# Axial exchange currents in nuclear physics

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A chiral-invariant Lagrangian of a system containing nucleons and $\pi$, $\rho$, and $A_1$ mesons is used to construct the operator of the axial exchange current in the $S$-matrix approach. Its connection with the soft-pion exchange current obtained from low-energy theorems is analyzed. It is shown that in the region of light nuclei there exist transitions for which allowance for the contribution of the axial exchange current leads to satisfactory agreement between the theory and experimental data.

## INTRODUCTION

The nucleon–nucleon interaction at large ($r \gtrsim 1.5$ F) and intermediate ($0.8$ F $\lesssim r \lesssim 1.5$ F) distances can be described on the basis of the picture of multipion exchange.[1–3] This is well demonstrated by the example of the Paris potential,[4] which in the region $r \gtrsim 0.8$ F is a $\pi + 2\pi + \omega$ potential sum. Moreover, in the core region the Paris group has clearly identified a contribution of a three-pion resonance with quantum numbers $J^P = 1^+$, $T = 1$ ($A_1$ meson). The remaining part of the potential, describing the contribution of the core, was fitted to the experimental data on nucleon–nucleon ($NN$) scattering up to energies $\sim 350$ MeV. The mechanism of multipion exchange between two nucleons as the source of the $NN$ interaction is used even more systematically as a hypothesis by the Bonn group in constructing the modern variant of their potential.[5]

Thus, the picture of pion exchange as the source of nuclear forces, proposed 50 years ago by Yukawa, is still being developed, since there are as yet no possibilities for describing fully the nuclear interaction in the framework of the more fundamental theory—quantum chromodynamics.

However, bearing in mind the fundamental mechanism of the interaction between the fundamental particles proposed in quantum chromodynamics, we conclude that, generally speaking, the picture of the nucleus may be quite different from the one to which we are accustomed. Therefore, testing of our ideas about the nucleus is a very important problem, a key part of which is the clarification of the part played by virtual pions in nuclear physics. This problem was recently discussed in detail by Ericson.[6] In particular, he considered the extent to which one can accurately describe the quadrupole moment $Q$ of the deuteron and the $D$–$S$-wave ratio $\eta \equiv D/S$ for its wave function on the basis of the potential description of the $NN$ interaction just discussed. These two quantities, $Q$ and $\eta$, warrant particular attention, since[6] they can be measured very accurately[1)]:

$$Q = 0.2859\ (3)\ \mathrm{F}^2, \quad \eta = 0.0271\ (4). \tag{1}$$

Ericson's analysis showed[6] that these data can be reproduced with high accuracy in the framework of pion exchange. Therefore, there is no need to introduce any corrections to our ideas about deuteron structure at intermediate and large distances ($r \gtrsim 0.8$ F).

Of course, we have so far considered only some static properties of the deuteron, the simplest nuclear system. The deuteron magnetic moment has not yet been satisfactorily described in the framework of the potential model,[9] its calculation requiring knowledge of the wave function at short distances.

The study of static characteristics of more complicated nuclei, for example, the magnetic moments, is a difficult problem, the solution of which, although still far from complete resolution, nevertheless gives valuable information on the manifestation of virtual mesons.[10]

Much information about nuclear structure can be obtained by studying the interaction of a nuclear system with leptons. The classical description of such a process is based on summation of the scattering (reaction) amplitudes for a test particle on individual nucleons (impulse approximation). The existence of mesons in the nucleus is taken into account solely through the nuclear wave function. In fact, however, the lepton particle also interacts with the virtual pions that maintain the nucleus in the bound state. Thus, we have here a possibility of studying directly the mesonic degrees of freedom (exchange currents) in nuclei. This part of physics began to be developed strongly only from the beginning of the seventies, when there appeared a sufficiently developed formalism for describing the interaction of leptons with a nuclear system containing explicitly not only nucleons but also mesons.[11]

The most detailed studies have been made of electromagnetic exchange currents of isovector type in light nuclei. This fact is explained by several favorable circumstances.

a) There exist many fairly accurate data obtained in electron and photon reactions on light nuclei.

b) The transition operator is basically determined by the spatial component of the isovector electromagnetic current. This component is of order $O(1/M)$ ($M$ is the nucleon mass) for both the single-particle current and the exchange

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current.[12] In other words, the amplitude of the exchange current has the same "strength" as the amplitude of the impulse approximation.

c) The main part of the exchange-current contribution is determined by a low-energy theorem and is therefore model-independent.

Mesonic degrees of freedom are most clearly manifested in the deuteron electrodisintegration reaction[13,14]

$$e + d \to e' + n + p. \quad (2)$$

In the kinematic region in which the energy transfer is small (a few MeV) and the momentum transfer is large, the contribution of the pionic exchange currents to the differential cross section can exceed the contribution of the impulse approximation by an order of magnitude. Overall, the theory describes very well the cross section in the complete region of momentum transfers in which there are data ($k^2 \lesssim 18\ \mathrm{F}^{-2}$). The relativistic corrections, the exchange currents with excitation of the $\Delta(1236)$ isobar and with $\rho$-meson exchange, and also allowance for the form factors in the $BNN$ vertices ($B = \pi, \rho$) cannot significantly distort the quality of the description.[15]

There are further, rather convincing examples of the manifestation of the virtual-pion effect induced by the isovector electromagnetic current in the lightest nuclei, a detailed discussion of which can be found in Refs. 10, 12, 16, and 17.

In the considered case of the reaction (2) it is remarkable that the theory correctly describes the data, although it is manifestly used outside the region of its applicability. Discussing this point, Rho[18] put forward the hypothesis of a chiral filter.

If the amplitude for production of the pion $\pi$ by the current $J_\mu$ on the nucleon $N$, $J_\mu + N \to \pi + N$, is enhanced relative to the amplitude of the vertex without the pion, then the contribution of the soft pions is dominant. In the opposite case, the effect of the soft pions is completely masked by the contribution of all the remaining possible processes.

We recall that precisely the vertex for pion production by the current on a nucleon occurs in the exchange-current operator.[11,16] In the case when $J_\mu$ is the isovector electromagnetic current the spatial part of the amplitude of current pion production is of order $O(1)$, while the spatial part of the amplitude of the exchange and single-particle currents is $\sim O(1/M)$.

The predictive power of the chiral-filter hypothesis can be tested for the deuteron photodisintegration reaction,

$$\gamma + d \to n + p, \quad (3)$$

when the energy of the incident photons is relatively low ($20 \lesssim E_\gamma \lesssim 100$ MeV) and the protons are emitted at zero angle. The transition operator is basically determined by the time component of the current. By virtue of the parity selection rule, the main $E1$ transition is strongly suppressed in the impulse approximation. As a result, both the relativistic corrections to the single-particle charge density and the contribution of the exchange density are manifested. Both types of correction are of order $O(1/M^2)$, whereas the single-particle charge density is of order $O(1)$ (see Ref. 12). In the given case, the calculation of the differential cross section for protons scattered forward is more complicated and more uncertain[19–21] than the case of the reaction (2) already discussed.

Besides the electromagnetic interaction, there is a further form of fundamental interaction responsible for slowly occurring processes with parity nonconservation. It has become known as the weak interaction. The Lagrangian density of the strangeness conserving weak interaction has the form[22]

$$\mathscr{L}(x) = \frac{G \cos \theta_C}{2\sqrt{2}} [Y_\lambda^+(x) Y_\lambda(x) + \mathrm{h.\,c.}], \quad (4)$$

with the universal weak-interaction constant $G \approx 10^{-5}/M^2$ and the experimental Cabibbo angle $\theta_C \approx 15^0$. The charged weak current $Y_\lambda(x)$ is divided into two parts: the leptonic $l_C(x)$ and the hadronic $J_\lambda(x)$. The leptonic current is constructed from the fundamental fields of the leptons as follows:

$$l_\lambda(x) = \sum_l i\bar{\psi}_{\nu_l}(x) \gamma_\lambda (1 + \gamma_5) \psi_l(x),$$

where $\psi_l$ is the field operator of the free lepton ($l = e, \mu, \ldots$).

In accordance with the Feynman–Gell-Mann theory, the charged weak hadronic current is a linear combination of the polar ($V$) and axial ($A$) vector currents of the form

$$J_\lambda(x) = (V_\lambda^1 + iV_\lambda^2) - (A_\lambda^1 + iA_\lambda^2). \quad (5)$$

For the neutral neutrino–hadron interaction[23]

$$\mathscr{L}(x) = \frac{G}{\sqrt{2}} l_\lambda(x) J_\lambda^0(x). \quad (6)$$

Now

$$l_\lambda(x) = \sum_l i\bar{\psi}_{\nu_l}(x) \gamma_\lambda (1 \pm \gamma_5) \psi_{\nu_l}(x); \quad (7)$$
$$J_\lambda^0(x) = V_\lambda^0 - A_\lambda^0 - 2 \sin \theta_W J_\lambda^{\mathrm{e.m.}}.$$

In (7), the plus and minus signs correspond to neutrinos and antineutrinos; $V_\lambda^0(x)$ and $A_\lambda^0(x)$ are the currents obtained from $V_\lambda^j(x)$ and $A_\lambda^j(x)$ ($j = 1, 2$) by rotation in the isotopic space; $J_\lambda^{\mathrm{e.m.}}$ is the electromagnetic current, $\theta_W \approx 35°$ is the Weinberg angle.

Since part of the nuclear hadronic current $J_\lambda(x)$ or $J_\lambda^0(x)$ relates to the mesonic degrees of freedom present in the nucleus, they can be excited through the interaction (4) or (6). This excitation can be manifested in a change of various nuclear characteristics. In this review, we have set ourselves the task of critically reviewing and summarizing the results obtained recently in the study of the manifestation of virtual mesons in nuclei using weakly interacting test particles—the leptons $e$, $\mu$, $\nu_1$, $\hat{\nu}_1$. All the calculations that we have so far made must facilitate a deeper understanding of the problems discussed in the review.

In the case of the weak interaction, the circumstances are less favorable than for the isovector electromagnetic current, for the following reasons.

1. Accurate data for the lightest nuclei are as yet sparse. Further, the weak interactions in these nuclei are determined by the spatial part of the axial current, but now it is of order $O(1)$ for the single-particle current and of order

$O(1/M^2)$ for the exchange current. The part played by the exchange current with excitation of the Δ isobar is here significantly more important than in the case of the isovector electromagnetic current, and this makes the analysis harder.

However, there already exists a theoretical analysis of numerous reactions on the lightest nuclei aimed basically at quantitative study of the amplitude of the exchange effect and, hence, aimed at finding transitions in which this effect can be clearly observed.

2. There are data for several transitions in the region of light nuclei ($A = 12, 14, 16, 18$), for whose description the time component of the axial exchange current is manifested significantly. For it, we have a situation analogous to that which obtains for the spatial part of the isovector electromagnetic current, since it is $\sim O(1/M)$. The impulse-approximation amplitude is also of order $O(1/M)$, and in accordance with the chiral-filter hypothesis the effect of the soft-pion exchange currents must be more strongly manifested in the reactions induced by the time component of the axial current.[24] Generally speaking, because of the uncertainties in the wave function of nuclei with $A > 3$, the analysis is more complicated and it is difficult to obtain reliable conclusions. However, tracing what has already been done, we can assert that the existence of virtual-meson effects has also been proved for the excitation of nuclei by the weak interaction. The effect is more strongly manifested in reactions in which the main transition operator is the time component of the axial exchange current, and it can reach values comparable with the contribution of the impulse approximation.

The present review consists of three sections.

Section 1 presents the theory of the construction of the operator of the axial exchange current. Since the construction of this operator using the low-energy theorem has already been considered in Refs. 1, 10, and 16, we give its construction only in the framework of the hard-pion method.[25] Section 2 discusses the manifestation of the mesonic degrees of freedom when the lightest nuclei are excited by the weak interaction. Section 3 considers the analogous problem in the region of light nuclei.

## 1. OPERATOR OF THE AXIAL EXCHANGE CURRENT

### Basic principles

The general criteria that the axial-current operator $A^j_\mu$ must satisfy can be formulated by analogy with the case of the electromagnetic interaction.[26]

a) The current and Hamiltonian of the system must satisfy the restrictions imposed by the special theory of relativity.

b) The current and Hamiltonian of the system must satisfy the restrictions imposed by chiral invariance.

c) The current must satisfy the condition of partial conservation of the axial current (PCAC),

$$\partial_\mu \mathbf{A}_\mu = m_\pi^2 f_\pi \boldsymbol{\pi}, \tag{8}$$

where $f_\pi = 92$ MeV, and $\boldsymbol{\pi}$ is the operator of the pion field.

In contrast to Ref. 26, our explicit point of departure is that the chiral invariance, broken by the nonvanishing pion mass, is a good symmetry of a system of strongly interacting particles.[27] The exact chiral invariance has a remarkable property—it is spontaneously broken. This means that the true chiral-transformation operator (the axial charge) does not annihilate the vacuum (the ground state of the system) but transforms it into a different ground state containing an arbitrary number of massless particles (Goldstone bosons), which can be identified with pions. In reality, the pion mass is finite, but small in the scale of hadronic masses, and therefore the situation does not differ strongly from the ideal one.[22] The idea of spontaneously broken chiral symmetry has proved to be very fruitful for the description of external electromagnetic interaction and weak interaction with nuclei,[11] for which low-energy theorems for the amplitude for production of a soft pion by a current are used essentially. The structure of the amplitude for soft-pion production by the weak current is considered in Ref. 16. An amplitude of this type was used to construct the operator of the weak exchange current in Refs. 11 and 28.

### Phenomenological chiral Lagrangians

Another way of constructing the current operator is based on the use of phenomenological chiral Lagrangians.[22,29] In this language, the exchange current operator corresponds to the set of all loopless Feynman graphs admissible in the framework of the chiral Lagrangian of the considered system.

