Microscopic description of collective excitations of rotating nuclei

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Fiz. Elem. Chastits At. Yadra **17**, 613–666 (July–August 1986)

A method is proposed for describing the collective excitations of rotating nuclei that combines the cranking model and the random-phase approximation. The symmetries that are preserved for rotating nuclei and their connection with the physical states are discussed. The results of analysis of the properties of collective excitations in the framework of different models are presented.

INTRODUCTION

The investigation of rotational states provides an important source of information about nuclear structure.[1] Interest in the study of rotational states is still undiminished more than a quarter of a century after the discovery of rotational bands in nuclei.[2] To a large degree, this is due to the progress of experimental techniques, which has made it possible to investigate nuclei in the region of limiting angular momenta (see, for example, Ref. 3) for which the nucleus still exists as a single object.

Many features in the spectra and electromagnetic transitions of rotating nuclei can be explained by means of the generalized nuclear model.[4] It is based on the assumption that it is possible to separate the intrinsic and rotational motions in a nucleus. However, with increasing angular momentum, when the centrifugal and Coriolis forces become large, rotation has a strong influence on the intrinsic degrees of freedom.[3,5–9] At the same time, study of nuclear structure at small values of the angular momentum has shown that the intrinsic motion of the nucleons is determined largely by the average field and by correlations of superconducting type.[10] It is natural to assume that the approximation of an average field is fairly reliable even under extremal conditions when the angular momentum is so great that the fission channel begins to be dominant in the process of nuclear de-excitation. Simple estimates for nuclei in the rare-earth region (the record angular momenta achieved experimentally were obtained in this region[6,11]) show that at maximal spins $I \sim 60\hbar$ for $A = 150$ the rotational energy per nucleon does not exceed 5% of the binding energy (about 0.4–0.6 MeV). One can therefore hope that the conditions governing the majority of the nucleons (the core of the nucleus) are changed very little compared with those that obtain at small spins.[12,13] This is then obviously sufficient to justify the use of the models that are successfully employed to investigate nuclear structure at small angular momenta.

One such model is the cranking model, which was first proposed by Inglis.[14] In this model, it is assumed that the average field of the nucleus rotates with a time-independent angular frequency around a distinguished direction. The cranking model can be used to analyze many properties of rotating nuclei, diagrams of quasiparticle energies determined in the intrinsic coordinate system being used.[15] However, if one remains in the framework of the model, it is possible to analyze states of the yrast line and of single- and two-quasiparticle nature. But the experimental information obtained by analyzing the γ spectra of rapidly rotating de-exciting nuclei indicates a collective nature of the states not only near the yrast line but also at appreciable excitation energies (about 15–20 MeV above the yrast line at $I \sim 60\hbar$; see Ref. 16).

In order to describe the γ spectra near the yrast line, theoretical calculations have been made[17] in which the lowest collective excitations are described as small oscillations of the transverse axes (precession of the rotation axis) of a triaxial rotator.[1] It is obvious that such a treatment is phenomenological in nature. It is therefore worth developing microscopic approaches. One such approach is to use a self-consistent cranking model and the random-phase approximation (RPA). This approach was proposed for the first time in Refs. 18 and 19. One should also mention other microscopic approaches[20–22] based on the use of a generalized density matrix. However,they have been less developed than the model described below.

Our aim in this review is to describe the method that combines the cranking model with the RPA and its applications to the calculation of specific characteristics of a number of nuclei. In Sec. 1 we describe the method with allowance for the influence of the symmetry conditions on the properties of the physical states. Then, in Sec. 2, taking a simple model of a rotating anisotropic oscillator, we investigate the properties of coherent excitations: isoscalar quadrupole excitations and isovector dipole excitations. Finally, in Sec. 3 we use the combination of the cranking model and the RPA (which we call the CM + RPA method) to describe the various characteristics of a number of nuclei. In the conclusions, we briefly summarize the results. Appendix A gives some properties of the operators used in the models.

 0090-4759/86/040265-25$04.40

1. CRANKING MODEL AND RANDOM-PHASE APPROXIMATION

The cranking-model Hamiltonian and its symmetries

As already noted in the Introduction, it is assumed in the cranking model that the nucleus rotates around a distinguished direction; usually, this is the x axis of the intrinsic and laboratory coordinate systems. The Hamiltonian corresponding to such rotation, expressed in the rotating coordinate system, has the form[1)]

$$H' = H - \sum_{\tau} \lambda_{\tau} \hat{N}_{\tau} - \Omega \hat{J}_x, \tag{1}$$

where H is the total nuclear Hamiltonian[2)]; $\hat{N}_{\tau}$ is the operator of the number of protons ($\tau = Z$) or neutrons ($\tau = N$); λ_{τ} is the corresponding chemical potential; Ω is the angular frequency of the rotation; and $\hat{J}_x$ is the x component of the total angular momentum. The Hamiltonian H contains intrinsic and collective degrees of freedom (associated with the rotation and the center-of-mass motion of the nucleus). In the general case, their separation is a nontrivial problem (see, for example, Refs. 20 and 25) that usually requires an approximation at some stage.

Of the Hamiltonian it is natural to require rotational and translational invariance. In addition, in the phenomena which we consider the number of particles does not change. Therefore,

$$[H, \hat{J}_i] = [H, \hat{P}_i] = [H, \hat{N}_{\tau}] = 0, \quad (i = x, y, z) \tag{2}$$

where $\hat{J}_i$ and $\hat{P}_i$ are the components of the total angular momentum and of the total momentum of the nucleus in the laboratory system. Since

$$\left.\begin{aligned} &[\hat{J}_i, \hat{P}_j] = i\varepsilon_{ijk}\hat{P}_k\,[\hat{J}_i, \hat{J}_j] = i\varepsilon_{ijk}\hat{J}_k; \\ &[\hat{N}_{\tau}, \hat{J}_i] = [\hat{N}_{\tau}, \hat{P}_i] = [\hat{P}_i, \hat{P}_j] = 0, \end{aligned}\right\} \tag{3}$$

it follows from the condition (2) for the cranking-model Hamiltonian that

$$\left.\begin{aligned} &[H', \hat{J}_x] = 0;\ [H', \hat{P}_x] = 0; \\ &[H', \hat{J}_y] = -i\Omega\hat{J}_z;\ [H', \hat{P}_y] = -i\Omega\hat{P}_z\ [H', \hat{N}_{\tau}] = 0; \\ &[H', \hat{J}_z] = i\Omega\hat{J}_y;\ [H'\ \hat{P}_z] = i\Omega\hat{P}_y. \end{aligned}\right\} \tag{4}$$

In the total nuclear Hamiltonian H, one usually separates the average field, which in explicit calculations is approximated by some phenomenological potential, this leading to breaking of the symmetry conditions (2). To restore the broken symmetries, it is necessary to impose some conditions on the residual interaction. If this is not done, the eigenvectors of the total Hamiltonian describing the intrinsic excitations of the nucleus acquire unphysical (from the point of view of the intrinsic degrees of freedom) components of a collective nature (see, for example, Refs. 26 and 29). Since, as a rule, the symmetries are restored in the framework of the approximation employed to find the eigenmodes of the Hamiltonian, we shall require below that the condiitons (2) hold in the framework of the RPA.

For simplicity of exposition, we shall use a spherically symmetric average field. We take the residual interaction in the form of separable multipole–multipole forces (long-range part) and monopole pairing (short-range part).[3)] Note that this choice of the average field and of the residual interactions in no way affects the generality of our presentation of the CM + RPA method. In the various applications of this method discussed below, the form of the average field and of the residual interactions is particularized, but all the calculations will be made in the framework of the scheme explained in this section.

Thus, we consider a nuclear Hamiltonian of the form

$$H = \sum_{k} e_k c_k^+ c_k - \frac{1}{4} \sum_{\tau} G_{\tau} \hat{P}_{\tau}^+ \hat{P}_{\tau} - \frac{1}{2} \sum_{\lambda=1,\,2,\,3\,\ldots} \sum_{m=-\lambda}^{\lambda} \varkappa_{\lambda} \hat{Q}_{\lambda m}^+ \hat{Q}_{\lambda m}, \tag{5}$$

where[4)]

$$\hat{P}_{\tau}^+ = \sum_{k \in \tau} c_k^+ c_{\bar{k}}^+; \quad \hat{Q}_{\lambda m} = \sum_{kl} q_{kl}^{\lambda m} c_k^+ c_{\bar{l}}, \qquad q_{kl}^{\lambda m} = \langle k \mid r^{\lambda} Y_{\lambda m} \mid l \rangle; \tag{6}$$

c_k^+ and c_k are, respectively, the operators of creation and annihilation of a particle in the state $|k\rangle$ of the spherical nuclear field, and $c_{\bar{k}}^+ = T c_k^+ T^{-1}$ (T is the operator of time reversal[31]); $\varkappa_{\lambda}$ and G_{τ} are, respectively, the coupling constants of the multipole–multipole and pairing interactions ($\tau = N, Z$). In (6), the summation is not only over the indices l and k but also over $\bar{l}$ and $\bar{k}$.

The experiments indicate that the majority of nuclei in the ground state and in the states of the yrast band possess the $\hat{P}$, $\hat{R}_k(\pi)$, and $\hat{S}_k \equiv \hat{P}\hat{R}_k^{-1}(\pi)$ symmetries ($\hat{P}$ is the operator of the intrinsic parity, and $\hat{R}_k(\pi) = \exp(-i\pi\hat{J}_k)$ is the operator of rotation around the k axis of the intrinsic coordinate system through angle π), i.e., deformed nuclei have deformations of even multipolarities. Because of the presence of the term $\Omega\hat{J}_k$, only the symmetries P, $R_1(\pi)$, and S_1 hold for the cranking-model Hamiltonian (1). Therefore, following Goodman,[32,33] we choose the basis of single-particle states by using the condition that these states be eigenvectors of the operator $\hat{R}_1^{-1}(\pi) = \hat{R}_x^{-1}(\pi)$:

$$e^{i\pi\hat{J}_x}\begin{pmatrix} c_k^+ \\ c_{\bar{k}}^+ \end{pmatrix} e^{-i\pi\hat{J}_x} = \mp i \begin{pmatrix} c_k^+ \\ c_{\bar{k}}^+ \end{pmatrix}. \tag{7}$$

Many experimental and theoretical studies (see, for example, Refs. 34–39 and the references in them) have demonstrated the possibility of a stable octupole deformation of a nucleus that breaks the intrinsic mirror symmetry of the nucleus. In this case, there is breaking of the P and $R_x(\pi)$ symmetries separately, but the S_x symmetry remains. The generalization of the CM + RPA method to the case of stable octupole deformation and the physical consequences are considered in Ref. 40.

As is shown by RPA calculations, if rotation is absent (see, for example, Ref.41), the quadrupole and octupole excitations play the most important part in the description of the observed spectrum. Therefore, we shall below restrict

ourselves in the Hamiltonian (5) to considering operators with multipolarity λ equal to 2 and 3. From the point of view of the intrinsic R_1 symmetry of the nucleus, we introduce the following combinations of multipole operators:

$$\left.\begin{aligned}
&\hat{Q}_0^{(+)}=\hat{Q}_{20};\ \hat{Q}_1^{(+)}=\frac{\mathrm{i}}{\sqrt{2}}(\hat{Q}_{21}+\hat{Q}_{2-1});\ \hat{Q}_1^{(-)}=\frac{1}{\sqrt{2}}(\hat{Q}_{21}-\hat{Q}_{2-1});\\
&\hat{Q}_2^{(+)}=\frac{1}{\sqrt{2}}(\hat{Q}_{22}+\hat{Q}_{2-2});\ \hat{Q}_2^{(-)}=\frac{\mathrm{i}}{\sqrt{2}}(\hat{Q}_{22}-\hat{Q}_{2-2});\\
&\hat{F}_0^{(-)}=\hat{Q}_{30};\ \hat{F}_1^{(+)}=\frac{1}{\sqrt{2}}(\hat{Q}_{31}-\hat{Q}_{3-1});\ \hat{F}_1^{(-)}=\frac{\mathrm{i}}{\sqrt{2}}(\hat{Q}_{32}+\hat{Q}_{3-1});\\
&\hat{F}_2^{(+)}=\frac{\mathrm{i}}{\sqrt{2}}(\hat{Q}_{32}-\hat{Q}_{3-2});\ \hat{F}_2^{(-)}=\frac{1}{\sqrt{2}}(\hat{Q}_{32}+\hat{Q}_{3-2});\\
&\hat{F}_3^{(+)}=\frac{1}{\sqrt{2}}(\hat{Q}_{33}-\hat{Q}_{3-3});\ \hat{F}_3^{(-)}=\frac{\mathrm{i}}{\sqrt{2}}(\hat{Q}_{33}-\hat{Q}_{3-3}),
\end{aligned}\right\} \tag{8}$$

which have the property

$$\mathrm{e}^{\mathrm{i}\pi\hat{J}_x}\begin{bmatrix}\hat{Q}_m^{(\pm)}\\ \hat{F}_m^{(\pm)}\end{bmatrix}\mathrm{e}^{-\mathrm{i}\pi\hat{J}_x}=\pm\begin{bmatrix}\hat{Q}_m^{(\pm)}\\ \hat{F}_m^{(\pm)}\end{bmatrix}. \tag{8a}$$

For what follows, it is convenient to represent the Hamiltonian (5) in the form

$$\begin{aligned}
H=&\sum e_k c_k^+ c_k-\frac{1}{4}\sum_\tau G_\tau \hat{P}_\tau^+\hat{P}_\tau\\
&-\frac{\varkappa_2}{2}\Big[\sum_{m=0}^{2}\hat{Q}_m^{(+)}\hat{Q}_m^{(+)}+\sum_{m=1}^{2}\hat{Q}_m^{(-)}\hat{Q}_m^{(-)}\Big]\\
&-\frac{\varkappa_3}{2}\Big[\sum_{m=1}^{3}\hat{F}_m^{(+)}F_m^{(+)}+\sum_{m=0}^{3}\hat{F}_m^{(-)}\hat{F}_m^{(-)}\Big].
\end{aligned} \tag{9}$$

Calculations in the CM + RPA method consist of two individual stages. First, one solves a Hartree–Fock–Bogolyubov (HFB) problem in the cranking-model approximation (see, for example, the review of Ref. 42 and the references there). This gives us a quasiparticle spectrum for the given rotational frequency Ω. The quasiparticle vacuum $|\Omega\rangle$ characterizes the state of the nucleus on the yrast line with spin I that corresponds to the mean value of the operator $\hat{J}_x$ with respect to the state $|\Omega\rangle$ af the given rotational frequency Ω. Second, in the RPA we seek the vibrational modes with respect to the states of the yrast line (see, for example, Refs. 18, 19, 24, 29, and 43–52). Below, both of these stages will be considered in detail. We note also that the entire treatment presented here applies to the case of even–even nuclei.

Hartree–Fock–Bogolyubov method for rotating nuclei (description of the yrast line)

The HFB method can be successfully used to describe many characteristics of rotating nuclei[53–59] and is described in detail in Ref. 42 (see also Ref. 60). We therefore give only the main ideas needed to understand what follows.

Using a Bogolybuov transformation, we go over to the quasiparticle operators α_i, α_i^+:

$$\alpha_i^+=\sum_k(A_k^i c_k^+ + B_{\bar{k}}^i c_{\bar{k}});\quad \alpha_{\bar{i}}^+=\sum_k(A_k^{\bar{i}}c_k^+ + B_k^{\bar{i}}c_k), \tag{10}$$

where by virtue of (7) the quasiparticle states are also eigenstates of the operator $\hat{R}_1^{-1}(\pi)$:

$$\mathrm{e}^{\mathrm{i}\pi\hat{J}_x}\begin{pmatrix}\alpha_i^+\\ \alpha_{\bar{i}}^+\end{pmatrix}\mathrm{e}^{-\mathrm{i}\pi\hat{J}_x}=\mp\mathrm{i}\begin{pmatrix}\alpha_i^+\\ \alpha_{\bar{i}}^+\end{pmatrix}. \tag{11}$$

The quasiparticle energies E_i, $E_{\bar{i}}$ and the transformation coefficients are determined from the solution of the equations[5)]

$$\begin{aligned}
&M\begin{pmatrix}A_k^i\\ B_{\bar{k}}^i\end{pmatrix}=E_i\begin{pmatrix}A_k^i\\ B_{\bar{k}}^i\end{pmatrix};\\
&M\begin{pmatrix}B_k^{\bar{i}}\\ A_{\bar{k}}^{\bar{i}}\end{pmatrix}=-E_{\bar{i}}\begin{pmatrix}B_k^{\bar{i}}\\ A_{\bar{k}}^{\bar{i}}\end{pmatrix};\quad M=\begin{pmatrix}h^{(1)} & \Delta\\ \Delta^+ & h^{(2)}\end{pmatrix},
\end{aligned} \tag{12}$$

in which the corresponding matrix elements have the form[6)]

$$\left.\begin{aligned}
&h_{kl}^{(1)}=\delta_{kl}(e_k-\lambda_\tau)-\Omega\langle k\mid\hat{J}_x\mid l\rangle\\
&\quad-\varkappa_2\langle\Omega\mid\hat{Q}_{20}\mid\Omega\rangle\langle k\mid\hat{Q}_{20}\mid l\rangle\\
&-\varkappa_2\langle\Omega\mid\hat{Q}_2^{(+)}\mid\Omega\rangle\langle k\mid\hat{Q}_2^{(+)}\mid l\rangle;\\
&h_{\bar{k}\bar{l}}^{(2)}=-\delta_{\bar{k}\bar{l}}(e_{\bar{k}}-\lambda_\tau)-\Omega\langle\bar{k}\mid\hat{J}_x\mid\bar{l}\rangle\\
&+\varkappa_2\langle\Omega\mid\hat{Q}_{20}\mid\Omega\rangle\langle\bar{k}\mid\hat{Q}_{20}\mid\bar{l}\rangle+\varkappa_2\langle\Omega\mid\hat{Q}_2^{(+)}\mid\Omega\rangle\langle\bar{k}\mid\hat{Q}_2^{(+)}\mid\bar{l}\rangle;\\
&\Delta_{k\bar{l}}=-\frac{G_\tau}{4}\langle\Omega\mid\hat{P}_\tau\mid\Omega\rangle\delta_{k\bar{l}}.
\end{aligned}\right\} \tag{13}$$

Here, $\langle k\mid\hat{A}\mid l\rangle$ is the single-particle matrix element of the corresponding operator, and $\langle\Omega|\hat{A}|\Omega\rangle$ is the expectation value of the corresponding operator with respect to the quasiparticle vacuum $|\Omega\rangle$, i.e., with respect to the yrast-line state at the definite rotational frequency Ω. The method of determining the expectation values of the single-particle operators is given in Appendix A. The self-consistent equations (12) and (13) must be augmented by the conditions

$$\langle\Omega\mid N_{\tau=N}\mid\Omega\rangle=N_0,\quad\langle\Omega\mid\hat{N}_{\tau=Z}\mid\Omega\rangle=Z_0,\quad\langle\Omega\mid\hat{J}_x\mid\Omega\rangle=J_0 \tag{14}$$

for a nucleus with N_0 neutrons and Z_0 protons that at the given angular frequency are in a state with spin J_0.

The solution of the HFB problem makes it possible to rewrite the cranking-model Hamiltonian (1) in the form

$$\begin{aligned}
H'=&\langle\Omega\mid H'\mid\Omega\rangle+\sum_i E_i(\alpha_i^+\alpha_i+\alpha_{\bar{i}}^+\alpha_{\bar{i}})\\
&-\sum_\tau\frac{G_\tau}{4}:(\hat{P}_\tau-\langle\Omega\mid\hat{P}_\tau\mid\Omega\rangle)^+(\hat{P}_\tau-\langle\Omega\mid\hat{P}_\tau\mid\Omega\rangle):\\
&-\frac{\varkappa_2}{2}\sum_{m=0}^{2}:(\hat{Q}_m^{(+)}-\langle\Omega\mid\hat{Q}_m^{(+)}\mid\Omega\rangle)(\hat{Q}_m^{(+)}-\langle\Omega\mid\hat{Q}_m^{(+)}\mid\Omega\rangle):\\
&-\frac{\varkappa_2}{2}\sum_{m=1}^{2}:\hat{Q}_m^{(-)}\hat{Q}_m^{(-)}:-\frac{\varkappa_3}{2}\Big[\sum_{m=0}^{3}:\hat{F}_m^{(-)}\hat{F}_m^{(-)}:+\sum_{m=1}^{3}:\hat{F}_m^{(+)}\hat{F}_m^{(+)}:\Big],
\end{aligned} \tag{15}$$

where the symbol : : denotes the normal product with respect to the quasiparticle vacuum $|\Omega\rangle$. Since the Hamiltonian (1) is invariant with respect to the $\hat{R}_1(\pi)$ transformation and commutes with the parity operator $\hat{P}$, the quasiparticle vacuum can be determined as follows

$$\mathrm{e}^{\mathrm{i}\pi\hat{J}_x}\mid\Omega\rangle=\hat{P}\mid\Omega\rangle=\mid\Omega\rangle. \tag{16}$$

Therefore, nonvanishing expectation values $\langle\Omega|\hat{A}|\Omega\rangle$ are obtained only for operators that do not change the parity and have positive signature.[7)]

It should be noted that the consistent solution of Eqs. (12) and (13) under the conditions (14) is quite difficult. Usually, the average field is approximated by a phenomenological nuclear potential of the type of the Nilsson potential (see, for example, Refs. 53 and 57) or the Woods–Saxon potential (see Refs. 56 and 61), the parameters of which are determined by the requirement that the experimental single-particle characteristics in the ground state be reproduced. Further, using Strutinskiĭ's method for a rotating nucleus,[12,57] we find the minimum of the deformation energy of the nucleus at the given rotational frequency in order to find the yrast-line state. For this, consistency with respect to rotation and pairing is obtained.

