Vacuum of the electroweak interactions in strong external fields

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Studies of the vacuum state of the electroweak interactions in the presence of strong magnetic and electric fields are discussed. The following topics are considered: introduction of an external electromagnetic field in renormalizable gauges, calculation of the effective potential in the single-loop approximation and its investigation, the connection with Yang-Mills theory in the presence of a covariantly constant field, and the problem of tachyon instability of the vacuum in a magnetic field. The renormalization-group method is used to study the effective potential near the instability threshold, and also the tachyon problem with allowance for radiative effects. The phase transition in a magnetic field associated with tachyons is described, and some features of the new vacuum state are noted. The case of an external electric field is briefly discussed.

INTRODUCTION

The modern theories of the weak and electromagnetic interactions are based on the spontaneous breaking of non-Abelian gauge symmetry groups. In these theories, the symmetry and properties of the vacuum, which is the condensate of a neutral scalar field, are not given once and for all and may change in phase transitions brought about by external factors. In this review, we discuss the results obtained in recent years from study of the electroweak vacuum in the presence of a strong electromagnetic field. The behavior of Abelian gauge symmetry in external fields has been discussed in particular in the review of Ref. 1.

Interest in such investigations was stimulated in the first place by cosmological problems and developed after Kirzhnits[1] had shown that an increase in the temperature leads to restoration of the symmetry. At that time, it was assumed that the physics of the electroweak vacuum is similar to the physics of the superconductivity phenomenon, and it was therefore natural to expect that a strong magnetic field, and also a combination of a field and temperature, would lead to analogous results in agreement with the superconductor case. Such macroscopic conditions are important for understanding the processes that took place in the early universe. Investigations in a field at zero temperature are a necessary stage in their study. Such investigations, in particular, make it possible to determine the different possible phases of the system and the properties of the phases and the response of the vacuum to external influences; they also make it possible to gauge the extent to which the analogy with superconductivity is in fact justified.

There are also other questions posed by the development of the theory that can be considered in the framework of the problem in a field. A characteristic feature of investigations at the present stage is the fact that they are made in the framework of individual specific models that combine different fundamental interactions and include, as a rule, experimentally undiscovered particles. It is important to clarify the general vacuum properties inherent in the majority of the models, and also to determine the consequences of the differences between the particle contents and of the dependence on the coupling constants and masses. This will make it possible to carry out a certain classification of the models. Results of this kind, obtained for a wide range of external fields and the relevant parameters, have a great importance extending beyond the specific study of the electroweak interactions.

The first investigation of the effects of an external field in non-Abelian gauge theories was made by Salam and Strathdee,[2] who studied the possible restoration of symmetry by a magnetic field H. The restoration of the symmetry is expressed in the disappearance of the scalar-field condensate by analogy with the disappearance of the Cooper-pair condensate and the destruction of superconductivity in a magnetic field. Salam and Strathdee[2] concluded that this effect is possible, so that in a strong field H the weak interaction acquires a long range like the electromagnetic interaction, and the symmetry of the system is identical to the symmetry of the original Lagrangian. However, the approximation used in Ref. 2 did not take into account adequately the vacuum dynamics in a field in non-Abelian theories, this dynamics being very different from the case of the Ginzburg-Landau theory[3] or the Higgs model. Further investigations were therefore needed. In addition, the following question arose: If the scalar condensate disappears, to what extent is the electroweak vacuum similar to the vacuum of massless non-Abelian theories? This question is nontrivial because the perturbation-theory vacuum in a magnetic field is unstable[4,5] with or without spontaneous symmetry breaking. Thus, the solution to the problem of restoration of the symmetry was found to be intimately related to the problem of finding a stable vacuum for non-Abelian gauge theories in a field.

The superposition of external fields is one of the effective methods of going beyond the framework of perturbation theory and uses solutions that are exact with respect to the field. The external field is a classical solution of the corresponding equation of motion on the background of which quantum effects are studied. It need not necessarily correspond to a real physical field but may serve as an auxiliary object in the investigation. The method has been widely used in quantum electrodynamics (QED), in which important results have been obtained.[6-9] We mention some of them.

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There is the production of electron-positron pairs by an electric field, study of the asymptotic properties of QED, the establishment of the connection between strong-field electrodynamics and short-distance electrodynamics,[9,10] and more. In non-Abelian theories too, such investigations have been and are still being made for different fields. The literature on this subject is very extensive. Therefore, we mention only the results relating directly to the questions under discussion.

In the first place, there is the study of asymptotic freedom as a problem in a field,[11-13] the occurrence of spontaneous magnetization,[14] and the discovery of instability of the perturbation-theory vacuum in a magnetic field,[4,5] these being intimately related to one another. Below, we shall mention other results.

The Higgs model in a magnetic (or, rather, quasimagnetic[1]) field H was studied comprehensively in Refs. 1 and 15–17. The original Lagrangian is $U(1)$-symmetric and has the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + P_\mu^* \varphi^* P^\mu \varphi - m_\varphi^2 \varphi^* \varphi - \frac{\lambda}{4} (\varphi^* \varphi)^2, \quad (1)$$

where $F_{\mu\nu} = \delta\mu A_\nu - \delta_\nu A_\mu$, $P_\mu = i\delta_\mu + eA_\mu$, and φ is a complex scalar field. If $m_\varphi^2 < 0, \lambda > 0$, then spontaneous symmetry breaking occurs, and φ acquires a vacuum expectation value $\delta(0) = \langle \varphi \rangle_0 = \sqrt{2|m_\varphi^2|/\lambda}$ and the gauge field A_μ acquires a mass $M_A = e\delta(0)$. In the model, there is a complete analogy with the Ginzburg-Landau theory of superconductivity.[1,16] In particular, for large H the symmetry is restored and the mass M_A vanishes. In the broken phase, the gauge field is completely screened and its measured value is zero. The effect of the external field reduces to an increase in the vacuum energy and is manifested at the tree level. Therefore, the critical field strength $H_{\rm cr}$ is $H_{\rm cr}^2/2 \sim O(M_A^4)$, this estimate showing that the symmetry is restored when the energy density of the magnetic field is equal to the energy density of the scalar condensate, which is $\sim M_A^4$.

In theories of the electroweak interaction, the gauge symmetry corresponding to an external electromagnetic field remains unbroken. Therefore, the measured field is nonzero in the broken phase as well. From general arguments it is to be expected that the restoration of symmetry must here occur at the single-loop level. For the scalar condensate is electrically neutral, and a magnetic field acts on it in the tree approximation. The effect of the field H reduces to a change in the spectrum of the vacuum oscillators of the charged fields and, as a result of this, a change in the vacuum energy density, and this may lead to a phase transition. An estimate of the critical field gives $e^2 H_{\rm cr}^2/2 \sim O(M^4)$, where M is the mass of the charged vector bosons.

Such arguments were the basis of Ref. 2, in which a single-loop calculation of the effective potential $V^{(1)}(\Phi_c, H)$ of the scalar field Φ_c in the presence of a field H made it possible to study the behavior of the symmetry. Using the effective potential in the approximation of small H, Salam and Strathdee noted that, because non-Abelian theories with spontaneous symmetry breaking have asymptotic freedom, the vacuum expectation value of the scalar field, determined from the position $\delta(H)$ of the minimum of the effective potential, decreases with increasing H. On the basis of this tendency, it was concluded that the symmetry may be restored. Thus, despite the difference in the mechanisms by which a field influences the vacuum in the Abelian and non-Abelian cases, it was assumed that the asymptotic freedom of the latter leads to the expected final result—restoration of the symmetry.

The approximate expression for the effective potential was later used by Midorikawa[18] to determine the critical fields in a number of models, and it was shown that symmetry is restored by a phase transition of the first kind.

Estimates showed that the critical fields must be fairly strong. Therefore, the weak-field approximation may be invalid. In Ref. 4, the present author studied the behavior of the exact expression for the single-loop effective potential $V^{(1)}(\Phi_c, H)$ and showed that the vacuum expectation value $\delta(H)$ does not vanish. In addition, a new phenomenon was noted in Ref. 4—instability of the electroweak vacuum in a magnetic field, manifested by an imaginary part, Im $V^{(1)}(\Phi_c, H) \neq 0$, in the effective potential. Nielsen and Olesen[5] discovered an analogous instability for Yang-Mills theory in the presence of a covariantly constant magnetic field $\overline{F}^a = Hn^a$, where n^a is a vector fixing a direction in the isotopic space. A difference between the massless and spontaneously broken non-Abelian gauge theories is that in the latter there is a threshold for the appearance of Im $V^{(1)}$, i.e., the imaginary part, this threshold being determined by the mass M of the charged vector particles, $H_0 = M^2/e$, while for massless theories $H_0 = 0$. The physical reason for the instability is that the spectrum of the charged vector particles in a magnetic field includes a tachyon, a consequence of the structure of the interaction in non-Abelian theories.

The discovery of vacuum instability made it necessary to re-examine the behavior of the symmetry in the field H. It became clear that the use of only the single-loop effective potential for this purpose is inadequate, more comprehensive investigations being needed. In Ref. 19, the renormalization-group method was used to study $V^{(1)}(\Phi_c, H)$ near the instability threshold H_0, and a singularity in the behavior of the effective electric charge $\overline{e}^2(H)$ was discovered. Solution of the classical equations of motion in Refs. 20 and 21 showed that the behavior and properties of the electroweak vacuum in a field are determined by, not one Φ_c, but two order parameters, Φ_c and $\langle W_i^+ W_i^- \rangle_0$, and the interaction between them ($\langle W_i^+ W_i^- \rangle_0$ is the condensate of the components of the W bosons). The qualitative arguments and estimates of the critical fields given above did not take into account this circumstance. It was also found that the properties of the vacuum are very different for theories with "light" and "heavy" Higgs mesons.

It can be seen that the problem of spontaneous symmetry breaking is rather complicated and involves several stages. In addition, the results obtained by different authors in their studies of the effective potential $V^{(1)}(\Phi_c, H)$ (see Refs. 4, 18, 22, and 23) frequently differ or even completely contradict each other. For this there are various reasons, given below. In the present review, the exposition is therefore arranged in such a way as to combine the currently available results for two specific models—the Weinberg-Salam (WS) theory and the Georgi-Glashow (GG) model[25]—and to correct inadequacies of the quoted studies when they exist.

The paper is arranged as follows. In Sec. 1, after a brief description of the theories, we consider the introduction of an external electromagnetic field in renormalizable (R) and unitary (U) gauges. We then describe the general formalism for calculating the effective potential by means of Green's functions. In Sec. 2, we study the effective potential in the complete range of magnetic fields, and also the dependence of the effective potential on the ratio of the squares of the masses of the Higgs and W bosons: $K = K(0) = m^2/M^2$. We show that the behavior of Re $V^{(1)}(\Phi_c, H)$ is different for $K < 9$–10 and for $K \gtrsim 9$–10. There is therefore a natural limit $K \approx 9$–10 in the definition of the concepts of light and heavy Higgs mesons. We consider the connection with massless Yang-Mills theory and also the phenomenon of spontaneous magnetization of the vacuum in their dependence on the given scalar field Φ_c. It can be shown that at sufficiently large Φ_c spontaneous magnetization does not occur.[23] We also discuss the tachyon instability in a magnetic field. Section 3 is devoted to investigations at the instability threshold $H_0 = M^2/e$. It is shown by the renormalization-group method that the effective charge $\bar{e}^2(H)$ exhibits "zero-charge" behavior as $H \to H_0$. The influence of radiative effects in the tachyon problem in a magnetic field is studied. A renormalization-invariant mass of the charged vector boson in the field H is introduced, and by means of this mass it is shown that in the Georgi-Glashow model for $K(H) < 1.66$ the mass square $M^2_{\text{eff}}(H)$ can become negative in fields $H = H_{\text{ch}} < H_0$, $K(H) = m^2(H)/M^2(H)$ determining the ratio of the mass squares in the presence of the field. At large $K(H)$, $M^2_{\text{eff}}(H)$ is positive. The Weinberg-Salam theory is also considered. In Sec. 4, phase transitions associated with the tachyon instability are discussed. The formation of a condensate of the W-boson field for $K < 1$ and $K > 10$ is described, together with some properties of the new vacuum. In the conclusions, we summarize the results and briefly consider the case of an external electric field and the state of the problem of the role of fermions in the presence of a field.

1. BACKGROUND-FIELD FORMALISM AND THEORIES WITH SPONTANEOUS SYMMETRY BREAKING

Models of electroweak interactions

We briefly describe the Weinberg-Salam[24] and Georgi-Glashow[25] models. The first of these is the realistic theory of electroweak interactions, in good agreement with the experimental data, while the other, which is simpler, is of interest for the following reasons. First, it is close to massless Yang-Mills theory and is actually its massive regularization.[26,27] Second, in the single-loop approximation investigated in what follows for the effective potential and the mass operator of the W boson all the results obtained in the Georgi-Glashow model can be extended to the Weinberg-Salam theory by a renaming of the parameters and some simple algebraic manipulations. Therefore, many studies have been made in the framework of this model. For this reason, and also to shorten the exposition, most of the calculations will be made for the Georgi-Glashow model, and the corresponding results for the Weinberg-Salam and Yang-Mills theories will be given without calculations.

The new and most important properties of the electroweak interactions discussed in the review are due to interaction with the external field of charged vector particles. Therefore, we consider only the bosonic part of the Lagrangians of the corresponding theories. The fermionic part is described in many sources (see, for example, Refs. 24, 25, 28, and 58).

The Georgi-Glashow model is based on spontaneous breaking of the gauge symmetry: $SU(2) \to U(1)$. It is assumed that the theory is quantized by the method of functional integration over classes of fields.[29] We represent the part of the Lagrangian in which we are interested in the form

$$\mathscr{L} = \mathscr{L}_b + \mathscr{L}_g + \mathscr{L}_{\text{F. P}}, \tag{1}$$

where $\mathscr{L}_b$ describes the Yang-Mills field, a triplet of scalar fields, and their interaction:

$$\mathscr{L}_b = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} D_\mu \chi^a D^\mu \chi^a - V(\chi), \tag{2}$$

$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g\varepsilon^{abc} V^b_\mu V^c_\nu$ is the field strength of the gauge field, $a = 1, 2, 3$ is the isotopic index,

$$V(\chi) = \frac{1}{2} m^2_\chi \chi^a \chi^a + \frac{\lambda}{8} (\chi^a \chi^a)^2, \tag{3}$$

$\mathscr{L}_g$ is the gauge-fixing term, and $\mathscr{L}_{\text{F.P}}$ is the Lagrangian of the Faddeev-Popov ghosts.

