Double charge exchange of pions on nuclei

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An analysis is made of the results of experimental and theoretical studies during the last ten years of a unique process—double charge exchange of pions on nuclei. On the one hand, the commissioning of meson factories and the use of new and more accurate experimental methods have led to the accumulation of rich and reliable material for not only light but also medium and heavy nuclei. On the other hand, the development and application of new microscopic theoretical approaches have led to a deepening of our ideas about the mechanism of these exotic reactions at low, intermediate, and high energies. The currently existing methods of theoretical investigation and all possible reaction mechanisms are considered. Particular attention is devoted to double charge exchange of pions on the lightest nuclei, for the theoretical description of which it is possible to use the well-developed methods for investigating few-particle systems. The experimental investigations of double-isobar analog and nonanalog transitions in the reaction of double charge exchange are systematized.

INTRODUCTION

During the last decade, interest in double charge exchange of pions on nuclei has revived. This was due to the commissioning of meson factories, which provided new possibilities for investigating the reaction, the increase in the intensity of pion beams, and the improvement in the angular and energy resolution of the facilities. All these developments made it possible to carry out high-precision measurements with the separation of the nuclear levels.

Double charge exchange of pions on nuclei occupies a particular position among all known nuclear reactions. It is unique because through the reaction one can obtain nuclei for which the Z component of the isospin differs by two units from that of the original nuclei:

$$\pi^{\pm} + A\ (Z,\ N) \to \pi^{\mp} + B\ (Z \pm 2,\ N \mp 2). \qquad (1)$$

This is made possible by double isospin flip of the pion, whose isospin is equal to unity and has three charge states: $+1$, 0, and -1. The existence of double charge exchange for pions was predicted by de Shalit, Drell, and Lipkin in 1961.[1] Experimentally, this process was discovered in the Laboratory of Nuclear Problems at the JINR in 1963.[2] A complete review of both experimental and theoretical studies of the process made up to 1971 can be found in Ref. 3.

What makes it attractive to study double charge exchange of pions on nuclei? In the reactions (1), it is possible to obtain and study nuclei far from the stability region. The reaction makes it possible to obtain information about double-isobar states of nuclei. The hopes of studying the expected difference between the neutron and proton densities in nuclei rest on this reaction. It offers the possibility of direct study of effects of short-range two-nucleon correlations in nuclei and their manifestation, for example, in high terms of the optical π–nucleus potential. This can be seen by noting that in an interaction with one nucleon the pion can change its charge by unity. Therefore, in double charge exchange at least two nucleons must participate. By this token, this reaction is more sensitive to the two-nucleon effects, manifested here already in the first order, than are reactions in which there is no need to consider two nucleons in the first order and in which the effects of the short-range correlations are manifested indirectly. Therefore, the process can give new information on two-nucleon aspects of nuclear dynamics. The reaction is one of those in which it is possible to study the effects of meson currents; moreover, the charge conservation law automatically eliminates first-order effects, which usually mask the effects of meson exchange currents. Finally, exotic nuclear states can be studied. For example, the double charge exchange of pions on the nuclei ^{3}He and ^{4}He makes it possible to investigate exotic states of three and four neutrons.

The progress in the experimental and theoretical study of the reaction in the period after the publication of the review of Ref. 3 has been partly reflected in Refs. 4–6. The time is now ripe for systematizing and summarizing the accumulated experimental and theoretical material because of the new experimental data on the reaction, a considerable fraction of which has been obtained by the operating LAMPF and SIN meson factories. The meson factories at the Institute of Nuclear Research in the Soviet Union and TRIUMF in Canada are also involved in the program of investigation of double charge exchange. Further investigations of the reaction on the lightest and heaviest nuclei are planned at the JINR. There have been many theoretical studies of the reaction; on the one hand, progress has been made, but on the other the difficulties in understanding the process and interpreting the mass of experimental facts have been demonstrated.

The present paper reviews the theoretical and experimental studies of the reaction made during the last decade.

1. MODELS AND APPROXIMATIONS IN THE THEORY OF DOUBLE CHARGE EXCHANGE

We consider the double charge exchange of pions on nuclei. Suppose the nuclear system is described by the Ha-

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miltonian H_0, and Ψ_i $(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N)$ is the wave function of the initial nucleus. As a result of the double exchange reaction, the nuclear system goes over into the new state described by the wave function Ψ_f $(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N)$.

We represent the Hamiltonian that describes the nucleus and the pion in the form

$$H = H_0 + h + \sum_{i=1}^{N} v_i, \tag{2}$$

where h is the kinetic-energy operator of the pion, and v_i is the potential of the interaction between the pion and the ith nucleon of the nucleus. In what follows, we shall assume that H_0 contains only two-particle NN interactions and that three-particle forces are excluded from the π-nucleus interaction. To describe pion-nucleus interactions it is customary to use the theory of multiple scattering, it being assumed that the pion is rescattered by the nucleons that constitute the nucleus. In such a treatment the point of departure is the Watson expansion for the T matrix of the π-nucleus interaction. This expansion has the form

$$T = \sum_{i=1}^{N} \tau_i + \sum_{i=1}^{N}\sum_{j\neq i}^{N} \tau_i G_0 \tau_j + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq j}^{N} \tau_i G_0 \tau_j G_0 \tau_k + \dots, \tag{3}$$

where the propagator G_0 at energy E of the pion-nucleus system has the form

$$G_0 = (E - H_0 - h + i\varepsilon)^{-1}, \quad \varepsilon = 0^+. \tag{4}$$

In (3), the matrix τ_i describes scattering of the pion by the ith bound nucleon in the nucleus and satisfies the equation

$$\tau_i = v_i + v_i G_0 \tau_i. \tag{5}$$

In general form, the amplitude of pion double charge exchange on the nucleus can be written in the form

$$M_{i\to f} = \langle \mathbf{k}_f, f \mid T \mid \mathbf{k}_i, i \rangle, \tag{6}$$

where $|\mathbf{k}_i, i\rangle$ and $|\mathbf{k}_f, f\rangle$ are the state vectors describing the pion-nucleus system before and after the reaction, respectively.

The Watson expansion (3) makes it possible to obtain the transition matrix element (6) in the form of an infinite series. Therefore, the use of the expansion (3) poses for us two problems:

1) the determination of the off-shell τ matrix of the πN interaction, which cannot be obtained from the free two-particle problem;

2) the summation of the multiple scattering series and the establishment of the rate of its convergence.

In the general case, it is in practice impossible to solve these problems, and therefore one must consider the approximate summation of the series (3) and the determination of approximations for the T matrix of the π-nucleus interaction. In turn, it is then possible to obtain corresponding expressions for the amplitude (6) for double charge exchange and to draw conclusions about the various reaction mechanisms.

Impulse approximation

If in (3) we replace the matrix τ_i, which describes scattering of a pion by a nucleon bound in the nucleus, by the matrix t_i of pion interaction with the free nucleon i, we obtain the T matrix in the impulse approximation. In the range of energies up to the threshold for production of two pions in the reaction of double charge exchange, it follows from the charge conservation law that at least two nucleons participate. Therefore, in the impulse approximation the first term in the expansion of the T matrix, corresponding to single scattering, does not contribute to the cross section for double charge exchange. Therefore, the main contribution to the cross section of the process will be made by the term corresponding to double scattering, and in the impulse approximation the transition amplitude (6) takes the form

$$M_{i\to f} = \langle \mathbf{k}_f, f \mid \sum_{i=1}^{N}\sum_{j\neq i}^{N} t_i G_0 t_j + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq j}^{N} t_i G_0 t_j G_0 t_k + \sum_{i=1}^{N}\sum_{j\neq i}^{N}\sum_{k\neq j}^{N}\sum_{n\neq k}^{N} t_i G_0 t_j G_0 t_k G_0 t_n + \dots \mid \mathbf{k}_i, i \rangle. \tag{7}$$

With each term in the expression (7) it is possible to associate one of the diagrams shown in Fig. 1, from which it can be seen that beginning with the second term in (7) the double charge exchange is accompanied by rescattering of the initial, intermediate, and final pions by nucleons of the nucleus. Therefore, the contribution of these terms is responsible for distortion of the pion waves.

In practical calculations in the impulse approximation, only the first term of the expansion (7) is usually taken into account:

$$M_{i\to f} = \langle \mathbf{k}_f, f \mid \sum_{i=1}^{N}\sum_{j\neq i}^{N} t_i G_0 t_j \mid \mathbf{k}_i, i \rangle = \langle \mathbf{k}_f, f \mid \sum_{n}\sum_{i=1}^{N}\sum_{j\neq i}^{N} t_i G_0 \mid n \rangle \langle n \mid t_j \mid \mathbf{k}_i, i \rangle. \tag{8}$$

The expression (8) makes it possible to obtain the cross section for double charge exchange in the impulse approximation with plane waves for the initial and final pions. If in (8) $|\mathbf{k}_i\rangle$ and $|\mathbf{k}_f\rangle$ are replaced by pion wave functions distorted by the optical potentials of the initial and final nuclei, the cross section of double charge exchange in the distorted-wave impulse approximation is obtained.

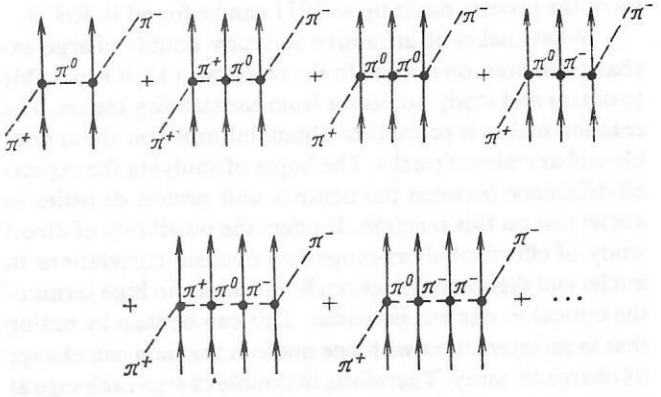

FIG. 1. Diagram expansion for the two-step mechanism.

The summation in (8) over the intermediate states is usually considered in the following two approximations: 1) *the summation is replaced by consideration of just one intermediate state*; 2) *the completeness condition is used.* In the first case, the matrix element (8) will be expressed in terms of the integral of the product of matrix elements of single-particle operators proportional to the density. If at the same time the intermediate state is an analog state, the propagator G_0 in (8) is replaced by the free-pion propagator. On the other hand, if it is borne in mind that the pion in the process of double charge exchange transfers a comparatively small energy (not more than 30 MeV) compared with its kinetic energy, the excitation energy ΔE_n can be replaced in the propagator by a certain averaged energy ΔE. At the same time, it is possible to sum in (8) over the intermediate states by using the completeness condition. Therefore, to calculate the matrix element it is necessary to know the two-particle density and, therefore, in such an approach one can directly investigate the part played by the two-particle short-range correlations. The influence of the nuclear medium on the propagation of the intermediate pion has been studied on several occasions. For this, one uses the propagator

$$\frac{1}{E-E_n-K_\pi-U_n(E)}, \tag{9}$$

where $U_n(E)$ is the optical potential that describes the propagation of the pion in the intermediate state.

Diffraction approximation

The study of pion double charge exchange on nuclei is simplified appreciably at high pion energies, when the wavelength of the incident particle is short compared with the nuclear diameter. In this case, the process can be described in the Glauber-Sitenko approximation.[7,8] The connection between Watson's theory of multiple scattering and the Glauber-Sitenko approximation for potential scattering was analyzed in detail in Ref. 9. Following Ref. 9, we can obtain the Glauber-Sitenko approximation in the theory of multiple scattering. For this, it is necessary in the Green's function (4) to ignore the Hamiltonian H_0 and use the eikonal approximation in the obtained propagator. Then in the Glauber-Sitenko approximation the amplitude of the process is written in the form

$$M_{i\to f}=\frac{ik_f}{2\pi}\int d^2b e^{iqb}\langle \mathbf{k}_f,\ f|1-\prod_{j=1}^{N}(1-\Gamma_j(\mathbf{b}-\mathbf{s}_j))|\,\mathbf{k}_i,i\rangle, \tag{10}$$

where $\mathbf{q}=\mathbf{k}_i-\mathbf{k}_f$, $\mathbf{b}$ is the impact parameter, $\mathbf{s}_j$ is the projection of the vector $\mathbf{r}_j$ onto the plane of the impact parameter, and the profile function is defined by

$$\Gamma_j(\mathbf{b}-\mathbf{s}_j)=(2\pi i k_j)^{-1}\int d^2q e^{-iq(\mathbf{b}-\mathbf{s}_j)}t(\omega_j,\ \mathbf{q}). \tag{11}$$

In the expression (11), $t(\omega_j \mathbf{q})$ is the on-shell πN scattering amplitude calculated at total energy ω_j and momentum transfer $\mathbf{q}$ in the center-of-mass system of the pion and nucleon j. It is clear that the term corresponding to single scattering will not make a contribution in (10) to the double charge exchange. We note that in the diffraction approximation contributions to the cross section for double charge exchange will be made by the diagrams in Fig. 1 and that in this case, in contrast to the impulse approximation, the graphical series is finite and contains diagrams up to those with N nucleon lines. The expression (10) can be simplified by ignoring in the πN scattering amplitude the spin-dependent terms, whose contribution to the process does not, according to Ref. 10, exceed 5%. In this case,

$$t(\omega,\mathbf{q})=t_1(\omega,\mathbf{q})+t_2(\omega,\mathbf{q})(\mathbf{I}\cdot\boldsymbol{\tau}), \tag{12}$$

where $\mathbf{I}$ and $\boldsymbol{\tau}$ are the isospin operators of the pion and nucleon, respectively, and for the profile function we obtain

$$\Gamma_j(\mathbf{b})=\Gamma_{1j}(\mathbf{b})+\Gamma_{2j}(\mathbf{b})(\mathbf{I}\cdot\boldsymbol{\tau}). \tag{13}$$

Using (13) in the matrix element for double charge exchange, we can separate the terms that contribute to the double charge exchange from the terms that distort the pion waves:

$$M_{i\to f}=\frac{ik_f}{2\pi}\int d^2\mathbf{b}e^{iq\mathbf{b}}\Gamma_{\rm tot}(\mathbf{b}), \tag{14}$$

where

$$\Gamma_{\rm tot}(\mathbf{b})=\langle \mathbf{k}_f,\ f|\,[-2\sum_{i<j}^{N}\Gamma_{2i}(\mathbf{b}-\mathbf{s}_i)\,\Gamma_{2j}(\mathbf{b}-\mathbf{s}_j) \times(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)]\Big[\prod_{l\neq i,\ j}^{N}(1-\Gamma_{1l}(\mathbf{b}-\mathbf{s}_l))\Big]|\,\mathbf{k}_i,\ i\rangle. \tag{15}$$

The first factor in (15) shows that two neutrons (or protons) go over into two protons (respectively, neutrons), whereas the second describes all possible elastic rescatterings of the initial, intermediate, and final pions. The last factor is particularly important in the resonance region of energies. The diffraction approximation in the double charge exchange of pions on nuclei was realized in Refs. 10–15.

