Canonical quantization of gauge theories with a scalar condensate and the problem of spontaneous symmetry breaking

V. V. Vlasov, V. A. Matveev, A. N. Tavkhelidze, S. Yu. Khlebnikov, and M. E. Shaposhnikov

Institute of Nuclear Research, USSR Academy of Sciences, Moscow

Fiz. Elem. Chastits At. Yadra **18**, 5–38 (January–February 1987)

The construction of physical variables in gauge theories is considered in the framework of canonical quantization. It is shown that a consistent treatment requires specification of a class of functions for the Lagrangian multiplier A_0, the time component of the vector potential. In theories with a nontrivial minimum of the scalar potential, vacuum degeneracy is possible only under certain restrictions on A_0. However, this degeneracy does not bear any relation to the appearance of a mass of the vector bosons and is not manifested at all in perturbation theory. In non-Abelian theories, the boundary conditions for A_0 must be taken into account when the realization of the algebra of global charges is studied.

INTRODUCTION

In this paper, we study some general problems of the quantum theory of gauge fields. We concentrate our attention on a gauge-invariant formulation and on the possibility of vacuum degeneracy in gauge theories.

The treatment is based on the formalism of the canonical quantization of systems with constraints developed by Dirac.[1] In particular, the concept of a physical, i.e., gauge-invariant, quantity is due to Dirac, as is the idea of formulating the theory in terms of gauge invariants.

It is true that there is an extensive literature[2–18] on the quantization of gauge theories, but in our view numerous problems still require further discussion.

1. The traditional scheme of quantization[3] regards the zeroth component A_0 of the vector field as a Lagrangian multiplier. From what class of functions is A_0 to be chosen?

2. In canonical quantization, the class of the functions A_0 is the class of gauge transformations with respect to which all physical quantities must be invariant. In other words, the very concept of gauge invariance is based on a definite A_0 class. Thus, if A_0 is regarded as a completely arbitrary function, then locally or globally invariant, i.e., uncharged or colorless states, will be physical. On the other hand, in electrodynamics one operates with charged states. Does it follow from this that the Lagrangian multiplier A_0 cannot be arbitrary?

3. Suppose that some restrictions are imposed on A_0. For example, one can consider the theory in a finite volume and require A_0 to vanish on the boundary. This corresponds to the requirement of local gauge invariance of all the physical states. Such a restriction on A_0 allows the existence of physical variables on the boundary of the volume in addition to the ordinary degrees of freedom. Are there any real manifestations of such variables? Indeed, does the physics depend on the boundary conditions for A_0?

4. In gauge theories, the appearance of a mass of the vector fields is frequently attributed to spontaneous symmetry breaking and degeneracy of the vacuum with respect to the corresponding charges.[8,9] However, there are indications that the occurrence of massive vector particles is in no way related to the phenomenon of spontaneous breaking.[12–14] If the situation is to be clarified, it is necessary to find a gauge-invariant characterization of the Higgs mechanism. What are the conclusions with regard to vacuum symmetry breaking in this case? Is indeed the appearance of massive vector bosons impossible without spontaneous symmetry breaking?

All these questions can be fully investigated in theories with weak coupling. This is the subject of the present paper. We shall see that in the framework of perturbation theory the choice of the boundary conditions for A_0 does not affect physical processes. In particular, in the formalism with arbitrary A_0 it is possible to reproduce the ordinary electrodynamical results. Vacuum degeneracy with respect to charges in theories of Higgs type is possible because of the presence of additional (not related to the local fields) variables, but it is in no way manifested in the framework of perturbation theory. In addition, it bears no relation to the appearance of a mass of the vector particles.

Section 1 describes the formalism, part of which is, in essence, standard. Only the restriction to the case of finite volume warrants discussion. In infinite volume, there are difficulties in the definition of the charges (for a review, see Ref. 16). In a finite volume one need not worry about the convergence of series of improper integrals, such as one has for the charge, Hamiltonian, etc. Of course,[9] in a finite volume spontaneous symmetry breaking—the presence of unitarily inequivalent representations of the algebra of observables—is not possible at all. Therefore, in all that follows we shall, as usual, mean by spontaneous symmetry breaking the presence of vacuum degeneracy with respect to the corresponding charges in the infinite-volume limit.

In Sec. 2, we consider scalar electrodynamics with zero-value boundary conditions on A_0.

Section 3 treats the same theory, but A_0 is now regarded as an entirely arbitrary function. Non-Abelian theories with a scalar potential of Higgs type are discussed in Sec. 4. The main attention is devoted here to investigation of the realization of the algebra of non-Abelian charges in the space of physical states in its dependence on the boundary conditions imposed on $A_0(x)$.

1. DESCRIPTION OF THE FORMALISM

In this section, we present the classical theory of gauge models, concentrating mainly on the way in which the properties of the physical quantities depend on the postulated

0090-4759/87/010001-13$04.40

class of allowed gauge transformations. As already noted, we consider the theory in a finite volume V, eliminating thereby the problem of convergence of the integrals that determine the charges.

Our point of departure is the Lagrangian of an arbitrary gauge theory with scalar fields:

$$\mathscr{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu \varphi - igT^a A^a_\mu \varphi)^2 - V(\varphi). \quad (1)$$

Without loss of generality, we have here chosen for the scalar fields a real and, in general, reducible representation. As usual,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu; \quad [T^a, T^b] = if^{abc}T^c.$$

The Lagranian $L = \int \mathscr{L}(x) d^3x$ can be represented in the form characteristic of a generalized Hamiltonian system (Ref. 3),[1)]

$$L = \int d^3x\, (\mathbf{p}\cdot\dot{\varphi} + B^a_i \dot{A}^a_i) - H - \int d^3x\, (B^a_i \partial_i A^a_0 + J^a_0 A^a_0), \quad (2)$$

where

$$\mathbf{p} = \partial_0\varphi - igT^a A^a_0 \varphi; \quad B^a_i = F^a_{0i}; \quad (3)$$

$$H = \int d^3x \left(\frac{1}{2}(B^a_i)^2 + \frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}\mathbf{p}^2 + \frac{1}{2}(\partial_i\varphi - igT^a A^a_i \varphi)^2 + V(\varphi) \right). \quad (4)$$

In Eq. (2), J^a_0 are the zeroth components of the Noether currents associated with the global color rotations:

$$J^a_0 = ig(\mathbf{p}T^a\varphi) + gf^{abc}B^b_i A^c_i. \quad (5)$$

The corresponding charges are

$$Q^a = \int d^3x J^a_0. \quad (6)$$

It can be seen from (2) that $(\varphi, \mathbf{p})$ and (A, B^a_i) are canonically conjugate pairs, H is the Hamiltonian and

$$\xi(A_0) = \int d^3x\, (-B^a_i \partial_i A^a_0 - J^a_0 A^a_0) \quad (7)$$

is a constraint on the dynamical variables, the functions A_0 playing the part of Lagrangian multipliers.

Following Dirac,[1] we write the constraint (7) in the form of the weak condition

$$\xi(A_0) \approx 0. \quad (8)$$

Such conditions can be used only after calculation of the Poisson brackets

$$\{f(p, q), g(p, q)\} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q}.$$

The constraints (7) satisfy the relation

$$\{\xi(A_0), \xi(A'_0)\} = g\xi(A''_0), \quad (9)$$

where

$$A''^a_0 = f^{abc}A^b_0 A'^c_0.$$

We emphasize that the expression of the weak condition (8) contains explicitly the A_0. Since we regard the A_0 as Lagrangian multipliers, the question arises of the class Ω of functions from which they are chosen. This class of functions plays a central part throughout the following treatment.

Dirac's scheme[1] presupposes the introduction of a generalized Hamiltonian

$$H_T = H + \xi(A_0). \quad (10)$$

The evolution of the dynamical variable X is given by

$$\dot{X} = \{X, H_T\}$$

and does not depend on the choice of $A_0 \in \Omega$ only if

$$\{X, \xi(A_0)\} \approx 0.$$

In this case, X is said to be physical.[1]

Note that $\xi(A_0)$ is simply the generator of gauge transformations:

$$\left.\begin{aligned} &\{\xi(A_0), A^a_i(x)\} = \partial_i A^a_0(x) + gf^{abc}A^b_i A^c_0; \\ &\{\xi(A_0), \varphi(x)\} = igA^a_0(x)T^a\varphi(x). \end{aligned}\right\} \quad (11)$$

Therefore, quantities invariant with respect to gauge transformations with parameters given by the class of functions for A_0 are physical.

In particular, if the A_0 are assumed to be entirely arbitrary, then, setting $A_0 = \text{const}$ in (8), we obtain for the charges

$$Q^a \approx 0. \quad (12)$$

Exactly the same results are obtained if the method proposed by Dirac,[1] in which A_0 is regarded as a dynamical variable like any other, is followed.

Time derivatives of A_0 being absent from the Lagrangian, there is a constraint on the conjugate momentum,

$$B^a_0 \lambda^a(x)\, d^3x \approx 0,$$

where λ^a are arbitrary functions.

In such an approach, (8) is regarded as a secondary constraint; it can be obtained by analyzing the condition of consistency of

$$\frac{d}{dt}\int B^a_0(x, t)\lambda^a(x)\, d^3x \approx 0.$$

Therefore, in (8) completely arbitrary functions are also allowed. From this global invariance of all the physical quantities follows.

In Abelian theory—scalar electrodynamics—this reduces to the vanishing of the electric charge of all physical quantities:

$$Q = 0. \quad (13)$$

Scalar electrodynamics with arbitrary A_0 is considered in Sec. 3.

To retain in the theory nonvanishing charges Q^a, it is sufficient to restrict from the beginning the class of functions from which the A_0 are chosen in such a way as to exclude from it all nonvanishing constants. In what follows, we shall concentrate on the case when this is achieved by imposing on A_0 zero-value boundary conditions:

$$A^a_0(x) = 0, \quad x \in S. \quad (14)$$

Integrating in (7) by parts, we obtain a constraint in the form

$$\int (\partial_i B^a_i - J^a_0)\, A^a_0 d^3x \approx 0, \quad (15)$$

i.e.,

$$\xi^a(x) = \partial_i B^a_i - J^a_0 \approx 0$$

for all interior points.

