Anisotropy of neutron elastic scattering and properties of nuclei

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Studies of the angular dependence of the scattering of neutrons with energies mainly below 0.5 MeV are reviewed. The first half of the review covers studies in which measurements were made of the resonance curves of the yield of neutrons scattered in a narrow interval of angles. Analysis of the shapes and areas of the individual resonances as functions of the scattering angle made possible: 1) reliable determination of the orbital angular momenta and the spins of about 700 resonances of seven nuclei; 2) the determination of the contributions of the spin channels in a number of p resonances of ^{19}F and ^{89}Y. The second half of the review summarizes the results of studies of the averaged differential cross sections of elastic scattering. Given here are: 1) the results of a search for the effect of one-pion exchange in the scattering of p neutrons; 2) the first systematic determination of the p-scattering radii; 3) the results of separate determination of the neutron strength functions for $p_{1/2}$ and $p_{3/2}$ neutrons, and the first observation of spin–orbit splitting of an unbound single-particle state.

INTRODUCTION

The elastic scattering of neutrons with energies above 0.5 MeV as a function of the scattering angle has already been investigated for more than 40 years. The reasonable set of neutron energies, scattering angles, and target nuclei is so great that such measurements are still being made today in many laboratories in the world despite the fact that the accumulated material is already very great. Having served as a good experimental basis of the optical model of the nucleus in the past, this material still serves today for its refinement, and it also finds applications in various applied nuclear problems. However, in this review we do not dwell on the traditional studies of neutron scattering with their reflection in the optical model, restricting ourselves merely to references to one of the first[1] and one of the most recent[2] studies in this direction, to a more complete compilation of the experimental data on the differential scattering cross sections,[3] and to a very recent review of studies on the neutron optical potential.[4]

The aim of this review is to summarize results of studies with neutrons of lower energies (mainly less than 0.5 MeV) used to determine nuclear properties different from the parameters of the neutron optical potential, namely, the parameters of neutron resonances, strength functions, phase shifts, or potential-scattering radii.[1)] The data on the scattering of low-energy neutrons are as yet few for the following reasons. First, with decreasing energy the intensity of the employed simple neutron sources decreases and they are less monoenergetic; one can therefore work only with powerful sources possessing neutron moderators. Second, at energies below 1–2 MeV optical analysis of the data is made difficult by the need to take into account scattering through a compound nucleus, and this is neither simple nor unambiguous. Third, with decreasing energy the angular dependence of the differential cross section becomes simpler, and, on the one hand, it carries little information for the optical model, while, on the other, it can for practical needs be obtained by extrapolation from higher energies.

But precisely this last circumstance—the simplicity of the structure of the cross section—permits the extraction from it of varied information. At energies below several hundred kilo-electron-volts, it is effectively only orbital waves with $l>1$ that participate in the scattering, and the differential cross section is given by the first three terms of the series

$$\sigma(E,\vartheta)=\frac{\sigma_s(E)}{4\pi}\sum_{i=0}^{\infty}\omega_i(E)P_i(\cos\vartheta), \tag{1}$$

where E and ϑ are the energy and scattering angle of the neutron, σ_s is the integrated scattering cross section, P_i are Legendre polynomials, and ω_i are dimensionless parameters of the anisotropy, with $\omega_0=1$ at all energies. In the isotropic first term of (1) the main contribution is made by s-wave scattering; the third term is entirely determined by p-wave scattering; and the second, by the interference of the s and p waves. Thus, knowledge of the angular distribution of the scattered neutrons makes possible an unambiguous separation of the contributions of the s and p neutrons and, as a consequence of this, one can obtain quite a lot of new information about the nucleus.

Using the recommendations of Ref. 5, we write down the detailed formula for $\sigma(E,\vartheta)$; this is the theoretical foundation of all the studies reviewed below:

$$\sigma(E,\vartheta)=\sigma_{\rm pot}+\sum(\sigma_{sr}+\sigma_{si})+\sum(\sigma_{pr}+\sigma_{pi}); \tag{2}$$

$$\sigma_{\rm pot}=\frac{1}{k^2}[\sin^2\delta_0+3\sin^2\delta_1+6\sin\delta_0\sin\delta_1 \times\cos(\delta_0-\delta_1)\cos\vartheta+6\sin^2\delta_1P_2(\cos\vartheta)]; \tag{3}$$

$$\sigma_{sr}=\frac{1}{4k_0^2}\frac{g\Gamma_n^2}{\Delta E^2+\Gamma^2/4}; \tag{4}$$

$$\sigma_{si}=-\frac{1}{kk_0}\frac{g\Gamma_n}{\Delta E^2+\Gamma^2/4}\Big\{\sin\delta_0\Big(\Delta E\cos\delta_0+\frac{\Gamma}{2}\sin\delta_0\Big) +3\sin\delta_1\Big[\Delta E\cos(2\delta_0-\delta_1)+\frac{\Gamma}{2}\sin(2\delta_0-\delta_1)\Big]\cos\vartheta\Big\};$$

$$\sigma_{pr}=\frac{1}{4k_0^2}\Big(\frac{E}{E_0}\Big)^2\frac{g}{\Delta E^2+\Gamma^2/4} \tag{5}$$

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$$\times \begin{cases} \Gamma_n^2 [1+(2J+1)\rho^2 P_2(\cos\Omega)] & \text{for } J=I\pm 3/2, \\ [\Gamma_n^2+(2J+1)(\rho_-\Gamma_{n-}-\rho_+\Gamma_{n+})^2 P_2(\cos\vartheta)] & \text{for } J=I\pm 1/2; \end{cases} \quad (6)$$

$$\sigma_{pl}=-\frac{1}{kk_0}\frac{E}{E_0}\frac{g\Gamma_n}{\Delta E^2+\Gamma^2/4}\left\{\sin\delta_1\left(\Delta E\cos\delta_1+\frac{\Gamma}{2}\sin\delta_1\right)\left[1+2P_2(\cos\vartheta)\right]+\sin\delta_0\left[\Delta E\cos(2\delta_1-\delta_0)+\frac{\Gamma}{2}\sin(2\delta_1-\delta_0)\right]\cos\vartheta\right\}, \quad (7)$$

where $\Delta E = E - E_0$; $g = 2J+1/2(2I+1)$, and

$$\rho(J,s) = \begin{cases} 0 & \text{for } J<1; \\ \frac{2}{\sqrt{5}}\frac{\sqrt{(2J-1)2J(2J+1)(2J+2)(2J+3)}}{(J+s)(J+s+1)(J+s+2)(1-s+J)!(1+s-J)!} & \text{for } J\geqslant 1. \end{cases} \quad (8)$$

The remaining notation is standard: E and k are the energy and wave number of the neutron (the subscript 0 is appended to the resonance values), δ_l are the phase shifts of the potential scattering, Γ and Γ_n are the total and the neutron widths of the resonance, I and J are the spins of the target nucleus and the resonance, s is the spin of the channel, taking the two possible values $s_- = I - 1/2$ and $s_+ = I + 1/2$, and $\Gamma_{n\pm}, \rho_\pm$ are the values that relate to the corresponding $s_\pm$. For the p resonance with $J = I \pm 3/2$, only one of the values of s is realized, and for it $\Gamma_n = \Gamma_{n-}$ or Γ_{n+}; but if $J = I \pm 1/2$, then both values of s are realized, and for such a resonance $\Gamma_n = \Gamma_{n+} + \Gamma_{n-}$. In the expressions (4)–(7), Γ_n are the constants relating to $E = E_0$, while in the total widths Γ it is necessary to include the same Γ_n but multiplied by $(E/E_0)^{1/2}$ and $(E/E_0)^{3/2}$ for s and p resonances, respectively. When allowance is made for the contribution of a p resonance very far from E_0, its Γ_n must everywhere be multiplied also by the factor $(1+(k_0R)^2/(1+(kR)^2)$ (R is the radius of the nucleus).

We note that the foregoing formulas refer to the center-of-mass system (c.m.s.) and describe the cross section well only in the case of well-isolated resonances, since the formulas do not take into account inter-resonance interference; the sums in (2) give only the incoherent contributions of the resonances, the limits of the summation over which must be determined on the basis of the accuracy required.

Sections 1 and 2 of the review cover studies with high energy resolution, when individual resonance lines can be clearly identified in the spectra of the detected neutrons. One group of studies determined the orbital angular momenta and spins of the resonances,[6–14] while another determined the mixtures of the spin channels in "two-channel" p resonances.[15–17]

Sections 3 and 4 are devoted to analysis of the differential scattering cross sections averaged over the resonances; this analysis makes it possible to determine the parameters of the potential scattering and the neutron strength functions for s, $p_{1/2}$, and $p_{3/2}$ neutrons.[18–28] The studies of Refs. 15–28 were made using the IBR-30 reactor at the Joint Institute for Nuclear Research.[29]

1. DETERMINATION OF THE ORBITAL ANGULAR MOMENTA AND SPINS OF RESONANCES

To study the properties of individual resonances by means of neutron scattering at different angles, only powerful pulsed sources of neutrons in conjunction with sufficiently long bases for time-of-flight neutron spectrometry have been used. In contrast to previous studies (see footnote 1), which were concerned with the angular distributions of the neutrons at different energies, the studies now considered analyzed the energy spectra of the scattered neutrons obtained at different scattering angles.

The choice of the investigated nuclei and neutron energies was dictated by the need to obtain clearly distinguishable individual resonances. For this, one requires not only a high energy resolution of the spectra but also, on the one hand, that the neutron widths of the resonances should not be too small compared with the radiative widths (or otherwise very thin targets are required) and, on the other, that the resonances should not overlap. The fulfillment of both conditions determined the investigated range of nuclear masses and neutron energies: The energies have as yet not gone beyond the range 1–1800 keV, while the set of target nuclei is restricted to comparatively light or nearly magic nuclei.

Method

The orbital angular momentum l of the resonance can be readily determined from the variation with scattering angle ϑ of the shape of the resonance resulting from its interference with the potential scattering. Since the s wave is dominant in the latter, and the resonance scattering amplitude contains the factor $P_l(\cos\vartheta)$, the angular dependence of the corresponding term of the cross section $\sigma(E,\vartheta)$ is also determined by $P_l(\cos\vartheta)$. For an s resonance, this term is expressed by the isotropic part of σ_{si} in Eq. (5); for a p resonance we have the term with $\cos\vartheta$ in σ_{pi} (7), while for a d resonance this term has the form (with neglect of σ_2 and with energy-dependent Γ_n)

$$-\frac{1}{k^2}\frac{g\Gamma_n\sin\delta_0}{\Delta E^2+\Gamma^2/4}\left(\Delta E\cos\delta_0-\frac{\Gamma}{2}\sin\delta_0\right)P_2(\cos\vartheta). \quad (9)$$

The second term (with $\Gamma/2$) in all these terms is symmetric with respect to E about E_0 and is of no interest here (it is usually smaller than the resonance term). In contrast, the first term is a characteristic odd function of $\Delta E = E - E_0$; it makes the resonance asymmetric. Bearing in mind that $\delta_0 < 0$, it is readily seen that for an s resonance there must at all ϑ be a "dip" (destructive interference) at the "wing" with $E < E_0$ but at $E > E_0$ there must be a slower decrease (constructive interference), i.e., the picture observed for s resonances and in the total cross sections. For resonances with $l > 0$, this same picture is additionally "modulated" with respect to the angle by the factor $P_l(\cos\vartheta)$. Therefore, a p resonance is symmetric at $\vartheta = 90°$, looks like an s resonance at $\vartheta < 90°$, and for $\vartheta > 90°$ has a "dip" at $E > E_0$. The "visiting card" of a d resonance is recognized by the symmetric shape at angles around 55 and 125°, the shape of an s resonance outside the interval between these angles, and a

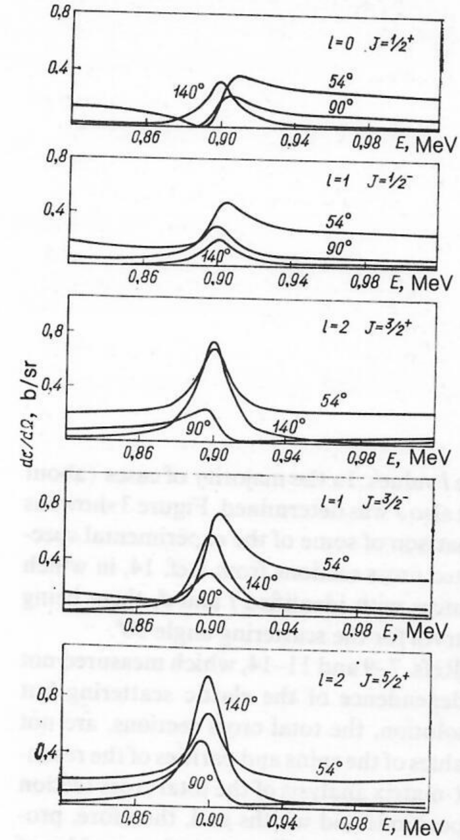

FIG. 1. Shapes of "standard" resonance in ^{40}Ca with $E_0 = 900$ keV, $\Gamma_n = 20$ keV for potential scattering of 1.5 b for the given l, J, and ϑ.[7]

shape that is the inverse of an s resonance for the angles within this interval.