For the example of the chiral Lagrangian for a system consisting of pions and nucleons we shall discuss an important point that makes it possible to understand correctly the significance of chiral symmetry as applied to calculations in nuclear physics. The Lagrangian of the $\pi N$ system[30]

$$\mathscr{L}(\psi, \pi) = -\bar{\psi}\gamma_\mu \partial_\mu \psi - M\bar{\psi} e^{-i\gamma_5 \bar{\boldsymbol{\pi}}\cdot\boldsymbol{\tau}} \psi + \mathscr{L}(\pi) \tag{9}$$

is invariant with respect to the transformation

$$\psi' = e^{i\gamma_5 \frac{\boldsymbol{\tau}}{2}\boldsymbol{\alpha}} \psi, \tag{10}$$

if simultaneously the field $\bar{\pi} \equiv \pi/f_\pi$ is changed in accordance with the nonlinear law

$$e^{-i\gamma_5 \bar{\boldsymbol{\pi}}'\cdot\boldsymbol{\tau}} = e^{-i\gamma_5 \frac{\boldsymbol{\tau}}{2}\boldsymbol{\alpha}} e^{-i\gamma_5 \bar{\boldsymbol{\pi}}\cdot\boldsymbol{\tau}} e^{-i\gamma_5 \frac{\boldsymbol{\tau}}{2}\boldsymbol{\alpha}} \tag{11}$$

The term $\mathscr{L}(\pi)$ in (9) is the Lagrangian of the free pion field,

$$\mathscr{L}(\pi) = -\frac{1}{2}(D_\mu \pi^i)(D_\mu \pi^i), \tag{12}$$

with covariant derivative of the pion field[22,29]

$$D_\mu \pi^i = \partial_\mu \pi^i + \partial_\mu \pi^k \left(\delta_{ik} - \frac{\pi^i \pi^k}{\boldsymbol{\pi}^2}\right)\left(\frac{\sin z}{z} - 1\right), \quad z^2 \equiv \pi^2. \tag{13}$$

Expanding in (9) the second term in powers of $1/f_\pi$, we have

$$\mathscr{L}(\psi, \pi) = -\bar{\psi}\gamma_\mu \partial_\mu \psi - M\bar{\psi}\psi + i\frac{M}{f_\pi}\bar{\psi}\gamma_5(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi + \frac{M}{f_\pi^2}\bar{\psi}\psi\boldsymbol{\pi}^2 + \mathscr{L}(\pi) + \ldots \tag{14}$$

We see that the second term in (9) contains besides the nucleon mass term not only a pseudoscalar $\pi N$ interaction (when the chiral symmetry is exact, the axial coupling constant is $g_A=1$ and the Goldberger-Treiman relation takes the form $M=gf_\pi$) but also an infinite series of contact terms. The contact term which we have retained in (14) is necessary for the correct description of the $\pi N$ interaction in the $s$ wave.

By the equivalence transformation

$$N=\exp\left(-i\gamma_5\frac{\tau}{2}\cdot\pi\right)\psi \qquad (15)$$

we transform the chiral Lagrangian (9) to the form

$$\mathscr{L}(N,\pi)=-\bar{N}\gamma_\mu\left\{\partial_\mu+\left[\exp\left(-i\gamma_5\frac{\tau}{2}\cdot\pi\right)\right]\right.$$
$$\left.\times\partial_\mu\left[\exp\left(i\gamma_5\frac{\tau}{2}\cdot\pi\right)\right]\right\}N-M\bar{N}N+\mathscr{L}(\pi). \qquad (16)$$

Expanding the second term in the square brackets with respect to the generators of the group SU(2)×SU(2),

$$e^{-i\gamma_5\frac{\tau}{2}\cdot\bar{\pi}}\partial_\mu e^{i\gamma_5\frac{\tau}{2}\cdot\bar{\pi}}\equiv\frac{i}{2}\left[\frac{1}{f_\pi}D_\mu\pi^j\gamma_5\tau^j+\theta^j_\mu(\pi)\tau^j\right], \qquad (17)$$

we obtain instead of (16)

$$\mathscr{L}(N,\pi)=-\bar{N}\gamma_\mu D_\mu N+\frac{i}{2f_\pi}\bar{N}\gamma_5\gamma_\mu\tau^j D_\mu\pi^j N$$
$$-M\bar{N}N+\mathscr{L}(\pi), \qquad (18)$$

where the covariant derivative $D_\mu N$ of the nucleon field is determined by

$$\left.\begin{aligned}D_\mu N&\equiv\left(\partial_\mu+\frac{i}{2}\theta^j_\mu(\pi)\tau^j\right)N;\\ f_\pi^2\theta^j_\mu(\pi)&=\varepsilon_{ijk}\pi^i\partial_\mu\pi^k\frac{\cos z-1}{z^2}.\end{aligned}\right\} \qquad (19)$$

Equation (17) serves to determine the covariant derivatives of the fields $\pi^i$ and $N$. In the lowest approximation in the pion field,

$$\mathscr{L}(N,\pi)=-\bar{N}\gamma_\mu\partial_\mu N+\frac{i}{4f_\pi^2}\bar{N}\gamma_\mu(\tau\cdot\pi\times\partial_\mu\pi)N$$
$$-\frac{i}{2f_\pi}\bar{N}\gamma_\mu\gamma_5(\tau\cdot\partial_\mu\pi)N-M\bar{N}N-\frac{1}{2}(\partial_\mu\pi)^2+\ldots \qquad (20)$$

The $\pi N$ coupling is now pseudovector, and the series of contact terms is different. It can be verified that the chiral Lagrangian (20) describes the $\pi N$ interaction in the $s$ wave as well as the chiral Lagrangian (14).

Such a situation is common—the physical content of the two unitarily equivalent chiral Lagrangians is the same. Recognition of this is decisive for testing the consistency of nuclear-physics calculations.[9]

To describe lepton interaction with a nuclear system, it is necessary to know the hadronic (vector and axial) currents. They can be obtained from (9) or (18) by using the Gell-Mann–Lévy method.[22] We shall not yet do this but will discuss the following circumstance, which is important from the point of view of the application of the formalism to physical problems.

The current-algebra method and the chiral-Lagrangian method, which is equivalent to it, are, strictly speaking, applicable only in the soft-pion regime. However, the real situation that arises in the description of particle–nucleus interactions may be far from this regime, since the particles may have high energy and the momentum transfer may reach large values. Therefore, our formalism must be changed accordingly. The generalization of the theory is not unique and can be implemented in at least two ways.

1. By proceeding from the current-algebra result for the amplitude of soft-pion production by the current and taking seriously the dependence on the momentum transfer that is introduced through the form factors of the currents and the vertices. Such a procedure is used, for example, in Ref. 31 to determine the mass of the axial meson from data on pion electroproduction.

2. By using the hard-pion model.[32] It contains the ideas of current algebra, the PCAC hypothesis, and vector dominance.[33] In such an approach, the coupling of not only the pion but also the heavier mesons ($\rho$, $A_1$, $\omega$) is fixed. In this way, the diagrams with their participation are correctly determined. This method can also be formulated in the language of chiral Lagrangians.[34,35]

In the case of pion production by the electromagnetic current, the amplitudes of the approaches 1 and 2 differ to the extent that the axial form factor $F_A[(k-q)^2]/g_A$ and the pion form factor $F^V_\pi(k^2)$ differ from the form factor $F^V_1(k^2)$.[36]

### Hard pions

The hard-pion model was used to study the weak exchange current in Ref. 25 and for the electromagnetic isovector current in Ref. 36. For the construction of the covariant derivatives, it is convenient to proceed from the nucleon part of the canonical form of the chiral Lagrangian (18), augmented by a term that describes the anomalous nucleon interaction:

$$\mathscr{L}^{\rm str}_N=\bar{N}\gamma_\mu D_\mu N-M\bar{N}N-i\frac{g_A}{f_\pi}\bar{N}\gamma_\mu\gamma_5\tau N D_\mu\pi$$
$$-\frac{\varkappa_V}{8M}g_\rho\bar{N}\sigma_{\mu\nu}\tau N\tilde{\rho}'_{\mu\nu}-\frac{g_{A_1}g_T}{2}\bar{N}\sigma_{\mu\nu}\gamma_5\tau N\tilde{a}'_{\mu\nu}. \qquad (21)$$

In Eq. (21), $g_A=1.25$, $\varkappa=3.7$, $g_\rho$ and $g_{A_1}$ are the universal coupling constants of the $\rho$ and $A_1$ mesons, and $g_T$ is the coupling constant of the weak tensor interaction. In place of (17), the covariant derivatives of the nucleon and pion fields are now determined by the procedure[37]

$$\exp\left(-i\gamma_5\frac{\tau}{2}\cdot\pi\right)\left[\partial_\mu+\frac{i}{2}(g_\rho\rho_\mu\cdot\tau+g_{A_1}\gamma_5 a_\mu\cdot\tau)\right]$$
$$\times\exp\left(i\gamma_5\frac{\tau}{2}\cdot\pi\right)\equiv\frac{i}{2}\left[\frac{1}{f_\pi}D_\mu\pi\cdot\tau\gamma_5+\theta_\mu\cdot\tau\right]. \qquad (22)$$

Here, $\rho_\mu$ and $a_\mu$ are the fields of the $\rho$ and $A_1$ mesons. From (22), it is possible to obtain the covariant derivatives to any required order in the pion field.[25,36] They now also contain the fields of the heavy mesons. In this manner one can also obtain expressions for $\tilde{\rho}'_{\mu\nu}$ and $\tilde{a}'_{\mu\nu}$.[25,36]

In order to describe fully the interaction of an external field with a nucleus, it is also necessary to consider the Lagrangian of the $A_1\pi\rho$ system.[34,35] For the construction of the operator of the exchange currents in Refs. 25 and 36, the minimal Lagrangian of this system constructed in Ref. 35 was taken. Since[38] not only $g^2_\rho=g^2_{A_1}$ but also $g_\rho=g_{A_1}$, the total interaction Lagrangian of the $N\pi\rho A_1$ system has the form[36]

$$\mathcal{L}_{N\pi\rho A_1} = -\bar{N}\gamma_\mu\partial_\mu N - M\bar{N}N$$
$$-(i/2)\, g_\rho \bar{N}\gamma_\mu \boldsymbol{\tau} N\left(\boldsymbol{\rho}_\mu + \frac{1}{f_\pi}\,\boldsymbol{\pi}\times\mathbf{a}_\mu\right)$$
$$-i\,(g_A/2f_\pi)\,\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot[\partial_\mu\boldsymbol{\pi} - g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\pi} + 2f_\pi g_\rho \mathbf{a}_\mu]$$
$$-(\varkappa_V/8M)\, g_\rho\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N\cdot\tilde{\boldsymbol{\rho}}_{\mu\nu} - (g_\rho g_T/2)\,\bar{N}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}N\cdot\tilde{\mathbf{a}}_{\mu\nu} + O(\pi^2). \tag{23}$$

The interaction with the $\Delta$ isobar can also be included similarly.[25]

Assuming breaking of the chiral invariance by the meson masses and using the Gell-Mann–Lévy method, it is possible to obtain the hadronic currents and their divergences in the hard-pion model[35]:

$$\mathbf{V}_\mu = (m_\rho^2/g_\rho)\,\boldsymbol{\rho}_\mu;$$
$$\mathbf{A}_\mu = (m_\rho^2/g_\rho)\,\mathbf{a}_\mu - f_\pi\partial_\mu\boldsymbol{\pi} + f_\pi g_\rho \boldsymbol{\rho}_\mu\times\boldsymbol{\pi} + O(\pi^2); \tag{24}$$
$$\partial_\mu\mathbf{V}_\mu = 0,\quad \partial_\mu\mathbf{A}_\mu = m_\pi^2 f_\pi\boldsymbol{\pi}. \tag{25}$$

The Lagrangian (23) contains pseudovector $\pi N$ coupling and numerous contact terms. The fourth term plays an important part in describing pion production by the axial current, and the sixth term performs in the case of $s$-wave $\pi N$ scattering the same role as the second term in the chiral Lagrangian (20).

The transition in the chiral Lagrangian (23) to pseudoscalar $\pi N$ coupling can be realized by the transformation that is the inverse of (14). As a result, we obtain a chiral Lagrangian with the same physical content:

$$\mathcal{L}_{N\pi\rho A_1} = -N\gamma_\mu\partial_\mu N - M\bar{N}N$$
$$-(i/2)\, g_\rho\bar{N}\gamma_\mu\boldsymbol{\tau}N\cdot\boldsymbol{\rho}_\mu + ig\bar{N}\gamma_5\boldsymbol{\tau}N\cdot\boldsymbol{\pi}$$
$$-i\,(g_\rho/2f_\pi)\,(1-2g_A^2)\,\bar{N}\gamma_\mu\boldsymbol{\tau}N\cdot(\boldsymbol{\pi}\times\mathbf{a}_\mu) - ig_A g_\rho\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\cdot\mathbf{a}_\mu$$
$$-(\varkappa_V/8M)\, g_\rho\bar{N}\sigma_{\mu\nu}\,(\boldsymbol{\tau} - i(g_A/f_\pi)\,\gamma_5\boldsymbol{\pi})\,N\cdot\tilde{\boldsymbol{\rho}}_{\mu\nu}$$
$$-(g_\rho g_T/2)\,\bar{N}\sigma_{\mu\nu}\,(\gamma_5\boldsymbol{\tau} + i\,(g_A/f_\pi)\,\boldsymbol{\pi})\,N\cdot\tilde{\mathbf{a}}'_{\mu\nu} + O(\pi^2). \tag{26}$$

Note here the renormalization of the contact ($NN\pi A_1$) interaction, the disappearance of the normal contact ($NN\pi\rho$) interaction, and the appearance of the anomalous contact ($NN\pi A_1$) and ($NN\pi\rho$) interactions. The part played by the latter in the construction of the operator of the electromagnetic charge was discussed in Ref. 36.

The amplitude of pion production by the axial current obtained in the hard-pion model was analyzed in Ref. 25, where it was shown that only correct allowance for the contribution of the $A_1$ meson leads to results consistent with current algebra, while allowance for the contribution of the $\rho$ meson alone is insufficient.