The approximation of fixed centers

The diffraction approximation as considered above was most successfully employed at small pion scattering angles. A theory of multiple scattering of a particle by a nucleus in which there are no restrictions on the scattering angles was developed in Ref. 16. In this approach, the nucleons in the nucleus are assumed to be fixed at the positions $\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N$ in the scattering process, and the T matrix of the π-nucleus interaction, which is a function of the nucleon coordinates $\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N$, is averaged over the corresponding nuclear wave functions. It was shown in Ref. 17 that if one uses the free-pion Green's function, i.e., one ignores the excitation and recoil energy of the nucleus compared with the kinetic energy of the incident pion, and writes down Eq. (5) for the τ matrix for a fixed nucleon, then from (3) it is possible to obtain the T matrix in the fixed-centers approximation. On the basis of this approximation for the T matrix, an approach for investigating double charge exchange was developed in Ref. 18. In this model, the transition matrix element (6) is written in the form

$$M_{i\to f}=\langle \mathbf{k}_f, f\,|\,T(\mathbf{k}_i,\mathbf{k}_f;\ \mathbf{r}_1,\ \mathbf{r}_2,\ \ldots,\ \mathbf{r}_N)\,|\,\mathbf{k}_i,\ i\rangle, \tag{16}$$

where

$$T(\mathbf{k}_i, \mathbf{k}_f; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \sum_{j=1}^{N} e^{-i\mathbf{r}_j\mathbf{k}_f} G_j(\mathbf{k}_i, \mathbf{k}_f; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N). \quad (17)$$

Here, $G_j(\mathbf{k}_i, \mathbf{k}_f; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ describes the multiple scattering in all orders, the final scattering taking place on nucleon j. This amplitude satisfies the system of equations[19]

$$\begin{aligned} G_j(\mathbf{k}_i, \mathbf{k}_f; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) &= t_j(\mathbf{k}_i, \mathbf{k}_f)\, e^{i\mathbf{k}_i\mathbf{r}_j} \\ &+ \sum_{n\neq j} \int d\Omega_p t_n(\mathbf{p}, \mathbf{k}_f)\, e^{i\mathbf{p}(\mathbf{r}_j-\mathbf{r}_n)}\, \Theta[\mathbf{p}\cdot(\mathbf{r}_j-\mathbf{r}_n)] \\ &\times G_n(\mathbf{k}_i, \mathbf{p}; \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N), \\ |\mathbf{k}_i| &= |\mathbf{k}_f| = |\mathbf{p}|, \quad j = 1, 2, \ldots, N, \end{aligned} \quad (18)$$

where $t_j(\mathbf{p}, \mathbf{k}_f)$ is the on-shell free πN scattering matrix. Equation (18) can be written in the symbolic form

$$G_j = t_j + t_j \sum_{n\neq j} G_n. \quad (19)$$

Using the form of expression (19), we can write down corresponding equations for the amplitude of double charge exchange. We consider the process for positive ions. Let G_j^+ be the elastic scattering amplitudes, G_j^- and G_i^0 be the amplitudes for production of π^- and π^0 mesons, respectively, t_j^+, t_j^-, t_j^0 be the amplitudes of free elastic πN scattering, and $t_j^{+0}, t_j^{0-}, t_j^{0+}, t_j^{-0}$ be the charge-exchange πN amplitudes. Then the system of coupled integral equations for the double charge exchange of π^+ mesons takes the form

$$\left.\begin{aligned} G_j^+ &= t_j^+ + t_j^+ \sum_{i\neq j} G_i^+ + t_j^{+0} \sum_{i\neq j} G_i^0; \\ G_j^0 &= t_j^{0+} + t_j^0 \sum_{i\neq j} G_i^0 + t_j^{0+} \sum_{i\neq j} G_i^+ + t_j^{0-} \sum_{i\neq j} G_i^-; \\ G_j^- &= t_j^- \sum_{i\neq j} G_i^- + t_j^{-0} \sum_{i\neq j} G_i^0. \end{aligned}\right\} \quad (20)$$

In (20), the term $t_i^0 \Sigma_{i\neq j} G_j^0$, for example, describes the process in which a π^0 meson propagates in the nucleus, the final interaction being elastic scattering on nucleon i. The term $t_i^{-0} \Sigma_{i\neq j} G_j^0$ describes a π^- meson created from a π^0 meson in its last charge-exchange scattering t_i^{-0} on nucleon i. The system of coupled equations (20) was solved in Ref. 18 using a separable form for the πN scattering amplitude. We note that the method described above makes it possible to investigate coherent double charge exchange, i.e., analog transitions. This follows from the requirement $|\mathbf{k}_i| = |\mathbf{k}_f| = |\mathbf{p}|$ in Eq. (18).

Optical-potential method

Besides the methods described above for constructing the T matrix for the process of double charge exchange in the theory of multiple scattering, the reaction on medium and heavy nuclei can be successfully described by the introduction of an optical potential. The optical model reduces the problem of pion scattering by the many-particle system, the nucleus, to the simpler problem of pion scattering in the field of a complex optical potential. In this approach, the T matrix of the elastic π-nucleus interaction is obtained by solving the Lippmann-Schwinger equation

$$T = U + UG_0T. \quad (21)$$

The optical potential U in (21) is selected phenomenologically or is constructed in the framework of the theory of multiple scattering. In the two different approaches of the theory,[20,21] the optical potential is expressed as a series expansion with respect to the πN interaction matrix.[17] The πN interaction being weaker than the NN interaction at low energies, it is assumed that for pion-nucleus scattering this series converges more rapidly than for nucleon-nucleus scattering.

The optical-potential method was used to investigate the reactions of elastic or coherent single and double pion charge exchange,

$$\pi^+ + {}_NX_Z \to \pi^0 + {}_{N-1}Y_{Z+1}; \quad (22)$$

$$\pi^+ + {}_NX_Z \to \pi^- + {}_{N-2}W_{Z+2}. \quad (23)$$

In the reactions (22) and (23), the nuclei ${}_NX_Z, {}_{N-1}Y_{Z+1}$, and ${}_{N-2}W_{Z+2}$ are isotopic multiplets. This means that in the charge-exchange reaction (22) ${}_{N-1}Y_{Z+1}$ is the isobar analog state of the ground state of the nucleus ${}_NX_Z$, while the nucleus ${}_{N-2}W_{Z+2}$ is the analog state of the nucleus ${}_{N-1}Y_{Z+1}$ or the double-isobar analog state of the target nucleus.

The cross section of double pion charge exchange on medium and heavy nuclei is usually calculated in the first approximation in the optical potential. The corresponding transition matrix element can be written in the form

$$M_{i\to f} = \langle \varphi_{\pi^-} | UG_0U | \varphi_{\pi^+} \rangle. \quad (24)$$

The optical potentials of both the first[22–24] and second[23,25,26] orders have been used to study double charge exchange on nuclei. The first-order optical potential has the form

$$U_{\alpha\beta}(\mathbf{k}_i, \mathbf{k}_f) = \sum_l \langle \mathbf{k}_i, \alpha | t_l | \mathbf{k}_f, \beta \rangle, \quad (25)$$

where the indices $\alpha, \beta, \ldots$ denote the elastic channel, the channel of single charge exchange, and the channel of double charge exchange. Using the expression for the optical potential, we write down the Klein-Gordon equation for the coupled channels, this making it possible to determine the pion functions in (24),

$$(-\nabla^2 + 2EV_c^\alpha - k_\alpha^2)\varphi_\alpha = 2E \sum_\beta U_{\alpha\beta}\varphi_\beta, \quad (26)$$

where k_α is the momentum of the pion in channel α, and V_c^α is the corresponding Coulomb potential of the nucleus. We note that whereas there are two distortions of the pion wave (of the incident pion and the π^0 meson) in the charge-exchange reaction, in the reaction of double charge exchange there are three such distortions: those of the π^+ and π^- waves in the entrance and exit channels, and the distortion of the π^0 meson that enters the expression (24) through the Green's function.

Using the coupled-channel method in the momentum

representation, Liu[26] constructed the diffraction theory of double charge exchange. His point of departure was a system of three integral equations which can be expressed formally as

$$\mathcal{T} = \tilde{v} + \tilde{v} G_0 \mathcal{T}, \tag{27}$$

where $\mathcal{T}$ is the scattering operator, and $\tilde{v}$ is the effective interaction, which is decomposed into the strong and Coulomb interactions:

$$\tilde{v} = v_s + v_c. \tag{28}$$

In (28), v_s is a complex, energy-dependent interaction which for nuclei with isospin $T \geqslant 1$ can be written in the general form

$$v_s = V_0 + (\mathbf{I}\cdot\mathbf{T})\, V_1 + (\mathbf{I}\cdot\mathbf{T})^2 V_2. \tag{29}$$

In (29), **I** and **T** are the isospin operators of the pion and nucleus, respectively, and V_0, V_1, and V_2 are usually called the isoscalar, isovector, and isotensor interactions, respectively. If the optical potential for the π^i-nucleus system ($i = +, 0, -$) is introduced by

$$U^i = \langle \pi^i, \text{nucleus} \mid v_s \mid \pi^i, \text{nucleus} \rangle, \tag{30}$$

then, using the standard angular-momentum algebra, we obtain the simple result[26]

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{T(2T-1)} \begin{bmatrix} -T & T(2T+1)-T^2 \\ 1-T & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} T \\ T^2 \\ 1 \end{bmatrix} \begin{bmatrix} U^+ \\ U^0 \\ U^- \end{bmatrix}, \tag{31}$$

where T is the nuclear isospin. Equation (31) relates the interactions V_0, V_1, and V_2 to the nth order optical potential. With the introduction of the second-order optical potential, which depends on the square of the nuclear density, this approach makes it possible to treat double charge exchange that takes place either on two valence nucleons or on two core nucleons through nonanalog transitions. The interference of these amplitudes with the amplitude including the analog transitions changes the cross section for double charge exchange qualitatively. The reason for this is that the values of these amplitudes are comparable.

Meson-current model

The study of meson currents in nuclear physics made it possible to explain the nuclear magnetic moments, some effects in electron-nucleus scattering, and also various aspects of the theory of photonuclear reactions and meson absorption by nuclei. However, in all the processes considered the meson currents were considered together with the single-particle effect. An example of a reaction in which the meson currents play the leading role, i.e., the effect of the currents must be manifested already in the first order, is the double charge exchange of pions. Indeed, if one proceeds from the fact that the nucleons are surrounded by a "meson cloud," the knocking out of a π^- (respectively, π^+) meson from the

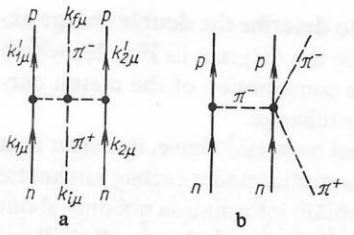

FIG. 2. Diagram of the meson-current mechanism.

meson cloud of a nucleon by a π^+ (respectively, π^-) meson leads to double charge exchange on the nucleus. The main hypothesis of the meson-current model for double charge exchange proposed in Ref. 27 reduced to the assumption that the pion is scattered by an off-shell pion exchanged by nucleons in the nucleus. The $\pi^+ NN \to \pi^- NN$ amplitude was constructed on the basis of the $\pi\pi$ scattering amplitude. For this mechanism, the amplitude of the elementary event can be found from the diagram in Fig. 2a,

$$A(\mathbf{k}_i, \mathbf{k}_f) = (i\sqrt{2}g)^2 \bar{u}_n(k_{1\mu})\, \gamma_5 \tau_- u_p(k'_{1\mu})\, \bar{u}_n(k_{2\mu}) \gamma_5 \times \tau_- u_p(k'_{2\mu}) \frac{M_{\pi\pi}(k_{i\mu}, q_{1\mu}; q_{2\mu}, k_{f\mu})}{(q_{1\mu}^2 - \tilde{m}^2)(q_{2\mu}^2 - \tilde{m}^2)}, \tag{32}$$

where $k_{1\mu}, k_{2\mu}, k_{i\mu}, k_{f\mu}$ are the 4-momenta of the nucleons and the pions, $q_{1\mu} = k_{1\mu} - k_{f\mu}, q_{2\mu} = k_{2\mu} - k_{i\mu}$, g is the πN coupling constant, $\tilde{m}$ is the pion mass, and $M_{\pi\pi}(k_{i\mu}, q_{1\mu}; q_{2\mu}, k_{f\mu})$ is the transition matrix element for $\pi^+\pi^- \to \pi^+\pi^-$ scattering. Because the energy of the $\pi\pi$ system in the center-of-mass system is fairly low, the off-shell $\pi\pi$ scattering can be described either by Weinberg's model[28] or by the modified Veneziano model.[29] If, following Ref. 27, the energy transfer and the momenta of the nucleons within the nucleus are ignored, the expression (32) can be written in the nonrelativistic limit in the form

$$A(\mathbf{k}_i, \mathbf{k}_f) = (i\sqrt{2}g)^2 \frac{(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)}{4m} \times \frac{M_{\pi\pi}(k_{i\mu}\mathbf{k}_1; \mathbf{k}_2, k_{f\mu})}{(k_1^2 + \tilde{m}^2)(k_2^2 + \tilde{m}^2)}. \tag{33}$$

Using the expression (33), it is possible to obtain the cross section of double charge exchange of pions on nuclei under the assumption of the meson-current mechanism. The contribution of the meson currents to double charge exchange was investigated in Refs. 15, 27, and 30–32. Consideration of the diagram in Fig. 2a poses a question: Is this diagram the only one that describes the contribution of the meson currents to the double charge exchange? If in Fig. 2a we remove one nucleon line, we obtain the diagram for the process $\pi N \to \pi\pi N$. On the basis of this, it was suggested in Ref. 15 that when one considers the meson currents in the reaction of double charge exchange it is necessary to use a model that describes the $\pi N \to \pi\pi N$ process well. About 30 diagrams that contribute to this process were constructed (it is true that the contributions of some of these cancel each other). If to these diagrams we add one nucleon line, we obtain the graphs that describe the contributions of the meson currents to the double charge exchange. For example, if the $\pi N \to \pi\pi N$ process is described by the chiral-invariant Ha-

miltonian of Ref. 28, then to describe the double charge exchange it is necessary to use the diagram in Fig. 2b, which also takes into account the contribution of the meson currents to the double charge exchange.

On the basis of what has been said above, it is clear that if double charge exchange is studied under certain kinematic conditions it is possible to obtain information not only about the $\pi\pi$ scattering amplitude but also about the off-shell extrapolation of the amplitude of the $\pi N \to \pi\pi N$ process.

2. MICROSCOPIC APPROACH IN THE THEORY OF DOUBLE CHARGE EXCHANGE

Double charge exchange of pions on three- and four-nucleon nuclei

From the theoretical point of view, the greatest interest attaches to study of pion double charge exchange on three- and four-nucleon nuclei:

$$\pi^- + {}^3\mathrm{He} \to \pi^+ + 3n; \quad (34)$$

$$\pi^+ + {}^3\mathrm{H} \to \pi^- + 3p; \quad (35)$$

$$\pi^- + {}^4\mathrm{He} \to \pi^+ + 4n; \quad (36)$$

$$\pi^+ + {}^4\mathrm{He} \to \pi^- + 4p. \quad (37)$$

There are several reasons for this interest. First, the ^{3}H, ^{3}He, and ^{4}He nuclei have the smallest number of nucleons for which the process under consideration takes place. Second, the reactions (34)–(37) proceed through a single channel—complete breakup of the nucleus. Third, it is precisely in the study of the reaction on three- and four-nucleon nuclei that one can obtain more or less unambiguous conclusions about the reaction mechanism and establish which reaction mechanism is dominant in which energy region. Indeed, the well-known model-free methods of the Faddeev equations and hyperspherical functions have been developed for the description of three and four nucleons in the discrete and continuous spectra. Using these methods in conjunction with only an assumption about the reaction mechanism, it is possible to construct the theory of double charge exchange of pions on three- and four-nucleon nuclei. Fourth, investigation of the processes (34) and (36) provides a unique possibility for studying systems consisting of three and four neutrons. Fifth, microscopic model-free description of the intitial and final nuclear states in the reactions (34)–(37) using NN potentials makes it possible to relate the experimentally established characteristics of double charge exchange to the properties of the NN interaction and to obtain more or less unambiguous conclusions about these properties.

Up to now, the study of the reactions (34)–(37) in the region of intermediate energies has been based on the consideration of two mechanisms: the two-step mechanism in which the pion is scattered successively by two nucleons of the nucleus and changes its sign twice and the meson current mechanism.

For the nuclei ^{3}H and ^{4}He the reaction was considered under the assumption of the two-step mechanism in Refs. 11 and 31–33. In Ref. 11, the diffraction approximation was used to calculate the differential cross section of the process (34) for forward scattering using the model of Chew and Low[34] for the πN interaction. The ^{3}He nucleus was described by a spatially symmetric Gaussian function of the s state, and the three neutrons by a Slater determinant formed from plane waves. The final-state interaction between the three neutrons was thus ignored. To investigate the reaction (34), Phillips[33] used a semimicroscopic approach, describing the three-neutron state by the solution of the Faddeev equation in the first iterative approximation and the ^{3}He nucleus by a variational Gaussian function of the s state. Although the final-state interaction between the two neutrons was taken into account, orthogonality of the initial and final nuclear states was not ensured. In Refs. 31 and 32, a microscopic theory was developed for the processes (34) and (35) under the assumption of both the two-step mechanism and the meson-current mechanism. In this microscopic approach, the wave functions of the initial and final nuclear states are eigenfunctions of the same Hamiltonian, so that orthogonality of these states is ensured.

The reaction on the ^{4}He nucleus was considered in Refs. 32 and 35–38 in the framework of the two-step mechanism using the impulse approximation. Becker and Schmit[35] constructed the πN scattering amplitude on the basis of a static model of pion-nucleon interaction and included the Δ resonance in the πN amplitude. In a recent study,[36] Gibbs *et al.* also treated the reaction on ^{4}He as a two-step process that used a more complete parametrization of the t matrix of the πN interaction. The approaches of Refs. 35 and 36 differ appreciably in the off-shell continuation of the πN scattering amplitude, and the difference between the results of these studies indicates that double charge exchange is very sensitive to off-shell effects. Variational Gaussian functions were used to describe the initial state in the reactions (36) and (37) in Refs. 35 and 36, and although the later paper of Ref. 36 took into account the Pauli principle and included an exact calculation of the five-particle phase space, the final-state interaction between the nucleons was not considered. It was shown in Refs. 32, 37, and 38 that such a treatment is rather crude, since the cross section of the process contains spurious contributions due to the nonorthogonality of the initial and final nuclear states, and allowance for the final-state interaction changes the results not only quantitatively but also qualitatively.