The l identification of the resonances obtained in this manner is unambiguous and reliable. It only needs to be said that with increasing neutron energy the contributions of the higher angular momenta in the potential scattering increase, and the simple behavior of the shapes of the resonances becomes gradually more complicated. Difficulties can also arise from the mutual interference of closely spaced resonances.

In principle, measurement of the resonance curve at several angles also makes it possible to determine unambiguously the resonance spin J, but success here depends on the accuracy of the experiment, since it is now necessary to follow not merely the qualitative but the quantitative features. The value of J significantly influences (through g and ρ) the resonance and interference terms of $\sigma(E, \vartheta)$. We note, in particular, that it is easiest to determine the spin of a p resonance on an even–even target ($I = 0$)[2]: The resonance value is $\omega_2 = 0$ if $J = 1/2$ and $\omega_2 = 1$ if $J = 3/2$, i.e., with increase in ϑ from 90° the resonance peak does not increase in the first case but does in the second.

In practice, to avoid a laborious exact fitting of a calculation to experimental points, one calculates several "standard" shapes of the resonances for different E_0, l, J, ϑ and compares the experimental peaks with them. An example of such standards, taken from Ref. 7, is shown in Fig. 1.

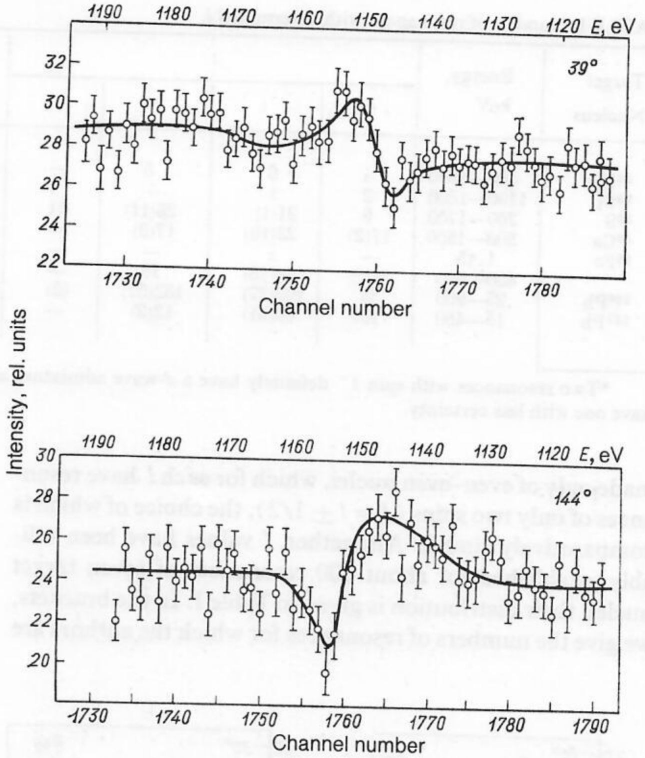

FIG. 2. The first p-wave resonance (in ^{56}Fe) identified through the angular dependence of the scattering.[6]

Results

The results of the majority of studies give only the relative neutron yields for 2–10 scattering angles, and these are not exactly proportional to $\sigma(E, \vartheta)$ because of the finite thickness of the scatterers, multiple scattering, and the finite resolution of the spectrometers. The first study[6] was made using a 54-m time-of-flight base of a neutron source at Harwell that gave neutron bursts with a duration of about 200 nsec. The scattered neutrons were detected by detectors with lithium glasses. The results of this study, shown in Fig. 2, made it possible to establish without doubt that the 1.15-keV resonance in the nucleus ^{56}Fe is a p-wave resonance, and this settled a dispute about this question in a number of studies.

This pioneering work was followed by similar investigations at a higher technical level at three laboratories possessing pulsed sources in the nanosecond range. Methodologically, these studies combine the use of fast detectors with plastic scintillators; the neutrons are detected through the recoil protons, beginning with energies of tens of kilo-electron-volts. In the case of the isochronous cyclotron at Karlsruhe, the measurements were made with a base of 58 m and a time uncertainty of 4 nsec simultaneously for three[7] and ten[8,9] scattering angles. For the electron linac at Oak Ridge the corresponding parameters of the experiments were 200 m and 12 nsec for three[10,11] and five[12,13] angles, while for the electron accelerator at Geel they were 400 m and about 6 nsec for six angles.[14]

Since the greatest interest attaches to determination of the orbital angular momentum l together with the total J, so far, apart from the ^{207}Pb nucleus,[11] investigations have been

TABLE I. Number of resonances with determined l.

Target Nucleus	Energy, keV	l: 0	1	2	3	Reference
^{24}Mg	400—1800	4	6	5	—	[10]
^{28}Si	1100—1300	2	1	—	1	[9]
^{32}S	200—1700	9	21(4)	26(11)	11	[14]
^{40}Ca	500—1500	17(2)	22(10)	17(3)	—	[7]
^{56}Fe	1.15	—	1	—	—	[6]
	450—900	21(2)	64(13)	16	—	[8]
^{206}Pb	25—900	35	109(27)	132(57)	(2)	[12, 13]
^{207}Pb	15—480	10*	60(31)	12(2)	—	[11]

*Two resonances with spin 1^- definitely have a d-wave admixture and two others have one with less certainty.

made only of even–even nuclei, which for each l have resonances of only two spins ($J = l \pm 1/2$), the choice of which is comparatively simple. Altogether, l values have been reliably established for about 600 resonances of seven target nuclei; their distribution is given in Table I. In the brackets, we give the numbers of resonances for which the authors are uncertain about the l values. In the majority of cases (about 70%) not only l but also J was determined. Figure 3 shows as an example a comparison of some of the experimental spectra and the calculated cross sections from Ref. 14, in which one can see resonances with identified l and J, these being identified on the curves for the scattering angle 30°.

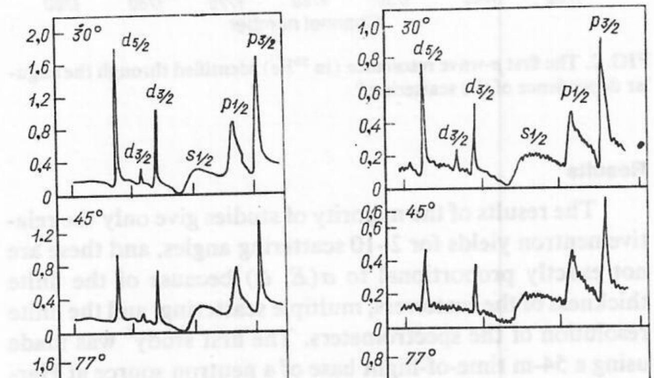

FIG. 3. Comparison of calculated differential cross sections with experimental neutron yields for different scattering angles for ^{32}S nuclei.[14]

The results of Refs. 7–9 and 11–14, which measured not only the angular dependence of the elastic scattering but also, with high resolution, the total cross sections, are not exhausted by the values of the spins and parities of the resonances. Multilevel R-matrix analysis of the total cross section also yielded their positions and widths and, therefore, provided the authors with fairly pure[3)] statistical ensembles of levels for five nuclei. In the following section, we shall briefly consider some of the physical conclusions drawn by the authors of these studies; here, we give only the experimental values of the mean distances between the levels and of the neutron strength functions (Table II). For the sufficiently pure systems of levels, although frequently only few, the authors give the mean distances $D_l(J)$ and the strength functions $S^l(J)$ for levels with given J^π, and in the remaining cases the values of D_l and S^l averaged over J. We note that the requirements on the purity of the ensemble are less stringent for the determination of $S^l(J)$, since it is usually the weak resonances that are not noted or do not have a J identification, and these make a small contribution to the strength functions. In conclusion, we recall the connections between the quantities given in Table II:

$$S^l(J) = \frac{\overline{\Gamma_n^l(J)}}{D_l(J)}; \qquad (10)$$

$$D_l(J) = \frac{D_l}{2J+1} \sum_J (2J+1); \qquad (11)$$

$$S^l(J) = \frac{1}{2l+1} \sum_J g S^l(J), \qquad (12)$$

where $\overline{\Gamma_n^l(J)}$ is the mean reduced neutron width for the resonances with the given l and J and $g = (1/2)(2J+1)/(2I+1)$. Equation (11) is correct if for the nucleus the density of levels with the given J^π at the given excitation is proportional to $2J+1$. As can be seen from Table II, this agrees with the data for ^{32}S but not with the data for ^{40}Ca.

TABLE II. Mean distances between resonances and neutron strength functions (references are given in square brackets).

Parameter	l	J^{π}	^{32}S[14]	^{40}Ca[7]	^{56}Fe[8]	^{206}Pb[12, 13]	^{207}Pb[11]
D_l, keV	0	1/2+	173^{+18}_{-15}	45^{+4}_{-6}	19,6±1,8	23±5*	
	0	1−					58±15
	1	1/2−	195^{+26}_{-21}	55^{+10}_{-24}			
	1	3/2−	$83^{+5}_{-4,5}$	54^{+10}_{-17}			
	1			28^{+5}_{-12}(22)**	4,8±0,8	6,2±0,7*	4,3±0,6
	2	3/2+	133^{+16}_{-13}	61^{+16}_{-32}			
	2	5/2+	91^{+6}_{-5}	82^{+16}_{-42}			
	2			35^{+10}_{-17}(19)**		4,2±0,4*	
	3		146±62*				
$10^4 S^l$	0	1/2+	$0.56^{+0.37}_{-0.20}$	$2,92^{+1.25}_{-0.67}$	2,6±0,8	1,06±0.25	
	0	1−					<0,5
	1	1/2−	$0,72^{+0.56}_{-0.36}$	$0.47^{+0.38}_{-0.14}$			
	1	3/2−	$0.48^{+0.21}_{-0.13}$	$0.29^{+0.23}_{-0.09}$			
	1	0+					0,42±0,10
	1	1+					0,68±0,06
	1	2+					0,28±0.04
	1			$0.36^{+0.15}_{-0.08}$	0,59±0,25	0.32±0.04	0.35±0.06
	2	3/2+	$0.55^{+0.36}_{-0.19}$	$3,4^{+3.2}_{-1.1}$		1.81±0,27	
	2	5/2+	$1,40^{+0.63}_{-0.39}$	$0,9^{+0.9}_{-0.3}$		1,24±0,15	
	2			$1,9^{+1.0}_{-0.5}$			2.8±1,0
	3		$2,29^{+1.51}_{-0.81}$				

*Estimated by us as $(\Delta E/N) \pm \sqrt{2/N}\,(\Delta E/N)$ (N is the number of resonances in the interval ΔE).

**The value corrected for the omission of levels is given in the brackets.

Discussion

In the studies which we have considered, numerous observations quite new for neutron spectrometry were made.

1. In Ref. 9 there appears to have been the detection of the first neutron f resonance for the nucleus ^{28}Si, while for ^{32}S 11 such resonances have already been found.[14]

2. It has long been known from theory that for a nucleus with spin $I > 0$ neutron resonances with many of the possible J^{π} can be excited through channels with several l values of the same parity. However, because of the strong influence of the centrifugal barrier it was assumed that at energies below 1 MeV only neutrons with the minimal angular momentum $l_{\min}$ form a resonance, and in practice no-one reckoned with the possibility of a contribution of neutrons with $l = l_{\min} + 2$. But in Ref. 11 and in the communication[30] that preceded it, it was unexpectedly found that the "s resonances" at 181.49 and 256.43 keV in ^{207}Pb ($I = 1/2^-$), which have spin 1^-, contain admixtures of d waves to the extent of 53 and 81% of the s wave. It is possible that the same applies to two further resonances of ^{207}Pb,[11] and that the same effect has been manifested in other experiments on other nuclei but not recognized. The possibility of such mixtures must now be borne in mind.

3. In the same investigation[11] of the ^{207}Pb + n system, a further important observation was made; this concerned the spin dependence of the neutron strength function S_j^1 for p neutrons with total angular momentum $j = 1/2$ and 3/2. Whereas p resonances with spins 0 and 2 of a nucleus with $I = 1/2$ are excited through one channel (neutrons with $j = 1/2$ form spin 0, and those with $j = 3/2$ form spin 2), resonances with spin 1 are excited through both channels. Therefore, if there are no doorway states with spin 0^+ or 2^+ in the vicinity, resonances with spin 1 must on the average be

stronger than resonances with spins 0 and 2, and if the spin-spin interaction is actually negligibly small, we must observe the equality $S^1(1) = S^1(0) + S^1(2)$, as was indeed observed in Ref. 11 in a region of energies free of doorway states (see also Table II). We note that, as was shown in Ref. 31, the realization of this equality does not, in contrast to what the authors of Ref. 11 assumed, require the signs of the amplitudes of the neutron widths to be random.

