MeV). For $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$, satisfactory agreement with the gross structure of the reaction cross section is obtained.

These phenomenological approaches require some comments:

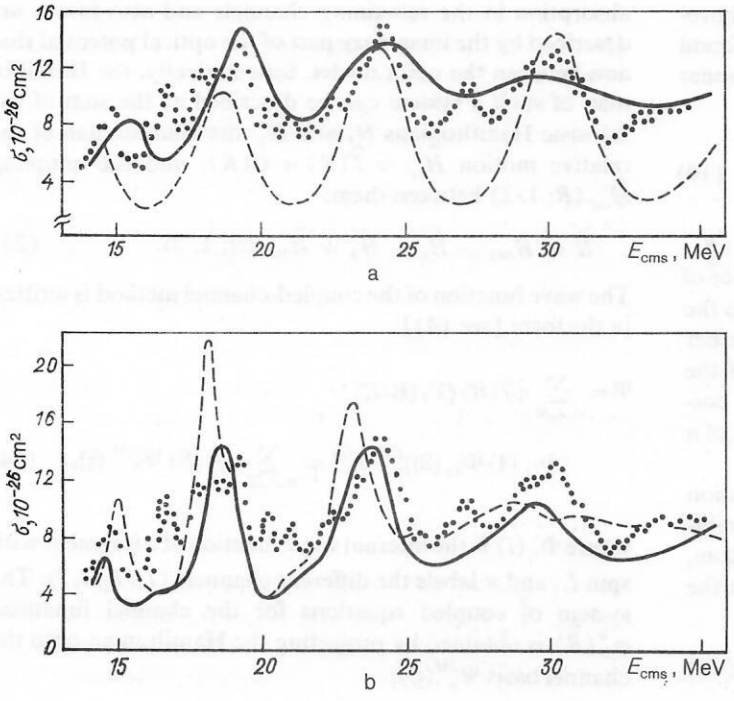

FIG. 12. Cross section of inelastic $^{12}C + {}^{12}C(2^+; 4.43$ MeV) scattering.[93] The points everywhere represent the experimental data. a) The continuous curve represents calculations in the DWBA, and the broken curve shows calculations in the diffraction model; b) the continuous curve gives the results of strong channel coupling, and the broken curve shows the results of the band-crossing model.

1. In both cases, kinematically advantageous conditions hold for the $^{12}C + {}^{12}C \to {}^{12}C + {}^{12}C^*(2^+)$ system—in the region $L \sim 10$–16 the positions of the resonances in the elastic (L) and inelastic ($L-2$) channels are very close together, and this leads to a dominant role of the width of the inelastic channel ($L-2$) in the formation of the resonance (see Imanishi's model[7]).

2. In both cases, the amplitudes admit a Breit–Wigner parametrization,

$$T_{LL'} \sim \text{const} \cdot [(E - E_L - i\Gamma_L)(E - \tilde{E}_{L'} - i\Gamma_{L'})]^{-1};$$

$$\tilde{E}_{L'} = E_{L'} + E^*(2^+),$$

which clearly demonstrates the role of the kinematic favoring in the isolated resonances of the entrance and exit channels.

3. In both cases, a decisive role is played by the "l window" of width Δ in the "diffraction" model and $\sim\Gamma_l$ in the Regge-pole model.

To conclude this subsection, we should mention Ref. 67, in which a mechanism of interference between the barrier and internal waves is proposed in order to improve the description of the gross structure. The barrier wave is described by an amplitude of the "diffraction type" (10) with an Ericson parametrization for η_l,

$$\eta_l = \frac{1}{1 + e^{i\gamma - (l-T)/\Delta}};$$

$$\Delta \sim 0.8;\ \gamma \leqslant \frac{\pi}{2} \text{ from the unitarity condition,} \qquad (18)$$

while the internal wave is described by the model of a moving Regge pole. This leads to better agreement with experiment for $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$ and it is concluded that the picture of a "pocket" in the potential and, therefore, a quasimolecular nature of the resonances matches the experimental data better than the diffraction model.

Fusion models

In the study of fusion reactions in the region of above-barrier energies for the majority of heavy ions it was found that the cross section can be expressed in a universal manner in terms of a critical angular momentum l_{cr}:

$$\sigma_{fus} = \pi\lambda^2 \sum_{l=0}^{l_{cr}} (2l+1) \simeq \pi\lambda^2 l_{cr}^2. \qquad (19)$$

As determined from the condition $l_{cr} \simeq kR_{cr}$ ($R_{cr} = r_{cr}(A_1^{1/3} + A_2^{1/3})$, $r_{cr} = 1 \pm 0.07$ F). The appearance of this quantity is due to the existence of a repulsive core in the interaction potential of the nuclei, which prevents a further increase of the density in the region of overlapping of the nuclei, after which fusion commences. We note that R_{cr} corresponds approximately to the minimum in the pocket of a potential of quasimolecular type.

