Interaction of low-energy nucleons with nuclei in a semimicroscopic approach

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A semimicroscopic approach to the description of the interaction of low-energy nucleons with nuclei is presented. The optical potentials and form factors of inelastic transitions are obtained in a closed form on the basis of effective nucleon-nucleon forces, which depend on the density of the matter distribution in the nucleus, and allowance for the Pauli principle in the density-matrix formalism. The influence of exchange effects on the properties of the nucleon–nucleus interaction is analyzed. A semimicroscopic coupled-channel method is formulated and used to analyze experimental data on the elastic and inelastic scattering of protons by nuclei. The connection between the deformation parameters of the nucleon potential and the parameters of the isoscalar and isovector deformation of the nuclei is investigated in the isospin formalism. The contribution of exchange effects to the energy dependence of the isobaric-spin potential is established.

INTRODUCTION

Study of the interaction of low-energy nucleons with nuclei is a topical problem of low-energy nuclear physics. Study of the scattering of low-energy protons and neutrons by nuclei is still an important source of information about the nuclear-reaction mechanisms and nuclear-structure properties. In a number of recent investigations (see, for example, Refs. 1–3), experimental data have been obtained with so-called high resolution, the angular distributions of inelastically scattered protons being measured in a wide range of angles for several tens of excitation levels of the target nuclei at a proton energy around 25 MeV. These experimental data are analyzed in the framework of the standard version of the coupled-channel method. As has already been noted in the literature, such a method of analysis is subject to some shortcomings, which include the existence of a large number of free parameters [optical-potential parameters, deformation parameters, etc.], neglect of the antisymmetrization effects, and the absence of a connection with semimicroscopic nuclear models.

At the same time, to describe nuclear-structure properties, semimicroscopic approaches are being developed strongly in nuclear theory: the self-consistent theory of finite Fermi systems,[4] the quasiparticle–phonon model,[5] nuclear field theory,[6] microscopic variants of the model of interacting bosons and fermions,[7] and so forth. As a rule, these models are used to analyze inelastic nuclear transitions associated with electromagnetic interactions. It is of interest to use the semimicroscopic nuclear models to describe the inelastic scattering of nucleons by nuclei. Such possibilities do not exist in the framework of the standard version of the coupled-channel method.

In the theory of nuclear reactions there has been extensive development of semimicroscopic methods (resonating-group method,[8] energy-density functional method,[9] folding model,[10–13] and others), in which, on the one hand, the shortcomings inherent in the microscopic analysis of experimental scattering data are eliminated and, on the other, it is possible to use semimicroscopic nuclear models to analyze nuclear-reaction mechanisms. Among these approaches, the folding model possesses the greatest simplicity and universality. It is equally applicable to spherical and deformed nuclei, to simple and composite projectile particles, and to the description of elastic and inelastic scattering. Generalization of the folding model and allowance for many-particle and exchange nucleon–nucleon correlations make it possible to include explicitly in the treatment the effects associated with the density dependence of the effective nucleon-nucleon interaction and the Pauli principle. The part played by the latter is particularly important in the description of the interaction of low-energy particles with nuclei.[14–16] Apart from the study of the part played by the Pauli principle, investigation of the interaction of low-energy nucleons with nuclei is of great interest in connection with the fact that at low energies the coupling of the channels is more important and, as a consequence of this, there is a greater probability (as compared with high energies) of excitation of target-nucleus states of complicated structure.

In Refs. 17 and 18, the folding model and the density-matrix formalism were used to develop a semimicroscopic approach in which the optical potentials and the form factors of inelastic transitions are constructed in a closed form with allowance for many-particle and exchange nucleon–nucleon correlations for the case of strong channel coupling.

Study of the inelastic scattering of low-energy nucleons by nuclei is an important source of information about the parameters describing the deformation of the matter distribution in nuclei and about transition densities. Experimental investigations with high-energy protons, and also with pions and other particles, have suggested that the deformations of the neutron and proton distributions in nuclei can exhibit significant differences.[19–21] These differences lead in their turn to differences in the parameters of the isoscalar and isovector deformation of the optical potential. From this point of view, it is of interest to analyze the isospin structure of inelastic transitions in nuclei manifested in the inelastic scattering of low-energy nucleons by nuclei in the frame-

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work of a semimicroscopic approach; for in this approach the form factors of the inelastic transitions are directly related to the proton and neutron transition densities. The formalism of the semimicroscopic approach includes in a natural manner a description on a unified basis of inelastic scattering of nucleons by nuclei and quasielastic and quasiinelastic scattering, the amplitude of the process in the latter case being basically determined by the parameter of the isovector deformation of the optical potential; in the semimicroscopic approach this parameter is directly related to the parameters describing the deformation of the proton and neutron distributions.

For the description of peripheral processes in the interaction of heavy ions, wide use is made of optical potentials constructed in the double-folding model on the basis of the effective M3Y interaction.[22] However, this interaction does not contain a density dependence, and the Pauli principle is taken into account in it effectively by the introduction of a zero-range pseudopotential. As a result, the use of the M3Y interaction to describe the scattering of light particles (nucleons and α particles) by nuclei is unsuccessful. It is therefore necessary to construct potentials for the interaction of nuclei with both simple and composite particles on the basis of a unified effective nucleon–nucleon interaction in the framework of a unified scheme for taking into account nucleon–nucleon correlations.

I. POTENTIALS AND FORM FACTORS OF INELASTIC TRANSITIONS

In the folding model, the potential of the interaction between two colliding particles A and B is determined by

$$U_{AB}(\mathbf{R}) = \int\int \rho(\mathbf{r}_A)\,\rho(\mathbf{r}_B)\,V(|\mathbf{r}_A + \mathbf{R} - \mathbf{r}_B|)\,d\mathbf{r}_A d\mathbf{r}_B. \quad (1)$$

Here, $\rho(\mathbf{r}_A)$ and $\rho(\mathbf{r}_B)$ are the densities of the nucleon distributions in the nuclei, and V is the potential of the effective nucleon–nucleon interaction. If the structure of the incident particle is not taken into account, the double-folding potential (1) is replaced by the single-folding potential

$$U(\mathbf{r}) = \int \rho(\mathbf{r}')\,V(|\mathbf{r} - \mathbf{r}'|)\,d\mathbf{r}'. \quad (2)$$

Expanding $\rho(\mathbf{r}')$ and $V(|\mathbf{r} - \mathbf{r}'|)$ in multipole series, we obtain

$$U(\mathbf{r}) = \sum_{\lambda} U_{\lambda 0}(r)\,Y_{\lambda 0}(\theta, \varphi); \quad (3)$$

$$U_{\lambda 0}(r) = \int v_{\lambda}(r, r')\,\rho_{\lambda 0}(r')\,(r')^2\,dr'. \quad (4)$$

In the collective model, $\rho_{\lambda 0}(r')$ and $U_{\lambda 0}(r)$ are, respectively, the transition densities and form factors of inelastic transitions with transfer of angular momentum a to the target nucleus, and $U_{00}(r)/\sqrt{4\pi}$ is the central part of the optical potential. In the optical model and in the macroscopic methods of analyzing inelastic scattering (DWBA and coupled-channel method), $U_{\lambda 0}(r)$ are parametrized and include optical-potential parameters and the deformation parameters $\beta_{\lambda 0}^{V}$ of the potential. The optical-potential parameters are subject to both discrete and continuous nonuniqueness. In the case of nucleons or high energies, this nonuniqueness can be eliminated, but in the case of composite particles or low energies it is not possible to eliminate the nonuniqueness of the optical-potential parameters in the framework of the optical model.

In the folding model, the potentials and form factors of the inelastic transitions are determined in accordance with (4) by specifying the central and transition densities, which can be determined either from an experiment that is independent with regard to the interaction of low-energy nucleons with nuclei (electron or pion scattering, interaction of high-energy protons with nuclei, electromagnetic transitions, etc.) or from calculations in the framework of a nuclear model. An effective nucleon–nucleon interaction can be constructed on the basis of the vacuum interaction with allowance for the nuclear medium[23] or can be taken from calculations of nuclear-structure properties. Thus the potentials and form factors of the inelastic transitions in the folding model do not contain free parameters and are not subject to the nonuniqueness inherent in the macroscopic approaches. In addition, they are directly related to the nuclear-structure properties—the matter distribution in the nucleus and the transition densities, and also the effective nucleon–nucleon interaction.

The potential in the form (2) is the so-called direct term in the interaction of a nucleon with a target nucleus. To take into account the Pauli principle, we introduce the density matrix $\rho(\mathbf{r}, \mathbf{r}')$; then a potential[1)] including direct and exchange terms can be expressed in the form[11]

$$U(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \int V_D(|\mathbf{r} - \mathbf{r}_1|)\,\rho(\mathbf{r}_1)\,d\mathbf{r}_1 + V_E(|\mathbf{r} - \mathbf{r}'|)\,\rho(\mathbf{r}, \mathbf{r}'). \quad (5)$$

Here, $V_D(|\mathbf{r} - \mathbf{r}'|)$ and $V_E(|\mathbf{r} - \mathbf{r}'|)$ are, respectively, the contributions of the direct and exchange parts of the effective nucleon–nucleon forces to the nucleon–nucleus interaction. As is well known, the optical-model potentials also contain spin–orbit, Coulomb, and imaginary terms. With regard to the Coulomb term, it can always be calculated by means of the folding procedure (usually, the charge distribution is specified in the form of a uniformly charged sphere, but diffuseness of the charge distribution can also be taken into account[25]). The spin–orbit term in the optical potential can be constructed on the basis of two-particle spin–orbit forces (see, for example, Ref. 26). The situation with regard to the imaginary part of the optical potential is more complicated. According to Refs. 27 and 28, the absorption potential has a dynamical nature and its complete calculation requires knowledge of the complete spectrum of states of the target nucleus and the compound system. What is actually possible is the calculation of the contribution of individual states to the imaginary part of the optical potential, the states of collective nature making the most important contribution. The contribution of these states to the absorption potential was calculated for the first time in Refs. 29–31.

Two approaches have been developed for the calculation of the imaginary part. The first of them is based on explicit allowance for the structure of the target-nucleus states.[32,33] Comparison of the absorption potentials calculated for the ^{40}Ca and ^{208}Pb nuclei in Refs. 34 and 35 with

phenomenological optical potentials shows that the theoretical calculations reproduce about 50% of the strength of the absorption potential. An alternative approach is based on the results of the theory of nuclear matter obtained in the approximation of a local density.[36-38] In this approach, the imaginary part of the optical potential is found to be greater than for the phenomenological potential. Thus, in both approaches it is necessary to renormalize the depth of the absorption potential. However, the underestimation or overestimation of the imaginary part can be due not only to the value of the strength constant of the absorption potential but also to the "geometrical" parameters of the potential. Therefore, although calculation of the imaginary part of the optical potential is an independent interesting problem in nuclear theory, to analyze scattering mechanisms in the framework of the semimicroscopic approach it is more expedient to use parameters of the absorption potential extracted from the description of elastic scattering in the optical model.

The expression (5) is written down in the first order in the effective nucleon–nucleon interaction. There are also second-order corrections (the imaginary part of which has been discussed above), whose real part is related to the so-called core-polarization effects, i.e., the effects of the rearrangement of the motion of the nucleons in the target nucleus under the influence of the incident particle. The importance of these polarization terms is essentially determined by two factors—the structure of the incident particle and the closeness to which the colliding nuclei approach each other. For composite particles and appreciable overlapping of the nuclear densities, the second-order effects can have a significant influence on the strength and shape of the potential in the interior and surface regions. However, because of the strong absorption the scattering cross sections are weakly sensitive to the behavior of the potential in the interior region of the nucleus. In the case of nucleons, the contribution of the terms of second order in the effective interaction to the real part of the nucleon potential was calculated for the ^{40}Ca and ^{208}Pb nuclei in Refs. 39 and 40. It was shown that near the Fermi energy this contribution is localized on the surface of the nucleus, while further from the Fermi energy it makes a correction to the interior part of the potential not exceeding 10%. Thus, for the description of the scattering by nuclei of nucleons with energy exceeding 15–20 MeV the second-order terms can be ignored. Another approach to this problem exploits the fact that (5) contains an effective interaction, so that the polarization terms can be taken into account by renormalizing the parameters of this interaction.

Through the allowance for the Pauli principle, the semimicroscopic potential is nonlocal, and, in its turn, has the consequence that the Schrödinger equation for the description of elastic scattering becomes an integro-differential equation, and the system of the coupled-channel method is transformed into a system of integro-differential equations, so that much computing time is needed to solve such a system. The characteristic magnitude of the nonlocality is directly related to the range of the effective nucleon–nucleon forces, which have a short range. By virtue of this, it is possible to go over from the nonlocal to a local, but energy-dependent, potential in accordance with the procedure explained in Ref. 41. Thus, instead of (5) we have

$$U(\mathbf{r}) = \int V_D(|\mathbf{r}-\mathbf{r}'|)\,\rho(\mathbf{r}')\,d\mathbf{r}' + \int V_E(|\mathbf{r}-\mathbf{r}'|)\,\rho(\mathbf{r},\mathbf{r}')\,j_0(k(\mathbf{r})s)\,d\mathbf{s}; \quad (6)$$

$$\mathbf{s} = \mathbf{r}' - \mathbf{r};$$

$$k^2(\mathbf{r}) = \frac{2m}{\hbar^2}[E - U(\mathbf{r}) - V_C(\mathbf{r})]. \quad (7)$$

Here, $J_0[k(\mathbf{r})s]$ is a spherical Bessel function which appears in the localization procedure, and $V_C(\mathbf{r})$ is the Coulomb potential.