As was already noted, the phenomenologically chosen deformed field breaks the symmetries (2) and (4). To restore them, it is necessary to choose a residual interaction. The matching of the residual interaction to the form of the phenomenological potential of the average field has been discussed in detail in a number of studies (see, for example, Refs. 26–28 and 62–66).

Random phase approximation for rotating nuclei

Hamiltonian of the RPA *and equations of motion*

The combination of the RPA with the HFB method for describing the properties of the states of rotating nuclei was first proposed by Marshalek (see Ref. 18, 30, 43, 67, and 68) and by Mikkhaĭlov and Janssen.[19,44] The basic ideas of the CM + RPA approach were formulated in these studies. Its further development is associated with the construction of definite models for quantitative and qualitative study of nuclear characteristics (Refs. 69, 45–52, and 70–76) and also with refinement of the theoretical scheme with a view to restoration of the symmetries of the total Hamiltonian[24,29] and the description of the RPA solutions by means of the strength-function method (Ref. 77).[8)] As already noted, the possibility of generalizing this approach to the case of an octupole deformation of the average nuclear field was considered in Ref. 40.

In the CM + RPA approach, the nuclear states near the yrast line are described by means of phonons, which at given rotational frequency Ω are represented by a linear combination of two-quasiparticle bonsons (see, for example, Ref. 43):

$$b^+_{k\bar{l}} = \alpha^+_k\alpha^+_{\bar{l}}; \quad b^+_{kl} = i\alpha^+_k\alpha^+_l; \quad b^+_{\bar{k}\bar{l}} = i\alpha^+_{\bar{k}}\alpha^+_{\bar{l}}; \tag{17a}$$

$$\left.\begin{aligned}\alpha^+_k\alpha_l &= \sum_m (b^+_{km}b_{lm} + b^+_{k\bar{m}}b_{l\bar{m}});\\ \alpha^+_k\alpha_{\bar{l}} &= i\sum_m (b^+_{k\bar{m}}b_{\bar{l}m} - b^+_{km}b_{\bar{l}\bar{m}});\end{aligned}\right\} \tag{17b}$$

these are antisymmetric with respect to the indices, $b_{ik} = -b_{ki}$, and satisfy the approximate commutation relations

$$\left.\begin{aligned}&[b_{kl}, b^+_{mn}] = \delta_{km}\delta_{ln} - \delta_{kn}\delta_{lm}; \quad [b_{k\bar{l}}, b^+_{m\bar{n}}] = \delta_{km}\delta_{\bar{l}\bar{n}};\\ &[b_{kl}, b_{mn}] = [b_{kl}, b^+_{m\bar{n}}] = 0.\end{aligned}\right\} \tag{18}$$

At the same time, it is assumed that the average number of quasiparticles with respect to the vacuum state $|\Omega\rangle$ is zero.

$$\langle\Omega \mid \alpha^+_k\alpha_l \mid \Omega\rangle \approx 0.$$

From the relation (11) for the bosons it follows that

$$e^{i\pi\hat{J}_x}\begin{bmatrix} b^+_{i\bar{k}(\bar{i}\bar{k})} \\ b^+_{i\bar{k}}\end{bmatrix} e^{-i\pi\hat{J}_x} = \mp\begin{bmatrix} b^+_{i\bar{k}(\bar{i}\bar{k})} \\ b^+_{i\bar{k}}\end{bmatrix}. \tag{19}$$

Any single-particle operator contained in the Hamiltonian (15) can be expressed in the form of an expansion with respect to bosons of the type (17). The specific form of this expansion depends on the symmetry of the operator with respect to Hermitian conjugation, time reversal, the operation $R_1(\pi)$, etc. (see Appendix A).

Introducing the expansion (17) in the cranking-model Hamiltonian (15), we obtain its boson representation. The presence of the $R_x(\pi)$ symmetry of the Hamiltonian and, accordingly, the fulfillment of the commutation relations (18) makes it possible to divide the cranking-model Hamiltonian into four mutually commuting parts:

$$H' = \langle\Omega \mid H' \mid \Omega\rangle + H^{(+)}_{(+)} + H^{(-)}_{(+)} + H^{(+)}_{(-)} + H^{(-)}_{(-)}, \tag{20}$$

where

$$H^{(+)}_{(+)} = \sum_{ik} E_{i\bar{k}}b^+_{i\bar{k}}b_{i\bar{k}} - \frac{1}{4}\sum_\tau G_\tau P^+_\tau(1)\,P_\tau(1) - \frac{\varkappa_2}{2}\sum_{m=0}^{2} Q^{(+)}_m(1)\,Q^{(+)}_m(1); \tag{21a}$$

$$H^{(-)}_{(+)} = \sum_{ik}\frac{1}{2}(E_{ik}b^+_{ik}b_{ik} + E_{\bar{i}\bar{k}}b^+_{\bar{i}\bar{k}}b_{\bar{i}\bar{k}}) - \frac{\varkappa_2}{2}\sum_{m=1}^{2} Q^{(-)}_m(1)\,Q^{(-)}_m(1); \tag{21b}$$

$$H^{(+)}_{(-)} = \sum_{ik} E_{i\bar{k}}b^+_{i\bar{k}}b_{i\bar{k}} - \frac{\varkappa_3}{2}\sum_{m=1}^{3} F^{(+)}_m(1)\,F^{(+)}_m(1); \tag{21c}$$

$$H^{(-)}_{(-)} = \sum_{ik}\frac{1}{2}(E_{ik}b^+_{ik}b_{ik} + E_{\bar{i}\bar{k}}b^+_{\bar{i}\bar{k}}b_{\bar{i}\bar{k}}) - \frac{\varkappa_3}{2}\sum_{m=0}^{3} F^{(-)}_m(1)\,F^{(-)}_m(1). \tag{21d}$$

Here, $E_{ik} = E_i + E_k$ and the symbols $P_\tau(1)$, $Q^{(\pm)}_m(1)$, $F^{(\pm)}_m(1)$ represent the parts of the corresponding operators linear in the bosons (see Appendix A). The subscript of the symbol $H^{\sigma=\pm}_{P=\pm}$ characterizes the parity of the single-particle operators that determine the Hamiltonian; the superscript characterizes their signature.

Substituting the boson expansion (A5) in the symmetry relations (4), we obtain in the framework of the RPA

$$[H^{(+)}_{(+)}, J_x(1)] = [H^{(+)}_{(+)}, N_\tau(1)] = 0; \tag{22}$$

$$[H^{(-)}_{(+)}, J_y(1)] = -i\Omega J_z(1); \quad [H^{(-)}_{(+)}, J_z(1)] = i\Omega J_y(1); \tag{23}$$

$$[H^{(+)}_{(-)}, P_x(1)] = 0; \tag{24}$$

$$[H^{(-)}_{(-)}, P_y(1)] = -i\Omega P_z(1); \quad [H^{(-)}_{(-)}, P_z(1)] = i\Omega P_y(1); \tag{25}$$

$$[J_i(1), P_j(1)] = [J_i(1), N_\tau(1)] = [P_i(1), N_\tau(1)] = [P_i(1), P_j(1)] = [J_x(1), J_y(1)] = [J_x(1), J_z(1)] = 0; \tag{26}$$

$$[J_y(1), J_z(1)] = i\langle\Omega \mid \hat{J}_x \mid \Omega\rangle. \tag{27}$$

Since all four parts of the Hamiltonian (20) commute with each other, the RPA equations of motion[79]

$$[H', \mathscr{P}_\nu] = i\omega_\nu^2 X_\nu; \quad [H', X_\nu] = -i\mathscr{P}_\nu; \quad [X_\nu, \mathscr{P}_{\nu'}] = i\delta_{\nu\nu'} \tag{28}$$

(X_ν and $\mathscr{P}_\nu$ are the generalized coordinate and generalized momentum of the given state ν with energy ω_ν) can be solved independently for each of them.

The Hamiltonian H', expressed in terms of the canonical variables X_ν and $\mathscr{P}_\nu$, takes the form

$$H' = \frac{1}{2}\sum_\nu (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) = \frac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) + \frac{1}{2}\sum_{\substack{\nu_0 \\ \omega_{\nu_0}=0}} \mathscr{P}_{\nu_0}^2$$

$$= \sum_{\substack{\nu \\ \omega_\nu \neq 0}} \omega_\nu \left(O_\nu^+ O_\nu + \frac{1}{2}\right) + \frac{1}{2}\sum_{\substack{\nu_0 \\ \omega_{\nu_0}=0}} \mathscr{P}_{\nu_0}^2. \tag{29}$$

Here, the phonon operator

$$O_\nu^+ = \frac{1}{\sqrt{2}}\left(\sqrt{\omega_\nu} X_\nu - \frac{i}{\sqrt{\omega_\nu}}\mathscr{P}_\nu\right) \tag{30}$$

corresponds to nonzero solutions $\omega_\nu \neq 0$. The equations of motion (28) for $\omega_\nu \neq 0$ take the form

$$[H', O_\nu^+] = \omega_\nu O_\nu^+; \quad [H', O_\nu] = -\omega_\nu O_\nu; \quad [O_\nu, O_{\nu'}^+] = \delta_{\nu\nu'}. \tag{31}$$

Comparing the RPA equations (28) or (31) with the symmetry conditions (22)–(25), we can determine all the "unphysical" (ghost) modes of Goldstone type (see Ref. 79) for all four parts of the Hamiltonian (20):

1. It follows from comparison of (22) with (28) that the RPA solutions of the Hamiltonian $H^{(+)}_{(+)}$ include one associated with the operator $J_x(1)$ and two associated with the operator $N_\tau(1)$ ($\tau = N, Z$). Therefore,

$$\left.\begin{aligned} \mathscr{P}_{\nu_0 = J_x} &= \sqrt{g_{J_x}}\, J_x(1) \\ \mathscr{P}_{\nu_0 = N_\tau} &= \sqrt{g_{N_\tau}}\, N_\tau(1) \end{aligned}\right\} \to H^{(+)}_{(+)} = \frac{1}{2}\sum_{\omega_\nu \neq 0} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2)$$
$$+ \frac{1}{2} g_{J_x} J_x^2(1) + \frac{1}{2}\sum_\tau g_{N_\tau} N_\tau^2(1). \tag{32}$$

The method of determining the mass parameters g_{J_x} and g_{N_τ} is discussed below. It follows from (32) that $\mathscr{J}_x = 1/g_x$ is the moment of inertia of the nucleus with respect to the rotation axis x.

2. Comparison of (23) with (28) shows that from the operators $J_y(1)$ and $J_z(1)$ it is possible to construct a mode of the Hamiltonian $H^{(-)}_{(+)}$ with energy $\omega = \Omega$:

$$\left.\begin{aligned} &[H^{(-)}_{(+)}, \Gamma^+] = \Omega\Gamma^+ \\ &[H^{(-)}_{(+)}, \Gamma] = -\Omega\Gamma \\ &[\Gamma, \Gamma^+] = 1 \end{aligned}\right\} \to H^{(-)}_{(+)}$$
$$= \frac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0, \Omega}} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) + \Omega\left(\Gamma^+\Gamma + \frac{1}{2}\right), \tag{33}$$

where we have introduced the operator $\Gamma^+ = (J_y(1) - iJ_z(1))/\sqrt{\langle\Omega|\hat{J}_x|\Omega\rangle}$.

3. Comparison of (24) with (28) leads to the expression

$$\mathscr{P}_{\nu_0 = P_x} = \sqrt{g_{P_x}}\, P_x(1) \to H^{(+)}_{(-)}$$
$$= \frac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) + \frac{1}{2} g_{P_x} P_x^2(1). \tag{34}$$

Here, $M = 1/g_{P_x}$ is the effective mass of the nucleus.

4. From comparison of (25) with (28) one can expect that the mode $[P_y(1), P_z(1)]$ is a solution of the Hamiltonian $H^{(-)}_{(-)}$ with energy $\omega = \Omega$. However, because of the commutation $[P_y(1), P_z(1)] = 0$ it is not possible to construct the corresponding phonon operator by means of these operators. One can show that the mode due to the momentum components $P_y(1)$ and $P_z(1)$ [and the corresponding coordinates $X_y(1)$ and $X_z(1)$] is orthogonal to all solutions of the RPA equations and therefore is not mixed to them.[24] Therefore, the Hamiltonian $H^{(-)}_{(-)}$, defined on the space of its solutions, has the form

$$H^{(-)}_{(-)} = \frac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0}} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) + \Omega(X_y(1) P_z(1) - X_z(1) P_y(1)), \tag{35}$$

where the last term in (35) ensures fulfillment of the conditions (25). Thus, using the symmetry conditions imposed on the cranking-model Hamiltonian, one can separate in the RPA the modes due to the rotation of the nucleus and the motion of the center of mass and the modes associated with conservation of the particle number.

In Refs. 19, 44, 45, 47, 49, and 50, solutions of the RPA equations are determined for the Hamiltonian of the purely intrinsic excitations. We shall demonstrate the connection between the solutions for the intrinsic excitations of the Hamiltonian satisfying all the symmetries and the cranking-model Hamiltonian (see Ref. 40). Formally, the Hamiltonian that describes only the intrinsic excitations can be written in the form

$$H'' = H - \mu J^2 - bP^2 - \sum_\tau a_\tau N_\tau^2, \tag{36}$$

where from the total nuclear Hamiltonian H we have separated the terms responsible for the collective motion (rotation and motion of the center of mass) and those associated with the conservation of the particle number.[9] The coefficients μ, b, a_τ are certain inertial parameters.

Between the Hamiltonians H' and H'' it is easy to establish a connection by comparing (1) and (36). To second order in the bosons [see (A5)], we have

$$H'' = H' + \Omega\hat{J}_x + \sum_\tau \lambda_\tau \hat{N}_\tau - \mu(\langle\hat{J}_x\rangle + J_x(1) + J_x(2))^2$$
$$- \mu(J_y(1) + J_y(2))^2 - \mu(J_z(1) + J_z(2))^2$$
$$- \sum_\tau a_\tau(\langle\hat{N}_\tau\rangle + N_\tau(1) + N_\tau(2))^2 - b(P_x(1) + P_x(2))^2$$
$$- b(P_y(1) + P_y(2))^2 - b(P_z(1) + P_z(2))^2, \tag{37}$$

where $\langle\hat{A}\rangle \equiv \langle\Omega|\hat{A}|\Omega\rangle$. Ignoring in (37) terms of the type $J_x(1)J_x(2)$ and $J_x(2)J_x(2)$ and using the relations (20), (29), and (32)–(35), we obtain

$$H'' = \langle\Omega|H'|\Omega\rangle + \frac{1}{2}\sum_{\substack{\nu \\ \omega_\nu \neq 0, \Omega}} (\mathscr{P}_\nu^2 + \omega_\nu^2 X_\nu^2) + \frac{1}{2} g_{J_x} J_x^2(1)$$

$$+\frac{1}{2}\sum_{\tau} g_{N_\tau} N_\tau^2(1)+\Omega\left(\Gamma^+\Gamma+\frac{1}{2}\right)+\frac{1}{2} g_{P_x} P_x^2(1)$$
$$+\Omega\left(X_y(1)\,P_z(1)-X_z(1)\,P_y(1)\right)+\Omega\hat{J}_x$$
$$+\sum_{\tau}\lambda_\tau\hat{N}_\tau-\mu\left(2\langle\hat{J}_x\rangle\hat{J}_x+J_x(1)\,J_x(1)\right.$$
$$\left.-\langle\hat{J}_x\rangle^2\right)-\mu\left(J_y^2(1)+J_z^2(1)\right)$$
$$-\sum_{\tau} a_\tau\left(2\langle\hat{N}_\tau\rangle\hat{N}_\tau+N_\tau(1)\,N_\tau(1)-\langle\hat{N}_\tau\rangle^2\right)$$
$$-b\,P_x^2(1)-b\left(P_y^2(1)+P_z^2(1)\right). \tag{38}$$

Taking into account the relations[44,80]

$$2\mu\langle\hat{J}_x\rangle=\Omega;\quad \mu=\frac{1}{2}g_{J_x};\quad b=\frac{1}{2}g_{Px};\quad 2a_\tau\langle\hat{N}_\tau\rangle=\lambda_\tau;$$
$$a_\tau=\frac{1}{2}g_{N_\tau} \tag{39}$$

and the fact that if the kinetic energy of the motion of the center of mass goes over into orbital motion of the mass

$$\Omega\left(X_y(1)\,P_z(1)-X_z(1)\,P_y(1)\right)=b\left(P_y^2(1)+P_z^2(1)\right),$$

we obtain

$$H''=\langle\Omega\mid H'\mid\Omega\rangle+\mu\langle\hat{J}_x\rangle^2+\sum_{\tau}\alpha_\tau\langle N_\tau\rangle^2$$
$$+\frac{1}{2}\sum_{\substack{\nu\\ \omega_\nu\neq 0,\ \Omega}}\left(\mathscr{P}_\nu^2+\omega_\nu^2 X_\nu^2\right). \tag{40}$$

Therefore, the Hamiltonian H'' describes in the framework of the RPA purely intrinsic excitations under the conditions (39), which ensure the correct separation of the collective and intrinsic degrees of freedom. We note that for the Hamiltonian H'', in contrast to the Hamiltonian H',

$$[H'',\ J_i(1)]=[H'',\ N_\tau(1)]=[H'',\ P_i(1)]=0.$$

Diagonalization of the Hamiltonian in the RPA

In this subsection, we give the general prescription for diagonalizing the Hamiltonian in the RPA and finding the "mass" parameters g; the prescription is described in detail in Refs. 29 and 77.

It follows from the expressions (20) and (21) that each of the four parts of the Hamiltonian H′ has the structure

$$\tilde{H}_{\mathrm{B}}=\sum_{\mu} E_\mu b_\mu^+ b_\mu+\sum_{s_1=1}^{n_1}\varkappa_{s_1}V_{s_1}V_{s_1}+\sum_{s_2=1}^{n_2}\varkappa_{s_2}W_{s_2}W_{s_2} \tag{41}$$
$$(\mu=ik,\ \bar{i}\bar{k}\ \text{ or }\ i\bar{k}),$$

where V_{s_1} and W_{s_2} are the parts of the corresponding operators linear in the bosons:

$$\left.\begin{aligned} V_{s_1}&=\sum_{\mu} V_\mu^{s_1}\left(b_\mu^+ + b_\mu\right),\quad s_1=1,\ \ldots,\ n_1;\\ W_{s_2}&=\sum_{\mu} W_\mu^{s_2}\left(b_\mu^+ - b_\mu\right),\quad s_2=1,\ \ldots,\ n_2.\end{aligned}\right\} \tag{42}$$

Therefore, the diagonalization procedure, which is carried through below for the example of the Hamiltonian $\tilde{H}_{\mathrm{B}}$ (41), can be used for any of the four parts of the Hamiltonian H'_{RPA}.

The Hamiltonian $\tilde{H}_{\mathrm{B}}$ possesses definite symmetries [see (22)–(25)], and this is expressed formally by the commutation of the Hamiltonian $\tilde{H}_{\mathrm{B}}$ with the corresponding operators [for example, $J_x(1)$ and $N_\tau(1)$ in the case of the Hamiltonian $H\begin{Bmatrix}+\\+\end{Bmatrix}$]. Suppose that for the Hamiltonian $\tilde{H}_{\mathrm{B}}$ there exist L_1 operators of the type $(b_\mu^+ + b_\mu)$ and L_2 operators of the type $(b_\mu^+ - b_\mu)$,

$$\left.\begin{aligned} D_{l_1}&=\sum_{\mu} D_\mu^{l_1}\left(b_\mu^+ + b_\mu\right),\quad l_1=1,\ \ldots,\ L_1;\\ K_{l_2}&=\sum_{\mu} K_\mu^{l_2}\left(b_\mu^+ - b_\mu\right),\quad l_2=1,\ \ldots,\ L_2,\end{aligned}\right\} \tag{43}$$

these being such that for all l_1 and l_2

$$[\tilde{H}_{\mathrm{B}}\,D_{l_1}]=[\tilde{H}_{\mathrm{B}}\,K_{l_2}]=0; \tag{44}$$
$$[D_{l_1},\ D_{l_1'}]=[K_{l_2},\ K_{l_2'}]=[D_{l_1},\ K_{l_2}]=0. \tag{45}$$

Substituting (41)–(43) in (44) and using (28), we obtain

$$\left.\begin{aligned} D_\mu^{l_1}E_\mu&=\sum_{s_2=1}^{n_2}\varkappa_{s_2}a_{s_2}^{l_1}W_\mu^{s_2},\quad l_1=1,\ \ldots,\ L_1;\\ K_\mu^{l_2}E_\mu&=\sum_{s_1=1}^{n_1}\varkappa_{s_1}b_{s_1}^{l_2}V_\mu^{s_1},\quad l_2=1,\ \ldots,\ L_2,\end{aligned}\right\} \tag{46}$$

where

$$\left.\begin{aligned} a_{s_1}^{l_2}&=2\left[W_{s_2},\ D_{l_1}\right]=4\sum_{\mu} D_\mu^{l_1}W_\mu^{s_2};\\ b_{s_1}^{l_2}&=2\left[V_{s_1},\ K_{l_2}\right]=4\sum_{\mu} K_\mu^{l_2}V_\mu^{s_1}.\end{aligned}\right\} \tag{47}$$

We note that in definite models $a_{s_2}^{l_1}$ and $b_{s_1}^{l_2}$ can be expressed in terms of expectation values of the type $\langle\Omega|\hat{Q}^+_{\tau=0,2}|\Omega\rangle$, $\langle\Omega|\hat{P}_\tau|\Omega\rangle$, etc. (see Refs. 19, 40, 43, and 44), i.e., the relations (46) are conditions of self-consistency between the HFB quasiparticle field and the residual interaction for the Hamiltonian of the cranking model.