In the region of the below-barrier energies, the fusion probability is basically determined by the barrier penetrability and the influence of the internal degrees of freedom, which facilitate fusion, and for the cross section we have the semiclassical formula

$$\sigma_{fus} = \pi R_B^2 \left(1 - \frac{V(R_B)}{E}\right), \qquad (20)$$

where $R_B = r_B(A_1^{1/3} + A_2^{1/3})$, $r_B = 1.4$ F; this formula is also valid for above-barrier fusion with the substitution $R_B \to R_{cr}$. Both of these expressions are limiting cases of the more general expression obtained in closed form in Ref. 68, in which the fusion and reaction cross sections

$$\sigma_{fus} = \pi\lambda^2 \sum_{l=0}^{l_{cr}} (2l+1) T_l;\ \sigma_r = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) T_l. \qquad (21)$$

were investigated. Note that the summation over l in the expression for σ_r is restricted by l_{max}, at which T_l is reduced by a factor of 2. For an inverted parabolic barrier with characteristic frequency ω, we can, by replacing the summation over l by an integration, obtain from (21)

$$\sigma_{fus} = \frac{\hbar\omega R_B^2}{2E} \ln \left\{ \frac{1 + \exp[2\pi(E - V(R_B))/\hbar\omega]}{1 + \exp[2\pi(E - V(R_B) - \frac{R_{cr}^2}{R_B^2}(E - V(R_{cr})))/\hbar\omega]} \right\}. \qquad (22)$$

The different behavior of $l_{cr}(E)$ and $l_{max}(E)$ divides the complete region of angular momenta at above-barrier energies into the region of strong absorption ($l \lesssim 10$), where shape resonances are not seen,[69] and the region of moderate absorption, where they may appear. This gives a qualitative interpretation of the disappearance of the clearly defined gross structure in the experimental data at energies above the Coulomb barrier for angular momenta $l \leqslant 8$.

Models using simplified microscopic representations

Coupled-channel method, DWBA, band-crossing model, and double-resonance model

Essentially, all calculations that take into account intrinsic excitations of one or two fragments are constructed in the framework of the same philosophy—the mechanism of double excitation (see Imanishi's model), in which one nucleus (or both), excited to low-lying collective states, loses kinetic energy and falls into the region of quasibound states in the same (or a slightly different) potential. The process of absorption in the remaining channels and also fusion are described by the imaginary part of the optical potential that acts between the point nuclei. Schematically, the Hamiltonian of such a system can be described as the sum of the intrinsic Hamiltonians $\hat{H}_1$ and $\hat{H}_2$, the Hamiltonian of the relative motion $\hat{H}_{rel} = \hat{T}(R) + U(R)$, and the coupling $\hat{H}_{int}(R; 1, 2)$ between them:

$$\hat{H} = \hat{H}_{rel} + \hat{H}_1 + \hat{H}_2 + \hat{H}_{int}(R; 1, 2). \qquad (23)$$

The wave function of the coupled-channel method is written in the form [see (4)]

$$\Psi = \sum_{\varkappa, J, M} \varphi_J^{\varkappa}(R) \{Y_l(\mathbf{R}/R) [\Phi_{I_1}(1)\Phi_{I_2}(2)]^{I M_I}\}_{\varkappa}^{JM} \equiv \sum_{\varkappa, J, M} \varphi_J^{\varkappa}(R) \Psi_{\varkappa}^{JM}(\xi), \qquad (24)$$

where $\Phi_{I_i}(i)$ is the internal wave function of a fragment with spin I_i, and $\varkappa$ labels the different channels (I_1, I_2, I, l). The system of coupled equations for the channel functions $\varphi_J^{\varkappa}(R)$ is obtained by projecting the Hamiltonian onto the channel basis $\Psi_{\varkappa}^{JM}(\xi)$:

$$\left\{E_{\text{cms}} - \left(\varepsilon_\varkappa - \frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{R^2} + V_\varkappa(R) + \mathrm{i}\,W_\varkappa(R)\right)\right\}\varphi_J^\varkappa(R) = \sum_{\varkappa'\neq\varkappa} V_{\varkappa\varkappa'}(R)\,\varphi_J^{\varkappa'}(R), \quad (25)$$

where $\varepsilon_\varkappa$ is the excitation energy of the two ions.

In the limit of weak channel coupling, ignoring the influence of the inelastic channel on the elastic channel, we obtain the formulas of the DWBA and the two-step DWBA.

In the framework of this approach, a great deal of work has been done by groups of theoreticians in the German Federal Republic (double-resonance model)[70–73] and Japan (band-crossing model).[74–76] Many forms of diagonal internuclear potentials were investigated; they follow from two extreme approaches:

(1) The adiabatic approach, in which the nuclear shells have time to readjust themselves during the interaction and the potential is close to the Woods–Saxon potential or an oscillator potential. This type can be realized at below-barrier energies, when the incident ions are decelerated in the long-range Coulomb field and at the time when the nuclear forces come into play lose kinetic energy that is expended on intrinsic excitations.