If $m^2_\chi < 0$, $\lambda > 0$, then the potential (3) has a minimum at $(\chi^a \chi^a)_c = 2|m^2_\chi|/\lambda$. A condensate of the scalar field is formed in the vacuum: $\chi^a_c = \langle \chi^a \rangle_0 \neq 0$. To specify the system above this nonsymmetric vacuum, it is necessary to fix a direction in the isotopic space, this leading to spontaneous symmetry breaking, and to go over to new fields φ^a that have a vanishing vacuum expectation value: $\langle \varphi^a \rangle_0 = 0$. We set

$$\langle \chi^a \rangle_0 = \delta(0) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad \delta(0) = \sqrt{\frac{2|m^2_\chi|}{\lambda}};$$

$$\chi^a = \langle \chi^a \rangle_0 + \varphi^a$$

and introduce in place of V^a_μ, φ^a the new fields

$$\left. \begin{aligned} W^\pm_\mu &= \frac{1}{\sqrt{2}} (V^1_\mu \pm i V^2_\mu); \quad A_\mu = V^3_\mu; \\ \varphi^\pm &= \frac{1}{\sqrt{2}} (\varphi^1 \pm i\varphi^2); \quad \Phi = \varphi^3. \end{aligned} \right\} \tag{4}$$

We then obtain

$$\begin{aligned} \mathscr{L}_b = &- P^*_\mu W^+_\nu P^\mu W^{-\nu} + P^*_\mu W^+_\nu P^\nu W^{-\mu} - ieF_{\mu\nu} W^{+\mu} W^{-\nu} \\ &+ (M + e\Phi)^2 W^+_\mu W^{-\mu} + P^*_\mu \varphi^+ P^\mu \varphi^- \\ &- (M + e\Phi)(W^+_\mu P^\mu \varphi^- - W^-_\mu P^{*\mu} \varphi^+) \\ &- \frac{1}{4} F^2_{\mu\nu} - \frac{e^2}{2} [(W^+_\mu W^{-\mu})^2 - W^-_\mu W^{-\mu} W^+_\nu W^{+\nu}] \\ &+ \frac{1}{2} |\partial_\mu \Phi + ie(W^+_\mu \varphi^- - W^-_\mu \varphi^+)|^2 - \frac{m^2}{2} \Phi^2 \\ &- q\left(\varphi^+\varphi^- + \frac{1}{2}\Phi^2 + \delta(0)\Phi\right) - \frac{\lambda}{8}[\Phi^4 + 4(\varphi^+\varphi^-)^2] \\ &- \frac{\lambda}{2} \varphi^+\varphi^- (\Phi^2 + 2\Phi\delta(0)) - \frac{1}{2}\frac{\lambda}{e} M\Phi^3, \end{aligned} \tag{5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and Φ is the Higgs scalar. Its mass m and the W-boson mass $M = e\delta(0)$ are related by

$$K = m^2/M^2 = \lambda/e^2. \tag{6}$$

The constant g is identified with the electric charge: $g \equiv e$; $q \equiv -|m_\chi^2| + \frac{1}{2}\lambda\delta^2(0)$ is the condition which ensures the vanishing $\langle \varphi^a \rangle_0 = 0$ in each order of perturbation theory. The field V_μ^3 remains massless and is identified with the electromagnetic field.

The Lagrangian (5) is invariant with respect to the gauge transformations

$$\left.\begin{aligned} &W_\mu^\pm \to W_\mu^{\pm\prime} = W_\mu^\pm \pm ie\,(A_\mu\alpha^\pm - W_\mu^\pm\alpha^0) + \partial_\mu\alpha^\pm;\\ &A_\mu \to A'_\mu = A_\mu + ie\,(W_\mu^+\alpha^- - W_\mu^-\alpha^+) + \partial_\mu\alpha^0;\\ &\varphi^\pm \to \varphi^{\pm\prime} = \varphi^\pm \pm ie\,(\Phi\alpha^\pm - \varphi^\pm\alpha^0) \mp iM\alpha^\pm;\\ &\Phi \to \Phi' = \Phi + ie\,(\varphi^+\alpha^- - \varphi^-\alpha^+), \end{aligned}\right\} \tag{7}$$

where $\alpha^\pm$ and α^0 are parameters of the transformations.

We also consider the variant of the Georgi-Glashow model in which the field χ^a is not initially massless and the symmetry breaking and acquisition of masses by the particles are due to radiative corrections. The corresponding mechanism was proposed by Coleman and Weinberg[30] and is called dynamical symmetry breaking. The expression for the potential in the tree approximation is obtained for $m_\chi^2 = 0$ in the expression (3). The Lagrangian (5) is degenerate. We consider the term $\mathscr{L}_g$ that fixes the gauge for the quantization in our context of applied external fields.

External electromagnetic field in non-Abelian gauge theories with spontaneous symmetry breaking

In QED, an external electromagnetic field $\overline{F}_{\mu\nu}$ can be introduced by decomposing the electromagnetic potential into a quantized part A_μ^R and a classical part $\overline{A}_\mu$: $A_\mu = A_\mu^R + \overline{A}_\mu$. The Lagrangian will be invariant with respect to gauge transformations $A_\mu \to A'_\mu = A_\mu + \partial_\mu\alpha^0$ realized by virtue of the external field:

$$A_\mu^R \to A_\mu^{R\prime} = A_\mu^R;\quad \overline{A}_\mu \to \overline{A}'_\mu = \overline{A}_\mu + \partial_\mu\alpha^0. \tag{8}$$

In non-Abelian theories, the situation is more complicated, since the electromagnetic field is separated from the multiplet of fields V_μ^a that transform together under the group SU(2) and in the quantization it is necessary to fix the gauge for $W_\mu^\pm$.

We consider the most general linear R gauge R_ξ,[31]

$$\partial_\mu W^{\pm\mu} \mp i\frac{M}{\xi}\varphi^\pm = C^\pm(x), \tag{9}$$

where ξ is a parameter. The conditions (9) completely lift the degeneracy (7) for the broken subgroups described by the parameters α^+ and α^-, and this makes it possible to quantize the theory. Formally, it all reduces to the addition to $\mathscr{L}_b$ of two terms:

$$\mathscr{L}_g = -\xi\left|\partial_\mu W^{+\mu} - i\frac{M}{\xi}\varphi^+\right|^2; \tag{10}$$

$$\begin{aligned}\mathscr{L}_{\mathrm{F.P}} = {}&\overline{\eta}^0\Box\eta^0 + \overline{\eta}^+\Box\eta^+ + \frac{M^2}{\xi}\overline{\eta}^+\eta^+ + \frac{eM}{\xi}\Phi\overline{\eta}^+\eta^+\\ &- \frac{eM}{\xi}\overline{\eta}^+\varphi^+\eta^0 + ie\overline{\eta}^+(\partial_\mu A^\mu + A_\mu\partial^\mu)\eta^+\\ &- ie\overline{\eta}^+(\partial_\mu W^{+\mu} + W_\mu^+\partial^\mu)\eta^0\\ &+ ie\overline{\eta}^0(\partial_\mu W^{+\mu} + W_\mu^+\partial^\mu)\eta^- + \text{h.c.},\end{aligned} \tag{11}$$

where η^+ and η^0 are scalar Fermi fields of the Faddeev-Popov ghosts. The first term in (11) arises if the gauge of the electromagnetic field is fixed, $\partial_\mu A^\mu = C^0(x)$.

If we now make the decomposition $A_\mu = A_\mu^R + \overline{A}_\mu$, then $\mathscr{L}_b + \mathscr{L}_g$ and also $\mathscr{L}_{\mathrm{F.P}}$ will not be invariant with respect to the transformations (8). This makes it difficult to interpret $\overline{A}_\mu$ as the potential of an external electromagnetic field. When studying processes due to interaction with $\overline{A}_\mu$, it is necessary to use a complicated system of equations and Ward identities for the complete set of fields corresponding to both the unbroken and the broken subgroups. Most problems are then effectively unsolvable. In this connection, and also because the effective potential depends on ξ (see below), the first studies in non-Abelian theories with spontaneous symmetry breaking involving an external field were made in the U gauge, in which there are only physical fields and only the $U(1)$ symmetry of the photon is explicitly preserved. Accordingly, the external field can be introduced in this case as in QED. However, the U gauge has the shortcoming that the Green's functions in it are nonrenormalizable, only the S matrix being renormalizable. This leads to difficulties in the calculations.

The quantization of theories possessing an arbitrary local symmetry by the introduction of classical fields satisfying certain equations of motion was implemented by DeWitt,[32] and his method has been widely used in massless non-Abelian theories. The main advantage of the method, the background-field method, is the possibility of maintaining invariance with respect to a gauge transformation of the external field under the imposed gauge conditions. Various aspects of gauge invariance and renormalization in this approach are discussed in Refs. 33–35.

We shall use the background-field method to quantize a theory with spontaneous symmetry breaking. Instead of (9), we impose a condition covariant with respect to $\overline{A}_\mu$,[25]

$$\partial_\mu W^{\pm\mu} \pm ie\overline{A}_\mu W^{\pm\mu} \mp i\frac{M}{\xi}\varphi^\pm = C^\pm(x), \tag{12a}$$

and this solves the problem of introducing the electromagnetic field in the R gauge. The addition $\mathscr{L}_g$ in (1) has the form

$$\mathscr{L}_g = -\xi\left|P_\mu^* W^{+\mu} - \frac{M}{\xi}\varphi^+\right|^2. \tag{13}$$

We shall discuss the transformation properties of the fields in the presence of $\overline{A}_\mu$. Under SU(2) transformations, $A_\mu = A_\mu^R + \overline{A}_\mu$ transforms in accordance with (7). Under transformations of the external field, we have (8). The SU(2) transformations can be realized in two ways: 1) by the homogeneous transformation $A_\mu^{R\prime} = A_\mu^R + ie(W_\mu^+\alpha^- - W_\mu^-\alpha^+)$ and $\overline{A}_\mu \to \overline{A}'_\mu$; 2) for fixed $\overline{A}_\mu$,

$$A_\mu^{R\prime} = A_\mu^R + ie\,(W_\mu^+\alpha^- - W_\mu^-\alpha^+) + \partial_\mu\alpha^0. \tag{14}$$

The law of transformation of the fields $W_\mu^\pm$ is obtained by replacing $\partial_\mu \alpha^+$ in (7) by the covariant derivative $D_\mu \alpha^\pm = \partial_\mu \alpha^\pm \pm ie\bar{A}_\mu \alpha^\pm$:

$$W_\mu^\pm \to W_\mu^{\pm\prime} = W_\mu^\pm \pm ie\,[(A_\mu^R + \bar{A}_\mu)\,\alpha^\pm - W_\mu^\mp \alpha^0] + \partial_\mu \alpha^\pm]. \tag{15}$$

For $\varphi^\pm$ and Φ the expressions (7) still stand. Using (12a), (14), and (15), we can readily find the ghost Lagrangian $\mathscr{L}_{\rm F.P}$. For its calculation, $\bar{A}_\mu$ is assumed fixed. This Lagrangian differs from (11) by the presence of additional terms describing the interaction with $\bar{A}_\mu$:

$$\Delta\mathscr{L}_{\rm F.\,P} = ie\bar{\eta}^+ (\partial_\mu \bar{A}^\mu + 2\bar{A}_\mu \partial^\mu)\,\eta^+ + e^2 \bar{A}_\mu W^{+\mu} \bar{\eta}^+ \eta^0 - e^2 \bar{\eta}^+ (\bar{A}_\mu A_\mu^R + \bar{A}_\mu \bar{A}^\mu)\,\eta^+ + \text{h.c.} \tag{16}$$

As a result, the Lagrangians $\mathscr{L}_g + \mathscr{L}_b$ and $\mathscr{L}_{\rm F.P}$ are invariant with respect to the gauge transformations (8). Therefore, for the renormalization constants we have the Ward identity $Z_1 = Z_2$, as in QED, and, in particular, to calculate the β function it is sufficient to calculate the polarization operator. In what follows, it is also necessary to take into account the presence of the external scalar field Φ_c besides the electromagnetic field. This is readily done by making the substitution $M \to e(\delta(0) + \Phi_c)$ in (12a).

We have considered the introduction of external fields in the R gauge for the example of the simplest Georgi-Glashow model. For an arbitrary gauge symmetry group, this question was studied by Shore,[22] who considered in detail constant external electromagnetic, $\bar{F} = \text{const}$, and scalar, $\Phi_c = \text{const}$, fields and formulated a number of simple theorems that ensure their specification. Shore took the Lagrangian before the symmetry breaking, in accordance with which he used the condition (12a), in which M/e is replaced by an arbitrary "running" parameter of the scalar field Φ_c:

$$\partial_\mu W^{\pm\mu} \pm ie\bar{A}_\mu W^{\pm\mu} \mp i\frac{e\Phi_c}{\xi}\varphi^\pm = C^\pm(x). \tag{12b}$$

The gauge (12b) is specialized for calculations of the effective potentials of the fields $\bar{F}$ and Φ_c, since it does not require a preliminary fixing of the vacuum of the theory. The choice of a particular gauge is dictated by convenience in solving the problem. Below, we shall use both of the gauges (12a) and (12b).

We shall establish the connection between the formulations of the theory in the R gauge (12a) and the U gauge. The subsidiary condition (12a) differs from (9) only by terms describing the interaction with $\bar{A}_\mu$. The propagators of the free particles are unchanged:

$$\left.\begin{aligned} D_{\mu\nu}^W &= -i\,\frac{g_{\mu\nu} - k_\mu k_\nu/M^2}{k^2 - M^2 + i\varepsilon} + i\,\frac{k_\mu k_\nu/(M^2/\xi)}{k^2 - M^2/\xi + i\varepsilon}\,;\\ D_{\eta^+,\,\varphi^+} &= i\,\frac{1}{k^2 - M^2/\xi + i\varepsilon}, \end{aligned}\right\} \tag{17}$$

while the vertices differ from the vertices in the U gauge by the terms

$$ie\xi\bar{A}_\mu\,(W_\mu^+ \partial_\nu W^{-\nu} - W^{-\mu}\partial_\nu W^{+\nu}) - e^2\xi W_\mu^+ W_\nu^- \bar{A}^\mu \bar{A}^\nu. \tag{18}$$

It follows from (17) that in the limit $\xi \to 0$ the mass of the fields $\eta^\pm$, $\varphi^\pm$, and also the mass of the unphysical quanta transmitted by the vector field $W_\mu^\pm$, tends to infinity. Therefore, their contribution to the matrix elements vanishes. So do the vertices (18). Thus, the gauge (12) contains the unitary gauge as the limiting case when $\xi = 0$. The background field $\bar{A}_\mu$ introduced in (12) and the external electromagnetic field in the U gauge are identical.

Weinberg-Salam model

The Weinberg-Salam theory is based on spontaneous breaking of the gauge symmetry: $SU(2) \otimes U(1) \to U(1)$.[24] The bosonic part of the Lagrangian is

$$\mathscr{L}_b^{\rm WS} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + (D_\mu\chi)^+ (D^\mu\chi) - V(\chi), \tag{19}$$

where $f_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$, $\chi = \binom{\chi^+}{\chi^0}$ is an isodoublet, $D_\mu\chi = \partial_\mu\chi - 1/2ig' b_\mu \chi - 1/2ig\tau^a V_\mu^a \chi$, $a = 1, 2, 3$ is the isotopic index,

$$V(\chi) = m_\chi^2 \chi^+\chi + \lambda(\chi^+\chi)^2; \tag{20}$$

and g and g' are the SU(2) and $U(1)$ gauge constants. For $m_\chi^2 < 0$, $\lambda > 0$, spontaneous symmetry breaking occurs and the field χ acquires in the tree approximation the vacuum expectation value

$$\langle\chi\rangle_0 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\,\delta(0) \end{pmatrix}$$

(for a suitable orientation of the internal symmetry space). After the shift $\chi = \langle\chi\rangle_0 + \Phi$, the gauge fields V_μ^a, b_μ acquire a mass, except for the combination

$$A_\mu = \frac{gb_\mu + g'V_\mu^3}{\sqrt{g^2 + g'^2}}, \tag{21}$$

which is identified with the photon. We also introduce the fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(V_\mu^1 \pm iV_\mu^2);\quad Z_\mu = \frac{gV_\mu^3 - g'b_\mu}{\sqrt{g^2 + g'^2}},$$

which describe the W and Z bosons. Their masses are expressed in terms of $\delta(0)$ by $M = g\delta(0)/2$, $M_Z = (\delta(0)/2)\sqrt{g^2 + g'^2}$. The electric charge e is related to g and g' by $e = g\sin\theta$, $\tan\theta = g'/g$, where θ is the Weinberg angle.

