Tunneling in an expanding universe: Euclidean and Hamiltonian approaches

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The theory of the decay of the false vacuum in de Sitter space and in an inflationary universe, and also the theory of the creation of the universe "from nothing" are discussed. The reasons why tunneling in an inflationary universe differs from tunneling in de Sitter space and cannot be completely homogeneous are explained. It is shown that in a number of important cases the Euclidean approach must be significantly modified or is even completely invalid for the description of tunneling in an expanding universe and for the description of quantum creation of the universe. A Hamiltonian approach for describing tunneling processes with allowance for expansion of the universe is developed. The results of this method are compared with the results obtained by means of the Euclidean approach.

INTRODUCTION

Many studies in recent years have been devoted to tunneling with allowance for expansion of the universe.[1-11] There are two reasons for this interest. First, the theory of tunneling from a supercooled "vacuum" state at the time of the high-temperature phase transitions in the early universe was one of the necessary elements in the inflationary-universe scenario.[12-14] Second, the question was posed of whether our entire universe might not have arisen in a process of tunneling "from nothing" or from some "other universe."[15-23] The first reason is today somewhat less pressing, since it has been understood that the scenario of an inflationary universe can be realized much more simply without any recourse to tunneling theory[24,25] and even without recourse to the theory of high-temperature phase transitions.[26] Nevertheless, the question of tunneling in an expanding universe continues to remain important and interesting not only for the development of the inflationary-universe scenario and the scenario of quantum creation of the universe but also for the general development of our understanding of quantum processes in cosmology.

The problem is extremely complicated, and its analysis requires simultaneous understanding of quantum field theory and the general theory of relativity in different situations, frequently rather nontrivial, in which the standard methods developed earlier do not work. For this reason, about half of the studies on tunneling in an expanding universe contained errors, whereas the correct results obtained in an appreciable fraction of the remaining studies of this subject were found on closer examination to have no direct relationship to the problems studied in these investigations. As a result, the literature now contains not only correct results but a great many assertions relating to tunneling in an expanding universe that are either completely incorrect, or not fully correct, or correct but irrelevant, etc. The present study arose as an attempt to come to grips with this situation and, if possible, develop methods suitable for the problem.

The standard approach to tunneling in quantum field theory is based on the observation that tunneling in the majority of the cases hitherto studied can be represented as motion with an imaginary momentum or in imaginary time. In these cases, the study of tunneling reduces to solving a problem of motion in Euclidean space. Practically all studies on tunneling in an expanding universe (except for the recent work of Starobinskiĭ[27,28]) have in fact been based on the use of this Euclidean approach. But the Euclidean approach is by no means always valid for the description of tunneling. The simplest example is the problem of the motion of a particle in a plane (x,y) in a potential $V(x,y)$ that has the form of a barrier only in the x direction. In this case, a particle that encounters the barrier tunnels in the x direction, but nothing prevents it moving along a classical trajectory in the y direction. To solve this problem, one cannot simply go over to an imaginary *time* (to an imaginary momentum); rather, one must solve the Schrödinger equation properly for the wave function $\Psi(x,y)$ with allowance for the fact that some *components* of the particle's momentum may acquire an imaginary part.[29] An analogous situation arises when tunneling is considered in an expanding universe, in which an increase in the scale factor R of the universe is classically allowed, so that in certain cases the process of rearrangement of the scalar field φ may take place without simultaneous tunneling with respect to both φ and R. In such a situation, the Euclidean approach becomes invalid, and instead one must use the much more complicated approach based on solution of the Schrödinger–Wheeler–DeWitt equation for the wave function $\Psi(a, \varphi)$ of the universe.

One might expect that the corresponding problems disappear when one is studying processes for which classical evolution of the scale factor R is also forbidden, as is the case, for example, in the process of quantum creation of the universe.[15-23] However, in this case too the naive use of the Euclidean approach may lead to incorrect results. This is so because the quantization of the scale factor R, which, in contrast to ordinary physical fields, has negative energy, is not entirely standard.[19-23]

Thus, the Euclidean approach, which has very great heuristic value and is very convenient for solving numerous important and interesting problems,[1,30,31] is not an "absolute weapon" and must be augmented by other methods of investigation. This conclusion is one of the main results of our paper.

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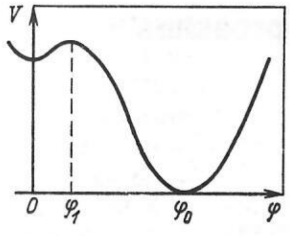

FIG. 1. Form of the effective potential $V(\varphi)$ with a false vacuum $\varphi = 0$.

As we have already said, the initial aim of our investigation was to develop a theory of tunneling in an expanding universe. The most interesting result so far obtained in this field is due to Hawking and Moss.[5] They considered tunneling in a universe with vacuum energy density $V(\varphi)$, where $V(\varphi)$ is the effective potential of the scalar field φ. In the case shown in Fig. 1, the tunneling takes place from the point $\varphi = 0$. According to Ref. 5, tunneling takes place to the point of the extremum of $V(\varphi)$ at $\varphi = \varphi_1$, and the probability of tunneling per unit volume per unit time is proportional to[a)]

$$P \sim \exp\left[\frac{3M_P^4}{8\pi}\left(\frac{1}{V(\varphi_1)} - \frac{1}{V(0)}\right)\right]. \tag{1}$$

Unfortunately, the expression (1) is not derived in Ref. 5, in which it is merely asserted that this result can be obtained by means of the Euclidean approach used by Coleman and De Luccia.[1] In addition, in Ref. 5 there is no discussion at all of the limits of applicability of the expression (1). But, for example, in the theory with the potential $V(\varphi)$ shown in Fig. 2 it would follow in accordance with (1) that the tunneling should take place from the point $\varphi = 0$ to any of the extrema of the potential φ with greater probability than to the nearest extremum $\varphi = \varphi_1$. It is obvious that such a conclusion would be physically incorrect. Therefore, the limits of the validity of the expression (1) requires careful analysis.

Even greater confusion arose in the physical interpretation of the expression (1). Since simultaneous tunneling in the complete universe to the point $\varphi = \varphi_1$ was being considered, the field that results should be strictly homogeneous in the complete universe. This conclusion, which was subsequently "confirmed" in a number of studies,[6,7] was criticized by one of the present authors in Ref. 32, in which it was noted that the probability of homogeneous tunneling in an expanding universe is extremely strongly suppressed. This question is discussed in detail in the present paper. Later, Hawking and Moss noted that the expression (1) must relate to the probability of a tunneling that appears homogeneous on the scale of the horizon in the de Sitter space, i.e., at distances $l \gtrsim H^{-1}$, where $H = \sqrt{8\pi V(\varphi)/3M_p^2}$ is the Hubble constant.[9] Until recently, their remark, like the expression (1) itself, remained unproven. It is only very recently that Starobinskiĭ has succeeded in proving the validity of this assertion for the special case of the potential $V(\varphi)$ shown in Fig. 1 under the condition $|d^2V(0)/d\varphi^2| \ll H^2$.[27,28] In some other cases, the result of Hawking and Moss is incorrect.

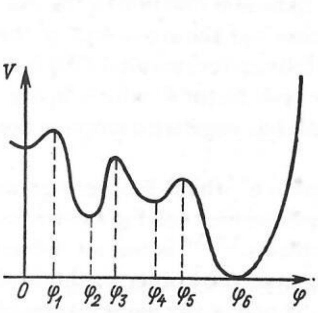

FIG. 2. Potential with several false vacua $\varphi = 0, \varphi_2, \varphi_4$.

The present paper, being an independent investigation, can at the same time be regarded as complementing the review in Ref. 20 of the current status of the inflationary universe scenario.

The theory of tunneling in Minkowski space and in an expanding universe is discussed in the first five sections of our paper. In Sec. 1, we discuss the theory of decay of the false vacuum (tunneling theory in quantum field theory) in ordinary Minkowski space, i.e., without allowance for gravitational effects. The exposition is developed on the basis of the Hamiltonian approach, which in the given simplest case is equivalent to the Euclidean approach.

In Sec. 2, we discuss the equations of motion, the question of stability, and the concept of an effective potential in an expanding universe. In Sec. 3, we discuss a number of attempts to reduce the problem of tunneling in an expanding (inflationary) universe to the solution to the problem of tunneling in a Minkowski world. In Sec. 4, we describe the Euclidean approach to tunneling in a de Sitter space as proposed by Coleman and De Luccia, we analyze (and generalize) Hawking and Moss's results for homogeneous tunneling in de Sitter space, and we discuss the question of the applicability of these results to the theory of an inflationary universe.

In Sec. 5, we develop the Hamiltonian approach to the problem of tunneling in an inflationary universe, discuss the alternative approach to this problem developed by Starobinskiĭ, and compare the results of both approaches with the results of the Euclidean approach. In Sec. 6, we consider the problem of quantum creation of the universe by tunneling from the state with scale factor a = 0. Finally, we summarize the main results of our investigation in the Conclusions.

There are also two appendices. In Appendix A, we recall the main properties of de Sitter space, which are frequently used in the main text of the paper. Appendix B contains a brief review of the inflationary-universe scenario, knowledge of which is desirable for a more complete understanding of the aim of the present paper and the physical formulation of the problem, and also for understanding the subtle differences that arise in the description of tunneling in de Sitter space and in the Friedmann universe in the stage of its exponential expansion.

1. FORMATION OF BUBBLES IN MINKOWSKI SPACE

The phase transition from the metastable state $\varphi = 0$ to the stable state $\varphi \neq 0$ corresponding to the absolute minimum of the effective potential $V(\varphi)$ is realized by the creation and subsequent expansion of $\varphi \neq 0$ field bubbles. Before we describe tunneling from the state $\varphi = 0$ with formation of bubbles of the field φ in curved space, we recall the main facts relating to the formation of bubbles in ordinary Minkowski space.

Let the system be described by the action functional

$$S=\int dt\, d^3x\left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)\right] \quad (2)$$

where $\eta^{\mu\nu}=\mathrm{diag}(-1,\ +1,\ +1,\ +1)$ is the metric of Minkowski space]. The state of the system can be described by a state vector that is a functional $\Phi[\varphi(\mathbf{x})]$ of $\varphi(\mathbf{x})$. In this (the coordinate) representation the canonical variables $\varphi(\mathbf{x})$ (the coordinates) and $\pi(\mathbf{x})$ (their conjugate momenta) correspond after quantization to operators $\varphi(\mathbf{x})$ and $\pi(\mathbf{x})$ (which we shall not identify by the usual "cap" symbol—no confusion will arise), which act on the state vector $\Phi[\varphi(\mathbf{x})]$ in accordance with the formulas

$$\left.\begin{aligned}&\varphi(\mathbf{x})\Phi[\varphi(\mathbf{x})]=\varphi(\mathbf{x})\Phi[\varphi(\mathbf{x})],\\&\pi(\mathbf{x})\Phi[\varphi(\mathbf{x})]=\frac{\hbar}{i}\frac{\delta}{\delta\varphi(\mathbf{x})}\Phi[\varphi(\mathbf{x})].\end{aligned}\right\} \quad (3)$$

The change of the state vector in time is determined by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Phi_t[\varphi(\mathbf{x})]=\mathscr{H}\Phi_t[\varphi(\mathbf{x})]$$
$$=\int d^3x\left[\frac{\pi^2(\mathbf{x})}{2}+\frac{1}{2}\partial_k\varphi(\mathbf{x})\partial_k\varphi(\mathbf{x})+V(\varphi(\mathbf{x}))\right]\Phi_t[\varphi(\mathbf{x})].$$
$$k=1,\ 2,\ 3, \quad (4)$$

with the Hamiltonian operator

$$\mathscr{H}=\int d^3x\left[-\frac{\hbar^2}{2}\frac{\delta^2}{\delta\varphi^2(\mathbf{x})}+\frac{1}{2}(\partial_k\varphi)^2+V(\varphi)\right]. \quad (5)$$

Suppose that the effective potential $V(\varphi)$ has the characteristic form shown in Fig. 1 and that at a certain time $t=0$ the wave function $\Phi_0[\varphi(\mathbf{x})]$ of the system is significantly nonvanishing only when the function $\varphi(\mathbf{x})$ has a small norm $\|\varphi(\mathbf{x})\|_\Omega$. As the norm, one can take, for example, the norm of the Sobolev space $W^1(\Omega)$:

$$\|\varphi\|^2_\Omega=\int_\Omega d^3x\,[\varphi^2(\mathbf{x})+|\nabla\varphi(\mathbf{x})|^2].$$

Intuition developed for the quantum mechanics of a single particle suggests that after a short time the time dependence of the wave function $\Phi_t[\varphi(\mathbf{x})]$ near $\varphi(\mathbf{x})\sim 0$ goes over to the asymptotic behavior

$$\Phi_t[\varphi(\mathbf{x})]=\exp\left(-i\frac{E_0}{\hbar}t-\frac{\Gamma}{2\hbar}t\right)\tilde{\Phi}[\varphi(\mathbf{x})]\equiv U_t\tilde{\Phi}[\varphi(\mathbf{x})], \quad (6)$$

where the evolution operator U_t has, thus, the form of multiplication by a numerical function of the time:

$$U_t=\exp\left[-\frac{i}{\hbar}\left(E_0-i\frac{\Gamma}{2}\right)t\right]. \quad (7)$$

We see that the wave function of the system is damped with the time:

$$|\Phi_t[\varphi(\mathbf{x})]|^2\propto\exp\left(-\frac{\Gamma}{\hbar}t\right). \quad (8)$$

The physical reason for the damping is clear. The state of the system near $\varphi=0$, corresponding to proximity of the system to the local minimum of the effective potential, is unstable by virtue of below-barrier jumping to the region of smaller values of $V(\varphi)$. The quantity Γ, which determines the probability of decay of the false vacuum, is proportional to the probability of below-barrier transition from the configuration $\varphi(\mathbf{x})=0$ to the configuration of a $\varphi(\mathbf{x})\neq 0$ bubble of the new phase. We shall now show how this coefficient of below-barrier transition can be found in the semiclassical (as $\hbar\to 0$) approximation.

