Radiative polarization of electrons and positrons in storage rings

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The radiative polarization of electrons and positrons resulting from their synchrotron emission as they move in storage rings is studied. The spontaneous orientation of the particle spins is analyzed by rigorous methods of quantum theory on the basis of a model of the motion of electrons and positrons in a homogeneous magnetic field. The semiclassical equation for the evolution of the spin is used to study the influence of the focusing properties of the field on the spin dynamics. The influence of the orbital motion of the particles on the polarization process is studied for the example of a realistic model of the motion of electrons and positrons in storage rings. The experimental methods of observing the effect are reviewed, and the applications of polarized particle beams in physics experiments, including the most recent advances, are discussed.

INTRODUCTION

Interest in the interaction of the electron spin with an external electromagnetic field arose at the beginning of the thirties in connection with the first attempts to explain the anomalous Zeeman effect and a number of other experimentally observed phenomena, including the well-known Stern–Gerlach experiments. Since then, the theory of spin and experimental observation of spin phenomena have developed in parallel, complementing each other.

The original development of the theory of spin was based on the hypothesis of Uhlenbeck and Goudsmit (1925), in accordance with which it was assumed that the electron possesses spin mechanical and magnetic moments that are not related to the displacement of the particle in space. Pauli's equation (1927) laid the foundations of the description of spin, and then Dirac's theory (1928) confirmed the complete unity of the spin and orbital properties of the motion of relativistic particles.

For the further development of theory and experiment, investigations into the influence of an external electromagnetic field on the spin of a particle were of great importance. An important part was played here by the magnetic-resonance method based on Rabi's idea (1938) of applying simultaneously to the magnetic moment a constant magnetic field and, perpendicular to it, a weaker variable radio-frequency magnetic field that changes the orientation of the particle's magnetic moment. The resonance nature of this phenomenon opened up the possibility of achieving a high accuracy of measurement in physics investigations—electron paramagnetic resonance and nuclear magnetic resonance.

The magnetic-resonance method of investigation was used, in particular, to study the splitting of the hyperfine-structure levels of atomic hydrogen and deuterium, and this made it possible to detect a deviation from Dirac's theory, which had seemed to be the most perfect. Despite the smallness of the deviations from the theory that were observed, these experiments (J. E. Nafe, E. Nelson, I. I. Rabi, 1947) were of fundamental importance. They suggested that the electron possesses an anomalous magnetic moment in addition to the ordinary spin magnetic moment, which is numerically equal to the Bohr magneton. In 1948, Schwinger developed a theory of this phenomenon, which explained the anomalous magnetic moment of the electron as the result of interaction of the particle with the electromagnetic vacuum.

It was subsequently found that the anomalous magnetic moment of the electron has a significant influence on the behavior of the particle's spin in an external electromagnetic field and can, under certain conditions, cause beam depolarization.

It should be remarked that the problem of creating polarized particle beams arose long ago and attracted attention from the point of view of the study of different external influences of the spin of a particle capable of giving it a preferred orientation. In 1929, Mott[1] used the Dirac wave equation to study the problem of the scattering of electrons by the Coulomb field of nuclei and found for the first time that an initially unpolarized beam of electrons in which the particle spin is oriented randomly must become polarized after the scattering. The Mott scattering subsequently became the basis of one of the methods of determining the spin state of electrons, since the scattering cross section of the electrons depends on the orientation of their spins.

In the course of the solution of the problem of creating beams of polarized electrons and positrons, the possibility of polarizing these particles as they move in storage rings stimulated serious interest. An important factor explaining this rather complicated process of establishment of a preferred spin orientation was synchrotron emission, which is capable of giving rise to quantum transitions that involve a change in the spin orientation and possess a particular asymmetry depending on the polarization of the particle.

As a result of the effect of synchrotron emission during prolonged circulation of electrons and positrons in the magnetic field of a storage ring, a directed process of orientation of the particle spin arises: The electron spins are oriented in the opposite direction to the magnetic field, the positron spins along it (Fig. 1).

The kinetics of the process of radiative polarization of an initially unpolarized electron beam is determined by

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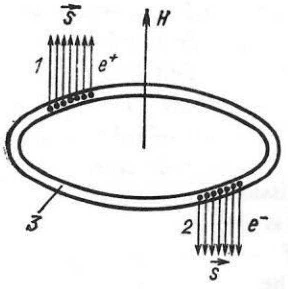

FIG. 1. Radiative polarization of bunches of positrons (*1*) and electrons (*2*) moving in the chamber of a storage ring (*3*). The arrows $\vec{s}$ show the direction of the spin.

$$P(t)=\frac{8\sqrt{3}}{15}\left(1-e^{-t/\tau}\right),$$

in which the polarization time is

$$\tau=\frac{8\sqrt{3}}{15}\,\frac{\hbar^2}{mce^2}\left(\frac{mc^2}{E}\right)^2\left(\frac{H_0}{H}\right)^3.$$

(Here, E is the electron energy, and $H_0=m^2c^3/e\hbar=4.4\times10^{13}$ Oe is the so-called Schwinger value of the magnetic field.) Thus, an initially unpolarized electron beam becomes polarized after a comparatively long time interval τ, its degree of polarization $P(t)$ tending to the limiting value

$$P(\infty)=8\sqrt{3}/15=0{,}924.$$

A simple estimate of the polarization time shows that for 1-GeV electrons it is of the order of 1 h, decreasing rapidly with increasing energy. Therefore, actual observation of the polarization process is possible in storage rings in which the particles move for a long time under conditions in which the energy loss through radiation is compensated (see Ref. 2).

The effect of radiative polarization of electrons and positrons in storage rings was predicted by the present author[3] (1961) (see also Ref. 5, published after the defending of the doctoral dissertation[3]) and was rigorously established theoretically in collaboration[4] with A. A. Sokolov (1963).[a)] Since then, more than 20 years have already passed, but nevertheless interest in this unusual and unexpected phenomenon has not only not disappeared but has in fact increased in recent years.

This is explained above all by the fact that storage rings were a natural (and at the present time unique) source of relativistic electrons and positrons with oriented spin. The radiative polarization effect opened up new possibilities of physics experiments to observe phenomena associated with an internal degree of freedom of particles—the spin.

In the early stage of the study of spin phenomena it was assumed that the part played by the spin in particle interactions is not significant. However, this point of view changed subsequently. In fact, in the recent past, particle polarization opened up the possibility of obtaining quite unique information about the laws of nature. It is sufficient to recall the well-known experiments on the β decay of polarized nuclei, which revealed the fundamental phenomenon of parity nonconservation in weak interactions. After β-decay experiments on polarized nuclei in many physics laboratories of the world, great interest was shown in polarized targets and polarized particle beams as new sources of information on the mechanism of particle interaction through the study of the spin dependence of phenomena.

In this connection it should be mentioned that experiments with polarized electrons and positrons make the process of their transformation into hadrons particularly informative, since this process depends strongly on the polarization of the colliding beams. The study of spin correlations makes it possible to investigate hadron structure functions, and also assists in the choice between different interaction models (see Ref. 6). The importance of this direction of investigations is emphasized by the present development of quantum chromodynamics (QCD) and the great experimental achievements.

The use of electron and positron beams with orientated spin has great importance in experiments in weak-interaction physics, since these processes also depend strongly on the spin correlations. There are now solid grounds for assuming that spin effects at high particle energies make a major contribution to their interaction dynamics. The increasing interest in the physics of polarization phenomena is reflected by the holding of several international symposia devoted to this subject (Fifth International Symposium on Spin Phenomena in High Energy Physics at Brookhaven in 1982, the International Seminar on Spin Phenomena at the Institute of High Energy Physics at Serpukhov in 1984, and the Sixth International Symposium on Polarization Phenomena in Nuclear Physics at Osaka, Japan, in 1985).

In connection with the development of the study of polarization phenomena, a new branch of high-energy physics, the creation of sources of fast polarized particles has become particularly important. For a long time no progress was made in the solution of this problem. The existing methods of polarization worked only in the nonrelativistic region of energies. We have here in the first place β-decay electrons, which are longitudinally polarized, and further, as already noted, the polarization of particles produced by their scattering; finally, there is orientation of the spin magnetic moment of particles to which combinations of electric and magnetic fields are applied. All these methods were restricted in their possibilities; they were applicable only in the nonrelativistic range of energies and, in addition, did not give a sufficient degree of polarization of the beams, which also had only a weak intensity.

It was obvious that the important problem of obtaining relativistic beams of polarized electrons and positrons remained unsolved and that any progress in this direction would be of great importance. In this connection, the idea of accelerating a beam of initially polarized particles to high energies arose. However, this proposal too has its difficulties—one would need a source of particles with given polarization, which must, moreover, be a source of intense beams. Further, since the particles in an accelerator travel a huge distance, it would be necessary to analyze the possible depolarizing factors—resonances, the influence of inhomogeneity of the field along the particle trajectory, and the part played by the anomalous magnetic moment of the particles.

The discovery of the effect of radiative polarization of electrons and positrons in storage rings[4] was a significant advance in the solution of the problem of obtaining beams of light polarized particles of high energy. For as the electrons and positrons move in the storage ring, the particles become polarized, having already a high energy. Moreover, the very process of polarization becomes effective only in the region of relativistic particle energies. This eliminates the need to solve the problem of accelerating nonrelativistic particles with oriented spin to high energies. It should also be pointed out that radiative polarization does not introduce any changes in the properties of particle beams—it does not influence the particle density in the beam or produce a spread of the energy parameters. In the process of radiative polarization, electron and positron beams acquire an oriented spin with hardly any changes in the characteristics of their orbital motion.

Several years after its theoretical establishment, the radiative polarization of electrons was confirmed experimentally in France (Orsay), at Novosibirsk in the Institute of Nuclear Physics of the Siberian Branch of the USSR Academy of Sciences, and somewhat later in the United States and the German Federal Republic. The experimental observations acted as a stimulus to the study and development of the theory of this interesting phenomenon under the real conditions of motion of particles in storage rings.

In the problem considered by the present author in collaboration with Sokolov,[4] and also with Bagrov and Rzaev,[7] the effect of radiative polarization was established and studied for the case of the motion of particles in a constant and homogeneous magnetic field. For the study, we chose the "exact-solution" method, in accordance with which the states of an electron in a magnetic field are described by exact solutions of the Dirac equation, while the interaction of the electron with the electromagnetic field of the radiation is treated by the methods of quantum electrodynamics. This approach to the solution of the problem revealed a number of fundamentally new physical effects, including the radiative polarization effect.

The formulation of the problem of the motion and emission of a charge in a constant and homogeneous magnetic field is, to a certain degree, a reasonable model for the motion of electrons and positrons in a storage ring. However, under the conditions of motion of particles in real storage facilities we encounter many additional factors that influence the stability of the spin orientation and can cause beam depolarization. In a number of studies made by scientists at the Institute of Nuclear Physics at Novosibirsk under the leadership of A. N. Skrinskiĭ (see Refs. 8 and 9), a method of semiclassical description of the phenomenon of radiative polarization was developed and used to make a detailed analysis of the influence of the particular conditions of the motion of a charge in a storage ring on the polarization process.

Subsequent theoretical and experimental investigations showed that if certain conditions are satisfied one can ensure effective operation of the mechanism of radiative polarization of electron and positron beams in real storage rings. In fact, the overall view obtained from these investigations is very optimistic: Up to electron energies of 100 GeV it is possible to ensure stable polarization of particle beams in close agreement with the model of a homogeneous field.

Thus, far-reaching possibilities have been revealed and established for obtaining polarized high-energy particles and also for controlling the polarization process and using fast particles with oriented spin. All this creates good prospects for the development of the physics of polarization phenomena, since the use of polarized particles greatly extends the possibilities of the experimentalist. In Table I we give brief data on the existing storage facilities in which experiments using polarized electron and positron beams are being made.

1. QUANTUM THEORY OF RADIATIVE POLARIZATION

The discovery of radiative polarization of electrons was made possible by a program of systematic and extensive investigation of quantum effects under conditions of macroscopic motion of relativistic electrons in a magnetic field. In developing the quantum theory of synchrotron emission (see Ref. 2), it is expedient to base the treatment on the general foundations of relativistic quantum mechanics and quantum electrodynamics, using the method of exact solutions of the Dirac equation. In accordance with this method, the wave function that describes the quantum states of an electron satisfies the Dirac equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},\ t)=\{c(\boldsymbol{\alpha}\hat{\mathbf{P}})+\rho_3mc^2+e\Phi^{\text{ext}}\}\ \Psi(\mathbf{r},\ t),$$

where $\hat{\mathbf{P}}=-i\hbar\nabla-(e/c)(\mathbf{A}^{\text{ext}}+\mathbf{A}^{\text{qu}})$ is the generalized momentum, and $\mathbf{A}^{\text{ext}}$ and $\mathbf{A}^{\text{qu}}$ correspond, respectively, to the external electromagnetic field $\mathbf{H}=\text{curl}\ \mathbf{A}^{\text{ext}}$, $\mathbf{E}=-\text{grad}\ \Phi^{\text{ext}}-(1/c)\dot{\mathbf{A}}^{\text{ext}}$ and the quantized transverse

TABLE I. Operating storage rings used as sources of polarized electrons and positrons.

Designation	Location	E, GeV	R, m	J, mA
AÇO	France, Orsay	0.55	1.1	100
VÉPP-2M	USSR, Novosibirsk	0.67	2	100
SPEAR	USA, Stanford	4	12.7	60
VÉPP-4	USSR, Novosibirsk	6	33	100
DORIS	FGR, Hamburg	7.5	12.1	100
PEP	USA, Stanford	15	170	100
PETRA	FGR, Hamburg	22.5	200	90

radiation field $\mathbf{A}^{qu}$ (div $\mathbf{A}^{qu}=0$).