Local approximation for deformed nuclei

We expand all quantities on the right- and left-hand sides of (6) in multipole series. We obtain[42]

$$U_{L0}(r) = \int_0^\infty \rho_{L0}(r')\,v^D_{L0}(r, r')\,(r')^2\,dr' + U^E_{L0}(r); \quad (8)$$

$$U^E_{L0}(r) = \sum_{\lambda\lambda'} S_{L\lambda\lambda'} \int_0^\infty v_E(s)\,\rho_{\lambda 00}(r, s)\,J_{\lambda'}(k(r)s)\,s^2\,ds; \quad (9)$$

$$S_{L\lambda\lambda'} = \left[\frac{(2\lambda+1)(2\lambda'+1)}{2L+1}\right]^{1/2} C^2(\lambda 0 \lambda' 0 \mid L0); \quad (10)$$

$$\rho_{\lambda 00}(r, s) = \frac{1}{\sqrt{4\pi}} \int \rho(\mathbf{r}, \mathbf{r}+\mathbf{s})\,Y_{\lambda 0}(\omega_{\mathbf{r}})\,d\omega_{\mathbf{r}}\,d\omega_{\mathbf{s}}; \quad (11)$$

$$J_{\lambda'}(k(r)s) = \int j_0(k(\mathbf{r})s)\,Y_{\lambda' 0}(\omega)\,d\omega. \quad (12)$$

In Eqs. (8)–(12), all quantities except $\rho(\mathbf{r}, \mathbf{r}')$ have been determined. The density matrix can be calculated for spherical or deformed nuclei on the basis of the single-particle formula (see, for example, Ref. 42):

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_i \varphi_i^*(\mathbf{r})\,\varphi_i(\mathbf{r}').$$

However, instead of this exact but rather cumbersome procedure a closed expression is, as a rule, employed for $\rho(\mathbf{r}, \mathbf{r}')$. The simplest such expression is the Slater approximation:

$$\rho(\mathbf{r}, \mathbf{r}') = \rho(\mathbf{R})\,\frac{3}{sk_F}\,j_1(sk_F); \quad (13)$$

$$k_F(\mathbf{R}) = \left[\frac{3}{2}\pi^2\rho(\mathbf{R})\right]^{1/3}; \quad (14)$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}+\mathbf{r}'). \quad (15)$$

A more realistic expression for $\rho(\mathbf{r}, \mathbf{r}')$ with, however, a structure as simple was proposed in Ref. 43:

$$\rho(\mathbf{r}, \mathbf{r}') = \rho(\mathbf{R})\,\frac{3}{s\hat{k}}\,j_1(s\hat{k}); \quad (16)$$

$$\hat{k}(\mathbf{R}) = \left|\frac{5}{3\rho(\mathbf{R})}\left[\tau(\mathbf{R}) - \frac{1}{4}\nabla^2\rho(\mathbf{R})\right]\right|^{1/2}. \quad (17)$$

Here, $\tau(\mathbf{R})$ is the kinetic-energy density of the motion of the nucleons in the nucleus.

Equations (8)–(12) in conjunction with (13), (14) or (16), (17) constitute a formalism by means of which it is possible, using information on the effective nucleon–nucleon forces, the central density, and the transition densities, to construct semimicroscopic optical potentials and the form factors of inelastic transitions for deformed nuclei. The

application of this formalism directly to the analysis of experimental scattering data encounters the following difficulties. We note that in the case of deformed nuclei the parameters β_λ^ρ of the static deformation of the matter distribution in the nucleus occur [see Eq. (7)] in the integrand of the exchange term, but the same parameters occur on the left-hand side of the relation (6). Thus, to find the potentials and the form factors of the inelastic transitions it is necessary to employ a cumbersome iterative procedure, and this makes it difficult to extract the parameters β_λ^ρ from analysis of experimental scattering data. The second difficulty is associated with the description of scattering by nuclei with vibrational states. In this case, the parameters β_λ^ρ have the meaning of dynamical deformation parameters, and on the transition to the phonon representation $k(\mathbf{r})$ becomes dependent on the phonon creation and annihilation operators. Integration in (6) becomes impossible and, thus, the formalism does not apply directly to the description of the excitation of vibrational states.

In Refs. 17 and 18, a semimicroscopic approach was developed that makes it possible to overcome both of these difficulties. To obtain closed expressions for the quantities $U_{L0}(r)$, we use the multiplication theorem for Bessel functions.[44] In the case of the spherical function $f_0(\mu z)$,

$$j_0(\mu z)=\sum_{n=0}^{\infty}\frac{1}{n!}j_n(z)\left(\frac{1-\mu^2}{2}z\right)^n. \tag{18}$$

We determine z and μ by the relations

$$\mu=[1-k_1^2(\mathbf{r})/k_0^2(r)]^{1/2}; \tag{19}$$

$$k_0^2(r)=\frac{2m}{\hbar^2}[E-U_0^D(r)-V_C(r)]; \tag{20}$$

$$\left.\begin{aligned}&U_0^D(r)=\frac{1}{\sqrt{4\pi}}U_{00}^D(r);\\ &k_1^2(\mathbf{r})=\frac{2m}{\hbar^2}\Big[U_0^E(r)+{\sum_{L'}}'U_{L'0}^E(r)\,Y_{L'0}(\omega)\\ &+{\sum_{L'}}'\beta_{L'}^\rho\tilde{U}_{L'0}^D(r)\,Y_{L'0}(\omega)\Big];\\ &z(r)=k_0(r)\,s.\end{aligned}\right\} \tag{21}$$

To separate explicitly the dependence on β_L^ρ, we also introduce

$$\tilde{U}_{L0}^D(r)=U_{L0}^D(r)/\beta_L^\rho;$$

$$\tilde{\rho}_{L0}(r)=\rho_{L0}(r)/\beta_L^\rho.$$

Using (19) and (21), we obtain for the nth term in the expansion (18)

$$\frac{1}{n!}j_n(k_0(r)s)\left(\frac{k_1^2(\mathbf{r})}{k_0^2(r)}\frac{k_0(r)s}{2}\right)^n. \tag{22}$$

It is readily seen that the expansion (22) converges rapidly, since the factor multiplying $j_n[k_0(r)s]$ in (22) is appreciably less than unity for all r and, in addition, $j_n(x)$ decreases rapidly with increasing n. Retaining in (18) the first three terms, substituting in (9), and taking into account (12), we obtain for $U_{L0}^E(r)$ the system of equations

$$\begin{aligned}U_{L0}^E(r)&=\beta_L^\rho I_{L0}(r)+\beta_L^\rho I_{L1}(r)\,\varkappa(r)\sqrt{4\pi}\,U_0^E(r)\\ &+\beta_L^\rho I_{L2}(r)\,\varkappa^2(r)\sqrt{4\pi}\,[U_0^E(r)]^2+{\sum_\lambda}{\sum_{\lambda'}}'\beta_\lambda^\rho S_{L\lambda\lambda'}\\ &\times\varkappa(r)\,[U_{\lambda'0}^E(r)+\beta_{\lambda'}^\rho\tilde{U}_{\lambda'0}^D(r)]\,I_{\lambda1}(r)\\ &+\frac{1}{\sqrt{4\pi}}(1-\delta_{L0})\,I_{02}(r)\,\varkappa^2(r)\\ &\times{\sum_{\lambda\lambda'}}S_{L\lambda\lambda'}[2U_{\lambda0}^E(r)\,\beta_{\lambda'}^\rho\tilde{U}_{\lambda'0}^D(r)\\ &+U_{\lambda0}^E(r)\,U_{\lambda'0}^E(r)+\beta_\lambda^\rho\beta_{\lambda'}^\rho\tilde{U}_{\lambda0}^D(r)\,\tilde{U}_{\lambda'0}^D(r)];\\ &\varkappa(r)=m/(k_0(r)\,\hbar^2).\end{aligned} \tag{23}$$

Here $I_{\lambda0}(r)$ for $\lambda\neq0$ is determined by

$$I_{\lambda0}(r)=\sqrt{4\pi}\int_0^\infty v_E(s)\,\tilde{\rho}_{\lambda00}(r,\,s)\,j_0(k_0(r)\,s)\,s^2\,ds$$

and in all the remaining cases $I_{\lambda n}(r)$ is determined by the relations

$$I_{\lambda n}(r)=\frac{1}{n!}\int_0^\infty v_E(s)\,\tilde{\rho}_{\lambda00}(r,\,s)\,j_n(k_0(r)\,s)\,s^{n+2}\,ds. \tag{24}$$

Equations (23) contain as small parameters the nuclear deformation parameters β_L^ρ, and also $\varkappa^n(r)I_{Ln}(r)$, which are less than unity for all r. Thus, the solutions of (23) can be sought as series in β_L^ρ and $\varkappa^n(r)I_{Ln}(r)$; in these series, the number of terms must be matched to the method by which the system (23) is obtained. As a result, solving the system (23) to terms of second order in β_L^ρ and $\varkappa(r)$, we obtain

$$\begin{aligned}U_0(r)&=U_0^D(r)+I_{00}(r)\,[1+\varkappa(r)\,I_{01}(r)]\\ &+\frac{1}{4\pi}{\sum_\lambda}'(\beta_\lambda^\rho)^2\,I_{\lambda1}(r)\,\varkappa(r)\,[\tilde{U}_{\lambda0}^D(r)+I_{\lambda0}(r)]+\delta_0(\varkappa^2);\end{aligned} \tag{25}$$

$$\begin{aligned}U_{L0}(r)&=\beta_L^\rho[\tilde{U}_{L0}^D(r)+I_{L0}(r)]+\beta_L^\rho I_{L1}(r)\,\varkappa(r)\sqrt{4\pi}\,I_{00}(r)\\ &+\frac{1}{\sqrt{4\pi}}{\sum_\lambda}{\sum_{\lambda'}}'\beta_\lambda^\rho\beta_{\lambda'}^\rho S_{L\lambda\lambda'}I_{\lambda1}(r)\,\varkappa(r)\,[\tilde{U}_{\lambda'0}^D(r)\\ &+I_{\lambda'0}(r)]+\delta_L(\varkappa^2).\end{aligned} \tag{26}$$

To simplify these expressions, we have not written down explicitly the terms quadratic in $\varkappa(r)$. In addition, the corresponding direct terms are added to the solutions of (23).

Equations (25) and (26) give, when allowance is made for exchange effects, closed expressions for the spherically symmetric part of the potential $U_0(r)$, which can be regarded as the main contribution to the real part of the optical potential, and for the inelastic-transition form factors $U_{L0}(r)$ with angular-momentum transfer L. In contrast to the expression (9), the expressions (25) and (26) can be directly used in the coupled-channel method to analyze experimental data and extract from this analysis the parameters β_L^ρ. In addition, after the transition to the phonon representation the expression (26) becomes suitable for the description of elastic scattering with the excitation of vibrational states.

2. EFFECTIVE NUCLEON–NUCLEON FORCES AND THE PAULI PRINCIPLE

Nuclei are fermion systems, and it is therefore important to take into account the identity of nucleons in describing nuclear reactions. The Pauli principle has a significant influence on the interaction of low-energy particles, especially composite particles, with nuclei. Thus, a consequence of the Pauli principle is the appearance at short distances of a core in the ion–ion potential.[8,45] In the case of nucleons, exchange effects significantly determine the energy dependence of the nucleon optical potential and also other properties of the optical potential and the form factors of inelastic transitions.

To take into account the Pauli principle in the description of inelastic nucleon scattering by nuclei, wide use has been made of the microscopic distorted-wave Born approximation (microDWBA).[46] In this approach, the interaction responsible for the inelastic transitions is taken to be an effective nucleon–nucleon interaction, and the wave functions of the initial and final states of the target nucleus are constructed with allowance for antisymmetrization, while the "distorted" wave functions of the scattered nucleons are found in the framework of a phenomenological optical model. Thus, in the microDWBA the Pauli principle is not taken into account in a consistent manner for the elastic and inelastic channels. More recently, the microDWBA has been developed further. In a number of studies,[47–50] an interaction constructed on the basis of realistic nucleon–nucleon potentials has been used as the effective nucleon–nucleon interaction. In this way, unity is achieved in the description of the interaction of free nucleons and the interaction of nucleons with nuclei. We note that in the microDWBA channel coupling is not taken into account, and it is used mainly at intermediate nucleon energies. Generalization of the folding model to take into account exchange nucleon–nucleon correlations[42] makes it possible to treat the Pauli principle in a unified manner in the description of elastic and inelastic scattering of nucleons by nuclei, and the formalism of the semimicroscopic approach presented in the previous section makes it possible to implement this description at low energies too, where the channel-coupling effects are important. It should, however, be noted that in the microDWBA the antisymmetrization is taken into account exactly, whereas in the semimicroscopic approach in the presence of channel coupling it is taken into account in the local approximation of the density-matrix formalism. Thus, these two approaches to the description of inelastic scattering of nucleons by nuclei can be regarded as complementing each other. Note also that much work has been done (see, for example, Refs. 34, 37, and 51–56) on the construction of a microscopic optical potential with allowance for antisymmetrization effects, but in the corresponding studies the description of only elastic scattering of nucleons by nuclei is considered.