Making in the Schrödinger Eq. (4) the usual semiclassical ansatz for the wave function,

$$\Phi=e^{iS/\hbar},$$

we arrive as a result at the Hamilton–Jacobi equation

$$\frac{\partial S}{\partial t}+\int d^3x\left[\frac{1}{2}\left(\frac{\delta S}{\delta\varphi(\mathbf{x})}\right)^2+\frac{1}{2}(\nabla\varphi(\mathbf{x}))^2+V(\varphi(\mathbf{x}))\right]$$
$$\equiv\frac{\partial S}{\partial t}+\mathscr{H}\left[\frac{\delta S}{\delta\varphi(\mathbf{x})},\ \varphi(\mathbf{x})\right]=0. \quad (9)$$

In contrast to classical mechanics, the requirement of reality is not imposed on the action function $S[\varphi(\mathbf{x})]$. The variables t and $\varphi(\mathbf{x})$ separate. Therefore, we seek the solution in the form

$$S[t,\ \varphi(\mathbf{x})]=-Et+\tilde{S}[\varphi(\mathbf{x})]. \quad (10)$$

The functional $\tilde{S}[\varphi(\mathbf{x})]$ satisfies the equation

$$E=\int d^3x\left[\frac{1}{2}\left(\frac{\delta\tilde{S}}{\delta\varphi(\mathbf{x})}\right)^2+\frac{1}{2}(\nabla\varphi(\mathbf{x}))^2+V(\varphi(\mathbf{x}))\right]. \quad (11)$$

If we are interested in below-barrier motion that "begins" at the configuration $\varphi(\mathbf{x})=0$, we obtain the boundary condition

$$\pi(\mathbf{x})\equiv\frac{\delta S}{\delta\varphi(\mathbf{x})}=0\quad\text{for}\quad\varphi(\mathbf{x})=0. \quad (12)$$

We substitute (12) in (11) and see that

$$E=\int d^3xV(0). \quad (13)$$

Therefore, the functional $\tilde{S}[\varphi(\mathbf{x})]$ satisfies the equation equivalent to Eq. (11) with allowance for (13)]

$$\int d^3x\left[\frac{1}{2}\left(\frac{\delta\tilde{S}}{\delta\varphi(\mathbf{x})}\right)^2+\frac{1}{2}(\nabla\varphi)^2+v(\varphi)\right]=0. \quad (14)$$

Because the function $v(\varphi)\equiv V(\varphi)-V(0)$ is positive definite for φ near zero, Eq. (14) will not have real solutions. We therefore set

$$\tilde{S}=-iW. \quad (15)$$

Instead of (14), we obtain

$$\tilde{\mathscr{H}}\left[\frac{\delta W}{\delta\varphi(\mathbf{x})},\ \varphi(\mathbf{x})\right]=0, \quad (16)$$

where

$$\tilde{\mathscr{H}}(\pi(\mathbf{x}),\ \varphi(\mathbf{x})]=\int d^3x\left[-\frac{\pi^2}{2}+\frac{1}{2}(\nabla\varphi)^2+v(\varphi)\right]. \quad (17)$$

It is well known from classical mechanics how one can solve the Cauchy problem for Eq. (16), with the initial condition $W[\varphi(\mathbf{x})]=W_0[\varphi(\mathbf{x})]$ for $\varphi(\mathbf{x})=\varphi^0(\mathbf{x})$:

1. It is necessary to solve the characteristic system of the equations

$$\frac{d\varphi(\mathbf{x})}{d\tau}=\frac{\delta\tilde{\mathscr{H}}}{\delta\pi(\mathbf{x})}, \tag{18}$$

$$\frac{d\pi(\mathbf{x})}{d\tau}=-\frac{\delta\tilde{\mathscr{H}}}{\delta\varphi(\mathbf{x})} \tag{19}$$

with the initial conditions

$$\varphi(\mathbf{x})|_{\tau_0}=\varphi^0(\mathbf{x}),\ \pi(\mathbf{x})|_{\tau_0}=\frac{\delta W_0}{\delta\varphi(\mathbf{x})}. \tag{20}$$

2. On the solution of the system (18)–(19) it is necessary to calculate the functional

$$\int\limits_{\varphi^0(\mathbf{x})}^{\varphi(\mathbf{x})} d\tau L[\varphi(\mathbf{x}),\ \dot{\varphi}(\mathbf{x})], \tag{21}$$

where the "Lagrangian" $L[\varphi(\mathbf{x}), \dot{\varphi}(\mathbf{x})]$ is the Legendre transform with respect to $\pi(\mathbf{x})$ of the "Hamiltonian" $\tilde{\mathscr{H}}$.

3. Then the solution of the Cauchy problem can be written in the form

$$W[\varphi(\mathbf{x})]=W[\varphi^0(\mathbf{x})]+\int\limits_{\varphi^0(\mathbf{x})}^{\varphi(\mathbf{x})} d\tau L[\varphi(\mathbf{x})\,\dot{\varphi}(\mathbf{x})]. \tag{22}$$

We realize steps 1–3 for our problem (16)–(17) as follows:

1. Equations (18) and (19) take the form

$$\frac{d\pi}{d\tau}\equiv\dot{\pi}=\nabla\varphi-v'(\varphi);\quad \frac{d\varphi}{d\tau}\equiv\dot{\varphi}=-\pi. \tag{23}$$

It follows from this that

$$\ddot{\varphi}+\nabla\varphi-v'(\varphi)=0. \tag{24}$$

As initial conditions (20) we take

$$\varphi^0(\mathbf{x})=0;\ \frac{\delta W}{\delta\varphi(\mathbf{x})}\bigg|_{\varphi^0(\mathbf{x})}=0. \tag{25}$$

2. We find the "Lagrangian":

$$\dot{\varphi}=\delta\tilde{\mathscr{H}}/\delta\pi=-\pi.$$

Therefore

$$L=\int d^3x\,[\pi\dot{\varphi}-\tilde{\mathscr{H}}]$$

$$=-\int d^3x\left[\frac{1}{2}\dot{\varphi}^2+\frac{1}{2}(\nabla\varphi)^2+v(\varphi)\right].$$

3. The solution of the Cauchy problem (16)–(17) is

$$W[\varphi(\mathbf{x})]=W[\varphi^0(\mathbf{x})=0]$$

$$-\int\limits_0^{\varphi(\mathbf{x})} d\tau\int\left[\frac{\dot{\varphi}^2}{2}+\frac{1}{2}(\nabla\varphi)^2+v(\varphi)\right]d^3x.$$

Thus, we arrive at the semiclassical expression for the coordinate part of the wave function:

$$\Phi[\varphi(\mathbf{x})]=\Phi[0]$$

$$\exp\left(-\int\limits_0^{\varphi(\mathbf{x})} d\tau\int d^3x\left[\frac{\dot{\varphi}^2}{2}+\frac{1}{2}(\nabla\varphi)^2+v(\varphi)\right]\right), \tag{26}$$

where, we repeat, the integral in the argument of the exponential must be calculated on the solution of Eq. (24). For some values of $(\tau,\mathbf{x})$ for the solution $\varphi(\tau,\mathbf{x})$ of Eq. (24) the equation $\dot{\varphi}=0$ will hold, i.e., $\pi=\delta S/\delta\varphi=0$.

This is the point in the function space at which the classically forbidden motion ends and the classically allowed motion begins. Thus, we find the configuration of the field $\varphi(\mathbf{x})$ that is the beginning of the classical motion (and corresponds to the incipient bubble) if we solve the equation of motion

$$\ddot{\varphi}+\nabla\varphi-v'(\varphi)=0, \tag{27}$$

and find that $\dot{\varphi}(\tau,\mathbf{x})=0$ for some fixed $\tau=\tau_c$. Then $\varphi_c(\mathbf{x})\equiv\varphi(\tau_c,\mathbf{x})$ is the configuration of the classical field that arises. If it is found that the "velocity" $\dot{\varphi}(\tau_c,\mathbf{x})$ is not identically equal to zero with respect to $\mathbf{x}$ for any τ_c, then we do not know how to relate the result of the calculations to the configuration of the developing bubble.

As the solution of Eq. (24), we can take the spherically symmetric instanton solution with center at the point $\tau=\mathbf{x}=0$ described, for example, in Ref. 30. Then by virtue of the spherical symmetry we have $\dot{\varphi}(0,\mathbf{x})=0$. The boundary conditions (25) are also satisfied, since on such a solution $\varphi(\mathbf{x})=0$ and $\dot{\varphi}(\mathbf{x})=0$ at $\tau=-\infty$. From Eq. (26) and the assertions formulated with regard to the solution of Eq. (24), we then find that the coefficient of below-barrier tunneling is

$$\frac{|\Phi[\varphi(\mathbf{x})|_{t=0}]|^2}{|\Phi[\varphi=0]|^2}=\exp\left\{-\frac{1}{\hbar}\int\limits_{-\infty}^{+\infty}dt\int d^3x\left[\frac{\dot{\varphi}^2}{2}\right.\right.$$

$$\left.\left.+\frac{(\nabla\varphi)^2}{2}+v(\varphi)\right]\right\}\equiv\exp\left\{-\frac{S_E}{\hbar}\right\}. \tag{28}$$

As we have already said, Γ is proportional to this coefficient and, of course, to the volume Ω of the system—the larger the volume, the sooner it will contain a region Ω' in which $\|\varphi\|_{\Omega'}$ is significantly nonzero, i.e., tunneling has occurred:

$$\Gamma=\gamma\Omega e^{-\frac{S_E}{\hbar}}.$$

The calculation of the pre-exponential factor γ, which obviously has dimension L^{-4}, is a difficult task and will not be done here. One can also calculate Γ by means of a functional integral,[30] the same result (28) being, of course, then obtained. When the functional integral is used, S_E plays the part of the Euclidean action on the solution of the Euclidean equations of motion—the instanton $\varphi(\tau,x)$. It is here appropriate to note that the transition to an imaginary time[30] is not a description of the dynamics of the field below the barrier. It is only a formal method of calculating Γ with a physical meaning that has been completely determined in advance.

We consider the special case of the effective potential

$$V(\varphi)=V(0)-\frac{\lambda\varphi^4}{4}. \tag{29}$$

In this case, the equation of motion (27) has the well-known solution (called the Fubini instanton)

$$\varphi(\tau,\ \mathbf{x})=\sqrt{\frac{8}{\lambda}}\,\frac{\rho_0}{\rho^2+\rho_0^2}, \tag{30}$$

where $\rho^2=\mathbf{x}^2+\tau^2\equiv\mathbf{x}_\alpha^2$, $\alpha=1,...,4$; $\mathbf{x}=(x_1,x_2,x_3)$, $x_4=\tau$, and ρ_0 is an arbitrary parameter with the dimensions of a length. The Euclidean action (28) does not depend on ρ_0 and is

$$S_E = 8\pi^2/3\lambda.$$

Adding the mass term to the effective potential (29), we obtain

$$V(\varphi) = V(0) + \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4. \qquad (31)$$

It can be shown that the Euclidean equations of motion (27) do not in this case have a nontrivial solution that tends to zero as $x_\alpha^2 \equiv \rho^2 \to \infty$. Indeed, suppose otherwise, i.e., let $\tilde{\varphi}(x)$ be such a solution. This means that the variation

$$\delta S_E = \delta \int d^4x \left[\frac{1}{2}(\partial_\alpha\varphi)^2 + \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4\right] \qquad (32)$$

of the action vanishes on the configuration $\tilde{\varphi}(x)$. We subject the solution $\tilde{\varphi}(x)$ to the scale transformation $\varphi_a(x) = a^{-1}\tilde{\varphi}(x/a)$, and then

$$\begin{aligned} S_E[\varphi_a(x)] &= \int d^4x \left[\frac{1}{2a^2}\left(\frac{\partial}{\partial x_\alpha}\tilde{\varphi}\left(\frac{x}{a}\right)\right)^2 + \frac{m^2}{2a^2}\tilde{\varphi}^2\left(\frac{x}{a}\right) - \frac{\lambda}{4a^4}\tilde{\varphi}^4\right] \\ &= \int d^4x \left[\frac{1}{2}(\partial_\alpha\tilde{\varphi})^2 - \frac{\lambda}{4}\tilde{\varphi}^4\right] + \frac{m^2}{2}a^2 \int d^4x \tilde{\varphi}^2(x). \end{aligned} \qquad (33)$$

Since, by hypothesis, $\dot{\varphi}(x)$ is a solution, we must have

$$dS_E[\varphi_a(x)]/da = 0,$$

but this is possible [for $\dot{\varphi}(x) \neq 0$] only if $m^2 = 0$ or $a = 0$.

The absence of solutions does not mean that tunneling becomes impossible. Indeed, even in a theory in which solutions exist [for example, in the theory with the effective potential (29)] one calculates the functional integral using configurations $\varphi(x)$ that are not solutions, namely, one uses many-instanton configurations. Such configurations are not stationary points of the action. The action $S_E^{(n)}$ of an n-instanton configuration becomes extremal (and equal to $nS_E^{(1)}$) only when the distances between the instantons tend to infinity. Nevertheless, such configurations are taken into account, and one speaks of an "approximate stationary point."

The situation here is entirely analogous. The action has an extremum on the edge of the function space when the configuration has zero width, i.e., $a \to 0$. Therefore, the method of stationary phase has the consequence that the imaginary part of the energy is, as before, determined by an exponential,

$$\Gamma \propto \exp(-8\pi^2/3\lambda\hbar). \qquad (34)$$

For a more detailed discussion of this question, see Refs. 36 and 37.

2. EQUATIONS OF MOTION, THE STABILITY PROBLEM, AND THE EFFECTIVE POTENTIAL IN AN EXPONENTIALLY EXPANDING UNIVERSE

It is well known that to describe the interaction of a scalar field with the gravitational field it is necessary to replace the ordinary derivatives in the Lagrangian of the scalar field by covariant derivatives, and one can also add to the Lagrangian the term $(\xi/2)^{(4)}R\varphi^2$, where ${}^{(4)}R$ is the scalar curvature and ξ is an arbitrary coefficient. There exist two distinguished forms of the theory, namely, the theory with $\xi = 0$, which is called the theory with minimal coupling, and the theory with $\xi = 1/6$, which is distinguished by the fact that when the term $(1/12)^{(4)}R\varphi^2$ is added the inclusion of gravity does not destroy the property of conformal invariance of the theory of the massless scalar field. We shall return later to a discussion of this question. For simplicity, in this section we shall study the theory in a flat Friedmann universe with metric

$$dS^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2), \qquad (35)$$

which can also be represented in the form

$$dS^2 = R^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2), \qquad (36)$$

where R is the scale factor of the universe, and

$$dt/d\eta = R. \qquad 37)$$

We shall be particularly interested in expansion of the universe in the case when the energy–momentum tensor reduces to the vacuum energy–momentum tensor $g_{\mu\nu}V(0)$. In this case, the universe expands exponentially:

$$R(t) = R_0 e^{Ht}, \qquad (38)$$

where H is the Hubble constant,

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{8\pi G}{3}V(0) = \frac{8\pi}{3M_P^2}V(0). \qquad (39)$$

The scalar curvature ${}^{(4)}$R in the universe (35), (38) is

$${}^{(4)}R = 6\left(\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2\right) = 12H^2. \qquad (40)$$

The equation of motion for the scalar field φ has in this case the form

$$\ddot{\varphi} + 3H\dot{\varphi} + R_0^2 e^{-2Ht}\Delta\varphi = -\frac{dV_0}{d\varphi} - 12\xi H^2\varphi, \qquad (41)$$

where V_0 is the effective potential without allowance for the term $(\xi/2)^{(4)}R\varphi^2$; the final term in (41) is equal to $\xi^{(4)}R\varphi$, and for a theory of the type (31), when

$$V_0(\varphi) = V(0) + \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4, \qquad (42)$$

Eq. (41) takes the form

$$\ddot{\varphi} + 3H\dot{\varphi} + R_0^2 e^{-2Ht}\Delta\varphi = -(m^2 + 12\xi H^2)\varphi^2 - \lambda\varphi^3. \qquad (43)$$

It can be seen from this that the term $(\xi/2)^{(4)}R\varphi^2$ leads to a modification of the effective mass of the field φ:

$$m_{\text{eff}}^2 = \frac{d^2V}{d\varphi^2} = \frac{d^2\left(V_0 + \frac{\xi}{2}{}^{(4)}R\varphi^2\right)}{d\varphi^2} = m^2 + \xi^{(4)}R. \qquad (44)$$

It will sometimes be convenient to write Eq. (43) in the conformal coordinates (36) and (37), making also the following transformation of the field:

$$\varphi = \psi/R(\eta). \qquad (45)$$

In this case, Eq. (43) takes the somewhat more familiar form

$$\frac{d^2}{d\eta^2}\psi - \Delta\psi = -R^2(\eta)(m^2+2(6\xi-1)H^2)\psi - \lambda\psi^3. \quad (46)$$

Then in the exponentially expanding universe $R(\eta)$ takes the form

$$R(\eta) = R_0/(1 - R_0 H\eta), \quad (47)$$

so that $R(\eta) = R_0$ for $\eta = 0$ and $R(\eta) \to \infty$ as $\eta \to (R_0 H)^{-1}$. We note that in the language of the variable η the region $0 < t < +\infty$ corresponds to the region

$$0 < \eta < (R_0 H)^{-1}. \quad (48)$$

It can be seen from (46) that the theory appears to be particularly simple in the case $m^2 = 0, \xi = 1/6$, when it appears as the massless $-\lambda\psi^4/4$ theory in Minkowski space. However, this simplicity is insidious and leads to a large number of confusions.

Let us investigate, for example, the question of the stable states of the field in the theory (42). To do this in the Minkowski world, we simply looked for states that are local minima of $V(\varphi)$. Should one proceed in this manner too in an expanding universe? In order to understand the answer to this question, which is important for us, we investigate the theory (42) when $m^2 = -M^2 < 0$ for the case $\xi = 1/6$. In this case, one could conclude from (46) that study of the stability problem in this theory at the point $\varphi = 0$ is equivalent to study of the stability problem in the theory with an effective potential $V(\psi)$ with negative curvature $V''(\psi) = -R^2(\eta)M^2\psi^2$ whose modulus increases continuously with the time, from which it would follow that the point $\psi = 0$ ($\varphi = 0$) is not a stability point of the field φ. However, this conclusion is not entirely correct—generally speaking, instability of the field ψ does not entail instability of the field φ, since in accordance with (45) $\varphi = e^{-Ht}\psi$. Therefore, the state $\varphi = 0$ will be unstable only if the field ψ increases faster than e^{Ht}. It would seem that this will be the case, since the effective mass of the field increases unboundedly with the time: $m_\psi^2 = -M^2R^2(\eta)$. However, it must also be borne in mind that the "time" η itself is bounded above by $(R_0H)^{-1}$, and this prevents the instability of the field ψ from developing sufficiently.