Ordinary perturbation theory, which requires solution of the Dirac equation in the form of an expansion of the wave function in a series in the external field ($\mathbf{A}^{ext}$, Φ^{ext}), is not valid here, since the effects associated with the external field (especially in the case of a strong field) can be essentially nonlinear. However, the processes that influence the electron in a bound state as it interacts with the electromagnetic field of the radiation (coupling constant $\alpha=e^2/\hbar c=1/137$) can be treated by perturbation theory, provided one has an exact determination of the wave function of the "zeroth" approximation in the bound state. Thus, in the Dirac equation the external electromagnetic field $\mathbf{A}^{ext}$, Φ^{ext} is taken into account exactly, while $\mathbf{A}^{qu}$, the quantized radiation field, is taken into account by perturbation theory.

In such an approach, all the expansions in the perturbation theory are with respect to a complete system of wave functions $\Psi(\mathbf{r},t)$ that are exact solutions of the Dirac equation for an electron in a bound state. In the problem of synchrotron radiation, this method was proposed for the first time by Sokolov[10] in 1949. Later, the method of exact solutions became known as the Furry picture (1951); Furry showed that the Feynman–Dyson formalism can be generalized to the case when the electron is not free but is in a bound state.[11]

The exact-solution method makes it possible to take into account all values of the external field strength. In particular, in the case of a magnetic field one can, because the vacuum is stable, even consider field values above the critical value, $H>H_{cr}=m^2c^3/e\hbar=4.41\times10^{13}$ Oe, which can be assumed to exist in the interior of pulsars.

Motion of an electron in a homogeneous magnetic field. The wave function

We consider the wave function of an electron moving in a constant and homogeneous magnetic field, which, for definiteness, we shall assume is oriented along the z axis of a cylindrical system of coordinates (r,φ,z) related in a very natural manner to the nature of the electron motion. As is well known, it is necessary in quantum mechanics to have a set of four independent quantities to characterize the state. It is therefore necessary to choose four operators that all commute with the Hamiltonian operator and are integrals of the motion; then all these operators will have common wave functions.

In the problem of the motion of an electron in a homogeneous magnetic field, one can require that the wave function be an eigenfunction for the following operators:

1. The energy,

$$\hat{\mathscr{H}}\Psi=E\Psi\quad \hat{\mathscr{H}}=c\,(\boldsymbol{\alpha}\hat{\mathbf{P}})+\rho_3mc^2. \tag{1}$$

2. The projection of the momentum onto the direction of the field,

$$\hat{P}_3\Psi=p_3\Psi. \tag{2}$$

3. The projection of the total angular momentum onto the direction of the field,

$$\hat{J}_3\Psi=\hbar\left(l-\tfrac{1}{2}\right)\Psi. \tag{3}$$

To determine the spin state, i.e., to separate the solutions of the Dirac equation with respect to the polarization states, we require a fourth operator that commutes with the Hamiltonian—the polarization operator. As this operator, we consider here a three-dimensional vector, the spin operator, which for a free particle has the form

$$\boldsymbol{\sigma}^0=\rho_3\boldsymbol{\sigma}-(\boldsymbol{\sigma}\mathbf{p})\,\mathbf{p}\,(\rho_3-\hat{\mathscr{H}}/E)/p^2. \tag{4}$$

This operator, which was introduced first by Stech,[12] is a unit operator, i.e., its projection onto any direction $\mathbf{s}$ in space ($|\mathbf{s}|=1$) satisfies the requirement

$$(\boldsymbol{\sigma}^0\mathbf{s})\,(\boldsymbol{\sigma}^0\mathbf{s})=1. \tag{5}$$

As can be seen from (4), in the rest frame of the particle the three-dimensional spin vector $\boldsymbol{\sigma}^0$ is equal to $\boldsymbol{\sigma}$ in the direction of motion of the particle (longitudinal polarization) and $\rho_3\boldsymbol{\sigma}$ in the direction perpendicular to the motion (transverse polarization). Since thus $\boldsymbol{\sigma}^0$ (4) is a unitary transformation of the ordinary spin operator, the eigenvalues $(\boldsymbol{\sigma}^0\mathbf{s})$ are equal to the eigenvalues of the spin operator in the rest frame. Therefore, the wave function is transformed from the rest frame to the laboratory system by means of Lorentz transformations independently of the polarization states: The polarization remains unchanged in all frames of reference (see Ref. 13).

We note further that "on solutions" of the Dirac equation $\hat{\mathscr{H}}\Psi=E\Psi$ the form of the operator $\boldsymbol{\sigma}^0$ can be somewhat changed—by means of (1), one can obtain its generalization to the case of motion of a particle in a magnetic field. Indeed, replacing in accordance with the general rules $\mathbf{p}$ by the generalized momentum $\mathbf{P}=\mathbf{p}-(e/c)\mathbf{A}$, we obtain

$$\boldsymbol{\sigma}^0=\rho_3\boldsymbol{\sigma}+\rho_1c\hat{\mathbf{P}}/E-\rho_3\frac{c^2\hat{\mathbf{P}}\,(\boldsymbol{\sigma}\hat{\mathbf{P}})}{E+mc^2}. \tag{6}$$

However, for the motion of an electron in a magnetic field, in contrast to the motion of a free particle, $\boldsymbol{\sigma}^0$ does not now commute in the general case with the Hamiltonian. Nevertheless, one can find an integral of the motion: The projection of $\boldsymbol{\sigma}^0$ onto the direction of motion of the electron, $\boldsymbol{\sigma}\hat{\mathbf{P}}/P$, and the projection of $\boldsymbol{\sigma}^0$ onto the direction of the magnetic field are conserved:

$$\sigma_3^0=\rho_3\sigma_3+\rho_1c\hat{P}_3/E-\rho_3\frac{c^2\hat{P}_3\,(\boldsymbol{\sigma}\hat{\mathbf{P}})}{E+mc^2}. \tag{7}$$

Choosing now σ_3^0 as the polarization operator (transverse polarization) and requiring the wave function to be an eigenfunction of this operator [see (1)–(3)],

$$\sigma_3^0\Psi=\zeta\Psi, \tag{8}$$

where $\zeta=\pm1$ determines the orientation of the electron spin along the magnetic field and in the direction opposite to it, respectively, we obtain the possibility of determining all the necessary numbers that characterize the state: E_n (the number n), p_3, $l-\frac{1}{2}$, ζ. Therefore, the wave function, the exact solution of the Dirac equation, can be written in the form

$$\Psi_{nsk_3\zeta}(\mathbf{r}, t) = (2\pi L)^{-1/2}$$

$$\exp\left[-ic\mathcal{K}t + ik_3 z + i\left(l - \frac{1}{2}\right)\varphi\right] f, \quad (9)$$

where

$$f = \sqrt{\gamma(1 + k_0/\mathcal{K})}\begin{pmatrix} S_1 \\ S_2 \end{pmatrix}; \quad S_{1,2} = \begin{pmatrix} C_{1,3} I_{n-1,s}(\gamma r^2) e^{-i\varphi/2} \\ {}_1C_{2,4} I_{n,s}(\gamma r^2) e^{i\varphi/2} \end{pmatrix}. \quad (10)$$

In these expressions, the Laguerre function $I_{n,s}(x)$ is related to the Laguerre polynomials $Q_s^{n-s}{}_n(x)$ (see Ref. 2) by means of the relation

$$I_{n,s}(x) = \frac{1}{\sqrt{n!\,s!}} e^{-\frac{x}{2}} x^{\frac{n-s}{2}} Q_s^{n-s}(x), \quad (11)$$

where $\gamma = eH/2c\hbar$, $k_0 = mc/\hbar$, $E = c\hbar\mathcal{K} = c\hbar\sqrt{k_0^2 + k_3^2 + 4n\gamma}$ is the electron energy, $\hbar k_3$ is the electron momentum along the field, $n = l + s$ is the principal quantum number, equal to the sum of the orbital, $l = 0, 1, 2, \ldots$, and radial, $s = 0, 1, 2, \ldots$, quantum numbers. The spin coefficients can be determined from Eq. (9) and have the form (see also Ref. 14)

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \sqrt{\frac{1+\zeta}{2}}\begin{pmatrix} 1 \\ 0 \\ k_3/(\mathcal{K} + k_0) \\ \sqrt{4n\gamma}/(\mathcal{K} + k_0) \end{pmatrix} + \sqrt{\frac{1-\zeta}{2}}\begin{pmatrix} 0 \\ 1 \\ \sqrt{4n\gamma}/(\mathcal{K} + k_0) \\ k_3/(\mathcal{K} + k_0) \end{pmatrix}. \quad (12)$$

As can be seen from this expression for the spin coefficients, the solutions (12) go over on the transition to the rest frame into the Pauli wave functions that correspond to the two alternative orientations of the spin relative to the external magnetic field. Such an approach was developed in our early investigations of spin effects (see Ref. 3).

We note that the introduction of the operator $\boldsymbol{\sigma}^0$ in order to describe the spin properties of the electron is not the only possibility. Moreover, this operator itself creates a certain feeling of dissatisfaction, for it is noncovariant and transforms on the transition to a new Lorentz coordinate system in accordance with special rules. Besides this operator, to describe the spin properties of electrons and positrons one can also introduce covariant operators: the four-dimensional spin pseudovector S^ν and the polarization tensor $\Pi_{\nu\mu}$ (see Refs. 2 and 14). All of these operators are invariant generalizations of the unit spin vector $\boldsymbol{\sigma}^0$ and are obtained by transforming it from the rest frame to the laboratory system. Therefore, all three methods of describing the spin of a free electron, by $\boldsymbol{\sigma}^0$, S^ν, and $\Pi_{\mu\nu}$ are completely equivalent. In this connection we note that in the problem of the motion of an electron in a homogeneous magnetic field one can choose, besides σ_3^0, other polarization operators that make it possible to separate the solutions of the Dirac equation with respect to the spin states, namely, one can choose the following projections, which commute with the Hamiltonian,

$$\hat{\mu}_3 = \sigma_3 + \frac{1}{mc}\rho_2[\boldsymbol{\sigma}\hat{\mathbf{P}}]_3 \text{ and } S^3 = \rho_3\sigma_3 + \rho_1\hat{P}/mc, \quad (13)$$

and are integrals of the motion. In Ref. 4, the separation with respect to the spin was done by means of the operator $\hat{\mu}_3$, which is a component of the tensor $\Pi_{\mu\nu}$.

Quantum transitions with spin flip. The radiative polarization of electrons

We consider the interaction of an electron moving in a magnetic field with the electromagnetic radiation field. In the most consistent formulation of this problem, one must consider the interaction of two quantum systems—the electron, described by means of a wave function satisfying the Dirac equation, and the quantized photon field. Then the interaction energy $U^{\rm int}$ of the electron with the photon field has the form

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}^0 + U^{\rm int} = c(\boldsymbol{\alpha}\hat{\mathbf{P}}) + \rho_3 mc^2 + U^{\rm int}, \quad (14)$$

$$U^{\rm int} = e_0 L^{-3/2}\sum_{\varkappa}\sqrt{\frac{2\pi c\hbar}{\varkappa}}(\boldsymbol{\alpha}\mathbf{a}^+)\exp(ic\varkappa t - i\boldsymbol{\varkappa}\mathbf{r}). \quad (15)$$

where the photon creation and annihilation operators, $\hat{\mathbf{a}}^+$ and $\hat{\mathbf{a}}$, respectively, satisfy Bose-field commutation relations.

The Dirac equation (14) must then be solved by the method of perturbation theory under the assumption that the wave function in the "zeroth" approximation is an exact solution of the Dirac equation for a particle in a magnetic field. For the transitions $E_n \to E_{n'}$ $(E_n > E_{n'})$ associated with emission, the transition probability per unit time can be obtained by means of the general method:

$$dw_{nn'} = \frac{e^2}{2\pi\hbar}\int\frac{d^3\varkappa}{\varkappa}\delta(\varkappa - \varkappa_{nn'})\,\Phi, \quad (16)$$

where $c\hbar\varkappa_{nn'} = E_n - E_{n'}$ is the change in the energy in the quantum transition (Bohr's frequency rule), and $\Phi = \Phi_\sigma + \Phi_\pi$ is associated with the polarization of the radiation. In particular,

$$\Phi_\sigma = \bar{\alpha}_1\bar{\alpha}_1^*, \quad (17)$$

characterizes the σ component of the radiation. The electric vector of the radiation field in this component lies in the plane of the orbit of revolution of the electron. The other radiation component

$$\Phi_\pi = (\bar{\alpha}_2\cos\theta - \bar{\alpha}_3\sin\theta)^2 \quad (18)$$

corresponds to the π component and is characterized by the fact that the electric vector of the radiation field is perpendicular to the plane of revolution of the particle. The matrix elements $\bar{\boldsymbol{\alpha}}$ of the Dirac matrices are determined from the functions (9)–(11) of the unperturbed problem:

$$\bar{\boldsymbol{\alpha}} = \int\Psi^+_{n's'k_3'\zeta'}\boldsymbol{\alpha}\exp(-i\boldsymbol{\varkappa}\mathbf{r})\,\Psi_{nsk_3\zeta}\,d^3x. \quad (19)$$