### Exchange current

The operator of the weak axial exchange current is represented by the sum of all possible tree Feynman graphs allowed by the vertices (23) and (24) or (26) and (24). Figure 1 shows only the part of the operator important for practical applications. The corresponding formula for the current in the nonrelativistic approximation has the form

$$\mathbf{A}^{\pm}(2) = \frac{g^2 m_\pi(1+\varkappa_V)}{32\pi g_A M^3}\sum_{i<j}\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\, e^{-i\mathbf{k}\cdot\mathbf{r}_i}\,[\boldsymbol{\sigma}_i\times(\boldsymbol{\nabla} - i\mathbf{k})]$$
$$\times(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\, Y_0(x_\pi^{ij}) + (i\leftrightarrow j)\} + \frac{g^2 m_\rho^2(1+\varkappa_V)}{64\pi g_A M^3}$$
$$\times\sum_{i<j}\left\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\, e^{-i\mathbf{k}\cdot\mathbf{r}_i}\,[\boldsymbol{\sigma}_i\times(\boldsymbol{\nabla}-i\mathbf{k})]\left[1+\frac{(\boldsymbol{\nabla}-i\mathbf{k})^2}{m_\rho^2}\right]\right.$$
$$\left.\times(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\, J(\mathbf{r},\mathbf{k}) + (i\leftrightarrow j)\right\}; \tag{27}$$

$$A_4^{\pm}(2) = i\,\frac{g^2 m_\pi}{16\pi g_A M^2}\sum_{i<j}\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\, e^{-i\mathbf{k}\cdot\mathbf{r}_i}$$
$$\times(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\, Y_0(x_\pi^{ij}) + (i\leftrightarrow j)\}$$
$$+ i\,\frac{g^2 m_\rho^2}{32\pi g_A M^2}\sum_{i<j}\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\, e^{-i\mathbf{k}\cdot\mathbf{r}_i}\,[1 + C(\boldsymbol{\nabla}-i\mathbf{k})^2$$
$$+ D(\boldsymbol{\nabla}-i\mathbf{k})^4]\,(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\, J(\mathbf{r},\mathbf{k}) + (i\leftrightarrow j)\}; \tag{28}$$

$$Y_0(x_\pi^{ij}) = e^{-x_\pi^{ij}}/x_\pi^{ij};\quad x_\pi^{ij} = m_\pi r_{ij};$$
$$C = \frac{1}{m_\rho^2} + \frac{\varkappa_V}{4M^2};\quad D = \frac{\varkappa_V}{4m_\rho^2 M^2}, \tag{29}$$
$$(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm} = \frac{1}{2}\,[(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^1 \pm i\,(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^2];$$
$$J(\mathbf{r},\mathbf{k}) = \pi^2\int_0^1\frac{dt}{a}\, e^{-ar+it\mathbf{k}\cdot\mathbf{r}}; \tag{30}$$
$$a = [t\,(1-t)\,\mathbf{k}^2 + t\,(m_\rho^2 - m_\pi^2) + m_\pi^2]^{\frac{1}{2}}.$$

In the case of a soft current ($k\to 0$), the expressions (27) and (28) simplify:

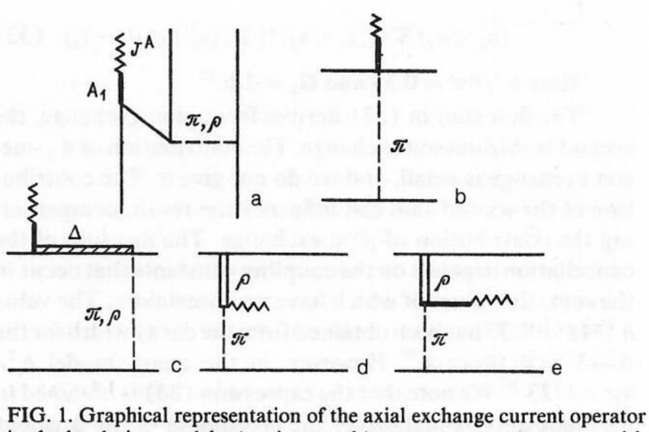


FIG. 1. Graphical representation of the axial exchange current operator in the hard-pion model: a) pair term; b) contact term; c) current with excitation of $\Delta$ isobar; d) current of the weak decay of the $\rho$ meson; e) the ($A_1$, $\pi\rho$) current.

$$A^{\pm}(2)=\frac{g^2 m_\pi(1+\varkappa_V)}{32\pi g_A M^3}\sum_{i<j}\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\,e^{-i\mathbf{k}\cdot\mathbf{r}_i}(\boldsymbol{\sigma}_i\times\boldsymbol{\nabla})(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})$$

$$\times Y_0(x_\pi^{ij})+(i\leftrightarrow j)\}+\frac{g^2(1+\varkappa_V)}{32\pi g_A M^3}\times\frac{m_\rho^2}{m_\rho^2-m_\pi^2}\sum_{i<j}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}$$

$$\times e^{-i\mathbf{k}\cdot\mathbf{r}_i}(\boldsymbol{\sigma}_i\times\boldsymbol{\nabla})\left(1+\frac{\boldsymbol{\nabla}^2}{m_\rho^2}\right)(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\,[m_\pi Y_0(x_\pi^{ij})$$

$$-m_\rho Y_0(x_\rho^{ij})]+(i\leftrightarrow j)\}; \qquad (31)$$

$$A_4^{\pm}(2)=i\frac{g^2 m_\pi}{16\pi g_A M^2}$$

$$\times\sum_{i<j}\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\,e^{-i\mathbf{k}\cdot\mathbf{r}_i}(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\,Y_0(x_\pi^{ij})+(i\leftrightarrow j)\}$$

$$+\frac{g^2}{16\pi g_A M^2}\frac{m_\rho^2}{m_\rho^2-m_\pi^2}\sum_{i<j}\left\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}\,e^{-i\mathbf{k}\cdot\mathbf{r}_i}\left(1+\frac{\varkappa_V}{4M^2}\boldsymbol{\nabla}^2\right)\right.$$

$$\left.\times\left(1+\frac{\boldsymbol{\nabla}^2}{m_\rho^2}\right)(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\,[m_\pi Y_0(x_\pi^{ij})-m_\rho Y_0(x_\rho^{ij})]+(i\leftrightarrow j)\right\}. \qquad (32)$$

We do not give the velocity terms here. It is precisely the form of expression (31) and (32) for $A^{\pm}(2)$ and $A_4^{\pm}(2)$ that we need in what follows. The second sum in (27) and (28) and also in (31) and (32) derives from the graphs d and e in Fig. 1. The first sum in these expressions is the sum of the pair (Fig. 1a) and contact (Fig. 1b) terms. In the approximation $m_\pi/M \ll 1$, $m_\pi/m_\rho \ll 1$, and also ignoring in (31) and (32) the contribution of the terms $\sim Y_0(x_\rho^{ij})$, we obtain the corresponding part of the operator of the soft-pion axial exchange current.[28]

We also give the spatial component of the operator of the axial exchange current corresponding to excitation of the $\Delta$ isobar (Fig. 1c):

$$A_\Delta^{\pm}(2)=-\frac{2g_A}{gm_\pi(M_\Delta-M)}\frac{h^2}{4\pi}$$

$$\times\sum_{i<j}e^{-i\mathbf{k}\cdot\mathbf{r}_i}[4\tau_j^{\pm}\boldsymbol{\nabla}-(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}(\boldsymbol{\sigma}_i\times\boldsymbol{\nabla})]$$

$$\times(\boldsymbol{\sigma}_i\cdot\boldsymbol{\nabla})\,Y_0(x_\pi^{ij})+(i\leftrightarrow j)\}-\frac{h}{72\pi}\,g_\rho^2 G_1\varkappa_V\,\frac{f_\pi m_\rho}{m_\pi M^2(M_\Delta-M)}$$

$$\times\sum_{i<j}\{e^{-i\mathbf{k}\cdot\mathbf{r}_i}[4(\boldsymbol{\nabla}^2\boldsymbol{\sigma}_j-(\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})\boldsymbol{\nabla})\tau_j^{\pm}+((\boldsymbol{\sigma}_j\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_i\times\boldsymbol{\nabla})$$

$$-(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\boldsymbol{\nabla}^2)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^{\pm}]\,Y_0(x_\rho^{ij})+(i\leftrightarrow j)\}. \qquad (33)$$

Here $h^2/4\pi = 0.35$ and $G_1 = 2.6$.[25]

The first sum in (33) derives from pion exchange, the second from $\rho$-meson exchange. The contribution of $A_1$-meson exchange is small, and we do not give it. The contribution of the second sum can influence the result, compensating the contribution of pion exchange. The measure of the cancellation depends on the coupling constants that occur in the sum, the values of which have an uncertainty. The value $h^2/4\pi = 0.35$ has been obtained from the decay width for the $\Delta \to N + \pi$ process.[39] However, in the quark model $h^2/4\pi = 0.23$.[39] We note that the expression (33) is obtained in the static approximation for the propagator of the $\Delta$ isobar (the momentum dependence of the propagator is ignored). More accurate allowance for this dependence leads to suppression of the contribution of the exchange current with excitation of the $\Delta$ isobar by 25–50%.[40–43] A current of the form (33) but with somewhat different constants was used in Refs. 44 and 45.

In contrast to the case of the isovector electromagnetic current, the relative contribution of the current (33) compared with the contribution of the current (31) in calculations of exchange effects is larger. From what was said above it can be seen that calculations of the effects of the spatial part of the axial current are more model-dependent than in the case of the isovector electromagnetic current.

We do not give the correction from the $\Delta$-isobar excitation current to the time component of the operator of the exchange current because it is 1–2% in the static approximation.[46] In Ref. 47, an attempt was made to improve this approximation by taking into account the momentum dependence in the numerator of the $\Delta$ propagator. The numerical estimate obtained in Ref. 47 for the contribution of about 10% for the $0^+ \to 0^-$ transition in nuclei with $A = 16$ is clearly overestimated; for in Ref. 47 no allowance was made for the momentum dependence of the $\Delta$ propagator, nor was the contribution of the current with $\rho$ exchange taken into account. As we have just seen, both effects strongly reduce the total contribution of the $A_\Delta^{\pm}$ component. It is obvious that the same arguments also remain valid for $A_{4,\Delta}^{\pm}$.

In Refs. 48 and 49, based on a chiral Lagrangian containing besides the pion only the $\rho$ meson, the operator of the axial exchange current was constructed in the soft-current limit. The spatial part of this operator is identical to (31), and the time part differs from (32) by a term proportional to

$$(\varkappa_V m_\rho^2/4M^2) \approx 0.6,$$

this being a consequence of the neglect, not justified in Refs. 48 and 49, of the contribution of the anomalous $\rho$-meson–nucleon coupling. Agreement between the results of Cheng's theory with the more general theory of Ref. 25 in the limit $k \to 0$ is to be expected on the basis of analysis of the amplitude of weak pion production.[25]

The effect of hard pions has been investigated numerically not only in weak reactions on deuterium[50] but also on nuclei with $A = 16$.[46,51–53] In the case of the deuteron, the effect is 15–20% of the total exchange effect. As was to be expected, for reactions on nuclei with $A = 16$ this effect depends on the quality of the employed nuclear wave functions—realistic wave functions and allowance for the renormalization of the vertices and the propagators reduce it. We shall discuss these results in more detail in the last section.

In all the approaches to the construction of the exchange-current operator that we have so far discussed the $S$-matrix method is employed. This method is a special case of a more general scheme of describing the problem of the interaction of a nuclear system with an external perturbation that was strongly developed for the case of the electromagnetic interaction.[10,16,54] Generally speaking, for systematic allowance of the mesonic degrees of freedom it is necessary to construct simultaneously the exchange-current operator and the nuclear potential. This process is unambiguous only in the static limit. Since the spatial part of the axial exchange current is a relativistic correction to the impulse approximation, from the point of view of the reliability of practical calculations it is advantageous to study reactions induced by the time component of the axial exchange current in the lightest nuclei. Calculations of its effect[55] in the nucleus with $A = 2$ have not yet been crowned with success. We shall discuss them later.

Finally, we give here expressions for the components of the single-particle axial current. They have the form

$$\mathbf{A}^{\pm}(1) = \sum_i \tau_i^{\mp} e^{-i\mathbf{k}\cdot\mathbf{r}_i} \left[ g_A \sigma_i + \frac{g_P}{2M} (\sigma_i \cdot \mathbf{k}) \mathbf{k} \right]; \quad (34)$$

$$A_4^{\mp}(1) \equiv i\rho^{A\pm}(1) = i \sum_i \tau_i^{\mp} e^{-i\mathbf{k}\cdot\mathbf{r}_i} \times \left[ \frac{g_A}{2M} (\sigma_i \cdot \mathbf{P}_i) + \frac{g_P}{2M} k_0 \sigma_i \cdot \mathbf{k} \pm g_T \sigma_i \cdot \mathbf{k} \right]. \quad (35)$$

Here, the pseudoscalar form factor $g_P$ is determined as follows:

$$g_P(k^2) = -g_A g f_\pi / (k^2 + m_\pi^2); \quad g f_\pi = M g_A,$$

and $\mathbf{P}_i = \mathbf{P}_i' + \mathbf{P}_i$ is the sum of the initial and final momenta of nucleon $i$.

With this we conclude the description of the operator of the axial current and turn to the exposition of the formalism needed to make practical calculations. In Refs. 23, 28, and 56, it was developed in a unified manner for the electromagnetic and weak interactions. Here, we follow the exposition of Delorme.[28] It is this formalism that is widely used to analyze data obtained in reactions with $1p$-shell nuclei.

#### Currents and matrix elements

By analogy with the current of an elementary particle, the amplitude of the weak nuclear current can be written down independently of its many-nucleon structure in the most general form, employing model-independent Lorentz-invariant form factors. As an example, we give the expression for the amplitude of the weak vector and axial currents for allowed Gamow-Teller transitions in which the spin changes by unity: $0^+ \to 1^+$. Transition of such type are observed in the $^6$He–$^6$Li doublet and in the $^{12}$B–$^{12}$C–$^{12}$N and $^{14}$C–$^{14}$N–$^{14}$O triads and have been the subject of numerous discussions in the literature. These amplitudes have the form

$$\langle p', 1^+, T_z = \alpha; M(f) | A_\lambda^\alpha(0) | p, 0^+; M(i) \rangle = F_A^\alpha(k^2) \xi_\lambda^+ + F_P^\alpha(k^2) k_\lambda (\xi^+ k) + F_T^\alpha(k^2) P_\lambda (\xi^+ k), \quad \alpha = \pm; \quad (36)$$

$$\langle p', 1^+, T_z = \alpha; M(f) | V_\lambda^\alpha(0) | p, 0^+; M(i) \rangle = F_M^\alpha(k^2) \varepsilon_{\lambda\mu\nu\rho} P_\mu k_\nu \xi_\rho, \quad \alpha = \pm, 0. \quad (37)$$

The nuclear states are characterized by the momenta $p$ and $p'$; $M(i)$ and $M(f)$ are the masses of the initial and final nuclei; $P_\lambda = (p + p')_\lambda$; $k_\lambda = (p - p')_\lambda$; $F_M$, $F_A$, $F_P$, and $F_T$ are, respectively, the magnetic, axial, pseudoscalar, and time form factors; $T_z$ specifies measurement of the isospin in the investigated transition; $\xi_\lambda$ is the polarization vector of a particle with spin 1; and $\varepsilon_{\lambda\mu\nu\rho}$ is the completely antisymmetric tensor. In the case of exact isotopic invariance, for the isobar-analog nuclear states $F_X^+ = F_X^0 = F_X^-$ ($X$ = V, M, A, P, S, T).

The expressions (36) and (37) are a special case of the general representation of current amplitudes in a Cartesian coordinate system with respect to a complete set of $K$ basis vectors $V_\lambda^{(K,X)}$ which transform as polar (axial) vectors and are constructed from $p$, $p'$ and bilinear combinations of polarization functions, which are eigenfunctions of the spin operator. The expansion has the form

$$\langle j'm'p' | J_\lambda(0) | jmp \rangle = \sum_{K,X} F_X^K V_\lambda^{(K,X)}(jmp; j'm'p'). \quad (38)$$

Here, $j$ and $j'$ are the angular momenta of the initial and final states of the nucleus with projections $m$ and $m'$ onto an axis perpendicular to the 4-momentum; $X$ = V, M, A, P, S, T.