Since we are ultimately interested in the instability of the field φ, we can attempt to investigate this question directly by means of Eq. (43), studying for simplicity homogeneous fields $\varphi = \varphi(t)$. We restrict ourselves to the case of small fields φ, ignore the term $\lambda\varphi^3$ in (43), and seek a solution of Eq. (43) in the standard form

$$\varphi = \varphi_0 e^{iwt}. \quad (49)$$

It is easy to show that such a solution exists and at the same time

$$w = \frac{3iH}{2} \pm i\sqrt{-m^2 + \left(\frac{9}{4} - 12\xi\right)H^2}. \quad (50)$$

It follows from (50) that $\operatorname{Im} w > 0$ when

$$m^2 > -12\xi H^2, \quad (51)$$

i.e., for $m^2 > -12\xi H^2$ the point $\varphi = 0$ is stable with respect to small perturbations. In this sense, to study the stability of the theory it would be sufficient to calculate the sign of m^2_{eff} (44), i.e., to investigate the stability of the effective potential

$$V(\varphi) = V_0(\varphi) + \frac{\xi}{2}\,{}^{(4)}R\varphi^2. \quad (52)$$

However, the growth of the perturbations of the field φ at the instability points of Eq. (43) differs from the growth of the perturbations of the field φ in the theory (52) in Minkowski space. Indeed, let us consider the important case $m_{\text{eff}} \ll H$. In this case, the growing solution has at small φ the form

$$\varphi = \varphi_0 \exp\left(-\frac{m^2_{\text{eff}}}{3H}t\right), \quad (53)$$

i.e., the field φ increases much more slowly than the field in the theory (52) in Minkowski space, which grows as

$$\varphi = \varphi_0 \exp(|m_{\text{eff}}|t). \quad (54)$$

This circumstance is very important for the question of the feasibility of the new inflationary scenario.[32]

The paradox with the stability considered above is the basis for numerous confusions associated with the transition to the metric (36) and the conformally transformed field (45). For example, in Ref. 33 the investigation of the stability of the theory (42) was reduced to investigation of the stability of the conformally transformed theory of the field ψ (45), and this, as we have seen, leads to mistakes. In Ref. 3, in which the same method of investigation was used, it was asserted that the conformal addition $(1/12)^{(4)}R\varphi^2$ to $V(\varphi)$ in no way affects the stability of the theory near $\varphi = 0$, in contradiction to our results (51).

Similar confusion associated with the conformal transformation (45) is encountered in the calculation of the effective potential $V(\varphi)$ and in the study of tunneling. The point is that in some cases it may be more convenient to calculate, not the effective potential $V(\varphi)$, but the effective potential $V(\psi)$, and then in the result obtained to make the substitution $\psi = R(t)\varphi$.[34] But it is not noted that to study the stability of the field φ we require the potential $V(\varphi)$ determined on fields φ varying weakly with the time. However, the effective potential $V(\psi)$ goes over under the substitution $\psi = R\varphi$ not into the effective *potential* $V(\varphi)$ but into the effective *Lagrangian* of the fields φ; the Lagrangian is defined on exponentially decreasing fields φ [since $V(\psi)$ was calculated for $\psi = \text{const}$], and thus has a very remote relation to the problem of stability with respect to growth of the field φ.

In Refs. 3 and 4, the investigation of the formation of bubbles of the field φ was reduced to the analogous problem for the field ψ. However, it was not noted that tunneling takes place mainly to bubbles of the field ψ in which the field after the substitution $\varphi = \psi/R(t)$ begins rapidly to *decrease* and not increase. In this connection, the formation of such bubbles does not lead to growth of the field and to the phase transition investigated in Refs. 3 and 4.

We have dwelt in such detail on the confusions that arise in the study of processes in an expanding universe in order to demonstrate the full complexity of the problem and to point out to anyone who continues to study this question the most dangerous traps into which even more experienced investigators than ourselves sometimes fall.

3. TUNNELING IN AN EXPONENTIALLY EXPANDING FRIEDMANN UNIVERSE

We now consider the question of tunneling in the theory (42) from the state $\varphi = 0$ with the formation of $\varphi \neq 0$ field bubbles in the exponentially expanding universe (35), (36). It is obvious in advance that for $m \gg H$ the description of the tunneling will in no way differ from the description of the tunneling in Minkowski space (see Sec. 1). For this reason, and also because the case $m \ll H$ is the most interesting one from the point of view of the new inflationary scenario,[20] we begin with the study of tunneling for $m = 0$. It is again clear that for $\rho_0 \ll H^{-1}$ (a more precise criterion will be obtained in the following section) the tunneling will be described by the Fubini instantons (30), which, as in flat space when $\rho_0 \ll m^{-1}$, also remain in this case almost exact solutions; see the discussion of this question in Sec. 1. However, this case is not for us the most interesting, since, in accordance with Ref. 20, the field φ then increases to large values in a time $t \lesssim H^{-1}$, which is not sufficient for realization of the new inflationary scenario.[13,14] In addition, for small λ the probability of formation of the corresponding bubbles will in accordance with (34) be suppressed very strongly.

Thus, we are interested in the creation of bubbles measuring $\rho_0 \gg H^{-1}$. Below, we shall discuss several different approaches to the solution of this problem and point out the difficulties in the way of its solution.

1. The first attempt to take into account the effects associated with a nonvanishing curvature ${}^{(4)}R$ in a hot universe was made in Ref. 2, in which it was noted that the effective mass

$$m_{\text{eff}}^2 = m^2 + \xi^{(4)}R$$

for $\xi \neq 0$ depends on ${}^{(4)}R$, and it was suggested that this circumstance should be taken into account by using m_{eff} instead of m in the expression $S_4(\varphi, T) = 19m/\lambda T$ (Refs. 35 and 36) for the action corresponding to tunneling at high temperature. However, this did not take into account the fact that Eq. (43) differs from the corresponding equation in Minkowski space not by the term $\xi\,{}^{(4)}R$ in m_{eff}^2 but also by other terms, so that for the description of bubble formation it is insufficient simply to replace m by m_{eff} in $S_4 = 19m/\lambda\, T$. To describe the tunneling, one could, as usual, attempt to make the substitution $t \to it$ and go over to Euclidean space in the metric (35). However, the metric (36) then becomes complex. It is not impossible that this could represent a danger only for the calculation of quantum corrections, provided the semiclassical approximation gave a sensible result. However, this is not the case, since after the continuation $t \to it$ Eq. (43) does not have real solutions $\varphi(t, \mathbf{x})$. In addition, the very applicability of the standard approach in an expanding universe becomes doubtful on account of the fact that the energy of the particles of the field φ is not conserved in this case, and the derivation of the standard expressions for the tunneling probability cannot be generalized to the considered case.

2. In our view, the situation is somewhat better if one makes the substitution

$$1 - R_0 H\eta = \tau; \quad R_0 H\mathbf{x} = \chi \tag{55}$$

and then makes the analytic continuation $\tau \to i\tau$, after which Eq. (46) takes the form

$$\frac{d^2\psi}{d\tau^2} + \Delta_\chi \psi = \frac{1}{H^2\tau^2}\left(m^2 + 2(6\xi - 1)H^2\right)\psi - \frac{\lambda}{R_0^2 H^2}\psi^3. \tag{56}$$

We consider for simplicity the case $m = 0, \xi = 1/6$ (conformally invariant theory). Then Eq. (56) has the same form as the equation for the tunneling in the theory with $V(\varphi) = -(\lambda/rR_0^2H^2)\psi^4$ (42) in Minkowski space, and the corresponding solution has the form

$$\psi = R_0 H\sqrt{\frac{8}{\lambda}}\frac{\rho_0}{\tau^2 + \chi^2 + \rho_0^2}. \tag{57}$$

It would seem that the problem has been solved. However, the analytic continuation $\tau \to i\tau$ describes the creation of a bubble at the time $\tau = 0$, i.e., as $t \to +\infty$. Thus, by the method considered above it is not possible to obtain a description of bubble creation at a finite time t.

4. TUNNELING IN CURVED SPACE. EUCLIDEAN APPROACH

A Euclidean approach to the description of tunneling in curved space was proposed by Coleman and De Luccia.[1] They proposed, as is done in flat space,[30] to find the analytic continuation of the metric to the Euclidean form (i.e., to the metric with signature $+ + + +$) and to consider the action of the scalar and gravitational fields in a curved Euclidean (or, rather, Riemannian) space:

$$S_E[\varphi, g] = \int d^4x \sqrt{g}\left\{\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi) + \frac{1}{2k}{}^{(4)}R\right\}, \tag{58}$$

where ${}^{(4)}R$ is the scalar curvature of the space with the metric $g_{\mu\nu}$.

Further, to estimate the rate of decay of the false vacuum it was proposed to use e^{-B}, where $B = S_E[\varphi(x)] - S_E[0]$, while $\varphi(x)$ is the scalar-field configuration that minimizes the action (58) to it there corresponds a definite metric $g_{\mu\nu}(x)$].

They then proposed that the functions $\varphi(x)$ and $g_{\mu\nu}(x)$ that minimize the action (58) should be analytically continued to values of the coordinates for which the metric is real and has the signature $- + + +$. Making the analytic continuation in different ways, it is possible to obtain different regions of the physical space.[8]

Since the majority of studies on tunneling in an expanding universe employ the method of Coleman and De Luccia,[1] in this section we shall briefly describe this method and apply it to the study of tunneling in de Sitter space. A brief description of de Sitter space can be found in the Appendix. We shall reproduce the well-known result of Hawking and Moss for homogeneous tunneling in the entire universe. At the end of the section we make some critical comments concerning the interpretation of these results and the possibility of applying the results obtained by the Euclidean method to the theory of tunneling in an expanding universe.

Following Coleman and De Luccia,[1] we assume that the functions $\varphi(x)$ and $g_{\mu\nu}(x)$ that minimize (58) have $O(4)$ symmetry:

$$dS^2 = d\xi^2 + \rho^2(\xi)\,d\Omega^2; \quad \varphi = \varphi(\xi). \tag{59}$$

Then the Lagrange equations that they satisfy have the form

$$\varphi''+3\frac{\rho'}{\rho}\varphi'-\frac{dV(\varphi)}{d\varphi}=0, \tag{60}$$

$$\rho'^2=1+\frac{k\rho^2}{3}\left(\frac{1}{2}\varphi'^2-V(\varphi)\right), \tag{61}$$

where the prime denotes differentiation with respect to ξ. On a solution of the system (60)–(61), the action is[1]

$$S_E=2\pi^2\int d\xi\left[\rho^3\left(\frac{1}{2}\varphi'^2+V\right)+\frac{3}{k}(\rho^2\rho''+\rho\rho'^2-\rho)\right]$$
$$=4\pi^2\int d\xi\left[\rho^3V-\frac{3\rho}{k}\right]. \tag{62}$$

As is shown in Ref. 8, $\rho(\xi)$ always has two zeros if $V(0)\neq 0$. The integral (62) is calculated between the points at which $\rho=0$. One of these points is $\xi=0$; we call the other Ξ.

We shall see later that the argument of the exponential that determines the tunneling probability,

$$B=S_E[\varphi]-S_E[0], \tag{63}$$

has zeroth order in k [and not the first negative order, as might appear from (62)]. To calculate the action (62) to terms of zeroth order in k, we must, as can be seen from (62), know $\rho(\xi)$ to terms of order k and $\varphi(\xi)$ to terms of order unity.

We consider tunneling from the state $\varphi=0$ in the theory with the effective potential

$$V(\varphi)=V(0)+\frac{m^2\varphi^2}{2}-\frac{\lambda\varphi^4}{4}\equiv V(0)+v(\varphi). \tag{64}$$

The system (60)–(61) has the trivial solution

$$\varphi=0,\ \rho=H^{-1}\sin H\xi, \tag{65}$$

where $H^2=kV(0)/3$.

We shall now seek solutions of the system (60)–(61) such that $\varphi(0)\neq 0, \varphi'(0)=0, \varphi(\Xi)=0, \xi\in[0,\Xi]$ [0 and Ξ are the zeros of $\rho(\xi)$].

We introduce the dimensionless variables

$$r=H\rho,\ \Phi=\sqrt{\lambda}\varphi/H,\ \zeta=H\xi,\ \mu=m/H. \tag{66}$$

Then instead of (60)–(61) we obtain the system

$$\ddot\Phi+3\frac{\dot r}{r}\dot\Phi-u'(\Phi)=0, \tag{60'}$$

$$\dot r^2=1-r^2+\varkappa r^2\left(\frac{1}{2}\dot\Phi^2-u(\Phi)\right),$$
$$u(\Phi)\equiv\frac{\mu^2}{2}\Phi^2-\frac{1}{4}\Phi^4, \tag{61'}$$

where $\varkappa=(1/3\lambda)kH^2\ll 1$, and the dot denotes differentiation with respect to ζ.

In accordance with the remark which we made, we assume $\Phi=O(1)$, and $r=r_0+\varkappa r_1$, with $r_0(0)=r_1(0)=0$. It then follows from (60)–(61) that[b)]

$$\dot r_0^2=1-r_0^2,\ \text{i.e.,}\ r_0=\sin\zeta, \tag{67}$$

$$\ddot\Phi+3\frac{\dot r_0}{r_0}\dot\Phi-u'(\Phi)=0,\ \text{i.e.,}\ \ddot\Phi+3\operatorname{ctg}\zeta\dot\Phi-u'(\Phi)=0, \tag{68}$$

$$\dot r_1+\operatorname{tg}\zeta r_1=\frac{\sin^2\zeta}{2\cos\zeta}E(\zeta), \tag{69}$$

where $E(\zeta)=\frac{1}{2}\dot\Phi^2-u(\Phi)$.

In solving Eq. (69), we assume that the dependence $\Phi(\zeta)$ is known from the solution of (58). Equation (69) can be integrated and, with allowance for the boundary condition $r_1(0)=0$, we obtain

$$r_1(\zeta)=\frac{1}{2}|\cos\zeta|\int_0^\zeta dt\,\frac{\sin^2 t}{\cos t\,|\cos t|}E(t). \tag{70}$$

On the solution, the action is

$$S_E=4\pi^2\int_0^\Xi d\xi\left[\rho^3(V(0)+v(\varphi)-\frac{3\rho}{k}\right]$$
$$=\frac{4\pi^2}{\lambda}\int_0^Z d\zeta\left[r^3(\lambda H^{-4}V(0)+u(\Phi))-\frac{r}{\varkappa}\right]$$
$$=\frac{4\pi^2}{\lambda}\int_0^Z d\zeta\left[\frac{r^3-r}{\varkappa}+r^3u(\Phi)\right], \tag{71}$$

where Z is the value of the variable ζ at which $r=0$. Obviously, $Z=\pi+\varkappa\delta$ where $\delta=O(1)$. Therefore

$$S_E\cong\frac{4\pi^2}{\lambda}\left\{\int_0^\pi d\zeta\left[\frac{r^3-r}{\varkappa}+u(\Phi)\right]+\left(\frac{r^3-r}{\varkappa}+r^3u(\Phi)\right)\Big|_{\zeta=\pi}\varkappa\delta\right\}$$
$$=\frac{4\pi^2}{\lambda}\left\{\int_0^\pi d\zeta\,\frac{r_0^3-r_0}{\varkappa}+\int_0^\pi d\zeta r_0^3u(\Phi)+\right.$$
$$\left.+\int_0^\pi d\zeta(3r_0^2-1)r_1+(r_0^3-r_0)|_{\zeta=\pi}\delta+O(\varkappa)\right\}.$$

We use the fact that $S_E[0]=(4\pi^2/\lambda)\int_0^\pi d\zeta[(r_0^3-r_0)/\varkappa]$, and $r_0\neq\sin\zeta$. Then

$$B\equiv S_E[\Phi]-S_E[0]$$
$$=\frac{4\pi^2}{\lambda}\left\{\int_0^\pi d\zeta\sin^3\zeta u(\Phi)+\int_0^\pi d\zeta(3\sin^2\zeta-1)r_1\right\}. \tag{72}$$

We now calculate the second integral in (72):

$$\int_0^\pi d\zeta(3\sin^2\zeta-1)r_1=\frac{1}{2}\int_0^\pi d\xi(3\sin^2\zeta-1)|\cos\zeta|\int_0^\zeta dt\,\frac{\sin^2 t}{\cos t\,|\cos t|}E(t).$$

We reverse the order of integration with respect to ζ and t and divide the region of integration into parts in each of which $\cos t$ has a definite sign. We then find that this integral is equal to

$$\frac{1}{2}\int_0^\pi d\zeta\sin^3\zeta\left(\frac{1}{2}\dot\Phi^2-u(\Phi)\right).$$

Thus, the argument of the exponential is

$$B=\frac{2\pi^2}{\lambda}\int_0^\pi d\zeta\sin^3\zeta\left(\frac{1}{2}\dot\Phi^2+u(\Phi)\right). \tag{73}$$

The result is perfectly natural: We have actually assumed implicitly that the field φ is small ($\frac{1}{4}\lambda\varphi^4\ll V$, i.e.,

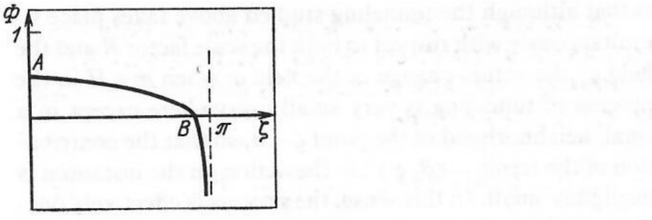

FIG. 3. Even solution of Eq. (68) in the case of the potential $u(\Phi) = -\Phi^4/4$ for $\Phi(0) < 1$.