(2) The approximation of a sudden collision, in which the incompressibility of nuclear matter on the macroscopic side or the Pauli principle on the microscopic side lead to an effective repulsive core[14] at distances less than the sum of the radii of the colliding nuclei. The region of attraction (the pocket) at large distances arises because of the nucleon–nucleon attraction. It is usually assumed that this type of quasimolecular potential is realized in the region of above-barrier energies.

In addition, different numbers of channels with excitation of one or two ^{12}C nuclei in 2^+, 3^-, and 0_2^+ states were included, and an investigation was also made of different forms of the imaginary parts—surface and volume absorption, and an imaginary part with a dependence on the energy and density of states of the ^{24}Mg compound nucleus.

Investigations were made of different variants of the nondiagonal parts of the potentials, which were obtained by the folding procedure with NN forces in the following forms: a sum of two potentials of Yukawa type, δ forces,[72] and a sum of a Gaussian function[77] and an expansion of the diagonal part of the potential in a Taylor series with respect to the parameters of the quadrupole deformation and the surface vibrations.

It was noted in Refs. 79–81 that the nondiagonal potentials calculated by the folding procedure give results very close to the microscopic results, in contrast to the diagonal potentials, for which the antisymmetrization of the total wave function has a strong influence on the radial dependence and the depth. In the models which we have mentioned,this dependence is taken into account only in the appearance of even spins for identical nuclei.

We now discuss the results.

All the listed calculations reproduce at least qualitatively the gross structure in the inelastic channels and the fusion cross section, while in the band-crossing model (see Fig. 12) and the double-excitation model (Fig. 13) they also reproduce the splitting of the resonances in approximate

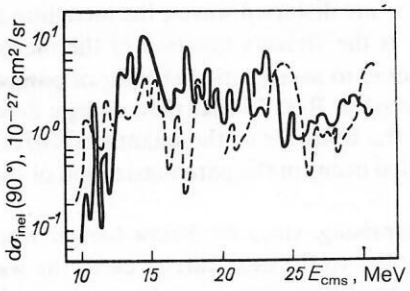

FIG. 13. Differential cross section of inelastic $^{12}C + {}^{12}C(2^+; 4.43\ \text{MeV})$ scattering in the double-resonance model.[73] The broken curve gives the experimental data, and the continuous curve shows the calculation with allowance for the channels $(0^+, 2^+)$, $(2^+, 2^+)$, $(0_2^+, 0_1^+)$, $(2_2^+, 0_1^+)$, $(4_1^+, 0_1^+)$.

correspondence with the number of channels taken into account. For example, the study of Ref. 73 took into account excitation of ^{12}C to 2^+, 4^+, 0_2^+ states and two-phonon states as a result of two-step processes (the terms of second order in the deformations of the ^{12}C nuclei were also retained in the expansion of the density), and a very rich intermediate structure was obtained. However, for spins greater than 14^+ such a structure is not observed experimentally. The band-crossing model is based essentially on weak channel coupling. The use of the method of strong channel coupling[77] leads to a similar form of the cross sections (Fig. 12) but with smoother resonances. Calculations in the DWBA[78,79] (Fig. 12) lead to a similar picture.

In Ref. 77 it is noted that although the behavior of the elastic element of the $\hat{S}$ matrix differs from that of a model of diffraction type,[62] the inelastic elements have the same characteristic resonance behavior. Despite the fact that the elastic phase shift does not exceed 60°, a fact which serves as the basis for the analysis of possible resonances, the enhancements in the cross section can be described by means of the Breit–Wigner resonance formula. Analysis of the elastic width Γ_{el} for $l \geqslant 10$ shows that $\Gamma_{\text{el}}/\Gamma \sim 0.2$. This indicates a quasimolecular nature of the resonances of the gross structure and gives a semimicroscopic justification for using a parametrization of the amplitude of DWBA type.[62,63]

It was noted in Refs. 70–72 that the elastic cross section at forward angles depends weakly on the employed form of the internuclear potential—adiabatic (without core) or in the sudden-collision approximation (with core)—and there was a discussion of the fact that the cross section of inelastic scattering with angular-momentum transfer Δl is more sensitive to the core, owing to the dominant contribution of the partial wave with $l = J - \Delta l$ due to the lower centrifugal barrier, which masks the core at large l.

It is also necessary to point out the direct genetic connection between the one- and two-step DWBA models (the latter is equivalent to the case of weakly coupled channels) and the phenomenological Austern–Blair model. It was shown in Ref. 78 that in the case of a coupling potential as the derivative of the density the matrix element $\langle u_f^{(-)}(r)|dU(r)/dr|u_i^{(+)}(r)\rangle$ which occurs in the elastic T matrix is proportional to $|u_f^{(-)}(R)u_i^{(+)}(R)|$, since $dU(r)/dr$ has a surface δ-function nature, and in the inelastic case (two-step process) $\sim|u_f^{(-)}(R)G_k^{(+)}(R,R)u_i^{(+)}(R)|$. Thus, the amplitudes are determined by the behavior of the wave function of the relative motion in the surface region.