For the quantization, it is necessary to add to (12) conditions that fix the gauge of the Z boson:

$$\partial_\mu Z^\mu - \frac{iM_Z}{\xi'}\varphi_Z = C_Z(x) \tag{22a}$$

or

$$\partial_\mu Z^\mu - \frac{i}{\xi'}(g^2 + g'^2)^{1/2}\,\Phi_c\varphi_Z = C_Z(x). \tag{22b}$$

The Lagrangian of the Weinberg-Salam theory is rather complicated. We give it in the U gauge, setting $\xi = \xi' = 0$:

$$\mathcal{L}_U^{\mathrm{WS}} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} Z_{\mu\nu}^2 + \frac{\left(M+\frac{1}{2} g\Phi\right)^2}{2\cos^2\theta} Z_\mu^2 - \widetilde{P}_\mu^* W_\nu^+ \widetilde{P}^\mu W^{-\nu}$$
$$+ \widetilde{P}_\mu^* W_\nu^+ \widetilde{P}^\nu W^{-\mu} + \left(M+\frac{1}{2} g\Phi\right)^2 W_\mu^+ W^{-\mu} - \mathrm{i}g N_{\mu\nu} W^{+\mu} W^{-\nu}$$
$$-\frac{1}{2} g^2 [(W_\mu^+ W^{-\mu})^2 - (W_\mu^+ W_\nu^-)^2]$$
$$+\frac{1}{2} (\partial_\mu \Phi)^2 - \frac{m^2}{2} \Phi^2 - \frac{\lambda}{4} \Phi^4$$
$$-2 \frac{\lambda}{g} M\Phi^3 - V^{(0)} (\delta(0)). \tag{23}$$

In (23), $\widetilde{P}^\mu = i\partial_\mu + gN_\mu = i\partial_\mu + g(\sin\theta A_\mu + \cos\theta Z_\mu)$; $F_{\mu\nu}$ and $Z_{\mu\nu}$ are the four-dimensional curls of the fields A_μ and Z_μ; and $V^0(\delta(0)) = -4(M^4/g^2)(\lambda/g^2)$ is the energy density of the condensate of the scalar field χ.

All aspects of the introduction of external fields in the R and U gauges discussed above remain the same.

Formalism for calculating the effective potential

To study the vacuum, the effective potential is used. The problem of calculating it in the case of an arbitrarily varying electromagnetic field is extremely complicated and has not hitherto been solved. Only for a constant and homogeneous electromagnetic field, $\overline{F}_{\mu\nu} = \mathrm{const}$, is the W-boson propagator known[11] and it is possible to obtain the effective potential $V(\overline{F}, \Phi_c)$ in a closed form. From the physical point of view, the restriction to constancy of the field is not so important in the study of the behavior of the symmetry. It is clear from general considerations that the fields of interest must be strong in order to produce a sufficiently strong vacuum polarization, which is what will determine the properties of the effective potential. In the case of strong fields, it is well known[36] that the results for the vacuum polarization obtained in a constant field remain valid for fields that vary rapidly from point to point. The condition of applicability of the approximation of a constant field for the study of inhomogeneous fields, $(\nabla F)^2/F \ll eF/M^2$,[36] shows that it is entirely suitable for the problem which we are considering and can correspond to a wide range of real physical conditions producing the field.

The physical meaning of the effective potential is that it gives the energy density of the classical field for a fixed value of it.[44] Let us clarify this assertion more formally. We write down the action for the Lagrangian (1) in the presence of external sources

$$S[J_\mu, J] = \int \{\mathcal{L}_{\mathrm{eff}}(A, \Phi) + J_\mu A^\mu + J\Phi\} d^4x$$

and the generating functional[2,22,37]

$$\exp W[J_\mu, J] = \int DW_\mu^\pm DA_\mu D\Phi D\varphi^\pm D\eta^0 D\eta^\pm \exp[S(J_\mu, J, V_\mu^a, \chi^a)]. \tag{24}$$

The integration on the right-hand side of (24) is over all fields, including ghosts. Making a Legendre transformation, we introduce the effective action

$$\Gamma(\overline{F}, \Phi_c) = W[J_\mu, J] - \int d^4x (J_\mu A^\mu + J\Phi). \tag{25}$$

The classical fields

$$\overline{A}_\mu = \delta W/\delta J_\mu; \quad \Phi_c = \delta W/\delta J \tag{26}$$

are defined as the vacuum expectation values of the quantized fields:

$$\overline{A}_\mu = \langle A_\mu \rangle_0, \quad \Phi_c = \langle \Phi \rangle_0.$$

The effective action can be expanded in powers of the external field momentum. In the coordinate representation,

$$\Gamma(\overline{F}, \Phi_c) = \int d^4x \Big[-V(\Phi_c, \overline{F}) + \frac{1}{2} Z(\Phi_c(x), \overline{F}) (\partial_\mu \Phi_c(x))^2 + \dots \Big]. \tag{27}$$

The first term in the expansion (27) is the effective potential. It is an ordinary function of Φ_c and $\overline{F}$.

The occurrence of spontaneous symmetry breaking is expressed in the existence of a nonvanishing vacuum expectation value of the scalar field, this being determined from the equation

$$\delta\Gamma(\Phi_c, \overline{A})/\delta\Phi_c = 0. \tag{28}$$

An analogous equation holds for the electromagnetic field in the presence of a classical source,

$$\frac{\delta\Gamma(\overline{A}, \Phi_c)}{\delta \overline{A}^\mu} = -J_\mu, \tag{29}$$

where J_μ must be expressed in terms of $\overline{A}_\mu$ and Φ_c by means of (26). Choosing the source J_μ, one can create any external field $\overline{F}_{\mu\nu}$. If we are interested in another phenomenon, the spontaneous production of an electromagnetic field in the presence of the scalar field, we must, in contrast, set $J_\mu = 0$ in Eq. (29), and on the right-hand side of (28) specify a source J that sustains a Φ_c of any value.

We note also that a constant electromagnetic field satisfies Maxwell's equation without a source. Therefore, such a field can be formally specified directly in Eq. (28), in which $\overline{A}_\mu$ is a free parameter. We choose the potential of the external field in the form[13,23]

$$\overline{A}_\mu = -\frac{1}{2} \overline{F}_{\mu\nu} x^\nu, \tag{30}$$

which takes into account the fact that the corresponding subgroup is not broken. For constant fields $\overline{F}$ and Φ_c, Eq. (28) is transformed from a functional equation into an equation for the extremum of the function $V(\Phi_c, \overline{F})$:

$$\partial V(\Phi_c, \overline{F})/\partial\Phi_c = 0. \tag{31}$$

The expression for $V(\Phi_c, \overline{F})$ is extremely complicated; it can be calculated only in a loop expansion with all possible numbers of external electromagnetic and scalar fields:

$$V(\Phi_c, \overline{F}) = \sum_{n=0}^{\infty} V^{(n)}(\Phi_c, \overline{F}).$$

In this review, we discuss the results of the investigation of the effective potential in the single-loop approximation, for which it is necessary to retain in the expansion the terms with $n = 0$ and 1:

$$V(\Phi_c, \overline{F}) = V^{(0)}(\Phi_c) + V^{(0)}(\overline{F}) + V^{(1)}(\Phi_c, \overline{F}). \quad (32)$$

Here, $V^{(0)}(\Phi_c)$ is the potential of the tree approximation determined in accordance with (3), $V^{(0)}(\overline{F})$ is the energy of the classical Maxwell field, and $V^{(1)}(\Phi_c, \overline{F})$ is the single-loop correction. The potential $V(\Phi_c, \overline{F})$ can be calculated by means of the Green's functions $G_{AB}(\Phi_c, \overline{F})$ of the quantized fields. They are also needed to calculate the change in the W-boson mass in a magnetic field due to the radiative corrections. For brevity, the letters AB in G_{AB} denote suitable combinations of the fields $W^{\pm}$, A^R, Φ, $\varphi^{\pm}$, $\eta^{\pm}$, η^0. We have the formula[34]

$$\int d^4x V^{(1)}(\Phi_c, \overline{F}) = -\frac{i}{2} \ln \det G_{AB}. \quad (33)$$

Using the identity

$$\ln \det G_{AB} = \mathrm{Sp} \ln G_{AB},$$

where the trace denotes summation over all pairs AB and integration over the coordinates x_μ, we obtain

$$V^{(1)}(\Phi_c, \overline{F}) = -\frac{i}{2} \sum_{(A,B)} \langle x | \ln G_{AB}(\Phi_c, \overline{F}) | x \rangle. \quad (34)$$

The Green's functions $G_{AB}(x, y)$ are determined from the operator G_{AB} as

$$G_{AB}(x, y) = (x \mid G_{AB} \mid y) = G_{AB}\delta(x-y)$$

and are calculated in accordance with the formula

$$G_{AB}^{-1}(x, y) = -\frac{\delta^2 S}{\delta A(x)\, \delta B(y)}, \quad (35)$$

which can be inverted by using the boundary conditions of causality. Thus, $V^{(1)}(\Phi_c, \overline{F})$ is found from the diagonal elements of the matrix $\ln G_{AB}$.

Conditions of normalization of the effective potential

We shall calculate the effective potential in the gauge (12b).[23] This makes it possible to consider simultaneously the cases of both spontaneous and dynamical symmetry breaking. In the expression for $V^{(1)}$ we separate the contribution of the Faddeev-Popov ghosts, obtaining

$$\int d^4x V^{(1)}(\Phi_c, \overline{F}) = -\frac{i}{2} \mathrm{Sp} \ln \left(\frac{\delta^2 S}{\delta A\, \delta B}\right) + i\, \mathrm{Sp} \ln D_{\mathrm{F.P}}. \quad (36)$$

The determinant $D_{\mathrm{F.P}}$ is equal to

$$D_{\mathrm{F.P}} = \det \begin{bmatrix} -P^*_\mu(\overline{A}) P^{*\mu}(\overline{A}) + \frac{(e\Phi_c)^2}{\xi} & 0 & 0 \\ 0 & -P_\mu(\overline{A}) P^\mu \overline{A} + \frac{(e\Phi_c)^2}{\xi} & 0 \\ 0 & 0 & \partial_\mu^2 \end{bmatrix}. \quad (37)$$

The matrix $\delta^2 S/\delta A \delta B$ can be written in the form

$$\frac{\delta^2 S}{\delta A\, \delta B} = \begin{bmatrix} G^{*-1}_{\mu\nu} & & & & & 0 \\ & G^{-1}_{\mu\nu} & & & & \\ & & D^{-1}_{\mu\nu} & & & \\ & & & I^{*-1} & & \\ & & & & I^{-1} & \\ 0 & & & & & K^{-1}_\Phi \end{bmatrix}, \quad (38)$$

where

$$C^{-1}_{\mu\nu} = \{[P_\lambda P^\lambda - (e\Phi_c)^2] g_{\mu\nu} - (1-\xi) P_\mu P_\nu + 2ie\overline{F}_{\mu\nu}\}; \quad (39)$$

$$D^{-1}_{\mu\nu} = \partial_\lambda \partial^\lambda g_{\mu\nu} - (1 - 1/\rho)\, \partial_\mu \partial_\nu; \quad (40)$$

$$I^{-1} = P_\lambda P^\lambda - \frac{(e\Phi_c)^2}{\xi} - \frac{\lambda}{2}\Phi_c^2 + m_\chi^2; \quad (41)$$

$$K^{-1}_\Phi = -\partial_\lambda^2 - \frac{3}{2}\lambda\Phi_c^2 + m_\chi^2. \quad (42)$$

The first term of (36) is given by the sum of the six terms on the diagonal of the matrix (38). The term $\sim D^{1}_{\mu\nu}$ does not depend on Φ_c and $\overline{F}$ and makes a trivial contribution. To obtain the potential corresponding to dynamical symmetry breaking, we must in (41) and (42) set $m_\chi^2 = 0$.

We write the propagators in the representation of the proper time s:

$$G_{AB} = -i \int_0^\infty ds\, e^{-isG_{AB}^{-1}}.$$

Then the trace is calculated in accordance with[32,34]

$$-\frac{i}{2} \mathrm{Sp} \ln G_{AB} = \frac{i}{2} \int_0^\infty \frac{ds}{s} \mathrm{tr} \exp(-isG_{AB}^{-1}). \quad (43)$$

To find the right-hand side, we use the eigenvalues of $F_{\mu\nu}$,

$$F_{1,2} = \left\{\left[\left(\frac{E^2 - H^2}{2}\right)^2 + (EH)^2\right]^{1/2} \pm \frac{E^2 - H^2}{2}\right\}^{1/2}. \quad (44)$$

The final result of the calculations is

$$V^{(1)}(\Phi_c, \overline{F}) = i \int_C \frac{ds}{s} \{\exp(-ise^2\Phi_c^2)$$
$$\times (1 - 2\,\mathrm{ch}\, 2eF_1 s - 2\cos 2eF_2 s)$$
$$+ \exp(-ise^2\Phi_c^2/\xi)\} \langle x(s) | x(0) \rangle$$
$$+ i \int_C \frac{ds}{s} \exp\left[-is\left(e^2\Phi_c^2/\xi + \frac{\lambda}{2}\Phi_c^2 - m_\chi^2\right)\right] \langle x(s) | x(0) \rangle$$
$$+ \frac{i}{2} \int_C \frac{ds}{s} \exp\left[-is\left(\frac{3}{2}\lambda\Phi_c^2 - m_\chi^2\right)\right] \frac{(-i)}{(4\pi s)^2}, \quad (45)$$

where[1)]

$$\langle x(s)|x(0)\rangle = -\frac{\mathrm{i}}{(4\pi s)^2}\exp[L(s)],$$

$$L(s) = \ln\left[\frac{\mathrm{sh}\,(eF_1 s)}{eF_1 s}\,\frac{\mathrm{sh}\,(\mathrm{i}eF_2 s)}{\mathrm{i}eF_2 s}\right],$$

and the contour C passes in the complex plane of s from the origin to infinity in the direction of convergence of the integral. The expression (45) shows that in the case of a theory with spontaneous symmetry breaking the effective potential for an arbitrary value of Φ_c depends on the gauge parameter ξ. This dependence vanishes for the equilibrium values $\Phi_c = \delta(H)$, at which the effective potential is gauge-invariant. In the calculations that follow, we choose the U gauge, setting $\xi = 0$. In a theory with dynamical symmetry breaking ($m_\chi^2 = 0$), the constants λ and e^4 are related by $\lambda \simeq 0(e^4)$, and the term of order λ in the second integral in (45) can be ignored. The terms of order λ^2 must also be omitted in the terms that do not depend on $\overline{F}$. After this, the terms containing ξ cancel each other, and the effective potential is gauge-invariant for arbitrary Φ_c.

The integrals in (45) diverge as $s \to 0$. To eliminate the divergences and fix the effective potential, we specify the renormalization conditions. They have different forms for spontaneous and dynamical symmetry breaking. In the former case, we choose the renormalization conditions in the form[18]

$$\left.\frac{\partial V(\Phi_c, \overline{F})}{\partial V^0(\overline{F})}\right|_{\Phi_c=\delta(0),\ \overline{F}=0} = 1; \tag{46}$$

$$\left.\frac{\partial V(\Phi_c, \overline{F})}{\partial \Phi_c}\right|_{\Phi_c=\delta(0),\ \overline{F}=0} = 0; \tag{47}$$

$$\left.\frac{\partial^2 V(\Phi_c, \overline{F})}{\partial \Phi_c^2}\right|_{\Phi_c=\delta(0),\ \overline{F}=0} = 2|m_\chi^2|, \tag{48}$$

where $V^{(0)} = 1/2(\mathbf{E}^2 + \mathbf{H}^2)$, this corresponding to renormalization at the position of the minimum of the tree potential (3).[1] In the case of dynamical symmetry breaking, the condition (46) is preserved, while the other two are replaced by the single condition

$$\left.\frac{\partial^2 V(\Phi_c, \overline{F})}{(\partial \Phi_c^2)^2}\right|_{\Phi_c=\delta(0),\ \overline{F}=0} = \frac{\lambda}{4}, \tag{49}$$

since for $m_\chi^2 = 0$ the divergences $\sim \Phi^2$ are absent (for a suitable method of regularization). The subtraction point $\delta(0)$ now corresponds to the position of the minimum when $\overline{F} = 0$ of only the complete effective potential that takes into account the radiative corrections as well [see the expression (60)]; the minimum of the tree-approximation effective potential $V^{(0)}(\Phi_c)$ is at the point $\Phi_c = 0$. On the renormalized potential one usually imposes the further requirement[23,30]

$$\left.\frac{\partial V^{(1)}(\Phi_c, \overline{F})}{\partial \Phi_c}\right|_{\Phi_c=\delta(0),\ \overline{F}=0} = 0,$$

which makes it possible to eliminate λ from $V^{(1)}$ by expressing it in terms of e^4:

$$\lambda = \frac{3}{4\pi^2}e^4. \tag{50}$$

A detailed study of the effective potential is made in the following section.