We now consider the matrix elements of the Dirac matrices, assuming transitions accompanied by spin flip, i.e., when the spin index of the wave function in Eq. (19) is chosen in the form $\zeta' = -\zeta$. The corresponding exact expressions for the case when the spin states are distinguished by means of the operator σ^0 were given for the first time in Ref. 3. Approximating the Laguerre functions $I_{nn'}$ by means of Bessel functions of index 1/3 (Macdonald functions K_μ) (see Ref. 2), we obtain

$$\left.\begin{aligned} |\bar{\alpha}_1|^{\uparrow\downarrow} &= |\bar{\alpha}_2|^{\uparrow\downarrow} = \frac{\sqrt{\varepsilon}}{2\pi\sqrt{3}}\left|\frac{k_3'}{\mathscr{K}}\right| K_{1/3}(z)\, I_{ss'}(x)\,\delta_{k_3',\,-\varkappa\cos\theta};\\ |\bar{\alpha}_3|^{\uparrow\downarrow} &= \frac{\sqrt{\varepsilon}}{2\pi\sqrt{3}}\,\xi y\,\{\sqrt{\varepsilon}K_{2/3}(z) + \zeta\sqrt{\varepsilon_0}\,K_{1/3}(z)\}\, I_{s,s'}(x)\\ &\qquad\times\delta_{k_3'-\varkappa\cos\theta}. \end{aligned}\right\} \tag{20}$$

In these expressions, the initial electron momentum along the magnetic field is assumed to be zero: $k_3 = 0$;

$$z = \frac{1}{2}y\left(\frac{\varepsilon}{\varepsilon_0}\right)^{3/2},\quad \xi = \frac{3}{2}\frac{H}{H_0}\frac{E}{mc^2},\quad y = \frac{2}{3}\nu\varepsilon_0^{3/2},$$

$$\frac{k_3'}{\mathscr{K}} = -\xi y\cos\theta$$

are associated with the harmonic of number $\nu = n - n'$, over which a summation is understood and is replaced here, the spectrum being quasicontinuous, by an integration over y. Further, we have here $\varepsilon = 1 - \beta^2\sin^2\theta$, $\varepsilon_0 = 1 - \beta^2$; the Kronecker delta symbol $\delta_{k_3',\,-\varkappa\cos\theta}$ reflects the law of momentum conservation along the field, i.e., the "recoil" effect associated with emission of a photon, while the Laguerre function $I_{ss'}(x)$ of the argument $x = \varkappa^2\sin^2\theta/4\gamma$ characterizes the recoil effect in the direction of the radius of the orbit. This factor disappears from the final formulas, since the matrix elements of the Dirac matrices do not depend on the radial quantum number s', and

$$\sum_{s'} I_{ss'}^2(x) = 1. \tag{21}$$

Further, bearing in mind that $\varkappa/\mathscr{K} = \xi y$, for the differential probability of transitions with spin flip we obtain

$$\frac{dw^{\uparrow\downarrow}}{d\Omega\, dy} = \frac{3}{32}\frac{1}{\pi^3}\frac{e^2}{\hbar R}\frac{1}{\varepsilon_0^3}\xi^2 y^2\varepsilon$$

$$\times\,[\cos^2\theta K_{1/3}^2 + (\sqrt{\varepsilon}K_{2/3} + \zeta\sqrt{\varepsilon_0}K_{1/3})^2]. \tag{22}$$

This expression is identical to those obtained when the polarization is determined by means of the operator $\hat{\mu}_3$ (see Refs. 2 and 4). Integration of it over the solid angle $d\Omega = \sin\theta\, d\theta\, d\varphi$ and the spectrum dy leads us to the final expression for the probability of transitions in 1 sec:

$$w^{\uparrow\downarrow} = \frac{1}{2\tau}\left(1 + \zeta\frac{8\sqrt{3}}{15}\right), \tag{23}$$

(see Refs. 4 and 7), where the polarization time τ was determined earlier.

As follows from the expression for the transition probability, the probability depends on the initial orientation ζ of the electron spin. This dependence shows that the emission of photons by the ultrarelativistic electron is the physical reason for the occurrence of the transverse polarization of the electrons. For in accordance with (23) the probability of transitions from the state $\zeta = 1$ (spin oriented along the magnetic field) will be greater than in the case of the opposite transition. Thus, as a result of the asymmetry of the emission we will observe a tendency of the electrons to go over into a state with preferred orientation of the spin in the opposite direction to that of the magnetic field (see Ref. 3). In contrast, the positrons must have a tendency to occupy the state with spin parallel to the magnetic field. Such states correspond to the smallest value of the potential energy of particles that possess a magnetic moment μ in the external magnetic field:

$$U = -\frac{e}{|e|}\mu\mathbf{H},\quad \mu = \frac{e_0\hbar}{2mc}\zeta. \tag{24}$$

It can be seen from this expression, which is valid in the nonrelativistic approximation, that the state with positron spin oriented along the direction of the magnetic field will be stable (see also Ref. 15).

Radiative polarization is an essentially quantum effect, since the probability of transitions with spin flip is proportional to $\hbar^2$, i.e., to the square of Planck's constant. It should be noted that this effect is manifested under conditions when the brightness temperature of the electron beam reaches very high values $T \sim 10^7$ °K.

It is obvious that so far the exposition does not give a picture of the kinetics of the polarization process, since the entire calculation has been made for a single electron. We now consider an ensemble of electrons, which, as before, we propose to describe by means of the function Ψ [see (9)–(12)]. However, we note that as a result of interaction with the radiation field the electron goes over from a pure to a mixed state, which can no longer be described by means of a wave function. In the light of this remark, to describe the spin, it is necessary to introduce a density matrix, using for its construction any of the polarization operators.

In particular, for electrons, particles with spin $\frac{1}{2}$, there exist only two possible and alternative projections of the spin relative to the quantization axis, which is defined by the magnetic field. If the beam contains particles with both spin orientations, then the beam will be partly polarized, and the degree of its polarization can be determined by means of the diagonal elements of the density matrix, which characterize the populations of the spin states: $N(-\frac{1}{2})$ and $N(\frac{1}{2})$.

Bearing this in mind, we now consider the statistical equations that characterize the change in the spin orientation of the particles in an electron beam (see Refs. 4 and 7). The kinetic equation for the polarization process has the form

$$\frac{d}{dt}n^{\downarrow} = n^{\uparrow}w_{\zeta=1} - n^{\downarrow}w_{\zeta=-1};\quad \frac{d}{dt}n^{\uparrow} = n^{\downarrow}w_{\zeta=-1} - n^{\uparrow}w_{\zeta=1}, \tag{25}$$

subject to the condition that $n^{\uparrow} + n^{\downarrow} = n = \text{const}$, where $n^{\uparrow}$ and $n^{\downarrow}$ are the numbers of electrons with spin oriented along the field and in the direction opposite to it, respectively. If at the initial time $t = 0$ the beam was unpolarized, i.e., $n_0^{\uparrow} = n_0^{\downarrow} = n/2$, then, integrating the kinetic equation (25), we find

$$\binom{n^{\downarrow}}{n^{\uparrow}} = \frac{15 \pm 8\sqrt{3}\,(1 - e^{-t/\tau})}{30}, \tag{26}$$

and then, taking into account the expression for the populations of the spin states,

$$N\left(-\frac{1}{2}\right)=\frac{n^{\downarrow}}{n^{\downarrow}+n^{\uparrow}},\ N\left(\frac{1}{2}\right)=\frac{n^{\uparrow}}{n^{\downarrow}+n^{\uparrow}},$$

we find that the degree of polarization of the beam is

$$P(t)=N\left(-\frac{1}{2}\right)-N\left(\frac{1}{2}\right)=\frac{8\sqrt{3}}{15}\left(1-e^{-t/\tau}\right), \quad (27)$$

and the limiting degree of polarization is

$$P(\infty)=\frac{8\sqrt{3}}{15}=0{,}924. \quad (28)$$

This expression was obtained for the first time by the present author in collaboration with Sokolov in 1963 (see Ref. 4). Later, it was found again in a number of studies of other authors[16–18] made by somewhat different methods.

One can arrive at the result (28) in a different way by considering the transition probability (22) and relating it to an electron ensemble. Then $\zeta(t)$, which appears as a quantity in this expression, will characterize the spin state of the electron beam, $\zeta(t)=\langle\zeta\rangle$, and for the variation of this quantity we can obtain a kinetic equation in the form

$$\frac{d}{dt}\zeta=\sum_{\zeta'}(\zeta'-\zeta)\,w^{\uparrow\downarrow}=-2\sum_{\zeta}\zeta w^{\uparrow\downarrow}. \quad (29)$$

After integration of this expression over the time under the assumption that at the initial time there are no polarized electrons, we find that

$$\zeta(t)=-\frac{8\sqrt{3}}{15}\left(1-e^{-t/\tau}\right). \quad (30)$$

Analysis of the polarization time τ [see Eq. (23)] shows that under the conditions of ordinary values of the magnetic field for accelerators, $H\approx 10^4$ Oe, radiative polarization can be observed only in the case of prolonged circulation of the particles in the magnetic field. Then the radiative energy losses must be compensated in order to ensure that the electron energy remains on the average constant. Such a situation is realized in storage rings, special constructions designed for colliding beams of accelerated particles traveling in opposite directions, at electron energies $E\gg 1$ GeV.

After the discovery of radiative polarization, attempts were made to explain it, i.e., to find a physical interpretation on the basis of a classical model of the emission of a magnetic dipole whose orientation relative to the magnetic field is changing (see Ref. 19). Although such discussions did lead to a certain qualitative explanation of radiative polarization, the attempts to explain this phenomenon on the basis of a simplified classical model were essentially illusory, giving an incorrect result for the degree of beam polarization and the polarization time. This is due to the fact that such an approach did not take into account the fluctuation nature of the effect.

Analysis of the expression for the probability of quantum transitions of electrons involving a change in the spin orientation shows that the quantum fluctuations of the synchrotron radiation make an important contribution to this process. Indeed, from the point of view of quantum theory the emission itself is regarded as due to the interaction of the electron with the fluctuations of the electromagnetic photon field, the recoil effect, i.e., the change in the electron momentum due to the emission of a photon, also being an important factor. In particular, if the electron does not initially have a component of the momentum along the field, $k_3=0$, then because of the recoil following photon emission it acquires in the final state a momentum $k'_3=-\varkappa$ (see Ref. 20).

In this connection, we draw attention to the construction of the expression (22) for the probability of quantum transitions accompanied by spin flip:

$$\frac{dw^{\uparrow\downarrow}}{d\Omega\,dy}=\frac{9}{8\pi}\frac{e^2}{\hbar R}\frac{1}{\varepsilon_0^3}\left(\bar{\alpha}_1^2+\bar{\alpha}_3^2\right). \quad (31)$$

In this expression, the π component of the radiation, represented by the square of the matrix element $\bar{\alpha}_3^2$, introduces a directionality into the spin orientation process. This quantity depends explicitly on the projection ζ of the spin onto the magnetic field. We note that the magnetic field of the radiation of this component lies in the plane of the orbit of revolution of the electron and, therefore, is at right angles to the external magnetic field, as is the case in Rabi's magnetic-resonance method).

At the same time, the probability (31) of quantum transitions accompanied by a change in the spin orientation also contains the σ component of the radiation, which is characterized by $\bar{\alpha}_1^2$. This component does not depend on the spin orientation, but it does make an important contribution to the kinetics of the polarization process. It is important to note that the matrix element $\bar{\alpha}_1^2$ is due to the recoil effect following emission of a photon; this follows directly from (20), since $\bar{\alpha}_1^2$ is proportional to $k'_3=-\varkappa\cos\theta$.

In order to give a clearer analysis of the part played by the recoil effect, we integrate the expression (31) for the transition probability over the spectrum (over dy) and obtain thus the probability of spin flip as a function of the angle of observation θ. Then in the ultrarelativistic limit $\varepsilon_0=1-\beta^2\ll 1$ we obtain

$$\frac{dw^{\uparrow\downarrow}}{d\Omega}=\frac{2\sqrt{3}}{9}\frac{e^2}{\hbar R}\frac{\xi^2}{\pi^2\varepsilon_0}\frac{1}{(1+\psi^2)^4}\left[1+\frac{35\sqrt{3}}{192}\frac{\pi}{\sqrt{1+\psi^2}}\zeta\right], \quad (32)$$

where $\psi=\cos\theta/\sqrt{\varepsilon_0}$.

The term containing the initial spin orientation in this expression attains its maximal value when $\psi=0$, i.e., in the plane of the orbit of revolution, when $\theta=\pi/2$. Further, its value, $35\sqrt{3}\pi/192=0.99$, is near unity. Thus, the maximum of the probability is attained where the recoil effect following photon emission is absent, since $\bar{\alpha}_1^2=0$ for $\theta=\pi/2$.

Thus, the quantum fluctuations of the synchrotron radiation appear, on the one hand, as the reason for the electron transitions accompanied by spin flip, the fluctuations of the radiation making possible, moreover, not only the one-sided process of ordering of the spin, as occurs in the classical theory, but also opposite transitions. At the same time, the quantum fluctuations of the electron momentum (the recoil effect), acting as a depolarizing factor, have a strong influence on the kinetics of the polarization process. Thus, the classical model of radiative polarization, which assumes continuity of the radiation process, is invalid (see also Ref. 20).

2. SEMICLASSICAL THEORY OF RADIATIVE POLARIZATION

Bargmann–Michel–Telegdi equation

The radiative polarization effect was established for the first time for the case of the motion of particles in a constant and homogeneous magnetic field; however, the importance of this phenomenon and its physical foundations as well as its quantitative description take us far beyond this restricted problem. Subsequent theoretical and experimental investigations showed that radiative polarization of electrons and positrons can in fact be obtained in real particle accelerators.

It is well known that in real storage rings the electrons and positrons move in a focusing axisymmetric magnetic field, which ensures the stability of the motion of the particles in closed orbits. Although the particles are subject to random forces, their motion remains stable, and they execute radial and vertical betatron oscillations around the equilibrium orbit.