Here, $u_i^{(+)}$ and $u_f^{(-)}$ are distorted waves, the incoming and the outgoing; G_k^+ is the Green's function of the inelastic channel corresponding to asymptotic behavior of outgoing waves. It is also noted in Ref. 78 that their energy dependence agrees with the behavior of the quantities $d\eta/dl$ in (10) and (11), which occur in the parametrization of the $\hat{S}$ matrix.[62]

This is not surprising, since for below-barrier resonances (this also applies to the near-barrier case) the wave function in the region of a resonance can be represented in factorized form by separating explicitly the energy dependence: $u(r) = A(E)f(r)$, where $f(r)$ is normalized to unity in the interior region $r < R_B$, and $|A(E)|^2 \simeq \Gamma/[(E-E_0)^2+\Gamma^2]$ ($\Gamma \sim e^{-\gamma(E)}$ is the barrier penetrability).

The Green's function in the neighborhood of a resonance can also be represented in pole form: $G_k^+(r,r') = f(r)f(r')/(E_k - E - i\Gamma_k)$.

Thus, the energy dependence of the T matrix is determined by the poles in the entrance and exit (or intermediate) channels and (for a two-step process) has the form

$$|T_{ij}|^2 \simeq \frac{|T_0|^2}{[(E-E_i)^2+\Gamma_i^2][(E-E_j)^2+\Gamma_j^2]}, \qquad (26)$$

which is characteristic of the general physical picture of double excitation that provides the basis of Imanishi's model and the double-resonance model.[70–72] Allowance for a large number of channels and their strong coupling does not change the qualitative conclusions [82] for the inelastic T matrix (an additional shift and broadening are obtained). Neglect of the effects of strong coupling leads to the band-crossing model,[74–76] according to which the intermediate structure is explained by the crossing of rotational bands of the elastic channel and the inelastic channels with excitation of one or both nuclei to states with spin $I(E^*)$, angular momentum of the bands $L = J-I,\ldots, J+I$, and energy $E' = E - E^*$.

Modeling of quasimolecular states in the interpolation approach

In Ref. 82, the general scheme of the interpolation approach was used to calculate the elastic and inelastic cross sections with allowance for the $^{12}C + {}^{12}C^*(2^+)$ channel in the range of energies $E_{cms} = 10$–23 MeV, where the states with $J = 10$ and 12 make the main contribution. Under the simplifying assumptions, the Green's function in the open channels was chosen in the form of a sum of two pole terms, and the nondiagonal potentials $V_{\varkappa\varkappa'}(\rho)$ of the coupling of the channels and of the coupling of the channels with states of compound-nucleus type, $D_\varkappa(\rho)$, were assumed to be localized [$\nu_{\varkappa\varkappa'}\delta(\rho-\rho_\varkappa)$ and $\eta_\varkappa\delta(\rho-\rho_0)$] in the neighborhood of the channel radius $\rho_\varkappa$ and a radius of the order of the compound-nucleus radius ρ_0; the quantities $\nu_{\varkappa\varkappa'}$ and $\eta_\varkappa$ were chosen on the basis of qualitative agreement with experiment. The results show (Fig. 14) that the model correctly reproduces the intermediate structure of the resonances as regards both the cross section and the width of the gross structure. Note that if the coupling with states of compound-nucleus type (doorway states) is switched off, then, naturally, the results of the methods of strong and weak channel coupling are obtained.

Other open channels can be taken into account by introducing two mechanisms: surface absorption iW_0 in the internuclear (diagonal) potential and the possibility of decay from states of compound-nucleus type: Γ_{dW}.

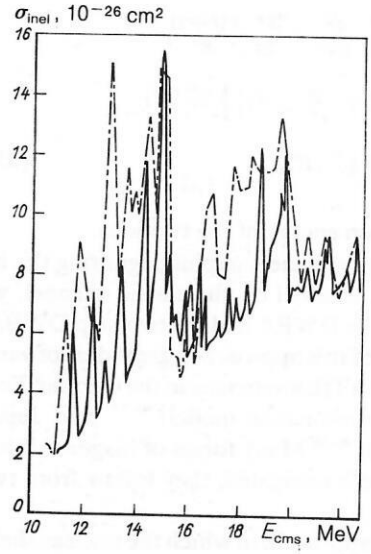

FIG. 14. Cross section of inelastic $^{12}C + {}^{12}C(2^+; 4.43$ MeV) scattering in the interpolation approach. The continuous curve gives the calculation for only $J = 10$ and 12 with allowance for the smooth background from all $J < 10$; the chain curve gives the experimental results.

