Gluon jets in hadron processes and confinement

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The confinement phenomena should impose certain restrictions on the gluon radiation length of quarks. The validity of this hypothesis can be verified by observing inelastic hadron–hadron processes in which an appreciable fraction of secondary particles are emitted at nearly equal polar angles (ring events). Electromagnetic analogs of this effect are discussed. Experimental results supporting the hypothesis are discussed.

INTRODUCTION

The theoretical description of strong interactions has undergone significant changes during the last decade. Instead of phenomenological models operating with hadrons as elementary objects, the center of the stage is now occupied by quantum chromodynamics (QCD), which treats hadrons as composite objects containing more fundamental constituents—quarks and gluons, which are characterized by a new quantum number, color. These constituents interact by exchanging color, i.e., by "color forces." Hadrons are colorless objects, and therefore their interactions (strong!) are merely an "echo" of the color interactions of the quarks and gluons, as, for example, the interaction of a dipole in electrodynamics is due to the mutually screened electromagnetic interactions of the charged objects (particle and antiparticle) that form it.

The analogy with electromagnetism goes much further than this simple example. First, the static properties (spin, mass, etc.) of quarks and gluons are similar to the corresponding characteristics of electrons and photons. Second, the law of interaction of two quarks at short distances, i.e., at large momentum transfers, is remarkably similar to the Coulomb law of interaction of two electric charges. In fact, it only differs from the Coulomb interaction in that the coupling "constant" which appears in place of the fine-structure constant decreases logarithmically with decreasing distance (this is the property of "asymptotic freedom").

However, there are more important differences. In the first place, electrons and photons can be directly observed, whereas it has not been possible to observe quarks and gluons in the free state. It is therefore assumed that they are confined by the color forces within hadrons (confinement), manifesting themselves as quasifree only at short distances less than the hadron dimensions. It has not yet been possible to prove the confinement property in the framework of QCD. It may be due to the fundamental difference between gluons and photons—photons are electrically neutral, whereas the gluons possess a color charge and can interact directly with one another (gluon self-interaction).

In this situation, in which quantum theory does not give unambiguous answers about the properties of interactions at distances of the order of the hadron dimensions, it is natural to attempt to model the confinement phenomenon in the framework of classical radiation theory and test experimentally the viability of such models. The following sections of the review are devoted to this.

At the same time, quantum theory is successfully used at short distances.

The similarity between quarks and electrons, on the one hand, and gluons and photons, on the other, is widely used by theoreticians in quantum-field calculations of the properties of the so-called hard processes characterized by large momentum transfers, in which asymptotic freedom is manifested and therefore (the coupling constant being small) perturbation theory can be used. Experimentally, hard processes are identified by the observation of hadron jets initiated by quarks or gluons, i.e., groups of hadrons emitted along a common axis determined by the direction of motion of the primary parton.

Only the quark–gluon stage of the process, in which quarks and gluons evolve, being transformed mutually and exchanging fairly large momenta, is amenable to a QCD calculation. The color currents of the quarks and gluons are effectively treated as unscreened in complete analogy with electromagnetic currents (apart from the gluon self-interaction, which is taken into account). However, hard processes test the hypothesis of unscreened color currents only at very short distances, of the order of the reciprocal momentum transfers in the given process. Nevertheless, it is important that the hypothesis of unscreened currents is completely justified at these distances, i.e., color currents do indeed exist. The final stage of hadronization, in which quarks and gluons with small relative momenta are combined into hadrons, is not amenable to consistent description in the framework of QCD because of the importance of the confinement effect in this stage. It is therefore necessary to introduce here phenomenological models capable of taking into account this effect and corresponding to the experimental data on the properties of the secondary hadrons.

One will expect the confinement effect to be most clearly manifested in the difference between the dimensions of the regions of space in which the electromagnetic and color currents act. It must have a particularly strong effect on the hadronic processes in which the formation regions are appreciably smaller than the corresponding longitudinal radiation formation lengths in the analogous electromagnetic processes.

For example, it is well known that photon bremsstrahlung of frequency ω by a charged particle of mass m and high energy E is formed over an appreciable length equal to

$$l_f \sim \omega^{-1}\gamma^2 \equiv \omega^{-1}(E/m)^2. \tag{1}$$

It is physically obvious that color currents cannot be manifested unscreened over such large distances. Confinement must significantly restrict the region of manifestation of the

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color currents compared with the case of electromagnetism. The decrease in the radiation formation length must, naturally, lead to the suppression of the low frequencies and increase the emission angle. Since at the same time the vector nature of the photons and gluons suppresses emission at small angles, the combined effect of these two factors must tend to produce a ring structure of the radiation of gluon jets at fairly large polar angles. Experimentally, one must observe inelastic hadron–hadron interactions in which a significant group of secondary hadrons fills a ring in the plane perpendicular to the collision axis of the initial hadrons, the ring being situated at an appreciable distance from this axis.

The aim of this review is to demonstrate the possibility of such an effect in hadron physics, point out ways in which it can be experimentally detected, discuss the physical information that can be extracted from the corresponding distributions, and consider the experimental data indicating a possible manifestation of a quark deconfinement effect over a restricted section of the path (so-called ring events).

The physical picture that we shall have in mind is as follows. The partons (quarks and gluons) of the initial hadron, encountering with high energy another hadron or nucleus, cease to screen each other when they arrive in the target, within which uncompensated color currents are formed, these being capable, over a certain length limited by the confinement phenomenon, of emitting gluons, which are then transformed into hadron jets. The main consequence of the hypothesis that the radiation length is limited will be the prediction of inelastic hadronic processes in which an appreciable fraction of the secondary particles are emitted at the same (and fairly large) polar angle (ring events).

Such a process is analogous to bremsstrahlung of a charged particle over a path of restricted length. In hadronic physics it is not the dominant process—the majority of processes take place through the direct interaction of partons and the production of new quarks and gluons. However, the characteristic angular distribution of the secondary hadrons will permit identification of such a process experimentally, and corresponding theoretical calculations show that valuable information can be obtained from analysis of these processes.

In Sec. 1, we consider a definite model of emission by a vector current manifested only over a finite length ("step" current). We also show that Cherenkov emission occurs if a medium is present. The possible properties of the hadronic medium are discussed. The characteristic properties of the gluon emission in the considered case are demonstrated, namely, the appearance of narrow hadronic rings at large angles, growth in the radiation intensity with increasing energy, and an influence of the refractive index of the medium.

Section 2 gives electromagnetic analogs of the step current manifested in definite physical situations.

In Sec. 3, we consider what information can be extracted from experiments by analyzing the characteristic ring events, and also what experimental data about ring events exist at the present time.

In the conclusions, we summarize the results of the study of the model and its comparison with experiment.

The Appendix contains a discussion of a generalized model that takes into account the spatial extension of the regions of manifestation of the color current and its "color screening." We show that the dimensions of these regions can be ignored if they are less than the range of action of the current by at least an order of magnitude.

1. EMISSION OF GLUON JETS AND THE CONFINEMENT EFFECT

Basic hypothesis

As already discussed in the Introduction, we shall use the similarity between the properties of electrons and quarks, and also of photons and gluons, in calculating the emission processes, making the assumption that the main difference between them is manifested in a very considerable reduction through the confinement effect of the length over which the gluon emission of the quarks is formed compared with the length available for the corresponding photon emission of electrons. Since there are no quarks and gluons in the free state in nature, the color currents are, if at all, manifested only over restricted lengths, in contrast to the electromagnetic currents of free charges, which can be displaced over arbitrary distances.

A clear model of the manifestation of color currents can be obtained by considering, for example, the inelastic interaction of a pion with a hadron or with a massive nucleus in the following manner. Color currents are present in neither the initial nor the final state—they are screened within the initial and final colorless hadrons. But the interaction process within the target hadron or nucleus can be regarded as a consequence of deconfinement of the quark and antiquark in the pion, which interact incoherently with the target partons, each of the partners "forgetting" each other and interacting independently, emitting gluons. This is particularly justified when the radiation wavelength is significantly less than the interquark distance in the hadron. It is this case in which we shall be interested.

We can then forget for a moment one of the pion's quarks and consider the problem of emission of gluons by a color current that appears only over an interval of length l within the nucleus; l is a parameter that can be determined subsequently from experimental data. Its physical meaning can be most readily understood by regarding the hadron or nucleus as a quark–gluon medium, a target of finite extension, through which the quark and antiquark of the incident pion pass. The appearance of the color current corresponds to deconfinement of the quark when the pion enters the target, and its disappearance is due either to an inelastic interaction of the quark in the target or to its screening in a secondary hadron emitted from the target. In such an interpretation, l must be of the order of the target dimensions or less. It is clear that, moving in such a medium, the quark may, besides the main inelastic interaction, emit over the length l gluons under the influence of the medium.