The boson representation of the canonically conjugate generalized coordinates X_ν and momenta $\mathscr{P}_\nu$ can be written in the form

$$\left.\begin{aligned} X_\nu&=\sum_{\mu} X_\mu^\nu\left(b_\mu^+ + b_\mu\right);\\ \mathscr{P}_\nu&=\mathrm{i}\sum_{\mu}\mathscr{P}_\mu^\nu\left(b_\mu^+ - b_\mu\right);\\ [X_\nu,\ \mathscr{P}_{\nu'}]&=2\mathrm{i}\sum_{\mu} X_\mu^\nu\mathscr{P}_\mu^{\nu'}=\mathrm{i}\delta_{\nu\nu'}.\end{aligned}\right\} \tag{48}$$

Substituting the expressions for X_ν and $\mathscr{P}_\nu$ from (48), and also the definition of the Hamiltonian $\tilde{H}_{\mathrm{B}}$ (41) in the RPA equations (28) and using (18), we obtain a matrix representation of these equations:

$$\left.\begin{aligned} X_\mu^\nu&=\sum_{s_2=1}^{n_2}\varkappa_{s_2}A_{s_2}^\nu\frac{E_\mu W_\mu^{s_2}}{E_\mu^2-\omega_\nu^2}-\sum_{s_1=1}^{n_1}\varkappa_{s_1}B_{s_1}^\nu\frac{V_\mu^{s_1}}{E_\mu^2-\omega_\nu^2};\\ \mathscr{P}_\mu^\nu&=\omega_\nu^2\sum_{s_2=1}^{n_2}\varkappa_{s_2}A_{s_2}^\nu\frac{W_\mu^{s_2}}{E_\mu^2-\omega_\nu^2}-\sum_{s_1=1}^{n_1}\varkappa_{s_1}B_{s_1}^\nu\frac{E_\mu V_\mu^{s_1}}{E_\mu^2-\omega_\nu^2}.\end{aligned}\right\} \tag{49}$$

Here

$$A^{\nu}_{s_2}=4\sum_{\mu}X^{\nu}_{\mu}W^{s_2}_{\mu};\quad B^{\nu}_{s_1}=4\sum_{\mu}\mathcal{P}^{\nu}_{\mu}V^{s_1}_{\mu}. \tag{50}$$

Using the definition of (50) by means of (49), we obtain a system of linear homogeneous algebraic equations for the n_1+n_2 unknowns $A^{\nu}_{s_2}$ and $B^{\nu}_{s_1}$:

$$\sum_{s_2=1}^{n_2}\varkappa_{s_2}A^{\nu}_{s_2}\left(S_{W_{s_2}W_{s_2'}}-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)-\sum_{s_1=1}^{n_1}\varkappa_{s_1}B^{\nu}_{s_1}U_{V_{s_1}W_{s_2'}}=0,$$

$$s_2'=1,\ \ldots,\ n_2; \tag{51a}$$

$$\sum_{s_2=1}^{n_2}\varkappa_{s_2}A^{\nu}_{s_2}\omega^2_{\nu}U_{V_{s_2'}W_{s_2}}-\sum_{s_1=1}^{n_1}\varkappa_{s_1}B^{\nu}_{s_1}\left(S_{V_{s_1}V_{s_1'}}+\frac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\right)=0,$$

$$s_1'=1,\ \ldots,\ n_1, \tag{51b}$$

where

$$S_{RT}=\sum_{\mu}\frac{E_{\mu}R_{\mu}T_{\mu}}{E^2_{\mu}-\omega^2};\quad U_{RT}=\sum_{\mu}\frac{R_{\mu}T_{\mu}}{E^2_{\mu}-\omega^2}, \tag{52}$$

for which the following useful relations hold:

$$S_{RT}=S_{RT}(0)+\omega^2W_{RT};\quad U_{RT}=U_{RT}(0)+\omega^2\mathcal{L}_{RT}; \tag{53}$$

$$W_{RT}=\sum_{\mu}\frac{R_{\mu}T_{\mu}}{E_{\mu}(E^2_{\mu}-\omega^2)};\quad \mathcal{L}_{RT}=\sum_{\mu}\frac{R_{\mu}T_{\mu}}{E^2_{\mu}(E^2_{\mu}-\omega^2)}. \tag{54}$$

Among the solutions of (51) there are solutions with zero energy, $\omega_{\nu}=0$, which are due to the requirement of fulfillment of the symmetry conditions [see Eqs. (22)–(27)]. These solutions must be separated, since from the physical point of view only solutions with $\omega_{\nu}\neq 0$ are of interest. From the matrix representation of the equations of motion (49) for the zero-energy solutions we readily obtain the system of equations

$$\left.\begin{aligned}&\sum_{s_2=1}^{n_2}\varkappa_{s_2}A^{l_1}_{s_2}(0)\left(S_{W_{s_2}W_{s_2'}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)=0,\quad s_2'=1,\ \ldots,\ n_2;\\&\sum_{s_1=1}^{n_1}\varkappa_{s_1}B^{l_2}_{s_1}(0)\left(S_{V_{s_1}V_{s_1'}}(0)+\frac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\right)=0,\quad s_1'=1,\ \ldots,\ n_1,\end{aligned}\right\} \tag{55}$$

for which the condition for the existence of solutions has the form

$$\left.\begin{aligned}&\det\left|S_{W_{s_2}W_{s_2'}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right|=0;\\&\det\left|S_{V_{s_1}V_{s_1'}}(0)+\frac{\delta_{s_1s_1'}}{4\varkappa_{s_1}}\right|=0.\end{aligned}\right\} \tag{56}$$

It is obvious that if the relations

$$A^{l_1}_{s_2}(0)=\sqrt{g_{l_1}}\,a^{l_1}_{s_2};\quad B^{l_2}_{s_1}(0)=\sqrt{g_{l_2}}\,b^{l_2}_{s_1}, \tag{57}$$

are satisfied, the self-consistency conditions (46) lead to (56), and these equations make it possible to determine the constants of the residual interaction. In addition, it follows from Eq. (51a) for the zero-energy solutions with allowance for (55) that

$$\sum_{s_1=1}^{n_1}\varkappa_{s_1}B^{l_1}_{s_1}(0)\,U_{V_{s_1}W_{s_2}}(0)=0. \tag{58}$$

We multiply each of the equations in (51a) by $\varkappa_{s_2}a^{l_1}_{s_2}$ and sum over the index s_2. Accordingly, we multiply each of the equations in (51b) by $\varkappa_{s_1}b^{l_2}_{s_1}$ and sum over the index s_1 and, using (55) in conjunction with (57), we obtain the equations

$$\left.\begin{aligned}&\sum_{s_2'=1}^{n_2}\sum_{s_2=1}^{n_2}\varkappa_{s_2}\varkappa_{s_2'}a^{l_1}_{s_2}A^{\nu}_{s_2'}\left(S_{W_{s_2}W_{s_2'}}-S_{W_{s_2}W_{s_2'}}(0)\right)\\&-\sum_{s_1=1}^{n_1}\sum_{s_2=1}^{n_2}\varkappa_{s_1}\varkappa_{s_2}a^{l_1}_{s_2}U_{V_{s_1}W_{s_2}}=0;\\&\omega^2_{\nu}\sum_{s_1=1}^{n_1}\sum_{s_2=1}^{n_2}\varkappa_{s_1}\varkappa_{s_2}b^{l_2}_{s_1}A^{\nu}_{s_2}U_{V_{s_1}W_{s_2}}\\&-\sum_{s_1'=1}^{n_1}\sum_{s_1=1}^{n_1}\varkappa_{s_1}\varkappa_{s_1'}b^{l_2}_{s_1}B^{\nu}_{s_1'}\left(S_{V_{s_1}V_{s_1'}}-S_{V_{s_1}V_{s_1'}}(0)\right)=0.\end{aligned}\right\} \tag{59}$$

By means of Eq. (58) and the relations (53) the system of equations (59) can be finally reduced to the form

$$\omega^2_{\nu}\sum_{s_2}\sum_{s_2'}\varkappa_{s_2}\varkappa_{s_2'}a^{l_1}_{s_2}A^{\nu}_{s_2'}W_{W_{s_2}W_{s_2'}}-\omega^2_{\nu}\sum_{s_1}\sum_{s_2}\varkappa_{s_1}\varkappa_{s_2}a^{l_1}_{s_2}B^{\nu}_{s_1}\mathcal{L}_{V_{s_1}W_{s_2}}=0; \tag{60a}$$

$$l_1=1,\ \ldots,\ L_1$$

$$\omega^2_{\nu}\sum_{s_1}\sum_{s_2}\varkappa_{s_1}\varkappa_{s_2}b^{l_2}_{s_1}A^{\nu}_{s_2}\mathcal{L}_{V_{s_1}W_{s_2}}-\omega^2_{\nu}\sum_{s_1}\sum_{s_1'}\varkappa_{s_1}\varkappa_{s_1'}b^{l_2}_{s_1}B^{\nu}_{s_1'}W_{V_{s_1}V_{s_1'}}=0, \tag{60b}$$

$$l_2=1,\ \ldots,\ L_2.$$

Thus, the complete system of equations for the n_1+n_2 unknowns $A^{\nu}_{s_2}$ and $B^{\nu}_{s_1}$ consists of the L_1 equations (60a), the L_2 equations (60b), n_1-L_1 arbitrarily chosen equations of the type (51a), and n_2-L_2 similarly chosen equations of the type (51b). It follows from the corresponding secular equation

$$\omega^{2(L_1+L_2)}_{\nu}\,|\,D(\omega_{\nu})\,|=0 \tag{61}$$

(here, $|D(\omega_{\nu})|$ is the determinant of the system of equations) that a solution with $\omega_{\nu}=0$ is $2(L_1+L_2)$-fold degenerate, and the energy of the solutions with nonzero energy is determined from the equation

$$|\,D(\omega_{\nu})\,|=0. \tag{62}$$

The structure of the corresponding phonon [i.e., the coefficients X^{ν}_{μ} and $\mathcal{P}^{\nu}_{\mu}$ in (48)] can be determined by means of the expressions (49) by using the normalization condition from (48) and the solution of Eq. (62). It remains to determine the mass parameters g_{l_1} and g_{l_2}. The method of their determination differs in the following two cases:

a) simultaneous existence of ghost states due to the operators K_{l_2} and D_{l_1};

b) the existence of ghost states generated only by the operators K_{l_2} or only by the operators D_{l_1}.

In the case of simultaneous existence of ghosts of both types, the relations (55) and (58) hold. Using the matrix representation (49) for the canonically conjugate quantities $X_{\nu}(0)$ and $\mathcal{P}_{\nu}(0)$ corresponding to the zero-energy solutions due, for example, to the operator k_{l_2}, we can write

[taking into account (57)]

$$\left.\begin{aligned}\mathscr{P}_\mu^{l_2}(0)&=-\sqrt{g_{l_2}}\sum_{s_1}\varkappa_{s_1}b_{s_1}^{l_2}\frac{V_\mu^{s_1}}{E_\mu};\\ X_\mu^{l_2}(0)&=\sum_{s_2}\varkappa_{s_2}A_{s_2}^{l_2}(0)\frac{W_\mu^{s_2}}{E_\mu}-\sqrt{g_{l_2}}\sum_{s_1}\varkappa_{s_1}b_{s_1}^{l_2}\frac{V_\mu^{s_1}}{E_\mu^2}.\end{aligned}\right\}\tag{63}$$

Substituting these expressions in the normalization condition (48),

$$[X_{l_2}(0),\ \mathscr{P}_{l_2}(0)]=2\mathrm{i}\sum_\mu X_\mu^{l_2}(0)\,\mathscr{P}_\mu^{l_2}(0)=\mathrm{i},$$

we obtain the equation

$$-\sqrt{g_{l_2}}\sum_{s_1}\sum_{s_2}\varkappa_{s_1}\varkappa_{s_2}A_{s_2}^{l_2}(0)\,b_{s_1}^{l_2}U_{\nu_{s_1}w_{s_2}}(0)$$
$$+g_{l_2}\sum_{s_1}\sum_{s_1'}\varkappa_{s_1}\varkappa_{s_1'}b_{s_1}^{l_2}b_{s_1'}^{l_2}\sum_\mu\frac{V_\mu^{s_1}V_\mu^{s_1'}}{E_\mu^2}=\frac{1}{2}.\tag{64}$$

From Eq. (58) with allowance for (57) there follows the vanishing of the first term of Eq. (64). Using (46), we obtain from (64) for the mass parameter g_{l_2}

$$\frac{1}{g_{l_2}}=2\sum_\mu\frac{K_\mu^{l_2}K_\mu^{z}}{E_\mu}.\tag{65}$$

We can similarly obtain an expression for g_{l_1}:

$$\frac{1}{g_{l_1}}=2\sum_\mu\frac{D_\mu^{l_1}D_\mu^{l_1}}{E_\mu}.\tag{66}$$

We consider case (b), where there exist ghosts of only one type, for example, ones due to the operator K_{l_2}. The relations (63) again hold. They yield the equation

$$\sum_{s_2}\varkappa_{s_2}A_{s_2}^{l_2}(0)\left(S_{W_{s_2}W_{s_2'}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)$$
$$-\sqrt{g_{l_2}}\sum_{s_1}\varkappa_{s_1}b_{s_1}^{l_2}U_{\nu_{s_1}w_{s_2'}}(0)=0,\tag{67}$$

which, using (46), we transform to

$$\sum_{s_2}\varkappa_{s_2}A_{s_2}^{l_2}(0)\left(S_{W_{s_2}W_{s_2'}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right)-\sqrt{g_{l_2}}S_{K_{l_2}w_{s_2}}(0)=0.\tag{68}$$

However, the relations (46) also make it possible to transform Eq. (64). Finally,

$$-\sqrt{g_{l_2}}\sum_{s_2}\varkappa_{s_2}A_{s_2}^{l_2}(0)\,S_{K_{l_2}W_{S_2}}(0)+g_{l_2}S_{K_{l_2}K_{l_2}}(0)=\frac{1}{2}.\tag{69}$$

Thus, we have obtained the system of inhomogeneous equations (68) and (69) for the unknowns $A_{s_2}^{l_2}(0)$ and g_{l_2}. By means of the Cramer rule it is easy to determine from this system the mass parameters

$$g_{l_2}=\frac{1}{2}\frac{\left|S_{W_{s_2}W_{s_2'}}(0)-\frac{\delta_{s_2s_2'}}{4\varkappa_{s_2}}\right|}{|D_{l_2}|}\tag{70}$$

where $|D_{l_2}|$ is the determinant of the system of equations (68)–(69).

We note in conclusion that the representation (48) can be interpreted as a transformation from the space of two-quasiparticle bosons to the space of the RPA modes $(X_\nu, \mathscr{P}_\nu)$ [or (O_ν, O_ν^+)], this transformation being realized by a unitary Hermitian matrix with elements X_μ^ν and $\mathscr{P}_\mu^\nu$. The number of two-quasiparticle bosons b_μ, b_μ^+ must correspond to the number of modes $(X_\nu, \mathscr{P}_\nu)$. This completeness of the two spaces can be expressed by the relation (see, for example, Ref. 40)

$$b_\mu^+=\mathrm{i}\sum_\nu\{[b_\mu^+,\ X_\nu]\,\mathscr{P}_\nu+[\mathscr{P}_\nu,\ b_\mu^+]\,X_\nu\},\tag{71}$$

where the summation is over the ghost as well as the non-ghost modes.

Eigenstates of the Hamiltonian

The solutions being orthogonal in the RPA, all the normal modes $(X_\nu, \mathscr{P}_\nu)$ with $\omega_\nu\neq0$ of the Hamiltonian (20) are also orthogonal to the Goldstone mode $[\theta_x(1), J_x(1)]$.[10] Then the expectation value of the angular-momentum operator $\hat{J}_x$ with respect to the phonon state in the first order of the RPA,[11]

$$\langle\Omega|O_\nu\hat{J}_xO_\nu^+|\Omega\rangle$$
$$=\langle\Omega|[O_\nu,\ \hat{J}_x]|\Omega\rangle+\langle\Omega|\hat{J}_xO_\nu O_\nu^+|\Omega\rangle\approx\langle\Omega|\hat{J}_x|\Omega\rangle,\tag{72}$$

corresponds to the expectation value of the operator $\hat{J}_x$ in the yrast-line state that corresponds to the given value of the rotational angular frequency Ω.

The yrast-line state $|\Omega\rangle$ is the vacuum for not only the quasiparticle but also the phonon operators, and therefore

$$O_\nu|\Omega\rangle=N_\tau(1)|\Omega\rangle=P_x(1)|\Omega\rangle=0;\tag{73a}$$
$$\Gamma\,|\Omega\rangle=J_x(1)\,|\,\Omega\rangle=0.\tag{73b}$$

Since $J_x^{(1)}\approx\hat{J}_x-\langle\Omega|\hat{J}_x|\Omega\rangle$, the relation (73b) means that the angular momentum in the state $|\Omega\rangle$ is almost completely aligned along the rotation axis, the x axis, this ensuring fulfillment of the cranking-model condition

$$\langle\Omega|\hat{J}_x|\Omega\rangle=\sqrt{I(I+1)}.\tag{74}$$

Therefore, the phonon creation operator O_ν^+ does not change the total angular momentum.[12] It follows from the relations (73a) and the condition $\langle\Omega|\hat{P}_x|\Omega\rangle=0$ that on the yrast line there are also no vibrations of the center of mass of the nucleus along the x axis. Generally speaking, such vibrations can arise as a result of excitation of the nucleus, but the separation of the ghost mode (X, P_x) from the solutions of the RPA equations ensures the absence of mixing of the normal modes and excitations due to vibrations of the center of mass along the x axis. As was already noted near the beginning of this section, vibrations of the center of mass along the y and z axes of the laboratory system are also not mixed with the normal modes.

In Refs. 30 and 43, Marshalek used the fact of alignment of the angular momentum in the states of the yrast line to construct eigenfunctions of the cranking-model Hamilto-

nian H' for positive-parity solutions. The negative-parity case is considered in Ref. 50.

Thus, we write the eigenvector of the Hamiltonian H' in the form

$$|\{n_{\nu-}\},\ \{n_{\nu+}\},\ N,\ Z,\ P_x,\ J,\ M\rangle$$
$$= \prod_{n_{\nu-}} \prod_{n_{\nu+}} \frac{(O^+_{\nu-})^{n_{\nu-}}}{\sqrt{n_{\nu-}!}} \frac{(O^+_{\nu+})^{n_{\nu+}}}{\sqrt{n_{\nu+}!}} \frac{e^{i(Z-Z_0)\theta_z}}{\sqrt{2\pi}}$$
$$\times \frac{e^{i(N-N_0)\theta_N}}{\sqrt{2\pi}} \frac{e^{iP_x X}}{\sqrt{2\pi}} \frac{e^{i(J-J_0)\theta_x}}{\sqrt{2\pi}}$$
$$\times \frac{(\Gamma^+)^{J-M}}{\sqrt{(J-M)!}} |0,\ 0,\ N_0,\ Z_0,\ 0,\ J_0,\ J_0\rangle, \qquad (75)$$

where n_{ν^-} and n_{ν^+} are the numbers of phonons of negative and positive signature irrespective of the parity; θ_τ ($\tau = N, Z$) and θ_x are the angles conjugate to the operators $\hat{N}_\tau$ and $\hat{J}_x$; and X is the projection of the center-of-mass coordinate onto the x axis. The vector $|0, 0, N_0, Z_0, 0, J_0, J_0\rangle$ describes the state of the nucleus on the yrast line having angular momentum J_0 (with projection $M = J_0$ onto the x axis), with number of neutrons N_0 and of protons Z_0, and with center of mass at the origin (the origins of the laboratory and intrinsic coordinate systems of the nucleus coincide). Whereas for the intrinsic wave function $R_x(\pi)$ symmetry holds, there follows by virtue of the condition (16) the following connection between the signature and the angular momentum of the state:

$$(-1)^J (-1)^{\sum_{\nu-} n_{\nu-}} = 1_s \qquad (76)$$

in accordance with which the single-phonon states of positive signature have an even value of the spin J, while the single-phonon states of negative signature have an odd value of the spin. The eigenvectors (75) can be represented in the form of a product of a rotational and an intrinsic wave function (see, for example, Refs. 30 and 40):

$$\left.\begin{aligned} |\alpha JM\rangle &= |\alpha J\rangle\, |JM\rangle; \\ |JM\rangle &= \frac{e^{i(J-J_0)\theta_x}}{\sqrt{2\pi}} \frac{(\Gamma^+)^{J-M}}{\sqrt{(J-M)!}} |J_0,\ M = J_0\rangle; \\ |\alpha J\rangle &= \frac{e^{i(N-N_0)\theta_N}}{\sqrt{2\pi}} \frac{e^{i(Z-Z_0)\theta_z}}{\sqrt{2\pi}} \frac{e^{iP_x X}}{\sqrt{2\pi}} \prod_{\substack{n_{\nu-} \\ n_{\nu+}}} \frac{(O^+_{\nu-})^{n_{\nu-}}}{\sqrt{n_{\nu-}!}} \\ &\times \frac{(O^+_{\nu+})^{n_{\nu+}}}{\sqrt{n_{\nu+}!}} |n_{\nu-} = 0,\ n_{\nu+} = 0,\ N = N_0,\ Z = Z_0,\ P_x = 0\rangle, \end{aligned}\right\} \qquad (77)$$

where α represents an ensemble of quantum numbers: n_{ν^+}, n_{ν^-}, N, Z, P_x. Note that the functions (75) or (77) are obtained, as in Refs. 30 and 43, by the substitution $J_x(1) \to \hat{J}_x - J_0$, $N_z(1) \to \hat{N}_z - Z_0$, $N_N(1) \to \hat{N}_N - N_0$, $P_x(1) \to \hat{P}_x$. The variables are defined on the intervals $(0, 2\pi)$, $(0, 2\pi)$, $(-\infty, \infty)$, respectively, and the wave function (75) or (77) is normalized.