The effects associated with the manifestation of the Pauli principle in nuclear reactions are essentially determined by the properties of the effective nucleon–nucleon interaction. In the general case, it contains central, tensor, and spin–orbit components. For the central component, we can write down the expression

$$v(r) = f(r)\{a_0 + a_\tau(\tau_1\tau_2) + a_\sigma(\sigma_1\sigma_2) + a_{\sigma\tau}(\tau_1\tau_2)(\sigma_1\sigma_2)\}. \quad (27)$$

For even–even nuclei, after application of Eq. (2), the first two terms in (27) make a nonzero contribution to the optical potential, the second term leading to the appearance of an isobaric-spin potential, the properties of which will be considered in detail below. We write the expression (27) in the alternative form

$$v(r) = V_0 f(r)\left\{W + B\frac{1+\sigma_1\sigma_2}{2} - H\frac{1+\tau_1\tau_2}{2} - M\frac{1+\sigma_1\sigma_2}{2}\frac{1+\tau_1\tau_2}{2}\right\}. \quad (28)$$

Applying the antisymmetrization operator to the right-hand side of (28), we obtain the following expressions for the strength constants of the direct and exchange parts of the effective interaction in Eq. (5):

$$V_D = \frac{V_0}{4}(4W + 2B - 2H - M);$$

$$V_E = \frac{V_0}{4}(4M + 2H - 2B - W).$$

Allowing for the Pauli principle in explicit form [the second term in Eq. (5)] involves cumbersome calculations which can be carried through to the end only in an approximation (for example, in the local approximation of the density-matrix formalism). An alternative is to take into account the Pauli principle effectively by a renormalization of the effective nucleon–nucleon interaction. Such a program can be realized by using the so-called M3Y interaction. In this case $V_E = 0$, and $V_D(r)$ has the radial dependence[22]

$$V_D(r) = 7999\frac{e^{-4r}}{4r} - 2134\frac{e^{-2.5r}}{2.5r} - 276(1 - 0.005E/A)\delta(r).$$

The last term is called the zero-range potential and effectively takes into account single-nucleon exchange.[23] The energy dependence of the pseudopotential reflects the energy dependence of the exchange term in the semimicroscopic optical potential [see Eqs. (6) and (7)]. The simple structure of the M3Y interaction makes it possible to develop an analytic variant of the folding model,[57–60] a representation in Gaussian form being used for the central density and transition densities. The analytic folding model was used in Ref. 58 to analyze the connection between the shape of the nuclear potential and the shape of the nuclear matter distribution, and in Refs. 59 and 60 it was used to investigate the radial dependence of the nucleon, α-particle, and ion–ion potentials. In Refs. 61 and 62, alternative variants of the analytic folding model were also developed.

The shape of the optical potential or the matter distribution in a nucleus can be characterized by the angle dependence of the corresponding half-decrease radius:

$$R = R_0^{v,\rho}\left[1 + \sum_\lambda \beta_\lambda^{v,\rho} Y_{\lambda 0}(\theta, \varphi)\right].$$

Here, the indices v and ρ relate to the potential and the matter distribution, respectively. For definiteness, an axisymmetric shape of the distribution is taken. The deformation

parameters $\beta_\lambda^{v,\rho}$ can be determined on the basis of the integral formula[58]

$$\beta_\lambda^g = \frac{4\pi}{2\lambda+1} \frac{\int g(\mathbf{r}) r^\lambda Y_{\lambda 0}(\theta, \varphi)\, d\mathbf{r}}{\int g(\mathbf{r}) r^\lambda\, d\mathbf{r}}; \quad g = (\rho, v). \tag{29}$$

Satchler[63] formulated a theorem from which, in particular, it follows that[64]

$$q_\lambda^v / J = q_\lambda^\rho / A; \quad q_\lambda^g = \int g_\lambda(r) r^{\lambda+2}\, dr; \quad J = \int U(\mathbf{r})\, d\mathbf{r}. \tag{30}$$

Applying this theorem and using (29), we obtain[58]

$$\beta_\lambda^v \langle r^\lambda \rangle_v = \beta_\lambda^\rho \langle r^\lambda \rangle_\rho; \tag{31}$$

$$\langle r^\lambda \rangle_g = \int g(\mathbf{r}) r^\lambda\, d\mathbf{r} / J_g \quad (J_v = J,\ J_\rho = A). \tag{32}$$

It follows from (31) that because the effective nucleon–nucleon forces have a finite range the deformation parameters β_λ^v and β_λ^ρ differ. This difference is greater, the larger is λ. As was shown in Ref. 58, allowance for the Pauli principle through a zero-range pseudopotential leads to a smoothing of these differences. The analytic folding model gives a simple analytic dependence of the parameters β_λ^v on the energy[58]:

$$\beta_\lambda^v(E) = \beta_\lambda^v(0)\{1 - \eta_\lambda E\}, \tag{33}$$

the coefficient η_λ depending on the parameters of the effective distribution and the parameters of the matter distribution in the nucleus. Calculation with the M3Y interaction for light nuclei gives a value of η_λ not greater than 0.001 MeV^{-1} for both nucleons and α particles.[58] Thus, in the interval $\Delta E = 100$ MeV the change in the deformation parameters of the potential is only a few percent. An analogous conclusion was drawn in other theoretical studies,[42,65] whereas analysis of experimental data on the scattering of low-energy protons leads to a strong energy dependence of the parameters β_λ^v (see, for example, Ref. 66), this being due to the inadequate description of the scattering mechanisms at low energies.

It was noted in the Introduction that the use of the M3Y interaction to describe scattering of nucleons was not so successful as in the case of heavy-ion interaction. This is the case because the nucleons are more sensitive to the details of the optical potential, which are determined not least by exchange nucleon–nucleon correlations, which are not explicitly considered when the M3Y interaction is used.

Energy dependence of the optical potential

We now consider the consequences of taking into account explicitly exchange effects in the density-matrix formalism of the semimicroscopic approach. We consider the energy dependence of the semimicroscopic optical potential due to the effect of the Pauli principle. Adding in (25) the contributions of the main terms containing $\varkappa^2$, we obtain for the central, β_λ^ρ-independent part of the optical potential[67]

$$U_0(r) = U_0^D(r) + I_{00}(r) \frac{1 + \varkappa^2(r) I_{02}(r) I_{00}(r)}{1 - \varkappa(r) I_{01}(r)}. \tag{34}$$

The fraction here gives a correction to $I_{00}(r)$ that does not exceed 15% at all r. Therefore, to study the energy dependence of the nucleon optical potential, we can regard $I_{00}(r)$ as the main contribution to it. We again use the multiplication theorem for the spherical Bessel functions [see Eq. (18)]. We now determine μ' and z' in this formula by

$$\mu' = [1 + E/|U_0^D(r)|]^{1/2};$$

$$z' = \xi(r) s; \quad \xi(r) = \left(\frac{2m}{\hbar^2} |U_0^D(r)|\right)^{1/2}.$$

Applying (18), we obtain

$$j_0(k_0(r) s) = j_0(\xi(r) s) - \frac{E}{|U_0^D(r)|} \frac{\xi s}{2} j_1(\xi(r) s) + \frac{1}{2}\left(\frac{E}{|U_0^D(r)|}\right)^2 \left(\frac{\xi s}{2}\right)^2 j_2(\xi(r) s) + \ldots \tag{35}$$

After substitution of (35) in the expression (24) for $I_{00}(r)$, we obtain an expansion of the exchange part of the potential in powers of the energy[17,67]:

$$U_0^E(E, r) = U_0^E(0, r) - \gamma(r) E + \delta(r) E^2 + \ldots, \tag{36}$$

where

$$\gamma(r) = \left(\frac{m}{2\hbar^2 |U_0^D(r)|}\right)^{1/2} I_{01}^\xi(r); \quad \delta(r) = \frac{m}{4\hbar^2 |U_0^D(r)|} I_{02}^\xi(r).$$

Here, $I_{0n}^\xi(r)$ differ from the previously defined quantities by the substitution $k_0(r) \to \xi(r)$. In the optical model, the energy dependence of the nucleon optical potential is established empirically. With increasing energy, the depth of the optical potential decreases, i.e., the potential becomes shallower.

For different sets of optical potentials there are different values of $\gamma(0)$.[68] It is important that in the optical model $\gamma(r)$ and $\delta(r)$ have the same radial dependence as $U_0^E(E, r)$, and a linear approximation is generally used. In Ref. 69, the experimental data on the elastic scattering of nucleons were analyzed in the framework of the phenomenological optical model in a wide range of energies and the mass number. In the linear approximation, a value equal to 0.24 was obtained for $\gamma(0)$.

One of the reasons for the appearance of an energy dependence of the nucleon optical potential in the semimicroscopic approach is the allowance made for the Pauli principle. With increasing energy, the effect of the Pauli principle becomes weaker, the exchange integrals $I_{0n}(r)$ become smaller, and at the same time the value of γ in (36) becomes smaller. The potential becomes shallower. For a quantitative estimate of the part played by the exchange effects in the formation of the energy dependence of the optical potential, it is necessary to fix the parameters of the effective nucleon–nucleon interaction. As effective forces, we use Wildermuth–Schmid forces.[70] These forces have been successfully used to describe NN and $\alpha\alpha$ scattering at low energies, and also to describe the cluster properties of light nuclei. The parameters of the effective potential are given in Table I. The calculation made in the semimicroscopic approach for the target nucleus ^{26}Mg gives $\gamma(0) = 0.15$. This value can be taken as the average for a large group of nuclei, since the values of the exchange integrals depend weakly on the mass number. Comparison with the empirical value shows that allowance for the Pauli principle explains 60% of the energy dependence of the optical potential. The energy dependence is also due to the contribution to the optical potential of the

TABLE I. Parameters of effective nucleon–nucleon forces.

V_0, MeV	a, F	W	B	M	H
−72.98	1.47	0.3742	0.1158	0.4408	0.0692

terms of second order in the effective interaction.[39,40,71–73] As noted earlier, this contribution is localized in the surface region of the nucleus and is maximal near the Fermi energy. In Refs. 71 and 72, a study was made of the relative importance of the two factors (Pauli principle and second-order terms) leading to the energy dependence of the optical potential. Numerical calculations for the ^{40}Ca and ^{208}Pb nuclei showed that allowance for the Pauli principle leads to a decrease in the depth of the potential (in agreement with the conclusions of the semimicroscopic approach), while the contribution of the second-order terms leads to an increase in the depth of the potential, i.e., gives the opposite effect in the energy dependence of the nucleon potential. However, with increasing distance from the Fermi energy the contribution of the second-order terms decreases, and the Pauli principle plays the dominant role in the energy dependence of the optical potential. It should also be noted that the effects of localization of the second-order terms influence the energy dependence of the optical potential. In addition, the effective nucleon–nucleon interaction can also depend on the energy.[38]

In the phenomenological optical model, a regular dependence of the geometrical parameters of the optical potential on the energy has not been established. The formalism of the semimicroscopic approach makes it possible, at least in the linear approximation, to obtain a simple analytic dependence of the "geometry" of the potential on the energy. Using the Slater approximation for the density matrix $\rho(\mathbf{r}, \mathbf{r}')$ and the short-range property of the effective nucleon–nucleon forces, we can write approximately

$$I_{01}(r) \simeq \frac{12\pi\rho_0(r)}{k_F(r)} V_E \int_0^\infty f(s)\, j_1(k_F(r)\, s)\, j_1(\xi(r)\, s)\, s^2\, ds.$$

The integral over s can be calculated analytically,[44] giving

$$I_{01}(r) \simeq 12\pi^{3/2} a^3 V_E \frac{\rho_0(r)}{k_F(r)} \exp\left[-\frac{k_F^2(r)+\xi^2(r)}{4} a^2\right] \times \left\{-\frac{\operatorname{Sinh}\sigma(r)}{\sigma^2(r)} + \frac{\cosh\sigma(r)}{\sigma(r)}\right\}; \quad (37)$$

$$\sigma(r) = \frac{k_F(r)\,\xi(r)}{2} a^2.$$

As the calculations show, the product of the exponential and the expression in the curly brackets in (37) depends weakly on r. Ignoring this dependence, we finally obtain

$$\gamma(r) \simeq J_0^E C \frac{\rho_0^{2/3}(r)}{(|U_0^D(r)|)^{1/2}}. \quad (38)$$

Here, J_0^E is the contribution to the strength of the two-particle effective forces from the exchange part of these forces. It follows from (38) that the strongest energy dependence of the optical potential corresponds to the interior and surface regions, the energy dependence becoming weaker than 50% at the "tail" of the potential. Since in accordance with (38) $\gamma(r)$ is approximately proportional to $\rho_0^{1/6}(r)$, the changes in the geometry of the potential with increasing energy occur smoothly.

We can consider similarly the energy dependence of the form factors $U_{l0}(r)$ of the inelastic transitions. Using the expressions (25) and (29), for low-energy nucleons we again obtain a linear energy dependence of the parameters β_λ^v [see Eq. (33)]. A calculation with Wildermuth–Schmid forces gives for the coefficients η_2 and η_4 the values 0.0016 and 0.0061, respectively. The value 0.0016 for η_2 is close to the estimate obtained in the approximation of a zero-range pseudopotential, and the larger value of η_λ for $\lambda = 4$ is due to the appreciable contribution of the exchange effects, which depend on the energy, to the form factor of an inelastic transition with transfer to the target nucleus of angular momentum $\lambda = 4$ (this question is discussed in more detail below). Note that, in contrast to the pseudopotential approximation, the energy dependence of the optical potential and of the inelastic-transition form factors is obtained explicitly in the semimicroscopic approach without additional parametrization of the effective interaction. In addition, the enhancement of the energy dependence of β_4^v cannot be described in the pseudopotential approximation.

Effect of mixing of multipoles

Besides the energy dependence of the optical potential, another manifestation of the Pauli principle in the interaction of nucleons with deformed nuclei is the so-called multipole-mixing effect.[17] A description of this effect is contained in the general expression (9), and it takes the form that the L component of the potential is determined not only by the L component of the density but also by λ components different from L. The expressions (25) and (26) of the semimicroscopic approach are convenient for analyzing this effect. Since it is determined by terms quadratic in β_λ^ρ, its manifestation requires fulfillment of the relation

$$(\beta_\lambda^\rho)^2 \geqslant \beta_L^\rho.$$

As a rule, this condition is satisfied for $\lambda < L$.