Effective Lagrangian of the electromagnetic field

As we have noted, the effective potential depends on the gauge. There is a further function closely related to it—the effective Lagrangian $\mathscr{L}_{\mathrm{eff}}(M^2, \overline{F})$ of the external electromagnetic field. It determines the nonlinear corrections to the Lagrangian of the Maxwell field,

$$\mathscr{L}^{(0)}(\overline{F}) = \frac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2),$$

due to the vacuum polarization of the charged field. In the single-loop approximation, $\mathscr{L}_{\mathrm{eff}}^{(1)}(M^2 \leqslant \overline{F})$ can be calculated in the Georgi-Glashow model with spontaneous[12,27] and dynamical[22] symmetry breakings and has the form

$$\mathscr{L}_{\mathrm{eff}}^{(1)}(M^2, F_{1,2})$$
$$= \frac{1}{16\pi^2}\int_C \frac{ds}{s}\,e^{-\mathrm{i}M^2 s}\,\frac{e^2F_1F_2(1 - 2\,\mathrm{ch}\,2eF_1 s - 2\cos 2eF_2 s)}{\mathrm{sh}\,eF_1 s\,\sin eF_2 s}, \tag{51}$$

where M is the mass of the W boson, F_1 and F_2 are determined by the expression (44), and C is the contour in the s plane. In contrast to the effective potential $\mathscr{L}_{\mathrm{eff}}$ is gauge-invariant, since the mass M is. The $\overline{F}$-dependent part of the effective potential in the U gauge is related in a simple way to $\mathscr{L}_{\mathrm{eff}}$: $V^{(1)}(\Phi_c, \overline{F}) = -\mathscr{L}^{(1)}(M \to e\Phi_c; \overline{F})$; this relation was used in Ref. 4.

Renormalizing $\mathscr{L}_{\mathrm{eff}}^{(1)}$ by a subtraction at $\overline{F} = 0$, we find the renormalization constant Z_3^{-1} of the external field and the electric charge:

$$Z_3^{-1} = 1 + \frac{7}{16}\frac{e^2}{\pi^2}\ln\left(\frac{1}{\mathrm{i}\gamma s_0 M^2}\right);$$

$$e_0^2 = e^2\left[1 + \frac{7}{16}\frac{e^2}{\pi^2}\ln\left(\frac{1}{\mathrm{i}\gamma s_0 M^2}\right)\right].$$

The subscript 0 is appended to the unrenormalized value, γ is Euler's constant, and $s_0 \to 0$ is the parameter of the cutoff at the lower limit in the integral (51): Knowing Z_3, we can readily find the β function of Callan and Symanzik, which regularizes the ultraviolet ($\overline{F} \to \infty$) asymptotic behavior[27]:

$$\beta = -\frac{7}{16}\frac{e^2}{\pi^2}. \tag{52}$$

The minus sign indicates the asymptotic freedom in the field.

We now discuss the passage to the limit of zero mass of the field $W_\mu^\pm$ in $\mathscr{L}_{\mathrm{eff}}^{(1)}$. Setting $M = 0$, we rewrite (51) as follows[27]:

$$\mathscr{L}_{\mathrm{eff}}^{(1)}(M = 0, F_1, F_2) = \mathscr{L}_{\mathrm{Y.M}}^{(1)} + \frac{1}{16\pi^2}\int_C \frac{ds}{s}\,\frac{e^2F_1F_2}{\mathrm{sh}\,eF_1 s\,\sin eF_2 s}. \tag{53}$$

In (53), $\mathscr{L}_{\mathrm{Y.M}}^{(1)}$ is the effective Lagrangian obtained by Batalin, Matinyan, and Savvidi[13] in massless Yang-Mills theory for the case of an external covariantly constant color field $\overline{F}^a = \overline{F}n^a$, where n^a is an isotopic vector. The second term on the right-hand side is equal to the contribution of the massless scalar particles.[8] The expression (53) indicates the existence of the transition to zero mass of the field $W_\mu^\pm$. In the limit $M \to 0$, the contribution of the charged scalar fields producing the longitudinal polarization of the massive W boson can be readily separated.

For the Weinberg-Salam theory, $\mathscr{L}^{(1)}_{\rm eff}$ is equal to (51), since the Z bosons do not contribute to the polarization in the single-loop approximation. All that is changed is the formula that relates M to the vacuum expectation value of the scalar field.

At the position of the minimum, the effective potential does not depend on the gauge and determines the energy density of the classical scalar field. The mass of the particles in the field $\overline{F}$ must be determined from the position of the minimum of the effective potential $\delta(\overline{F})$; in particular, this applies to the mass of the W boson. It is therefore clear that M in the expression (51) depends on $\overline{F}$, and precisely therefore $\mathscr{L}_{\rm eff}$ is gauge-invariant:

2. INVESTIGATIONS OF THE EFFECTIVE POTENTIAL IN A MAGNETIC FIELD

Expression for the effective potential

We shall assume that $E=0, H=\text{const}\neq 0$. In this case, the field invariants (44) are $F_1=0$ and $F_2=H$. The expression (32) takes the form

$$V(\Phi_c,\ H)=V^{(0)}(\Phi_c)+\frac{H^2}{2}+V^{(1)}(\Phi_c)+V^{(1)}(\Phi_c,\ H). \quad (54)$$

In (54), we have separated the tree and single-loop potentials $V^{(0)}(\Phi_c)$ and $V^{(1)}(\Phi_c)$, which depend on the specific model. The part containing the field, $V^{(1)}(\Phi_c, H)$, is the same in all the cases discussed (apart from the notation for the parameters).

We shall consider the Georgi-Glashow model. Where necessary, we shall denote the effective potential corresponding to the case of spontaneous breaking by $V_{\rm sb}(\Phi_c)$, and for dynamically broken symmetry by $V_{\rm db}(\Phi_c)$. From Eq. (45), we obtain the potential

$$V^{(1)}(\Phi_c,\ H)$$
$$=\frac{1}{16\pi^2}\Big[\int_0^\infty \frac{ds}{s^2}\,e^{-e^2\Phi_c^2 s}\frac{-3eH}{\operatorname{sh} eHs}-4\int_0^\infty \frac{ds}{s^2}\,e^{-ie^2\Phi_c^2 s}\,eH\sin eHs\Big]$$
$$+a_1\frac{H^2}{2}+a_2\Phi_c^2+a_3\Phi_c^4. \quad (55)$$

The first term in the brackets is thrice the contribution of the charged scalar particles,[8] while the second gives the contribution due to the magnetic moment of the W boson; a_1, a_2, a_3 are renormalization counterterms. We begin with the case of spontaneous symmetry breaking. Using the conditions (46)–(48), we fix the counterterms. We find the first integral in (55) by using the generalized Riemann ζ function,[2]

$$V^{(1)}_{sc}(\Phi_c,\ H)=V^{(1)}_{sc}(\Phi_c)+\frac{3}{16\pi^2}(eH)^2$$
$$\times\Big\{4\zeta_R'\Big[-1,\ \frac{1}{2}\Big(1+\frac{e^2\Phi_c^2}{eH}\Big)\Big]$$
$$+\frac{1}{6}+\frac{1}{4}\,\frac{e^4\Phi_c^4}{(eH)^2}+\Big[\frac{1}{6}-\frac{1}{2}\,\frac{e^4\Phi_c^4}{(eH)^2}\Big]$$
$$\times\ln\Big(\frac{e^2\Phi_c^2}{2eH}\Big)-\frac{1}{6}\ln\frac{e^2\Phi_c^2}{e^2\delta^2(0)}\Big\}. \quad (56)$$

The first term $V^{(1)}_{\rm sc}(\Phi_c)$ here is given in (57b) (the term in the second square brackets); $\delta(0)$ is the normalization point, taken at the position of the minimum of the tree potential (3). We recall that the mass of the W boson in the absence of a field H is equal to $M=e\delta(0)$. The second integral in the expression (55) can also be readily calculated.

Another representation for the effective potential, convenient for numerical calculations, can be obtained by using the relation[38]

$$\zeta'(-1,\ x)=\ln\Gamma_1(x)+\zeta'(-1),\ \ \zeta'(-1)=\frac{1}{12}-0.2487,$$
$$\ln\Gamma_1(x)=\int_0^x dy\ln\Gamma(y)+\frac{1}{2}x(x-1)-\frac{1}{2}x\ln(2\pi).$$

In what follows, we shall measure the effective potential in units of $H_0^2=(M^2/e)^2$ and employ the dimensionless variables $\Phi^2=e^2\Phi_c^2/M^2, h=eH/M^2, K=\lambda/e^2$. The effective potential $v(\Phi^2,h,K)=V(\Phi_c,H)/H_0^2$ is determined by the sum of the following terms:

$$v^{(0)}(\Phi^2,\ K)=K\Big(-\frac{1}{4}\Phi^2+\frac{1}{8}\Phi^4\Big); \quad (57a)$$

$$v^{(1)}(\Phi^2,\ K)=\frac{K^2}{32\pi}\alpha\Big[\Big(\frac{9}{2}\Phi^4-3\Phi^2+\frac{1}{2}\Big)\ln\Big(\frac{3\Phi^2-1}{2}\Big)$$
$$+\frac{1}{2}\ln(4\pi\alpha K)-\frac{27}{4}\Phi^4+\frac{21}{2}\Phi^2\Big]$$
$$+\frac{\alpha}{\pi}\Big[\frac{3}{8}\Phi^4\ln\Phi^2-\frac{9}{16}\Phi^4+\frac{3}{4}\Phi^2\Big]; \quad (57b)$$

$$v^{(0)}(h)+v^{(1)}(\Phi^2,\ h)=\frac{h^2}{2}+\frac{\alpha}{\pi}\Big\{3h^2\ln\Gamma_1\Big[\frac{1}{2}(1+\Phi^2/h)\Big]$$
$$+3h^2\zeta'(-1)$$
$$-\frac{3}{8}\Phi^4\ln(\Phi^2/2h)-\frac{1}{8}h^2\ln(2h)-\frac{3}{16}\Phi^4+\frac{1}{8}h^2\Big\}$$
$$+\frac{\alpha}{2\pi}\{h^2+h\Phi^2)\ln(h+\Phi^2)+(h^2-h\Phi^2)\ln|h-\Phi^2|-2h^2\}$$
$$+i\operatorname{Im}v^{(1)}(\Phi^2,\ h), \quad (57c)$$
$$\alpha=1/137.$$

In the expression (57b), the term in the first square brackets gives the contribution of the Higgs scalars. The expressions in the first and second curly brackets in (57c) describe the h-dependent contributions of the integrals (55), respectively:

$$\operatorname{Im}v^{(1)}(\Phi^2,\ h)=-\frac{\alpha}{2}\theta(h-\Phi^2)(h^2-h\Phi^2). \quad (58)$$

It can be seen that in fields $h>\Phi^2$ an imaginary part arises in the effective potential.[4,22,23] It is a characteristic feature inherent in non-Abelian gauge theories (massless and with spontaneous symmetry breaking). Investigations associated with it make up an appreciable fraction of the present paper.

We turn to the systematic study of $v(\Phi^2, h, K)$ as a function of all the parameters. We now see the advantage of measuring the effective potential in units of H_0^2. For then the energy of the scalar-field condensate can be expressed directly in terms of the ratio K of the squares of the particle masses. Since a Higgs meson has not so far been detected, it is natural to express the quantities in terms of a parameter that has been reliably fixed in experiments, namely, the mass of the W boson.

We shall find the values of K at which it is possible to have spontaneous symmetry breaking in the absence of the field h; this will give us the so-called Linde-Weinberg restriction.[1,39] The formation of a condensate is energetically advantageous if $v^{(0)}(\Phi^2=1) \leqslant v^{(1)}(\Phi^2=1)$. In the dimensionless variables, the point $\Phi^2=1$ corresponds to the position of the minimum when $h=0$. By means of (57a) and (57b), we find

$$K > K_0 = \frac{3}{2}\,\frac{\alpha}{\pi}. \tag{59}$$

With increasing K, the energy of the condensate Φ decreases, and accordingly the minimum of the effective potential becomes deeper (Fig. 1).

For a theory with dynamical symmetry breaking, the potential $v_{db}(\Phi^2)$ can be calculated from the expression (45) using the renormalization condition (49) and the relation (50), and it has the form[30]

$$v_{db}(\Phi^2) = -\frac{3}{8}\,\frac{\alpha}{\pi}\,\Phi^4\left(\ln\Phi^2 - \frac{1}{2}\right). \tag{60}$$

This expression must be used instead of (57a) and the first square brackets of (57b). In the given case, $K = 3\alpha/\pi$.

Asymptotic properties of the effective potential

The possible restoration of spontaneously broken symmetry by a magnetic field was raised in Ref. 2 in connection with analysis of the expansion of the expressions (57) in a weak field, $h/\Phi^2 \ll 1$. In this approximation, the expression (57c) becomes[2,18]

$$v^{(0)}(h) + v^{(1)}(\Phi^2, h) = \frac{h^2}{2} + \frac{7}{8}\,\frac{\alpha}{\pi}\,h^2\ln\Phi^2. \tag{61}$$

It is determined by the Φ^2-dependent part of the renormalization constant $Z_3 = 1 + (7/4)(\alpha/\pi)\ln\Phi^2$. The plus sign in front of the logarithm arises because of the asymptotic freedom of the theory and leads to a decrease in the vacuum expectation value of the scalar field at small h. It was this result that suggested the possible restoration of symmetry.

In Ref. 18, Midorikawa studied the restoration of symmetry in the Weinberg-Salam theory. He considered the case $K_0 < K \ll 1$ and used the approximation (61) for the effective potential. He found that the symmetry is restored and that the transition to the state $\Phi = 0$ is a phase transition of the first kind. The critical field h_c was calculated from the system of equations

$$\partial v(\Phi^2, h)/\partial\Phi = 0; \tag{62}$$

$$v(0, h) = v(\Phi = \delta(h), h). \tag{63}$$

The first of these determines the position $\delta(h)$ of the minimum of the effective potential, while the second is the condition for the absence of a condensate to be energetically advantageous.

As noted in the Introduction, to study the restoration of symmetry it is necessary to use the exact expression (57). The problem is complicated by the fact that the effective potential contains the imaginary part (58). We encounter a new physical situation which calls for a more comprehensive

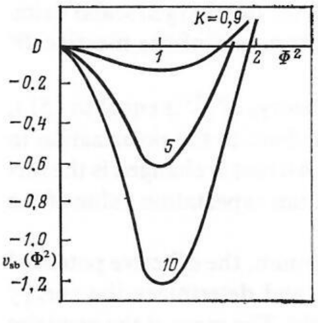

FIG. 1. Behavior of $v_{sb}(\Phi^2, K)$ as a function of Φ^2 for fixed K.

approach to the study of the effective potential. The same situation is encountered in massless non-Abelian theories in the presence of a covariantly constant magnetic field $\overline{F}^a = Hn^a$,[5] a question that has been discussed from various points of view.[5,40–42] In the literature, there has been a tendency to divide the investigation of the effective potential into two parts: 1) study of the properties of the real part Re $v(\Phi^2, h, K)$; 2) examination of the various questions associated with the presence of Im $v^{(1)}(\Phi^2, h)$. We shall follow this tradition.