It is convenient to go over in (38) to an expansion in a spherical basis, generalized to the relativistic case. In this basis, one chooses the timelike vector $e_\lambda^{(4)}$ along the direction of $P_\lambda$ and three spacelike vectors $e_\lambda^{(t)}$ ($t = \pm 1, 0$) orthogonal to it. In the Breit ($P = 0$) system the 4-momentum $\tilde{k}_\lambda = k_\lambda - (P^k/P^2) P_\lambda$ coincides with the three-dimensional momentum transfer, and the basis vectors are directed along the ordinary coordinate axes. The projections of the unit vector $\tilde{k}_\lambda$ onto $e_\lambda^{(t)}$ specify the spherical harmonics $Y_t^1(k)$:

$$\hat{\tilde{k}}_\lambda e_\lambda^{(t)} = (4\pi/3)^{1/2} Y_t^1(\hat{\tilde{\mathbf{k}}}),$$

by means of which one constructs a basis of spherical tensors[57]:

$$T_{\lambda,\mu}^{(J)} \equiv Y_\mu^J(\hat{\tilde{\mathbf{k}}}) e_\lambda^4 \text{ and } T_{\lambda,\mu}^{(L,J)} \equiv [Y^L(\hat{\tilde{\mathbf{k}}}) \otimes e_\lambda^{(t)}]_\mu^J.$$

The spherical and Cartesian bases are related by a matrix $\mathscr{K}$ by the relation

$$V_\lambda^{(K,X)}(jmp; j'm'p') = \sum_J (4\pi/(2j'+1))^{1/2} (jmj'm'/Jm'-m) \times [\mathscr{K}_{J;KX} T_{\lambda,m'-m}^{(J)+} + \sum_L \mathscr{K}_{LJ;KX} T_{\lambda,m'-m}^{(L,J)+}]. \quad (39)$$

Substituting (39) in (38), we can obtain a multipole expansion of the current amplitude with invariant multipole form factors $a^{(J)}$ and $a^{(L,J)}$, which play the part of reduced matrix elements in accordance with the Wigner-Eckart theorem,

$$\left. \begin{aligned} a^{(J)}(k^2) &= \sum_{K,X} \mathscr{K}_{J;KX} F_X^K(k^2); \\ a^{(L,J)}(k^2) &= \sum_{K,X} \mathscr{K}_{LJ;KX} F_X^K(k^2). \end{aligned} \right\} \quad (40)$$

Complete expressions for the matrix $\mathscr{K}$ were obtained by Delorme.[28] Using these expressions, we can obtain for the time form factor in the $0^+ \to 1^+$ transition the expression

$$F_T = \pm \frac{1}{M(i) + M(f)} \left[ \frac{1}{|\mathbf{k}|} a_A^{(1)} + \frac{M(f) - M(i)}{\mathbf{k}^2} a_A^{(2,1)} \right]. \quad (41)$$

Here, the subscript $A$ relates to the axial current, and the upper and lower signs correspond to the transitions $0^+ \to 1^+$ and $1^+ \to 0^+$.

In discussing the formalism needed for practical calculations, we have hitherto regarded the nucleus as an elementary particle. Such a model-independent approach to the nucleus is helpful in the case when the aim of the investigations is to identify the most general properties of the nuclear currents (conservation of the vector current, PCAC hypothesis, the presence or absence of currents of the second kind). However, this method leaves open the question of the nuclear structure of the form factors as matrix elements of transition operators between nuclear wave functions. It is clear that in such a treatment the nuclear form factors cease to be model-independent. We shall assume that the axial component of the current that determines the nuclear matrix elements is given by the expressions (27)–(35).

The concrete content of the multipole form factors is now determined by the form of the current operator. For the single-particle current (34), (35) we can define the bracket $[\hat{O}]$ as reduced matrix elements in the coordinate (but not in the isospin) space:

$$[\hat{O}]^{(L,J)} = (j' \| \sum_i \tau_i^{\pm} \sqrt{4\pi}\, j_L(kr_i) \{Y^L(\hat{r}_i) \otimes \hat{O}_i\}^J \| j);$$
$$[\hat{O}]^{(J)} = (j' \| \sum_i \tau_i^{\pm} \sqrt{4\pi}\, j_L(kr_i)\, Y^J(\hat{r}_i)\, \hat{O}_i \| j), \quad (42)$$

where $O$ is a tensor operator of the zeroth ($\hat{O}_i = 1, \boldsymbol{\delta}_i \cdot \mathbf{P}_i$) or first ($\hat{O}_i = \boldsymbol{\delta}_i, \mathbf{P}_i$) rank.

Using this expression, for the nonrelativistic form of the multipole form factors in (39) we obtain

$$a_A^{(1)} = -\frac{1}{2M}\Big\{g_A\,[i\sigma\cdot\mathbf{P}]^{(1)} - (g_P k_0 \pm g_T)\,|k| \times \sqrt{\frac{1}{3}}\big([\sigma]^{(0,1)} + \sqrt{2}\,[\sigma]^{(2,1)}\big)\Big\};$$

$$a_A^{(2,1)} = \Big\{g_A\,[\sigma]^{(2,1)} + g_P \frac{k^2}{2M}\frac{\sqrt{2}}{3}\big([\sigma]^{(0,1)} + \sqrt{2}\,[\sigma]^{(2,1)}\big)\Big\}. \quad (43)$$

Allowance for the mesonic degrees of freedom in the nuclear form factors now reduces to renormalization of the single-particle multipoles (43) through the addition of the exchange current. For example, the part of (32) corresponding to the soft-pion model leads to the following effective multipole $[\sigma\cdot\mathbf{P}]^{(J)}$, which determines the density of the axial charge:

$$\widetilde{[\sigma\cdot\mathbf{P}]}^{(J)} = [\sigma\cdot\mathbf{P}]^{(J)} + \frac{g^2 m_\pi}{4\pi g_A^2 M}\Big(j' \Big\| \sum_{i<j} \{(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^{\pm} \sqrt{4\pi}\, j_L(kr_i) \times Y^J(\hat{r}_i)\,(\boldsymbol{\sigma}_j\cdot\nabla)\; Y_0(x_\pi^{ij}) + (i \leftrightarrow j)\} \Big\| \Big). \quad (44)$$

The remaining effective multipoles are determined similarly. The time form factor (41) for $\beta$ decay of the $^{12}$B nucleus to the $^{12}$C ground state will now be

$$F_T \approx \frac{1}{M(i)+M(f)}\Big\{-\frac{g_A}{2M|\mathbf{k}|}\big[-\widetilde{i\sigma\cdot\mathbf{P}}\big]^{(1)} + g_A\sqrt{\frac{3}{2}}\,\frac{M(f)-M(i)}{\mathbf{k}^2}\,[\tilde{\sigma}]^{[2,1]}\Big\}. \quad (45)$$

### Calculation of observables

The weak-interaction Lagrangian density (4), (6) determines the $S$ matrix of the transition from the initial, $i$, to the final, $f$, nuclear state:

$$S_{fi} = \delta_{fi} - 2\pi i\delta(E_f - E_i)\,\mathcal{M}_{fi} \quad (46)$$

where

$$\mathcal{M}_{fi} = \langle f, \beta \mid \int d\mathbf{r}\,\mathcal{L}(\mathbf{r}, t=0) \mid i, \alpha\rangle.$$

The probability $\omega_{fi}$ of the transition is determined by

$$\omega_{fi} = 2\pi\delta(E_i - E_f)\,|\mathcal{M}_{fi}|^2. \quad (47)$$

For the reaction of muon capture by a nucleus,

$$\mu^- + (A, Z) \to \nu_\mu + (A, Z-1), \quad (48)$$

the transition rate is calculated in accordance with

$$\Lambda_\mu(i \leftrightarrow f) = \frac{1}{4\pi^2}|p_\nu|^2 \int d\hat{p}_\nu \frac{1}{(2J_i+1)} \sum_{m_i, m_f, \sigma_\mu} \frac{1}{2}|\mathcal{M}_{fi}|^2; \quad (49)$$
$$\hat{p}_\nu \equiv p_\nu/|p_\nu|, \quad |p_\nu| = m_\mu + M(i) - M(f);$$

here, $J_i$ is the spin of the initial nucleus. The summation in (49) is over the spin projections of the initial and final particles.

From the point of view of nuclear physics, the process of nuclear $\beta$ decay is the inverse of the muon capture process:

$$(A, Z) \to (A, Z+1) \begin{cases} \nearrow e^- + \bar{\nu}_e; \\ \searrow e^+ + \nu_e. \end{cases}$$

If by $p_e$ we denote the momentum of the charged lepton, then the transition rate will be

$$\Lambda_\beta(i' \to f') = \frac{1}{(2\pi)^5}\int d\hat{p}_{\nu_e}\, dp_e\,\big(\omega_0' - \sqrt{p_e^2 + m_e^2}\big)^2 \times \frac{1}{2J_{i'}+1}\sum_{m_{i'}, m_{f'}} |\mathcal{M}_{f'i'}|^2; \quad (50)$$
$$\omega_0' = M(i') - M(f').$$

Information about the dynamics of the process is contained in $\mathcal{M}_{fi}$. Since the matrix elements of the leptonic and nuclear currents separate,

$$\left.\begin{aligned} \mathcal{M}_{fi} &\sim \int d\mathbf{r}\,\langle\beta|l_\lambda(r,0)|\alpha\rangle\,\langle f|J_\lambda^+(r,0)|i\rangle; \\ \mathcal{M}_{f'i'} &\sim \int d\mathbf{r}\,\langle\beta|l_\lambda^+(r,0)|\alpha\rangle\,\langle f|J_\lambda(r,0)|i\rangle. \end{aligned}\right\} \quad (51)$$

The capture of muons by nuclei takes place from the $K$ orbit of the mesic atom, which is situated near the nucleus. For light and intermediate nuclei, the muon wave function varies little within the nucleus, and it is frequently replaced by the mean value over the volume of the nucleus: $\varphi_\mu^{av} = [\int d\mathbf{r}|\rho(r)\varphi_\mu(r)|^2]^{1/2}$, where $\rho(r)$ is the nuclear density.

In the case of $\beta$ decay, the electron wave function perturbed by the Coulomb field of the daughter nucleus with charge $Z$ is approximately equal to

$$|\psi_e(0)|^2 = F(Z, E_e) \approx \frac{\pi y}{\sinh y}e^{\pi y}; \quad y = \alpha Z E_e/|p_e|; \quad (52)$$
$$E_e = (p_e^2 + m_e^2)^{1/2}.$$

For inelastic neutrino scattering by a nucleus, the differential cross section is determined by

$$d\sigma_{i\to f} = 2\pi\delta(E_f - E_i)(2J_i+1)^{-1}\sum_{m_i, m_f} |\mathcal{M}_{fi}|^2\Phi(\mathbf{P}_f, \mathbf{p}_f \ldots), \quad (53)$$

where $\Phi(P_f, p_f\ldots)$ is the phase space for the final states, and $P_f$ and $p_f$ are the momenta of the final nucleus and the lepton. For the case of excitation of a nuclear level without emission of nucleons,

$$\Phi(P_f, p_f) = dP_f\, dp_f\, (2\pi)^{-6}.$$

Either the matrix element $\mathcal{M}_{fi}$ is specified as in (51) ($\beta$ relates to the charged final lepton $e^\pm, \mu^\pm$) or, for the neutral process $(\nu_e, \nu_e'), (\bar{\nu}_e, \bar{\nu}_e'), \mathcal{M}_{fi}$ contains the function $\mathcal{L}(x)$ from (6) with $l_\lambda(x)$ and $J_\lambda^0(x)$ from (7):

$$\mathcal{M}_{fi} \sim \int d\mathbf{r}\, \langle\beta'|l_\lambda(\mathbf{r}, 0)|\alpha'\rangle\, \langle f'|J_\lambda^0(\mathbf{r}, 0)|i'\rangle. \quad (54)$$

The most difficult part in the calculations of the observables is the calculation of the matrix element of the nuclear current,

$$\langle f|\tilde{J}_\lambda(k)|i\rangle \sim \int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} \langle f|J_\lambda(\mathbf{r}, 0)|i\rangle. \quad (55)$$

Using the representation of the nuclear current in terms of the form factors (36), we write the rate of the $\beta$ decay $^{12}B(1^+, T=1)\to {}^{12}C(0^+, T=0) + e^- + \bar{\nu}$ in the form

$$\Lambda_\beta(1^+\to 0^+) = K_\beta[F_A(0^+)]^2\left[1 + \frac{2(M(i)+M(f))}{3}\frac{F_T(0)}{F_A(0)}\right]^2. \quad (56)$$

In accordance with (41) and (43), the form factors $F_X$ can be expressed in terms of the matrix elements of multipole operators between the nuclear states. The constant $K_\beta$ is given in Ref. 45.

We have already given all the elements needed to calculate the observables. In the next two sections we discuss the results obtained by different authors for specific reactions.

## 2. AXIAL EXCHANGE EFFECT IN THE LIGHTEST NUCLEI

The spatial component of the axial exchange current is manifested in all known reactions on nuclei with $A = 2$ and 3.

$A = 2$. For this case there are numerous interesting processes. Important in astrophysics is the reaction

$$p + p \to d + e^+ + \nu_e. \quad (57)$$

The effective cross section for this reaction is directly related to the prediction for the flux of neutrinos incident on the surface of the Earth from nuclear processes taking place in the Sun. However, at the present time there is a definite discrepancy between the prediction 4.66 SNU (Ref. 58) for the rate of absorption of solar neutrinos in $^{37}Cl$ and its experimental value, which is $1.8 \pm 0.4$ SNU.[59] According to Ref. 60, the flux of the incident neutrinos is related to the effective cross section $S_{11}$ for the reaction (57) by

$$\text{neutrino flux} \approx S_{11}^{-2.5}. \quad (58)$$

It is therefore of interest to consider the change in $S_{11}$ due to exchange currents. Since the reaction (57) takes place within the Sun near the threshold (the proton energy is of the order of hundreds of keV), the impulse approximation can be reliably calculated in the effective-range approximation.[61] Effectively, it is only protons in the initial $^1S_0$ state that participate in the reaction. Therefore, the process (57) is an example of a pure Gamow-Teller transition and the analog of the well-studied capture of thermal neutrons by protons: $n + p \to d + \gamma$. It is well known that in this reaction there is a clear manifestation of the effect of about 10% of the electromagnetic isovector exchange current.[1,10,12,16]

The exchange effect for the reaction (57) was calculated in Refs. 62–66. In Refs. 62–65, the current operator was constructed using the $S$-matrix formalism. This operator corresponds to the expression (31) without the term $\sim Y_0(x_\rho^{ij})$ and to the first sum in (33). Numerically, the effect is situated in the range between 6.2 and 8.8% of the impulse approximation.

In contrast to Refs. 62–65, Bargholtz[66] took into account in $\mathbf{A}_\Delta(2)$ not only the current with $\pi$ exchange but also the current with $\rho$ exchange, i.e., both sums in (33) and the total current (31). He also included monopole form factors of the type

$$F_B(k^2) = (\Lambda_B^2 - m_\pi^2)/(\Lambda_B^2 + k^2) \quad (59)$$

in the $BNN$ and $BN\Delta$ vertices $(B = \pi, \rho)$, $\Delta_\pi = 1.2$ GeV, $\Delta_\rho = 1.5$ GeV. For the $\pi N\Delta$ coupling constant, he used the value predicted by the quark model, $h^2/4\pi = 0.23$, and for the $\rho NN$ coupling constant the value given in Ref. 67. In this way, Bargholtz reduced the exchange effect to the minimum, obtaining for it the value 2%, much less than in Refs. 62–65. However, this aim of reducing the exchange effect to a minimum is unjustified. For example, in analogous calculations for the $n + p \to d + \gamma$ reaction, Mathiot[68] also obtained an effect less (about 7.6%) than did other authors[69] who explained the established $\approx 10\%$ effect.

