Interaction of pions with ^{3}He and ^{4}He nuclei

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Experimental and theoretical results of the study of the interaction of intermediate-energy pions with ^{3}He and ^{4}He nuclei are considered. The topics discussed include the energy dependence of the total, elastic, and inelastic scattering cross sections and of the real part of the forward scattering amplitude, the behavior of the elastic scattering phase shifts, and various theoretical models. Some topical problems in the interaction of pions with the lightest nuclei are considered—the question of the pion absorption mechanism, the search for effects of charge-symmetry violation in pion scattering, and aspects of quasielastic pion scattering.

INTRODUCTION

The study of the interaction of pions with nuclei has been transformed in recent years into an extensive and independent branch of nuclear physics. The accuracy of experiments made with pions at the meson factories has achieved a level as high as hitherto was typical only for a traditional direction of nuclear physics such as the investigation of the nucleon–nucleus interaction.

In this review, we analyze the experimental and theoretical results on the study of the interaction of pions of intermediate energies with ^{3}He and ^{4}He nuclei. We have chosen precisely these nuclei, since, first, the available experimental information on πHe scattering is fairly complete and, second, the πHe interaction has long served as a proving ground for testing different theoretical ideas. Because the helium isotopes have a relatively simple nuclear structure, the mechanism of some characteristic reactions of the pion–helium interaction is understood better than the scattering of pions by heavier nuclei. For example, it is only in the case of pion absorption by ^{3}He that it has been possible to separate experimentally the absorption amplitudes in the different isotopic channels. Nevertheless, in contrast to pion scattering by deuterium, we encounter in πHe scattering practically all the complications due to the effects of the binding of the nucleons in the nucleus and the many-particle nature of the pion–nucleus interaction.

In Sec. 1, we discuss the basic characteristics of the interaction of pions with ^{4}He and ^{3}He: the relative magnitude and energy dependence of the total, elastic, and inelastic scattering cross sections and the real part of the forward scattering amplitude and the behavior of the elastic scattering phase shifts. We consider primarily the experimental situation as it has so far developed with regard to the measurement of these quantities, the most important that characterize pion–helium scattering. We shall discuss the most general physical results, which can be obtained in an almost model-independent manner.

The degree of development of theoretical models for describing elastic and inelastic scattering of pions by light nuclei is reflected in Sec. 2. The central theme of this section is the comparison of the physical content of the standard models based on the theory of multiple scattering (optical model, coupled-channel model) and the model of isobar excitations. It is shown how the many-particle nature of the pion–nucleus interaction is taken into account in the various theoretical schemes.

In Sec. 3, we discuss some of the most topical problems of pion interaction with the lightest nuclei, in the first place the study of the mechanisms of pion absorption by nuclei. This process is distinguished from all other inelastic-scattering reactions by its essentially nonpotential nature. It is well known that the absorption of nucleons by a nucleus takes place mainly in a two-nucleon cluster, for which it is necessary for both nucleons to be at short distances, of order 0.5–0.8 F. From this it is to be expected that nucleon–nucleon correlations will be strongly manifested in the absorption reaction.

Study of elastic scattering of pions of both signs by nuclei with zero isospin provides a good possibility for looking for effects associated with violation of charge symmetry in πA interactions. The existence of such effects follows naturally from quark models in which the u and d quarks have different masses. In Sec. 3, we discuss the present state of searches for the effects of charge-symmetry violation in pion scattering by the lightest nuclei. We also consider the main characteristics of the dominant channel of inelastic scattering processes—quasielastic pion scattering.

Finally, we discuss some interesting experiments that could be made in meson factories.

A detailed exposition of the theory of multiple scattering can be found in Refs. 1–3. Various aspects of pion–nucleus dynamics not considered in the present review are discussed in Refs. 4–8.

1. BASIC CHARACTERISTICS OF THE PION–HELIUM INTERACTION

Measurements of the total cross section of π^4He and π^3He scattering

The total cross section of pion–nucleus scattering is usually found in experiments that measure the attenuation of a beam passing through a nuclear target. Unfortunately, this method does not permit determination of the total cross section σ_{tot} without model assumptions. For when the attenuation of a beam passing through a target is measured, some particles that have undergone elastic or inelastic scattering through small angles also enter the counters placed

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behind the target and thus appear to increase the number of particles that have not interacted. In addition, we are interested in the total cross section of the purely nuclear interaction, but the Coulomb interaction also leads to a certain attenuation of the beam. Therefore, the experimental data must be corrected to take into acount all these effects. A detailed description of the procedure for finding the total cross sections can be found in Refs. 9–11. It is important to bear in mind that in the case of π^4He scattering the model-dependent corrections are quite appreciable, particularly at low energies ($T_\pi < 100$ MeV), where they may reach 10%–15%.[11]

As was noted in Ref. 9, a dependence on theoretical models in the determination of $\sigma_{\rm tot}$ could be avoided if the raw experimental data were corrected only for the purely Coulomb scattering. The optical theorem for the total cross section

$$\tilde{\sigma}_{\rm tot} =: \frac{4\pi}{k} \operatorname{Im} f_N(0) \tag{1}$$

relates it, not to the purely nuclear amplitude

$$f_{NP}(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) f_l P_l(\cos\theta), \tag{2}$$

but to the amplitude $f_N(\theta)$, which contains the Coulomb phase shifts σ_1:

$$f_N(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1)\, e^{2i\sigma_l} f_l P_l(\cos\theta). \tag{3}$$

In the expressions (2) and (3), k is the c.m.s. pion momentum, $P_1(\cos\theta)$ are Legendre polynomials, and f_1 are the partial-wave scattering amplitudes.

However, the $\tilde{\sigma}_{\rm tot}$ defined in (1) can differ quite appreciably from the ordinary total cross section,

$$\sigma_{\rm tot} = \frac{4\pi}{k} \operatorname{Im} f_{NP}(0), \tag{4}$$

and in the case of pion scattering by heavy nuclei $\tilde{\sigma}_{\rm tot}$ may even be a negative quantity.[9] Figure 1 shows the total cross section of pion scattering by ^{4}He as a function of the energy. It can be seen that the dependence of the total cross section has a characteristic resonancelike form with a peak at energy $T \sim 165$ MeV. This behavior of the total cross section is in no way a manifestation of some resonance in the π^4He system but is a reflection of the existence of the Δ resonance in the pion–nucleon interaction. Measurements of the total cross section of pion scattering for other nuclei show that with increasing mass number A the position of the peak in $\sigma_{\rm tot}$ occurs at ever lower energies, while the width of the resonancelike peak increases with increasing A. The physical reasons for the broadening of the peak for $\sigma_{\rm tot}$ are entirely clear—the Fermi motion of the nucleons, the large number of inelastic channels, the enhancement of pion absorption, etc. The shift in the position of the peak in pion–nucleus scattering to lower energies compared with the free πN interaction can be explained rather easily on the basis of simple kinematic considerations (see, for example, Refs. 4 and 5). However, the exact values of the shift and width of the peak in $\sigma_{\rm tot}$ depend on a delicate cancellation of the various corrections, and therefore they cannot be reproduced so simply in theoretical calculations.

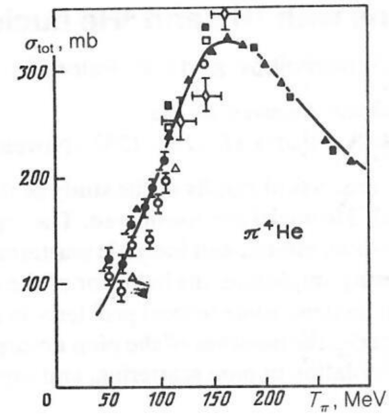

FIG. 1. Total cross section of $\pi^\pm$ ^{4}He scattering. The continuous curve is the result of fitting in accordance with a certain empirical formula; the experimental data for π^- are as follows: the black circles from Ref. 11, the black triangles from Ref. 15, the black squares from Ref. 10, and the open diamonds from Ref. 20. The corresponding open symbols are for π^+.

Unfortunately, the currently existing set of experimental data on the total cross sections of π^4He scattering is plagued by serious problems. There are practically no data on measurements of $\sigma_{\rm tot}$ at low energies T < 100 MeV (Johnson's results,[11] which are shown in Fig. 1, have not been published in scientific journals). There are no systematic measurements of $\sigma_{\rm tot}$ for π^+ and π^- scattering, which are so important for analyzing the problem of charge-symmetry violation in strong interactions. This question recently became particularly acute following the discovery, in not only measurements of the total cross section of $\pi^\pm d$ scattering[12] but also recent experiments on elastic $\pi^\pm d$ scattering,[13,14] of a difference between the cross sections of reactions with π^+ and π^- mesons, which could not be explained by the introduction of Coulomb corrections.[12–14] It would be very interesting to make similar measurements for $\pi^\pm$ ^{4}He scattering, especially since, if the data of Ref. 15 are to be believed, a very strong violation of charge symmetry can be seen at 110 MeV, where the total nuclear cross sections for π^{+4}He and π^{-4}He scattering differ by 30 mb with a characteristic measurement error of order 3 mb.

As can be seen from Fig. 1, the results of the different experimental groups do not agree very well with one another. For example, at $T = 110$ MeV the difference between the measurements of Refs. 15 and 10 is about nine standard deviations. At the same time, accurate knowledge of the total cross section in the resonance region $T \sim 50$–250 MeV is particularly necessary for calculations of the real part Re $f(0)$ of the forward scattering amplitude using dispersion relations. It is this interval of energies that makes the decisive contribution to the corresponding dispersion integral. The insufficiently good knowledge of the total cross sections leads to uncertainties in the calculation of Re $f(0)$. For example, attention was drawn in Ref. 16 to an appreciable discrepancy between the values of Re $f(0)$ determined from

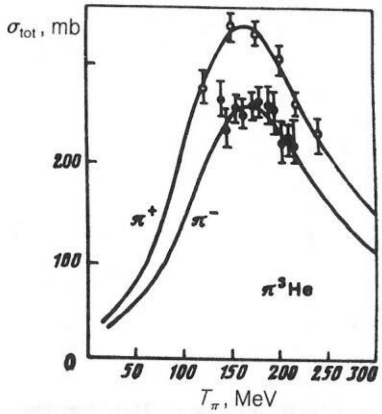

FIG. 2. Total cross section of $\pi^{\pm}\,{}^3$He scattering. The continuous curves are the result of fitting in accordance with a certain phenomenological formula; the experimental data are taken from Ref. 21—the black circles for π^- and the open circles for π^+.

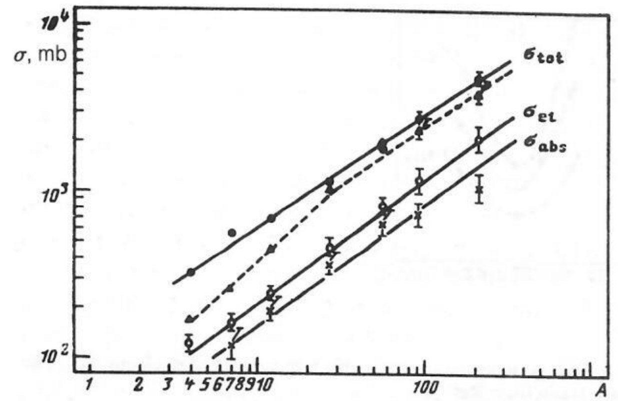

FIG. 3. Total cross sections of pion–nucleus scattering σ_{tot}, cross sections of elastic scattering σ_{el}, and absorption cross sections σ_{abs} as functions of the mass number A for the nucleus interacting with π^- mesons with energy $T = 165$ MeV. The data are taken from Refs. 22 and 24. The continuous and broken lines have been drawn only for convenience of perception. The triangles show σ_{tot} for π^+ scattering at energy 85 MeV.[22]

dispersion relations and from direct analysis of the differential cross sections of elastic scattering at small angles (this question will be considered in more detail below).

Knowledge of the total cross sections of π^4He scattering is very important for models that take into account the excitation of isobar states.[17–19] In these approaches, the interaction of the Δ isobar with the nuclear medium is described by a certain semiphenomenological potential, the parameters of which are chosen to obtain a description of the experimental total cross section of π^4He scattering (for more details about such models, see Sec. 2).

The experimental situation with regard to the measurement of the total cross section of π^3He scattering is considerably worse than for π^4He scattering. In the range of energies $T \sim 0$–300 MeV there exist only Spencer's data (Fig. 2),[21] which have not been published.

Relative values of the cross sections of the various channels of inelastic π^4He interaction

For correct understanding of the pion–nucleus interaction mechanism, the experimental data on the cross sections of the various channels of pion–nucleus scattering are very important. Numerous measurements of the cross sections of pion interaction with light and heavy nuclei have shown that three reaction channels are dominant: elastic scattering, disintegration of the nucleus induced by the pions, and absorption of the pions. In fact, in a first approximation one can assume that the cross sections of all three of these channels are approximately the same, each providing about 30% of the total cross section (more accurate values are given in Table I). It is important to emphasize that the cross section of true pion absorption is fairly large (see Refs. 6, 22, and 23).

Figure 3 shows the pion–nucleus cross sections of elastic scattering and absorption and the total cross sections as functions of the mass number A of the nucleus. In this figure, we show the results of measurements made for π^- mesons at 165 MeV.[22] It can be seen that at this energy the A dependence of the total cross sections for all nuclei from helium to bismuth can be satisfactorily described by the simple power law $\sigma_{tot} \sim A^n$ with an exponent $n = 2/3$. However, the A dependence for σ_{tot} measured at energy $T = 85$ MeV has a more complicated behavior (the corresponding cross sections are shown in Fig. 3 by the triangles). In Ref. 22 it was also noted that the exponent n depends rather strongly on the energy for the cross sections σ_{el} and σ_{abs} as well. All these facts are support for a resonance nature of pion–nucleus scattering. At the resonance energies, the very strong absorption of the pion from the elastic channel has the consequence that the interaction takes place predominantly in the

TABLE I. Relative contributions of the cross sections of the different πA interaction channels to σ_{tot} at energies in the region of the Δ_{33} resonance.

Nucleus	σ_{el}, %	$\sigma(\pi, \pi')$, %	σ_{abs}, %	σ_{ch}, %
^{2}H [$T_\pi = 182$ MeV (Ref. 25)]	~ 25	~ 60	~ 2.5	~ 12.5
Li $\to$ Bi [$T_\pi = 165$ MeV (Ref. 6)]	$35 \to 40$	$32 \to 20$	$20 \to 35$	$7 \to 4$*

*The cross section σ_{ch} was not measured; only a certain estimate is given in the table.

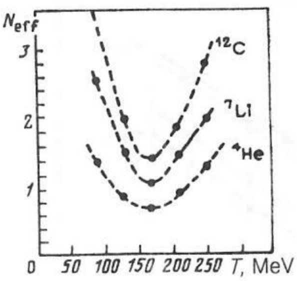

FIG. 4. Energy dependence of the effective number of nucleons N_{eff}. The data are taken from Ref. 22.

