The pion-nucleon σ term: state of the problem

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The problem of establishing the mechanism of breaking of SU(3) ⊗ SU(3) chiral symmetry and estimating the pion-nucleon σ term, which is the parameter of this breaking, is the subject of this state-of-the-problem review. The main methods of determining the pion-nucleon σ term are reviewed. The possible reasons for the large spread in the existing estimates of $\sigma_{\pi N}$ are investigated, and the main uncertainties of the various methods are established. The analysis leads to the establishment of an acceptable interval for $\sigma_{\pi N}$ from 30 to 50 MeV, this being in agreement with the (3,3*) ⊕ (3*,3) mechanism for breaking the SU(3) ⊗ SU(3) chiral symmetry in accordance with the Gell-Mann–Oakes–Renner model and QCD.

INTRODUCTION

The chiral group SU(3) ⊗ SU(3) as an approximate symmetry group for hadrons

The point of departure for models of the πN interaction in the framework of current algebra was the prediction, satisfactorily confirmed by experiment, by Weinberg[1] of the relationships $a_1 - a_3 = 0.3 m_\pi^{-1}$ and $a_1 + a_3 = 0$ for the s-wave scattering lengths, which he derived using the current commutation relations[2] and the hypothesis of partial conservation of the axial current (PCAC).[3] The idea of approximate SU(2) ⊗ SU(2) symmetry provides the simplest way of understanding why the pion masses are small. The picture of the strong interactions that is obtained in the light of this symmetry corresponds to a Hamiltonian containing two terms:

$$H = H_0 + \varepsilon H_1, \tag{1}$$

where H_0 is a scalar with respect to the group SU(2) ⊗ SU(2), and the term εH_1 is such that in the limit $\varepsilon \to 0$ the pion mass tends to zero, $m_\pi \to 0$. In this framework, the SU(2) ⊗ SU(2) algebra, generated by the vector, Q_α, and axial-vector, $\overline{Q}_\alpha$ ($\alpha = 1,2,3$), charges, can be regarded as the algebra of an approximate symmetry group; one can then speak of corrections proportional to

$$\overline{D}_\alpha = i\varepsilon\, [H_1, \overline{Q}_\alpha], \tag{2}$$

where $\overline{D}_\alpha = \partial^\mu A_\mu^\alpha(x)$ is the divergence of the axial-vector current. The characteristic scale of the SU(2) ⊗ SU(2) symmetry breaking is 10–15%. This estimate is determined with the accuracy to which the Goldberger-Treiman relation[3] is satisfied experimentally.

At the same time, an example of successful use of the symmetry-breaking principle for the description of elementary particles is provided by the SU(3) symmetry group.[2,4,5] Exact SU(3) symmetry would mean that the particle multiplets are degenerate with respect to the mass (examples: the baryon octet or the octets of vector and pseudoscalar mesons). In fact, the SU(3) symmetry must be broken to the extent needed to obtain the experimentally observed mass spectrum. An example of the impressive achievements of SU(3) symmetry is provided by the various mass formulas and classification schemes, and also Cabibbo's theory of weak decays.[6] This theory is a consequence of the fact that, as is shown experimentally, the weak and electromagnetic hadronic currents generate the algebra SU(3) ⊗ SU(3).[7,8]

It is thus natural to assume that the group SU(3) ⊗ SU(3) will be an approximate symmetry group for the hadrons.

In field theory, currents and charges are usually introduced in the standard manner. For a field theory with a Lagrangian density $L(\varphi, \partial\varphi/\partial x_\mu)$ the Euler-Lagrange equations (equations of motion) have the form

$$\partial_\mu \pi^\mu = \delta L/\delta\varphi,$$

where $\pi^\mu = \delta L/\delta(\partial\varphi/\partial x^\mu$ is the canonical 4-momentum.

We consider an infinitesimal transformation of the fields:

$$\varphi(x) \to \varphi(x) + \delta\varphi(x).$$

This transformation will be a symmetry if without explicit use of the equations of motion the variation of the Lagrangian is the total derivative of some function:

$$\delta L = \frac{\delta L}{\delta\varphi}\delta\varphi + \frac{\delta L}{\delta\left(\frac{\partial\varphi}{\partial x_\mu}\right)}\delta\left(\frac{\partial\varphi}{\partial x_\mu}\right)$$

$$= \frac{\delta L}{\delta\varphi}\delta\varphi + \pi_\mu \partial^\mu \delta\varphi = \partial_\mu \Lambda^\mu$$

(we consider transformations for which the operations δ and $\partial/\partial x$ commute). Then variation of the Lagrangian does not change the action $I = \int d^4x L(x)$, and in this case a conserved quantity can be introduced.

Using the equation of motion explicitly, we have

$$\delta L = \partial_\mu \pi^\mu \delta\varphi + \pi_\mu \partial^\mu \delta\varphi = \partial_\mu(\pi^\mu \delta\varphi),$$

or

$$0 = \partial_\mu(\pi^\mu \delta\varphi - \Lambda^\mu),$$

i.e.,

$$J_\alpha^\mu = \pi^\mu \delta\varphi/\delta\varepsilon^\alpha - \delta\Lambda^\mu/\delta\varepsilon^\alpha \tag{3}$$

is a conserved quantity. If the system possesses an integral symmetry, then $\Lambda^\mu = 0$.

For the Lagrangian of the free quark fields

$$L = i\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi$$

the transformations of the SU(3) ⊗ SU(3) chiral symmetry are $\psi \to e^{-i\varepsilon\alpha\lambda_\alpha/2}\psi$, $\delta\varphi/\delta\varepsilon^\alpha = -i\lambda_\alpha/2\psi$, and $\psi \to e^{i\varepsilon^\alpha \gamma_5 \lambda_\alpha/2}\psi$, $\delta\varphi/\delta\varepsilon^\alpha = -i\gamma_5\lambda_\alpha/2\psi$, where λ_α are ordinary matrices of the SU(3) algebra.[9]

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To these transformations there correspond eight vector, $V_\mu^\alpha = \bar{\psi}\gamma_\mu\lambda_\alpha/2\psi$, and eight axial-vector, $A_\mu^\alpha = \bar{\psi}\gamma_\mu\gamma_5\lambda_\alpha/2\psi$, currents (the canonical 4-momentum being $\pi_\mu = i\bar{\psi}\gamma_\mu$).

Using the canonical anticommutation relations

$$\left.\begin{aligned} &\{\psi_i(\mathbf{x}, t), \psi_j(\mathbf{x}, t)\} \\ &=\{\psi_i^\dagger(\mathbf{x}, t), \psi_j^\dagger(\mathbf{x}, t)\} = 0; \\ &\{\psi_i^\dagger(\mathbf{x}, t), \psi_i^\dagger(\mathbf{y}, t)\} = \delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}) \end{aligned}\right\} \tag{4}$$

for the quark fields, and for the matrices λ_α the commutation relations

$$[\lambda_\alpha, \lambda_\beta] = 2if_{\alpha\beta\gamma}\lambda_\gamma, \tag{5}$$

we can show that the charges

$$\left.\begin{aligned} Q^\alpha(t) &= \int d^3x\, V_0^\alpha(\mathbf{x}, t); \\ \bar{Q}^\alpha(t) &= \int d^3x\, A_0^\alpha(\mathbf{x}, t) \end{aligned}\right\} \tag{6}$$

are generators of the corresponding transformations, i.e.,

$$\exp(i\varepsilon_\alpha Q^\alpha)\,\psi\,\exp(-i\varepsilon_\alpha Q^\alpha) = \exp(-i\varepsilon^\alpha\lambda_\alpha/2)\,\psi;$$

$$\exp(i\varepsilon_\alpha \bar{Q}^\alpha)\,\psi\,\exp(-i\varepsilon_\alpha \bar{Q}^\alpha) = \exp(-i\varepsilon^\alpha\gamma_5\lambda_\alpha/2)\,\psi$$

and generate the $SU(3)\otimes SU(3)$ algebra

$$\left.\begin{aligned} &[Q^\alpha(t), Q^\beta(t)] = if_{\alpha\beta\gamma}Q^\gamma(t); \\ &[Q^\alpha(t), \bar{Q}^\beta(t)] = if_{\alpha\beta\gamma}\bar{Q}^\gamma(t); \\ &[\bar{Q}^\alpha(t), \bar{Q}^\beta(t)] = if_{\alpha\beta\gamma}Q^\gamma(t). \end{aligned}\right\} \tag{7}$$

This algebra is isomorphic to the product $SU(3)_L \otimes SU(3)_R$ of two mutually commuting $SU(3)$ algebras:

$$\left.\begin{aligned} &[Q_L^\alpha, Q_L^\beta] = if_{\alpha\beta\gamma}Q_L^\gamma; \\ &[Q_R^\alpha, Q_R^\beta] = if_{\alpha\beta\gamma}Q_R^\gamma; \\ &[Q_L^\alpha, Q_R^\beta] = 0, \end{aligned}\right\} \tag{8}$$

whose generators are

$$\left.\begin{aligned} Q_L^\alpha(t) &= 1/2\,\{Q^\alpha(t) - \bar{Q}^\alpha(t)\}, \\ Q_R^\alpha(t) &= 1/2\,\{Q^\alpha(t) + \bar{Q}^\alpha(t)\}, \end{aligned}\right\} \tag{9}$$

and basic vectors are, respectively, the left and right quarks

$$q_R^L = 1/2\,(1 \mp \gamma_5)\,q.$$

Further, q_R is invariant under the $SU(3)_L$ transformation and vice versa. Thus, the mass term of the Lagrangian of the free quark fields (and the Lagrangian of QCD) transforms in accordance with the $(3^*,3)\oplus(3,3^*)$ representation of the group $SU(3)\otimes SU(3)$:

$$\bar{q}Mq = \bar{q}_L M q_R + \bar{q}_R M q_L. \tag{10}$$

Exact $SU(3)\otimes SU(3)$ symmetry predicts the existence of $SU(3)$ multiplets degenerate with respect to the masses of the particles (baryons, vector mesons, etc.) and eight massless pseudoscalar mesons (Goldstone bosons) π, K, and η. These mesons must satisfy low-energy theorems,[7] by means of which one can, in particular, obtain certain conclusions about the nature of the meson-baryon interactions. The $SU(3)\otimes SU(3)$ summetry also leads to predictions such as the generalized Goldberger-Treiman relations[10,11] (relations between the masses of the baryons, the meson-baryon coupling constants, the axial-vector constants, and the meson decay constants), the Adler-Weisberger sum rule (renormalization of the axial-vector constants by the strong interactions), and the Callan-Treiman relation (which relates a certain combination of form factors for $K\to\pi + l + \bar{\nu}_1 (1 = e,\mu)$ to the K_{l_2} decay constants).

In the real world, chiral symmetry is approximate, so that there exist $SU(3)$ particle multiplets with mass splitting, and the eight low-mass mesons π, K, and η satisfy the approximate PCAC conditions, so that the low-energy theorems mentioned above are approximately valid. The most sensitive to the symmetry-breaking mechanism are the corrections to the low-energy theorems. These theorems, which relate the chiral-symmetry breaking part of the Hamiltonian to the scattering amplitudes of soft mesons (i.e., mesons of zero mass), become exact in the limit in which the meson masses tend to 0 and the axial-vector currents are conserved.

PCAC hypothesis

As was pointed out, the physical consequences of the partial conservation of the axial-vector currents $A_\mu^\alpha (\alpha = 1,2,3)$ are related to the small values of the pion masses. By virtue of Lorentz invariance, the matrix element of the axial-vector current between the single-pion and vacuum states has the form

$$\langle 0 \mid A_\mu^\alpha(x) \mid \pi^\beta\rangle = i\delta_{\alpha\beta} f_\pi q_\mu e^{-iqx}, \tag{11}$$

where f_π is the pion decay $(\pi\to\mu+\bar{\nu}_\mu, \pi\to e+\bar{\nu}_e)$ constant. Taking the divergence of both sides of Eq. (11), for on-shell pions we have

$$\langle 0 \mid \bar{D}^\alpha(0) \mid \pi^\beta\rangle = \delta_{\alpha\beta} f_\pi m_\pi^2. \tag{12}$$

The relation (12) is equivalent to the field-theory version of PCAC[3]:

$$\varphi_\alpha(x) = \frac{1}{f_\pi m_\pi^2}\frac{\partial A_\alpha^\mu(x)}{\partial x^\mu}, \tag{13}$$

where $\varphi_\alpha(x)$, the operator of a pseudoscalar field with the quantum numbers of the pion, is such that

$$\langle 0 \mid \varphi_\alpha(0) \mid \pi_\beta\rangle = \delta_{\alpha\beta}.$$

It can be expected that the error arising from the simultaneous use of PCAC and the formulas valid in the soft-pion limit is not large (about 10%).

Noninvariance of the vacuum and Goldstone's theorem

In the limit of exact $SU(3)\otimes SU(3)$ symmetry, all the 16 vector and axial-vector currents are conserved, i.e.,

$$[Q^\alpha, H] = 0 \text{ and } [\bar{Q}^\alpha, H] = 0.$$

Then, if the charges $\bar{Q}^\alpha$ do not leave the vacuum invariant, i.e., $\bar{Q}^\alpha|0\rangle \neq 0$, whereas $\bar{Q}^\alpha|0\rangle = 0$, Goldstone's theorem[12,13] predicts the existence of an octet of massless pseudoscalar mesons. In this case, the ground state is noninvariant with respect to the group of continuous transformations with the generators $\bar{Q}^\alpha$, and spontaneous symmetry breaking arises. Massless and spinless hadron states are generated by the operators which transform one vacuum into another in the degenerate set. In the case of approximate conservation of the axial-vector current (13), the experimentally observed octet of pseudoscalar low-mass mesons arises.

Low-energy theorem for the isosymmetric amplitude of πN scattering; the pion-nucleon σ term

Very important information about the way in which the chiral symmetry is broken can be obtained by using the low-energy theorems for meson-baryon scattering. Important in this connection is the pion-nucleon σ term, which arises naturally in the πN interaction amplitude in the soft-pion limit. Consider the scattering process

$$\pi^{\alpha}(q)+N(p)\rightarrow\pi^{\beta}+N'(p'), \tag{14}$$

where the 4-momenta of the particles are given in the brackets, and α and β are the isotopic indices of the pions. The kinematic invariants of the process are defined by

$$s=(p+q)^2,\ t=(q-q')^2, \tag{15}$$

this being equivalent to

$$\left.\begin{aligned}&\nu=(p+p')(q+q')/(4m_N)=(s-u)/(4m_N)=\omega_L+t/(4m_N);\\&\nu_B=-qq'/(2m_N)=(t-q^2-q'^2)/(4m_N),\end{aligned}\right\} \tag{16}$$

where m_N is the nucleon mass, ω_L is the total laboratory energy of the incident pion, and $u=(p-q')^2$.