It is clear from the magnitude of the exchange effect for the reaction (57) that it cannot significantly influence the value of $S_{11}$ or, therefore, the solar-neutrino problem.

We now consider the reaction of weak deuteron disintegration by reactor antineutrinos:

$$\nu_e + d \to 2n + e^+; \quad (60)$$

$$\bar{\nu}_e + d \to n + P + \bar{\nu}_e'. \quad (61)$$

In the reaction (61), the neutral current (7) is manifested. Since we are dealing with low-energy antineutrinos, it is again only the axial part of the current that is effective in the reactions (60) and (61). These reactions were recently studied experimentally by the group of Reines.[70] The effective cross section for the reaction (60) is

$$\sigma_c^{\text{exp}} = (1.5 \pm 0.4)\cdot 10^{-45}\,\text{cm}^2/\bar{\nu}_e, \quad (62)$$

and for the reaction (61) with the neutral current

$$\sigma_N^{\text{exp}} = (3.8 \pm 0.9)\cdot 10^{-45}\,\text{cm}^2/\bar{\nu}_e. \quad (63)$$

The reactions (60) and (61) are of considerable interest in connection with the problem of neutrino instability.[71]

Reactor antineutrinos have mainly low energies. At energy around 8 MeV, their number is approximately three orders of magnitude less than at the threshold energy values.[72] Because the antineutrinos have a spectral function $n(\varepsilon_\nu)$, it is necessary to calculate quantities averaged over this spectrum.

The effective cross section for the reactions (60) and (61) have been calculated in the impulse approximation by several authors.[72,73] The main difficulty is the uncertainty with which the spectral function $n(\varepsilon_\nu)$ is known.

Exchange effects for the reactions (60) and (61) were considered in Refs. 74 and 75. The first of these took into account completely the contribution of the current (31) and the contribution of the current $\mathbf{A}_\Delta(2)$ with $\pi$ exchange, calculated in a nonstatic approximation. The $\pi N\Delta$ coupling constant was taken to be $h^2/4\pi = 0.35$, and the $\rho NN$ coupling constant was taken from the vector-dominance model. The resulting value obtained for the exchange effect was about 8%. Including in $\mathbf{A}_\Delta(2)$ the contribution of the current with $\rho$ exchange, it is possible to reduce the value of the effect to about 6%.

The study of Ref. 75 used nuclear wave functions containing components of $\Delta$-isobar configurations, and therefore the $\mathbf{A}_\Delta(2)$ contribution was not calculated. Besides the graphs with $\pi$ and $\rho$ exchange, possible graphs with $\omega$ exchange were taken into account. The correction found for the exchange effect was 4%.

The uncertainty in $n(\varepsilon_\nu)$ and the large experimental error (about 20%) in the measurement of the cross sections (62) and (63) mean that definite conclusions about the magnitude of the exchange effect cannot be drawn.

We now turn to the following important reaction, the absorption of negative muons in deuterium:

$$\mu^- + d \to 2n + \nu_\mu. \quad (64)$$

Before the publication of the data for the reactions (60) and (61), it was the only weak reaction in the two-nucleon system measured under laboratory conditions.[76–78] However, the accuracy in the measurement of the doublet rate $\Lambda_d$ of the transition from the hyperfine state of the muonic atom with spin $F = 1/2$ is as yet poor[77]:

$$\Lambda_d = 445 \pm 60 \text{ sec}^{-1}. \quad (65)$$

The reaction (64) is potentially important, both for the extraction of the neutron–neutron scattering length $a_{nn}$ and for obtaining information about the constant $g_\text{P}$.

The doublet and quartet, $\Lambda_q$, transition rates were calculated in the impulse approximation in Ref. 79. Under the assumption $g_\text{P} \sim 7 g_\text{A}$,

$$\Lambda_d \approx 380 \text{ sec}^{-1}; \quad \Lambda_q \approx 10 \text{ sec}^{-1}. \quad (66)$$

The influence of exchange currents on the transition rate $\Lambda_d$ was considered in Refs. 64 and 80. In Ref. 64, the same transition operator as in the case of the reaction (57) was used—and with approximately the same result: the correction to $\Lambda_d$ from the effect of the axial exchange currents was found to be 6%. This means that $\Lambda_d$ acquires a correction of about 24 sec$^{-1}$. As a result,

$$\Lambda_d \approx 404 \text{ sec}^{-1}, \quad (67)$$

a value closer to the experimental value (65). However, the change in $\Lambda_d$ due to the uncertainty in $g_\text{P}$ (Refs. 64 and 79) actually masks the effect of the exchange currents, and therefore the results (65) and (67) cannot be directly compared.

The effect of the exchange currents for the reaction (64) was calculated in Ref. 80 using the current operator obtained in the hard-pion method. Compared with the calculation of Ref. 74 for the reactions (60) and (61), the dependence of the current operator on the momentum transfer was taken into account, i.e., instead of (31) Eq. (27) was used. The result was also found to be approximately 8%. This means that in the considered region of energies the exchange-current operator does not depend on the momentum transfer.

The influence of the effect of exchange currents on the neutron spectra in the reaction (64) was investigated in Ref. 81. It was found that there exists an interval of neutron momenta,

$$40 \lesssim p_1 \lesssim 60 \text{ MeV}/c,$$

for which the effect of the exchange currents on the extraction of $a_{nn}$ from the normalized differential neutron spectrum $(d^2\Lambda_d/d\Omega_1\, dp_1)$ can be ignored.

In Ref. 81, the dependence of the ratio $R = \Lambda_q/\Lambda_d$ on the value of $g_\text{P}$ was also considered. The calculation showed that the neglect of the exchange effect shifts the value of $g_\text{P}$ by an amount approximately equal to $0.5 g_\text{A}$ and that measurement of this ratio with an error of about 10% leads to a significant improvement in our knowledge of $g_\text{P}$. We recall the mean value of $g_\text{P}$ from the $\mu^- + p \to n + \nu_\mu$ reaction[82]: $g_\text{P} = (6.94 \pm 1.52) g_\text{A}$, while from the $\mu^- + {}^{12}\text{C} \to {}^{12}\text{B} + \nu_\mu$ reaction we have $g_\text{P} = (10.1 \pm {}^{2.4}_{2.6}) g_\text{A}$.[83] As the calculations show (see Table I), measurement of $R$ with an error of about 10% reduces the uncertainty in $g_\text{P}/g_\text{A}$ by 3–4 times.

The influence of the time component of the axial exchange current on some characteristics of the reactions (60) and (64) was considered in Ref. 55. In all cases, the calculated effect is not more than 2%, a very small correction compared with what is expected on the basis of the general considerations (24) for the magnitude of the effect (~40–50%).

**$A = 3$.** There is as yet only one case for which the experiment achieves an error of 1% in weak reactions in light nuclei, namely, the $\beta$ decay of the tritium nucleus:

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e. \quad (68)$$

The experimental value of the reduced Gamow-Teller matrix element is[84]

$$M_A = \sqrt{3}\,(0.970 \pm 0.008). \quad (69)$$

The accuracy in $M_\text{A}$ is restricted mainly by the accuracy of the data extracted from the neutron $\beta$-decay reaction.[85,86] The theoretical prediction for $M_\text{A}$ is given by[85]

TABLE I. The ratio $\Lambda_q/\Lambda_d$ with inclusion ($R$) and without inclusion ($R_0$) of the axial exchange current as a function of $g_P/g_A$ for the reaction (64). The calculation was made in Ref. 81.

| $g_P/g_A$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|
| $100R$ | 1.47 | 1.86 | 2.33 | 2.88 | 3.51 | 4.23 | 5.06 | 6.00 |
| $100R_0$ | 1.36 | 1.72 | 2.14 | 2.64 | 3.22 | 3.88 | 4.63 | 5.48 |

$$M_A=\sqrt{3}\left[1-\frac{2}{3}P(D)-\frac{4}{3}P(S')+\delta\right], \quad (70)$$

where $P(D)$ and $P(S')$ are the admixtures of the $D$ and $S'$ waves, respectively, in the three-nucleon bound-state wave function, and $\delta$ is the sum of all possible corrections to the impulse approximation.

In the case of realistic wave functions, $P(D)\approx 8–9\%$ and $P(S')\approx 1–2\%$. Then from (69) and (70) we obtain

$$\delta^{\text{exp}}=4-6\%. \quad (71)$$

Attempts to calculate $\delta$ have been made over many years (Refs. 39, 41, 43, 44, 85, and 87–92) and have already been discussed in the literature (Refs. 10, 17, 39, and 85). Therefore, we consider here only the most recent calculations.[43,44]

Following Ref. 92, Bargholtz[44] separated from $\delta$ in the impulse approximation the part corresponding to the $\Delta$-isobar component of the nuclear wave function, i.e.,

$$M_A=\sqrt{3}\left[1-\frac{2}{3}P(D)-\frac{4}{3}P(S')-\frac{2}{3}P(\Delta)+\delta'\right], \quad (72)$$

where

$$P(\Delta)=|\langle A_\Delta(2)\rangle|^2/P(D)\,\frac{128}{225} \quad (73)$$

and $\langle A_\Delta(2)\rangle$ is the matrix element of the current with excitation of the $\Delta$ isobar: In Ref. 44, the following corrections were included in $\delta'$:

$$\delta'=\delta(\text{rel})+\delta(\Delta)+\delta(\pi-\rho)+\delta(\text{rec}+\text{norm}). \quad (74)$$

Here, $\delta(\text{rel})$ is the relativistic correction to the single-particle current, $\delta(\Delta)$ is the contribution of the current with excitation of the isobar, $\delta(\pi-\rho)$ is the contribution of the $\rho$-meson weak decay current, and $\delta(\text{rec}+\text{norm})$ is the contribution of the recoil current and the renormalization of the wave function.

In fact, Bargholtz calculated the exchange effect using the same operator and the same approximations as in Ref. 66 for the reaction (57). For the correction $\delta(\text{rec}+\text{norm})$ he used the result of Ref. 11. The three-nucleon bound-state wave function was obtained in Ref. 93 and is a parametrization of the solution of Faddeev's equations with the realistic RSC potential.[94] It has the form $P(D)=8.08\%$, $P(S')=1.47\%$. Bargholtz's results for the individual terms in (74) are given in Table II. For the reduced Gamow-Teller matrix element we have in accordance with (72) the value

$$M_A=\sqrt{3}\times\begin{cases}0.941 & \text{with the form factors (59)},\\ 0.958 & \text{for point-like couplings}.\end{cases} \quad (75)$$

The values (75) for $M_A$ are somewhat lower than the experimental values (69).

Analogous calculations were made in Ref. 43 with the exchange operator (31), (33), which had already been used in the case of the reactions (60) and (61), though in this case the form factors (59) were taken into account. The three-particle wave function was taken from Ref. 95. It was obtained in a hyperspherical basis by solving the Schrödinger equation for three particles with the RSC potential[94] and $P(D)=9.02\%$ and $P(S')=0.84\%$. In the calculations of the contribution of the current $A_\Delta(2)$ both $h^2/4\pi=0.36$ and $h^2/4\pi=0.23$ were used; the value $h^2/4\pi=0.66$ was taken from Ref. 67, and the momentum dependence of the $\Delta$ propagator was taken into account. The correction $\delta'$ was represented in Ref. 43 as follows:

$$\delta'=\delta(\text{pair})+\delta(\text{c. t.})+\delta(\Delta)+\delta(\pi-\rho)+\delta(A_1-\pi-\rho). \quad (76)$$

The individual terms correspond to the contribution of the currents shown graphically in Fig. 1, and their numerical values are given in Table III.

Now

$$M_A=\sqrt{3}\cdot 0.967 \quad (77)$$

irrespective of the value of $h$, this being a consequence of the appreciable cancellation of the $P(\Delta)$ and $\delta(\Delta)$ contribu-

TABLE II. Results of calculations[44] for individual terms of the correction $\delta'$ (74) to the impulse approximation for the process (68). The first row gives the calculation with the form factors (59), and the second row is with pointlike couplings.

| $\delta$ (rel) | $\delta$ ($\Delta$) | $\delta$ ($\pi-\rho$) | $\delta$ (rec + norm) |
|---|---|---|---|
| −0.009 | 0.022 | 0.014 | −0.005 |
| −0.009 | 0.022 | 0.031 | −0.005 |

TABLE III. Corrections to the impulse approximation for the reaction (68), calculated in Ref. 43 in accordance with (76). They correspond to the exchange current in Fig. 1.

| $\delta$ (pair) | $\delta$ (c. t.) | $\delta$ ($\Delta$) | $\delta$ ($\pi-\rho$) | $\delta$ ($A_1-\pi-\rho$) |
|---|---|---|---|---|
| 0.0158 | 0.0066 | 0.0265 [a] <br> 0.0411 [b] | 0,0130 | −0,0130 |

[a] $h^2/4\pi = 0.23$, $\varkappa_V = 6.66$; [b] $h^2/4\pi = 0.35$, $\varkappa_V = 6.66$.

tions. The result for $\delta(\Delta) = 0.0265$ in Table III agrees with Bargholtz's result $\delta(\Delta) = 0.022$. Comparing the contribution of the remaining terms in Table III with $\delta(\pi - \rho)$ from Table II, we see that in Ref. 44 it is smaller by a factor of about 2. The reason for this discrepancy is as yet obscure. In Ref. 43, the relativistic correction to the single-particle current was not calculated. The values for $\delta$(rel) given in the literature[41,44,89] lie in the range $-1.5\% \lesssim \delta(\text{rel}) \lesssim -0.24\%$. It is obvious that this point warrants particular attention.[96]

The exchange effect obtained in Ref. 43 is 4.9–6.3%. From this, the part that arises as a consequence of the low-energy theorem is $\delta(\text{pair}) + \delta(\text{c.t.}) \approx 2.2\%$. The contribution $\delta(\Delta) \approx 2.7$–$4.1\%$ is somewhat uncertain. However, this uncertainty is compensated by the cancellation between $\delta(\Delta)$ and $P(\Delta)$.

With regard to the correction $\delta(\text{rec} + \text{norm})$, the following comment is needed. In Ref. 11, it was calculated in accordance with the projection method for constructing the exchange-current operator and the corresponding potential. However, this correction should be regarded only as the uncertainty of the calculations of Refs. 16 and 97, since there exist other approaches[89,97–99] in which to terms $\sim O(1/M^2)$ inclusive $\delta(\text{rec} + \text{norm}) = 0$ for all values of the momentum transfer.

The next reaction of undoubted interest from the point of view of studying the exchange effect is the capture of the negative muon by the $^3$He nucleus:

$$\mu^- + {}^3\text{He} \to {}^3\text{H} + \nu_\mu. \tag{78}$$

Generalizing the experience of the calculations of this effect for weak reactions in two-nucleon systems, one can assert that its magnitude will be approximately the same as in the case of the reaction (68), i.e., about 5%. As yet, the experimental data are at approximately the same level of accuracy.[100]

We now discuss the question of the use of the vertex form factors (59) in the calculations. For potential exchange currents, we obtain from the PCAC condition the relation[16,101,102]

$$A^j_{\text{int}}(2) = i\left[V(r_1, r_2), \sum_{s=1}^{2} \frac{1}{2}(r_s\rho_s^{A_j} + \rho_s^{A_j} r_s)\right]_-, \tag{79}$$

where $V(r_1, r_2)$ is the two-particle potential and $\rho_s^{A_j}$ is the single-particle density of the axial current.