$\Phi^4 \ll 1/\varkappa$) and that its gradients are too [$\dot{\varphi}^2/2 \ll V(0)$, i.e., $\dot{\Phi}^2 \ll 1/\varkappa$]. Therefore, the metric is determined by the value of the effective potential at $\varphi = 0$, and the evolution of the field φ takes place in the given curved space.

Computer calculations show that for $\mu = 0$ and small $\Phi(0)$ [$\Phi(0) \lesssim 1$] the solutions of Eq. (68) have a characteristic form (Fig. 3). Since $\Phi = 0$ is also a solution of Eq. (68), the function whose graph is shown in Fig. 4 is a solution everywhere except at the salient point $\Phi = 0$. In addition, it satisfies the boundary condition $\Phi(Z) \equiv |\Phi_{r=0} = 0$ and therefore has the usual form of a "bubble" in the sea of the false vacuum $\Phi = 0$.

The numerical calculation shows that the dependence of B (73) on the value of the field Φ at $\zeta = 0$ is well approximated by the formula

$$B = \frac{2\pi^2}{\lambda}\, 0{,}3\Phi^4, \tag{74}$$

where $\Phi_0 \equiv \Phi(0)$.

We have now a situation entirely similar to that described in Sec. 1. There, in the case $V(\varphi) = V(0) + (m^2/2)\varphi^2 - (\lambda/4)\varphi^4$ there were also no solutions of the equations of motion (27), and the action had a stationary point on the boundary of the range of variation of the variables. The quantity B in Eq. (74) is stationary with respect to the parameter Φ_0 at $\Phi_0 = 0$, i.e., when the solution $\Phi(\zeta)$ is trivial, $\Phi(\xi) = 0$. More interesting is the case of nonzero mass, $\mu \neq 0$. For $\Phi(0) \equiv \Phi_0$ only slightly exceeding the extremal point of the potential $u(\Phi)$, i.e.,

$$\Phi_0 \gtrsim \mu,$$

the solution has a step form (Fig. 5). As Φ_0 approaches μ from above, the step is drawn out and becomes flatter, and the normalized action B on this configuration tends to

$$\frac{2\pi^2}{\lambda}\int_0^{\pi} d\zeta \sin^3\zeta\, u(\Phi_0) = \frac{2\pi^2}{3\lambda}\mu^4 = \frac{2\pi^2}{3\lambda}\frac{m^4}{H^4} \tag{75}$$

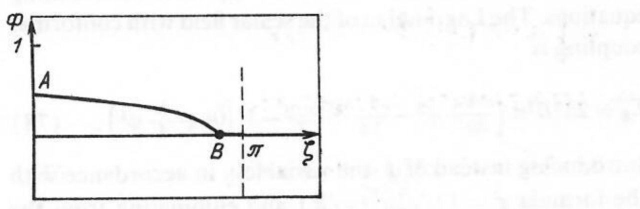

FIG. 4. "Almost-solution" of Eq. (68) with $u(\Phi) - \Phi^4/4$ satisfying the condition $\Phi(\pi) \equiv \Phi_{r=0} = 0$. Curve AB is the same as in Fig. 3.

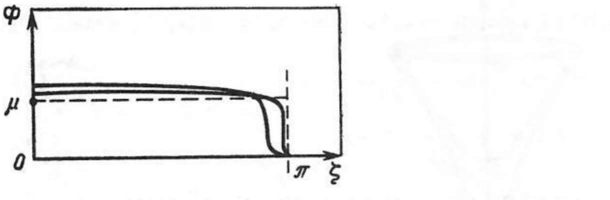

FIG. 5. "Almost-solutions" of Eq. (68) with $u(\Phi) = (u^2/2)\Phi^2 - \frac{1}{4}\Phi^4$ corresponding to two different values of $\Phi(0) > \mu$.

and agrees with the value of the action on the Hawking–Moss instanton.[5] This agreement is not remarkable. The solution (Fig. 5) differs from the Hawking–Moss instanton only at the edge of the interval $[0,\pi]$, where $\sin\zeta \simeq 0$ and the contribution from even the very steep wall of the instanton is suppressed by the smallness of the volume through the factor $\sqrt{g} \cong \sin^3\zeta$.

Thus, we can conclude that if the tunneling process in curved space were described by a functional integral in the Euclidean domain, then in the semiclassical approximation the rate of decay of the false vacuum would be determined by the almost-solutions which we have described and would be proportional to

$$\exp\left[-\frac{8\pi^2}{3\lambda}u(\Phi_0) + O(\varkappa)\right] = \exp\left[-\frac{2\pi^2}{3\lambda}\frac{m^4}{H^4} + O(\varkappa)\right]. \tag{76}$$

In reality, the connection between the action integral on the Riemannian manifold and the manner in which bubbles of the new phase are created in the expanding de Sitter universe is unclear. In the study of tunneling in a flat world (see Sec. 1), the transition to the imaginary time in the functional integral was used, not to describe the field dynamics, but to calculate the decay rate Γ of the wave function, i.e., to calculate a quantity whose physical meaning had already been determined. We are not aware of studies in which a functional integral has been related to an analogous quantity in a curved universe.

Further, the analytic continuation of the Euclidean de Sitter world S^4 could be either the throat of the de Sitter hyperboloid or any inclined section of it passing through its center (Fig. 6). It is in one such section that the inflationary universe should be "materialized" after tunneling. However, the inflationary universe began from a hot singular state, and no section of it coincided, even asymptotically, with the complete section of the de Sitter hyperboloid indicated above (Fig. 6). Therefore, it remains unclear what is the connection between the description of tunneling in the throat of the hyperboloid or in the inclined sections of the hyperboloid by means of the method of Coleman and De Luccia and the description of tunneling in a section very far from the throat of the hyperboloid as must be realized in an inflationary universe.

A further objection to the application of the described procedure to the real universe is the well-known fact relating to tunneling in the quantum mechanics of a single particle. Suppose that a particle in three-dimensional space with momentum **p** encounters a one-dimensional potential barrier. If

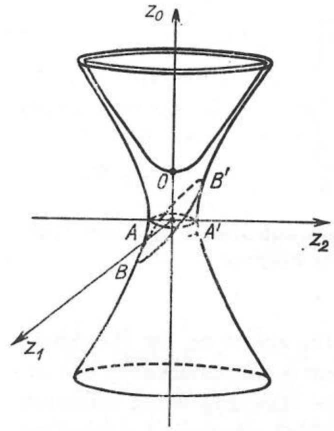

FIG. 6. De Sitter space corresponds to the hyperboloid $z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_0^2 = H^{-2}$ (see Appendix A). In the Coleman–De Luccia formalism, tunneling is realized in the throat AA' of the hyperboloid or in the "inclined throat" BB'. An inflationary universe that was initially hot and began at a singularity corresponds to the surface passing through the point 0 and approaches the hyperboloid asymptotically.

the momentum **p** is perpendicular to the barrier, the motion is effectively one-dimensional, and to find the coefficient of below-barrier transition one can speak of either an imaginary momentum below the barrier or an imaginary time. The two languages are equivalent. But if the particle does not impinge on the barrier at a right angle, only one language is possible, namely, the momentum component $p_\perp$ normal to the barrier becomes imaginary, while the component $\mathbf{p}_\parallel$ parallel to it remains real. Tunneling here takes place with respect to one direction (if you wish, with respect to one variable), and one cannot speak of a particle living in imaginary time.[29] Along the direction parallel to the barrier, classical motion is not forbidden, and it would be meaningless to describe the motion of the particle by the Euclidean action.

The situation is just the same in the case of an inflationary universe. With respect to the variable φ, the system is confined by the potential barrier $V(\varphi)$, and therefore, if we follow only the variable φ, we must expect to see something similar to a quasistationary state concentrated around $\varphi = 0$. The probability of measuring in the system a value of a field φ near zero will slowly decrease. At the same time, with respect to the variables of the gravitational field the system is not confined by any barrier, and this is reflected in the possibility of classical motion—increase of the radius of curvature R in accordance with the Friedmann law. If we look at the case from this point of view, then no quasistationary state with respect to R and slow decay of it through the below-barrier tunneling can in general be possible unless tunneling with respect to the field φ *necessarily* requires simultaneous tunneling with respect to the scale factor R.

Thus, in the general case the application of the Euclidean approach to tunneling in an expanding universe is unjustified and may lead to incorrect results. Nevertheless, Hawking and Moss's result[5] for the case $m \ll H$ is correct, although it applies not to tunneling that is strictly homogeneous in the whole of space but only to tunneling with the formation of bubbles measuring $l \gtrsim H^{-1}$. The reason for this is that although the tunneling studied above takes place simultaneously with respect to both the scale factor R and the field φ, the actual change of the field φ when $m \ll H$ in the process of tunneling is very small everywhere except in a small neighborhood of the point $\xi \sim \pi$, so that the contribution of the terms $\sim (\partial_\mu \varphi)^2$ to the action on the instanton is negligibly small. In this sense, the process is effectively one-dimensional, as in the case when a particle encounters a barrier almost at right angles. For such cases, the Euclidean approach can give the correct result.

There still remains the objection that in the Euclidean approach the tunneling should take place to the throat of the hyperboloid (see above). However, an analogous problem also occurred in the Euclidean theory of tunneling at a nonzero temperature, where the clarification of the question of the shape of the resulting bubble requires an additional investigation but the probability of bubble formation itself is determined uniquely and correctly[36] in the Euclidean approach. As will be seen from the results of the following section, the probability of formation of a bubble of diameter $l \gtrsim H^{-1}$ in de Sitter space is almost independent of the bubble shape. It is for this reason that the result obtained by Hawking and Moss is insensitive to the question of the section of the de Sitter space in which the bubble of the new phase arises.

5. TUNNELING IN AN EXPANDING UNIVERSE. HAMILTONIAN APPROACH

We begin this section by considering the so-called minisuperspace model, in which one considers a spatially homogeneous closed universe filled with a homogeneous scalar field. In this case, the metric can be written in the form

$$ds^2 = -\alpha^2(t)\, dt^2 + R^2(t)\, dl^2,$$

where $dl^2 = d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta d\varphi^2)$ is the line element on the three-dimensional sphere.

The Lagrangian of the gravitational field in the homogeneous case is[38]

$$L_G = \frac{6\pi^2}{k}[-\alpha^{-1}R\dot{R}^2 + \alpha R]. \qquad (77)$$

The Lagrangian (77) is singular, since it does not contain the derivative with respect to the time of $\alpha(t)$, which effectively determines the choice of the scale of the time t. Thus, $\alpha(t)$ is a gauge variable, and the canonical momentum π_α corresponding to it is zero: $\pi_\alpha = \partial L / \partial \dot\alpha = 0$. This result still remains true in the presence of matter, which we take to be a scalar field.

We consider first a scalar field coupled conformally to gravity, since in this case we can exactly solve the resulting equations. The Lagrangian of the scalar field with conformal coupling is

$$L_S = 2\pi^2 R^3 \alpha \left(\frac{1}{2\alpha^2}\dot{\varphi}^2 - \frac{1}{12}\,{}^{(4)}R\varphi^2 - V(0) + \frac{\lambda}{4}\varphi^4\right). \qquad (78)$$

Introducing instead of φ the variable χ in accordance with the formula $\varphi = (1/\sqrt{2\pi^2}(\chi/R)$ and eliminating from the Lagrangian the total derivative with respect to the time, we obtain

$$L_S = \frac{\alpha}{R}\left\{\frac{1}{2}\left(\frac{R}{\alpha}\dot{\chi}\right)^2 - \frac{1}{2}\chi^2 + \frac{\lambda}{8\pi^2}\chi^4 - 2\pi^2 R^4 V(0)\right\}. \tag{78'}$$

Making also the change of variables $\alpha = \sigma N$, $R = \sigma a$, where $\sigma^2 = k/12\pi^2$, we arrive at the total Lagrangian

$$L = L_G + L_S = \frac{N}{2a}\left\{\left(\frac{a}{N}\frac{d\chi}{dt}\right)^2 - \chi^2 + \frac{\nu}{2}\chi^4 - \left(\frac{a}{N}\frac{da}{dt}\right)^2 + a^2 - \Lambda a^4\right\}, \tag{79}$$

where $\Lambda \equiv 4\pi^2\sigma^4 V(0) = (k^2/36\pi^2)/V(0)$, $\nu = \lambda/2\pi^2$.

The canonical momenta are

$$\pi_\chi = \frac{\partial L}{\partial \dot{\chi}} = \frac{a}{N}\dot{\chi},\ \pi_a = \frac{\partial L}{\partial \dot{a}} = -\frac{a}{N}\dot{a},\ \pi_N = 0,$$

and the Hamiltonian is

$$\mathscr{H} = \frac{N}{2a}\left\{\pi_\chi^2 - \pi_a^2 - a^2 + \Lambda a^4 + \chi^2 - \frac{\nu}{2}\chi^4\right\}.$$

On the canonical variables π_a, π_χ, a_χ we must impose the constraint condition that arises from the vanishing of the momentum corresponding to the variable N:

$$0 = \frac{\partial \mathscr{H}}{\partial N} = \frac{1}{2a}\left\{-\pi_a^2 - a^2 + \Lambda a^4 + \pi_\chi^2 + \chi^2 - \frac{\nu}{2}\chi^4\right\}. \tag{80}$$

On quantization, the canonical variables are associated in the usual manner with operators:

$$\chi \to \chi,\quad \pi_\chi \to \frac{1}{i}\frac{\partial}{\partial \chi};$$
$$a \to a,\quad \pi_a \to \frac{1}{i}\frac{\partial}{\partial a},$$

and the constraint (80) becomes a condition imposed on the state vector of the system:

$$\frac{1}{2}\left\{\frac{\partial^2}{\partial a^2} - a^2 + \Lambda a^4 - \frac{\partial^2}{\partial \chi^2} + \chi^2 - \frac{\nu}{2}\chi^4\right\}\Psi(a, \chi) = 0. \tag{81}$$

(In the semiclassical approximation, in which we shall be interested, the order of the noncommuting operators is unimportant, and therefore we shall ignore such questions.)