The amplitude of the process (14) has the form

$$T^{\beta\alpha}(\nu,\ t,\ q^2,\ q'^2)=\langle\pi^{B}N'_{\text{out}}|j_{\pi}^{\alpha}(0)|N_{\text{in}}\rangle, \tag{17}$$

where j_{π}^{α} is the pion source,

$$(m_{\pi}^2-q^2)\,\varphi^{\alpha}(x)=j_{\pi}^{\alpha}(x), \tag{18}$$

and the amplitude $T^{\beta\alpha}$ is related to the S matrix of the process by

$$S^{\beta\alpha}=\delta_{\beta\alpha}+i(2\pi)^4\delta^4(p+q-p'-q')\,T^{\beta\alpha}. \tag{19}$$

Substituting (18) in (17) and using the PCAC relation (13), we obtain for the scattering amplitude

$$T^{\beta\alpha}=(m_{\pi}^2-q^2)\,\langle\pi^{\beta}N'_{\text{out}}|\varphi^{\alpha}(0)|N_{\text{in}}\rangle$$

$$=\frac{m_{\pi}^2-q^2}{f_{\pi}m_{\pi}^2}\left\langle\pi^{\beta}N'_{\text{out}}\left|\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right|N_{\text{in}}\right\rangle,$$

or, in the limit $q^2\rightarrow0$,

$$T^{\beta\alpha}=\frac{1}{f_{\pi}}\left\langle\pi^{\beta}N'_{\text{out}}\left|\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right|N_{\text{in}}\right\rangle. \tag{20}$$

Further, we reduce the "out" state $\pi^{\beta}(q')_{\text{out}}$,[14] replacing it by the corresponding field creation operator:

$$T^{\beta\alpha}=\frac{1}{f_{\pi}}\left\langle N'_{\text{out}}\left|a_{\text{out}}^{\beta}(q')\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right|N_{\text{in}}\right\rangle$$

$$=\frac{1}{f_{\pi}}\left\langle N'_{\text{out}}\left|a_{\text{out}}^{\beta}(q')\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}-\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}a_{\text{in}}^{\beta}(q')\right|N_{\text{in}}\right\rangle$$

$$+\frac{1}{f_{\pi}}\left\langle N'_{\text{out}}\left|\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right|N_{\text{in}},-q'\right\rangle. \tag{21}$$

The last term on the right-hand side of (21) contributes to pion scattering only at zero angle, and therefore we omit it ($|N_{\text{in}},-q'\rangle$ is the initial state, from which it is necessary to remove the pion with momentum equal to the final momentum q').

If we use the expression for the pion field,

$$\varphi_{\substack{\text{in}\\\text{out}}}(\mathbf{x},\ t)=\int\frac{d^3q}{\sqrt{(2\pi)^3 2E_q}}\left(a_{\substack{\text{in}\\\text{out}}}^{+}(\mathbf{q})\,e^{iqx}+a_{\substack{\text{in}\\\text{out}}}(\mathbf{q})\,e^{-iqx}\right), \tag{22}$$

we can obtain

$$a_{\substack{\text{in}\\\text{out}}}(q)=i\int\frac{e^{iqx}}{\sqrt{(2\pi)^3 2E_q}}\overleftrightarrow{\partial^0}\varphi_{\substack{\text{in}\\\text{out}}}(\mathbf{x},\ t)\,d^3x. \tag{23}$$

Substituting (23) in (21) and introducing the interpolating field $\varphi(x,t)$,

$$\varphi(\mathbf{x},\ t)\rightarrow\varphi_{\text{in}}(\mathbf{x},\ t),\ t\rightarrow-\infty;$$
$$\varphi(\mathbf{x},\ t)\rightarrow\varphi_{\text{out}}(\mathbf{x},\ t),\ t\rightarrow\infty,$$

we have

$$T^{\beta\alpha}=\frac{i}{f_{\pi}}\int d^4x\,\partial^0\left[e^{iq'x}\overleftrightarrow{\partial^0}\left\langle N'\left|T\left(\varphi^{\beta}(x)\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right)\right|N\right\rangle\right.$$

$$=\frac{i}{f_{\pi}}\int d^4x\,e^{iq'x}(\Box+m_{\pi}^2)\left\langle N'\left|T\left(\varphi^{\beta}(x)\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right)\right|N\right\rangle,$$

or, using (13) for $\varphi^{\beta}(x)$, we obtain in the limit $q'^2\rightarrow0$

$$T^{\beta\alpha}=\frac{i}{f_{\pi}^2}\int d^4x\,e^{iq'x}\left\langle N'\left|T\left(\frac{\partial A_{\beta}^{\nu}(x)}{\partial x^{\nu}}\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right)\right|N\right\rangle. \tag{24}$$

The expression (24) can be transformed by means of the generalized Ward-Takahashi identities,[15] and in the soft-pion limit ($q\rightarrow0,q'\rightarrow0$) this leads to the expression

$$T^{\beta\alpha}(\nu=0,\ t=0,\ q^2=0,\ q'^2=0)$$

$$=\frac{i}{f_{\pi}^2}\int d^4x\,\delta(x^0)\,e^{iq'x}\left\langle N'\left|\left[A_{\beta}^{0}(x),\frac{\partial A_{\alpha}^{\mu}(0)}{\partial x^{\mu}}\right]\right|N\right\rangle$$

$$+if_{\pi}^{-1}\int d^4x\,e^{iq'x}\langle N'|\{q'_{\nu}q_{\mu}T(A_{\beta}^{\nu}(x)\,A_{\alpha}^{\mu}(0))\}|N\rangle. \tag{25}$$

The nucleon pole with singularities as $q_{\nu}\rightarrow0,q_{\mu}\rightarrow0$ makes a nonvanishing contribution to the second term on the right-hand side of (25). Using for the nucleon vertex the simplified expression

$$\langle N'|A_{\nu}^{\beta}|N\rangle=g_A/2\bar{u}'\gamma_{\nu}\gamma_5\tau^{\beta}u_{\mathbf{s}} \tag{26}$$

we readily find that this contribution of the nucleon pole is $g_{\pi N}^2/m_N\delta_{\alpha\beta}\bar{u}_{N'}u_N$, where $g_{\pi N}$ is the pion-nucleon coupling constant, $g_{\pi N}^2/(4\pi)\approx14.3$,[16] and u_N is a Dirac spinor ($\bar{u}_N u_N=2m_N$).

The pion-nucleon σ term is defined as

$$\bar{u}_{N'}u_N\sigma_{\pi N}\delta_{\alpha\beta}=i\int d^4x\,e^{iq'x}\delta(x^0)\,\langle N'|[A_{\alpha}^{0}(x),\ \partial_{\mu}A_{\beta}^{\mu}(0)]\,N\rangle. \tag{27}$$

The expression (27) can be rewritten by means of the expression for the derivative of the axial-vector charge:

$$\dot{\overline{Q}}_{\beta}(t)=\int d^3x\,\partial^{\mu}A_{\mu}^{\beta}(\mathbf{x},\ t)=i\,[H,\ \overline{Q}_{\beta}(t)], \tag{28}$$

where H is the Hamiltonian of the system. It follows from (28) that the Hamiltonian density $\mathscr{H}_2(x)$, which breaks the $SU(2)\otimes SU(2)$ symmetry, is related to the divergence of the axial-vector current by

$$\partial^{\mu}A_{\mu}^{\beta}(0)=i\,[\mathscr{H}_2(0),\ \overline{Q}_{\beta}(0)]. \tag{29}$$

From this we obtain[1]

$$\bar{u}_{N'} u_N \sigma_{\pi N} \delta_{\alpha\beta} = \langle N | [\bar{Q}_\alpha (0), [\bar{Q}_\beta (0), \mathcal{H}_2 (0)]] | N \rangle. \quad (30)$$

Denoting $T^+ \bar{u}_{N'} u_N = (T^{\alpha\beta} + T^{\beta\alpha})/2$, we have

$$T^+ (0, 0, 0, 0) = -\frac{1}{f_\pi^2} \sigma_{\pi N} + \frac{g^2_{\pi N}}{m_N}. \quad (31)$$

The expression (31) is the basic relation connecting the pion-nucleon σ term to the isosymmetric amplitude of πN scattering at the Weinberg point ($\nu = 0, t = 0, q^2 = q'^2 = 0$).

Thus, the pion-nucleon σ term ($\sigma_{\pi N}$ (30) is a measure of the SU(2) ⊗ SU(2) symmetry breaking, and reliable determination of its value is particularly important from this point of view, since chiral symmetry leads to a number of important consequences, mentioned above, for strong-interaction physics.

Mechanisms of chiral-symmetry breaking and QCD

Interest in the value of $\sigma_{\pi N}$ is also due to the fact that it can be used to establish the mechanism of chiral-symmetry breaking. In accordance with the relations for the divergences of the axial-vector, $\bar{D}_\alpha$, and vector, D_α, currents

$$\bar{D}_\alpha (x) = \mathrm{i}\, [\mathcal{H}' (x), \bar{Q}_\alpha], \; D_\alpha (x) = \mathrm{i}\, [\mathcal{H}' (x), Q_\alpha],$$

where $\mathcal{H}'(x)$ is the part of the Hamiltonian density that breaks the SU(3) ⊗ SU(3) symmetry. These divergences belong to the same representation of the group SU(3) ⊗ SU(3) as $\mathcal{H}'$. Since there are experimental grounds for assuming that there are no "exotic" operators—carrying isospin $I = 2$ and hypercharge $Y = 2$—among the divergences and currents, it can be concluded that the operators D_α and $\bar{D}_\alpha$, and also $\mathcal{H}'$, transform in accordance with the representations (3,3*) ⊕ (3*,3) or (1,8) ⊗ (8,1) of the chiral group SU(3) ⊗ SU(3) (Gell-Mann–Oakes–Renner model[9]). At the same time, it has been shown[5] that the effect of the possible (1,8) ⊕ (8,1) admixture is small. It should be noted that the part of the Hamiltonian which transforms as (1,8) ⊕ (8,1) does not contribute to the pion-nucleon σ term.

Reliable establishment of the mechanism of the SU(3) ⊗ SU(3) symmetry breaking is even more topical because of the need to test standard QCD. The QCD Lagrangian density is[17]

$$\mathcal{L} = \mathrm{i} \bar{q}^A_\alpha \hat{D}_{\alpha\beta} q^A_\beta - 1/4 F^a_{\mu\nu} F^{a\mu\nu} - \bar{q}^A_\alpha M^A q^A_\alpha, \quad (32)$$

where

$$\hat{D}_{\alpha\beta} q^A_\beta = (\delta_{\alpha\beta} \partial_\mu - \mathrm{i} g \lambda^a_{\alpha\beta} A^a_\mu / 2)\, \gamma^\mu q^A_\beta;$$

q^A_α are the operators of the quark fields, α is the color index, which takes three values, A is the flavor index, g is the strong coupling constant, A^a_μ is the vector gauge field ($a = 1,2,...,8$), λ^a are the matrices of the color group $\mathrm{SU}_c(3)$, $F^a_{\mu\nu}$ is the tensor of the gluon field,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu;$$

and f^{abc} are the structure constants of the group $\mathrm{SU}_c(3)$. This Lagrangian density $\mathcal{L}$ is gauge-invariant under $\mathrm{SU}_c(3)$ and can be represented in the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}',$$

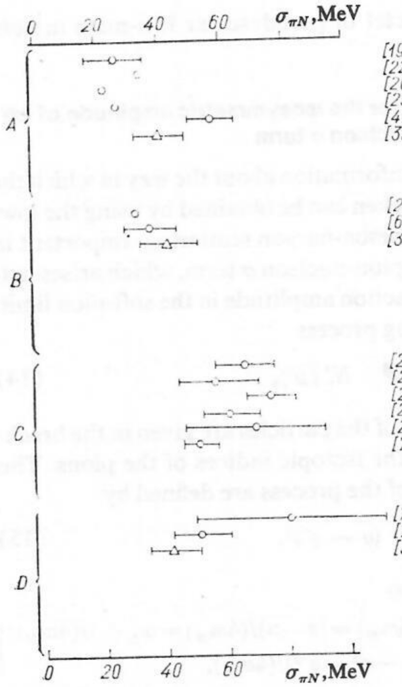

FIG. 1. Results of estimates of the pion-nucleus σ term by various methods: A) methods using the experimental pion scattering lengths and sum rules; B) calculations of $\sigma_{\pi N}$ using quark models and current algebra; C) calculations using dispersion relations and extrapolation of the πN amplitude to the Cheng–Dashen point; D) methods using expansion of the πN amplitude with respect to the variables t, ν, q_1^2, q_2^2 and data on πN scattering near the physical threshold. (References are given in square brackets.)

where $\mathcal{L}_0$ is also invariant under the chiral SU(3) ⊗ SU(3) transformations, and $\mathcal{L}' = -\bar{q} M q$ is a tensor operator, transforming in accordance with the representation (3*,3) ⊕ (3,3*) (10) as in the Gell-Mann–Oakes–Renner model.[9] Thus, the magnitude of the pion-nucleon σ term can be regarded as a serious test of standard QCD, just as of the well-known Gell-Mann–Oakes–Renner scheme.

This parameter being so important, numerous attempts have been made to estimate $\sigma_{\pi N}$ (see, for example, Ref. 15). Unfortunately, the literature data on the value of the pion-nucleon σ term have a large spread (25–80 MeV) (Fig. 1). Moreover, there is a contradiction between the results of estimating $\sigma_{\pi N}$ in the two main methods using experimental data: on the one hand, in methods using sum rules of the Fubini-Furlan type[18] and extrapolation of the isosymmetric pion-nucleon or pion-nucleus amplitude to the Weinberg point ($\nu = 0, t = 0, q_1^2 = 0, q_2^2 = 0$),[19–23] which give $\sigma_{\pi N} = 25$–35 MeV, and, on the other, in the methods which use dispersion relations and extrapolation of the isosymmetric πN amplitude to the Cheng-Dashen point ($\nu = 0, t = 2m_\pi^2, q_1^2 = m_\pi^2, q_2^2 = m_\pi^2$),[24–28] which give $\sigma_{\pi N} = 60$–70 MeV. There is also a contradiction between these last results and that of the standard calculation of Cheng[29] in the framework of the quark-gluon model ($\sigma_{\pi N} = 32$ MeV). Such a situation is itself a problem. Moreover, the spread in the estimates of $\sigma_{\pi N}$ means that preference cannot be definitely given to the (3,3*) ⊕ (3*,3) mechanism for breaking the SU(3) ⊗ SU(3) chiral symmetry as suggested by the Gell-Mann–Oakes–Renner model and QCD. In this review, in which we take into account Refs. 30–45, which relate to this problem, we attempt to establish the reason for the spread in the estimates of $\sigma_{\pi N}$ in the various methods with a view to determining an acceptable interval for $\sigma_{\pi N}$ sufficiently accurate to permit a conclusion to be drawn about the mechanism by which the $SU(3) \otimes SU(3)$ chiral symmetry is broken.

1. DETERMINATION OF THE PION-NUCLEON σ TERM USING SUM RULES OF FUBINI-FURLAN TYPE

Connection between the physical pion-nucleon and pion-nucleus amplitudes and the soft-pion amplitudes

As was pointed out above, the pion-nucleon σ term is related to the isosymmetric amplitude of πN scattering in the soft limit (25), (27), (31). It is often estimated by a method similar to the extrapolation technique of Fubini and Furlan,[18] which relates the pion-nucleon or pion-nucleus scattering amplitude in the low-energy limit to the corresponding amplitude at the threshold (the group of results A in Fig. 1). In this case, to derive the corresponding sum rules, which contain half-off-shell amplitudes (i.e., amplitudes in which one of the states, the "in" or the "out" state, is off the mass shell), off-shell dispersion relations or a reduction technique[14] are used.