We begin the detailed discussion of theories with the boundary condition (14) with the simplest model—scalar electrodynamics.

2. SCALAR ELECTRODYNAMICS WITH ZERO-VALUE BOUNDARY CONDITIONS FOR A_0

This section is devoted to the construction of the quantum theory of scalar electrodynamics in the case when $A_0(x)$ vanishes on the boundary of the volume:

$$A_0(x) = 0, \quad x \in S. \qquad (16)$$

The Lagrangian of scalar electrodynamics with a real representation of the scalar fields

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \quad \varphi^+ = \varphi^T$$

and generator

$$T = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is obtained from the general expression (1).

We shall be interested in the expressions for the asymptotic states in models with different forms of the scalar potential, and also the possibility of vacuum degeneracy with respect to the charge

$$Q = \int J_0 d^3x; \quad J_0 = ig\,(\mathbf{p}T\varphi). \qquad (17)$$

An important step in the solution of these questions will be the formulation of the classical theory in terms of gauge-invariant, physical, quantities. For this, it is sufficient to find a canonical transformation from the variables (φ, $\mathbf{p}$) and (A_r, B_r) to certain new variables such that in the new variables the constraint

$$\int (\partial_i B_i - J_0)\, A_0 d^3x \approx 0 \qquad (18)$$

is expressed solely in terms of the coordinate and momentum corresponding to just one degree of freedom ("diagonalization" of the constraint). Then all the remaining coordinates and momenta will certainly be physical.

We emphasize right away that there are different canonical transformations that diagonalize the constraints. The corresponding sets of new variables are, of course, themselves canonically equivalent. We first of all discuss the formulation of the theory in terms of the variables proposed by Dirac.[2]

The Dirac variables

Dirac variables are introduced by the canonical transformation

$$\left.\begin{aligned}
&(\varphi, \mathbf{p}, A_r, B_r) \to (\mathbf{\Phi}, \mathbf{P}, \bar{A}_r, \bar{B}_r);\\
&\mathbf{\Phi} = \exp\left(ig \int g_i(y, x)\, A_i(y)\, d^3yT\right)\varphi;\\
&\mathbf{P} = \exp\left(ig \int g_i(y, x)\, A_i(y)\, d^3yT\right)\mathbf{p};\\
&\bar{A}_i^{tr} = A_i^{tr}, \quad \bar{A}_i^{l} = A_i^{l};\\
&\bar{B}_i^{tr} = B_i^{tr};\\
&\bar{B}_i^{l} = B_i^{l}(x) - \int g_i(x, y)\, J_0(y)\, d^3y.
\end{aligned}\right\} \qquad (19)$$

We have here used the function $g_i(x, y)$, the gradient of the Green's function of the Laplacian with zero-value boundary conditions:

$$g_i(x, y) = -\partial_i G(x, y); \qquad (20)$$

$$\partial_i^x g_i(x, y) = -\Delta^x G(x, y) = \delta^3(x - y); \qquad (21)$$

$$G(x, y)\,|_{y \in S} = 0. \qquad (22)$$

The same function g_i is used when the longitudinal and transverse components of the vector field are separated, for example,

$$\left.\begin{aligned}
&A_i = A_i^l + A_i^{tr}\\
&A_i^l = \int g_i(x, y)\, \partial_k A_k(y)\, d^3y,
\end{aligned}\right\} \qquad (23)$$

where by virtue of the boundary condition (22)

$$\int d^3x A_i^l(x)\, A_i^{tr}(x) = 0.$$

The constraint (18) is then expressed solely in terms of the longitudinal field component $\bar{B}_i$:

$$\int \partial_i \bar{B}_i^l A_0(x)\, d^3x \approx 0.$$

Therefore, the variables $\mathbf{\Phi}$, $\mathbf{P}$, $\bar{A}_r^{tr}$, $\bar{B}_r^{tr}$ are physical. The longitudinal component $A_i^l(x)$ is obviously not physical for x within the volume. At the same time, since the constraint is absent on the boundary, one can in principle construct a physical variable from the boundary values of the field $A_i^l(x)$. It is sufficient to take the projection of this longitudinal component onto the plane tangent to the surface, i.e.,

$$\bar{A}_i^l(x) - n_i(x)\,(n \cdot \bar{A}^l)(x), \quad x \in S,$$

where $n(x)$ is the normal to the surface at the point x. However, for our choice of the boundary condition [see (22)] it simply vanishes. In other words, we have defined the longitudinal component of the vector field in such a way that it is entirely unphysical. Therefore, no locally gauge-invariant quantities arise apart from $\mathbf{\Phi}$, $\mathbf{P}\,\bar{A}_r^{tr}$, $\bar{B}_r^{tr}$.

Under global transformations of the original fields, only $\mathbf{\Phi}$ and $\mathbf{P}$ change, and therefore we say that they are charged, whereas $\bar{A}^{tr}$ and $\bar{B}^{tr}$ are neutral.

We rewrite the generalized Hamiltonian of the theory in terms of the variables (19). With allowance for the properties (20)–(22) of the function g_i it can be reduced to the form

$$\begin{aligned}
H_T = &\int d^3x \left\{\frac{1}{2}(\bar{B}_i^{tr})^2 + \frac{1}{4}(\bar{F}_{ij})^2 + \frac{1}{2}\mathbf{P}^2 + \frac{1}{2}(\partial_k \mathbf{\Phi} - igT\bar{A}_k^{tr}\mathbf{\Phi})^2 + V(\mathbf{\Phi})\right\}\\
&+ \frac{1}{2}\int d^3x d^3y J_0(x)\, G(x, y)\, J_0(y)\\
&+ \int d^3x \partial_i \bar{B}_i \left[A_0 - \int d^3y G(x, y)\left(J_0(y) + \frac{1}{2}\partial_i \bar{B}_i\right)\right].
\end{aligned} \qquad (24)$$

Since the last integral vanishes weakly, for physical quantities the Hamiltonian can be expressed solely in terms of $\mathbf{\Phi}$, $\bar{A}^{tr}$, and the conjugate momenta. The Coulomb interaction of localized objects is ensured by the well-known asymptotic behavior of $G(x, y)$ for large V:

$$G(x, y) = \frac{1}{4\pi\,|x - y|} + O\left(\frac{|x| + |y|}{V^{1/3}}\right).$$

Instead of diagonalizing the constraint, one can separate the physical variables by using a scheme with fixing of gauge conditions. This scheme reduces to choosing an arbitrary gauge-noninvariant functional $\chi(A_0)$, which depends on the generalized coordinates and momenta. Because $\chi(A_0)$ is gauge-noninvariant, i.e.,[3]

$$\det\{\chi(A_0),\ \xi(A_0')\}\neq 0, \tag{25}$$

it is possible to go over by means of a gauge transformation to variables for which simultaneously

$$\chi(A_0)=0;\quad \xi(A_0)=0. \tag{26}$$

The first of the conditions (26) is the gauge condition. By virtue of (25), the system (26) is, in the terminology of Dirac, a system of strong constraints and can be solved uniquely. As a result, the generalized Hamiltonian of the system is reduced to an ordinary Hamiltonian, containing one fewer degree of freedom, and all the remaining variables are physical. One can show that the variables (19) are equal to the fields in the Coulomb gauge

$$\int(\partial_i A_i)\,A_0(x)\,d^3x=0. \tag{27}$$

Colorless variables

The second choice of variables that we discuss is particularly interesting for the following reasons. In some recent studies[12–14] it has been argued that the asymptotic states in a theory with a nontrivial minimum of the scalar potential are invariant with respect to global charge transformations, i.e., they are colorless.

Colorless variables can be introduced by means of the canonical transformation

$$\begin{gathered}(\varphi,\ \mathbf{p},\ A_i,\ B_i)\to(\rho,\ p_\rho,\ \theta,\ p_\theta,\ \widetilde{A}_i,\ \widetilde{B}_i);\\ \varphi=\exp(i\theta t)\begin{pmatrix}0\\ \rho\end{pmatrix},\quad \rho=\sqrt{\varphi^2};\\ p_\rho=\frac{1}{\rho}(\mathbf{p}\varphi);\\ p_\theta=\frac{1}{g}\left(J_0-\partial_i B_i+\int ds_i(y)\,B_i(y)\,\delta(x-y)\right);\\ \widetilde{A}_i=A_i-\frac{1}{g}\partial_i\theta;\\ \widetilde{B}_i=B_i.\end{gathered} \tag{28}$$

Note that to ensure canonicity of the transformation everywhere, including the boundary, we have used in the expression for p_θ the surface integral $\int ds_i(y)B_i(y)\delta(x-y)$. Then the momentum p_θ becomes singular on the boundary. In the new variables, the constraint becomes

$$\int p_\theta(x)\,A_0(x)\,d^3x\approx 0,$$

i.e., $p_\theta(x)$ vanishes everywhere inside the volume. If $p_\theta(x)$ is continuous on the boundary, then we have $p_\theta\approx 0$ in the complete closed volume. Generally speaking, there are no reasons for extending the constraint to the points of the surface, so that in the general case

$$p_\theta(x)\approx\int ds(y)\,\gamma(y)\,\delta(x-y). \tag{29}$$

The form of the singularity is fixed by considerations relating to the integrability of p_θ. Since the charge can be expressed from (17), (28), and (29) as

$$Q=g\int p_\theta d^3x\approx g\int ds(y)\,\gamma(y),$$

γ acquires the significance of a surface density of charge.

By virtue of the boundary condition (22) the longitudinal component is orthogonal to the surface, and therefore, bearing in mind that in general $B_i^1(S)\neq\lim_{x\to S}B_i^1(x)$, we obtain

$$\partial_i B_i(x)=\partial_i B_i^{\mathrm{reg}}(x)+\int ds(y)\,(B_i n_i-\beta)(y)\,\delta(x-y);$$
$$\beta=\lim_{x\to S}(B_i\cdot n_i).$$

Here, $\partial_i B_i^{\mathrm{reg}}$ is the regular part, which can be assumed continuous from the interior.