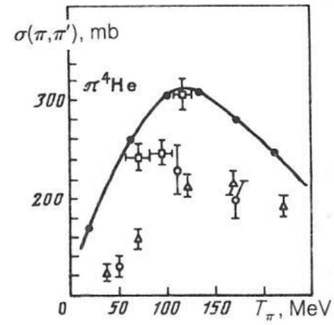

FIG. 5. Inclusive cross section $\sigma(\pi, \pi')$ of the inelastic π^{+4}He interaction. The experimental points are as follows: open circles from Ref. 29, open triangles from Ref. 28, and open squares from Ref. 20. The black circles represent the data of Ref. 15 on the total cross section of all inelastic channels in π^{-4}He scattering. The curve has been drawn only for convenience of perception.

surface layer of the nucleus, from which the $A^{2/3}$ dependence arises. Far from the energy of the Δ_{33} resonance, the nucleus becomes more "transparent" for pions, and the interaction with it takes place not only on the surface. For light nuclei, pion energies of order 80 MeV are already quite far from the energy of the peak in the total cross section, whereas for heavy nuclei such energies can, because of the shift of the peak in σ_{tot} to lower energies and its broadening, still be regarded as resonance energies.

To characterize pion–nucleus scattering, a helpful auxiliary parameter is the so-called effective number N_{ef} of nucleons, defined as

$$N_{ef} = \sigma_{tot}^{\pi A}/(N\sigma_{\pi n} + Z\sigma_{\pi p}). \tag{5}$$

Figure 4 shows the behavior of N_{ef} for various light nuclei. A strong dependence of N_{ef} on the pion energy can be seen. Noteworthy is the very small value of N_{ef} in the case of π^4He scattering at resonance energies. Thus, at $T = 165$ MeV the effective number of nucleons in π^4He scattering is $N_{ef} = 0.68$. This clearly demonstrates (Fig. 4) the increased screening of the nucleons in the nucleus with increasing cross section of the elementary πN interaction.

Thus, the scattering of pions by the lightest nuclei has a specific character, which can be manifested, for example, in changes in the relative contributions of the various inelastic scattering channels. Indeed, as can be seen from Table I, in πd scattering the deuteron breakup channel has the largest cross section, whereas the relative strength of the absorption channel is rather weak and about 3% of the total πd scattering cross section. Unfortunately, the situation that has developed with regard to measurements of the various channels of π^4He inelastic reactions is rather complicated. Thus, the Dubna–Turin collaboration measured the cross sections of individual channels of inelastic $\pi^{\pm}$ ^{4}He scattering by means of a diffusion chamber in a magnetic field[20] and a high-pressure streamer chamber without a magnetic field.[26,27] It was found that the relative value of the absorption cross section is of order 3% of the total cross section, i.e., in this sense pion absorption in helium resembles pion absorption in deuterium rather than in heavier nuclei.

On the other hand, measurements were made at the meson factories at SIN[28] and Los Alamos[29] of the inclusive cross section $\sigma(\pi^+, \pi^{+'})$ of pion scattering by ^{4}He (Fig. 5). These experiments were made using single-arm magnetic spectrometers. An exceptionally small cross section $\sigma(\pi, \pi')$ was obtained—of order 13%–15% of σ_{tot} in the region $T_\pi = 90$–120 MeV. This small value of $\sigma(\pi, \pi')$ requires that pion absorption processes make the main contribution to pion inelastic scattering by ^{4}He. According to the estimates of Ref. 28 (σ_{abs} was not measured directly in the experiments of Refs. 28 and 29), the pion absorption cross section is of order 30%–40% of σ_{tot} at $T_\pi = 90$–120 MeV and about 22% at higher energies. Therefore, according to the results of Refs. 28 and 29 the absorption of pions by helium resembles the absorption by heavier nuclei (see Table I) and is the dominant channel of inelastic scattering.

This existence of two contradictory experimental conclusions suggests that the truth may lie somewhere in the middle, and future measurements must undoubtedly clarify this uncertain situation.

We should like to point out that there exist some experimental indications that the relative value of the cross section of pion absorption by helium is nevertheless more similar to that observed in the case of the pion–deuterium interaction. This is indicated by the results of Källne's group,[30] which measured the differential cross section of the ^{3,4}He$(\pi, p)X$ absorption reaction for one angle of the emitted protons. These results will be discussed in more detail in Sec. 3, and we here merely mention that according to the estimates of Ref. 30

$$\sigma_{abs}\ (\pi^4\text{He}) \sim R\sigma_{abs}\ (\pi d),$$

where the coefficient of proportionality R is equal in order of magnitude to the number of possible np pairs in the helium: $R \sim 2$ for π^3He absorption and $R \sim 4$ for pion absorption in ^{4}He. The same conclusion was obtained in Ref. 31 from recent measurements of the pion absorption cross section in ^{3}He. Finally, the cross section of π^+ absorption by ^{3}He was measured at energy 145 MeV by means of a streamer chamber operating in a beam of pions from the synchrocyclotron at the Joint Institute for Nuclear Research, Dubna.[32] The value obtained was $\sigma_{abs} = 5.4 \pm 1.5 \pm 11$ mb (the first error is the statistical one, and the second is the systematic one), a value that is comparable with the cross section of absorption by the deuteron at these energies: $\sigma_{\pi+d}(\text{abs}) = 11.8 \pm 0.5$ mb.[33]

The real part Re $f(0)$ of the π^4He elastic scattering amplitude at zero angle

Information about the real part of the elastic scattering amplitude at zero angle has been obtained by various methods. In a number of studies (Refs. 10, 16, and 34–36) Re $f(0)$ has been calculated on the basis of dispersion relations, i.e., data on the total π^4He interaction cross sections were used as the basic experimental material for obtaining Re $f(0)$. In Ref. 15, Re $f(0)$ was extracted from an analysis of data on the differential cross sections of π^4He elastic scattering, which were measured in a wide range of angles, including the region of fairly small angles ($\theta \geqslant 4°$). It is true that in Ref. 15 a certain phenomenological formula for the π^4He scattering amplitude was used in order to determine Re $f(0)$. However, overall, the results agree with the conclusions of the more consistent treatment given in Ref. 37. We note that the method of extrapolation to the Coulomb pole in conjunction with the technique of conformal mapping used in Ref. 37 does not require knowledge of the pion–nucleus interaction amplitude, and in this sense the approach of Ref. 37 is the most suitable model-independent method for obtaining information about Re $f(0)$ from data on the differential elastic scattering cross sections. Finally, the energy dependence of Re $f(0)$ can also be obtained by means of phase-shift analysis.[38,39]

Figure 6 shows Re $f(0)$ for elastic π^4He scattering obtained by various methods. The continuous curve plots the results obtained from once-subtracted dispersion relations[39]:

$$\operatorname{Re} f(\omega) = \operatorname{Re} f(m_\pi) + \frac{2k_L^2}{\pi}\, \mathcal{P} \int\limits_{\omega_0}^{\infty} \frac{x \operatorname{Im} f(x)\, dx}{(x^2 - m_\pi^2)(x^2 - \omega^2)}, \qquad (6)$$

where k is the pion momentum in the laboratory system, $\omega = T_\pi + m_\pi$, Re $f(\omega)$ and Im $f(\omega)$ are the real and imaginary parts of the elastic scattering amplitude at zero angle, and $\omega_0 \simeq 0$ is the threshold of the cut due to pion absorption. As usual,[40] the point chosen at which to make the subtraction was $\omega = m_\pi$, where the real and imaginary parts of the π^4He scattering length are known from data on π-mesic atoms. The value of Im $f(\omega)$ was approximated by a diagonal Padé approximant (4×4). Approximately the same behavior of Re $f(\omega)$ is obtained in other dispersion-relation calculations, these differing from one another mainly by the method of continuing Im $f(\omega)$ outside the physical region (to $\omega < m_\pi$).

Figure 6 also shows the results obtained from analysis of the differential cross sections of elastic π^4He scattering. It can be seen that they are in good agreement with the dispersion-relation calculations. The only exception is the point at energy $T = 260$ MeV, at which the value of Re $f(0)$ obtained from the analysis of $d\sigma/d\Omega$ is somewhat smaller in absolute magnitude than the Re $f(0)$ predicted by the dispersion relations. This situation was discussed in detail in Ref. 16, in which it was shown that such a discrepancy cannot be eliminated either by different choices of Im $f(\omega)$ in the unphysical region ($\omega < m_\pi$) or by varying the real part of the π^4He scattering length. It was demonstrated that the results of the dispersion relations agree with the data of Ref. 15 at $T = 260$ MeV only if it is assumed that the total cross sections of π^4He scattering in the region $T = 200$–800 MeV differ very strongly (by 10–15 standard deviations!) from the measured σ_{tot} in Refs. 41 and 42. It is therefore proposed in Ref. 16 that the total cross sections of π^4He scattering should be remeasured in the range of energies given above in order to clarify finally the problem of the Re $f(0)$ discrepancy at 260 MeV.

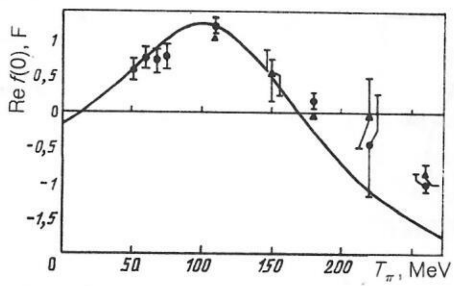

FIG. 6. Dependence of Re $f(0)$ on the pion energy. The continuous curve represents calculations in accordance with dispersion relations.[39] The values of Re $f(0)$ were obtained from analysis of the differential cross sections; the black circles are from Ref. 37, and the black triangles from Ref. 15.

Behavior of the phase shifts of π^4He elastic scattering

The procedure for extracting the phase shifts from data on elastic πA scattering—phase-shift analysis—has been fairly well developed, and in essence phase-shift analysis provides investigators with additional "quasiexperimental" information. However, compared with direct measurements of the differential and total scattering cross sections, phase-shift analysis makes it possible to obtain important information about the nature of the interaction between the particle and the target nucleus in states with different angular momenta. Determination of the π^4He phase shifts has a further aim. The point is that in some approaches the scattering of pions by α-particle nuclei, for example, ^{12}C or ^{16}O, is treated in such a way that the $\pi\alpha$ rather than the pion–nucleon interaction is taken as elementary. In such models, a good knowledge of the π^4He scattering amplitude is to a high degree necessary.

Two types of phase-shift analysis exist: (1) so-called energy-independent phase-shift analysis, in which the phase shifts are found by fitting to the experimental data at individual energies; (2) energy-dependent phase-shift analysis, in which a certain energy dependence of the scattering amplitude is specified and the parameters in this formula are fitted to obtain agreement with the experimental data simultaneously at all energies. In the case of π^4He scattering, both types of analysis have been made.

The energy-dependent variant was used for elastic

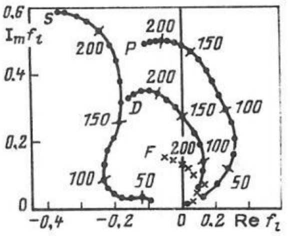

FIG. 7. Argand diagram for partial-wave amplitudes of elastic π^4He scattering obtained in Ref. 38. All the points on the curves are plotted at intervals of 10 MeV.

$\pi^{\pm}$ ^{4}He scattering in Ref. 38. It was assumed that the partial-wave S matrix

$$S_l(k) = 1 + 2\,i\,f_l/k \tag{7}$$

(here, f_l is the partial-wave scattering amplitude) can be represented in the form

$$S_l(k) = P_l(k)/Q_l(k), \tag{8}$$

where $P(k)$ and $Q_l(k)$ are certain polynomials with complex coefficients, these being adjusted parameters.

Figure 7 shows the Argand diagram obtained in this analysis for various partial-wave amplitudes of π^4He scattering. It can be seen that in the Argand diagram all the partial-wave amplitudes describe the characteristic resonancelike circles in the counterclockwise direction. However, such behavior by no means indicates the occurrence in each partial wave of a resonance in the pion–nucleus system (see, for example, Ref. 43). It is simply that in the interaction of the pion with the nucleus the pion–nucleon resonance P_{33} is projected onto the pion–nucleon states with different angular momenta. It was demonstrated in Ref. 39 that the contribution of the πN interaction in the P_{33} wave determines to a large degree the energy behavior of even S-wave π^4He scattering.

Figure 8(a) shows the energy dependence of the partial-wave total cross section $\sigma^l_{\rm tot}$ obtained by the energy-dependent analysis:

$$\sigma^l_{\rm tot} = \frac{4\pi}{k^2}(2l+1)\,\mathrm{Im}\,f_l. \tag{9}$$

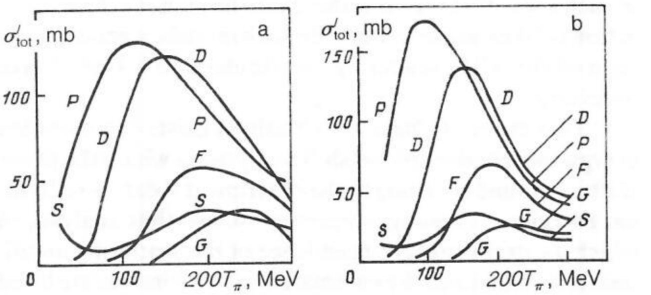

FIG. 8. Energy dependence of the total partial-wave cross sections $\sigma^l_{\rm tot}$ of π^4He scattering: (a) obtained in the energy-dependent phase-shift analysis of Ref. 38; (b) calculated in accordance with the optical model with a first-order potential.[50]

It can be seen that for practically all l the total partial-wave cross section behaves in a resonancelike manner. However, the maximum of $\sigma^l_{\rm tot}$ occurs at different energies. Whereas in the F wave the maximum of $\sigma^l_{\rm tot}$ occurs at $T_{\max} = 180$–200 MeV, in the D wave $T_{\max} = 150$ MeV and in the P wave $T_{\max} = 120$ MeV. It is interesting that the same tendency for $T_{\max}$ to be shifted has been observed in phase-shift analyses of π^{12}C and π^{16}O scattering.[44,45] In all cases, the largest shift in the resonance energy $T_{\max}$ occurred in the lowest partial waves. In the states with large l the pion–nucleus total partial-wave cross sections were found, in contrast, to peak at approximately the same energy as in free πN scattering.

Figure 8(b) shows the partial-wave total cross sections of π^4He scattering calculated in the optical model with a first-order potential (see Sec. 2). It can be seen that in this theoretical calculation as well the maximum of $\sigma^l_{\rm tot}$ in states with different l occurs at different energies, the dependence of $T_{\max}$ on l being qualitatively the same as in the energy-dependent phase-shift analysis. A very interesting physical result can be obtained by considering the behavior of the ratio $X_l = \sigma^l_{\rm el}/\sigma^l_{\rm tot}$, where

$$\sigma^l_{\rm el} = \frac{4\pi}{k^2}\,|f_l|^2.$$

The results of the energy-dependent analysis indicate that with increasing l the ratio X_l decreases, i.e., in the higher partial waves the scattering becomes progressively more inelastic. For example, at $T = 200$ MeV the decrease in X_l is

$$X_l = 0.64,\ 0.51,\ 0.38,\ 0.13 \quad \text{for} \quad l = 0,\ 1,\ 2,\ 3.$$

The possible existence of such a dependence was noted by Lenz,[7] who pointed out that the ratio X_l is determined in the impulse approximation mainly by the corresponding harmonic of the nuclear form factor:

$$X_l = \frac{\sigma^l_{\rm el}}{\sigma^l_{\rm tot}} \sim \frac{|f_l|^2}{\mathrm{Im}\,f_l} \sim F_l(q^2).$$

It is clear that the value of the partial-wave harmonic $F_l(q^2)$ must decrease with increasing l. From the physical point of view, the decrease in X_l must, according to Lenz, take place because quasielastic scattering processes are dominant at large l.