4. Among the first four studies in which some neutron resonances in light nuclei were recognized as isobar-analog states, two belong to the series of papers under discussion. One of the *s* resonances of ^{28}Si (Ref. 9) and two or three of the *d* resonances of ^{32}S (Ref. 14) were found to be analogs of known bound states of the nuclei ^{29}Al and ^{33}P with isospin $T = 3/2$. Since the formation of pure states with $T = 3/2$ would require nonconservation of the isospin (for the target $T = 0$ and for the neutron $T = 1/2$), they must be mixed with states having $T = 1/2$. The matrix elements of the isospin mixing, determined by the authors from the neutron widths of the resonances, are of appreciable interest as a measure of the deviation of nuclear forces from charge independence.

5. In Ref. 13, the experimental values of the neutron strength functions and phase shifts of potential scattering for the target nucleus ^{206}Pb were compared with optical-model calculations. Two sets of optical-potential parameters were found, one of which reproduced well the experimental values for $l = 0$ and 2 and badly for $l = 1$, while the other gave the opposite picture. An analogous picture was observed earlier for nuclei with *A* around 100 (Ref. 32) and ^{32}S.[33] This may be an indication that the optical potential depends on the orbital angular momentum.

We complete this list with two types of more traditional observations aimed at the search for nonstatistical effects.

6. The appreciable number of identified levels of ^{207}Pb (about 350) and the width of the excitation interval (about 900 keV) enabled the authors of Refs. 12 and 13 not only to verify the $2J + 1$ dependence of the level density but also to follow the dependence of the density with increasing excitation energy. The graphs of the number of resonances with energy less than a given energy are well described by a model with constant nuclear temperature 0.9 MeV, which corresponds to a growth of the density of about 12% for each 100 keV, but the $2J + 1$ dependence is significantly violated: The number of *p* resonances up to energy 200 keV is twice as large as it should be, but by the energy 900 keV this excess is reduced to 13%. A similar effect but with the opposite result was observed[14] for the compound nucleus ^{33}S, for which a ~30% shortfall of *p* resonances was found, admittedly with considerably less statistical certainty than in Refs. 12 and 13. We note that a difference between the number of compound levels with opposite parities has a theoretical explanation[34] in the framework of the semimicroscopic model of Ref. 35.

7. Another type of nonstatistical effect was also observed—concentrations of the "strength" of neutron resonances, which are interpreted as doorway states. They were clearly observed for lead isotopes.[11–13] The nature of the doorway state for the *p* resonances with $J^\pi = 1^+$ in the $^{207}Pb + n$ reaction at energy around 140 keV was analyzed in detail in Ref. 36. The details of the intermediate structure of the strength functions for the *s* and *d* neutrons in this reaction and in the $^{206}Pb + n$ reaction were qualitatively reproduced in the calculations of Ref. 37 using the quasiparticle–phonon model.[38]

This brief account of the physical results obtained in Refs. 9 and 11–14 shows how fruitful the investigation of neutron resonances can be if their spins and parities are determined.

2. INVESTIGATION OF MIXTURES OF SPIN CHANNELS IN *p* RESONANCES

Theoretical treatment

For target nuclei with spin $I \neq 0$, *p* resonances with spins $J = 1$ for $I = 1/2$ and $J = I \pm 1/2$ for $I > 1/2$ can, in contrast to *s* resonances and *p* resonances with spins $J = I \pm 3/2$, be excited by neutrons and emit neutrons through two different channels with different channel spins $s_\pm = I \pm 1/2$ or different total neutron angular momenta $j = 1/2, 3/2$ (depending on the chosen representation). For unpolarized neutrons and nuclei, the cross sections integrated over the solid angle contain only the total neutron width Γ_n of such two-channel resonances, while the partial neutron widths (or their amplitudes) corresponding to the individual channels occur only in the differential cross sections, with

$$\Gamma_n = \Gamma_{n-} + \Gamma_{n+} = \Gamma_{n\frac{1}{2}} + \Gamma_{n\frac{3}{2}}, \qquad (13)$$

where the partial widths are identified by the additional channel indices.

Between the amplitudes of the widths $\gamma_x = \pm\sqrt{\Gamma_{nx}}$ in the different representations there is a one-to-one connection (see, for example, Ref. 31):

$$\gamma_- = 2\sqrt{I}\left[-W\left(I\tfrac{1}{2}J1;\ I-\tfrac{1}{2}\tfrac{1}{2}\right)\gamma_{\frac{1}{2}} + \sqrt{2}\,W\left(I\tfrac{1}{2}J1;\ I-\tfrac{1}{2}\tfrac{3}{2}\right)\gamma_{\frac{3}{2}}\right]; \qquad (14)$$

$$\gamma_+ = 2\sqrt{I+1}\left[-W\left(I\tfrac{1}{2}J1;\ I+\tfrac{1}{2}\tfrac{1}{2}\right)\gamma_{\frac{1}{2}} + \sqrt{2}\,W\left(I\tfrac{1}{2}J1;\ I+\tfrac{1}{2}\tfrac{3}{2}\right)\gamma_{\frac{3}{2}}\right], \qquad (15)$$

where *W* is a Racah coefficient. Since the signs of the amplitudes are usually unknown, the connection between the corresponding widths is two-valued. The relations between Γ_{n^+} and Γ_{n^-} (or $\Gamma_{n\frac{1}{2}}$ and $\Gamma_{n\frac{3}{2}}$), i.e., the mixing of the channels, can be found only from experiment.

We turn to the differential cross section of elastic scattering. We consider the feature associated with the presence of two channels in the resonance part of the cross section—in the term containing $P_2(\cos\vartheta)$. In the channel-spin representation, this part of the cross section is given by Eq. (6) (for $J = I \pm 1/2$), while in the *j* representation it has the form

$$\sigma_{Pr} = \frac{1}{4k_0^2}\left(\frac{E}{E_0}\right)^2 \frac{g}{\Delta E^2 + \Gamma^2/4} \times \left\{\Gamma_n^2 + \frac{8\,(2J+1)\,(6J-4I+1)}{5\left(I+J+\frac{1}{2}\right)\left(I+J+\frac{3}{2}\right)}\right.$$

$$\times \frac{(I-J)(10I-8J+1)}{\left[\sqrt{2\left(I+J-\frac{1}{2}\right)\left(I+J+\frac{5}{2}\right)}\,\Gamma_{n\frac{3}{2}}+\gamma_{1\frac{1}{2}}\gamma_{\frac{3}{2}}\right]^2}$$

$$P_2(\cos\vartheta)\Big\}. \qquad (16)$$

We write the differential scattering cross section in an isolated p resonance, ignoring the energy dependence of Γ_n, in the form

$$\sigma(\vartheta)=\sigma_{\rm pot}(\vartheta)+\frac{1}{4k_0^2}\frac{g\Gamma_n^2}{\Delta E^2+\Gamma^2/4}\times[1+\omega_1\cos\vartheta+\omega_2 P_2(\cos\vartheta)], \qquad (17)$$

where in accordance with (6) and (7) the anisotropy of the scattering in the resonance is given by the parameters

$$\omega_1=\frac{2\sin\delta_0}{\Gamma_n}[2(E_0-E)\cos\delta_0+\Gamma\sin\delta_0]; \qquad (18)$$

$$\omega_2=(2J+1)\rho^2(J,\ s_\pm)\ \text{for}\ \ J=I\pm 3/2; \qquad (19)$$

$$\omega_2=(2J+1)[\rho(J,\ s_-)\beta_s-\rho(J,\ s_+)(1-\beta_s)]^2 \quad \text{for}\ \ J=I\pm 1/2. \qquad (20)$$

The phase shift δ_0 in (18) can be determined from the potential-scattering radius as $\delta_0=-kR'$, $\rho(J,s)$ can be calculated in accordance with Eq. (8), while β_s in (20) is the *fraction of the channel with spin* $s_-=I-1/2$ in the excitation and decay of the given resonance:

$$\beta_s=\Gamma_{n-}/(\Gamma_{n-}+\Gamma_{n+}). \qquad (21)$$

Note that the expressions (17)–(20) are written down in the approximation $|\delta_1|\ll|\delta_0|$, which is good to several tens of kilo-electron-volts.

TABLE III. Dependence of ω_2 in p-wave resonances on J and β_s.

I	J	ω_2	$\omega_2(0)$	$\omega_2(1)$	$\beta_s(0)$	$\Delta\beta_s$
0	1/2	0				
	3/2	1				
1/2	0	0				
	1		1/2	2	1/3	2/3—1
	2	7/10				
3/2	0	0				
	1		1/50	1/2	1/6	1/3—1
	2		7/10	7/10	1/2	None
	3	12/25				
5/2	1	1/50				
	2		2/35	7/10	2/9	4/9—1
	3		3/4	12/25	5/9	0—1/9
	4	11/28				
7/2	2	2/35				
	3		1/12	3/4	1/4	1/2—1
	4		77/100	11/28	7/12	0—1/6
	5	26/75				
9/2	3	1/12				
	4		28/275	77/100	4/15	8/15—1
	5		39/50	26/75	3/5	0—1/5
	6	7/22				

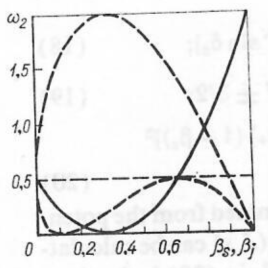

FIG. 4. Dependence of the anisotropy of the scattering for p-wave resonances with $I = 1/2$, $J = 1$ on the parameter measuring the mixing of the spin channels.

As can be seen from (19), the ω_2 for single-channel resonances are constants, and from their values one can determine the spins of the resonances (see the third column of Table III). But for two-channel resonances, ω_2 is determined not only by the spin but also by β_s, i.e., *if the spin J of the resonance is known*, then from ω_2, using Eq. (20), *the ratio* β_s *of the channel contributions can be determined.*

In Fig. 4, the continuous curve shows the function $\omega_2(\beta_s)$ for p resonances with spin $J = 1$ for a nucleus with spin $I = 1/2$. The broken curve shows the connection between ω_2 and the channel ratio of the j representation, $\beta_j = \Gamma_{n\frac{1}{2}}/(\Gamma_{n\frac{1}{2}} + \Gamma_{n\frac{3}{2}})$, which can be readily obtained from (17) by using the expression (16) instead of (6) for $J = I \pm 1/2$. It can be seen that the function $\beta_s(\omega_2)$ is single-valued only for $\omega_2 > 0.5$, while for $\omega_2 \leqslant 0.5$ it is two-valued. But the function $\beta_j(\omega_2)$ in the same regions of ω_2 takes two and four values, respectively. Therefore, despite the fact that in neutron physics the j representation has greater importance and the partial neutron widths are more necessary too in this representation (see the last subsection of Sec. 2), it is, because there is less ambiguity, more advantageous to study the behavior of the contributions of the different channels by analyzing $\sigma(\vartheta)$ in the s representation.

The connection of ω_2 to β_s and β_j depends qualitatively on the spins I and J, but is basically similar to the case $I = 1/2$, $J = 1$, represented in Fig. 4. The basic features of the connection between ω_2 and β_s can be seen in the last four columns of Table III. We give there the anisotropy of the scattering for 100% contributions of the channels [$\omega_2(0)$ and $\omega_2(1)$], the fraction of channel s_- giving zero anisotropy [$\beta_s(0)$], and the region of values of β_s ($\Delta\beta_s$) in which the function $\beta_s(\omega_2)$ is single-valued. Note that the neighborhood of the point $\beta_s(0)$, i.e., the point at which the abscissa axis touches the parabola, is an additional "region of single-valuedness," since there the two values of β_s are close to each other.

The distribution of β_s and β_j

Measurements of the partial neutron widths Γ_1 and Γ_2 for the spin channels 1 and 2 are not only needed for a more complete description of the interaction of neutrons with nuclei,[4)] but are also of independent physical interest. The main question here is the following: Do the reduced neutron widths Γ_1^1 and Γ_2^1 fluctuate independently from resonance to resonance or is there a correlation between them?

It is known that the amplitude of the neutron widths can be represented in the form of a surface integral of the product of the complicated wave function of the composite state and the simple wave function of the neutron state. In accordance with this, the expressions for the amplitudes γ_1 and γ_2 differ by the wave function of channels 1 and 2, for which only the values of the spin variables are different. Therefore, the answer to the question posed above concerning correlation of the widths, which was posed in Ref. 39 and stimulated the investigations described below, gives new information about the spin dependence of the nuclear forces and about the structure of the wave functions of highly excited nuclear states.