Of the variables (28), ρ and $\widetilde{A}_i$ together with the conjugate momenta are physical, i.e., locally gauge-invariant, and so are $\theta(y)$ and $\gamma(y)$ if $y\in S$. Further,

$$\{\theta(y),\ \gamma(y')\}=\delta_S(y-y'),\ y,\ y'\in S,$$

where δ_S is the surface δ function:

$$\int\delta_S(y-y')\,ds(y')=1,\quad y\in S.$$

Under global transformations, only $\theta(y)$ changes:

$$\theta(y)\to\theta(y)+\text{const}.$$

The Hamiltonian of the theory can be written down using only the listed physical variables:

$$H\approx\int d^3x\left\{\frac{1}{2}\widetilde{B}_i^2+\frac{1}{4}\widetilde{F}_{ij}^2+\frac{1}{2}p_\rho^2+\frac{1}{2}(\partial_i\rho)^2+\frac{1}{2}g^2\rho^2\widetilde{A}_i^2+V(\rho)+\frac{1}{2g^2\rho^2}\left(\partial_i\widetilde{B}_i^{\mathrm{reg}}+\int ds(y)\,(g\gamma(y)-\beta(y))\,\delta(x-y)\right)^2\right\}. \tag{30}$$

Note that the expression (30) contains a dependence on the variables on the boundary. One can therefore expect the energy of the states to depend on their charge. This problem is investigated in detail below.

Since all the singularities have already been taken into account in the derivation of (30), we shall in what follows be concerned solely with the regular parts of the corresponding expressions, omitting the symbol "reg," believing that this will not cause confusion.

Just as the Dirac variables in the previous subsection became identical to the fields in the Coulomb gauge, the variables (28) become identical to the fields in the unitary gauge

$$\int\theta(x)\,A_0(x)\,d^3x=0. \tag{31}$$

It must be emphasized that the gauge condition (31) contains explicitly a dependence on the choice of the class of functions for A_0. In particular, when

$$A_0(S)=0$$

the condition (31) does not impose any restrictions on the values on the boundary. By the introduction of A_0 in the gauge condition we ensure the complete equivalence of the scheme with gauge fixing and the approach using weak conditions.

But if we fix the unitary gauge by the condition[13,14]

$$\theta(x)=0,\quad x\in\overline{V} \tag{32}$$

which is independent of the allowed functions A_0, this equivalence is lost. In addition, in this case there is no canonical connection between the variables in the different gauges. This is already evident from the fact that in the gauge (32) there are no charged quantities, whereas in the Coulomb gauge, for example, they are in general present.

In summary, we have at our disposal two sets of variables, and in both the constraint can be expressed solely in terms of one of the generalized momenta. As a consequence, we have achieved a gauge-invariant formulation of the theory—the Hamiltonian has been rewritten entirely in terms of physical degrees of freedom. There is canonical equivalence between these sets of variables. Our next task is to quantize the theory and find its ground state in the framework of the approximation of weak coupling. As we shall see, the different sets of variables are not equally suited to the construction of perturbation theory.

Perturbation theory and Bogolyubov transformation

When systems with weak constraints are quantized, ordinary commutation relations are fixed[1]:

$$[q,\, p] = \mathrm{i}\,\{q,\, p\}.$$

The weak constraints are transformed into conditions that distinguish physical states, in particular

$$\xi\,(A_0)\mid\Phi> = 0.$$

In Abelian gauge theory with small coupling constants, perturbation theory is valid. We shall now consider which variables are best adapted to the construction of perturbation theory for a particular form of the scalar potential. The question of the choice of the variables is intimately related to the problem of constructing asymptotic fields in the considered model.

In the framework of perturbation theory, one of the necessary requirements on the choice of the variables is the possibility of interpreting the quadratic part of the Hamiltonian, expressed in terms of these variables, as the Hamiltonian of free particles. In other words, these variables must be classified by representations of the Poincaré group in accordance with the masses obtained from the quadratic Hamiltonian. In what follows, we shall call this condition the requirement of diagonality of the quadratic part of the Hamiltonian. We shall establish how this condition is satisfied in particular variables for different forms of the scalar potential.

We consider first the case when the scalar potential has a trivial minimum, $\varphi = 0$. From the expressions (24) and (30) we separate the quadratic parts. In the Dirac picture, we obtain the diagonal expression

$$H_0 = \int d^3x \left\{ \frac{1}{2}(\overline{B}_i^{\mathrm{tr}})^2 + \frac{1}{4}\overline{F}_{ij}^2 + \frac{1}{2}\mathbf{P}^2 + \frac{1}{2}(\partial_i\Phi)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial\Phi_i\,\partial\Phi_j}\Big|_{\Phi=0}\Phi_i\Phi_j \right\}.$$

At the same time, the colorless variables lead to an expression singular for $\rho = 0$. Therefore, in the case of the trivial minimum of the scalar potential the Dirac fields can correspond to the asymptotic states.[2)]

Determining the perturbation-theory vacuum in the usual manner by means of the annihilation operators constructed from the fields $\mathbf{\Phi}$ and $\overline{A}_i^{\mathrm{tr}}$, we obtain a spectrum that consists of a charged, in general, massive scalar and a neutral massless vector particle with two polarizations.

We now consider a potential with a nontrivial minimum, $\varphi^2 = v$. We begin with the analysis of the Hamiltonian (24) in the Dirac variables. We set

$$\mathbf{\Phi} = \begin{pmatrix} 0 \\ v \end{pmatrix} \equiv \mathbf{v}.$$

It is important to emphasize that by itself such a choice has nothing in common with spontaneous symmetry breaking. We merely fix the point of the classical equilibrium, in the neighborhood of which we shall quantize.

We separate from (24) the bilinear part in terms of $\mathbf{\Phi}' = \mathbf{\Phi} - \mathbf{v}$:

$$H_0 = \int d^3x \left\{ \frac{1}{2}(\overline{B}_i^{\mathrm{tr}})^2 + \frac{1}{4}\overline{F}_{ij}^2 + \frac{1}{2}\mathbf{P}^2 + \frac{1}{2}(\partial_i\mathbf{\Phi}')^2 + \frac{1}{2}g^2v^2(\overline{A}_i^{\mathrm{tr}})^2 + \frac{1}{2}\frac{\partial^2 V}{\partial\Phi_i\,\partial\Phi_j}\Big|_{\Phi=\mathbf{v}}\Phi_i'\Phi_j' \right\} + \frac{1}{2}\int d^3x\, d^3y\, g^2v^2 P_1(x)\, G(x,\, y)\, P_1(y).$$

This expression is nondiagonal but can be diagonalized by the linear transformation

$$\widetilde{A}_i^{\mathrm{l}} = \frac{\partial_i\Phi_1'}{gv},$$

$$P_1 = \frac{1}{gv}\left(\partial_i\overline{B}_i^{\mathrm{l}} + \int ds\,(y)\,(g\gamma\,(y) - \beta\,(y))\,\delta\,(x-y)\right),$$

which is actually a Bogolyubov transformation.[20] The Hamiltonian becomes

$$H_0 = \int d^3x \left\{ \frac{1}{2}(\overline{B}_i^{\mathrm{tr}})^2 + \frac{1}{4}\overline{F}_{ij}^2 + \frac{1}{2}g^2v^2(\widetilde{A}_i^{\mathrm{l}})^2 + \frac{1}{2}g^2v^2(\overline{A}_i^{\mathrm{tr}})^2 + \frac{1}{2}(\widetilde{B}_i^{\mathrm{l}})^2 + \frac{1}{2}\frac{\partial^2 V}{\partial\Phi_2^2}\Big|_{\Phi_2=v}(\Phi_2')^2 + \frac{1}{2}(\partial_i\Phi_2)^2 + \frac{1}{2}P_2^2 + \frac{1}{2g^2v^2}\left(\partial_i\widetilde{B}_i^{\mathrm{l}} + \int ds\,(y)\,(g\gamma\,(y) - \beta\,(y))\,\delta\,(x-y)\right)^2 \right\}.$$

This is identical to the bilinear part of the Hamiltonian (30) in the colorless variables if we also set

$$\widetilde{A}_i^{\mathrm{tr}} = \overline{A}_i^{\mathrm{tr}}, \quad \widetilde{B}_i^{\mathrm{tr}} = \overline{B}_i^{\mathrm{tr}},$$

$$\rho = \Phi_2, \quad p_\rho = P_2.$$

Thus, in the case of a potential of Higgs type the colorless fields can be asymptotic. However, we have seen that when zero-value boundary conditions are imposed on A_0 there appear in addition to the colorless fields physical variables on the boundary, and these are charged. Therefore, the physical state space in perturbation theory can be represented as the product of the Fock space constructed by means of the colorless fields and the state space of the quantum-mechanical system $[\theta(S), \gamma(S)]$:

$$|\,\mathrm{ph}\,\rangle = |\,\mathrm{Fock}\,\rangle \times F\,(\theta\,(S)).$$

We now discuss the vacuum structure. We pose the problem as follows: To find the eigenstates of the Hamiltonian with minimal energy in the sector in which the operator of the surface charge density $\gamma(S)$ takes a definite value, say, $(1/g)f(S)$. If for the states of the quantum system (θ, γ) we choose the coordinate representation, i.e., the representation by functionals of the boundary values of $\theta(x)$, then the most general state of this sector can be written in the form

$$|\,\Phi\rangle_f = \exp\left[\frac{i}{g}\int f\,(S)\,\theta\,(S)\,ds\right]|\,\mathrm{Fock}\rangle. \tag{33}$$

The question is now the following: What Fock state must be chosen in order to make the vector $|\Phi\rangle$ an eigenvector for the total Hamiltonian? This is most readily done by arguing initially at the classical level.