A phase-shift analysis of π^4He elastic scattering at individual energies was made in Refs. 15 and 46–49. The nuclear scattering amplitude was parametrized in the standard manner:

$$f_{CN}(\theta) = \frac{1}{k}\sum_{l=0}^{L_{\max}}(2l+1)\,e^{2i\sigma_l}f_lP_l(\cos\theta), \tag{10}$$

where σ_l is the phase shift for scattering by the purely Coulomb potential, and the partial-wave amplitude f_l is described by two parameters:

$$f_l = \frac{1}{2i}\left(\eta_l e^{2i\delta_l} - 1\right). \tag{11}$$

Here, δ_l is the real part of the nuclear phase shift as distorted by the presence of the long-range Coulomb potential, and η_l is the inelasticity parameter, related to the imaginary part of the distorted nuclear phase shift ω_l by

$$\eta_l = \exp(-2\omega_l). \quad (12)$$

A serious shortcoming of phase-shift analysis at individual energies is its ambiguity. One can always obtain several sets of phase shifts giving the same differential cross section. This problem is examined in detail in Refs. 51 and 52. However, it is already clear from the form of the scattering amplitude (10) that this is a polynomial in powers of $x = \cos\theta$. It can be represented in the form

$$f_{CN}(\theta) = f_{CN}(0) \prod_{i=1}^{L_{\max}} \frac{(x - x_i)}{(1 - x_i)}, \quad (13)$$

where $x = \cos(\theta)$, and x_i are complex numbers, the roots of the polynomial (10). Any replacement of x_i by its complex conjugate x_i^* does not change, $|f_{CN}(\theta)|^2$. However, the real and imaginary parts of $f_{CN}(\theta)$ are changed, and, therefore, so are the phase shifts δ_l and ω_l. By means of such an algorithm, one can, for given $L_{\max}$ construct $2^{L_{\max}}$ sets of phase shifts giving absolutely the same value of $|f_{CN}(\theta)|^2$ at all scattering angles. For example, to describe π^4He scattering at $T = 200$ MeV it is possible to construct 32 such sets of phases. Nevertheless, it was shown in Ref. 48 that in principle the problem of unique determination of the phase shifts of π^4He elastic scattering can be completely solved. Using additional information about the analytic structure of the scattering amplitude and the basic features of the pion–nucleus interaction, it is possible to distinguish one set of phase shifts, the one most acceptable from the physical point of view.

For example, at low energies $T < 75$–80 MeV there are two sets of solutions that describe the experimental data equally well. However, one of them corresponds to the absence of absorption in the S wave and to large absorption in the P wave, while the other corresponds to quite the opposite situation—no absorption in the P wave ($\eta_P = 1$) and practically all pions must be absorbed from the S-wave state. Careful analysis showed that in the first case the parameter η_S somewhat exceeds unity, and this excess is outside the limits of the error in the determination of η_S. Therefore, such a solution violates the unitarity principle and cannot be regarded as acceptable. The energy dependence of the inelasticity parameters η_S and η_P corresponding to the correct solution is shown in Fig. 9.

2. THEORETICAL MODELS OF PION–HELIUM SCATTERING

Besides the phenomenological analysis considered in Sec. 1 considerable efforts have been made during the last two decades to achieve a more fundamental understanding of pion interaction with light nuclei. Numerous theoretical models have been developed in which the pion–nucleus scattering has been treated as a succession of elementary interactions of pions with nucleons of the nucleus. As a rule, the point of departure for such approaches has been the (essentially nonrelativistic) theory of multiple scattering.[1,53] (For a more detailed account of the present status of this theory, see Refs. 2 and 3.)

It was usually assumed that the pion interacts with nucleon i in the nucleus through a potential v_i. Then the pion–

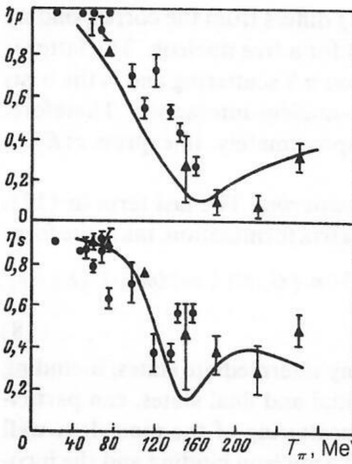

FIG. 9. Energy dependence of the inelasticity parameters η_S and η_P. The continuous curves are the results of calculations in accordance with the optical model with a first-order potential.[50] The points show the results of phase-shift analysis: the black triangles from Ref. 15, and the black circles from Ref. 49.

nucleus scattering matrix $T(E)$ can be written as

$$T(E) = \sum_{i=1} v_i + \sum_{i,j} v_i G(E) v_j + \sum_{i,j,k} v_i G(E) v_j G(E) v_k + \ldots, \quad (14)$$

where

$$G(E) = (E - K_\pi - H_A + i\varepsilon)^{-1}.$$

Here K_π is the operator of the pion kinetic energy, and H_A is the nuclear Hamiltonian. It is more convenient to redefine the summation in (14) as follows:

$$T(E) = \sum_{i=1} \tau_i(E) + \sum_{i \neq j} \tau_i(E) G(E) \tau_j(E) + + \sum_{\substack{i\neq j\\ j\neq k}} \tau_i(E) G(E) \tau_j(E) G(E) \tau_k + \ldots \quad (15)$$

and introduce the matrices $\tau_i(E)$,

$$\tau_i(E) = v_i + v_i G(E) \tau_i(E), \quad (16)$$

which describe the scattering of the pion by the i-th nucleon bound in the nucleus.

We need to know the matrix element $\langle m|T(E)|n\rangle$ between the completely antisymmetric nuclear states $|m\rangle$ and $|n\rangle$; for this, Eq. (15) can be rewritten in the more compact form

$$T(E) = A\tau(E) + (A-1)\,\tau(E)\,G(E)\,T(E), \quad (17)$$

where A is the number of nucleons in the nucleus. However, the entities that occur in (17) are still very complicated many-particle operators. The models of pion–nucleus scattering to be discussed in the present section essentially represent different approximation schemes for treating the many-particle aspects of pion–nucleus dynamics. The most important many-particle effects that we encounter in the study of pion–nucleus scattering are the following:

1. *Effects of the binding of the nucleons in the nucleus.* Since the pion scattering takes place on a nucleon bound in

the nucleus, the matrix $\tau(E)$ differs from the corresponding pion scattering matrix $t(E)$ for a free nucleon. The latter is deduced from experiments on πN scattering and is the basis for calculations of the pion–nucleus interaction. Therefore, we must be able, at least approximately, to express $\tau(E)$ in terms of $t(E)$.

2. Incoherence of the scattering. The last term in (17), written down in the exact matrix formulation, takes the form

$$(A-1)\sum_{m}\langle n' \mid \tau(E) \mid m\rangle\langle m \mid G(E) \mid m\rangle\langle m \mid T(E) \mid n\rangle . \tag{18}$$

It can be seen that very many intermediate states, including states different from the initial and final states, can participate in elastic or inelastic scattering of the pion. It is well known[2] that the effects of the nucleon binding and the incoherence of the scattering are much less important in the pion–nucleus interaction than, for example, in low-energy nucleon–nucleus scattering. However, in order to reproduce quantitatively the recent very accurate experimental data it is definitely necessary to take into account these effects.

3. Pion annihilation. Because of the nonrelativistic nature of multiple-scattering theory, the complete set of intermediate states in (18) contains only states with one pion and A nucleons. Therefore, the treatment of processes in which a pion is absorbed by one of the nucleons and emitted by another is already beyond the scope of the approaches based on purely potential ideas about the scattering. However, pion absorption is the dominant inelastic process at low energies, and this important reaction channel must be taken into account in the study of pion–nucleus scattering. Unfortunately, the corresponding effects are frequently treated only at the phenomenological level.

Optical model

One of the first theoretical schemes used to describe elastic pion–nucleus scattering was the so-called optical model. This term is not very good, since it recalls completely phenomenological models of nucleon–nucleus scattering, in which the form of the optical potential was chosen in a semiarbitrary manner. The point of departure of the optical model of pion–nucleus scattering is the relation (17). One introduces the projection operators

$$P = \mid 0\rangle\langle 0 \mid,\ Q = 1 - P, \tag{19}$$

which project onto the nuclear ground state $|0\rangle$ and onto all the remaining nuclear states, respectively. After this, (17) can be rewritten in the form

$$T'(E) = U(E)\,[1 + PG(E)\,T'(E)]; \tag{20}$$

$$U(E) = (A-1)\,\tau(E)\,[1 + QG(E)\,U(E)]; \tag{21}$$

$$T'(E) = \frac{A-1}{A}\,T(E). \tag{22}$$

In the standard approach, the effects of the binding of the nucleons are ignored, $\tau(E) \approx t(E)$ (impulse approximation), and, in addition, intermediate excitations of the nucleus are not taken into account, $U(E) \approx (A-1)t(E)$ (coherent-scattering approximation). Then Eq. (20) reduces to a two-particle equation of Lippmann–Schwinger type:

$$\langle \mathbf{Q}'0|T(E)|0\mathbf{Q}\rangle = (A-1)\,\langle \mathbf{Q}'0|t(E)|0\mathbf{Q}\rangle + \frac{(A-1)}{(2\pi)^3}\int \frac{\langle \mathbf{Q}'0 \mid t(E) \mid 0\mathbf{Q}''\rangle\langle \mathbf{Q}''0 \mid T(E) \mid 0\mathbf{Q}\rangle}{E - E(Q'') + i\varepsilon}\,d\mathbf{Q}'', \tag{23}$$

where $\mathbf{Q}'$ and $\mathbf{Q}$ are the relative momenta in the pion–nucleus center-of-mass system in the final and initial states, respectively.

The optical potential

$$\langle \mathbf{Q}'\ 0 \mid U(E) \mid \mathbf{Q}0\rangle = (A-1)\,\langle \mathbf{Q}'0 \mid t(E) \mid \mathbf{Q}0\rangle \tag{24}$$

in the static approximation, i.e., when the Fermi motion of a nuclear nucleon is ignored, has a very simple form:

$$\langle \mathbf{Q}'0 \mid U(E) \mid 0\mathbf{Q}\rangle = (A-1)\,\langle \mathbf{Q}' \mid t(E) \mid \mathbf{Q}\rangle * F_0(\mathbf{Q}' - \mathbf{Q}), \tag{25}$$

where $F_0(\mathbf{Q}' - \mathbf{Q})$ is the nuclear form factor, i.e., the Fourier transform of the nuclear density in the ground state.

In order to determine completely the optical potential (25), it is necessary to specify the extrapolation of the pion–nucleon scattering matrix off the energy shell. The reason for this is that solution of Eq. (23) requires knowledge of the t matrix for the πN interaction for $|\mathbf{Q}'| \neq |\mathbf{Q}|$. Restricting ourselves to only the S- and P-wave πN interactions,

$$\langle \mathbf{Q}' \mid t(E) \mid \mathbf{Q}\rangle = b(E) + c(E)\,\mathbf{Q}'\mathbf{Q}, \tag{26}$$

we arrive in the coordinate representation at the well-known potential of Kisslinger type.[54] The coefficients $b(E)$ and $c(E)$ can be expressed in terms of the pion–nucleon scattering phase shifts.

A later development was the model of pion–nucleon separable potentials,[55] which was used to obtain a more realistic extrapolation of the t matrix off the energy shell for the πN interaction. The pion–nucleon partial-wave amplitude $f^{j}_{lt}(Q', Q, E)$, which is characterized by the values of the angular momentum l, the total spin j, and the isospin t, was expressed in the form

$$f^{j}_{lt}(Q', Q, E) = \frac{g^{j}_{lt}(Q')\,g^{j}_{lt}(Q)}{[g^{j}_{lt}(k)]^2}\,f^{j}_{lt}(k, k, E), \tag{27}$$

where the momentum k corresponded to the energy E, and $g^{j}_{lt}(Q)$ are the separable pion–nucleon potentials. The use of the separable potentials leads to the appearance in the expression (25) for the optical potential of an additional nonlocality [besides the term $\mathbf{Q}'\cdot\mathbf{Q}$ in (26)], this being a manifestation of the finite range of the pion–nucleon interaction.

It was subsequently established that the corrections to the static approximation significantly improve the agreement between the optical-model calculations and the experimental data. Instead of ignoring the dependence of the matrix elements $\langle 0\mathbf{Q}'|t(E)|\mathbf{Q}0\rangle$ on the nucleon momentum, these matrix elements were calculated at several effective momenta of the nucleon in the initial and final states.[56-59] After this so-called angular transformation the optical potential is then again expressed in factorized form (nuclear form factor multiplied by the πN amplitude), but now the error of the factorization can be estimated; it is equal in order of magnitude to the square of the ratio of the pion mass to the nucleon mass ($\sim 1/50$).[60]

Detailed calculations of the elastic scattering of pions by ^{4}He in the framework of such an approach were made by Landau and Thomas[58,59] and other authors.[61] The three-particle kinematics for the pion–nucleon–core system was used to calculate the matrix element $\langle 0\mathbf{Q}'|t(\widetilde{E})|\mathbf{Q}0\rangle$. Although the calculations were made in the impulse approximation, some of the effects of the nucleon binding in the nucleus were nevertheless taken into account by a so-called "energy shift," i.e., the energy $\widetilde{E}$ was shifted somewhat relative to the reaction energy E. This shift was selected using experimental data on π^4He scattering. In the region of low energies ($E\leqslant 100$ MeV) there is added to the optical potential a certain phenomenological term, which describes the influence of the processes of creation and absorption of a pion in the intermediate state.

The results of calculations[59] of the total elastic cross section $\sigma_{\rm el}$ and the total reaction cross section $\sigma_{\rm tot}$ in accordance with such a model are given in Fig. 10. It can be seen that the theoretical calculations reproduce rather well the experimental data in the region of resonance energies. Knowing the imaginary part of the forward elastic scattering amplitude $F(0)$, we can, using the optical theorem, write down the relation

$$\frac{4\pi}{k}F(0)=\sigma_{\rm el}+\sigma_{\rm in}=\sigma_{\rm el}-\frac{2}{k}\langle\Psi^{(-)}|\,{\rm Im}\,U\,|\Psi^{(+)}\rangle. \qquad (28)$$

The following question arises: which processes of inelastic π^4He scattering are taken into account when the cross section $\sigma_{\rm in}$ is calculated in terms of the matrix element of the imaginary part Im U of the optical potential (25) between the wave functions $\Psi^{(\pm)}$ of elastic pion–nucleus scattering? In Refs. 62 and 63 it was established that this procedure basically describes the processes of quasielastic pion scattering ($\pi+A\rightarrow\pi+N+A'$) without allowance for the interaction of the knocked-out nucleon N in the final state. Since quasielastic scattering is the main channel of inelastic scattering in the resonance region of energies (see Sec. 1), it is not surprising that the cross sections $\sigma_{\rm el}$, $\sigma_{\rm in}$, and $\sigma_{\rm tot}$ are described fairly well in the optical model. We emphasize that, as a rule, calculations in the impulse approximation overestimate the cross sections of quasielastic scattering, and it is this that compensates the absence in the optical model of the pion annihilation channel.