Probabilities of electric transitions

In the limit of large angular momenta, $E2$ transitions were analyzed by Mikhaĭlov[83] and Marshalek.[43] In the case of broken intrinsic reflection symmetry of the nucleus, $E1$ and $E3$ transitions were analyzed in Ref. 40. Quantitative estimates of $E1$ transitions for specific nuclei in the case of rotation can be found in Refs. 70–73 and 82.

As is well known, the reduced probability of a given transition is defined as[2]

$$B(E\lambda,\ \alpha_1 J_1 \to \alpha_2 J_2) = \frac{|\langle \alpha_2 J_2 \| \hat{\mathfrak{M}}(E\lambda) \| \alpha_1 J_1\rangle|^2}{2J_1 + 1}, \qquad (78)$$

where $\hat{\mathfrak{M}}(E\lambda, \mu)$ is the operator of the transition of multipolarity λ with projection μ. An aribitrary tensor operator $\mathfrak{M}_{\lambda\mu}$, defined in the laboratory coordinate system, can be expressed by means of the standard transformation (see, for example, Ref. 84)

$$\hat{\mathfrak{M}}_{\lambda\mu} = \sum_\nu \hat{D}^\lambda_{\mu\nu} \hat{\mathfrak{M}}'_{\lambda\nu} = \frac{1}{2} \sum_\nu \{\hat{\mathfrak{M}}'_{\lambda\nu},\ \hat{D}^\lambda_{\nu\mu}\} \qquad (79)$$

in terms of the components of the tensor operator $\hat{\mathfrak{M}}'_{\lambda\nu}$, which is defined in the intrinsic coordinate system, the Wigner function $D^\lambda_{\nu\mu}$ being expressed in the operator form (see Refs. 30 and 85). For $I \gg 1$, when the quantization axis coincides with the rotation axis, the reduced matrix element has the form[83,30]

$$\langle \alpha_2 J + \nu \| \hat{\mathfrak{M}}_\lambda \| \alpha_1 J\rangle = \sqrt{2J+1}\, \langle J\lambda J\nu | J + \nu J + \nu\rangle\, \langle \alpha_2 J + \nu | \hat{\mathfrak{M}}'_{\lambda\nu} | \alpha_1 J\rangle. \qquad (80)$$

Transition operators. Since all projections of the angular momenta in the wave functions (75) and (77) are defined in the system of quantization with the axis x, we use the transformation

$$\hat{Q}_{\lambda\mu_x} = \sum_{\mu_z} D^\lambda_{\mu_x\mu_z} \left(\frac{\pi}{2}\right) \hat{Q}_{\lambda\mu_z} \qquad (81)$$

to redefine the operators that occur in the Hamiltonian H'.

Using in (81) the boson expansions of the operators $\hat{Q}_{\lambda\mu z}$ (see Appendix A) and the explicit form of the Wigner functions (see Ref. 84), we obtain to second order in the bosons

$$\left.\begin{aligned} \hat{Q}^{(+)}_{\lambda\mu_x=0,\ \pm 2} &= \langle\Omega|\hat{Q}^{(+)}_{\lambda\mu_x=0,\ 2}|\Omega\rangle\, \delta_{\lambda,\ 2} \\ &+ c \sum_{k\bar{l}} \{\mathfrak{M}^{\lambda\pm\mu_x}_{k\bar{l}} b^+_{k\bar{l}} + (-1)^\lambda \mathfrak{M}^{\lambda\mp\mu_x}_{k\bar{l}} b_{k\bar{l}}\}; \\ \hat{Q}^{(-)}_{\lambda\mu_x=\pm 1,\ \pm 3} &= c \sum_{kl} \{\mathfrak{M}^{\lambda\pm\mu_x}_{kl} b^+_{kl} + \mathfrak{M}^{\lambda\mp\mu_x}_{kl} b_{kl} \\ &+ \mathfrak{M}^{\lambda\pm\mu_x}_{\bar{k}\bar{l}} b^+_{\bar{k}\bar{l}} + \mathfrak{M}^{\lambda\mp\mu_x}_{\bar{k}\bar{l}} b_{\bar{k}\bar{l}}\}, \end{aligned}\right\} \qquad (82)$$

where $-\lambda \leqslant \mu_x \leqslant \lambda$ and

$$c = \begin{cases} +1, & \lambda = 2, \\ -1, & \lambda = 1,3 \end{cases}; \quad \mathfrak{M}^{\lambda\mu}_{kl},\ \mathfrak{M}^{\lambda\mu}_{\bar{k}\bar{l}},\ \mathfrak{M}^{\lambda\mu}_{k\bar{l}}$$

are the quasiparticle matrix elements of the operators $Q^{(\pm)}_{\lambda\mu x}$.

The expressions (82) must be augmented by the condition

$$\hat{Q}^{(\pm)+}_{\lambda\mu_x} = (-1)^{\mu_x} \hat{Q}^{(\pm)}_{\lambda-\mu_x}. \qquad (83)$$

Here, the symbol ($\pm$) as superscript to an operator determines its signature. Substituting in (82) the relation (71), we express the operators $\hat{Q}^{(\pm)}_{\lambda\mu x}$ in terms of the modes of the RPA equations $X_{\nu\pm}$, $\mathscr{P}_{\nu\pm}$, the ghost modes $[X(1), P_x(1)]$, $[\theta_x(1), J_x(1)]$, $[\theta_\tau(1), N_\tau(1)]$, $[J_y(1), J_z(1)]$, and the modes $[Y(1), P_y(1)]$, $[Z(1), P_z(1)]$. The commutators of the types $[Q^{(\pm)}_{\lambda\mu x}, J_x(1)]$, $[Q^{(\pm)}_{\lambda\mu x}, J_{y,z}(1)]$, $[Q^{(\pm)}_{\lambda\mu x}, P_x(1)]$ and $[Q^{(\pm)}_{\lambda\mu x}, P_{y,z}(1)]$ that appear in these expressions can be replaced by he corresponding linear combination of the expectation values of the multipole operators in the yrast-line states (see Refs. 19, 40, 43, and 44). Further, following Marshalek,[30] we must in these expresisons make the substitutions $\Theta_x(1)\to\Theta_x$, $\Theta_\tau(1)\to\Theta_\tau$, $X(1)\to X$, $P_x(1)\to P_x$, $N_N(1)\to N-N_0$, $N_z(1)\to Z-Z_0$, $J_x(1)\to J-J_0$ in order to obtain the expressions for the operators $\hat{O}^{(\pm)}_{\lambda\mu x}$ in the laboratory coordinate system, or, more precisely, in the space of the functions (75) or (77). The resulting expressions can be used to determine the operators $\hat{Q}^{(\pm)}_{\lambda\mu x}$ that act in the intrinsic coordinate system [i.e., in the space of the functions $|\alpha J\rangle$; see (77)]. To this end, it is necessary to express the Wigner function $D^{\lambda}_{\mu\nu}$ in operator form in terms of the operators $J_x(1)$, $\theta_x(1)$, Γ, Γ^+, and also make the substitution $\Theta_x(1)\to\hat{\Theta}_x$, $J_x(1)\to\hat{J}-J_0$. As a result, we obtain[13)]

$$\left.\begin{aligned}
\hat{Q}^{(+)'}_{\lambda\mu_x=0} &= \sum_{\nu}\Lambda^{(\lambda 0)}_{\nu}(O^+_{\nu}+(-1)^{\lambda}O_{\nu})+\delta_{\lambda,2}\{\langle\Omega|\hat{Q}^{(+)}_{2\mu_x=0}|\Omega\rangle\\
&+[\hat{Q}^{(+)}_{2\mu_x=0},\ \mathrm{i}\hat{\Theta}_N](N-N_0)+[\hat{Q}^{(+)}_{2\mu_x=0},\ \mathrm{i}\hat{\Theta}_z](Z-Z_0)\\
&+[\hat{Q}^{(+)}_{2\mu_x=0},\ \mathrm{i}\hat{\Theta}_x](J-J_0)\};\\
\hat{Q}^{(\sigma)'}_{\lambda\mu_x=1,2,3} &= \sum_{\nu}(\Lambda^{\lambda\mu_x}_{\nu}O^+_{\nu}+\Lambda^{\lambda-\mu_x}_{\nu}O_{\nu})\\
&+\delta_{\lambda,2}\{\langle\Omega|\hat{Q}^{(+)}_{2\mu_x=2}|\Omega\rangle+[\hat{Q}^{(+)}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_N](N-N_0)\\
&+[\hat{Q}^{(+)}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_z](Z-Z_0)+[\hat{Q}^{(+)}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_x](J-J_0)\}
\end{aligned}\right\} \tag{84}$$

($\sigma = +$ for $\mu_x = 2$; $\sigma = -$ for $\mu_x = 1, 3$), where the matrices $\Lambda^{\lambda\mu x}_{\nu}$ have the form

$$\left.\begin{aligned}
\Lambda^{\lambda\mu_x=0}_{\nu} &= \sum_{ik}\mathfrak{M}^{\lambda\mu_x=0}_{i\bar{k}}(\varphi^{\nu}_{i\bar{k}}+\psi^{\nu}_{i\bar{k}});\\
\Lambda^{\lambda\mu_x=\pm1,\pm2,\pm3}_{\nu} &= 2\sum_{ik}(\mathfrak{M}^{\lambda\mu_x}_{ik}\varphi^{\nu}_{ik}-\mathfrak{M}^{\lambda-\mu_x}_{ik}\varphi^{\nu}_{ik}\\
&+\mathfrak{M}^{\lambda\mu_x}_{i\bar{k}}\varphi^{\nu}_{i\bar{k}}-\mathfrak{M}^{\lambda-\mu_x}_{i\bar{k}}\psi^{\nu}_{i\bar{k}});\\
(\Lambda^{\lambda\mu}_{\nu})^* &= (-1)^{\lambda+\mu}\Lambda^{\lambda\mu}_{\nu},\quad -\lambda\leqslant\mu\leqslant\lambda,
\end{aligned}\right\} \tag{85}$$

and the amplitudes $\psi^{\nu}_{ik(\bar{ik})}$, $\varphi^{\nu}_{ik(\bar{ik})}$, $\psi^{\nu}_{i\bar{k}}$, $\varphi^{\nu}_{i\bar{k}}$ follow from the definition of the phonon operators:

$$\left.\begin{aligned}
O^+_{\nu+} &= \sum_{ik}(\psi^{\nu}_{ik}b^+_{ik}+\varphi^{\nu}_{ik}b_{i\bar{k}});\\
O^+_{\nu-} &= \sum_{ik}(\psi^{\nu}_{ik}b^+_{ik}+\varphi^{\nu}_{ik}b_{ik}+\psi^{\nu}_{i\bar{k}}b^+_{i\bar{k}}+\varphi^{\nu}_{i\bar{k}}b_{i\bar{k}}).
\end{aligned}\right\} \tag{86}$$

Nuclear multipole moments. The static nuclear moments are determined by the expectation values of the operators $\hat{O}^{(\pm)'}_{\lambda\mu x}$ with respect to the state $|\alpha J\rangle$. Using (77) and (84), we can write the nuclear moments in the form of a Taylor expansion:

$$\begin{aligned}
eQ_{2\mu_x=0,2} &= \left\langle\alpha J\left|\sqrt{\frac{16\pi}{5}}\hat{Q}^{(+)'}_{2\mu_x=0,2}\right|\alpha J\right\rangle\\
&= \left\langle\Omega\left|\sqrt{\frac{16\pi}{5}}\hat{Q}^{(+)}_{2\mu_x=0,2}\right|\Omega\right\rangle+\left(\frac{\partial Q_{2\mu_x}}{\partial N}\right)_{N=N_0}(N-N_0)\\
&+\left(\frac{\partial Q_{2\mu_x}}{\partial Z}\right)_{Z=Z_0}(Z-Z_0)+\left(\frac{\partial Q_{2\mu_x}}{\partial J}\right)_{J=J_0}(J-J_0).
\end{aligned} \tag{87}$$

We have here introduced the notation

$$\begin{aligned}
\left(\frac{\partial Q_{2\mu_x}}{\partial N}\right)_{N=N_0} &= \sqrt{\frac{16\pi}{5}}[\hat{Q}^{(+)}_{2\mu_x},\ \mathrm{i}\hat{\Theta}_N];\quad \left(\frac{\partial Q_{2\mu_x}}{\partial Z}\right)_{Z=Z_0}\\
&= \sqrt{\frac{16\pi}{5}}[\hat{Q}^{(+)}_{2\mu_x},\ \mathrm{i}\hat{\Theta}_z];\\
\left(\frac{\partial Q_{2\mu_x}}{\partial J}\right)_{J=J_0} &= \sqrt{\frac{16\pi}{5}}[\hat{Q}^{(+)}_{2\mu_x},\ \mathrm{i}\hat{\Theta}_x].
\end{aligned} \tag{88}$$

We note that the moments in (87) are defined with respect to the x axis. By means of the transformation (81), we can obtain the expressions for the nuclear moments in the system with quantization axis z.

Reduced probabilities of electric transitions. Substituting (80) in (78) and using the asymptotic behavior of the Clebsch-Gordan coefficients ($J_1, J_2 \gg \lambda$), we obtain [here $\mathfrak{M}(E\lambda,\mu) = e^{\lambda}_{\mathrm{eff}}Q^{(\pm)}_{\lambda\mu x}$]

$$B(E\lambda;\ \alpha_1 J\to\alpha_2 J-\nu) = |\langle J-\nu\alpha_2|e^{\lambda}_{\mathrm{eff}}\hat{Q}^{(\pm)'}_{\lambda\mu_x=-\nu}|J\alpha_1\rangle|^2, \tag{89}$$

where we have reflected the dependence of the nature of the electric transition on the sign of the signature. The formal appearance of an effective charge in the expression implies multiplication of the additive components of the total moment from the different particle species (neutrons or protons) by the corresponding effective charges. Substituting (84) in (89), we obtain all the required reduced transition probabilities.

For transitions that do not change the number of phonons, i.e., transitions along the rotational bands, including the yrast line, we have

$$\begin{aligned}
&B(E2;\ \alpha J\to\alpha J-\nu)\\
&= |\langle\Omega|\tilde{Q}^{(+)}_{2\mu_x=2}|\Omega\rangle+[\tilde{Q}^{(+)}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_N](N-N_0)\\
&+[\tilde{Q}^{(+)}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_z](Z-Z_0)+[\tilde{Q}^{+}_{2\mu_x=2},\ \mathrm{i}\hat{\Theta}_x](J-J_0)|^2,
\end{aligned} \tag{90}$$

where $\hat{Q}^{(\pm)}_{2\mu x} = e^{(2)}_{\mathrm{eff}}\hat{Q}^{(\pm)'}_{2\mu x}$. It is obvious that when the intrinsic structure of the states changes slowly with increasing J it is possible to set $N = N_0$, $Z = Z_0$, $J = J_0$ (see Ref. 30).

Transitions that change the number of phonons by 1 (interband transitions) can be separated in accordance with the change in the angular momentum:

a) for transitions with $\Delta J = 0$

$$B(E\lambda;\ n_{\nu}J_0\to n_{\nu}\pm1\ J_0) = |\tilde{\Lambda}^{\lambda 0}_{\nu+}|^2\times\begin{cases} n_{\nu+}+1;\\ n_{\nu+}.\end{cases} \tag{91}$$

Here $\lambda = 1, 2, 3$ and $\hat{\Lambda} = e_{\mathrm{eff}}\Lambda$;

b) for transitions with $\Delta J = 1$

$$B(E\lambda;\ n_{\nu}J_0\to n_{\nu}\pm1\ J_0-1) = |\tilde{\Lambda}^{\lambda\mp1}_{\nu\sigma}|^2\times\begin{cases} n_{\nu\sigma}+1;\\ n_{\nu\sigma}.\end{cases} \tag{92}$$

Here $\lambda = 1, 2, 3$; $\sigma = +$ for $\lambda = 3$ and $\sigma = -$ for $\lambda = 1, 2$;

c) for transitions with $\Delta J = 2$

$$B(E\lambda;\ n_\nu J_0 \to n_\nu \pm 1\ J_0 - 2) = |\tilde{\Lambda}_{\nu+}^{\lambda\mu_x = \mp 2}|^2 \times \begin{cases} n_{\nu+} + 1; \\ n_{\nu+}. \end{cases} \tag{93}$$

Here $\lambda = 2, 3$;

d) for transitions with $\Delta J = 3$

$$B(E3;\ n_\nu J_0 \to n_\nu \pm 1\ J_0 - 3) = |\tilde{\Lambda}_{\nu-}^{3\mp 3}|^2 \times \begin{cases} n_{\nu-} + 1; \\ n_{\nu-}. \end{cases} \tag{94}$$

The index $\nu\sigma$ in the expressions (91)–(94) characterizes the signature of the phonon state from which the particular transition takes place.

It is to be expected that all transitions in which the number of phonons changes (interband transitions) are weaker than the transitions within a rotational band, since the reduced probability of the single phonon transitions is proportional to the amplitudes $|\Lambda_{\nu\sigma}^{\lambda\mu x}|^2$ [see (91)–(94), which are smaller by the factor of the boson expansion than the expectation value of the quadrupole operator $\langle\Omega|\hat{Q}_{2\mu x}^{(\pm)}|\Omega\rangle$, which characterizes the intraband transitions.

In conclusion, we note that the expressions (90)–(94) have been obtained in the limit of large spins $J_1, J_2 \gg \lambda$. However, in a number of studies, in particular in Refs. 9 and 86, arguments have been given for the possibility of using the approximations contained in the cranking model for low spins. In this case, there is no need for the transition to the asymptotic values of the Clebsch-Gordan coefficients, and the corresponding expressions for the reduced transition probabilities, which for $I \gg 1$ go over to (90)–(94), can be found in Ref. 87.

Strength-function method

To obtain information about the probabilities of transitions between discrete levels in the framework of the CM + RPA method, it is necessary to know the structure of the corresponding initial and final states. For this, it is necessary to solve the corresponding secular equations (62) and the system of equations (51) in order to obtain the two-quasiparticle amplitudes X_μ^ν, $\mathcal{P}_\mu^\nu$ (49) or ψ_{ik}^ν, φ_{ik}^ν (86). However, the investigation of individual solutions of secular equations at high excitations (for example, in the region of giant resonances) becomes meaningless, since the density of levels in this part of the spectrum increases strongly and the physical information extracted by means of an experiment also has an averaged nature. Therefore, for quantitative analysis of electromagnetic transitions in the region of excitations with high energy, it is natural to use the strength-function method,[31] which can be successfully used to describe giant resonances in the absence of rotation.[41,88,89] We give the main details of the strength-function method, which is used below to describe solutions of the CM + RPA mehod (see Ref. 77).