Equation (81) admits separation of the variables, so that its solution can be represented in the form

$$\Psi(a, \chi) = \int d\varepsilon c_\varepsilon(a)\, u_\varepsilon(\chi), \tag{82}$$

where the functions $c_\epsilon(a)$ and $u_\epsilon(\chi)$ satisfy the equations[18]

$$\frac{1}{2}\left\{-\frac{\partial^2}{\partial a^2} + a^2 - \Lambda a^4\right\} c_\varepsilon(a) = \varepsilon c_\varepsilon(a), \tag{83}$$

$$\frac{1}{2}\left\{-\frac{\partial^2}{\partial \chi^2} + \chi^2 - \frac{\nu}{2}\chi^4\right\} u_\varepsilon(\chi) = \varepsilon u_\varepsilon(\chi). \tag{84}$$

In the problem of the decay of the false vacuum we are interested in the evolution of the wave function Ψ at large a. For $a \gg 1$, Eq. (83) has the fundamental system of solutions

$$c_\varepsilon^{(1,2)}(a) = q_\varepsilon^{-1/4}(a) \exp\{+iS_\varepsilon^{(1,2)}(a, a_0)\}\left[1 + O\left(\frac{1}{\sqrt{\Lambda a^3}}\right)\right], \tag{85}$$

where $q_\epsilon(a) - \Lambda a^4 - a^2 + 2\epsilon$ and $S_\epsilon^{(1,2)}(a, a_0) = \pm\int_{a_0}^a \sqrt{q_\epsilon(a')}\, da'$ is the classical action, which satisfies the Hamilton–Jacobi equation

$$\left(\frac{\partial S_\varepsilon}{\partial a}\right)^2 + a^2 - \Lambda a^4 - 2\varepsilon = 0. \tag{86}$$

The wave function (82) contains, thus, positive- and negative-frequency parts corresponding to the upper and lower signs in front of the radical in the expression for $S_\epsilon^{(1,2)}$ and describing a universe that expands and contracts, respectively, with the passage of time.

Between these two types of universe there is the same connection as between a particle and an antiparticle, the first of which moves forward in time, while the second moves backward. In this sense, one can call the expanding world a universe and the contracting world an antiuniverse.[39] Of course, such a choice is just as nominal as the stipulation that a particle with a certain charge is to be called a "particle," in contrast to an "antiparticle." The theory is symmetric with respect to time reversal, $t \to -t$, under which the universe goes over into the antiuniverse.

We consider the positive-frequency component of the wave function (82) corresponding to the expanding universe, $a > 0$; in the semiclassical approximation the corresponding canonical momentum is

$$\pi_a = \frac{\partial S}{\partial a} = -\frac{a}{N}\dot{a} < 0 \tag{87}$$

and the function $c_\epsilon(a)$ is asymptotically equal, when $a \gg 1$, to

$$c_\varepsilon(a) = \left(\frac{\Lambda a_0^4 - a_0^2 + 2\varepsilon}{\Lambda a^4 - a^2 + 2\varepsilon}\right)^{1/4} \exp\left[-i\int_{a_0}^{a} \sqrt{\Lambda a'^4 - a'^2 + 2\varepsilon}\, da'\right] c_\varepsilon(a_0). \tag{88}$$

One can assume that at $a = a_0$ the initial wave packet is formed from functions $u_\epsilon(\chi)$ with bounded separation parameter ϵ, since functions with a large parameter ϵ evolve extremely rapidly, and a wave packet formed from them leaves the neighborhood of the point $\chi = 0$ in a time negligibly short compared with the time of existence of the quasistationary state in which we are interested. We recall the situation in quantum mechanics: A quasistationary state is characterized by a narrow band of energies whose width decreases with decreasing penetrability of the potential barrier. Therefore, the integral (82) is effectively taken over a finite interval of ϵ, and there exists a large a for which to a given accuracy we can restrict ourselves to the first term in the expansion of (88) in powers of $\epsilon/\Lambda a^4$ for any ϵ in the interval of integration of (82):

$$c_\varepsilon(a) \simeq \left(\frac{\Lambda a_0^4 - a_0^2}{\Lambda a^4 - a^2}\right)^{1/4} \exp\left[-i\int_{a_0}^{a} \sqrt{\Lambda a'^4 - a'^2}\, da' - i\varepsilon\int_{a_0}^{a} (\Lambda a'^4 - a'^2)^{-1/2}\, da'\right] c_\varepsilon(a_0). \tag{89}$$

In addition, we assume that $a \gg \Lambda^{-1/2}$ (i.e., $R \gg H^{-1}$ in the dimensional variables), and we introduce the conformal time $\eta - 1/\sqrt{\Lambda}a$. Under this condition,

$$c_\varepsilon(a) \simeq \frac{a_0}{a}\exp\left[-i\int_{a_0}^{a} \sqrt{\Lambda}\, a'^2\, da' - i\varepsilon(\eta - \eta_0)\right] c_\varepsilon(a_0), \tag{90}$$

and the expansion (82) takes the form

$$\Psi(a,\chi)=\frac{a}{a_0}\exp\left[-\frac{i\sqrt{\Lambda}}{3}(a^3-a_0^3)\right]W(a,\chi), \quad (91)$$

where

$$W(\eta,\chi)=\int d\varepsilon\exp\left[-i\varepsilon(\eta-\eta_0)\right]c_\varepsilon(a_0)u_\varepsilon(\chi), \quad (92)$$

and this function satisfies the equation

$$i\frac{\partial W}{\partial\eta}=\frac{1}{2}\left[-\frac{\partial^2}{\partial\chi^2}+\chi^2-\frac{\nu}{2}\chi^4\right]W, \quad (93)$$

which is identical to the Schrödinger equation for a particle in the potential $V(\chi)=\chi^2/2-(\nu/4)\chi^4$.

It is known from quantum mechanics that if at a certain time η_0 is specified by a wave function $W_0(\chi)$ localized near the point $\chi=0$, then with increasing η the square of the modulus of the wave function $W(\eta,\chi)$ at small χ decreases exponentially,

$$|W(\eta,\chi)|^2\propto\exp\left[-\Gamma(\eta-\eta_0)\right], \quad (94)$$

owing to below-barrier tunneling. The decay constant Γ in (94) is proportional to the square of the coefficient of below-barrier transmission of a particle with energy $\epsilon=0$ [χ_0 is the turning point of the potential $V(\chi)$: $V(\chi_0)=V(0)=0$]:

$$\Gamma\sim\exp\left[-2\sqrt{2}\int_0^{\chi_0}\sqrt{\frac{\chi^2}{2}-\frac{\nu}{4}\chi^4}\,d\chi\right]=\exp\left[-\frac{8\pi^2}{3\lambda}\right]. \quad (95)$$

We note that we have obtained for the decay rate the same expression as in the flat world for the $-\lambda\varphi^4/4$ theory, for which the tunneling can be described, for example, by means of Fubini instantons, on which the action is precisely $8\pi^2/3\lambda$. This leads to the conclusion that in the semiclassical approximation the field with conformal coupling is not sensitive to the curvature of the universe (and in fact, strictly speaking, this was why the conformal coupling was introduced) if the conformal time is used instead of the time.

There is, however, a fundamental difference between the situation which we have considered and the case of decay of the false vacuum in a flat universe. This difference is that the conformal time is bounded above, $\eta<0$. Since, by hypothesis, the time of formation η_0 of the wave packet is small, $|\eta_0|\equiv1/a_0\sqrt{\Lambda}\ll1$, so is the increment of the time: $\eta-\eta_0<1/a_0\sqrt{\Lambda}\ll1$. But it is well known (see, for example, Ref. 29) that the wave function of a metastable state is damped exponentially with the time, in accordance with the expression (94), only when $\eta-\eta_0\gg1$. Thus, in our case there simply is insufficient time for decay of the wave function of a formally metastable state, i.e., tunneling homogeneous in the whole of space is not realized. This conclusion is in disagreement with Hawking and Moss's assertion for the theory under consideration.

We now consider a scalar field coupled minimally to gravity. In this case, the Lagrangian

$$L_S=2\pi^2R^3\alpha\left[\frac{1}{2\alpha^2}\dot{\varphi}^2-V(\varphi)\right] \quad (96)$$

takes after the change of variable $\varphi=(1/\sigma\sqrt{2\pi^2})\Phi$ the form (the remaining notation is the same as in the case of the field with conformal coupling considered above)

$$L_S=\frac{N}{2\alpha}\left[a^4\left(\frac{1}{N}\frac{d\Phi}{dt}\right)^2-a^4\Lambda(\Phi)\right];\quad \Lambda(\Phi)\equiv4\pi^2\sigma^4V. \quad (97)$$

Proceeding as before, we arrive at the Hamiltonian

$$\mathscr{H}=\frac{N}{2a}\left\{\frac{\pi_\Phi^2}{a^2}-\pi_a^2-a^2+\Lambda(\Phi)a^4\right\}. \quad (98)$$

On the transition to the quantum theory, we obtain an equation analogous to (81):

$$\left\{a^2\frac{\partial^2}{\partial a^2}-\frac{\partial^2}{\partial\Phi^2}-a^4+\Lambda(\Phi)a^6\right\}\Psi(a,\Phi)=0. \quad (99)$$

Separating from $\Lambda(\Phi)$ the part that does not depend on the field, $\Lambda(\Phi)=\Lambda+w(\Phi)$ [$w(0)=0$] and considering evolution of Ψ for $\sqrt{\Lambda}a\gg1$, we make, as before, the ansatz (91). The function $W(a,\Phi)$ now satisfies the equation

$$i\sqrt{\Lambda}a^4\frac{\partial W}{\partial a}=\frac{1}{2}\left\{-\frac{\partial^2}{\partial\Phi^2}+w(\Phi)a^6\right\}W. \quad (100)$$

We consider first the case of the simplest potential $V(\varphi)=V(0)+m^2\varphi^2/2$, i.e., $w(\Phi)=\mu^2\Phi^2$, where $\mu^2=\sigma^2m^2$. In the corresponding Schrödinger equation

$$i\sqrt{\Lambda}a^4\frac{\partial W}{\partial a}=\frac{1}{2}\left(-\frac{\partial^2}{\partial\Phi^2}+\mu^2\Phi^2a^6\right)W, \quad (100')$$

it is convenient to make a change of the independent variables Φ: $a\to\xi=a^{3/2}\Phi$, and go over from the function $W(t,\xi)$ to the function $u(t,\xi)$ in accordance with the formula

$$W(t,\xi)=\exp\left[\frac{3}{4}\sqrt{\Lambda}(t-i\xi^2)\right]u(t,\xi). \quad (101)$$

The function $u(t,\xi)$ introduced in this manner satisfies the Schrödinger equation for a harmonic oscillator:

$$i\frac{\partial}{\partial t}u=-\frac{1}{2}\frac{\partial^2}{\partial\xi^2}u+\frac{\omega^2}{2}\xi^2u, \quad (102)$$

where $\omega^2=\mu^2-9\Lambda/4$.

The ground-state wave function of the oscillator, equal to

$$u(t,\xi)=\left(\frac{\omega}{\pi}\right)^{1/4}\exp\left[-i\frac{\omega}{2}t-\frac{\omega}{2}\xi^2\right], \quad (103)$$

leads to the following dependence of the function W on a and Φ:

$$W(a,\Phi)=\left(\frac{\omega}{\pi}\right)^{1/4}a^{3/4}\exp\left[-i\left(\frac{\omega}{2\sqrt{\Lambda}}\ln a+\frac{3}{4}\sqrt{\Lambda}a^3\Phi^2\right)\right]\exp\left[-\frac{\omega}{2}a^3\Phi^2\right]. \quad (104)$$

The square of the modulus of the wave function $W(a,\Phi)$ corresponds to a Gaussian packet that contracts with increasing a:

$$|W(a,\Phi)|^2=\sqrt{\frac{\omega}{\pi}}a^{3/2}\exp(-\omega a^3\Phi^2). \quad (105)$$

The normalization of the wave function (105) is conserved with increasing a: $\int_{-\infty}^{+\infty}d\Phi|W(a,\Phi)|^2=1$. This is as it must be, since we have a Hermitian operator on the right-hand side of Eq. (101).

Note that we can speak of a ground state of the oscillator only when $\omega^2 > 0$, i.e., when $\mu^2 > 9\Lambda/4$. We now consider why the value $\mu^2 = 9\Lambda/4$ is distinguished. For this, we examine the classical equation of motion for the field Φ:

$$\ddot{\Phi} + 3\sqrt{\Lambda}\,\dot{\Phi} + \mu^2\Phi = 0, \tag{106}$$

where the dot denotes differentiation with respect to $t = (1/\sqrt{\Lambda})\ln a$. The fundamental system of solutions of this equation is

$$\Phi_{1,2} = \exp\left[\left(-\frac{3}{2}\sqrt{\Lambda} \pm i\sqrt{\mu^2 - \frac{9}{4}\Lambda}\right)t\right]. \tag{107}$$

Thus, for $\mu^2 > 9\Lambda/4$ the solution has a damped periodic nature, whereas for $\mu^2 < 9\Lambda/4$ there are no oscillations and the motion is aperiodic. We can see why the width of the Gaussian packet (105) decreases with increasing a—in the classical treatment, the amplitude of the oscillations decreases, as follows from Eq. (107). In connection with the decay of the false vacuum, we shall be interested in only the case $\mu^2 > 9\Lambda/4$ This is due to the fact that the decay of the quasistationary state occurs because, using the terminology of single-particle mechanics, the particle periodically strikes the potential barrier and in each collision there is a certain probability of tunneling. When we go over from the potential $V(\varphi) = V(0) + (m^2/2)\varphi^2$ to the potential with barrier $V(\varphi) = V(0) + (m^2/2)\varphi^2 - (\lambda/4)\varphi^4$, decay of a state concentrated near $\varphi = 0$ will occur only if the field "hits" the potential barrier in its oscillations, i.e., only if the regime of damped periodic motion is realized, i.e., for $\mu^2 > 9\Lambda/4$.

For $w(\Phi) = \mu^2\Phi^2 (\mu^2 > 9\Lambda/4)$, we found the ground-state wave function (104). Now suppose $w(\Phi) = \mu^2\Phi^2 - (\nu/2)\Phi^4$, $\nu = \lambda/2\pi^2 \ll 1$. By virtue of below-barrier tunneling, the ground state becomes quasistationary. The decay rate is determined by the wave function beyond the barrier, for $\Phi^2 \gg \mu^2/\nu$, relative to its value at $\Phi = 0$. We consider the situation in the semiclassical approximation, when $W = \exp[iS(a,\Phi)\sqrt{\Lambda}]$, where the function $S(a,\Phi)$ satisfies the Hamilton–Jacobi equation that follows from (100):

$$a^4\frac{\partial S}{\partial a} + \frac{1}{2}\left(\frac{\partial S}{\partial \Phi}\right)^2 + \frac{1}{2}\frac{w(\Phi)}{\Lambda}a^6 = 0, \tag{108}$$

or, in more detail,

$$a^4\frac{\partial S}{\partial a} + \frac{1}{2}\left(\frac{\partial S}{\partial \Phi}\right)^2 + \left(\frac{\tilde{\mu}^2}{2}\Phi^2 - \frac{g}{4}\Phi^4\right)a^6 = 0, \tag{109}$$

where $\tilde{\mu}^2 = \mu^2/\Lambda$, $g = \nu/\Lambda$.

As can be seen from Eq. (104), in the case $g = 0$ the function $S(a,\Phi)$ is

$$S = a^3\Phi^2\left(-\frac{3}{4} + \frac{i}{2}\tilde{\omega}\right), \tag{110}$$

where $\tilde{\omega} = \omega/\sqrt{\Lambda} = \sqrt{\tilde{\mu}^2 - 9/4}$. This suggests seeking a solution of Eq. (109) in the form

$$S(a, \Phi) = a^3\Phi^2\sigma(\eta), \tag{111}$$

where $\eta = \eta(\Phi) = \sqrt{g/2}\Phi$.

The function $\sigma(\eta)$ satisfies the ordinary differential equation

$$(2\sigma + \eta\sigma')^2 + 6\sigma + (\tilde{\mu}^2 - \eta^2) = 0, \tag{112}$$

from which it follows that for $\eta = 0$

$$\sigma \equiv \sigma_0 = -\frac{3}{4} \pm \frac{i}{2}\sqrt{\tilde{\mu}^2 - \frac{9}{4}}. \tag{113}$$

To achieve agreement with (110), it is necessary to choose here the upper sign. For this choice, Eq. (112) is equivalent to the equation

$$\eta\sigma' = -\sqrt{\eta^2 - \tilde{\mu}^2 - 6\sigma} - 2\sigma, \tag{114}$$

in which it is assumed in calculating the square root that the cut in the complex plane is taken along the negative real half-axis.