Basic formulas

We shall study the properties of such a process. In the classical treatment, we restrict ourselves to radiation frequencies ω much less than the primary energy $E(\omega \ll E)$, when the motion of the initial quark can be assumed given and recoil can be ignored. In this sense, the process can be regarded as "soft." However, the gluon energy must be sufficiently high ($\omega \gg m_\pi$, where m_π is the pion mass) for one to be able to speak subsequently of gluon jets and a fairly

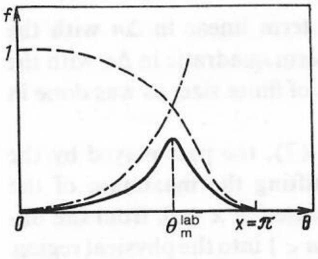

FIG. 1. Schematic representation of the angular distribution of gluon jets in accordance with Eq. (6). The chain curve shows the behavior of the factor $\sin^2\theta$ in (6) due to the vector nature of the gluon; the broken curve, the behavior of the factor $\sin^2x/x^2$ in (6) due to the restricted radiation length; the continuous curve, the total angular distribution obtained by multiplying these two factors. It has a well-defined peak at the angle θ_m^{lab} (8).

"hard" nature of the process, so that perturbation theory is valid in the quantum approach.

The analogous electromagnetic problem of the emission of photons by an electron that, after instantaneous acceleration, traverses a path of length l in a medium with refractive index n and is then instantaneously stopped was solved by Tamm in 1939.[1] Here, we use a somewhat different device, namely, the well-known[2] transition to the classical limit for the treatment of soft radiation ($m_\pi \ll \omega \ll E$) in quantum electrodynamics, when the S matrix can be written in the form

$$S=\exp\left\{-\frac{1}{2}\int d^4x' \int d^4x'' j_\mu(x') D^c_{\mu\nu}(x'-x'') j_\nu(x'')\right\} \times N\exp\left\{\mathrm{i}\int j_\mu(x) A_\mu(x)\, d^4x\right\}, \quad (2)$$

where in the case of QCD j_μ is the quark color current, $D^c_{\mu\nu}$ is the gluon propagator, and A_μ is the vector potential. Following the standard method,[2] we obtain the mean number of emitted (with $\omega \ll E$) gluons:

$$\bar{n}_g=\int |j^\perp_\mu(k)|^2 \frac{d^3k}{2\omega(2\pi)^3} \quad (3)$$

or the invariant inclusive cross section of gluon production[3,4]:

$$\frac{\omega}{\sigma}\frac{d^3\sigma}{d^3k}=\frac{|j^\perp_\mu(k)|^2}{2(2\pi)^3}=\frac{\sin^2\theta\,|j_\mu(k)|^2}{2(2\pi)^3}, \quad (4)$$

where

$$j_\mu(k)=\int j_\mu(x)\,\mathrm{e}^{-\mathrm{i}kx} d^4x. \quad (5)$$

The symbol $\perp$ indicates that only the components transverse with respect to the direction of emission of the radiated gluon contribute; θ is the angle of emission of the gluon relative to the current vector j.

In accordance with the very simple model explained above, the current j is chosen in the form of a "step" of length l, i.e., it is a product of two θ functions, which bound its action above and below.[1)] The Fourier transform of such a function can be readily calculated and is well known, for example, from the theory of Fraunhofer diffraction by a slit of finite size.[2)] The inclusive cross section for the total radiation then has in accordance with (4) and (5) the form[3,4]

$$\frac{\omega}{\sigma}\frac{d^3\sigma}{d^3k}=\frac{\alpha_s C_F l^2}{4\pi^2}\sin^2\theta\,\frac{\sin^2 x}{x^2}, \quad (6)$$

where

$$x=\omega l(1-\beta n\cos\theta)/2. \quad (7)$$

Here, α_s is the chromodynamical running coupling constant, $C_F=4/3$, θ and ω are the angle and energy of the emitted gluon in the laboratory coordinate system, $\beta=v/c$, and n is the refractive index of the gluon within the nucleus. This formula can, naturally, be directly obtained from the electromagnetic expressions (7.6)–(7.9) of Ref. 1 by making there the substitution $\alpha \to \alpha_s C_F$.

Consequences

Thus, the expressions (6) and (7) are the basis of the entire following treatment, and we therefore study them in more detail.

The structure (6) is entirely clear. The factor $\sin^2\theta$ reflects the vector nature of the gluon. It suppresses small emission angles. The factor $\sin^2x/x^2$ is due to the choice of the current in the "step" form, is analogous to the sharp slit in the study of Fraunhofer diffraction, and allows emission angles and frequencies up to values determined from the condition $x\approx\pi$.[3)] For high initial energies ($\beta\approx 1$) and fixed radiation frequency $\omega \gg l^{-1}$, and setting for the moment $n=1$ (see below), we find that the inclusive cross section has a peak at an angle in the laboratory system given by

$$\theta_m^{lab} \approx (2\pi/\omega l)^{1/2}. \quad (8)$$

We recall that a secondary relativistic particle produced in the interaction of two particles of mass m at energy E and angle $(2m/E)^{1/2}$ in the laboratory system is emitted at angle $\pi/2$ in their center-of-mass system. From this it is clear that the angle determined by the condition (8) corresponds to fairly large angles in the center-of-mass system. Even if we fix frequencies proportional to the primary energy, for fixed length l the angle θ_m^{lab} (8) decreases only as $E^{-1/2}$ in the laboratory system. This means that most of the gluons will be emitted at such a polar angle, forming a ring in the target diagram.[4)] The width of this ring can also be readily estimated in accordance with Eq. (6) and is comparatively small on the scale of pseudorapidities [$\eta_c=-\log\tan(\theta_c/2)$, where θ_c is the angle in the center-of-mass system]:

$$\Delta\eta \sim 0.5. \quad (9)$$

Figure 1 shows graphically the general structure of Eq. (6) and the part played by the individual factors.

The number of emitted gluons is obtained by integrating the expression (6) over the angles and frequencies [we here take the integral up to $\omega\sim E$; although at such frequencies Eq. (6) is incorrect, the error is small, since the dependences are logarithmic]:

$$\bar{n}_g \approx \frac{\alpha_s C_F}{\pi}\ln^2 El. \quad (10)$$

At high energies, the number of gluon jets is sufficiently large for a ring of hadrons to be actually formed in the target diagram.[5)] The distribution with respect to the number of jets must be a Poisson distribution.

It must be emphasized immediately that the expressions (6)–(10) are to be regarded as model expressions, valid only for the simplest estimates. A change of the deconfinement law to one smoother than a step can lead to replacement of the factor $\sin^2x/x^2$ by others and to a change in the numerical coefficient in Eq. (8). Allowance for the spherical shape of the target when averaging over the impact parameters does not strongly affect the result. The restriction of the integration over the frequencies to $\omega \ll E$ changes the argument of the logarithm in Eq. (10). But none of these modifications are fundamental for order-of-magnitude estimates of the basic parameters, and the step-current model (which seems at the first glance artificial) also proves itself for the study of a number of exactly solvable electrodynamical problems (see the following section).

The most important qualitative conclusions are: 1) the increase in the angle at which the radiation is maximal; 2) the narrowness of the maximum, so that one can speak of a ring structure of the target diagram of inelastic events at large angles in the center-of-mass system; 3) the significant change in the threshold of the effect, which is now determined by the condition $x \sim \pi$; 4) the hardening of the radiation energy spectrum. For example, integrating Eq. (6) over the angles, we obtain the spectrum (for $\omega l\,|1-\beta n| \ll 1$)

$$dN = \frac{2\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\left[\ln(\omega l) - \mathrm{ci}(2\omega l) + \frac{\sin 2\omega l}{2\omega l} - 1\right], \quad (11)$$

which at low frequencies does not have an infrared singularity (it is of the type $\omega d\omega$) and at high frequencies behaves as $\ln \omega d\omega/\omega$; but, as we shall discuss below,[6)] this behavior can again be changed by replacement of the abrupt law of deconfinement by a very smooth one [in this connection, there may also be a change in Eq. (10), which was obtained by integrating the law (11) over the frequencies].

When Eqs. (6) and (7) are used, it is easy to obtain the distributions, say, with respect to the angles and transverse energies $E_{\rm tr}$, frequently used by experimentalists, and also with respect to other variables. For example, it readily follows from the condition (8) that the emission will be maximal for jets satisfying the condition $E_{\rm tr}\theta l = 2\pi$. Here, we shall not describe these modifications in detail.

The part played by the refractive index

We now discuss the part played by the refractive index n, which occurs in Eqs. (6) and (7).[3,4] It can be seen that for finite l, in both QCD and QED, it is not possible to distinguish strictly bremsstrahlung, Cherenkov radiation, and transition radiation. The expressions (6) and (7) describe them all together.

There are two limiting cases in which a separation can be made—for an infinitely thick target and for a rarefied target. In the first case, $l\to\infty$ and

$$l\,\frac{\sin^2 x}{x^2} \to \frac{2\pi}{\omega}\,\delta(1-\beta n\cos\theta), \quad (12)$$

i.e., there is no bremsstrahlung or transition radiation, and there remains only Cherenkov radiation[5,6] at the definite angle

$$\cos\theta = 1/\beta n, \quad (13)$$

which was already well known from the work of Tamm and Frank,[7] who first derived Eq. (13). Accordingly, we can also readily obtain the well-known formulas for the intensity of this radiation. It must be emphasized that the real condition for validity of the limit is

$$\omega l\,(n-1) \gg 1, \quad (14)$$

which is usually satisfied at optical frequencies for real media (see, however, Example 4 in Sec. 3).