It is also important to note that the motion of a particle in a storage ring is accompanied by the effect on it of a high-frequency electric field, which compensates the radiative losses. This field acts on an electron synchronously with its motion, and in the case of a random deviation from the equilibrium phase the particle also executes phase oscillations (slower than the betatron oscillations and described by the autophasing principle of Veksler and McMillan).

To obtain a more detailed treatment of radiative polarization, particularly for realistic conditions of the motion of particles in storage rings, it was found to be effective to use the semiclassical theory of spin based on the Bargmann–Michel–Telegdi (BMT) equation.[21] The BMT equation for the evolution of the spin in a magnetic field (we omit here the external electric field, assuming it to be equal to zero) can be written in the form

$$\frac{d}{dt}\boldsymbol{\zeta}=[\boldsymbol{\Omega\zeta}]=-\frac{ec}{E}\left(1+\frac{\alpha}{2\pi}\frac{E}{mc^2}\right)[\mathbf{H}\boldsymbol{\zeta}] +\frac{\alpha}{2\pi}\frac{e}{mc}\frac{[\boldsymbol{\beta\zeta}](\boldsymbol{\beta}\mathbf{H})}{1+mc^2/E}, \qquad (33)$$

where $\boldsymbol{\beta}E=\langle c\mathbf{P}\rangle$ is the mean value over the wave packet, $\mathbf{H}$ is the external magnetic field, and the term proportional to $\alpha=e^2/\hbar c$ takes into account the electron's anomalous magnetic moment:

$$(g-2)/2=\alpha/2\pi. \qquad (34)$$

This equation of the spin evolution in time is frequently called a classical equation because it does not contain Planck's constant h. It can be obtained on the basis of the general idea of transition from the classical nonrelativistic equation for the evolution of the magnetic moment, taken in the electron rest frame, to the laboratory system with allowance for the well-known "Thomas precession," which is associated physically with the Lorentz contraction of the circumference of a circle on the transition to the laboratory coordinate system (for more details, see Ref. 22). From the point of view of quantum mechanics, the BMT equation is a rigorous consequence of Dirac's theory with allowance for the vacuum anomalous magnetic moment. It can be obtained on the basis of the equation for the evolution of the spin in Heisenberg's form for the operator $\boldsymbol{\sigma}^0$ with transition to a definite parity of the operators (see Refs. 23 and 24) and to the semiclassical limit $\hbar\to 0$. Thus, the BMT equation is valid if the external field is far from the critical value and thus permits a single-particle approach to the solution of the problem.

It is interesting in this connection to note[25] that the BMT equation has an intimate connection with Frenkel's theory,[26] in which it is assumed that a point particle with spin has not only the three ordinary degrees of freedom but must also have "rotational" degrees of freedom corresponding to $2|s|+1$ possible orientations of its spin (rotating point top). If one introduces phenomenologically in Frenkel's equations an arbitrary magnetic moment associated with the spin and differing in the rest frame from the Bohr magneton (taking into account, for example, the anomalous magnetic moment of the electron), then in the approximation of constant and homogeneous fields Frenkel's equation goes over into the BMT equation for the spin $\boldsymbol{\zeta}$.

We note that the BMT equation takes into account the interaction of the electron with the electromagnetic radiation field in its vacuum state. Indeed, the terms of this equation proportional to $\alpha/2\pi$ correspond to the anomalous magnetic moment of the electron, $\Delta\mu=(\alpha/2\pi)\mu_0$, and are explained theoretically as the result of interaction of the electron with the vacuum fluctuations of the field [Schwinger, 1948 (Ref. 27)].

The BMT equation takes into account the change in the g factor of the electron only in the approximation linear in the external field H. In the case of strong magnetic fields, the dynamical nature of the anomalous magnetic moment is revealed, and it becomes a complicated nonlinear function of the field and the electron energy (see Ref. 28). The change in the electron gyromagnetic factor significantly influences the dynamics of the particle spin, giving it an additional precession in the magnetic field [see (30)], as a result of which the spin precession period and the electron revolution period are no longer equal to each other.

We now consider the generalization of the BMT equation to take into account the reaction forces that arise when real photons are emitted, and to this end we first of all write down an expression for the probability of quantum transitions of the electron accompanied by a change in the spin projection:

$$w=\frac{1}{2\tau}\left\{1-\frac{2}{9}(\boldsymbol{\zeta\beta})^2+\frac{8\sqrt{3}}{15}\boldsymbol{\zeta}[\boldsymbol{\beta}\dot{\boldsymbol{\beta}}]/|\dot{\boldsymbol{\beta}}|\right\}. \qquad (35)$$

In such a form, this expression was obtained for the first time by the methods of the semiclassical treatment.[29] Somewhat later, using rigorous quantum theory, we found expressions for the probabilities of transitions with change of the state with longitudinal, $w^{\rightleftarrows}$, and transverse, $w^{\uparrow\downarrow}$, polarization (see Refs. 4 and 7):

$$w^{\rightleftarrows}=\frac{1}{2\tau}\,\frac{7}{9} \qquad (36)$$

(projection of the spin onto the velocity, $\boldsymbol{\zeta}\cdot\boldsymbol{\beta}$), and

$$w^{\uparrow\downarrow}=\frac{1}{2\tau}\left(1+\zeta\,\frac{8\sqrt{3}}{15}\right) \qquad (37)$$

(projection of the spin onto the direction of the magnetic field, $\zeta \cdot \mathbf{H}/H$, or perpendicular to the velocity $\zeta \times \beta$). Both these expressions follow from (35) if the acceleration $\dot{\beta}$ is eliminated by means of the equations of motion. Note that the probability (36) does not depend on the initial orientation of the spin—the change in the longitudinal polarization as a result of the emission does not have the nature of a directed process and occurs with equal probability for both spin states. In contrast, the probability (37) of transitions with a change of the transverse polarization depends on the initial orientation of the spin: The quantum transitions of an electron in a state with orientation of the spin in the opposite direction to the magnetic field ($\zeta = -1$) are preferred (see also Refs. 3 and 4). Thus, only transverse polarization arises in the radiation process.

If we now give the vector ζ the meaning of the spin corresponding to the ensemble of electrons, then for the change in time of this quantity we obtain the equation

$$\frac{d}{dt}\zeta = \sum_{\zeta'} (\zeta' - \zeta)\, w = -\frac{1}{\tau}\mathbf{Q}, \tag{38}$$

where

$$\mathbf{Q} = \zeta - \frac{2}{9}\beta(\zeta\beta) + \frac{8\sqrt{3}}{15}\frac{[\beta\dot{\beta}]}{|\dot{\beta}|}. \tag{39}$$

This change of the spin projections is due to the radiation reaction forces. One can then arrive at the following generalization of the BMT equation, the equation of the polarization kinetics (see Ref. 8):

$$\frac{d}{dt}\zeta = [\Omega\zeta] - \frac{1}{\tau}\mathbf{Q}. \tag{40}$$

To analyze the motion of the electron spin, we consider the case of the motion of a particle in a constant and homogeneous magnetic field directed along the z axis, $\mathbf{H} = (0, 0, H)$. To solve Eq. (40), we introduce a coordinate system rotating with angular velocity equal to the electron velocity $\omega_0 = ecH/E$. Then, introducing the new spin vector ζ' by means of the transformation matrix,

$$\zeta' = \begin{pmatrix} \zeta_\rho \\ \zeta_\varphi \\ \zeta_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \zeta, \tag{41}$$

where $\varphi = -\omega_0$ (for an electron $\omega_0 = -|\omega_0|$), we find that the system of equations for the spin components then has the form

$$\left.\begin{aligned} &\dot{\zeta}_\rho = \omega_a\zeta_\varphi - \frac{1}{\tau}\zeta_\rho; \quad \dot{\zeta}_\varphi = -\omega_a\zeta_\rho - \frac{1}{\tau}\frac{7}{9}\zeta_\varphi; \\ &\dot{\zeta}_z = -\frac{1}{\tau}\left(\zeta_z + \frac{8\sqrt{3}}{15}\right); \quad \omega_a = \frac{\alpha}{2\pi}\frac{eH}{mc}. \end{aligned}\right\} \tag{42}$$

The solution of these equations without allowance for the radiation reaction forces,

$$\zeta_\rho = \zeta_\perp \sin\omega_a t, \; \zeta_\varphi = \zeta_\perp \cos\omega_a t, \; \zeta_z = \zeta_\parallel, \tag{43}$$

shows that in the coordinate system associated with the electron the spin precesses by virtue of the anomalous magnetic moment around the direction of the magnetic field, maintaining a constant value of its projection onto this direction. The complete solution of the system of equations (42) with allowance for damping has the form

$$\left.\begin{aligned} &\zeta_\rho = \zeta_\perp \sin\omega_a t e^{-8t/9\tau}; \; \zeta_\varphi = \zeta_\perp \cos\omega_a t e^{-8t/9\tau}; \\ &\zeta_z = -\frac{8\sqrt{3}}{15} + \left[\zeta_\parallel + \frac{8\sqrt{3}}{15}\right] e^{-t/\tau}. \end{aligned}\right\} \tag{44}$$

It can be seen from these equations that with the passage of time the longitudinal polarization is damped, while the transverse polarization (along the field) tends to its limiting value, which is independent of the initial spin orientation:

$$|\zeta_z(\infty)| = 8\sqrt{3}/15 = 0.924. \tag{45}$$

It is important to emphasize that the radiative damping, as can be seen from the expressions (44), changes not only the orientation of the spin but also its absolute value:

$$(\zeta_\rho^2 + \zeta_\varphi^2 + \zeta_z^2)_{t\to\infty} = 0.85. \tag{46}$$

This shows clearly that, owing to the interaction of the electron with the radiation field, the final quantum state is a mixed state, in contrast to the original pure state. Therefore, in quantum theory rigorous description of the spin kinetics is possible only by means of a density matrix.

The expressions (44) describe the kinetics of the radiative polarization process. The kinetics of this process was considered for the first time in Refs. 4 and 7 on the basis of the equations expressing the statistical balance of the populations of the spin states of the electron ensemble; this corresponds to analysis of the diagonal elements of the density matrix. The kinetics of the radiative polarization process was later also analyzed by the density-matrix method in Refs. 16 and 17.

Such are the features of the motion of the electron spin in a homogeneous magnetic field. Using the BMT equation, we now consider the motion of the electron spin in an inhomogeneous axially focusing magnetic field with soft focusing:

$$\mathbf{H} = \left\{-\frac{qxz}{R^2}H_0, \; -\frac{qyz}{R^2}H_0, \; H_0\right\}. \tag{47}$$

Here, $0 < q < 1$ is the exponent of the decrease of the field (the field gradient), which ensures focusing of a particle moving in a circle of radius R in the plane $z = 0$ perpendicular to the symmetry axis of the field. If random forces deflect the electron from the plane $z = 0$ and radius R ($r = R + \rho$), it executes so-called radial and vertical betatron oscillations with frequencies $\omega_\rho = \sqrt{1-q}\,\omega_0$, $\omega_z = \sqrt{q}\,\omega_0$, $\omega_0 = ecH/E$.

We now consider how these oscillations influence the spin orientation process, restricting ourselves to vertical oscillations in order to simplify the problem. Then $z = Ra_z \sin(\omega_z t + \alpha)$, where a_z is the dimensionless amplitude of the oscillations, and α is an arbitrary phase.

We use the BMT equation (40), ignoring the damping, i.e., setting $Q = 0$. Going over then to the rotating coordinate system by means of the transformation matrix (41), we can write the magnetic field (47) in the form

$$\mathbf{H}(H_\rho, H_\varphi, H_z) = \left(-\frac{qz}{R}H_0, \; 0, \; H_0\right). \tag{48}$$

It is convenient to solve the BMT equation by perturbation theory, making the assumption that the amplitude of the vertical oscillations is very small compared with the radius of the electron orbit: $a_z = z/R \ll 1$. Then, assuming that $\zeta = \zeta_0 + \zeta^1$, we find that the projection of the spin onto the direction of the magnetic field has the form

$$\zeta_z = \zeta_{\parallel} - \frac{1}{2}\zeta_{\perp} q a_z \omega_a \cos[(\omega_z - \omega_a)t + \alpha - \beta]/(\omega_z - \omega_a)$$
$$+ \text{ nonresonance terms } . \qquad (49)$$

This result was obtained in Ref. 8 using the methods associated with the BMT equation (see also Ref. 34); resonance depolarization phenomena were considered by the methods of quantum theory in Ref. 33.

For the longitudinal polarization, we find under the same assumptions

$$\zeta_\varphi = \zeta_\perp \cos(\omega_a t + \beta) + \frac{1}{2}\zeta_\parallel q a_z \omega_a \cos(\omega_z t + \alpha)/(\omega_z - \omega_a)$$
$$+ \text{ nonresonance terms } . \qquad (50)$$

It can be seen from these solutions that the anomalous magnetic moment of the electron already begins to have an influence in a homogeneous magnetic field, when the exponent of the decrease of the field is $q = 0$. At the same time, the longitudinal polarization loses stability—it varies rapidly in accordance with a periodic law (see also Ref. 30). In the case of an inhomogeneous focusing magnetic field $(0 < q < 1)$, the vacuum magnetic moment of the electron also affects the stability of the transverse polarization. As long as there is no resonance of the frequencies, $\omega_z \neq \omega_a$, the betatron oscillations are superimposed on the stable motion of the spin only as a small perturbation. However, if the spin precession frequency ω_a is equal to the frequency ω_z of the betatron oscillations or its harmonics,

$$\omega_z = \omega_a, \quad E/mc^2 = \sqrt{q}\; 2\pi/\alpha , \qquad (51)$$

a characteristic resonance occurs, and this may cause beam depolarization.