The relation (79) is the analog of Siegert's theorem[103] for the electromagnetic current. A change in the left-hand side of (79) must be accompanied by a corresponding change in the potential. Usually, this is not done, with detriment, in general, to the consistency of the calculations. As yet, this problem has not been investigated in more detail.

We note that a characteristic feature of the realistic wave functions of nuclei with $A = 2$ and 3 is the admixture of the $D$ state. Although it is a small fraction $[P(D) \lesssim 10\%]$, its presence is decisive for the calculation of the effect of the axial exchange currents in the reactions discussed here.

## 3. AXIAL-VECTOR EXCHANGE CURRENT IN PARTIAL TRANSITIONS OF $\mu^-$ CAPTURE AND $\beta$ DECAY IN NUCLEI WITH A PARTIALLY CLOSED $1p$ SHELL

The transition rates in processes of muon capture and $\beta$ decay in complex nuclei are determined mainly by the weak axial-vector hadronic current. The part played by the axial exchange current in weak processes on nuclei with a partially closed $1p$ shell was analyzed systematically in Refs. 104–111.

### Allowed transitions in $\mu^-$ capture

For this process, there exist 119 allowed transitions in nuclei of the $1p$ shell. If transitions with a low rate ($\sim 300$ sec$^{-1}$) and transitions leading to excited levels above the neutron-emission threshold are ignored, only seven measured transitions remain (Table IV).[105,106] In Ref. 105, stable nuclei with $A = 6,\ldots,14$ were studied. The positive-parity levels were described by means of the shell model in a restricted $1p$ subspace. The axial exchange current was obtained on the basis of the Adler-Dothan low-energy theorem.[112] The most important result of Ref. 105 was that the relative contribution of the exchange current for the individual transitions depends little on the nuclear model and that it varies smoothly with the mass number. Since the relative contribution of the exchange current is comparable with the relative uncertainty that derives from the model dependence of the impulse approximation, the contribution of the exchange current should be taken into account in improved calculations The exchange current always suppresses the transition rate beginning at the level 2% for $A = 6$ and reaching 20% for $A = 14$.

### The *ft* values for allowed $\beta$ transitions

Further possibilities for elucidating the part played by the axial exchange current are provided by study of allowed $\beta$ transitions in the region of $1p$ nuclei, for a number of which the *ft* values have been reliably measured. They were calcu-

TABLE IV. Partial rates of $\mu^-$ capture (in $\sec^{-1}$), calculated in Ref. 105 with (8–16)POT wave functions[133] in the impulse approximation ($\Lambda_{(0)}$) and with allowance for the soft-pion exchange current ($\Lambda_{(\pi)}$), $\Delta\Lambda^{\mu}(\%) = 100 \cdot (\Lambda_{(\pi)} - \Lambda_{(0)})/\Lambda_{(0)}$. A summary of the data for $\Lambda_{\rm exp}$ is given in Ref. 105.

| $A$ | Transition | $\Lambda^{\mu}_{\rm exp}$, sec$^{-1}$ | $\Lambda^{\mu}_{(0)}$ | $\Lambda^{\mu}_{(\pi)}$ | $\Delta\Lambda^{\mu}$, % |
|---|---|---|---|---|---|
| 6 | Li ($1^+$, 0; g. s.) → He ($0^+$, 1; g. s.) | $1600^{+330}_{-129}$ | 1566 | 1526 | −2.4 |
| $10_1$ | B ($3^+$, 0; g. s.) → Be ($2^+$, 1; 3.37 MeV) | ~ 1000 | 914 | 804 | −12 |
| $10_2$ | B ($3^+$, 0; g. s.) → Be ($2^+$, 1; 5.96 MeV) | ~ 4000 | 5808 | 5361 | −7.7 |
| 11 | B ($3/2^-$, 1/2; g. s.) → Be ($1/2^-$, 3/2; 0.32 MeV) | 1000±100 | 850 | 764 | −10.1 |
| 12 | C ($0^+$, 0; g. s.) → B ($1^+$, 1; g. s.) | 5700±800 or 6000±400 | 5907 | 5097 | −13.7 |
| 13 | C ($1/2^-$; 1/2; g. s.) → B ($3/2^-$, 3/2; g. s.) | 10 000±300 | 6329 | 5448 | −13.9 |
| 14 | N ($1^+$, 0; g. s.) → C ($2^+$, 1, 17.01 MeV) | 8000 6000±1500 | 23 380 | 18 943 | −19.0 |

lated with allowance for the exchange current in Ref. 109 for 40 allowed transitions—for nuclei from $^6$He to $^{14}$O. The nuclear states were described by the model of Cohen and Kurath[13] with the interaction (8–16)POT. The corrections $\Delta ft$ (%) for the one-pion exchange effect,

$$\Delta ft = 100\,[ft_{(\pi)} - ft_{(0)}]/ft_{(0)},$$

where $ft_{(0)}$ is the result of the impulse approximation, were given separately by the authors for the scalar and tensor parts of the spatial component of the current. In the case of pure Gamow-Teller transitions, the contribution of the scalar part to the $ft$ values varies smoothly with the mass number, beginning at 2% for the system with $A = 6$ and reaching 10% for nuclei with $A = 14$. The contribution of the tensor part of the exchange current depends strongly on the nuclear state. For transitions with $\log ft < 3.5$ it is always small ($\approx 4\%$), and it is positive for pure Gamow-Teller transitions and negative for mirror transitions. For $\log ft > 4.0$, the contribution of the tensor component is sometimes positive and sometimes negative and for some hindered transitions reaches unexpectedly large values:

| | $\Delta ft$, % |
|---|---|
| $^9$Li → $^9$Be ($3/2^-$, 1/2) | +84 |
| $^{12}$B (or N) → $^{12}$C ($2^+$, 0) | −54 |
| $^{13}$B → $^{13}$C ($5/2^-$, 1/2) | −76 |
| $^{13}$B → $^{13}$C($1/2^-$, 1/2)* | +300 |
| $^{14}$C (or O) → $^{14}$N ($1^+$, 0) | −63 |

Calculations with different variants of the Cohen-Kurath shell model confirm the conclusion drawn from the study of muon capture processes—that the relative contribution of the exchange corrections depends much less on the nuclear model than does the impulse approximation.

### Axial form factor for weak transitions in the $^{14}$C–$^{14}$N–$^{14}$O triad

Particular attention has been paid in the literature to $\beta$ decay of the $^{14}$C nucleus with transition to the ground state of nitrogen:

$$^{14}\mathrm{C}\,(0^+,\ 1) \to {}^{14}\mathrm{N}\,(1^+,\ 0) + e^- + \bar{\nu}_e. \tag{80}$$

It is well known that this decay is anomalously weak, although the angular momentum, parity, and isospin of the levels correspond to an allowed Gamow-Teller transition. The large value $ft = 10^{9.04}$ can be understood in the impulse approximation as due to the small overlap of the nuclear wave functions of the carbon and nitrogen ground states in the presence of the spin-flip operator into which the operator of the axial current transforms at small momentum transfers. Data on the related reactions

$$^{14}\mathrm{O} \to {}^{14}\mathrm{N} + e^+ + \nu_e;$$
$$\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C};$$
$$\gamma + {}^{14}\mathrm{N} \to \pi^+ + {}^{14}\mathrm{C};$$
$$\pi^- + {}^{14}\mathrm{N} \to \gamma + {}^{14}\mathrm{C}$$

also indicate anomalously slow transitions, the last two reactions taking place at large momentum transfers.

We now represent the axial form factor in the impulse approximation:

$$\{F_A^+(0)\}_0 = \langle {}^{14}\mathrm{N}| - g_A \sum_{i=1}^{14} \tau_i^+ (\sigma_z)_i |{}^{14}\mathrm{C}\rangle = g_A\,(N_{01}C_{00} - N_{10}C_{11}/\sqrt{3}), \tag{81}$$

where $N_{LS}$ and $C_{LS}$ are the weights of the $|LST\rangle$ configurations in $^{14}$N and $^{14}$C. Since all parametrizations[114] of the residual interaction in the $0\hbar\omega$ space determine $N_{01}$ to be near $1/\sqrt{3}N_{10}C_{11}/C_{00}$, it follows that $\{F_A^+(0)\}_{(0)}$ oscillates around zero. Thus, residual interactions of the type of Ref. 114 can be used only for a qualitative estimate of the correction $\{F_A^+(0)\}^{\rm MEC}$ to the axial form factor with the aim of establishing the most general tendencies in the behavior of the exchange effect. This correction is determined as follows[107,108]:

$$\{F_A^+(0)\}^{\rm MEC} = \langle {}^{14}N| - g_A \sum_{i \neq j} A^+_{(\pi)}(i,\ j) |{}^{14}C\rangle = -\sum_{LS,\ L'S'} N_{LS} C_{L'S'} h^{(j)}_{LS,\ L'S'}. \tag{82}$$

The rather cumbersome expressions for $h^{(j)}_{LS,L'S'}$ are given in Ref. 107. The numerical calculation showed that the matrix element of the exchange current is determined mainly ($\approx 90\%$) by the term $\sim h^{(1)}$ and remains stable. The main contribution to it is made by the graphs with excitation of the $\Delta$ isobar and $\rho$-meson exchange in the intermediate states. For the MIT wave functions,[115] which, in contrast to the others,[114] were not chosen to describe the $\beta$ decay of the $^{14}$C nucleus (and therefore lead to a nonvanishing $\{F_A^+(0)\}_{(0)}$), destructive interference is observed between the single- and two-particle contributions to the axial form factor:

$$\{F_A^+(0)\}_{(0)} + \{F_A^+(0)\}^{\rm MEC} = +0.055 - 0.032.$$

Thus, the anomalously small transition rate for the process (80) is due to a coherent superposition of the contributions of the single-particle and exchange currents.

### Effect of the spatial component of the axial exchange current in light mirror nuclei

The experimentally observed rates of Gamow-Teller $\beta$ transitions in the mirror nuclei with $A = 15$, 17, 39, 41 are less than can be obtained in calculations made in the framework of the simple shell model using the value of the axial coupling constant $g_A$ extracted from neutron $\beta$ decay ($g_A = 1.25$). To explain this hindrance, Towner and Khanna[89] made a consistent calculation of the contributions of the nuclear core polarization and the exchange current, and also took into account the relativistic corrections to the nuclear wave functions. They obtained good agreement with the data for the nuclei with $A = 15$ and 17, but for the nuclei with $A = 39$ and 41 the results of their calculations were below the data by about 25%. The calculations are sensitive to short-range correlations and to the length of the oscillator parameter. Appreciable cancellation between the configuration-mixing and exchange-current contributions is also observed.

The description of this same suppression of the $\beta$-transition rate in the framework of Landau-Migdal theory was considered with approximately the same success by Oset and Rho.[116] Their approach was based on the assumption that at least for nuclei with a closed $LS$ shell the change in the Gamow-Teller matrix element under the influence of the non-nucleon degrees of freedom is effectively due to the same mechanism that is dominant in $p$-wave pion scattering by nuclei, i.e., the formation of isobar-hole excitations. At the same time, the effect of the polarization of the nuclear core and the exchange currents is reduced to a secondary role. Oset and Rho found support for such treatment in the tritium $\beta$-decay process (68). For $P(D) = 9.3\%$ and $P(S') = 1.6$–$2\%$, $M_A = \sqrt{3}(0.917$–$0.911)$, a value that differs appreciably from the experimental data (69). Assuming that in (70) $[\delta - 2P(D)/3] \approx 0$, we obtain $M_A \approx \sqrt{3}(1 - 4P(S')/3) \approx \sqrt{3}(0.979$–$0.973)$, in good agreement with (69). However, as a detailed calculation[43,44] showed, the cancellation between the contribution of $P(D)$ and $\delta$ is only partial. When the values of $P(D)$ and $P(S')$ differ from those assumed by Oset and Rho, it is necessary to take into account all the terms in (70) in order to obtain agreement between the theory and the exact data (69).

### Nuclear density of the axial charge—chiral-symmetry filter

One of the most interesting tasks of nuclear physics is to seek general symmetry principles for the behavior of elementary particles in a baryon environment whose properties may be very different from the vacuum properties. In the light of the spontaneous breaking of chiral symmetry, the pion is clearly distinguished from the family of the remaining mesons. This circumstance has far-reaching consequences. Calculating the axial-charge density, i.e., the generator of the true chiral transformations, as the sum of the contributions of the individual nuclear nucleons (impulse approximation), one unjustifiably omits the contribution of the pionic degrees of freedom.

There are two nuclear processes in which the time component $A_4$ of the axial current is manifested and which can be experimentally measured with accuracy sufficiently high to test the chiral-filter hypothesis. The first of them corresponds to allowed Gamow-Teller $\beta$ transitions with $\Delta T = 1$, which we have mentioned in the previous parts. The second process corresponds to first-forbidden weak transitions, which we shall consider below.

Although in allowed Gamow-Teller transitions the spatial component of the axial current $\mathbf{A}$ is dominant, its time component $A_4$ can be separated by measuring electron–neutrino correlations or by using exactly measured mirror $\beta$ spectra ($T = 1$, $T_z = \pm 1 \xrightarrow{\beta\pm} T = 0$, $T_z = 0$). Low-energy data for the $0^+ \leftrightarrow 1^+$, $\Delta T = 1$ transitions in the $^{12}$B–$^{12}$C–$^{12}$N triad were studied in detail by Guichon and Samour in Ref. 45. They analyzed the reactions

$$^{12}\mathrm{N}\ (1+;\ \mathrm{g.\ s.}) \to {}^{12}\mathrm{C}\ (0+;\ \mathrm{g.\ s.}) + e^+ + \nu_e;$$
$$^{12}\mathrm{B}\ (1+;\ \mathrm{g.\ s.}) \to {}^{12}\mathrm{C}\ (0+;\ \mathrm{g.\ s.}) + e^- + \nu_e;$$
$$\mu^- + {}^{12}\mathrm{C}\ (0+;\ \mathrm{g.\ s.}) \to {}^{12}\mathrm{B}\ (1+;\ \mathrm{g.\ s.}) + \nu_\mu;$$
$$e^- + {}^{12}\mathrm{C}\ (0+;\ \mathrm{g.\ s.}) \to {}^{12}\mathrm{C}\ (1+;\ 15,\ 11\ \mathrm{MeV}) + e^-;$$
$$^{12}\mathrm{C}\ (1+;\ 15,\ 11\ \mathrm{MeV}) \to {}^{12}\mathrm{C}\ (0+;\ \mathrm{g.\ s.}) + \gamma.$$

The experimental data for $\beta^\mp$ decay determine the $\Lambda_{\beta\mp}$ decay rates and the correlation parameters $\alpha^\pm$, which contain information about the axial-charge density:

$$\alpha^\pm = (\pm F_M^\pm(0) - F_T^\pm(0)/(3MF_A^\pm(0)). \qquad (83)$$

In Ref. 45, the time form factor $F_T$ (45) was calculated initially in the impulse approximation for all variants of the phenomenological Cohen-Kurath model[113] and for the effective residual interaction of Hauge and Maripuu.[117] Comparison of the values $\{F_T(0)\}_{(0)}$ calculated for six different models of the residual interaction showed that to achieve agreement with experiment the single-particle form factor must be multiplied by $1.26 \pm 0.19$. In Ref. 45, Guichon and Samour analyzed systematically tbe significance of the various sources of inaccuracy in the description of the investigated processes. They paid particular attention to taking into account the core polarization, i.e., the contribution of the configurations lying outside the model space of the $(1s)^4(1p)^8$ configurations. They also estimated the relative effect ($\Delta^{\rm c.p.}$) of allowing for the core polarization at the level of the particle–hole approximation for the nuclear states. The numerical calculation of $\Delta^{\rm c.p.}$ showed that the core-polarization contribution suppresses the current matrix element by about 13% and thus increases the difference between the impulse approximation and the experimental $F_T(0)$ value. The calculations made at this stage also showed that $\{F_T(0)\}_{(0)}$ behaves in a stable manner with respect to significant changes in the residual interaction in the $1p$-model space.