Using the characteristic linear dependence of the real part of the pion-nucleus scattering length $a_{\pi^- A}$ on A for nuclei with $2 \leqslant A \leqslant 24$,[46] Gensini[19,47] derived within the framework of current algebra and PCAC a linear formula for the pion-nucleus scattering lengths, this following from the sum rule

$$(1+m_\pi/M)\,\mathrm{Re}\,a(\pi^-A)=\frac{m_\pi}{2\pi f_\pi^2}\Big[T^3-\frac{\Sigma(\pi A)}{2m_\pi}-(C^+(\pi^-A)$$
$$+m_\pi C^-(\pi^-A))/(4m_\pi^3M)$$
$$+\frac{1}{16\pi M}\left(\frac{m_\pi}{M}\right)^2 P\int_{-\infty}^{\infty} d\zeta\,\frac{F_{\pi^-A}(\zeta)}{\zeta^2(\zeta-m_\pi/M)}\Big], \tag{33}$$

where $F_{\pi^- A}(\zeta)$ is the imaginary part of the pion-nucleus off-shell forward scattering amplitude

$$[F_{\pi^-A}(\zeta)=F_{\pi^-A}(s=M^2(1+\zeta)^2,\ q^2=m_\pi^2),\ \zeta>0,$$
$$F_{\pi^-A}(\zeta)=F_{\pi^-A}(u=M^2(1+\zeta)^2,\ q^2=m_\pi^2),\ \zeta<0];$$

$\Sigma(\pi A)=A\sigma_{\pi N}$, M is the mass of the nucleus, and $C^+(\pi^- A)$ and $C^-(\pi^- A)$ are the equal-time commutators, equal to 0 in models in which $\overline{D}$ is proportional to the canonical pion field:

$$\int d^3x\,\langle A|[\overline{D}_{\pi^+}(\mathbf{x},\ 0),\ \overline{D}_{\pi^-}(0)]|A\rangle=C^+(\pi^-A);$$
$$\int d^3x\,\langle A|[\overline{D}_{\pi^+}(\mathbf{x},\ 0),\ \dot{\overline{D}}_{\pi^-}(0)]|A\rangle=-iC^-(\pi^-A).$$

An important point in this approach is the estimate of the integral of the imaginary part of the pion-nucleus amplitude in (33), which is represented in the form of the existing singel-particle contribution of the double pion pole using PCAC and a certain continuum of pion-nucleus states. The approximations of this integral in Refs. 19 and 47 are different, and the results for $\sigma_{\pi N}$ accordingly differ appreciably:

$$\sigma_{\pi N}=(26\pm 8)\ \text{MeV (Ref. 19)} \tag{34}$$

and

$$\sigma_{\pi N}=(57\pm 6)\ \text{MeV (Ref. 47)}. \tag{35}$$

This spread of the results (34) and (35) for $\sigma_{\pi N}$ is to be seen primarily as reflecting the uncertainty in the estimate of the dispersion integral in (33).

In the method analogous to that of Refs. 19 and 47 with the assumption that the imaginary part of the pion-nucleus scattering amplitude with an off-shell pion can be approximated by the imaginary part of the physical amplitude near the threshold, Hakin[20] obtained

$$\sigma_{\pi N}=(22\pm 1)\ \text{MeV}. \tag{36}$$

In an approach analogous to that of Chew and Low,[48] but without the static approximation, Banerjee and Cammarata[23] obtained sum rules for the physical πN scattering amplitude and for the amplitude with one off-shell pion:

$$F_{\beta\alpha}(k)=F_{\beta\alpha}(0)+\mathrm{i}\int d^4x e^{ikx}\langle P_f|T(j_\beta(x)\,j_\alpha(0))|P_i\rangle$$
$$-\mathrm{i}\int d^4x\,\langle P_f|T(j_\beta(x)\,j_\alpha(0))|P_i\rangle, \tag{37}$$

where

$$F_{\beta\alpha}(0)=\lim_{k_0\to 0}[\lim_{\mathbf{k}\to 0}F_{\beta\alpha}(k)],$$

i.e., $F_{\beta\alpha}(0)$ is the limit in which one pion is soft and the other has momentum $k'=P_f-P_i$. The quantity $F_{\beta\alpha}(0)$ includes a part proportional to the pion-nucleon σ term, and also a nucleon pole in the soft-pion limit. The integrals (37) take into account the intermediate states $|N\rangle$ and $|\pi N\rangle$ and Z diagrams, which arise from the intermediate states $|\overline{N}NN\rangle$. The approximate solution of the system of integral equations (37) with expansion of the amplitude with respect to c.m.s. partial waves gives an estimate for the pion-nucleon σ term:

$$\sigma_{\pi N}\simeq 26\ \text{MeV (Ref. 23)}. \tag{38}$$

To estimate $\sigma_{\pi N}$, Ericson and Rho[22] used a sum rule different from that of Refs. 19, 20, and 47 and relating the isosymmetric amplitude of pion scattering by nuclei in the low-energy limit $T^+(0)$ to the same amplitude $T^+(m_\pi)$ at the physical threshold. This sum rule is derived by means of a reduction technique. We consider pion elastic scattering by a nucleus B:

$$\pi^\alpha+\mathrm{B}\to\pi^\beta+\mathrm{B}. \tag{39}$$

If double charge exchange is ignored, we obtain the amplitudes T^- and T^+:

$$(Z-N)\,T^-=1/2\,(T_{\pi^-}-T_{\pi^+}),\quad T^+=1/2\,(T_{\pi^-}+T_{\pi^+}), \tag{40}$$

where T^- and T^+ are, respectively, the charge-exchange and charge-symmetric amplitudes, Z and N are the proton and neutron numbers, and $T_{\pi^\pm}$ are the amplitudes for $\pi^\pm$ scattering by the nucleus. The S matrix of the process (39) can be expressed as

$$S^{\beta\alpha}=\mathrm{i}\,(2\pi)^4\delta^4(k+p_f-q-p_i)\,T^{\beta\alpha}, \tag{41}$$

where q,p_i and k,p_f are the 4-momenta, respectively, of the initial and final pion and nucleon, and

$$T^{\beta\alpha} = 1/2\,[\tau_\beta,\ \tau_\alpha]_+ T^+ + 1/2\,[\tau_\beta,\ \tau_\alpha]_- T^-, \quad (42)$$

where τ_α are isospin matrices.

In the reduction formalism of Lehmann, Symanzik, and Zimmermann,[14] we have by analogy with (20) and (24)

$$T^{\beta\alpha} = \mathrm{i}\,(m_\pi^2 - k^2)\,(m_\pi^2 - q^2)\int d^4x e^{\mathrm{i}kx}\theta(x_0)\,\langle f|[\Phi^\beta(x)\,\Phi^\alpha(0)]|\mathrm{i}\rangle. \quad (43)$$

Using for the pion field the PCAC relation (13) and integrating by parts, we find at zero pion momenta, $\mathbf{q} = \mathbf{k} = 0, q_0 = k_0$,

$$T^{\beta\alpha}(q_0) = \mathrm{i}\,\frac{(m_\pi^2 - q_0^2)^2}{f_\pi^2 m_\pi^4}\int d^4x e^{\mathrm{i}q_0x_0}\,\langle f|\{q_0^2\theta(x_0)\,[A_0^\beta(x),\ A_0^\alpha(0)]$$
$$-\delta(x_0)\,[A_0^\beta(x),\ \dot{A}_0^\alpha(0)] - \mathrm{i}q_0\delta(x_0)\,[A_0^\beta(x),\ A_0^\alpha(0)]\}|\mathrm{i}\rangle. \quad (44)$$

Noting further that

$$(m_\pi^2 - q_0^2)\,e^{\mathrm{i}q_0x_0} = (m_\pi^2 + \partial_0^2)\,e^{\mathrm{i}q_0x_0},$$

and integrating by parts for a second time in (44), we obtain

$$T^{\beta\alpha}(q_0) = \mathrm{i}\,\frac{m_\pi^2 - q_0^2}{f_\pi^2 m_\pi^4}\Big\{q_0^2\int d^4x$$
$$\times e^{-\mathrm{i}q_0x_0}\theta(-x_0)\,\langle f|[A_0^\beta(0),\ (m_\pi^2 + \partial_{x_0}^2)\,A_0^\alpha(x)]|\mathrm{i}\rangle$$
$$-\langle f|(m_\pi^2[\overline{Q}^\beta(0), \dot{A}_0^\alpha(0)] + \mathrm{i}q_0m_\pi^2[\overline{Q}^\beta(0),\ A_0^\alpha(0)])|\mathrm{i}\rangle\Big\}. \quad (45)$$

The integration in (45) can be performed by substituting a complete set of intermediate states $\Sigma_n|n\rangle\langle n|$ in the commutator using a relation of the type

$$\int d^3x\,\langle n|(m_\pi^2 + \partial_0^2)\,A_0^\alpha(x)|m\rangle$$
$$= \frac{f_\pi m_\pi^2}{\mathrm{i}\,(E_n - E_m)}\int d^3x\langle n|(\Box_x + m_\pi^2)\,\Phi(x)|m\rangle$$

Then the symmetric combination of πN amplitudes can be written as

$$\frac{T^{\beta\alpha}(q_0) + T^{\alpha\beta}(q_0)}{2q_0^2\,(m_\pi^2 - q_0^2)} = -\frac{\mathrm{i}}{f_\pi^2 m_\pi^2 q_0^2}\langle f|[\overline{Q}^\beta(0),\ \dot{A}_0^\alpha(0)]|\mathrm{i}\rangle$$
$$-\Big\{\sum_n (2\pi)^3\,\delta(\mathbf{P}_n - \mathbf{P}_i)\,\frac{\langle f\mid j_\pi^\beta(0)\mid n\rangle\langle n\mid\Phi^\alpha(0)\mid i\rangle}{[q^2 - (E_n - E_i)^2\,]\,](E_n - E_i)} - \mathrm{c.t.}\Big\}, \quad (46)$$

where E_n and $\mathbf{P}_n$ are the energy and momentum of the intermediate system, and c.t. corresponds to the crossed term with $j_\pi^\beta(0)\rightleftarrows\Phi^\alpha(0)$ and $E_n - E_i\rightleftarrows E_f - E_n$. The second term on the right-hand side of (46) has a pole at $q_0 = m_\pi$. This pole can be subtracted by means of the model-dependent commutator $C'^{\beta\alpha}$:

$$C'^{\beta\alpha} = \int d^3x\,\langle f|[\dot{\Phi}^\beta(0),\ \Phi^\alpha(\mathbf{x},\ 0)]|i\rangle.$$

Subtracting from the right-hand side of (46) the quantity

$$\frac{\mathrm{i}}{m_\pi^2}C'^{\beta\alpha} = \sum_n\frac{E_n - E_i}{m_\pi^2}(2\pi)^3\delta^3(\mathbf{P}_n - \mathbf{P}_i)$$
$$\times\frac{\langle f\mid j_\pi^\beta(0)\mid n\rangle\langle n\mid\Phi^\alpha(0)\mid i\rangle}{m_\pi^2 - (E_n - E_i)^2} - \mathrm{c.t.}$$

and simultaneously adding it to the second term of (46), we obtain

$$\frac{T^{\beta\alpha}(q_0) + T^{\alpha\beta}(q_0)}{2q_0^2\,(m_\pi^2 - q_0^2)} = -\frac{\mathrm{i}}{m_\pi^2}C'^{\beta\alpha}$$
$$-\frac{\mathrm{i}}{f_\pi^2 m_\pi^2 q_0^2}\langle f|[\overline{Q}^\beta(0),\ \dot{A}_0^\alpha(0)]|i\rangle$$
$$-\Big\{\frac{1}{m_\pi^2}\sum_n (2\pi)^3\delta(\mathbf{P}_n - \mathbf{P}_i)$$
$$\frac{\langle f\mid j_\pi^\beta(0)\mid n\rangle_c\langle n\mid j_\pi^\alpha(0)\mid i\rangle_c}{(E_n - E_i)\,[m_\pi^2 - (E_n - E_i)^2]} - \mathrm{c.t.}\Big\} + \mathrm{D.C.}, \quad (47)$$

where $\langle f|f_\pi^\beta(0)|n\rangle_c$ is the connected part of the pion-nucleon amplitude,[18] and D.C. denotes the half-connected part of the term

$$-\sum_n (2\pi)^3\delta(\mathbf{P}_n - \mathbf{P}_i)\Big[\frac{1}{[q_0^2 - (E_n - E_i)^2]\,(E_n - E_i)}$$
$$-\frac{E_n - E_i}{m_\pi^2\,[m_\pi^2 - (E_n - E_i)^2]}\Big]\langle f|j_\pi^\beta(0)|n\rangle\langle n|\Phi^\alpha(0)|i\rangle - \mathrm{c.t.}$$

It can be seen from (47) that none of the terms on the right-hand side apart from D.C. have a pole at $q_0 = \mathrm{m}_\pi$, so that

$$\mathrm{D.C.} = \frac{1}{m_\pi^2}\,\frac{T^{\beta\alpha}(m_\pi) + T^{\alpha\beta}(m_\pi)}{2\,(m_\pi^2 - q_0^2)} \quad (48)$$

Substituting (48) in (47), we obtain a sum rule for $T^+(m_\pi)$ at the threshold (as $q_0\to m_\pi$):

$$1/2\,[\tau_\beta,\ \tau_\alpha]_+ T^+(m_\pi) = -\mathrm{i}m_\pi^2 C'^{\beta\alpha} - \frac{\mathrm{i}}{f_\pi^2}\langle f|[\overline{Q}^\beta,\ \dot{A}_0^\alpha(0)]|i\rangle$$
$$-\Big\{m_\pi^2\sum_n (2\pi)^3\delta(\mathbf{P}_n - \mathbf{P}_i)\,\frac{\langle f\mid j_\pi^\beta(0)\mid n\rangle_c\langle n\mid j_\pi^\alpha(0)\mid i\rangle_c}{(E_n - E_i)\,[m_\pi^2 - (E_n - E_i)^2]} - \mathrm{c.t.}\Big\}. \quad (49)$$

Thus,

$$T^+(m_\pi) = T^+(0) + \frac{\mathrm{i}}{2m_\pi^2 f_\pi^2}\int d^3x\,\langle B|[\dot{\overline{D}}^+(0),\ \overline{D}^-(\mathbf{x},\ 0)]|B\rangle$$
$$-\Big\{m_\pi^2\sum_n (2\pi)^3\delta(\mathbf{P}_n - \mathbf{P}_B)\frac{\langle B\mid j_\pi^-(0)\mid n\rangle\langle n\mid j_\pi^+(0)\mid B\rangle}{(E_n - E_B)[\,m_\pi^2 - (E_n - E_B)^2]} - \mathrm{c.t.}\Big\}, \quad (50)$$

where

$$T^+(0) = -\mathrm{i}/(2f_\pi^2)\,\langle B|[\overline{Q}^+(0),\ \dot{A}_0^-(0)]|B\rangle \quad (51)$$

or

$$T^+(0) = -\frac{1}{2f_\pi^2}\langle B|[\overline{Q}^+(0),\ [\overline{Q}^-(0),\ \mathscr{H}'(0)]]|B\rangle. \quad (51\mathrm{a})$$

In Ref. 22, the summation over the intermediate states $|n\rangle$ took into account states without a pion with an excited nucleus B' and with one pion and the nucleus in the ground state and the excited state. In Refs. 34 and 35, it was not necessary to introduce such a restriction on the intermediate states (see below). The commutator in (50) in a model in which the divergence $\overline{D}$ is proportional to the canonical pion field [see (13)] is equal to zero.