So far, we have considered only vertical betatron oscillations; however, resonance beam-depolarization phenomena can also be produced by the radial betatron oscillations and the phase oscillations. The conditions of beam depolarization due to resonance effects in the motion of polarized particles was analyzed for the first time by Froissart and Stora[31] in 1960. They did this for the case of the motion of polarized protons. Subsequently, the resonance influence of the orbital motion on proton polarization was considered by Cohen[32] and then resonance conditions were also formulated for electrons.[8]

These conditions do not depend on the rest mass of the particle and have the form

$$\omega_a = N_0\omega_0 + N_1\omega_z + N_2\omega_\rho + N_3\omega_s, \qquad (52)$$

where N_0, N_1, N_2, and N_3 are integers; $\omega_0, \omega_z, \omega_\rho, \omega_s$ are the frequencies of the orbital motion of the electron: respectively, the revolution frequency ω_0, the frequencies ω_z and ω_ρ of the vertical and radial betatron oscillations, and the frequency ω_s of the phase oscillations; $\omega_a = (\alpha/2\pi)eH/mc$.

Thus, the motion of the electron spin loses stability when the spin precession frequency (in the laboratory coordinate system, this is equal to $\omega_0 + \omega_a$) is close to a combination of frequencies of the orbital motion. This creates a danger of beam depolarization and requires special study.

To analyze in more detail the passage of an electron through a resonance point (see also Ref. 8), we consider the motion of the spin of a particle in a variable magnetic field that depends on the time in accordance with the law[b)]

$$\mathbf{H} = \{H_1 \cos\tilde{\omega}t, \; -H_1 \sin\tilde{\omega}t, \; H\}, \qquad (53)$$

where H is a homogeneous field and it is assumed that $H_1 \ll H$; H_1 is the amplitude of the rotating field. Such a choice of the field is frequently made in the study of problems associated with magnetic resonance phenomena. In this case, the BMT equation (33) admits an exact solution. To simplify the problem, we can assume that in this equation the electron factor is equal to 2, and the anomalous magnetic moment can be taken into account additionally. Further, it is convenient to go over to a coordinate system rotating with angular velocity $\tilde{\omega}$ by means of a transition matrix analogous to the one in (41). Then the BMT equation takes the form

$$\frac{d}{dt}\zeta' = [\Omega'\zeta'], \quad \Omega' = (-\omega, \; 0, \; \delta), \qquad (54)$$

where $\delta = \omega_R - \tilde{\omega}$ is the difference of the frequency from resonance, $\omega_R = e_0H/mc\gamma$, $\omega = e_0H_1/mc\gamma$, $\gamma = E/mc^2$. It is important to emphasize that the spin evolution equation is again independent of the time. The physical picture of the motion of the spin in the rotating coordinate system is that the spin moves under the influence of the effective field

$$\mathbf{H}^{\text{eff}} = (-H_1, \; 0, \; H - mc\gamma\tilde{\omega}/e_0). \qquad (55)$$

At the same time, the spin executes precession around the direction of $\mathbf{H}^{\text{eff}}$ with angular frequency $\Omega' = \sqrt{\omega^2 + \delta^2}$, describing a cone.

Solving (54), we find in particular that

$$\zeta_3' = -\zeta_{01}'\frac{\delta\omega}{\Omega'^2}(1 - \cos\Omega' t) - \zeta_{02}'\frac{\omega}{\Omega'}\sin\Omega' t$$
$$+ \zeta_{03}'(\delta^2 + \omega^2\cos\Omega' t)/\Omega'^2. \qquad (56)$$

For the case of the initial spin state $\zeta_0' = (0, 0, 1)$ we obtain from this a relativistic generalization of Rabi's formula:

$$\zeta_3' = 1 - \frac{2\omega^2}{\omega^2 + \delta^2}\sin^2\frac{\Omega' t}{2}. \qquad (57)$$

If the resonance condition is strictly satisfied, i.e., if $\delta = 0$, the spin of a particle oriented initially along the field will precess, remaining oriented at right angles to the field $\mathbf{H}_1$. The spin will periodically take the direction opposite to that of the field $\mathbf{H}$.

If the frequency difference δ varies with the time adiabatically slowly, then for $\delta \ll \omega$ the angle between the vectors ζ' and Ω' hardly changes, and the spin vector ζ' follows $\mathbf{H}^{\text{eff}}$ smoothly, precessing around it. If now the field $\mathbf{H}_1$ is switched on for a brief interval (rapid passage through the resonance), the spin is rotated through only a small angle. In the other limiting case, when the time of passing through the resonance is long ($\mathbf{H}_1$ is switched on for a long interval of time), the spin may be flipped. Indeed, in this case, using the solution of (54) for $\zeta'(0) = (0, 0, \zeta_3^0)$, we find that $\zeta'(\infty) = (0, 0, -\zeta_3^0)$.

We consider the passage of the spin through the resonance in more detail by means of the solutions of Eq. (56). We specify the dependence of the difference of the frequency from resonance in the form $\delta = kt$. Under these assumptions, we can set approximately

$$\Omega' t \rightarrow \int_{-\infty}^{t} \Omega'(t')dt'. \qquad (58)$$

The integration in this formula can be readily performed in the two limiting cases by means of the integrals

$$J(t) = \int_{0}^{t} \Omega'\, dt' = \begin{cases} kt^2/2\ (\delta \gg \omega), \\ \omega t\ (\delta \ll \omega), \end{cases} \qquad (59)$$

with $\int_{-\infty}^{t} \Omega' dt' = J(t) + \varphi$, where $\varphi = \int_{-\infty}^{0} \Omega' dt'$. In the first case ($\delta \gg \omega$), there is a rapid passage through the resonance; in the second ($\delta \ll \omega$), a slow one. The characteristic time of passage through the resonance can be estimated on the basis of the following simple considerations. Since

$$\overline{\Omega}' T = \lim_{T\rightarrow\infty} \int_{0}^{T} \Omega'\, dt', \qquad (60)$$

it follows that $T \sim 1/\omega$ when $\delta \ll \omega$, while $T \sim 1/\sqrt{k}$ when $\delta \gg \omega$. Thus, it follows from the generalizations of (56) that in (56) it is everywhere necessary to replace Ω' in accordance with (58). Assuming further that

$$\zeta_3'(\infty) = \pm \zeta_3'(-\infty) + \delta\zeta_3', \qquad (61)$$

we attempt to estimate the nonadiabatic correction

$$\delta\zeta_3' = \int_{-\infty}^{\infty} \frac{\partial \zeta_3}{\partial t}\, dt. \qquad (62)$$

In the case of rapid passage through the resonance, it is necessary in (56) to take the derivative with respect to the oscillating term in ζ_3'. Then

$$\delta\zeta_3' = \zeta_{01}'\omega \int_{-\infty}^{\infty} \sin(kt^2/2 + \varphi)\, dt = \zeta_{01}' \sqrt{\frac{2\pi\omega^2}{k}} \sin\left(\varphi + \frac{\pi}{4}\right). \qquad (63)$$

Since $\omega^2 \ll k$, this correction is negligibly small: $\delta\zeta_3' \rightarrow 0$. Thus, in the case of rapid passage through the resonance small oscillations are superimposed on the regular precession, and $\zeta_3'(\infty) = \zeta_3'(-\infty)$.

In the other limiting case (slow passage through the resonance), the derivative in (56) must be taken with respect to the slowly varying coefficients $\cos\chi$ and $\sin\chi$: $\cos\chi = \delta/\Omega'$, $\sin\chi = \omega/\Omega'$. Then

$$\delta\zeta_3' = \zeta_{01}' \int_{-\infty}^{\infty} \frac{k\omega}{\omega^2 + k^2 t^2} \cos(\omega t + \varphi)\, dt = \zeta_{01}' \pi e^{-\omega^2/k} \cos\varphi, \qquad (64)$$

or $\zeta_3'(\infty) = -\zeta_3'(-\infty)$, since the nonadiabatic correction is very small in this case too: $\delta\zeta_3' \rightarrow 0$. Thus, in the case of slow passage through the resonance the spin of the particle is reversed.

Evolution of the spin of a particle moving in a storage ring

The studies made at the Institute of Nuclear Physics at Novosibirsk led by A. N. Skrinskiĭ[35] were of great importance for the analysis of the motion of the spin of a particle in a real storage ring. By solution of the BMT equation it was established that in an arbitrary electromagnetic field of a storage ring that ensures the existence of a periodic closed electron orbit the radiative polarization of a particle beam is as stable as in a homogeneous magnetic field, and for a closed electron orbit, when $\mathbf{\Omega}(\theta) = \mathbf{\Omega}(\theta + 2\pi)$, the spin of the particle conserves its projection onto a certain direction $\mathbf{n}(\theta)$ (precession axis), being a periodic function of the particle's azimuth $\theta = \omega t$.

To determine this vector $\mathbf{n}$, we multiply the left- and right-hand sides of Eq. (33) by $\mathbf{n}$, obtaining

$$\dot{\boldsymbol{\zeta}}\mathbf{n} = [\Omega\boldsymbol{\zeta}]\mathbf{n} = -\boldsymbol{\zeta}[\Omega\mathbf{n}], \qquad (65)$$

from which it follows that

$$\frac{d}{dt}(\boldsymbol{\zeta}\mathbf{n}) = \boldsymbol{\zeta}\left(\frac{d\mathbf{n}}{dt} - [\Omega\mathbf{n}]\right), \qquad (66)$$

and thus the vector $\mathbf{n}$ satisfies the equation

$$\dot{\mathbf{n}} = [\Omega\mathbf{n}]. \qquad (67)$$

The existence and uniqueness of solutions of this equation follow from the general theory of solutions of systems of homogeneous equations with periodic coefficients.

Thus, the electron spin precesses around the precession axis $\mathbf{n}$ (periodic vector), conserving its projection onto this axis. The general solution for $\boldsymbol{\zeta}$ can be represented in the form of a decomposition with respect to a periodic system of orthogonal vectors $\mathbf{n}$, $\mathbf{e}_1$, and $\mathbf{e}_2$:

$$\boldsymbol{\zeta}(t) = \zeta_n \mathbf{n} + \zeta_\perp \mathrm{Re}(\boldsymbol{\eta} e^{i\nu\theta}), \qquad (68)$$

where

$$\mathbf{n}(\theta + 2\pi) = \mathbf{n}(\theta), \quad \boldsymbol{\eta}(\theta + 2\pi) = e^{-2\pi i\nu}\boldsymbol{\eta}(\theta); \qquad (69)$$

$\zeta_n = \zeta_\parallel = \mathrm{const}$; $\zeta_\perp = \sqrt{\zeta^2 - \zeta_\parallel^2}$, and $2\pi\nu$ is the angle of rotation of the spin around $\mathbf{n}$ during a period of the motion of the particle in the equilibrium orbit.

The theorem proved here is important, since the existence of a stable periodic solution (polarization axis) makes it possible to create the conditions necessary for polarization of particles at a given point of their trajectory, in particular, at the point at which the particle beams collide in the storage ring. The motion of the spin of particles that are near the equilibrium orbit is stable, and this stability can break down only in a narrow region of resonances, when the spin precession frequency is equal to frequencies of the orbital motion [see (52)].

If we introduce $\varepsilon_k = \omega_a - \omega_k$, the "frequency detuning" where ω_k is a multiple of any of the resonance frequencies of the orbital motion, and w_k is the power of the resonance (of the Fourier harmonic), then, as was shown in Ref. 36,

$$\zeta_z^{\mathrm{fin}} = \langle (2e^{-2J} - 1)\, \zeta_z^{\mathrm{in}} \rangle, \qquad (70)$$

where the angular brackets $\langle\ \rangle$ denote the average over the phases: $J = (\pi/4)|\omega_k|^2/|\dot{\varepsilon}_k|$, and $\dot{\varepsilon}_k$ is the rate of passing through the resonance, assumed to be a constant quantity: $\dot{\varepsilon}_k = \mathrm{const}$. We note here that the expression (70) for an isolated resonance was obtained earlier by Froissart[31] by exact integration of the BMT equation using hypergeometric functions. From the expression (70), we immediately obtain two limiting cases for the passage through the resonance. If $J \ll 1$, there is rapid passage through the resonance, and $\dot{\varepsilon}_k$ is large, then the polarization does not change. In the other extreme case, when the passage through the resonance is slow, $\dot{\varepsilon}_k \to 0$, $J \gg 1$, the polarization changes sign adiabatically, but the degree of polarization is hardly changed. The change in the degree of polarization (beam depolarization) is appreciable if J is of the order of unity—this is the intermediate and most unfavorable case.

Various recommendations[36] were made for suppressing the harmful influence of depolarization. Essentially, they either compensate dangerous harmonics of the disturbing fields or increase the rate at which the passage through the resonance takes place. However, there is a further possibility—to switch on in the gap between magnets a constant field directed along the particle velocity (suppression of $\omega_a = k\omega_0$ resonances) and to introduce analogous time-dependent fields with frequencies near ω_z and ω_ρ (suppression of the $\omega_a = k\omega_0 \pm \omega_z$ and $\omega_a = k\omega_0 \pm \omega_\rho$ resonances). The introduction of such perturbations raises the power of the harmonics and leads as a result to adiabatic passage through the resonances, for which the degree of polarization remains unchanged.