Calculations in the framework of Feshbach's method

In Refs. 83 and 84, this method was used in a quantitative study of the $^{12}C + {}^{12}C$ elastic cross section and the cross section of the reaction with excitation of ^{12}C to a 2^+ state with $J = 8$–14 and $E_{cms} = 10$–30 MeV, the following simplifying assumptions being made: (1) the diagonal (optical) potential is chosen to be local in the Woods–Saxon form with a variable depth; (2) as doorway states the vibrational 2^+ and 3^- states of one of the ^{12}C nuclei are considered.

Thus, the scheme reduces to a variant of the coupled-channel method. It is pointed out in Ref. 83 that the main contribution to the cross section is made by the amplitudes T_3 and T_4 (8) with T_3 responsible for the envelope of the gross structure. The results agree poorly with experiment as regards both the number of intermediate resonances and the cross section.

In Ref. 85 the same method was used to describe the inelastic cross section with excitation of the 0_2^+ and 3^- states in the same range of energies. The results also deviate appreciably from the experiments.

Fusion in semimicroscopic calculations

In more realistic calculations made using the coupled-channel method and the DWBA, the fusion cross section σ_{fus} is determined as the difference between the absorption cross section and the total inelastic cross section (reaction cross section) and depends on the imaginary part of the ion–ion potential in the elastic and inelastic channels. If there is no imaginary part, $\sigma_{fus} = 0$. It should be noted that σ_{fus} defined in this way also includes the effects of other open channels possessing intrinsic structure, for example, channels with rearrangement, and can serve only as an estimate for its value.

Nevertheless, the results of the calculations made in

Refs. 73–79 reveal an oscillating behavior of σ_{fus} close to the experimentally observed gross structure (see Fig. 8). Moreover, the cross section and the positions of the characteristic maxima and minima are satisfactorily described.

3. ADDITIONAL METHODS OF STUDYING QUASIMOLECULAR STATES

Both the experimental and theoretical investigations made during the last few years indicate that channels with excitation of one or both nuclei to their low-lying states play an important part in the formation of the quasimolecular resonances. As a rule, the excited states of the colliding nuclei have nonzero spins. This makes it possible to measure the populations of the magnetic substates (the alignments), which are sensitive to the nuclear reaction mechanism.

The presence of a clearly expressed rotational nature of the gross structure of the quasimolecular resonances stimulated experiments to measure γ transitions within this band. Such measurements can give new additional information about the wave-function structure of the quasimolecular states.

Population of magnetic substates (alignment)

A new aspect of the study of reaction mechanisms is measurement of the alignment[86,87] of a state with spin I:

$$P_{zz}(\Phi)=\frac{1}{I(2I-1)}\left[\sum_m 3m^2P_m(\Phi)-I(I+1)\right], \quad (27)$$

where $P_m(\Phi)$ is the population of the magnetic state m for momentum transfer ΔL, and Φ is the azimuthal angle between $\mathbf{k}_i$ and $\mathbf{k}_f$. The population is defined as $(d\sigma(\Phi)/d\Omega)_m/[\Sigma_m (d\sigma(\Phi)/d\Omega)_m]$, where

$$\left(\frac{d\sigma(\Phi)}{d\Omega}\right)_m=\frac{2(4\pi^2)^2}{k_i^2}\left|\sum_{JML'M'} i^{J-L'}e^{i(\sigma_J+\sigma_{L'})}\right.$$
$$\left.\times(L'M'\Delta Lm/JM)\,T_{L'J}Y^*_{JM}(\pi/2,\,0)\,Y_{L'M'}(\pi/2,\Phi)\right|^2. \quad (28)$$

The paper of Ref. 86 gives the results of calculations of the alignment for the $^{12}C+{}^{12}C\to{}^{12}C+{}^{12}C(2^+)$ reaction with $\Delta L=2$ in the DWBA, the strong channel coupling scheme, the band-crossing model, and the diffraction model. It is noted that these models do not reproduce well the alignment measured at several angles, particularly the dips between the peaks. In addition, for $E>20$ MeV the alignment tends to the statistical limit $P_{zz}=1/3$ (for identical nuclei with $I=2$ states with $m=\pm2$ and 0 are populated). Attention is also drawn to the need to measure the angular distribution of the alignment, which is very sensitive to the spin–angular-momentum coupling scheme. Measurement of the alignment in a reaction with mutual excitation of the two ^{12}C nuclei to the 2^+ state in the interval of energies 15–35 MeV shows that its behavior is correlated with the peaks in the total cross section for E_{cms} equal to 25 and 30 MeV.[88] It is noted that the main contribution to these resonances is made by the state with $(m_i, m_j)=(2, 2)$, while the main contribution to the regions between the peaks is made by the (2, 0) and (1, 1) states. The (0, 0) substate makes a negligibly small contribution at all energies.