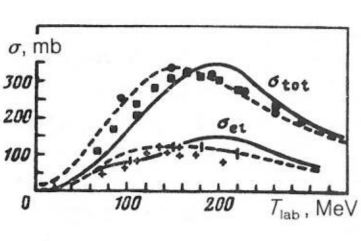

FIG. 10. Results of calculations in Ref. 59 of $\sigma_{\rm tot}$ and $\sigma_{\rm el}$ for π^4He scattering. The continuous curves correspond to the three-particle choice of the energy; the broken curves, to the two-particle choice without allowance for corrections for nucleon binding in the nucleus.

A quite different situation is observed in the region of very low energies ($E\leqslant 30$ MeV), where very accurate experiments have recently been made.[64] It is found that the existing phenomenological methods of taking into account pion annihilation and the prescriptions for imitating the effects of nucleon binding by means of an "energy shift" are too crude and are incapable of correctly describing the experimental data.

The nucleon-binding effects play a fairly important part in π^4He scattering at very low energies,[65–68] and also in the determination of the π^4He scattering lengths.[69–71] In this region, the impulse approximation must be very carefully formulated, since the principle of elastic unitarity must be exactly satisfied below the threshold of the inelastic reactions.

The optical model considered above was also used to analyze pion scattering by ^{3}He. In this case, the optical potential has a more complicated structure because the ^{3}He spin and isospin are nonzero ($J=T=\frac{1}{2}$). Then a procedure of averaging over the nuclear wave functions analogous to (24) leads to the optical potential

$$\langle \mathbf{Q}'0|\,U(E)\,|0\mathbf{Q}\rangle=(A-1)\Big\{B_0F_0(q)+\frac{2}{A}(\mathbf{tT})\,B_TF_T(q)$$
$$+\frac{2}{A}\,\mathrm{i}\mathbf{J}\cdot\mathbf{Q}\times\mathbf{Q}'\,[B_SF_S(q)+2(\mathbf{tT})\,B_{ST}F_{ST}(q)]\Big\}, \qquad (29)$$

where B_i, $i=O, S, T, ST$, are the corresponding combinations of πN partial-wave amplitudes, and $F_i(q)$ are the nuclear form factors associated with the mass, charge, and spin distributions in the ^{3}He nucleus. Because the spin and isospin effects depend on the mass number as $1/A$, they are more important for π^3He scattering than for pion scattering by heavier nuclei.

There have been several attempts to investigate the sensitivity of π^3He scattering to the details of the nuclear structure and even to obtain some information about the ^{3}He nuclear structure from the reaction with pions. It was established[72] that elastic π^{-3}He scattering and the charge-exchange reactions to the analog state, $\pi^-+{}^3\mathrm{He}\rightarrow\pi^0+{}^3\mathrm{H}$, are fairly sensitive to admixtures of the S' and D states in the ^{3}He wave function (this situation does not occur for π^{+3}He scattering). This circumstance was used by Landau[73] to determine the magnetic radius $R_{\rm m}$ of the ^{3}He nucleus from elastic-scattering and charge-exchange data. These studies aimed to resolve the existing contradiction between the $R_{\rm m}$ values obtained in different electron–nucleus scattering experiments.[74] Unfortunately, a more careful analysis showed[75,76] that the sensitivity of the calculations of the studied cross sections to $R_{\rm m}$ is quite insufficient to resolve the problem.

The connection between the scattering of pions and electrons by the three-nucleon system was studied in Ref. 77. Whereas the form factors $F_0(q)$ and $F_T(q)$ in Eq. (29) can be determined with sufficient accuracy from analysis of the form factors measured in elastic e^3He and e^3H scattering, information from electron–nucleus scattering cannot in general be directly used for the form factors $f_S(q)$ and $F_{ST}(q)$. The problem is that an important contribution to the magnetic form factors of electron–nucleus scattering is made by

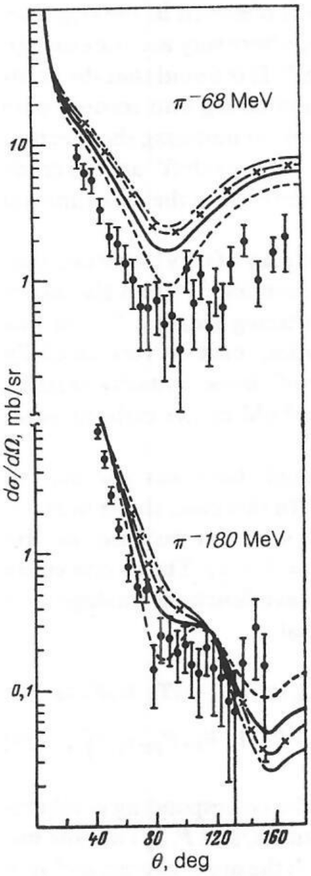

FIG. 11. Differential cross sections of elastic π^{-3}He scattering for different values of the ^{3}He magnetic radius R_m : $R_m = 1.95$ F for the continuous curve, 1.75 F for the broken curve, 2.15 F for the chain curve, and 1.95 F for the curve with crosses, but without allowance for meson exchange effects. The experimental data are taken from Ref. 77.

the effects of meson exchange currents, which must be separated if this information is to be used to calculate pion–nucleus characteristics. Such a procedure for separating the effects of meson exchange currents was carried out in Ref. 77, and calculations in accordance with the optical model with the potential (29) led to quite reasonable agreement with the experimental data on pion elastic scattering in the resonance region (Fig. 11). For comparison, Fig. 11 gives the results of calculations with three values of R_m; the part played by the meson exchange currents is also indicated.

Allowance for effects of higher orders

The most direct way to take into account approximately some many-particle effects is to retain in the expansion of the optical potential (21) the first two terms:

$$U(E) = (A-1)\,\tau(E) + (A-1)^2\,\tau(E)\,QC(E)\tau(E). \tag{30}$$

If exactly the same procedure is carried out in the equation that connects the matrices $\tau(E)$ and $t(E)$,

$$\tau(E) = t(E) + t(E)\,[G(E) - d(E)]\,\tau(E) \tag{31}$$

[here, $d(E)$ is the Green's function for pion scattering by the free nucleon], we arrive at an optical potential that not only contains corrections to the coherent-scattering approximation but also takes into account some nucleon binding effects quadratic in powers of the free scattering t matrix. It is important to take into account these corrections simultaneously, since they have a tendency to cancel each other.[50,78]

The completeness approximation is usually employed, i.e., it is assumed that all the intermediate states are degenerate and have the energy of the ground state. Then the second-order optical potential can be written in the form

$$\langle \mathbf{Q}'0|\,U^{(2)}(E)\,|\mathbf{Q}0\rangle = (A-1)\,\langle \mathbf{Q}'0|\,t(E)\,|0\mathbf{Q}\rangle + \frac{(A-1)^2}{(2\pi)^3}\int'' \frac{\langle \mathbf{Q}'0|\,t(E)\,|0\mathbf{p}\rangle\langle \mathbf{p}0|\,t(E)\,|0\mathbf{Q}\rangle''}{E-E(p)+i\varepsilon}\,C \times (\mathbf{Q}'-\mathbf{p},\ \mathbf{p}-\mathbf{Q})\,d^3p. \tag{32}$$

Here, the symbol 0 means that only the scalar and isoscalar parts of the operator 0 contribute to the optical potential in the case of nuclei with zero spin and isospin; $C(\mathbf{q}_1\mathbf{q}_2)$ is the nuclear correlation function for the ground state. In the case of π^3He scattering, the second-order potential has a somewhat different and more complicated form[72,80] because of the more complicated spin and isospin structure of ^{3}He.

Correlations of various types have been considered in the calculation of the optical potential (32). For example, Wakamatsu[79] showed that in the region of energies which we consider the short-range correlations associated with the nucleon–nucleon repulsion can be ignored. It was found that even for a light nucleus such as ^{4}He the correlations associated with recoil are rather small (Fig. 12). The processes of

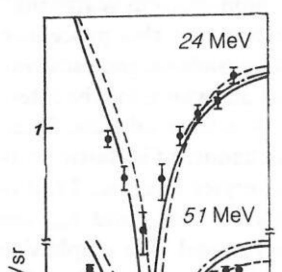

FIG. 12. Differential cross sections of π^{-4}He elastic scattering. The broken, continuous, and chain curves correspond to optical-model calculations with the potentials $U^{(1)}$, $U^{(1)} + U^{(2)}$, and $U^{(1)} + U^{(2)}$ without allowance for the recoil corrections.[50]

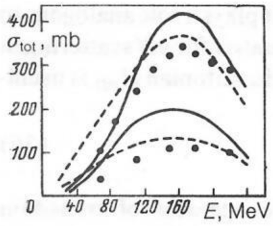

FIG. 13. Results of calculations[50] of the total and elastic cross sections of π^{-4}He scattering in the optical model with allowance for the second-order potential $U^{(2)}$ (continuous curves) and without $U^{(2)}$ (broken curves). The experimental points, the black circles, are taken from Refs. 10 and 15.

double charge exchange in the intermediate state make the most important contribution to the second-order potential for π^4He scattering.[50] In the region of the Δ_{33} resonance, double spin-flip processes in the intermediate state also become important.

Allowance for the second-order effects gives better agreement between the theoretical calculations and the experimental data at energies $T<100$ MeV than in the case of the calculations with the first-order optical potential. However, in the resonance region, as can be seen from Fig. 13, the addition of the second-order potential makes the description of the experimental data less good. This situation can evidently be attributed to the slow convergence (or even divergence) of the iterative expansion (30) in this energy range.

Recently, π^4He scattering was described by a method based on considering the evolution of the system with respect to the coupling constant.[81] In its physical content, this approach is analogous to the optical model with a second-order potential but differs from that model in the manner in which two-particle unitarity is taken into account.

Significant progress has been achieved in the description of π^3He scattering in the framework of the four-particle Faddeev–Yakubovskiĭ equations[82] and their approximate solution.[83]

Coupled channel method

Another possible way of taking into account the excitation of the nucleus in the intermediate state is to extend the space of states, which are taken into account exactly, the projection operators P and Q in (20) and (21) being redefined accordingly. Using the impulse approximation and ignoring the states in the Q space, we then arrive (see Ref. 85) at the system of coupled integral equations

$$\langle Q'n|\,T(E)\,|0Q\rangle=\langle Q'n|\,U(E)\,|0Q\rangle + \frac{1}{(2\pi)^3}\sum_{m\in P}\int\frac{\langle Q'n|\,U(E)\,|m\mathbf{p}\rangle\langle\mathbf{p}m|\,T(E)\,|0Q\rangle}{E-E_m(\mathbf{p})+i\varepsilon}\,d^3p, \qquad (33)$$

where the matrix elements of the optical potential $U(E)$ are analogous to (25). By comparison with the second-order optical model, this approach takes into account only a few low-lying excited states but does this in all powers of the t matrix.

Calculations in which a P space consisting of all 1p1h excited states was used for the ^{4}He nucleus demonstrated better agreement with the experimental data than optical-model calculations with a first-order potential.[84] This applies particularly to the improvement in the description of the differential cross sections of π^4He elastic scattering.

Glauber's model

Although the natural field of application of Glauber's model is high-energy physics, this approach has been used several times to describe π^4He elastic scattering, even at energies as low as $T\sim30$ MeV. Glauber's model provides an approximation scheme for treating the multiple-scattering series (15) that is quite different from the theoretical schemes discussed so far. Besides the completeness approximation, Glauber's model uses the eikonal approximation, which consists of ignoring the terms quadratic in the momentum transfer in the Green's function $G(E)$. As a result, the series (15) becomes finite, containing only terms corresponding to scattering up to order A. However, in the intermediate states no restrictions on the multiplicity of the scattering arise, and therefore Glauber's model takes into account all the same correlations as the optical model with a second-order potential together with correlations of higher orders.

Detailed Glauber-model calculations were made for pion scattering by ^{4}He and ^{3}He at energies in the interval $100\leqslant T\leqslant400$ MeV.[86] They yielded rather remarkable agreement with the experimental data (Fig. 14). The applicability of Glauber's model was investigated, and it was shown that its use is completely justified only at energies above $T\sim180$–200 MeV. This can be clearly seen in Fig. 15, which shows the results of calculations by the optical model and Glauber's model, the part played by the eikonal approximation being demonstrated explicitly.

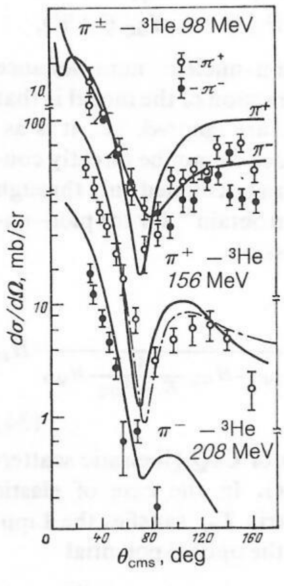

FIG. 14. Differential cross sections of $\pi^{\pm}$ ^{3}He scattering. The continuous curves are the results of calculations using Glauber's model in Ref. 86; the chain curve (156 MeV) shows the results of calculations in which the recoil corrections were ignored; the experimental data are taken from Ref. 87.

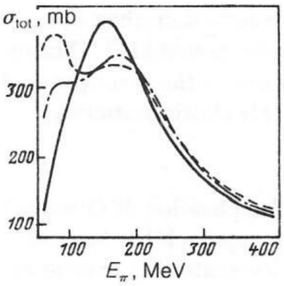

FIG. 15. Total cross section of π^4He scattering from Ref. 86. The continuous curve shows the results of optical-model calculations, the broken curve is for Glauber-model calculations, and the chain curve is for optical-model calculations using the eikonal approximation for the Green's function $G(E)$.

Model of isobar-hole excitations

In recent years, the model of isobar–hole excitations has gained steadily in popularity. In this theoretical scheme, it is assumed that the Δ isobar is a stable particle with its own Hilbert space, acquiring its actual mass and finite width only through coupling to the πN space.[88] Let us introduce the following projection operators: P, onto the subspace of states of the pion and nucleus in the ground state; D, onto the space of states with one Δ isobar and $A-1$ nucleons; and Q, onto all the remaining states. Then, using the Schrödinger equation

$$(E-H)\mid\Psi\rangle=0$$

and the standard algebra of the projection operators ($P+Q+D=1$, and $P^2=P$, etc.), we can obtain the system of coupled equations

$$\begin{aligned}(E-H_{PP})\,P\mid\Psi\rangle&=H_{PD}D\mid\Psi\rangle+H_{PQ}\,Q\mid\Psi\rangle;\\(E-H_{QQ})\,Q\mid\Psi\rangle&=H_{QD}\,D\mid\Psi\rangle+H_{QP}\,P\mid\Psi\rangle;\\(E-H_{DD})\,D\mid\Psi\rangle&=H_{DP}\,P\mid\Psi\rangle+H_{DQ}\,Q\mid\Psi\rangle,\end{aligned}$$

where $H_{PP}=P\cdot H\cdot P$ is the pion–nucleus nonresonance Hamiltonian, etc. The main assumption of the model is that the Hamiltonians H_{PQ} and H_{QP} are ignored, i.e., it is assumed that the states in the Q space cannot be directly coupled to states in the P space and can be coupled only through states of the D space. One can then obtain[88] for the pion–nucleus scattering matrix the expression

$$T_{CP}=H_{CD}\frac{1}{E-H_{DD}-H_{DP}\dfrac{1}{E-H_{PP}}H_{PD}-H_{DQ}\dfrac{1}{E-H_{QQ}}H_{QD}}H_{DP},\tag{34}$$

where $C=P$ (elastic scattering) or $C\in Q$ (inelastic scattering or pion absorption process). In the case of elastic pion–nucleus scattering, the matrix T_{PP} satisfies the Lippmann–Schwinger equation with the optical potential

$$U_{PP}=H_{PD}\frac{1}{E-H_{DD}-H_{DQ}\dfrac{1}{E-H_{QQ}}H_{QD}}H_{DP}+U^{NR}.\tag{35}$$

Calculations in the isobar–hole model begin with construction of the H_{PQ} matrix element, which describes the coupling between the P and D spaces. It plays a role analogous to that of the knowledge of the free t matrix for πN scattering in the case of the optical model. The Hamiltonian H_{PD} is usually chosen to have the form

$$H_{PD}=f\,\mathbf{S}\mathbf{q}g(q)\,\mathbf{T},\tag{36}$$

where $\mathbf{S}$ (respectively, $\mathbf{T}$) is the operator of transition between the object with spin (respectively, isospin) 1/2 and the object with spin (isospin) 3/2; $g(q)$ is the $\pi N\Delta$ vertex function [similar to the one in (27)]; $\mathbf{q}$ is the relative momentum in the pion–nucleon system; and the coupling constant f is determined by fitting the shape of the free Δ_{33} resonance by means of a Breit–Wigner expression. The nonresonance optical potential $U^{\rm NR}$ is constructed in the same way as $U(E)$ in (25) under the assumption that the resonance pole has been removed from the t matrix of πN scattering.