We consider in more detail the fourth term in the expression (26):

$$U_{L0}(\beta_\lambda^\rho;\ r) = \sum_{\lambda\lambda'}{}' \beta_\lambda^\rho \beta_{\lambda'}^\rho S_{L\lambda\lambda'} \times I_{\lambda 1}(r)\, \varkappa(r)\, [\tilde{U}_{\lambda' 0}^D(r) + I_{\lambda' 0}(r)]. \quad (39)$$

Setting $L = 4$, $\lambda = \lambda' = 2$ in (39) and using for $S_{L\lambda\lambda'}$ Eq. (10), we obtain

$$U_{40}(\beta_2^\rho;\ r) = \frac{6}{7}(\beta_2^\rho)^2 \varkappa(r)\, I_{21}(r)\, [\tilde{U}_{20}^D(r) + I_{20}(r)].$$

The direct term $U_{40}^D(r)$ is proportional to β_4^ρ, so that under

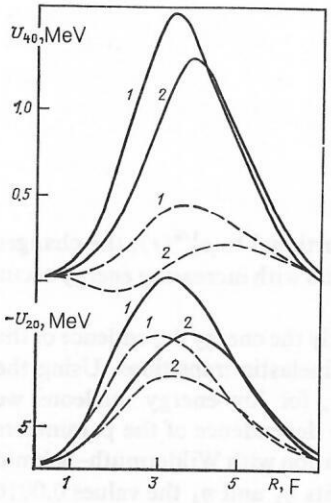

FIG. 1. Form factors of inelastic transitions with $L = 2$ and $L = 4$ (continuous curves) and contribution to their direct terms (broken curves): 1) calculation with Wildermuth-Schmid forces; 2) with Volkov V1 forces.

the condition $(\beta_2^p)^2 \gg \beta_4^p$ the exchange part of the inelastic-transition form factor can make an appreciable contribution to the total form factor and, thus, lead to an increase in the cross section of inelastic scattering with transfer of angular momentum $L = 4$ to the target nucleus. For example, for the ^{26}Mg nucleus the values of β_2^p and β_4^p obtained in Ref. 74 are, respectively, 0.44 and -0.024, and one can therefore expect an appreciable contribution of the exchange effects to the form factor of an inelastic transition with $L = 4$. The results of calculations made using the Wildermuth-Schmid forces, and also with the Volkov V1 forces[75] in Ref. 17, are given in Fig. 1. For the transition with $L = 2$, the direct term is more important than the exchange term for both Wildermuth-Schmid and Volkov forces, whereas for $L = 4$ the multipole-mixing effect is manifested in both cases, the exchange term dominating over the direct term on account of mixing of terms with $L = 2$ in the form factor.

Allowance for this effect can lead to an increase in the cross section of scattering with transfer $L = 4$ (the experimental data on the scattering of low-energy neutrons by heavy deformed nuclei contain indications of an enhancement of such transitions[76]). Note that even for $\beta_4^p \simeq 0$ the amplitude of the transition with transfer $L = 4$ calculated in the DWBA is nonzero. When inelastic scattering is described in the framework of the coupled-channel method, allowance for the multipole-mixing effect leads to an additional channel coupling, which, in contrast to the standard version of the coupled-channel method, is realized without transitions to continuum states. We note also that in the pseudopotential approximation there is no multipole-mixing effect, and the enhancement of the inelastic transitions can be described only by introducing an additional parametrization of the effective nucleon–nucleon interaction. The zero-range pseudopotential must now depend (through the constant d) on the angular-momentum transfer L. This dependence can be established not only empirically but also by calculations in the semimicroscopic approach. Equating the optical-potential moments q_λ^v calculated in the semimicroscopic approach to the moments calculated in the pseudopotential approximation, we can determine $d(\lambda)$.

As was noted in the previous section, the closed expressions for the inelastic-transition form factors constructed in the semimicroscopic approach make it possible to apply this approach to the description of the interaction of low-energy nucleons with vibrational excitations of nuclei. We shall now consider how allowance for the Pauli principle affects the properties of this interaction.

Nucleon–phonon interaction

To describe the properties of low-lying states of odd nuclei, wide use is made of the particle–vibrator model (see, for example, Ref. 77), in which the interaction of nucleons with core vibrations is considered. During the last three decades, this model has evolved from the very simplest forms of the model of an excited core to modern semimicroscopic models such as the quasiparticle–phonon model and the model of interacting bosons and fermions. A topical problem is that of taking into account the Pauli principle in the description of the interaction of the particles (or quasiparticles) with the phonons. A number of investigations have demonstrated the important part that the Pauli principle plays in forming the properties of low-lying nuclear states. In particular, in Ref. 78, which was based on a study of the part played by the Pauli principle in the structure of deformed nuclei, it was concluded that there are no collective two-phonon states in deformed nuclei. It was noted that the exchange term has an important influence on the order of the levels and on the splitting of multiplets in the model of interacting bosons and fermions.

In the macroscopic approach of Ref. 79 the static potential is augmented by interaction of the particle with the vibrational field of multipolarity λ by a correction having the form

$$\delta V = -\chi_\lambda(r) \sum_\mu Y^*_{\lambda\mu}(\theta, \varphi)\, \alpha_{\lambda\mu}, \tag{40}$$

where $\chi_\lambda(r)$ is determined by the derivative of the static potential:

$$\chi_\lambda(r) = R_0\, \partial V(r)/\partial r.$$

The expression (40) contains the dynamical variables $\alpha_{\lambda\mu}$ only to the first power, but the terms of second order in $\alpha_{\lambda\mu}$ can also be included in it. For the transition in the expression (40) to the phonon representation, one usually makes the substitution

$$\alpha_{\lambda\mu} \to \frac{\beta_\lambda}{2\lambda+1}\,[\hat{b}_{\lambda\mu} + (-)^\mu \hat{b}^+_{\lambda,-\mu}]. \tag{41}$$

Here, β_λ is a dynamical deformation parameter.

We list the difficulties associated with allowing for the Pauli principle in the interaction of incident particles with phonons. The use of (40) in conjunction with (41) does not make it possible to do this, since the expression (40) does not contain the coordinates of the core nucleons, and therefore the antisymmetrization procedure cannot be implemented. One of the ways of solving the problem is to go over from the

phonon operators to quasiparticle operators and then take into account the Pauli principle. Such a program can be realized in the quasiparticle–phonon model, though this method of solving the problem cannot be applied directly in problems with a continuum.

In the semimicroscopic approach, the Pauli principle is taken into account in the following manner in the particle–phonon interaction.[80] The matter distribution density in the nucleus can be represented in the form

$$\rho(\mathbf{r}) = \rho_0(\mathbf{r}) + \delta\rho(\mathbf{r}), \qquad (42)$$

$$\delta\rho(\mathbf{r}) = \sum_{\lambda}{}' \sum_{\mu} \rho_{\lambda\mu}(r)\,\alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi).$$

Here, the correction $\delta\rho(\mathbf{r})$ contains oscillations of the nuclear matter density about the equilibrium position. Using Eqs. (3) and (26), in which β_λ is replaced by the parameters $\alpha_{\lambda\mu}$, and carrying out the substitution (41), we obtain for the nucleon–phonon interaction the expression

$$\begin{aligned} &V(\mathbf{r};\ \hat{b}^{+}_{\lambda\mu},\ \hat{b}_{\lambda\mu}) \\ &= \sum_{\lambda}{}' \sum_{\mu} f_\lambda(r)\, Y_{\lambda\mu}(\theta, \varphi)\, \beta^{\rho}_{\lambda}\hat{\lambda}^{-1}[\hat{b}_{\lambda\mu} + (-)^{\mu}\hat{b}^{+}_{\lambda,-\mu}] \\ &+ \sum_{\lambda\mu}{}' \sum_{\lambda_1\lambda_2\mu_1\mu_2} F_{\lambda_1\lambda_2}(r)\, Y_{\lambda\mu}(\theta, \varphi) \\ &\times S(\lambda_1\lambda_2\lambda;\ \mu_1\mu_2)\, \beta^{\rho}_{\lambda_1}\hat{\lambda}_1^{-1}[\hat{b}_{\lambda_1\mu_1} \\ &+ (-)^{\mu_1}\hat{b}^{+}_{\lambda_1,-\mu_1}]\, \beta^{\rho}_{\lambda_2}\hat{\lambda}_2^{-1}[\hat{b}_{\lambda_2\mu_2} + (-)^{\mu_2}\hat{b}^{+}_{\lambda_2,-\mu_2}]; \end{aligned} \qquad (43)$$

$$S(\lambda_1\lambda_2\lambda;\ \mu_1\mu_2) = \frac{\hat{\lambda}_1\hat{\lambda}_2}{\hat{\lambda}\sqrt{4\pi}}(\lambda_1\lambda_2 00 \mid \lambda 0)(\lambda_1\lambda_2\mu_1\mu_2 \mid \lambda\mu); \qquad (44)$$

$$\hat{\lambda} \equiv (2\lambda+1)^{1/2}. \qquad (45)$$

The inelastic-transition form factors of first and second order, $f_\lambda(r)$ and $F_{\lambda_1\lambda_2}(r)$, are determined by

$$f_\lambda(r) = [\tilde{U}^{D}_{\lambda 0}(r) + I_{\lambda 0}(r)]\,[1 + I_{01}(r)\,\varkappa(r)] + I_{\lambda 1}(r)\,\varkappa(r)\,I_{00}(r); \qquad (46)$$

$$F_{\lambda_1\lambda_2}(r) = I_{\lambda_1 1}(r)\,\varkappa(r)\,[(1-\delta_{\lambda_2 0})\,\tilde{U}^{D}_{\lambda_2 0}(r) + I_{\lambda_2 0}(r)]. \qquad (47)$$

In contrast to the macroscopic approach, the terms of second and higher order in the phonon operators appear in the semimicroscopic approach only through the allowance made for the Pauli principle. For this reason, the particle–phonon interaction depends in all orders on the nucleon energy [through $k_0(r)$]. Finally, in contrast to the macroscopic approach, the inelastic-transition form factors depend explicitly on the angular-momentum transfer. Analysis of the radial dependence of the second-order form factors reveals that there are differences here too—in the macroscopic approach $F(r) \sim \delta^2\rho(r)/\partial r^2$, whereas in the semimicroscopic approach $F_{\lambda_1\lambda_2}(r) \sim (\partial\rho/\partial r)^2$. This circumstance can lead to differences in the magnitude and shape of the angular distributions of particles scattered inelastically by nuclei with excitation of vibrational states.

The expressions (43)–(47) solve in closed form the problem of taking into account the Pauli principle in the particle–vibrator model in the semimicroscopic approach. It follows from the foregoing exposition that allowance for the Pauli principle gives rise to a renormalization of the particle–phonon interaction, and this renormalized effective interaction can be used equally to solve discrete-spectrum problems (calculation of the energy spectra of odd nuclei) as well as continuum problems (description of inelastic scattering of nucleons by nuclei). The basic assumption for the derivation of the expressions (43)–(47) came from hydrodynamic representations of the density of the nucleon distribution in a nucleus [see Eq. (42)]. However, the sphere of application of Eqs. (43)–(47) can be extended by bearing in mind that the phonon states can be described in the language of different semimicroscopic nuclear models. In this case and under the assumption that the form of the interaction (43)–(47) is preserved, the transition from the hydrodynamic to the semimicroscopic description of the target nucleus consists of replacing the hydrodynamic transition densities $\rho_{\lambda 0}(r)$ in the form factors (46) and (47) by semimicroscopic densities calculated in the framework of nuclear models. Thus, unity is achieved in the method by which the Pauli principle is taken into account in the description of the properties of nuclear structure and nuclear reactions, and it becomes possible to extract information about the nuclear model from analysis of experimental data on the inelastic scattering of low-energy nucleons by nuclei.

Since allowance for the Pauli principle significantly determines the properties of the optical potential and the inelastic-transition form factors, one must also expect it to have a considerable influence on elastic and inelastic scattering cross sections. It is known that the elastic scattering of low-energy α particles by light nuclei is characterized by an anomalously large cross section at large angles (see, for example, Ref. 81). This effect has become known as anomalous backward scattering. Many hypotheses have been advanced for its causes; one is that it is due to manifestation of the Pauli principle in the interaction of the α particles with light cluster nuclei. In the case of nucleons, such anomalies are not observed, but here too allowance for the Pauli principle leads to a rise in the cross sections at large angles. This is illustrated by Figs. 2 and 3, which give the cross sections calculated in the semimicroscopic approach for elastic scattering of 17.5-MeV protons by the ^{26}Mg nucleus[18] and the

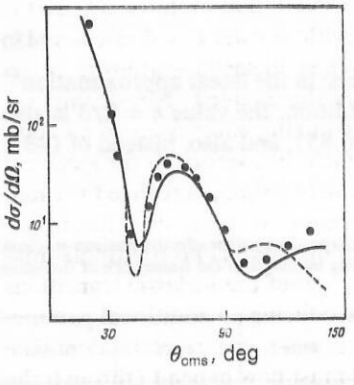

FIG. 2. Angular distributions of 17.5-MeV protons elastically scattered by ^{26}Mg. The continuous curve is the calculation with allowance for exchange effects, and the broken curve is the one without exchange effects; the experimental data (black circles) are taken from Ref. 82.