It is readily seen that at large η Eq. (114) has the solution $\sigma = -\alpha\eta + i\beta/\eta$. The value of the coefficient α does not depend on $\tilde{\mu}^2$ and is determined directly from Eq. (114): $\alpha = 1/3$. To determine the coefficient β, it is necessary to use a numerical calculation, which shows, in particular, that at large μ ($\mu \gtrsim 10$) the dependence of β on μ^3 can be excellently approximated by the straight line

$$\beta = 0{,}7\mu^3. \tag{115}$$

Thus, at large Φ, $\Phi^2 \gg 1/g$, i.e., for $\varphi^2 \gg H^2/\lambda$, the imaginary part of the action function (111) becomes a constant with respect to φ:

$$\operatorname{Im} S_\infty = \frac{2}{g}a^3\beta = \frac{4\pi^2}{\lambda}\Lambda\beta a^3. \tag{116}$$

For $\Phi \gg 1/\sqrt{g}$, the wave function $W(a,\Phi)$ is suppressed with respect to its value at $\Phi = 0$ as

$$\exp(-\sqrt{\Lambda}\operatorname{Im} S_\infty) = \exp\left(-\frac{4\pi^2}{\lambda}\beta\Lambda^{3/2}a^3\right) = \exp\left(-\frac{4\pi^2}{\lambda}\beta(HR)^3\right). \tag{117}$$

We note that the volume Ω of the three-dimensional world is $2\pi^2R^3$ and find that the probability of detecting the field beyond the barrier is suppressed as

$$\exp\left(-\frac{4\beta}{\lambda}H^3\Omega\right), \tag{118}$$

a result that is entirely natural from the physical point of view, since we consider tunneling in the entire volume of the universe. Thus, the probability of homogeneous tunneling can be completely ignored at large R, as was expected earlier on the basis of qualitative considerations.[32]

The significance of the expression (118) is in fact more far-reaching than was assumed in its derivation. The expression (118) can be regarded as the probability of tunneling with formation of a field that is homogeneous in a physical volume $\Omega \gtrsim H^{-3}$ not necessarily equal to the volume of the complete closed universe. At large μ ($\mu \gtrsim 10$), when the expression (115) is valid, we find that the probability of homogeneous tunneling in the volume Ω is proportional to

$$\exp\left(-\frac{2{,}8}{\lambda}m^3\Omega\right).$$

A result of just this structure can be obtained by using the methods of Sec. 1 and considering tunneling in a flat universe with formation of a field homogeneous in the volume Ω. Such agreement is also to be expected in the limit $m \gg H$, when the neglect of gravitational effects appears valid.

The results (117) and (118) obtained above related only to the case of large mass, $m^2 > 9H^2/4$. In the opposite case of small masses, $m^2 \ll H^2$, there is no homogeneous tunneling, as we saw above. But if we are interested in a field $\varphi(\mathbf{x})$ that is homogeneous, not in the complete (closed) universe, but on the scale of physical lengths, say of order H^{-1}, then it is necessary to consider the harmonics of the field $\varphi(\mathbf{k})$ with wave numbers $|\mathbf{k}_{\text{phys}}| \lesssim H$. Important here is the circumstance that in an expanding universe the physical wavelength of each mode increases with increasing scale factor R. By virtue of this, an ever increasing number of modes of the field φ contribute to the field averaged over the physical volume of order H^{-3}. We shall consider this process, following Starobinskiĭ's studies.[27,28]

The basic idea is that when the field $\varphi(\mathbf{x})$ is divided into long-wave, Φ, and short-wave parts the influence of the short-wave part on Φ takes the nature of a random force possessing simple statistical properties.

In quantum field theory, the quadratic fluctuation of the field $\varphi(\mathbf{x})$ is infinite because of the superposition of the zero-point vibrations of an infinite number of modes. However, the mean value $\bar{\varphi}(\mathbf{x})$ in a finite volume of space has a finite quadratic fluctuation. We determine the average over the coordinate volume b^3 by the formula

$$\varphi_b = \frac{1}{(2\pi)^{3/2}}\,\frac{1}{b^3}\int d^3x\, e^{-x^2/2b^2}\varphi(\mathbf{x}),\quad x = |\mathbf{x}|.$$

Using an expansion in the momentum space (we assume that the space is flat),

$$\varphi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[a_{\mathbf{k}}\varphi_{\mathbf{k}}e^{i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}}^{+}\varphi_{\mathbf{k}}^{*}e^{-i\mathbf{k}\mathbf{x}}\right], \tag{119}$$

we obtain for φ_b the result

$$\varphi_b = \int \frac{d^3k}{(2\pi)^{3/2}}\, e^{-\frac{k^2b^2}{2}}\left[a_{\mathbf{k}}\varphi_{\mathbf{k}} + a_{\mathbf{k}}^{+}\varphi_{\mathbf{k}}^{*}\right]. \tag{120}$$

Here, $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{+}$ are ordinary annihilation and creation operators satisfying the commutation relations $[a_{\mathbf{k}}, a_{\mathbf{k}}^{+}] = \delta^3(\mathbf{k}-\mathbf{q})$. As expected, modes with $k \lesssim b^{-1}$ make an effective contribution to the value of the field φ averaged over the volume b^3. For convenience, we replace (120) by the formula

$$\varphi_b = \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(-k+b^{-1})\left[a_{\mathbf{k}}\varphi_{\mathbf{k}} + a_{\mathbf{k}}^{+}\varphi_{\mathbf{k}}^{*}\right]. \tag{121}$$

We shall now consider the evolution of the field φ on the background of the given metric $dS^2 = -dt^2 + R^2(t)(dx^2+dy^2+dz^2)$, the metric of a de Sitter space with Hubble constant $H = \sqrt{(k/3)V(0)}$. Since we are interested in macroscopic effects that hold on the scale of physical lengths of order H^{-1}, the corresponding b, which characterizes the coordinate volume, is

$$b = \frac{1}{\varepsilon R}H^{-1},\quad \varepsilon \ll 1.$$

A more precise restriction on the value of ϵ will be obtained below. Denoting the value of the field φ averaged over this volume by the letter Φ,

$$\Phi = \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(-k+\varepsilon RH)\left[a_{\mathbf{k}}\varphi_{\mathbf{k}} + a_{\mathbf{k}}^{+}\varphi_{\mathbf{k}}^{*}\right], \tag{122}$$

we obtain for the rate of change of Φ

$$\begin{aligned}\dot{\Phi} &= \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(-k+\varepsilon RH)\left[a_{\mathbf{k}}\dot{\varphi}_{\mathbf{k}} + a_{\mathbf{k}}^{+}\dot{\varphi}_{\mathbf{k}}^{*}\right]\\ &+ \varepsilon RH^2\int \frac{d^3k}{(2\pi)^{3/2}}\,\delta(k-\varepsilon RH)\left[a_{\mathbf{k}}\varphi_{\mathbf{k}} + a_{\mathbf{k}}^{+}\varphi_{\mathbf{k}}^{*}\right]\\ &\equiv \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(-k+\varepsilon RH)\left[a_{\mathbf{k}}\dot{\varphi}_{\mathbf{k}} + a_{\mathbf{k}}^{+}\dot{\varphi}_{\mathbf{k}}^{*}\right] + f(t).\end{aligned} \tag{123}$$

The equation of motion of the field $\varphi(\mathbf{x})$ on the background of the de Sitter space $[v(\varphi) \equiv V(\varphi) - V(0) = (m^2/2)\varphi^2 - \lambda\varphi^4/4]$ is

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{R^2}\nabla^2\varphi + m^2\varphi - \lambda\varphi^3 = 0, \tag{124}$$

so that the mode $\varphi_{\mathbf{k}}$ satisfies approximately the equation

$$\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{R^2}\varphi_{\mathbf{k}} + m^2\varphi_{\mathbf{k}} - \lambda\langle\varphi^2\rangle\varphi_{\mathbf{k}} = 0. \tag{124'}$$

For the modes with $k \geqslant \epsilon RH$, the last two terms in Eq. (124') can be ignored if two conditions are satisfied:

1) $m^2 \ll k^2/R^2$, i.e., $m/H \ll \epsilon$;
2) $\lambda\langle\varphi^2\rangle \ll k^2/R^2$.

Condition 2 is always valid when one can speak of a metastable state in the neighborhood of $\varphi = 0$; for the effective square of the mass

$$M^2 = m^2 - \lambda\langle\varphi^2\rangle$$

must in this case be positive, and, taking into account condition 1, we obtain

$$\lambda\langle\varphi^2\rangle < m^2 \ll k^2/R^2.$$

At the same time, in accordance with Ref. 32,

$$\langle\varphi^2\rangle = \frac{3H^4}{8\pi^2m^2}$$

and for λ we obtain the bound

$$\lambda < \frac{8\pi^2}{3}\,\frac{m^4}{H^4} <\!<\!< \frac{8\pi^2}{3}\,\varepsilon^4.$$

Note that our condition 1, $m/H \ll \epsilon \ll 1$, is somewhat more stringent than the condition discussed by Starobinskiĭ: $m^2/H^2 \ll \epsilon \ll 1$.

Thus, when conditions 1 and 2 are satisfied the modes with $k \geqslant \epsilon RH$ satisfy the equation

$$\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{R^2}\varphi_{\mathbf{k}} = 0\quad (R = H^{-1}e^{Ht}), \tag{125}$$

and therefore we can use the results of Ref. 41 and in the expansion (119), (122) set

$$\varphi_{\mathbf{k}} \equiv \varphi_k = \frac{H}{\sqrt{2k}}\left(\eta + \frac{1}{ik}\right)e^{-ik\eta},$$

where $\eta = -1/HR$. But $|k\eta| = k/HR = \epsilon \ll 1$, so that

$$\varphi_k \simeq -i\,\frac{H}{\sqrt{2}\,k^{3/2}}\quad \text{for}\quad k = \varepsilon RH. \tag{126}$$

We substitute in the equation of motion (124) the expansion (120) of the field $\varphi(\mathbf{x})$ in the momentum space:

$$\int \frac{d^3k}{(2\pi)^{3/2}}\left\{a_{\mathbf{k}}\left(\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{R^2}\varphi_{\mathbf{k}}\right)e^{i\mathbf{k}\mathbf{x}} + \text{h.c.}\right\} + m^2\varphi - \lambda\varphi^3 = 0.$$

Dividing the region of integration into two parts, we obtain

$$\int \frac{d^3k}{(2\pi)^{3/2}} \theta(\varepsilon RH - k) \left\{ a_{\mathbf{k}} \left(\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{R^2} \varphi_{\mathbf{k}} \right) e^{i\mathbf{k}\mathbf{x}} + \text{h.c.} \right\}$$

$$+ \int \frac{d^3k}{(2\pi)^{3/2}} \theta(k - \varepsilon RH)$$

$$\left\{ a_{\mathbf{k}} \left(\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \frac{k^2}{R^2} \varphi_{\mathbf{k}} \right) e^{i\mathbf{k}\mathbf{x}} + \text{h.c.} \right\}$$

$$+ m^2\varphi(\mathbf{x}) - \lambda\varphi^3(\mathbf{x}) = 0.$$

The second of the integrals is equal to zero, since for all $k > \epsilon RH$ Eq. (125) holds. In the first integral, the term containing the first derivative is the most important. As a result, we find that

$$\int \frac{d^3k}{(2\pi)^{3/2}} \theta(\varepsilon RH - k) \{a_{\mathbf{k}} \dot{\varphi}_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}} + \text{h.c.}\}$$

$$= -\frac{1}{3H} (m^2\varphi(\mathbf{x}) - \lambda\varphi^3(\mathbf{x})). \tag{127}$$

Averaging this equation over the volume b^3, and taking into account Eq. (123), we find that

$$\dot{\Phi} = -\frac{1}{3H} \frac{\partial V(\Phi)}{\partial \Phi} + f(t), \tag{128}$$

where

$$f(t) = \varepsilon RH^2 \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k - \varepsilon RH) [a_{\mathbf{k}} \varphi_{\mathbf{k}} + a_{\mathbf{k}}^+ \varphi_{\mathbf{k}}^*],$$

and $\varphi_{\mathbf{k}}$, in its turn, is determined by the expression (126):

$$\varphi_{\mathbf{k}} = \varphi_k = -i \frac{H}{\sqrt{2}\, k^{3/2}}, \quad k = \varepsilon RH.$$

Equation (128) is actually a Langevin equation with random force $f(t)$. The statistical properties of $f(t)$ are characterized by the correlation functions $\langle f(t) \rangle$, $\langle f(t_1) f(t_2) \rangle$, $\langle f(t_1) f(t_2) f(t_3) \rangle$, etc. If the force $f(t)$ describes Gaussian random noise, i.e., if

$$\langle f(t_1) \ldots f(t_n) \rangle = 0, \quad n \text{ odd},$$

$$\langle f(t_1) \ldots f(t_n) \rangle = \sum_{(i,j)} \prod \langle f(t_i) f(t_j) \rangle, \quad n \text{ even}.$$

$$\langle f(t_1) f(t_2) \rangle = 2D\delta(t_1 - t_2),$$

then the Langevin equation (128) leads to the ordinary Fokker–Planck equation for the distribution function $\rho(t,\Phi)$:

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \Phi} \left(\frac{1}{3H} \frac{\partial V}{\partial \Phi} \rho \right) + D \frac{\partial^2 \rho}{\partial \Phi^2}. \tag{129}$$

We find the first correlation functions of the force $f(t)$ by averaging over the vacuum, i.e., over the state $|\rangle$ that satisfies the equation $a_{\mathbf{k}} |\rangle = 0$:

$$\langle f(t) \rangle = 0,$$

$$\langle f(t_1) f(t_2) \rangle = \varepsilon^2 H^4 R_1 R_2 \int \frac{d^3k dq}{(2\pi)^3} \delta(k - \varepsilon R_1 H)$$

$$\times \delta(q - \varepsilon R_2 H) \langle a_{\mathbf{k}} a_{\mathbf{q}}^+ \rangle \varphi_{\mathbf{k}}(t_1) \varphi_{\mathbf{q}}^*(t_2)$$

$$= \varepsilon^2 H^4 R_1 R_2 \int \frac{d^3k}{(2\pi)^3} \delta(k - \varepsilon R_1 H) \delta(k - \varepsilon R_2 H) \varphi_k(t_1) \varphi_k^*(t_2)$$

$$= \frac{\varepsilon^2 H^6 R_1 R_2}{(2\pi)^2} \int \frac{dk}{k} \delta(k - \varepsilon R_1 H) \delta(k - \varepsilon R_2 H)$$

$$= \frac{\varepsilon^2 H^6 R_1 R_2}{(2\pi)^2} \frac{1}{\varepsilon R_1 H} \delta(\varepsilon R_1 H - \varepsilon R_2 H)$$

$$= \frac{\varepsilon H^5 R_2}{(2\pi)^2} \frac{1}{\varepsilon R_2 H^2} \delta(t_1 - t_2) = \frac{H^3}{4\pi^2} \delta(t_1 - t_2).$$

It is also readily seen that

$$\langle f(t_1) f(t_2) f(t_3) \rangle = 0,$$

$$\langle f(t_1) f(t_2) f(t_3) f(t_4) \rangle$$

$$= \frac{H^6}{(2\pi)^4} [\delta(t_1 + t_2) \delta(t_3 - t_4)$$

$$+ \delta(t_1 - t_3) \delta(t_2 - t_4) + \delta(t_1 - t_4) \delta(t_2 - t_3)].$$

The force $f(t)$ does indeed have a Gaussian nature for the assumption that has been made concerning the state $|\rangle$.

Thus, the distribution function $\rho(\Phi,t)$ of the values of Φ satisfies the Fokker–Planck equation (129) with $D = H^3/8\pi^2$. For Fokker–Planck processes, the solution to the probelm of the "time of the first jump" is well known.[40] In the case when Φ is localized at the initial time in the neighborhood of a metastable equilibrium point separated from the point of true equilibrium by a high potential barrier (of height ΔV), the mean time over which Φ passes through the barrier is exponentially long[27,28]

$$\Delta t \sim \exp\left[\left(\frac{\Delta V}{3H} \right) \Big/ D \right] = \exp\left[\frac{8\pi^2 \Delta V}{3H^4} \right]. \tag{130}$$

This result is true when the argument of the exponential is large. In the case of the potential that we consider, $v(\varphi) = (m^2/2)\varphi^2 - \lambda\varphi^4/4$, this means that we must have the inequality

$$\frac{8\pi^2}{3\lambda} \frac{m^4}{4H^4} \gg 1,$$

in agreement with requirement 2 considered earlier.