In the second case, for $n - 1 \ll 1$, the expressions (6) and (7) can be expanded in series with respect to $\Delta n \equiv n-1$ and the term that does not depend on Δn can be associated with the bremsstrahlung, the term linear in Δn with the Cherenkov radiation, and the term quadratic in Δn with the transition radiation in a target of finite size, as was done in Refs. 3 and 4.

As follows from (6) and (7), the part played by the refractive index reduces to shifting the maximum of the function $\sin^2 x/x^2$, which is situated at $x=0$, from the unphysical region of angles with $\beta n<1$ into the physical region with $\beta n>1$, this explaining why there is a threshold of Cherenkov emission in an infinitely extended target. For a finite target length [see the condition (14)] there is, in general, no such threshold, and this is a natural result, since the main part is played by the emission from the ends of the interval (as noted above, the nominal threshold now lies at $x\sim\pi$).

It is readily seen that the width of the angular distribution in (6) depends on the parameter $\omega l\Delta n$, which determines the width of the function $\sin^2 x/x^2$, and also on the part played by the factor $\sin^2\theta$ within this region of angles. In the case $l\to\infty$ [see (14)], as already noted, the first function reduces to a δ function, and $\sin^2\theta$ determines the factor $1-\beta^{-2}n^{-2}$ in the radiation intensity, i.e., the proportionality of the Cherenkov emission to Δn when $\Delta n \ll 1$.

We shall give arguments below which indicate that the interaction of hadrons and nuclei in which we are interested is characterized by a small value of the required parameter,

$$\omega l\,\Delta n \ll 1, \quad (15)$$

and therefore the function $\sin^2 x/x^2$ is fairly broad, and in conjunction with it the effect of the factor $\sin^2\theta$ leads to a maximum of the radiation at the angles determined by the condition (8).

Of course, a small uncertainty in the estimate of the parameter $\omega l\Delta n$ resides in the concept of the hadronic refractive index. Purely theoretically, to determine the refractive index, one should calculate the dispersion curve for passage of a gluon through the quark–gluon medium. Such calculations at finite temperature T have been made in the chiral limit of QCD, in which the quark mass is ignored.[8] However, the analogous calculations made for an ordinary plasma led to the conclusion that there is no Cherenkov radiation for the passage of an electron through a plasma, a result in disagreement with experiment, and it was only allowance for the electron mass that restored agreement between theory and experiment.

Therefore, we shall not rely on the results of Ref. 8 but use a phenomenological approach based on analogies and certain assumptions. In the electrodynamics of continuous media, the refractive index $n(\omega)$ of a photon of high frequency ω is related to the amplitude $F(\omega)$ of its forward elastic scattering by particles of the medium by[9]

$$n(\omega) = 1 + \frac{2\pi N}{\omega^2}F(\omega). \quad (16)$$

Here, N is the density of the scatterers (inhomogeneities) of the medium, and the amplitude $F(\omega)$ is normalized by the optical theorem:

$$\mathrm{Im}\,F(\omega) = \frac{\omega}{4\pi}\sigma(\omega), \quad (17)$$

where $\sigma(\omega)$ is the total cross section for interaction of the radiation with the particles of the medium.

In the case of high-frequency photons, the real part of the Thomson elastic scattering amplitude is $-e^2/m$ and for

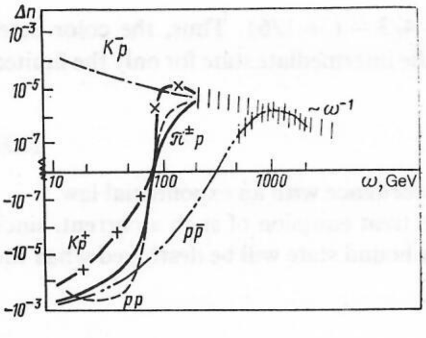

FIG. 2. Deviation from unity of the refractive index of the hadronic medium. In accordance with Eq. (19), it is small and positive at sufficiently high energies if one substitutes in (19) the experimental data on the total cross sections and ratios of the real to the imaginary parts of the forward elastic scattering amplitude for all studied hadronic processes (identified next to the corresponding curves).

permittivity $\varepsilon \equiv n^2$ we obtain the well-known formula

$$\varepsilon = 1 - 4\pi N e^2/(m\omega^2) = 1 - \omega_0^2/\omega^2, \qquad (18)$$

where ω_0 is the Langmuir (plasma) frequency.

In the hadronic case in which we are interested, we rewrite Eq. (16) in the form[3,4]

$$n(\omega) = 1 + \frac{N\sigma(\omega)}{2\omega}[\rho(\omega) + i] \qquad (19)$$

where $\rho(\omega) = \mathrm{Re}\,F(\omega)/\mathrm{Im}\,F(\omega)$, and n in Eq. (7) is simply $\mathrm{Re}\,n(\omega) = n$. The imaginary part $\mathrm{Im}\,n(\omega)$ of the refractive index indicates the presence of absorption in the medium.

Naturally, for a gluon moving in a nuclear-active medium we do not know any of the quantities in (19). We therefore attempt to estimate the right-hand sides of (19), substituting in them experimentally known data on hadronic interactions (and also dispersion-relation calculations that describe them). It is found that all hadronic reactions lead to very similar results at very high energies (above 10 GeV) for the value determined by (19) (Fig. 2), and, moreover, estimates show that the additional term in (19), $\Delta n \equiv n - 1$ is very small:

$$\Delta n \approx \frac{3m_\pi^3 \sigma(\omega)\rho(\omega)}{8\pi\omega} \leqslant 10^{-4} \qquad (20)$$

(for the numerical estimate, we took the concrete value $N \approx 3m_\pi^3/4\pi$, where m_π is the pion mass, but we emphasize that a change in N by even an order of magnitude does not affect the conclusion that Δn is small).

This universality in the behavior of $n(\omega)$ makes it possible to assume that for gluons too Δn is small, and the parameter $\omega l \Delta n$ is small as well ($\omega l \Delta n \sim 0.1\rho \lesssim 10^{-2}$, for $l \sim m_\pi^{-1}$, $\sigma \sim m_\pi^{-2}$). Thus, the properties of the quark–gluon medium must be weakly manifested in Eqs. (6) and (7) for a small deviation of n from unity, and the position of the radiation maximum hardly depends on n [as can also be seen from Eq. (8)].

It also follows from (19) that the absorption may be ignored to lengths $l \sim 10m_\pi^{-1}$ (Ref. 4), and Δn becomes positive at energies ω exceeding hundreds of giga-electron-volts, since it is only at such energies that the real parts of all the forward hadron scattering amplitudes become positive. This fixes the threshold of gluon emission of Cherenkov type determined as the term linear in Δn in the expansion of Eq. (6) (see the discussion above).

Although the conclusion that the quark–gluon medium has an appreciable transparency for the passage of high-energy gluons appears, at the first glance, unexpected, it is worth mentioning that it is actually confirmed by models of hard processes too. In fact, it is assumed in them that the scattered colored partons transmit all kinematic characteristics of the secondary hadron jets. Neither the color-screening process nor the possible influence of the hadronic medium (scattering or absorption) affects the passage of these partons. In our case, the produced gluon is also fairly hard ($\omega \gg m_\pi$) and makes itself a way similarly.

Ring structure

The ring structure of the target diagram of the events also seems unusual. We have become used to such a structure in electrodynamics for Cherenkov radiation and pay little attention to the fact that it is *typical* of all photon emission processes. The point is that for the majority of processes it is manifested at very small angles $\theta \sim m/E$, which in practice cannot be measured at high energies, and one therefore says that the radiation is "directed forward." In the following section, we shall also consider electrodynamical examples in which the radiation appears at fairly large angles. Here, we should like to discuss the physical reason why certain polar angles may be distinguished. The reason is to some extent a reflection of the uncertainty relation, the transverse momentum (and angle) of the radiation increasing when the length over which the emitting current acts is decreased. The expressions (6) and (7) clearly reflect this. The fundamental quantity here is the length

$$l_f = \omega^{-1}(1 - \beta n \cos\theta)^{-1}. \qquad (21)$$

Equation (1) is obtained from (21) for $\theta \sim m/E$ and $n = 1$.

As already mentioned, such a quantity appeared in Tamm's study,[1] in which a criterion for transition to the limiting case of an infinitely extended medium was determined by comparison with it. It was widely used in Frank's study of Ref. 10. The physical meaning of this quantity is that it is the length of the Fresnel zone, i.e., the region of coherent emission of the current. Ter-Mikaelyan[11,12] first drew attention to its singular properties and important role for emission processes in crystals at high energies. Landau and Pomeranchuk showed[13] that the properties of radiation in an amorphous medium at high energies are also determined by such a length. Subsequently, it was used in many studies (see the review of Ref. 14, and also Refs. 15–17).