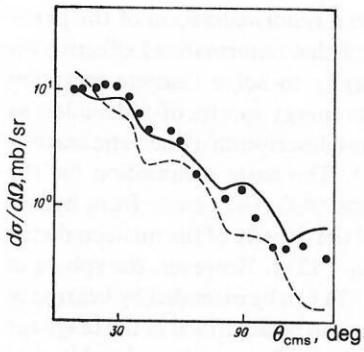

FIG. 3. Angular distributions of 27.1-MeV protons inelastically scattered by the ^{58}Ni nucleus with excitation of the state $|2_1^+\rangle$ ($E_x = 1.45$ MeV, $\beta_2 = 0.18$). The notation is the same as in Fig. 2.

cross sections of inelastic scattering of 27.1-MeV protons by the ^{58}Ni nucleus.[83]

Density dependence of the effective forces

Besides the exchange nucleon–nucleon correlations, an important part in forming the properties of nuclear structure and the nucleon–nucleus interaction is also played by many-particle nucleon–nucleon correlations, which are usually simulated by a density dependence of the effective nucleon–nucleon forces. The dependence of the effective interaction on the matter distribution density in the nucleus can be found in the approximation of a local density in the theory of nuclear matter.[38] Another approach to the solution of this problem is to parametrize the density dependence of the effective interaction and to find the corresponding parameters by calculating the nuclear-structure properties or the characteristics of nucleon–nucleus scattering. Usually, a factorized form of the effective nucleon–nucleon forces is assumed:

$$V_{\text{eff}}(|\mathbf{r}-\mathbf{r}'|;\rho) = V_{\text{eff}}(|\mathbf{r}-\mathbf{r}'|;0)\,F(\rho).$$

The factor $F(\rho)$ takes into account the saturation of the effective forces, i.e., the fact that with increasing matter distribution density in the nucleus the interaction force decreases and vice versa. The form for $F(\rho)$ most frequently employed is

$$F(\rho) = C(1-\alpha\rho^n). \tag{48}$$

Here, C and α are parameters; in the linear approximation[84] in the density, $n = 1$. In addition, the value $n = 2/3$ is also used (see, for example, Ref. 85), and also, instead of (48), an exponential form of the density dependence of the interaction.[86]

It was shown in Ref. 87 that the dependence of the effective forces on the matter distribution density in the nucleus is due mainly to the short-range part of the nucleon–nucleon interaction. In accordance with this, and also using the linear dependence in $F(\rho)$, the density term in the effective interaction can be expressed in the form

$$V_\rho(\mathbf{r},\mathbf{r}') = d\rho\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\delta(\mathbf{r}-\mathbf{r}'). \tag{49}$$

Substituting (49) in Eq. (2), we obtain for the contribution of the density term to the effective interaction in the optical potential

$$\delta U_{\rho 0}(r) = d\rho_0^2(r) + \frac{1}{4\pi}\sum_\lambda{}' (\beta_\lambda^\rho)^2\,\tilde{\rho}_{\lambda 0}^2(r). \tag{50}$$

In Refs. 18 and 88–90, the influence of the density dependence of the effective interaction on the angular distributions of elastically scattered protons was studied. It was established that this influence is more important for medium and heavy nuclei than for light nuclei. With increasing value of d, the cross sections for elastic scattering through large angles decrease, as a rule. When channel coupling is included, the angular distributions become less sensitive to the density-dependence parameter d.

With the aim of establishing the dependence of d on the mass number, an analysis was made in Ref. 88 of the angular distributions measured in Ref. 91 of 30.3-MeV protons elastically scattered by the nuclei ^{20}Ne, ^{40}Ca, ^{58}Ni, ^{120}Sn, and ^{208}Pb. The analysis was made in the framework of the optical model with potentials calculated in the semimicroscopic approach with allowance for the Pauli principle and the density dependence of the effective interaction. For the spin–orbit term and the absorption potential, the corresponding expressions from the optical model with parameters (fixed in the process of the calculations) taken from Ref. 92 were used. Analysis of elastic scattering of electrons by these nuclei yielded the parameters of the charge distribution in the nuclei. Further, it was assumed that the neutron distribution in the nucleus is identical to the proton distribution. For the parameters V_D and V_E values the same for all nuclei and corresponding to the Wildermuth–Schmid parameters were taken (see Table I). Thus, the only free parameter in the calculations was d. It was determined for each nucleus by the best description (minimal χ^2 was used as the criterion) of the experimental angular distribution. The results are given in Table II and in Fig. 4. It follows from the data of Table II

TABLE II. Parameters of the density dependence of the effective nucleon–nucleon forces determined by analysis of proton elastic scattering by nuclei in the framework of the semimicroscopic (s.m) approach.

Nucleus	^{20}Ne	^{40}Ca	^{58}Ni	^{120}Sn	^{208}Pb
d, MeV · F^6	860	600	650	740	650
$\chi^2_{\text{s.m}}/\chi^2_{\text{opt.mod}}$	2,0	0,67	0,80	1,2	1,8

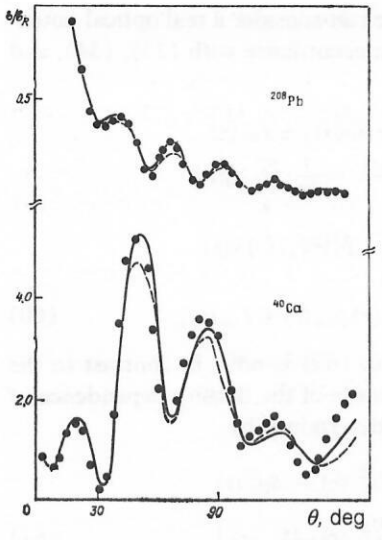

FIG. 4. Angular distributions of 30.3-MeV protons elastically scattered by the ^{40}Ca and ^{208}Pb nuclei. The continuous curves are calculated in the semimicroscopic approach, and the broken curves in the optical model; the points are the experimental data.

(second row) that for all nuclei the description obtained for the experimental data is overall not worse than in the macroscopic optical model. In Fig. 4, the results are presented for two nuclei, though analogous results were obtained for other nuclei as well.

As follows from Table II (first row), the density-dependence parameter of the effective interaction changes by only 20% compared with the mean value over all nuclei. It must here be borne in mind that, for example, for ^{20}Ne the angular distributions are not very sensitive to the values of d. As the calculations showed, for $d = 700$ MeV · F^6 the description of the angular distribution of protons scattered by ^{20}Ne is also fairly good. Analysis of the angular distributions of 25.7-MeV protons elastically scattered by Mo and Pd protons made in Refs. 89 and 90 led to the same values of the parameter d. Thus, it may be concluded that for a single value of the density-dependence parameter of the effective interaction (with a small spread) a good description has been obtained for the elastic scattering of protons by nuclei in a wide range of mass numbers. It should be noted that allowance for a number of factors, such as channel coupling, a difference between the proton and neutron distributions, and more exact allowance for exchange effects, may lead to a certain renormalization of the optimal value of d.

It is of interest to compare the semimicroscopic optical potentials with the optical-model potentials. Figure 5 shows potentials calculated in the semimicroscopic approach at proton energy $E_p = 30.0$ MeV for the ^{58}Ni and ^{120}Sn nuclei together with the optical-model potentials. It can be seen that the potentials agree from a certain value of r less than the half-decrease radius (indicated by the arrow in Fig. 5). The exchange effects make a significant contribution to the values of the potentials, particularly in the surface region of the nucleus. Allowance for the Pauli principle leads, in particular, to a radial dependence of the potential that differs from the radial dependence of the Woods–Saxon potential.

3. ALLOWANCE FOR CHANNEL COUPLING

When low-energy nucleons interact with nuclei, an important contribution to the cross section of the processes is made by multistep transitions due to the strong coupling of the open inelastic channels. There are indications that at intermediate proton energies ($E_p \simeq 800$ MeV) channel coupling is important for the description of inelastic proton scattering by nuclei.[93] Among the inelastic channels, the most important contribution to the multistep transitions is made by processes with excitation of collective states of the target nucleus. In this case, the channel coupling can be taken into account in the framework of the well-known formalism of the coupled-channel method[94] (coupling of collective states of a deformed target nucleus was taken into account for the first time in the adiabatic approximation in Ref. 95).

In the semimicroscopic description, the difficulty of taking into account the channel coupling arises because of the Pauli principle, which causes the system of differential equations to become a system of integro-differential equations. However, in the local approximation of the density-matrix formalism, at least for the collective model, this system can be reduced to a system of differential equations in which the optical potential and the inelastic-transition form factors are renormalized. The formalism of the semimicroscopic approach presented in the previous section makes it possible to write the system of the coupled-channel method in a closed form and apply it to the description of elastic and inelastic nucleon scattering using both rotational and vibrational models of the nucleus.

Following the standard version of the coupled-channel method,[94] we write down the Hamiltonian of the system, which consists of the incident nucleon and the target nucleus, in the form

$$H = T + H_t + V_{\text{diag}} + V_{\text{coupl}}. \qquad (51)$$

Here, T is the kinetic-energy operator of the nucleon, H_t is the Hamiltonian of the target nucleus, V_{diag} is the diagonal part of the operator of the interaction of the nucleon with the target nucleus, which does not give rise to inelastic transitions, and V_{coupl} is the part of the interaction operator responsible for the channel coupling.

We represent the total wave function of the complete system in the form

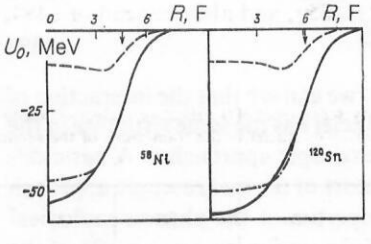

FIG. 5. Proton optical potentials for the ^{58}Ni and ^{120}Sn nuclei. The continuous curves are calculated in the semimicroscopic approach, the chain curves correspond to the potentials of the optical model, and the broken curves to the contribution of the exchange terms.

$$\Psi_{JM}=\frac{1}{r}\sum_{I_n l_n j_n} R_{JI_n l_n j_n}(r)$$

$$\times\sum_{m_j M_n}(j_n m_j I_n M_n \mid JM)\,\Omega_{l_n j_n m_j}\Phi_{I_n M_n}, \qquad (52)$$

where $\Phi_{I_nM_n}$ are the functions of the collective states of the target nucleus, and $\Omega_{l_nj_nm_j}$ is a spherical spinor:

$$\Omega_{l_n j_n m_j}=\sum_{m_l m_s}(l_n m_l s m_s \mid j_n m_j)\,\mathrm{i}^l Y_{l_n m_l}\chi_{sm_s}.$$

Substituting the expansion (52) in the Schrödinger equation with the Hamiltonian (51) and eliminating the functions $\Phi_{I_nM_n}$, we obtain the system of equations[94]

$$\left\{\frac{d^2}{dr^2}-\frac{l_n(l_n+1)}{r^2}-\frac{2m}{\hbar^2}V_{\mathrm{diag}}+k_n^2\right\}R_{JI_nl_nj_n}(r)$$

$$=\frac{2m}{\hbar^2}\sum_{I_n'l_n'j_n'}\langle(\Omega_{l_nj_n}\otimes\Phi_{I_nM_n})_{JM}\mid V_{\mathrm{coupl}}\mid$$

$$\times(\Omega_{l_n'j_n'}\otimes\Phi_{I_n'M_n'})_{JM}\rangle$$

$$\times R_{JI_n'l_n'j_n'}(r); \qquad (53)$$

$$k_n=\left[\frac{2m}{\hbar^2}(E-E_n)\right]^{1/2}.$$

In the standard version of the coupled-channel method, two forms of collective model are usually considered: rotational and vibrational. For the rotational variant,

$$V_{\mathrm{coupl}}=\sum_{\lambda}v_\lambda(r)\,Y_{\lambda0}(\theta,\varphi); \qquad (54)$$

$$v_\lambda(r)=\int\frac{V_0Y_{\lambda0}(\omega)\,d\omega}{1+\exp\{(r-R_0[1+\sum_{\lambda'}\beta_{\lambda'}^v Y_{\lambda'0}(\omega)])/a\}}. \qquad (55)$$

In the case of the vibrational variant, the integral formula (55) does not apply, and after expansion of (54) in a Taylor series, retaining the first two expansion terms, we obtain

$$V_{\mathrm{coupl}}=\sum_{\lambda}'\sum_{\mu}f_\lambda(r)\,\alpha_{\lambda\mu}Y_{\lambda\mu}(\theta,\varphi)$$

$$+\sum_{\lambda}'\sum_{\mu}\sum_{\lambda_1\lambda_2\mu_1\mu_2}F_{\lambda_1\lambda_2}(r)\,S(\lambda_1\lambda_2\lambda;\mu_1\mu_2)\,\alpha_{\lambda_1\mu_1}\alpha_{\lambda_2\mu_2}Y_{\lambda\mu}(\theta,\varphi), \qquad (56)$$

where

$$f_\lambda(r)=\frac{V_0R_0}{a}\frac{\mathrm{e}}{(1+\mathrm{e})^2}; \qquad (57)$$

$$F_{\lambda_1\lambda_2}(r)=-\frac{V_0R_0^2}{2a^2}\frac{\mathrm{e}(1-\mathrm{e})}{(1+\mathrm{e})^3}; \qquad (58)$$

$$\mathrm{e}\equiv\exp\frac{r-R}{a}. \qquad (59)$$

Comparing (54) with (3), we can see that the interaction of a nucleon with deformed nuclei has the same structure in the macroscopic and semimicroscopic approaches. A basic difference is that the form factors of the macroscopic approach are determined by the properties of the phenomenological optical potential and the deformation parameters β_λ^v of the potential, whereas the form factors of the semimicroscopic approach are determined by the parameters of the effective interaction and the transition densities. As V_{diag} in the semimicroscopic approach we can consider a real optical potential, for which we shall in accordance with (25), (34), and (50) have

$$U_{\rho0}(r)=U_0^D(r)+d\rho_0^2(r)+I_{00}(r)$$

$$\times\frac{1+\varkappa^2(r)\,I_{02}(r)\,I_{00}(r)}{1-\varkappa(r)\,I_{01}(r)}+\frac{1}{4\pi}\sum_{\lambda}'(\beta_\lambda^\rho)^2$$

$$\times\tilde{\rho}_{\lambda0}^2(r)+\frac{1}{4\pi}\sum_{\lambda}'(\beta_\lambda^\rho)^2 I_{\lambda1}(r)\,\varkappa(r)$$

$$\times[\tilde{U}_{\lambda0}^D(r)+2d\rho_0(r)\,\tilde{\rho}_{\lambda0}(r)+I_{\lambda0}(r)]. \qquad (60)$$

The momentum $k_{\rho0}(r)$ in (60) is now, in contrast to the expression (20), and because of the density dependence of the effective interaction, determined by

$$k_{\rho0}^2(r)=\frac{2m}{\hbar^2}\Big[E-U_0^D(r)-d\rho_0^2(r)$$

$$-\frac{1}{4\pi}\sum_{\lambda}'(\beta_\lambda^\rho)^2\tilde{\rho}_{\lambda0}^2(r)-V_C(r)\Big]. \qquad (61)$$