We consider the function $b(\omega_i)$, which has physical meaning at the points ω_i determined by the equation

$$\mathcal{F}(\omega_i) = 0 \tag{95}$$

and representing the excitation energy of the system.[14] By the strength function of the given quantity $b(\omega_i)$ we understand the weighted mean value of this quantity over some energy interval Δ in the neighborhood of the point ω_i, i.e.,[31]

$$b_\Delta(\omega) = \sum_i b(\omega_i)\rho_\Delta(\omega - \omega_i), \tag{96}$$

where $\rho_\Delta(\omega - \omega_i)$ is a weight function normalized to unity and having its maximum at the point $\omega - \omega_i = 0$. It is usually taken in the form

$$\rho_\Delta(\omega - \omega_i) = \frac{1}{2\pi}\frac{\Delta}{(\Delta - \omega_i)^2 + \Delta^2/4}. \tag{97}$$

Regarding all solutions of Eq. (95) as nondegenerate, i.e., $(\partial F/\partial\omega)_{\omega \neq \omega i} \neq 0$, we introduce a function $P(\omega)$ such that

$$b(\omega_i) = (P(\omega)/\partial F/\partial\omega)_{\omega = \omega_i}. \tag{98}$$

If the function $P(z)/F(z)$, regarded as a function of a complex variable, has no other singularities apart from the simple poles corresponding to the zeros of $F(z)$, and if

$$\lim_{z \to \infty} P(z)/F(z) \to 0,$$

then, using Cauchy's theorem, we can obtain the key relation

$$b_\Delta(\omega) = \frac{1}{\pi}\operatorname{Im}\frac{P(\omega + i\Delta/2)}{F(\omega + i\Delta/2)}. \tag{99}$$

We consider the problem of constructing the strength function for the probability of transitions from single-phonon states to states of the yrast line. To this end, we rewrite the equations of motion (49) in a more symmetric form. Instead of the unknowns (50), we introduce new ones (for simplicity, we restrict ourselves to the case of normal modes with $\omega_\nu \neq 0$):

$$\mathcal{R}_{s_2}^{(+)}(\nu) = \sqrt{\omega_\nu}\,\varkappa_{s_2} A_{s_2}^\nu;\quad \mathcal{R}_{s_1}^{(-)}(\nu) = -\frac{1}{\sqrt{\omega_\nu}}\varkappa_{s_1} B_{s_1}^\nu. \tag{100}$$

Then (49) can be rewritten in the form

$$\left.\begin{aligned} X_\mu^\nu &= \frac{E_\mu}{E_\mu^2 - \omega_\nu^2}\frac{1}{\sqrt{\omega_\nu}}\sum_{s_2}\mathcal{R}_{s_2}^{(+)}(\nu)\,W_\mu^{s_2} \\ &\quad + \frac{1}{E_\mu^2 - \omega_\nu^2}\sqrt{\omega_\nu}\sum_{s_1}\mathcal{R}_{s_1}^{(-)}(\nu)\,V_\mu^{s_1}; \\ \mathcal{P}_\mu^\nu &= \frac{\omega_\nu^2}{E_\mu^2 - \omega_\nu^2}\frac{1}{\sqrt{\omega_\nu}}\sum_{s_2}\mathcal{R}_{s_2}^{(+)}(\nu)\,W_\mu^{s_2} \\ &\quad + \frac{E_\mu}{E_\mu^2 - \omega_\nu^2}\sqrt{\omega_\nu}\sum_{s_1}\mathcal{R}_{s_1}^{(-)}(\nu)\,V_\mu^{s_1}. \end{aligned}\right\} \tag{101}$$

The system of equations (51) with allowance for the relations (100) takes the form

$$\left.\begin{aligned} &\sum_{s_2}\mathcal{R}_{s_2}^{(+)}(\nu)\left(S_{W_{s_2}W_{s_2'}} - \frac{\delta_{s_2 s_2'}}{4\varkappa_{s_2}}\right) + \omega_\nu\sum_{s_1}\mathcal{R}_{s_1}^{(-)}(\nu)\,U_{V_{s_1}W_{s_2'}} = 0; \\ &\omega_\nu\sum_{s_2}\mathcal{R}_{s_2}^{(+)}(\nu)\,U_{V_{s_1'}W_{s_2}} + \sum_{s_1}\mathcal{R}_{s_1}^{(-)}(\nu)\left(S_{V_{s_1}V_{s_1'}} + \frac{\delta_{s_1 s_1'}}{4\varkappa_{s_1}}\right) = 0, \end{aligned}\right\} \tag{102}$$

i.e., we have obtained a system of homogeneous equations for finding the vectors $\mathcal{R}^{(\pm)}$:

$$\sum_{s'}\mathcal{J}_{ss'}\mathcal{R}_{s'} = 0,\quad s = 1, \ldots, n_1 + n_2, \tag{102a}$$

where

$$\mathcal{J}=\begin{pmatrix} S_{W_{s_2}W_{s_2'}}-\dfrac{\delta_{s_2s_2'}}{4\varkappa_{s_2}} & \omega U_{V_{s_1}W_{s_2}} \\ \omega U_{V_{s_1}W_{s_2}} & S_{V_{s_1}V_{s_1'}}+\dfrac{\delta_{s_1s_1'}}{4\varkappa_{s_1}} \end{pmatrix},\quad \mathcal{R}=\begin{pmatrix} \mathcal{R}^{(+)}_{s_2} \\ \mathcal{R}^{(-)}_{s_1} \end{pmatrix}.$$

The normalization condition from (48) has with allowance for (101) the form

$$\sum_{\mu} X^{\nu}_{\mu}\mathcal{P}^{\nu}_{\mu}=\sum_{\mu}\frac{1}{(E^2_{\mu}-\omega^2_{\nu})^2}\Big\{E_{\mu}\omega_{\nu}\Big(\sum_{s_2s_2'}\mathcal{R}^{(+)}_{s_2}(\nu)\,\mathcal{R}^{(+)}_{s_2'}(\nu)\,W^{s_2}_{\mu}W^{s_2'}_{\mu}$$
$$+\sum_{s_1s_1'}\mathcal{R}^{(-)}_{s_1}(\nu)\,\mathcal{R}^{(-)}_{s_1'}(\nu)\,V^{s_1}_{\mu}V^{s_1'}_{\mu}\Big)$$
$$+(E^2_{\mu}+\omega^2_{\nu})\sum_{s_1s_2}\mathcal{R}^{(+)}_{s_2}(\nu)\,\mathcal{R}^{(-)}_{s_1}(\nu)\,V^{s_1}_{\mu}W^{s_2}_{\mu}\Big\}$$
$$=\frac{1}{2}\sum_{s,\,s'=1}^{n=n_1+n_2}\mathcal{R}_s(\nu)\,\mathcal{R}_{s'}(\nu)\,\partial\mathcal{J}_{ss'}/\partial\omega|_{\omega=\omega_{\nu}}=\frac{1}{2}. \tag{103}$$

We transfer to the right-hand side of (102a) one term, containing, for example, $\mathcal{R}_n(\nu)$:

$$\sum_{s'=1}^{n-1}\mathcal{J}_{ss'}(\nu)\,\mathcal{R}_{s'}(\nu)=-\mathcal{J}_{sn}\mathcal{R}_n(\nu)\to\sum_{s'=1}^{n-1}\mathcal{J}_{ss'}a_{s'}=-\mathcal{J}_{sn}, \tag{104}$$

where

$$a_{s'}=\frac{\mathcal{R}_{s'}}{\mathcal{R}_n}\to\mathcal{R}_{s'}=a_{s'}\mathcal{R}_n. \tag{105}$$

We substitute (105) in the normalization condition (103):

$$\mathcal{R}^2_n\sum_{ss'}a_sa_{s'}\frac{\partial\mathcal{J}_{ss'}}{\partial\omega}\Big|_{\omega=\omega_{\nu}}=1. \tag{106}$$

The secular equation corresponding to the system of equations (102a) can be represented in the form of an expansion of the determinant $|\mathcal{J}|=0$ with respect to its cofactors:

$$|\mathcal{J}|=\sum_{s,\,s'=1}^{n}\mathcal{J}_{ss'}A_{ss'}=0\to\sum_{s'=1}^{n-1}\mathcal{J}_{ss'}\frac{A_{ss'}}{A_{sn}}=-\mathcal{J}_{sn}, \tag{107}$$

where s is an arbitrary index from the set 1, ..., n. Comparing (107) and (104), we obtain

$$a_{s'}=A_{ss'}/A_{sn}, \tag{108}$$

and since s is arbitrary, we choose $s=n$:

$$a_{s'}=A_{ns'}/A_{nn}. \tag{108a}$$

From (108) and (108a) we have

$$a_sa_{s'}=A_{ss'}/A_{nn}. \tag{109}$$

Substituting (109) in the condition (106), we obtain

$$\mathcal{R}^2_n=\frac{A_{nn}}{\sum_{ss'}A_{ss'}\,\partial\mathcal{J}_{ss'}/\partial\omega}. \tag{110}$$

We consider the complete determinant of the system

$$|\mathcal{J}|=\sum_{p}(-1)^p\mathcal{J}_{1p_1}\dots\mathcal{J}_{sp_s}\dots\mathcal{J}_{np_n} \tag{111}$$

and differentiate it with respect to ω:

$$\frac{\partial|\mathcal{J}|}{\partial\omega}=\sum_{sp}(-1)^p\mathcal{J}_{1p_1}\dots\frac{\partial\mathcal{J}_{sp_s}}{\partial\omega}\dots\mathcal{J}_{np_n}$$
$$=\sum_{s}\begin{vmatrix}\mathcal{J}_{11} & \dots & \mathcal{J}_{1n}\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \partial\mathcal{J}_{s1}/\partial\omega & \dots & \partial\mathcal{J}_{sn}/\partial\omega\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \mathcal{J}_{n1} & \dots & \mathcal{J}_{nn}\end{vmatrix}=\sum_{ss'}A_{ss'}\frac{\partial\mathcal{J}_{ss'}}{\partial\omega}. \tag{112}$$

Then finally

$$\mathcal{R}^2_n=\frac{A_{nn}}{\partial|\mathcal{J}|/\partial\omega}. \tag{113}$$

The remaining unknowns $\mathcal{R}_s$ can be determined from (108a) and (105). We express the generalized coordinates X^{ν}_{μ} and momenta $\mathcal{P}^{\nu}_{\mu}$ in terms of $\mathcal{R}_n$:

$$\left.\begin{aligned} X^{\nu}_{\mu}&=\mathcal{R}_n\Big\{\frac{E_{\mu}}{E^2_{\mu}-\omega^2_{\nu}}\frac{1}{\sqrt{\omega_{\nu}}}\sum_{s_2}a_{s_2}(\nu)\,W^{s_2}_{\mu}\\ &\quad+\frac{\sqrt{\omega_{\nu}}}{E^2_{\mu}-\omega^2_{\nu}}\sum_{s_1}a_{s_1}(\nu)\,V^{s_1}_{\mu}\Big\};\\ \mathcal{P}^{\nu}_{\mu}&=\mathcal{R}_n\Big\{\frac{\omega^2_{\nu}}{E^2_{\mu}-\omega^2_{\nu}}\frac{1}{\sqrt{\omega_{\nu}}}\sum_{s_2}a_{s_2}(\nu)\,W^{s_2}_{\mu}\\ &\quad+\frac{E_{\mu}\sqrt{\omega_{\nu}}}{E^2_{\mu}-\omega^2_{\nu}}\sum_{s_1}a_{s_1}(\nu)\,V^{s_1}_{\mu}\Big\}. \end{aligned}\right\} \tag{114}$$

The corresponding phonon amplitudes ψ^{ν}_{μ} and φ^{ν}_{μ} [see (86)] have the form

$$\left.\begin{aligned} \psi^{\nu}_{\mu}&=\frac{1}{\sqrt{2}}\Big(\sqrt{\omega_{\nu}}X^{\nu}_{\mu}+\frac{1}{\sqrt{\omega_{\nu}}}\mathcal{P}^{\nu}_{\mu}\Big)\\ &=\frac{1}{\sqrt{2}}\frac{\mathcal{R}_n}{E_{\mu}-\omega_{\nu}}\Big[\sum_{s_2}a_{s_2}(\nu)\,W^{s_2}_{\mu}+\sum_{s_1}a_{s_1}(\nu)\,V^{s_1}_{\mu}\Big];\\ \varphi^{\nu}_{\mu}&=\frac{1}{\sqrt{2}}\Big(\sqrt{\omega_{\nu}}X^{\nu}_{\mu}-\frac{1}{\sqrt{\omega_{\nu}}}\mathcal{P}^{\nu}_{\mu}\Big)\\ &=\frac{1}{\sqrt{2}}\frac{\mathcal{R}_n}{E_{\mu}+\omega_{\nu}}\Big[\sum_{s_2}a_{s_2}(\nu)\,W^{s_1}_{\mu}-\sum_{s_1}a_{s_1}(\nu)\,V^{s_1}_{\mu}\Big]. \end{aligned}\right\} \tag{115}$$

Since the amplitudes ψ and φ of the wave function of the single-phonon states are proportional to $\mathcal{R}_n$, any quantity quadratic in these amplitudes, for example, the probability of electric transitions [see (90)–(94)] from single-phonon states, contains the factor (113), which makes possible the use of the strength-function method [see Eq. (98) and Ref. 77]. For example, in Ref. 90, which uses the model of quadrupole excitations of rotating nuclei, expressions are given for calculating the characteristics of $E2$ transitions from single-phonon states to states of the yrast line on the basis of the strength-function method.

The CM + RPA method described above makes it possible to obtain quantitative information about the structure of the states of rotating nuclei in the framework of a microscopic theory based on a unified model. However, the practi-

cal realization of the method involves difficult calculations and the introduction of further simplifications and assumptions. At the same time, the qualitative aspects of the influence of rotation on the collective properties of the nuclei can be traced by means of a comparatively simple model of a self-consisent anisotropic oscillator potential, which was investigated in Refs. 95–97 (see also Ref. 6) in the approximation of the cranking model. In Sec. 2, using this model, we shall follow all stages in the solution of the problem of finding eigensolutions of the RPA equations for the residual interactions (isovector dipole forces[70,98] and isoscalar quadrupole forces[48,49,51]) and discuss the physical consequences. Further, in Sec. 3, in which we shall already use realistic potentials, we make a detailed analysis on the basis of the CM + RPA method of both the energy and the electric characteristics of some specific nuclei in the rare-earth region in their dependence on the angular momentum.

2. ANALYSIS OF COLLECTIVE EXCITATIONS OF ROTATING NUCLEI AND THE SIMPLE MODEL

Cranking-model Hamiltonian in the simple model

The changes in the shape and in the inertial parameters of a nucleus when it rotates can be modeled on the basis of the Hamiltonian of a harmonic oscillator with effective quadrupole forces,[96,97] from which one separates the Hamiltonian of the averaged deformed field and the residual quadrupole–quadrupole interaction responsible for the collective excitations of positive parity.[48,49] To describe the collective excitations of negative parity, it is necessary to introduce into the Hamiltonian a residual interaction of negative parity.

We consider a Hamiltonian of the form

$$H'=\sum_{\nu=1}^{A}\left(\frac{p_\nu^2}{2m}+\frac{m\omega_0^2}{2}r_\nu^2\right)-\frac{\varkappa}{2}\sum_{i,\,k=x,\,y,\,z}\hat{Q}_{ik}^2+H_{\text{res}}^{(-)}-\Omega\hat{L}_x, \tag{116}$$

where $\hat{Q}_{ik}=\Sigma_\nu\hat{q}_{ik}(\nu)$ are the components of the tensor of the quadrupole moment of the system, $\hat{L}_x=\Sigma_\nu\hat{l}_x(\nu)$ is the projection of the operator of the orbital angular momentum onto the x axis, and Ω is the rotational angular frequency. The single-particle Hamiltonian of a nucleus rotating with angular frequency Ω, made consistent with the Hamiltonian (116) by the Hartree method, can be written in the form[97]

$$\left.\begin{aligned}&H_{\text{AF}}(\Omega)=\sum_{\nu=1}h_\nu(\omega);\\&h_\nu(\Omega)=\frac{p^2}{2m}+\frac{m}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)-\Omega l_x.\end{aligned}\right\} \tag{117}$$

Following Ref. 96, we can conveniently determine the eigenfunctions and eigenvalues of the Hamiltonian (117) from the equation

$$[H_{\text{AF}}\ a_\lambda^+]=\omega_\lambda a_\lambda^+ \tag{118}$$

for the creation operators a_λ^+ of the oscillator quanta; these operators are linear combinations of the particle coordinates $\mathbf{r}_i$ and the conjugate momenta $\mathbf{p}_i$. Equations (118) correspond to the linear transformation

$$\begin{pmatrix}x\\p_x\end{pmatrix}=\begin{pmatrix}(2m\omega_x)^{-1/2} & (2m\omega_x)^{-1/2}\\ i\left(\frac{m\omega_x}{2}\right)^{1/2} & -i\left(\frac{m\omega_x}{2}\right)^{1/2}\end{pmatrix}\begin{pmatrix}a_x^+\\a_x\end{pmatrix}; \tag{119a}$$

$$\begin{pmatrix}y\\z\\p_y\\p_z\end{pmatrix}=\begin{pmatrix}Y_+ & Y_+^* & Y_- & Y_-^*\\ Z_+ & Z_+^* & Z_- & Z_-^*\\ P_+^y & P_+^{y*} & P_-^y & P_-^{y*}\\ P_+^z & P_+^{z*} & P_-^z & P_-^{z*}\end{pmatrix}\begin{pmatrix}a_+\\a_+^+\\a_-\\a_-^+\end{pmatrix}. \tag{119b}$$

The explicit form of the expressions for the coefficients of the transformation (119) can be found in Ref. 97. In terms of the operators a_λ^+ and a_λ, the Hamiltonian (117) has the form

$$H_{\text{AF}}=\sum_{\substack{\nu=1\\ \sigma=x,\,+,\,-}}^{A}\omega_\sigma\left(\hat{n}_\sigma+\frac{1}{2}\right)_\nu\equiv\sum_\sigma\omega_\sigma W_\sigma. \tag{120}$$

Here

$$\left.\begin{aligned}&\hat{n}_\sigma=a_\sigma^+(\nu)\,a_\sigma(\nu),\ \ [a_\sigma(\nu),\ a_{\sigma'}^+(\nu')]=\delta_{\sigma\sigma'}\delta_{\nu\nu'};\\&\omega_\pm^2=\frac{\omega_y^2+\omega_z^2}{2}+\Omega^2\pm\frac{1}{2}[(\omega_y^2-\omega_z^2)^2+8\Omega^2(\omega_y^2+\omega_z^2)]^{1/2}.\end{aligned}\right\} \tag{121}$$

The changes in the shape of a nucleus when it rotates were analyzed in detail in this model in Ref. 97. A typical hodograph showing the dependence of the parameters of the equilibrium deformation of the potential on Ω is shown in Fig. 1. At comparatively low rotation frequencies, the deformation of the potential decreases with increasing Ω but the nonaxiality increases. At rotation frequencies exceeding a certain critical value $\Omega_{\text{cr}}^{(1)}$, the rotation regime of an oblate nucleus around the symmetry axis is energetically advantageous. We note that the behavior of the deformation parameters is determined by the conditions of minimum of the energy of the system, and to semiclassical accuracy these can be expressed in the form[97]

$$\omega_xW_x=\omega_+W_+=\omega_-W_-=C=\text{const}.$$

In the absence of rotation, these conditions ensure that the nucleon density is consistent with the shape of the oscillator potential,[1] whereas in the more general case they correspond to the requirement of an isotropic distribution of the nucleon velocities in the considered system.[6,96,97] In addition, the fixing of the relationship between the populations along the various axes leaves open various possibilities corresponding

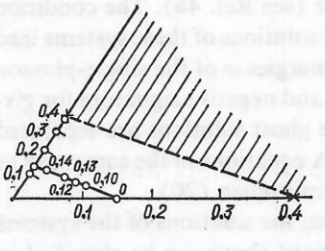

FIG. 1. Change in the oscillator frequencies of the average field as functions of Ω. The numbers next to the points determine the corresponding values of Ω/ω_0. In the hatched part of the figure, there are no solutions to the problem of minimizing the energy of the system.

to different rotational bands. It is obvious that among these the one with the lowest energy will be the yrast line.

Appreciable changes in the shape of the nucleus occur in the given model at too high rotation frequencies. In this respect the estimates of the rotational deformability of nuclei in the liquid-drop model[99] are more realistic. In the framework of the simple model, the collective excitations are also described by means of phonons, which can be expressed in terms of the genralized coordinates and momenta [see (30)] determined by the equations of motion (28). Since the Hamiltonian (116) is invariant with respect to rotation through angle π about the x axis, $R_x(\pi)$, the phonon operators in the RPA are characterized in accordance with Eq. (19) by the signature quantum number $\pm$:

$$R_x^{-1}(\pi)\,\hat{O}_\nu^+ R_x(\pi) = \pm\hat{O}_\nu^+, \tag{122}$$

i.e., the residual interactions can be divided into four mutually commuting parts $H^{(\pm)}_{(\pm)}$ [as in the case of (20)], where

$$H^{(+)}_{(+)} = -\frac{\varkappa}{2}\sum_{i=0,\,1,\,2}\hat{Q}_i^{(+)^2};\quad H^{(-)}_{+} = -\frac{\varkappa}{2}\sum_{i=1,\,2}\hat{Q}_i^{(-)^2}; \tag{123}$$

$$H^{(-)} = H^{(+)}_{(-)} + H^{(-)}_{(-)}. \tag{123a}$$

Isoscalar quadrupole excitations

Analysis of the spectrum

The generalized coordinates and momenta for the Hamiltonian $H^{(\pm)}_{(+)}$ (the Hamiltonian of the isoscalar quadrupole forces) can be represented in the form

$$X_\nu^{(\pm)} = \sum_s X_s^\nu(\pm)\,\hat{q}_s^{(\pm)};\quad \mathcal{P}_\nu^{(\pm)} = i\sum_s \mathcal{P}_s^\nu(\pm)\,\hat{p}_s^{(\pm)}, \tag{124}$$

where $\hat{q}_s^{(\pm)}$ and $\hat{p}_s^{(\pm)}$ are bilinear combinations of the operators a_λ^+ and a_λ, whose form can be found in Ref. 48. Linearization of the equations of motion is achieved by means of the substitution

$$[\hat{q}_s^{(\pm)},\ \hat{p}_{s'}^{(\pm)}] \to [\hat{q}_s^{(\pm)},\ \hat{p}_{s'}^{(\pm)}]_{\mathrm{RPA}} \equiv \langle\Omega|[\hat{q}_s^{(\pm)},\ \hat{p}_{s'}^{(\pm)}]|\Omega\rangle = V_s^{(\pm)}\delta_{ss'}, \tag{125}$$

where $V_s^{(\pm)}$ is a c number. Following the scheme of solution of the equations of motion of the RPA set forth in Sec. 1, we obtain for the unknowns $X_s^\nu(\pm)$ and $\mathcal{P}_s^\nu(\pm)$ a system of algebraic equations of the type (51); it is of the third order ($n_1 = 2, n_2 = 1, n_1 + n_2 = 3$) in the case of positive signature and of the second order ($n_1 = 1, n_2 = 1, n_1 + n_2 = 2$) in the case of negative signature (see Ref. 48). The condition for the existence of nontrivial solutions of these systems lead to secular equations for the energies ω of the single-phonon quadrupole states of positive and negative signature for given rotation frequency Ω. The ghost solutions are separated from the solutions of the RPA equations in the same way as was done in Sec. 1 for the Hamiltonian (20).

In the absence of rotation, the solutions of the systems of algebraic equations mentioned above can be classified in the usual manner in accordance with the projection K^π of the angular momentum onto the symmetry axis. The spectrum of positive-parity excitations will be determined for each of the possible values of $K^\pi = 0^+, 1^+, 2^+$ by the equation

$$1 - 2\varkappa S_{kk}(\omega) = 0. \tag{126}$$

Here, $S_{kk}(\omega) = S_{Q^{(\pm)}_k Q^{(\pm)}_k}$ is a relation of the type (52), and the corresponding matrix elements can be obtained by means of the coefficients of the transformation (119). The solution of these equations is determined by the formulas

$$\left.\begin{aligned}
&\left.\begin{matrix}\omega_2(0^+_{\sigma=+})/\omega_0\\ \omega_1(0^+_{\sigma=+})/\omega_0\end{matrix}\right\} = \left[3 - \tfrac{4}{3}\delta \pm \left[1 + \tfrac{8}{3}\delta + \tfrac{208}{9}\delta^2\right]\right]^{1/2}\\
&\approx \begin{cases} 2\\ \sqrt{2}\left(1 - \tfrac{2}{3}\delta\right)\end{cases};\\
&\omega(1^+_{\sigma=\pm})/\omega_0 = \sqrt{2\left(1 - \tfrac{1}{3}\delta\right)} \approx \sqrt{2}\left(1 - \tfrac{1}{6}\delta\right);\\
&\omega(2^+_{\sigma=\pm})/\omega_0 = \sqrt{2\left(1 + \tfrac{8}{3}\delta\right)} \approx \sqrt{2}\left(1 + \tfrac{4}{3}\delta\right),
\end{aligned}\right\} \tag{127}$$

where to describe the oscillator frequencies we have used the Nilsson deformation parameter δ. The state 0_2^+ with energy $\omega \approx 2\omega_0$ does not possess collective properties. The remaining solutions characterize the positions of the corresponding branches of the giant quadrupole resonance (GQR) in the deformed nuclei. In the limit of zero deformation, the solution $\omega = \sqrt{2}\omega_0$ agrees well with the available experimental data on the isoscalar GQR.[1] We note that the considered model does not reproduce the low-lying collective excitations of nuclei (β and γ bands), since the spectrum of the corresponding quasiparticle excitations has an oversimplified nature.[49]

The dependence of the energies of the GQR states on the deformation parameter in the absence of rotation ($\Omega = 0$) is shown in Fig.2. It can be seen that the excitation energy of the phonon state 0_1^+ decreases monotonically with increasing δ and vanishes at $\delta = \frac{1}{4}(\sqrt{7}-1) = 0.411$. Below, we restrict ourselves to considering the characteristics of the model in the region of δ and Ω for which the frequencies of the phonon excitations in the RPA are nonzero.

