# Experimental studies of the stopping powers of various types of matter for protons and helium ions

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The experimental studies of the stopping powers of various types of matter for protons and helium ions are reviewed. The experimental methods of determining the stopping power are analyzed and the ranges where they are applicable are discussed. A comparison is made with the experimental data cited in the most commonly used tables of stopping powers. Empirical formulas for the stopping power and the rule of additivity of the stopping powers of multicomponent matter are discussed.

## INTRODUCTION

The study of matter in the solid state and the modification of its properties by means of accelerated ion beams is currently an important area in both applied and theoretical science. This is a consequence of its numerous applications in science and technology: semiconductor technology, radiation aspects of materials science, controlled thermonuclear fusion, element analysis of the composition of materials, and so on. It is therefore necessary to organize and analyze the large amount of data on the diverse physical processes occurring when ions pass through matter.

The present review is devoted to a relatively minor problem, that of the stopping powers of various types of matter for protons and helium ions. We do not attempt to give a complete review, but focus only on the relevant experimental studies at the energies most important for applications (energies above several tens of keV). We analyze the commonly employed experimental methods of determining the stopping powers of various types of matter and discuss the possibilities that they offer and their ranges of applicability. We make comparisons with the experimental data contained in the most commonly used tables of stopping powers and discuss the empirical formulas for the stopping powers. The material in this review was selected primarily on the basis of its relevance to the practical requirements in the use of accelerated ion beams.

The limitations on the length of this article unavoidably forced us to omit many important physical questions in the subject of stopping powers. We apologize to those authors whose work is not mentioned here.

## 1. DEFINITION OF FUNDAMENTAL QUANTITIES

When an ion passes through matter, the combined action of various physical processes (elastic scattering on the atoms of the matter, ionization and excitation of electron shells, charge exchange, all sorts of collective effects, etc.), causes it to lose energy, decelerate, and finally stop. The energy loss of an ion in matter is quantitatively characterized by the quantity $\varepsilon(E)$, referred to as the stopping cross section and defined as

$$\varepsilon(E) = \sum_i \int T_i \, d\sigma_i. \tag{1}$$

Here $T_i$ is the energy lost by the ion in an individual collision with an atom of the matter via a process of type $i$ and $d\sigma_i$ is the cross section for a process of type $i$ differential in the energy transfer. The quantity

$$\varepsilon_i(E) = \int T_i d\sigma_i \tag{2}$$

is the partial stopping cross section.

The physical meaning of $\varepsilon(E)$ can be understood as follows. The ratio $d\sigma_i/\sigma_i$, where $\sigma_i = \int d\sigma_i$ is the total cross section for a process of type $i$, is the probability for a process of type $i$ and the quantity

$$Q_i = \int T_i d\sigma_i/\sigma_i \tag{3}$$

is the average energy lost by the ion in a single ion-atom collision as the result of a process of type $i$. It then follows from (2) and (3) that the partial stopping cross section

$$\varepsilon_i(E) = Q_i\sigma_i \tag{4}$$

is the product of the average energy $Q_i$ lost by the ion via a process of type $i$ in a single ion-atom collision and the total cross section for this process, $\sigma_i$. The total stopping cross section $\varepsilon(E)$ is then the sum of the partial cross sections. It is measured in units of $10^{-15}$ eV·cm$^2$/atom or $10^{-15}$ eV·cm$^2$/molecule.

If $\varepsilon(E)$ is multiplied by the density of atoms (or molecules) in the material, $n_0$, we obtain

$$n_0\varepsilon(E) = \sum_i Q_i/L_i, \tag{5}$$

where $L_i$ is the mean free path for a process of type $i$, since

$$L_i = 1/(n_0\sigma_i).$$

Each term in Eq. (5) is the average energy lost by the ion per unit of mean free path as the result of a process of type $i$ and altogether $n_0\varepsilon(E)$ is the total average energy lost by the ion per pathlength. This quantity is denoted as $dE/dx$ and referred to as the linear stopping power of the matter. The quantity $dE/(\rho dx)$, where $\rho$ is the matter density, is called the mass stopping power of the matter. Denoting the latter quantity by $S(E)$ and the former by $T(E)$ (which we shall use in addition to the notation $dE/dx$), we can express the relation between these quantities as

$$\left.\begin{aligned} T(E) &= 0.1\rho S(E); \\ S(E) &= \frac{602.5}{A}\varepsilon(E); \\ T(E) &= \frac{60.25\rho}{A}\varepsilon(E). \end{aligned}\right\} \tag{6}$$

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Here $T(E)$ and $S(E)$ are measured in keV/$\mu$m and keV·mg$^{-1}$·cm$^2$, respectively, the density $\rho$ is in g/cm$^3$, and $A$ is the mass number of the atoms of the matter. In the case of multi-component matter $A$ in formulas (6) is replaced by the sum $\Sigma m_i A_i$, where $m_i$ is the number of atoms of type $i$ in a molecule of the matter and $A_i$ is the mass number of the atoms of type $i$.