We consider further the possibility of obtaining beams of longitudinally polarized particles, which are of great interest for modern experiments in high-energy physics. As was shown by the present author in collaboration with Tumanov,[30] the direct acceleration of longitudinally polarized particle beams encounters serious difficulties. Because of the anomalous magnetic moment of the electron, the longitudinal polarization is rapidly destroyed, the spin precessing in the plane of the electron's orbit. Therefore, a different method was found to be more effective, namely, to rotate the transverse polarization into longitudinal polarization by means of a system of magnets that turn the spin in a small region of space near the equilibrium orbit.[37,38]

In the simplest case, the rotation of the spin can be readily achieved by introducing an additional radial magnetic field in a straight interval of the storage ring. By choosing the magnetic-field configuration appropriately, one can create at a given point of the orbit the necessary direction of the polarization.

Let us consider an example. Let $\mathbf{B}(t)$ be the additional magnetic field switched on as the electron passes through the straight section. It then follows from the BMT equation that the change in the projection of the spin onto the direction of the electron momentum satisfies the equation

$$\frac{d}{dt}(\zeta\mathbf{P}) = \dot{\zeta}\mathbf{P} = \frac{\alpha}{2\pi}\frac{e_0}{mc}\mathbf{B}[\mathbf{P}\zeta]. \tag{71}$$

Introducing now the angle θ between the spin vector ζ and the electron momentum $\mathbf{P}$ and the angle φ between $\mathbf{P}\times\zeta$ and $\mathbf{B}$, we find that

$$\theta = \frac{\alpha}{2\pi}\frac{e_0}{mc}\cos\varphi\int_0^t B\,dt. \tag{72}$$

Thus, by introducing the radial magnetic field in the straight section it is possible to turn the spin through an angle $\pi/2$ relative to the velocity, i.e., in the plane of the electron's orbit. As a result, the transverse polarization due to the radiative effects becomes a longitudinal polarization. The subsequent motion of the particle will be accompanied by a restoration of the transverse polarization, since the longitudinal spin orientation is not maintained. From the point of view of the possible influence of depolarizing factors, it is advantageous to preserve for the greater part of the time the natural transverse polarization except in the small section of the particle trajectory in which the beams must collide. This can be done by restoring the transverse polarization from the longitudinal by switching on an additional magnetic field that acts in the opposite order (Fig. 2).[37,38]

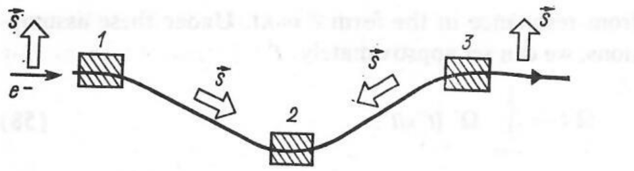

FIG. 2. Rotation of the spin vector. The radial magnetic field is perpendicular to the plane of the diagram and is introduced in regions 1, 2, and 3.

Thus, great possibilities are opened up for creating any given spin orientation. In particular, one can even obtain colliding electron–positron beams with the same spin direction, something that is of interest for experiments with e^+e^- beams of the same helicity, since in this case the electrodynamic single-photon channel is forbidden, so that the importance of the other annihilation processes is enhanced.

Influence of radiative reaction on the radiative polarization of electrons

The development of the quantum theory of synchrotron radiation (see Ref. 2) showed that at electron energies of the order of $E_{1/5} = mc^2(mcR/\hbar)^{1/5}$ (around 500 MeV) the discrete properties of the radiation must be manifested and lead to quantum fluctuations of the electron trajectory; the theoretical predictions of Ref. 39 were soon experimentally confirmed.[40] As the present author showed in collaboration with Sokolov,[39] the motion of an electron in a homogeneous magnetic field is accompanied by a singular excitation of the radial degrees of freedom of the electron—there are quantum fluctuations in the radius of the orbit of its revolution in the magnetic field. The growth of the quadratic fluctuation of the orbit radius is determined by

$$\frac{d}{dt}\left\{\frac{eH}{c\hbar}\left(\overline{r^2}-(\bar{r})^2\right)\right\} = \frac{ds}{dt} = \frac{55}{48\sqrt{3}}\frac{e^2}{mcR^2}\gamma^6,\quad \gamma = E/mc^2, \tag{73}$$

where $s = 0, 1, 2, \ldots$ is the radial quantum number. The electron begins to move like a Brownian particle, being subject to random jumps of the radius of the orbit, which are caused by the recoil following photon emission.

There are also accordingly quantum fluctuations of the electron energy:

$$E = \sqrt{m^2c^4 + c^2p_z^2 + 2eHc\hbar(l+s)}. \tag{74}$$

The radiative losses associated with the change in the orbital quantum number l (the rotational degree of freedom) are compensated by the external high-frequency field of the cyclotron (storage ring), while the quantum fluctuations of the energy $\overline{\Delta E^2} = e^2H^2[\overline{R^2} - (\overline{R})^2]$ increase with the time as

$$\frac{1}{\gamma^2}\frac{d}{dt}\delta\gamma^2 = \frac{2\hbar}{mcR}\frac{1}{\gamma}\dot{s}, \tag{75}$$

where $\dot{s}$ is determined by Eq. (73). Simultaneously, the recoil also changes the momentum of the electron along the field.

The quantum fluctuations in the radius of the electron trajectory were found to be an important factor determining the dynamics of particles in an accelerator. In the case of the motion of an electron in a focusing magnetic field, the general picture of the motion is complemented by radiative damping of the betatron oscillations.[41,42] In the case of an axisymmetric magnetic field with soft focusing, we have

$$H = H_0\,(R/\rho)^q, \quad 0 < q < 1, \quad \rho = R + x. \tag{76}$$

Equation (75) is modified when the damping forces are taken into account:

$$\frac{1}{\gamma^2}\frac{d}{dt}\delta\gamma^2 = \frac{2\hbar}{mcR}\sqrt{1-q}\,\frac{1}{\gamma}\dot{s}, \tag{77}$$

where

$$s = \frac{55}{48\sqrt{3}}\frac{e^2}{mcR^2}\frac{\gamma^6}{(1-q)^{3/2}} - \frac{q}{1-q}s\frac{W}{E}, \tag{78}$$

in which $W = (2/3)(e^2c/R^2)\gamma^4$ is the power of the synchrotron emission.

The simultaneous influence of the radiative damping and the quantum fluctuations leads to a stabilization of the amplitude of the betatron oscillations, and after the damping time

$$\tau_d = \left(\frac{q}{1-q}\frac{W}{E}\right)^{-1} \tag{79}$$

the oscillation amplitude takes the equilibrium value corresponding to an extremum for the variation of the number s, i.e.,

$$\dot{s} = 0 \tag{80}$$

(see Ref. 42, p. 190). The phenomena of radiative damping and quantum fluctuations described here were observed experimentally by a group of physicists at the Moscow State University led by F. A. Korolev using the 680-MeV synchrotron of the P. N. Lebedev Physics Institute[40] and were found to be in good agreement with the theory.

The quantum fluctuations of the particle trajectory must have a depolarizing influence on the electron spin, giving rise to a "jitter" of the precession axis, as was pointed out by Baĭer (see Ref. 8).

To make a more detailed analysis of the influence of the radiation reaction on the radiative polarization, we now consider the total change in the projection of the spin onto the precession axis $\mathbf{n}$, $|\mathbf{n}| = 1$,[36]

$$\frac{d}{dt}(\zeta\mathbf{n}) = \frac{d}{dt}\zeta(\mathbf{n}+\delta\mathbf{n}) = \dot{\zeta}_n + \zeta\dot{\mathbf{n}} + \zeta\frac{d}{dt}\delta\mathbf{n}. \tag{81}$$

We here take into account the direct influence of the radiation on the state of the spin orientation, $\dot{\zeta}_n$, the effect of the radiation reaction on the precession axis, $\zeta\cdot\dot{\mathbf{n}}$, and the quantum fluctuations of the precession axis: $\zeta(d/dt)\delta\mathbf{n}$.

If the field of the storage ring differs appreciably from the ideal situation, the precession axis depends on the particle energy: $\mathbf{n}(\gamma)$, $\gamma = E/mc^2$. Therefore, going over in (81) to values averaged over the particle ensemble, we obtain

$$\left\langle \frac{d}{dt}\zeta\mathbf{n}\right\rangle = \dot{\zeta} + \zeta\mathbf{n}\left\langle\left(\gamma\frac{d\mathbf{n}}{d\gamma}\right)\frac{1}{\gamma}\frac{d}{dt}\gamma\right\rangle - \frac{1}{2}\zeta\left\langle\left(\gamma\frac{d\mathbf{n}}{d\gamma}\right)^2\frac{1}{\gamma^2}\frac{d}{dt}\delta\gamma^2\right\rangle, \tag{82}$$

where $\zeta = \zeta_n$ is the projection of the spin onto the precession axis. The general picture of the influence of the radiation on the process of radiative polarization is to a certain degree analogous to this influence on the orbital motion. We note first of all that, as the present author showed in collaboration with Bagrov and Rzaev,[43] the power of the synchrotron emission without spin flip depends explicitly on the spin orientation of the particle:

$$W = W^{\text{cl}}\left(1 - \frac{55\sqrt{3}}{24}\xi\right) - \zeta\xi W^{\text{cl}}, \quad \xi = \frac{3}{2}\frac{\hbar}{mcR}\gamma^2. \tag{83}$$

Because of this, the change in the energy of the electron with allowance for the compensation of the radiative losses in the storage ring has the form

$$\frac{1}{\gamma}\frac{d}{dt}\gamma = \zeta\left(\frac{W}{\gamma}\right)\xi = \frac{8\sqrt{3}}{15}\frac{1}{\tau_n}\zeta, \tag{84}$$

where the characteristic polarization time is determined by the expression (23). This change in the energy, which depends on the spin ζ, is analogous to the radiative damping of the betatron oscillations and characterizes the spin–orbit coupling, which can produce an additional polarization of the electron beam (see Refs. 20 and 36).

The change in the quadratic fluctuation of the energy (75), which characterizes the quantum depolarization process, is determined for the case of a homogeneous magnetic field by [see Eqs. (73) and (75)]

$$\frac{1}{\gamma^2}\frac{d}{dt}\delta\gamma^2 = \frac{11}{9}\frac{1}{\tau_n}. \tag{85}$$

Substituting the previously obtained values (77), (84), and (85) in Eq. (82) and taking into account the direct influence of the radiation on the spin, which can be found by means of the probability $w^{\uparrow\downarrow}$ (22) of quantum transitions, we obtain for the mean polarization the expression

$$\zeta = -\frac{8\sqrt{3}}{15}\frac{\left\langle 1 - \mathbf{n}\left(\gamma\frac{d\mathbf{n}}{d\gamma}\right)\right\rangle}{\left\langle 1 + \left(\gamma\frac{d\mathbf{n}}{d\gamma}\right)^2 Q\right\rangle}, \tag{86}$$

where

$$Q = \frac{\tau_n}{2}\frac{1}{\gamma^2}\frac{d}{dt}\delta\gamma^2, \tag{87}$$

and the angular brackets denote the average over the particle ensemble.

It can be seen from these expressions that the synchrotron radiation, being the physical reason for the ordering of the spin orientation of the electrons, also acts on the precession axis. This gives rise to depolarization of the electron beam and is a further mechanism that enhances the polarization effect.

The influence of the radiation on the electron polarization was calculated in general form in Ref. 36:

$$\zeta=-\frac{8\sqrt{3}}{15}\frac{\left\langle |\bar{v}|^2[\mathbf{v}\dot{\mathbf{v}}]\left(\mathbf{n}-\gamma\frac{d\mathbf{n}}{d\gamma}\right)\right\rangle}{\left\langle |\dot{\mathbf{v}}|^3\left[1-\frac{2}{9}(\mathbf{n}\mathbf{v})+\frac{11}{18}\left(\gamma\frac{dn}{d\gamma}\right)^2\right]\right\rangle}. \quad (88)$$

It should, however, be noted, as can be seen from (85), that Q in the case of a homogeneous magnetic field takes the value $Q=11/18$. In general it is a function of the time, and in a focusing magnetic field and for values of t greater than the time τ_d of radiative damping,

$$t>\tau_d=\frac{1-q}{q}\frac{E}{W} \quad (89)$$

[see (79)], the value of Q tends to zero ($Q\to 0$) (steady process). Therefore, in the case when the quantum fluctuations are restricted by radiative damping the depolarizing influence of these fluctuations requires additional study.

We now dwell briefly on the details of the radiative polarization of colliding beams. The motion of colliding beams has a number of particular features; it is accompanied by a strong nonlinearity of the collective field and by an increase in the power of the spin resonances—by an increase in the dependence of the stability of the polarization on the betatron and synchrotron oscillations. Under these conditions, the density of resonances is related to the density of the particles in the beams, and it is therefore necessary to consider whether one can ensure simultaneously a high polarization percentage and sufficient luminosity. The stability of the polarization of colliding beams was analyzed in Ref. 44. The depolarizing influence of any source of stochastic perturbations, including quantum fluctuations, was considered. To this end, a depolarization time was introduced, this being the reciprocal of a probability:

$$\tau_g^{-1}=w_g=\sum_k w_k=\sum_k \pi\langle |U_k|^2\delta(\nu-\nu_k)\rangle. \quad (90)$$

Here, ν is the frequency of the spin precession around the polarization axis $\mathbf{n}$, ν_k is an integral combination of the frequencies of the orbital motion, and the delta function characterizes the effect on the spin of the stochastic source. The equation for the evolution of the spin with allowance for the depolarizing effects,

$$\dot{\zeta}=-\zeta w^{\uparrow\downarrow}-\zeta\,\omega_{\rm dep}, \quad (91)$$

has the following solution for the equilibrium depolarization:

$$\zeta_{\rm eq}=-\frac{8\sqrt{3}}{15}\frac{1}{1+\tau_n/\tau_{\rm dep}}, \quad (92)$$

which is established after the time $T=\tau_n\tau_{\rm dep}/(\tau_n+\tau_{\rm dep})$.