Sum rules of Fubini-Furlan type and potential model of pion-nucleus scattering

If in the sum in (50) we separate explicitly the coherent multiple scattering, i.e., $|n\rangle = |B\pi\rangle$, we can rewrite the expression (50) in the form[22]

$$T^+(m_\pi) = T_B^+ + \frac{m_\pi^2}{(2\pi)^3\, 2m_B} \int \frac{d^3q}{q^2\omega_q^2} [2|T_{0q}^-|^2 + |T_{0q}^+|^2]. \quad (52)$$

In the integral on the right, the coherent multiple scattering has been explicitly separated, q and ω_q are the momentum and energy of the pion in the intermediate state, and T_B^+ includes the soft-pion part of the amplitude and the contributions to the amplitude of the other (i.e., in addition to the coherent) intermediate states n, including incoherent multiple scattering $|n\rangle = |B'\pi\rangle$ and pion absorption $|n\rangle = |B'\rangle$; T_{0q}^+ and T_{0q}^- are the charge-symmetric and charge-exchange parts of the expression

$$T_{0q}^{\alpha\beta} = \langle B(0)|j_\pi^\alpha(0)|B(-q)\,\pi^\beta(\mathbf{q})\rangle. \quad (53)$$

Note that in the amplitude (53) the state $|B(-\mathbf{q})\pi^\beta(\mathbf{q})\rangle$ is an off-shell state.

Determining the scattering amplitude

$$f = 1/(2ik) \sum_l (2l+1)(e^{2i\delta_l} - 1) P_l(\cos\theta)$$

in the usual manner, we obtain for s-wave scattering of pions with zero momentum

$$(1 + m_\pi/m_B) f_{00} = -4\pi^2 m_\pi \langle\chi_0|V_{\mathrm{opt}}|\psi_0^+\rangle, \quad (54)$$

where the indices in f_{00} denote the initial and final momenta of the pions, χ_0 is a plane wave, ψ_0^+ is the wave function of an outgoing pion with zero momentum, V_{opt} is the potential of the pion-nucleus interaction, with

$$H_0\chi_0 = E_0\chi_0, \ (E_0 - H_0 + i\varepsilon)\psi_0^+ = V_{\mathrm{opt}}\psi_0^+$$

and $H = H_0 + V_{\mathrm{opt}}$ is the total Hamiltonian. Substituting in (54) ψ_0^+ from the corresponding Lippmann–Schwinger equation,

$$\psi_0^+ = \chi_0 + \frac{1}{E_0 - H + i\varepsilon} V_{\mathrm{opt}}\chi_0,$$

we obtain

$$(1 + m_\pi/m_B) f_{00} = -4\pi m_\pi \langle\chi_0|V_{\mathrm{opt}}|\chi_0\rangle - 4\pi^2 m_\pi \left\langle \chi_0 \left| V_{\mathrm{opt}} \frac{1}{E_0 - H + i\varepsilon} V_{\mathrm{opt}} \right| \chi_0 \right\rangle. \quad (55)$$

Introduction of a complete set of "out" states in the second term on the right-hand side of (55),

$$\sum_q |\psi_q^+\rangle\langle\psi_q| = 1,$$

makes it possible to transform this term to the form

$$a = a_B + \frac{1}{2\pi^2}(1 + m_\pi/m_B) \int \frac{d^3q}{q^2} |f_{0q}|^2, \quad (56)$$

where $a \equiv f_{00}$,

$$(1 + m_\pi/m_B)\, a_B = \langle\chi_0| - 4\pi^2 m_\pi V_{\mathrm{opt}}|\chi_0\rangle \quad (57)$$

and

$$(1 + m_\pi/m_B) f_{0q} = \langle\chi_0| - 4\pi^2 m_\pi V_{\mathrm{opt}}|\psi_q^+\rangle.$$

The analogy of (56) with the expression (52) and the corresponding expression for the charge-exchange amplitude T^-,[22]

$$T^-(m_\pi) = T_B^- - \frac{m_\pi^3}{(2\pi)^3 2m_B} \int \frac{d^3q}{q^2\omega_q^3} \mathrm{Re}\, [T_{0q}^- (T_{q0}^- + 2T \quad (58)$$

becomes clear after the transition from the relativistically invariant amplitudes $T^\pm$ to the nonrelativistic amplitudes $f^\pm$:

$$T^\pm = 8\pi\,(m_B + m_\pi)\, f^\pm.$$

In this case, we obtain

$$\left.\begin{aligned} a^+ &= a_B^+ + \frac{1}{2\pi^2}(1 + m_\pi/m_B) \int \frac{d^3q}{q^2} [2|f_{0q}^-|^2 + |f_{0q}^+|^2]; \\ a^- &= a_B^- + \frac{1}{2\pi^2}(1 + m_\pi/m_B) \int \frac{d^3q}{q^2} f_{0q}^- [f_{0q}^- + 2f_{q0}^+]. \end{aligned}\right\} \quad (59)$$

On the transition in (59) from $a^\pm$ to scattering lengths with a definite isospin channel, for example, in the case of an isospin-1/2 target.

$$(Z - N) f^- = (Z - N)(f_1 - f_3)/3 \text{ and } f^+ = (f_1 + 2f_3)/3$$

Eqs. (59) can be written as

$$a_T = (a_B)_T + \frac{1}{2\pi^2}(1 + m_\pi/m_B) \int \frac{d^3q}{q^2} |(f_{0q})_T|^2. \quad (60)$$

Thus, $T_B^\pm$ correspond to the Born amplitudes of the potential model of pion-nucleus scattering, which are known from analysis of the data on the levels of pionic atoms. This makes it possible to reduce the uncertainties that arise in the calculation of integrals which include the unphysical pion-nucleus amplitudes $T_{0q}^\pm$ and makes the method of Ref. 22 preferable to the use of sum rules of Fubini-Furlan type in Refs. 19–21, 23, and 47. In Ref. 22, Ericson and Rho, making the assumption that the contribution of absorption to (50) is small, and estimating the contribution of incoherent multiple scattering $|n\rangle = |B'\pi\rangle$ in the Fermi-gas model, obtained the value

$$\sigma_{\pi N} \approx 34 \text{ MeV}. \quad (61)$$

However, the estimate used here for the contribution of the incoherent processes is too crude. Therefore, it is difficult to speak of the uncertainty of the result (61).

Scattering of slow pions by light nuclei and the pion-nucleus σ term

The sum rule (52) can be written down for the isosymmetric scattering length a^+ in the form[30–35]

$$a^+(m_\pi) = a_B^+ + a_{\mathrm{coh}}^+; \quad (62)$$

$$a_B^+ = a^+(0) + a_{\mathrm{incoh}}^+, \quad (63)$$

where $a^+(0)$ is the soft-pion scattering length, a_{coh} is the contribution of coherent multiple scattering, and a_{incoh} describes the contributions of the intermediate states apart from $|n\rangle = |B\pi\rangle$.

The sum rules (62) were tested for self-consistency in Refs. 30–32 for nuclei with $A \leqslant 4$, and it was shown that (62) is satisfactorily fulfilled for these nuclei; the same form factor for this group of nuclei was used to calculate the unphysical amplitudes (53).

To determine $\sigma_{\pi N}$ from the scattering lengths in the case of scattering by light nuclei ($4 \leqslant A \leqslant 24$) the studies of Refs. 33–35 used the circumstance that the experimental data on s-wave scattering of pions by a number of these nuclei[49] indicate that in the s-wave amplitude $f_0 = (\eta_0 e^{2i\delta_0} - 1)/(2i)$ the absorption parameter η_0 is near zero. Now if $\eta_0 = 0$, the cross sections of inelastic and elastic scattering are equal, so that we may assume

$$a^{+}_{\text{incoh}} \approx a^{+}_{\text{coh}} \tag{64}$$

for this range of nuclei. Then it follows from (62)–(64) that

$$a^{+}(0) = 2a^{+}_{B} - a^{+}(m_\pi). \tag{65}$$

Equation (65) makes it possible to calculate $a^+(0)$ for these nuclei [without the calculation involving considerable uncertainties in the integrals with the unphysical amplitudes in (50)], since the Born scattering lengths are known from analysis of the level shifts of pionic atoms in the potential model. It is convenient to use nuclei with isospin $I = 0$, since for them the experiment gives directly the isosymmetric scattering lengths $a^+(m_\pi)$.

In the calculation of the pion-nucleon σ term by such a method, the A dependence of $a^+(0)$ is important, and for this it is desirable to investigate the A dependence of $\text{Re}\, a^+(m_\pi) = \text{Re}\, a(\pi^- A)$ and a^+_B.[33–35]

The values of $a(\pi^- A)$ are usually calculated by analyzing the experimental data on pionic atoms by means of the modified method of Deser.[50,51] However, Gensini recently proposed a new method[52] for extracting $a(\pi^- A)$ from the same experimental data, based on solution of the static Klein–Gordon equation for a pion-nucleus bound state in a simple approximation. The $\text{Re}\, a^+(m_\pi)$ values as obtained in both methods exhibit a well-defined linear dependence on A,[33–35]

$$\text{Re}\, a^{+}(m_\pi) \propto A - 1. \tag{66}$$

A dependence of the type (66) is rather natural, since the optical pion-nucleus potential in the approximation of a "frozen" nucleus can be represented[53] in the form $V_{\text{opt}} = (A-1)\langle\tau\rangle$, where $\langle\tau\rangle$ is the expectation value of the pion-nucleon scattering operator in the nuclear medium. At the same time, the Born amplitude of pion-nucleus scattering must also have an A dependence of the type (66), as shown in Ref. 54: The s-wave part of the effective pion-nucleus potential has the form

$$V_s(r) = (A-1)\,\frac{2\pi}{m_\pi}\,b_0\rho(r), \tag{67}$$

where $\rho(r)$ is the density of the nucleons, normalized to unity. Note that the A dependence of $a^+(m_\pi)$ must also include a small quadratic term[55]; for the experimental values[52] of $\text{Re}\, a^+(m_\pi)$ can be very satisfactorily described by the curve[35] shown in Fig. 2:

$$\text{Re}\, a^{+}(m_\pi) = [-0.0320\,(A-1) + {} + 0.0002A^2]\, m_\pi^{-1}. \tag{68}$$

In the considered range of nuclei, the quadratic term plays an unimportant part and can be ignored in the calculation of $\sigma_{\pi N}$. Thus, bearing in mind the relations (65)–(67), we can assume that $a^+(0)$ is also proportional to $A-1$, and this is important for the determination of $\sigma_{\pi N}$ as the single-nucleon contribution to $a(0)$. Therefore, taking into account the renormalization of the soft-pion part of the πN amplitude in the nuclear medium, we have[33–35]

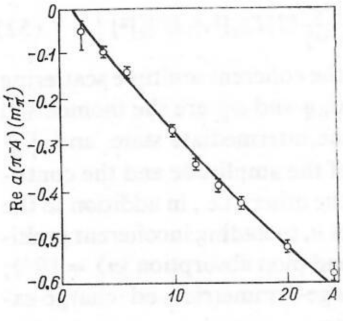

FIG. 2. Pion scattering lengths for scattering by light nuclei with isospin $I = 0$. The experimental values are taken from Ref. 52. The continuous curve corresponds to Eq. (68).

$$\sigma_{\pi N} = -\frac{4\pi f_\pi^2 a^{+}(0)}{A-1}\, c(\rho), \tag{69}$$

where $c(\rho)$ is a coefficient that takes into account this renormalization and, in general, depends on the nuclear density.

To estimate $c(\rho)$, we use the circumstance that in the linear σ model[56] the commutator in (51) is equivalent to the divergence $\partial^\mu A^\alpha_\mu(0) = \overline{D}^\alpha(0)$ of the axial-vector current. In the limit $q^2 \to 0$, we have the matrix element

$$\langle N|\overline{D}^{+}(0)|N\rangle = 2m_N g_A \bar{u} i\gamma^5 \tau^+ u, \tag{70}$$

where $|N\rangle$ is a free-nucleon state, m_N is the nucleon mass, and g_A is the axial-vector constant of neutron β decay. In the nuclear medium, the relation (70) is renormalized in such a way that the product $m_N g_A$ becomes $\tilde{m}_N \tilde{g}_A$, where $\tilde{m}_N$ and $\tilde{g}_A$ are, respectively, the effective nucleon mass and the renormalized axial-vector constant in the nuclear medium. Thus, $c(\rho)$ is determined by the renormalization of the product $m_N g_A$, i.e.,[35]

$$c(\rho) = \frac{m_N g_A}{\tilde{m}_N \tilde{g}_A}. \tag{71}$$

In the theory of finite Fermi systems,[57] the effective nucleon mass in a nucleus is given by the estimate $\tilde{m}_N = (0.8\text{–}0.9)m_N$. For $\tilde{g}_A$ in this theory one obtains the expression (see also Ref. 58) $\tilde{g}_A = (1 - 2\zeta_s)g_A$ with the parameter $\zeta_s = 0.05\text{–}0.1$. For our estimates in the considered range of nuclei ($A \leqslant 24$), it is natural to use $\tilde{m}_N = 0.9m_N$ and $\zeta_s = 0.05$.

Thus, using the relations (65), (68), (69), and (71), we can calculate the pion-nucleon σ term. Moreover, the Born scattering lengths a_B for these nuclei have been found in terms of the parameter b_0 of the pion-nucleus interaction potential.[59] which successfully describes a large set of experimental data on pionic atoms and pion-nucleus scattering. We note that in these calculations the parameter b_0 was converted to the form (67).

Such a procedure for calculating a_B reduces to the replacement of V_{opt} in (57) by a phenomenological potential of the Kisslinger–Ericsons type used in Ref. 59. Of course, in this calculation too there is uncertainty. However, it can be hoped that this is not great, since the phenomenological optical potential, although not rigorously derived in multiple-scattering theory, nevertheless describes well a large set of experimental data.

As a result, for the pion-nucleon σ term we obtain[35]

$$\sigma_{\pi N} = (40 \pm 7)\ \text{MeV}. \tag{72}$$

The uncertainty in the result given in (72) is a consequence of the procedure for fitting the experimental data to (68), and also the uncertainty in the estimate of $c(\rho)$ in accordance with (71).

In this calculation of $\sigma_{\pi N}$, the parameter b_0 (67) of the effective pion-nucleus potential is very important. This parameter is determined from analysis of the level shifts of pionic atoms. It is found that in the procedure for determining the parameters of this potential it is difficult to separate unambiguously in the optical potential the contributions proportional to $b_0\rho$ and $\mathrm{Re}\,B_0\rho^2$.[59–60] Therefore, in practice it is necessary to fix the value of $\mathrm{Re}\,B_0$ using additional arguments. Theoretical estimates in different approaches give for the ratio $\mathrm{Re}\,B_0/\mathrm{Im}\,B_0$ a fairly wide range of values: $\pm$ 0–2.[61] In Ref. 59, in accordance with the theoretical calculation of Ref. 62, $\mathrm{Re}\,B_0$ was taken to have a small positive value $(\sim 0.002 m_\pi^{-4})$, and this leads to the value $b_0 = 0.033 m_\pi^{-1}$ used to obtain (72). Such a choice of $\mathrm{Re}\,B_0$ is also confirmed by the calculation of Ref. 63. In contrast to this, in Refs. 50 and 64 the ratio $\mathrm{Re}\,B_0/\mathrm{Im}\,B_0$ is taken to be equal to -1, and this gave for b_0 the value $-0.017 m_\pi^{-1}$. The use of this value of b_0 in our calculations would lead to a too small value of the pion-nucleon σ term (smaller by 15 MeV), contradicting the existing estimates of this quantity. This circumstance must be regarded as an argument for using the parameters of the pion-nucleus optical potential in Ref. 59 rather than those of Refs. 50 and 64, and equally as an indication that the order of magnitude of $\mathrm{Re}\,B_0$ obtained in Ref. 63 is correct.