The range of an ion in the matter, $R(E)$, is related to the energy loss. By "range" we generally mean the depth to which an ion penetrates into the matter, that is, the projection of the path traversed by the ion in the matter onto the original direction of incidence. Of course, owing to the curvature of the trajectory the actual range is somewhat different from the range as defined above, but the higher the ion energy, the smaller this difference. The stopping power of the matter and the range are related as

$$R(E) = R(E_1) + \int_{E_1}^{E} \frac{dE}{T(E)}. \tag{7}$$

This equation holds only for energies greater than a certain value $E_1$, since as the energy decreases, firstly, the curvature of the ion trajectory in the matter increases and, secondly, the role played by fluctuations in the energy loss becomes more important, so that the actual concept of the range as defined above basically becomes meaningless and is replaced by a distribution of ranges. The actual value of $E_1$ is not clearly defined. The value $E_1 = 1$ MeV was recommended for the case of protons in Ref. 1.

The range is measured either in units of length ($R_r$, $\mu$m) or in units of the amount of matter per square centimeter ($R_s$, mg/cm$^2$). The relation between these two quantities is

$$R_s(E) = 0.1\rho R_r(E).$$

As in (6), here $\rho$ is the matter density in g/cm$^3$.

The existence of an energy loss leads to a relation between the penetration depth $x$ of the ions in the matter and the ion energy $E$ at the depth $x$:

$$x = \int_{E}^{E_0} \frac{dE}{T(E)}. \tag{8}$$

Here $E_0$ is the initial ion energy.

In conclusion, we note that the definition (1) is based on the representation of the matter as a mechanical mixture of atoms, so that the energy of the ion is lost exclusively in individual ion-atom collisions. Strictly speaking, this is true only for gaseous targets. In the case of solids, the processes occurring in individual ion-atom collisions are accompanied by collective effects and, in addition, effects due to the zonal nature of the electronic structure, so that the overall picture of the stopping process becomes more complicated, as does the definition of the quantity $\varepsilon(E)$. We shall not discuss these fine points in detail.

## 2. THE CONTRIBUTION OF VARIOUS STOPPING MECHANISMS TO THE TOTAL ENERGY LOSS

The relative roles played by the various ion stopping mechanisms are different at different energies. The case of protons and gaseous targets has been analyzed theoretically

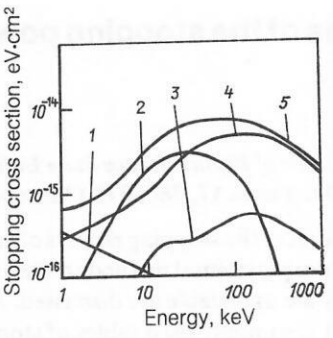


FIG. 1. Energy losses of protons in helium (Ref. 2): 1—elastic scattering; 2—to charge exchange; 3—to excitation; 4—to ionization; 5—total energy losses.

in Refs. 2 and 3. In Fig. 1 we show the results of Ref. 2 on the proton stopping cross section in helium. We give both the total and the partial cross sections corresponding to various stopping mechanisms. It is clear that for proton energies in the range from 1 to 10 keV the main contribution to the stopping cross section comes from elastic scattering, charge exchange, and ionization. At $E > 10$ keV the contribution from elastic scattering is negligible and up to 200 keV the energy loss is due to three inelastic processes—ionization, charge exchange, and excitation. For $E > 200$ keV ionization is the dominant stopping mechanism. Purely elastic scattering dominates for $E < 1$ keV. The numerical estimates are consistent with this: for $E > 5$ keV roughly 90% of the energy loss is due to ionization (both charge exchange and direct ionization) and 10% is due to excitation.

Similar results were obtained in the analysis of proton energy losses in molecular hydrogen in Ref. 3: for proton energies above 100 keV roughly 90% of the energy loss can be attributed to ionization (30% goes into the stripping of electrons from molecules and 60% goes into kinetic energy transferred to these electrons) and 10% goes into excitation.

Therefore, when a sufficiently energetic proton travels through a gaseous medium the pure ionization of the atoms of the medium is initially the dominant process. As the proton slows down, first excitation and then charge exchange begin to become important and over part of the proton trajectory all three mechanisms of inelastic energy loss are operating. Near the end of the trajectory excitation becomes less important and energy loss via elastic scattering is enhanced. At the very end of the trajectory electronic energy losses play essentially no role and energy losses via purely elastic scattering dominate. As a consequence of this, the maximum energy transfer from the ion to the atoms of the medium occurs at the end of the trajectory.

It therefore follows that at the ion energies generally used in experiments (several tens of keV and higher) the energy losses are almost exclusively determined by electronic processes. The energy dependence of the total electronic energy loss contains a maximum: for protons it is located in the range 80–150 keV and for helium ions it is in the range 0.4–1.2 MeV.

## 3. EXPERIMENTAL METHODS OF DETERMINING THE STOPPING POWERS OF VARIOUS MATERIALS FOR LIGHT IONS

All of the methods commonly used at present to extract information on the stopping powers of materials from experiments fall into one of the following categories: a) passage through an absorber; b) back-scattering; c) other methods.