The reactions (36) and (37) were considered in Refs. 27 and 32 under the assumption of the meson-current mechanism. In Ref. 27, the Pauli correlations and the final-state interaction were not considered, and the five-particle phase space was not completely taken into account. The use of completely antisymmetrized wave functions of the initial and final states, allowance for the final-state interaction between all the nucleons, and the exact calculation of the five-particle phase space made in Ref. 32 strongly changed the results and conclusions of Ref. 27 and showed that in the low-energy region the main mechanism of the double charge exchange is the successive scattering of the pion on two nucleons of the nucleus.

A microscopic theory of the reactions (34)–(37) was developed in Refs. 31, 32, 37, and 38 in the framework of the method of hyperspherical functions for three and four parti-

cles. This approach made it possible to determine the wave functions of the target nuclei and the wave functions of three and four interacting nucleons in the continuum on the basis of the same realistic NN potentials. On the one hand, this ensured orthogonality of these states; on the other, it made it possible to relate the experimental facts on double charge exchange to the properties of the NN potential. In addition, the use of the hyperspherical basis made it possible to find the four- and five-particle phase spaces exactly. Below, following Refs. 31, 32, 37, and 38, we present the fundamentals of the microscopic approach in the theory of the reactions (34)–(37).

Differential cross section of the reactions

Differential cross section of double charge exchange on three-nucleon nuclei. For the kinematic description of the processes (34)–(37), we shall regard the pions as relativistic particles and describe the motion of the nucleons in the nonrelativistic approximation. If we denote the pion 4-momenta in the initial and final states by $k_{i\mu} = (\mathbf{k}_i, E_i)$ and $k_{f\mu} = (\mathbf{k}_f, E_f)$, respectively, then the 4-momentum conservation law for the reactions (34) and (35) takes the form

$$k_{i\mu} + Q_\mu = k_{f\mu} + \sum_{j=1}^{3} K_{j\mu}, \tag{38}$$

where $Q_\mu = (0, M_3)$ is the 4-momentum of the three-nucleon nucleus, and $K_{j\mu} = (\mathbf{K}_j, E_j)$ is the 4-momentum of nucleon j in the final state.

In the three-nucleon system, we introduce Jacobi momenta:

$$\left.\begin{aligned} \mathbf{k}_1 &= \frac{1}{\sqrt{2}}(\mathbf{K}_1 - \mathbf{K}_2),\ \mathbf{k}_2 = \frac{1}{\sqrt{6}}(\mathbf{K}_1 + \mathbf{K}_2 - 2\mathbf{K}_3); \\ \mathbf{K} &= \frac{1}{\sqrt{3}}(\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3). \end{aligned}\right\} \tag{39}$$

In these variables, the differential cross section of the processes (34) and (35) in the center-of-mass system of the nucleus takes the form

$$d\sigma = 2\pi |M_{i\to f}|^2\, \delta\left(E_i - E_f - \frac{k_1^2 + k_2^2}{2m} - B_3\right) \frac{E_i}{k_i}\, \frac{d\mathbf{k}_f d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^9}, \tag{40}$$

where B_3 is the binding energy of the three-nucleon nucleus, and m is the nucleon mass.

We go over from the Jacobi momenta (39) to hyperspherical coordinates in the momentum space:.

$$k^2 = k_1^2 + k_2^2,\ k_1 = k \sin\alpha,\ k_2 = k\cos\alpha;$$
$$d\mathbf{k}_1 d\mathbf{k}_2 = k^5\, dk\, d\Omega_k,\ d\Omega_k = \sin^2\alpha \cos^2\alpha\, d\alpha d\hat{\mathbf{k}}_1 d\hat{\mathbf{k}}_2. \tag{41}$$

In the hyperspherical coordinates, the angular and energy distributions of the pions produced by the double charge exchange take the form

$$\frac{d^2\sigma}{dE_f d\Omega_\pi} = 2\pi\rho_f \int |M_{i\to f}|^2\, d\Omega_k, \tag{42}$$

where ρ_f is the four-particle density of the final states,

$$\rho_f = (2\pi)^{-9}\, \frac{E_i E_f k_f}{k_i}\, \frac{(2m)^3}{2}\, (E_i - E_f - B_3)^2. \tag{43}$$

Differential cross section of double charge exchange on the ^{4}He nucleus. We now consider the processes (36) and (37). We denote by $P_{j\mu} = (\mathbf{P}_j, E_j)$ and $P_\mu = (0, M_4)$ the momenta of nucleon j in the continuum and in the ^{4}He nucleus, respectively. In this notation, the 4-momentum conservation law for the reactions (36) and (37) becomes

$$k_{i\mu} + P_\mu = k_{f\mu} + \sum_{j=1}^{4} P_{j\mu}. \tag{44}$$

In the four-nucleon system, we go over from the momenta $\mathbf{P}_j$ to the Jacobi momenta

$$\left.\begin{aligned} \mathbf{p}_1 &= \frac{1}{\sqrt{2}}(\mathbf{P}_1 - \mathbf{P}_2),\ \mathbf{p}_2 = \frac{1}{2}(\mathbf{P}_3 + \mathbf{P}_4 - \mathbf{P}_1 - \mathbf{P}_2); \\ \mathbf{p}_3 &= \frac{1}{\sqrt{2}}(\mathbf{P}_3 - \mathbf{P}_4),\ \mathbf{P} = \frac{1}{4}(\mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3 + \mathbf{P}_4) \end{aligned}\right\} \tag{45}$$

and in these variables we write down the differential cross section of double charge exchange of pions on the ^{4}He nucleus. After integration over the four-nucleon c.m.s. momentum, the cross section becomes

$$d\sigma = 2\pi\, |\tilde{M}_{i\to f}|^2\, \delta\left(E_i - E_f - \frac{p_1^2 + p_2^2 + p_3^2}{2m} - B_4\right) \frac{E_i E_f k_f}{k_i}\, \frac{dk_f\, d\mathbf{p}_1\, d\mathbf{p}_2\, d\mathbf{p}_3}{(2\pi)^{12}}, \tag{46}$$

where B_4 is the binding energy of the ^{4}He nucleus. As in the three-body case, we go over from the Jacobi momenta $\mathbf{p}_i$ to hyperspherical coordinates in the momentum space. The hypermomentum in the nine-dimensional momentum space is determined by the formula $p = (p_1^2 + p_2^2 + p_3^2)^{1/2}$, and the hyperspherical angles β_1 and β_2 are introduced as follows:

$$p_1 = p\cos\beta_1 \sin\beta_2,\ p_2 = p\sin\beta_1 \sin\beta_2,\ p_3 = p\cos\beta_2. \tag{47}$$

The remaining six angles determine the directions of the vectors $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$, and $\hat{\mathbf{p}}_3$. Using the hyperspherical variables and bearing in mind that $d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 = p^8 dp d\Omega_p$, we obtain from (46) for the differential cross section

$$\frac{d^2\sigma}{dE_f\, d\Omega_\pi} = 2\pi\tilde{\rho}_f \int |\tilde{M}_{i\to f}|^2\, d\Omega_p, \tag{48}$$

where

$$d\Omega_p = \sin^2\beta_1 \cos^2\beta_1 \sin^5\beta_2 \cos^2\beta_2\, d\beta_1\, d\beta_2\, d\hat{\mathbf{p}}_1\, d\hat{\mathbf{p}}_2\, d\hat{\mathbf{p}}_3; \tag{49}$$

$$\tilde{\rho}_f = (2\pi)^{-12}\, \frac{E_i E_f k_f}{k_i}\, \frac{(2m)^{9/2}}{m}\, (E_i - E_f - B_4)^{7/2}. \tag{50}$$

Wave functions of the initial and final nuclear states

We obtain the wave functions of the initial and final nuclear states for the processes (34)–(37) by using the method of hyperspherical functions for three and four bodies. We expand the ground-state wave function of the three-nucleon nucleus with $L = 0$, $S = 1/2$, and $T = 1/2$ with respect to symmetrized three-particle hyperspherical functions:

$$\Psi_i(\eta_1, \eta_2) = -\sum_K \varphi_{K0}^s(r)\, \Phi_{K0}^s(\Omega_r)\, V_i^a(\sigma, \tau) + \frac{1}{\sqrt{2}} \sum_K \varphi_{K0}(r) \left(\Phi_{K0}^{m1}(\Omega_r)\, V_i^{m2}(\sigma, \tau) - \Phi_{K0}^{m2}(\Omega_r)\, V_i^{m1}(\sigma, \tau)\right), \tag{51}$$

where $\varphi^s_{K0}(r)$ and $\varphi_{K0}(r)$ are hyperradial functions. The indices s and a correspond to symmetric and antisymmetric states, and the indices $m1$ and $m2$ to mixed-symmetry states of the group S_3. In (51), $r^2 = \eta_1^2 + \eta_2^2$, $\eta_1 = r\cos\beta$, $\eta_2 = r\sin\beta$, $\Omega_r \equiv (\beta, \hat{\eta}_1, \hat{\eta}_2)$, where η_1 and η_2 are Jacobi coordinates and $V_i^p(\sigma,\tau)$ $(p = s,a,m1,m2)$ are the spin-isospin functions of the three nucleons in the state $S = 1/2$, $T = 1/2$. The symmetrized hyperspherical functions $\Phi^p_{KL}(\Omega_r)$ are defined as follows:

$$\Phi^p_{KL}(\Omega_r) = \sum_{l_1 l_2 m_1 m_2} C^p_{KI}(l_1 l_2)\langle l_1 l_2 m_1 m_2 | LM\rangle \Phi_K^{l_1 l_2 m_1 m_2}(\Omega_r). \quad (52)$$

In (52), the symmetrization coefficients $C^p_{KL}(l_1 l_2)$ are constructed in accordance with Ref. 39, and the hyperspherical functions $\Phi_K^{l_1 l_2 m_1 m_2}(\Omega_r)$ are eigenfunctions of the angular part of the six-dimensional Laplacian. Substituting the expansion (51) in the three-particle Schrodinger equation, we obtain an infinite system of coupled differential equations for the hyperradial functions $\varphi^s_{K0}(r)$ and $\varphi_{K0}(r)$.

We calculate the wave functions of the initial nuclear state for the processes (36) and (37) in a basis of four-particle hyperspherical functions. To this end, we expand the wave function of the ^{4}He nucleus with respect to symmetrized hyperspherical functions,

$$\Psi_i(\xi_1, \xi_2, \xi_3) = \sum_{\mu[f]\lambda L} \chi^{\mu[f]L\lambda}(R)\,\Gamma^{LMST}_{\mu[f]\lambda}(\Omega_R, \sigma, \tau), \quad (53)$$

where [f] and λ are a Young diagram and a Yamanouchi symbol; μ is the generalized angular momentum in the nine-dimensional space; L, M and S, T are the total orbital angular momentum and its projection and the total spin and isospin, respectively; $R = (\xi_1^2 + \xi_2^2 + \xi_3^2)^{1/2}$ is the length of the radius vector in the nine-dimensional coordinate space; $\Omega_R \equiv (\gamma_1, \gamma_2, \hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3)$ is a set of eight angles, six of which ($\hat{\xi}_1$, $\hat{\xi}_2$, and $\hat{\xi}_3$) determine the directions of the unit vectors corresponding to the Jacobi coordinates, while the angles γ_1 and γ_2 are introduced by analogy with the relations (47) using the hyperradius R. The symmetrized hyperspherical functions $\Gamma^{LMST}_{\mu[f]\lambda}(\Omega_R, \sigma, \tau)$ are constructed from the spin-isospin and hyperspherical functions:

$$\Gamma^{LMST}_{\mu[f]\lambda}(\Omega_R, \sigma, \tau) = \frac{1}{\sqrt{h_{[f]}}} \sum_{\nu} \Phi^{[f]\nu\lambda}_{\mu LM}(\Omega_R)\Phi^{ST}_{[\tilde{f}]\tilde{\nu}}(\sigma, \tau), \quad (54)$$

where $h_{[f]}$ is the dimension of the representation of the group S_4, and ν labels the rows of this representation; $\Phi^{ST}_{[\tilde{f}]\tilde{\nu}}(\sigma,\tau)$ are four-particle spin-isopin functions. The symmetrized four-particle hyperspherical functions $\Phi^{[f]\nu\lambda}_{\mu LM}(\Omega_R)$ are constructed in accordance with Ref. 40. Substituting the expansion (53) in the four-particle Schrödinger equation, for the hyperspherical functions $\chi^{\mu[f]L\lambda}(R)$ we obtain an infinite system of coupled differential equations.

We also seek the final-state wave functions in the hyperspherical formalism. The reactions (34)–(37) are examples of many-particle nuclear reactions in which three and four nucleons are produced at the end in the continuum. To what extent is one justified in describing such states in a hyperspherical basis?

In fact, the description of the final state in many-particle nuclear reactions gives rise to great difficulties, some of which are fundamental in nature. If one is working in the configuration space, the scattering wave function has a complicated asymptotic behavior[41,42] even in the three-body case, while in the momentum representation the corresponding integral equations contain singularities associated with the single-scattering components.[41–43] It is natural to assume that for a larger number of particles the situation will be even more complicated, although the asymptotic behavior of the four-particle scattering wave functions has not yet been investigated. The only thing that one can conclude with confidence is that a multidimensional spherical wave is certainly present in the asymptotic behavior of the many-particle function. In the three-body case, it has the form $\exp\{i(\mathbf{k}_1\eta_1 + \mathbf{k}_2\eta_2)\}/r^{5/2}$, and in the four-body case $\exp\{i(\mathbf{p}_1\cdot\xi_1 + \mathbf{p}_2\cdot\xi_2 + \mathbf{p}_3\cdot\xi_3)\}/R^4$, where r and R are the lengths of the radius vectors in the corresponding multidimensional spaces. Can one separate processes in which the principal part is played by the asymptotic behavior of the many-particle wave function in the form of a multidimensional spherical wave? Of this kind, we may assume, are complete breakup reactions (Refs. 37–39 and 44–48) when bound states of subsystems cannot be formed in the final state (the states are "democratic" and there are no on-shell two-particle scatterings) and the three-particle scattering does not contain a "ridge." A ridge (the concept was introduced by Newton and Shtokhamer[49]) is a direction of the six-dimensional radius vector r corresponding to double on-shell scattering. This does not mean that double scattering cannot be described in the method of hyperspherical functions. It is simply that near a ridge the convergence of the method is not nearly so good. However, far from the ridge the asymptotic form of the wave function that corresponds to double off-shell scattering behaves as a six-dimensional spherical wave,[41] and, therefore, this scattering can be investigated by the method of hyperspherical functions. Thus, in the study of breakup processes in which bound systems cannot be formed in the final state and a multidimensional spherical wave plays the main part in the asymptotic behavior of the wave function, one can successfully use the method of expanding the nuclear final-state function with respect to hyperspherical functions,[48] since this method ensures the necessary asymptotic behavior.

As a result of the reactions (34) and (35), there is formed a system of three nucleons in the continuum in the state $L = 1$, $T = 3/2$ and two different spin states with $S = 1/2$ and $S = 3/2$. Spin flip making a small contribution to the cross section of the process, the treatment can be restricted to the three-nucleon state with $S = \frac{1}{2}$. We expand the final-state wave function with $L = 1$, $S = \frac{1}{2}$, $T = 3/2$ with respect to symmetrized hyperspherical functions,

$$\Psi_f(\eta_1, \eta_2; \mathbf{k}_1, \mathbf{k}_2) = \frac{1}{2}\sum_K \varphi_{K1}(r; k)$$

$$\times [\Phi^{*m2}_{K1}(\Omega_h) - \Phi^{*m1}_{K1}(\Omega_h)]\,[\Phi^{m1}_{K1}(\Omega_r)\,V_f^{m2}(\sigma,\tau) - \Phi^{m2}_{K1}(\Omega_r)\,V_f^{m1}(\sigma,\tau)], \quad (55)$$

where $V_f^{m_1}(\sigma,\tau)$, $V_f^{m_2}(\sigma,\tau)$ are spin-isospin functions of the

three nucleons in the state $S = 1/2$, $T = 3/2$.

We now consider the reactions (36) and (37). We represent the wave function of the four-particle system in the continuum in the form of the expansion

$$\Psi^{[f]\beta\beta'}_{f\bar{\beta}\bar{\beta}'}(\xi_1, \xi_2, \xi_3; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \sum_{\mu LM} \frac{\chi^{[f]}_{\mu L}(R; p)}{R^4} \Psi^{[f]\beta\beta'}_{\mu LM}(\Omega_R, \sigma, \tau)\, \Psi^{*[f]\bar{\beta}\bar{\beta}'}_{\mu LM}(\Omega_p), \quad (56)$$

where β, β' and $\bar{\beta}, \bar{\beta}'$ correspond to the symbols s or a from Ref. 50 and label in pairs the functions of the five irreducible representations of the group S_4: the symmetric [4], the antisymmetric [1111], the two three-dimensional [31] and [211], and the one two-dimensional [22]. In particular, for the three-dimensional representations we have the functions ([31], *ss*), ([31], *sa*), ([31], *as*), ([211], *aa*), ([211], as), ([211], *sa*). The functions $\Psi^{[f]\beta\beta'}_{\mu LM}(\Omega_R, \sigma, \tau)$ are composed of four-particle spin-isospin and hyperspherical functions in accordance with the expression (54). For the ^{4}He nucleus, we use the spin-isospin functions found in Ref. 51, and for the four-nucleon states with $T = 2$ the functions from Ref. 50.