We note that the form of the dependence (68) is very important for calculating $\sigma_{\pi N}$ in the approach of Refs. 33–35. It would therefore be very desirable to determine more accurately the experimental data on the 1s-level shifts of pionic atoms. Also desirable is further improvement in the procedure for extracting the pion-nucleus scattering lengths from these data. An interesting attempt in this direction was, as we mentioned above, made by Gensini.[52]

2. METHODS OF DETERMINING THE PION-NUCLEON σ TERM USING DISPERSION RELATIONS

Extrapolation of the isosymmetric πN amplitude to the Cheng–Dashen point (Cheng–Dashen method)

The expression (31) relates the value of the pion-nucleon σ term to the off-shell amplitude. Cheng and Dashen[65] proposed a method of relating $\sigma_{\pi N}$ to the πN amplitude on the mass shell ($q_1^2 = q_2^2 = m_\pi^2$); this would make it possible to use dispersion relations to extrapolate this parameter. In this method, one usually employs the amplitude

$$F^+(\nu,\ t,\ q_1^2,\ q_2^2) = T^+(\nu,\ t,\ q_1^2,\ q_2^2)$$

$$-\frac{g_{\pi N}^2}{m}\,\frac{\nu_B^2}{\nu_B^2-\nu^2} = A^+ + \nu B^+ - \frac{g_{\pi N}^2}{m_N}\,\frac{\nu_B^2}{\nu_B^2-\nu^2}. \tag{73}$$

The amplitude F^+ is equal to the ordinary isosymmetric amplitude $T^+ = A^+ + \nu B^+$ of πN scattering with isospin $I = 0$ in the t channel, from which the nucleon pole term is subtracted to obtain a smooth function near the Weinberg point; A^+ and B^+ are the ordinary invariant amplitudes.[66] In accordance with (31),

$$F^+(0,\ 0,\ 0,\ 0) = -\frac{1}{f_\pi^2}\,\sigma_{\pi N}. \tag{74}$$

Cheng and Dashen[65] assumed that the amplitude $F^+(0, 2m_\pi^2, m_\pi^2, m_\pi^2)$ can be expanded with respect to q_1^2 and q_2^2

$$F^+(0,\ 2m_\pi^2,\ m_\pi^2,\ m_\pi^2) = F^+(0,\ 0,\ 0,\ 0)$$

$$+ m_\pi^2\frac{\partial}{\partial q_1^2}F^+(0,\ 0,\ 0,\ 0) + m_\pi^2\frac{\partial}{\partial q_2^2}F^+(0,\ 0,\ 0,\ 0) + 0\,(m_\pi^4). \tag{75}$$

Adler's consistency condition[67] gives

$$F^+(0,\ m_\pi^2,\ m_\pi^2,\ 0) = F^+(0,\ m_\pi^2,\ 0,\ m_\pi^2) = 0. \tag{76}$$

Then the use of the expansions

$$\begin{aligned} &F^+(0,\ m_\pi^2,\ m_\pi^2,\ 0) = F^+(0,\ 0,\ 0,\ 0) \\ &\quad + m_\pi^2\frac{\partial}{\partial q_1^2}F^+(0,\ 0,\ 0,\ 0) + 0\,(m_\pi^4); \\ &F^+(0,\ m_\pi^2,\ 0,\ m_\pi^2) = F^+(0,\ 0,\ 0,\ 0) \\ &\quad + m_\pi^2\frac{\partial}{\partial q_2^2}F^+(0,\ 0,\ 0,\ 0) + 0\,(m_\pi^4 \end{aligned} \tag{77}$$

leads to the relations

$$\left.\begin{aligned} F^+(0,\ 0,\ 0,\ 0) &= -m_\pi^2\frac{\partial}{\partial q_1^2}F^+(0,\ 0,\ 0,\ 0) + 0\,(m_\pi^4); \\ F^+(0,\ 0,\ 0,\ 0) &= -m_\pi^2\frac{\partial}{\partial q_2^2}F^+(0,0,0,0) + 0\,(m_\pi^4). \end{aligned}\right\} \tag{78}$$

Substitution of (78) in (75) leads to

$$F^+(0,\ 2m_\pi^2,\ m_\pi^2,\ m_\pi^2) = -F^+(0,\ 0,\ 0,\ 0) + 0\,(m_\pi^4), \tag{79}$$

or, in accordance with (74)

$$F^+(0,\ 2m_\pi^2,\ m_\pi^2,\ m_\pi^2) = \frac{1}{f_\pi^2}\,\sigma_{\pi N} + 0\,(m_\pi^4). \tag{80}$$

Note that an important part of the derivation of (80) was the assumption that the πN scattering amplitude does not have singularities with respect to t, q_1^2, q_2^2 at $\nu = 0$. Also important is the question of the corrections needed in (80). This question was investigated in Refs. 68–70 but is still topical. The question is evidently intimately related to the possible nonanalyticity of the πN amplitude with respect to t at $\nu = 0$.[36,38,69]

The group of results C in Fig. 1 includes the methods of determining $\sigma_{\pi N}$ by extrapolating the πN scattering amplitude to the Cheng–Dashen point ($\nu = 0$, $t = 2m_\pi^2$, $q_1^2 = q_2^2 = m_\pi^2$).[24–28,71] These methods include four basic elements:

1) Data on the pion-nucleon coupling constant;

2) the data of phase-shift analysis;

3) the use of dispersion relations to determine the amplitude for spacelike t and $\nu = 0$, and also frequent use of the amplitude

$$\widetilde{C}^+(\nu,\ t) = A^+(\nu,t) + \frac{\nu}{1+t/(4m_N^2)}\,B^+(\nu,\ t)$$

$$-\frac{g_{\pi N}^2}{m_N\,(1-t/(4m_N^2))}\,\frac{\nu_B^2}{\nu_B^2-\nu^2}; \tag{81}$$

4) extrapolation with respect to t to the Cheng–Dashen point for $\widetilde{C}^+(0,t)$ and $F^+(0,t,m_\pi^2,m_\pi^2)$ or expansion of $\widetilde{C}^+(0,t)$ and $F^+(0,t,m_\pi^2,m_\pi^2)$ or expansion of $\widetilde{C}^+(0,t)$:

$$F^+(0,\ t,\ m_\pi^2,\ m_\pi^2) \approx C_1^+ + C_2^+(t/m_\pi^2) + C_3^+(t/m_\pi^2)^2. \quad (82)$$

The pion-nucleon σ term is calculated from $\widetilde{C}(0,2m_\pi^2)$ by means of the relation[28]

$$\sigma_{\pi N}/f_\pi^2 = \widetilde{C}^+(0,\ 2m_\pi^2) + g_{\pi N}^2 m_\pi^3/(2m_N^3) + 0\,(m_\pi^4) \quad (83)$$

or from (80).

In Refs. 26 and 27, to make the extrapolation with respect to t the amplitude $\widetilde{C}^+(\nu,t)$ was expanded in a series with respect to ν^2 and t about $\nu = 0, t = 0$:

$$\widetilde{C}^+(\nu,\ t) = \sum_{m,\ n=0}^{M,\ N} x_{mn}(\nu^2)^m t^n. \quad (84)$$

The values of the coefficients x_{mn} calculated in the two quoted papers agree with one another. To determine these coefficients, subtracted dispersion relations were used for $\widetilde{C}(\nu,t)$ for the first derivative of $\widetilde{C}(\nu,t)$ with respect to t and also for the derivatives of the amplitude with respect to ν^2. Nielsen and Oades [26] used contour dispersion relations at fixed t of the form

$$\mathrm{Re}\,\widetilde{C}^+(\nu,\ t) = \frac{2}{\pi} P \int_{m_\pi + \frac{t}{4m_N}}^{\nu_1(t)} d\nu' \frac{\nu' \,\mathrm{Im}\, C^+(\nu',\ t)}{\nu'^2 - \nu^2} + C_c^+(\nu,\ t), \quad (85)$$

where $\nu_1(t)$ corresponds to the highest energy at which phase-shift data are available; $C_c^+(\nu,t)$ is the contribution from the finite contour. As a result, they obtained

$$\sigma_{\pi N} = (66 \pm 9)\ \text{MeV}. \quad (86)$$

A systematic analysis was made by Höhler *et al.*[27] using all the available data on πN and $\pi\pi$ scattering, and they obtained

$$\sigma_{\pi N} = (61 \pm 8)\ \text{MeV}. \quad (87)$$

In Ref. 24, to estimate $\sigma_{\pi N}$ Chao *et al.* used an extrapolation of $\widetilde{C}^+(\nu,t)$ along hyperbolic curves in the νt plane below the physical threshold. The lower branches of these curves are in the physical region for energies above the elastic threshold. They found

$$\sigma_{\pi N} = (57 \pm 12)\ \text{MeV}. \quad (88)$$

The given error was estimated from the spread of $\sigma_{\pi N}$ values found by the different methods of extrapolation. Moir *et al.*[25] determined $\sigma_{\pi N}$ by means of so-called interior dispersion relations. These dispersion relations do not require knowledge of the πN coupling constant and are not sensitive to the contribution of the higher partial waves. In this method, the pion-nucleon σ term is obtained by extrapolation to the Cheng–Dashen point only with respect to one variable, and the result was

$$\sigma_{\pi N} = (74 \pm 7)\ \text{MeV}. \quad (89)$$

Langbein[28] extrapolated the πN amplitude to the Cheng–Dashen point, using an expansion of the amplitude with respect to polynomials of the third degree,

$$\widetilde{C}^+(0,\ t) = \sum_{i=0}^{3} c_i q^i, \quad (90)$$

where $q(t) = m_\pi^2 - t/4)^{1/2}$.

To reduce the errors of the extrapolation, he used a restriction that follows from unitarity in the t channel:

$$c_1/c_0 = -a_0^0 = (-0.28 \pm 0.05)\, m_\pi^{-1},$$

where a_0^0 is the $\pi\pi$ scattering length for $I = J = 0$. The best fit for $\sigma_{\pi N}$ gives

$$\sigma_{\pi N} = (69 \pm 22)\ \text{MeV (Ref. 28)}. \quad (91)$$

Thus, in the studies in this direction the most probable values of the pion-nucleon σ term are in the interval 60–70 MeV.[24–28] On this basis, discussing the results of this group, Jaffe[72] gave as their final result for $\sigma_{\pi N}$ the value

$$\sigma_{\pi N} = (65 \pm 5)\ \text{MeV}. \quad (92)$$

In fact, the most careful calculation was the recent one of Koch,[71] who, using hyperbolic dispersion relations in the (ν^2,t) plane, obtained

$$\sigma_{\pi N} = (64 \pm 8)\ \text{MeV}. \quad (93)$$

We note that the derivation of the results (92) and (93) did not take into account any corrections in the relations (80) and (83), i.e., it was assumed that the amplitude F^+ at the Cheng-Dashen point did not effectively differ from the amplitude at the Weinberg point taken with the opposite sign.

Series expansion of the isosymmetric πN amplitude near the physical threshold (Altarelli–Cabibbo–Maiani method)

The group of results D in Fig. 1 corresponds to methods that employ expansion of the ampltitude with respect to the variables t, ν, q_1^2, q_2^2 and πN scattering data near the threshold (Refs. 36–38 and 73–75). In the Altarelli–Cabibbo–Maiani method,[73] the amplitude is represented in the form of the expansion

$$F^+(\nu,\ t,\ q_1^2, q_2^2) = A + Bt$$
$$+ C(q_1^2 + q_2^2) + D\nu^2 + R(\nu,\ t,\ q_1^2,\ q_2^2), \quad (94)$$

where R is a nonlinear function of t, ν^2, q_1^2, and q_2^2, and $R(0,0,0,0) = 0$. To determine the unknown coefficients A, B, C, and D, the following were used: Adler's consistency condition

$$F^+(0,\ m_\pi^2,\ m_\pi^2,\ 0) = 0; \quad (95)$$

the connection between the pion-nucleon σ term and the amplitude F^+ at the Weinberg point (74),

$$F^+(0,\ 0,\ 0,\ 0) = -\sigma_{\pi N}/f_\pi^2; \quad (96)$$

the experimentally known threshold values of

$F^+(m_\pi,\ 0,\ m_\pi^2,\ m_\pi^2)$ and $d/dt\, F^+(m_\pi,\ t,\ m_\pi^2,\ m_\pi^2)\,|_{t=0}$;

and, finally, the dispersion relation for $F^+(\nu,t,m_\pi^2,m_\pi^2)$ in the forward direction ($t = 0$) with a subtraction at the point $\nu = 0$,

$$\mathrm{Re}\,F(\nu_1, t, m_\pi^2, m_\pi^2) = \mathrm{Re}\,F(0, t, m_\pi^2, m_\pi^2) + \frac{2\nu_1^2}{\pi}\int_0^\infty \frac{dk}{\nu^2}\sigma^+(k, t), \quad (97)$$

where $\nu_1 = m_\pi + t/(4m_N)$, k is the momentum of the primary pion in the laboratory system, and $\sigma^+(\nu,t) \equiv k^{-1}\,\mathrm{Im}\,T^+(\nu,t)$. The relation (97) follows from the dispersion relations proposed by Chew, Goldberger, Low, and Nambu[76] for the invariant amplitudes $A^\pm(\nu,t)$ and $B^\pm(\nu,t)$ $(T^\pm = A^\pm + \nu B^\pm)$:

$$\mathrm{Re}\,A^\pm(\nu, t) = \frac{1}{\pi}P\int_{m_\pi + t/(4m_N)}^\infty d\nu'\,\mathrm{Im}\,A^\pm(\nu', t)\left(\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu}\right); \quad (98)$$

$$\mathrm{Re}\,B^\pm(\nu, t) = \frac{g_{\pi N}^2}{2m_N}\left(\frac{1}{\nu_B - \nu} \mp \frac{1}{\nu_B + \nu}\right) + \frac{1}{\pi}P\int_{m_\pi + t/(4m_N)}^\infty d\nu'\,\mathrm{Im}\,B^\pm(\nu', t)\left(\frac{1}{\nu' - \nu} \mp \frac{1}{\nu' + \nu}\right). \quad (99)$$

Among these, the dispersion relations for $A^\pm(\nu,t)$ and $B^\pm(\nu,t)$ and their derivatives in the forward direction, $(\partial/\partial t)\mathrm{Re}\,A^\pm(\nu,t)|_{t=0}$ and $(\partial/\partial t)\mathrm{Re}\,B^\pm(\nu,t)|_{t=0}$, were derived the most thoroughly, by Hamilton and Woolcock[77] (for $\nu \neq 0$). They showed that unsubtracted dispersion relations exist for $A^-(\nu,t)$, $B^+(\nu,t)$, $B^-(\nu,t)$, while for $A^+(\nu,t)$ one subtraction is needed.