In the case of complete correlation between Γ_1^1 and Γ_2^1, the values of $\beta = \Gamma_1/(\Gamma_1 + \Gamma_2)$ would be the same for different resonances (at least, for a given spin $J = I \pm 1/2$). But in the opposite case of independent fluctuations, β will be continuously distributed in the interval from 0 to 1 in accordance with the distributions of the widths in the two channels. Thus, if Γ_1^1 and Γ_2^1 follow independent χ^2 distributions with number $\nu = 1$ of degrees of freedom and with mean values $\overline{\Gamma}_1^1$ and $\overline{\Gamma}_2^1$, then, as is easily seen, the differential of the probability of a given β is given by the expression[15,40]

$$P(\beta)\, d\beta = \frac{\sqrt{\varkappa}}{\beta + \varkappa(1-\beta)} \frac{d\beta}{\pi\sqrt{\beta(1-\beta)}}, \qquad (22)$$

where $\varkappa = \overline{\Gamma}_1^1/\overline{\Gamma}_2^1$. The distribution (22), normalized to unity, has mean value $\langle\beta\rangle = \sqrt{\varkappa}/(1+\sqrt{\varkappa})$, a minimum at

$$\beta_{\min} = \frac{5\varkappa - 3 - \sqrt{9\varkappa^2 - 14\varkappa + 9}}{8(\varkappa - 1)},$$

and tends to infinity as $\beta \to 0$ and 1. For equal mean widths in both channels, $\varkappa = 1$ and $\langle\beta\rangle = \beta_{\min} = 0.5$. As $\varkappa$ is increased (decreased) from 1, $\langle\beta\rangle$ and $\beta_{\min}$ are slowly displaced in the direction of 1 (0) and 0 (1), respectively.

Under conditions of maximally complicated nature of the excitation, the properties of the nuclei in the compound states can be well described by a statistical model of the nucleus, according to which, in particular, there should be no correlations between the partial widths of any two reaction channels. Nevertheless, a great many exceptions to this rule are known, these occurring when the excitation is due to a small number of nucleons or has a collective nature. Such are the cases of direct capture of a valence neutron and the cases of doorway and analog states; at the same time, for many nuclei correlations have been observed between different pairs of nucleon and radiative channels.

The theoretical analysis of the correlations between widths made in Ref. 41 made it possible to prove formally a rule: If in a certain interval of energies compound states are excited and decay through a doorway state common to several channels, then between the partial widths of the corresponding resonances for these channels there must be correlations. For a pair of spin channels that are mixed, any doorway state is evidently always a common state, and therefore investigation of mixtures of such channels may be a very sensitive tool for discovering and studying nonstatistical effects at high nuclear excitations.

As was shown in Ref. 42, if certain conditions are satisfied, there can still be a correlation between the partial widths in the spin channels even when one of the simple configurations (a doorway state) is not dominant in the compound states. Moreover, approximate analysis of some experimental results led Nikolenko[42] to the conclusion that for the nuclei Ag and Tl such a correlation exists.

Measurements and results

To determine the parameters β_s, it is necessary to make measurements of the type described in Sec. 1, i.e., to obtain resonance peaks of the neutrons at different scattering angles. This was done using the pulsed reactor IBR-30 with an injector at the Laboratory of Neutron Physics at the Joint Institute for Nuclear Research. The comparatively low energy resolution (time uncertainty of about 5 μsec) was responsible for the small number of p resonances studied in the three experiments of Refs. 15–17.

Two measuring arrangements, in which the neutrons were detected by proportional counters filled with ^{3}He to a pressure of 10 atm, were used. The first had a time-of-flight base of 250 m and is shown schematically in Fig. 5. A flat scatterer was set up in a beam of cross section 22×12 cm, and the scattered neutrons were detected by one battery of counters with total working volume of about 8 liters, this being situated in a massive mobile shield of paraffin with B_4C at a distance of about 1 m from the scatterer. In the arrangement over a base 1000 m from the reactor (Fig. 6), the cross section of the neutron beam had the shape of a ring with outer diameter 80 cm and inner diameter 60 cm, and the scatterer was made in the shape of the side surface of a truncated cone, its dimensions corresponding to those of the beam. Two detectors of volumes 3 and 8 liters detected neutrons scattered by the sample at angles near 90° and 150°. A more detailed description of the equipment and method of measurement with it can be found in Refs. 15–17 and 23.

With regard to the procedure for extracting the values of ω_2 from the experimental spectra, it is necessary to say the following. After subtraction of the smooth base below the

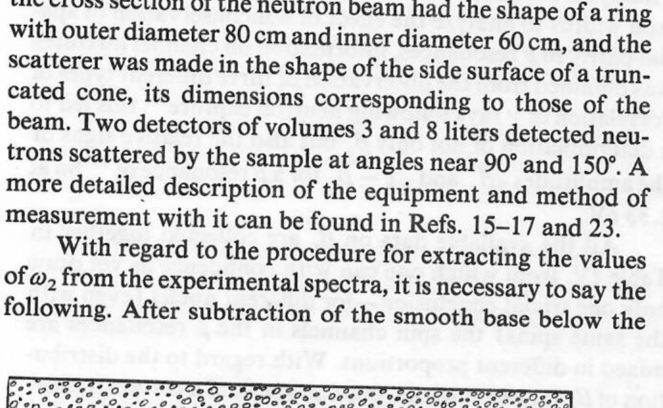

FIG. 5. Schematic arrangement of experiment with a 250-m base: 1) mobile neutron detector in shield; 2) scatterer; 3) collimator; 4) evacuated neutron guide; 5) fixed platform of detector; 6) monitor counters; 7) electromechanical drive for displacing the detector and the scatterer; 8) detector electronics; 9) control block for the electrical drive.[23]

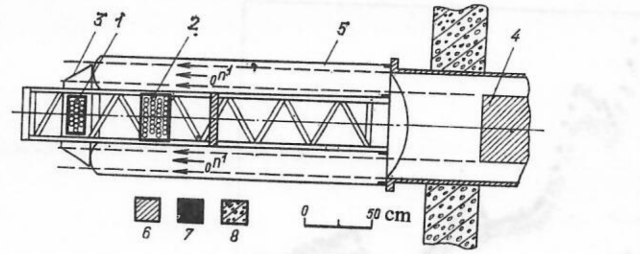

FIG. 6. Schematic arrangement of the experiment with a 1000-m base: 1) 90° detector; 2) 150° detector; 3) scatterer; 4) collimator; 5) polyethylene "bag" filled with argon; 6) paraffin with B_4C; 7) B_4C; 8) concrete.[17]

peaks corresponding to potential scattering by nuclei of the sample and the background, it is necessary to calculate the areas of the resonances, correct them for parasitic effects[5] that depend on the scattering angle, and make a transformation to the center-of-mass system. Then the area of each resonance can be expressed as

$$B(\vartheta)=B_0\,[1+\bar{\omega}_1 P_1(\cos\vartheta)+\bar{\omega}_2 P_2(\cos\vartheta)], \qquad (23)$$

where B_0, $\bar{\omega}_1$, and $\bar{\omega}_2$ are obtained by integrating the yield of scattered neutrons over the energy. The E independence [in accordance with (19) and (20)] of ω_2 in (17) ensures the equality $\bar{\omega}_2=\omega_2$, while in $\bar{\omega}_1$ there appears only the second term of the expression (18) (the first gives zero on integration and when $E_0\gg\Gamma$), so that

$$\bar{\omega}_1=2\sin^2\delta_0\frac{\Gamma}{\Gamma_n}\approx\frac{E\sigma_{\rm pot}}{1300}\,\frac{\Gamma}{\Gamma_n}, \qquad (24)$$

where E is measured in kilo-electron-volts and $\sigma_{\rm pot}$ in barns. Thus, for the determination of ω_2 (and $\bar{\omega}_1$) measurements at three different angles are needed. But if $\bar{\omega}_1$ is small (at small E) or is assumed known [in accordance with (24)], then measurements at two angles are sufficient.

The results of all three experiments proved to be rather intriguing. The first experiment[15] was made with a 250-m base with scatterers made of metallic yttrium with thickness along the beam of 0.0113 and 0.0221 b^{-1}. The values of ω_2 were determined for six p resonances or unresolved groups of them. Since the spin of ^{89}Y is $I=1/2$, it follows in accordance with Table III that resonances with spins 0 and 2 have $\omega_2=0$ and 0.7, respectively, while resonances with spin 1 and mixed channels can have any ω_2 between 0 and 2. In accordance with this, not less than three resonances were found to have spin 1 and β_s greater than 0.8, and the ω_2 of two resonances also admit the possibility of spin 2, i.e., are close to 0.7. Thus, three out of the five β_s values were concentrated around 0.8.

In the second experiment,[16] made with the same arrangement, p resonances of ^{19}F were studied (Teflon samples of thickness 0.6 and 1 mm); for determination of the areas of the resonances additional use was made of a self-indication method, which made it easier to separate the resonances from the background. The two upper spectra in Fig. 7 are measurements with a sample without a filter (black circles) and with a filter (open circles) of Teflon of thickness 2.4 mm; the lower spectrum is their difference. In the upper half of the figure, at the bottom, we show the back-

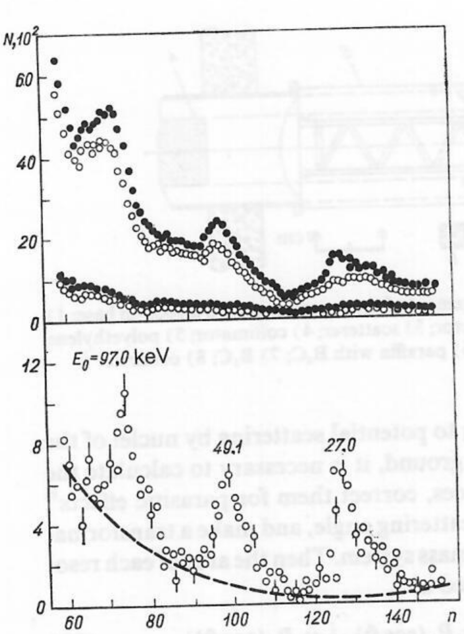

FIG. 7. Neutron spectra from ^{19}F at $\vartheta = 37°$: n is the number of the 1-μsec wide channel, and N is the number of counts in the channel.[16]

ground spectra without a sample (open circles with a filter; black circles without it). To within the errors of the measurements, allowance for the background does not affect the result. The result of this experiment is that for the two-channel resonances of ^{19}F at 49 and 97 keV the values of ω_2 were found to be zero, and, therefore, both resonances have $\beta_s \simeq 1/3$ ($I = 1/2$).

With the 1000-m base in the third experiment[17] the investigation of the yttrium resonances was continued with improved resolution. The scatterer consisted of 16 trapezoidal aluminum containers filled with about 7 kg of Y_2O_3. The thickness of yttrium in the direction of the beam was ~ 0.034 b^{-1}. In the energy interval from 2.6 to 30 keV, the areas of 20 resonances out of a total number of 30 resonances, according to Ref. 43, were determined at two angles by means of the UPEAK program.[44] The areas of the eight s resonances

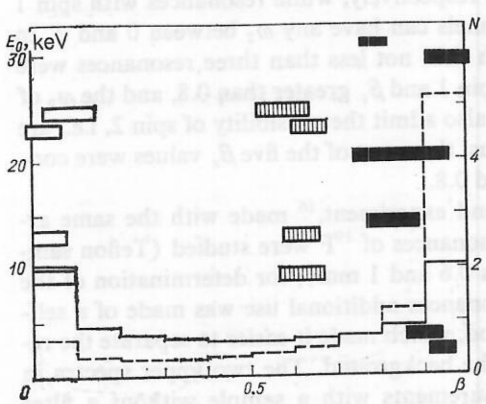

FIG. 8. Values of β_s for ten resonances of ^{89}Y. The number N corresponds to the histograms and shows the number of resonances in the β_s interval of width 0.1.[17]

made it possible to normalize the spectra of the two detectors to each other, and the areas of the 12 p resonances, in conjunction with Eq. (24), made it possible to determine the values of ω_2. For seven p resonances, ω_2 deviated from 0 and 0.7 by not less than two standard deviations, for three by not less than one, and for two by less than one. Therefore, spin 1 was ascribed to ten resonances, the β_s of which are shown graphically in Fig. 8. The centers of the rectangles correspond to the experimental values of β_s on the abscissa and to the energies of the resonances on the left-hand ordinate, while the horizontal dimensions of the rectangles correspond to the standard deviations of β_s. For six resonances, the β_s are single-valued ($\omega_2 > 0.5$) and are shown by the black rectangles, while for the other four $\omega_2 < 0.5$ and the β_s each have two possible values, which are shown by the open and hatched rectangles. It can be seen that a concentration of *all* β_s around 0.8 is possible but can be questioned.

Other data and discussion

An attempt to investigate the contributions of spin channels in neutron p resonances by measuring neutron scattering was made already in 1958,[45] but the first definite results were obtained in measurements of the angular dependence of the emission of γ rays of the radiative capture of neutrons for three resonances of ^{93}Nb,[40] and later for a resonance of ^{35}Cl.[46] The study of Ref. 47 analyzed the real possibilities for solving the problem in measurements of the total cross sections for polarized neutrons and oriented nuclei. Finally, in connection with the need to know the partial neutron widths to analyze the effect of nonconservation of spatial parity in p resonances, information on channel mixtures was obtained from the observation of three different types of correlation of γ rays following neutron capture[48]; this led to a determination of not only β_s but also the relative signs of the amplitudes $\sqrt{\beta_s}$ and $\sqrt{1-\beta_s}$ for a p resonance of ^{117}Sn at 1.33 eV.