Its appearance in the problem of gluon emission too is natural. When the available radiation length is limited, one can only form radiation with frequencies and at angles for which the length of the Fresnel zone does not exceed the available length ($l_f \lesssim l$), i.e., in this sense the length of the Fresnel zone is the radiation formation length. It can be seen from (21) that for a short length only fairly hard radiation at fairly large angles can be emitted. The expressions (6) and (7) describe this effect quantitatively. Equation (8), which follows from them, shows that for the large lengths $l \sim \omega^{-1}(E/m)^2$ indicated in (1) the emission angles will be small, of order m/E, whereas for finite lengths $l \sim m_\pi^{-1}$ these angles are large, and thus the ring structure of the events is more readily observed.

We emphasize that it is the formation of a ring structure due to the predominant emission of gluon jets at the same polar angle that is the most characteristic feature of the ef-

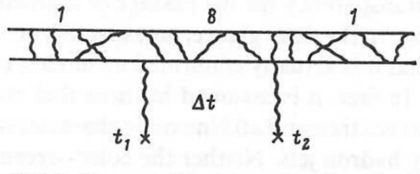

FIG. 3. Diagram of the interaction of quarkonium with the vacuum gluon condensate. The wavy lines are gluons, and the straight lines are quarks. The numbers 1 and 8 identify color singlet and octet quarkonium states.

fect. The distribution with respect to frequencies and, hence, with respect to the transverse momenta depends much more sensitively on the assumptions about the manner in which the color current is screened.[7)] Nevertheless, low frequencies and small transverse momenta will be suppressed, since introduction of a finite length eliminates the infrared singularities.

The numerical coefficient in (8) also depends on the way in which the color screening process occurs. Therefore, Eq. (8) can be used, say, to determine l only in order of magnitude. In the initial stage, this will be sufficient if it is found (see Sec. 3) that this length differs by many orders of magnitude from the ordinary formation length (1) of electromagnetic radiation. It is this aim that we have for the moment.

2. ELECTROMAGNETIC RADIATION FROM FINITE PATH INTERVALS

In electrodynamics there are numerous examples when it is necessary to consider the emission of photons by a current that acts over some limited length. They all confirm the qualitative conclusions about the properties of the radiation drawn in the previous section. At the same time, they demonstrate that the model of emission by a current over a restricted path interval is perfectly viable and realizable in practice. In QCD, the number of such examples is much smaller and, in addition, their practical value still requires experimental confirmation. Nevertheless, for completeness of the picture, we give first some examples with color currents.

Quarkonium

As a first example of manifestation of a color object over a finite length, we consider the interaction of quarkonium with the vacuum gluon condensate.[18] In the lowest approximation, it is described by the diagram in Fig. 3. The wavy lines of the gluons exchanged between the quarks (straight lines) show the presence of a constant interaction of the quarks in quarkonium. In the initial and final states there is ordinary quarkonium, in a color-singlet state, and its potential at short distances is given by $-4\alpha_s/3R$, where R is the distance between the quarks. At the same time, in the intermediate state after interaction with one of the external gluons the quarkonium must go over into a color-octet state, in which the interaction potential is $+\alpha_s/6R$. This color-octet state is again transformed into a singlet state by interaction with another gluon. If the emission times of the two gluons are denoted by t_1 and t_2, respectively, the contribution of the intermediate state to the amplitude of the Wilson loop at short distances will be proportional to the factor[19]

$$\exp\left[-(t_2-t_1)\,\frac{3\alpha_s}{2R}\right], \tag{22}$$

where $-3/2=-4/3-(+1/6)$. Thus, the color octet current appears in the intermediate state for only the limited time

$$\Delta t\sim 2R/3\alpha_s, \tag{23}$$

being damped in accordance with an exponential law.

It is difficult to treat emission of such a current, since the coherence of the bound state will be destroyed when the emission occurs.

Jets

A second example was proposed in Ref. 20 and in spirit is very close to what was considered in Sec. 1. There is a discussion of the possibility of emission of gluons by color currents of a quark pair produced in electron–positron annihilation when allowance is made for the possible limitation of the region in which the color currents act. The linear growth of the interaction potential between quarks suggested[20] the idea of screening of the color currents at distances proportional to the energy of the quarks. Therefore, gluon emission takes place at small angles of order $m/(\omega E)^{1/2}$. Even emission at such small angles can increase the mean transverse momenta. Unfortunately, there are several factors which imitate this phenomenon, for example, additional screening of the emitted color currents moving at small angles relative to each other, etc.

Neutral jets

In principle, the hypothesis of a limited length over which a color current acts can be tested experimentally by using the fact that quarks always carry an electric charge, whereas hadrons can be neutral. Therefore, when an electrically neutral hadron is produced the screening lengths of the color and electric currents must be the same. If we choose, for example, the electron–positron annihilation channel with production of two neutral jets (or pions) accompanying a bremsstrahlung photon, we may suppose that the photon was emitted by one of the produced quarks before the screening of this quark occurred. When the neutral jet or pion is formed, the screenings of the color and electric charges take place simultaneously, and therefore the angular distribution of the photons in such a process must be described by the expressions (6) and (7) (with $\alpha_s C_F\to\alpha$), and from its characteristic properties the length l in (6) and (7) can be determined.

In practice, for a process with two neutral pions we must observe events with the production of five photons, four of which are emitted pairwise in opposite directions, and the fifth at a comparatively large angle to them. If the radiation length is limited, then in the process of neutral-jet production the distributions with respect to the thrust and the photon emission angle are changed.[50]

Processes with production of neutral particles have not as yet been studied experimentally in detail from this point of view.

We now turn directly to electromagnetic analogs of the effect. Here, the situation is much clearer and more amenable to experimental verification. Since the number of possible examples is here very large, and for our purposes they have only an illustrative nature, we restrict ourselves to an exposition of them at a descriptive level, giving only a minimal number of formulas and referring the interested reader to the original studies.

Cherenkov radiation in thin plates

The radiation of electrons passing through a thin plate is one of the clearest examples of the effect of electromagnetic currents acting over a finite path length. The qualitative features of this phenomenon are readily explained by the formulas obtained for the first time by Tamm[1] and developed by Frank for this case.[21] A detailed quantitative theory was given by Pafomov.[22] Besides the Cherenkov radiation within the plate, a part is also played by emission in the vacuum before and after the plate, reflection and refraction of light on the boundaries of the plate, and multiple scattering of the electron within it. Because they are cumbersome, we shall not give here the final expressions.

In a series of experiments made by Kobzev in 1978–1981,[23–25] all the theoretical conclusions were confirmed. Mica plates of thickness 1240 nm were used as a target. The radiation of electrons with energies from 100 to 300 keV at wavelength 400 nm, for which the refractive index of the mica is 1.58, was investigated. Thus, if we examine the conditions (14) and (15) considered above, we see that we have the intermediate case

$$\frac{l}{\lambda}\Delta n \sim 1 \tag{24}$$

(it is true that we must here also bear in mind that the energies of the electrons are not high, i.e., $\beta \neq 1$).

Kobzev observed an appreciable shift of the angle at which the radiation intensity was maximal as compared with the case of an infinite (thick) plate, and also a displacement of the emission threshold to lower energies (recall the condition $x \sim \pi$) compared with the usual condition for thick targets. Observation at a given angle must, if the energy is increased, i.e., β is varied, lead in accordance with (6) and (7) (for fixed ω, l, n) to a characteristic curve with a maximum, as was observed experimentally.

Emission in accelerators

Analogous maxima in the emission of electrons passing through gaps between two magnets in cyclic acclerators have been observed at a fixed angle for given frequency as the electron energy is increased during the acceleration process [i.e., as β in (6) varies as the particles are accelerated]. They are also due to the finite length of the path traversed by the electrons in the gaps between the magnets.

Emission in solids

An interesting thought example is the radiation of electrons in a metal under the influence of pumping of an intense electromagnetic wave. If the electron is regarded as a relativistic oscillator, then, as is well known, in the problem of a relativistic oscillator under the influence of a wave the amplitude of its oscillations increases and the velocity at the center also grows. For sufficiently strong pumping, this velocity may exceed the velocity of light in the medium, and Cherenkov radiation becomes possible in the limited interval of the motion of the oscillator in which the corresponding condition is satisfied. This example is interesting in that the length of the interval can be continuously regulated by changing the wave intensity. It can be shown that in the classical theory for nondispersive media such a situation is impossible because of the sharp rise in the emission intensity of the oscillator when its velocity at the center approaches the velocity of light (this result is readily obtained by generalizing the results of Ref. 27 to the case when a medium is present). However, in media with dispersion it is possible to have such a case; moreover, at the transition point an increase in the emission intensity of the oscillator will be noted.