From the ordinary calculation in the framework of Nilsson's model with a choice of the parameters appropriate to the $^{12}$C nucleus, one can obtain single-particle deformed $1p$ orbitals:

$$\left.\begin{aligned} &|1,\ k=3/2\rangle=|1p_{3/2},\quad m=3/2\rangle;\\ &|2,\ k=1/2\rangle=\alpha|1p_{3/2},\quad m=1/2\rangle-\beta|1p_{1/2},\quad m=1/2\rangle;\\ &|3,\ k=1/2\rangle=\beta|1p_{3/2},\quad m=1/2\rangle+\alpha|1p_{1/2},\quad m=1/2\rangle, \end{aligned}\right\} \tag{84}$$

where $\alpha^2+\beta^2=1$, $\alpha\approx\beta\approx0.7$–$0.8$.

In the limit $\alpha\to1$, $\beta\to0$ we obtain the case of a spherical nucleus. The matrix element of the $0^+\to1^+$ transition is now determined primarily by the matrix element $\langle 3,k\,|A_4(1)|\,2,k\rangle$. Bearing in mind the Hermitian-conjugation property of the operator

$$A_4^{\alpha}(1)=\sum_{i=1}^{12} r_i\,(\sigma_i\cdot\mathbf{P}_i)\,\tau_i^{\alpha},$$

one can readily show that

$$\begin{aligned}&\langle 3,\ k\mid A_4^{\alpha}(1)\mid 2,\ k\rangle\\ &=(\alpha^2+\beta^2)\,\langle 1p_{1/2},\ k\mid A_4^{\alpha}(1)\mid 1p_{3/2},\ k\rangle.\end{aligned} \tag{85}$$

Thus, being proportional to $\alpha^2+\beta^2$, the matrix element of the single-particle operator of the axial-charge density depends weakly on the model of the residual interaction. Therefore, the large difference between the measured axial-charge density and the density calculated in the impulse approximation cannot have a nuclear-structure origin and most probably reflects the specific nature of the transition operator. This operator is the axial charge, which in the ground state generates (in the limit $m_\pi\to0$) pions. They appear in order to ensure conservation of the axial charge.

It is now intuitively clear that $F_T(0)$ cannot be given by the sum of the nucleon contributions alone but that it is also necessary to take into account the mesonic degrees of freedom in the nucleus. In the $^{12}$B$(1^+;$ g.s.$)\to{}^{12}$C$(0^+;$ g.s.$)$ transition, allowance for long-range one-pion exchange does indeed lead to an increase in $\langle F_T(0)\rangle_{(0)}$ by about 37% at once; and this is the required effect (Table V).

Thus, it was shown in Ref. 45 that the experimental value of $F_T$ cannot be explained without inclusion of the axial exchange current in the weak nuclear Hamiltonian.

TABLE V. Time form factor $F_T(0)$ calculated in Ref. 45 with the (8–16)POT wave functions of Cohen and Kurath (CK)[113] and Hauge and Maripuu (HM)[117] in the impulse approximation (I) or with inclusion of the time component of the axial exchange current (II).

| CKI | CKII | $\Delta F_T(0)$, % | HMI | HMII | $\Delta F_T(0)$, % | Experiment |
|---|---|---|---|---|---|---|
| −2.214 | −3.025 | +36.6 | −2.051 | −2.821 | +37.5 | −2.61±0.33 |

## The ratio $g_P/g_A$ in a nuclear medium

One of the processes suitable for extracting $g_P/g_A$ is the capture of polarized muons by the $^{12}$C nucleus:

$$\mu^-+{}^{12}\mathrm{C}\,(0^+;\ \mathrm{g.\,s.})\to{}^{12}\mathrm{B}\,(1^+;\ \mathrm{g.\,s.})+\nu_\mu. \tag{86}$$

The polarization $P_{av}$ of the residual nucleus along the direction of the muon spin is very sensitive to $g_P/g_A$. The ratio $g_P/g_A$ was extracted with allowance for the contribution of the one-pion exchange current and with allowance for the effect of the core polarization in Refs. 118 and 119. The wave functions were taken from Ref. 117. The resulting value $g_P/g_A=(10.1\pm^{2.4}_{2.6})$ is in reasonable agreement with the prediction $g_P/g_A\approx7$ of the PCAC hypothesis. This result was confirmed in Ref. 111, in which $g_P/g_A$ was determined from data obtained in the muon-capture reaction in the $^{11}$B, $^{13}$C, and $^{14}$N nuclei.

## First-forbidden weak transitions in systems with $A=16$ and 18

In first-forbidden $\beta$ transitions (or in the inverse transitions in $\mu^-$ capture), the time component $A_4$ of the nuclear current is the main transition operator. As the calculation in Refs. 120 and 121 of the $ft$ values in the processes

$$^{11}\mathrm{Be}\,(1/2+;\ \mathrm{g.\,s.})\xrightarrow{\beta^-}{}^{11}\mathrm{B}\,(1/2^-;\ 2{,}125\ \mathrm{MeV})\ \text{and}$$
$$^{15}\mathrm{C}\,(1/2+;\ \mathrm{g.\,s.})\xrightarrow{\beta^-}{}^{15}\mathrm{N}\,(1/2^-;\ \mathrm{g.\,s.})$$

showed, the matrix element of the transition operator is determined mainly by its tensor component of zeroth rank. In Refs. 120 and 121, the contributions of the zeroth, $f^{(0)}$, and first, $f^{(1)}$, rank to the $ft$ values were calculated in the impulse approximation. In them there is a detailed discussion of the strong dependence of $f^{(0)}$ on the choice of the radial dependence of the single-particle wave function. The radial dependence that arises for the Woods-Saxon potential leads, in contrast to the harmonic-oscillator radial functions, to $f^{(0)}$ values that are 2–3 times smaller.

*Weak isovector $0^+\to0^-$, $\Delta T=1$ transition in the $A=16$ system.* Very valuable information about the contribution of meson exchange effects in $A_4$ can be obtained by studying the weak processes

$$\left.\begin{aligned} &^{16}\mathrm{N}\,(0^-;\ 1)\to{}^{16}\mathrm{O}\,(0^+,\ 0)+e^-+\bar\nu_e;\\ &\mu^-+{}^{16}\mathrm{O}\,(0^+,\ 0)\to{}^{16}\mathrm{N}\,(0^-,\ 1)+\nu_\mu;\\ &^{16}\mathrm{C}\,(0^+,\ 2)\to{}^{16}\mathrm{N}\,(0^-,\ 1)+e^-+\bar\nu_e;\\ &^{18}\mathrm{Ne}\,(0^+,\ 1)\to{}^{18}\mathrm{F}\,(0^-,\ 0)+e^++\nu_e. \end{aligned}\right\} \tag{87}$$

Possible here are only two single-particle operators ($\gamma_5$ and $\boldsymbol{\sigma}\cdot\mathbf{r}$), corresponding to the time and spatial parts of the axial current. Characteristic of the $0^+ \leftrightarrow 0^-$ transition in $\beta$ decay is the fact that the matrix elements of these operators tend to cancel each other, thus increasing the relative contribution of the exchange current to the transition rate $\Lambda_\beta$. In $\mu^-$ capture, we have the opposite situation, since the single-particle contributions are added.

The $0^+ \leftrightarrow 0^-$ transition has the further feature that $\Lambda_\mu$ is very sensitive to the ratio $g_P/g_A$.[122]

*Results of the soft-pion model.* The $0^+ \leftrightarrow 0^-$ processes were considered for the first time with allowance for the mesonic degrees of freedom in the $A=16$ system by Guichon, Giffon, and Samour.[123] Using the simplest scheme for describing the nuclear states (closed $1p$ shell for the $^{16}O$ ground state and particle–hole configuration $2s^1_{1/2}\,1p^{-1}_{1/2}$ for the $0^-$ state in $^{16}N$), they showed that the soft-pion exchange current leads to an increase in the contribution of the axial-charge density by 60%. In this way, the $\Lambda_\mu/\Lambda_\beta$ ratio is brought into agreement with the data for $g_P/g_A \approx 7$–8. Although the ratio $\Lambda_\mu/\Lambda_\beta$ is described satisfactorily, the values of $\Lambda_\mu$ and $\Lambda_\beta$ are too large. In Ref. 124, an attempt was made to improve the $^{16}O$ ground-state wave function by adding to the closed $|0p\text{–}0h\rangle$ core a $2p$–$2h$ admixture in accordance with the scheme of Brown and Green.[125] As a result, $\Lambda_\mu$ and $\Lambda_\beta$ were somewhat reduced, but so was the relative contribution of the exchange current. At the same time, a Japanese group[126] suggested that the ratio $\Lambda_\mu/\Lambda_\beta$ can be correctly described by using only a more accurate description of the leptonic part of the weak current and taking into account the correlations in the ground state.

Using the technique of expansion with respect to bound valence clusters, Brandow, Towner, and Khanna[127] considered the influence on $\Lambda_\mu$ and $\Lambda_\beta$ of six different models for the effective residual interaction. Effectively, they calculated the renormalization of the operator of the axial-charge density with allowance for the soft-pion exchange current in the language of perturbation theory in second order, extending thus the $0\hbar\omega$ basis to the subspace of $2\hbar\omega$ excitations. In Ref. 127, it was also shown that systematic description of $2p$–$2h$ admixtures in the wave function of the $0^+$ state leads to the elimination of the destructive interference of the $|0p\text{–}0h\rangle$ and $|2p\text{–}2h\rangle$ contributions[124] and to the reestablishment of a large relative contribution of the one-pion exchange current for all forms of interaction. For one of them (OBEP), it is possible to describe not only the ratio but also the partial transition rates for $g_P/g_A = 10$. The leptonic part of the current then plays a secondary role.

The calculation of Towner and Khanna was improved in Refs. 47 and 53 by extension of the subspace of $2\hbar\omega$ excitations and inclusion in the treatment of processes with excitation of the $\Delta$ isobar. The OBEP potential from Ref. 127 was used. The upshot was that the result of Towner and Khanna was not changed, although the estimate of the contribution of the graph with $\Delta$ excitation in Ref. 47 was clearly too large (see Sec. 1).

In Ref. 127, the important question of the part played by the exchange of heavy vector mesons was not considered at all. It was first attacked in Ref. 48, which estimated the possible contribution of the graph with $\rho$-meson exchange at the level of about 20% of the main pair term. We now turn to a detailed consideration of this question.

*Short-range correlations and hard-pion model.* The influence of short-range effects on the two-particle axial-charge density in the $^{16}O(0^+,1) \leftrightarrow {}^{16}N(0^-,0)$ transition was studied in Refs. 46, 51, and 52. To describe consistently the exchange of the vector mesons $\rho$ and $A_1$, the hard-meson model (Refs. 25, 32, 34, and 35) is here used. The axial current constructed in the framework of this model is given in Eqs. (27)–(33).

We denote the long-range one-pion exchange corresponding to the first sum in (32) by $A^4_{(\pi')}$. We denote the heavy-meson exchange corresponding to the second sum in (32) by $A^4_{(\rho A_1)}$. Then the total current (32) can be written in the form

$$A^4_{(\rho A_1\pi)} = A^4_{(\pi')} + A^4_{(\rho A_1)}.$$

For the soft-pion current, we retain the notation $A^4_{(\pi)}$.

In Ref. 46, the matrix elements of the current were calculated using the wave functions of the shell model with configuration mixing that were obtained in Ref. 128 after diagonalization of the separable Tabakin interaction. The $^{16}O$ ground state contains all possible $0\hbar\omega + 2\hbar\omega$ excitations. The $0^-$ state contains two very strong (each about 1%) $2p$–$2h$ components. The calculation showed that the destructive interference of the small $|2p\text{–}2h\rangle$ contributions leads to the simple relation

$$\begin{aligned}&{}_{(1\hbar\omega+3\hbar\omega)}\langle 0^-,\ 1|A^4|0^+,\ 0\rangle_{(0\hbar\omega+2\hbar\omega)}\\ &\approx \alpha_0\beta_0\,\langle 2s^1_{1/2}1p^{-1}_{1/2},\ 0^-,\ 1|A_u|0p-0h\rangle,\end{aligned} \tag{88}$$

which contains only the principal components of the wave functions with weights $\alpha_0$ and $\beta_0$, respectively. One can now demonstrate the sensitivity of the observables to the different parts of the operator $A^4$, studying only the right-hand side of (88).

The relative contribution of heavy-meson exchange is found to be 10% for $\mu^-$ capture and 20% for $\beta$ decay[46]; these are rather large values. However, in the model $0\hbar\omega + 2\hbar\omega$ subspace of configurations there are no two-particle correlations resulting from the core in the $NN$ potential. They can be introduced by multiplying the shell wave functions by the Miller-Spencer[129] factor $(1-\Sigma_{i<j} f_{ij})$. The function $f_{ij}$ is chosen for the nuclei of the $1p$ shell in the form[129]

$$f_{ij} = f\,(r \equiv |\mathbf{r}_i - \mathbf{r}_j|) = \exp(-\alpha r^2)\,(1-\beta r^2),$$
$$\alpha = 1.1\ \mathrm{F}^{-2}, \qquad \beta = 0.68\ \mathrm{F}^{-2}.$$

Recalculation of the nuclear matrix element with inclusion of the two-particle correlations[51] shows that the contribution of heavy-meson exchange to $\Lambda_\mu$ and $\Lambda_\beta$ is reduced by an order of magnitude, i.e., to 3 and 6%, respectively (Table VI). At the same time, the matrix element $\langle A^4_{(\pi')}\rangle$ is reduced under the influence of the two-particle correlations by about 25%, and the sum

$$\langle A^4_{(\rho A_1\pi)}\rangle > \langle A^4_{\pi'}\rangle + \langle A^4_{(\rho A_1)}\rangle$$

TABLE VI. Contributions to the matrix element $\langle 0^-, T=1|A^4|0^+, T=0\rangle$ with and without allowance for the correlation function $f(r)$, the hadronic form factors $F$, and the polarization $\Pi$ of the medium for $b = 1.7$ F.[52]

| Matrix element | Term of operator | | |
|---|---|---|---|
| | without allowance for $f$ | with allowance for $f$ | with allowance for $f+F+\Pi$ |
| $\langle A^4_{(\pi')}\rangle$ | −0,151 | −0,113 | −0.106 |
| $\langle A^4_{(\rho A_1)}\rangle$ | 0.037 | 0.009 | 0.003 |
| $\langle A^4_{(\rho A_1 \pi)}\rangle$ | −0.114 | −0.104 | −0.102 |
| $\langle A^4_{(\pi)}\rangle$ | −0,151 | — | −0,127 |

remains fairly stable with respect to variations in the wave functions (Table VI). We note that the results of Ref. 51 confirm Towner and Khanna's conclusion concerning the value of $g_P/g_A$. It is also equal to 10.