All the available data on β_s are collected together in Table IV, from which one can with confidence as yet draw only one trivial conclusion—for different nuclei (even with the same spins) the spin channels in the p resonances are mixed in different proportions. With regard to the distribution of β_s for one nucleus, one can speak of a certain indication of a correlation between Γ_{n^-} and Γ_{n^+}. Indeed, the hypothesis of a grouping of the β_s in a narrow range of values is contradicted only by the results of Ref. 40 for ^{93}Nb. However, this contradiction is significantly weakened if the spin of the resonance at 94.3 eV is not 4 but 3 (in accordance with Ref. 43); then this resonance does not have two channels, and its "β_s" must be ignored. The other two ^{93}Nb resonances have different spins and correspond to different systems of levels, for which the "preferred" β_s can be different. The equality of the β_s of the ^{19}F resonances speaks for a strong correlation, but there are just two resonances and the coincidence could be fortuitous.

The β_s situation for the ten resonances of ^{89}Y (Fig. 8), four of which may have any of the two possible values, is curious. If for all four resonances the smaller values of β_s are correct (there are no hatched rectangles), then the observed

TABLE IV. List of known mixtures of spin channels of p-wave resonances (references are given in square brackets).

Target nucleus	E_0, keV	J	β_s
^{19}F [16] ($I=1/2$)	49.1	1	0.33±0.14
^{35}Cl [46] ($I=3/2$)	97.0	1	0.33±0.13
^{89}Y [17] ($I=1/2$)	0.398	2	∽0
	2.61	1	0.94±0.03
	3.38	1	0.90±0.04
	9.41	1	(0.62 or 0.05)±0.05
	12.99	1	(0.62 or 0.04)±0.04
	14.21	1	0.83±0.06
	20.27	1	0.85±0.11
	23.00	1	(0.64 or 0.03)±0.04
	24.56	1	(0.58 or 0.09)±0.06
	29.26	1	0.96±0.05
	30.05	1	0.79±0.03
^{93}Nb [40] ($I=9/2$)	0.0358	5	0.30±0.08
	0.0422	4	0.73±0.17
	0.0943	4	0.16±0.13
^{117}Sn [48] ($I=1/2$)	0.00133	1	0.82±0.07

distribution agrees with the theoretical distribution (22) shown in the figure by the continuous curve and corresponding to two independent Porter–Thomas distributions $\Gamma^1_{n^-}$ and $\Gamma^1_{n^+}$ with equal mean values. But if the larger values are realized (no open rectangles), then all the β_s lie in the interval 0.5–1, the probability of which for a symmetric distribution is only 0.2%. For an appreciable probability of such a grouping of the β_s, it is necessary to assume the existence of either a large difference of the mean widths (the broken line in Fig. 8 corresponds to $\overline{\Gamma^1_{n^-}} = 10\,\overline{\Gamma^1_{n^+}}$) or an appreciable correlation between Γ_{n^-} and Γ_{n^+}. Other versions of the four resonances lead, naturally, to less definite conclusions.

Concluding the discussion of the results of the investigation of neutron spin channels, we must not fail to mention, at least briefly, the significant experimental material obtained for proton channels. Because of the high energy resolution (~300 eV) at Duke University at Durham, North Carolina in the United States, spectroscopy of proton resonances can be successfully developed. Investigation of the angular dependence of the emission of protons and photons in the (p,p') reaction makes it possible to determine for p and d resonances the amplitudes of the widths in the exit spin channels and their relative signs.[49] For the target nuclei ^{44}Ca, ^{48}Ti, ^{50}Cr, and ^{56}Fe there are now eight sets, each containing several tens of resonances with spins $J=3/2^-$, $3/2^+$, $5/2^+$, for which the amplitudes of the widths in mixed channels are known. These data contain (see Ref. 50 and the bibliography there) a rich diversity of the presence and absence of correlations associated with analog states and not associated with such states. Figure 9 gives two β_s distributions for the partial proton widths, one of which agrees well with the distribution (22) (no correlations) and the other badly (strong correlations).

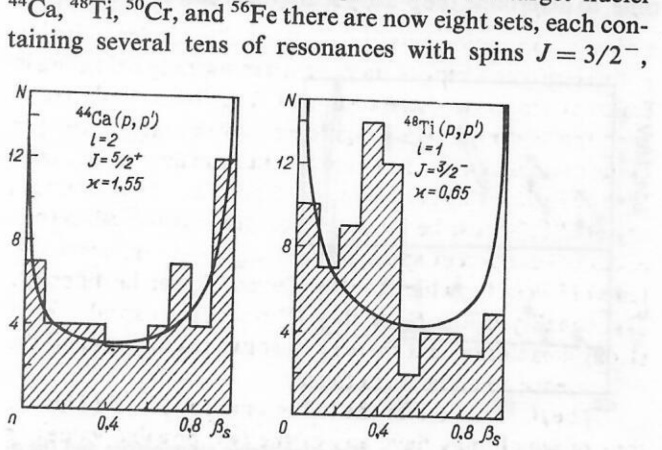

FIG. 9. Comparison of the experimental β_s distributions with the theoretical distributions for Ca and Ti.

Finally, data have very recently appeared on spin channels in the elastic scattering of protons by nuclei with nonzero spin, these data being completely analogous to the neutron data discussed above. For the nuclei ^{25}Mg, ^{27}Al, and ^{29}Si values of β_s have been obtained for, respectively, 14 (Ref. 51), 22 (Ref. 52), and nine (Ref. 53) proton p resonances. As yet, none of them contradict the statistical assumption that there are no correlations between the channels, as can be seen from Fig. 10. The hatched part of the histograms for ^{25}Mg and ^{27}Al relates to the resonances with spin $J=2$; the unhatched, to those with $J=3$; the ^{29}Si resonances have $J=1$.

3. SEARCHES FOR THE EFFECT OF ONE-PION EXCHANGE

This and the following section review studies in which nuclear properties were determined from the angular distributions of neutrons in energy intervals containing many resonances. For such measurements, a high energy resolution is not required, and in the atlas of differential cross sections[3]

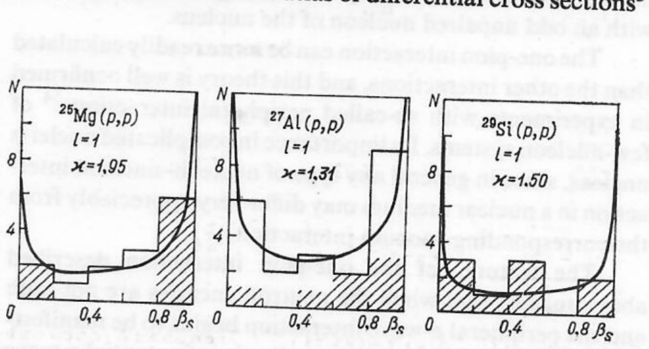

FIG. 10. Comparison of the experimental β_s distributions with the theoretical distributions for Mg, Al, and Si.

one can find quite a substantial body of data for neutron energies less than 0.5 MeV, the majority of which were obtained at Argonne National Laboratory in the United States by means of nuclear reactions as sources of monoenergetic neutrons with an uncertainty of tens of kilo-electron-volts. However, the lower limit of the energies in these data is, as a rule, around 300 keV; the authors analyzed them only from the point of view of the optical model, and therefore we shall not consider these studies.

All the investigations reviewed below were made at the Joint Institute for Nuclear Research using the IBR-30 reactor and the time-of-flight method. The arrangement employed[15,23] has already been briefly described in a subsection of Sec. 2 (see Fig. 5). Measurements at scattering angles 45, 90, and 135° with and without a sample were made in an automatic regime using a MERA-60-30 computer; after the introduction of corrections[21] and calibration to a carbon standard the parameters σ_s, ω_1, and ω_2 of the differential cross sections were obtained in accordance with the expansion (1) for energies in the range 2–450 keV. The samples employed were of pure elements of the natural isotopic composition and samples enriched with one isotope; the mass of the samples varied in the range 70–300 g.

The absence of interaction at orbital angular momenta $l > 1$ and the possibility of separating the weak *p*-wave scattering on the background of the strong *s*-wave scattering made it possible in the study of the scattering of neutrons with kilo-electron-volt energies to test experimentally one of the hypothetical possible "mechanisms" of *p*-wave scattering.[18–20]

As is well known, the nuclear forces acting between nucleons can be interpreted as due to the exchange of quanta of the nuclear field—mesons of various species, with one, two, etc., being exchanged. The effective range of the interaction corresponding to exchange of a given species and number of mesons is then inversely proportional to the total mass of the mesons. Therefore, the interaction with the longest range, characterized by about 1.4 F, is obtained for exchange of one of the lightest mesons—the pion. Exchanges of two pions or more massive mesons give ranges less than 0.7 F. But the one-pion interaction Hamiltonian contains as a factor the product of the spins of the interacting particles, and therefore if one of them is represented by two "paired" nucleons with zero total spin, the contribution of one-pion exchange with this pair will also be zero. Hence, for the interaction of a nucleon with a nucleus one-pion exchange is important only with an odd unpaired nucleon of the nucleus.

The one-pion interaction can be more readily calculated than the other interactions, and this theory is well confirmed in experiments with so-called peripheral interactions[54] of few-nucleon systems. Its importance in complicated nuclei is unclear, since in general any type of nucleon–nucleon interaction in a nuclear medium may differ very appreciably from the corresponding vacuum interaction.

The features of the one-pion interaction described above suggest that when the neutron energies are not high and the peripheral *p*-wave interaction begins to be manifested the difference between the amplitudes of potential *p*-wave scattering by an odd nucleus and the neighboring even–even nucleus will to a large degree be determined by the contribution of one-pion exchange, while the contribution of this exchange to the difference of the *s*-wave amplitudes must be many times less. A convenient measure of this effect is the forward–backward scattering asymmetry ω_1 in the expansion (1), which is, when resonances are ignored, essentially equal to twice the ratio of the *p* and *s* scattering amplitudes and is given in accordance with (3) by

$$\omega_1 = 6 \sin \delta_1 \cos (\delta_0 - \delta_1)/\sin \delta_0.$$

In the limit $kR \ll 1$, this can be represented[19] in the form

$$\omega_1 = 2\,(kR)^2\, R_1'/R_0', \tag{25}$$

where R_0' is the widely used radius of potential scattering, usually denoted by R', and R_1' is the analogous quantity for *p* neutrons introduced in Ref. 19 (see also the third subsection of Sec. 4). Thus, ω_1 is a linear function of the neutron energy, and the ratio of the *p*- and *s*-wave ranges in which we are interested is characterized by the slope of the straight line $\omega_1(E)$ relative to the abscissa axis, i.e., by ω_1/E.

In Refs. 18 and 19, a chain of ten elements in succession from Ru to I was investigated in the IBR-30 reactor regime. To neutron energies of 40–60 keV, the experimental values of $\omega_1(E)$ were found to lie well on straight lines emanating from the coordinate origin, the slopes ω_1/E of which were determined by the method of least squares. The results are presented in Fig. 11. In the upper half of this figure, the open circles show the values of ω_1/E for five elements with even Z containing predominantly even–even nuclei. It can be seen that the black circles, corresponding to nuclei with even A-Z and odd Z, lie systematically above the line connecting the points for the even–even nuclei. To eliminate the influences of *s* and *p* resonances by the approximate method described in Ref. 55, their contributions to ω_1 at $E = 40$ keV were calculated. It was found that the resonances lower the "potential" ω_1 by 10–20% without exhibiting a correlation with the parity of the number of protons in the nuclei. The ω_1/E values corrected in this manner are shown by the crosses above the uncorrected values. In order to eliminate possible fluctuations of the *s*-wave scattering from nucleus to nucleus, Eq. (25) and the available data on R_0' were used, for the first time, to determine the *p-wave scattering radii* R_1', which are

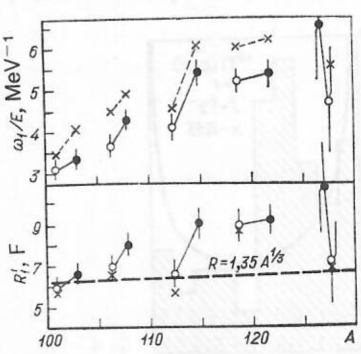

FIG. 11. Experimental values of the scattering asymmetries and the *p*-wave scattering radii for the nuclei Ru, Pd, Cd, Sn, Te (open circles) and Rh, Ag, In, Sb, I (black circles).[19]

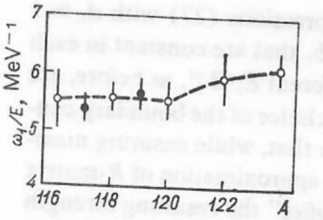

FIG. 12. Experimental values of the scattering asymmetries for even (open circles) and odd (black circles) tin isotopes.[20]

shown in the lower part of Fig. 11 (the crosses show the values "corrected" for the presence in Z-even elements of isotopes with an odd number of neutrons calculated by assuming that these elements would have R_1' values as for the Z-odd element of the given pair). The same effect as for ω_1/E is found; the R_1' values are on the average appreciably above the line $R = 1.35A^{1/3}$ F, as they must be in this region of A in accordance with the optical model of the nucleus (see also the third subsection of Sec. 4).

In Ref. 20, the values of ω_1/E were obtained for a chain of seven tin isotopes (Fig. 12). It can be seen that to within the errors of the measurements these quantities do not exhibit a correlation with the parity of the number of neutrons, and a more detailed analysis of them was not made.