For the actual realization of this situation, enormous intensities of the initial wave are required. In practice, evidently, one can realize only a situation similar to that considered above in which the velocity of the oscillator at the center exceeds the velocity of sound, and then there is a sharp rise in the emission of phonons, and Mach waves can be formed from a restricted section of the path. Experimental results that could apparently be interpreted in this manner were obtained in Ref. 28. Such a process is of interest from the point of view of the creation of acoustic amplifiers.

Fraunhofer diffraction

Finally, we mention Fraunhofer diffraction by a small slit [described by a function of the type $\sin^2 x/x^2$; see (6)[8)]] and droplets of small (micron) diameter [described by the function $J_1^2(x)/x^2$, which is obtained by averaging over the impact parameters]. The only difference from the case considered above is that the characteristic factor $\sin^2\theta$, which suppresses the emission at small angles, is absent. Intense laser radiation makes it possible to observe the complete diffraction structure with secondary maxima for diffraction by droplets with dimensions of tens of microns.[29] These diameters can be determined by means of an ordinary microscope and from the diffraction pattern. The agreement is very good. It is interesting that if two particles that scatter the light are close to each other, the diffraction pattern acquires a characteristic structure (azimuthal nonuniformity of the filling of each ring), from which the distance between the droplets can be established.

The corresponding expressions of the type (6) must here too be modified. In the hadron case one could, for example, hope to obtain information about the distance between the partons—the valence quarks in the hadron—were an azimuthal structure of the gluon rings detected. But here it can also be imitated by the fact that each gluon is transformed into a hadron jet.

All the examples given above demonstrated the manifestation of the effect of a step current either on the background of other, sometimes more powerful effects or (Fraunhofer diffraction) without the important first factor in Eq. (6). Below, we shall describe situations in which the background effects can be made small, and the combined effect of the two factors in Eq. (6) is most clearly manifested.

Radiation in a cut wave guide

We consider the radiation of an electron passing along the axis of a semi-infinite wave guide into another such wave guide separated from the first by a small gap. This recalls the problem described a little earlier, in the subsection on emission in accelerators, but without a magnetic field and with the additional possibility of varying the gap and the angle of observation. We do not specify any step current but simply solve Maxwell's equations with the given boundary conditions, calculating the Poynting vector and the radiation flux in the given direction. It is found[30] that the result reproduces Eqs. (6), i.e., indicates an increase of the emission angle with decreasing gap (for given energies of the electron and photon frequency). An experimental verification has not yet been made.

It should be emphasized that such a correspondence between the formal specification of a step current and the solution of Maxwell's equations with appropriate boundary conditions could already be deduced from the original paper of Tamm.[1]

Scattering by two centers

From many points of view, the example of photon emission in the case of classical scattering of an electron by two centers[14,31,32] is very instructive. The intensity of radiation with frequency $\omega \ll E$ in the direction $\mathbf{n}=\mathbf{k}/\omega$ is given by

$$\frac{dN}{d\omega\, d\Omega}=\frac{\alpha}{4\pi^2\omega}\,|M_{12}+M_{23}e^{il/l_{f_2}}|^2, \tag{25}$$

where

$$M_{ij}=\frac{[\mathbf{v}_i\mathbf{n}]}{1-\mathbf{v}_i\mathbf{n}}-\frac{[\mathbf{v}_j\mathbf{n}]}{1-\mathbf{v}_j\mathbf{n}}; \tag{26}$$

$$l_{f_2}=\omega^{-1}(1-\mathbf{v}_2\mathbf{n})^{-1}; \tag{27}$$

l is the distance between the scattering centers, l_{f_2} is the formation length in Section 2 after the first scattering, and the indices 1, 2, 3 label, respectively, the velocities of the particle before the first scattering, in the interval between the first and the second scattering, and after the second scattering.

If $l \ll l_{f_2}$, then in Eq. (25) the expression inside the modulus signs reduces to $M_{13}=M_{12}+M_{23}$, i.e., the scattering occurs as if by one center (which is trivial if $l=0$). This circumstance provides the basis for explaining the Landau–Pomeranchuk effect.[13]

For $l \gg l_{f_2}$, the scattering by two centers takes place independently, since the interference term in (25) disappears because of the strong oscillations and $|M_{12}|^2+|M_{23}|^2$ remains.

For $\mathbf{v}_1=\mathbf{v}_3=0$, the case of a step current, leading to Eq. (6), is naturally obtained. However, the aim of our treatment here is to show that Eq. (6) really is reproduced in a process in which it is not required to change the velocity. For this it is sufficient if both electron scatterings are through fairly large angles, and this is something that can be realized in an experiment, whereas the instantaneous acceleration or stopping of a particle requires special conditions.

Indeed, for clarity let us consider the emission process in the case of scattering of a relativistic electron ($v_i \approx 1$) when both scattering and the emission take place in one plane. We shall be interested in angles $\theta_i=\cos^{-1}(\mathbf{n}\cdot\mathbf{v}_i)$ that, despite being small, are still appreciably larger than the typical angles for bremsstrahlung, i.e., $m/E \ll \theta_i \ll 1$. Then from (25)–(27) we obtain

$$\frac{dN}{d\omega\, d\Omega}=\frac{\alpha}{\pi^2\omega}\left[(\theta_1^{-1}-\theta_3^{-1})^2+4(\theta_2^{-1}-\theta_1^{-1})(\theta_2^{-1}-\theta_3^{-1})\sin^2\frac{\omega l\theta_2^2}{4}\right]. \tag{28}$$

If the photon detectors are placed in such a way that $\theta_2 \ll \theta_1, \theta_3$, then we can readily recover Eq. (6) from (28) in the limit of small angles. Thus, the emissions from the first and third stages of the electron's path take place effectively along the direction of its motion in them, whereas the emission from the intermediate Section 2 is at a fairly large angle to it, the angle being greater the shorter the interval. Of course, when the length of this interval is reduced, the intensity falls rapidly (quadratically).

In fact, when the length of the radiation path is limited, the bremsstrahlung at small angles is "eaten away," but at large angles it remains. As a result, the peak is displaced to ever larger angles, getting lower and lower. Large angles and enhanced transverse momenta are characteristic for radiation from limited path intervals. The quantum treatment does not change the conclusions [the numerators and denominators in (26) are replaced by the 4-products (ep) and (kp), where p and k are the 4-momenta of the electron and photon, and e is the polarization of the photon].

Analogous effects are manifested in the radiation obtained when relativistic particles pass through a short section of a force field[33] or through two plates separated from one another.[34]

The role of the transition region

Of course, even in the considered case of emission as a result of scattering by two centers a step current arises as a certain limiting case, whereas in general the edges of the "step" are usually smoothed to some degree.[9)] If the law of decrease at the edges is exponential (or steeper), then no basic changes compared with Eq. (6) occur. But if, for example, the law of decrease is as a power, then it is clear that the Fourier transform of such a function will differ appreciably from (6), above all in the appearance of an additional exponential cutoff of the high frequencies.[35,36,53] The total number of emitted quanta is also changed.

If the current is damped exponentially over length l, its radiation is given by[37]

$$\frac{dN}{d\omega\, d\Omega}=\frac{\alpha v^2}{4\pi^2\omega}\,\frac{\sin^2\theta}{(1-v\cos\theta)^2+(\omega l)^{-2}}, \tag{29}$$

i.e., the shape of the distribution is not strongly changed compared with (6). In particular, the position of the maximum in the angular distribution differs only by a numerical factor from (8) for the same functional dependence on the frequency ω and length l.

We emphasize that this case is not merely of academic interest but is applicable to the radiation of unstable particles. For example, the radiation of a particle of mass m and energy E having decay width Γ is given by[38]

$$d\sigma=d\sigma_0\,\frac{\theta^3\, d\theta}{\left(\theta^2+\dfrac{m^2}{E^2}\right)^2+\dfrac{m^2\Gamma^2}{E^2\omega^2}}\,\frac{d\omega}{\omega}, \tag{30}$$

where $d\sigma_0$ is the cross section for production of the particle in the given process (for example, $\nu_\mu p \to \mu X$). As can be seen from (30), the role of the length l is played here by the quantity $(1/\Gamma)(E/m)$, i.e., the decay length of the particle. One can attempt to use[39] the properties of such radiation reflected in a pronounced increase of the photon emission angles compared with ordinary bremsstrahlung angles when

$$\Gamma/\omega \gg m/E, \tag{31}$$

to measure lifetimes that are not amenable to measurement by other methods. This will evidently be of interest in the

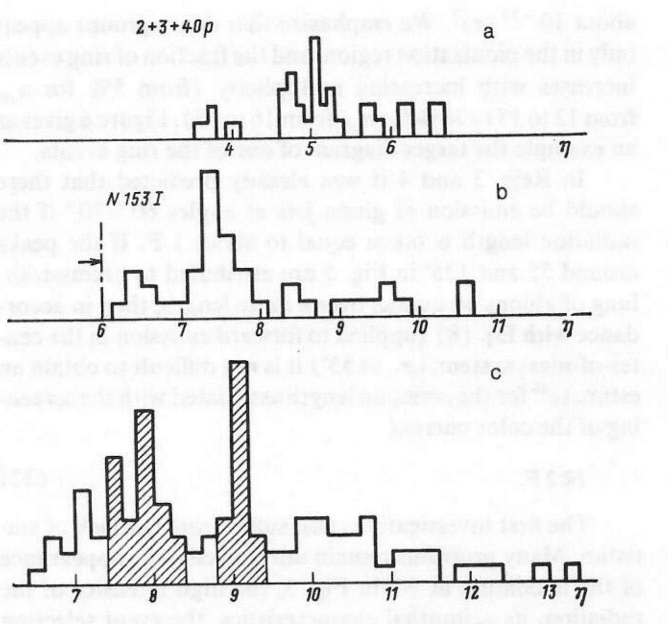

FIG. 4. Histograms of secondary-particle distributions with respect to the pseudorapidity in events found in cosmic rays: a) event at energy 10^{13} eV obtained in a photoemulsion exposed in an aircraft[41,42]; b) event of the Japanese–Brazilian collaboration[40] at 10^{15} eV (the arrow indicates the limit of the region of angles accessible for measurement in this event); c) stratospheric event at 10^{16} eV.[39] The rings are hatched.

case of top particles (containing top quarks), whose lifetime is about 10^{-19} sec.