### Passage through an absorber

Up to now, most of the experimental information on the stopping powers of various types of matter has been obtained by this method. There are several variations of it, but in the most widely used one the beam is shot through a series of thin, self-supporting foils. The details are as follows. The ion beam is shot through a sufficiently thin, self-supporting layer of thickness $d$ of the material in question. The ion energy is measured before ($E_1$) and after ($E_2$) the beam passes through the material. The ratio

$$\Delta E/d = (E_1 - E_2)/d \tag{9}$$

is then taken to be the value of $dE/dx$ at the energy

$$E = (E_1 + E_2)/2. \tag{10}$$

The justification for this technique is discussed in Ref. 4. If the stopping power of the material in the range $[E_1, E_2]$ is accurately given by the expression

$$T(E) = \frac{1}{a_1 + 2a_2E}, \tag{11}$$

this method gives a quite accurate value of $dE/dx$ independently of the absorber thickness. Deviation of the actual energy dependence of $dE/dx$ in the range $[E_1, E_2]$ from the form (11) leads to systematic error.

When the ion energy is sufficiently far from the maximum in $dE/dx$ as a function of energy, expression (11) is accurate over a fairly broad energy range. As the energy is decreased and the maximum is approached the range over which the approximation (11) accurately reproduces $dE/dx$ becomes smaller. For example, when protons pass through silicon with $E_1 = 2.1$ MeV and $E_2 = 2.0$ MeV ($d = 3.9\,\mu$m) the systematic error is 0.006%, and when helium ions pass through aluminum with $E_1 = 0.6$ MeV and $E_2 = 0.4$ MeV ($d = 0.56\,\mu$m) the systematic error is already 5.7%. In the latter case, the systematic error can be reduced to less than 1% if the thickness of the aluminum foil is of order $10^{-5}$ cm or smaller. This is technically quite difficult to achieve and, in addition, considerably increases the error due to uncontrollable factors such as nonuniformity of the foil thickness, the condition of the surface of the target, and so on. This is why the spread in the data of various authors is rather large near the maximum of $dE/dx$. The error in the reproducibility due to this spread is $\pm$ 10% and even larger.

In addition to the usual method, based on Eqs. (9) and (10), of extracting information on $dE/dx$ from experiments on the passage of ions through self-supporting foils, there is the method used in Refs. 5 and 6 in which the ion beam is shot through a thick foil. This goes as follows. We consider a fairly large energy range $[E_{\min}, E_{\max}]$. We assume that in this interval the range of the ion in the matter is described by the polynomial

$$R(E) = a_0 + a_1E + \ldots + a_nE^n. \tag{12}$$

Then

$$dE/dx = 1/R'(E) = 1/\sum_{k=1}^{n} ka_kE^{k-1} \tag{13}$$

and

$$\frac{d}{\Delta E} = y = a_1F_1 + a_2F_2 + \ldots + a_nF_n. \tag{14}$$

Here

$$\left.\begin{aligned} &F_1 = 1;\ F_2 = E_1 + E_2;\ F_3 = E_1^2 + E_1E_2 + E_2^2; \\ &F_n = E_1^{n-1} + E_1^{n-2}E_2 + \ldots + E_1E_2^{n-2} + E_2^{n-1}. \end{aligned}\right\} \tag{15}$$

Given a set of experimental data

$$d_i,\ E_{1ij},\ E_{2ij},$$

we can calculate the values of $F_k$ and $y$,

$$F_{2p},\ F_{3p},\ \ldots,\ F_{np},\ y_p,\ p = 1,\ 2,\ \ldots,\ N$$

and, using (14), find the parameters $a_k$ by the method of least squares. Here it is advisable to apply the method of least squares not in the standard form (which leads to an ill-conditioned system of linear algebraic equations), but instead to first use the Gram-Schmidt orthogonalization procedure. A FORTRAN program that does this is given in Ref. 7.

The numerical experiment and data analysis by this method in Refs. 6 and 8 show that the method gives correct results on the whole, but that the choice of experimental technique is nontrivial. The possible experimental techniques are: a) variation of the beam energy for a fixed absorber thickness; b) variation of the absorber thickness for a fixed beam energy; c) a combination of these, variation of the beam energy for several foil thicknesses. Since the energy straggling increases with the foil thickness, thus leading to additional errors in $dE/dx$, there must be an optimal range of thicknesses. All of these fine points should be studied in detail by the methods used in planning experiments. No such analysis has yet been carried out, but a preliminary analysis of the data available in the literature and the numerical experiment of Refs. 6 and 8 indicate that the third variant listed above is the optimal one, with the energy loss in the foils reaching 50% of the primary beam energy. The fact that such large foil thicknesses can be used is a decisive argument in favor of the method of thick foils, since this permits the error in the final results to be decreased considerably.

A method of finding $dE/dx$ from experimental data obtained when an ion beam is shot through self-supporting foils which is similar to that described above was used in Refs. 9 and 10. However, the question of the choice of the optimal experimental conditions was not addressed in those studies.

A variant of the method that we have described is that in which the ion beam hits a target consisting of a film of the material under study on a substrate of a different material. In this case the energy loss is determined either by means of a resonance nuclear reaction on the nuclei of the substrate,[11] or from the known energy dependence of the emitted x rays produced in collisions of the beam ions with the atoms of the substrate,[12] or from back-scattering of the ions. Energy losses to recoil contribute to the total energy loss in the latter

case. Various methods have been proposed for taking this effect into account—both approximate methods[13] and more accurate but more complicated methods, like scattering on two different types of atom[5] and scattering on only one type of atom, but with two different experimental geometries.[14]

In the case of different geometries, the energies of ions scattered on atoms of a single type are measured at the same angle of incidence but at two different angles of reflection. After reflection the ions travel along paths of different length, so that their energy losses will be different. The difference between the energy losses is then the energy loss in the path equal to the difference of the paths traversed by the ions after reflection.