In Ref. 52, the short-range effects were augmented by considering the influence of the vertex form factors $F$. In addition, the influence of the polarization of the nuclear medium was considered by means of the scheme proposed in Ref. 130. The following polarization operator was introduced into the meson propagators:

$$\Pi(q) = \Pi_0(q)/(1 + g'U(q)), \quad \Pi_0(q) = -q^2 U(q).$$

Here, $q$ is the momentum of the virtual meson, and $g' \approx 0.6$. Ignoring the $q$ dependence of the function $U(q)$ and taking values corresponding to the case of nuclear matter, $U^\pi(0) \approx 0.88$, $U^\rho(0) \approx 1.1$, we obtain for the monopole form factors the expression[130]

$$F_B(q) = (\Lambda_B^2 - m_B^2)/[(\Lambda_B^2 + q^2)(1 + g'U^B(0))], \quad B = \pi, \rho. \tag{89}$$

In the actual calculations, the values $\Lambda_\pi = 1.18$ GeV and $\Lambda_\rho = 2.0$ GeV were taken. In this scheme, the two particle operator of the axial-charge density of the hard-pion model is obtained from (32) by the substitution

$$\left.\begin{aligned}
&\phi(m_\pi^2)\,Y_1(m_\pi r) \to \frac{m_\pi^{*2}}{m_\pi^2(1-(1-g')U^\pi(0))}\\
&\times\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - m_\pi^{*2}}\right)\Big[\phi(m_\pi^{*2})\,Y_1(m_\pi^* r) - \frac{\Lambda_\pi^2}{m_\pi^{*2}}\,\phi(\Lambda_\pi^2)\,Y_1(\Lambda_\pi r)\\
&-\frac{\Lambda_\pi^2 - m_\pi^{*2}}{2m_\pi^{*2}}\,\phi(\Lambda_\pi^2)\,e^{-\Lambda_\pi r}\Big];\\
&C(m_\rho^2)\,Y_1(m_\rho r) \to \frac{m_\pi^2 C(\Lambda_\rho^2)(\Lambda_\rho^2 - m_\rho^2)^2}{2m_\pi^{*2} m_\rho^4 (1+g'U\rho(0))}\,e^{-\Lambda_\rho r};\\
&m_\pi^{*2} = \frac{1+g'U^\pi(0)}{1-(1-g')U^\pi(0)}\,m_\pi^2;\quad \phi(y^2) = \frac{1}{2} + \frac{1}{2}C(y^2);\\
&C(y^2) = \frac{m_\rho^2}{m_\rho^2 - m_\pi^2}\left(1 + \frac{\varkappa_V y^2}{4m_\rho^2}\right)\left(1 + \frac{y^2}{m_\rho^2}\right).
\end{aligned}\right\} \tag{90}$$

It can be seen from (90) that the long-range part of the hard-pion current differs from the soft-pion part by factors $\phi(y^2) \neq 1$. Allowance for polarization of the medium and introduction of hadronic form factors leads to an increase of this difference, since $\phi(m_\pi^{*2}) \approx 1.1$ and $\phi(\Lambda_\pi^2) \approx 5$. As can be seen from Table VI, the total effect consists of a reduction of $\langle A^4_{(\pi')}\rangle$ by about 30%, compared with 15% for $\langle A^4_{(\pi)}\rangle$ and 10% for $\langle A^4_{(\rho A_1 \pi)}\rangle$.

The calculation of the absolute values of the partial transition rates and their comparison with the data (Table VII) are in this case too somewhat difficult on account of the

TABLE VII. Partial rates ($sec^{-1}$) for the isovector transition $^{16}O(0^+, 0) \underset{\beta^-}{\overset{\mu^-}{\rightleftarrows}} {}^{16}N(0^-, 1)$, obtained by different authors. The experimental value of the ratio is $(\Lambda_\mu/\Lambda_\beta)_{exp} = (3.8 \pm 0.8)\cdot 10^3$ in accordance with Refs. 131a and 133 or $(\Lambda_\mu/\Lambda_\beta) = (2.62 \pm 0.35)\cdot 10^3$ in accordance with Refs. 132 and 133, and $R \equiv (\Lambda_\mu/\Lambda_\beta)_{theor}$. The residual interaction is given in the notation of the authors. For the transition rates there are the following data: $\Lambda_\mu = (1.57 \pm 0.10)\cdot 10^3\ sec^{-1}$ (Ref. 133), $\Lambda_\beta = (0.45 \pm 0.05)\ sec^{-1}$ (Ref. 131a), $(0.43 \pm 0.10)\ sec^{-1}$ (Ref. 131b), $(0.60 \pm 0.07)\ sec^{-1}$ (Ref. 132).

| Reference | Residual interaction | $\Lambda_\mu/10^3$ | $\Lambda_\beta$ | $R/10^3$ | $g_P/g_A$ | $b$, F |
|---|---|---|---|---|---|---|
| [127] | $\delta+\pi+\rho$ | 0,52 | 0,08 | 6,4 | 7.0 | 1,769 |
| | OBEP | 2,00 | 0,42 | 4,8 | 7.0 | 1.769 |
| | *G* matrix a | 1,40 | 0,26 | 5,3 | 7.0 | 1,769 |
| | OBEP | 1,58 | 0,42 | 3.8 | 10.0 | 1.769 |
| [52] | Tabakin | 1,92 | 0,53 | 3,6 | 10,5 | 1,769 |
| [53] | OBEP | 1,8 | 0,48 | 3.8 | 7.0 | 1,769 |
| [52] | Tabakin | 2,05 | 0,64 | 3,2 | 10.5 | 1.7 |

uncertainties of nuclear origin. In the case of $\beta$ decay, a change in the oscillator length parameter $b$ by 10% leads to a change in $\Lambda_\beta$ by 50%. For this reason it is not easy on the basis of the calculation in the single-particle oscillator basis to distinguish the data of Ref. 131 from the results of the measurement[132] for the $\beta$-transition rate, which differ by 50%.

The stability of the matrix element of the exchange current with respect to changes in the model of the nuclear structure (see Table VI) is in no doubt, and this is a strong argument for the existence of an effect of virtual pions in weak processes induced by the time component of the axial current, as assumed by the chiral-filter hypothesis.

#### Isovector $0^+ \to 0^-$, $\Delta T = 1$ transition in the $A = 18$ system

Evidence for an exchange effect is also provided by the study of the contribution of the axial-charge density to the $\beta^+$-transition rate

$$^{18}\mathrm{Ne}\ (0^+,1;\ \mathrm{g.\ s.}) \to {}^{18}\mathrm{F}\ (0^-,0;\ 1.081\,\mathrm{MeV})$$

made in Refs. 134 and 135. The calculation included all $0\hbar\omega + 1\hbar\omega + 2\hbar\omega$ configurations and the two-particle Miller–Spencer correlations.[129] It was shown that the contribution of the soft-pion exchange current increases the transition rate $\Lambda_\beta^{\mathrm{th}} = 4.8 \cdot 10^{-6}\ \mathrm{sec}^{-1}$ by about a factor 2.5 compared with the result of the impulse approximation:

$$\Lambda_\beta^{IA} = 1.8 \cdot 10^{-6}\ \mathrm{sec}^{-1}.$$

The experimental value of the transition rate is

$$\Lambda_\beta^{\mathrm{exp}} = (8.6 \pm 1.2) \cdot 10^{-6}\ \mathrm{sec}^{-1}\ (\text{Refs. 135 and 136}).$$

We note that the rate measured in Ref. 135 for the $\beta^+$ transition $^{19}\mathrm{Ne}(1/2^+,\ 1/2;\ \mathrm{g.s.}) \to {}^{19}\mathrm{F}(1/2^-,\ 1/2;\ 110\ \mathrm{keV})$,

$$\Lambda_\beta^{\mathrm{exp}} = (4.8 \pm 0.8) \cdot 10^{-6}\ \mathrm{sec}^{-1},$$

is suppressed by an order of magnitude compared with calculations made only in the $0\hbar\omega + 1\hbar\omega$ subspace.[135] By analogy with the transition in $A = 18$, one can expect that the inclusion of $2\hbar\omega$ configurations will reduce significantly the matrix elements of the single- and two-particle operators of the axial charge and bring the calculation into reasonable agreement with the data.

The isobar-analog state of the neon ground state in fluorine is somewhat below the $(0^-,\ 0;\ 1.081\ \mathrm{MeV})$ state, namely, at energy $E_{(0^+,1)} = 1.042$ MeV. When this state is de-excited, one observes experimentally an asymmetry of the circular polarization of the $\gamma$ rays, $P_\gamma \sim \langle 0^+,\ 1;\ 1.042\ \mathrm{MeV}\ |V_{\mathrm{PNC}}|0^-,\ 0;\ 1.081\ \mathrm{MeV}\rangle$. Using the circumstance that[134,135]

$$\langle {}^{18}\mathrm{F}\ (0,^-0;\ 1.081\ \mathrm{MeV})\,|\,A_{(\pi)}^4\,|\ {}^{18}\mathrm{Ne}\ (0^+,1;\ \mathrm{g.\ s.})\rangle$$
$$\sim\ \langle {}^{18}\mathrm{F}\ (0^-,0;\ 1.081\ \mathrm{MeV}\ |\ V_{\mathrm{PNC}}\ |\ {}^{18}\mathrm{F}\ (0^+,1;\ 1.042\ \mathrm{MeV})\rangle\,,$$

it is possible from the data for the exchange current in the $\beta^+$-decay process to determine the circular polarization for the photodecay process. However, in the more realistic approach of the hard-pion model the current $A^4_{(\rho A_1\pi)}$ has a structure different from that of the soft-pion current $A^4_{(\pi)}$. This difference becomes appreciable after allowance has been made for the finite size of a nucleon and the polarization of the medium. As a result, the analogy with the matrix element of the parity-nonconserving potential $V_{\mathrm{PNC}}$ is somewhat lost. Using the matrix element of the current $A^4_{(\pi)}$ to determine $P_\gamma$, we can obtain only certain bounds for the fundamental constants of the weak $NN$ potential:

A large effect of the time component of the axial exchange current can be expected in the transition $^{16}\mathrm{C}(0^+,\ 2) \to {}^{16}\mathrm{N}(0^-,\ 1) + e^- + \bar{\nu}_e$. A simple estimate of the transition rate in the impulse approximation and using simple shell wave functions reproduces[137] only a sixth of the experimental value of the transition rate measured in Ref. 137.

#### CONCLUSIONS

We give here the main conclusions obtained by the critical analysis of the material discussed in the review:

1. In the lightest nuclei, there is just one well-studied case of a weak process in which the spatial component of the axial current is clearly manifested. The accurately measured Gamow-Teller matrix element in the $\beta$ decay of the tritium nucleus can be explained only by taking into account the contribution of this component. It is about 5% of the contribution of the impulse approximation. An effect of approximately the same magnitude can also be expected in other weak reactions in the lightest nuclei.

2. The study of the influence of the spatial component of the axial exchange current in allowed transitions in $\mu^-$ capture and in the $\beta$ decay of nuclei of the $1p$ shell $(6 \leqslant A \leqslant 14)$ using simple shell wave functions is qualitative in nature. It has been shown that the relative contribution of this component to the transition rate depends weakly on the employed nuclear model. The contribution of the exchange current always suppresses the transition rate, beginning at 2% for $A = 6$ and reaching 20% and 10% for $A = 14$ in the case of $\mu^-$ capture and $\beta$ decay, respectively.

In the more realistic calculation of the supression of the $\beta$-transition rate in mirror nuclei with $A = 15, 17, 39, 41$ we observe an appreciable cancellation between the configuration-mixing contribution and the effect of the exchange current, the agreement with the data being satisfactory. However, similar results can be obtained by using the mechanism of formation of isobar-hole excitations and configuration mixing in the space of $2\hbar\omega$ excitations.

3. The time form factor separated by measuring the electron–neutrino correlations in allowed Gamow-Teller transitions with $\Delta T = 1$ in the $^{12}\mathrm{B}$–$^{12}\mathrm{C}$–$^{12}\mathrm{N}$ traid cannot be explained without allowance for a 40% contribution of the time component of the axial exchange current. An effect of the same component is also manifested in the first-forbidden $1^+ \leftrightarrow 1^-$, $\Delta T = 1$ weak transitions. In $\mu^-$ capture in the $^{16}\mathrm{O}(0^+,\ 0)$ nucleus, it increases the transition rate by about 40%; in the inverse process of $\mu^-$ decay of the $^{16}\mathrm{N}(0^-,\ 1)$ nucleus and in the $\beta$ decay of the $^{18}\mathrm{Ne}(0^+,\ 1)$ nucleus, this effect is greater than 150%, and the exchange correction significantly improves the agreement between the theory

and the data. In all the considered cases, the relative contribution of the exchange corrections behaves in a stable manner with respect to changes in the model of the nuclear structure.

4. The conclusions of the hard-pion model have shown that the dominant contribution to the axial-charge density is made by the current of long-range one-pion exchange. For this reason, weak processes in nuclei present a rare possibility of studying the manifestation of pions in a baryon environment.

The current of the long-range one-pion exchange in the hard-pion model differs in its structure from the soft-pion current. This difference becomes important after consideration of the finite dimensions corresponding to the meson–nucleon vertices and the polarization of the nuclear medium. It is responsible for the somewhat smaller (by about 20%) renormalization of the single-particle axial-charge density compared with the low-energy theorems.

5. Allowance for the mesonic degrees of freedom is necessary to achieve an accuracy of description of the processes permitting extraction from the data of reliable information about the weak-interaction constants. Namely, separation of the ratio $g_P/g_A$ from experiments measuring the polarization of the recoil nuclei in $\mu^-$ capture in the $^{12}C$, $^{13}C$, and $^{14}N$ nuclei leads to the value $g_P/g_A = 10.1 \pm^{2.4}_{2.6}$, which differs only slightly from the vacuum value $g_P/g_A = 7$–8.

[1)] Note that in accordance with Ref. 7 the error in the determination of $\eta$ is much greater and the model dependence of $\eta$ is stronger than was assumed in Ref. 8. However, the exact value of $Q$ alone is sufficient to justify the conclusions drawn in Ref. 6.


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