For $v_\lambda(r)$ we obtain in the semimicroscopic approach in accordance with Eqs. (26) and (49)

$$U_{\rho L0}(r)=\beta_L^\rho[1+I_{01}(r)\,\varkappa(r)]\,[\tilde{U}_{L0}^D(r)$$

$$+2d\rho_0(r)\,\tilde{\rho}_{L0}(r)+I_{L0}(r)]$$

$$+\beta_L^\rho I_{L1}(r)\,\varkappa(r)\,I_{00}(r)$$

$$+\frac{1}{\sqrt{4\pi}}d\sum_{\lambda}'\sum_{\lambda'}'\beta_\lambda^\rho\beta_{\lambda'}^\rho S_{L\lambda\lambda'}\tilde{\rho}_{\lambda0}(r)\,\tilde{\rho}_{\lambda'0}(r)$$

$$+\frac{1}{\sqrt{4\pi}}\sum_{\lambda}'\sum_{\lambda'}'S_{L\lambda\lambda'}I_{\lambda1}(r)\,\varkappa(r)$$

$$\times[\tilde{U}_{\lambda'0}^D(r)+2d\rho_0(r)\,\tilde{\rho}_{\lambda'0}(r)+I_{\lambda'0}(r)]. \qquad (62)$$

An analogous situation obtains for the vibrational variant. In accordance with Eqs. (43) and (56), the structure of the interaction is the same in the two approaches, but the form factors in the semimicroscopic approach, in contrast to (58), are determined, as in the rotational variant, by the parameters of the effective interaction and the transition densities. In accordance with (46), (47), and (49), we obtain for $f_\lambda(r)$ and $F_{\lambda_1\lambda_2}(r)$

$$f_{\rho\lambda}(r)=[1+I_{01}(r)\,\varkappa(r)]\,[\tilde{U}_{L0}^D(r)+2d\rho_0(r)\,\tilde{\rho}_{L0}(r)+I_{L0}(r)]$$

$$+I_{L1}(r)\,\varkappa(r)\,I_{00}(r); \qquad (63)$$

$$F_{\rho\lambda_1\lambda_2}(r)=d\tilde{\rho}_{\lambda_10}(r)\,\tilde{\rho}_{\lambda_20}(r)+I_{\lambda_11}(r)\,\varkappa(r)\,[\tilde{U}_{\lambda_20}^D(r)$$

$$+2d\rho_0(r)\,\tilde{\rho}_{\lambda_20}(r)$$

$$+I_{\lambda_20}(r)]. \qquad (64)$$

The relations (54), (56), (60)–(64) in conjunction with the system (53) constitute the formal basis of the semimicroscopic coupled-channel method (SCCM).

We now consider the advantages that the SCCM possesses over the microscopic coupled-channel method for the analysis of experimental data on nucleon–nucleus scattering. The real part of the optical potential and the form factors of inelastic collective transitions in the SCCM are determined by the parameters of the effective nucleon–nucleon

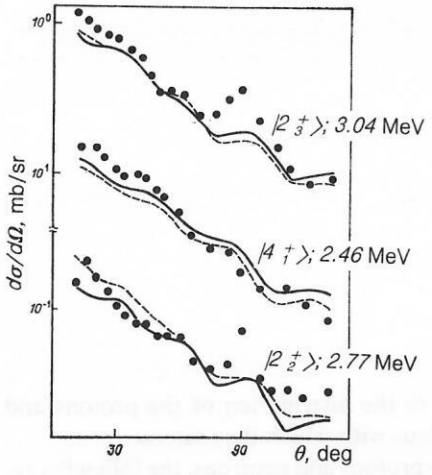

FIG. 6. Angular distributions of 27.1-MeV protons inelastically scattered by the ^{58}Ni nucleus with excitation of the states $|2_2^+\rangle$ ($E_x = 2.77$ MeV, $\beta_2 = 0.022$), $|2_3^+\rangle$ ($E_x = 3.04$ MeV, $\beta_2 = 0.051$), and $|4_1^+\rangle$ ($E_x = 2.46$ MeV, $\beta_4 = 0.096$). The continuous curves are calculated in the harmonic approximation of the semimicroscopic coupled-channel method, and the broken curves in the anharmonic approximation.

interaction, which depends on the matter distribution density in the nucleus, the central density, and the transition densities, and, thus, does not contain free parameters. In the presence of strong channel coupling, allowance is made in the local approximation of the density-matrix formalism for the Pauli principle simultaneously for the elastic channel and the inelastic channels. Finally, information about the nuclear-structure properties enters the SCCM formalism through the transition densities. This information can be extracted from experiments that are independent of the scattering of low-energy nucleons by nuclei (for example, from scattering of electrons or high-energy protons). On the other hand, the transition densities can be calculated in the framework of the semimicroscopic nuclear model. In this case, using the SCCM to analyze the experimental cross sections, it is possible to establish how well the nuclear model describes the transition densities.

With the aim of realizing numerically the SCCM, the ECIS program was modified in Ref. 83. This was achieved by replacing the phenomenological form factors and the real part of the optical potential, (57)–(59), by semimacroscopic quantities in accordance with Eqs. (60), (61), (63), and (64). In addition, the form factors $F_{\lambda_1\lambda_2}(r)$ were introduced under the summation sign. It should be noted that in the standard ECIS version there is the so-called folding procedure. However, its implementation makes it possible to take into account only terms of the type $U_0^D(r)$ and $U_{L0}^D(r)$. All the effects associated with exchange and many-particle nucleon–nucleon correlations are ignored, but they strongly influence the cross sections of both elastic and inelastic processes.

We note that the system of equations (53) contains only coupling of open channels, and therefore its solution makes possible description of the contribution of direct mechanisms to the scattering cross section. The coupling of the open channels with the closed channels, which is ignored in (53), leads to the formation of bound states of the compound system. At low energies ($E_N < 10$ MeV) of the nucleons, the contribution of such processes to the scattering cross section is appreciable, but with increasing energy it falls. Therefore, the use of the SCCM to analyze experimental cross sections is preferable at nucleon energies exceeding 15–20 MeV.

In Ref. 83, the modified version of the ECIS program was used to describe the inelastic scattering of 27.1-MeV protons by the ^{58}Ni target nucleus. The calculated cross sections together with the experimental angular distributions obtained in Ref. 1 are given in Figs. 3 and 6. It follows from these results that in the semimicroscopic approach it is possible to give an overall successful description of the cross section of inelastic proton scattering with excitation of the lowest levels of the ^{58}Ni nucleus. As regards the nature of the target-nucleus states, analysis made in the framework of the harmonic and anharmonic approximations of the vibrational model showed that in the states $|2_1^+\rangle$ ($E_x = 1.45$ MeV), $|2_3^+\rangle$ ($E_x = 3.04$ MeV), and $|4_1^+\rangle$ ($E_x = 2.46$ MeV) the single-phonon component is dominant. This conclusion is also confirmed by investigations made earlier.[1,20] The state $|2_2^+\rangle$ ($E_x = 2.77$ MeV) cannot be described in the framework of the vibrational model in either the harmonic or the anharmonic approximation.

The interaction of 25.6-MeV protons with the isotopes ^{94,96,100}Mo was analyzed in the SCCM in Refs. 96 and 97. The experimental angular distributions of elastically and inelastically scattered protons were obtained using the isochronous cyclotron at the University of Hamburg.[2] Figure 7 shows the excitation cross sections of the states $|2_1^+\rangle$ ($E_x = 778$ keV), $|4_1^+\rangle$ ($E_x = 1628$ keV), and $|3_1^-\rangle$ ($E_x = 2235$ keV) of ^{96}Mo calculated in the semimicroscopic

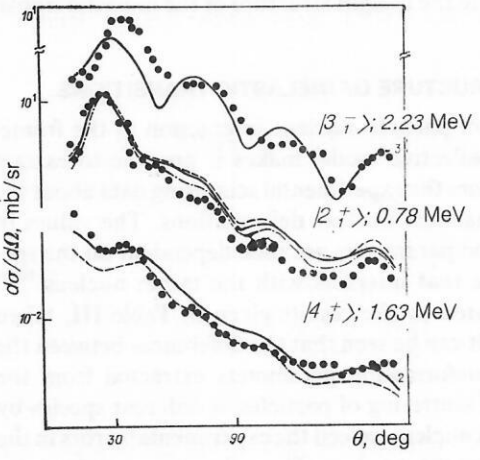

FIG. 7. Angular distributions of 25.6-MeV protons inelastically scattered by the ^{96}Mo nucleus: 1) $|2_1^+\rangle$ ($E_x = 778$ keV, $\beta_2 = 0.16$); the continuous curve is the calculation in the semimicroscopic coupled-channel method with allowance for the channel coupling $|0_1^+\rangle \rightarrow |2_1^+\rangle$; the broken curve is the same for $|0_1^+\rangle \rightarrow |2_1^+\rangle \rightarrow |3_1^-\rangle$; the chain curve the same for $|0_1^+\rangle \rightarrow |2_1^+\rangle \rightarrow |4_1^+\rangle$; 2) $|4_1^+\rangle$ ($E_x = 1628$ keV, $\beta_4 = 0.058$), the continuous curve for $|0_1^+\rangle \rightarrow |4_1^+\rangle$, the broken curve for $|0_1^+\rangle \rightarrow |2_1^+\rangle \rightarrow |4_1^+\rangle$; 3) $|3_1^-\rangle$ ($E_x = 2235$ keV, $\beta_3 = 0.167$), the continuous curve for $|0_1^+\rangle \rightarrow |2_1^+\rangle \rightarrow |3_1^-\rangle$.

TABLE III. Values of deformation parameters determined from the analysis of different processes for the ^{232}Th nucleus.

Process	E, MeV	β_2	β_4
Coulomb excitation	16—17	0.238±0.006	0.130±0.02
(e, e')	45—100	0.238±0.002	0.101±0.003
(n, n')	0.7—3.4	0.190±0.010	0.071±0.012
(p, p')	20.26	0.215±0.005	0.060±0.005
(p, p')	23	0.23±0.01	0.050±0.015
(p, p')	35	0.210±0.003	0.069±0.003
(α, α')	50	0.152±0.009	0.041±0.008

approach together with the experimental angular distributions. For all three states, it was assumed that the single-phonon components make the main contribution to them. It can be seen that in this case too a good overall description of the experimental results is obtained. Allowance for the channel coupling somewhat improves the description of the cross sections for the excitation of the states $|2_1^+\rangle$ and $|4_1^+\rangle$. In the case of the state $|3_1^-\rangle$ it is also necessary to consider the coupling with high-lying states. Analogous results were obtained in Ref. 97 for the description of proton interactions with the isotopes ^{94,100}Mo.

The numerical calculations presented in this and the previous section show that in the semimicroscopic approach it is possible, using the same effective interaction, which depends on the matter distribution density in the nucleus, to describe elastic and inelastic scattering of protons by nuclei in the presence of channel coupling. We note that the calculations made in Refs. 83, 96, and 97 used the transition densities of the Tassie model. It is of interest to go beyond the framework of this model and use transition densities constructed in semimicroscopic nuclear models. It will then be important to take into account the differences between the proton and neutron transition densities, since these differences determine the isospin structure of the inelastic transitions.

4. ISOSPIN STRUCTURE OF INELASTIC TRANSITIONS

Analysis of particle–nucleus interaction in the framework of the collective model makes it possible to extract information from the experimental scattering data about the static and dynamical nuclear deformations. The values of the deformation parameters are then dependent on the species of particle that interacts with the target nucleus.[19,98] This is illustrated by the results given in Table III, taken from Ref. 99. It can be seen that the differences between the values of the deformation parameters extracted from the analysis of the scattering of particles of different species by the same target nucleus exceed the experimental errors in the determination of these values. Thus, there are certain systematic variations in the relationships between the various deformation parameters. They are due to the difference between the proton and neutron distributions in the nuclei, and also in the corresponding transition densities. In addition, the projectile particles interact with different strengths with the protons and neutrons of the target nucleus, and this leads to different sensitivities of the interaction of particles of different species to the distribution of the protons and neutrons in the nucleus with which they interact.

For low-energy protons and neutrons, the following relation holds by virtue of charge invariance and the Pauli principle: $V_{nn} = V_{pp} = (1/3)V_{np} = (1/3)V_{pn}$. Therefore, low-energy protons are more sensitive to the neutron distribution in the nucleus and vice versa. As was noted in Ref. 21, this circumstance makes it preferable to use low-energy nucleons rather than high-energy nucleons to investigate the differences between the proton and neutron transition densities. However, the nonuniqueness of the optical potential and the existence of channel coupling at low energies make the implementation of such an investigation much more difficult.

In Ref. 98, the effective-charge model was used to study the connection between the deformation parameters $\beta_{pp'}$ and $\beta_{nn'}$ of the potential and the target-nucleus structure. Nuclei with vibrations of n type (with a closed proton shell) and p type (with a closed neutron shell) were considered. The following relations were obtained:

$$\frac{\beta_{pp'}}{\beta_p} = \frac{1 \pm 0.106 - 0.306\xi}{1 \mp 0.212 + 0.612\xi}; \quad (65)$$

$$\frac{\beta_{nn'}}{\beta_p} = \frac{1 \mp 0.106 + 0.306\xi}{1 \mp 0.212 + 0.612\xi}; \quad (66)$$

$$\xi = (N - Z)/A.$$

In Eqs. (65) and (66), the + sign corresponds to nuclei with n-type vibrations, and the − sign to nuclei with p-type vibrations. In accordance with (65) and (66), $\beta_{pp'}/\beta_{nn'} > 1$ for nuclei with a closed proton shell but $\beta_{pp'}/\beta_{nn'} < 1$ for nuclei with a closed neutron shell. This conclusion agrees well with the data given in Table IV (taken from Ref. 100). Comparing the third and fourth rows of this table, we can see that there is not only qualitative but also quantitative agreement between the predictions of the effective-charge model and the data extracted from analysis of the cross sections of inelastic nucleon–nucleus scattering. In Ref. 101, the effective-charge model was generalized to nuclei with open shells. In a number of cases, the model correctly reproduces the relationships between the deformation parameters obtained from analysis of the scattering data. However, to unify the description of the connections between the deformation parameters of the potential and the matter, on the one hand, and the scattering mechanisms on the other, it is necessary to have analogs of the relations (65) and (66) containing the parameters of the effective interaction used to describe the

TABLE IV. Values of deformation parameters determined from the analysis of inelastic nucleon scattering by nuclei and as calculated in the effective-charge model, $\lambda = 2$.