In the case of scattering on two different types of atom we have the system of equations

$$\left.\begin{aligned}\Delta_1 E &= p_1 \Delta' E + \Delta'' E;\\ \Delta_2 E &= p_2 \Delta' E + \Delta'' E.\end{aligned}\right\} \qquad (16)$$

Here $\Delta' E$ is the ion energy loss in the medium before reflection, $\Delta'' E$ is that after reflection, $p_1$ and $p_2$ are the kinematical factors in the reflection, and $\Delta_1 E$ and $\Delta_2 E$ are the total energy losses, that is, the difference between the ion energies before entering the target and after exiting it. The solution of the system of equations (16) determines the energy losses $\Delta' E$ and $\Delta'' E$ due exclusively to stopping.

### The back-scattering method

The method of back-scattering from a massive target has recently been used more and more often to find the stopping powers of materials. In this approach the energy spectrum of ions scattered by a massive target is measured and then some sort of mathematical analysis of the experimental data is used to extract information on $dE/dx$. Various questions pertaining to the theory of this method and the technique of extracting information on the stopping power from the back-scattering spectra have been considered in Ref. 15–25. We shall not dwell on the details of this method; they can be found in the studies cited above and in our earlier review.[26]

### Other methods

Of the several other methods, we shall mention only the one which seems to us to be the most promising.[27]

Using Eq. (8) and the expression for the x-ray yield due to the flux of ions of energy $E$ in the medium,

$$Y(E) = n \int_0^E \sigma(\varepsilon)\, e^{-\mu x(E,\varepsilon)} \frac{d\varepsilon}{T(\varepsilon)}, \qquad (17)$$

and its derivative with respect to the beam energy $E$, we obtain

$$\sigma(E) = \frac{dY}{dE}\varepsilon(E) + \frac{\mu}{n} Y(E). \qquad (18)$$

Here $\sigma(E)$ is the cross section for excitation of x-ray radiation by the ions, $\mu$ is the x-ray attenuation factor, $n$ is the density of atoms whose x-ray emission is measured, and $E$ is the energy of the ions incident on the target.

The x-ray yield of silicon atoms excited by protons of energy 60–150 keV in two different matrices, Si and $SiO_2$, was measured in Ref. 27. Eliminating $\sigma(E)$ for Si from the two versions of (18) and knowing $\varepsilon(E)$ for Si, we find $\varepsilon(E)$ for $SiO_2$. The authors estimate the error to be 6–8%.

### Comparison of the various experimental methods of determining the stopping powers of materials

The method of passage through an absorber certainly gives the most reliable information, since when it is used the extraction of information on $dE/dx$ does not require any additional information on the elementary interactions between the ions and the matter and no overly complicated mathematical analysis of the experimental data is needed. The statistical error of the method varies from $\pm 1\%$ to $\pm 5\%$ and in most experiments is $\pm 2$–$3\%$. At sufficiently high ion energies the results of various authors are usually in good agreement with each other within the statistical error. In addition to the statistical error, near the maximum in the energy dependence of $dE/dx$ there is the reproducibility error, so that the total error reaches $\pm 10\%$. This makes it impossible to obtain reliable results on some of the fine points of stopping physics related to the effect of the electronic structure of a solid material on the energy loss (the rule of additivity of the stopping powers of multicomponent matter, the effect of the aggregate state of the matter on its stopping power, and so on), since it is these effects which are most important at low ion energies and their numerical values lie within the range of error.

The use of the method of passage through thick foils permits the experimental errors in the region of the maximum to be reduced considerably, so that the effects mentioned above can be studied more reliably. Estimates indicate that if the minimum foil thickness is of order $10^{-5}$ cm, the use of the method of passage through thick foils in the case of protons, for example, can give information on $dE/dx$ up to energies of order 50 keV.

The strong point of the back-scattering method is that it does not require the preparation of thin films, so that it is especially attractive at low ion energies. However, the need to take into account multiple scattering leads to additional errors which are difficult to estimate.

The method used in Ref. 27 seems promising for the study of stopping in multi-component matter, particularly at low energies.

## 4. TABLES OF THE STOPPING POWERS OF VARIOUS MATERIALS AND THE RANGES OF HYDROGEN AND HELIUM IONS

At the present time there is a rather large amount of tabulated data on the stopping powers of materials and the ranges of ions in matter. We shall not analyze all the existing tables, but mention only those which have become the most widely known in the last few years.

The two studies of Chu and Ziegler[28,29] are devoted to helium ions. They present the results of the analysis of the experimental data on $dE/dx$ for the media for which data were available and the results of theoretical calculations for other media. The energy range is 0.4–4.0 MeV. They also give the coefficients of the polynomial used to approximate $dE/dx$.

We should also mention Refs. 30 and 31, where the results of an analysis of the experimental and theoretical information on $dE/dx$ are given for protons and helium ions. The parameters of the approximation formulas are also quoted. The energy range is 1 keV to 100 MeV.