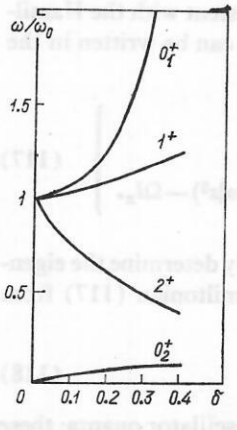

FIG. 2. Dependence of the excitation energy of positive-parity states on the deformation parameter δ at $\Omega = 0$.

As was noted above, for $\Omega \geqslant \Omega_{cr}^{(1)}$ the angular momentum is directed along the symmetry axis, and the RPA phonons have an exact value of the quantum number of the angular-momentum projection onto this axis (denoted in what follows by τ). The dispersion relations for the frequencies of the normal vibrations also separate in this case into three independent equations; two of them determine the excitations of positive signature with $\tau = 0$ and ± 2, while the third determines the excitations of negative signature with $\tau = \pm 1$. When we determined the solutions of the RPA equations, we made the assumption that the coupling constant $g = 18\varkappa C/m^2\omega_0^4$ of the model depends on the rotation frequency Ω. The condition needed to determine this dependence can be found by requiring that the spectrum of intrinsic excitations contain no ghost solutions due to the breaking of the rotational invariance of the Hamiltonian (see Refs. 48 and 49). This condition has the form

$$g = \nu_+\nu_- \tag{128}$$

and in conjunction with the self-consistency conditions[97] completely determine the function $g(\Omega)$. We note that there are alternative approaches to the analysis of the collective excitations of rapidly rotating nuclei,[100] in which it is assumed that the coupling constants of the model are independent of the rotation frequency.

When the matrix elements of the quadrupole operators are expressed in a spherical coordinate system oriented along the symmetry axis, Eq. (126) for the state $\tau = 0$ takes the form

$$\frac{2}{3}(\nu_y - \lambda)^2 \left\{ \frac{2}{\nu_x(4\nu_x - \nu)} + \frac{1}{(\nu_y - \lambda)(4\nu_y - \lambda)} \right\} = 1. \tag{129}$$

For a state with $\tau = \pm 2$ we can, after some manipulations and separation of the ghost solution, write the dispersion relation in the form

$$\nu^2 + 6\sqrt{\lambda}\nu + 10\lambda - 2\nu = 0. \tag{130}$$

For negative-signature states with $\tau = \pm 1$ we can, after the separation of the ghost solutions and a number of simple calculations, represent the dispersion relation in the form

$$Z^3 - Z\left[2\left(1 + \frac{\sqrt{\lambda}}{6}\right) + \frac{2\sqrt{\lambda}\left(1 + \frac{2}{3}\sqrt{\lambda}\right)}{1 + \sqrt{\lambda}}\right] + \frac{2\sqrt{\lambda}\left(1 + \frac{2}{3}\sqrt{\lambda}\right)}{1 + \sqrt{\lambda}} = 0, \tag{131}$$

where $Z = \sqrt{\nu} + \sqrt{\lambda}$. We give approximate solutions obtained by expansion in powers of Ω:

$$\omega^{\tau=\pm 1} \approx \sqrt{2}\omega_0 + \frac{7}{6\sqrt{2}}\Omega \pm \frac{3}{2}\Omega; \tag{132}$$

$$\omega_{\text{prec}}^{\tau=+1} \approx \frac{3}{2}\frac{\Omega^2}{\omega_0}\left(1 - \frac{8}{9}\frac{\Omega}{\omega_0}\right). \tag{133}$$

The analogous expansions for the positive-signature solutions have the form

$$\left.\begin{aligned} \omega^{\tau=0} &\approx \begin{cases} 2\omega_0; \\ \sqrt{2}\,\omega_0 + \frac{5\sqrt{2}}{6}\Omega; \end{cases} \\ \omega^{\tau=\pm 2} &\approx \sqrt{2}\omega_0 - \frac{\Omega}{3\sqrt{2}} \pm 3\Omega. \end{aligned}\right\} \tag{134}$$

The relations (132) and (134) characterize the splitting of the different branches of the isoscalar GQR in the rotating nucleus, while (133) determines the frequency of the low-frequency precession mode. The adiabatic estimate for the frequency of the precession oscillations,[1]

$$\omega_{\text{prec}}^{\text{ad}} = \Omega\left[(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)/(\mathcal{J}_y\mathcal{J}_z)\right]^{1/2},$$

where $\mathcal{J}_i$ are the rigid-body values of the moments of inertia, leads for the axial rotation regime to the relation

$$\omega_{\text{prec}}^{\text{ad}} = \Omega\frac{\nu_x - \nu_y + \lambda}{\nu_x + \nu_y + \lambda} \approx \frac{\Omega^2}{2\omega_0}\left(1 - \frac{5}{6}\frac{\Omega}{\omega_0} + \ldots\right). \tag{135}$$

This result differs appreciably from the analogous estimate in the RPA corresponding to (133). For the axial rotation regime $(\omega^{\text{RPA}}/\omega^{\text{ad}})_{\text{prec}} \approx 3$ for low rotation frequencies, and the ratio of the precession frequencies is reduced to approximately 2 in the region of higher Ω. This ratio is also close to 2 for the nonaxial rotation regime.

The general behavior of the reduced probabilities is in good agreement with the results of the analysis of the intrinsic matrix elements based on the sum rules of Ref. 49. The soft branch of the GQR retains collectivity at all values of the angular momentum up to the critical value. For $\Omega \geqslant \Omega_{cr}^{(1)}$, this branch is characterized by an exact value of the projection of the angular momentum onto the rotation axis ($\tau = 2$), transitions with a decrease of a phonon excitation of given species causing a reduction in the angular momentum of the nucleus by two units.

Transitions between yrast states and states of a precessional nature have an appreciable reduced probability. Calculation shows that the dependence of the probability of such transitions on the angular momentum found in the RPA does not have the same behavior as the estimates in accordance with the model of a rigid rotator. There is a certain similarity between the RPA results and the phenomenological rigid-rotator model only for $\Omega \geqslant 0.2\omega_0$.

Isovector dipole resonance

As an example of residual interactions of negative parity, we consider the case of isovector dipole forces.[70] Then $H_{\text{res}}^{(-)}$ takes the form

$$H_{\text{res}}^{(-)} = \eta \sum_{i=x,\,y,\,z} \frac{m\omega_i^2}{2A}\left(\sum_{\nu=1}^{A} \tau_3(\nu)\,\hat{x}_i(\nu)\right)^2, \tag{136}$$

where $\tau_3(\nu)$ is the third projection of the Pauli isospin matrix $\tau_3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ and η is the parameter that characterizes the isovector component of the neutron and proton average fields:

$$V_{\binom{n}{p}}(\nu) = \frac{m}{2}\left(1 \mp \eta\frac{N-Z}{A}\right)\sum_{i=x,\,y,\,z} \omega_i^2 x_i^2(\nu). \tag{137}$$

Our parametrization of the dipole interaction (136) corresponds to the prescriptions for constructing the effective forces that restore the translational invariance of the model Hamiltonian (see Ref. 93). The value of the parameter η, determined from the experimental data on the position of the giant dipole resonance (GDR), for the oscillator potential is $\eta \simeq 3$.

The symmetry of the average field with respect to the operation $R_x(\pi)$ makes it possible to divide $H_{\text{res}}^{(-)}$ into two parts [see (123)]:

$$\left.\begin{aligned} H_{(-)}^{(+)} &= \eta \frac{m\omega_x^2}{2A}\left(\sum_{\nu=1}^{A} \tau_3(\nu)\,\hat{x}(\nu)\right)^2; \\ H_{(-)}^{(-)} &= \eta \sum_{i=y,z} \frac{m\omega_i^2}{2A}\left(\sum_{\nu=1}^{A} \tau_3(\nu)\,\hat{x}_i(\nu)\right)^2. \end{aligned}\right\} \tag{138}$$

The RPA equations with the residual interaction (136) can be solved in the same way as for the quadrupole interactions (see the section headed "Isoscalar quadrupole excitations"). However, the fairly simple form of the dipole interactions (138) makes it possible to use a simpler alternative method. In the investigated case, the RPA equations can be written in the form (we are interested in only the nonghost solutions)

$$[H_{\text{AF}} + H_{\text{res}}^{(-)},\ D_\lambda^+] = \tilde{\omega}_\lambda D_\lambda^+, \tag{139}$$

where the dipole-phonon creation operator has the structure

$$D_\lambda^+ = \frac{1}{\sqrt{A}} \sum_{\nu=1}^{A} \tau_3(\nu)\, a_\lambda^+(\nu). \tag{140}$$

Here, a_λ^+ ($\lambda = +, -, x$) are the creation operators of the oscillator quanta [see (118) and (119)]. Thus, for the spectrum of frequencies of the GDR in the rotating nucleus we obtain

$$\left.\begin{aligned} &\tilde{\omega}_x = \sqrt{1+\eta}\,\omega_x; \\ &\tilde{\omega}_\pm^2 = (1+\eta)\,\frac{\omega_y^2+\omega_z^2}{2} + \Omega^2 \\ &\pm \sqrt{(1+\eta)^2(\omega_y^2-\omega_z^2)^2 + 8\Omega^2(1+\eta)(\omega_y^2+\omega_z^2)}. \end{aligned}\right\} \tag{141}$$

The probabilities of electromagnetic transitions that de-excite the states (λ, I) of the GDR to states of the yrast line are determined by the relations

$$\begin{aligned} |\hat{X}_\lambda^\mu|^2 &= |\langle \text{yr}|\,\hat{\mathfrak{M}}(E1,\ -\mu)\,D_\lambda^+|\,\text{yr}\rangle|^2 \\ &= |[\hat{\mathfrak{M}}(E1,\ -\mu),\ D_\lambda^+]_{\text{RPA}}|^2. \end{aligned} \tag{142}$$

Here,

$$\hat{\mathfrak{M}}(E1,\ -\mu) = \begin{cases} \sum_{\nu=1}^{A} \tau_3(\nu)\, x(\nu), & \mu = 0 \\ \mp \frac{1}{\sqrt{2}} \sum_{\nu=1}^{A} \tau_3(\nu)\,(y(\nu) \pm iz(\nu))^2 & \mu = \pm 1 \end{cases} \tag{143}$$

are the spherical components of the dipole-moment vector of the nucleus in the system of axes in which the quantization axis coincides with the rotation axis. Using the results of Ref. 97 for the coefficients of the transformation (119), we can write the explicit form of the relations for the transition amplitudes $|\tilde{X}_\lambda^\mu|^2$:

$$|\tilde{X}_\lambda^0|^2 = (2m\omega_x\sqrt{1+\eta})^{-1} \quad \text{for} \quad \lambda = x; \tag{144}$$

$$\begin{aligned} |\tilde{X}_\lambda^{\pm 1}|^2 &= |Y_\lambda \pm iZ_\lambda|^2 = \\ &= \lambda \frac{\omega_\lambda^2 - \omega_z^2 + \Omega^2/(1+\eta)}{2m\omega_\lambda \sqrt{(1+\eta)(\omega_+^2-\omega_-^2)}} \\ &\left\{1 \pm \frac{2\omega_\lambda\Omega/\sqrt{1+\eta}}{\omega_\lambda^2 - \omega_z^2 + \Omega^2/(1+\eta)}\right\} \quad \text{for} \quad \lambda = \pm. \end{aligned} \tag{145}$$

In the absence of rotation ($\Omega = 0$), the vibrations are, as in the case of the quadrupole excitations, classified by the quantum number K^π. For the longitudinal, $K^\pi = 0^-$, and transverse, $K^\pi = 1^-$, modes of the dipole vibrations we have in this case

$$\left.\begin{aligned} &\tilde{\omega}(0^-) = \sqrt{1+\eta}\,\omega_z = \sqrt{1+\eta}\,\omega_0\left(1 - \frac{4}{3}\delta\right)^{1/2}; \\ &\tilde{\omega}(1^-) = \sqrt{1+\eta}\,\omega_\perp = \sqrt{1+\eta}\,\omega_0\left(1 + \frac{2}{3}\delta\right)^{1/2}. \end{aligned}\right\} \tag{146}$$

The squares of the intrinsic matrix elements for $\Omega = 0$ will determine the reduced probability of the dipole transitions to the corresponding states,

$$B(E1,\ 0 \to K^\pi) = \frac{q_1}{2m\tilde{\omega}(K^\pi)} \frac{2}{1+\delta_{k0}}, \tag{147}$$

where $q_1 = 3A(e\hbar^2)/16\pi$.

The estimates obtained for the splitting of the GDR in deformed nuclei are in fairly good agreement with the experimental data on photoabsorption cross sections.[1] When $\Omega \neq 0$, there is an additional splitting of the frequency of the transverse dipole vibrations and a shift in the frequency of the longitudinal vibrations. The corresponding excitations are then classified by means of the signature quantum number. In this model, when the nucleus acquires an axial oblate shape and the rotation axis coincides with the symmetry axis, there appears an additional quantum number $\lambda = \pm 1$, which distinguishes the excitations of negative signature. To the different values there corresponds projection of the angular momentum of the phonon onto the rotation axis equal to $\mu = \pm 1$.

Figure 3 gives the dependence of the GDR energy on the rotation frequency. The inset to the figure shows the hodograph of the change in the deformation of the nucleus in the process of rotation. The dependence of the reduced tran-

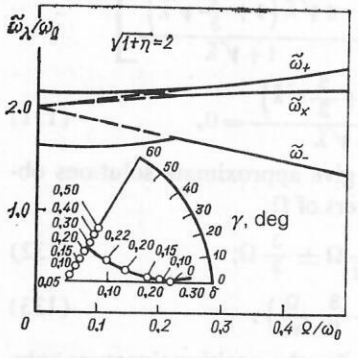

FIG. 3. Dependence of the excitation energy of the GDR modes $\tilde{\omega}_\lambda$ ($\lambda = x, +, -$) on the rotational angular frequency Ω. In the hodograph in the lower part of the figure the continuous thick points show the values of the equilibrium deformation parameters as functions of Ω/ω_0 (the values of Ω/ω_0 are shown next to the points). The broken lines correspond to a nucleus that has a spherical shape at $\Omega = 0$, whereas the continuous curves are calculated for a nucleus with initial deformation $\delta = 0.25$ at $\Omega = 0$ (see Ref. 70).

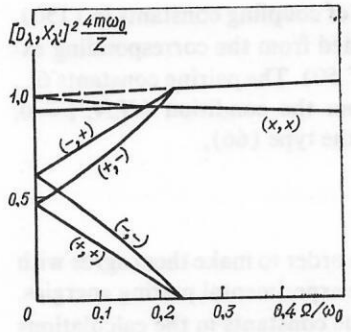

FIG. 4. Reduced probabilities of dipole transitions $[B(E1),\lambda I_i \to \mathrm{yr} I_f]$ from GDR states to states of the yrast line for different values of $\mu = I_i - I_f$ in units of q_1. The first symbol in the brackets next to the curve identifies the type of GDR state ($\lambda = x, +, -$), while the second index corresponds to $\mu = 0$ for $\lambda = x$ and $\mu = +1$ for $\lambda = \pm$.

sition probabilities on the rotation frequency is shown in Fig. 4. The selection rules for the states with respect to the signature,

$$(-1)^I \sigma = 1, \tag{148}$$

forbid dipole transitions $\lambda = x \to \mathrm{yr}$ with a change in the angular momentum. Thus, the partial sum

$$\sum_\lambda \tilde{\omega}_\lambda |\langle \mathrm{yr}| \hat{\mathfrak{M}}(E1,\ \mu=0)|\lambda\rangle|^2 = \tilde{\omega}_\lambda |[\hat{\mathfrak{M}}(E1,\ 0), D^+_\lambda]_{\mathrm{RPA}}|^2 = \frac{q_1}{2m} \tag{149}$$

completely determines the integrated characteristics of the transitions in which the spin of the state does not change. In the case of rotation about the symmetry axis, the possible transitions are ($\lambda = \pm, I \mp 1 \to \mathrm{yr}, I$), i.e., transitions that change the angular momentum by $+1$ or by -1, depending on the initial state λ. The reduced probability of such transitions is the same for all spins (Fig. 4) if the population of the state with $\lambda = \pm 1$ is the same. In the case of collective rotation, both types of dipole transition ($\lambda = \pm, I \to \mathrm{yr}, I \pm 1$) are allowed, and the changes in the reduced probabilities of such transitions with the rotation frequency are shown in Fig. 4 by the continuous curves. The energy-weighted sum rule for these transitions is determined by two terms, the sum of which remains practically the same as in the case of transitions without a change in the spin; see Eq. (149).

3. USE OF THE CM+RPA METHOD TO DESCRIBE CERTAIN CHARACTERISTICS OF COLLECTIVE STATES OF ROTATING NUCLEI

A. Low-lying states of ^{158}Dy and ^{168}Er

For the description of the low-lying states, we shall use a total Hamiltonian of the type (5), restricting ourselves to a residual interaction with multipolarity $\lambda = 1,2,3$. In addition, the condition of translational invariance of the Hamiltonian leads to the appearance of an additional residual dipole–octupole interaction. It is known[10] that for the description of low-lying quadrupole and octupole states it is sufficient to take into account the isoscalar part of the residual quadrupole–quadrupole and octupole–octupole interactions. In the study of $E1$ transitions between low-lying states, the tails of the isovector dipole resonance play an important part. Therefore, the Hamiltonian of the residual interaction has the form

$$H_{\mathrm{res}} = -\frac{1}{2}\sum_{m=-2}^{2} \varkappa_{2m}\hat{Q}^+_{2m}Q_{2m} - \frac{1}{2}\sum_{\tau=0,1}\sum_{m=-1}^{1} \varkappa^{[\tau]}_{1m}\hat{Q}^{[\tau]+}_{1m}Q^{[\tau]}_{1m} - \frac{1}{2}\sum_{m=-3}^{3} \varkappa_{3m}\hat{Q}^+_{3m}\hat{Q}_{3m} - \frac{1}{2}\sum_{\tau=0,1}\sum_{m=-1}^{1} \chi^{[\tau]}_m [\hat{Q}^{[\tau]+}_{1m}Q^{[\tau]}_{3m} + \hat{Q}^{[\tau]+}_{3m}\hat{Q}^{[\tau]}_{1m}], \tag{150}$$

where, in contrast to (5), we have introduced a dependence of the coupling constants on the projection μ ($\varkappa_{\lambda\mu} = \varkappa_{\lambda-\mu}$, $\chi_m = \chi_{-m}$). Here, $\varkappa^{[\tau]}_{\lambda\mu} = \varkappa^{[0]}_{\lambda\mu}$ and $\varkappa^{[1]}_{\lambda\mu}$ are, respectively, the isoscalar and isovector coupling constants. To solve the HFB problem for the cranking-model Hamiltonian, we approximate the average field by an axially deformed Woods–Saxon potential with parameters from Refs. 61 and 101. The deformation parameters were obtained by means of Strutinskii's method: $\beta_2 = 0.265$, $\beta_4 = 0.044$ for ^{158}Dy and $\beta_2 = 0.284$, $\beta_4 = -0.001$ for ^{168}Er.

To solve the HFB equations, the method described in Refs. 102 and 103 was used. It is known (see, for example, Ref. 104) that the HFB method without allowance for projection onto the exact particle number for the cranking-model Hamiltonian in the region of angular momenta corresponding to the disappearance of pairing does not permit the obtaining of a reliable description. Since the energy gap begins to decrease at spins $J \sim 8\hbar$ for ^{168}Er and $J \sim 6\hbar$ for ^{158}Dy, the yrast-line spectrum was calculated for these nuclei up to $J \leqslant 8\hbar$ and $J \leqslant 6\hbar$, respectively.

The dependence of the spin J on the rotation frequency on the yrast line is determined in the standard manner: the HFB equations are solved for several increasing values of Ω, in accordance with which $\langle\Omega|\hat{J}_x|\Omega\rangle$ is calculated. Using the cranking-model condition $\langle\Omega|\hat{J}_x|\Omega\rangle = \sqrt{J(J+1)}$, we determine the function $J(\Omega)$.