Since in (30) the square of the δ function is a nonintegrable infinity, for states with finite energy it is necessary to set the difference $\beta - g\gamma$ equal to zero. Thus, we shall now seek the point of the classical minimum for the Hamiltonian (30) with allowance for the boundary condition

$$\widetilde{B}_i n_i \,|_S = f\,(S).$$

To this point there obviously correspond zero values of all the colorless variables except for the longitudinal component of the field $\widetilde{B}_i$. Substituting this component in the form

$$\widetilde{B}_i^1 = \partial_i u$$

for the "potential" u, we obtain, from the minimality of the quadratic Hamiltonian, the equation

$$\Delta u - m^2 u = 0; \quad m^2 = g^2 v^2 \tag{34}$$

with the boundary condition

$$\partial_n u \,|_S = f\,(S). \tag{35}$$

Indeed, the quadratic part of the Hamiltonian is

$$\Delta H = \int d^3x \left\{ \frac{1}{2} (\widetilde{B}_i^1)^2 + \frac{1}{2m^2} (\partial_i \widetilde{B}_i)^2 \right\}$$
$$= \int d^3x \left\{ \frac{1}{2} (\partial_i u)^2 + \frac{1}{2m^2} (\Delta u)^2 \right\}. \tag{36}$$

Varying with respect to u with the fixed conditions (35) on the boundary, we arrive at Eq. (34).

The solution of the boundary-value problem (34)–(35) is

$$u\,(x) = \int F\,(x,\,y)\,\psi\,(y)\,ds\,(y),$$
$$F\,(x,\,y) = \frac{\exp\,(-m\,|\,x-y\,|)}{4\pi\,|\,x-y\,|}, \tag{37}$$

where the function ψ is determined from the integral equation corresponding to the boundary condition (35).[19] Substituting (37) in (36), we obtain for the energy of the considered classical configuration

$$\Delta H = \int ds\,(y)\,ds\,(y')\,f\,(y)\,\psi\,(y')\,F\,(y,\,y'). \tag{38}$$

This expression admits a simple interpretation. We have here the energy of the Yukawa interaction of the surface charge having density $f(S)$ with the potential of the simple layer with density $\psi(S)$.

In the leading order in $mV^{1/3}$,

$$\psi\,(y) = f\,(y),$$

so that we obtain the possibility of estimating ΔH as a function of the total charge

$$Q = \int dsf\,(S).$$

Restricting ourselves, for example, to $f(S) = \text{const}$, we have

$$\Delta H \sim Q^2/(mV^{2/3}) \tag{39}$$

apart from terms of higher order in $mV^{1/3}$.

Our classical investigation makes it possible to recover the Fock state in (33) directly. In the coordinate representation for the longitudinal vector field, we have

$$|\,\Phi\rangle_f = \exp\left(\frac{i}{g}\int f\,(S)\,\theta\,(S)\,ds\right)$$
$$\times \exp\left(i\int \widetilde{A}_i^1\,(x)\,\partial_i u\,(x)\,d^3x\right)|\,\Phi\rangle_{f=0},$$

where u is determined in (37). The classical energy (38) can be identified with the difference between the energies of the "vacuum" with the given value $f(S)$ and the vacuum with $f(S) = 0$, the difference being calculated in the tree approximation. Since for $f(S) \neq 0$ the energy (38) is certainly positive, there is no vacuum degeneracy in the finite volume. However, if we first go to infinite volume and then consider the choice of the vacuum, degeneracy of the vacuum with respect to the charge does arise, as can be seen, for example, from (39).

Traditionally,[8,9] vacuum degeneracy and spontaneous symmetry breaking are regarded as an invariable sign of theories with massive vector bosons. However, we see that, strictly speaking, vacuum degeneracy and a mass of the vector particles are in no way related.[12–14] Indeed, the vacuum degeneracy that we have obtained in the limit $V \to \infty$ is directly related to the existence of physical variables on the boundary, and it, in its turn, is due to the postulated class of functions for A_0. For the formalism with arbitrary A_0 vacuum degeneracy with respect to the formal charge is obviously impossible.

At the same time, irrespective of whether or not there are any additional charged physical variables, the colorless Fock space of states above the vacuum $|0\rangle$ ensures the presence in the spectrum of a massive vector boson and a scalar particle.

The Higgs regime is characterized by the formation of a vacuum condensate of the colorless quantity ρ,[14]

$$\langle \text{vac}\,|\,\rho\,|\,\text{vac}\rangle = v \neq 0,$$

but this by itself does not, of course, indicate that there is symmetry breaking.

Nevertheless, vacuum degeneracy with respect to the charge in theories with definite restrictions on A_0 warrants an independent study. A natural question that arises in this case is this: Does the existence of many vacua lead to new physical effects compared with the theory that uses one of them? As examples we may take the degeneracy with respect to the topological number in non-Abelian theories and the related θ structure of the vacuum.

It is clear that in our case there are no transitions between the different vacua, since the operator γ commutes with the Hamiltonian. In addition, in the framework of perturbation theory the dynamics of the colorless fields in the limit of infinite volume does not depend on the choice of the vacuum state. This is an obvious consequence of the fact that all interactions in a theory with massive vector bosons vanish exponentially with the distance, and therefore the influence of a distant surface charge density on physical processes can be ignored.

With this we conclude the discussion of the theory with A_0 that vanish on the boundary and now turn to the case of arbitrary A_0.

3. SCALAR ELECTRODYNAMICS WITH ARBITRARY A_0

As already noted, a distinctive feature of the theory with arbitrary A_0 is the vanishing of the global electric charge of all physical states:

$$Q = \int J_0 d^3x \approx 0. \tag{40}$$

In the case when the scalar potential has a nontrivial minimum, no problems with the gauge-invariant formulation arise. The theory can be formulated entirely in terms of colorless variables, and there is no vacuum degeneracy. The vector bosons acquire mass by the formation of a condensate $\langle 0|\rho|0\rangle$, where the field ρ is introduced in (28).

In what follows, we shall therefore concentrate our attention on theories with a trivial minimum of the scalar potential. Here, a very interesting situation arises.

For the formulation of the theory without a condensate in Sec. 2, we used the Dirac variables (19), which include among their number the charged fields $\mathbf{\Phi}$ and $\mathbf{P}$. However, these expressions are now not physical. Therefore, the problem arises of finding variables suitable for constructing perturbation theory in the case of a trivial scalar potential.

We now show that any choice of locally and globally invariant variables that depend on only one spatial coordinate leads to singularities in the equations of motion. This is most readily seen in the formalism with gauge fixing. We consider the constraint (40) and choose a function $\chi(\varphi, A_i,...)$ that satisfies the condition of nondegeneracy

$$\int d^3x \left(\frac{\delta\chi}{\delta\varphi_2(x)} \varphi_1(x) - \frac{\delta\chi}{\delta\varphi_1(x)} \varphi_2(x) \right) \neq 0, \quad (41)$$

at least near the points $\varphi = 0$ of the classical equilibrium. From (41), we obtain

$$\{Q, \chi\} \neq 0$$

i.e., χ is nonanalytic at the point $\varphi = 0$:

$$\frac{\delta\chi}{\delta\varphi_2} \sim \frac{1}{\varphi_1}, \quad \frac{\delta\chi}{\delta\varphi_1} \sim \frac{1}{\varphi_2}.$$

The singularity of the gauge condition leads to nonanalyticity at the point $\varphi = 0$ of the equations of motion for the variables that could play the part of a colorless "electron" or "positron."[17] Thus, a gauge-invariant formulation of the theory in terms of local (i.e., dependent only on one spatial coordinate) quantities having nonsingular equations of motion is impossible. Ultimately, this is due to the fact that the global transformations in no way rotate the point of the classical equilibrium.

But there are physical variables that depend on several spatial coordinates for which singularities do not arise in the equations of motion. Consider, for example,

$$u(x, y, t) = \varphi(x, t) \times \exp\left[-ig\int g_i(x, y, z) A_i(z, t)\, d^3z\right] \varphi^*(y, t), \quad (42)$$

where

$$\varphi = i\varphi_1 + \varphi_2;$$
$$g_r(x, y, z) = \partial_r^z G(x, y, z);$$
$$\Delta^z G(x, y, z) = -\delta^3(x-z) + \delta^3(y-z);$$
$$g \cdot n(z)\,|_{z\in S} = 0.$$

The expression (42) can be regarded as a pair consisting of a particle and an antiparticle at the points x and y, respectively.

Our next task will be to obtain a number of ordinary electrodynamical results in a scheme in which the concept of a charged particle is absent. The idea of what follows is that when the spatial coordinates x and y are separated the expression (42) retains its formal uncharged nature, while at the same time the principle of correlation weakening comes into play.[3)] We define the vacuum by the conditions

$$Q\,|\,\text{vac}\,\rangle = 0; \quad H\,|\,\text{vac}\,\rangle = 0.$$

We note that the vacuum introduced in this manner is identical to the vacuum in the formalism with $A_0(S) = 0$.

We now show how it is possible to define the total propagator of the "electron," although asymptotic fields corresponding to one particle do not exist in the theory. We consider the Green's function

$$G(x, x', y, y', t_1, t_2) = \langle \text{vac}\,|\,Tu(x, x', t_1)\,u(y', y, t_2)\,|\,\text{vac}\rangle.$$

The assertion of correlation weakening (cluster decomposition property)[20,21] is as follows:

$$G(x, x', t_1, y, y', t_2) \to G_1(x, y, t_1, t_2) \times G_2(x', y', t_1, t_2) \text{ as } |x - x'| \to \infty, \ |y - y'| \to \infty. \quad (43)$$

The transition of the distances in (43) to infinity is understood in the sense that the points x and y remain fixed while their "partners" x' and y' tend to the boundary of the volume, so that (43) is valid to terms $O(V^{-1/3})$. The assertion (43) can be verified explicitly in the framework of perturbation theory. We now define the total electron propagator by setting

$$\Pi(x, t_1, y, t_2) = G_1(x, y, t_1, t_2). \quad (44)$$

We can define similarly any other Green's functions. One can show[17] that they are identical to the Green's functions with $A_0(S) = 0$ in the large-volume limit. This identity holds, of course, only in the case when the calculations in the theory with $A_0(S) = 0$ and in the theory with arbitrary A_0 are made in the same gauge. For example, the total electron propagator determined in (44) is equal to the analogous Green's function for the Dirac variables (19), since in both (19) and (42) the functions g_i are chosen in purely longitudinal form (Coulomb gauge).