Of our own tables we should first mention Refs. 32 and 33. Tables of the stopping powers and ranges of hydrogen and helium ions are given in Ref. 32 for all media from hydrogen to uranium in the energy range 1–100 MeV. The minimum energy is 100 keV for a number of media. The ranges of hydrogen and helium ions in the same media in the energy range 0.1–8 MeV are tabulated in Ref. 33. The tables in these two studies were obtained by analyzing the experimental data and theoretical calculations.

We should also mention the tables of Refs. 34 and 35. Reference 34 quotes the results of an analysis of the experimental data on the stopping powers of various materials for protons and helium ions, with the quantity $1/\varepsilon(E)$ approximated by a polynomial including the term proportional to $E^{-1}$. Reference 35 gives the parameters of a power-law approximation of the ranges of hydrogen, helium, lithium, and beryllium ions for all single-component media in the energy range 0.0125–12 MeV/ nucleon.

The reliability of the tables of stopping powers can be checked by comparing the tabulated data with the available experimental data. For this we have chosen the energy range of most interest for applications—the range from 0.4 to 2.5 MeV. The experimental data for protons are taken from Refs. 36 and 37 and those for helium ions are from Ref. 38.

In the case of protons we carried out the comparison as follows. We chose a certain number of cases (156 in the analysis of the tables in Ref. 30, and 82 in the analysis of the tables in Ref. 32) and divided these cases into groups corresponding to the error $\Delta$ being less than 5%, in the range from 5 to 10%, and greater than 10%. The results are given in Table I.

We see from Table I that in the tables of Refs. 30 and 32 the probability of obtaining a value of the stopping cross section with an error greater than 10% is no more than 5%. The slight skew toward better than 10% accuracy in the use of the tables of Ref. 30 should apparently not be taken seriously, since the statistics are not very large and the errors are not distributed uniformly over the tables.

In the case of helium ions, the results given in Ref. 38 coincide almost exactly with the results of Refs. 28 and 29 and somewhat worse with the tables of Refs. 31 and 32, with the difference amounting to 10% in the case of the tables of Ref. 31 and 14–15% in the case of the tables of Ref. 32.

TABLE I. Results of the comparison of the experimental data and the tabulated stopping powers of various media in Refs. 30 and 32 for protons.

| Error range, % | Data from Tables in Ref. 30 | Data from Tables in Ref. 32 |
|---|---|---|
| $\Delta < 5$ | 77.6 % | 70.7 % |
| $5 < \Delta < 10$ | 18.0 % | 24.3 % |
| $\Delta > 10$ | 4.4 % | 4.8 % |

## 5. EMPIRICAL FORMULAS FOR THE STOPPING POWERS

The lack of a simple analytic expression for the stopping power of a material derived from a theory of stopping, on the one hand, and the need for such an expression in the solution of various applied problems, on the other, has led to the development of a large number of empirical formulas. We shall not discuss all of those known at the present time (see Ref. 39, for example), but shall mention only a few of them.

The formula obtained by Brice[40] has become widely known in recent years. This formula reproduces $dE/dx$ with an error of less than 2–3% over fairly wide energy ranges including the region of the maximum and is often used to present experimental data in a compact form, especially for light ions.

We should also mention the formulas obtained by Gott[39] and Pucherov and Chesnokova,[41] which, however, are applicable only for protons. Empirical formulas were also employed in Refs. 30 and 31 and Refs. 34 and 35 to represent a wide range of tabulated data.

In addition to the compact representation of a broad spectrum of tabulated data, empirical formulas are also necessary for solving various applied problems. Here the main difficulty is how to obtain the ion energy in the matter as a function of the penetration depth in a more or less simple analytic form. To obtain such a dependence, the empirical formula for $dE/dx$ must be such that the integral in (8) can be done analytically and the resulting expression can be solved for the energy.

From the formulas given above, this condition can be satisfied only for

$$R(E) = aE^b; \tag{19}$$

$$T(E) = \alpha E^\beta; \tag{20}$$

and formula (11) for $T(E)$. The dependences of the ion energy $E$ on the depth $x$ which follow from these formulas are, respectively,

$$E = E_0\left(1 - \frac{x}{R(E_0)}\right)^{1/b}; \tag{21}$$

$$E = E_0[1 - \alpha(1-\beta)E_0^{\beta-1}x]^{1/(1-\beta)}; \tag{22}$$

$$E = \frac{1}{2a_2}[\sqrt{a_1^2 - 4a_2(x - a_1E_0 - a_2E_0^2)} - a_1]. \tag{23}$$

In all of these expressions $E_0$ is the energy of the incident beam.

Formulas (11), (19), and (20) reproduce the initial values [the range $R(E)$ and the stopping power $T(E)$] in sufficiently large energy ranges and with the required accuracy only for $E \ll E_{max}$ and $E \gg E_{max}$, where $E_{max}$ is the energy at which $dE/dx$ reaches its maximum value.