Solving the HFB equations and bearing in mind the symmetry of the single-particle operators in (150), we can write the cranking-model Hamiltonian H' with the residual interaction (150) in the form (20) (see Ref. 50). The ghost states are separated and the solutions of the RPA equations are found for all four parts of the Hamiltonian (150) in accordance with Sec. 1. The dimensions of the systems of equations for the Hamiltonians $H^{(+)}_{(+)}$, $H^{(-)}_{(+)}$, $H^{(+)}_{(-)}$, $H^{(-)}_{(-)}$, are, respectively, 7, 2, 6, 10. Their explicit form can be found in Ref. 50. The existence of the ghost mode $[\theta_x(1), J_x(1)]$ among the solutions of the RPA equations for the Hamiltonian $H^{(+)}_{(+)}$ makes it possible to determine the moment of inertia of the nucleus with respect to the x axis [see (32)] by means of the relation (70) (because the formula is cumbersome, we do not give it here; see Ref. 50). An analogous expression can also be obtained from g_{N_τ}. From the condition that the mode $[X(1), P_x(1)]$ has a zero solution of the RPA equations for the Hamiltonian $H^{(+)}_{(-)}$, we obtain for

the mass parameter g_{P_x} the expression [see (34) and (70)]

$$g_{P_x}=\frac{1}{M}=\frac{1}{2}\frac{S_{F_2^{(+)}F_2^{(+)}}(0)-\frac{1}{2\varkappa_{32}}}{\left[S_{F_2^{(+)}F_2^{(+)}}(0)-\frac{1}{2\varkappa_{32}}\right]S_{P_xP_x}(0)-S^2_{F_2^{(+)}P_x}(0)}. \tag{151}$$

It follows from the symmetry of the single-phonon wave function of the Hamiltonian (150) with respect to the operation $R_x(\pi)$ [see (76)] that to even values of the total angular momentum J there correspond the solutions of the RPA equations of the Hamiltonians $H^{(+)}_{(\pm)}$, and to odd values of J the solutions of the RPA equations of the Hamiltonians $H^{(+)}_{(\pm)}$, and to odd values of J the solutions of the RPA equations of the Hamiltonians $H^{(-)}_{(\pm)}$.

For comparison with the experimental data, we used the following definition of the energy of an excited state:

$$E_\lambda(J)=E_{\rm yr}(J_0)+\frac{\hbar^2}{2\mathscr{J}_x(J_0)}\times[J(J+1)-J_0(J_0+1)]+\hbar\omega_\lambda(J_0), \tag{152}$$

where $J=J_0$ for even values of the spin and $J=J_0\pm1$ for odd values of the spin; $\hbar\omega_\lambda(J_0)$ is the energy of the RPA phonon $O^+_\lambda(\pm)|\Omega_{J_0}\rangle$, and $\mathscr{J}_x(J_0)$ is the moment of inertia. It should be noted that this definition is more accurate, the higher the angular momentum. For small angular momenta, the solutions must be corrected [see footnote 12].

The results of the calculations obviously depend on the coupling constants. For their determination, we used the following requirements:

a) the fulfillment on the average for the Hamiltonian (150) of the symmetry conditions (2), this leading for each part of the total Hamiltonian H' to self-consistency conditions of the type (46) (for the actual form, see Ref. 50);

b) since the number of conditions (2) in the considered case is less than the number of coupling constants in (150), some of them were determined from the corresponding experimental energies (see Ref. 50). The pairing constants G_τ, which were determined from the condition $[H,\hat{N}_\tau]=0$, which leads to a relation of the type (66),

$$G_\tau=\Big(\sum_\mu\frac{p^{(-)}_\mu p^{(-)}_\mu}{E_\mu}\Big)^{-1},$$

needed to be renormalized in order to make them agree with the values obtained from the experimental pairing energies. We note that the values of the constants in the calculations hardly depend on the spin (see Ref. 50).

The theoretical and experimental spectra are compared in Figs. 6 and 8 for ^{166}Er and in Figs. 5 and 7 for $^{158}_{66}Dy_{92}$. To determine the correspondence between the calculated and experimental energies, it was required that the structure of the phonon that forms the given rotational band should change slowly with the spin. The experimental data are taken from Refs. 105–115. It can be seen from Fig. 6 that up to energies ~2.4 MeV practically all the solutions of the RPA equations correspond to experimental levels. An exception is the band $K^\pi_\nu=3^+_1$ in ^{168}Er observed in the experiment of Ref. 105, for which a theoretical partner was not found in the model, and therefore it is omitted in Fig. 6. In the bands $K^\pi_\nu=3^-_2$, 4^-_2, 3^-_3, and 4^-_3 in ^{168}Er at $J=5\hbar$, theoretical partners corresponding to the experimental levels were also not found. In the band $K^\pi_\nu=4^-_1$ (Fig. 8) in ^{168}Er a shift of the theoretical levels relative to the experimental levels is observed. In this connection, it is interesting to note that the solutions of the RPA equations associated with this band disappear from the energy interval up to 2.4 MeV when in the Hamiltonian (150) $\varkappa_{1m}=\chi_m=0$, i.e., only the octupole–octopule interaction is taken into account. It appears that the nature of this band is due to the dipole interaction and its coupling to the rotation. Thus, our model developed

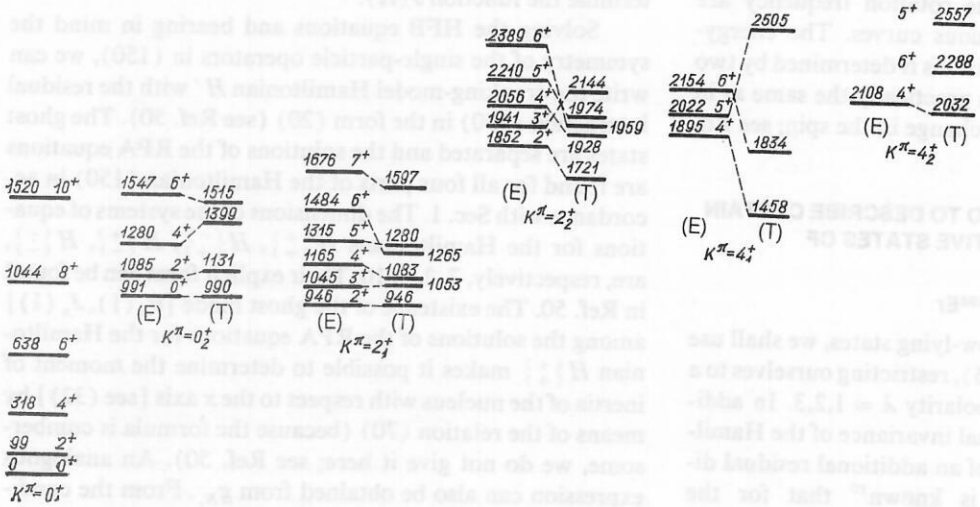

FIG. 5. Comparison of calculated (T) and experimental (E) rotational bands of positive parity for $^{158}_{66}Dy_{92}$.

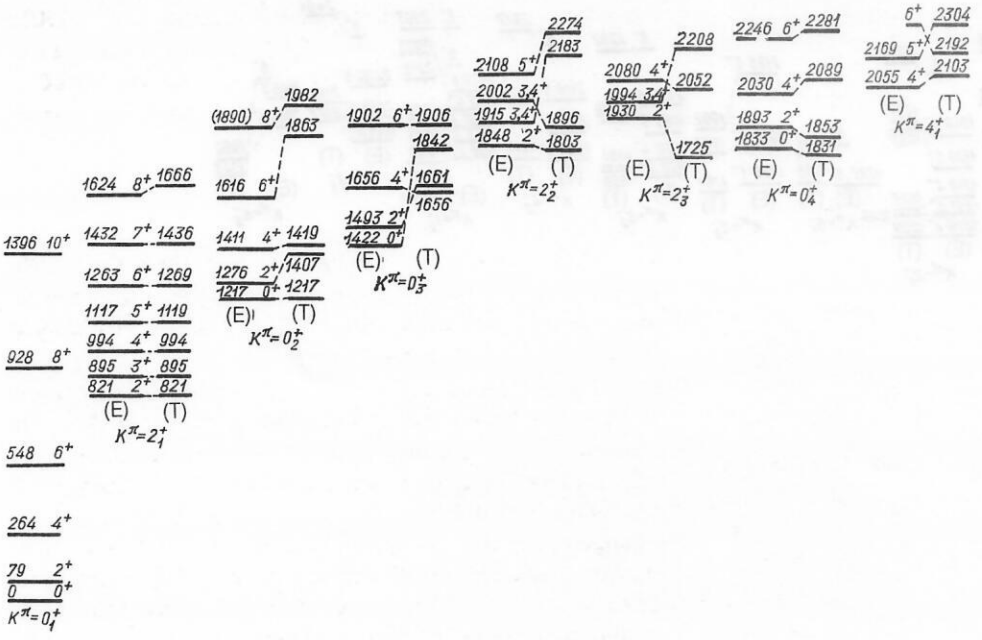

FIG. 6. The same as in Fig. 5 for $^{168}_{68}\mathrm{Er}_{100}$.

in the framework of the CM + RPA method makes it possible to obtain a good description of the experimental data in the low-energy region of the spectrum.

The theoretical values of the $B(E2)$ transitions in ^{168}Er and ^{158}Dy were obtained in accordance with Eqs. (91)–(93) and were compared with the experimental values (see Ref. 50). In the calculations in accordance with Ref. 116 for the $E2$ transitions $e_{\mathrm{eff}}^{\lambda=2}=0.2$ for the neutrons and $e_{\mathrm{eff}}^{\lambda=2}=1.2$ for the protons. The absolute values $B(E2)^{\mathrm{exp}}$ were obtained from the relative $B(E2)^{\mathrm{exp}}$ by means of the Alaga rules within the given band under the assumption that the intrinsic quadrupole moment Q_0 for the given band is equal to the ground-state moment[117] [$Q_0=760\ \mathrm{F}^2$ for ^{168}Er (Ref. 108) and $Q_0=694\ \mathrm{F}^2$ for ^{158}Dy (Ref. 114)]. We note that the agreement with the experimental $B(E2)$ values is better for ^{156}Dy than for ^{168}Er. In the first place, this is evidently due to the appreciable anharmonic effects in ^{168}Er,[117,118] which can only be taken into account in the CM + RPA approach by going beyond the RPA. It is interesting that the employed model reflects the experimentally observed reduction by one or two orders of magnitude in the values of the $B(E2)$ transitions from the β band to the ground-state band as compared with the $B(E2)$ transitions from the γ band to the ground-state band for ^{168}Er compared with ^{158}Dy.

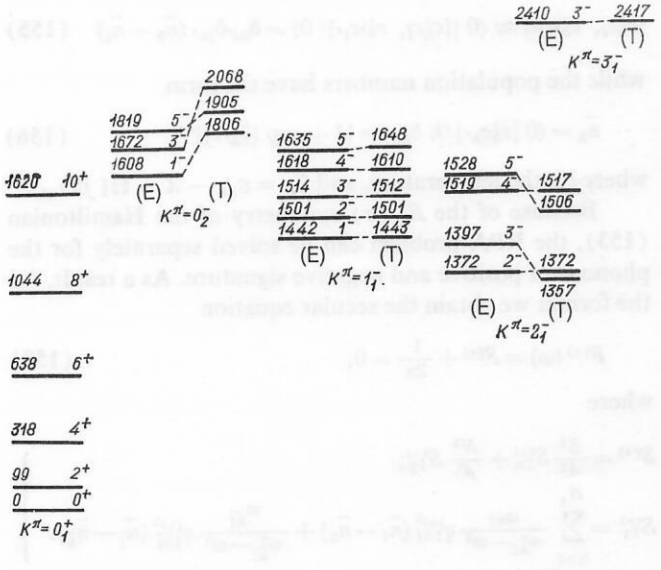

FIG. 7. Comparison of calculated (T) and experimental (E) rotational bands of negative parity for $^{158}_{66}\mathrm{Dy}_{92}$.

The large difference between the ratios of the reduced probabilities of $E2$ transitions taking place from single-phonon states (β or γ) to all possible (compatible with the selection rules) states of the yrast lines compared with the corresponding Alaga ratios indicates a large ΔK mixing.

The values of the theoretical ratios $B(E1, I\nu \to I+1_{\mathrm{yr}})/B(E1, I\nu \to I-1_{\mathrm{yr}})$ were compared with the results of Ref. 119 obtained on the basis of the phenomenological model of Ref. 120. The following values were used for the effective charges: $e_{\mathrm{eff}}^{\lambda=1}=-0.405$, $e_{\mathrm{eff}}^{\lambda=3}=0.2$ for the neutrons and $e_{\mathrm{eff}}^{\lambda=1}=0.595$, $e_{\mathrm{eff}}^{\lambda=3}=1.2$ for the protons for ^{158}Er; $e_{\mathrm{eff}}^{\lambda=1}=-0.418$, $e_{\mathrm{eff}}^{\lambda=3}=0.2$ for the neutrons and $e_{\mathrm{eff}}^{\lambda=1}=0.582$, $e_{\mathrm{eff}}^{\lambda=3}=1.2$ for the protons in the case of ^{158}Dy.

It should be noted that the agreement with the experimental probabilities of $E1$ transitions is not as good as for the $E2$ transitions. The probabilities of the $E1$ transitions are more sensitive to the anharmonic effects, which are not taken into account in the framework of the RPA, and also to the errors due to the mixing of states with different signatures at low spins.

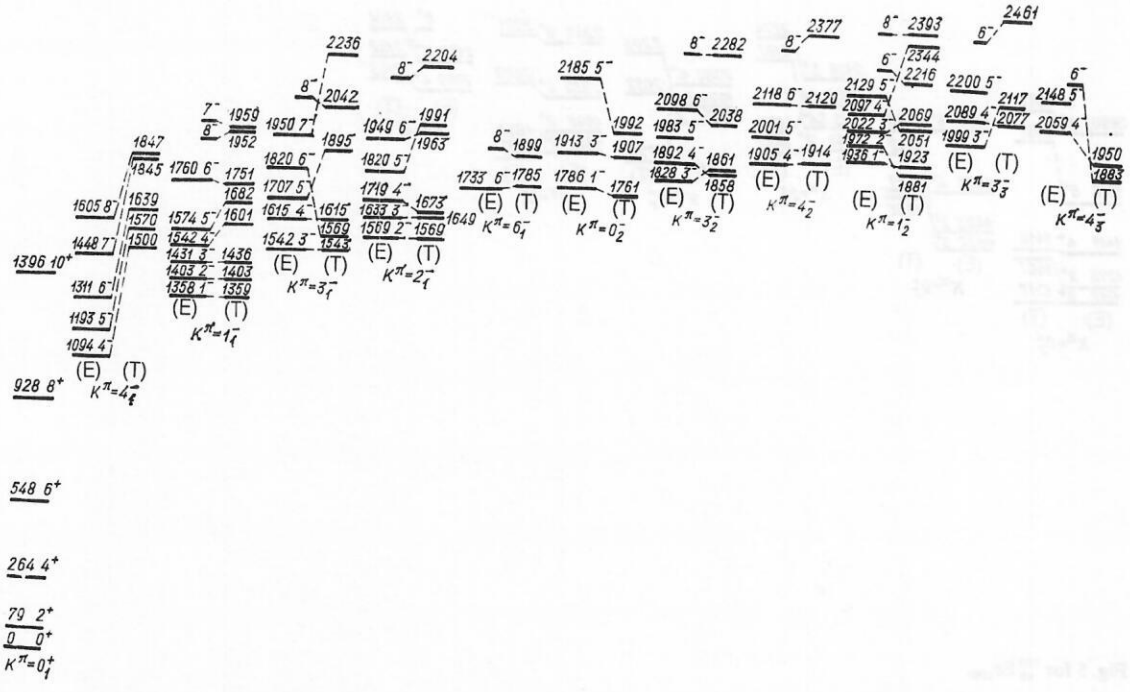

FIG. 8. The same as in Fig. 7 for $^{168}_{68}\mathrm{Er}_{100}$.

Highly excited states (giant dipole resonance at high spins)

A simple model of the GDR of rapidly rotating nuclei was considered in Sec. 2. Here, we investigate in the CM + RPA framework the properties of the GDR in a realistic model. First, we approximate the average field by the Nilsson potential. Calculations in the cranking-model approximation on the basis of this potential make it possible to follow the evolution of the shape of rapidly rotating nuclei in a wide range of values of the angular momentum and the excitation energies.[121,122] Second, under real experimental conditions one measures averaged quantities for the description of which we use a statistical version of the CM + RPA method. Such an approach for the investigation of the energy spectrum of the quadrupole excitations of "heated" rotating nuclei was proposed for the first time in Ref. 123. In addition, for the description of the electromagnetic transitions from highly excited states we use the strength-function method considered in Sec. 1.

The Hamiltonian permitted study of the GDR in rotating nuclei can be written in the form

$$H' = \sum_{kl} (\varepsilon_{kl} c_k^+ c_l + \varepsilon_{\bar{k}\bar{l}} c_{\bar{k}}^+ c_{\bar{l}}) - \Omega \hat{J}_x - \frac{1}{2} \varkappa (\hat{D}_0^{(-)^2} + \hat{D}_1^{(-)^2} + \hat{D}_1^{(+)^2}), \tag{153}$$

where ε_{kl} are the matrix elements of the potential of the average field, Ω is the rotational angular frequency, and $\varkappa$ is the coupling constant of the residual isovector dipole–dipole interaction. The operators $\hat{D}_{(0)}^{(-)}$ and $\hat{D}_{(1)}^{(\pm)}$ can be expressed in terms of the components $\hat{Q}_{1m}$ of the isovector dipole moment (see Appendix A):

$$\hat{Q}_{1m} = \frac{Z}{A} \sum_{k=1}^{N} r_k Y_{1m}(\Omega_k) - \frac{N}{A} \sum_{k=1}^{Z} r_k Y_{1m}(\Omega_k). \tag{154}$$

The sign as an index of the operators determines their signature. The basis states $|k\rangle$, $|\bar{k}\rangle$ are chosen as eigenstates of the operator $\hat{j}_x$; $\hat{j}_x |k\rangle = m_k |k\rangle$, $|\bar{k}\rangle = T|k\rangle$ ($|k\rangle \equiv |nljm\rangle$).

To investigate the properties of coherent excitations, we go over to the finite-temperature variant of the RPA formulated in Ref. 124, which differs from the variant described in Sec. 1 by the use of the commutation relations

$$[c_k^+ c_l,\ c_{k'}^+ c_{l'}] \approx \langle 0 | [c_k^+ c_l,\ c_{k'}^+ c_{l'}] | 0 \rangle = \delta_{kl'} \delta_{lk'} (\bar{n}_k - \bar{n}_l) \tag{155}$$

while the population numbers have the form

$$\bar{n}_k = \langle 0 | c_k^+ c_{k'} | 0 \rangle \delta_{kk'} = [1 + \exp(\xi_k / t)]^{-1}, \tag{156}$$

where t is the temperature, and $\xi_k = \varepsilon_{kk} - \lambda - \Omega (jx)_{kk}$.[71]

Because of the $R_x(\pi)$ symmetry of the Hamiltonian (153), the RPA problem can be solved separately for the phonons of positive and negative signature. As a result, for the former we obtain the secular equation

$$F^{(+)}(\omega) = S^{(+)} + \frac{1}{2\varkappa} = 0, \tag{157}$$

where

$$\left.\begin{aligned} S^{(+)} &= \frac{Z^2}{A^2} S_{1n}^{(+)} + \frac{N^2}{A^2} S_{1p}^{(+)}; \\ S_{1\tau}^{(+)} &= \sum_{k>l}^{N_\tau} \frac{\omega_{kl}}{\omega_{kl}^2 - \omega^2} q_{1kl}^{(+)^2} (\bar{n}_l - \bar{n}_k) + \frac{\omega_{\bar{k}\bar{l}}}{\omega_{\bar{k}\bar{l}}^2 - \omega^2} q_{1\bar{k}\bar{l}}^{(+)^2} (\bar{n}_{\bar{l}} - \bar{n}_{\bar{k}}). \end{aligned}\right\} \tag{158}$$

Here $\omega_{kl} = \xi_k - \xi_1$. For the latter, we have

$$F^{(-)}(\omega) = \left(S_{00} + \frac{1}{2\varkappa}\right)\left(S_{11} + \frac{1}{2\varkappa}\right) - \omega^2 S_{01} = 0, \quad (159)$$

where the quantities S_{ij} are determined by relations of the type (158) and have the structure

$$S_{ii} = \sum_{kl}^{N_\tau} [\omega_{k\bar{l}}/(\omega_{k\bar{l}}^2 - \omega^2)]\,(q_{k\bar{l}}^i)^2\,(\bar{n}_{\bar{l}} - \bar{n}_k);$$

$$S_{ij} = \sum_{kl}^{N_\tau} [q_{k\bar{l}}^i q_{k\bar{l}}^j/(\omega_{k\bar{l}}^2 - \omega^2)]\,(\bar{n}_{\bar{l}} - \bar{n}_k).$$

We introduce partial strength functions of the reduced probabilities of the electric dipole transitions from the single-phonon states to the yrast line:

$$b(E1,\ \tau.\ \omega) = \sum_i B(E1,\ \omega_i^\lambda,\ \tau)\,\rho(\omega - \omega_i^\lambda). \quad (160)$$

The weight function $\rho(\omega - \omega_i)$ is determined by Eq. (97). Using for the calculation of the probability of the dipole transitions the expression for the reduced matrix element of the tensor operator (80) and following the scheme explained in Sec. 1, we have for the partial strength functions, now defined in accordance with (99), the following results:

a) for positive signature

$P_{\tau=0} = 1/\varkappa$, $F^{(+)}$ is determined by Eq. (157);

b) for negative signature

$P_{\tau=+1} = [(2\varkappa S_{00} + 1)/2\varkappa]\ [2\varkappa S_{01}\omega/(2\varkappa S_{00} + 1) - 1]^2;$
$P_{\tau=-1} = [(2\varkappa S_{11} + 1)/2\varkappa]\ [2\varkappa S_{01}\omega/(2\varkappa S_{11} + 1) + 1]^2$

and $F^{(-)}$ is determined by Eq. (159).