We consider the real part. We write down the asymptotic behavior in a strong field $h/\Phi^2 \gg 1$:

$$\begin{aligned}\mathrm{Re}\,v(\Phi^2, h, K) &= v^{(0)}(\Phi^2) + v^{(1)}(\Phi^2) + h^2/2 \\ &+ \frac{\alpha}{\pi}\left\{h^2\left[\frac{7}{8}\ln h + 3\int_0^{1/2}\ln\Gamma(y)\,dy - \frac{3}{4}\ln\pi\right.\right. \\ &\left. - \frac{7}{8}\ln 2 + 3\zeta'(-1) - \frac{5}{4}\right] - \frac{3}{4}\Phi^2 h\ln 2 \\ &\left. + \Phi^4\left[\frac{17}{16} + \frac{3}{8}\psi(1/2) + \frac{3}{8}(\ln(2h) - \ln\Phi^2)\right]\right\}.\end{aligned} \tag{64}$$

Note the appearance in (64) of a term linear in h, which plays the part of a field correction to the mass of the scalar field. Assuming $K<1$ and omitting the contributions $\sim K^2$ in (57b), we solve Eq. (62):

$$\delta^2(h) = \frac{K + 3\frac{\alpha}{\pi}(h\ln 2 - 1)}{K + 3\frac{\alpha}{\pi}(\ln 2h + \psi(1/2) + 4/3)}. \tag{65}$$

Substitution of (65) in (64) gives $\mathrm{Re}[v(\delta^2(h), h) - v(0, h)] < 0$. Therefore, in strong fields the vacuum expectation value of the scalar field does not decrease but increases. The asymmetry of the vacuum increases (of course, if the existence of Im $v^{(1)}$ is ignored).

The expression (64) makes it possible to discuss one further interesting possible phenomenon—spontaneous magnetization of the vacuum. It was discovered by Savvidi[14] for massless Yang-Mills theory. We consider the potential for $\Phi = 0$:

$$v(h, 0) = \frac{h^2}{2} + \frac{7}{8}\,\frac{\alpha}{\pi}\,h^2(\ln h - 0{,}916). \tag{66}$$

It has a minimum at $h = h_0$,

$$H_0 = \frac{M^2}{e}\exp\left(-\frac{4\pi}{7\alpha} + 0{,}416\right). \tag{67}$$

The occurrence of a level with a nonzero average field has also been widely discussed in the literature[40-49] and is regarded as an important feature of quantum chromodynamics. We note that the argument of the exponential in (67) differs somewhat from the corresponding argument in Ref. 14, this being due to the contribution of the additional term in the theory with spontaneous symmetry breaking [see the expression (53)]. It can be seen that M^2 plays the part of the normalization point in the field for the massless theory. The existence of a smooth limit $\Phi^2 \to 0$ is a very important feature of non-Abelian gauge theories with spontaneous symmetry breaking, and it has a double significance. It gives rise to a similarity between the vacuum properties of the electroweak interactions and chromodynamics (for corresponding external conditions). On the other hand, theories with spontaneous symmetry breaking can be regarded as a mass regularization permitting the elimination of infrared divergences in massless theories. The possibility of spontaneous magnetization at $\Phi = 0$ poses the following question: If in some manner a value $\Phi \neq 0$ is maintained in the vacuum, how will this affect the occurrence of spontaneous magnetization? The asymptotic behavior (64) makes it possible to consider this question for the case of small Φ^2. Differentiating (64) with respect to h and equating the derivative to zero, we obtain the equation

$$\frac{1}{h}\frac{\partial v(\Phi^2, h)}{\partial h} = 1 + 2\frac{\alpha}{\pi}\left[\frac{7}{8}\ln h + \frac{7}{8} - B\right] - \frac{3}{4}\left(\frac{\Phi^2}{h}\right)\ln 2 = 0,$$

where B is a constant that is here unimportant. The solution of the equation in the adopted approximation has the form

$$h(\Phi^2) = h_0\left[1 + \frac{3}{7}\frac{\pi}{\alpha}\left(\frac{\Phi^2}{h}\right)\ln 2\right], \qquad (68)$$

where h_0 is given by (67). Thus, at small Φ^2 the Savvidi level arises. It is shifted to fields $h > h_0$. The depth of this level is less than the depth at $\Phi = 0$.

Spontaneous magnetization of the vacuum when $\Phi^2 \neq 0$

Ghoroku[23] investigated the possibility of spontaneous magnetization of the vacuum for different values of $\Phi_c = \text{const}$ in the framework of the Georgi-Glashow model with dynamical symmetry breaking. He used numerical integration of $v(\Phi^2, h)$. His results are shown in Fig. 2.

The abscissa is the ratio h/h_0, where h_0 is the magnetic field for $\Phi = 0$ (68), and the ordinate is the ratio Re v_{db}/h_0^2 for fixed values of Φ^2. It can be seen that for $h \neq 0$ the local minimum is shifted with increasing Φ^2 to the region of larger values of h. Simultaneously, the minimal value of Re v_{db} decreases. When $\Phi^2 = 1.2h_0$, the nontrivial minimum is shifted to $h = 1.5h_0$, and its depth at this point becomes equal to the minimum at the point $h = 0$. For $\Phi^2 \gtrsim 1.2h_0$, the absolute minimum is at the point $h = 0$. With increasing Φ^2, the nontrivial minimum continues to be displaced to the right and disappears at sufficiently large Φ^2. Conversely, the small initially minimal value of Re v_{db} at $h = 0$ is preserved at large Φ^2 and becomes the bottom of the potential when $\Phi = \Phi_0 = 1$ in accordance with (60). Thus, if the scalar field acquires a large vacuum value, spontaneous magnetiza-

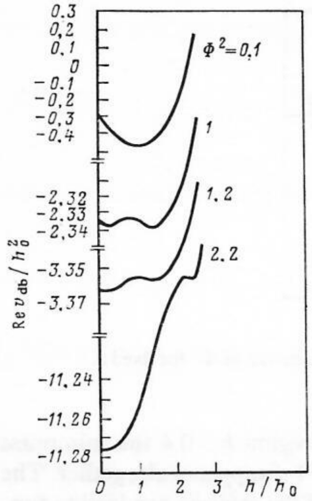

FIG. 2. Behavior of $v_{db}(\Phi^2)$ as a function of h/h_0 for fixed Φ^2.

tion of the vacuum is impossible. This conclusion also holds for theories with spontaneous symmetry breaking.[2)]

Behavior of the symmetry in a magnetic field

We investigate the behavior of Re $v(\Phi^2, h)$ as a function of Φ^2 for fixed h and K. It is necessary to make numerical calculations because of the presence of $\ln \Gamma_1[1/2(1 + \Phi^2/h)]$ in the expression (57c). For a theory with spontaneous symmetry breaking when $K > 1$, this question was considered in Ref. 4, and for $K \ll 1$ in Ref. 23. In Ref. 23, and also in Ref. 22, the case of the Georgi-Glashow model with dynamical symmetry breaking was considered. The conclusions reached about the behavior of the symmetry in Refs. 4, 23 and Ref. 22 are contradictory. Shore,[22] like Midorikawa,[18] concluded that there is a phase transition to the state with $\Phi = 0$. In this case, problems associated with the presence of Im $v^{(1)}(\Phi = 0, h)$ arise, as in Yang-Mills theory. In Refs. 4 and 23, it is asserted that restoration of the symmetry does not occur, since $\delta(h) \neq 0$, though it is noted that the presence of the imaginary part in the effective potential (but for $\Phi^2 \neq 0$) means that this conclusion is not final.[3)]

We begin with the case of dynamical symmetry breaking. We consider the function $v'_{db}(\Phi^2, h) = \text{Re } v_{db}(\Phi^2, h) - \text{Re } v_{db}(0, h)$, which is of interest for studying the behavior of the symmetry. The results of the numerical calculations are shown in Fig. 3.

It can be seen that with increasing h the position of the initial minimum at $\Phi^2 = 1$ is shifted to smaller values. In the range of fields $0.1 < h < 0.4$ there are two minima. The minimum at the larger Φ^2 is shifted in the direction of decreasing Φ^2. The second minimum at smaller Φ^2 is due to the effect of the magnetic field. It is produced by the term with the magnetic moment of the W boson [second integral in the expression (55)]. Characteristically, this minimum is in the region $\Phi^2 \lesssim h$. At $h \simeq 0.3$ and for $h > 0.3$ the depth of the new minimum becomes equal to and greater than the depth of the displaced original minimum. The position of the field-induced minimum is displaced with increasing h in the direc-

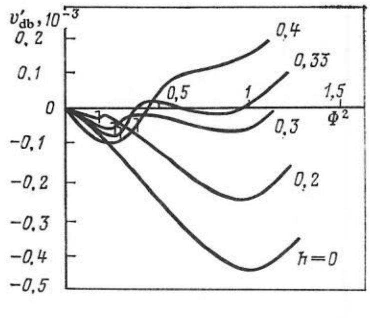

FIG. 3. Behavior of $v'_{db}(\Phi^2, h)$ as a function of Φ^2 for fixed h.

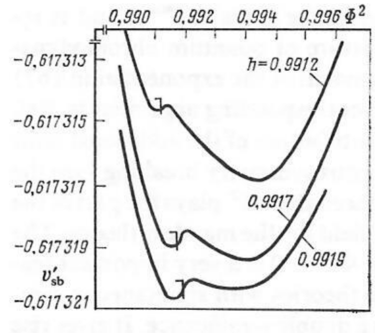

FIG. 5. Behavior of $v'_{sb}(\Phi^2, h, K = 5)$ as a function of Φ^2 for fixed h.

tion of increasing Φ^2. In the region $h > 0.4$ the minimum displaced from the point $\Phi^2 = 1$ disappears altogether. The change in the relative depths of the minima can lead to tunneling to the deeper minimum, for which $\mathrm{Im}\, v^{(1)}(\Phi^2, h) \neq 0$. In the figures, the marks on the curves indicate the values of Φ^2 to the left of which the effective potential has an imaginary part. As will be shown below, the presence of the imaginary part is an indication of instability of the system at the new minimum, and therefore further phase transitions are possible.

This behavior of the effective potential occurs in the Georgi-Glashow model with spontaneous symmetry breaking when $K < 1$. In the interval $1 < K < 9$–10, only the mutual position of the curves in the graph of $v'_{sb}(\Phi^2, h, K)$ for different h values changes. The curves corresponding to larger h are here situated lower. Figures 4 and 5 give graphs of $v'_{sb}(\Phi^2, h, K)$ for $K = 0.06$ and $K = 5$, respectively. It can be seen that at the larger K the field-induced minimum arises in stronger fields, the distance between the minima when their depths become equal is decreased, and the field minimum becomes less pronounced on the background of the complete curve. For example, for $K = 0.06$, the depths of the minima become equal at $h = 0.7$, while the distance between them is $\Delta\Phi^2 = 0.12$. For $K = 5$, however, $h = 0.9919$ and $\Delta\Phi^2 = 0.002$. In Ref. 22, the field minimum was not taken into account, and it was said that there is a phase transition of the first kind to the state with $\Phi^2 = 0$ beginning with fields $h = 0.336$. However, the neglect of the deeper minimum is unjustified. This is obvious in the case of the theory with spontaneous symmetry breaking, for which $v'_{sb}(\Phi^2, h, K) < 0$ until the disappearance of the original minimum with increasing h. Further increase in K qualitatively changes the properties of the effective potential. Figure 6 shows the behavior of $v'_{sb}(\Phi^2, h, K)$ for $K = 10$. The most important thing is that the field minimum remains shallower than the original one at all h. Therefore, tunneling is impossible, and the system always remains in one minimum, which is slightly shifted to smaller Φ^2. The imaginary part appears with increasing h in the original minimum. Such behavior of the effective potential was described in Ref. 4, in which the field minimum was therefore ignored altogether.[4)]

This behavior of the effective potential as a function of K leads to important differences between theories with light ($K < 9$–10) and heavy ($K \gtrsim 9$–10) Higgs mesons, since the effective potential is the main characteristic of the system that describes its properties. These differences will largely become clear from the following exposition. If $\mathrm{Im}\, v^{(1)}$ is not taken into account, then with increasing h the system goes over to a new field minimum when $K < 9$–10 or remains in the original minimum when $K > 9$–10. Thus, the existence of $\mathrm{Im}\, v^{(1)}(\Phi^2, h)$ determines all the further properties of the non-Abelian gauge theories in the field h. It can be seen that the vacuum state of the system and its properties are not determined by $V(\Phi_c, H)$ alone and remain unknown. In the Weinberg-Salam theory, the effective potential differs from the one considered here by the additional contribution of the Z bosons, which does not depend on K and h. This somewhat changes the depth of the potential $v(\Phi^2, h = 0)$, but all the results obtained apply fully to the Weinberg-Salam theory.

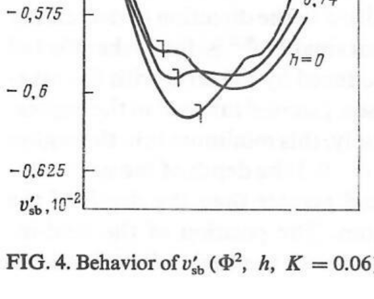

FIG. 4. Behavior of $v'_{sb}(\Phi^2, h, K = 0.06)$ as a function of Φ^2 for fixed h.

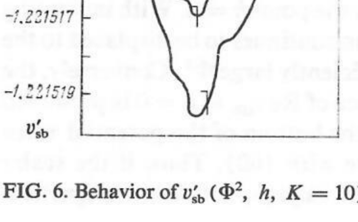

FIG. 6. Behavior of $v'_{sb}(\Phi^2, h, K = 10)$ as a function of Φ^2 for fixed h.

Imaginary part of the effective potential

The physical meaning of the effective potential as the energy density of the vacuum state determines the interpretation of the imaginary part (58). Namely, $2\,\mathrm{Im}\,v^{(1)}(\Phi^2, h)$ is the probability of decay of the vacuum in unit time.[8,44] The large numerical value of $\mathrm{Im}\,v^{(1)}$ leads to rapid damping of the vacuum amplitude.

The reason for the instability is that the spectrum of the charged vector boson in the magnetic field contains a state with negative square of the mass—a tachyon. We consider the linearized equation of the W boson in the field H. The corresponding vacuum is the perturbation-theory vacuum, since $\langle W_\mu^\pm\rangle_0 = 0, \langle Z_\mu\rangle_0 = 0$. The equation of motion that follows from the Lagrangian in the U gauge (23) has the form

$$[(P^2 - M^2)\,g_{\mu\nu} - P_\mu P_\nu - ie\overline{F}_{\mu\nu}]\,W^{-\mu} = 0, \qquad (69)$$

where $P_\mu = i\partial_\mu + e\overline{A}_\mu$. Taking into account the commutation relations for P_μ, we transform (69) into the following equation and subsidiary condition:

$$\left.\begin{aligned} &[(P^2 - M^2)\,g_{\mu\nu} + 2ie\overline{F}_{\mu\nu}]\,W^{-\nu} = 0;\\ &P_\mu W^{-\mu} = 0. \end{aligned}\right\} \qquad (70)$$

Its spectrum is[2,45]

$$P_0^2 = p_\parallel^2 + M^2 + (2n + 1 - 2\sigma)\,eH, \qquad (71)$$

where p is the projection of the momentum along the field H, σ is the spin variable, taking values 0, ± 1, and $n = 0, 1, 2, \ldots$. Without loss of generality, we set $p_\parallel = 0$. The ground state $n = 0, \sigma = +1$ of the spectrum,

$$P_0^2 = M^2 - eH = M^{2(0)}(H), \qquad (72)$$

becomes a tachyon state when $H > H_0 = M^2/e$. This is what leads to the appearance of an imaginary part in the effective potential. The same phenomenon occurs in Yang-Mills theory.[5] The corresponding spectrum and $\mathrm{Im}\,v^{(1)}$ are obtained from (71) and (58) for $\Phi_c = 0$. The value $\sigma = 0$ must be eliminated.