A very promising method for detailed study of the reaction mechanisms is the search for correlations using 4π geometry of γ counters and particle detectors in the scattering plane ("crystal sphere"), when it is possible to measure not only the alignment but also to separate the angular distributions of the products in different final states. The results of measurements for $^{12}C+{}^{12}C$ in the interval of energies $E_{lab}=38.6$–69.4 MeV (Ref. 89) also indicate a distinguished role of the (2, 2) substate (Fig. 15) in the region of the resonances 25.8 MeV ($l_i=14$) and 31.5 MeV ($l_i=16$) due to favorable kinematic conditions—the difference between the centrifugal energies E_l^{Mg} and E_{l-4}^{Mg} is close to twice the excitation energy of the 2^+ state in ^{12}C. From the classical point of view, the configuration of the two deformed ^{12}C nuclei with axes perpendicular to the direction of the relative motion plays the main part in the formation of these resonances. This can serve as an indication of the consistency of the directions of the intrinsic spins with the direction of the orbital angular momenta (alignment) and casts light on the dynamics of the formation of the quasimolecular states.

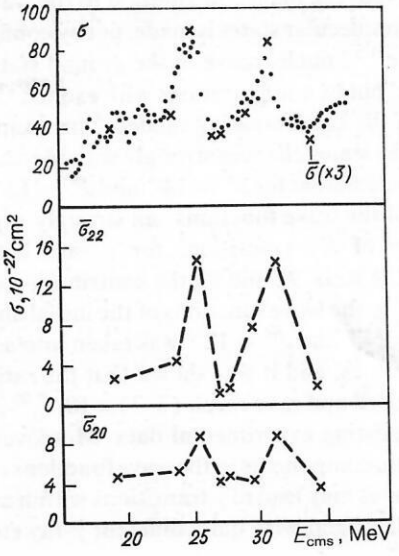

FIG. 15. Decomposition of the total cross section with excitation of both ^{12}C nuclei to the 2^+ (4.43 MeV) state (points) from the populations of the magnetic substates (broken line). The crosses give the positions of the measurements of the populations.

Gamma transitions within a quasimolecular band

In recent years, the attention of investigators has been drawn to the study of $E2$ transitions within the band of quasimolecular states in the ^{12}C–^{12}C system. Estimates show that in the case of a γ transition between the resonances with $J^\pi=14^+$ ($E_{cms}\approx25.8$ MeV) and $J^\pi=12^+$ ($E_{cms}\simeq19.3$ MeV) the ratio of the γ-decay width (Γ_γ) to the total width may reach $\Gamma_\gamma/\Gamma\sim2\times10^{-5}$ (under the simple assumption that the considered resonances have a structure characterized by the rotation of a rotator consisting of two touching ^{12}C nuclei in the ground states).

The papers of Refs. 90 and 91 report detailed experimental investigations that gave an upper bound $\sim8\times10^{-6}$ for the ratio Γ_γ/Γ, i.e., several times less than expected from the theoretical estimates. At the first glance, these results indicate the absence of quasimolecular states at such energies. However, it is here necessary to take into account the important circumstance, which we have noted frequently al-

ready in this review, that the main contribution to the wave functions of the quasimolecular states is made, not by configurations in which the ^{12}C nuclei move in the ground states relative to each other, but by configurations with excited ^{12}C nuclei. Moreover, in the band-crossing model,[74] for example, the structure of the states changes strongly with increasing energy and may be different for $J^\pi = 14^+$ and $J^\pi = 12^+$. Thus, the structure of the wave functions can strongly suppress the probability of $E2$ transitions for γ rays with $E_\gamma \simeq 6.2$ MeV. Thus, in Refs. 92 and 93 the contribution of the inelastic channels to the wave functions of the initial and final states with $J^\pi = 14^+$ and $J^\pi = 12^+$ was taken into account in different schemes, and it was shown that the ratio Γ_γ/Γ is strongly reduced and may reach $(3\text{–}7)\times 10^{-7}$,[93] in agreement with the existing experimental data. Moreover, the presence of inelastic components in the wave functions of the quasimolecular states may lead to γ transitions within an inelastic quasimolecular band with quite different γ-ray energies.[92]

4. CONCLUDING REMARKS. MAIN CONCLUSIONS

The intensive experimental and theoretical investigations of heavy-ion reactions during the last 25 years have made it possible to discover a new class of phenomena—nuclear molecular states or nuclear quasimolecules. The overwhelming majority of the studies in this direction have been based on collisions of two ^{12}C ions, though there have been a number of experiments on other systems, up to $^{56}Ni + {}^{56}Ni$.