The list of experimental processes in which the restricted length over which the radiation is formed is important could be continued, and we could also go into further details of their theoretical description. Our aim here was merely to show the topicality and reality of such processes, to demonstrate their characteristic features, and, in particular, to emphasize the very complete analogy with the treatment of color currents whose action is restricted by the confinement effect. More precisely, rapid change of a color current may be due not only to a variable magnitude or direction of the parton velocity but also to its screening by another parton.

3. PREDICTIONS AND EXPERIMENTAL RESULTS

Main conclusions

The confinement phenomenon may affect the bremsstrahlung of gluons by a color current, leading to a characteristic ring structure of the target diagram of inelastic hadron events, i.e., to the emission of groups of hadrons with nearly equal polar angles (and, hence, nearly equal pseudorapidities). The presence of this ring structure can be most readily identified by a maximum in the scale of pseudorapidities in an individual event or by looking in an individual event for a dense group of particles on this scale. Of course, this immediately imposes the requirement of a high multiplicity of the secondary particles, i.e., a high initial energy.

The distribution with respect to the polar angles of the centers of these groups (which can be associated with the directions of emission of the primary gluons) must be described qualitatively by Eq. (6). From this it is clear what information can be obtained about the properties of quarks and gluons from the analysis of such events.

First, of greatest interest is the determination of the length l the region of action of the color current, which is directly related to the size of the quark confinement region.

Second, there is a possibility of obtaining information about the refractive index n of the gluons in the quark–gluon medium. Hitherto, these two quantities, l and n, have not been determined experimentally, and the methods of determining them proposed here are as yet unique.

Third, it is of interest to find the number of emitted jets at different energies [see Eq. (10)], from which one could deduce, besides the chromodynamical coupling constant α_s, the manner in which the color current is screened (see the discussion in the last subsection of Sec. 2).

Fourth, the presence of ring events will confirm once more the vector nature of the gluon, which leads to the factor $\sin^2\theta$ in (6).

Fifth, one can hope to obtain information about less studied questions, for example, information on the parton structure of the hadron, if it is possible to study the azimuthal structure of the rings (see the discussion on Fraunhofer diffraction in Sec. 2), on the possibility of reflection and refraction of gluons at the boundary with the vacuum (see the subsection on Cherenkov radiation in thin plates in Sec. 2), on the structure of the transition region, i.e., on the unfolding of the deconfinement process (see the Appendix), etc.

In addition, it will be of interest to study ring events in interactions of hadrons with nuclei, from which one can learn how the radiation length changes when the target is increased (in a naive approach, one must have $l \sim A^{1/3}$, where A is the mass number of the nucleus), the distances between the partons in the nucleus, etc.

Other qualitative consequences of the discussed effect—the growth of the mean transverse momentum, the jet structure at low multiplicities, etc.—are less pronounced and can be described by other processes, and therefore we shall not discuss them here.

Data from cosmic rays

The first indications from experiments of the appearance of ring events were obtained from cosmic rays,[40–43] in which high particle densities were observed on the scale of pseudorapidities in individual interaction processes of hadrons of very high energies [at 10^{13} eV (Refs. 42 and 43), 10^{15} eV (Ref. 41) and 10^{16} eV (Ref. 40)]. The distributions of the particles in these events are shown in Fig. 4. The number of particles in the peaks is large (sometimes about 50), and their azimuthal distribution is fairly uniform.

In the event with lowest energy[42] (photoemulsion experiments in an aircraft) one can see [Fig. 4(a)] four dense groups of particles, which could be interpreted as two ordinary cones with small transverse momenta and two cones at large angles (forward and backward) given by the condition (8) for perfectly reasonable estimates $\omega l \sim 10^4$–10^5 ($\theta \sim 10^{-2}$).

In the second event,[41] which was obtained by a Japanese–Brazilian collaboration, one can see one dense group of particles at angle $\theta \sim 10^{-3}$, which again agrees with (8) if one takes into account the growth of ω with the initial energy. The conditions of observation cut out (as indicated by the arrow in the figure) the region of possible appearance of other dense groups of particles.

The third event[40] stimulated the idea of Cherenkov gluon radiation,[3] work on which then led to the idea of emis-

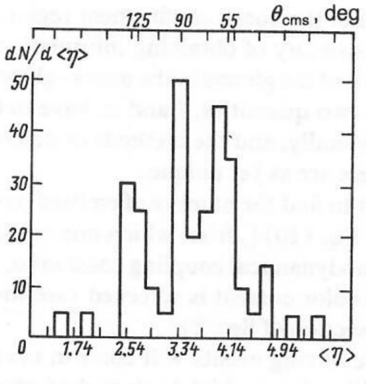

FIG. 5. Histogram of the distribution of the centers of dense groups of particles with respect to the pseudorapidity in ring events identified in inelastic proton–nucleon collisions with energy 400 GeV in photoemulsion.[45]

sion due to deconfinement.[31] One can see here two groups of particles at fairly large polar angles $\theta_1 \approx 3\cdot 10^{-4}$ and $\theta_2 \approx 7\cdot 10^{-4}$ (hatched in the figure), which again agree with Eq. (8) and the estimates given above in the case of linear extrapolation of ω with increasing primary energy.

At approximately the same time, individual events were observed[44] in photoemulsion experiments in accelerators at energy 200 GeV. The fraction of such events was estimated at 1%. The emission angles were close to $\theta \sim 10^{-1}$, in agreement with (8). However, the much lower multiplicities at this energy made it difficult to identify ring events.

Accelerator data

After a 5-year interruption, interest in ring events recently revived in connection with the appearance of two studies.

Ring events were studied in detail in Ref. 45, which used photoemulsion data on interactions of 400-GeV protons with quasifree nucleons of nuclei. The accuracy of the measurement of the angles was very high, but the statistics comparatively poor. Since ring events can be identified only in the case of high multiplicities, 284 events with the number of secondary particles between 12 and 18, i.e., with multiplicity exceeding the mean by two or three times, were chosen for analysis.

Events were deemed to be of the ring type when there were groups of not less than six charged particles ($K_0 \geqslant 6$) such that the mean distance in pseudorapidity between the particles of the group was less than 0.15 and no one distance in the group exceeded either 0.2, if the mean distance was less than 0.1, or twice the mean for a mean distance in the interval from 0.1 to 0.15.[10)] This criterion was met by 59 of the 284 investigated events. In each of them, the position of the center (arithmetic mean) of the group's pseudorapidities was determined. The distribution of the centers of the groups with respect to the pseudorapidity is shown in Fig. 5. It was found that all dense groups appear at large angles in the center-of-mass system, and a tendency was observed for more frequent appearance of groups of particles near the angles 55, 90, and 125° in the center-of-mass system (pseudorapidities in the laboratory system of about 2.7, 3.34, and 4.0). The cross section for the forward and backward generation of ring events in the center-of-mass system (near the angles 55 and 125°) is, after subtraction of the background, about 10^{-27} cm^2. We emphasize that dense groups appear only in the pionization region, and the fraction of ring events increases with increasing multiplicity (from 5% for n_{ch} from 12 to 15 to 30% for n_{ch} from 16 to 18). Figure 6 gives as an example the target diagram of one of the ring events.

In Refs. 3 and 4 it was already predicted that there should be emission of gluon jets at angles 60°–70° if the radiation length is taken equal to about 1 F. If the peaks around 55 and 125° in Fig. 5 are attributed to bremsstrahlung of gluons by quarks over a finite length, then in accordance with Eq. (8) (applied to forward emission in the center-of-mass system, i.e., at 55°) it is not difficult to obtain an estimate[48] for the emission length associated with the screening of the color current

$$l \gtrsim 2\ \text{F}. \tag{32}$$

The first investigations still suffer from the lack of statistics. Many problems remain unresolved—the appearance of the maximum at 90° in Fig. 5, the high intensity of the radiation, its azimuthal characteristics, the event selection criterion, and so forth.