Thus, the presence of the constraint (40) does not lead to difficulties in the calculation of the observables, since this constraint can be satisfied by introducing "compensating" charges in a very distant region, so that their influence on the physics on ordinary scales will be negligible.

Following the same philosophy, we define the physical charge

$$Q_{\text{ph}} = \int_{V_0} J_0 d^3x,$$

where V_0 is a certain macroscopic interior region. The significance of such a definition is very transparent: Q_{ph} is the charge contained in the region V_0. The physical charge Q_{ph} does not vanish weakly and has a number of properties necessary from the practical point of view:

1. $$[Q_{\text{ph}}(t), u(x, y, t)] = \begin{cases} gu, & x \in V_0, \ y \notin V_0, \\ -gu, & x \notin V_0, \ y \in V_0, \\ 0, & x, y \in V_0. \end{cases}$$

We recall that the commutator of the "formal" charge Q with u is equal to zero independently of x and y.

2. To calculate the physical charge, we can use Gauss's theorem

$$Q_{\text{ph}} = \int_{S_0} ds_i B_i.$$

3. Generally speaking, Q_{ph} is not conserved, but this

nonconservation is entirely due to the possibility of particles leaving the region V_0:

$$\dot{Q}_{\rm ph} = \frac{g}{2i}\int\limits_{S_0} ds_i\,(D_i\varphi^*\cdot\varphi - D_i\varphi\cdot\varphi^*).$$

We note that states with nonvanishing physical charge also exist in the theory with a scalar condensate. Consider, for example, the state

$$|\Psi\rangle = \exp\{ig\int d^3y g_i(y, x)\,A_i(y)\}\,|\,0\,\rangle \equiv \Psi(x)\,|\,0\rangle,$$

which is a coherent superposition of longitudinal components of the massive vector field with Coulomb behavior of the electric field intensity,

$$\langle\Psi\,|\,B_i(z)\,|\,\Psi\rangle \sim \frac{(z-x)_i}{|\,z-x\,|^3},$$

and for $x\in V_0\,|\Psi\rangle$ has physical charge equal to g.

However, since $|\Psi\rangle$ is not an eigenstate of the free part of the Hamiltonian, such a state is unstable. In addition, the amplitude for production of such states in local collisions is exponentially suppressed because the vector field is massive. Indeed, let the vector $|\text{in}\rangle$ denote a set of colorless particles that enter into a reaction near the origin. We calculate the mean physical charge of the scattering state at the time t:

$$\langle\text{in}\,|\,\exp(iHt)\,Q_{\rm ph}\exp(-iHt)\,|\,\text{in}\rangle$$
$$= \int ds_i\,(y)\,\langle\text{in}\,|\,B_i\,(y, t)\,|\,\text{in}\rangle \sim \frac{\exp(-mr)}{r},$$

where r is the radius of the region V_0 and m is the mass of the vector particle.

Summarizing the content of this section, we emphasize once more that the vanishing of the generator Q on the complete physical space does not contradict the existence of a local electric charge $Q_{\rm ph}$. It is important to note that the possibility of defining $Q_{\rm ph}$ in the Abelian theory is related to the fact that the zeroth component J_0 of the current is gauge-invariant. In the non-Abelian theories, to the discussion of which we now come, the J_0^a are not invariant even with respect to local transformations. This leads to certain difficulties when "physical" non-Abelian charges are introduced. However, we shall not dwell on this question, since in what follows we shall discuss exclusively non-Abelian theories with weak coupling, in which an emitted charge is always Abelian.

4. NON-ABELIAN GAUGE THEORIES

In this section, we consider non-Abelian theories with weak coupling, so that all the methods used earlier for scalar electrodynamics apply. In the non-Abelian models, weak coupling is ensured by the formation of a large scalar condensate,

$$\langle\varphi^+\varphi\rangle \gg \Lambda^2,$$

where Λ is the reciprocal confinement radius; this can be achieved, for example, by choosing a scalar potential that has a nontrivial minimum.[4] In addition, the "surviving" massless vector bosons must belong to the Abelian subgroup.

Although the last requirement strongly restricts the class of theories that come within our ambit, the generalization of the Abelian results is nevertheless not entirely direct on account of the possible presence in the non-Abelian case of massless vector bosons. In what follows, theories with massless vector bosons will in fact be the main subject of our consideration. More precisely, we shall be interested in realization of the algebra of the non-Abelian charges Q^a,

$$\{Q^a,\ Q^b\} = gf^{abc}Q^c, \tag{45}$$

in the physical state space.

Since in the formalism with arbitrary A_0 we have

$$Q^a \approx 0,$$

the realization of the algebra (45) is nontrivial only for certain restrictions on A_0. As in the Abelian case, we impose on A_0 zero-value boundary conditions. We note that the charges Q^a are locally gauge-invariant. We shall see that the validity of (45) is ensured by the presence of physical variables on the boundary of the volume.

It is helpful to begin the discussion of the non-Abelian theories with a general group-theoretical analysis.

General treatment

In this subsection, we shall mainly follow Weinberg's paper,[5] but we draw particular attention to the part played by the adopted restrictions on A_0. If the scalar potential $V(\varphi)$ has a nontrivial minimum, then by virtue of the symmetry this minimum is necessarily degenerate. We choose some vector φ_0 that minimizes $V(\varphi)$. The degeneracy is manifested in the fact that in the space of the vectors φ there exist directions in which motion from the point φ_0 is not accompanied by a change in $V(\varphi)$. These flat directions are specified by the vectors $T^a\varphi_0$, where $a = 1,\ldots,N$, N being the dimension of the group. In principle, some of these vectors may be linearly dependent. In other words, one can form orthogonal linear combinations θ_i of the generators T^a such that

$$\begin{aligned}\theta_i\varphi_0 &\neq 0,\quad i = 1,\ \ldots,\ m;\\ \theta_i\varphi_0 &= 0,\quad i = m+1,\ \ldots,\ N.\end{aligned} \tag{46}$$

The projections of an arbitrary $\varphi(x)$ onto the first m vectors (46)

$$\Pi_i = (\varphi\theta_i\varphi_0) \tag{47}$$

for x strictly within the volume are gauge-noninvariant quantities—they are "Goldstone" fields. Therefore, one can choose a local gauge transformation

$$\varphi \to \tilde{\varphi} = U\varphi,$$

in such a way that everywhere within the volume

$$(\varphi\,(x)\,\theta_i\varphi_0) = 0,\quad i = 1,\ \ldots,\ m. \tag{48}$$

This is equivalent to fixing the unitary gauge for all the interior points.

In contrast, for x lying on the boundary the fields Π_i are not affected by the local gauge transformation and, thus, are included among the physical variables. As in the Abelian case, we now introduce the vector field

$$\left.\begin{aligned}\tilde{A}_i &= U(\varphi)\left(A_i - \frac{1}{ig}\,\partial_r\right)U^{-1}(\varphi),\\ \hat{A} &= A^n\theta^n\end{aligned}\right\} \tag{49}$$

by means of the matrix U. The mass matrix of the vector fields is[5]

$$M_{ab} = g^2\,(\theta_a\varphi_0,\ \theta_b\varphi_0),$$

i.e., m fields are massive, and the remainder are massless.

In the Abelian theory, the field (49) was invariant with respect to both the local and the global transformations. Now there are two cases:

1. The matrix $U(\varphi)$ for given φ is determined uniquely. Then for any matrix Λ of the gauge group

$$U(\Lambda\varphi)\,\Lambda\varphi = \tilde{\varphi},$$

i.e.,

$$U(\Lambda\varphi)\,\Lambda = U(\varphi). \tag{50}$$

From (50), we find that $\tilde{A}_i$ is invariant with respect to all transformations.

2. There is a complete family of $U(\varphi, \alpha)$ such that for any α

$$(U(\varphi, \alpha)\,\varphi(x),\ \theta_i\varphi_0) = 0,\ x \in V\setminus S,\ i = 1, \ldots, m.$$

A necessary and sufficient condition for the existence of a nontrivial α set is that in the gauge group the subgroup that carries φ_0 into itself should be not equal to the identity. The generators of this subgroup are the θ_i for which[5]

$$\theta_i\varphi_0 = 0,$$

i.e., precisely the $N-m$ generators to which massless vector bosons correspond. The presence of massless vector bosons is, thus, a criterion for the realization of the given case.

In the simple examples that we shall consider in what follows, there exists a unique vector $\tilde{\varphi}$ satisfying (48). In this case, for all α

$$U(\varphi, \alpha)\,\varphi = \tilde{\varphi},$$

and instead of (46) we can consider the equivalent condition

$$\theta_i\tilde{\varphi} = 0, \quad i = m+1, \ldots, N. \tag{46a}$$

Instead of (50), we now obtain

$$U(\Lambda\varphi, \alpha)\,\Lambda = U(\varphi, \alpha'), \tag{51}$$

where α and α' are in general different. As a consequence, the vector field (49) [without loss of generality, it can be assumed that it is introduced by $U(\varphi) = U(\varphi, \alpha = 0)$] will no longer be either globally or locally invariant. Under the gauge transformation Λ it is changed by means of the matrix $U(\varphi, \alpha')U^+(\varphi)$, where α' is determined as a function of Λ from (51) with $\alpha = 0$.

In specific models, having determined $\alpha'(\Lambda)$, one can then construct physical (locally invariant) fields, which, in general, will not be colorless.

Putting it in other words, in the presence of massless vector bosons the number m of nonvanishing fields Π_i (47) is less than the dimension of the group, and therefore the unitary gauge (48) cannot fix the complete existing arbitrariness. We require additionally $N-m$ further conditions, and these can be imposed now only on the vector fields. As is well known,[14] conditions of this type are not capable of ensuring global invariance.

Of course, charged variables are also present when all the vector bosons are massive. However, in this case, as in the Abelian theory, the entire charge is concentrated in the variables $\Pi_i(S)$ on the boundary, and these bear no relationship to the ordinary Fock space.

Thus, in the non-Abelian theories, depending on the group and the representation of the scalar fields, either the Fock space may be made completely colorless or it may contain color states. In the following subsections of this section we shall consider examples that illustrate this.