The instability of the perturbation-theory vacuum for $\Phi \neq 0$ gives grounds for re-examining the question of restoration of the symmetry: Could the classical scalar field disappear by a mechanism different from the one originally expected, namely, by evolution of the system caused by the tachyon? In the following sections we shall attack this and other related questions. It can be seen that instability of the vacuum in an Abelian field is a characteristic feature of non-Abelian gauge theories. Thus, the problem arises of finding the stable ground state and determining its properties and structure. In QCD, many studies have been devoted to these problems. In particular, the series of studies of Refs. 40–42, which considered the evolution of the perturbation-theory vacuum in the chromomagnetic field, led to the construction of the so-called Copenhagen vacuum. The case of spontaneously broken theories was considered in Refs. 19–21. There are here a number of features associated with the presence of the threshold for the occurrence of instability, to the discussion of which we now turn.

3. INVESTIGATIONS NEAR THE INSTABILITY THRESHOLD

The tachyon appears in the spectrum of the equation of motion of the tree approximation. It is important to establish how the situation is changed if allowance is made for the radiative corrections to the mass $M^{(0)}(H)$ determined by the expression (72). It is also important to establish the properties of the effective potential outside the framework of the single-loop approximation. The presence of a threshold for the occurrence of instability makes it possible in theories with spontaneous symmetry breaking, for $H < H_0$ at least, to use the traditional methods of field theory—calculation of the radiative corrections and solution of the renormalization-group equations. In massless non-Abelian theories, systematic allowance for the radiative corrections is not yet possible, since the perturbation-theory vacuum is unstable and the true vacuum state is not known. At the same time, as we have seen, theories with spontaneous symmetry breaking have a regular limit $M \to 0$ and it is possible to obtain some results corresponding to massless non-Abelian theories.

Renormalization-group equation for $\mathscr{L}'(H)$

We investigate the properties of the effective Lagrangian $\mathscr{L}'(H)$ by the renormalization-group method as $H \to H_0$.[19] The study of $\mathscr{L}'(H)$ rather than the effective potential is simpler, since M^2 is a fixed number.[5)] We choose the gauge (12a). We recall that in this gauge the action is invariant with respect to the gauge transformations $\overline{A}_\mu \to \overline{A}_\mu + \partial_\mu \alpha^0$. The electric charge e and $\overline{A}_\mu$ are renormalized by the single renormalization constant $Z_3^{1/2}$. Therefore, the effective Lagrangian $\mathscr{L}'(M^2, H)$ is gauge- and renormalization-invariant. The use of a covariant gauge greatly simplifies the derivation of the renormalization-group equations.[46]

We use the renormalization scheme of subtraction in a field proposed by Matinyan and Savvidi[46] in the investigation of $\mathscr{L}'(H)$ in Yang-Mills theory. Essentially, it is the analog of the ordinary momentum renormalization-group scheme used to analyze the function $\mathscr{L}'(H)$. We impose the renormalization condition

$$2\,\frac{\partial L}{\partial H^2}\bigg|_{H=\mu^2/e} = -1, \qquad (73)$$

where $L = -H^2/2 + \mathscr{L}'(H)$. The expression for $\mathscr{L}'(H)$ is found in the form of an expansion $\mathscr{L}' = \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots$, where $\mathscr{L}^{(1)}, \mathscr{L}^{(2)}, \ldots$ contain one, two, etc., loops and has the form

$$\mathscr{L}' = \mathscr{L}'(H, M, m, e, \lambda).$$

In the Lagrangians of the theories of the electroweak interactions there are no vertices coupling the Higgs and electromagnetic field. Therefore, a dependence of $\mathscr{L}'$ on λ first appears in the three-loop approximation with respect to the radiation field in the vacuum. In the leading approximation, it can be ignored.

The renormalization-group equations simplify and will be the same as in the massless case[46]:

$$\left[\mu^2\,\frac{\partial}{\partial\mu^2} + \beta(e)\left(\frac{\partial}{\partial e} - \frac{H}{e}\,\frac{\partial}{\partial H}\right)\right] L = 0. \qquad (74)$$

In (74), we have taken into account the connection between the β and γ functions: $\beta(e) = -e\gamma(e)$, which holds in covariant gauges. We define the dimensionless function

$$L' = 2\frac{\partial L}{\partial H^2},$$

which depends on dimensionless parameters. The form of the parameters and the nature of the dependence of the function $\mathscr{L}'$ on them are determined by the range of H values that are to be investigated.

To analyze the behavior as $H \to H_0$, we choose the normalization point $\mu^2 = (M^2 - \tau^2)/e$, $\tau^2 \to 0$. Then in the limit $H \to H_0$ the function $\mathscr{L}'$ takes the form

$$L' = \overline{L}'\,[(M^2 - eH)/\tau^2,\ e,\ \lambda] + O\,(\tau^2/M^2,\ \tau^2/m^2,\ (M^2 - eH)/\tau^2).$$

Therefore, in the limit $H \to H_0$ we can make the substitution $L' \to \overline{L}'(t,e,\lambda), t = \ln((M^2 - eH)/\tau^2)$. The solution of Eq. (74) for the function $\overline{L}'(t,e)$ with the boundary condition

$$\overline{L}'(t,\ e)\mid_{t=0} = -1$$

has the form[46]

$$\overline{L}'(t,\ e) = -e^2/\bar{e}^2, \tag{75}$$

where $\bar{e}^2(t,e)$ is the invariant charge, which satisfies the condition $\bar{e}(0,e) = e$. The β function is determined by

$$\bar{\beta}(e) = \lim_{H\to H_0} \frac{e}{2}\,\frac{\partial \overline{L}'(t,\ e)}{\partial t}\bigg|_{t=0}. \tag{76}$$

Effective charge $\bar{e}^2$ at the instability threshold

We now use the result of perturbation theory. The single-loop effective Lagrangian for the considered theories is given by the expression (51). Setting $E = 0$ and renormalizing in accordance with (73), we find for $L'^{(1)}$ as $H \to H_0$

$$\left.\begin{aligned} &L'^{(1)} = \overline{L}'^{(1)} + O\,(\tau^2/M^2,\ (M^2 - eH)/\tau^2);\\ &\overline{L}'^{(1)} = -1 - \frac{e^2}{8\pi^2}\ln\left(\frac{M^2 - eH}{\tau^2}\right). \end{aligned}\right\} \tag{77}$$

From (75) and (76), taking into account (77), we obtain

$$\overline{e^2} = e^2\left[1 + \frac{e^2}{8\pi^2}\ \ln\left(\frac{M^2 - eH}{\tau^2}\right)\right]^{-1}; \tag{78}$$

$$\bar{\beta}(e) = e^3/16\pi^2. \tag{79}$$

The expressions (78) and (79) determine the effective charge and the β function for field averages of order $\sim H_0$. It can be seen that as $H \to H_0$ the value of $\bar{e}^2$ initially increases, and then tends to zero. This is known as zero-charge behavior ("ghost-pole" problem). Analogous behavior of $\bar{e}^2$ occurs for massless non-Abelian theories at H less than the normalization point in a field μ^2/e. In QED, the same phenomenon occurs in the ultraviolet region $dH/m_e^2 \gg 1$.[9,47] Growth of $\bar{e}^2$ takes us out of the weak-coupling region and means that reliable conclusions about the properties of the system cannot be obtained in the corresponding regions. Moreover, the establishment of the very fact of the zero-charge phenomenon is an extremely difficult problem. In Ref. 48, taking the example of QED, Kirzhnits and Linde showed that the presence of the phenomenon leads to rearrangement of the vacuum. The true difficulty of QED, if the phenomenon is realized in it, is the spontaneous generation of an electric field of unbounded strength.

In non-Abelian gauge theories with spontaneous symmetry breaking, zero-charge behavior of $\bar{e}^2$ is in general absent. However, in an external magnetic field it arises for fields $H \sim H_0$. There is a region of H in which $\bar{e}^2 \gtrsim 1$. The transition from fields $H < H_0$ to fields $H > H_0$ cannot be controlled by perturbation theory. The threshold for the appearance of the tachyon is in this region.

As we have noted, proof of the zero-charge phenomenon is an unresolved problem. In such a situation, all we can do is to analyze the physical consequences of its possible presence and to find the region of parameters of the theory for which the existence of the phenomenon is unimportant for the system. Below, we shall consider the situation that arises near H_0 from this point of view.

Both the tachyon and the zero-charge phenomenon lead to instability of the system. The nature of the phase transition in a field will depend sensitively on the reality or otherwise of the zero-charge phenomenon discovered in perturbation theory. If it exists, then the magnetic permeability of the vacuum, determined in the single-loop approximation by means of (77), $\mu^{(1)} = 1 + \varkappa^{(1)} = -\overline{L}'^{(1)} = 1 + (e^2/8\pi^2)\ln((M^2 - eH)/\tau^2)$ where $\varkappa^{(1)}$ is the magnetic susceptibility, vanishes at $eH = M^2 - \tau^2\exp(-8\pi^2/e^2)$ and then becomes negative, this leading to vacuum instability (in this connection, see Ref. 48).

Since $0 \leqslant M^2 - eH < \tau^2$, it can be seen that instability arises in the region of H for which $M^{2(0)}(H) > 0$. Further, the manifestations of the instability associated with the zero-charge phenomenon in non-Abelian theories with spontaneous breaking may differ in principle from those described in QED,[48] since the zero-charge behavior of $\bar{e}^2$ occurs in different regions of H in the different theories. A field $H_0 = M^2/e$ may be unattainable altogether, since the external magnetic moment needed to produce it tends to infinity: $M_{\rm ext}^{(1)}(H) = \partial\mathscr{L}^{(1)}/\partial H\,|_{H\to H_0} \to \infty$. The consequences of the zero-charge phenomenon in spontaneously broken theories have not been studied in more detail. One could, however, imagine that the phenomenon is due to an incorrect choice of the perturbation-theory vacuum for $H \sim H_0$, as in massless non-Abelian theories when $H < \mu^2/e$. In this connection, it appears natural to use the results of perturbation theory in the region in which it applies, i.e., for $H < H_0$ and $H > H_0$, and for $H \sim H_0$ to use methods not related to perturbation theory. The results obtained determine the expansion parameter in intermediate fields: $e_V^2(M^2,\ H) = e^2\ln(1 - eH/M^2) < 1$.

The function $\overline{L}'^{(1)}$, determined in accordance with (51) and (77), makes it possible to establish the nature of the phase transition in the neighborhood $H \sim H_0$. For $H < H_0$, the magnetic susceptibility is negative, $\varkappa^{(1)} < 0$, and the vacuum possesses the properties of a diamagnet. For $H > H_0$, the susceptibility is positive, $\varkappa^{(1)} > 0$

$[\mu^{(1)}(H) = 1(7e^2/16\pi^2) + \ln(eH/M^2)$ as $H \to \infty$ in accordance with (52)], and the vacuum becomes a paramagnet. A diamagnet-paramagnet phase transition takes place in the system. The presence of the imaginary part in $\mathscr{L}^{(1)}$ indicates instability of the new phase. The applicability of perturbation theory for $H > H_0$, $H \to \infty$ ensures that the relation $\varkappa \gtrsim 0$ persists. It can therefore be supposed that a stable vacuum state has the properties of a ferromagnet.

In studying $\mathscr{L}^{(1)}(H)$, we treated M^2 as a fixed parameter. In reality, the mass in a field must be determined from the position $\delta(H)$ of the minimum of the effective potential: $M^2(H) = e^2\delta^2(H)$. If $K \gtrsim 9$–10, then the effective potential has one essential minimum (see Sec. 2). In this case, all that was said above about $\mathscr{L}'(H)$ remains valid. It is only necessary to make the substitution $M^2 \to M^2(H)$, which somewhat changes the absolute values of the parameters but not the conclusions.

A different situation arises for $K < 9$–10, when the effective potential has two essential minima in a certain interval of fields H, this interval depending on K. Moreover, as we have seen, the new field-induced minimum is in the region $H \gtrsim \Phi_c^2/e$, while the instability threshold $H_0 = e^2\Phi_c^2/e$ corresponds to the nonequilibrium value of Φ_c in the effective potential (is outside the position of the minimum). In accordance with (78), the zero-charge phenomenon occurs in the immediate proximity of H_0 and also does not correspond to the equilibrium value of Φ_c. In this connection, a phase transition due to the zero-charge phenomenon is altogether impossible. In addition, in the interval of H values for which the effective potential has two minima the field minimum is in the region in which $e_V^2(\Phi^2, H) \lesssim 1$.

These circumstances lead to the conclusion that the methods of perturbation theory become invalid for the analysis of the behavior of the symmetry in the case $K < 9$–10. If $K > 9$–10, the threshold of instability occurrence can be approached at the necessary values of H, so that one does not leave the position of the minimum of the effective potential or go beyond the region of applicability of perturbation theory. In the latter case, one can consider the tachyon problem with allowance for radiative effects.

Concluding the investigation of the effective potential, we note that the region in which the single-loop approximation is valid is restricted to certain intervals of the variables Φ_c^2 and H. Since powers of $\ln[(eH + (e\delta(0))^2/(e\delta(0))^2]$ appear in the higher approximations in the number of loops, the suitable region is restricted to values $eH + (e\delta(0))^2 \sim O(e\delta(0))^2$. The neighborhood near $\Phi_c \simeq 0$ and $H = H_0 \exp(-\mathrm{const}/e^2) \ll (e\delta(0))^2$ [see the expression (67)], where $\mathrm{Re}\, v_{\mathrm{db}}(\Phi_c^2, H)$ has a nontrivial minimum when investigated as a function of H, is outside the region of applicability of the single-loop approximation. Therefore, strictly speaking, the minimal structure cannot be reliably determined in this region. An analogous situation occurs in the massless theory, in which the Savvidi level occurs at $eH = \mu^2 \exp(\int_e^\infty dx/\beta(x))$, where $\beta(e) = d\,\bar{e}(t)/dt$, $t = \ln(eH/\mu^2)$, in which μ^2 is a certain mass (see footnote 2). The single-loop approximation is valid in the region $\Phi_c \simeq 0$ and $H \simeq e^2\delta^2(0)/e$, and its reliability is here emphasized by the approximate expression (64). For the study of the effective potential as a function of Φ_c^2, there is an additional field minimum in a certain interval of H for $(e\Phi_c)^2 \sim eH$. In this region, as we have seen, the condition of applicability of the single-loop approximation is also violated. If $|eH - (e\Phi_c)^2| \sim O((e\Phi_c)^2)$, then the results of the single-loop approximation are reliable. In particular, this makes it possible to establish the vanishing of the original minimum with increasing H in the case of small $K(0)$ and a phase transition to a state of the studied form of $V(\Phi_c^2, H)$ which cannot be determined from the effective potential, and also to establish the absence of a transition from this minimum when $K(0) \gtrsim 10$.

Radiative spectrum of the *W* boson in a magnetic field

The square of the mass of the W boson in a magnetic field is determined by

$$M_{\mathrm{eff}}^2(H) = M^{2(0)}(H) + \Delta M^2(H),$$

where $\Delta M^2(H)$ is the correction due to quantum radiative effects in the field. The important part that these play in the study of the tachyon problem is due to the fact that the square of the mass of the tree approximation tends to zero, $M^{2(0)}(H) = M^2 - eH \to 0$, as $H \to H_0$.[6)] To find $\Delta M^2(H)$, it is necessary to calculate the mass operator of the W boson in the field and find its expectation value in the ground state of the spectrum (71), which becomes a tachyon state: $\Delta M^2(H) = \langle M_{n=0}^{\sigma=+1}\rangle$. This problem was solved in the single-loop approximation in the field in Ref. 49. The calculations were made in the Georgi-Glashow model with spontaneous symmetry breaking. The gauge (12a) and the proper-time representation for the propagators (39)–(42) were used. It was shown in Ref. 50 that the state $n = 0$, $\sigma = +1$ is an eigenstate for the mass operator as a whole.