The Hamiltonian of the isobar has the form

$$H_{DD}=T_\Delta^{\rm kin}+U_\Delta,\tag{37}$$

where U_Δ corresponds to the distortion of the propagation of the isobar in the field of the $A-1$ nucleons. The term

$$U_{\rm in}=H_{DQ}\frac{1}{E-H_{QQ}}H_{QD}\approx R(E)-{\rm i}\Gamma(E)/2-\delta W+U_{sp}(E)\tag{38}$$

is the contribution of all the remaining channels of inelastic pion–nucleus scattering. There exist two main channels of inelastic pion–nucleus scattering in the range of energies $80\leqslant T\leqslant 300$ MeV: quasielastic pion scattering and pion annihilation. The contribution of the first of these is approximated in (38) by the term $R(E)-i\Gamma(E)/2-\delta W$, and the contribution of the second by the term $U_{\rm sp}$, which is sometimes called the "smearing potential." With regard to quasielastic scattering ($\pi+A\to\pi+N+A'$ processes), the following assertions are usually made. If the isobar were an entirely free particle, the contribution of the πN channel would be described simply by some resonance with mass $R(E)$ and width $\Gamma(E)$. Both of these quantities could be expressed in the form of a certain vertex function $g(q)$. However, by virtue of the Pauli principle the states populated by the core nucleons are forbidden. This is reflected in the addition of the correction δW. Thus, in the model of isobar–hole excitations quasielastic scattering is to a large degree similar to the treatment in the optical model. For example, in the case of π^4He scattering the correction δW is the same quantity as the spin–isospin part of the second-order optical potential (32).

The form of U_Δ, and even more of $U_{\rm sp}$, is not known very well. There is general agreement that U_Δ must have the same value as the nucleon–core interaction potential; for calculations of π^4He scattering, it was taken in a Gaussian form.[18,89]

The potential $U_{\rm sp}(E)$ is determined purely phenomenologically, its parameters being simply fitted to reproduce $\sigma_{\rm tot}$ and the differential cross section of forward elastic scattering at all the considered energies. It must be emphasized that the correction for the coupling of the isobar in the field of the nucleus, U_Δ, and the "smearing potential" $U_{sp}(E)$ occur in the optical potential (35) on completely the same basis, and there is no possibility of determining them separately.

The form of the denominator in Eq. (34) having being fixed, this equation is diagonalized, for example, in a basis of harmonic-oscillator functions in the space of the Δ and $A-1$ nucleons. Then the elastic scattering amplitude takes the form

$$\langle \mathbf{Q}'0 \mid T_{PP} \mid 0\mathbf{Q}\rangle = \sum_\alpha \frac{\langle \mathbf{Q}'0 | H_{PD} | \alpha\rangle \langle \alpha | H_{DP} | 0\mathbf{Q}\rangle}{E-E_\alpha - \mathrm{i}\Gamma_\alpha/2}, \quad (39)$$

where $|\alpha\rangle$ are eigenstates. It is found that for every nonperipheral ($l<kR_A$) pion–nucleus partial wave there exists a single state that exhausts practically the entire strength of the transition between the pion–nucleus ground state and the Δ–hole states. One can put this differently by saying that in the region of the resonance strong collective pion–nucleus states are formed in the lowest partial waves. This important result has been obtained for pion scattering by all the nuclei with closed shells hitherto studied.[89-91] It can in no case be regarded as a consequence of the assumptions put into the theory.

It is helpful to make a more detailed comparison of the physical content of the isobar–hole model and the optical model. For this, we define in the D space the following set of basis functions:

$$|d_{ik}\rangle = |\Delta_i\rangle \otimes |\psi_k^{(A-1)}\rangle, \quad (40)$$

where the subscripts i and k label the states of the isobar and the core of $A-1$ nucleons, respectively. The set of functions (40) is chosen to diagonalize the isobar Hamiltonian:

$$(E_i - H_{DD}) \mid d_{ik}\rangle = 0.$$

Such a diagonalization procedure would, of course, be rather impracticable for real calculations, but we shall use it only in order to follow the connection between the formalism of the isobar–hole model and multiple scattering theory. The interconnection between them is not manifested particularly clearly if one considers directly Eq. (39), which was obtained by a different diagonalization procedure.

To simplify the following arguments, we ignore the contribution of the nonresonance pion–nucleus scattering and make the separation

$$E - H_{DD} - H_{DQ} - H_{DQ}\frac{1}{E-H_{QQ}} H_{QD} = e + h, \quad (41)$$

where e and h are the diagonal and nondiagonal parts of the denominator of Eq. (35), respectively. Then

$$e \mid d_{ik}\rangle = (E - E_i - R_{ik} + \mathrm{i}\Gamma_{ik}/2) \mid d_{ik}\rangle$$

and $\langle d_{i'k'} | h | d_{ik}\rangle \neq 0$ for all cases except $i'=i$, $k'=k$. The use of the decomposition (41) leads for the optical potential (35) to an iterative series of the form

$$U_{PP} = H_{PD}\frac{1}{e} H_{DP} + H_{PD}\frac{1}{e} h \frac{1}{e} H_{DP} + \dots \quad (42)$$

Let us consider in more detail the first term in (42):

$$\langle \mathbf{Q}'0 | U_{PP}^{(1)} | 0\mathbf{Q}\rangle = \sum_{ik} \frac{\langle \mathbf{Q}'0 | H_{PD} | d_{ik}\rangle \langle d_{ik} | H_{DP} | 0\mathbf{Q}\rangle}{E-E_i-R_{ik}-\mathrm{i}\Gamma_{ik}/2}. \quad (43)$$

It is usually assumed that the vertex operator H_{PD} is not renormalized in the nuclear medium, i.e., its expression (36) for free scattering is not changed in the nucleus. One can show that such an assumption is much weaker than the assumption of validity of the impulse approximation. Since H_{PD} [see (36)] acts only on the isobar variables,

$$\langle \mathbf{Q}'0 | U_{PP}^{(1)} | \mathbf{Q}0\rangle = A \sum_{ik} \int \frac{\langle \mathbf{Q}'\mathbf{p}' | H_{PD} | \Delta_i\rangle \langle \Delta_i | H_{DP} | \mathbf{Q}\mathbf{p}\rangle}{E-E_i-R_{ik}+\mathrm{i}\Gamma_{ik}/2} \rho^{(k)}(\mathbf{p}'\mathbf{p})\, d^3p'\, d^3p. \quad (44)$$

Here, $\mathbf{Q}'$ (respectively, $\mathbf{Q}$) and $\mathbf{p}'$ (respectively, $\mathbf{p}$) are the momenta of the pion and nucleon in the final (respectively, initial) states. Further,

$$\rho^{(k)}(\mathbf{p}'\mathbf{p}) = \langle 0 \mid \mathbf{p}'k\rangle \langle k\mathbf{p} \mid 0\rangle \quad (45)$$

is the matrix of the nuclear density when the core occupies the state k. The expression

$$\langle \mathbf{Q}'\mathbf{p}' | \tau(E) | \mathbf{p}\mathbf{Q}\rangle = \sum_i \frac{\langle \mathbf{Q}'\mathbf{p}' | H_{PD} | \Delta_i\rangle \langle \Delta_i | H_{DP} | \mathbf{Q}\mathbf{p}\rangle}{E-E_i-R_{ik}+\mathrm{i}\Gamma_{ik}/2} \quad (46)$$

must be interpreted as the pion–nucleon scattering matrix as modified in the nuclear medium.

Although the τ matrix determined by the relation (46) is less general than the exact $(A+1)$-particle τ matrix in (16), it nevertheless takes into account the distortion in the Δ propagation due to the potential U_Δ. It is at this level that the effects of the binding are taken into account in the isobar–hole model. After the completeness approximation with respect to all the isobar states has been made in (46), we arrive at the free pion–nucleon scattering matrix:

$$\langle \mathbf{Q}'\mathbf{p}' | t(E-\Delta E_k) | \mathbf{p}\mathbf{Q}\rangle = \langle \mathbf{Q}'\mathbf{p}' | H_{PQ} | \mathbf{Q}'+\mathbf{p}'\rangle \times \langle \mathbf{p}+\mathbf{Q} | H_{QP} | \mathbf{Q}\mathbf{p}\rangle (E-\bar{E}-\bar{R}_k+\mathrm{i}\bar{\Gamma}_k/2)^{-1}. \quad (47)$$

In the impulse approximation, the first-order potential will now have the form

$$\langle \mathbf{Q}'0 \mid U_{PP}^{(1)} \mid 0\mathbf{Q}\rangle = A \sum_k \int \mathbf{Q}'\mathbf{p}' \mid t(E-\Delta E_k) \mid \mathbf{p}\mathbf{Q}\rangle \rho^{(k)}(\mathbf{p}'\mathbf{p}) \times d^3p' d^3p. \quad (48)$$

We again use the completeness approximation, setting $\Delta E_k = \Delta\bar{E}$ for k. Then the first-order optical potential takes the factorized form

$$\langle \mathbf{Q}'0 \mid U_{PP}^{(1)} \mid 0\mathbf{Q}\rangle = A \langle \mathbf{Q}' \mid t(E-\overline{\Delta E}) \mid \mathbf{Q}\rangle F_0(\mathbf{q}). \quad (49)$$

Similarly, the second term in (42) can also be rewritten in the form

$$U_{PP}^{(2)} = A(A-1)\tau \frac{Q}{E-H_{QQ}} \tau. \quad (50)$$

All the remaining terms of higher orders in (42) can also be paired against terms of the iterative expansion of the optical potential (21), which have a transparent physical meaning.

We now discuss the results of recent calculations of pion–nucleus scattering in the framework of the isobar–hole model. Compared with the first-order optical potential (24), the optical potential (35) of the isobar–hole model has an additional nonlocality, which arises through the distortion of the isobar propagation by the potential $U_\Delta + U_{sp}(E)$. The part played by this nonlocality was analyzed in Ref. 92, in which it was shown that such nonlocality must be introduced if the differential cross sections of π^4He, π^{12}C, and π^{16}O elastic scattering are to be correctly reproduced.

A detailed study of π^4He scattering in the formalism of the isobar–hole model was made in Refs. 18 and 89. Figure 16 shows the part played by allowance for the "smearing potential" in calculations of the total cross sections of π^4He scattering. Further investigations[90,91] revealed the need to introduce a spin-dependent part in the smearing potential, for otherwise $U_{sp}(E)$ becomes strongly energy dependent. It is obvious that if $U_{sp}(E)$ is ignored, it is no longer possible to obtain even qualitatively the correct behavior of the total, σ_{tot}, and elastic, σ_{el}, cross sections of π^4He scattering. In this respect, the optical model with a first-order potential has an advantage over calculations in the isobar–hole approach. This can be seen by comparing the results shown in Figs. 10 and 16. Since the two models differ only in the presence of the distorting potential U_Δ in (35), it may be concluded that in the isobar–hole theory the correction to the impulse approximation obtained is too large, and it must to a large degree be compensated by the expression $U_{sp}(E)$.

In the standard version of the isobar–hole model, there is a further dangerous point. The simple nuclear shell model, which is used in the calculations in (39) or (44), in no way guarantees that the nuclear form factors $F_0(q)$ calculated with it agree well with the data on electron–nucleus scattering. Our experience of working with the optical model suggests that correct specification of the nuclear form factors is a necessary condition, particularly if one is to obtain a good description of the differential cross sections of elastic scattering.

If the distorting potential U_Δ is fixed and the parameters of the potential $U_{sp}(E)$ are fitted, then in principle the various inelastic scattering channels can be calculated by

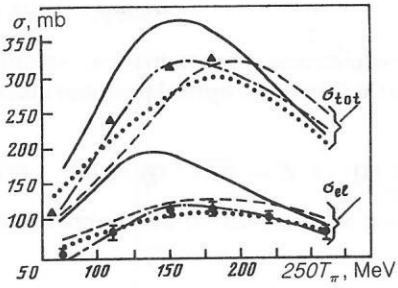

FIG. 16. Total and elastic cross sections of π^4He scattering. The theoretical curves correspond to different choices of W_0 in the smearing potential $U_{sp}(r) = W_0 \exp(-\beta r^2)$: the continuous curves to $W_0 = 0$, the broken curves to $W_0 = 60$ MeV, the dotted curves to $W_0 = 40 - i30$ MeV, and the chain curve to fitting of W_0 using the experimental data for σ_{tot} and Re $f(0)$ (from Ref. 18).

means of Eq. (34). We consider in somewhat more detail the predictions made by the isobar–hole model in the case of description of the channel of pion annihilation on ^{4}He. The direct use of Eq. (34) leads to the very difficult problem of approximating the matrix element H_{QD}, which is a very complicated object. Instead of this, one usually[18,89] calculates the expression

$$\sigma_a = -\frac{2}{k} \langle \Psi^{(-)} | \operatorname{Im} U_{sp}(E) | \Psi^{(+)} \rangle, \tag{51}$$

where $\Psi^{(+)}$ are the wave functions of elastic pion–nucleus scattering. However, the expression (51) corresponds, generally speaking, to the absorption spectrum only in the approximation of single scattering (see, for example, Ref. 93), in which the contributions to the absorption cross section from the terms of higher order were also obtained). In addition, calculations of the absorption cross section in accordance with (51) lead to rather large values of σ_{abs} (about 100 mb at $T \sim 100$ MeV). This agrees with the data of Refs. 28 and 29, but contradicts the experiments of Refs. 20, 26, and 27 (see the discussion of the relative cross section of the absorption channel in Sec. 1).

There are more complicated schemes of the model of isobar excitations, in which it is assumed that other resonances (for example, the Roper resonance) or other poles (for example, the πN pole) have their own Hilbert space. As a rule, they ensure a more microscopic treatment of the pion absorption mechanism in nuclei.[94,95]

Low-energy pion–nucleus scattering and π-mesic atoms

Negative pions can be bound by the Coulomb field of the nucleus and form π-mesic atoms. The highly excited states of such atoms populate almost pure Bohr orbits. As a rule, one almost always has one or several states with energies different from the corresponding energies of the Bohr orbit. In addition, because of the strong interaction, the widths of these states become somewhat greater. Practically all pions are absorbed from these orbits and virtually no pions reach the low-lying states. However, in the case of the lightest nuclei the pions can reach even the ground state of the π-mesic atom, and the energy shift ε_{1S} of the level and its width Γ_{1S} are an important source of information about the pion–nucleus interaction. Direct measurement of Γ_{1P} is impossible, but the width of a $1P$ level can be obtained by calculations in the cascade model.