Thus, in connection with the hypothesis advanced in Ref. 18 concerning a possible influence of one-pion exchange on the p-wave scattering of neutrons by nuclei with an odd nucleon, the following can be said. First, if such an influence does exist, it is by no means decisive. Second, there are grounds for believing that in the interaction of neutrons with nuclei there exists some mechanism by which an odd proton of a nucleus introduces into p-wave neutron scattering a contribution that is approximately an order of magnitude greater than that of each of the remaining nucleons (the jump in ω_1 for Z-odd nuclei is about 10%, while the number of nucleons in the nucleus is about 100). An odd neutron does not have this property or has it at most only to a much lower degree. It is not possible to attribute this effect unambiguously to one-pion exchange, though there are qualitative arguments[20] that support this (including the difference of the effect for a proton and a neutron).

Unfortunately, a serious theoretical investigation of the hypothesis, which could cast light on the results and stimulate further experimental efforts, is lacking. It would evidently be desirable to at least estimate the correction to the nuclear potential obtained by "spreading" the one-pion potential over the nucleus by means of the wave function of the odd nucleon.

4. DETERMINATION OF THE SCATTERING RADII AND THE NEUTRON STRENGTH FUNCTIONS

Development of the method

In the development of the analysis of averaged differential scattering cross sections, several stages can be distinguished.

1. As is well known, the statistically most accurate method of determining neutron strength functions is to extract them from the cross sections averaged over the resonances. For this purpose, use has long been made of the average total cross sections $\bar{\sigma}_{tot}$ and the cross sections $\bar{\sigma}_\gamma$ of radiative capture, which can be expressed in terms of the strength functions as parameters. Analysis of the averaged differential cross sections of elastic scattering became a natural development of these methods. The advantages of this method, noted more than 20 years ago,[56] are as follows. First, being a sum of three terms (one isotropic, one with cos ϑ, and one with $\cos^2\vartheta$), the cross section $\overline{\sigma(\vartheta)}$ contains more information than $\bar{\sigma}_{tot}$ or $\bar{\sigma}_\gamma$, since each of the terms is expressed differently in terms of the phase shifts and the strength functions. Second, at energies up to 50 keV for spinless target nuclei, the third term (or the parameter ω_2) contains the neutron strength function only for $p_{3/2}$ neutrons, while the other two terms depend on the total strength function for the p neutrons; this makes it possible to decompose the total strength function into its components and observe the spin–orbit splitting of its maximum in the region $A \sim 100$.

2. The expression in terms of the strength functions of the $\overline{\sigma(\vartheta)}$ term with cos ϑ (Ref. 55) initiated the development of a new method that gave the first practical results in Ref. 22: For five tin isotopes the experimental values of the parameters σ_s, ω_1, and ω_2 of the cross section (1) were described by functions containing seven adjustable parameters: the neutron strength functions S^0, $S^1_{1/2}$, and $S^1_{3/2}$ for s, $p_{1/2}$, and $p_{3/2}$ neutrons, the radiative strength functions S^0_γ and S^1_γ, and the contributions R_0^∞ and R_1^∞ of distant levels for the s and p neutrons. In these calculations and subsequently in calculations with other nuclei it was found that the radiative strength functions are determined from scattering data with a large error and that, if their values are fixed within reasonable limits, they do not influence the determination of the remaining parameters (since, beginning at tens of kilo-electron-volts, $\Gamma_n \gg \Gamma_\gamma$). Therefore, in all subsequent studies S^0_γ and S^1_γ were assumed to be fixed and were estimated from the data given in Ref. 43.

The parameters R_1^∞ of the R-matrix formalism determine for the chosen radius R of the nucleus the phase shifts of potential scattering:

$$\delta_l = \varphi_l + \tan^{-1}\frac{P_l R_l^\infty}{1-(S_l-b_l)R_l^\infty}, \quad (26)$$

where φ_l, P_l, and S_l are the phase shifts for scattering by a hard sphere, the penetrability factor, and the shift factor, respectively, these being known functions of kR, and b_l is a constant of the boundary conditions; R_l^∞ were assumed to be independent of the energy.

The essence of the method, which was developed further in Refs. 24 and 25, in which scattering by 12 Z-even elements from Ti to Te was investigated, is explained in Fig. 13, which contains the real quantities for neutron scattering by cadmium nuclei. The broken curves represent the contributions to σ_s, ω_1, and ω_2 determined by the parameters or parameter combinations indicated next to the curves, while the sums of all the contributions (continuous curves) must describe the experimental points. The parameters $S^1_{3/2}$ and δ_1 (i.e., R_1^∞) are determined better than the others, while

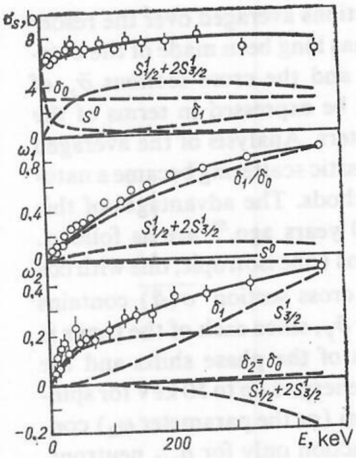

FIG. 13. Contributions of different parameters to the formation of the differential cross section of neutron scattering by cadmium nuclei.

between $S^1_{1/2}$, δ_0, and S^0 there are certain correlations.

3. The correlations and errors of the adjustable parameters were reduced by a combined analysis of the data on $\overline{\sigma(\vartheta)}$ with data on the widths $g\Gamma_n$ of individual resonances.[24–27] The experimental material included not only the values of σ_s, ω_1, and ω_2 but also the values of the sums $\Sigma_g \Gamma_n$ in 3–10 energy intervals ΔE, into which the region of the known resonances for each sample was divided. From the compilation of Ref. 43 the widths of all resonances in succession were taken (independently of l), averaging over the isotopes was done for each element, and the resulting experimental points were described by means of the formula[6)]

$$\Sigma g\Gamma_n/\Delta E = \sqrt{E}\left[S^0 + \frac{(kR)^2}{1+(kR)^2}(S^1_{1/2}+2S^1_{3/2})\right].$$

4. The energy-dependent quantities $\overline{\Gamma_{nlj}/D_{nlj}}$ that appear when $\overline{\sigma(\vartheta)}$ are obtained from $\sigma(\vartheta)$ are expressed in terms of the strength functions in accordance with

$$\frac{\overline{\Gamma}_{nlj}}{D_{nlj}} = \sqrt{E}\,\frac{P_l}{kR}\,\frac{S^l_j}{d_l}, \tag{27}$$

where

$$d_l = (P_l R_l^\infty)^2 + [1-(S_l-b_l)\,R_l^\infty]^2. \tag{28}$$

Allowance for the factor d_l of Ref. 57 in Refs. 24–26 showed that the values obtained for the strength functions S^l_j depend strongly on the chosen boundary conditions. Thus, for the Cd isotopes[26] the values of S^1_j obtained for $b_1 = 0$ and $b_1 = -1$ differ by a factor 1.5–2. This uncertainty was eliminated in Ref. 27 by choosing values of b_l which ensure the equality $d_l = 1$. Solving Eq. (28) with $d_l = 1$ for b_l, we obtain

$$b_l = -\frac{1}{R_l^\infty} + S_l \pm \sqrt{\frac{1}{(R_l^\infty)^2} - P_l^2}, \tag{29}$$

and by substituting (29) in (26) we arrive at a simpler expression for the phase shift[7)]:

$$\delta_l = \varphi_l + \sin^{-1} P_l R_l^\infty. \tag{30}$$

Thus, the description of $\overline{\sigma(\vartheta)}$, the average over the interval ΔE with center E, using the expressions (27) with $d_l = 1$ and (30) amounts to the use of b_l that are constant in each ΔE but slightly different for different E; R_l^∞, as before, are constant fitted parameters. The choice of the boundary conditions described here is good in that, while ensuring maximal accuracy of the single-level approximation of R-matrix theory,[58] it simultaneously "unifies" the resulting strength functions, making them compatible with those obtained from the resonance parameters without allowance for the factors d_l.

5. The next step in refining the analysis of the differential scattering cross sections is to take into account the spin-orbit splitting in the potential scattering, i.e., the addition to the index l of the index $j = l \pm 1/2$ in the phase shifts for $l > 0$, as was done from the very beginning for the strength functions. The inequality of δ_+ and δ_-, as, for brevity, we shall denote the p-wave phase shifts with $j = 3/2$ and $1/2$, respectively, is due to the inequality of the contributions to the p-wave scattering of the tails of the resonances outside the averaging interval ΔE and formed through channels with different j. All this is expressed formally by the introduction of R_+^∞ and R_-^∞ instead of R_l^∞ and the calculation of δ_+ and δ_- by means of the same expressions (26), (29), and (30). The corresponding expression for $\overline{\sigma(\vartheta)}$ was obtained in Ref. 59 and was tested in the process of the preparation of Refs. 24 and 25. However, decomposition of δ_1 into δ_+ and δ_- did not succeed, since in the new $\sigma(\vartheta)$ the expressions of the type $3\sin\delta_1$ and $3\sin^2\delta_1$ are replaced by the expressions $\sin\delta_- + 2\sin\delta_+$ and $\sin^2\delta_- + 2\sin^2\delta_+$, and at small δ_1 a strong correlation arises between δ_+ and δ_-—for a given sum $\delta_- + 2\delta_+$, $\overline{\sigma(\vartheta)}$ is almost insensitive to the difference $\delta_+ - \delta_-$.

For separate determination of δ_+ and δ_-, it was proposed in Ref. 60 to use data on the polarization of scattered neutrons that arises from allowance for the spin–orbit interaction. In Ref. 60, an expression was obtained for the polarization as a function of δ_0, δ_+, δ_-, S^0, $S^1_{1/2}$, $S^1_{3/2}$ and containing terms with $\sin(\delta_+ - \delta_-)$.

Preliminary calculations showed that it described well the polarization data for neutron energy 400 keV for reasonable values of all the parameters.

6. The final refinement of the method was its extension to target nuclei with spin $I \neq 0$, the details of which will be discussed in the next section.

General expression for $\overline{\sigma(\vartheta)}$

An expression for the average differential scattering cross section for even–even nuclei was given in Ref. 22. The dependence of the phase shifts on j for $l > 0$ was taken into account in Ref. 59, and we give below a general expression for $\overline{\sigma(\vartheta)}$ for arbitrary target spin I obtained, like the expressions in Refs. 22 and 59, under the following assumptions:

1) the widths Γ of the resonances, the distances D between them, and the interval of averaging ΔE over them satisfy the inequalities

$$\Gamma \ll D \ll \Delta E \ll E,$$

where E is the mean energy of the interval;

2) the fluctuations of the reduced neutron widths Γ_n^l satisfy a Porter–Thomas distribution, while for the radiative widths Γ_γ they are negligibly small;

3) the density of levels with given spin J is proportional to $2J+1$;

4) the strength function for s neutrons, S^0, and its j components for p neutrons, $S^1_{1/2}$ and $S^1_{3/2}$, do not depend on J.

Using the expression (4.19) of Ref. 61 and the simplification (24.6) of Ref. 62, which simplifies the multiple summations, and averaging over the resonances under the assumptions given above, we can obtain expressions for the coefficients that determine the cross section in the form

$$\overline{\sigma(\vartheta)} = B_0 + B_1 \cos\vartheta + B_2 P_2(\cos\vartheta); \tag{31}$$

$$k^2 B_0 = \sin^2\delta_0 + 2\sin^2\delta_+ + \sin^2\delta_- - f_0 \sin^2\delta_0 - 2f_{3/2}\sin^2\delta_+ - f_{1/2}\sin^2\delta$$

$$+\begin{cases} \frac{1}{2} f_0 F(a_0) + \frac{1}{2} f_{1/2} F(a_{1/2}) + f_{3/2} F(a_{3/2}) & \text{for } I=0; \\ \frac{1}{2} f_0 \sum\limits_{J=0,1} gF(a_0^J) + \frac{1}{8} f_{1/2} F(a^0_{1/2}) + \frac{5}{8} f_{3/2} F(a^2_{3/2}) + \frac{3}{8} f_{1/2} \times G(\rho,\ a^1_{3/2}) & \text{for } I=\frac{1}{2}; \\ \frac{1}{2} f_0 \sum\limits_{J=I-1/2,\ I+1/2} gF(a_0^J) + \frac{1}{2} f_{3/2} \sum\limits_{J=I-3/2,\ I+3/2} gF(a^J_{3/2}) + \frac{1}{2} f_{3/2} \times \sum\limits_{J=I-1/2,\ I+1/2} gG(\rho,\ a^J_{3/2}) & \text{for } I>\frac{1}{2}; \end{cases} \tag{32}$$

$$k^2 B_1 = 2\sin\delta_0 [\sin\delta_- \cos(\delta_0-\delta_-) + 2\sin\delta_+ \cos(\delta_0-\delta_+)] - f_0 [\sin\delta_- \sin(2\delta_0-\delta_-) + 2\sin\delta_+ \sin(2\delta_0-\delta_+)] - \sin\delta_0 [f_{1/2}\sin(2\delta_- - \delta_0) + 2f_{3/2}\sin(2\delta_+ - \delta_0)] \quad \text{for all } I; \tag{33}$$