To estimate $R(\nu,t,q_1^2,q_2^2)$ in the method of Ref. 73, the resonance model with the main contribution given by the isobar $\Delta(1232)$ is used. However, as was shown in Ref. 37, the contribution of only one isobar in the calculation of the dispersion integrals in (97)–(99) is not sufficient to describe the experimental values of the πN scattering amplitudes at the threshold, or the s- and p-wave scattering lengths (and volumes). Since the contribution from the nonlinear terms $R(\nu,t,q_1^2,q_2^2)$ to $\sigma_{\pi N}$ in the calculation of Ref. 73 was large (about 50%), a large uncertainty (about 50%) arises in the Altarelli–Cabibbo–Maiani resonance model, and as a result one cannot in practice rely on the calculated value of $\sigma_{\pi N}$:

$$\sigma_{\pi N} = (80 \pm 30)\ \text{MeV (Ref. 73)}. \quad (100)$$

Influence of nonanalyticity of the isosymmetric πN amplitude with respect to t at $\nu = 0$ on the dispersion estimates of $\sigma_{\pi N}$

Olsson and Osypowski[74] attempted to avoid the difficulty of the Altarelli–Cabibbo–Maiani method by considering from the very beginning, not the expansion (94), but an expansion of F^+ at $\nu = 0$.

$$F^+(0, t, q_1^2, q_2^2) \equiv F^+(t, q_1^2, q_2^2) = -\sigma_{\pi N}/f_\pi^2 + B(q_1^2 + q_2^2) + Ct + R(t, q_1^2, q_2^2). \quad (101)$$

According to the estimates of Ref. 74, the contribution of the isobar to $R(t,q_1^2,q_2^2)$ is small, but it was here assumed that the functional dependence of the resonance contribution of the isobar on t does not change on the transition from physical values of ν to $\nu = 0$. However, this assumption does not correspond to the results of Ref. 78, in which it was shown that the t dependence of the isobar contribution at $\nu = 0$ must be assumed to differ appreciably from the ordinary parametrization for physical values of ν.

Application to (101) of Adler's condition gives

$$Cm_\pi^2 = \sigma_{\pi N}/f_\pi^2 - Bm_\pi^2 - R(m_\pi^2, m_\pi^2, 0).$$

Hence, on the pion mass shell the expansion (101) takes the form

$$F^+(t, m_\pi^2, m_\pi^2) = -\sigma_{\pi N}/f_\pi^2 + 2Bm_\pi^2 + \left(\frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2} - B\right)t + [R(t, m_\pi^2, m_\pi^2) - R(m_\pi^2, m_\pi^2, 0)\,t/m_\pi^2]. \quad (102)$$

To find the unknown terms of the expansion in (102), the dispersion relation (97) and its derivative with respect to t at $t = 0$ were used. This yielded a system of two equations with the two unknowns $\sigma_{\pi N}$ and B:

$$F^+(0, m_\pi^2, m_\pi^2) = T^+(m_\pi, 0) + \frac{g_{\pi N}^2}{m_N}\frac{\nu_B^2(m_\pi - \nu_B)}{(m_\pi - \nu_B)^2(m_\pi + \nu_B)} - I_0; \quad (103)$$

$$d/dt\,F^+(t, m_\pi^2, m_\pi^2)_{t=0} = d/dt\,T^+_{\substack{t=0\\ \nu=m_\pi}} - \frac{g_{\pi N}^2}{m_N m_\pi}\frac{\nu_B^2(m_\pi - \nu_B)}{(m_\pi - \nu_B)^2(m_\pi + \nu_B)^2} - I_1/(2m_N m_\pi) - I_2, \quad (104)$$

where

$$I_0 = \frac{2m_\pi^2}{\pi}\int_0^\infty \frac{dk}{\omega^2}\sigma^+(k); \quad (105)$$

$$I_1 = \frac{2m_\pi^2}{\pi}\int_0^\infty \frac{k^2\,dk}{\omega^4}\sigma^+(k); \quad (106)$$

$$I_2 = \frac{2m_\pi^2}{\pi}\int_0^\infty \frac{dk}{\omega^2}\left.\frac{d\sigma^+(k, t)}{dt}\right|_{t=0}; \quad (107)$$

$$1/(4\pi)\,T^+(m_\pi, 0) = (1 + m_\pi/m_N)\,a^+_{0\ 1/2}; \quad (108)$$

$$1/(4\pi)\,d/dt\,T^+|_{(m_\pi=0)} = 1/(8m_N^2)\,a^+_{0\ 1/2} + 1/2a^+_{1\ 1/2} + (1 + 3m_\pi/(2m_N))\,a^+_{1\ 3/2}, \quad (109)$$

$$\omega = (m_\pi^2 + k^2)^{1/2}.$$

Using for I_0 and I_1 the values obtained by Höhler *et al.*[79] from experimental data on the πN scattering cross sections,

$$\left.\begin{aligned} I_0 &= (1.46 \pm 0.02)\,m_\pi^{-1};\\ I_1 &= (1.12 \pm 0.01)\,m_\pi^{-1} \end{aligned}\right\} \quad (110)$$

and calculating the integral I_2 under the assumption that the isobar makes the leading contribution,

$$I_2 = (0.28 \pm 0.01)\,m_\pi^{-3}, \quad (111)$$

Olsson and Osypowski obtained[74]

$$\sigma_{\pi N} = (70 \pm 6)\ \text{MeV}. \quad (112)$$

But when Olsson[75] used a dispersion relation for $T(\omega,0)$ with a subtraction at the threshold and the s-wave parametrization near the threshold[77]

$$\sqrt{s}/(m_N + m_\pi)\,\mathrm{Re}\,f^+_{0\ 1/2} = a^+_{0\ 1/2} + k^2C^+(k),$$

this making it possible to replace the combination $1/2(a^+_{1,1/2} + 2a^+_{1,3/2})$, which makes the main contribution to (109), by an expression that depends on $C^+(k)$, he obtained for the pion-nucleon σ term the value[75]

$$\sigma_{\pi N} = (51 \pm 9)\ \text{MeV}. \qquad (113)$$

This value was found from the expression

$$\sigma_{\pi N}/f_\pi^2 = 4\pi\,[(1+m_\pi/(2m_N))^2 a_{0\ 1/2}^+ + 3m_\pi^3/m_N a_{1\ 3/2}^+ - (1+m_\pi/m_N)^2 m_\pi^2 C^+]$$
$$+ 2/\pi m_\pi^4 (1+m_\pi/m_N) \int_0^\infty \frac{dk}{\omega^2 k^2} (\sigma^+(k) - \sigma^+(0))$$
$$- m_\pi (1+m_\pi/m_N)\sigma^+(0) + m_\pi^4 I_R - 2m_\pi^2 I_2 + 0\,(R\,(t,\ m_\pi^2,\ m_\pi^2)), \qquad (114)$$

where

$$I_R = \frac{2m_\pi}{\pi m_N} \int_0^\infty \frac{dk}{\omega^4} \sigma^+(k).$$

To estimate $\sigma_{\pi N}$ in accordance with (114), Olsson[75] used $C^+ = (0.032 \pm 0.008)\, m_\pi^{-3}$, calculated in Ref. 77, the πN scattering lengths from the energy-independent phase-shift analysis of Ref. 80, and the parametrization of the phase shifts from this phase-shift analysis to an energy-dependent form at low energies.[81]

The spread in the results (112) and (113) of the calculation of $\sigma_{\pi N}$ induces doubt, if not in the procedure, then at least in the uncertainty in the estimates of $\sigma_{\pi N}$ given by the authors of Refs. 74 and 75. Important here is the circumstance[36–38] that the validity of differentiating $F^+(0,t,m_\pi^2,m_\pi^2)$ with respect to t was assumed in Refs. 74 and 75. For differentiation of this function (or for interpolation with respect to the variable t from $t=0$ to $t=2m_\pi^2$), analyticity of $F^+(0,t,m_\pi^2,m_\pi^2)$ with respect to t must be established. But in 1971 Pagels and Pardee[69] showed that the function $\Sigma(t) = \sigma_{\pi N}(t)/(f_\pi m_\pi^2)$, which arises in the expansion of $A^+(\nu,t)$,

$$A^+(\nu,\ t) = m_\pi^2/f_\pi \Sigma(t) + g_{\pi N}^2/m_N\,[1+(m_\pi/m_N)^4 a + (2\nu_B/m_N)\, b + (\nu/m_N)^2 c], \qquad (115)$$

is nonanalytic with respect to t in the sense that

$$\Sigma(2m_N^2) - \Sigma(0) = 3/(8\pi)\,(g_A/(2f_\pi))^2 m_\pi/f_\pi + 0\,(m_\pi^2 \ln m_\pi^2), \qquad (116)$$

i.e., the difference is proportional to $\sqrt{t}$. The fact that the difference is proportional to m_π, and not m_π^2, emphasizes the singular nature of the point $t=2m_\pi^2$. The importance of such nonanalyticity increases with increasing contribution $\Sigma(t)$ in (115) and in the amplitude $T^+(\nu,t)$, and this occurs as $\nu \to 0$ and $\nu_B \to 0 (t \to 2m_\pi^2)$. At the physical threshold, the relative importance of the terms with the coefficients a, b, c and $\Sigma(t)$ will be comparable, whereas at the Cheng–Dashen point the contribution $m_\pi^2/f_\pi \Sigma(t)$ will be dominant. Thus, it can be seen from (116) that, $\sigma_{\pi N}(t)$ being nonanalytic in the neighborhood of the Cheng–Dashen point, $\sigma(2m_\pi^2)$ will differ from $\sigma_{\pi N}(0) = \sigma_{\pi N}$. The value of the correction, calculated in chiral perturbation theory in accordance with the estimate of Ref. 69, is -14 MeV. If one simply subtracts this correction from the results (92), as was done in Ref. 72, then (92) is replaced by

$$\sigma_{\pi N} \equiv \sigma_{\pi N}(0) = (51 \pm 5)\ \text{MeV}. \qquad (117)$$

A more accurate determination of the correction used here would be very valuable.

Determination of the pion-nucleon σ term from phase-shift analysis of πN scattering near the physical threshold

To reduce the influence of the nonanalyticity of $F^+(0,t,m_\pi^2,m_\pi^2)$ with respect to t in the estimate of the pion-nucleon σ term, the amplitude $F^+(\nu,t,q_1^2,q_2^2)$ was expanded[36–38] near the physical threshold ($\nu = m_\pi, t=0$), and the coefficients of this expansion were determined from the results of the energy-dependent phase-shift analysis of Ref. 82. Since the isosymmetric amplitude F^+ must be an even function of ν, its expansion can be written in the form

$$F^+(\nu,\ t,\ q_1^2,\ q_2^2) = -\sigma_{\pi N}/f_\pi^2 + Bt + C\,(q_1^2+q_2^2) + D\nu^2 + R_1 t^2 + R_2 t\nu^2 + R_3 \nu^4. \qquad (118)$$

Taking into account (73) and representing the amplitude T^+ by means of the phase-shift data,[82] we obtain the possibility of determining the coefficients of the expansion (118).

Strictly speaking, if the expansion (118) is to be used, its convergence must be proved. In the absence of such a proof, the restriction in the expansion of F^+ to the terms given in (118) is justified by the smallness of the following terms. Namely, the coefficient C in the earlier analyses[73–75] was approximately $0.3 m_\pi^{-3}$, and therefore terms with higher powers of (q_1^2,q_2^2) were not included in the expansion. As was shown in Ref. 23, the coefficient R_1 is small, so that there is no need to take into account the terms of higher powers of t. The direct calculation of Ref. 37 showed that the coefficients of the terms with $t\nu^4$ and ν^6 are small in modulus ($\lesssim 0.1 m_\pi^{-7}$). Therefore, bearing in mind that in the phase-shift analysis of Ref. 82 the expansion of the amplitude $T^+(\omega,0)$ was known with good accuracy only to terms $\sim q^4$ (q is the c.m.s. momentum), a restriction was made in Refs. 36–38 to the terms in the expansion of F^+ given in (118).

The requisite number of equations for determining the coefficients of the expansion (118) was obtained by using Adler's consistency condition and also by equating the terms with equal powers of q^2 on the right- and left-hand sides of Eq. (73) at $t=0$ and, respectively, Eq. (73) differentiated once and twice with respect to t at $t=0$. The result obtained was

$$F^+(\nu,\ t,\ q_1^2,\ q_2^2) = -0.69 + 1.05t - 0.37\,(q_1^2+q_2^2) + 1.06\nu^2 + 0.02t^2 + 0.48t\nu^2 + 0.48\nu^4 \qquad (119)$$

(here, a system of units in which $m_\pi = 1$ is used). For the pion-nucleon σ term there was then obtained[38]

$$\sigma_{\pi N} = (42 \pm 8)\ \text{MeV}, \qquad (120)$$

where the given error includes the inaccuracy due to the ignored terms in (118) and the uncertainty in the results of the phase-shift analysis of Ref. 82. Using the coefficients of the expansion (119), it is not difficult to estimate independently the dispersion integral I_0 (105):

$$I_0 = D + R_3 \approx 1.5\ m_\pi^{-1},$$

this giving a value close to the result (110) of the direct calculation of this integral by Höhler *et al.*[79] It was also shown in Ref. 37 that the dispersion relation (97), differentiated with respect to t (at $t=0$) contradicts the original relation if for the calculation of the dispersion integrals that arise the experimental data for $\sigma^+(k)$ and $d\sigma^+(k,t)/dt|_{t=0}$ are used. This gives a possibility for understanding the reason for the spread of the results (112) and (113) of the $\sigma_{\pi N}$ estimates.

An analogous procedure for calculating the coefficients in the expansion of F^+ using the parametrization of Ref. 81 of the results of the energy-independent phase-shift analysis of Ref. 80 leads for $\sigma_{\pi N}$ to a result close to (120):

$$\sigma_{\pi N} \approx 40 \text{ MeV (Ref. 38)}.$$

However, it must be borne in mind that the parametrization of Ref. 81 reproduces the energy dependence of the πN amplitude near the physical threshold somewhat less well than the results of the energy-dependent phase-shift analysis of Ref. 82. One must therefore give preference to the result (120), obtained using the results of Ref. 82. This follows, in particular, from the fact that the value of the integral obtained by means of Ref. 81 ($I_0 \approx 1.2 m_\pi^{-1}$) agrees less well with the result of Ref. 79. If the result (120) is to made more accurate, it is necessary to raise the accuracy in determining the coefficients in the expansion of the purely nuclear phases with respect to q^2 in the energy-dependent phase-shift analysis of πN scattering at low energies.