SU(2) model with scalars in the fundamental representation

The first example that we shall discuss—$SU(2)$ theory with a complex doublet—realizes the case of complete "decoloring" and in many respects is analogous to Abelian theory.

Since the general treatment (as in Sec. 1) has been given for real representations of the scalar fields, we shall find it convenient here to adopt a form of expression for φ that differs somewhat from the usual one, namely, we represent it in the form of a four-component column (all the components are real):

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}.$$

Thus, we have gone over to the fundamental representation of the group $SO(4)$, and we shall be interested in its $SU(2)$ subgroup with generators

$$\left.\begin{aligned} &T^1 = \frac{1}{2}\begin{pmatrix} \cdot & \cdot & \cdot & \mathrm{i} \\ \cdot & \cdot & -\mathrm{i} & \cdot \\ \cdot & \mathrm{i} & \cdot & \cdot \\ -\mathrm{i} & \cdot & \cdot & \cdot \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} \cdot & \cdot & \mathrm{i} & \cdot \\ \cdot & \cdot & \cdot & \mathrm{i} \\ -\mathrm{i} & \cdot & \cdot & \cdot \\ \cdot & -\mathrm{i} & \cdot & \cdot \end{pmatrix}, \\ &T^3 = \frac{1}{2}\begin{pmatrix} \cdot & -\mathrm{i} & \cdot & \cdot \\ \mathrm{i} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \mathrm{i} \\ \cdot & \cdot & -\mathrm{i} & \cdot \end{pmatrix}. \end{aligned}\right\} \tag{52}$$

The other three generators of $SO(4)$ also satisfy the $SU(2)$ algebra. The corresponding $SU_f(2)$ group is a global symmetry group of the Lagrangian. This is due to the fact that the fundamental representation of $SU(2)$ is pseudoreal, i.e., equivalent to its adjoint. In the given case, we can realize a canonical transformation to colorless variables that is analogous to the one discussed in Sec. 2 for the case of scalar electrodynamics:

$$(\varphi, \mathbf{p}, A_r^a, B_r^a) \to (\rho, \theta^a, p_\rho, \pi^a, \tilde{A}_r^a, \tilde{B}_r^a), \quad a = 1, 2, 3;$$

$$\left.\begin{aligned} &\varphi = \exp(2\mathrm{i}\theta^a T^a)\begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho \end{pmatrix} \equiv U^+\begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho \end{pmatrix}; \\ &p_\rho = (\mathbf{p}\varphi)/\rho; \\ &\hat{\tilde{A}}_r = \exp(-2\mathrm{i}\theta T)\left(\hat{A}_r - \frac{1}{\mathrm{i}g}\partial_r\right)\exp(2\mathrm{i}\theta T); \\ &\hat{\tilde{B}}_r = \exp(-2\mathrm{i}\theta T)(\hat{B}_r)\exp(2\mathrm{i}\theta T) \end{aligned}\right\} \tag{53}$$

where θ^a is determined from the equation

$$\frac{\theta^a}{|\theta|}\tan|\theta| = -\frac{\varphi_a}{\varphi_4}, \quad a = 1, 2, 3,$$
$$|\theta| = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}, \tag{54}$$

and π^a from the linear system

$$f^{abc}\theta^b\pi^c - \frac{\theta^a(\pi^b\theta^b)}{|\theta|^2} + \frac{\theta^a(\pi^b\theta^b)}{|\theta|}\cot|\theta| - \pi^a|\theta|\cot|\theta|$$
$$= \frac{1}{g}\left(\xi^a - \int ds_i(y)\,B_i(y)\,\delta(x-y)\right);$$
$$\xi^a = \partial_i B_i^a - J_0^a. \qquad (55)$$

The rather cumbersome expressions (54) and (55) are derived in the Appendix.

In connection with the foregoing general treatment, we emphasize that the matrix $U(\varphi)$ that carries φ into ρ,

$$U(\varphi) = \exp(-2i\theta T) = \frac{1}{\rho}\begin{pmatrix} \varphi_4 & \varphi_3 & -\varphi_2 & -\varphi_1 \\ -\varphi_3 & \varphi_4 & \varphi_1 & -\varphi_2 \\ \varphi_2 & -\varphi_1 & \varphi_4 & -\varphi_3 \\ \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{pmatrix}, \qquad (56)$$

is in the given case unique, so that the fields ρ and $\widetilde{A}_r$ and their conjugate momenta are colorless. The Hamiltonian (4) can be entirely rewritten in terms of the variables (52). The only difficulty here—and it is a purely technical one—is the expression in terms of (52) of the momentum contribution

$$\Delta H = \int \frac{1}{2}\mathbf{p}^2 d^3x. \qquad (57)$$

It is convenient to go over from the momenta p_i ($i = 1, 2, 3, 4$) to the momenta p'_i:

$$\mathbf{p}' = U\mathbf{p}$$

[p'_i must not be confused with (p_ρ, π^a)!], so that ΔH has the form

$$\Delta H = \int d^3x \frac{1}{2} p_i'^2.$$

Instead of the system of constraints (7), we can now introduce the equivalent system of constraints $\tilde{\xi}^a$, where $\tilde{\xi}^a(\varphi', \mathbf{p}', \widetilde{A}_r, \widetilde{B}_r)$[4)] are the same functions (but of different variables) as $\xi^a(\varphi, \mathbf{p}, A_r, B_r)$. These systems of constraints are equivalent because the transition $(\varphi, \mathbf{p}, A_r, B_r) \to (\varphi', \mathbf{p}', \widetilde{A}_r, \widetilde{B}_r)$, which is realized by the matrix U, is simply a gauge transformation.[5)] Therefore, $\tilde{\xi}$ are more or less complicated linear combinations of the ξ.

Using now the concrete form

$$\varphi' = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho \end{pmatrix},$$

we obtain from the equations

$$\tilde{\xi}^a = \partial_i \widetilde{B}_i^a - g f^{abc} \widetilde{B}_i^b \widetilde{A}_i^c + \frac{g\rho}{2} p^{a\prime}$$

and

$$p_\rho = p'_4$$

the following expression for ΔH (57):

$$\Delta H = \int d^3x \frac{1}{2}\frac{(\tilde{\xi}^a - \partial_i\widetilde{B}_i^a + g f^{abc}\widetilde{B}_i^b\widetilde{A}_i^c)^2}{g^2\rho^2} + \int d^3x \frac{1}{2} p_\rho^2,$$

where $\tilde{\xi}^a \approx 0$ within the volume. This is completely analogous to the corresponding contribution in Abelian theory. The total Hamiltonian H is

$$H = \int d^3x \left\{ \frac{1}{2}(\widetilde{B}_r^a)^2 + \frac{1}{4}(\widetilde{F}_{ij}^a)^2 + \frac{1}{2}(\partial_i\rho)^2 + \frac{1}{8} g^2\rho^2 (\widetilde{A}_i^a)^2 + V(\rho) \right\} + \Delta H. \qquad (58)$$

The physical variables in this picture are ρ and $\widetilde{A}_r$ (and the conjugate momenta), and they are, moreover, globally invariant, i.e., colorless[6)]; besides them, as in the Abelian case, there are the variables θ and π on the boundary. In general, the latter are charged and lead in the limit $V \to \infty$ to vacuum degeneracy. The question of degeneracy is studied in exactly the same way as was done in Sec. 2, in view of the above-mentioned analogy (58) with the corresponding Abelian expression.

Thus, the $SU(2)$ model with scalars in the fundamental representation is completely analogous to scalar electrodynamics, and, without going into details, we formulate the results. In the theory, the physical variables forming the Fock space are colorless, so that all the charges in the Fock space are screened.[14] There are (in connection with the choice of the class of the functions A_0) additional physical variables, charged in general, and these lead to vacuum degeneracy in the limit $V \to \infty$. However, this phenomenon is in no way related to the fact that the vector bosons have a mass, since the Higgs mechanism is characterized[14] by the fact that the colorless field ρ has a vacuum condensate:

$$\langle 0 \mid \rho \mid 0 \rangle \neq 0.$$

To realize the less trivial case of incomplete "decoloring," it is necessary to choose the group and representation of the scalar fields in such a way that the number of "phases" of the scalar field is less than the number of group parameters. This can be readily achieved by a simple modification of the already considered example. Namely, we propose to extend the group by adding to it an invariant $U(1)$ factor.

Incomplete "decoloring" in the case of the group $SU(2)\times U(1)$

The theory that we now consider is actually the boson sector of the standard Weinberg–Salam model.

Besides the three generators (52) there is also the generator of the $U(1)$ subgroup,

$$Y = \frac{1}{2}\begin{pmatrix} \cdot & -i & \cdot & \cdot \\ i & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -i \\ \cdot & \cdot & i & \cdot \end{pmatrix},$$

to which there corresponds the Abelian gauge field A_i^0. This makes it possible to construct the linear combination

$$Q = T^3 + Y,$$

which vanishes on the vector

$$\tilde{\varphi} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho \end{pmatrix}, \quad Q\tilde{\varphi} = 0,$$

i.e., (46) holds.

Thus, the matrix of the transition from φ to $\tilde{\varphi}$ contains a one-parameter freedom,

$$U(\varphi, \alpha) = \exp(i\alpha Q)\,U(\varphi),$$

where $U(\varphi)$ is the matrix (56).

The matrix $U(\varphi, \alpha)$ factorizes:

$$U(\varphi, \alpha) = U_1(\alpha)\,U_2(\varphi, \alpha),$$

so that $U_1(\alpha) = \exp(i\alpha Y)$ belongs to the $U(1)$ subgroup and $U_2(\varphi, \alpha) = \exp(i\alpha T^3) U$ to the $SU(2)$ subgroup.

The formulas for the transition to the new variables (49) must be modified to take into account the semisimple structure of the group:

$$\left.\begin{aligned} \hat{A}_r &= U_2(\varphi, \alpha)\left(A_r - \frac{1}{ig}\partial_r\right)U_2^+; \\ \widetilde{A}_r^0 &= A_r^0 + \frac{1}{g'}\partial_r\alpha, \end{aligned}\right\} \quad (59)$$

where g and $g'$are, respectively, the non-Abelian and the Abelian coupling constants.