We propose an empirical formula which to a certain degree is free from this deficiency:

$$\frac{dE}{dx} = aE^{1-p}(E^p + q)^{1-b}. \tag{24}$$

Using (24) with (7), we find the range

$$R(E) = A(E^p + q) + c. \tag{25}$$

Here

$$\left.\begin{aligned} A &= 1/(apb); \\ c &= R(E_1) - A(E_1^p + q). \end{aligned}\right\} \tag{26}$$

The energy as a function of the penetration depth $x$ is

$$E = \{[(E_0^p + q)^b - apbx]^{1/b} - q\}^{1/p}. \quad (27)$$

An important applied problem is how to find the scattering depth $d$ as a function of the energy $E$ of the particles leaving the target after scattering. Let $\theta_1$ and $\theta_2$ be the ion angle of incidence and the exit angle measured from the normal to the target surface. In addition, let $k$ be the kinematical factor for reflection and $E_1$ be the energy of a particle incident on the target at a depth $d$ before the scattering event. Then the scattering depth $d$ as a function of the known energy $E$ of the particles leaving the target is determined from the solution of the following two equations derived from the dependence (27):

$$d = \frac{\cos\theta_2}{apb}\{[(kE_1)^p + q]^b - (E^p + q)\}; \quad (28)$$

$$E_1 = \{[(E_0^p + q)^b - apbd/\cos\theta_1]^{1/b} - q\}^{1/p}. \quad (29)$$

Since the scattering depth $d$ is usually small compared to the range, the second term in the square brackets in (29) is small compared to the first. We can therefore find $d$ from Eqs. (28) and (29) by iteration: setting $d = 0$ in (29), we compute the zeroth-order approximation to $E_1$ and then use (28) to find the zeroth-order approximation to $d$. We then use this value of $d$ in (29) to find an improved value of $E_1$, after which we use (28) to find an improved value of $d$, and so on. The process converges very rapidly, usually after one or two iterations.

In Table II we give the values of the parameters in formula (24) for protons. The parameters reproduce the stopping cross sections $\varepsilon(E)$ in units of $10^{-15}$ eV·cm$^2$/atom in the range from 200 keV to 100 MeV with an error of $\pm$ 2–3%. The parameters were obtained on the basis of the tables of Ref. 30 and the experimental data of Refs. 36 and 37.

## 6. THE RULE OF ADDITIVITY OF THE STOPPING POWERS FOR MULTICOMPONENT MEDIA

The rule of additivity of the stopping powers of multicomponent media was formulated in Ref. 42: for a molecule

TABLE II. Values of the parameters of the approximation (24) for protons.

| Target | | $a$ | $p$ | $q$ | $b$ |
|---|---|---|---|---|---|
| Atomic number | Symbol | | | | |
| 1 | H | 1.154 | 1.351 | 0.036 | 1.355 |
| 2 | He | 2.000 | 1,582 | 0,451 | 1,149 |
| 3 | Li | 2.891 | 1.261 | 0.226 | 1.438 |
| 4 | Be | 3.795 | 1.187 | 0.317 | 1.533 |
| 5 | B | 4.533 | 1.176 | 0.242 | 1.543 |
| 6 | C | 5.412 | 1.338 | 0.478 | 1.355 |
| 7 | N | 5.464 | 1.378 | 0.292 | 1.296 |
| 8 | O | 6.305 | 1.309 | 0.315 | 1.364 |
| 9 | F | 6.910 | 1.107 | 0.255 | 1.616 |
| 10 | Ne | 7.458 | 1.153 | 0.335 | 1.549 |
| 11 | Na | 7.826 | 1.181 | 0.286 | 1.504 |
| 12 | Mg | 8.263 | 1.091 | 0.188 | 1.623 |
| 13 | Al | 8.802 | 1.246 | 0.404 | 1.418 |
| 14 | Si | 8.636 | 1.013 | 0.167 | 1.719 |
| 15 | P | 10.298 | 1.443 | 0.978 | 1.230 |
| 16 | S | 10.768 | 1.380 | 0.727 | 1.283 |
| 17 | Cl | 11.153 | 1.006 | 0.176 | 1.760 |
| 18 | Ar | 11.619 | 1.202 | 0.311 | 1.467 |
| 19 | K | 12.312 | 1.360 | 0.634 | 1.297 |
| 20 | Ca | 13.443 | 1.247 | 0.461 | 1.423 |
| 21 | Sc | 12.837 | 1.300 | 0.486 | 1.348 |
| 22 | Ti | 13.549 | 1.266 | 0.545 | 1.390 |
| 23 | V | 13.744 | 1.465 | 1.571 | 1.193 |
| 24 | Cr | 13.025 | 1.286 | 0.332 | 1.346 |
| 25 | Mn | 14.659 | 1.208 | 0.453 | 1.452 |
| 26 | Fe | 14.816 | 1.295 | 0.664 | 1.350 |
| 27 | Co | 15.152 | 1.247 | 0.643 | 1.400 |
| 28 | Ni | 16.723 | 1.295 | 1.241 | 1.363 |
| 29 | Cu | 16.275 | 0.929 | 0.363 | 1.883 |
| 30 | Zn | 17.079 | 1.214 | 0.870 | 1.446 |
| 31 | Ga | 17.372 | 1.084 | 0.525 | 1.614 |
| 32 | Ge | 19.985 | 1.112 | 0.741 | 1.600 |
| 33 | As | 17.701 | 1.229 | 0.714 | 1.417 |
| 34 | Se | 18.501 | 1.131 | 0.595 | 1.542 |
| 35 | Br | 18.161 | 1.317 | 0.911 | 1.318 |
| 36 | Kr | 18.882 | 1.366 | 1.449 | 1.274 |
| 37 | Rb | 19.114 | 1.096 | 0.454 | 1.582 |
| 38 | Sr | 19.674 | 1.335 | 1.065 | 1.302 |
| 39 | Y | 18.915 | 1.314 | 0.816 | 1.309 |
| 40 | Zr | 18.924 | 1.185 | 0.436 | 1.444 |
| 41 | Nb | 19.227 | 1.176 | 0.335 | 1.458 |
| 42 | Mo | 20.705 | 1.472 | 2.658 | 1.175 |
| 43 | Tc | 18.406 | 1.288 | 0.330 | 1.331 |
| 44 | Ru | 20.596 | 1.440 | 1.626 | 1.194 |
| 45 | Rh | 19.530 | 1.073 | 0.235 | 1.584 |
| 46 | Pd | 20.279 | 1.303 | 0.683 | 1.311 |
| 47 | Ag | 19.990 | 1.283 | 0.887 | 1.319 |
| 48 | Cd | 21.808 | 1.240 | 0.716 | 1.385 |
| 49 | In | 21.778 | 1.115 | 0.400 | 1.535 |
| 50 | Sn | 22.794 | 1.447 | 2.848 | 1.189 |