The essence of the phenomenon is the existence of a large number of resonances of nonstatic nature (in particular, for the $^{12}C + {}^{12}C$ system with spins 0^+–14^+) that are well correlated in different channels and fit (for the gross structure) on a rotational band corresponding to the moment of inertia of two weakly overlapping nuclei. Although the widths of decay of these resonances to the elastic channel are smaller than the single-particle widths, they are much greater than follows from a statistical model and constitute an appreciable fraction of their total widths.

In recent years, the unique experiments on γ transitions within the quasimolecular band and the measurement of alignment have made it clear that a fundamentally important role in the formation of the quasimolecular states is played by channels with excitation of the ^{12}C nuclei to their low-lying states, particularly with excitation of one or both nuclei to the state 2^+ (4.43 MeV) (this same fact is indicated by the great success of Imanishi's model and its wide use in many subsequent theoretical investigations of quasimolecular phenomena). In this case, the system remains for an appreciable fraction of the time in the form $^{12}C + {}^{12}C^*(2^+)$, and this makes it possible to propose the existence of a quasimolecular band of negative parity of the same structure, which is not forbidden by the condition of identity of the ^{12}C nuclei and has not hitherto been sought. It should be noted that although such a band is in the same region of excitation energies as the well studied quasimolecular band in the $^{12}C + {}^{12}C$ system, it need not be excited by the collision of two ^{12}C nuclei (parity selection rule), and it should be sought in other entrance channels, in particular, in the $^{12}C(^{14}N, d)$ reaction, in which it is possible to have transfer of the ^{12}C nucleus in both the ground state and an excited state.[41]

The next important feature of quasimolecular states that must be taken into account in theoretical investigations and in the construction of corresponding models is the clear manifestation of α-cluster degrees of freedom of light nuclei in the formation of quasimolecular resonances. It is also necessary to point out general features of the excitation spectrum of both α-cluster states and quasimolecular states at high excitation energies—the presence of splitting or several states with the same spin found in the study of high-lying α-cluster states in the ^{16}O, ^{20}Ne, ^{24}Mg nuclei.[94]

One further important feature of the quasimolecular states is the high, on nuclear scales, angular velocity of the system. If one could rotate quasimolecular systems consisting of heavy nuclei with the angular velocity characteristic of the band of quasimolecular states in the ^{24}Mg nucleus, angular momenta of order $200\hbar$–$300\hbar$ would be readily obtainable. For this reason, quasimolecular states could also be of considerable interest for specialists in the field of rapidly rotating nuclei.

From the point of view of traditional nuclear physics, the quasimolecular states can be regarded as extremal shape isomers characterized by rapid rotation and maximal octupole deformation (among the other $^{24}Mg^*$ states, which have predominantly quadrupole deformation).

The foregoing analysis of the theoretical methods and models used to describe the quasimolecular phenomena in the $^{12}C + {}^{12}C$ system show convincingly that we have a qualitatively and quantitatively correct description of the gross structure of the elastic cross sections and individual inelastic cross sections and a qualitative understanding of the part played by the inelastic channels in the formation of the quasimolecular resonances. The theoretical calculations and comparison with the experimental data indicate a strong coupling of the inelastic channels to the elastic channel (allowance for weak coupling of the channels in the band-crossing model) does not make it possible to explain the structure of the inelastic cross section in the channel with excitation of one of the ^{12}C nuclei to the state 0_2^+ (7.65 MeV). However, we are still far from understanding all the physics of this phenomenon, especially the fine structure. A good qualitative picture is provided by Feshbach's approach, the resonating-group method, and the interpolation approach, though it is still not yet entirely clear which states can ensure the fine structure of the quasimolecular resonances. In this sense, study of the alignment and γ transitions within the quasimolecular band will make it possible to "feel" in more detail the structure of the wave functions of the quasimolecular states.

In this review, we have so far considered the quasimolecular phenomena predominantly in the $^{12}C + {}^{12}C$ system and have hardly mentioned other systems. The investigations of the last ten years show convincingly that phenomena of the same class are observed for a large number of colliding ions (and, very importantly, not only α-cluster ions). These phenomena are observed in the systems $^{12}C + {}^{16}O$ (Ref. 95), $^{14}C + {}^{14}C$ (Ref. 9), $^{16}O + {}^{16}O$ (Ref. 8), $^{12}C + {}^{14}N$ (Ref. 96), $^{12}C + {}^{28}Si$, $^{16}O + {}^{28}Si$ (Ref. 97), $^{14}C + {}^{16}O$ (Ref. 98), $^{24}Mg + {}^{24}Mg$, $^{28}Si + {}^{28}Si$ (Ref. 99), and a number of others (Figs. 16 and 17). In particular, the structures found in the $^{28}Si + {}^{28}Si$ system for E_{cms} equal to 53.56 and 59 MeV have well-defined spins J equal to 36, 38, and 40 (Ref. 99). We mention specially once more that the excitation energies