Nucleus	^{90}Zr	^{92}Mo	^{118}Sn	^{120}Sn	^{122}Sn	^{124}Sn
$\beta_{pp'}$	0.070(5)	0.080(6)	0.134(10)	0.119(10)	0.112(7)	0.108(7)
$\beta_{nn'}$	0.085(8)	0.099(5)	0.109(7)	0.106(5)	0.100(6)	0.092(6)
$\beta_{nn'}/\beta_{pp'}$ (experiment)	1.2	1.3	0.81	0.90	0.90	0.85
$\beta_{nn'}/\beta_{pp'}$ (model)	1.33	1.31	0.888	0.896	0.903	0.911

scattering mechanisms. Such relations can be readily obtained in the formalism of the folding model by using Satchler's theorem. We obtain[102]

$$\beta_{aa',\lambda} = \frac{V_{ap}Z\beta_{p\lambda} + V_{an}N\beta_{n\lambda}}{V_{ap}Z + V_{an}N}. \quad (67)$$

Here, $\beta_{aa'}$ is the deformation parameter of the potential of the interaction of particle a with the nucleus, V_{ap} and V_{an} are the strengths of the effective interaction of a with the proton and neutron, respectively, and λ is the angular-momentum transfer. In (67), it is assumed that $\langle r^\lambda \rangle_n = \langle r^\lambda \rangle_p = \langle r^\lambda \rangle_{aa'}$, i.e., the geometrical parameters of the corresponding distributions are the same. To go beyond the framework of this restriction, we determine the quantities $\delta_{f\lambda}$:

$$\delta_{f\lambda} \equiv \beta_{f\lambda} \langle r^\lambda \rangle_f. \quad (68)$$

It was shown earlier (in Sec. 2) that on account of Satchler's theorem the quantities $\delta_{f\lambda}$ are invariants, i.e., do not depend on the shape of the radial dependence of the effective interaction or the radial dependence of the matter distribution density in the nucleus. Thus, this relation, in contrast to the usually adopted relation $(\beta_\lambda R)_v = (\beta_\lambda R)_p$,[19] is model-independent. Using (68) and Satchler's theorem, we obtain instead of (67)

$$\delta_{aa',\lambda} = \frac{V_{ap}Z\delta_{p\lambda} + V_{an}N\delta_{n\lambda}}{V_{ap}Z + V_{an}N}. \quad (69)$$

The allowance for the difference between the geometrical parameters has been reduced to the substitution

$$\beta_{f\lambda} \to \delta_{f\lambda}. \quad (70)$$

This rule is universal. In what follows, we shall use the relationships between the parameters β, bearing in mind that it is always possible to go over to relations of the type (69) with allowance for the substitution (70).

It is readily seen that for $\beta_{p\lambda} = \beta_{n\lambda} = \beta_{0\lambda}$ the quantity $\beta_{aa',\lambda}$ in (67) does not depend on the particle species and is determined by the matter deformation parameter $\beta_{0\lambda}$. Thus, all the differences in $\beta_{aa',\lambda}$ are due to the differences in $\beta_{p\lambda}$ and $\beta_{n\lambda}$. Making a combined analysis at a given energy (as was shown above, the deformation parameters of the potential depend on the energy) for the data on the scattering of particles of different species (for example, protons and neutrons, or protons and electrons) by nuclei, it is possible to determine the parameters $\beta_{n\lambda}$ and $\beta_{p\lambda}$ from the two relations (67). There is a much larger amount of data on the electromagnetic interaction (inelastic scattering of electrons, probabilities of electromagnetic transitions, etc.) than on the interaction of neutrons with nuclei, and therefore, having in mind a combined analysis of the data on proton scattering by nuclei and experiments with electromagnetic interaction, it is helpful to obtain the relationship between $\beta_{n\lambda}/\beta_{p\lambda}$ and $\beta_{pp',\lambda}/\beta_{\mathrm{em},\lambda}$. Inverting the relation (67) in the case $a \equiv p$, we obtain[103]

$$\frac{\beta_{n\lambda}}{\beta_{p\lambda}} = 1 + \frac{4}{3}\left(1 - \frac{1}{2}\xi\right)\left(\frac{\beta_{pp',\lambda}}{\beta_{\mathrm{em},\lambda}} - 1\right). \quad (71)$$

The expression (71) makes it possible, using experimental data on the scattering of low-energy protons by nuclei and the probabilities of electromagnetic transitions, to determine the differences between the proton and neutron deformations. Thus, scattering of low-energy protons by nuclei complements the scattering of high-energy protons and pions as a source of information about the isospin structure of the transition densities. An appreciable amount of data has already been accumulated on the differences between the proton and neutron deformations, which have been investigated theoretically and experimentally. They are reflected in the recently published review of Ref. 21, and therefore we shall not dwell here in detail on this question.

Note that, as follows from (71), when $\beta_{pp',\lambda}/\beta_{\mathrm{em},\lambda} > 1$ we have $\beta_{n\lambda}/\beta_{p\lambda} > 1$, i.e., the target nucleus is a nucleus with n-type vibrations, and, conversely, when $\beta_{pp',\lambda}/\beta_{\mathrm{em},\lambda} < 1$ we have $\beta_{n\lambda}/\beta_{p\lambda} < 1$, i.e., the target nucleus is a nucleus with p-type vibrations. These conclusions agree with the predictions of the effective-charge model.[98]

Isoscalar and isovector deformations

The differences between the proton and neutron transition densities can be conveniently described in the isospin formalism. The distinctive feature of this formalism is the unified treatment of both inelastic particle–nucleus scattering and charge-exchange reactions. We define the isoscalar, $\rho_{0\lambda}(r)$, and isovector, $\rho_{1\lambda}(r)$, components of the transition densities:

$$\rho_{0\lambda}(r) = \rho_{p\lambda}(r) + \rho_{n\lambda}(r); \quad \rho_{1\lambda}(r) = \rho_{p\lambda}(r) - \rho_{n\lambda}(r).$$

In accordance with (29), (30), and (32), we obtain for the parameters of the isoscalar, $\beta^{\rho}_{0\lambda}$, and isovector, $\beta^{\rho}_{1\lambda}$, matter deformations the expressions

$$\beta^{\rho}_{0\lambda}=\frac{4\pi}{2\lambda+1}\frac{q_{0\lambda}}{\langle r^{\lambda}\rangle_0 A};\quad \beta^{\rho}_{1\lambda}=\frac{4\pi}{2\lambda+1}\frac{q_{1\lambda}}{\langle r^{\lambda}\rangle_1 (Z-N)}.$$

The parameters of the isoscalar and isovector deformations of the potential are related to $\beta^{\rho}_{0\lambda}$ and $\beta^{\rho}_{1\lambda}$ by (31):

$$\beta^{v}_{i\lambda}\langle r^{\lambda}\rangle_{iv}=\beta^{\rho}_{i\lambda}\langle r^{\lambda}\rangle_{i\rho}\quad (i=0,\ 1).$$

In the folding model, it is easy to establish the connection between the parameters $\beta^{\rho}_{0\lambda}$, $\beta^{\rho}_{1\lambda}$ and $\beta_{n\lambda}$, $\beta_{p\lambda}$ (Ref. 104):

$$\frac{\beta_{n\lambda}}{\beta_{p\lambda}}=\frac{Z}{N}\frac{1+(\beta^{\rho}_{1\lambda}/\beta^{\rho}_{0\lambda})\,\xi}{1-(\beta^{\rho}_{1\lambda}/\beta^{\rho}_{0\lambda})\,\xi}. \tag{72}$$

If it is assumed that the matter isovector deformation parameter vanishes, $\beta^{\rho}_{1\lambda}=0$, then it follows from (72) that $q_{p\lambda}=q_{n\lambda}$. This result corresponds to the standard collective model, which does not take into account the differences between the proton and neutron transition densities.

It is of interest to find the connection between the parameters of the isoscalar and isovector deformations of the potential and the potential deformation parameters extracted from analysis of data on the scattering of low-energy nucleons by nuclei. Using (67) and (72), we obtain[103]

$$\frac{\beta^{v}_{1\lambda}}{\beta^{v}_{0\lambda}}=\frac{1-\left(1-\frac{3}{2}\xi\right)\beta_{nn',\lambda}/\beta_{pp',\lambda}}{1+\frac{1}{2}\left(1-\frac{3}{2}\xi\right)\beta_{nn',\lambda}/\beta_{pp',\lambda}}\frac{1}{\xi}. \tag{73}$$

The ratios $\beta_{nn',\lambda}/\beta_{pp',\lambda}$ are determined by analyzing the experimental data on scattering with certain errors, but these errors, as follows from the data in Table III, are less than the actual differences between $\beta_{nn',\lambda}$ and $\beta_{pp',\lambda}$ due to the differences between the proton and neutron transition densities. Thus, (73) makes it possible to study the isospin structure of the interaction potential by analyzing the experimental data on inelastic scattering.

Quasielastic scattering cross sections provide other experimental data from which the potential isovector deformation parameter $\beta^{v}_{1\lambda}$ can be determined directly. Lane[105] proposed a charge-invariant form of the nucleon–nucleus interaction potential:

$$U(\mathbf{r})=U_0(\mathbf{r})+U_1(\mathbf{r})\frac{\mathbf{tT}}{A}. \tag{74}$$

Here, $U_0(\mathbf{r})$ is the isoscalar part of the potential, and the second term is the isovector part. In the isospin representation, the diagonal part of the isovector potential makes a contribution to the nucleon optical potential responsible for elastic scattering, while the nondiagonal part corresponds to an interaction leading to a "rotation" of the isotopic spin $\mathbf{t}$ of the incident nucleon, i.e., to a quasielastic (p, n) scattering reaction. Thus, Lane's potential (74) makes it possible to describe on a unified basis, without the introduction of additional parameters, the elastic scattering of protons and neutrons by nuclei and also the quasielastic (p, n) scattering reaction.

In Ref. 106, Lane's model was generalized to the case of the interaction of nucleons with deformed nuclei. For the interaction giving rise to the quasi-inelastic scattering reaction, we now have in accordance with Ref. 106

$$U_{pn,\lambda}(r,\ \theta,\ \varphi)=\beta^{v}_{1\lambda}\frac{d}{dr}[U_{pn,0}(r)]\,Y_{\lambda 0}(\theta,\ \varphi); \tag{75}$$

$$U_{pn,0}(r)=2\,(N-Z)^{1/2}A^{-1}V_1 f(r). \tag{76}$$

Here, V_1 is the strength constant of the isobaric-spin potential, $f(r)$ is its radial dependence, and $2(N-Z)^{1/2}A^{-1}$ is the nondiagonal part of the operator $(\mathbf{tT})/A$. In the case $\lambda=2$ and for the interaction of protons with an even–even nucleus, the interaction (75) leads to the excitation in the residual nucleus of the state $|2^+\rangle$, the isobar analog with respect to the state $|2_1^+\rangle$ in the target nucleus. In accordance with (75), the probability of excitation of this state is directly determined by the isovector deformation of the potential (in accordance with its origin, it can also be called the deformation of the isobaric-spin potential). There have been various investigations into the quasi-inelastic scattering reaction with excitation of isobar analog states, and the potential isovector deformation parameters $\beta^{v}_{1\lambda}$ have been extracted in the framework of Lane's model. In one of the recent studies,[107] the angular distributions of the neutrons in the (p, n) reaction on the nuclei ^{54,56}Fe and ^{58,60,62,64}Ni were studied at proton energy $E_p=35$ MeV. For all the nuclei, the potential isovector deformation parameters were extracted in the framework of Lane's model and the DWBA by analyzing the cross sections for excitation of states that are the isobar analogs with respect to the $|2_1^+\rangle$ states of the target nuclei. These parameters are given in Table V together with the values of the parameter V_1 and the isoscalar deformation parameters β^{v}_{02} (which are represented by the $\beta_{\mathrm{em},2}$ values from Ref. 108; the differences between β^{v}_{02} and $\beta_{\mathrm{em},2}$ are insignificant compared with those between β^{v}_{12} and β^{v}_{02}). It can be seen that the ratios $\beta^{v}_{12}/\beta^{v}_{02}$ lie in the interval 3–5.

In Ref. 107 the ratio $\beta^{v}_{12}/\beta^{v}_{02}$ was analyzed for the given nuclei on the basis of the relationships of the effective-charge theory,[98] and it was concluded that the ratio $\beta^{v}_{12}/\beta^{v}_{02}$ extracted by analyzing the quasi-inelastic scatter-

TABLE V. Values of parameters of the isoscalar and isovector deformation of the potential and strength parameters of the isobaric-spin potential.

Reaction	β_{02}	β_{12}	V_1, MeV
^{54}Fe (p, n) ^{54}Co	0,18	0,74	−16,55
^{56}Fe (p, n) ^{56}Co	0,23	0,69	−18,0
^{58}Ni (p, n) ^{58}Cu	0,187	0,94	−16,76
^{60}Ni (p, n) ^{60}Cu	0,211	0,92	−16,86
^{62}Ni (p, n) ^{62}Cu	0,193	0,67	−18,96
^{64}Ni (p, n) ^{64}Cu	0,192	0,64	−19,11

ing reaction agrees with this theory. On the other hand, the ratio $\beta_{12}^{v}/\beta_{02}^{v}$ can be determined, as noted above, from data on the inelastic scattering of protons and neutrons by nuclei. Using the corresponding data for Ni isotopes from Ref. 109 and (73), we obtain for $\beta_{12}^{v}/\beta_{02}^{v}$ values in the range 1.5–2.0. Thus, the values of the isovector deformation parameters extracted from the scattering data are 2–2.5 times smaller than the corresponding values obtained from analysis of the quasi-inelastic scattering reaction.