TABLE II. (Continued)

| Target | | $a$ | $p$ | $q$ | $b$ |
|---|---|---|---|---|---|
| Atomic number | Symbol | | | | |
| 51 | Sb | 21.714 | 1.062 | 0.312 | 1.602 |
| 52 | Te | 22.873 | 1.441 | 2.090 | 1.188 |
| 53 | I | 23.760 | 1.500 | 4.439 | 1.145 |
| 54 | Xe | 22.349 | 1.506 | 2.676 | 1.126 |
| 55 | Cs | 25.252 | 1.475 | 4.031 | 1.168 |
| 56 | Ba | 25.969 | 1.450 | 2.848 | 1.189 |
| 57 | La | 26.881 | 1.499 | 5.622 | 1.153 |
| 58 | Ce | 26.885 | 1.462 | 3.711 | 1.181 |
| 59 | Pr | 26.891 | 1.474 | 4.534 | 1.169 |
| 60 | Nd | 27.488 | 1.460 | 4.375 | 1.182 |
| 61 | Pm | 24.781 | 1.203 | 0.499 | 1.410 |
| 62 | Sm | 29.490 | 1.482 | 9.596 | 1.172 |
| 63 | Eu | 24.729 | 1.244 | 0.594 | 1.356 |
| 64 | Gd | 29.278 | 1.477 | 7.647 | 1.171 |
| 65 | Tb | 26.270 | 1.274 | 0.914 | 1.335 |
| 66 | Dy | 26.910 | 1.231 | 0.934 | 1.384 |
| 67 | Ho | 26.692 | 1.261 | 1.131 | 1.348 |
| 68 | Er | 27.658 | 1.206 | 0.935 | 1.416 |
| 69 | Tm | 27.115 | 1.216 | 1.021 | 1.397 |
| 70 | Yb | 27.535 | 1.257 | 1.442 | 1.352 |
| 71 | Lu | 27.322 | 1.205 | 0.963 | 1.407 |
| 72 | Hf | 29.614 | 1.356 | 3.105 | 1.263 |
| 73 | Ta | 28.559 | 1.242 | 1.383 | 1.369 |
| 74 | W | 31.191 | 1.243 | 1.805 | 1.383 |
| 75 | Re | 28.662 | 1.239 | 1.157 | 1.369 |
| 76 | Os | 30.188 | 1.345 | 2.754 | 1.269 |
| 77 | Ir | 28.359 | 1.209 | 1.022 | 1.397 |
| 78 | Pt | 28.231 | 1.278 | 1.840 | 1.319 |
| 79 | Au | 28.710 | 1.172 | 0.882 | 1.439 |
| 80 | Hg | 30.005 | 1.195 | 1.241 | 1.418 |
| 81 | Tl | 29.171 | 1.228 | 1.018 | 1.372 |
| 82 | Pb | 32.990 | 1.452 | 9.355 | 1.179 |

$X_m Y_n$

$$\varepsilon\,(X_m Y_n) = m\varepsilon\,(X) + n\varepsilon\,(Y). \qquad (30)$$

A large number of studies have been devoted to the experimental verification of the Bragg-Kleeman rule (30) (see, for example, Ref. 43 and the literature cited therein). In several cases for proton energies below 0.5 MeV and helium ion energies below 2 MeV there has been found to be a marked deviation from (30) which reaches especially large values (10–15%) at energies near the region of the maximum. However, these deviations are roughly the same size as the experimental errors.

In the preceding discussion we were concerned only with energies close to the region of the maximum (for example, $E > 150$–$200$ keV for protons). Lower energies are dealt with in Ref. 27, where the region of the maximum was apparently treated for the first time: the stopping of protons of energies 60–150 keV in $SiO_2$ was investigated. Experiments showed that the maximum in the curves $dE/dx$ found experimentally and that obtained using the rule (30) are very different: the experimental maximum is located at an energy above 150 keV, but the rule (30) puts the maximum at 60 keV. At $E = 150$ keV the discrepancy from the rule (30) is more than 20%. The regularities found in Ref. 27 are apparently of a general nature: near the maximum of $dE/dx$ the rule (30) does not hold, but it is satisfied more and more accurately with increasing energy.