$$k^2 B_2 = 2\sin^2\delta_+ + 4\sin\delta_+ \sin\delta_- \cos(\delta_+ - \delta_-) + 10\sin\delta_0 \sin\delta_2 \cos(\delta_0 - \delta_2) - 2f_{1/2}\sin\delta_+ \sin(2\delta_- - \delta_+) - 2f_{3/2}[\sin^2\delta_+ + \sin\delta_- \sin(2\delta_+ - \delta_-)]$$

$$+\begin{cases} f_{3/2} F(a_{3/2}) & \text{for } I=0; \\ \frac{5}{2} r f_{3/2} F(a^2_{3/2}) + \frac{3}{2} p f_{3/2} H(\rho,\ q,\ a^1_{3/2}) & \text{for } I=\frac{1}{2}; \\ 2f_{3/2} \sum\limits_{J=I-3/2,\ I+3/2} rgF(a^J_{3/2}) + 2f_{3/2} \sum\limits_{J=I-1/2,\ I+1/2} pgH(\rho,\ q,\ a^J_{3/2}) & \text{for } I>\frac{1}{2}, \end{cases} \tag{34}$$

where

$$g = \frac{2J+1}{2(2I+1)}; \quad p = \frac{2(2J+1)(6J-4I+1)}{5(I+J+1/2)(I+J+3/2)};$$

$$q = \frac{(I-J)(10I-8J+1)}{\sqrt{2(I+J-1/2)(I+J+5/2)}};$$

$$r = \frac{(3J-I+3/2)(3J-I+1/2)}{20(I+J+3/2)(I+J+1/2)}.$$

In the expressions (32)–(34), the neutron strength functions are contained in the factors[8]

$$f_0 = \pi\sqrt{E}\,\frac{S^0}{d_0}, \quad f_j = \pi\sqrt{E}\,\frac{(kR)^2}{1+(kR)^2}\,\frac{S^1_j}{d_{1j}},$$

in which all $d \equiv 1$ for the boundary conditions (29), and in $\rho = S^1_{1/2}/S^1_{3/2}$. The functions F, G, and H were formed in the calculation of the mean values[9] $\langle \Gamma_n^2/\Gamma\rangle$, $\langle (\Gamma_{n\frac{1}{2}} + \Gamma_{n\frac{3}{2}})^2/\Gamma\rangle$ and $\langle (q\Gamma_{n\frac{3}{2}} \pm \sqrt{\Gamma_{n\frac{1}{2}}\Gamma_{n\frac{3}{2}}})^2/\Gamma\rangle$; their argument a is the ratio of the width Γ_γ, assumed to be independent of E, to the mean $\overline{\Gamma}_n$ at the given E. For all possible values of I, l, J, and j, it is expressed as

$$a_0 = \frac{\pi S_\gamma^0}{f_0}; \quad a_{1/2} = \frac{\pi S^1_\gamma}{3f_{1/2}}; \quad a_{3/2} = \frac{2\pi S^1_\gamma}{3f_{3/2}}; \quad a^0_{1/2} = \frac{\pi S^1_\gamma}{9f_{1/2}};$$

$$a^1_{3/2} = \frac{\pi S^1_\gamma}{3f_{3/2}}; \quad a^2_{3/2} = \frac{5\pi S^1_\gamma}{9f_{3/2}}; \quad a^J_0 = \frac{\pi g S^0_\gamma}{f_0}; \quad a^J_{3/2} = \frac{\pi g S^1_\gamma}{2f_{3/2}},$$

where $S^l_\gamma = \Gamma^l_\gamma/D_l$ are the radiative strength functions. Here, the subscripts of a and f are the same: 0 for s-wave resonances and the j value for p-wave resonances. The superscript of a labels the spin J of the resonances (for nuclei with $I=0$ it is omitted, while for p resonances of nuclei having $I=1/2$ the expressions for a are given for each J).

Of the three mean values given above, the first can be found unambiguously, and the function F has the form

$$F(a) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{x\sqrt{x}}{x+a}\, e^{-x/2}\, dx = 1 - a + a\sqrt{\frac{\pi a}{2}}\, e^{a/2}\left[1 - \operatorname{erf}\left(\sqrt{\frac{a}{2}}\right)\right].$$

The other two mean values, which occur, respectively, in B_0 and B_2 and relate to two-channel p resonances (see Sec. 2), can be calculated only if one knows the joint distribution function of the two random variables $\Gamma_{n1/2}$ and $\Gamma_{n3/2}$. Since *a priori* this function is unknown, analysis of the cross sections $\overline{\sigma(\vartheta)}$ for nuclei with $I>0$ is meaningful in two cases: Either the parameters obtained in this analysis do not depend on the distribution function of the two widths, or their dependence on it is so strong that, making different hypotheses about this function, one can choose the one that is most suitable among them. As a criterion for correctness of a particular hypothesis, one can take, for example, identity of the strength functions for even–even and odd nuclei having nearly equal masses. Thus, one can obtain information about the behavior of the mixtures of spin channels discussed in Sec. 2. An analogous but simplified approach to this problem was proposed in Ref. 42.

We consider here, as earlier, two extreme hypotheses. If $\Gamma_{n1/2}$ and $\Gamma_{n3/2}$ are completely independent, then their joint distribution function is the product of two Porter–Thomas distributions, and to this hypothesis there correspond G and H given by

$$G(\rho,\ a) = \frac{1}{2\pi} \int_0^\infty\int_0^\infty \frac{(\rho x + y)^2}{\rho x + y + a}\, \frac{e^{-\frac{1}{2}(x+y)}}{\sqrt{xy}}\, dx\, dy;$$

$$H(\rho,\ q,\ a)=\frac{1}{2\pi}\int_0^\infty\int_0^\infty \frac{q^2y^2+\rho xy\pm 2qy\sqrt{\rho xy}}{\rho x+y+a}\,\frac{e^{-\frac{1}{2}(x+y)}}{\sqrt{xy}}\,dx\,dy.$$

Because the signs of the amplitudes of the widths are random, the third term in the numerator of H must be equal to zero; we retain it for generality, bearing in mind certain unexpected results for the signs of the amplitudes of the proton widths (see the final subsection of Sec. 2 and the references quoted in it). If the signs of the amplitudes did not change from resonance to resonance, we would obtain a third term with a plus or minus sign according as the signs in the channels with different j are the same or opposite.

The second hypothesis is that the two widths are completely correlated and satisfy the same Porter–Thomas distribution but with different mean values $\overline{\Gamma}_{n1/2}$ and $\overline{\Gamma}_{n3/2}$. Then G and H reduce to F:

$$G(\rho,\ a)=(1+\rho)\,F\left(\frac{a}{1+\rho}\right);$$

$$H(\rho,\ q,\ a)=\frac{q^2+\rho\pm 2q\sqrt{\rho}}{1+\rho}\,F\left(\frac{a}{1+\rho}\right).$$

The meaning of the third term in the numerator of H is the same as in the previous case, but its vanishing is here less probable. Thus, of the six possibilities obtained for H, three appear to be the most plausible: a vanishing third term in the first hypothesis and a nonvanishing term with opposite signs in the second.

As in Ref. 59, we introduce in the coefficient B_2 a correction for the neutron d wave—the third term in (34), in which the phase shift δ_2 is calculated in accordance with Eqs. (26) or (30) with $\varphi_2=-x+\tan^{-1}[3x/(3-x^2)]$, $P_2=x^5/(x^4+3x^2+9)$ $(x=kR)$, and $R_2^\infty=R_0^\infty$. This correction to B_2 increases with the energy approximately quadratically and for the majority of nuclei does not exceed 10% at energy 400 keV. Only in the region $60<A<80$, where the R_1^∞ are maximal (the $|\delta_1|$ are small) and the $S^1_{3/2}$ are not large and where, therefore, the B_2 are small, can the correction be tens of percent.

Scattering radii

At low energies, neutron potential scattering can be characterized by a single phase shift $\delta_0=-kR'$, where R', which up to tens of kilo-electron-volts and more is a constant, is called the scattering radius, or length; thus, in the limit $k\to 0$ the cross section takes the simple form

$$\sigma_{\rm pot}=4\pi R'^2. \tag{35}$$

At high energies, it is necessary to introduce many phase shifts δ_{lj}, which have different energy dependences. In the intermediate case, when two or three partial waves are important, the scattering of each of them at different energies can be conveniently characterized for each nucleus by a single constant parameter. In the R-matrix description of the cross sections, such a parameter is R_{lj}^∞, which determines δ_{lj} through (26).

It is convenient to use a more transparent quantity like R' for $l=0$ and weakly dependent on the choice of the channel radius R. For p neutrons, such a quantity was introduced in Ref. 19 and was estimated experimentally for ten elements (see Sec. 3).

Writing the partial cross section of potential scattering in the limit $k\to 0$ in the following form,[59] which generalizes (35),

$$\sigma_{\rm pot}^{lj}=4\pi\frac{(kR)^{4l}}{(2l+1)^2[(2l-1)!!]^4}\,\frac{2j+1}{2}\,R_{lj}'^2,$$

we obtain for the scattering radii with given l and j the expression

$$R'_{lj}=R\left[1-\frac{(2l+1)\,R_{lj}^\infty}{1+(l+b_l)\,R_{lj}^\infty}\right]. \tag{36}$$

Neutrons with $l=0$. The experimental values of R_0^∞,[27] converted to $R_0'=R(1-R_0^\infty$ (as in all that follows, we have used $R=1.35A^{1/3}$ F), exhibit in general reasonable agreement with the data recommended in Ref. 43. The individual discrepancies can be attributed to the natural fluctuations of the potential cross section,[63] which are different in different energy intervals.

Neutrons with $l=1$. Prior to the investigations reviewed here, there were extremely few data on p-wave potential scattering. The reason for this is that in the total cross section the quantity δ_1 begins to be manifested on the background of the overwhelming contribution δ_0 only at energies of hundreds of kilo-electron-volts, when the p resonances acquire an interference asymmetry [the first term in (7), which contains $\Delta E\sin 2\delta_1$, multiplied by 4π], and the potential scattering acquires a p-wave contribution $(\sim\sin^2\delta_1/E)$ that increases as $\sim E^2$. The first of these effects can be used only for light and nearly magic nuclei and in spectrometers with very high resolution.[13,64] It is also worth mentioning Ref. 32, in which from high-precision measurements of the averaged total cross section[10)]

$$\bar\sigma_{\rm tot}=4\pi B_0+\bar\sigma_\gamma=\frac{4\pi}{k^2}(\sin^2\delta_0+3\sin^2\delta_1)$$
$$+\frac{2\pi^2}{k^2}\sqrt{\bar E}\left[S^0\cos 2\delta_0+(S^1_{1/2}+2S^1_{3/2})\frac{(kR)^2\cos 2\delta_1}{1+(kR)^2}\right] \tag{37}$$

for 12 elements in the energy interval 1–600 keV the values of the parameters R_1^∞ and $S^1=(S^1_{1/2}+2S^1_{3/2})/3$ were obtained by using in (37) the known S^0 and $\delta_0=-kR_0'$.

If the spin–orbit splitting of the phase shifts is ignored and the boundary conditions $b_l=-l$ or (29), which give $b_l\to -l$ as $E\to 0$, are used, then Eq. (36) gives for the *p-wave scattering radius*

$$R_1'=R(1-3R_1^\infty), \tag{38}$$

i.e., a more accentuated dependence on A than in the case of the radius R_0' (since the nature of the dependence on A for R_0^∞ and R_1^∞ is the same).

The results of the latest fitting of the five parameters[27] to the data on the differential cross sections,[26,28] made in accordance with Eqs. (31)–(34) for $\delta_+=\delta_-$ with the boundary conditions (29), made it possible to obtain the complete set of p-wave scattering radii shown in Fig. 14. This set contains the results for nuclei from Ti to Te, the black circles belonging to Z-even elements and to ten even–even isotopes of Cd and Sn, and the open circles to Z-odd elements and isotopes of ^{117}Sn and ^{119}Sn. The crosses show the values of R'_1 obtained in accordance with (38) from Ref. 32. All the data were obtained using the formulas for $I = 0$, but nevertheless the open circles do not seem to deviate from the dependence determined by the black circles. This is not surprising if one bears in mind that δ_1 (i.e., R'_1) is determined largely by the term of the cross section B_1, whose form (33) does not depend on the spin of the target (see also Fig. 13).

As one would expect, R'_1, like R'_0 in the regions $30 < A < 100$ and $100 < A < 170$, exhibits a characteristic dispersion shape, this being a consequence of the connection between R^∞_l and the strength function S^l (in the energy scale)[11]:

$$R^\infty_l = \frac{1}{2kR}\sqrt{E}\int_{-\infty}^{+\infty}\frac{S^l(E')}{E'-E}\,dE',$$

where in practice it is sufficient to integrate over one or two of the nearest maxima of S^l. At the maximum of S^1 ($A \simeq 102$), $R^\infty_1 = 0$ and $R'_1 = R$. To the left of this position $R'_l < R$, and to the right $R'_1 > R$; the extremal values of R'_1 correspond to the sections of steepest decrease of S^1.