A number of studies have considered the possible reasons for this discrepancy, and it has been suggested that one of the main ones is a significant contribution of multistep processes to the quasi-inelastic scattering reaction.[110,111] Their neglect in the DWBA leads to overestimated values of the parameter β_{12}^{v}. Allowance for the multistep processes in the coupled-channel method leads to values of β_{12}^{v} close to the β_{12}^{v} values extracted from the scattering data.[110] In this connection, a more suitable source of information about the parameter β_{12}^{v} could be the (^{3}He, t) reaction, for which, as was noted in Ref. 111, the multistep processes are less important. However, in the case of this reaction the problem of constructing an optical potential for composite particles arises.

It was noted in the Introduction that the analysis of experimental data on nucleon scattering by nuclei in the framework of standard collective models is subject to various shortcomings. Everything that we have said also applies to the use of Lane's potential in the framework of the collective model for describing charge-exchange reactions. Transition to the isospin formalism of the semimicroscopic approach described in the previous sections makes it possible, using a single effective interaction and taking into account exchange and many-particle correlations, and also the strong channel coupling, to describe in a unified manner proton and neutron scattering by nuclei and also charge-exchange reactions.

Exchange effects and isobaric-spin potential

Taking into account explicitly the isovector components of the effective nucleon–nucleon forces, we obtain instead of (5) the following expression for the nonlocal nucleon optical potential:

$$U(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\left[\int V_{D0}(|\mathbf{r}-\mathbf{r}_1|)\,\rho_0(\mathbf{r}_1)\,d\mathbf{r}_1 - \tau_{0z}\int V_{D1}(|\mathbf{r}-\mathbf{r}_1|)\,\rho_1(\mathbf{r}_1)\,d\mathbf{r}_1\right] + V_{E0}(|\mathbf{r}-\mathbf{r}'|) \times \rho_0(\mathbf{r}, \mathbf{r}') - \tau_{0z} V_{E1}(|\mathbf{r}-\mathbf{r}'|)\,\rho_1(\mathbf{r}, \mathbf{r}').$$

Here, $V_{D0}(s), V_{E0}(s), V_{D1}(s), V_{E1}(s)$ are, respectively, the contributions of the isoscalar and isovector components of the effective interaction to the direct and exchange parts of the potential; in the case of protons $\tau_{0z} = -1$, and in the case of neutrons $\tau_{0z} = 1$; $\rho_0(\mathbf{r}, \mathbf{r}')$ and $\rho_1(\mathbf{r}, \mathbf{r}')$ are the corresponding components of the density matrix:

$$\rho_0(\mathbf{r}, \mathbf{r}') = \rho_p(\mathbf{r}, \mathbf{r}') + \rho_n(\mathbf{r}, \mathbf{r}');$$

$$\rho_1(\mathbf{r}, \mathbf{r}') = \rho_p(\mathbf{r}, \mathbf{r}') - \rho_n(\mathbf{r}, \mathbf{r}').$$

Using the local approximation, for the main terms in the real part of the optical potential we obtain

$$U_0(r) = U_0^D(r) + I_{00,0}(r) - \tau_{0z}[U_1^D(r) + I_{00,1}(r)]. \quad (77)$$

In (77), $I_{00,0}(r)$ and $I_{00,1}(r)$ are determined by the expressions (24), in which we have substituted the isoscalar and isovector densities, respectively, and the strength constants are related to the parameters of the effective forces by the relations

$$V_{D0} = a_0;\ V_{D1} = a_\tau;\ V_{E0} = a_0';\ V_{E1} = a_\tau'$$

(here, a_i' differ from a_i by the allowance for the antisymmetrization). To cast the expression (77) into the form employed in the optical model, we assume equality of the geometrical parameters of the proton and neutron distributions. We then obtain

$$U_0(r) = U_0^D(r)\left\{1 + \tau_{0z}\frac{a_\tau}{a_0}\frac{N-Z}{A}\right\} + I_{00,0}(z)\left\{1 + \tau_{0z}\frac{a_\tau'}{a_0'}\frac{N-Z}{A}\right\}. \quad (78)$$

In the case of effective Wildermuth-Schmid forces, a_i and a_j' take in accordance with the data in Table I the values $a_0 = -20.97$ MeV, $a_\tau = 10.57$ MeV, $a_0' = -23.64$ MeV, $a_\tau' = 11.05$ MeV. Substituting these values in (78) and expanding τ_{0z}, we obtain

$$U_0(r) = U_0^D(r)\left\{1 \mp 0.50\frac{N-Z}{A}\right\} + I_{00,0}(r)\left\{1 \mp 0.47\frac{N-Z}{A}\right\}. \quad (79)$$

The $-$ sign corresponds to neutrons, and the $+$ sign to protons. It can be seen that the Wildermuth-Schmid forces give a value of the isobaric-spin potential close to the value established in the analysis of the experimental data. The relative contribution of the exchange effects associated with allowance for the Pauli principle to the isovector part of the optical potential is approximately the same as in the direct terms. This is due to the fact that $a_\tau/a_0 \simeq a_\tau'/a_0'$.

We now investigate the energy dependence of the isobaric-spin potential. In this approach, as noted above, it is due to the effect of the Pauli principle, as a result of which the nucleon optical potential is nonlocal, its energy dependence arising from localization. In accordance with (77), for the isovector part of the optical potential we have

$$U_1(r) = U_1^D(r) + I_{00,1}(r).$$

Using for the Bessel function in $I_{00,1}(r)$ the multiplication theorem, we can expand $I_{00,1}(r)$, as in Sec. 2, in a series in powers of the energy. In the linear approximation for the isovector part of the optical potential, we obtain

$$U_1(E, r) = U_1(0, r) - \frac{N-Z}{A}\gamma_1(r)\,E; \quad (80)$$

$$\gamma_1(r) = \frac{1}{2}\frac{\xi(r)}{|U_0^D(r)|}\frac{a_\tau'}{a_0'}I_{01}^{\xi}(r). \quad (81)$$

Here, $\xi(r) = [(2m/\hbar^2)|U_0^D(r)|]^{1/2}$. As in the case of the isoscalar part of the optical potential [see (36)], Eqs. (80) and (81) make it possible to obtain the energy dependence of not only the depth of the isobaric-spin potential but also of its geometrical parameters. The energy dependence of the

isobaric-spin potential is less reliably established by analyzing the empirical material than is the energy dependence of the isoscalar part of the optical potential. The results obtained in Ref. 112 by analyzing the (p, n) reaction suggested that this dependence is nonlinear. At the same time, in the review of Ref. 113, it was concluded on the basis of an analysis of data on the elastic scattering of protons and neutrons by nuclei that there is a linear dependence of the isobaric-spin potential on the energy. A calculation in accordance with (81) gives the value $\gamma_1(0) = 0.08$. Comparing this value with the value $\gamma_1 = 0.16$ given in Ref. 113, we conclude that allowance for the Pauli principle is responsible for 50% of the energy dependence of the isobaric-spin potential.

It follows from (79) and from the analysis of the calculation of the values of $I_{00,0}(r)$ for different nuclei that the contribution of the exchange part of the isovector potential to the optical potential is about 3% of the optical potential even for heavy nuclei with a large neutron excess. Therefore, for analysis in the semimicroscopic approach of the experimental data on proton and neutron scattering by nuclei it is sufficient to take into account only the direct part of the isobaric-spin potential; effectively, the contribution of the exchange correlations to the isobaric-spin potential will be taken into account by the renormalization of the parameter d, the parameter of the density dependence of the effective interaction. At the same time, the influence of the exchange correlations on the isovector part of the potential is important for the description of charge-exchange reactions in the semimicroscopic approach. In this case, instead of the expressions of Lane's phenomenological model [see Eqs. (75) and (76)] we shall have for the interactions responsible for the quasielastic and quasi-inelastic reactions, respectively,

$$U_{pn,0}(r) = 2(N-Z)^{1/2}A^{-1}[U_1^D(r) + I_{00.1}(r)]; \qquad (82)$$

$$U_{pn,\lambda}(\mathbf{r}) = 2(N-Z)^{1/2}A^{-1}\beta_{1\lambda}^{\rho}[U_{1\lambda}^D(r) + I_{\lambda 0,1}(r)]Y_{\lambda 0}(\theta, \varphi). \qquad (83)$$

The contribution of $I_{00,1}(r)$ to the interaction $U_{pn,0}(r)$ is up to 30%; in addition, as was shown above, $U_{pn,0}(r)$ and $U_{pn,\lambda}(\mathbf{r})$ depend on the energy of the incident nucleon because of allowance for the Pauli principle. The expressions (82) and (83) in conjunction with the formalism of the semimicroscopic approach presented in the previous sections makes it possible to describe in the framework of a unified approach the elastic and inelastic scattering of protons and neutrons by nuclei with allowance for strong coupling of the inelastic channels and also quasielastic and quasi-inelastic reactions. The channel coupling through the charge-exchange interaction is weaker than the coupling of the inelastic channels, and it can be taken into account by perturbation theory.

At the present time, there is no systematic microscopic description of processes of scattering and charge-exchange reactions in the presence of strong coupling of the inelastic channels. In Ref. 114, an attempt was made to describe the (p, n) quasi-inelastic reaction on a large group of nuclei at proton energies in the interval 22.8–45 MeV using an interaction analogous to the interaction in the form (82). However, the parameter a_τ was free and varied from nucleus to nucleus. In addition, the real part of the optical potential was not calculated with effective forces but was taken from the optical model; thus, a reconciliation at the microscopic level in the description of scattering and the (p, n) reaction was not implemented. In Ref. 115, the folding model with allowance for exchange correlations was used for a combined analysis of the inelastic scattering of protons and neutrons with energy $E = 11$–26 MeV on the isotopes ^{54,56}Fe with the aim of extracting information about the differences between the proton and neutron transition densities. The calculation was made in the DWBA, but for these nuclei it is important to take into account channel coupling at low energies. As noted earlier, its neglect can lead to distorted information about the relationship between the parameters $\beta_{n\lambda}$ and $\beta_{p\lambda}$. In the light of what we have said, it is of interest to implement the semimicroscopic approach to the description of the interaction of low-energy nucleons with nuclei as presented in the present review in a combined analysis of experimental data on elastic and inelastic scattering of neutrons and protons by nuclei and on quasielastic and quasi-inelastic reactions. This is also important from the point of view of verifying the nuclear-model ideas about the isospin structure of the transition densities in the description of nuclear reactions.

CONCLUSIONS

We formulate the main conclusions and results set forth in the review. In the semimicroscopic approach to the description of the interaction of low-energy nucleons with nuclei the optical potentials and form factors of inelastic transitions are constructed on the basis of information about the effective nucleon–nucleon forces, the central density, and the transition densities with allowance for the Pauli principle in the local approximation of the density-matrix formalism. In contrast to the standard versions of the optical model and the coupled-channel method, such a description does not contain free parameters; on the other hand, it makes it possible to use information about nuclear-structure properties to analyze experimental scattering data.

Analysis of the part played by the Pauli principle in the interaction of low-energy nucleons with nuclei leads to the following conclusions. The optical potentials and the form factors of the inelastic transitions depend on the energy of the incident particle, and exchange correlations are responsible for more than 60% of this dependence. In the case of the interaction of nucleons with deformed nuclei, we encounter the so-called multipole-mixing effect, according to which transitions associated with transfer to the target nucleus of angular momenta $\lambda \geqslant 4$ are enhanced. The allowance for the Pauli principle renormalizes the nucleon–phonon interaction, terms nonlinear in the phonon operators appearing; in all orders, the interaction becomes energy-dependent. The calculations which we have made showed that allowance for the exchange effects leads to an increase in the cross sections for elastic and inelastic scattering of nucleons by nuclei in the backward hemisphere.

At low energies, it is important to take into account channel coupling. In the semimicroscopic approach, there exists such a possibility for nuclei with both static and dy-

namical deformation. Calculations made for even–even nuclei demonstrated a good description with the same effective interaction, dependent on the matter distribution density in the nucleus, for both the elastic and inelastic scattering of low-energy protons by nuclei. In this connection, it is of interest to generalize the formalism of the semimicroscopic approach and to use it to analyze scattering of nucleons by odd nuclei.

Transition to the isospin representation makes it possible to establish a number of simple relationships between the deformation parameters that characterize the differences between the proton and neutron transition densities. The introduction of the parameters of the isoscalar and isovector deformation of a nucleus leads to a unified description of the various processes of nucleon–nucleus interaction. On the one hand, these parameters determine the isospin structure of the inelastic transitions; on the other, they are directly related to the cross section of the charge-exchange quasi-inelastic reactions. In this connection, it is of interest to use the semimicroscopic approach for the description of the scattering of protons and neutrons and also the charge-exchange (p, n) reaction on the same nucleus. From the experimental point of view, it is also important to investigate charge-exchange reactions with composite particles [for example, $(^3\text{He}, t)$], since in this case the multistep processes are less important and it is possible to obtain more reliable information about the isovector deformation parameters.

The nucleon optical potentials and the form factors of inelastic transitions constructed in the semimicroscopic approach with allowance for the exchange and many-particle nucleon–nucleon correlations can be used as the basis for finding by the folding method the corresponding quantities for composite particles with their subsequent use to analyze the interaction of composite particles with nuclei in the peripheral region. In this way, it would be possible to achieve unity in the description of the interaction of nucleons and composite particles with nuclei.

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[1]Note that in the literature this potential is also called a folding potential, although, strictly speaking, the second term in (5) is not a convolution integral in accordance with the mathematical definition.[24] In what follows, we shall call only the first term in (5) a folding potential.

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