The expression for the effective mass in the limit $H \to H_0$ has the form[49]

$$M_{\mathrm{eff}}^2(H) = M^{2(0)}(H) + \Delta M_W^{2(1)}(H) + \Delta M_\Phi^{2(1)}(H); \tag{80}$$

$$\Delta M_W^{2(1)}(H) = 2eH\frac{\alpha}{4\pi}\ln\left(\frac{1}{M^2/eH - 1}\right); \tag{80a}$$

$$\Delta M_\Phi^{2(1)}(H) = -4M^2\frac{\alpha}{4\pi}e^{m^2/2eH}\left|\mathrm{Ei}\left(-\frac{m^2}{2eH}\right)\right| \times \ln\left(\frac{1}{M^2/eH - 1}\right), \tag{80b}$$

where $\Delta M_W^{2(1)}$ describes the contribution due to the radiation field of the vector particles, and $\Delta M_\Phi^{(1)}$ is the contribution of the radiation field of the Higgs scalar. It can be seen that in this limit $M_W^{2(1)}(H)$ has an infrared singularity. The reason for its occurrence and the associated important role of the radiative effects in the field are to be found in the fact that the propagator (39) of the W boson contains a mode with $M^{2(0)}(H) \to 0$. The asymptotic behavior of $\Delta M_\Phi^{2(1)}$ is negative, its value depending on the ratio $m^2(H)/eH \approx K(H)$. It follows from the expressions (80) that if $K(H) < 2$, then the threshold for the appearance of the tachyon state is in a region of weaker fields than prescribed by the tree approximation. If $K(H) > 2$, then the contribution

ΔM_W^2 is dominant. The square of the effective mass $M_{\rm eff}^{2(1)}(H)$ increase logarithmically, remaining positive, as $H \to H_0$. Therefore, tachyon instability does not occur. The results (80), (80a), and (80b) are valid provided

$$1-\frac{eH}{M^2}<\frac{\alpha}{4\pi}\ln\left(\frac{1}{M^2/eH-1}\right)<1.$$

The left-hand side of the inequality determines the values of H at which radiative effects are dominant, while the right-hand side determines the applicability of the weak-coupling approximation.

The expression (80a) describes the asymptotic behavior of the correction to the ground-state energy for massless Yang-Mills theory, M^2 playing the part of a normalization point. The sign of the expression (80a) and also the existence of the zero-mass limit for the Georgi-Glashow model indicate a possible elimination of the instability in massless non-Abelian gauge theories by radiative corrections.

Renormalization-invariant mass of the *W* boson

The result of the previous subsection can be generalized by renormalization-group methods. It is worth noting that the actual existence of a nontrivial renormalization group for the mass in a field is an interesting feature. In non-Abelian gauge theories, the unrenormalized mass operator contains divergences proportional to $(P^2 - M^2 + 2ie\overline{F})_{\mu\nu}$ which are eliminated by a renormalization of the wave function $W_\mu^{\pm}$ of the field and disappear on the mass shell (Ref. 49).[7)] The external field $\overline{F}_{\mu\nu}$ together with the kinetic momentum P_μ determines the mass shell in accordance with the expression (70) and participates in the choice of the subtraction point in the renormalization. This makes it possible to write down a renormalization-group equation, varying $\overline{F}_{\mu\nu}$ as a renormalization-group parameter, and still remain on the mass shell by fixing the connection between the field and the momentum:

$$[(P^2 - M^2)\, g_{ik} + 2ie\overline{F}_{ik}]\, W^{-k} = 0,\ i,\ k = 1,\ 2,\ 3.$$

By virtue of this, the renormalization-group procedure for quantities on the mass shell in the external field corresponds to the well-known renormalization-group procedure for scattering amplitudes described by Blank.[53,54]

The renormalization-invariant mass of the vector boson in an external field has the form

$$\overline{M}^2(H) = M_{\rm ph}^2\,(H) - eH, \tag{81}$$

where $M_{\rm ph}(H)$ is the physical mass in the field as determined from the two-point Green's function of the *W* boson. As the tree approximation for the renormalization-group summation, it is necessary to choose M^2, since eH is a renormalization-invariant quantity. To analyze the $H \to H_0$ asymptotic behavior, we introduce the dimensionless function

$$f\,(x,\ y,\ \alpha,\ K) = M_{\rm ph}^2\ (H)/M^2. \tag{82}$$

The variables x and y are given by

$$x = (M^2 - eH)/\tau^2;\ y = M^2/\tau^2.$$

We use the renormalization procedure for masses developed in the momentum space by Lowenstein[55] and generalized in Ref. 56 to the case when an external field is present. The role of the momentum in the standard renormalization-group scheme (see, for example, Ref. 54) is played by $M^2 - eH$; the mass argument y is present in the renormalization-group scheme irrespective of the presence of an external field.[55]

The renormalization-group equations for f as $H \to H_0$ and the initial condition have the form[56]

$$[x\partial_x + (1-\beta_M)\, y\partial_y - \beta_\alpha\partial_\alpha - \beta_K\partial_K - \beta_M]\, f = 0; \tag{83}$$

$$f\,((x) = 0,\ \ y,\ \alpha,\ K) = 1. \tag{84}$$

Omitting the details of the calculations, we give the final expression for $\overline{M}^2(H)$ in the leading approximation[56]:

$$\overline{M}^2\,(t,\ \alpha,\ K) = M^2\,(1+t\beta_\alpha^{(1)})^{\beta_M^{(1)}/\beta_\alpha^{(1)}} - eH, \tag{85}$$

where $t = \ln x$ and $\beta_\alpha^{(1)} = \alpha/2\pi$ in accordance with (79). The value of $\beta_M^{(1)}$ is determined from the asymptotic behaviors (80a) and (80b) and is equal to

$$\beta_M^{(1)} = \beta_{M_W}^{(1)} + \beta_m^{(1)}, \tag{86a}$$

$$\beta_{M_W}^{(1)} = -\alpha/2\pi,\ \ \beta_m^{(1)} = \frac{\alpha}{\pi}\, e^{K/2}\, \mathrm{E}_1\,(K/2), \tag{86b}$$

where $E_1\,(x)$ is the exponential integral. The region of applicability of the expression (85) is determined by that of the asymptotic behavior of the effective charge $\overline{\alpha}(H)$, and this is found from the well-known condition[57]

$$1 + t\beta_\alpha^{(1)} \gg \alpha.$$

Depending on the value of K, the renormalization-group function $\beta_M^{(1)}$ may be positive or negative. The boundary value K_0 can be calculated from the equation $\beta_M^{(1)}(K_0) = 0$. For the function (86), $K_0 \approx 1.66$.

Tachyon problem with allowance for the renormalization-invariant mass

The behavior of $\overline{M}^2\,(H)$ depends in a fundamental manner on $K(H)$. If $K(H) < 1.66$, the contribution of the Higgs sector to the Georgi-Glashow model with spontaneous symmetry breaking is dominant, and $\beta_M^{(1)} > 0$. The first term in (85) tends to zero as $H \to H_0$, and the value of $\overline{M}^{(2)}\,(H)$ becomes negative. Therefore, with allowance for radiative effects the tachyon appears in the region of fields weaker than H_0. For example, if $K(H) = 1.1$, then $H_{\rm cr} \approx 0.995 H_0$. In such fields, the effective expansion parameter is $|t\beta_\alpha^{(1)}| \approx 6.1\times 10^{-3} \ll 1$ and the threshold region of values of the field $H_{\rm cr}$ can be controlled by perturbation theory. For $K(H) \geqslant 1.66$, the vector sector is dominant, $\beta_M^{(1)} < 0$, and $\overline{M}^2\,(H)$ is greater than zero until after the weak-coupling region has been left.

The results of the previous subsections can be readily extended to the Weinberg-Salam theory. For this, new calculations are hardly needed. The contribution due to the *Z*-boson radiation field is simply found from the photon calculation. The asymptotic behavior for the ground state as $H \to H_0$ has the form[56]

$$\langle M^{\sigma=+1}_{n=0} \rangle \approx \beta^{(1)\mathrm{WS}}_M M^2 \ln (M^2/eH - 1), \tag{87}$$

where

$$\beta^{(1)\mathrm{WS}}_M = -\frac{\alpha}{4\pi}\, \frac{R}{R-1} \left[2 + e^{R/2}\, \mathrm{E}_1 (R/2) - e^{K/2}\, \mathrm{Ei}\, (K/2) \right], \tag{88}$$

$R = M_Z^2/M^2 = 1/\cos^2\theta$. Taking $\sin^2\theta = 0.238$,[58] i.e., $R \approx 1.312$, we find that $\beta^{(1)\mathrm{WS}}_M$ is negative for $K < 0.08$ and positive for the remaining K. We recall that in the two previous subsections we wrote K instead of $K(H)$ for brevity, and we also recall that it is meaningful to take into account the properties of $\overline{M}^2(H)$ in the tachyon problem in the case $K = K(0) > 9$–10, when there is one essential minimum in the effective potential. Thus, if we ask whether the radiative corrections will stabilize the spectrum of the W boson in a field or strengthen the instability, the answer depends on the extent to which the ratio $K(H)$ of the mass squares changes with increasing field and on whether it can become less than 1.66 for the Georgi-Glashow model and 0.08 for the Weinberg-Salam theory before an imaginary part appears at the minimum of the effective potential.[8)]

The square of the mass of the Higgs meson in a field is determined by the curvature of the effective potential at the position of the minimum,

$$m^2(H) = \frac{\partial^2 v\,(\Phi^2,\, h,\, K)}{\partial \Phi^2}\bigg|_{\Phi=\delta(h)} M^2(0), \tag{89}$$

while the square of the mass of the W boson is determined by the position of the minimum of the effective potential:

$$M^2(h) = M^2(0)\,\delta^2(h).$$

Using the explicit form of $v^{(1)}(\Phi^2, h, K)$ in the single-loop approximation (57), we obtain

$$m^2(h) = M^2(0)\,(v_1'' + v_2'' + v_3'' + v_4'')\,|_{\Phi=\delta(h)}; \tag{90}$$

$$v_1'' = K\left(\frac{3}{2}\Phi^2 - \frac{1}{2}\right); \tag{90a}$$

$$v_2'' = \frac{\alpha}{\pi}\left\{\frac{3}{16}K^2\left[(9\Phi^2 - 1)\ln\left|\frac{3\Phi^2-1}{2}\right| + 3 - 3\Phi^2\right] + \frac{3}{2}(3\Phi^2\ln\Phi^2 + 1 - \Phi^2)\right\}; \tag{90b}$$

$$v_3'' = \frac{\alpha}{\pi}\left\{3h\ln\Gamma\left[\frac{1}{2}(1+\Phi^2/h)\right] + 3\Phi^2\psi\left[\frac{1}{2}(1+\Phi^2/h)\right] + \frac{3}{2}\Phi^2 - \frac{9}{2}\Phi^2\ln(\Phi^2/2h) - \frac{3}{2}h\ln(2\pi)\right\}; \tag{90c}$$

$$v_4'' = \frac{\alpha}{\pi}\left[h\ln\left|\frac{h+\Phi^2}{h-\Phi^2}\right| - 4\,\frac{\Phi^2}{(\Phi^2/h^2)^2 - 1}\right]. \tag{90d}$$

The position $\delta^2(h)$ of the minimum of the effective potential can be determined by using the graphs shown in Figs. 4–6. In Table I, we give the results for the case $K(0) = 10$.

It can be seen from the expression (90d) that the expression (90) for $m^2(h)$ is invalid near $\delta^2(h) \approx h$ because of a divergence. Overall, the shape of the graphs is such that the minimum of the effective potential is made shallow and the curvature at the position of the minimum is fairly small. This has the consequence that $K(h)$ becomes less than 1.66, and the square of the renormalization-invariant mass of the W boson in the original minimum can become negative in sufficiently strong fields. In the Weinberg-Salam theory, it is more difficult to draw such a conclusion, since a value $K(h) < 0.08$ can be obtained only by definitely going outside the region of applicability of the expressions (90). Here, the most probable value is $K(h) > 0.08$.

Thus, our results show that in the Weinberg-Salam theory with heavy Higgs mesons stabilization of the W-boson spectrum by radiative corrections is in principle possible. The definitive solution to this problem entails difficulties in the employed method and requires a more significant break with perturbation theory.

We give a partial summary. The tachyon problem in non-Abelian gauge theories with spontaneous symmetry breaking requires us to take into account a number of circumstances and to a large extent is model-dependent. The appearance of the tachyon is expected above the perturbation-theory vacuum in a field H determined from an effective potential of the form $V(\Phi_c^2, H, K)$. In the case $K < 9$–10, the problem does not really arise, since the given effective potential becomes inadequate for the problem by virtue of tunneling of the system to the field minimum, which is unstable with respect to the generation of $W_c^{\pm}$ fields (see Sec. 2). Therefore, to determine the vacuum it is necessary to use a more general effective potential: $V(\Phi_c^2, W_c^{\pm}, H, K)$ (see Sec. 4). For $K \gtrsim 10$, a tachyon can arise. Since at the threshold there are singularities in both the effective potential and the mass operator, we consider in more detail the conditions of applicability of perturbation theory. We note that it is necessary to take into account two perturbation-theory expansion parameters. For the effective potential, the parameter of the expansion with respect to the number of loops is $e_V^2(\Phi_c^2, H) = (\alpha/\pi)\ln(\Phi_c^2 e^2 - eH)|_{|eH \to e^2\Phi_c^2| \to 0} \ll 1$. For the mass operator, $e_M^2[\delta^2(H), H] = (\alpha/\pi) \times \ln[e^2\delta^2(H) - eH]|_{eH \to e^2\delta^2(H)} \ll 1$ is the parameter of the expansion with respect to the radiation field in the presence

TABLE I. Masses of the W and Higgs bosons in a field for $K(0) = 10$.

h	$M^2(h)/M^2(0)$	$m^2(h)/M^2(0)$	$K(h)$
0,9956	0,99645	4.5431	4.5590
0.9957	0.9964	3.3380	3,3392
0.9958	0.9963	0,7518	0,7547

of H. As we have seen, the threshold for the appearance of $\mathrm{Im}\, V^{(1)}(\Phi_c, H)$ in the effective potential is in the region in which $e_V^2(\Phi_c^2, H) \gtrsim 1$. However, this does not mean that the occurrence of a tachyon also cannot be controlled by perturbation theory. In the case when $eH \to e^2\delta^2(H)$, the condition $e^2\Phi_c^2 = eH < e^2\delta^2(H)$ is satisfied. Therefore, although $e_V^2 > e_M^2$, the slow growth of the logarithms means that the square (80) of the effective mass can become negative in the region of applicability e_V^2, $e_M^2 \ll 1$ of perturbation theory. Thus, with allowance for radiative effects the appearance of the tachyon can be reliably established.

4. DETERMINATION OF A STABLE VACUUM

In this section, we consider two methods employed to describe the phase transition in a magnetic field in connection with the instability of the perturbation-theory vacuum. In the first of them, one introduces an effective Lagrangian $\mathscr{L}^{\text{unst}}$ for the unstable mode and studies its properties. The minimal-energy classical solutions found for $\mathscr{L}^{\text{unst}}$ are taken as the new vacuum. Essentially, this reproduces the approach used in the construction of the Copenhagen vacuum[40–43] but with allowance for renormalization-group summation.[56] The second method is based on investigation of the classical equations of motion in a given external field and was used in Refs. 20 and 21 for the Georgi-Glashow and Weinberg-Salam theories.

The main idea used to describe phase transitions in quantum field theory was formulated by Migdal in his work on π condensation (see Ref. 36 and the literature quoted there): if under certain conditions unstable modes arise in the linearized system, then allowance for the nonlinearities of the system may lead to condensation of these modes and the formation of a new vacuum state. The actual realization of this principle can, of course, take different forms.