In the considered case $m \ll H$, the expression (130) agrees with the result of Hawking and Moss.[5] We wish to emphasize that this agreement holds only for the case of tunneling that is not completely homogeneous but appears so in a certain bounded region of diameter $l \gtrsim H^{-1}$. As we have already noted, completely homogeneous tunneling is not realized in the inflationary-universe scenario. In addition, the Hawking–Moss result (as can already be seen from the restriction $m \ll H$) is valid for by no means all potentials $V(\varphi)$. The difficulties associated with using the Hawking–Moss result are manifested particularly clearly when one considers tunneling in the potential shown in Fig. 2 (see the Introduction). For the entire Hawking–Moss "derivation" is based on the existence of a constant solution $\varphi = \varphi_1$ of the Euclidean equations (60) corresponding to a maximum of the potential $V(\varphi)$. And therefore according to Hawking and Moss the tunneling takes place from the minimum $\varphi = 0$ of the potential $V(\varphi)$, which is also a solution of Eq. (60), to a maximum of $V(\varphi)$. For the potential shown in Fig. 1, the conclusion is unambiguous. But in the case of the potential shown in Fig. 2, there are as many solutions of the Euclidean equations (60) as the potential $V(\varphi)$ has minima and maxima, and to each of them the Hawking–Moss argument applies. Moreover, at the maximum φ_3 the field appears earlier than at φ_1, and at the maximum φ_5 earlier than at φ_3. But the field should appear even more rapidly at any of the minima, and the time necessary for this should be not exponentially large but, in accordance with the expression (1), exponentially small. But why is there then any tunneling to the maximum with subsequent classical evolution of the field to a minimum? Even stranger is the fact that according to the result of Hawking and Moss the tunneling to any of the local extrema in no way depends on the behavior of $V(\varphi)$ between $\varphi = 0$ and the extremum to which the tunneling takes place. Such a conclusion certainly seems to be incorrect.

In contrast, in the approach proposed by Starobinskiĭ it can be clearly seen that the field φ (122) leaves the neighborhood of the minimum $\varphi = 0$ through "excitation" by short-wave fluctuations and, naturally, initially arrives at the maximum φ_1 closest to the original minimum $\varphi = 0$, after which it falls classically into the minimum φ_2. The process is then repeated, i.e., the field is excited in the neighborhood of the minimum φ_2, reaches the neighboring and lower maximum (φ_3), etc. At the same time, the tunneling probability is, in accordance with Starobinskiĭ's approach, given by the expression (130) and is valid only subject to important restrictions on the form of $V(\varphi)$ [the barrier must be sufficiently low and the potential must be sufficiently flat; see the discussion after Eq. (124′)]. It follows from our results that completely homogeneous tunneling can occur only when even more stringent restrictions on $V(\varphi)$ hold.

Summarizing, we may say that for the study of tunneling in an expanding universe the Euclidean approach can play an important heuristic role, and in some cases it may help us to guess the correct answer. However, by itself this method is not always sufficiently justified, and the results obtained by means of it are sometimes physically invalid. In such cases, it is necessary to use better methods, such as the general Hamiltonian approach or the method developed by Starobinskiĭ.

6. QUANTUM CREATION OF THE UNIVERSE "FROM NOTHING"

The idea of quantum creation of the universe "from nothing," which was put forward in Ref. 15, has been widely discussed in recent years in the literature; see, for example, Refs. 16–23. The theory of the corresponding processes has not yet been fully developed, and even the very notion of creation "from nothing" or "from another universe" requires more detailed elaboration. Nevertheless, some features of the theory of quantum creation of the universe are already beginning to take shape.

Namely, in accordance with the quantum theory of gravity, the quantum fluctuations of the metric and of all physical fields on small scales $\Delta l \lesssim M_p^{-1}$ are extremely large.[42] Suppose that as a result of such fluctuations there arises a region filled with a slowly varying scalar field φ with energy density $V(\varphi)$. If the scale Δl of this region exceeds the scale of the event horizon in the de Sitter space with energy density $V(\varphi)$, $\Delta l \gtrsim H^{-1} = \sqrt{3M_p^2/8\pi V}$, then the interior part of this region will expand exponentially, independently of the events outside the region.[43] Since the characteristic scale at which the fluctuations of the metric are large is $l_P \sim M_p^{-1}$, the quantum fluctuations can lead to the creation of an inflationary universe only if $H^{-1} = \sqrt{3M_p^2/8\pi V} \lesssim M_p^{-1}$, from which there follows the condition

$$V(\varphi) \gtrsim M_P^4. \tag{131}$$

At the same time, the probability of creation of a universe with $V(\varphi) \ll M_P^4$ must be strongly suppressed. On the other hand, it follows from the condition $\Delta l \lesssim M_P^{-1}$ that if a Friedmann universe is created, it must necessarily be closed.[16]

The first attempt at a qualitative description of quantum creation of a universe "from nothing" was made by Vilenkin.[17] However, from our point of view his approach was not entirely justified, and his result for the probability $P \sim \exp[3M_P^4/8V(\varphi)]$ of creation of the universe would, contrary to the generally accepted point of view, imply that the quantum-gravitational effects are stronger, the lower the energy density $V(\varphi)$.

Recently, an interesting approach to the problem of quantum creation of the universe was proposed by Hartle and Hawking.[18] This approach is based on calculation of the ground-state wave function $\Psi_0(a,\varphi)$ of a universe with scale factor a filled with a homogeneous field φ,[38] this wave function having in accordance with Ref. 18 in the semiclassical approximation the form

$$\Psi_0(a, \varphi) \propto \exp[-S_E(a, \varphi)]. \tag{132}$$

Here, $S_E(a,\varphi)$ is the Euclidean action corresponding to the solutions of the equations of motion for $a(\tau)$ and $\varphi(\tau)$ with boundary conditions $a(0) = a$, $\varphi(0) = \varphi$ in a space with Euclidean signature of the metric. The basic idea of the derivation of the relation (132) is as follows. We consider the Green's function of a particle that moves from the point $(0,t')$ to the point $(\mathbf{x},t)$:

$$\langle \mathbf{x}, 0 \mid 0, t' \rangle = \sum_n \Psi_n(\mathbf{x}) \Psi_n^*(0) \exp(iE_n t')$$
$$= \int D\mathbf{x}(t) \exp(iS[\mathbf{x}(t)]), \tag{133}$$

where $\Psi_n(\mathbf{x})$ are the time-independent eigenfunctions of the energy operator with eigenvalues $E_n \geqslant 0$. We now make the rotation $t \to -i\tau$ and go to the limit $\tau' \to -\infty$. In this case, there survives in the sum (133) only the term corresponding to the smallest value of E_n (normalized to zero), i.e.,

$$\Psi_0(\mathbf{x}) \sim \int D\mathbf{x}(\tau) \exp\{-S_E[\mathbf{x}(\tau)]\}. \tag{134}$$

The generalization of this formula in the semiclassical approximation to the case in which we are interested should be the expression (132).[18] For the case of a slowly varying field φ (and it is this case that is of the greatest interest to us from the point of view of realization of the inflationary-universe scenario) the corresponding action is $S_E(a,\varphi) = 3M_P^4/16V(\varphi)$. It should follow from this that the probability of quantum creation of the universe is $P \sim |\Psi_0|^2 \sim \exp[3M_P^4/8V(\varphi)]$, which agrees with Vilenkin's result.[17] Fortunately, in the framework of this treatment it is not difficult to find the reason for the appearance of a physically incorrect result. The point is that the effective action of the scale factor a has the "incorrect" sign, as can be seen by examining (79):

$$S(a) = -\frac{1}{2} \int d\eta \left[\left(\frac{da}{d\eta} \right)^2 - a^2 + \Lambda a^4 \right] \tag{135}$$

(in the notation of Sec. 5), where η is the conformal time, $\eta = \int N dt/a(t)$, and Λ is the dimensionless cosmological constant, with $\Lambda = k^2 V(\varphi)/36\pi^2$ for a slowly varying field φ. It is clear from comparison of (135) and (2) that the "excitations" of the field a near $a=0$ must, in contrast to the particles of the field φ, have negative energy, $E_n \leqslant 0$. The physical reason for this is that the total energy of a closed universe, the creation of which we consider, is always equal to zero, and, therefore, the sign of the gravitational energy is opposite to the sign of the matter energy. But in this case one should, in the formula analogous to (133), make the rotation $t \to i\tau$, and not $t \to -i\tau$, in order to calculate $\Psi_0(a,\varphi)$. This leads to a corrected semiclassical expression for $\Psi_0(a,\varphi)$,[19]

$$\Psi_0(a,\ \varphi) \sim \exp\left[S_E(a,\ \varphi)\right] = \exp\left[-\frac{3M_P^4}{16V(\varphi)}\right]. \quad (136)$$

As we saw above, uncritical application of the Euclidean approach to the question of quantum creation of the universe leads to a physically incorrect result. Since the justification of the Euclidean approach in Ref. 18 (and, therefore, the justification of its corrected variant in Ref. 19) was to a large degree intuitive, we consider the Hamiltonian description of quantum creation of the universe. For simplicity, we restrict the treatment to a scalar field coupled conformally to gravity, although our results are also valid in the more important case of a nonconformal slowly varying field φ, when the contribution of the derivatives of this field to the action can be ignored, so that the entire part played by this field reduces to the appearance of a cosmological constant $\Lambda \propto V(\varphi)$ in Eq. (135).

In the framework of the minisuperspace model, the wave function of the universe in the theory with the conformal scalar field satisfies Eq. (81), which admits separation of the variables; see Eqs. (82) and (84). If the cosmological constant is zero, $\Lambda = 0$, then Eq. (83) is identical to the Schrödinger equation for a harmonic oscillator. It is natural to assume that the physical state of the universe is described by a normalizable wave function,[18] and that the ground state of the universe, which is of the greatest interest, is simply the oscillator ground state determined by Eq. (83). In this case, the probability that the universe has a large radius R is suppressed as $\exp(-a^2) = \exp[-(3\pi/2)(R/l_P^2)]$. Therefore, for example, in a universe with zero cosmological constant, in which we now live, the space at Planck scales is far from homogeneity and evidently consists of a space–time foam, the "bubbles" of which never expand to sizes at which we could observe them.

But if the cosmological constant is $\Lambda > 0$, then a universe that oscillates around $a=0$ could tunnel below the barrier of the "potential energy" $W(a) \equiv a^2/2 - \Lambda a^4/2$ to the value $a = 1/\sqrt{\Lambda}$ and then expand classically, reaching a macroscopic size. The probability of formation of a macroscopic universe from a microscopic bubble of the space–time foam is, in this case, equal to the probability of tunneling and has in accordance with Eq. (83) a value of the order[21–23]

$$\exp\left(-\frac{2}{3\Lambda}\right) = \exp\left(-\frac{8\pi^2}{3kH^2}\right) = \exp\left(-\frac{3M_P^4}{8V}\right), \quad (137)$$

in agreement with the results of the earlier treatment.[19]

The expression (137) can be interpreted as the relative probability of creation of the universe for different vacuum energy densities, so that the ratio of the probabilities of creation of universes with different V is

$$\frac{P_1}{P_2} = \exp\left[-\frac{2}{3\Lambda_1} + \frac{2}{3\Lambda_2}\right] = \exp\left[-\frac{3M_P^4}{8}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)\right].$$

It is important that the creation of a universe with $V(\varphi) \gtrsim M_P^4$ is not exponentially suppressed, in agreement with the results of the qualitative analysis made at the beginning of this section.[c)] This, as was shown in Ref. 19, leads to a natural realization of the random inflation scenario.[26]

CONCLUSIONS

In this paper, we have considered numerous questions related to the theory of classical fields in an expanding universe, to the theory of tunneling in de Sitter space and in an inflationary universe, and to the problem of quantum creation of the universe. These questions are extremely complicated, and by no means always have we been able to achieve a final success in their study. Nevertheless, we hope that we have succeeded in clarifying a number of points for which a large number of contradictory opinions can be found in the literature. Particularly important is the conclusion that we have drawn for ourselves from the investigation, namely, that the Euclidean approach is subject to a certain limitation and that it is necessary to develop new methods in the theory of tunneling and in the quantum theory of gravity.

In recent years, the interest of physicists in the theory of the very earliest stages in the evolution of the universe and in questions of quantum cosmology has grown considerably. The number of publications on this subject is rapidly increasing, and it frequently occurs that investigators who join in this work for the first time repeat the errors of their predecessors instead of learning from them. Our understanding of quantum cosmology and the theory of tunneling in an expanding universe is also rather incomplete and limited. But we still hope that the present paper will be helpful for those who wish to attack seriously the situation that now exists in this interesting branch of science.

APPENDIX A. de SITTER SPACE

After Minkowski space, de Sitter space is the simplest geometrical space corresponding to a ten-parameter symmetry group: $O(4,1)$. From the mathematical point of view, the simplicity of de Sitter space resides precisely in this high degree of symmetry. It is as symmetric as Minkowski space, which also has a ten-parameter symmetry group: the Poincaré group. However, from the physical point of view de Sitter space has very nontrivial properties—an event horizon, singular behavior of the fluctuations of a scalar field in such a space, etc.

De Sitter space is a solution of the Einstein equations with maximally symmetric energy–momentun tensor: $T_{\mu\nu} = g_{\mu\nu}V$, where $V > 0$ is the vacuum energy density. In fact, V has the meaning of the well-known Λ term in Einstein's equations.

The symmetry of de Sitter space makes it possible to represent its metric in the Friedmann–Robertson–Walker form:

$$ds^2 = -dt^2 + R^2(t)\, dl^2, \quad (A.1)$$

where dl^2 is the line element of a three-dimensional space of constant curvature K. Without loss of generality, we can choose $K = +1, 0, -1$ if in (A.1) we replace R^2 by R^2K (for $K>0$) or by $-R^2K$ (for $K<0$).

The dependence of the scale factor R on the time is determined from the Einstein equations with $T_{\mu\nu} = g_{\mu\nu}V$:

$$\left(\frac{\dot{R}}{R}\right)^2+\frac{K}{R^2}=H^2. \quad (A.2)$$

where $H^2 = kV/3$, and $k = 8\pi G$ is the gravitational constant.

In the case of a closed three-dimensional space, for $K = 1$, the solution of Eq. (A.2) is

$$R(t) = H^{-1}\,\mathrm{ch}\,Ht. \quad (A.3)$$

For $K = 0$, i.e., in the case of a flat three-dimensional space,

$$R(t) = H^{-1}e^{Ht}. \quad (A.4)$$

At large times, $t \gg H^{-1}$, the scale factors ("radii") of the closed and flat universes are almost equal. However, it by no means follows from this that the corresponding spaces are nearly the same; for a start, the closed universe has volume $2\pi^2R^3$, while the volume of the flat universe is infinite.

A convenient way of representing the Friedmann space (A.1) is to embed it in a flat five-dimensional space with the metric

$$ds^2 = -dz_0^2+dz_1^2+\ldots+dz_4^2. \quad (A.5)$$

Let $K = 1$. The corresponding three-dimensional space can be represented in the form of a three-sphere $u_1^2 + \ldots + u_4^2 = 1$ embedded in a four-dimensional space with metric $du^2 = du_1^2 + \ldots + du_4^2$. Then, after introduction of the coordinates $z_0 = \int dt\sqrt{1+\dot{R}^2}$, $z_k = Ru_k = 1,\ldots, 4$, the Friedmann space (A.1) can be represented in the form of the surface of revolution $z_1^2 + \ldots + z_4^2 = R^2(z_0)$ in the five-dimensional space with the metric (A.5) in the sense that the line element on this surface is determined by the expression (A.1) [for $dl^2 = d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)$, corresponding to the line element on the three-sphere].