The properties of π-mesic atoms are usually interpreted in terms of the optical model. The optical potential of Kisslinger type has in the coordinate representation the form

$$\langle \mathbf{r} \mid U\Psi \rangle = V_{\text{Coul}} + 4\pi A\,[-(b_0\rho(r) + b_1\delta\rho(r) + B_0\rho^2(r)) + \nabla(c_0\rho(r) + c_1\delta\rho(r) + C_0\rho^2(r))\nabla]\langle \mathbf{r} \mid \Psi \rangle, \tag{52}$$

where the coefficients b_0 (respectively, c_0) and b_1 (c_1) are certain combinations of the pion–nucleon scattering lengths (respectively, volumes) in the S and P waves. The nuclear structure enters (52) through

$$\left.\begin{aligned} \rho(r) &= (N\rho_n(r) + Z\rho_p(r))/A; \\ \delta\rho(r) &= (N\rho_n(r) - Z\rho_p(r))/A, \end{aligned}\right\} \tag{53}$$

TABLE II. Experimentally measured shifts and widths of states of π-mesic atoms.

Nucleus	ε_{1S}, eV	Γ_{1S}, eV	Γ_{1P}, eV	Reference
^{3}He	34 ± 4 32 ± 3 44 ± 12	36 ± 7 28 ± 7 65 ± 12	$(1.6 \pm 0.8) \cdot 10^{-3}$	[96] [97] [98]
^{4}He	-79 ± 5 -76 ± 2	45 ± 21 45 ± 3	$(7.2 \pm 3.3) \cdot 10^{-4}$	[99] [100]

where $\rho_n(r)$ and $\rho_p(r)$ are the neutron and proton densities, respectively, normalized to unity. The $\rho^2(r)$ terms are introduced in (52) purely phenomenologically, and therefore the complex coefficients B_0 and C_0 contain information about the pion absorption as well as the higher order effects of multiple-scattering theory.

Existing experimental values for ε_{1S}, Γ_{1S}, and Γ_{1P} for the ^{3}He and ^{4}He nuclei are given in Table II. It can be seen that the level widths Γ_{1P}, and also Γ_{1S} in the case of ^{3}He, are known with rather poor accuracy. The attractive nature of the πN interaction in the S wave ($b_0 > 0$) has the consequence that the shift ε_{1S} becomes negative for all nuclei except ^{3}He. Here, because of the fact that the isospin-dependent term $b_1 \delta\rho(r)$ is negative and has a fairly large value, the resulting energy shift becomes positive.

The experimental values of the shifts of the levels and their widths are used to fix the coefficients B_0 and C_0 in the optical potential (52). Optical models with such potentials give a fairly good description of the pion scattering up to energies $E \sim 50$ MeV for nuclei with $J = T = 0$.[101-104] Figure 17 shows the results of calculations of π^4He elastic scattering in the optical model with first- and second-order potentials to which there is also added a term $U_{\rm abs}$, which takes into account effectively pion absorption.[50] The parameters C_0 and B_0 from (52) in the expression $U_{\rm abs}$ were determined from the data on π-mesic atoms. Calculations of π^3He elastic scattering have not been made in such an approach because the spin and isospin structure of the absorbing terms of the optical potential (52) are poorly known. In this direction, only the first steps have been taken.[105,106]

It has by now become clear that all microscopic calculations of pion absorption constitute a very difficult problem. It has been found that such processes depend strongly on the effects of the binding of the nucleons in the nucleus,[107] on the nuclear correlations,[108] and on the behavior of the pion–nucleon scattering matrix off the energy shell.[109] In fact, this is by no means the complete list of all the many-particle effects that play a part in the description of pion absorption by nuclei.

3. TOPICAL PROBLEMS OF PION INTERACTION WITH THE LIGHTEST NUCLEI

In this section, we discuss some of the most topical problems of pion scattering by the lightest nuclei. In the first place, we consider pion absorption processes. It is interesting to study these reactions on light nuclei, since it is in this case possible to obtain information about the isotopic structure of the amplitude of absorption by a pair of nucleons. It is well known[110,111] that absorption by a two-nucleon cluster is the dominant mechanism of pion annihilation. However, by studying the reaction $\pi d \to pp$ it is not possible to obtain the amplitude of absorption in the state with isospin $T = 1$ (for the simple reason that the deuteron is a system with $T = 0$). Recent experiments[31,112-117] on pion absorption by the ^{3}He and ^{4}He nuclei have led to the discovery of interesting features of pion annihilation in the channel with $T = 1$.

Another important problem that can be solved by means of pion–nucleus physics at intermediate energies is that of the violation of charge symmetry in strong interactions. It is well known that one of the consequences of the

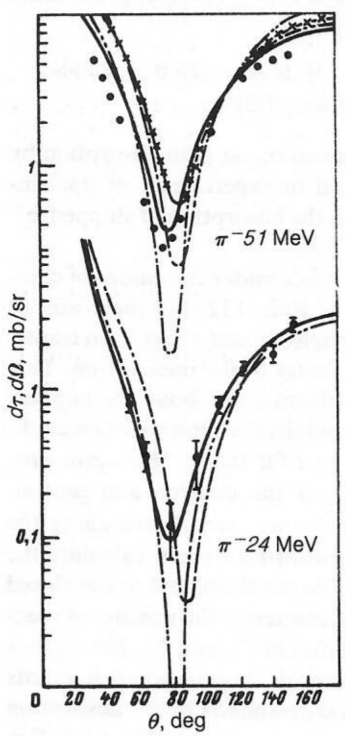

FIG. 17. Differential cross section of elastic π^4He scattering at low energies. The continuous curves are the results of optical-model calculations with potentials $U^{(1)} + U^{(2)} + U^{\rm abs}$ (Ref. 50) (the parameters of $U^{\rm abs}$ were taken from data on π-mesic atoms); the chain curves give the results of calculations with the potential $U^{(1)}$; the chain curve with two dots gives the results with the potential $U^{(1)} + U^{(2)}$; and the curve with the crosses corresponds to $U^{(1)} + U^{\rm abs}$. The experimental data are taken from Refs. 46 and 47.

principle of charge symmetry is equality of the cross sections of π^+ and π^- scattering by nuclei with zero spin and isospin. (Of course, all the effects of the Coulomb interaction in such processes must be taken into account.) However, in recent experiments on $\pi^\pm$ elastic scattering by deuterium[12-14] and the ^{3}He and ^{3}H nuclei[129] indications of violation of charge symmetry were obtained.

Finally, in this section we shall consider the main characteristics of the dominant channel of inelastic pion scattering—quasielastic nucleon knockout.

Pion absorption by ^{3}He and ^{4}He nuclei

As already noted above, the main mechanism of pion absorption is absorption by a two-nucleon cluster. But what fraction of the total absorption cross section is due to this process? It is clear that three, four, or more nucleons can participate in the absorption reaction on heavy nuclei. The probability of three- and four-nucleon absorption can be extracted from data on pion absorption by ^{4}He and ^{3}He.

Pion absorption by ^{3}He nuclei was investigated experimentally for the first time at Dubna.[130] In this work, measurements were made for stopped π^- mesons of the relative probability W of the channels with emission of a proton or a deuteron; the Panofsky ratio was measured:

$$P\,(^3\mathrm{He}) = \frac{\sigma\,(\pi^-\,{}^3\mathrm{He}\to{}^3\mathrm{H}+\pi^0)}{\sigma\,(\pi^-\,{}^3\mathrm{He}\to{}^3\mathrm{H}+\gamma)} = 2.28 \pm 0.18.$$

This work already showed that pion absorption takes place predominantly on an *np* pair:

$$W\,(pnn) = (57.8 \pm 5.4)\,\%; \quad W\,(dn) = (15.9 \pm 2.3)\,\%; \quad W\,(dn\gamma) = (3.6 \pm 1.2)\,\%.$$

The most complete information on pion absorption by nucleon clusters was obtained in experiments of Backenstoss's group at SIN, in which the absorption of stopped π^- mesons in ^{3}He was studied.[112-115]

These experiments were made under conditions of complete kinematics. Moreover, in Refs. 112–115 there was simultaneous detection of the nucleons and γ rays from transitions of the pion in different states of the mesic atom. This made it possible to obtain information about the angular momentum of the state from which the pion was captured.

Figure 18 is the Dalitz plot for the $\pi^-\,{}^3\mathrm{He}\to pnn$ process.[113] The kinetic energies of the neutron and proton, which were detected in coincidence, are plotted along the axes. Knowing both these quantities, we can calculate the energy of the third particle. The numbers next to the closed contours in the Dalitz plot characterize the number of reaction events with the given values of T_n and T_p. Six regions are clearly distinguished: The peak D corresponds to events with low proton energy and corresponds to π^- absorption by an *np* pair [the $\pi^-(pn)\to nn$ reaction]. The peak F is characterized by a low kinetic energy of the neutron and corresponds to absorption by a *pp* pair [the $\pi^-(pp)\to np$ reaction]. If after absorption by a *pp* pair a high-energy proton and a neutron are detected, then in the Dalitz plot such events occupy the region B. Region A corresponds to the maximally possible proton energy. It follows from the kinematics of the reaction that in this case two neutrons must be

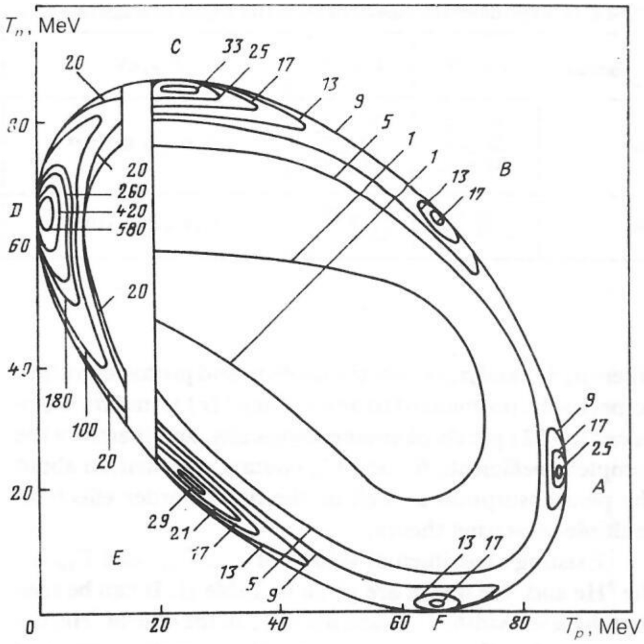

FIG. 18. Dalitz plot for the nnp system in the $\pi^-\,{}^3$He reaction.[113] Region A corresponds to final-state interaction in the nn system; regions B and F to absorption by a *pp* pair, $\pi pp\to np$; regions C and E to final-state interaction in the np system; and D corresponds to absorption by an *np* pair, $\pi np\to nn$.

emitted in the direction opposite to that of the proton and must have relative momentum near zero, i.e., this means that there is an interaction in the final state between the neutrons. The analogous configuration for an *np* pair corresponds to the region C. Here, the effects of the final-state interaction between the proton and neutron are strong. The region E is the mirror reflection of the peak C; it also characterizes interaction in the final state between an *np* pair, but in this case a low-energy neutron is detected.

Analysis of the Dalitz plot in Fig. 18 makes it possible to determine the partial probabilities of the various absorption reaction mechanisms. It is found that the two-particle absorption is about 70% of all the cases; the reaction $\pi^-\,{}^3\mathrm{He}\to nd$ with production of a deuteron provides about 10%; the remainder is taken up by the fraction of events with final-state interaction. The results of Backenstoss's experiments demonstrate very clearly the important part played by the processes of interaction in the final state. It is clear that without allowance for such effects one could not obtain a satisfactory quantitative description of the pion absorption processes. Figure 18 also clearly exhibits another fundamental fact that characterizes the pion absorption processes. It can be seen that the events are mainly grouped in the region D, where the absorption takes place on an *np* pair. There are far fewer pion absorptions by a *pp* pair (the regions F and B). The branching ratio for these two cases is

$$R = \frac{(\pi,\ np)}{(\pi,\ pp)} = 10.1 \pm 1.5.$$

An indication of a strong difference between the *np* and *pp* pair absorption channels was obtained in Ref. 116, in which π^+ and π^- absorption by ^{3}He and ^{4}He was studied at

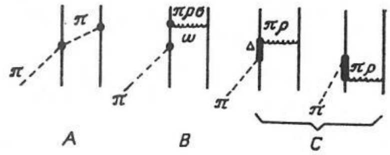

FIG. 19. Diagrams of the pion absorption process.

$T = 165$ MeV. A colossal discrepancy between the differential cross sections of the π^+, pp and π^-, np reactions was found.[116] According to the data, np absorption exceeded pp absorption by 100 times. Although subsequent experiments,[114,117] and also a different study of the same authors,[118] yielded a value of R approximately three times smaller, this value differed strongly from what could have been expected on the basis of simple isotopic considerations.

For suppose the absorption takes place on two nucleons either in a state with total isospin $T = 0$ and total spin $S = 1$ (np pair) or in a state with $T = 1$, $S = 0$ (pp and np pairs). Suppose also that the dominant process is excitation of the Δ isobar with its subsequent decay (the corresponding diagram is shown in Fig. 19). Then it follows from simple isotopic relations that the ratio of the cross section of absorption in the pure state with $T = 0$ to the cross section of absorption in the state with $T = 1$ must be $R_T = 2$.[119] And then the observed ratio of the cross sections of absorption in ^{3}He (with allowance for the total number of np pairs with isospin $T = 0$ and T = 1) must be

$$R = \frac{6R_T + 1}{2} = 6.5.$$

This value is appreciably smaller than what is actually observed in experiments (Fig. 20).

The suppression of absorption by a pair of nucleons with isospin $T = 1$ can be explained qualitatively as follows. Investigations of pion absorption by deuterons showed that absorption from the state $l_{\pi - 2N} = 1$ is dominant.[120] When this P-wave absorption of the pion ($J^\pi = 1^+$) takes place on a pair of nucleons with $T = 1$ and total spin $S = 0$ and $L = 0$, the quantum numbers of the final state must be $J^\pi = 1^+$ and $T = 1$. But the Pauli principle prevents two nucleons from being in such a state, i.e., absorption by a pair of nucleons with $T = 1$ (Fig. 19) cannot take place in the state $l_{\pi - 2N} = 1$, but it is precisely this state that is dominant in the region of energies of the Δ_{33} resonance. The quantum numbers that two nucleons in the absorption reaction can have are given in Table III. It can be seen that absorption by a pair of nucleons with $T = 1$ must take place from the S and D states, the contribution of which is relatively small.

Arguments similar to those given above were used for quantitative analysis in Refs. 94 and 119. The results of these calculations are shown in Fig. 20 by the continuous[94] and broken[119] curves. It can be seen that as yet the theory describes only the basic features of the energy dependence of R. One of the reasons for the discrepancy may be the need to take into account not only rescatterings through the Δ isobar (of the type of the graphs C in Fig. 19) but also other processes (see graphs A and B in Fig. 19). It is asserted in Ref. 121 that although the type-C graphs are the most basic ones for pion absorption by a pair of nucleons with $T = 0$, they are not sufficient for a description of absorption by a pair with $T = 1$. It is precisely the small cross section of the channel with $T = 1$ obtained as a result of taking into account only Δ rescatterings that leads to the overestimation of the ratio R. In Ref. 121 it is shown that diagrams A and B are indeed important for absorption by a pair with $T = 1$, and allowance for them makes it possible to reduce significantly the value of R.