The traditional method of checking the rule (30) is that of direct verification. In the overwhelming majority of cases one of the components of a multicomponent medium in the free form is in the gaseous phase, and it is this case to which the tables of stopping powers refer. This does not lead to any problems in those situations where the multi-component matter in question is also in the gaseous phase under ordinary conditions. When the matter is a solid, the use of the method described above is not, strictly speaking, giving information on the rule (30), but on a combination of two effects—the deviation from the rule (30) and the effect of the solid phase on the stopping cross section for the component which is a gas under ordinary conditions.

The method proposed in Ref.43 for checking the additivity rule is free from this deficiency. However, that method is not universal, but can be applied only in certain special cases. It goes essentially as follows. Using the experimental values of the stopping cross section $\varepsilon(E)$ for protons in LiF and Li, we calculate the difference

$$\varepsilon_1 = \varepsilon\,(\mathrm{LiF}) - \varepsilon\,(\mathrm{Li}).$$

The result $\varepsilon_1$ is the sum

$$\varepsilon_1 = \varepsilon\,(\mathrm{F}) + \Delta_1\varepsilon$$

of the stopping cross section for fluorine bound in the molecule LiF in the solid phase and the correction $\Delta_1\varepsilon$ which takes into account the possible deviation from the rule (30) for protons in LiF. Doing the same for $CaF_2$ and Ca, we find

$$\varepsilon_2 = \frac{1}{2}\,[\varepsilon\,(\mathrm{CaF_2}) - \varepsilon\,(\mathrm{Ca})] = \varepsilon\,(\mathrm{F}) + \Delta_2\varepsilon.$$

Let us now compare $\varepsilon_1$ and $\varepsilon_2$. Analysis of the published experimental data has shown[44] that for protons of energy

below 0.5 MeV the difference $\varepsilon_2 - \varepsilon_1$ reaches 25% of $(\varepsilon_1 + \varepsilon_2)/2$, which is considerably higher than the estimated experimental errors. One of the possible reasons why the difference $\varepsilon_2 - \varepsilon_1$ is so large is the following: it could be that for LiF the correction $\Delta_1\varepsilon$ is negative, while for $CaF_2$ the correction $\Delta_2\varepsilon$ is positive, so that the corrections $\Delta_1\varepsilon$ and $\Delta_2\varepsilon$ add in the difference $\varepsilon_2 - \varepsilon_1$. If this is true and we assume that $|\Delta_1\varepsilon| \approx |\Delta_2\varepsilon|$ (i.e., each correction is about 12–13%, in good agreement with the data of other authors for other compounds), then $(\varepsilon_1 + \varepsilon_2)/2$ gives an estimate of the stopping cross section for protons in fluorine existing in the form of bound molecules in the solid phase. For protons of energy 1 MeV the value of this quantity is $7.7\times10^{-15}$ eV·cm$^2$/atom (see Table 1 in Ref. 43), in good agreement with the value of the stopping cross section for protons in fluorine obtained using the results for $CaF_2$ and the additivity rule ($7.57\times10^{-15}$ eV·cm$^2$/atom; see Table 2 of Ref. 43). The assumption made above therefore seems reasonable.

The values of the stopping cross section for protons in fluorine in the bound form in the solid phase obtained using the method discussed above and the additivity rule are considerably greater than the values for gaseous fluorine, and the difference grows from 12.5% to 27% as the energy increases from 0.3 to 2.0 MeV. However, this result contradicts those of Ref. 31, where the stopping cross section for helium ions in fluorine, bound and in the solid phase, is smaller than in gaseous fluorine. The opposite situation occurs in the case of protons.

We have thus demonstrated the contradictory nature of the presently available information on the effect of the solid phase on the stopping cross section in the case of media which are gaseous under ordinary conditions. More refined and accurate experiments are needed to resolve this problem.

## 7. ENERGY LOSSES BY HEAVY IONS

According to the results of Ref. 45, the energy loss of heavy ions of energy greater than 200 keV/nucleon is rather simply related to the proton energy loss:

$$\langle S \rangle = S_{\mathrm{HI}}(V, Z_2)/Z_{\mathrm{HI}}^2 S_p(V, Z_2).$$

Here $S_{\mathrm{HI}}(V, Z_2)$ is the stopping power of a material with atomic number $Z_2$ for a heavy ion with speed $V$, $S_p(V, Z_2)$ is the analogous quantity for protons, $Z_{\mathrm{HI}}$ is the atomic number of the heavy ion, and $\langle S \rangle$ is the reduced stopping power.

Analysis of a wide variety of experimental data has shown that for ions of energy exceeding 200 keV/nucleon the reduced stopping power is accurately given by a universal function of form determined only by the type of ion and independent of the medium. The approximation of this function is of the form

$$\langle S \rangle = [1 - e^{-V_2}(1{,}034 - 0{,}1777e^{-0.08114Z_1})]^2,$$

with

$$V_2 = V_1 + 0{,}0378 \sin\left(\frac{\pi}{2} V_1\right)$$

and

$$V_1 = 0{,}886\,(V/V_0)\,Z^{-2/3};$$
$$V/V_0 = [E/25M_1]^{1/2}$$

($E$ is in keV and $M$ is in amu).

The standard error of this method is less than 5%. The method has been verified for ions of atomic number 6–92 and targets of atomic number 4–79.


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