It is nevertheless worth noting that indications supporting the nonuniformity noted above in the grouping of the particles with respect to the pseudorapidity can be found in data with good statistics on pp interactions at energy $\sqrt{s} = 31$ GeV (i.e., 480 GeV in the laboratory system) given in Ref. 46. Measurements were made of the two-particle correlation function

$$R_2 = \frac{\sigma\partial^2\sigma/\partial\eta_1\,\partial\eta_2}{(\partial\sigma/\partial\eta_1)\,(\partial\sigma/\partial\eta_2)} - 1 \tag{33}$$

of two charged secondary particles for different positions of the rapidity interval of the first trigger particle. In each case, close correlations were observed (Fig. 7). But the height of the correlation function (i.e., the strength of the correlations) varied for different positions of the first particle. As can be seen from Fig. 7, two particles have a greater tendency to have nearly equal angles when they are emitted near 90 and near 50° in the center-of-mass system and have smaller correlations in the interval between these angles, and also at smaller angles (estimating roughly the successive intervals lying near 90°, 70°, and 50°, we obtain from Fig. 7 maxima of the two-particle correlations in them roughly in the ratios

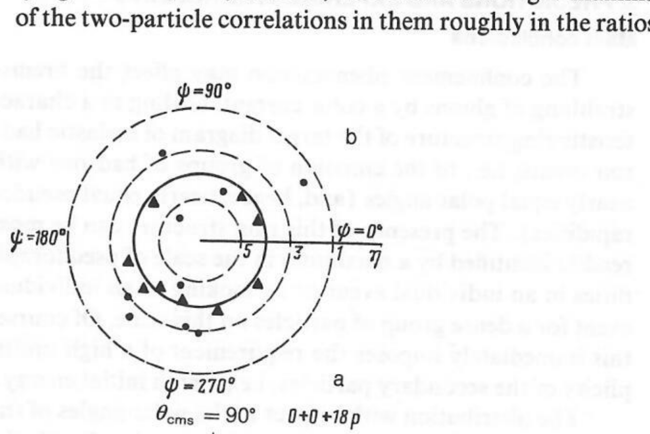

FIG. 6. Distribution with respect to the pseudorapidity (a) and target rapidity–azimuthal diagram (b) of a pp event[45] of the type $0+0+18p$ (ring: $K_0 = 9$, $\langle\eta\rangle = 4.02$, $\langle\Delta\eta\rangle = 0.066$). The triangles identify the particles of the ring, and the small black circles are the remaining particles.

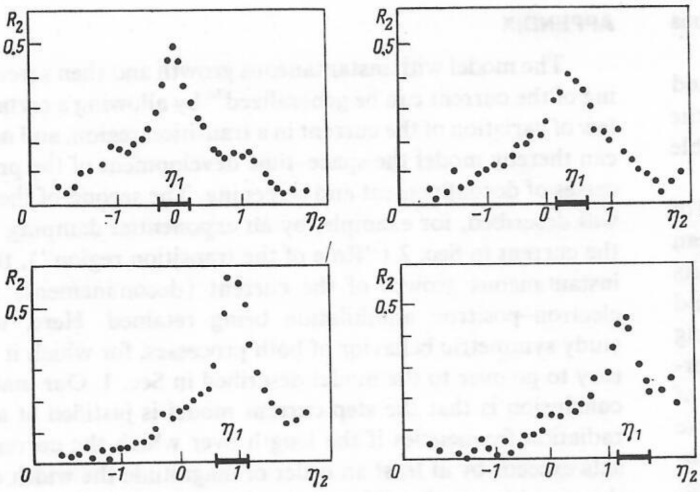

FIG. 7. Two-particle correlation function in pp interactions at energy $\sqrt{s} = 31$ GeV (Ref. 46) for different choices of the pseudorapidity interval of the trigger particle (shown by the heavy line on the abscissa in each graph).

0.48:0.42:0.58). This indicates indirectly a tendency of the particles to be more closely grouped where the corresponding correlations are large, and this is also confirmed by the photoemulsion data at 400 GeV, for which the positions of the centers of the groups were also found to be nonuniformly distributed, depending on the polar angle. Unfortunately, the width of the rapidity interval covered by the trigger detector in this experiment was rather large [$(\Delta\eta)_{\rm tr} = 0.5$], so that there was in fact an averaging over such an interval, and, as can be seen from Fig. 5, an averaging can appreciably smooth the observed structure. The optimal choice would be $(\Delta\eta)_{\rm tr} \lesssim 0.2$. Such an experiment has not yet been performed.

The UA-5 group working on the $Sp\bar{p}S$ collider at CERN found ring events (Fig. 8) in proton–antiproton interactions at $\sqrt{s} = 540$ GeV. A report of this has appeared in three review papers,[47–49] but no detailed paper has been published.

Ring events were identified as those having at least one pseudorapidity interval of width 0.5 in which the number of charged particles satisfies $K_0 > (0.1n_{\rm ch} + 6.7)$, where $n_{\rm ch}$ is the total number of charged particles in the given event.[11)] This choice of K_0 is due to the requirement that the peak in the particle distribution in the pseudorapidity scale should be sufficiently high in conjunction with a large total number of particles. Among 6339 examined events, 47 were found that satisfied the criterion. In them, the mean values of K_0 and $n_{\rm ch}$ were, respectively, 12.9 and 47, the K_0 and $n_{\rm ch}$ distributions being fairly broad.

Thus, about 0.7% of the events had peaks exceeding the mean background by 3–4 times. Within the peaks, the particles had a practically isotropic distribution with respect to the azimuthal angles.

The largest peak contained 18 charged particles in the 0.5 pseudorapidity interval. The reviews[47–49] give only one of the 47 events (see Fig. 8), in which 15 out of 44 tracks lie in such a narrow interval. The emission angle can be estimated as $\sim 10°$ in the center-of-mass system, i.e., it is very large when compared with the typical bremsstrahlung angles at such energies (of order m/E). It is asserted in Ref. 47 that a Monte Carlo cluster model developed by the UA-5 group leads to such fluctuations when ordinary clusters are superimposed, but details of the model and comparisons of it with the experiment are not given. In Ref. 49, such events are interpreted as indicating the formation of "droplets" in the hot quark–gluon plasma, though the ring structure of the ejection of these droplets from the plasma does still remain obscure. (The explanation of the azimuthal symmetry in Ref. 49 by the "atomizing" of one droplet is incorrect, since one droplet, emitted at an appreciable angle to the collision axis, leads to a spot, and not to a ring. We note also that an isotropically decaying droplet will give a broad pseudorapidity distribution.) If Eq. (8) is used, it can be estimated that the radiation length for such event characteristics (emission angle in the center-of-mass system of about $2\cdot10^{-3}$) must be not less than 10 F. The color screening mechanism evidently occurs outside the hadron but at comparatively short distances that increase with the energy by no means linearly [and, *a fortiori*, not quadratically (!); see (1)].

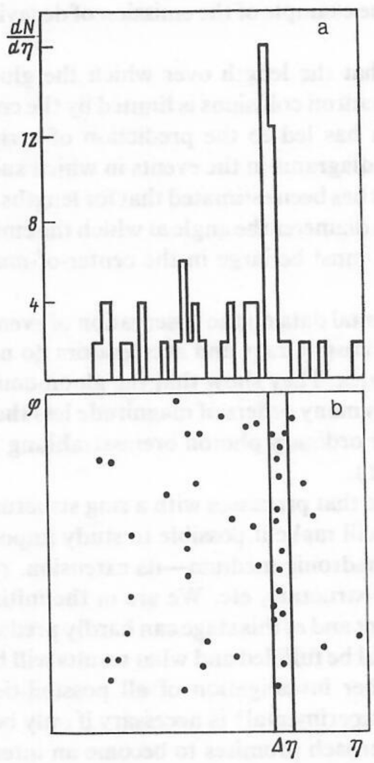

FIG. 8. Histogram of the distribution of charged particles with respect to the pseudorapidity (a) in one of the events observed in the collider at energy $\sqrt{s} = 540$ GeV (Ref. 47) (the peak is due to the presence of 15 particles in the 0.5 pseudorapidity interval) and the azimuthal–rapidity distribution of the particles in this event (b).

Unfortunately, a systematic analysis of these events has not been made.

A structured nature of inelastic events at angles around 60° in the center-of-mass system has also been found in the analysis of proton–proton interactions at 205 GeV in bubble chambers.[52]

Thus, summarizing our comparison with experiment of the model of gluon jet emission over a finite length, we can say that the initial stage has been completed, i.e., the first bits of evidence in support of such a model have been found, and the estimate obtained for the length over which the screening of the color current takes place exhibits a significant difference between this length and the distances characteristic of ordinary electromagnetic processes. However, for definitive conclusions we require further confirmations in experimental material with better statistics, a comparison with the fluctuations in ordinary models, a fuller elaboration of the model and the event selection criteria, and testing of other characteristic properties of the radiation, particularly their dependence on the energies of the gluon jets, the azimuthal characteristics, etc.