The power $|U_k|^2$ of the resonance, which occurs in the expression (90), depends, in particular, on the amplitude of the betatron oscillations and, in the first place, on the amplitude of the z oscillations. We note that the amplitude of the z oscillations may change in a head-on encounter of the two beams (of electrons and positrons), since the encountered beam has an influence on the vertical oscillations. The effective amplitude is found to depend on the number of particles in a bunch, and as a result of this the number of particles N occurs in the expression for the depolarization time $\tau_{\rm dep}$. The calculated estimates showed that the conditions that limit the particle number N, i.e., limit the luminosity, are not unrealistic, being the same as the ordinary requirements on the beam that ensure stability of the colliding beams in the orbital motion.

The study of the stability of the polarization and the part played by the depolarizing factors[45] showed that for appropriate requirements on the accuracy of construction of the magnetic system of the storage ring one can ensure polarization of colliding beams even in the region of superhigh electron energies 10–100 GeV. These requirements relate in the first place to the accuracy of alignment of the magnetic system, which must ensure that the maximal deviation of the beam from the equilibrium orbit is limited, and also to the choice of the storage-ring parameters, which must guarantee that the spin precession frequency is far from the resonances due to the random deviations of the magnetic field.

As was shown in Ref. 45, there exists a possibility for accelerating the polarization process by introducing in the straight sections of the storage ring a strong vertical sign-variable magnetic field: B_+ and $B_-(B_+\gg B_-)$—a "snake." Then polarizing processes will take place when the electrons radiate in the snake section. This shortens the time of establishment of the polarization by $2\pi\langle B_0^3\rangle/B_+^3\theta_+$ times ($B_0\ll B_+$ is the guiding field of the accelerator, and θ_+ is the length of the section over which the vertical B_+ field acts). The degree of polarization is then

$$|\zeta|=\frac{8\sqrt{3}}{15}\frac{B_+^2-B_-^2}{B_+^2-B_-^2}. \quad (93)$$

This method of enhancing the role of the polarizing processes due to the radiation is particularly effective in large rings with relatively small $\langle B_0\rangle$ used to store particles with very high energies.

3. THE RADIATIVE POLARIZATION EFFECT IN EXPERIMENTS

Experimental methods of observing radiative polarization

As we have already noted, the effect of radiative polarization of electrons and positrons in storage rings was established theoretically in 1963.[4] The first indication of the existence of this effect in storage rings was obtained in 1968—radiative polarization of electrons (natural polarization) was observed experimentally in the ACO storage ring (536 MeV) by a group of physicists in France (Orsay).[46]

In 1971, the effect was studied experimentally at the Institute of Nuclear Physics of the Siberian Branch of the USSR Academy of Sciences at Novosibirsk using the VÉPP-2 storage ring (625 MeV),[8] and also in France using the ACO storage ring.[47] Later, observations were made at higher energies: in 1975 at Stanford (USA) using the SPEAR storage ring (2.4 and 3.7 GeV,[48–50] and in 1978 in the German Federal Republic using the PETRA storage ring (15.2 GeV) (Refs. 51 and 52; see also Ref. 53). In 1975, detailed observations of the effect were made using the VÉPP-2M storage ring (0.5–0.7 GeV) at Novosibirsk.[54] These observations, which were based on different methods, not only proved qualitatively the existence of the radiative polarization effect in storage rings but also established good agreement with the theoretical predictions for the degree and time of polarization.[4]

We shall consider the basic methods of measuring the polarization of electrons and positrons used in the experiments.

A. Intrabeam scattering (Tousek effect)

The first experiments to investigate the radiative polarization of electrons and positrons[8,47,53,54] used a method of measurement based on the Møller scattering of particles by each other in one and the same bunch. This scattering, which is associated with the betatron oscillations of the particles, can lead to transformation of the radial momentum into longitudinal momentum,[55] i.e., to different values of the energies of two particles, the energies also differing from the mean value of the beam energy. Intrabeam scattering may lead to particles being knocked out of the beam if the additional momentum acquired by a particle in scattering exceeds the maximally allowed value.

It is an important circumstance that the scattering cross section depends on the spin orientation of the particles. Therefore, the number of particles taken out of the acceleration regime will increase in pairs, as is clear from the essence of the internal scattering effect, and can be detected by appropriate particle detectors. The formula for the differential cross section for elastic scattering of the electrons and positrons in the center-of-mass system of the beam has, with allowance for the polarization, the general form

$$d\sigma_c = d\sigma_0\,(p,\ \theta)d\,\theta\,[1 - \zeta\zeta' G\,(p,\ \theta)]. \quad (94)$$

Here, $d\sigma_0\,(p,\theta)$ is the well-known Møller differential cross section for unpolarized particles, $p = p_c/mc$ is the momentum in the center-of-mass coordinate system of the beam, θ is the scattering angle, and ζ and ζ' are the spins of the particles (transverse polarization).

The maximally allowed momentum is determined by the particular parameters of the storage ring. The integration of (94) with respect to the angle θ, and also with respect to the momentum p gives, when allowance is made for the given momentum distribution function of the particles, the following expression for the "counting rate" of the particles knocked out of the beam:

$$Y = Y_{\max} - b\,(1 - e^{-t/\tau})^2, \quad (95)$$

where $Y_{\max}$ and b are positive coefficients, and τ is the polarization time.

At the initial time $t = 0$, when the beam is unpolarized, the number of pairs knocked out is maximal: $Y = Y_{\max}$; then, as the radiative polarization is established, the rate at which the particles are knocked out of the beam decreases exponentially and tends to the limiting value $Y = Y_{\max} - b$. Then, by switching on an additional magnetic field, the beam energy is changed to make the spin precession frequency equal to one of the eigenfrequencies of the orbital motion. The beam is depolarized, and a sharp increase in the counting rate of the ejected pairs to its maximal value is observed.

Resonance depolarization of the beam opens up the possibility of determining the main characteristics of the radiative polarization effect, namely, by comparing the jump in the counting rate of the ejected electron pairs with the expression (95) for determining the time, one can find the coefficients $Y_{\max}$ and b, and, therefore, the degree of polarization of the beam and the time τ.

B. Annihilation method

The method of measuring the polarization based on the Tousek effect is not suitable for measuring the polarization of colliding particle beams. In addition, it is very sensitive to the details of the beam structure, and this lowers the necessary accuracy in the measurement of the polarization.

The annihilation method, which is free of these shortcomings, is based on the interaction of high-energy particles, since the cross section of two-particle reactions is sensitive to the spin orientation of the colliding particles—the electrons and positrons. In particular, in the case of annihilation of electrons and positrons, muon pairs may be produced, and the cross section of this process ($e^+ e^- \to \mu^+ \mu^-$), calculated by the methods of quantum electrodynamics, has the form

$$\frac{d\sigma_{\mu\mu}}{d\Omega} = \frac{r_0^2}{16}\left(\frac{mc^2}{E}\right)^2 \{1 + \cos^2\theta + p^2 \sin^2\theta \cos 2\varphi\}, \quad (96)$$

where $p = |\zeta_1| = |\zeta_2|$ is the degree of polarization of the electrons and the positrons, θ is the polar angle of the μ^+ meson with respect to the incident positron, and φ is the azimuthal angle made by the production plane (the plane passing through the momenta of the initial and the final particle) with the horizontal plane. Thus, the process of production of a muon pair depends on the polarization of the electrons and positrons, and this must be observed experimentally in a dependence of the process on the angle φ. An azimuthal asymmetry must also be observed in the annihilation processes $e^+e^- \to e^+e^-, \pi^+\pi^-, K^+K^-, K_L^0 K_S^0$.

In experiments made using the SPEAR storage ring[49] at energy 3.7 GeV, anisotropy was observed in the production of the muon pairs, and this made it possible to obtain in the observations of the polarization good agreement between the theory and experiment. At $E = 1.55$ GeV, the spin precession frequency was equal to harmonics of the orbital frequency, and therefore, owing to the depolarizing resonance, no polarization was observed. The experimental investigations of the azimuthal asymmetry at 3.7 GeV were compared with the theoretical formula, and good agreement was found (Fig. 3). With good agreement with theory, the polarization was investigated by the same method using the VÉPP-2M storage ring.[54] We note that the annihilation method does not destroy the polarization of the beams and permits continuous observations.

C. The Compton scattering method

One of the methods of observing the radiative polarization effect uses the Compton scattering of circularly polarized photons by electrons. The Compton effect exhibits a characteristic dependence of the cross section on the circular polarization of the photons and on the transverse polarization of the electrons.

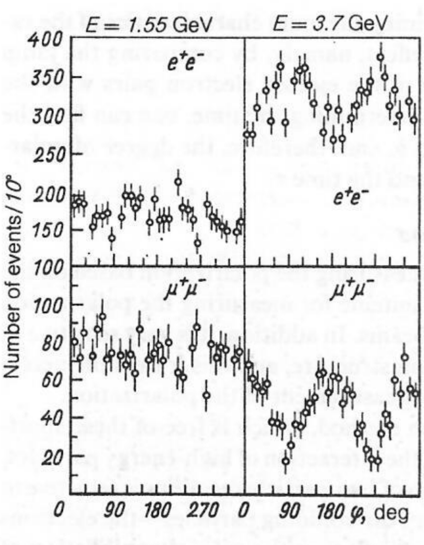

FIG. 3. Azimuthal angular distribution of events at beam energies 1.55 and 3.7 GeV for $e^+e^- \to e^+e^-, \mu^+\mu^-$ (in accordance with the data of Ref. 49).

Because of the relativistic properties of synchrotron radiation in a head-on collision of a photon with an electron, the final photons are emitted in a narrow cone around the electron velocity vector with opening angle mc^2/E. At the same time, the scattering cross section

$$d\sigma = d\sigma_0 + d\sigma_1 P_\varphi P_e \sin\varphi \tag{97}$$

depends on P_φ, the degree of circular polarization of the photons, on P_e, the polarization of the electrons and the positrons, and on φ, the angle between the plane perpendicular to the electron polarization vector and the scattering plane. For scattering of circularly polarized photons by transversely polarized electrons, an asymmetry must therefore be observed in the angular "up–down" splitting (Fig. 4; see Ref. 56).

Measurement by the Compton scattering method does not destroy the beam polarization—the investigation of the interaction of the photons with the electrons leads to determination of the spin orientations of individual electrons, and a conclusion about the complete beam is drawn from the individual sampling tests. In the measurements, the polarized laser beam is directed in the opposite direction to the particle beam. Individual laser photons interact with the electrons and are reflected backward ("inverse Compton effect"), acquiring a very high energy. They then enter detectors. The characteristic distribution of the photon scattering makes it possible to determine the degree of polarization of the electrons. The method developed using the PETRA storage ring[5] was perfected to such a degree that the process of measuring the polarization lasted 10 min, and therefore the degree of polarization of the e^+e^- beams could be measured continuously without destroying the spin orientation.

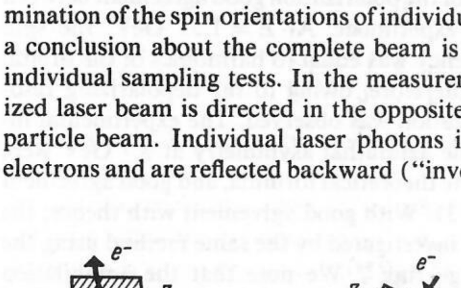

FIG. 4. Arrangement of experiment with Compton scattering of circularly polarized photons by electrons: 1) laser; 2) polarizer; 3) mirror; 4) region of collision of photons with electrons; 5) backward-scattered photons; 6) photon detector; 7) bending magnets.

D. Method of observing the spin dependence of synchrotron radiation

All the methods listed here for observing and measuring the polarization of electron—positron beams are in a certain sense indirect, that is, they permit a judgment to be drawn about the orientation of the particle spin as a result of certain interactions—collisions of particles, annihilation, or interaction of laser photons with the beam of electrons and positrons.

In this connection, it is interesting to mention one further possibility for measurement—the observation of the polarization of electrons and positrons using the spin dependence of the synchrotron radiation spectrum. This method of observation, proposed at the Institute of Nuclear Physics at Novosibirsk,[57] is in a certain sense direct—it does not require other particles to interact with the electron beam.

Indeed, as was shown by the present author in collaboration with Bagrov and Rzaev,[43] the power of the synchrotron radiation emitted by polarized electrons depends on the spin orientation of the particles. Using the results of Ref. 43 to study the σ component of the synchrotron radiation in the linear approximation in $\hbar$, one finds that a naturally polarized electron beam will radiate somewhat more than an unpolarized beam. In the short-wavelength part of the spectrum, which is the most favorable for observation, the additional power of the synchrotron radiation due to the spin orientation has the form

$$\Delta = \frac{2\xi y \mid \zeta \mid K_{1/3}(y)}{\int\limits_y^\infty K_{5/3}\,dx + K_{2/3}(y)}. \tag{98}$$

Using the asymptotic behavior of the Macdonald function,

$$\underset{(y\to\infty)}{K_\mu(y)} = \sqrt{\frac{\pi}{2y}}\, e^{-y}, \tag{99}$$

we find that

$$\Delta = \mid \zeta \mid y\,\xi = 0.92\, y\xi, \tag{100}$$

i.e., for fixed observation wavelength, Δ is proportional to ξ, i.e., to the magnetic field intensity and the energy of the particles. The quantity Δ is the jump in the energy when the depolarizer is switched on.