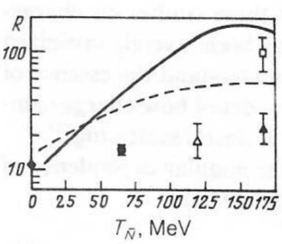

FIG. 20. The ratio $R = \sigma(\pi^+, pp)/\sigma(\pi^-, pn)$ for pion absorption in ^{3}He. The experimental points are from Refs. 113, 116, and 117, the continuous curve is the result of the theoretical calculations of Ref. 94, and the broken curve is from Ref. 119.

A detailed study of the spectra of the protons in ^{3,4}He($\pi^\pm$, p) reactions at energies 50–250 MeV was made in Refs. 30 and 122. Absorption by two nucleons is clearly manifested in these spectra in the form of a peak at large values of the momenta. The width of the peak is 54 and 100 MeV for ^{3}He and ^{4}He, respectively. The authors also note that the absorption cross section for the lightest nuclei is directly related to the number of np pairs in the nucleus. Thus, the absorption cross sections for ^{2}H, ^{3}He, and ^{4}He bear the ratios 1:2:4. However, and this is very important, the law is violated already for absorption by ^{6}Li and heavier nuclei. For example, the absorption cross section for ^{27}Al is only eight times greater than σ_{abs} (πd). The cross section for the absorption of pions by ^{6}Li should exceed σ_{abs} (π^4He) by a factor 2.25 but in fact exceeds it by only 20%. These facts receive a natural explanation if one bears in mind that pion absorption must occur when the nucleons are at small relative distances. But when the p shell begins to be filled in the nuclei, the effective number of np pairs that can absorb the pion is decreased. This leads to a quite different nature of the A dependence of the absorption cross section than in the case of the lightest nuclei, for which all the nucleons are in the s state.

Searches for violation of charge symmetry in πA scattering

There are numerous indications that charge symmetry can be violated in strong interactions. There is the difference between the nn and pp scattering lengths in the same isotopic state,[123,124] the difference between the ^{3}H and ^{3}He binding energies, which cannot be explained by purely Coulomb effects,[125,126] and the fact that different πN scattering phase shifts are extracted from data on $\pi^+ p$ and $\pi^- p$ scattering.[127,128] In recent years, the validity of the charge-symmetry principle has been subjected to more sustained testing in pion–nucleus reactions.

TABLE III. Quantum numbers of the πNN system that are allowed in pion absorption reactions. The indices i, ΔN, and f label the pair of nucleons in the initial, intermediate, and final states, respectively. The angular momentum in the $\pi 2N$ system is denoted by $l(\pi-2N)$.

J_i^π, T_i $L_i=0$	$l(\pi-2N)$	$J_{\Delta N}^\pi$; $L_{\Delta N}$ $T_{\Delta N}=1$	J_f^π ; L_f; T_f	${}^{2(s+1)}L_J$
1^+, 0	0	1^-; 1	1^-; 1; 1;	${}^3P_1$
	1	0^+; 2	0^+; 0; 1;	${}^1S_0$
		2^+; 0,2	2^+; 2; 1;	${}^1D_2$
0^+; 1	0	0^-; 1	0^-; 1; 1;	${}^3P_0$
	1	Δ not formed	1^+; 0; 0;	${}^3S_1$
			1^+; 2; 0;	${}^3D_1$

For example, an experiment at Los Alamos studied the elastic scattering of $\pi^\pm$ mesons by the ^{3}He and ^{3}H nuclei at energy $T = 180$ MeV.[29] If charge symmetry holds, then after subtraction of the Coulomb effects the differential cross sections of $\pi^\mp$ elastic scattering by ^{3}He must be equal to the differential cross sections of $\pi^\pm$ ^{3}H scattering, i.e., the ratios $r_1 = d\sigma(\pi^{+3}\mathrm{H})/d\sigma(\pi^{-3}\mathrm{He})$ and $r_2 = d\sigma(\pi^{+3}\mathrm{He})/d\sigma(\pi^{-3}\mathrm{H})$ must be equal to unity.

The Los Alamos study investigated the angular dependence of tne ratios r_1 and r_2, and also the "super-ratio" R, defined as

$$R = \frac{r_1}{r_2} = \frac{d\sigma\,(\pi^+\,{}^3\mathrm{H})/d\sigma\,(\pi^-\,{}^3\mathrm{He})}{d\sigma\,(\pi^-\,{}^3\mathrm{He})/d\sigma\,(\pi^-\,{}^3\mathrm{H})}. \tag{54}$$

The results of this study are shown in Fig. 21. It can be seen that at angles $\theta = 60$–$80°$ the super-ratio R differs appreciably from unity, the deviation being outside the limits of the systematic errors of the experiment, which are shown in Fig. 21 by the broken lines. The largest value of R is reached at $\theta = 65°$, where $R = 1.31 + 0.09$. It is interesting that if the angular dependences of r_1 and r_2 are analyzed separately, then it is seen to be the "weak" nonresonance ratio r_1 that differs from unity. At the same time, the "strong" resonance ratio r_2 for the processes with the largest π^{+3}He and π^{-3}H scattering cross sections behaves normally and does not exhibit any violation of charge symmetry.

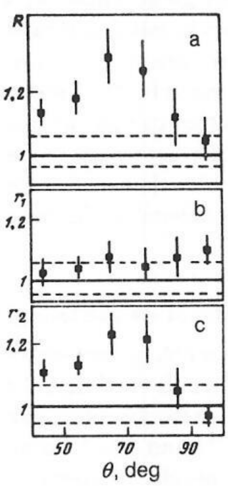

FIG. 21. Angular dependence of the "super-ratio" R in (54) for elastic $\pi^\pm$ ^{3}He and $\pi^\pm$ ^{3}H scattering at $T_\pi = 180$ MeV (a)[129]; the angular dependence of the ratio $r_1 = d\sigma(\pi^{+3}\mathrm{H})/d\sigma(\pi^{-3}\mathrm{He})$ (b); and angular dependence of $r_2 = d\sigma(\pi^{+3}\mathrm{He})/d\sigma(\pi^{-3}\mathrm{H})$ (c). The systematic errors are shown by the broken lines.

Unfortunately, no allowance was made for Coulomb effects in Ref. 129, the authors of which express the hope that in the ratios r_1 and r_2 and especially in the super-ratio R, the Coulomb effects must cancel strongly. However, this question undoubtedly requires detailed elaboration. This is all the more necessary in view of the fact that the greatest deviations are observed in the region of angles in which there is a minimum in the differential scattering cross sections. In this angular interval, the purely nuclear amplitude is small, and, as is well known, the interference between the Coulomb and nuclear amplitudes introduces quite sensible changes.

Detection of charge-symmetry violation was announced in Ref. 12, in which measurements were made of the total cross sections of $\pi^\pm$ scattering by deuterium, and also in Refs. 13 and 14, in which the differential cross sections of $\pi^\pm d$ elastic scattering were analyzed. The measured differences were interpreted in terms of splitting with respect to the masses of the components of the isobar charge multiplet as predicted by quark models.

It was found in Ref. 12 that the characteristic splitting parameter

$$C_W = m\,(\Delta^-) - m\,(\Delta^{++}) + \frac{1}{3}\,[m\,(\Delta^0) - m\,(\Delta^+)] \tag{55}$$

has the value $C_W = 4.6 \pm 0.2$ MeV. In Ref. 13, the value $C_W = 4.35 \pm 0.5$ MeV was obtained. Both of these values are in good agreement with the predictions of the quark models: $C_W = 4.47$ MeV.[131]

However, the conclusions of these studies on charge-symmetry violation have frequently been severely criticized (see, for example, Ref. 133). To understand the essence of the objections, we consider in more detail how charge-symmetry violation was sought in $\pi^\pm d$ elastic scattering.[13,14]

The experiments measured the angular dependence of the asymmetry parameter $A(\theta)$, defined as

$$A\,(\theta) = \frac{d\sigma^{(-)}/d\Omega - d\sigma^{(+)}/d\Omega}{d\sigma^{(+)}/d\Omega + d\sigma^{(-)}/d\Omega}. \tag{56}$$

The experimental data were then compared with theoretical calculations in which the nuclear amplitude $f(\theta)$ was de-

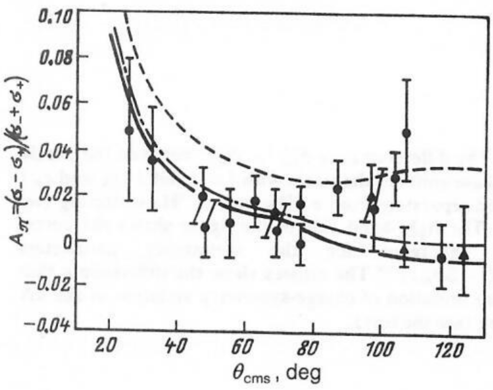

FIG. 22. Asymmetry parameter $A_\pi(\theta)$ in elastic $\pi^\pm d$ scattering at $T_\pi = 143$ MeV.[13] The broken curve shows the results of calculations for the case when complete absence of charge-symmetry violation was assumed, i.e., $C_W = 0$; the continuous curve corresponds to a splitting parameter C_W in (55) equal to 4.35 MeV, and the chain curve to $C_W = 3.6$ MeV.

termined from calculations based on Faddeev's equations without allowance for the Coulomb interaction. After this, a phenomenological distortion of the calculated amplitudes, intended to take into account the Coulomb effects, was introduced. The results obtained at energy $T = 143$ MeV are shown in Fig. 22 by the broken curve; the experimental data are reproduced rather poorly. It is from this that the conclusion of charge-symmetry violation is drawn.

It is clear that such a procedure may be criticized. In Ref. 133, in which the Coulomb corrections were taken into account more carefully, the behavior of the asymmetry $A(\theta)$ at $T = 143$ MeV could be completely reproduced. From this it was included that there is in fact no charge-symmetry violation in pion–nucleus scattering. However, even the authors of Ref. 133 could not obtain a good description of the asymmetry measured at the lower energy $T = 65$ MeV by the Saclay group.[134] They nevertheless assume that there is still no charge-symmetry violation; instead, they believe that in their calculations at low energies the purely nuclear scattering amplitudes are poorly determined.

In our view, the very method of seeking charge-symmetry violation through analysis of the angular dependence of the parameter $A(\theta)$ introduces a high degree of subjectivity. It is necessary to be able to calculate small (about 2%) deviations of $A(\theta)$ from zero. For this, the theoretical model must guarantee an accuracy of calculation of the differential cross section better than 1%. Unfortunately, no contemporary theory of the pion–nucleus interaction is capable of doing this, the reason being the existence of various types of important approximation.

It would be good to look for charge-symmetry violation using model representations as little as possible. A suitable method for this is phase-shift analysis of π^4He scattering. The scattering phase shifts that occur in Eqs. (11) and (12) can be decomposed as follows:

$$\left.\begin{aligned} \delta_l^{(\pm)} &= \delta_{H,\,l} + \delta_{R,\,l}^{(\pm)};\\ \omega_l^{(\pm)} &= \omega_{H,\,l} + \omega_{R,\,l}^{(\pm)}, \end{aligned}\right\} \qquad (57)$$

where $\delta_{H,l}$ and $\omega_{H,l}$ are the purely nuclear phase shifts that would be obtained if we had the possibility of "switching off" the Coulomb interaction, while the phases $\delta_{R,l}^{(\pm)}$ and $\omega_{R,l}^{(\pm)}$ take into account effectively the distortion of the nuclear potential in the Coulomb field. Physically, the phase shifts $\delta_R^{(\pm)}$ and $\omega_R^{(\pm)}$ correspond to some kind of allowance for radiative corrections. The diagrams of the corresponding processes are shown in Fig. 23. In Refs. 132 and 135, an approximate formalism was developed for calculating the Coulomb corrections $\delta_R^{(\pm)}$ and $\omega_R^{(\pm)}$ in terms of the purely nuclear phase shifts δ_H and ω_H and their derivatives. (If we now make a phase-shift analysis of π^{+4}He and π^{-4}He scattering separately and determine the purely nuclear phase shifts, then in the case of charge symmetry they must be equal.)

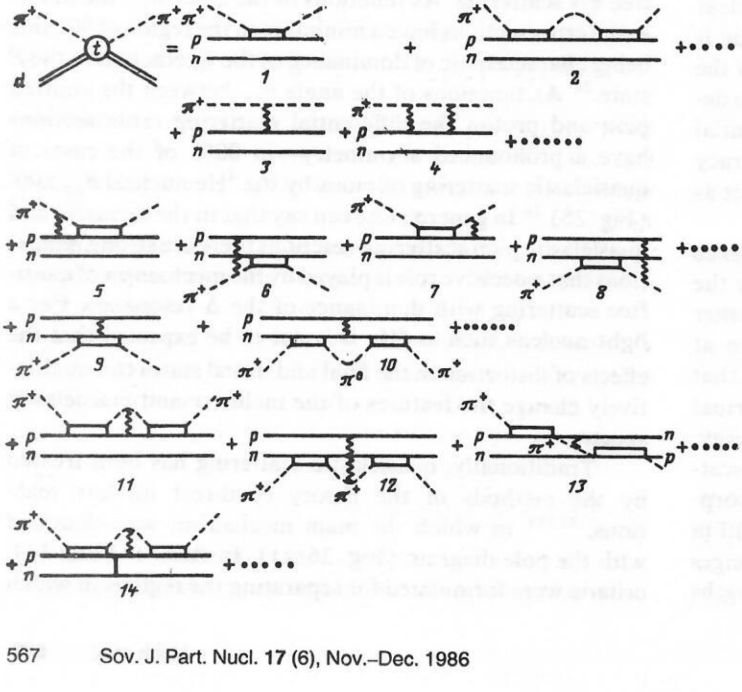

FIG. 23. Diagrams of processes corresponding to Coulomb effects for the example of $\pi^+ d$ scattering. Graphs 1 and 2 and their iterations correspond to the purely nuclear interaction; they form the amplitude $f_{\rm NP}(\theta)$ in (2); graphs 3 and 4 describe purely Coulomb scattering; the processes 5–8 correspond to external Coulomb corrections, taken into account by means of the phase shifts $\delta_{R,l}^{(\pm)}$ and $\omega_{R,l}^{(\pm)}$; diagrams 9–12 and 14 correspond to the "internal" Coulomb corrections, which contribute to the mass difference of the states of the Δ isobar; diagram 13 corresponds to three-particle "internal" Coulomb corrections (grad).