Using the initial- and final-state wave functions (53)–(56), we can calculate the transition matrix elements for the processes (34)–(37). The corresponding expressions for the transition matrix elements under the assumption of the two-step reaction mechanism and the meson current mechanism are given in Refs. 31, 32, 37, and 38. According to these studies, the calculation of the cross section of double charge exchange for the processes (34)–(37) in accordance with (42) and (48) reduces to finding the three- and four-particle hyperradial functions of the discrete and continuous spectra.

Equations for the three- and four-particle hyperradial functions of the continuum

The well-known variable-phase approach developed by Babikov and Calogero[52] for the two-body problem was generalized in Ref. 53 for three and four bodies. The method was developed further in Ref. 54.

If we substitute the expansions (55) and (56) in the three and four-particle Schrödinger equations, respectively, then for the hyperradial functions $\varphi_{K1}(r;k)$ and $\chi^{[f]}_{\mu L}(R;p)$ we obtain a system of coupled differential equations. We write this system in the matrix form

$$\frac{d^2u(\rho;\varkappa)}{d\rho^2} + \frac{1}{\rho}\frac{du(\rho;\varkappa)}{d\rho} + \left[\varkappa^2 - \frac{\nu^2}{\rho^2}\right] u(\rho;\varkappa) = W(\rho)\, u(\rho;\varkappa), \quad (57)$$

where $\varkappa^2 = k^2$, $\nu = K + 2$ in the three-particle case and $\varkappa^2 = p^2$, $\nu = \mu + 7/2$ in the four-particle case. In Eq. (57), $u(\rho;\varkappa)$ is a hyperradial column composed, respectively, of the functions $\varphi_{K1}(r;k)$ or $\chi^{[f]}_{\mu L}(R;p)$, and

$$W(\rho) = \begin{Vmatrix} W^{\alpha\alpha'}_{11}(\rho) & W^{\alpha\alpha'}_{12}(\rho) & W^{\alpha\alpha'}_{13}(\rho) \dots W^{\alpha\alpha'}_{1n}(\rho) \dots \\ W^{\alpha\alpha'}_{21}(\rho) & W^{\alpha\alpha'}_{22}(\rho) & W^{\alpha\alpha'}_{23}(\rho) \dots W^{\alpha\alpha'}_{2n}(\rho) \dots \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ W^{\alpha\alpha'}_{n1}(\rho) & W^{\alpha\alpha'}_{n2}(\rho) & W^{\alpha\alpha'}_{n3}(\rho) \dots W^{\alpha\alpha'}_{nn}(\rho) \dots \\ \dots\dots\dots\dots\dots\dots\dots\dots \end{Vmatrix} \quad (58)$$

is a square matrix composed of the effective three-particle (for three nucleons) and four-particle (for four nucleons) potential energies obtained by averaging the *NN* potential over the three- and four-particle hyperspherical functions, respectively. For simplicity, the index α denotes the set of quantum numbers on which the three- and four-particle hyperspherical functions depend.

We seek a solution of the matrix equation (57) in the form

$$u(\rho, \varkappa) = \frac{1}{\rho}\left(J(\varkappa\rho)\, U^{-1}(n;\rho) - N(\varkappa\rho)\, U^{-1}(n;\rho)\, T(\rho)\right) A(\rho), \quad (59)$$

where

$$J(\varkappa\rho) = \begin{pmatrix} J_\nu(\varkappa\rho) & 0 & 0 & \dots \\ 0 & J_{\nu+2}(\varkappa\rho) & 0 & \dots \\ 0 & 0 & J_{\nu+4}(\varkappa\rho) & \dots \\ \dots\dots\dots\dots\dots\dots \end{pmatrix}; \quad (60)$$

$$N(\varkappa\rho) = \begin{pmatrix} N_\nu(\varkappa\rho) & 0 & 0 & \dots \\ 0 & N_{\nu+2}(\varkappa\rho) & 0 & \dots \\ 0 & 0 & N_{\nu+4}(\varkappa\rho) & \dots \\ \dots\dots\dots\dots\dots\dots \end{pmatrix} \quad (61)$$

are diagonal matrices whose elements are Bessel and Neumann functions, and $A(\rho)$ is an amplitude function, a column matrix. In (59), the inverse matrix $U^{-1}(n;\rho)$ is obtained from the orthogonal matrix of nth order constructed by the recursive procedure

$$U(n;\rho) = \begin{pmatrix} 1 & 0 \\ 0 & U(n-1;\rho) \end{pmatrix} O^T(n;\rho), \quad (62)$$

where the matrix $O^T(n;\rho)$ is obtained by transposing the matrix

$$O(n;\rho) = \begin{Vmatrix} c_1 & -s_1 & 0 & \dots & 0 & 0 \\ s_1c_2 & c_1c_2 & -s_2 & \dots & 0 & 0 \\ s_1s_2c_3 & c_1s_2c_3 & c_2c_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ & & & \dots & -s_{n-2} & 0 \\ s_1s_2 \dots s_{n-2}c_{n-1} & c_1s_2 \dots s_{n-2}c_{n-1} & c_2s_3 \dots s_{n-2}c_{n-1} & \dots & c_{n-2}c_{n-1} & -s_{n-1} \\ s_1s_2 \dots s_{n-2}s_{n-1} & c_1s_2 \dots s_{n-2}s_{n-1} & c_2s_3 \dots s_{n-2}s_{n-1} & \dots & c_{n-2}s_{n-1} & c_{n-1} \end{Vmatrix} \quad (63)$$

$$c_j \equiv c_j(\rho) = \cos\varepsilon_j(\rho); \quad s_j \equiv s_j(\rho) = \sin\varepsilon_j(\rho); \quad j = 1, 2, 3 \dots;$$

$T(\rho)$ is a diagonal matrix of the form

$$T(\rho)=\begin{pmatrix}\tan\delta_{\nu}(\rho) & 0 & 0 & \dots\\ 0 & \tan\delta_{\nu+2}(\rho) & 0 & \dots\\ 0 & 0 & \tan\delta_{\nu+4}(\rho) & \dots\\ \dots & \dots & \dots & \dots\end{pmatrix}. \quad (64)$$

We shall call the functions $\delta_\nu(\rho)$ and $\varepsilon_\nu(\rho)$ introduced in (63) and (64) the phase functions and mixing functions, respectively.

We impose the following condition on the derivative:

$$\frac{du(\rho;\varkappa)}{d\rho}=\left(\frac{dJ(\varkappa\rho)}{d\rho}U^{-1}(n;\rho)-\frac{dN(\varkappa\rho)}{d\rho}U^{-1}(n;\rho)T(\rho)\right)A(\rho). \quad (65)$$

This condition is equivalent to

$$\left[J(\varkappa\rho)\frac{dU^{-1}(n;\rho)}{d\rho}-N(\varkappa\rho)\frac{dU^{-1}(n;\rho)}{d\rho}T(\rho)-N(\varkappa\rho)U^{-1}(n;\rho)\frac{dT(\rho)}{d\rho}\right]A(\rho)+[J(\varkappa\rho)U^{-1}(n;\rho)-N(\varkappa\rho)U^{-1}(n;\rho)T(\rho)]\frac{dA(\rho)}{d\rho}=0. \quad (66)$$

Substituting (59) in (57), taking into account the relations (65) and (66) and the expression for the Wronskian of the Bessel and Neumann functions, we obtain after simple manipulations

$$\frac{dT(\rho)}{d\rho}+T(\rho)\frac{dU^{-1}(n;\rho)}{d\rho}U^{-1}(n;\rho)-\frac{dU(n;\rho)}{d\rho}U^{-1}(n;\rho)T(\rho)=-\frac{\pi}{2}\rho[U(n;\rho)J(\varkappa\rho)-T(\rho)U(n;\rho)N(\varkappa\rho)]W(\rho)[J(\varkappa\rho)U^{-1}(n;\rho)-N(\varkappa\rho)U^{-1}(n;\rho)T(\rho)]. \quad (67)$$

The nonlinear first-order matrix differential equation (67) makes it possible to determine the phase functions $\delta_\nu(\rho)$ and the mixing functions $\varepsilon_\nu(\rho)$. For the amplitude function $A(\rho)$, we obtain the equation

$$\frac{d}{d\rho}(U^{-1}(n;\rho)A(\rho))=-\frac{\pi\rho}{2}N(\varkappa\rho)\dot{W}(\rho)[J(\varkappa\rho)U^{-1}(n;\rho)-N(\varkappa\rho)U^{-1}(n;\rho)T(\rho)]A(\rho). \quad (68)$$

Equation (68) can be formally integrated in matrix form,

$$A(\rho)=U(n;\rho)\exp\left\{-\frac{\pi}{2}\int_0^{\rho}d\rho'\,\rho'N(\varkappa\rho')W(\rho')\times[J(\varkappa\rho')U^{-1}(n;\rho')-N(\varkappa\rho')U^{-1}(n;\rho')T(\rho')]U(n;\rho')\right\}A_0(\rho), \quad (69)$$

where $A_0(\rho)$ is an amplitude column with elements $A_{0\nu+2i}$ ($i=1,2,3,\dots$) determined by the choice of the boundary conditions.

Knowing the solution of Eqs. (67) and (68) and using the expression (59) and the expansions (55) and (56), we can find the wave function of three or four particles in the continuum in the approximation of an arbitrary number of harmonics, the resulting functions having asymptotic behavior as $\rho\to\infty$ corresponding to a six- or nine-dimensional outgoing spherical wave, i.e., the asymptotic behavior discussed above.

Comparison of the results of theory and experiment

Experimental data. An experimental investigation of the reaction (34) was made in 1970.[55] A beam of 140-MeV π^0 mesons was used, and the π^+ meson was detected in the interval of angles 15–40°. The differential cross section of the reaction was measured as a function of the kinetic energy of the three neutrons.

The first experimental investigations of pion double charge exchange on the ^{4}He nucleus were made in 1964.[56] The differential cross section of the process (36) was later measured in Refs. 57 and 58. The experiment of Ref. 57 analyzed the energy spectrum of the emitted π^+ at emission angle $\theta=20°$ and fixed initial beam energy 140 MeV. The CERN experiment of Ref. 58 measured $d^2\sigma/dE_f d\Omega_\pi$ as a function of the initial energy of the π^- mesons, which was varied in the range 210–270 MeV. The corresponding kinematic parameters were $\theta=0°$, $T_f=176$ MeV. With the aim of detecting the tetraneutron, a measurement was made[118] of the differential cross section of π^- double charge exchange on the ^{4}He nucleus at $T_{\pi^-}=165$ MeV and π^+ emission angle $\theta=0°$. The differential cross section was investigated in this experiment as a function of the π^+ momentum. Recently, the differential cross sections of the reaction (37) have been measured as functions of the kinetic energy of the four protons.[59] The total cross sections for double charge exchange at 140, 200, and 295 MeV were determined from the obtained spectrum. The reaction (37) was also investigated in Refs. 60–64. These experiments measured the total cross section of the process. Because all the secondary particles in the reaction (37) are charged, they can be readily detected in chamber experiments, so that a complete kinematic analysis of the experiment is possible. In Refs. 60, 63, and 64, the measurements were made with a bubble chamber; in Refs. 61 and 62, with a streamer chamber. Table I gives the total cross sections of the reaction (37). These experimental data give the total cross section of π^+ double charge exchange on the ^{4}He nucleus as a function of the energy (or momentum) of the pions. Examination of Table I shows that it is necessary to make a further experimental investigation in order to identify correctly the reaction mechanisms at the different energies.

Differential cross sections of double charge exchange. For the calculation of the differential cross sections of the reactions (34)–(37), the wave functions of the initial and final nuclear states were found using the same NN potentials. This ensured orthogonality of the initial and final nuclear states. Below, we investigate the part played by the interaction in the continuum of the nucleons produced as a result of the reactions (34)–(37). To establish whether the final state interaction is in principle always important or only so in the case of particular NN forces, different NN potentials were used in Refs. 31, 32, 37, and 38. The differential cross sections were calculated using the NN potentials of Volkov (V),[65] Baker (B),[66] Eikemeir and Hackenbroich (EH),[67]

TABLE I. Total cross sections of the $\pi^+ + {}^4\text{He} \to \pi^- + 4p$ reaction.

Kinetic energy of π^+ meson, MeV	Momentum of π^+ meson, MeV/c	Reaction cross section, mb	Reference
98	192	0.30±0.10	[62]
100	195	0.30±0.15	[61]
135	226	0.29±0.11	[62]
140	242	0.0899±0.0147	[59]
145	248	0.34±0.17	[62]
156	260	0.13±0.07	[62]
200	310	0.398±0.0626	[59]
295	412	1.2874±0.1583	[59]
486	610	1.20±0.21	[60]
1327	1460	0.41±0.14	[63]
1576	1710	0.25±0.05	[64]
1586	1720	0.24±0.08	[63]
1725	1860	0.14±0.07	[63]

GPT,[68] and Afnan and Tang (S1),[69] and a potential well (SW).[70]

We now discuss the results of the calculation of the differential cross sections for the processes (34)–(37). We first consider the sensitivity of the cross section of the process to the initial-state wave function. Figure 3a shows the results of calculations of the differential cross section of the reaction (34) to the kinematic conditions of the experiment of Ref. 55. The calculation was made under the assumption of the two-step reaction mechanism without allowance for the final-state interaction between the three neutrons (plane-wave approximation). Comparison of these curves indicates that the reaction cross section is strongly sensitive to the form of the initial-state wave function. The investigation made in Ref. 31 of the convergence of the cross section with respect to the hypermoment (generalized angular momentum) showed that the contribution of the harmonic $K = 6$ to the cross section of the process is negligibly small. In addition, the contribution of the S' state to the cross section of the process (34) is slight, and restriction to the harmonics $K = 0, 2, 4, 6$ in the expansion of the initial-state wave function leads to satisfactory convergence of the cross section. The main contribution (not less than 80%) to the cross section is made by the harmonic $K = 0$ in the initial-state wave function.

In Fig. 3b, the results of calculating the cross section of π^- double charge exchange with allowance for the final-state interaction for the potentials S1, V, and EH are compared with the experimental data of Ref. 55. The results exhibit a strong sensitivity of the cross section to the form of the NN potential. The final-state interaction changes qualitatively the results of the calculations obtained in the plane-wave approximation (cf. Fig. 3a). In the region of low energies transferred to the three-neutron system, the interaction between the three neutrons leads to a strong increase in the cross section compared with the calculation in the plane-wave approximation, this being observed for all the considered potentials. Thus, the assumption of a two-step mechanism and the microscopic description of the initial and final nuclear states satisfactorily reproduces the behavior of the cross section as a function of the energy. The experimentally observed resonance form of the differential cross section is reproduced with allowance for the final-state interaction and therefore hardly indicates the formation of a three-neutron resonance state.

Figure 3c gives the results of calculations of the cross section of the process (34) under the assumption of the mechanism of Germond and Wilkin[27] in the plane-wave approximation and with allowance for the final-state interaction. The calculations were made for the potential S1. It can be seen that in the considered energy range the contribution of the mechanism of Ref. 27 is much less than that of the two-

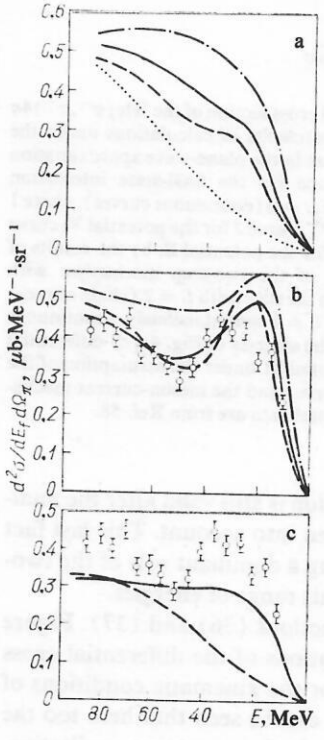

FIG. 3. Dependence of the differential cross section of the $^3\text{He}(\pi^-,\pi^+)3n$ reaction on the kinetic energy of the neutrons: a) calculation under the assumption of the two-step mechanism in the plane-wave approximation for the potentials S1 (continuous curve), V (broken curve), and EH (chain curve); the dotted curve from Ref. 33 corresponds to the four-particle phase space; b) calculation under the assumption of the two-step mechanism with allowance for the final-state interaction between the three neutrons; the notation is the same as in Fig. 3a; c) calculation under the assumption of the meson-current mechanism in the plane-wave approximation (broken curve) and with allowance for the final-state interaction (continuous curve) for the potential S1. The experimental data are from Ref. 55.