We draw attention to the negative values of R'_1 in the range $A \simeq 70$–90, which, according to (38), correspond to $R^\infty_1 > 1/3$. They mean that for the nuclei in this mass range the phase shift of the neutron p wave has an anomalous positive sign [at small kR, as is readily seen from (30), since $\delta_1 \simeq -(kR)^2 kR'_1/3$, while for nuclei near the place at which δ_1 changes sign nonresonance p-wave scattering is altogether absent. In our set of nuclei, Ge, Se, and Y have $\delta_1 > 0$,[12] and Ni and Cu have very small $|\delta_1|$.

This curious phenomenon, which has no analog in the case of the s-wave R'_0 and as yet has not been subjected to a serious theoretical analysis, has a simple qualitative explanation if one replaces the scattering nucleus by a rectangular potential well with radius R. If the nucleus is transparent for the neutron wave, then the well depth $V_0 \simeq 50$ MeV is real, and δ_1 for given neutron energy $E \ll V_0$ is determined by KR, where K is the wave number of the neutron within the well (i.e., at energy $E + V_0$). In particular, between the points $KR \simeq 9.095$ and $KR = 3\pi$ we have $\delta_1 > 0$, while outside this interval (and outside the analogous intervals near $n\pi$) δ_1 has a "normal" negative sign. For $V_0 = 46.3$ MeV and $R = 1.35A^{1/3}$ F the point 3π (jump $\delta_1 = \pm\pi/2$, $3p$ level) occurs at $A \simeq 102$, and the point 9.095 ($\delta_1 = 0$, analog of the Ramsauer effect for the p wave) occurs at $A \simeq +92$. As the transparency of the nucleus decreases, i.e., as ζ increases in the complex potential $-V_0(1 + i\zeta)$, this behavior of δ_1 is "smeared," and the regions of anomalous signs of δ_1 become narrower and then disappear altogether. This is illustrated by the two curves in Fig. 14 (we recall that $R'_1 \simeq -3\delta_1/k^3R^2$), calculated by means of (38) and the well-known relation

$$R^\infty_1 \simeq \mathrm{Re}\,(1/f_1),$$

where the expression for f_1, the value of the logarithmic derivative of the wave function at the edge of the well, can be found in Ref. 65. It can be seen that for $\zeta = 0.025$ the curve has the necessary span and that it can be "stretched" along the A axis by rounding the corners of the well and introducing a spin–orbit interaction [the measured dependence $R^\infty_1(A)$ is a superposition $(1/3)R^\infty_- + (2/3)R^\infty_+$ of two identical functions displaced relative to each other by $\Delta A \sim 10$–15].

It should be emphasized that the data in Fig. 14 represent the first *systematic experimental information* on potential p-wave scattering.[13] It can be used to calculate the various neutron cross sections up to hundreds of kilo-electron-volts [for example, for calculating the total cross section in accordance with Eq. (37)]. The fairly smooth dependence of R'_1 (and δ_1) on A makes it possible to use interpolation for all target nuclei in the range $A \sim 50$–130.

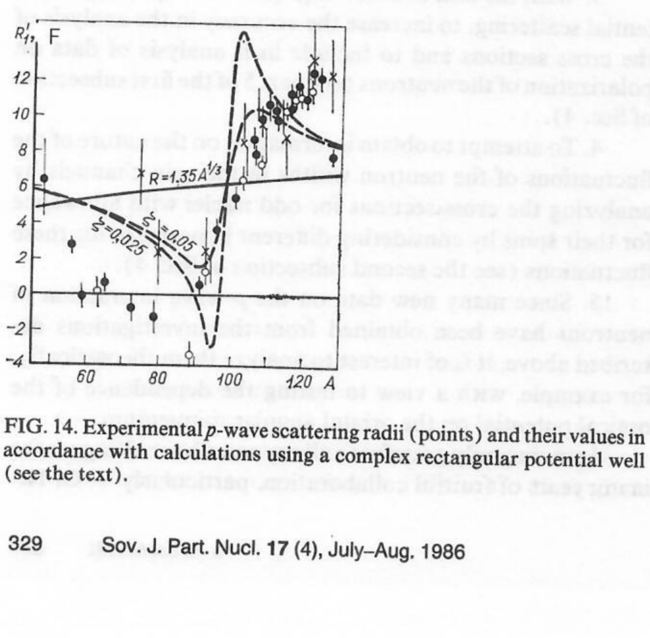

FIG. 14. Experimental p-wave scattering radii (points) and their values in accordance with calculations using a complex rectangular potential well (see the text).

Neutron strength functions

Neutrons with $l = 0$. The information on the strength functions S^0 obtained by analyzing data on the cross sections $\overline{\sigma(\vartheta)}$ is not of great value. First, it is not much compared with the existing information. Second, it refers mainly to mixtures of many isotopes. Third, for nuclei with $A \lesssim 80$ and, possibly, some heavier nuclei, the values of S^0 are too low. The point is that scatterers with transmission 0.90–0.95 are not thin for strong s resonances, and a blocking effect occurs for them. But fixed literature values of S^0 cannot be used because of the blocking and also because of a possible intermediate structure of S^0. Thus, the values of S^0 in Refs. 22, 24, and 26 are to be regarded primarily as "working parameters" for the true determination of R'_0 and $S^1_{1/2}$. Nevertheless, with the exception of the nuclei lighter than Zr, they are close to the literature data.

Neutrons with $l = 1$. The history and state of the problem with regard to the components of the neutron strength function $S^1 = (1/3)S^1_{1/2} + (2/3)S^1_{3/2}$ before the appearance of the reviewed studies are considered in Ref. 31. Many years of measurement of S^1 led to the recognition that the $S^1_{1/2}$ and $S^1_{3/2}$ peaks are shifted in the A scale by an amount ΔA less than their widths ~ 25–30. Moreover, if the spin–orbit part of the potential of the neutron–nucleus system is as in the shell model, then ΔA must be ~ 7–10.

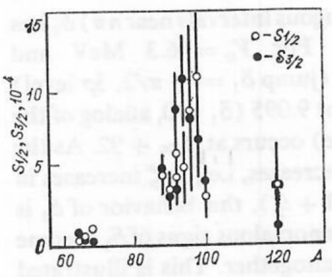

FIG. 15. Components of the strength function S^1 obtained using the parameters of individual resonances.[31]

In accordance with the most recent data of Ref. 43 on the p-resonance spins, the two components of S^1 were determined[31] for 12 even–even target nuclei. The results are shown in Fig. 15, which on account of the inaccuracy and incompleteness of the data does not cast much light on the situation with regard to the spin–orbit splitting of the "strength" of the p resonances. This figure together with some rather diverse data of the type of Refs. 13 and 64 exhausted the information on $S^1_{1/2}$ and $S^1_{3/2}$.

The recent data of Ref. 27, obtained simultaneously with R'_1 (Fig. 14), are shown graphically in Fig. 16. The black circles correspond to spinless targets, and the open circles to targets with $I \neq 0$. As can be seen from the figure, neglect of the spin in the description of the cross sections of the odd nuclei does not lead to large errors,[14)] as in the case of the p-wave scattering radii (see Fig. 14). Therefore, the nature of the fluctuations of the neutron widths in the spin channels (see Sec. 2) is such that the exact expression (34) for B_2 is close to the case $I = 0$. For more definite conclusions, a special analysis is required.

The main result, demonstrated in Fig. 16 and published for the first time in Refs. 24 and 25, consists of the direct observation of *spin–orbit splitting of the maximum of the strength function* for p-wave neutrons; there have been disputes about this since the end of the fifties. The $S^1_{3/2}$ peak around $A = 95$ is described better, and the $S^1_{1/2}$ peak around $A = 110$ less well. As we have already said, the $S^1_{1/2}$ parameters are obtained with less accuracy because of the correlations with R'_0 and S^0 and the possible additional errors due to this reason. Therefore, the values of $S^1_{1/2}$ for Ni and Se are probably overestimated.

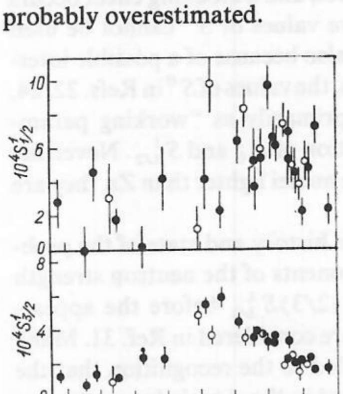

FIG. 16. Experimental values of the components of the p-wave neutron strength function.

Augmenting the data on the differential cross sections by accurate data on the total cross sections will help to make the $S^1_{1/2}$ results more accurate.

CONCLUSIONS

The main results of the investigations which we have described can be briefly formulated as follows:

1. For seven nuclei, the orbital angular momenta have been determined for about 700 neutron resonances, for the majority of which the spins have also been determined.

2. Several new interesting observations have been made: f resonances, mixtures of s and d waves, analog states, a relationship between the p-wave strength functions for different spins, and more (see the last subsection in Sec. 1).

3. The mixtures of spin channels for 12 p-wave resonances have been determined, out of a total number of 17 resonances with known mixtures.

4. Searches have been made for the effect of one-pion exchange, and these have shown that if such an effect is manifested in p-wave scattering, it is not decisive.

5. Systematic information has been obtained for the first time on the phase shifts of p-wave potential scattering of neutrons with energies up to ~400 keV by nuclei in the region $A = 50$–130.

6. For the same nuclei, the neutron strength functions have been determined for the $p_{1/2}$ and $p_{3/2}$ neutrons, leading to the first direct observation of spin–orbit splitting of an unbound single-particle state.

In our view, it would be interesting to continue these investigations in the following directions:

1. To continue the measurements with high energy resolution at different scattering angles in order to determine the spins and mixtures of the spin channels, particularly for spin-$\frac{1}{2}$ nuclei. We recall that such investigations have hitherto been made only with nine nuclei.

2. To extend the measurements of the averaged differential cross sections to nuclei with $A > 130$ in order to investigate the behavior of the scattering radii and the strength functions for p-wave neutrons in the region of deformed nuclei.

3. With the aim of observing spin–orbit splitting in potential scattering, to increase the accuracy in the analysis of the cross sections and to include in it analysis of data on polarization of the neutrons (see part 5 of the first subsection of Sec. 4).

4. To attempt to obtain information on the nature of the fluctuations of the neutron widths in the spin channels by analyzing the cross sections for odd nuclei with allowance for their spins by considering different hypotheses for these fluctuations (see the second subsection of Sec. 4).

5. Since many new data on the p-wave interaction of neutrons have been obtained from the investigations described above, it is of interest to analyze them theoretically, for example, with a view to testing the dependence of the optical potential on the orbital angular momentum.

I am sincerely grateful to all my coauthor colleagues for many years of fruitful collaboration, particularly V. G. Ni-

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[1)]The review does not include some rather old investigations with light nuclei in which analysis of the angular distributions was used to determine the parameters of resonances and the phase shifts at energies 1–5 MeV.

[2)]For the influence of J on the resonance cross section in the case $I > 0$, see also the first subsection of Sec. 2.

[3)]A pure ensemble is formed by *all* levels with given spin J and parity π in a given interval of energies.

[4)]By the number 1 we denote the channel with $s = I - 1/2$ or $j = 1/2$, and by the number 2 the channel with $s_+ = I + 1/2$ or $j = 3/2$, and we omit the index n of the neutral widths.

[5)]The following quantities vary with the scattering angle: 1) the neutron energy and the efficiency of its detection by the detector; 2) the probability of emission of a neutron from a thick sample; 3) the degree of blocking of the potential scattering by the resonance, i.e., it is as if the "hole" in the background below the resonance changes.

[6)]Here and in similar places below, $\sqrt{E}$ is a dimensionless factor, and E is assumed to be expressed in electron volts.

[7)]For $R_l^\infty > 0$ it is necessary to take in (29) the plus sign; for $R_l^\infty < 0$, the minus sign.

[8)]See footnote 6.

[9)]Apart from an unimportant factor, the numerator of the last expression is analogous to the expression $(\rho_-\Gamma_{n^-} - \rho_+\Gamma_{n^+})^2$ in (6) but is expressed in the representation of the total angular momentum of the neutron [see Eq. (16)].

[10)]See footnote 6.

[11)]See footnote 6.

[12)]This feature is reflected in the nature of the scattering. For the Ge, Se, and Y nuclei the forward–backward scattering asymmetry (i.e., the coefficient ω_1) is negative at low energies. With increasing energy, ω_1 becomes positive because of the growth in the positive contribution of the p-wave resonances [see the last term in (33)].

[13)]The previously found[19] values of R_1' (see Fig. 11) are undoubtedly less accurate and reliable since: 1) they were obtained by an approximate method; 2) they relate to a narrow interval 1–40 keV; 3) they were determined using R_0', S^0, and S^1 from different sources.

[14)]The points for ^{93}Nb are a possible exception to this.

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