The polarization was measured by this method[58] using the VÉPP-4 storage ring. The radiation powers of two positron bunches—polarized and unpolarized (one of the bunches was subjected to selective depolarization)—were compared. The polarization of the particles was found by the method of maximal likelihood from the measurement in the jump of the radiation energy due to the polarization of the positrons.

Thus, observation of the spin dependence of the synchrotron radiation proved to be an effective method for investigating the radiative polarization in storage rings. Since the effect increases with the particle energy ($\Delta \sim EH$), this method is of particular interest for storage rings for high-energy particles (10–100 GeV).

In conclusion, we note that the measurement of the radiative polarization of electrons and positrons by the method which uses observation of the spin dependence of the synchrotron radiation introduces a new concept in the problem of measuring the spin of a free electron (quantum theory of measurements). Here, the source of information about the spin orientation is the electron itself, which is free and not bound in an atom. Moreover, the observation of the polarization is not directly related to the effect of a macroscopic instrument on a microscopic object (as is the case, for example, in Stern–Gerlach experiments).

The radiative polarization effect and new possibilities of physics experiments

The natural process of radiative polarization of electrons and positrons in storage rings is at the present time unique and by virtue of this represents a unique method for obtaining polarized particles of high energies. Since the polarization process can be readily controlled, it is possible to vary freely the particle energy and the polarization time and also to change the spin orientation in accordance with the requirements of the experiment and, in particular, to go over from transverse to longitudinal polarization. This has opened up new possibilities for physics experiments with polarized particles.

a) Resonance depolarization. High-precision measurement of the beam energy in an accelerator

This method[59,60] is based on exact measurement of the spin precession frequency by resonance depolarization of the electron beam. As we have already noted, the frequency Ω_0 of spin precession around the guiding magnetic field is related to the energy of the particle by

$$\Omega_0 = \omega_0 \left(1 + \frac{\alpha}{2\pi} \gamma\right), \tag{101}$$

where $\gamma = E/mc^2$ is the relativistic factor, and ω_0 is the relativistic cyclotron frequency: $\omega_0 = ec\overline{H}_z/E$. It is here assumed that averaging over the fast betatron oscillations has been performed. The (slow) synchrotron oscillations of the energy of the particle near the mean value γ_0 with frequency ω_γ have the consequence that the spin precession frequency is modulated, as a result of which the frequency spectrum of the spin precession has a fundamental frequency Ω_0 and displaced side frequencies

$$\Omega_0 \pm n\omega_\gamma, \tag{102}$$

where n is an integer. However, in the practically important cases these side frequencies make an exponentially small contribution.

The resonance depolarization of the beam is achieved by means of a depolarizer (current loop), which creates an alternating longitudinal magnetic field that varies with the frequency $\omega_{\text{dep}} = \omega_0((\alpha/2\pi)\gamma - 1)$. If the resonance condition

$$\omega_{\text{dep}} = \omega_0 \frac{\alpha}{2\pi} \gamma \pm \omega_\gamma \tag{103}$$

is satisfied, the beam polarization is destroyed, and this can be established, for example, by measuring the polarization by the intrabeam scattering method.

The resonance depolarization method makes it possible to solve the problem of calibrating the beam energy in a storage ring to an accuracy much higher than 10^{-4}. This is of great importance for measuring the masses of particles that participate in reactions obtained by means of colliding beams. High-precision measurement of the energy of the electrons and positrons was used at the Institute of Nuclear Physics at Novosibirsk in experiments with the VÉPP-2 storage rings to determine the masses of mesons in reactions in which decay of the Φ meson was detected by means of two charged pions:

$$e^+e^- \to \Phi \to K_L K_S; \quad K_S \to \pi^+\pi^-.$$

It was found that $M_\Phi = (1019.4 \pm 0.3)$ MeV.[60,61]

In experiments made using the VÉPP-4 storage ring and the resonance depolarization method, the mass of the Υ meson in the inclusive reactions $e^+e^- \to \Upsilon + h$ was measured with high accuracy. As a result of accurate calibration of the energy and analysis of the experimental data in accordance with the Breit–Wigner curve, the value $M_\Upsilon = (9459.7 \pm 0.6)$ MeV was obtained[62] for the mass of the Υ meson.

The resonance depolarization method developed at the Institute of Nuclear Physics was also used successfully in experiments with the Cornell storage ring CESR,[63] and also DORIS (in the German Federal Republic)[64] for a precision measurement of the masses of the Υ mesons.

An interesting experiment to make an accurate determination of the anomalous magnetic moment of the electron and positron was made by means of the radiative polarization method using the VÉPP-2 storage ring.[65] In these experiments, in contrast to the previously known experiments, the anomalous magnetic moments were measured under identical conditions, i.e., in the same storage ring, and this created the best possibilities for comparing them.

b) Polarized electron and positron beams in high-energy physics

In many of the largest laboratories in the world, there is now a growing interest in performing experiments with polarized electron and positron beams. As was already noted, the cross sections of two-particle reactions (scattering by a polarized target, the role of which is played by one of the particles in a collision of colliding beams, photon production, annihilation) exhibit a dependence on the polarization of the electrons and positrons. In this connection, inclusive processes accompanied by hadron production have acquired particular interest. Study of annihilation reactions of polarized electrons and positrons has made possible a detailed investigation of electroweak interactions and high-precision measurements for testing various theoretical models, in par-

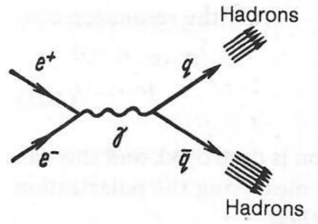

FIG. 5. Hadron production in e^+e^- annihilation.

ticular, the standard Weinberg–Salam gauge theory. These investigations yielded many new interesting results previously inaccessible to experiments with unpolarized particles.

We shall consider briefly some of them, bearing in mind that the reader can find a complete exposition of these questions in Isaev's monograph,[6] which is devoted to modern problems of quantum electrodynamics in the region of high energies.

1. Jet structure of hadron production. In 1975, jet structure in hadron production in e^+e^- annihilation was observed[66–68] using the SPEAR storage ring at Stanford in the United States. It is well known (see Ref. 6) that in the quark–parton model of elementary particles hadrons are produced in e^+e^- annihilation by a sequence in which an e^+e^- pair is transformed into a virtual photon, the photon into a quark–antiquark pair, and each of the quarks then subsequently fragments into an individual hadron jet (Fig. 5).

It is conjectured that the transverse component of the momentum of each hadron relative to the direction of emission of the quark (or antiquark) has a bounded value. The reason for such a restriction is still unclear (see Ref. 6); however, if the transverse momentum of the hadrons is bounded, the distribution of the produced hadrons must correspond to the initial direction of emission of the quarks. Under such conditions, the hadrons will be produced in the e^+e^- annihilation process in the form of oppositely directed jets (Fig. 6)—the hadrons appear to remember the direction in which the quarks were emitted.

In accordance with the considerations presented here, the angular distribution of the jets must have the form

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_0}{d\Omega}\left[1+F\left(\cos^2\theta+P^2\sin^2\theta\cos 2\varphi\right)\right]. \qquad (104)$$

Here, θ is the polar angle of the jet axis relative to the corresponding direction of the incident positron, φ is the azimuthal angle with the plane of the storage ring, and P is the transverse polarization of each electron and positron beam.

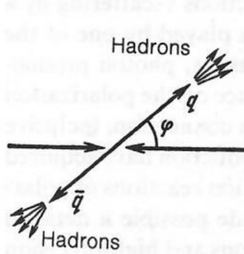

FIG. 6. Jet model of hadron production.

A very important aspect of the jet model is the dependence of the cross section of the produced hadrons on the orientation of the e^+e^- spins. This opened up the possibility of determining the spins of the quarks from the angular distribution of the jets.

In the pioneering experiment study[67,68] made using the SPEAR storage ring at 7.4 GeV, the hadron inclusive distribution exhibited a clear azimuthal asymmetry. This confirmed the quark–parton model and established that the quark spin is more likely to be $\frac{1}{2}$ than zero.

2. Three-jet events. Discovery of the gluon. An increase of the energy of the colliding electron–positron beams to values greater than 10 GeV opens up a possibility for observing the part that gluons play in changing qualitatively the jet behavior (see Ref. 6).

In 1979, three-jet events were observed for the first time at DESY (in the German Federal Republic) using the PETRA storage ring. These provided evidence of the experimental discovery of gluons and a very strong experimental confirmation of the basic ideas of quantum chromodynamics.

Quantum chromodynamics predicts the possibility of three-jet hadron-production events in e^+e^- annihilation through the process $e^+e^-\to q\bar{q}g\to 3$ hadrons, in which the virtual gluon is emitted by a quark (or antiquark) with subsequent fragmentation into hadrons (Fig. 7; see Ref. 6).

Thus, the three-jet events are a direct way of studying the properties of gluons, including the determination of the quark–gluon coupling constant.

In May 1979, five such events were observed in the PETRA storage ring (27 GeV in the center-of-mass system); by now,[69] several thousand have been observed. The observation of three-jet events in hadron production (Fig. 8) provides a method for determining the spin of the gluon[70,71] by measuring the position of the jet axes and finding the azimuthal asymmetry due to the transverse polarization of the e^+e^- in the same way as was done in the case of the two-jet events; moreover, the polarization of the initial particles provides a basis for determining the nature of the produced hadron jets—quark–antiquark or gluon. At the present time, the observation of three-jet events with longitudinally polarized electrons is continuing. Study of the annihilation of longitudinally polarized e^+e^- will make it possible to extend still further the information about these interesting phenomena.

3. Observation of the interference of weak and electromagnetic interactions. The process of muon production in e^+e^- annihilation already attracted interest long ago because of the possibility of observing interference of the weak and electromagnetic interactions. In accordance with the

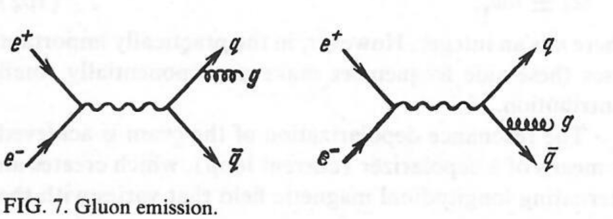

FIG. 7. Gluon emission.

FIG. 8. Three-jet events.

Salam–Weinberg unified gauge model of electroweak interactions, the annihilation of e^+e^- into $\mu^+\mu^-$ muons can proceed either through a virtual photon or through a virtual intermediate Z^0 boson (weak neutral current) (Fig. 9; see Ref. 6). The two processes are indistinguishable, and by virtue of this they interfere with each other. Since the boson vertex contains both an axial and a vector interaction, the production of muons through a Z^0 boson leads to the presence of charge asymmetry in the $\mu^+\mu^-$ angular distribution. The observation of the charge asymmetry of the muon distribution makes it possible to establish the existence of the effect of the neutral currents. It is important in this connection to emphasize that the dependence of these processes on the electron and positron polarization makes it possible to carry out experiments under conditions of a relative enhancement of the asymmetry in the angular distribution of the muons that is due to the weak interaction, the reaction in the electromagnetic channel being suppressed.

Such experiments were made with the PETRA storage ring and 2×19-GeV colliding e^+e^- beams. At energy 15 GeV, the beam polarization reached 60% in 20 min. The observed asymmetry of the muon production confirmed the Salam–Weinberg model. These experiments were one of the great achievements of experimental high-energy physics.[69]

From the point of view of the theory of electroweak interactions, great interest attaches to the prospects for carrying out experiments with polarized particles in electron–proton collisions (HERA at DESY: 30-GeV electrons and 800-GeV protons; CHEER at Fermilab: 10-GeV electrons and 150–800-GeV protons). Under these conditions, the spin effects becomes particularly important; moreover, at such energies photon exchange becomes as probable as Z^0-boson exchange.

To conclude this review, we should emphasize once more that polarized beams of high-energy particles are a very important tool of physics investigations—both now and for the future. The interest in polarized electron and positron beams has been manifested recently in connection with the imminent commissioning of new e^+e^- colliders designed to obtain high particle energies (~ 100 GeV): TRISTAN, SLC, LEP.

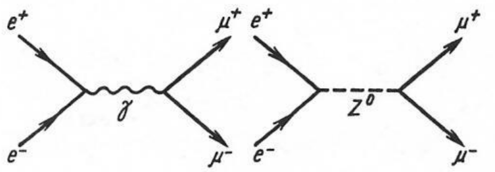

FIG. 9. Diagrams of e^+e^- annihilation into muons.

It is expected that in these colliders it will be possible to observe the theoretically predicted Higgs boson, which plays an important part in the unified theory of the electromagnetic and weak interactions. The use of polarized e^+e^- beams that annihilate into a lepton pair and a Higgs boson will make it possible to separate this reaction from the background of other processes.[72]

Polarized electron and positron beams may have great importance for the experimental confirmation of supersymmetric theories (SUSY), the idea of which arose comparatively recently and has become particularly popular in the last few years. In accordance with the basic SUSY ideas, fermions and bosons appear in the theory symmetrically, and for each known particle there must be a supersymmetric partner. During the last two or three years, much attention has been devoted to searches for the supersymmetric partners. However, despite the experimental efforts the existence of these particles has not been confirmed.

In the analysis of the experimental possibilities of detecting SUSY partners, an important role is played by their polarization properties.[73,74] These properties may be helpful for the detection of supersymmetric partners in extremely delicate experiments using polarized e^+e^- beams. It is expected that the polarization properties of the particles will play an important part in the discovery of new physical laws at the energies of the future e^+e^- colliders.

[a] In 1973, this effect was registered in the USSR state register of discoveries as No. 131 with priority from 1963 (the publication date of Ref. 4).
[b] This calculation was made with the participation of V. A. Bordovitsyn.

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