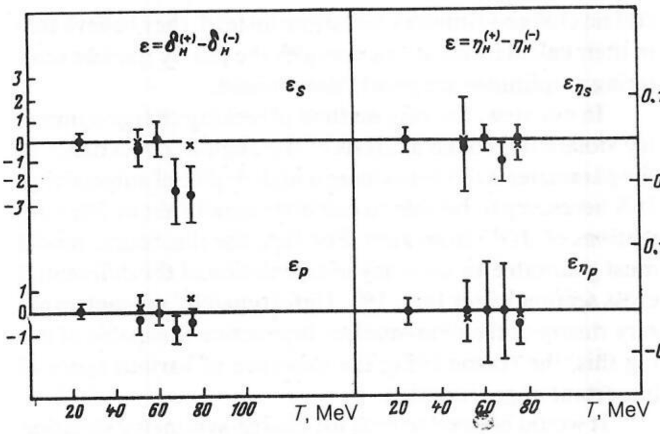

FIG. 24. The difference $\varepsilon_l = \delta_{H,l}^{(+)} - \delta_{H,l}^{(-)}$ between the purely nuclear phase shifts in the states with $l = 0$ and 1 (ε_S and ε_P) determined separately from π^{+4}He and π^{-4}He scattering (on the left). The right-hand side of the figure shows the corresponding difference for the inelasticity parameters $\eta_l = \exp(-2\omega_{H,l})$.[49] The crosses show the difference ε that arises from imitation of charge-symmetry violation in the πN interaction (see the text).

Such a program was implemented in Ref. 49. Figure 24 shows the difference ε between the purely nuclear phase shifts δ_H and ω_H determined from π^{+4}He and π^{-4}He scattering. It can be seen that no statistically significant difference between the purely nuclear phases obtained from the π^{+4}He and π^{-4}He scattering was found. This is a weighty indication that in this region there are no effects of charge symmetry violation.

In Ref. 49 the authors also considered what difference between the purely nuclear phase shifts of π^4He scattering could be produced by the differences between the masses of the components of the Δ-isobar multiplet. For this, a calculation was made in the unitary approach by the method of evolution with respect to the coupling constant,[81,136] differences between the masses and widths of the Δ isobars being introduced in the description of the amplitudes of the elementary πN scattering event. It was assumed that the mass difference of the Δ isobars is $\Delta m = 6$ MeV, and that the difference of the widths is $\Delta\Gamma = 8$ MeV. Then, using the parametrization proposed in Ref. 137, one can show that the difference between the πN scattering phase shifts in the P_{33} wave at energy $T = 75$ MeV is 0.35°. In the case of π^4He scattering, this difference is enhanced and the purely nuclear phase shifts differ by 0.72° at $T_\pi = 75$ MeV. This value is significant; it is more than twice the correction due to the external Coulomb corrections and, in principle, could be detected by means of phase-shift analysis of the differential cross sections of elastic scattering measured to accuracy $\sim 1\%$–2%. Unfortunately, such measurements have not as yet been made for π^4He scattering.

It should be said that the existence of a mass difference in the components of the charge multiplets of not only the isobar but also the nucleon and the pion can lead to rather interesting consequences in pion–nucleus physics even at very low energies. For example, it was noted in Ref. 138 that because of the $\pi^{\pm,0}$ mass differences the process of virtual pion charge exchange is possible even at zero pion energy. This must lead to the appearance in the pion–nucleus scattering length of an imaginary part not related to the absorption process. These questions were investigated in detail in Refs. 138–140, in which it was shown that certain changes are obtained in the πd, π^{-3}He, and π^{+3}H scattering lengths on account of the mass differences of the pions, nucleons, and nuclei. They have a particularly strong effect on the real parts of the π^{-3}He and π^{+3}He scattering lengths, changing them, for a certain class of πN potentials, by 100% and more.[140]

Quasielastic and inclusive pion scattering

Although quasielastic pion scattering has been studied for a rather long time, the experimental situation in this field is still far from perfection. Basically, one measures only individual characteristics of these reactions (excitation functions and double differential cross sections), and, as a rule, experiments with complete kinematics are not made (see Refs. 20, 26, 28–30, 32, 42, 142, and 147).

The main feature of the inclusive scattering of pions is the resonance peak in the energy spectra of the emitted pions. It is due to quasielastic scattering and is observed at all energies of the initial pions. The center of the peak is approximately at the energy $\Delta E = E'_\pi - E_\pi = q^2/2M$, where q is the momentum transfer and M is the nucleon mass (see, for example, Ref. 30). Such an energy balance occurs in free πN scattering. As functions of the angle $\theta_{\pi'}$, the differential cross sections have a minimum in the region of 90°, this being characteristic of dominance of the interaction in the P state.[28] As functions of the angle $\theta_{\pi p}$ between the emitted pion and proton the differential scattering cross sections have a pronounced asymmetry—in 80% of the cases of quasielastic scattering of pions by the ^{4}He nucleus $\theta_{\pi p} \geqslant 90°$ (Fig. 25).[26] In general, one can say that in the inclusive and quasielastic pion scattering reactions there are strong indications that a decisive role is played by the mechanism of quasifree scattering with dominance of the Δ resonance. For a light nucleus such as ^{4}He it is not to be expected that the effects of distortion in the final and initial states can qualitatively change the features of the inclusive and quasielastic spectra.

Traditionally, quasielastic scattering has been treated by the methods of the theory of direct nuclear reactions,[143,144] in which the main mechanism was identified with the pole diagram [Fig. 26(a)]. In Refs. 143 and 144, criteria were formulated for separating the regions in which

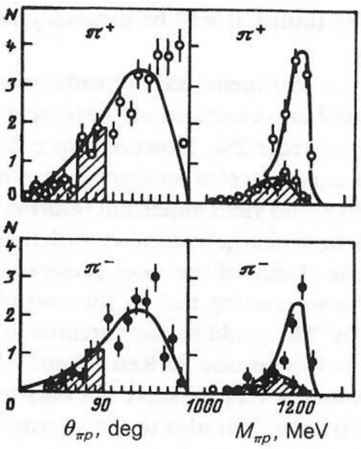

FIG. 25. Distribution of events from the ^{4}He$(\pi^\pm,\pi^\pm p)^3$H reaction at energy 156 MeV with respect to the angle $\theta_{\pi p}$ between the emitted pion and proton, and also with respect to the effective mass of the πp system (on the right). The curves correspond to phase-space calculations with allowance for excitation of the Δ resonance in the intermediate state; the areas below the distributions of events with $\theta_{\pi p} < 90°$ are hatched. The experimental data are taken from Ref. 26.

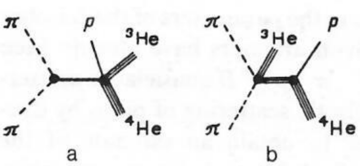

FIG. 26. Pole diagrams of quasielastic pion scattering by ^{4}He.

the pole diagram is dominant. The calculations of quasielastic scattering of pions by ^{4}He made in Ref. 145 showed that the pole diagram alone is not sufficient for even qualitative reproduction of the differential cross sections of ^{4}He$(\pi,\pi p)$ reactions. However, antisymmetrization of the final-state wave function [which is equivalent to taking into account the diagram of Fig. 26(b)] made it possible to achieve a significant improvement in the description of the experimental data (Fig. 27). Nevertheless, these calculations, made in the plane-wave approximation, do not give the correct absolute value of the reaction cross section.

One of the greatest surprises in pion–nucleus physics of recent years was the discovery of a very strong difference between the cross sections of quasielastic π^+ and π^- scattering by the ^{16}O nucleus.[146] It was found that at $T = 180$ MeV

$$R = \frac{\sigma(\pi^+ {}^{16}\mathrm{O} \to \pi^+ p\, {}^{15}\mathrm{N})}{\sigma(\pi^- {}^{16}\mathrm{O} \to \pi^- p\, {}^{15}\mathrm{N})} \sim 40$$

(it is well known that for the elementary πN interaction this ratio is of order 9). It is interesting that in the case of quasielastic scattering by ^{4}He the corresponding ratio is less than for free πN scattering. Figure 28 shows the angular dependence of the ratio R in the $\pi^\pm\, {}^4\mathrm{He} \to \pi^\pm p\, {}^3\mathrm{H}$ reactions.[147] To explain the large isotopic ratio in the case of π^{16}O quasielastic scattering the mechanism of nucleon knockout from the nucleus by means of the Δ isobar was used,[146] but in the case of quasielastic scattering by ^{4}He this process does not play an important part.

Reactions of inclusive scattering of pions by ^{4}He can give information about the renormalization of the character-

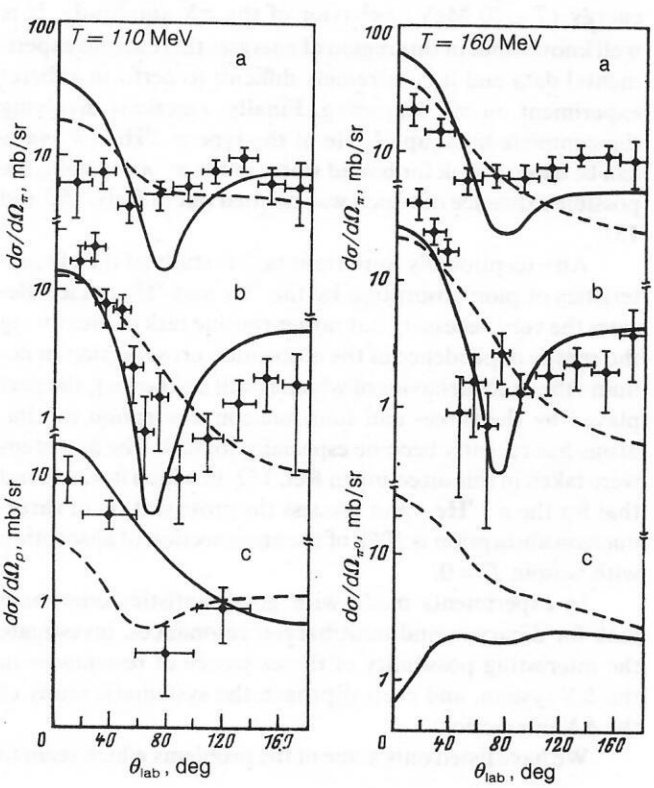

FIG. 27. Differential cross sections of quasielastic pion scattering by ^{4}He at different energies[145]: a) $\pi^+ + {}^4\mathrm{He} \to \pi^+ + p + {}^3\mathrm{H}$; b) $^4\mathrm{He}(\pi^+,\pi^+ n)^3\mathrm{He}$; c) $^4\mathrm{He}(\pi^+,\pi^0 p)^3\mathrm{He}$. The broken curves show the results of calculations with allowance for just the one pole diagram of Fig. 26(a); the continuous curves correspond to allowance for the diagrams of Figs. 26(a) and 26(b). The experimental data are from Ref. 20.

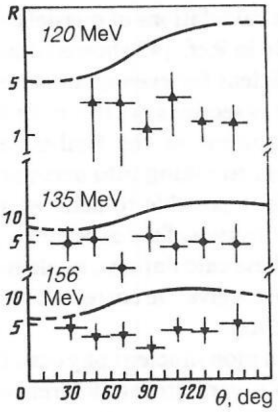

FIG. 28. Dependence of the ratio $R(\pi^+/\pi^-)$ of the differential cross sections of the $\pi^{+4}He \to \pi^+ p^3H$ and $\pi^{-4}He \to \pi^- p^3H$ reactions on the angle θ_π of the scattered pion.[147] The continuous curves show the behavior of the same ratio for free πN scattering.

istics of the elementary πN scattering event in the nuclear medium. In the framework of the isobar–hole model (see Sec. 2) this process was studied in Ref. 28, in which it was assumed that a certain average field U_Δ influences the propagation of the isobar in the nucleus.[37] In addition, the part played by the channel of true pion absorption was taken into account by means of the "smearing" potential U_{sp} [see Eq. (38)]. Allowance for binding effects of this type also had the consequence that the peak in the energy spectrum of the pions is broadened and its height becomes less dependent on the energy of the incident pion than in the case of calculations without renormalization of the πN interaction in the medium. The calculations in the isobar–hole model describe the experimental data in the region of energies $T > 150$ MeV much better than calculations in the plane-wave approximation. However, at $T < 150$ MeV there are certain discrepancies between the predictions of the isobar–hole model and the experimental data, and as yet it is not clear whether these are due to the presence of more complicated reaction mechanisms or whether the isobar–hole model is simply invalid in this region of reactions.

CONCLUSIONS

We should like to conclude by considering what interesting characteristics of pion interaction with the helium isotopes it would be helpful to measure in future experiments.

Above all, of course, it is necessary to have experimental data for the total interaction cross section σ_{tot}, a fundamental quantity. We noted in Sec. 1 that there are as yet virtually no measurements of σ_{tot} for the π^3He interaction. In the case of π^4He, the total cross section has not been measured at low energies ($T < 100$ MeV), and there are no systematic measurements of σ_{tot} for π^+ and π^- mesons. Such data are important for investigating the problem of charge-symmetry violation in strong interactions. In addition, as was noted in Sec. 1, testing of the dispersion relations for π^4He scattering gave rise to a suspicion that the total cross sections σ_{tot} have been inaccurately measured at high energies ($T > 300$ MeV). In the future, it will be necessary to clarify this question finally.

At the meson factories, experiments have already been made to study the differential cross sections of elastic scattering with a relative error of order 2%. However, for πHe scattering measurements at such a level of accuracy have not yet been made. Their results could yield important information not only for the problem of charge-symmetry violation (see Sec. 3) but also for the choice of the most preferable theoretical conceptions. An interesting task in the case of elastic scattering of pions by ^3He would be measurement of the vector polarization t_{11} in this process. In Refs. 72 and 73 it was shown that this quantity is very sensitive not only to the details of the nuclear structure but also to the approximations used to describe the reaction mechanism.

The $^3He(\pi^-,\pi^0)^3H$ charge-exchange cross sections are also about as sensitive to these effects, and it is much simpler to measure them than to carry out the polarization experiment. Unfortunately, in this field too the experimental situation is far from perfect.

A complete kinematic experiment to investigate the quasielastic scattering of pions by both ^3He and ^4He is very important for many reasons. For example, it would be of interest to trace the change in the parameters of the Δ isobar in the nuclear medium. Such attempts have already been made in a study of $^{12}C(\pi^+, \pi^+ p)^{11}B$ quasielastic scattering.[148] In Ref. 149, quasielastic scattering of pions by deuterium was used in order to obtain an estimate of the $\Delta^- - \Delta^{++}$ mass difference. Such data are of very great interest for quark models of elementary particles. V. B. Belyaev has made the interesting suggestion that the effects of pion interaction with the nucleon in the final state of quasielastic knockout reactions should be used to investigate the low-energy ($T < 20$ MeV) behavior of the πN amplitude. It is well known that in this region of energies there are no experimental data and it is extremely difficult to perform a direct experiment on πN scattering. Finally, reactions involving the complete breakup of ^4He of the type $\pi^- {}^4He \to \pi^- nnpp$ can be used to look for bound states in the $\pi^- nn$ system, the possible existence of which was pointed out in Refs. 150 and 151.

An exceptionally important task is study of the characteristics of pion absorption by the ^3He and ^4He nuclei. Besides the very necessary but rather routine task of measuring the energy dependence of the absorption cross section in helium (the exact behavior of which is still unknown), the part played by the three- and four-nucleon absorption mechanisms has recently become especially topical. The first steps were taken in this direction in Ref. 152, in which it was found that for the $\pi^- {}^3He \to pnn$ process the cross section of three-nucleon absorption is 20% of the cross section of absorption with isospin $T = 0$.

In experiments made with good statistics one could look for dibaryon and multibaryon resonances, investigate the interesting possibility of the existence of resonances in the ΔN system, and even approach the systematic study of the ΔN interaction.

We have listed only some of the problems which seem to us fairly topical. However, it is clear that the list can be significantly extended and made more varied. It is what is required for further successful development in the study of pion interactions with light nuclei.

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