We now clarify the properties of the variables (59) under the gauge transformation

$$\Lambda = \exp(iT^a\lambda^a + iY\lambda_0). \quad (60)$$

As we have pointed out, for this it is necessary to determine $\alpha'(\lambda)$ from (51) (in what follows, without loss of generality, we take $\alpha = 0$). Since from (51)

$$\alpha' = \lambda_0,$$

we have under the transformation (60)

$$\widetilde{A}_r^a T^a \to \exp(iT^3\lambda_0)\left(\widetilde{A}_r^a T^a - \frac{1}{ig}\partial_r\right)\exp(-iT^3\lambda_0),$$

$$a = 1, 2, 3;$$

$$\widetilde{A}_r^0 \to \widetilde{A}_r^0 + \frac{1}{g'}\partial_r\lambda_0.$$

In other words, the variables (59) are completely invariant with respect to the $SU(2)$ subgroup, but they are not invariant with respect to $U(1)$ and are therefore unphysical.

Note that the third component $\widetilde{A}_r^3$, like the Abelian field $\widetilde{A}_r^3$ under (60), undergoes only a gradient stretching,

$$\widetilde{A}_r^3 \to \widetilde{A}_r^3 + \frac{1}{g}\partial_r\lambda_0,$$

so that the linear combination

$$Z_r = \frac{g\widetilde{A}_r^3 + g'A_r^0}{\sqrt{g^2 + g'^2}} \quad (61)$$

is invariant with respect to all transformations. The linear combination

$$E_r = \frac{1}{\sqrt{g^2 + g'^2}}(g\widetilde{A}_r^0 + g'\widetilde{A}_r^3),$$

which is orthogonal to (61), changes in accordance with the law

$$E_r \to E_r + \frac{1}{e}\partial_r\lambda_0, \quad e = \frac{gg'}{\sqrt{g^2 + g'^2}}. \quad (62)$$

By means of the fields (61) and (62) it is possible to construct a set of physical variables:

$$\left.\begin{aligned} &\rho,\ Z_r,\ E_r^{tr}; \\ &W_r^i T^i = V(\widetilde{A}_r^i T^i)V^+, \quad i = 1, 2, \end{aligned}\right\} \quad (63)$$

where

$$V = \exp\left[ie\int g_r(y, x)\,E_r(y)\,Qd^3y\right].$$

We have here used the same function g_r as in Sec. 2 in the analysis of Abelian theory. This function g_r is the gradient of the Green's function of the Laplacian with zero-value boundary conditions, so that

$$\partial_r^x g_r(x, y) = \delta(x - y).$$

The conjugate momenta can be obtained similarly. In addition, the set (63) must be augmented by the physical variables on the boundary.

Note that the quantities (63) are equal to the fields in the "unitary–Coulomb" gauge:

$$\varphi_1(x) = \varphi_2(x) = \varphi_3(x) = 0; \quad \partial_i E_i(x) = 0$$

for all x strictly within the volume.

We now consider the charge properties of the Fock states obtained on quantization of the theory in terms of the variables (63). The perturbation-theory spectrum consists of three massive gauge bosons, one massless particle, and a Higgs particle. The charges corresponding to the $SU(2)$ generators are completely screened in the Fock space. Only the charge corresponding to the generator Y is nonzero. It takes unit value for the linear combinations

$$W^- = \frac{1}{\sqrt{2}}(W^1 - iW^2); \quad W^+ = \frac{1}{\sqrt{2}}(W^1 + iW^2).$$

The remaining particles are neutral. Therefore, this charge can be identified with the electric charge.[7)]

The appearance of a mass of the $W^{\pm}$ and Z bosons is explained solely by the presence of the condensate $\langle 0|\rho|0\rangle$ and has nothing in common with spontaneous symmetry breaking.

Thus, in the $SU(2)\times U(1)$ model only one out of the four charges survives in the Fock space. This one charge is well defined in perturbation theory and takes definite values on the Fock states. It should be noted that this picture is largely due to the simplifications that arise from the group's semisimple structure. In a certain sense, the most general scenario is realized in the case of $SU(2)$ theory with a triplet of scalars.

SU(2) theory with a triplet of scalars

To the triplet (real) representation

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

there correspond the generators

$$T_1 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & -i \\ \cdot & i & \cdot \end{pmatrix}; \quad T_2 = \begin{pmatrix} \cdot & \cdot & i \\ \cdot & \cdot & \cdot \\ -i & \cdot & \cdot \end{pmatrix}; \quad T_3 = \begin{pmatrix} \cdot & -i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}.$$

Since for the standard vector

$$\widetilde{\varphi} = \begin{pmatrix} 0 \\ 0 \\ \rho \end{pmatrix}, \quad \rho = \sqrt{\varphi^2}$$

we have

$$T_3\widetilde{\varphi} = 0,$$

we have in this case too an arbitrariness in the matrix of the transition from φ to $\widetilde{\varphi}$:

$$U(\varphi, \alpha) = \exp(iT_3\alpha) \begin{pmatrix} \frac{\varphi_2^2}{l^2} + \frac{\varphi_1^2}{l^2}\frac{\varphi_3}{\rho} & \frac{\varphi_1\varphi_2}{l^2}\left(\frac{\varphi_3}{\rho} - 1\right) & -\frac{\varphi_1}{\rho} \\ \frac{\varphi_1\varphi_2}{l^2}\left(\frac{\varphi_3}{\rho} - 1\right) & \frac{\varphi_1^2}{l^2} + \frac{\varphi_2^2}{l^2}\frac{\varphi_3}{\rho} & -\frac{\varphi_2}{\rho} \\ \frac{\varphi_1}{\rho} & \frac{\varphi_2}{\rho} & \frac{\varphi_3}{\rho} \end{pmatrix} \equiv \exp(iT_3\alpha) B(\varphi),$$
$$l^2 = \varphi_1^2 + \varphi_2^2, \tag{64}$$

in which α is an arbitrary parameter, and therefore the quantities analogous to (49),[8)]

$$\rho = \sqrt{\varphi^2}; \quad p_\rho = \frac{1}{\rho}(\mathbf{p}\varphi);$$
$$\hat{\tilde{A}}_r = B(\varphi)\left(\hat{A}_r - \frac{1}{ig}\partial_r\right)B^+(\varphi);$$
$$\hat{\tilde{B}}_r = B(\varphi)\hat{B}_r B^+(\varphi) \tag{65}$$

have nontrivial transformation properties with respect to the gauge group:

$$\left.\begin{aligned} &\rho, \ p_\rho \text{ — are invariant;} \\ &\hat{\tilde{A}}_r \to X(\varphi, \alpha')\left(\hat{\tilde{A}}_r - \frac{1}{ig}\partial_r\right)X^+; \\ &\hat{\tilde{B}}_r \to X(\varphi, \alpha')\tilde{B}_r X^+; \\ &X(\varphi, \alpha') = U(\varphi, \alpha')U^+(\varphi, \alpha = 0). \end{aligned}\right\} \tag{66}$$

From the general form of (64) we directly obtain $X(\varphi, \alpha') = \exp(iT_3\alpha')$, where α' is determined from the condition (51):

$$U(\varphi, \alpha') = U(\Lambda\varphi, \alpha = 0)\,\Lambda. \tag{67}$$

It is simplest to solve (67) for $\Lambda = \Lambda_3 = \exp(iT_3\lambda_3)$. At the same time,

$$\alpha' = \lambda_3.$$

When $\Lambda = \Lambda_1 = \exp(iT_1\lambda_1)$ or $\Lambda = \Lambda_2 = \exp(iT_2\lambda_2)$, the expression for α' is much more complicated. For our purposes, it will be sufficient to give it for small λ_1 and λ_2, restricting ourselves to the first approximation.

We obtain, respectively,

$$\alpha' = -\lambda_1 \frac{\varphi_1\rho}{l^2}\left(-1 + \frac{\varphi_3}{\rho}\right);$$
$$\alpha' = -\lambda_2 \frac{\varphi_2\rho}{l^2}\left(-1 + \frac{\varphi_3}{\rho}\right).$$

We emphasize that the vector fields in (65) are noninvariant not only globally but also even with respect to local transformations, i.e., they are not physical. However, physical variables can be constructed from them by repeating the procedure of the previous section. For this, we note that under the transformations (66) the variable $\tilde{A}_r^3$ undergoes only a gradient stretching:

$$\tilde{A}_r^3 \to \tilde{A}_r^3 + \frac{1}{g}\partial_r\alpha',$$

whereas $\tilde{A}_r^{\frac{1}{2}}$ is subject only to an isotopic rotation.

Therefore, the physical variables can be defined as

$$\left.\begin{aligned} &\rho, \ p_\rho; \\ &\hat{A}_r^{\text{ph}} = W\left(\hat{\tilde{A}}_r - \frac{1}{ig}\partial_r\right)W^+; \\ &\hat{\tilde{B}}_r^{\text{ph}} = W(\tilde{B}_r^i T^i + (\tilde{B}_r^{\text{tr}})^3 T^3)\,W^+, \end{aligned}\right\} \tag{68}$$

where

$$W = \exp\left\{ig\int g_r(y, x)\,\tilde{A}_r^3 T^3 d^3y\right\};$$
$$\hat{A}_r^{\text{ph}} = A_r^{\text{ph}^1}T^1 + A_r^{\text{ph}^2}T^2 + (A_r^{\text{ph}^3})^{\text{tr}}\,T^3,$$

and similarly

$$\tilde{B}_r^{\text{ph}} = B_r^{\text{ph}^1}T^1 + B_r^{\text{ph}^2}T^2 + (B_r^{\text{ph}^3})^{\text{tr}}\,T^3.$$

We also give the expression for the tensor of the "electromagnetic" field,

$$G_{\mu\nu} = \partial_\mu A_\nu^{\text{ph}^3} - \partial_\nu A_\mu^{\text{ph}^3},$$

in terms of the original fields. This tensor arises in the magnetic-monopole problem.[22] Using the matrices (64) and (68), we obtain $G_{\mu\nu} = F^a_{\mu\nu}(\eta_a/\rho) - (1/g)\varepsilon^{abc}(\varphi^a/\rho)D_\mu(\varphi^b/\rho)D_\nu(\varphi^c/\rho)$, which agrees with the formula given in Ref. 22.