In the calculations, the single-particle energies in the rotating system were obtained by diagonalizing the Nilsson potential. The equilibrium configurations (ε,γ) determined from the extremum of the thermodynamic potential,[121,122] were used to calculate the partial and total strength functions. The constant of the residual interaction, $\varkappa = 1200A^{-5/3}$ MeV/F^2, was determined from the experimental position of the GDR.

To check the results in the Nilsson potential, the terms of the type **l**·**l** and **l**·**s** were omitted. In this case, the analytic solution for the positions of the different branches of the GDR (see Sec. 2) were exactly reproduced in the calculation. Another effective control method is the fulfillment of the sum rules. In our calculations of the single-particle energies, there is no coupling between different shells. Using an Elliott oscillator (see Ref. 71), in which there is also no mixing of the configuration spaces from different shells, it is easy to calculate energy-weighted sum rules for the dipole operators of both signatures:

$$\left.\begin{aligned} &S_0^{(-)} = S_0\alpha A_z,\ S_1^{(-)} = S_0\alpha A_y,\ S_1^{(+)} = S_0\alpha A_x; \\ &S_0 = \frac{3}{8\pi}\frac{\hbar^2}{m}\frac{NZ}{A}e^2. \end{aligned}\right\} \quad (161)$$

In the numerical calculations of the transition sum rules in the absence of **l**·**l** and **l**·**s** terms in the Nilsson potential, the deviation from the estimates (161) did not exceed 1%.

Figure 9 shows the change in the partial strength functions of the isovector dipole excitations as a function of the rotational angular frequency for the ^{160}Yb nucleus at $t = 0$ MeV. The arrows and numbers in the figure indicate the position of the maximum of the strength function and the value corresponding to it in the oscillator case.

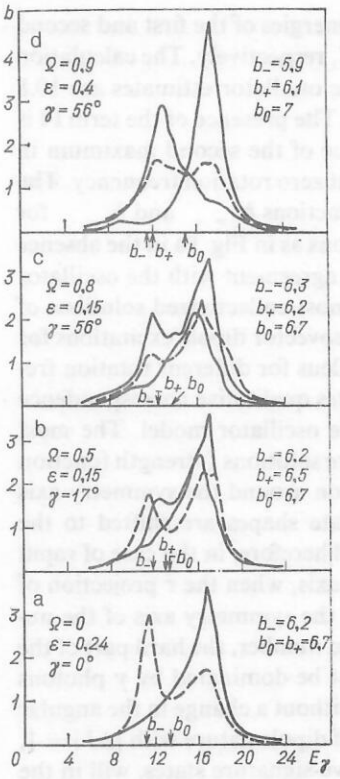

FIG. 9. Partial strength functions for the ^{160}Yb nucleus in the case of the Nilsson potential for different rotation frequencies Ω, MeV. To the left of each figure we show the conditions for the calculation and to the right the values of the strength functions in the case of an oscillator; the positions corresponding to them are indicated by the arrows on the energy axis. The thin curve corresponds to the strength function $b_{\tau=0}$, the thick curve to $b_{\tau=+}$, and the broken curve to $b_{\tau=-}$.

In deformed nuclei with axial symmetry, the dipole resonance is split into two components. In the absence of rotation, they can be classified in the usual manner by means of the projection of the angular momentum onto the symmetry axis. In Fig. 9a for $\Omega = 0$ for both the oscillator and the Nilsson potential the strength functions of the two branches of the isovector dipole excitations (strength functions $b_{\tau=0}$ and $\beta_{\tau=\pm}$) behave in exactly the same way as functions of the energy of the γ photons. They are due to transverse vibrations and together form the mode $K^\pi = 1^-$. The most collectivized solutions of the strength function of the third branch of isovector dipole excitations, $b_{\tau=-}$, due to the longitudinal vibrations, are less hard and have a lower energy. This excitation branch forms the mode $K^\pi = 0^-$.

In the realistic calculation, in contrast to the oscillator estimates, the presence in the Nilsson potential of the term proportional to **l**·**l** has a significant influence on the width of the strength function as well as on the positions of its maxima. The maxima of the partial strength functions are shifted to higher energies. In addition, the splitting of the branches of the GDR is increased. For example, for the ^{152}Sm nucleus

the experiment gives for the energies of the first and second maxima 12.45 and 15.84 MeV, respectively. The calculation gives 11.8 and 16.8 MeV (the oscillator estimates are 10.8 and 13.8 MeV, respectively). The presence of the term l·l is responsible for the appearance of the second maximum in the strength function $b_{\tau=-}$ at zero rotation frequency. The calculation of the strength functions $b_{\tau=+}$ and $b_{\tau=-}$ for ^{160}Yb under the same conditions as in Fig. 9a in the absence of the l·l term indicates close agreement with the oscillator result. The evolution of the most collectivized solutions of the different branches of the isovector dipole excitations for the example of the ^{160}Yb nucleus for different rotation frequencies ($t = 0$ MeV) indicates qualitative correspondence with the results of the simple oscillator model. The most collectivized positive-signature solutions (strength function $b_{\tau=0}$) is the regime of rotation around the symmetry axis corresponding to axially oblate shapes are shifted to the harder part of the spectrum. Therefore, in the case of rapid rotation about the symmetry axis, when the τ projection of the angular momentum onto the symmetry axis of the nucleus becomes a good quantum number, the hard part of the photoemission spectrum must be dominated by γ photons which de-excite the nucleus without a change in the angular momentum. The transitions of dipole nature with $|\Delta I| = 1$, which take place from negative-signature states, will in the case of rapid rotation essentially form the soft part of the γ spectrum. Comparison of Figs. 9a and 9d ($\Omega = 0$ and 0.9 MeV, respectively) shows that the rotation significantly influences the branch of isovector dipole excitations characterized by the strength function $b_{\tau=+}$. On the transition to the axially oblate rotation regime ("noncollective" rotation), the most collectivized solutions of the strength function $b_{\tau=+}$ are shifted to the softer part of the γ spectrum. Thus, in the regime of noncollective rotation, photons that de-excite the rapidly rotating nucleus with decrease of the angular momentum by $1\hbar$ will be dominant in the soft part of the spectrum of the dipole photons.

The more the nucleus is deformed, the greater the splitting of the components of the isovector dipole excitations and, therefore, the broader the region of localization of the isovector dipole excitations that contribute to the γ spectrum. Thus, in the case of rapid rotation, when the compound nucleus that is formed is strongly deformed, the region of localization of the isovector dipole excitations and, accordingly, the GDR must be greater than at small angular momenta.

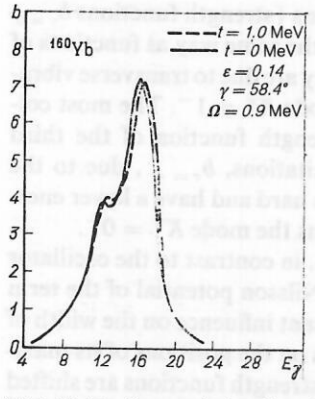

FIG. 10. Total strength functions for the ^{160}Yb nucleus at different temperatures. The conditions of the calculation and the explanations are given in the figure.

Figure 10 shows the total strength functions of the ^{160}Yb nucleus for t equal to 0 and 1.0 MeV. The deformation parameters for $t \neq 0$ are taken from Ref. 122. As in the absence of rotation,[125] the "heating" of the nucleus does not have a significant influence on the isovector dipole excitations and the giant dipole resonance. However, the temperature effects, weakening the shell effects, significantly influence the evolution in the shape of the rotating nuclei.[122] Therefore, the influence of the temperature effects on the isovector dipole excitations occurs through the change in the characteristics of the average field (the deformation parameters), which determine the nature of the strength functions of the isovector dipole excitations.

CONCLUSIONS

The investigation of high-spin nuclear states is a new and rapidly developing field in low-energy nuclear physics. The study of the properties of nuclei under conditions different from traditional nuclear spectroscopy has stimulated the appearance of new theoretical approaches, one of which is the CM + RPA method. Its development is to a large degree associated with the determination of the symmetries that remain when there is rotation, since strong Coriolis mixing leads to the impossibility of the ordinary classification of nuclear levels by means of the quantum number K^{π}. The preservation of the symmetry of the nucleus with respect to the rotation axis [in the case of the cranking model, this is the $R_x(\pi)$ symmetry] led to the introduction of a new quantum number, the signature, by means of which it is possible to understand many features in the experimental data for rotating nuclei.[15,126] The possibility of classifying states by means of the signature made it possible on the theoretical side to solve serious technical problems in the solution of the HFB equations in the cranking model and then in the solution of the RPA equations.

The CM + RPA method described in this review, which makes essential use of the concept of signature, makes it possible to investigate both qualitatively and quantitatively the collective excitations of single-phonon nature in a wide range of values of the angular momentum and excitation energies. Although the method was designed for studying high-spin states, the description of the low-lying states of the ^{158}Dy and ^{168}Er nuclei on its basis (see the beginning of Sec. 3) offers hope of its universality. However, to obtain more detailed information about the nature of the transitions (for example, dipole transitions) it is evidently necessary to go beyond the RPA or to take into account in the Hamiltonian terms of higher order in the boson expansion,[15] and this leads to the mixing of states with different signatures. Importance of these effects must be expected in the description of low-spin states, since only when $I \gg 1$ does the signature become an asymptotically good quantum number.[130] The quality of the calculations in the models can also be improved by taking into account residual interactions

corresponding to exact restoration of the rotational and translational symmetries of the total Hamiltonian.

On the basis of the CM + RPA method one can in the framework of a comparatively simple model not only obtain the position and the dependence on the quadrupole deformation of the GDR and GQR states at low spins but also follow the change in the properties of these states when there is rotation. From the results of the analysis it is to be expected that the GDR states are less subject to the influence of rotation than the GQR states. The destruction of pairing when a nucleus rotates leads to the disappearance of the collective states whose existence is due to the superconducting properties of nuclear matter. Therefore, the absence in the simple model of the soft β and γ excitation modes at low spins can hardly have a strong effect on conclusions relating to collective states with $I \gg 1$.

Calculations of the properties of the GDR in the framework of the realistic model are, despite the more complicated nature of the spectrum, in fairly close agreement with the predictions of the simple model. The behavior of the isovector dipole excitations in the presence of rotation is largely determined by the evolution of the shape of the rotating nucleus or of the parameters of the equilibrium deformation of the average field. The latter can change appreciably when the pairing is destroyed[73] or there is weakening of the shell effects in the presence of "heating" of the nucleus in the absence of pairing.[122] It is obvious that allowance for all these effects is important in study of the mechanisms of electromagnetic de-excitation of rapidly rotating nuclei.

We are sincerely grateful to I. N. Mikhaĭlov for critical comments and helpful discussions of the questions considered in the review.

APPENDIX A: BOSON REPRESENTATIONS OF SINGLE-PARTICLE OPERATORS

In the language of second quantization, each single-particle operator can be expressed in the form

$$\hat{G}=\sum_{kl}\{\langle k\mid\hat{G}\mid l\rangle c_k^+c_l+\langle k\mid\hat{G}\mid\bar{l}\rangle c_k^+c_{\bar{l}} +\langle\bar{k}\mid\hat{G}\mid l\rangle c_{\bar{k}}^+c_l+\langle\bar{k}\mid\hat{G}\mid\bar{l}\rangle c_{\bar{k}}^+c_{\bar{l}}\}. \tag{A1}$$

In what follows, we shall assume that the operator $\hat{G}$ has the symmetries

$$\left.\begin{aligned} &T\hat{G}T^{-1}=\gamma_T\hat{G};\quad T=U_TK,\quad T^2=(-1)^{2J};\\ &\langle k\mid\hat{G}\mid l\rangle^*=r\langle k\mid\hat{G}\mid l\rangle;\\ &R_x(\pi)\hat{G}R_x^{-1}(\pi)=\gamma_x G,\quad G^+=h\hat{G}, \end{aligned}\right\} \tag{A2}$$

where T is the operator of time reversal,[31] U_T is a unitary operator, K is the operator of complex conjugation and the numbers $\gamma_T=\pm1$, $r=\pm1$, $\gamma_x=\pm1$, $h=\pm1$ characterize the symmetry of the given operator with respect to the corresponding operation. Because of the $R_x(\pi)$ symmetry of the nuclear Hamiltonian (5) and of the single-particle and quasiparticle vacuum, we obtain, combining (7) and (A2),

$$\left.\begin{aligned} &\langle\bar{k}\mid\hat{G}\mid\bar{l}\rangle=\gamma_T r\langle k\mid\hat{G}\mid l\rangle;\\ &\langle\bar{k}\mid\hat{G}\mid l\rangle=-\gamma_T r\langle k\mid\hat{G}\mid\bar{l}\rangle;\\ &\langle k\mid\hat{G}\mid\bar{l}\rangle=-\gamma_x\langle k\mid\hat{G}\mid\bar{l}\rangle;\\ &\langle k\mid\hat{G}\mid l\rangle=\gamma_x\langle k\mid\hat{G}\mid l\rangle, \end{aligned}\right\} \tag{A3}$$

and as a result of this we have

$$\begin{aligned} &\hat{G}=\sum_{kl}\langle k\mid\hat{G}\mid l\rangle(c_k^+c_l+\gamma_T rc_{\bar{k}}^+c_{\bar{l}})\quad\text{for}\quad\gamma_x=+1;\\ &G=\sum_{kl}\langle k\mid\hat{G}\mid\bar{l}\rangle(c_k^+c_{\bar{l}}-\gamma_T rc_{\bar{k}}^+c_l)\quad\text{for}\quad\gamma_x=-1. \end{aligned} \tag{A4}$$

In what follows, the notation $\hat{G}$ will be used for the operators $\hat{G}$ with $\gamma_x=\pm1$. Using the Bogolyubov transformation (10) and introducing the two-quasiparticle bosons b_{kl}^+, $b_{\bar{k}\bar{l}}^+$, $b_{k\bar{l}}^+$ [see (17)], we obtain for the operators $\hat{G}^{(+)}$ ($\gamma_x=+1$)

$$\begin{aligned} &\hat{G}^{(+)}=\langle\Omega\mid\hat{G}^{(+)}\mid\Omega\rangle+G^{(+)}(1)+G^{(+)}(2);\\ &\langle\Omega\mid\hat{G}^+\mid\Omega\rangle=\sum_{jkl}\langle k\mid\hat{G}^{(+)}\mid l\rangle(\gamma_T rB_k^jB_{\bar{l}}^j+B_{\bar{k}}^{\bar{j}}B_l^{\bar{j}});\\ &G^{(+)}(1)=\sum_{ij}g_{ij}^{(+)}(b_{i\bar{j}}^++hrb_{i\bar{j}});\\ &g_{ij}^{(+)}=\sum_{kl}\langle k\mid\hat{G}^{(+)}\mid l\rangle(A_k^iB_{\bar{l}}^{\bar{j}}-\gamma_T hB_k^iA_{\bar{l}}^{\bar{j}});\\ &G^{(+)}(2)=\sum_{ijkl}\langle k\mid\hat{G}^{(+)}\mid l\rangle\{(A_k^iA_l^j-\gamma_T hB_k^iB_l^j)\sum_m(b_{i\bar{m}}^+b_{jm}+b_{i\bar{m}}^+b_{j\bar{m}})\\ &\quad+(\gamma_T rA_{\bar{k}}^{\bar{i}}A_{\bar{l}}^{\bar{j}}-hrB_{\bar{k}}^{\bar{i}}B_l^{\bar{j}})\sum_m(b_{\bar{i}\bar{m}}^+b_{\bar{j}\bar{m}}+b_{\bar{i}m}^+b_{\bar{j}m})\} \end{aligned} \tag{A5a}$$

and for $\hat{G}^{(-)}$ ($\gamma_x=-1$)

$$\begin{aligned} &\hat{G}^{(-)}=G^{(-)}(1)+G^{(-)}(2);\\ &G^{(-)}(1)=-\frac{\mathrm{i}}{2}\sum_{ij}\{g_{ij}^{(-)}(b_{i\bar{j}}^+-rhb_{ij})-\gamma_T rg_{ij}^{(-)}(b_{ij}^+-rhb_{i\bar{j}}^+)\};\\ &g_{ij}^{(-)}\equiv\sum_{kl}\langle k\mid\hat{G}^{(-)}\mid\bar{l}\rangle(A_k^iB_l^j+\gamma_T hA_l^jB_k^i);\\ &g_{\bar{i}\bar{j}}^{(-)}\equiv\sum_{kl}\langle k\mid\hat{G}^{(-)}\mid\bar{l}\rangle(\gamma_T hA_{\bar{l}}^{\bar{j}}B_{\bar{k}}^{\bar{i}}+A_{\bar{k}}^{\bar{i}}B_l^{\bar{j}});\\ &G^{(-)}(2)=\mathrm{i}\sum_{ijkl}\langle k\mid\hat{G}^{(-)}\mid\bar{l}\rangle\{(A_k^iA_{\bar{l}}^{\bar{j}}+\gamma_T hB_l^{\bar{j}}B_{\bar{k}}^i)\sum_m(b_{i\bar{m}}^+b_{\bar{j}\bar{m}}-b_{i\bar{m}}^+b_{\bar{j}m})\\ &\quad+\mathrm{i}hr(\gamma_T hA_{\bar{k}}^{\bar{i}}A_l^j+B_{\bar{k}}^{\bar{i}}B_{\bar{l}}^j)\sum_m(b_{\bar{i}\bar{m}}^+b_{\bar{j}\bar{m}}-b_{\bar{i}m}^+b_{\bar{j}m})\}. \end{aligned} \tag{A5b}$$

In (A5), the symbols $G^{(\pm)}(1)$ and $G^{(\pm)}(2)$ denote the parts of the given operator $\hat{G}^{(\pm)}$ that are linear and quadratic in the bosons; $g_{ij}^{(+)}$ and $g_{ij}^{(-)}$ are the quasiparticle matrix elements of the operator $\hat{G}^{(\pm)}$. It follows from (A5) that the negative-signature operators $\hat{G}^{(-)}$ have zero expectation values with respect to the quasiparticle vacuum $|\Omega\rangle$.

[1)]More details of the theoretical and experimental aspects of the cranking model can be found in the recent review of Ref. 23.

[2)]The total nuclear Hamiltonian H in (1) is the Hamiltonian of the nucleus in the laboratory system but expressed in terms of the variables of the rotating coordinate system. In the same way, the momentum components P_i below are to be understood as the variables of the rotating coordinate system,[24] i.e., $\hat{P}\to\exp(i\hat{J}_x\Omega t)\hat{P}\exp(-i\hat{J}_x\Omega t)$.

[3)]In Ref. 30, some aspects of the CM + RPA method using nonseparable forces are discussed.

[4)]All operators are defined in the system with quantization axis z.

[5)]Equations (12) can be obtained, for example, by the variational method[42,44]:

$$\delta\{\langle\Omega\mid H'\mid\Omega\rangle+\sum_{ij}E_i\,[\sum_l(A_l^iA_l^j+B_l^iB_l^j+A_l^{\bar{i}}A_l^{\bar{j}}+B_l^{\bar{i}}B_l^{\bar{j}})-2]\}=0,$$

where the total nuclear Hamiltonian in (1) is chosen in the form (5).

[6)]In the derivation of the expressions for the matrix elements, the exchange terms are as a rule ignored, and therefore this is to be regarded rather as a Hartree–Bogolyubov approximation. The connection between allowance for the exchange terms at the level of the average field (AF) and at the level of the RPA is discussed in Ref. 30.

[7)]The operator $\hat{A}$ has positive signature under the condition $\hat{R}_1(\pi)\hat{A}\hat{R}_1^{-1}(\pi)=\hat{A}$ and negative signature if $\hat{R}_1(\pi)\hat{A}\hat{R}_1^{-1}(\pi)=-\hat{A}$ (see Refs. 18 and 19).

[8)]Therefore, it is sufficient to transform the general HFB + RPA approach to a concrete realization—the CM + RPA method, which was used for the first time to find low-lying excitations of precession type in Refs. 78 and 45.

[9)]In the general case, these terms may have a more complicated dependence (see Ref. 80). For example, nonadiabatic effects due to a more complicated dependence on J^2 were considered in Refs. 81 and 82.

[10)]The symbol $\theta_x(1)$ represents the part of the operator of the angle $\hat{\theta}_x$, which is canonically conjugate to the operator $\hat{J}_x$ which is linear in the bosons.

[11)]In the absence of complete alignment, the part of the operator $\hat{J}_x$ quadratic in the bosons may be important and then (see Ref. 46) $\langle\Omega|O_\nu\hat{J}_xO_\nu^+|\Omega\rangle=\langle\Omega|[O_\nu^+,[O_\nu,J_x(2)]]|\Omega\rangle+\langle\Omega|\hat{J}_x|\Omega\rangle$.

[12)]In the absence of complete alignment (low spins) the expression (74) must be replaced in accordance with $\langle\nu|\hat{J}_x|\nu\rangle\approx\sqrt{I(I+1)-K_\nu^2}$, where K_ν is the projection of the total angular momentum onto an axis perpendicular to the rotation axis in the state $|\nu\rangle$.

[13)]Below, we shall not consider the terms associated with the modes $[X(1),P_x(1)]$, $[Y(1),P_y(1)]$, $[Z(1),P_z(1)]$, since the modes $[Y(1),P_y(1)]$, $[Z(1),P_z(1)]$ do not occur in the space of the functions (77), and for the study of nuclear vibrations we can assume $P_x=0$. It is important that in our treatment these modes are not mixed with the normal modes $(X_\nu,\mathscr{P}_\nu)$ or with other ghost modes.

[14)]The function $b(\omega_i)$ can, for example, be the probability of a transition of a given type and given multipolarity from single-phonon states to the yrast line. In this case, ω_i are the energies of the phonons, and Eq. (95) is identical to (62).

[15)]The development beyond the RPA (to take into account the Pauli principle) has been studied in the absence of rotation in the framework of the quasiparticle–phonon model.[127–129]

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Translated by Julian B. Barbour