In the special case of de Sitter space, we find, using the expression (A.3), that

$$z_0 = H^{-1}\,\mathrm{sh}\,Ht \text{ and } R^2 = H^{-2} + z_0^2.$$

Therefore, the geometrical image of de Sitter space is the single-sheeted hyperboloid

$$-z_0^2+z_1^2+\ldots+z_4^2=H^{-2}, \quad (A.6)$$

embedded in the flat five-dimensional space (A.5). Omitting two dimensions ($z_2 = z_3 = 0$), we show the de Sitter space in Fig. 7. The three-dimensional world corresponding to the surface $t =$ const will be shown in Fig. 7 in the form of a circle. As the time t increases, the radius R of the three-dimensional world first decreases to the minimal value H^{-1} corresponding to the throat of the hyperboloid and then increases in accordance with (A.3).

The coordinate grid corresponding to the closed three-dimensional world with line element

$$ds^2 = -dt^2 + H^{-2}\,\mathrm{ch}^2\,Ht\,(d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta d\varphi^2)) \quad (A.7)$$

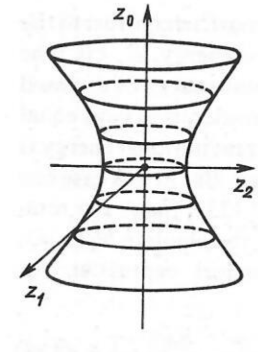

FIG. 7. The de Sitter hyperboloid $z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_0^2 = H^{-2}$ embedded in a flat five-dimensional space $(z_0,\ldots,z_4)$ (in the figure, two dimensions are omitted) with the coordinate grid (A.7) corresponding to a closed space.

covers the complete de Sitter space. At the same time, the coordinate grid corresponding to the flat three-dimensional world in which the line element is written in the form [see (A.4)]

$$ds^2 = -dt^2 + H^{-2}e^{2Ht}\,(dx^2 + dy^2 + dz^2), \quad (A.8)$$

covers for all possible t ($-\infty < t < +\infty$) only the half of de Sitter space bounded by two parallel generators L, i.e., by straight lines lying on the hyperboloid. Such lines appear in the section of the hyperboloid with the plane that touches the hyperboloid at its throat and are the world lines of light emitted from a point in the throat. In the coordinate grid (A.8) of the space, the lines $t =$ const will be represented by the parabolas into which the hyperboloid is cut by planes parallel to the plane determined by the generators L (Fig. 8).

We shall not consider the coordinate grid corresponding to the open Friedmann universe ($K = -1$) or the grid corresponding to the "static de Sitter universe," since we do not use them in the main text of the paper.

We note in conclusion that if the substitution $t = i\tau$ is made in (A.7), the line element becomes

$$ds^2 = d\tau^2 + H^{-2}\cos^2 H\tau\,(d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta d\varphi^2)), \quad (A.9)$$

and this is the metric of a 4-sphere embedded in five-dimensional Euclidean space. The coordinate grid in which the metric (A.9) is expressed covers the complete sphere if $H\tau$ varies in the interval $[-\pi/2, \pi/2]$, and the spaces t = const

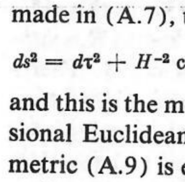

FIG. 8. The de Sitter hyperboloid with parabola PP corresponding to the space $t =$ const in the coordinate grid (A.8) (L and L' are straight-line generators lying in the section of the hyperboloid with the two parallel planes tangent to the throat of the hyperboloid at the diametrically opposite points M).

are (if, as before, two dimensions are dropped for clarity) circles parallel to a fixed equator.

This property of de Sitter space is frequently expressed by a sentence of the type: "On the transition to the Euclidean domain, the de Sitter hyperboloid goes over into the sphere S^4."

APPENDIX B. INFLATIONARY-UNIVERSE SCENARIO

One of the most interesting directions that has arisen in recent years at the frontier of elementary-particle theory and cosmology is the so-called inflationary-universe scenario. This scenario has made it possible to solve numerous problems that existed in the standard theory of a hot universe and has forced cosmologists to look quite differently on the physical processes that take place in the earliest stages in the evolution of the universe and also at the question of the properties of the universe at scales exceeding by many orders of magnitude the scales of the observable part of the universe with radius $l \sim 10^{28}$ cm.

The most general feature of the different forms of the inflationary-universe scenario is the assumption that during the time of its expansion the universe must pass through a stage in which it is in an unstable quasivacuum state with energy–momentum tensor $T_{\mu\nu} \approx g_{\mu\nu} V$. In this stage, the universe expands approximately as a de Sitter space (A.4), i.e., exponentially (or quasiexponentially, the Hubble parameter $H = \dot{R}/R$ decreasing very slowly, $\dot{H} \ll H^2$). Then V is either constant or decreases slowly. The quasi de Sitter stage then terminates (stage of inflation) and the energy of the quasivacuum state goes over into thermal energy. The temperature of the universe after its heating is almost independent of the duration of the inflationary stage. At the same time, all the scales that characterize the inhomogeneities in the universe, the radius R of its three-dimensional curvature, etc., become exponentially large. This makes it possible to explain why the geometry of the three-dimensional space is now nearly Euclidean, why the universe is almost homogeneous and isotropic, why it contains few superheavy monopoles, which must have been created in the earliest stages in the evolution of the universe, etc. As we here have no possibility of discussing this in detail, we merely mention that the inflationary-universe scenario, in its present form, makes it possible to solve more than ten different problems at the frontier of the theory of elementary particles and cosmology, and that at the present time no other possible solution to the greater part of these problems exists.

The idea of expansion of the universe in an unstable vacuumlike state was first advanced by Gliner[44] (see also Ref. 45). An important stage in the development of this idea is associated with Starobinskiĭs model,[48] which, in a somewhat modified form, is now one of the variants of the inflationary-universe scenario. The actual expression "inflationary-universe scenario" was proposed by Guth,[12] who assumed that the energy $V(\varphi)$ of a scalar field must play the part of the vacuum energy and that the inflation must take place in a supercooled unstable vacuumlike state $\varphi = 0$ $[V(0) > 0]$ until a phase transition to the stable state $\varphi = \sigma$ $[V(\sigma) = 0]$. This scenario led to two large inhomogeneities of the density after the phase transition and was replaced by the "new" inflationary scenario,[13,14] according to which the inflation takes place not only to the phase transition but also *after* it in a process of slow "rolling" of the field φ to its equilibrium value σ. In both Guth's scenario and the new scenario, the phase transition is realized by below-barrier creation of bubbles (or spherically asymmetric regions) filled with a nonvanishing field, $\varphi \neq 0$. It was to describe this transition that the theory of tunneling in an expanding (inflationary) universe was required.

At the present time, it appears that the idea of inflation can be most naturally realized in the random-inflation scenario.[26] In accordance with this scenario, the universe passes through a stage of inflation in regions of it that for some reason or other were originally filled with a fairly large and fairly homogeneous nonequilibrium field φ. This mechanism works in a large class of elementary-particle theories, including all theories for which $V(\varphi) \sim \varphi^n$, $n > 0$, when $\varphi \gtrsim M_P$. As a simplest example, one can consider the theory with $V(\varphi) = \lambda\varphi^4/4$. As is shown in Refs. 26 and 20, when $\varphi \gg M_P$ the Einstein equations and the equation of motion for the scalar field have the solution

$$\left.\begin{aligned} \varphi(t) &= \varphi_0 \exp\left(-\frac{\sqrt{\lambda}}{\sqrt{6\pi}} M_P t\right), \\ R(t) &= R_0 \exp\left(\frac{\pi}{M_P^2}\,\varphi_0^2 - \varphi^2(t)\right). \end{aligned}\right\} \tag{B.1}$$

During the time that the field φ rolls down to zero from its initial value $\varphi(0)$, the universe undergoes inflation by $\exp[\pi\varphi^2(0)/M_P^2]$ times. If one assumes that, in order of magnitude, the most probable initial value of the field φ is determined by the relation $V[\varphi(0)] = \lambda\varphi^4(0)/4 \sim M_P^4$ (in this connection, see Refs. 26 and 49), then the degree of inflation of the universe will have the order $\exp(1/\sqrt{\lambda})$. Then $V(\varphi)$ at the time when the inflation ends will have the order λM_P^4, and the subsequent stages in the evolution of the universe will take place independently of the initial value of the field φ and the duration of the inflation.

In itself, the scenario of random inflation (the degree of which in each particular region of the universe depends on the value of the initial field φ randomly distributed in it) is not based on the theory of tunneling in an inflationary universe. Nevertheless, the theory of tunneling in an expanding universe is still an important element in the scenario, since the corresponding phase transitions during the expansion of the universe may take place not only for the scalar field φ responsible for the inflation but also for numerous other scalar fields present in the theory. We merely mention that a quite unexpected effect such as tunneling from the absolute minimum to a local minimum of the potential $V(\Phi)$ during the time of inflation may provide a basis for solving the problem of symmetry breaking in supersymmetric grand unification theories[50] The inhomogeneities that arise when bubbles of the new phase are created and expand exponentially during the time of inflation can play an important part in forming the large-scale structure of the universe observed today.[51] It is also not impossible that quantum creation of the universe occurs through a process similar to tunneling; see

Sec. 6. All this makes the study of tunneling in an expanding universe an important and interesting problem, the solution of which is needed for the further development of the inflationary-universe scenario.

The reader can find a more detailed discussion of the present status of the inflationary-universe scenario in the reviews of Refs. 20, 52, and 53.

[a] Throughout the paper, we use a system of units in which $\hbar = c = 1$, while the Planck mass M_P is equal to $G^{-1/2} = (k/8\pi)^{-1/2}$, where G is the gravitational constant.

[b] *Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the article in the displayed equations.

[c] It should be noted that the degree of suppression of the probability of creation of a universe with $V \lesssim M_P^4$ may be somewhat weakened when allowance is made for the effects associated with the creation of elementary particles.[22]

[1] S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
[2] L. F. Abbott, Nucl. Phys. **B185**, 233 (1981).
[3] Y. Fujii, Phys. Lett. **103B**, 29 (1981).
[4] Y. Fujii, Phys. Lett. **107B**, 51 (1981).
[5] S. W. Hawking and I. G. Moss, Phys. Lett. **110B**, 35 (1982).
[6] E. Mottola and A. Lapedes, Phys. Rev. D **27**, 2285 (1983).
[7] L. F. Abbott and C. J. C. Burges, Phys. Lett. **131B**, 49 (1983).
[8] A. H. Guth and E. Weinberg, Nucl. Phys. **B212**, 321 (1983).
[9] S. W. Hawking and I. G. Moss, Nucl. Phys. **B224**, 180 (1983).
[10] L. G. Jensen and P. J. Steinhardt, Nucl. Phys. **B237**, 176 (1984).
[11] D. Lindley, Nucl. Phys. **B236**, 522 (1984).
[12] A. H. Guth Phys. Rev. D **23**, 347 (1981).
[13] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[14] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[15] P. I. Fomin, Preprint ITF-73-1379 [in Russian], Institute of Theoretical Physics, Kiev (1973); Dokl. Akad. Nauk Ukr. SSR Ser. A **9**, 831 (1975); E. P. Tryon, Nature **246**, 396 (1973); R. Brout, F. Englert, and E. Gunzig, Ann. Phys. (N.Y.) **115**, 78 (1978).
[16] Ya. B. Zel'dovich, Pis'ma Astron. Zh. **7**, 579 (1981) [Sov. Astron. Lett. **7**, 322 (1981)]; L. P. Grishchuk and Ya. B. Zel'dovich, in *Quantum Structure of Space-Time* edited by M. Duff and C. Isham (Cambridge University, Cambridge 1982), p. 409.
[17] A. Vilenkin, Phys. Lett. **117B**, 25 (1982); Phys. Rev. D **27**, 2848 (1983).
[18] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); S. W. Hawking, Nucl. Phys. **B239**, 257 (1984).
[19] A. D. Linde, Zh. Eksp. Teor. Fiz. **87**, 369 (1984) [Sov. Phys. JETP **60**, 211 (1984)]; Lett. Nuovo Cimento **39**, 401 (1984).
[20] A. D. Linde, Usp. Fiz. Nauk **144**, 177 (1984); Expanded version in English: Rep. Prog. Phys. **47**, 925 (1984).
[21] Ya. B. Zel'dovich and A. A. Starobinskiĭ, Pis'ma Astron. Zh. **10**, 323 (1984) [Sov. Astron. Lett. **10**, 135 (1984)].
[22] V. A. Rubakov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 89 (1984) [JETP Lett. **39**, 107 (1984)].
[23] A. Vilenkin, Phys. Rev. D **30**, 509 (1984).
[24] A. D. Linde, Phys. Lett. **114B**, 431 (1982); *ibid.* **116B**, 225, 340 (1982).
[25] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[26] A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 149 (1983) [JETP Lett. **38**, 176 (1983)]; Phys. Lett. **129B**, 177 (1983).
[27] A. A. Starobinskiĭ, Fundamental'nye vzaimodeĭstviya (Fundamental Interactions), Moscow State Pedagogical Institute, Moscow (1984), pp. 55–79.
[28] A A. Starobinskiĭ, in Tezisy dokladov Vsesoyuznoĭ konferentsii po sovremennym problemam teorii otnositel'nosti i gravitatsii (Abstracts of Papers at the All-Union Conference on Modern Problems of the Theory of Relativity and Gravitation), Moscow (1984), p. 39.
[29] A. I. Baz', Ya. B. Zel'dovich, and A. M. Perelomov, Rasseyanie, reaktsii i raspady v nerelyativistskoĭ kvantovoĭ mekhanike, Nauka, Moscow (1971); English translation: Scattering, Reactions and Decay in Nonrelativistic Quantum Mechanics, Jerusalem (1969).
[30] S. Coleman, Phys. Rev. D **15**, 2929 (1977); C. Collen and S. Coleman, Phys. Rev. D **16**, 1762 (1977).
[31] S. W. Hawking, Phys. Rev. D **18**, 1747 (1978).
[32] A. D. Linde, in *The Very Early Universe* edited by G. W. Gibbons, S. W. Hawking, and S. T. Siklos (Cambridge University, Cambridge 1983), p. 205.
[33] A. A. Grib, S. G. Mamaev, and V. M. Mostepanenko, Kvantovye éffekty v intensivnykh vneshnikh polyakh (Quantum Effects in Strong External Fields), Atomizdat, Moscow (1980).
[34] G. Denardo and E. Spallucci, Nuovo Cimento **64A**, 15 (1981).
[35] E. Witten, Nucl. Phys. **B177**, 477 (1981).
[36] A. D. Linde, Nucl. Phys. **B216**, 421 (1983).
[37] J. Affleck, Nucl. Phys. **B191**, 429 (1981).
[38] B. DeWitt, Phys. Rev. **160**, 1113 (1967).
[39] C. Teitelboim, Phys. Rev. D **25**, 3159 (1982).
[40] H. A. Kramers, Physica **7**, 284 (1940).
[41] T. S. Bunch and P. C. W. Davies, Proc. R. Soc. London Ser. A **360**, 117 (1978).
[42] J. A. Wheeler, in *Relativity, Groups and Topology* edited by B. DeWitt and C. DeWitt (Gordon and Breach, New York 1964); S. W. Hawking, Nucl. Phys. **B144**, 349 (1978).
[43] G. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[44] É. B. Gliner, Zh. Eksp. Teor. Fiz. **49**, 542 (1965) [Sov. Phys. JETP **22**, 378 (1966)]; Dokl. Akad. Nauk SSSR **192**, 771 (1970) Sov. Phys. Dokl. **15**, 559 (1970)].
[45] B. L. Al'tshuler, in: Tezisy 3-ĭ sovetskoĭ gravitatsionnoĭ konferentsii (Abstracts of the Third Soviet Gravitational Conference), Erevan (1972).
[46] É. B. Gliner and I. G. Dymnikova, Pis'ma Astron. Zh. **1**, 7 (1975).
[47] L. E. Gurevich, Astrophys. Space Sci. **38**, 67 (1975).
[48] A. A. Starobinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 719 (1979) [JETP Lett. **30**, 682 (1979)]; Phys. Lett. **91B**, 99 (1980).
[49] A. D. Linde, Phys. Lett. **162B**, 281 (1985).
[50] A. D. Linde, Phys. Lett. **131B**, 330 (1983).
[51] L. A. Kofman and A. D. Linde, Inst. Astrophys. Atmosph. Physics Preprint (1985).
[52] R. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985).
[53] A. D. Linde, Comments Astrophys. **10**, 229 (1985); A. D. Linde, Proc. of the Symposium "Meson 50," Kyoto (1985).

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