CONCLUSIONS

The model describes the emission of particles over a finite path length in the framework of classical physics. In quantum theory, there does not exist a consistent description of the properties of radiation from a finite path length, but there are various semiphenomenological approaches which confirm at a qualitative level the estimates obtained in the classical theory (see the example of the emission of decaying particles in Sec. 2).

The suggestion that the length over which the gluon emission is formed in hadron collisions is limited by the confinement phenomenon has led to the prediction of a ring structure of the target diagrams in the events in which such emission is observed. It has been estimated that for lengths of the order of the hadron diameter the angle at which the emission maximum occurs must be large in the center-of-mass system.

The first experimental data on the observation of events with such structure in cosmic rays and accelerators do not contradict these estimates. They show that the gluon-emission formation length is many orders of magnitude less than the length available for ordinary photon bremsstrahlung in accordance with Eq. (1).

Thus, there is hope that processes with a ring structure of the target diagram will make it possible to study important properties of the hadronic medium—its extension, refractive index, internal structure, etc. We are in the initial stage of this development and at this stage can hardly predict which of these hopes will be fulfilled and what results will be obtained, but the further investigation of all possibilities (both theoretical and experimental) is necessary if only because the proposed approach promises to become an interesting source of information about important characteristics of hadronic matter.

I am grateful to E. L. Feĭnberg, who read the review and made some valuable comments, and B. M. Bolotovskiĭ for discussing a number of electrodynamical problems.

APPENDIX

The model with instantaneous growth and then screening of the current can be generalized[36] by allowing a certain law of variation of the current in a transition region, and one can thereby model the space–time development of the processes of deconfinement and screening. The second of these was described, for example, by an exponential damping of the current in Sec. 2 ("Role of the transition region"), the instantaneous growth of the current (deconfinement) in electron–positron annihilation being retained. Here, we study symmetric behavior of both processes, for which it is easy to go over to the model described in Sec. 1. Our main conclusion is that the step-current model is justified at all radiation frequencies if the length over which the current acts exceeds by at least an order of magnitude the width of the transition region. However, when this condition is not satisfied, radiation with high frequencies will be emitted at smaller angles than in Eq. (8), but at a given frequency a ring structure of the events will still persist.

Thus, we consider a specific current model[12)] that generalizes the model (6)[13)]:

$$j=\frac{g_s v}{2}\left\{\operatorname{th}\frac{l'}{\Delta}+\operatorname{th}\frac{l-l'}{\Delta}\right\}, \tag{A.1}$$

where l determines the region in which the current acts, l' is the running coordinate, and Δ is the width of the transition region. The invariant inclusive cross section for the emission of gluons with frequency ω at angle θ has the form

$$\frac{\omega}{\sigma}\frac{d^3\sigma}{d^3k}=\frac{\alpha_s c_F}{16\pi^2}\sin^2\theta|I|^2, \tag{A.2}$$

where

$$I=\frac{\pi\omega\Delta^2\operatorname{sh}(l/\Delta)}{\operatorname{sh}(\pi\omega\Delta/2)}e^{i\frac{\omega\beta\cos\theta}{2}l-\frac{l}{\Delta}}F$$

$$\times\left(1-\frac{i\omega\Delta\beta\cos\theta}{2},\ 1+i\frac{\omega\Delta}{2};\ 2;\ z\right), \tag{A.3}$$

$$z\equiv 1-e^{-2l/\Delta}.$$

Here, F is the hypergeometric function.

To obtain from (A.2) the case of a step current, the transition region must be small compared with the extension of the region in which the current acts, i.e., we consider the condition

$$\Delta\ll l. \tag{A.4}$$

Using in (A.2) and (A.3) the expressions for the limiting values of the hypergeometric and gamma functions,[51] we obtain

$$|I|^2=4l^2\frac{\sin^2 x}{x^2}\frac{\zeta}{\operatorname{sh}\zeta}\frac{\operatorname{sh}\frac{\pi\Delta\omega\beta\cos 0}{2}}{\beta\cos\theta\operatorname{sh}(\pi\Delta\omega/2)}, \tag{A.5}$$

where

$$\zeta=\pi x\Delta/l=\pi\Delta/l_f. \tag{A.6}$$

Thus, we see that (A.5) contains the three very important parameters l/l_f, Δ/l_f, and $\Delta/\lambda\!\!\!^-$, which determine the various ratios of the length of action of the current, the length of the Fresnel zone, the wavelength of the radiation ($\lambda\!\!\!^-=\omega^{-1}$), and the width of the transition region.

Naturally, Eq. (6) for an abrupt step is obtained from (A.5) by simply setting $\Delta=0$. However, the expression

(A.5) makes it possible to particularize the condition under which $\Delta \to 0$.

First, it is clear that this condition is satisfied for sufficiently soft gluons having wavelengths appreciably greater than the width of the transition region, i.e.,

$$\lambda \gg \Delta, \text{ and hence also } l_f \gg \Delta. \tag{A.7}$$

Such gluons do not "feel" the shape of the transition region.

Second, for hard gluons having wavelengths appreciably less than the width of the transition region,

$$\omega^{-1} = \lambda \ll \Delta, \tag{A.8}$$

the ratio of the length of the Fresnel zone to the width of the transition region is important. Indeed, in this limiting case we obtain from (A.5)

$$\frac{\omega}{\sigma}\frac{d^3\sigma}{d^3k} = \frac{\alpha_s C_F}{4\pi^2} l^2 \sin^2\theta \frac{\sin^2 x}{x^2} \frac{\zeta e^{-\zeta}}{\beta \cos\theta \operatorname{sh}\zeta}. \tag{A.9}$$

From this it can be seen that we again arrive at an abrupt step if we consider gluons for which the length of the Fresnel zone appreciably exceeds the width of the transition region, i.e., $\zeta \ll 1$ (bearing in mind that the quark is relativistic, $\beta \approx 1$, and the radiation emerges at small angles, $\cos\theta \approx 1$).

The condition $\zeta \ll 1$ must be satisfied in the first place in the region of the maximum of the distribution given by Eq. (6), i.e., at $\theta_{\max} = (2\pi/\omega l)^{1/2}$ or at $x \approx \pi/2$. From this we immediately obtain the requirement

$$l \gg \frac{\pi^2}{2}\Delta, \tag{A.10}$$

which ensures fulfillment of the consequences of the model of an abrupt step for radiation with any frequency and thus particularizes the condition (A.4). Thus, if the length l over which the current acts exceeds by at least an order of magnitude the width Δ of the transition region, we can without hesitation ignore the width of the transition region.[14)]

Only when the width of the transition region appreciably exceeds the length of the Fresnel zone is it necessary to take into account the additional suppression (of the type $\zeta e^{-2\zeta}$) of high radiation frequencies at fixed angle. This leads to a displacement of the radiation maximum to smaller angles and to a decrease in the height of the maximum, but it does not change the general nature of the distribution with a peak at a certain polar angle, i.e., with a ring structure of the target diagram of the inelastic hadronic event.

Thus, the presence of a transition region can only displace the position of the ring to the region of smaller angles, playing, naturally, the same part as an increase in the length l over which the current acts in the case of an abrupt step (with some differences in the region of high frequencies). The general conclusion that the target diagram has a ring structure remains, and the actual parameters l and Δ characterizing it can be determined from experiments if ring events are identified (for example, by means of the methods employed in Refs. 45 and 48).

[1)]See also the last two subsections in Sec. 2 and the Appendix.

[2)]See the subsection on Fraunhofer diffraction in Sec. 2.

[3)]The secondary maxima are, first, small and, second, may be absent altogether if deconfinement does not take place so rapidly as assumed. This last possibility will be discussed below.

[4)]The target diagram is the plane perpendicular to the collision axis of the initial particles on which the positions (polar and azimuthal emission angles) of the secondary particles are indicated.

[5)]The multiplicity of the hadrons in the ring depends on the jet fragmentation model, but it is clear that if its ratio to the mean multiplicity does differ from a constant, then it is only by logarithmic (in the energy) factors.

[6)]See the last two subsections in Sec. 2 and the Appendix.

[7)]See the last two subsections in Sec. 2 and the Appendix.

[8)]In ordinary Fraunhofer diffraction $x = \omega l \sin\theta$. It is evident that expressions of the type (6) and (7) are obtained for diffraction of light that moves parallel to a plane with a slit.

[9)]See the Appendix.

[10)]Thus, dense and fairly uniform groups of particles that did not break up into two or more clearly separated subgroups were identified.

[11)]Qualitatively similar results are obtained if one selects peaks of unit pseudorapidity width.

[12)]Such a model was applied by I. I. Abbasov to some electrodynamical problems.

[13)]*Translator's Note*. The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the article in the displayed equations.

[14)]Incidentally, a condition of the same type explains, for example, why we see a clear diffraction pattern for the light passing the moon despite the high mountains on its surface (this remark was made by E. L. Feĭnberg in a discussion of Ref. 36).

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