3. THEORETICAL METHODS OF ESTIMATING THE PION-NUCLEON σ TERM

The calculation of $\sigma_{\pi N}$ made by Cheng in the quark-gluon model

As was noted in the Introduction, the kinetic part of the QCD Lagrangian density (32) is invariant under $SU(3) \otimes SU(3)$ transformations, while the mass term $\bar{q}Mq$ [see (10)] transforms in accordance with the $(3^*,3) \oplus (3,3^*)$ representation of the chiral group as in the Gell-Mann–Oakes–Renner model.[9] Denoting the mass matrix of the "bare" (current) quarks by

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = c_0\lambda^0 + c_3\lambda^3 + c_8\lambda^8,$$

where

$$\lambda^0 = (2/3)^{1/2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and equating the corresponding terms, we find

$$\begin{aligned} c_0 &= 1/\sqrt{6}\,(m_u + m_d + m_s); \\ c_3 &= 1/2\,(m_u - m_d); \\ c_8 &= 1/(2\sqrt{3})\,(m_u + m_d - 2m_s). \end{aligned} \tag{121}$$

Then, ignoring the mass difference of the nonstrange quarks, we can write the $SU(3) \otimes SU(3)$ symmetry-breaking mass term $\mathscr{H}' = qMq$ as

$$\mathscr{H}' = c_0 u^0 + c_8 u^8, \tag{122}$$

where

$$u^\alpha(x) = \bar{q}(x)\,\lambda^\alpha q(x), \ \alpha = 0, 1, \ldots, 8.$$

It is easy to show that the pion-nucleon σ term (30) gets a contribution from only the part $\mathscr{H}_2$ of the density $\mathscr{H}'$ that breaks $SU(2) \otimes SU(2)$

$$\mathscr{H}_2 = 1/3\,(\sqrt{2}\,c_0 + c_8)\,(\sqrt{2}\,u_0 + u_8), \tag{123}$$

the other part

$$\mathscr{H}_1 = 1/3\,(c_0 - \sqrt{2}\,c_8)\,(u_0 - \sqrt{2}\,u_8) \tag{124}$$

breaking the SU(3) symmetry but not $SU(2) \otimes SU(2)$. Substituting (123) in (30) and taking into account the relations

$$\left.\begin{aligned} [\bar{Q}^\alpha, u^\beta] &= i d_{\alpha\beta\gamma} v^\gamma; \\ [\bar{Q}^\alpha, v^\beta] &= -i d_{\alpha\beta\gamma} u^\gamma, \end{aligned}\right\} \tag{125}$$

where $v^\alpha(x) = i\bar{q}(x)\lambda^\alpha\gamma^5 q(x)$ $(a = 0,1,\ldots,8)$, and $d_{\alpha\beta\gamma}$ are the structure constants of the SU(3) anticommutation relations

$$\{\lambda^\alpha, \lambda^\beta\} = 2d_{\alpha\beta\gamma}\lambda^\gamma,$$

we obtain

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N} \langle N \mid \bar{u}u + \bar{d}d \mid N\rangle, \tag{126}$$

where $\hat{m} = 1/2(m_u + m_d)$, and the nucleon states $|N_p\rangle$ with momentum p are normalized covariantly:

$$\langle N_p \mid N_{p'}\rangle = (2\pi)^3\, 2p_0 \delta\,(\mathbf{p} - \mathbf{p}'). \tag{127}$$

In a different notation,

$$\sigma_{\pi N} = \frac{1}{2m_N}\,\frac{1}{3}(\sqrt{2}\,c_0 + c_8)\,\langle N \mid \sqrt{2}\,u_0 + u_8 \mid N\rangle, \tag{128}$$

from which it can be seen that the contribution $\Delta M_N(\mathscr{H}_2)$ to the nucleon mass of the $SU(2) \otimes SU(2)$ breaking part (123) of the Hamiltonian density is equal to the pion-nucleon σ term, while the contribution to the nucleon mass due to $\mathscr{H}_1$ has the form

$$\Delta M_N(\mathscr{H}_1) = \frac{m_s}{2m_N} \langle N \mid \bar{s}s \mid N\rangle. \tag{129}$$

To estimate the pion-nucleon σ term from (128), it is necessary to know only the ratio

$$\alpha = \frac{\langle N \mid u_0 \mid N\rangle}{\langle N \mid u_8 \mid N\rangle}. \tag{130}$$

In fact, the ratio c_8/c_0 can be calculated in the framework of current algebra and PCAC. In Ref. 9, it is estimated to be

$$c_8/c_0 = -1.25, \tag{131}$$

while the matrix elements $\langle B_i | c_8 u^8 | B_i\rangle = \Delta M_i$ are fixed by the experimentally known splitting of the masses of the baryon octet.

As is well known, SU(3) symmetry leads to the relation

$$c_8 \langle B_j \mid u_j \mid B_i\rangle = i f_{jji} F + d_{jji} D,$$

where

$$F = 1/\sqrt{3}\,(m_N - m_\Xi), \quad D = \sqrt{3}/2\,(m_\Sigma - m_\Lambda).$$

Taking into account the covariant normalization (127) of the states, we obtain

$$\langle N \mid c_8 u^8 \mid N\rangle = -m_\Xi^2 + m_\Lambda^2. \tag{132}$$

The consistency of such an estimate of the matrix element (132) with allowance for the covariant normalization (127) [in contrast to the other approaches to estimation of the matrix element (132) encountered in the literature] can be confirmed in the framework of chiral perturbation theory,[70] in which in the first order in the quark masses the following relations are obtained for the squares of the masses of the baryon octet:

$$\left.\begin{aligned} m_N^2 &= A+\hat{m}\,(B^u+B^l)+m_sB^s+0\,(m^{3/2}); \\ m_\Sigma^2 &= A+\hat{m}\,(B^u+B^s)+m_sB^d+0\,(m^{3/2}); \\ m_\Xi^2 &= A+\hat{m}\,(B^d+B^s)+m_sB^u+0\,(m^{3/2}), \end{aligned}\right\} \quad (133)$$

where $B^u=\langle p|\bar{u}u|p\rangle$, $B^d=\langle p|\bar{d}d\,|p\rangle$, and $B^s=\langle p|\bar{s}s|p\rangle$. A formula for the square of the singlet mass of the baryon octet (Λ) can be derived from (133) by assuming the validity of the Gell-Mann–Okubo formula for the mass squares of the baryon octet,

$$(3m_\Lambda^2-m_\Sigma^2)/4=(m_N^2+m_\Xi^2)/2, \quad (134)$$

which is satisfied with an error up to 1.5%. Namely,

$$m_\Lambda^2=A+\frac{1}{3}\,\hat{m}\,(B^u+4B^d+B^s)$$
$$+\frac{1}{3}\,m_s\,(2B^u-B^d-2B^s)+0\,(m^{3/2}). \quad (135)$$

Then from (133) and (135) we have

$$m_\Lambda^2-m_\Xi^2=-\frac{1}{\sqrt{3}}\,(m_s-\hat{m})\,\frac{1}{\sqrt{3}}$$
$$\times(B^u+B^l-2B^s)=\langle N\mid c_8u^8\mid N\rangle. \quad (136)$$

Thus, the expression for the pion-nucleon σ term is a function of the coefficient α:

$$\sigma_{\pi N}=\frac{1}{3}\,\frac{\sqrt{2}-c_8/c_0}{c_8/c_0}\,(\sqrt{2}\,\alpha+1)\,\frac{(-m_\Xi^2+m_\Lambda^2)}{2m_N}. \quad (137)$$

In Ref. 30, it was assumed that the relation $\langle\pi|c_0u^0+c_8u^8|\pi\rangle\approx m_\pi^2$ (Ref. 9) implies smallness of the total mass shift due to $\mathscr{H}'$ for any quark system not containing strange quarks, i.e.,

$$\langle N\mid c_0u^0+c_8u^8\mid N\rangle\sim m_\pi^2 \text{ and}$$
$$\langle N\mid c_0u^0\mid N\rangle\approx-\langle N\mid c_8u^8\mid N\rangle.$$

From this the value $\alpha=1.25$ was obtained. In Ref. 29, Cheng, assuming exact fullfilment of the Okubo-Zweig-Iizuka (OZI) rule,[83] found that

$$\Delta M_N\,(\mathscr{H}_1)=\frac{m_s}{2m_N}\,\langle N\mid\bar{s}s\mid N\rangle=0,$$

and then, with allowance for (123)

$$\langle N\mid u_0\mid N\rangle=\sqrt{2}\,\langle N\mid u_8\mid N\rangle,$$

i.e., $\alpha=\sqrt{2}$. With this value of α and the value from Eq. (137) for c_8/c_0 (131), we obtain

$$\sigma_{\pi N}=32\ \text{MeV (Ref. 29)}. \quad (138)$$

The pion-nucleon σ term and the Okubo-Zweig-Iizuka rule

The OZI rule is an important property of strong-interaction dynamics and states that processes with corresponding Feynman diagrams in which two ends of one and the same quark line belong to one particle are forbidden. The experimental testing of the rule showed that it is well satisfied for the vector mesons ($\omega,\varphi,J/\psi,\psi'$), less well for the tensor mesons (f,f'), and not at all well for the pseudoscalar mesons (η,η'). Two simple sources of violation of the OZI rule are known: quark-antiquark annihilation processes

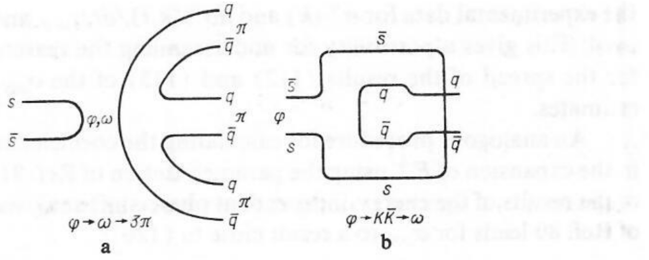

FIG. 3. Basic diagrams for the violation of the OZI rule: a) annihilation diagram of the decay $\varphi\to3\pi$; b) diagram with "twisted" quark lines for the transition $\varphi\to K\bar{K}\to\omega$.

(Fig. 3a) and "twisted" diagrams with physical intermediate hadronic states (Fig. 3b).

Introducing the phenomenological parameter

$$\beta=\frac{\langle N\mid\bar{s}s\mid N\rangle}{\langle N\mid(\bar{u}u+\bar{d}d)/\sqrt{2}\mid N\rangle}, \quad (139)$$

which measures the violation of the OZI rule, we obtain

$$\alpha=(\beta+\sqrt{2})/(1-\beta\sqrt{2}) \quad (140)$$

and for the value of the pion-nucleon σ term[39]

$$\sigma_{\pi N}=\frac{\sqrt{2}+c_8/c_0}{c_8/c_0}\,\frac{1}{1-\beta\sqrt{2}}\,\frac{M_\Lambda^2-M_\Xi^2}{2m_N}. \quad (141)$$

From experiments on the production of ω and φ mesons,[84,85] it can be concluded that $|\beta|\lesssim0.1$ if it is assumed that the contribution of the diagrams in Fig. 3 dominates over the ω–φ mixing effects. But if, following the quoted investigations, it is assumed that the violation of the OZI rule for the physical states ω and φ occurs only through the mixing of the states $|(\bar{u}u+\bar{d}d)/\sqrt{2}\rangle$ and $|\bar{s}s\rangle$, then β is near zero. The estimate of β in the parton model[40] (see below) gives for this parameter the interval

$$\beta=0.09\pm0.13. \quad (142)$$

In Refs. 43–45, the following interval for the ratio c_8/c_0 was obtained in the framework of current algebra and PCAC with allowance for breaking of the SU(3) symmetry of the vacuum:

$$-1.26<c_8/c_0<-1.22. \quad (143)$$

Taking into account (142) and (143), from the expression (141) for $\sigma_{\pi N}$ we can obtain[38,40]

$$\sigma_{\pi N}=(42\pm8)\ \text{MeV}, \quad (144)$$

where the confidence limits reflect the variation of the parameters β (142) and c_8/c_0 (143).

The violation of the OZI rule gives rise to a positive shift (about 70 MeV) of the nucleon mass from the chiral-invariant mass of the baryon octet (about 830 MeV) due to the breaking of the SU(3) symmetry in addition to the shift, equal to the pion-nucleon σ term, which arises because of the nonvanishing pion mass.

Dominguez and Langacker[68] obtained the estimate

$$y=\frac{\langle N\mid\bar{s}s\mid N\rangle}{\langle N\mid(\bar{u}u+\bar{d}d)/2\mid N\rangle}=\sqrt{2}\,\beta$$

under the assumption that the matrix elements in this ratio satisfy unsubtracted dispersion relations which can be saturated by pairs of Goldstone bosons. Then $y \simeq 0.36$ and

$$\sigma_{\pi N} = (36 \pm 8)\ \text{MeV (Ref. 68)}, \tag{145}$$

this being close to (144).

Estimate of the parameter of the OZI-rule violation in the parton model

The value of the parameter β (139) can be calculated[40] in the parton model using the distribution functions of the valence quarks, quark-antiquark pairs, and gluons in hadrons; these are known from deep inelastic lepton-nucleon scattering experiments. The parton model is based on the assumption that the cross section of deep inelastic processes is equal to the sum of the cross sections on the quasifree quark-partons. In the framework of this model, the electroproduction and neutrino-production structure functions can be expressed in terms of the corresponding distribution functions $f_\alpha(x)$, which are the probability that the partons of species α have longitudinal component of the momentum equal to xp (p is the nucleon momentum):

$$0 \leqslant f_\alpha(x) \leqslant 1, \quad \int_0^1 dx\, f_\alpha(x) = 1.$$

Since nucleons can emit virtual mesons, a quark-antiquark sea is present in nucleons even at the smallest Q^2 (primordial quark sea). The production of dimuon pairs of opposite sign in deep inelastic 'neutrino and antineutrino scattering was analyzed in Ref. 86 in order to determine the sea of strange quarks in the nucleon. The results of this analysis do not contradict the assumption of SU(3) symmetry of the $q\bar{q}$ sea. On the other hand, from neutrino experiments[87] the second moments[88] of the quark and antiquark distributions of the sea were found to be $4\xi_2 = 0.06$ and $\xi_2' = 0.5\xi_2$ (ξ for u and d, and ξ' for s). It should be said that the value 0.5 for the SU(3) symmetry-breaking factor was chosen in Ref. 87 rather arbitrarily.

Therefore, one can make the assumption of SU(3) symmetry of the $q\bar{q}$ sea in the nucleon, as was done in Refs. 88 and 89. In addition, it is natural to assume that the $c\bar{c}$ pairs in the wave function of the incident nucleon are not "internal" but arise in the process of QCD evolution at sufficiently large $Q^2 \sim m_c^2$. The distribution functions of the valence quarks and the quark-antiquark pairs of the sea vary by virtue of the violation of scaling with variation of Q^2; to estimate the parameter β (139), it is natural, in accordance with the meaning of its definition, to use these distribution functions at comparatively small Q^2, corresponding to the start of the asymptotic region. And to determine the initial distribution of the quark sea in the nucleon it is necessary to choose $Q^2 < Q_0^2 = m_c^2$. But to investigate the quark-gluon structure of the nucleon, one requires fulfillment of the condition $Q^2 \gg m_p^2$. We note that the numerous current attempts to solve the evolution equations of QCD[89–91] are rather uninformative in the region $Q^2 \lesssim m_p^2$ in the sense that they do not reflect the initial quark-gluon distribution but, in the best case, give evidence of the possibilities of a "test" virtual particle with the given Q^2. Because of this, for our estimate of the parameter it is sensible to use the parton distribution functions at $Q^2 \approx 2\ \text{GeV}^2$ at the point frequently taken as a reference point, at the beginning of the asymptotic region.[89,90,92]

Following Refs. 47 and 72, we can assume that the matrix elements in (139) are proportional to the contributions of the quarks of the corresponding flavor to the longitudinal nucleon momentum,

$$\beta = 1/3 \int_0^1 xq^p(x)\,dx \times \left\{ \frac{1}{\sqrt{2}} \left[\int_0^1 xq^v(x)\,dx + \frac{2}{3} \int_0^1 xq^p(x)\,dx \right] \right\}^{-1}, \tag{146}$$

where $xq^v(x)$ is the distribution of the valence quarks with respect to the longitudinal momentum, and $xq^p(x)$ is the corresponding distribution of the quarks and antiquarks of the $q\bar{q}$ pairs. The expression (146) would be exact if the distributions with respect to the longitudinal momentum of the valence quarks and sea quarks and antiquarks were the same. In fact, the quark distribution in the nucleon with respect to the fraction x of the longitudinal momentum is defined[88] as

$$\langle N \mid \bar{\psi}_q \psi_q \mid N \rangle = q(x) + \tilde{q}(x),$$

where $q(x)dx$ is the expected number of quarks of species q having fraction of the longitudinal nucleon momentum from x to $x + dx$. In our case, the relation (146) can be regarded as an order-of-magnitude estimate.