We now study the global (charge) properties of the variables (68). In other words, we are now interested in the case when $\lambda_1, \lambda_2, \lambda_3$ in (67) do not depend at all on the point of space. Under such global transformations, we obtain for the quantities (68)

$$\left.\begin{aligned} &\rho \text{ is invariant;} \\ &A_r^{\text{ph}^i}T^i \to V(A_r^{\text{ph}^i}T^i)V^+, \quad i = 1, 2; \\ &(A_r^{\text{ph}^3})^{\text{tr}} \text{ is invariant} \end{aligned}\right\} \tag{69}$$

with corresponding expressions for the generalized momenta. The matrix V is

$$V = \begin{cases} \exp(i\lambda_3 T_3), & \Lambda = \Lambda_3; \\ \exp\left\{i\left(\alpha'(x) - \int g_r(y, x)\,\partial_r\alpha'(y)\,d^3y\right)T_3\right\}, & \Lambda = \Lambda_{1,2}. \end{cases} \tag{70}$$

In quantum language, (69) means that the charge Q_3 is defined on a Fock space constructed in the given case from the Fourier components of the fields (68). The Higgs particle and the massless vector boson are neutral, while the massive bosons have a definite charge. The situation is different with regard to the operators Q_1 and Q_2. The argument of the exponential (70) obtained for $\Lambda = \Lambda_{1,2}$ is simply the additional physical variable on the boundary, and this cannot be expressed in terms of the Fock degrees of freedom. Therefore Q_1 and Q_2 carry us out of the Fock space and cannot be considered in perturbation theory. Nevertheless, their commutator

$$[Q_1, Q_2] = igQ_3 \tag{71}$$

leaves the Fock space invariant overall. Thus, the algebra (71) is realized in the given case in a very nontrivial way. We emphasize that the impossibility of determining Q_1 and Q_2 in perturbation theory has nothing in common with the well-known fact that the generators of spontaneously broken symmetries are not well defined.[11,12] The point is that in this model too the Higgs mechanism is characterized by the presence of a condensate of the colorless field:

$$\langle 0 \,|\, \rho \,|\, 0 \rangle \neq 0.$$

It is obvious that this is, quite generally, a property of all theories of Higgs type.

5. CONCLUSIONS

Thus, the analysis of gauge theories from the point of view of a gauge-invariant formulation, the possibility of vacuum degeneracy, and the realization of the algebra of the charges requires specification of the class of functions for the Lagrangian multipliers A_0.

If the A_0 are arbitrary functions, then the theory can be formulated in terms of colorless variables and vacuum degeneracy with respect to the charge is impossible.

But if the A_0 satisfy zero-value boundary conditions, then there exist physically charged states. In models with a nontrivial minimum of the scalar potential, the charged variables are concentrated on the boundary of the volume and are additional with respect to the colorless Fock degrees of freedom. The presence of these additional variables leads to vacuum degeneracy, which, however, is not in any way manifested in the framework of perturbation theory. In addition, it has nothing in common with the appearance of a mass of the vector bosons. In the non-Abelian theories, the variables on the boundary are important in order to ensure the correct commutation relations of the charges.

We thank N. N. Bogolyubov and A. A. Logunov for their interest in the work and valuable discussions. We are also grateful to colleagues in the theoretical section of the Institute of Nuclear Research of the USSR Academy of Sciences for a number of valuable comments.

APPENDIX

In this Appendix, we obtain the expressions (54) and (55) for the unphysical sector in the $SU(2)$ model with scalars in the fundamental representation.

We first of all establish what restrictions are imposed on the functions of the original variables, which are unphysical coordinates and momenta. These restrictions derive, first, from the requirement that the transformation from the original coordinates and momenta to the new coordinates (both physical and unphysical) be canonical. In particular, the constraint algebra (9) must be preserved.

Second, this transformation must diagonalize the constraints in the sense that the constraints must be expressible exclusively in terms of the unphysical new variables.

It is convenient to choose for the scalar field the parametrization (for the notation, see Sec. 4)

$$\varphi = \exp(2i\theta_a T_a)(000\rho)^T, \quad a = 1, 2, 3. \qquad \text{(A.1)}$$

We denote the momentum conjugate to θ_a by p_θ^a. In terms of the original variables, the constraints have the structure

$$\xi^a = F^a + G^a(\theta, p_\theta),$$

where the function F^a does not depend at all on the scalar degrees of freedom, while G^a is the zeroth component of the current of the scalar fields. In the parametrization (A.1), G^a depends only on θ_a and p_θ^a, so that ρ is a charge singlet. This circumstance makes it possible to express the constraints ξ^a in terms of the new variables $(\tilde{\theta}, \pi)$ by means of the same function G^a:

$$\xi^a = G^a(\tilde{\theta}, \pi) + \int ds_i(y)\, B_i^a(y)\, \delta(x-y).$$

At the same time, from the fact that F^a does not depend on the scalar variables it follows that one can consistently set

$$\tilde{\theta}^a = \theta^a.$$

If G^a is now found from Noether's theorem, and θ is expressed in terms of $\varphi_1, ..., \varphi_4$, we arrive at the expressions (54) and (55).

[1)] The indices i and j take only the spatial values 1, 2, 3.

[2)] It is interesting to note that variables analogous to (19) were obtained in Ref. 15 by requiring the perturbation theory to be infrared-finite.

[3)] Of course, the arguments presented below make no pretence at a rigorous construction of the quantum theory. We are speaking rather of a scenario of such a construction.

[4)] To unify the notation, we have introduced $\boldsymbol{\varphi}' = (0\,0\,0\,\rho)^T$. We emphasize that $\boldsymbol{\varphi}'$ and $\mathbf{p}'$ are not canonically conjugate.

[5)] We emphasize once more the inadmissibility of identifying this *gauge* transformation, introduced for purely technical requirements, with the *canonical* transformation (53)–(55).

[6)] With respect to the additional $SU_f(2)$ group, the field ρ is a singlet, while the fields $\tilde{A}_i^a$ form a triplet. The Hamiltonian H is $SU_f(2)$-invariant.

[7)] Traditionally, the electric charge is associated with the generator $Q = T^3 + Y$. However, we shall see that on the Fock space this reduces to our definition.

[8)] To determine the new variables, we have used the matrix U with $\alpha = 0$.

[1] P. A. M. Dirac, Lectures on Quantum Mechanics, Yeshiva University, New York (1964) [Russian translation published by Mir, Moscow (1968)].

[2] P. A. M. Dirac, *The Principles of Quantum Mechanics*, (Clarendon Press, Oxford, 1958) [Russian translation published by Mir, Moscow (1979)].

[3] A. A. Slavnov and L. D. Faddeev, Vvedenie v kvantovuyu teoriyu kalibrovochnykh poleĭ, Nauka, Moscow (1978); English translation: Gauge Fields, Introduction to Quantum Theory, Benjamin/Cummings, Reading, Mass. (1980).

[4] P. W. Higgs, Phys. Lett. **12**, 132 (1964); T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967).

[5] S. Weinberg, Phys. Rev. D **7**, 1068 (1973).

[6] E. S. Fradkin and G. A. Vilkovisky, Phys. Rev. D **8**, 4241 (1973).

[7] E. S. Abers and B. W. Lee, Phys. Rep. **9**, 1 (1973).

[8] J. Bernstein, Rev. Mod. Phys. **46**, 7 (1974).

[9] A. A. Grib, Problema neinvariantnosti vakuuma v kvantovoĭ teorii polya (The Problem of Vacuum Noninvariance in Quantum Field Theory), Atomizdat, Moscow (1978).

[10] J. Goldstone and R. Jackiw, Phys. Lett. **74B**, 81 (1978); A. G. Izergin, V. E. Korepin, M. A. Semenov-Tyan-Shanskiĭ, and L. D. Faddeev, Teor. Mat. Fiz. **38**, 3 (1979); Yu. V. Novozhilov, Teor. Mat. Fiz. **60**, 372 (1984).

[11] J. Fröhlich, G. Morchio, and F. Strocchi, Phys. Lett. **89B**, 61 (1979).

[12] J. Fröhlich, G. Morchio, and F. Strocchi, Phys. Lett. **97B**, 249 (1980); Nucl. Phys. **B190**, 553 (1981).

[13] T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).

[14] V. A. Matveev, M. E. Shaposhnikov, and A. N. Tavkhelidze, Preprint R-0325 [in Russian], Institute for Nuclear Research, Moscow (1983); V. A. Matveev, A. N. Tavkhelidze, and M. E. Shaposhnikov, Teor. Mat. Fiz. **59**, 323 (1984).

[15] E. d'Emilio and M. Mintchev, Fortschr. Phys. **32**, 473 (1984).

[16] M. Minchev and I. T. Todorov, Fiz. Elem. Chastits At. Yadra **16**, 59 (1985) [Sov. J. Part. Nucl. **16**, 26 (1985)].

[17] V. V. Vlasov, V. A. Matveev, A. N. Tavkhelidze, *et al.*, Preprint R-0418 [in Russian], Institute of Nuclear Research, USSR Academy of Sciences, Moscow (1985).

[18] V. V. Vlasov, V. A. Matveev, A. N. Tavkhelidze, *et al.*, Preprint R-0424 [in Russian], Institute of Nuclear Research, USSR Academy of Sciences, Moscow (1985).

[19] V. S. Vladimirov, Uravneniya matematicheskoĭ fiziki (The Equations of Mathematical Physics), Nauka, Moscow (1981).

[20] N. N. Bogolyubov, Izbrannye trudy po statisticheskoĭ fizike (Selected Studies in Statistical Physics), Nauka, Moscow (1981).

[21] H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta **35**, 164 (1962).

[22] G. 'tHooft, Nucl. Phys. **B79**, 276 (1974).

Translated by Julian B. Barbour