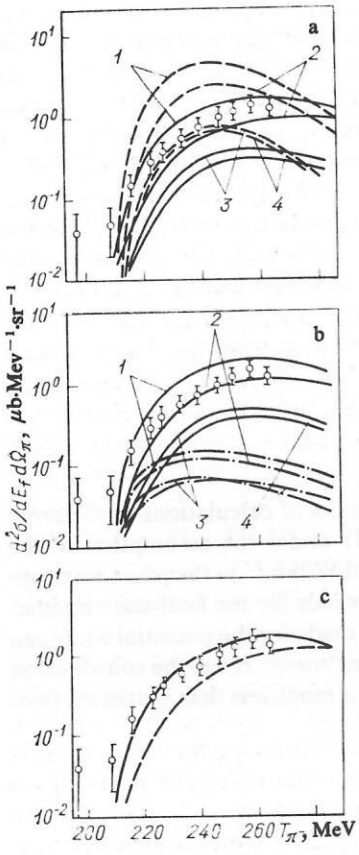

FIG. 4. Dependence of the differential cross section of the $^4He(\pi^-,\pi^+)4n$ reaction on the π^- kinetic energy: a) results of calculations under the assumption of the two-step mechanism in the plane-wave approximation (broken curves) and with allowance for the final-state interaction between the four neutrons in the state $L = 0$ (continuous curves); curve 1 is the calculation for the potential GPT, curve 2 for the potential V, curve 3 for the potential SW, and curve 4 for the potential B; b) the results of calculations under the assumption of the two-step mechanism with allowance for final-state interaction in the state with $L = 2$ (chain curves) and in the states with $L = 0$ and $L = 2$ simultaneously (continuous curves). The remaining notation is the same as in Fig. 4a; c) differential cross section calculated for the potential S1 under the assumption of the two-step mechanism (continuous curve) and the meson-current mechanism (broken curve). The experimental data are from Ref. 58.

step mechanism. This conclusion is still valid after the final-state interaction has been taken into account. This last fact can be interpreted as indicating a dominant role of the two-step reaction mechanism in this range of energies.

We now consider the reactions (36) and (37). Figure 4a gives the results of calculations of the differential cross section of the process (36) for the kinematic conditions of the experiment of Ref. 58. It can be seen that here too the final-state interaction between the four neutrons qualitatively changes the results of the calculations in the plane-wave approximation. It is interesting to note that for the potentials of Refs. 66 and 70 the cross section approaches the experimental points already in the plane-wave approximation, whereas allowance for the final-state interaction leads to the opposite result. This fact indicates once more how unreliable are the conclusions drawn about the cross section for double charge exchange in the plane-wave approximation.

It was assumed above that the four nucleons in the continuum are in a state with total orbital angular momentum $L = 0$. We now consider the contribution to the cross section made by states with $L = 2$. In Fig. 4b, the chain curves show the results of calculating the differential cross section of the process (36) for different NN potentials when the system in the final state has $L = 2$. It can be seen that in this case the final-state interaction strongly suppresses the cross section. The results of the calculations with allowance for $L = 0$ and $L = 2$ in the region of pion energies 220–260 MeV differ by almost an order of magnitude. From these calculations it can be concluded that in the $^4He(\pi^-,\pi^+)4n$ reaction the four nucleons are formed mainly in a state with total orbital angular momentum $L = 0$. In the same figures, the continuous curves are the results of calculations of the differential cross section for which terms with $L = 0$ and $L = 2$ are taken into account in the expansion of the final-state wave function. It can be seen that allowance for the state with $L = 2$ leads to an increase in the cross section for all forms of the NN potentials. Figures 4c and 5 show the results of calculations of the differential cross sections of the reactions (36) and (37) for the kinematic conditions of the experiments of Refs. 58 and 59. The calculations were made under the assumption of the two-step mechanism and the meson-current mechanism with allowance for the final-state interaction. It can be seen from these figures that with increasing pion energy the importance of the meson-current mechanism increases, though the two-step mechanism makes a large contribution to the cross section.

Summarizing the results of the calculations of the differential cross section of double charge exchange on the ^4He nucleus, we can conclude that the theoretical curves without allowance for the final-state interaction are far from the experimental data and in the majority of cases do not even qualitatively reproduce the behavior of the cross section. The inclusion of interaction simultaneously between all the nucleons in the final state changes the plane-wave calculations both qualitatively and quantitatively and leads to new results that correctly reproduce the behavior of the experimental curve of the cross section, this being achieved for practically all forms of the employed NN potentials. However, the results differ quantitatively for the different potentials. The contribution of the final-state interaction depends strongly on the energy transferred to the four-nucleon sys-

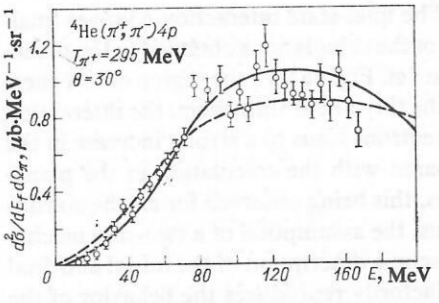

FIG. 5. Dependence of the differential cross section of the $^4He(\pi^+,\pi^-)4p$ reaction on the kinetic energy of the fourprotons. The continuous curve is the calculation for the potential S1 under the assumption of the two-step mechanism, and the broken curve is for the meson-current mechanism. The experimental data are taken from Ref. 59.

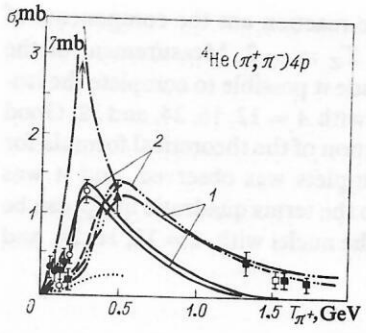

FIG. 6. Total cross section of the ^{4}He$(\pi^+,\pi^-)4p$ process. The chain curve gives the results of the calculations of Ref. 35, the dotted curve those of Ref. 36, the broken curve those of Ref. 27, and curves 1 and 2 those of Ref. 32; the continuous curves correspond to the two-step mechanism; the chain curves with two dots are for the meson-current mechanism. The experimental data are as follows: black circles from Refs. 61 and 62, black triangle from Ref. 60, open square from Ref. 64, black squares from Ref. 63, and open circles from Ref. 59. The diagram is taken from Ref. 32.

tem in the process of double charge exchange. Although the term with $L = 0$ in the final-state wave function is dominant, it is necessary to include the term $L = 2$ to get better agreement with experiment.

Total cross section. Integrating the differential cross section over dE_f and $d\Omega_\pi$, we obtain the total cross section of the double charge exchange. The calculations made in Refs. 37 and 38 of the total cross section of π^+ double charge exchange on the ^{4}He nucleus in the plane-wave approximation and with allowance for the final-state interaction showed that the latter strongly changes the total cross section. Satisfactory agreement with the experimental data of Refs. 60–62 is observed. It is interesting to note that the recently obtained experimental data of Ref. 59 are in good agreement with the theoretical predictions of Refs. 37 and 38.

Figure 6 shows the results of calculations of the total cross section of double charge exchange under the assumption of the two-step mechanism and the meson-current mechanism made by various authors. In the calculations of Refs. 35 and 36, made under the assumption of the two-step mechanism, different parametrizations of the πN scattering amplitude were used, and these amplitudes have different off-shell continuations. The strong difference between these calculations indicates the extreme sensitivity of double charge exchange to off-shell effects at intermediate energies. At the same time, the comparison of the results of the calculations of Refs. 36 and 32 made using the same πN scattering amplitude reveal strong sensitivity to the form of the initial-state wave function and the effect of the final-state interaction. It may be concluded from the results shown in Fig. 6 that at pion energies $T_\pi < 300$ MeV the two-step mechanism plays the dominant part, and the amplitude of the elementary event of double charge exchange obtained in Ref. 36 can be successfully used to investigate the double charge exchange of pions at intermediate energies. Beginning at energies $T_\pi > 250$ MeV, the meson-current mechanism becomes more important, and at high energies it is decisive.

Figure 7 shows the results of the calculations of the total cross section of the process (37) under the assumption of the meson-current mechanism. Curves 1 and 2 were obtained in Ref. 71 using Weinberg's model[28] for $\pi\pi$ scattering and the modified Veneziano model,[29] respectively. Curve 3 was calculated under the assumption that the $\pi^+ n \to \pi^- \pi^+ p$ process occurs and the off-shell π^+ meson is absorbed by the second neutron (the diagram of Fig. 2b). In Ref. 64, a different model of double charge exchange in the reaction (37) at high energies was proposed; in the first stage, a single meson is produced, $\pi^+ n \to \pi^+ \pi^- p$, this being absorbed in the second stage by an np pair (it is kinematically impossible for an on-shell π^+ meson to be absorbed by a single nucleon). The estimate corresponding to this mechanism is shown by curve 4. The results given in Fig. 7 show that the process is strongly sensitive to the $\pi\pi$ coupling constant and the off-shell continuation of the $\pi\pi$ amplitude. In addition, correct treatment of the initial and final nuclear states is important even at pion energies around 1 GeV. Despite the fact that the approximate curve 4 correctly describes the experimental data, this result may be strongly changed in a microscopic description of the process in the framework of the described mechanism.

Thus, in the region of intermediate energies the two-step mechanism makes the main contribution to the cross section of the reactions (34)–(37). At higher pion energies, the meson-current mechanism becomes dominant. In these reactions, the final-state interaction is decisive, changing the reaction cross sections not only quantitatively but also qualitatively. It is only after allowance for the final-state interaction that the cross section of the reaction (34) acquires a maximum at low energies and the curve of the cross section takes a resonance shape. The same qualitative result is obtained for all the considered potentials. It is only after the final-state interaction has been taken into account that one obtains a consistent description of the differential and total cross sections of double charge exchange on the ^{4}He nucleus. The cross sections of pion double charge exchange on the ^{3}He and ^{4}He nuclei are very sensitive to the form of the NN potential. This fact enhances the interest of further experimental investigations of such reactions, particularly on few-particle systems.

It is interesting to note that the cross section of pion double charge exchange on the ^{4}He nucleus can be calculat-

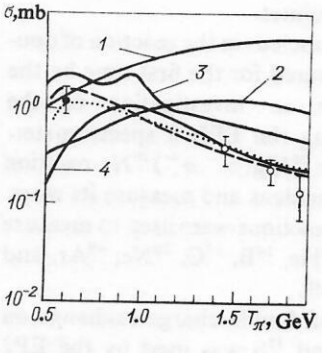

FIG. 7. Total cross section of the ^{4}He$(\pi^+,\pi^-)4p$ process. The dotted curve is the calculation for the potentials GPT, and the broken curve for the potential S1; the experimental data: black circle from Ref. 60, open circles from Ref. 63. The remaining curves are explained in the text.

ed by using the cross section of the crossing-symmetric reaction of two-pion emission accompanying antiproton annihilation in ^{4}He.[72] Therefore, a comparative study of the two reactions leads, on the one hand, to a deeper understanding of the mechanisms of double charge exchange at different energies and, on the other, makes it possible to establish the relative importance of the various diagrams in contributing to two-pion annihilation of antiprotons in ^{4}He.

3. ANALOG AND NONANALOG TRANSITIONS IN DOUBLE CHARGE EXCHANGE

Experimental study of double charge exchange began in the middle of the sixties. However, up to 1976 neither analog transitions ($\Delta T = 0$) nor transitions to the ground state ($\Delta T \geqslant 0$) had been observed in such reactions. It was only with the commissioning of the meson factories and the achievement of a high intensity and good quality of the pion beams that such investigations became possible. The first measurements of pion double charge exchange on nuclei using meson-factory beams were made using the LEP low-energy pion channel at LAMPF. Detailed investigations of double charge exchange were then made at this meson factory with high-energy pions, the EPICS spectrometer being used. Cross sections for double charge exchange were measured at the SIN meson factory from 1977 with the SUSY spectrometer. The experiments made at the meson factories studied reactions of double charge exchange in which the resulting nucleus has the same atomic number as the target nucleus. In these experiments, the final pions were detected, and the following characteristics were measured: 1) the masses of the nuclei produced by the reaction; 2) the excitation functions; 3) the angular distributions; 4) the A dependence of the cross sections for double charge exchange. We now discuss the results of the measurements and their theoretical explanation.

Measurement of the masses

The (π^-,π^+) and (π^+,π^-) reactions of double charge exchange make it possible to obtain the isobar nuclei. The isobars can also be obtained in reactions with nucleons. However, the isotopic spectrum in the pion reactions is much richer than the spectrum obtained in reactions with isospin-1/2 particles. For example, the (π^+,π^-) reaction of double charge exchange makes it possible to obtain their isotopic spectrum for an isobar quintet.

The mass of the resulting nucleus in the reaction of double charge exchange was measured for the first time by the EP1 group (Ref. 73)[1)] in an investigation of the $^{18}O(\pi^-,\pi^+)^{18}C$ reaction using the EPICS spectrometer. The same group[74] studied the $^{26}Mg(\pi^-,\pi^+)^{26}Ne$ reaction in order to observe the ^{26}Ne nucleus and measure its mass. The (π^-,π^+) and (π^+,π^-) reactions were used to measure the masses of the nuclei ^{7}H, ^{9}He, ^{14}B, ^{18}C, ^{26}Ne, ^{48}Ar, and ^{58}Zn in Refs. 6, 73–75, and 119.

The (π^+,π^-) reaction of double charge exchange on the nuclei ^{12}C, ^{16}O, ^{24}Mg, and ^{32}S was used by the EP2 group[76] to determine the masses of the nuclei ^{12}O, ^{16}Ne, ^{24}Si, and ^{32}Ar. The differential cross section was measured at $T_{\pi^+} = 180$ MeV and pion emission angle $\theta = 5°$. The nuclei obtained as a result of the reaction are the components of four isobar quintets with $T_Z = -2$. Measurement of the masses of these isobars made it possible to complete the isobar quintets for the nuclei with $A = 12$, 16, 24, and 32. Good agreement with the prediction of the theoretical formula for the masses of isobar multiplets was observed, and it was shown that a restriction to the terms quadratic in T_Z can be made in this formula for the nuclei with $A = 12$, 16, 24, and 32.

Excitation functions

We shall systematize the reactions of pion double charge exchange in accordance with the final states: a double-isobar state or a nonanalog state. We consider first the experiments on π^+ double charge exchange on nuclei with $T = 0$ that lead to nonanalog transitions. All the data were obtained for reactions in which a $T = 2$ nucleus is produced in the final state, i.e.,

$$\pi^+ + A\ (N, Z;\ T = 0) \to \pi^- + B\ (N - 2, Z + 2;\ T = 2). \tag{70}$$

The differential cross sections on $T = 0$ nuclei were measured for the first time in the LEP channel at LAMPF.[77,78] The forward differential cross section was measured at $T_{\pi^+} = 145$ MeV. Although the energy resolution of the facility was 4 MeV, the use of special geometry, the cross section being measured at $\theta = 0°$, made it possible to separate the reaction channels with transition of the final nucleus to the ground state. The cross sections of double charge exchange on the ^{12}C, ^{16}O, ^{24}Mg, and ^{28}Si nuclei were measured, for which the corresponding results were: $d\sigma/d\Omega(\theta = 0°) = 0.65 \pm 0.20$, 0.87 ± 0.21, 0.67 ± 0.20, 0.35 ± 0.10 μb/sr.[2)] A systematic study of double charge exchange on the $T = 0$ nuclei ^{12}C, ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, and ^{40}Ca was made using EPICS spectrometer by the EP2 group.[79–83] The excitation functions for these nuclei were measured at angle $\theta = 5°$ in the energy range 90–300 MeV. The emitted pions were detected by means of a special system comprising a magnetic spectrometer, multiwire drift chambers (readout by means of delay lines), and Cherenkov counters. The excitation functions for all the nuclei have a clear maximum at energies around 150–170 MeV. At higher pion energies, the cross section for the nonanalog transitions is smaller.

Double-isobar states in reactions of double charge exchange were investigated in the following processes: $^{14}C(\pi^+,\pi^-)^{14}O$, $^{18}O(\pi^+,\pi^-)^{18}Ne$, $^{26}Mg(\pi^+,\pi^-)^{26}Si$, $^{42}Ca(\pi^+,\pi^-)^{42}Ti$, $^{56}Fe(\pi^+,\pi^-)^{56}Ni$, $^{209}Bi(\pi^+,\pi^-)^{2090}At$. The reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$(DIAS) was investigated in several studies (Refs. 76, 79, 84, and 85). The differential cross section for this reaction was measured for the first time in Ref. 84 at $\theta = 0°$ and pion energies 95, 126, and 139 MeV. The data obtained indicated that the excitation function is constant for forward scattering. Subsequently, the differential cross section at $\theta = 18°$ and $T_{\pi^+} = 148$ and 187 MeV was measured at the SIN meson factory.[85] The energy resolution 1 MeV of the facility made it possible to separate transitions to double-isobar analog states. The angle 18° was chosen as the smallest for the given spectrometer.

TABLE II. Experimental investigations of analog and nonanalog transitions in the reaction of double charge exchange.