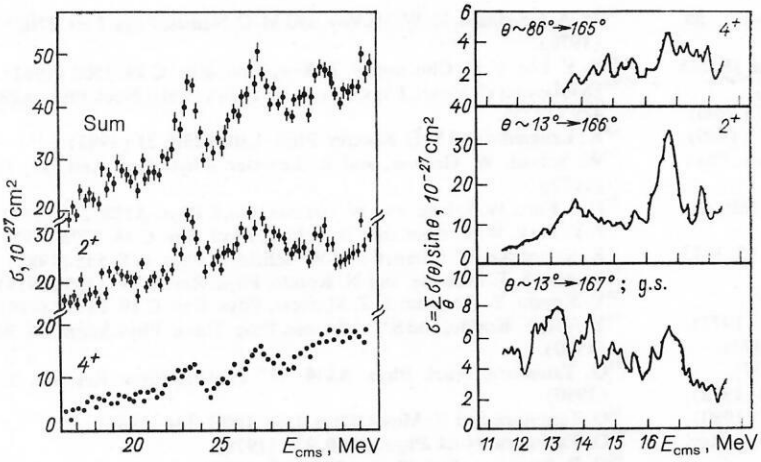

FIG. 16. Excitation function in the $^{14}C(^{16}O,^{12}C)^{18}O^*$ reaction[98] (a) and cross section of the $^{12}C(^{16}O, ^{8}Be)^{20}Ne^*$ reaction summed over the angles[95] (b).

of the $^{56}Ni^*$ compound system at which these structures are observed are of order 70 MeV. The idea of formation of a two-particle nuclear molecule consisting of heavy nuclei, U + U and U + Cm,[100] makes it possible to understand well the narrow peak in the spectrum of positrons observed in collisions of these nuclei with energies of several mega-electron-volts per nucleon.[101] The same idea, known by the name of double nuclear system, proved to be extremely helpful in the study of deep inelastic reactions, fusion, and fission in the interaction of heavy ions in the complete range of mass numbers and energies $\lesssim 10$ MeV/nucleon.[102]

The Pauli principle plays a fundamental role both in the mechanism of formation of long-lived quasimolecular states and in the phenomenon of anomalous backward scattering.[103,104] Consistent allowance for the Pauli principle in microscopic calculations of nucleus–nucleus potentials[14] leads to the appearance in this potential of a repulsive core that hinders the fusion of two nuclei and leads to the formation of a long-lived surface quasimolecular state or an α-cluster state. Simultaneously, the wave reflected from the core leads to an increase in the elastic scattering at backward angles (anomalous backward scattering).

There is much interest in studying not only two-particle nuclear quasimolecules but also quasimolecules consisting of a larger number of clusters or nuclei. The first steps in the theoretical study of such structures were taken in Ref. 105, while in Refs. 40 and 106 a model of three-particle nuclear molecules was constructed and the spectra and wave functions of a number of such systems were calculated. Recent experimental investigations of highly excited ^{20}Ne states in the $^{12}C(^{12}C, \alpha)^{20}Ne^*$ reaction,[107,108] and also of the ^{28}Si state 14^+ ($E^* \simeq 36.5$ MeV), which is excited in the $^{12}C + ^{16}O$ reaction,[109] give serious indications of a large admixture of a three-particle (^{12}C–α–^{12}C) configuration for the ^{28}Si state and the presence of configurations of (α–^{12}C–α) structure in the spectrum of the ^{20}Ne nucleus.[110] The question of many-particle nuclear quasimolecules is also interesting from the point of view of microscopic explanation of fragmentation mechanisms that are being intensively investigated in heavy-ion reactions.

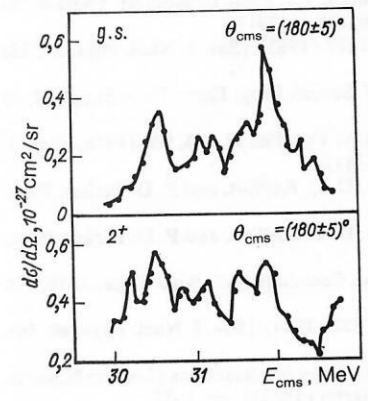

FIG. 17. Excitation functions for elastic and inelastic scattering[97] in the $^{28}Si(^{16}O, ^{16}O)^{28}Si(2^+)$ reaction.

Thus, nuclear molecular reactions can be regarded as a new class of nuclear phenomena and one that is fairly common for the whole of heavy-ion physics. The study of nuclear molecular states, both experimental and theoretical, is for this reason of great general interest and casts light on the mechanisms of nuclear reactions and the structure of nuclei at high excitation energies and high spins.

The huge volume of experimental and theoretical information so far accumulated on quasimolecular phenomena in the region of light and medium nuclei makes it impossible to eliminate all aspects and nuances of these phenomena in the framework of a single paper. For a more detailed knowledge of this interesting field of heavy-ion physics the reader can consult the reviews of Refs. 5, 53, 87, 99, 100, 111, and 112, which also contain detailed references to original studies.

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