In Ref. 40, the parameter β (146) was estimated by means of the distribution functions of the valence quarks (q^v), the sea quarks and antiquarks (q^p), and the gluons (g) in the nucleon from Ref. 92 for $Q^2 \simeq 2\ \text{GeV}^2$:

$$\left. \begin{aligned} & xq^v(x) = 3 \cdot 1.73 x^{0.65} (1-x)^3; \quad xq^p(x) = 0.87\,(1-x)^9; \\ & xg(x) = 0.493\,(1+\eta)\,(1-x)^\eta, \quad \eta = 3-9. \end{aligned} \right\} \tag{147}$$

The gluon distribution is not well known but affects the quark distributions comparatively weakly. Using (147) and (146), we obtain[40]

$$\beta \approx 0.09. \tag{148}$$

It should be noted that the second moments of the distribution (147) of the nonstrange sea agree with the results of neutrino experiments[87] and also with the results[90] of the solution of the QCD evolution equations. Confidence that the estimate (148) is correct in order of magnitude arises from using the parton distribution function at different Q^2, when these distributions vary. Indeed, calculation of β with the quark and gluon distribution functions at $Q^2 = 5\ \text{GeV}^2$ (Ref. 88) gives the value $\beta \simeq 0.12$. Use of the distribution functions[86] at $Q^2 = 5\ \text{GeV}^2$ gives $\beta \simeq 0.10$.

In fact, the representation of β in the form (146) means that there is no violation of the OZI rule. If this rule is violated, gluons contribute to the matrix element $\langle N | \bar{q}q | N \rangle$. Then the parameter β can be expressed by

$$\beta = \Delta_{\text{OZI}} \sqrt{2} \left[3 \left(1 + \frac{2}{3} \Delta_{\text{OZI}} \right) \right]^{-1}, \tag{149}$$

where

$$\Delta_{\rm OZI} = (C+G)/V; \tag{150}$$

V, C, G are, respectively, the contributions to the matrix element $\langle N|\bar{u}u+\bar{d}d+\bar{s}s|N\rangle$ of the valence quarks, sea quarks and antiquarks, and gluons. Allowance for the gluon contribution G in (150) at 10% violation of the OZI rule[84,85] gives for β (149) the value[40]

$$\beta \simeq 0.13. \tag{151}$$

Such values of the parameter β, (148) and (151), correspond to the β interval (142) used to obtain the result (144) for $\sigma_{\pi N}$.

Difficulties associated with the estimate of $\sigma_{\pi N}$ in the bag model

Bag models trace their origin to P. N. Bogolyubov's paper of Ref. 93 and are a variant of quark models based on the corresponding equations for bound states.[94]

In Ref. 95, the MIT bag model[96,97] was used to obtain a wide range of $\sigma_{\pi N}$ values corresponding to different nonstrange quark masses. The other hadron characteristics calculated in this model are weakly sensitive to the choice of m. The experimental value of the axial-vector constant g_A of β decay of the neutron [$g_A = 1.255 \pm 0.006$ (Ref. 98)] is not reproduced well in this model. To calculate $\sigma_{\pi N}$ in the MIT model, the interval allowed for the nonstrange quark mass in Ref. 39 was chosen by requiring simultaneous description of the experimental values of the proton mass, charge radius, and magnetic moment. It was shown that in the framework of the MIT model it is impossible to describe satisfactorily g_A simultaneously with these parameters. In Ref. 72, allowance was made for the chiral contributions to the pion-nucleon σ term in a "hybrid" model. However, in this case the calculation of $\sigma_{\pi N}$ was not coordinated to the simultaneous description of the parameters of the light hadrons, including g_A and the pion mass.

It should also be noted that on the background of the general success of the MIT bag model[96,97] in describing the masses and other parameters of the light hadrons by means of a small number of constants the pion is the greatest exception. A satisfactory description of the light hadrons together with the pion has also not proved possible in the chiral-invariant forms of the bag model.[99,100] Nor has it been possible to describe the axial-vector constant g_A in either the MIT bag model[96,97] or the two-phase chiral-invariant bag models.[101,102] Moreover, there is an indication that in models which allow the existence of the pion field both outside and inside the bag[100,103] the contribution of the pion field to the axial-vector constant g_A is equal to zero[104] for a fairly large class of functions describing this field.

For a reliable estimate of the pion-nucleon σ term, it is necessary to use a variant of the bag model which describes satisfactorily the experimental data on the light hadrons, including the pion, and also the axial-vector constant of neutron β decay. In Ref. 41, an attempt was made to formulate such a variant of the bag model.

For the description of the pion-nucleon σ term in the bag model, the choice of the nonstrange quark mass $m_u = m_d = m$, or the ratio m_s/m is very critical. In Ref. 41, this ratio was taken to be equal to the mass ratio of the corresponding current quarks by analogy with QCD. In Ref. 41, it was assumed that the failure to describe the pion in the bag models is due to an incorrect approach to the influence of spin-spin splitting, which in the case of π and ρ is particularly large. Helpful for the solution of this problem is the analogy with the nonrelativistic potential model of constituent quarks for light hadrons. In Ref. 43, it was shown that the masses of the light hadrons can be well described by the formula

$$m_A = \Sigma\tilde{m}_i + V_0 + T + V_s, \tag{152}$$

where $\tilde{m}_i$ is the constituent-quark mass, V_0 is a scalar interaction, T is the kinetic energy, and V_s is the chromomagnetic interaction responsible for the spin-spin splitting: for mesons

$$V_s = \beta_M \frac{\mathbf{S}_1\cdot\mathbf{S}_2}{\tilde{m}_1\tilde{m}_2}, \tag{153}$$

and for baryons

$$V_s = \beta_B\left(\frac{\mathbf{S}_1\cdot\mathbf{S}_2}{\tilde{m}_1\tilde{m}_2} + \frac{\mathbf{S}_2\cdot\mathbf{S}_3}{\tilde{m}_2\tilde{m}_3} + \frac{\mathbf{S}_1\cdot\mathbf{S}_3}{\tilde{m}_1\tilde{m}_3}\right).$$

The representation for the hadron mass in the MIT model[96,97] also includes the corresponding chromomagnetic interaction E_M:

$$E_M = \sum_{i>j} \lambda\sigma_i\sigma_j\alpha^s_{ij}\frac{M_{ij}(m_iR,\ m_jR)}{R}, \tag{154}$$

where α^s is the effective quark-gluon coupling constant, $\lambda = 1$ for baryons and $\lambda = 2$ for mesons, σ_i are the Pauli spin matrices, $M_{ij}(m_iR, m_jR)$ is a function of the masses of the interacting quarks and the bag radius, and the summation is over all the quarks of the hadron. Specifically,

$$m_A = \sum_i \frac{n_i\omega_i}{R} + \frac{4}{3}\pi R^3B + E_M - \frac{Z_0}{R}, \tag{155}$$

where n_i is the number of quarks (antiquarks) of the hadron of the corresponding flavor, ω_i/R is the self-energy of the given quark in the spherical bag of radius R, and Z_0 is the dimensionless constant of the vacuum contribution. By analogy with (152), it was assumed in Ref. 41 that E_M (in the absence of chiral contributions) is completely responsible for the spin-spin splitting. This analogy requires equality of the bag radii for the corresponding hadron pairs: $N\Delta$, $\pi\rho$, KK^*, etc. An important result is that in such an approach the constants α^s_{ij} determined from the experimental spin-spin splittings depend in practice only on the strangeness of the pair of interacting quarks, i.e., this splitting can be described by means of three constants.

Chiral invariance is introduced in Ref. 41 by representing the axial-vector current in the form

$$A^\mu = \bar{q}\gamma^\mu\gamma^5\tau/2q\theta_v - f_\pi\partial^\mu\Phi\bar{\theta}_v, \tag{156}$$

where θ_v (respectively, $\bar{\theta}_v$) is equal to 1 inside (respectively, outside) the bag and 0 outside (inside) the bag. The assumption of approximate chiral invariance leads to a discontinuity of the flux of the axial current through the bag surface, i.e., to the introduction of the parameter η, which measures the violation of the continuity of the axial-vector current, into the boundary condition on the bag surface:

$$f_\pi \frac{\partial \Phi}{\partial r} = -\frac{i}{2} \eta \bar{q} \gamma^5 \tau q. \tag{157}$$

This parameter η renormalizes the pion field Φ and the chiral contribution to the hadron energy:

$$\Delta E = +\frac{1}{2} \sum_{a,\, b} \int d\Omega\, R^2 \Phi_a \left. \frac{\partial \Phi_b}{\partial r} \right|_{r=R}. \tag{158}$$

The introduction of three parameters η in such an approach (η_B for the baryons, η_π for π and ρ, and η_K for K and K^*) makes it possible to describe the mass spectrum of the light hadrons, including the pion, and the experimental values of the axial-vector neutron β-decay constant g_A and also the charge radii of the hadrons and magnetic moments of the baryons. The estimate of the pion-nucleon σ term with allowance for the chiral contributions gives[41]

$$\sigma_{\pi N} \approx 30 \text{ MeV}. \tag{159}$$

Of course, this result is approximate because of the considerable uncertainties in not only the estimates of the nonstrange quark mass in the bag model but also the contribution of the pion field to $\sigma_{\pi N}$. However, it appears that allowance for these uncertainties cannot lead to a significant increase in the value of $\sigma_{\pi N}$ calculated in the framework of the bag model.

CONCLUSIONS

Acceptable interval for the values of the pion-nucleon σ term

Summarizing the results of our discussion of the calculations of the pion-nucleon σ term, we conclude that the range $\sigma_{\pi N} = 34$–50 MeV (144) calculated in the quark model[38,40] under the assumption of the $(3,3^*) \oplus (3^*,3)$ breaking mechanism of the $SU(3) \otimes SU(3)$ symmetry, with allowance for the violation of the OZI rule and the uncertainties in the masses of the current quarks,[43–45] matches the interval (120) for $\sigma_{\pi N}$ calculated in Ref. 38 by expanding the isosymmetric πN amplitude near the physical threshold and using the data of the energy-dependent phase-shift analysis. These intervals (144) and (120) for $\sigma_{\pi N}$ also agree with the interval $\sigma_{\pi N} = 33$–47 MeV (72) calculated in Ref. 35 using sum rules for the isosymmetric pion-nucleus amplitude and data on the scattering of slow pions by light nuclei, and also with allowance for the renormalization of the soft-pion part of the πN amplitude in the nuclear medium.

Thus, as the acceptable interval for $\sigma_{\pi N}$ we can take

$$\sigma_{\pi N} = 30 - 50 \text{ MeV}. \tag{160}$$

We emphasize that the interval (160) is a consistent result for $\sigma_{\pi N}$ obtained by very different methods. This result does not contradict the estimates obtained earlier by methods of the Fubini-Furlan type[19–23] ($\sigma_{\pi N} \approx 30$–$35$ MeV), and the results of theoretical calculations [$\sigma_{\pi N} \approx 32$ MeV (Ref. 29), $\sigma_{\pi N} = 36 \pm 8$ MeV (Ref. 68), and $\sigma_{\pi N} \simeq 35$ MeV (Ref. 70)].

However, it should be noted that the methods of estimating $\sigma_{\pi N}$ which use extrapolation of the πN amplitude to the Cheng–Dashen point ($\nu = 0$, $t = 2m_\pi^2$) give much larger values for $\sigma_{\pi N}$ [60–70 MeV (Refs. 24–28)].

Thus, there is a certain contradiction between the results of the studies that use extrapolation of the isosymmetric πN amplitude to the Cheng–Dashen point and the results of alternative methods of estimating $\sigma_{\pi N}$. In this connection, it must be emphasized that the value of $\sigma(2m_\pi^2)$ calculated at the Cheng–Dashen point does not in general differ from the value $\sigma(0)$ taken as the definition of the pion-nucleon σ term. It is, however, possible that the isosymmetric πN amplitude is nonanalytic near the Cheng–Dashen point.[37,39] To reduce the influence of this possible nonanalyticity, the result (120) was obtained in Ref. 38 by an expansion of the isosymmetric πN amplitude near the physical threshold. With regard to the methods that use extrapolation to the Cheng–Dashen point, for the calculation of $\sigma_{\pi N}$ it seems justified to take into account the difference between $\sigma(2m_\pi^2)$ and $\sigma(0)$. Unfortunately, it is difficult to estimate this difference correctly.[70] However, if one takes for it the value calculated in Ref. 69, -14 MeV, then for this group of studies it is found that $\sigma_{\pi N} = 51 \pm 5$ MeV [Eq. (117)]. Such a correction to a large degree eliminates the contradiction between the results of these studies for the interval (160) for the pion-nucleon σ term. We emphasize that the interval (160) for $\sigma_{\pi N}$ agrees with the $(3,3^*) \oplus (3^*,3)$ mechanism of breaking of the $SU(3) \otimes SU(3)$ chiral symmetry, in agreement with the Gell-Mann–Oakes–Renner model[9] and QCD. We note that the acceptable interval (160) for $\sigma_{\pi N}$ could be further reduced by more accurate experimental data on the 1s-level shifts of the pionic atoms of light nuclei and by an improvement in the procedure for extracting the pion–nucleus scattering lengths from these data. For a further improvement in the result (160) it is also necessary to raise the accuracy in the determination of the coefficients in the expansion of the purely nuclear phase shifts with respect to q^2 in the energy-dependent phase-shift analysis of πN scattering at low energies.

Pion-nucleon σ term and parameters of the quark model. Behavior of the πN amplitude outside the region of physical values of ν and t and its bearing on the pion-nucleon σ term

The pion-nucleon σ term is the parameter which measures the breaking of the $SU(3) \otimes SU(3)$ chiral symmetry and gives important information for determining the structure of this breaking. The explicit form of the QCD Lagrangian gives certain restrictions on the mechanisms of this breaking possible in the general case. Namely, the mass term of the "bare" quarks in the QCD Lagrangian, which breaks the $SU(3) \otimes SU(3)$ symmetry, is a $(3,3^*) \oplus (3^*3)$ representation of the chiral group. With such a mechanism for breaking the chiral symmetry, the pion-nucleon σ term in various theoretical estimates is a function of the mass ratio $m_s/\hat{m}$ of the current quarks, and also of the parameter β measuring the violation of the OZI rule. To reconcile, for example, the results of these theoretical estimates with the $\sigma_{\pi N}$ estimates in the methods that use extrapolation of the πN amplitude to the Cheng–Dashen point ($\sigma_{\pi N} = 60$–70 MeV),[24–28] it is necessary to assume a ratio $m_s/\hat{m}$ both in the framework of current algebra and in QCD. Another possibility for reconciling these results would be to assume that the OZI-rule violation parameter is $\beta \simeq 0.37$, which contradicts the experimental data of Refs. 84 and 85. Thus, a more accurate determination of $\sigma_{\pi N}$ would lead to a decrease of the uncertainties in the parameters $m_s/\hat{m}$ and β and also in the nonstrange-quark mass in the bag model.

We mention also the connection between the pion-nucleon σ term and assumptions about the behavior of the pion-nucleon and pion-nucleus amplitudes off the mass shell in the methods of Refs. 19–23 using sum rules of Fubini-Furlan type, and also the connection with the behavior of the πN amplitude outside the region of physical ν and t values in the methods of Refs. 24–28, which use extrapolation of the πN amplitude to the Cheng–Dashen point. A more accurate value of $\sigma_{\pi N}$ would make it possible to obtain information about these properties of the πN amplitudes and would also indicate whether the procedures used to extrapolate $\sigma_{\pi N}$ are justified. In particular, this is related to the need to justify the dispersion-relation method for the πN amplitudes at physical values of ν as well as at $\nu = 0$.

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