Reaction	Excitation function		Angular distribution		References
	Pion detection angle, deg	Range of variation of the energy, MeV	Initial pion energy, MeV	Range of variation of the angles, deg	
^{9}Be (π^+, π^-) ^{9}C	5	140—290	164	5—35	[6]
	5	180	—	—	[79]
^{12}C (π^+, π^-) ^{12}O	5	140—290	220	5—30	[6]
	5	180	—	—	[79]
	5	120—210	164	5—35	[83]
^{12}C (π^-, π^+) ^{12}B	—	—	220	5—30	[6, 74]
^{13}C (π^+, π^-) ^{13}O	5	180	—	—	[79]
	5	120—292	164, 292	0—50	[120]
^{14}C (π^+, π^-) ^{14}O	—	—	50	50—120	[90]
	5	120—292	164, 292	0—50	[121]
^{14}C (π^-, π^+) ^{14}B	5	164	—	—	[119]
^{16}O (π^+, π^-) ^{16}Ne	0	145	—	—	[77]
	5	100—292	—	—	[79]
	—	—	164	0—50	[89]
^{18}O (π^+, π^-) ^{18}Ne	18	148, 187	—	—	[85]
	0	139	—	—	[84]
	0	95—139	—	—	[78]
	—	—	164	13—45	[75, 88]
	5	80—292	164, 292	5—33	[79]
^{18}O (π^-, π^+) ^{18}C	5	140—240	164	5—45	[121]
^{24}Mg (π^+, π^-) ^{24}Si	5	140—292	—	—	[79]
	5	120—210	—	—	[83]
	0	145	—	—	[77]
^{26}Mg (π^+, π^-) ^{26}Si	5	120—310	292	5—33	[79]
	5	120—180	164	0—50	[121]
^{26}Mg (π^-, π^+) ^{26}Ne	5	162	—	—	[74]
^{32}S (π^+, π^-) ^{32}Ar	5	180	—	—	[79]
	5	120—210	—	—	[83]
^{40}Ca (π^+, π^-) ^{40}Ti	5	120—210	164	5—35	[83]
^{42}Ca (π^+, π^-) ^{42}Ti	5	130—292	180	5—15	[87]
^{56}Fe (π^+, π^-) ^{56}Ni	5	140—290	—	—	[82]
^{209}Bi (π^+, π^-) ^{209}At	5	292	—	—	[86]

A systematic measurement of the excitation functions for transitions to double-isobar analog states was made by the EP2 group in their investigation of the reactions $^{18}O(\pi^+,\pi^-)^{18}Ne$, $^{26}Mg(\pi^+,\pi^-)^{26}Si$ (Ref. 79), $^{56}Fe(\pi^+,\pi^-)^{56}Ni$ (Ref. 82), and $^{209}Bi(\pi^+,\pi^-)^{209}At$ (Ref. 86). The investigations showed that the differential cross section of the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction with transition to a double-isobar analog state at $\theta = 5°$ in the energy range 80–292 MeV has an oscillating shape with a maximum at 120 MeV and a minimum at 160 MeV. The differential cross sections for the $^{26}Mg(\pi^+,\pi^-)^{26}Si$ and $^{56}Fe(\pi^+,\pi^-)^{56}Ni$ reactions with transitions to double-isobar analog states have a similar tendency. At energies $T_{\pi^+} > 160$ MeV the excitation functions increase. In recent experimental studies of the reaction $^{42}Ca(\pi^+,\pi^-)^{42}Ti(DIAS)$ at $\theta = 5°$ and pion energies 130, 163, 180, 235, and 295 MeV,[87] a minimum was found in the excitation function in the region of the Δ_{33} resonance. At high energies, the excitation function increases.

Do the excitation functions differ for the transitions to double-isobar analog states and nonanalog states? In all the theoretical models, it was assumed that the transitions to the analog states are dominant. This was justified by the fact that the overlapping of the wave functions is greatest for an isobar-analog triplet. However, the experimental facts now available contradict such a proposition. As was first shown in Ref. 77, the ratio of the cross section of the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne(DIAS)$ to that of the reaction $^{16}O(\pi^+,\pi^-)^{16}Ne$ at 140 MeV is approximately 2. Measurements made by the EP2 group[79] showed that the ratio $d\sigma/d\Omega(^{18}O)/d\sigma/d\Omega(^{16}O)$ depends on the π^+ energy. Whereas $d\sigma/d\Omega(^{18}O)$ has a maximum at $T_{\pi^+} = 120$ MeV, $d\sigma/d\Omega(^{16}O)$ has a maximum at 160 MeV, where the cross section $d\sigma/d\Omega(^{18}O)$ has a minimum. This ratio is equal to 3/1 at 160 MeV and $\approx$20/1 at 292 MeV. A similar situation is observed for the ratio $d\sigma/d\Omega(^{26}Mg)/d\sigma/d\Omega(^{24}Mg)$. This ratio is of order 1 at 140 MeV and $\simeq$70/1 at 292 MeV. Above, we compared the analog and nonanalog transitions for the excitation function for different entrance channels, i.e., for target nuclei with different A. The energy dependence of the cross sections for transitions to double-isobar analog states and nonanalog states were studied in Ref. 82 for double charge exchange on the same target nucleus in the $^{56}Fe(\pi^+,\pi^-)^{56}Ni$ reaction. This reaction has two different exit channels with transitions to analog states and nonanalog states. The ground state of the ^{56}Ni nucleus is not the double-isobar analog state of the ^{56}Fe nucleus. The double-isobar analog state is the excited state of the ^{56}Ni nucleus with $E_{ex} = 9.6$ MeV. The measurements showed that the excitation function for the transition to the double-isobar analog state increases with increasing pion energy, while for the nonanalog transition it decreases. Table II gives the reactions of pion double charge exchange studied in order to measure the excitation functions for analog and nonanalog transitions.

Thus, summarizing the available experimental data, we can answer the question posed above. The cross section of

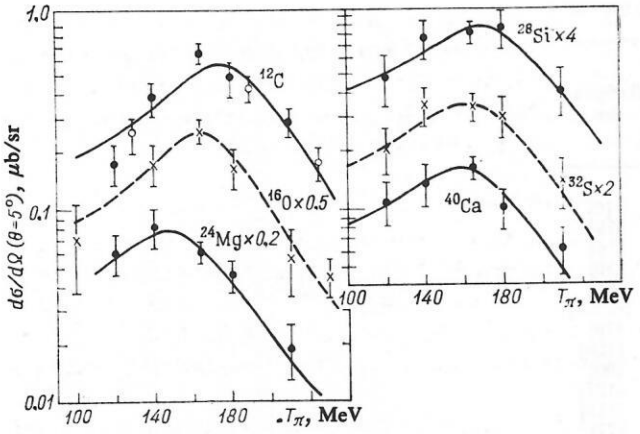

FIG. 8. Differential cross section of the (π^+,π^-) reaction for nonanalog transitions on different nuclei. The experimental data are as follows: black circles and crosses from Ref. 83, open circles from Ref. 6, and the data for the ^{16}O nucleus from Ref. 89. The data are fitted by the Breit-Wigner formula. The figure is taken from Ref. 83.

double charge exchange with transition to a double-isobar analog state increases with increasing pion energy and is dominant at high energies. The ratio of these cross sections for different nuclei changes little with increasing pion energy. The cross section of double pion exchange for transitions to nonanalog states is comparable with the cross section for transitions to the double-isobar analog states in the region of energies 140–160 MeV, and with a further increase in the pion energy it decreases strongly. The ratio of the excitation functions for nonanalog transitions in double charge exchange on different target nuclei remains constant with increasing pion energy.

Angular distributions

In various theoretical studies, it was assumed that measurement of the angular distributions would be a good critical test for the theory of double charge exchange. The angular distribution of the pions was measured for the first time by the EP1 group[88] at LAMPF in an investigation of the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction. The measurements were made at π^+ energy 164 MeV in the interval of angles 13–45° with formation of the double-isobar analog states (0^+ ground state, ^{18}Ne) and the first excited state of ^{18}Ne (2^+, 1.89 MeV). This experiment showed that the cross sections for the transitions to the analog and nonanalog states are approximately the same, but the differential cross section for the transition to the double-isobar analog state has a diffraction nature with a minimum at $\theta \approx 20°$. For the transition to the first excited 2^+ state the angular distribution has the Cupola shape characteristic of $L = 2$ transitions in surface direct reactions. Later, the angular distributions were measured for the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction by the EP2 group[79] in the interval of angles 5–33° and at energies 164 and 292 MeV. The data at 164 MeV agree well with the data of the EP1 group[88]: The cross section at $\theta = 5°$ is of order 1 mb/sr, has a minimum of 40 nb/sr at $\theta = 20°$, and then increases to 250 nb/sr at $\theta = 33°$. The differential cross section at 292 MeV for the transition to the double-isobar analog state also has a diffraction nature with a small shift of the minimum to the region of larger angles.

The recent data[121] on the angular distributions of negative pions in the reaction of π^+ double charge exchange on the ^{14}C and ^{26}Mg nuclei for double-isobar analog transitions at $T_{\pi^+} = 164$ and 292 MeV in the interval of angles 0–50° also indicate that they have a diffraction structure with a minimum in the angular distribution at $\theta \simeq 20°$ at 164 MeV and at $\theta \simeq 26°$ at 292 MeV, in good agreement with the data of Ref. 79 for double-isobar analog transitions in the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction. It is interesting to note that the ratio $d\sigma/d\Omega(^{18}O)/d\sigma/d\Omega(^{26}Mg)$ of the angular distributions for transitions to the double-isobar analog state is approximately constant in the interval 5–33°.

The angular distributions of the pions in the reaction of double charge exchange on $T = 0$ nuclei leading to nonanalog transitions were measured by the EP2 group.[83,89] It was found in Ref. 89 that the angular distribution of the pions in the $^{16}O(\pi^+,\pi^-)^{16}Ne$ reaction has a diffraction nature with a minimum at $\theta \approx 30°$. The investigation of the angular distribution of the π^+ mesons in the $^{18}O(\pi^-,\pi^+)^{18}C$ reaction[122] also revealed a minimum at $\theta \approx 30°$, this resembling more the angular distribution of the π^- mesons in the nonanalog $^{16}O(\pi^+,\pi^-)^{16}Ne$ double charge exchange than the angular distribution of the negative pions in the reaction $^{18}O(\pi^+,$
$\pi^-)^{18}Ne(DIAS)$. Measurement of the angular distributions for the ^{12}C and ^{40}Ca nuclei[83] also revealed their diffraction nature, but with a different position of the minimum. Table II gives the reactions of pion double charge exchange studied in order to measure the angular distributions for the analog and nonanalog transitions.

Thus, the experimental investigations of the angular distributions showed that in double charge exchange on nuclei with $T = 0$ (nonanalog transitions) and with $T \geqslant 1$ (double-isobar analog transitions) the angular distributions have a diffraction nature and are comparable. This suggests that in the theoretical description of double charge exchange on $T = 1$ nuclei it is necessary to take into account double nonanalog charge exchange on the nuclear core with isospin $T = 0$.

The A dependence of the cross sections of double charge exchange

In all the theoretical models, the cross section of pion double charge exchange is proportional to the excess of neutron or proton pairs. However, the increase in the cross section with increasing excess pairs is to some extent compensated by the increase in the effects of absorption with increasing A. The different theoretical approaches give different A dependences of the cross section. In the optical model of Ref. 23, the cross section is proportional to A^{-2}, while in the phenomenological model of Ref. 25, in which the absorption effects are determined by the absorption radius, this dependence has the form $A^{-10/3}$. The A dependence of the cross section for double charge exchange was investigated for the first time by the EP2 group,[79] but data for some nuclei were obtained at 164 MeV and for others at 180 MeV. The cross sections of double charge exchange on the $T = 0$ nuclei ^{12}C, ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, and ^{40}Ca for transitions to

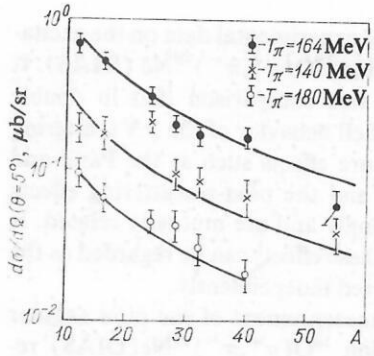

FIG. 9. The A dependence of the differential cross section of double charge exchange at $\theta = 5°$ for nonanalog transitions. The data at 140 and 180 MeV are divided by 2 and 5, respectively. The experimental data for the nuclei ^{12}C, ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca are from Ref. 83 and those for ^{56}Fe from Ref. 82. The curves show the dependence $A^{-4/3}$.

nonanalog states at different energies for forward scattering ($\theta = 5°$) were studied systematically in Ref. 82. As can be seen from Fig. 9, the experimental data at all the considered energies can be well approximated by a curve having the dependence $A^{-4/3}$. Just one point, at 140 MeV for the ^{24}Mg nucleus, departs from the general tendency. The data for the ^{56}Fe$(\pi^+,\pi^-)^{56}$Ni reaction for the nonanalog transition at 164 MeV are also in good agreement with the dependence $A^{-4/3}$.

We now consider the A dependence of the cross section of forward double charge exchange in the case of transitions to double-isobar analog states. Figure 10 gives the results of measurements of the differential cross sections of double charge exchange in the case of double-isobar analog transitions on the nuclei ^{14}C, ^{18}C, ^{26}Mg, ^{42}Ca, ^{56}Fe, and ^{209}Bi at different energies. At pion energy 292 MeV, the experimental points are well approximated by the dependence $A^{-10/3}$, whereas the behavior of the cross section at 164 MeV is anomalous. It followed from analysis of the excitation functions that in the region of energies below 160 MeV the cross sections for the analog and nonanalog transitions are comparable, while at the higher pion energies the nonanalog transitions are strongly suppressed. Therefore, the strong energy dependence for the A dependence in the case of transitions to double-isobar analog states can be understood if it is assumed that at low energies analog and nonanalog amplitudes of double charge exchange contribute to the double-isobar analog transitions. At the higher energies, the nonanalog transitions are suppressed, and only the analog amplitudes lead to double-isobar analog states in the case of pion double charge exchange. It is this last fact that leads to an A dependence of the typ $A^{-10/3}$.

The above systematization of the excitation functions, angular distributions, and A dependence permits the following conclusions to be drawn. First, the reaction of double charge exchange has a strong isospin dependence, but the process is determined to a greater extent by the reaction mechanism than it is by the isospin effects of the nuclear structure. Second, the mechanism of double charge exchange depends strongly on the pion energy. In the region of low energies, the double-isobar analog transitions must be due to both analog and nonanalog transitions, the corresponding amplitudes being comparable in magnitude. Therefore, it can be assumed that in double charge exchange on $T = 1$ nuclei in the case of transitions to double-isobar analog states double charge exchange on the nonanalog core with $T = 0$ must be taken into account.

Theoretical analysis

The excitation functions and angular distributions in reactions of double charge exchange have been studied using the diffraction approximation,[10,13–15] the approximation of fixed centers,[18,91] the optical-potential method,[22–26,92,93] the meson-current model,[15] and the dynamics of the Δ_{33} isobar.[94,95]

In Refs. 22 and 23, double charge exchange with transitions to double-isobar analog states was studied by the coupled-channel method. In this approach, the intermediate and initial states belong to the same isotopic multiplet. Both Kisslinger's optical potential[22,23] and the optical potential in the local-Laplacian model[23] were used. The investigations revealed a strong dependence of the cross section for double charge exchange on the shape of the optical potential and, therefore, on the off-shell effects in the πN scattering amplitude. The process of double charge exchange was shown to be strongly sensitive to Pauli and short-range correlations. In order to take into account short-range correlations in Ref. 23, the second-order optical potential was calculated and the usual formula for it was generalized to the case of coupled channels. The calculations demonstrated that in the region of pion energies around 100 MeV the effects of the short-range correlations strongly influence the cross section for double charge exchange, while at energies greater than 180 MeV these effects are small. The total and differential cross sections of double charge exchange were studied in their dependence on the difference $\Delta\rho(r) = \rho_p(r) - \rho_n(r)$ between the neutron and proton distributions in the target nucleus. To this end, the expression for Kisslinger's optical potential was generalized in Ref. 22 to the case $\Delta\rho(r) \neq 0$. The potential contained the term $\Delta\rho(r)(\tau\cdot\mathbf{T})$, and the amplitude for double charge exchange was found by solving the linearized Klein-Gordon equation. The investigation showed that the cross section for double charge exchange is extremely sensi-

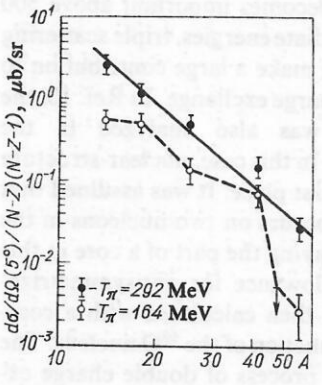

FIG. 10. The A dependence of the differential cross section of double charge exchange at $\theta = 5°$ for double-isobar analog transitions. The continuous curve is the dependence $A^{-10/3}$. The figure is taken from Ref. 82.

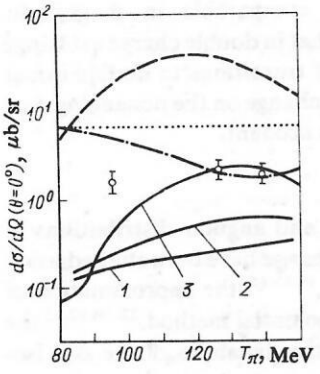

FIG. 11. Differential cross section of the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$ (DIAS). Curves 1–3 are the results of calculations in the optical model[23] using the potentials of Kisslinger, the local Laplacian, and the local Laplacian with allowance for short-range correlations, respectively; the broken curve is from Ref. 11, the dotted curve from Ref. 91, and the chain curve from Ref. 14. The experimental data are from Ref. 78.

tive to the different shapes of the proton and neutron distributions. However, the adopted approaches did not ensure the experimentally observed proximity of the analog and nonanalog transitions. As was shown in Refs. 24 and 96, the reason for this is the neglect of the intermediate nonanalog states. In Ref. 24, the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction was analyzed on the basis of the Kisslinger potential. The intermediate states $0^+, 2^+, 4^+$ of the ^{18}F nucleus were considered. The calculations showed that in the region of energies 100–200 MeV allowance for the intermediate 2^+ and 4^+ states is rather important, the cross section for double charge exchange due to the transition through the first excited 2^+ state of the ^{18}F nucleus being comparable to the cross section obtained by the transition through the intermediate analog state. This demonstrated the importance of taking into account intermediate nonanalog transitions in the case of double charge exchange to double-isobar analog states.

Investigation of the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction in theframework of the diffraction approximation[14] showed that multiple scattering of pions in double charge exchange is important and that the multiple-scattering series converges slowly. The calculations took into account the s, p, d, and f partial waves in the πN scattering amplitude. The d-wave contribution was found to be important at energies above 100 MeV, while the f wave becomes important above 500 MeV. In the region of intermediate energies, triple scattering and higher-order rescattering make a large contribution to the cross section for double charge exchange. In Ref. 10, the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction was also analyzed in the frameworkof Glauber theory. In this case, nuclear-structure effects are considered in the first place. It was assumed that the double charge exchange occurs on two nucleons in the $d_{5/2}$ shell, the ^{16}O nucleus playing the part of a core in this case (calculation without allowance for antisymmetrization). The cross section was then calculated with a completely antisymmetric wave function of the ^{18}O nucleus. The comparison showed that the process of double charge exchange is very sensitive to the Pauli correlations and the details of the nuclear surface in the considered energy range.

In Fig. 11, we compare the results of the different theoretical approaches with the experimental data on the excitation function for the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$ (DIAS). It may be concluded from this comparison that in double charge exchange the off-shell behavior of the πN scattering amplitude, nuclear-structure effects such as the Pauli and short-range correlations, and the pion-rescattering effects influence the process strongly and are mutually related. It would seem that none of these effects can be regarded as the leading effect and considered independently.

As was said above, measurement of the pion angular distribution in the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$(DIAS) revealed a first minimum at $\theta = 20°$.[88] In all the theoretical predictions, the position of this minimum was found to be at $\theta = 30$–$40°$ whether or not a single intermediate analog state or nonanalog intermediate states as well were taken into account in the calculations. As was shown in Refs. 10 and 97, the Pauli and short-range correlations do shift the minimum to smaller angles but do not lead to agreement with experiment. The experiment of Ref. 88 assisted further progress in the theoretical understanding of both the reaction mechanism and the importance of nuclear structure in the reaction of double charge exchange.

The large cross section of double charge exchange on nuclei with $N = Z$ led to the suggestion of a "single-step" mechanism of the reaction, this proceeding through interaction of the pion with the Δ_{33} resonance (Fig. 12a). In this case, the double charge exchange of pions on $N > Z$ nuclei is due to the interference between this "single-step" mechanism and the ordinary two-step mechanism. It was suggested that this interference is also responsible for the anomalous behavior of the angular distribution and the excitation function in the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction.

It was proposed in Ref. 94, in the framework of the optical-potential formalism, that the Δ_{33} isobar, formed within the nucleus, propagates as a quantum-mechanical particle with finite lifetime. The blocking effects due to the Pauli principle and true absorption of the pions were described by means of a Δ–nucleus interaction in which the parameters of the scalar part were determined using data on pion elastic scattering. A minimum in the differential cross section of the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$(DIAS) was obtained by interference between the contribution of the successive charge-exchange diagram and the contribution of the diagram describing the propagation of the Δ isobar in the nucleus. The results of the calculations showed that allowance for the motion of the Δ isobar and the Δ-nucleus interaction is very important and depends on the parameters

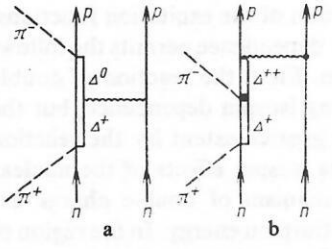

FIG. 12. Diagrams of the single-step mechanism. The wavy line represents $(\pi+\rho)$-meson exchange.

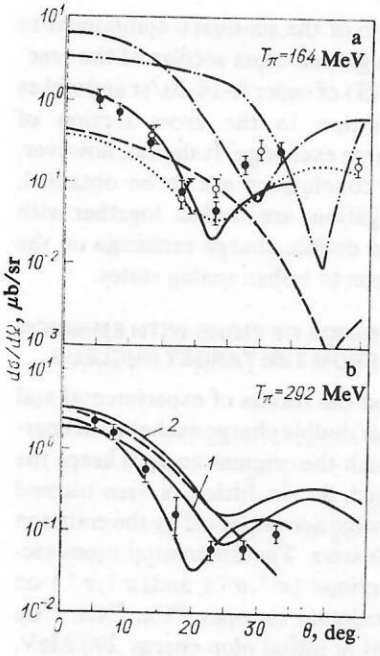

FIG. 13. Differential cross section of the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$ (DIAS) according to the data of different studies: the continuous curve from Ref. 26, the broken curve from Ref. 93, the dotted curve from Ref. 94, the chain curve from Ref. 10, and the chain curve with two dots from Ref. 92. Curves 1 and 2 were obtained in Ref. 15 in the diffraction approximation with and without allowance for the diagram in Fig. 2a, respectively. The calculation in Ref. 15 was made at energy $T_\pi = 250$ MeV. The experimental data: the black circles are from Ref. 79, and the open circles from Ref. 88.

of the Δ-nucleus interaction. However, the angular distribution at 164 MeV cannot be described by any choice of the depth parameters of the optical potential. The curve corresponding to the approach of Ref. 94 is shown in Fig. 13.

It was shown in Ref. 95 that double charge exchange on the Δ_{33} component of the ground-state wave function (Fig. 12b) makes a small contribution to the cross section of the process. The contribution of the diagram in Fig. 12a was found to be much more appreciable, this being due to the fact that the isobar charge exchange $\Delta^+ \to \Delta^0$ takes place off the mass shell by virtue of $(\pi + \rho)$-meson exchange. The interference of the contributions of the diagrams of Fig. 11a and the successive charge exchange explains the experimental data on the excitation function below the resonance, but with increasing pion energy the discrepancy with experiment increases strongly (Fig. 14).

The next attempt to explain the angular distributions and the oscillations of the excitation function in the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$(DIAS) was associated with consideration of the effect of the meson currents on the process of double charge exchange. In Ref. 15, the diffraction approximation was used to consider the successive charge-exchange mechanism and the meson-current mechanism in the process of double charge exchange. In the latter case, the contribution of the diagram in Fig. 2a was considered. The results were not very promising. It was shown that the interference of the two-step mechanism and the meson-current mechanism leads to a slight shift of the minimum in the angular distribution to smaller angles but does not describe the experimental data. For the excitation functions, allowance for the meson currents reduces the differential cross section in the region of energies below the resonance and increases it at high energies. Thus, neither allowance for the meson currents nor the dynamics of the Δ_{33} isobar can explain the complete set of experimental data. True, it should be noted that primitive nuclear models were employed in Refs. 15, 94, and 95 for the $^{18}0$ and ^{18}Ne nuclei, but a more realistic description of the structure of these nuclei can hardly change the situation.

It was shown above that in all the theoretical models successive charge exchange is the leading mechanism, and the amplitude for double charge exchange is determined by the single-particle density $\rho(r)$. Then the dramatic situation with regard to the position of the minimum of the angular distribution in the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$(DIAS) can be explained by interference between the amplitude of successive charge exchange and the other amplitude, which is comparable with the amplitude of the two-step mechanism. One proposal was that this latter amplitude must be due to the term in the π-nucleus potential proportional to $\rho^2(r)$. The importance of this term in double charge exchange to double-isobar analog states was also pointed out in Refs. 23 and 98. In Ref. 26, an attempt was recently made at not only a realistic description of the nuclear structure but also allowance for the term in the π-nucleus interaction proportional to $\rho^2(r)$ in order to describe the process $^{18}O(\pi^+, \pi^-)^{18}Ne$(DIAS) using the coupled-channel method. The system of three integral equations (27) was written down with allowance for the different components of the πN scattering amplitude—the isoscalar, isovector, and isotensor—and the constructed second-order optical potential included a term proportional to $\rho^2(r)$. In this approach, the strong interaction v_s is split into two parts. One, proportional to $\rho(r)$, describes the successive charge exchange on two valence nucleons through analog states. The other, proportional to $\rho^2(r)$, is responsible for double charge exchange not only on two valence nucleons but also on core nucleons through nonanalog intermediate states. This treatment

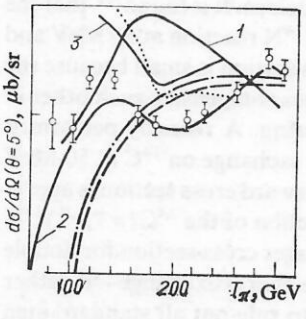

FIG. 14. Differential cross section of the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$ (DIAS) according to the data of differentstudies. The curves from Ref. 95 were obtained under the assumption of the mechanism of successive charge exchange (broken curve), the mechanism of successive charge exchange with allowance for the diagram of Fig. 12b (chain curve), and with allowance for the diagram of Fig. 12a (curve 1); curve 2 is from Ref. 26, curve 3 is from Ref. 15, and the dotted curve from Ref. 15 was obtained in the Glauber-Sitenko approximation with allowance for the diagram of Fig. 2a. The experimental data are taken from Ref. 79.

made it possible to explain the angular distribution and the excitation functions in the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$-(DIAS). Later, this method was successfully used to describe π^+ double charge exchange on the ^{42}Ca nucleus.[87] It was shown that the term proportional to $\rho(r)$ and the allowance for the core excitations lead to the same dependence of the excitation function for double charge exchange, while inclusion of the term proportional to $\rho^2(r)$ and allowance for the excited core states lead to a satisfactory description of the experimental data on the double charge exchange $^{42}Ca(\pi^+,\pi^-)^{42}Ti(DIAS)$.

Pion double charge exchange and quark degrees of freedom

The successes of modern nonrelativistic nuclear theory in describing the basic properties of nuclei indicate that nucleon degrees of freedom are dominant in the nuclear wave function. However, according to the ideas of quantum chromodynamics, nucleons are colorless three-quark formations. Therefore, multiquark objects may be manifested in nuclear processes. Which nuclear processes are the most favorable for manifesting quark degrees of freedom? Recent experiments on deep inelastic scattering of muons and electrons on nuclei (EMC effect)[99,100] indicate that a significant non-nucleon component is present in the nucleus. The effects of the hidden color components of the nuclear function must be manifested not only in electroweak interactions but also in processes in which hadrons participate. It seems evident that pion double charge exchange on nuclei at both low and high energies will yield information about the quark structure of the nucleus. We shall consider this possibility at low energies. Two nucleons participate in double charge exchange. On the basis of this, it seems possible that for certain kinematic conditions the double charge exchange will be sensitive to the six-quark component of the nuclear wave function. In particular, if at low energies the cross section of forward double charge exchange is appreciably greater than the cross section of single charge exchange on the same nucleus, then as one of the possible ways of explaining this strange fact one can consider the presence of hidden color components of the nuclear wave function.[101] Let us explain this for the example of the ^{14}C nucleus. It is found[102] that the cross section of the $^{14}C(\pi^+,\pi^0)^{14}N$ reaction at 50 MeV and $\theta = 0°$ is very small.[3)] The cross section is small because the contributions of the s and p waves compensate each other in the charge-exchange πN scattering. A recently performed experiment[90] on double charge exchange on ^{14}C at 50 MeV shows that the corresponding forward cross section is appreciably greater than the cross section of the $^{14}C(\pi^+,\pi^0)^{14}N$ reaction. Such a situation—a larger cross section for double charge exchange than for simple charge exchange—is rather unusual and makes it possible to rule out all standard step mechanisms.

Under the considered kinematic conditions, the following mechanism is proposed. Two valence nucleons of the ^{14}C nucleus at distances less than 1 F form a six-quark bag. The π^+ meson, interacting with a u quark in the six-quark bag, is absorbed, and through the participation of a d quark the π^- is emitted from the same bag. The calculations made in Ref. 101 show that the inclusion of the six-quark component in the nuclear wave function gives a cross section of the reaction $^{14}C(\pi^+,\pi^-)^{14}O(DIAS)$ of order 8–14 μb/sr andmakes a much smaller contribution to the cross section of $^{14}C(\pi^+,\pi^0)^{14}N$ single charge exchange. It should, however, be noted that if definite conclusions are to be obtained, further theoretical investigations are needed, together with experiments on single and double charge exchange on the same nucleus with transition to isobar analog states.

4. DOUBLE CHARGE EXCHANGE OF PIONS WITH EMISSION OF SEVERAL PARTICLES FROM THE TARGET NUCLEUS

In Sec. 3, we analyzed the results of experimental and theoretical investigations of double charge exchange at energies up to 300 MeV in which the original nucleus keeps the nucleon number unchanged. So far, little has been learned about double charge exchange accompanied by the emission of nucleons or nucleon clusters. The differential cross sections of the inclusive reactions (π^+,π^-) and (π^-,π^+) on the ^{12}C nucleus and the calcium isotopes ^{40}Ca, ^{44}Ca, ^{48}Ca were measured in Ref. 104 at initial pion energy 290 MeV. Produced pions with energy 175–225 MeV were detected at $\theta = 60°$. The inclusive cross sections for double charge exchange were found to be one or two orders of magnitude smaller than the cross sections of quasielastic scattering. The results for the calcium isotopes indicate a strong sensitivity of the double charge exchange to the difference between the proton and neutron distributions in the surface region of the nucleus. The measurements also showed that the π^+ and π^- cross sections on these nuclei are not equal. This fact cannot be explained by the difference between the Coulomb distortions for the π^+ and π^-. However, if we assume that in this experiment sufficient energy is transferred to the nucleus for its disintegration, then channels with strongly differing cross sections are open in the (π^+,π^-) and (π^-,π^+) reactions. In addition, nuclear-structure effects are not the least important. For example, the strong difference between the $(\pi^+,\pi^-)/(\pi^-,\pi^+)$ cross-section ratios for the ^{12}C and ^{40}Ca nuclei can be due to different core excitations as well as to the effects of α clustering of the core. Because nucleon clustering effects in nuclei are important, it is of interest to consider the mechanism of double charge exchange based on the assumption that the pion double charge exchange takes place on a quasi-α-particle association in the initial nucleus.

Quasi-α-particle mechanism of double charge exchange

We consider the following mechanism of pion double charge exchange on an α-cluster nucleus. The incident pion, interacting with the nucleus, "heats" locally the nuclear matter, this leading to the "evaporation" of four nucleons of the same kind from the nucleus and charge exchange of the pion. By local heating of the nuclear matter we understand double charge exchange of the pion on a quasi-α-particle formation-of the initial nucleus. What facts support such a reaction mechanism? The experimental study of various channels of π^+ double charge exchange on the ^{12}C nucleus revealed predominance of the channel

$$\pi^+ + {}^{12}C \to \pi^- + 4p + 2\alpha. \qquad (71)$$

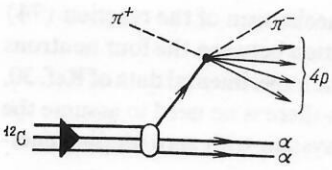

FIG. 15. Diagram of the quasi-α-particle mechanism of double charge exchange on the ^{12}C nucleus.

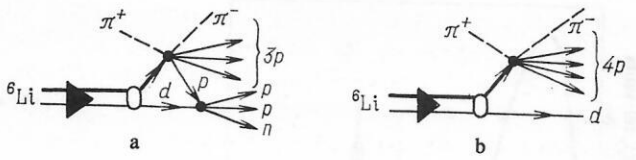

FIG. 17. Diagrams of the quasi-α-particle mechanism of double charge exchange on the ^{6}Li nucleus in the αd model.

According to Ref. 105, the cross section of the process (71) at π^+ energy 138 MeV is 62% of the total cross section of π^+ double charge exchange on the ^{12}C nucleus and equal to 0.77 ± 0.27 mb. In contrast, the cross section for π^+ double charge exchange on the ^{4}He nucleus [reaction (37)] at 135 MeV is 0.29 ± 0.11 mb.[62] It can be seen that the ratio of the cross section of the process (71) to the cross section of the process (37) is near 3. This fact can evidently be interpreted as an indication that in the reaction (71) of double charge exchange the process unfolds predominantly on substructures of the ^{12}C nucleus resembling α particles. On the basis of the α-cluster model of the ^{12}C nucleus, the mechanism of the reaction (71) can be illustrated by the diagram shown in Fig. 15.

We now consider pion double charge exchange on the ^{6}Li nucleus, which results in the production of four nucleons of the same kind. The following processes are possible:

$$\pi^+ + {}^6\mathrm{Li} \to \pi^- + 4p + n + p; \qquad (72)$$

$$\pi^+ + {}^6\mathrm{Li} \to \pi^- + 4p + d. \qquad (73)$$

It is clear from consideration of the reactions (72) and (73) that the double charge exchange on ^{6}Li can proceed only through the quasi-α-particle mechanism. Indeed, at energies below the pion production threshold at least two nucleons of the same kind must participate in the reaction of double charge exchange. The p shell of the ^{6}Li nucleus contains nucleons of different kinds. Therefore, the process must take place on s-shell nucleons. On the basis of the cluster structure of the ^{6}Li nucleus (in the two-particle cluster model, the ^{6}Li nucleus is represented in the form of an α particle and a deuteron—this is the αd model; in the three-particle cluster model it is represented in the form of an α particle, a neutron, and a proton—this is the αnp model), the four nucleons of the s shell form a quasi-α-particle association, on which the processes (72) and (73) will take place. If we start from the αnp model of the ^{6}Li nucleus, the reactions (72) and (73) will be described by the diagrams of Fig. 16, whereas in the two-particle cluster model the diagrams of Fig. 17 will be responsible for the processes (72) and (73). The diagrams in Figs. 16a and 17a describe the complete breakup of the ^{6}Li nucleus as a result of double charge exchange. The diagram in Fig. 17a represents a process in which complete breakup takes place because of rescattering of a nucleon by a deuteron. The contribution of this diagram will be of higher order than the contribution of the diagram in Fig. 16a. The diagrams of Figs. 16b, 16c, and 17b correspond to the process (73). The diagrams of Figs. 16b and 16c describe the process in which the deuteron is formed by the interaction of an np pair in the final state, and the contributions of these diagrams will also be of higher order than the contribution of the diagram of Fig. 17b. Thus, in the first approximation the process (72) is described by the diagram of Fig. 16a in the αnp model of the ^{6}Li nucleus and the process (73) by the diagram of Fig. 17b in the αd model of the ^{6}Li nucleus. However, whatever the cluster (two- or three-particle) configuration of the ^{6}Li nucleus, the double charge exchange takes place in both reactions as a result of interaction of the π^+ with a quasi-α-particle formation of the nucleus.

An experimental investigation was recently made of the reaction $\pi^- + {}^7\mathrm{Li} \to \pi^+ +$ anything.[30] On the basis of the cluster structure of the ^{7}Li nucleus, it must be assumed that on this nucleus too the pion double charge exchange can take place only on a quasi-α-particle nuclear formation. In the $\alpha{}^3$H model of the ^{7}Li nucleus, the assumption of such a mechanism leads to the reaction channel

$$\pi^- + {}^7\mathrm{Li} \to \pi^+ + 4n + {}^3\mathrm{H}. \qquad (74)$$

The papers of Refs. 106 and 107 developed a microscopic theory of the quasi-α-particle mechanism of double charge exchange for α-particle nuclei,[106] and also for nuclei consisting of α particles and clusters of other types. It was assumed that the $\pi^{\mp}$ meson is transformed into the $\pi^{\pm}$ meson by successive charge exchanges on two nucleons of the same kind in an α formation in the nucleus. This results in the breakup of the quasi-α-particle into four protons ($\pi^+ \to \pi^-$ reaction) or into four neutrons ($\pi^- \to \pi^+$ reaction). The ^{12}C nucleus was described as a collection of three α particles interacting through a realistic $\alpha\alpha$ potential. The three-particle cluster configuration of the ^{6}Li nucleus was described similarly with allowance for all interactions between the three particles. To treat the two-particle configurations of the ^{6}Li and ^{7}Li nuclei, the two-particle Schrödinger equation was solved. For each α association, its internal structure was

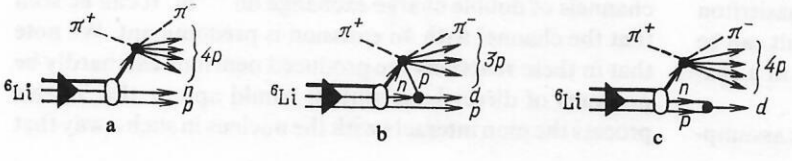

FIG. 16. Diagrams of the quasi-α-particle mechanism of double charge exchange on the ^{6}Li nucleus in the αpn model.

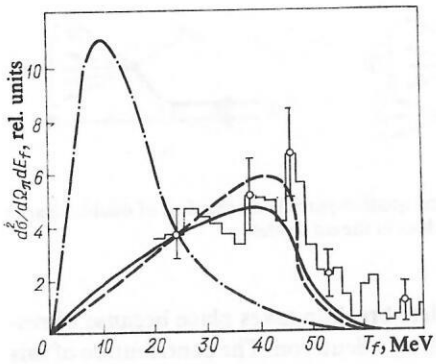

FIG. 18. Dependence of the differential cross section of the reaction (76) on the kinetic energy of the pions. The continuous curve represents the results of the calculation of the cross section under the assumption of the quasi-α-particle reaction mechanism using the NN potential V; the broken curve gives the results of calculations with the NN potential GPT; the chain curve and the experimental data are from Ref. 30. The figure is taken from Ref. 107.

taken into account. The wave functions describing the system of three particles ($\alpha\alpha\alpha$ or αnp) and the four nucleons in the α association were found by solving the three- and four-particle Schrödinger equations, respectively, by the hyperspherical-basis method. The treatment of the final state took into account all interactions between the four nucleons emitted from the nucleus as a result of the double charge exchange, the same realistic NN potential being used as for the description of the bound state of the four nucleons in the α association.

As an illustration of the quasi-α-particle mechanism, Fig. 18 shows the results of calculations of the differential cross section of the process (74). The broken curve shows the results of the calculations of Ref. 30 for the reaction (74) obtained under the assumption of the meson-current mechanism. The other two curves were obtained in Ref. 107 under the assumption that the ^{7}Li nucleus has a two-particle cluster structure and, therefore, the π^- double charge exchange can take place only on α particles. It can be seen that in this case the experimental data are satisfactorily described, and, therefore, we can probably conclude that the channel (74) makes the main contribution to the process that was studied in Ref. 30. It is interesting to note that the assumption of the quasi-α-particle mechanism of double charge exchange leads already in the plane-wave approximation (four noninteracting neutrons in the final state) to a shift of the maximum of the cross section and a lowering of this maximum. The inclusion of interaction between the four neutrons in the final state leads to an appreciable deviation of the curve of the distribution. The cross section can be described by a smooth curve with a maximum shifted to higher pion energies. The shift of the maximum of the cross section to the higher pion energies corresponds to its displacement to lower relative energies of the four nucleons. If it is assumed that comparatively little energy is transferred to the quasitriton (only the α particle is locally heated), this last result can be interpreted as a temporary delay of four neutrons in a spatially localized region.

Thus, it follows from the calculations that the assumption of the quasi-α-particle mechanism of the reaction (74) and allowance for the interaction between the four neutrons in the final state can explain the experimental data of Ref. 30, so that to interpret these data there is no need to assume the formation of a four-neutron system with zero binding energy.

It is well known that the results of investigating reactions of the type ^{6,7}Li$(\alpha,2\alpha)^{2,3}$H, ^{6,7}Li$(p,p\alpha)^{2,3}$H, ^{6}Li$(e,e'\alpha)^2$H, ^{6}Li$(e,e'd)^4$He, interpreted as processes of quasielastic knockout of α and d clusters, indicate a high degree of α clustering of these nuclei. If quasi-α-particles exist for an appreciable time in the ground state of the nuclei as certain spatially localized groupings of nucleons, then pion double charge exchange must manifest a process with kinematics like that of pion double charge exchange on free ^{4}He nuclei. If this is observed, it will support the quasi-α-particle mechanism of double charge exchange. However, it must be borne in mind that interaction in the final state may distort the picture.

Finally, the quasi-α-particle mechanism of double charge exchange may play an important part in reactions on other light nuclei, and also on heavy nuclei, in whose surface layer it is well known that quasi-α-particles can be formed.

Double charge exchange on heavy nuclei

There have been far fewer experimental studies of pion double charge exchange on heavy nuclei than on light and medium nuclei. This is due to the fact that double-isobar analog transitions with $\Delta T = 2$ in heavy nuclei must be predominant at energy transfers of 20–40 MeV. However, at such excitations it is expected that the double charge exchange will be accompanied by the knockout of nucleons and nucleon clusters into the continuum, so that it will not be an easy matter to separate the double-isobar analog states on the background of these channels. Therefore, the main investigations so far made have been of double charge exchange on heavy nuclei, accompanied by the emission of nucleons.

There have been three experimental studies of double charge exchange on the neutron-rich ^{209}Bi nucleus. Batusov *et al.*,[108] using the radiochemical method at pion energy 90 MeV, determined the cross section of the ^{209}Bi$(\pi^+,\pi^-2n)^{207}$At reaction, which they found to be 120 ± 30 μb, and an upper bound of 10 μb for the ^{209}Bi$(\pi^+,\pi^-)^{209}$At process. The next attempt was to measure spectrometrically double-isobar analog transitions in the ^{209}Bi$(\pi^+,\pi^-)^{209}$At reaction at 292 MeV (Ref. 86); this revealed a strong suppression of this channel. Clark *et al.*[109] recently reported the results of a radiochemical study of the ^{209}Bi$(\pi^+,\pi^-Xn)^{209-X}$At reaction at pion energies 100, 180, and 300 MeV. The cross sections for chemically separated astatine isotopes with $A = 205$–209 were determined by α and γ spectroscopy. Table III gives the cross sections of the different channels of double charge exchange on ^{209}Bi. It can be seen that the channel with 4n emission is predominant. We note that in these reactions the produced neutrons can hardly be products of direct knockout. It would appear that in this process the pion interacts with the nucleus in such a way that

TABLE III. Cross sections of pion double charge exchange on the ^{209}Bi nucleus, μb.

Reaction channels	Initial pion energy, MeV			
	90	100	180	300
^{209}Bi (π^+, π^-) ^{209}At	<10	<6	2.5 ± 3.6	1.0 ± 3.0
^{209}Bi (π^+, π^-2n) ^{207}At	120 ± 30	69 ± 18	22 ± 7	3.7 ± 4.8
^{209}Bi (π^+, π^-3n) ^{206}At	—	99 ± 29	—	10
^{209}Bi (π^+, π^-4n) ^{205}At	—	436 ± 105	60 ± 20	40 ± 10

there is sufficient time for part of its energy to be distributed between many nucleons with subsequent evaporation of secondary neutrons. The evaporation of charged particles from the excited ^{209}At nucleus must be significantly suppressed by the Coulomb barrier, although in principle it is possible at high excitations of the nucleus.

Experiments in which ^{209}Bi was bombardet with π^+ revealed evidence of ^{211}At, i.e., a manifestation of an isotope with atomic mass higher than that of the original target nucleus. This required a special study of the possible reactions in which it can be formed. Bearing in mind that the cross section of the $(\alpha, 2n)$ reaction on Bi is very large and ~ 1 b, it can be concluded that the ^{211}At is formed in the reaction $\alpha + {}^{209}\text{Bi} \rightarrow {}^{211}\text{At} + 2n$. Then α particles with the necessary energy must be formed in the double charge exchange $\pi^+ + {}^{209}\text{Bi} \rightarrow \pi^- + \alpha + \ldots$. In this connection, it was conjectured that in heavy nuclei pion double charge exchange takes place strongly on nucleon clusters. To test this hypothesis, it would be interesting to measure double charge exchange on heavy nuclei such as ^{208}Pb, ^{232}Th, ^{238}U with coincidence detection of the pions and α particles. The results of the recently performed radiochemical investigations of pion double charge exchange on the ^{209}Pb nucleus[110,111] aimed at obtaining the tetraneutron can also be interpreted as double charge exchange on an α particle in the ^{208}Pb nucleus.

Double charge exchange of pions at high energies

Pion-nucleus interactions at energies above the pion production threshold have mainly been regarded as a source of information about the interaction of pions with quasifree nucleons. The situation has now changed, and interest has arisen in processes that cannot take place on the free nucleon, for example, cumulative processes. Among these, one can include pion double charge exchange on nuclei. At high pion energies, one considers the isobar mechanism of double charge exchange,[112]

$$\pi^{\mp}2p\ (2n) \rightarrow \pi^{\pm}\Delta^- p\ (\Delta^{++}n) \rightarrow \pi^{\pm}2n\ (2p), \qquad (75)$$

which leads mainly to backward emission of the $\pi^{\pm}$ mesons from double charge exchange. Double charge exchange at energies $\gtrsim 2$ GeV at small angles is assumed to be due to the two-step process[113]

$$\pi^{\pm}2p\ (2n) \rightarrow X^0 pn \rightarrow \pi^{\pm}2n\ (2p), \qquad (76)$$

where $X^0 = \langle \pi, \rho, \eta, \omega, \text{f}, \ldots \rangle$ is any neutral meson whose production is allowed by the conservation laws at the given beam energy. The performed experiments on double charge exchange of π^+ mesons with momentum 2.9 GeV/c in a propane-xenon chamber[114] demonstrated the validity of the mechanism (76), the dominant intermediate states X^0 being the ρ, ω, and f mesons. Thus, in double charge exchange at high energies what actually occurs is the inclusive production of neutral resonances (ρ, f), each of which decays into a pair of pions (π^+ and π^-), and in the experiment one pion with a charge different from that of the incident pion is detected.[115] It is interesting to note that the study of the screening of the proton in the deuteron in the reaction of π^- double charge exchange made in Ref. 116 showed that the reaction also takes place predominantly through the formation and subsequent decay of the ρ^0 meson. The entire correction for the screening is made up of multiple scattering of the two quarks that comprise the incident π^- meson and the two quarks that comprise the ρ^0 meson.

Comparatively recently, a study was made[117] of strangeness-changing double charge exchange of π^- mesons on nuclei, i.e., the transitions $\pi^- \rightarrow K^0\pi^+$, which can take place through an intermediate resonance state K^{*+}. Such reactions have not hitherto been investigated either theoretically or experimentally. One of their advantages is that the neutral particle, carrying information about the nuclear target, may be a Λ hyperon, which is incomparably easier to study than the neutron in "ordinary" double charge exchange. In addition, by detecting strange particles, one can determine the cross sections of the individual channels by using the strangeness conservation law. Study of the processes

$$\left.\begin{aligned} \pi^- + A &\rightarrow \pi^+ + \Lambda\ (\Sigma^0) + K^0 + mp + n\pi^0 + A'; \\ \pi^- + A &\rightarrow \pi^+ + K + \bar{K}^0 + mp + n\pi^0 + A', \end{aligned}\right\} \qquad (77)$$

where $m, n = 0,1,2,\ldots$, showed that one of the possible mechanisms of strangeness-changing pion double charge exchange on nuclei is the "strange-isobar" mechanism. To it there corresponds the triangle diagram of the ordinary isobar mechanism,[112] the Δ isobar being replaced by the strange isobar and the π^+ by the $(K\pi)^+$ system.

So far, relatively few studies have been made of pion double charge exchange on nuclei at high energies. The further experimental and theoretical study of these processes will reveal the kinematic regions for the mechanisms of ordinary or strangeness-changing double charge exchange and will establish the effect of the nuclear structure on the process.

This review covers all investigations made during the last ten years on pion double charge exchange on nuclei. Comparing the situation today with what we had in 1971,[3]

we can conclude that important steps have been made in the detailed investigation of this exotic phenomenon. Can we foresee today the direction in which the further efforts will be made and what we shall have ten years from now? The decisive progress of quark physics suggests that quark models will play an important part in describing pion double charge exchange on nuclei.

[1]We shall denote by EP1 the group of K. K. Seth,[73] which was the first to begin the investigation of double charge exchange with the EPICS spectrometer, and by EP2 the second group[76] that studied the process with the same spectrometer.
[2]The data are taken from Ref. 6.
[3]The calculations made recently in Ref. 103 also indicate a very small cross section of the $^{13}C(\pi^+, \pi^0)^{13}N$ reaction under the same kinematic conditions.

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