Effective Lagrangian for the unstable mode

We consider the Georgi-Glashow model with spontaneous symmetry breaking for $K(0) > 9$–10, $K(H) < 1.66$. The square of the effective renormalization-invariant mass of the W boson becomes negative when $H_{\text{cr}} < H_0$. Condensation of tachyons occurs if allowance is made for their self-interaction. The condensate parameters are determined as follows.[36] We separate from the operator W_μ^-, expressed as an expansion with respect to the eigenfunctions corresponding to the spectrum (71),

$$W_\mu^- = \sum_{n,\sigma} a(n,\sigma)\, W_\mu^{-(n,\sigma)}(x),$$

the state that becomes a tachyon state:[56]

$$W_\mu^{-(n=0,\,\sigma=+1)}(x) = \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix} \sqrt{\frac{eH}{2\pi}} \times \exp\left(-\frac{eH}{4}x_\perp^2 + \frac{ieH}{2}x_1x_2\right) \times \psi(x_0, x_3), \tag{91}$$

where

$$x_\perp^2 = x_1^2 + x_2^2,$$

and the field H is directed along the x_3 axis. Ignoring the other modes, we write down the effective Lagrangian $\mathscr{L}^{\text{unst}}$ describing the tachyon and its self-interaction. The trial function $\psi(x_0, x_3)$ in (91) must be determined by minimizing the effective action S^{unst}.

The unrenormalized Lagrangian (1) contains a term describing the four-particle self-interaction of the field $W_\mu^\pm$ that has order α. This linearity was taken into account in the determination of the condensate structure in QCD.[40,43] The radiative corrections in a field are functions of H of the same order ($\sim\alpha$). Such corrections were not considered in Refs. 40–47, but an investigation self-consistent within the framework of perturbation theory requires them to be taken into account.

Using (91) and integrating over the transverse coordinates x_1, x_2 by means of the device described in Ref. 40, we obtain an expression for S^{unst}:

$$S^{\text{unst}} = \int d^4x\, \mathscr{L}^{\text{unst}} = \int dx_0\, dx_3\, [\psi^* P_\parallel^2 \psi - \overline{M}^2(H)\,|\psi|^2 - \alpha eH\,|\psi|^4], \tag{92}$$

where we are to understand by the wave function ψ its renormalized value. The gauge invariance has the consequence that the self-interaction of the fields $W_\mu^\pm$ and their interaction with the external field take place with a single coupling constant. Therefore, for the investigation in the neighborhood of the instability threshold we replace α by $\overline{\alpha}(H)$, determined by the expression (78).[9)] The effective potential of the unstable mode,[56]

$$V^{\text{unst}}(\psi) = \overline{M}^2(H)\,|\psi|^2 + \overline{\alpha}(H)\, eH\,|\psi|^4$$

has a minimum at $\psi = \psi_0(H)$:

$$|\psi_0|^2 = \frac{|\overline{M}^2(H)|}{2\overline{\alpha}(H)\, eH}. \tag{93}$$

In the magnetic field, the vacuum energy density is lowered by the amount

$$V(\psi_0) = -\frac{[\overline{M}^2(H)]^2}{4\overline{\alpha}(H)\, eH}. \tag{94}$$

The magnetic field H and the field of the condensed tachyon mode have one nonvanishing isotopic component,

$$F_{12}^3 = H - 2e\,|W_\mu^-|^2, \tag{95}$$

the second term determining the self-field of the condensate. The maximal amplitude of this field for $x_\perp^2 = 0$ is

$$H_{\max} = \frac{2}{e}\,|\overline{M}^2(H)|.$$

If spatially inhomogeneous structures of the domain type as described in Refs. 41 and 42 are to occur in the condensate, $H_{\max}$ must reach the value of the external field H. Then at the boundary of the region, $F_{12}^3 = 0$. However, this is possible for fields very close to H_0, when $\overline{\alpha}(h) \to \infty$ and perturbation theory becomes invalid.

Thus, in the Georgi-Glashow model with $H > H_{\text{cr}}$ condensation of vector bosons is possible, and this lowers the vacuum energy. The initial stage of the condensation is determined by the radiative corrections and takes place in the region of applicability of perturbation theory. Study of the further development of the condensate and its structure as $H \to H_0$ requires solution of the zero-charge problem.

We now turn to exposition of the second method, which is suitable when $K<1$. Before that, we discuss in more detail the mechanism of occurrence of the tachyon.

Interaction with the anomalous magnetic moment of the W boson

The Lagrangians (5) and (23) and the equation of motion (69) contain a term $\sim \overline{F}_{\mu\nu}$, which determines the anomalous magnetic moment of the W boson. The interaction with this moment has a long-range nature and plays the part of a field mass, leading to instability.[4,5,45] We shall show this by considering the motion along the field $H = H_3$. We take the potential in the form $\overline{A}_\mu = (0, 0, Hx_1, 0)$. In Eq. (69), we set the operator P_μ equal to $i\partial_\mu$, this eliminating the field-induced oscillations in the (x_1, x_2) plane. We arrive at the equation

$$[\partial^2_\lambda g_{\mu\nu} - \partial_\mu \partial_\nu + ie\overline{F}_{\mu\nu} + M^2 g_{\mu\nu}]\, W^{-\mu} = 0. \tag{96}$$

Using the p representation and setting the momenta p_1, p_2 equal to zero, we find the longitudinal spectrum for (96):

$$p^2_\parallel = M^2 \pm eH, \quad p^2_\parallel = p_0^2 - p_3^2 = M^2. \tag{97}$$

It contains the level $p^2_\parallel = M^2 - eH$, which corresponds to the ground state of the complete spectrum (71).

The distinguished nature of the interaction with the anomalous magnetic moment for the formation of the tachyon and the longitudinal nature of the oscillators in the W-boson ground state are very important. The mechanism and the region of formation of the tachyon are not the same as for the stable modes. Let us discuss this in more detail. It is well known[9,10] that the spectrum of a charged particle in a magnetic field is formed at distances of the order of the Larmor radius, $r \gtrsim r_0 = (eH)^{-\frac{1}{2}}$, in the (x_1, x_2) plane.[10)] If the particle has spin 1, the noncommutativity of P_μ results in the appearance of a "normal" magnetic moment. The gyromagnetic ratio determined by the terms with P_μ is $g_P = 1$, as in Proca theory.[60] The occurrence of an anomalous magnetic moment ($\varkappa = g_{\rm Y.M} - 1 = 1$) in non-Abelian gauge theories is not organically related to the fact that the charged vector particle has spin but is due to the structure of the triple vertex determined by the symmetry group. For the chosen potential, the operators P_μ commute for longitudinal displacements for which a spin magnetic moment "does not arise." Formally, the substitution $P_\mu \to i\partial_\mu$ in Eq. (69) is admissible in regions $x_1 \ll (eH)^{-\frac{1}{2}}$, where the term $\sim e\overline{F}_{\mu\nu}$ is dominant. In this region, there are no Larmor orbits, and only the longitudinal oscillations of the field $W^{\pm}_\mu$ are important. In speaking of the region of formation of the instability, we mean that the tachyon nature of the longitudinal oscillators is due to the anomalous magnetic moment, the interaction with which determines the properties of the theory for $r < r_0$, where one can ignore the effects associated with the charge and spin.

Description of the phase transition

We shall consider the Weinberg-Salam theory in the presence of a field H. In the calculations, we shall use the U gauge. The equations of motion for the fields W^-_μ and Φ follow from (23) and have the form

$$\left[\widetilde{P}^2 g_{\mu\nu} - \widetilde{P}_\mu \widetilde{P}_\nu - igN_{\mu\nu} - \left(M + \frac{1}{2} g\Phi\right)^2 g_{\mu\nu}\right] W^{-\mu} + g^2 (W^+_\mu W^{-\mu} W^-_\nu - W^-_\mu W^{-\mu} W^+_\nu) = 0; \tag{98}$$

$$\partial^2_\mu \Phi + m^2 \Phi + \lambda \Phi^3 + 6\frac{\lambda}{g} M\Phi^2 - g\left(M + \frac{1}{2} g\Phi\right)(W^+_\mu W^{-\mu} + Z^2_\mu/(2\cos^2\theta)) = 0. \tag{99}$$

The system (98), (99) must be augmented by the equations for the fields Z_μ, A_μ. The potential is $A_\mu = \overline{A}_\mu + \widehat{A}_\mu$, where $\overline{A}_\mu = (0, 0, Hx_1, 0)$ and $\widehat{A}_\mu$ is the potential of the electromagnetic field developed in the system.

We set ourselves the aim of studying the evolution of the perturbation-theory vacuum in a field, using the classical equations of motion. The external field H is assumed to be constant. It satisfies Maxwell's equations without sources and requires the use of appropriate boundary conditions. The system of equations (96), (97) (with the equations for Z_μ, A_μ) is very complicated. Its solutions in the general case are unknown. However, the results of the previous subsection make it possible to establish the existence of a phase transition in a certain region of space and to establish specific features of it. The circumstance that tachyons are linear oscillators, longitudinal with respect to the field, that are formed at distances $r < r_0$ in the direction perpendicular to the field makes it possible to find a stable vacuum state as follows. It is sufficient to obtain a solution for the complete coupled system of fields valid in the region of formation of the instability. Here the system simplifies and can be readily solved. The occurrence of the phase transition will be manifested in a change in the nature of the solution at a certain $H = H_{\rm cr}$. In such an approach, it is not the tachyon mode that is distinguished but the interaction with the anomalous magnetic moment; this interaction is dominant in this region and leads to the tachyon. The solution which we have found takes into account exactly the motion along the field due to the long-range nature of the interaction with the anomalous magnetic moment.

In the region $x_1 < (e, H)^{-1/2}$, we make the substitution $\widetilde{P}_\mu \to i\partial_\mu$. In the system (98) and (99) there are nonvanishing constant solutions determined by the expression (102). In Ref. 21 it was shown that if by ε we denote $\varepsilon = x_1(eH)^{-1/2} \ll 1$, then the fields $\widehat{A}_\mu, Z_\mu$ have in this region the estimate $\widehat{A}_\mu \approx Z_\mu \sim O(\varepsilon^3)$. Therefore, the solution of only two equations is justified.

For the determination of the constant solutions, we obtain the system

$$-ie\overline{F}_{\mu\nu} W^{-\mu} + \left(M + \frac{1}{2} g\Phi\right)^2 W^-_\nu + g^2 (W^+_\mu W^{-\mu} W^-_\nu - W^-_\mu W^{-\mu} W^+_\nu) = 0; \tag{100}$$

$$-m^2\Phi + g\left(M + \frac{1}{2} g\Phi\right) W^+_\mu W^{-\mu} - \lambda\Phi^3 - 6\frac{\lambda}{g} M\Phi^2 = 0. \tag{101}$$

Setting $W_0^{\pm}=0$ and multiplying (100) by W_i^-, we arrive at the relation $W_i^- W^{-i}=0$. For $i=3$ we find $W_3^{\pm}=0$, $W_{cr}^1 W^{-cr}\neq 0$, and therefore $(W_1^{\pm})^2+(W_2^{\pm})^2=0$. Using the explicit form of $\overline{F}_{\mu\nu}$, we readily obtain[21] the nonvanishing solutions[11)]

$$\left.\begin{array}{l}(W_i^+W_i^-)_c = i\,(W_1^+W_2^- - W_2^+W_1^-) = \dfrac{eH}{g^2} = 2\rho^2;\\ \Phi_c = -2M/g;\ Z_\mu = \hat{A}_\mu = 0.\end{array}\right\} \quad (102)$$

The calculation of the energy density of the fields Φ_c, $(W_i^+W_i^-)_c$ in the presence of H shows that the energy of the condensate Φ_c completely compensates the energy of the scalar condensate present after a symmetry breaking [see the expression (23)]: $V^{(0)}(\Phi_c)=|V^{(0)}(\delta(0))|$. Thus, the classical scalar field completely disappears. Instead, there is a condensate $(W_i^+W_i^-)_c$, the formation of which follows from Eqs. (100) and (101) under the condition of lowering of the energy[21]: $V^{(0)}\ [(W_i^+W_i^-)_c]=-(H^2/2)\sin^2\theta<V^{(0)}(\delta(0))$. The critical field of the phase transition is $H_{cr}=H_0\,(m/M)$. If $m/M=\sqrt{K}<1$, the threshold for the appearance of the tachyon is not attained. If $K\gtrsim 1$, the analysis of the equations of motion becomes much more complicated. Above, we have investigated this case by means of $\mathscr{L}^{\mathrm{unst}}$.

We shall discuss the result. Bearing in mind that the fields behave as $\hat{A}_\mu(\varepsilon)$, $Z_\mu(\varepsilon)\sim O(\varepsilon^3)$,[21] we arrive at this conclusion: In a small neighborhood near the origin in the (x_1, x_2) plane, the solution (102) satisfies to sufficient accuracy the equations of motion of all the fields and describes a phase transition that consists of the replacement of the scalar field by the classical field of the W boson. It can be seen that the transition occurs at the tree level, despite the fact that the magnetic field does not affect the neutral condensate $\Phi_c=\delta(0)$. The interaction of H with the field of the W boson and the coupling of the "order parameters" Φ_c and $(W_i^+W_i^-)_c$ determine the mechanism of the transition in the non-Abelian gauge theory.

Thus, the classical field of the W bosons determines all the vacuum properties of the electroweak interactions in a strong applied magnetic field. The problems that arise in this case are analogous to those in QCD in the presence of a covariantly constant magnetic field. The main problem among them is the determination of the macroscopic structure of the vacuum, which requires investigation of nonlinear equations of the type (98) and (99) for all fields in the whole of space. Thus, the solution (102) is not consistent with the equations for the fields $\hat{A}_\mu$ and Z_μ outside the region of formation of the instability, and the theory of the Copenhagen vacuum uses the linearized equation (98) (for M and Φ equal to zero), always in the same gauge.[12)]

We note that in the description of the phase transitions and the properties and structure of the vacuum the functions used in the majority of cases do not satisfy the exact equations of motion; rather, it is required in the first place that there be a lowering of the energy for some approximate solution (as a rule, the solution of the linearized equation of motion).[13)] The homogeneous solution (102) is one of the possible candidates for the description of the vacuum state. The external field H is constant, and the choice of the origin in the (x_1, x_2) plane is arbitrary. For $K<1$, the phase transition occurs in fields that are weaker than the threshold for the appearance of the tachyon. Therefore, there are no grounds for expecting disturbance of the homogeneity in the vacuum. The abrupt change of the condensate Φ_c recalls the disappearance of superconductivity in superconductors of the first kind or the analogous effect in the Abelian Higgs model when $m_\varphi/M_A<1/\sqrt{2}$.[1] However, the results of the phase transitions are different in the theories which we are comparing. The formation in the non-Abelian case of the W condensate leads to a partial, and possibly even a complete, screening of the external field within the system, since the condensate field is opposite in direction to the external field.

The result for the Georgi-Glashow model is obtained from (102) by setting $g\equiv e$, $2M\to M$, $\sin\theta\to 1$. In absolute magnitude, the energy $V^{(0)}\ [(W_i^+W_i^-)_c]$ of the W condensate is exactly equal to the energy of the external field. The classical field is $\overline{F}_{12}=-H+ie(W_1^+W_2^- - W_2^+W_1^-)=0$. The same result will hold for massless Yang-Mills theory if the solution with a homogeneous vacuum condensate is realized.

Concluding the exposition of the factual material, we discuss the question touched upon in the Introduction, namely, the relationship between theories of the electroweak interactions and also Yang-Mills theory, on the one hand, and superconductivity and its relativistic analog—the Higgs model—on the other. This will enable us to understand the results of the present section better. The case of Yang-Mills theory is simpler, and we therefore begin with it.

Because the charged gluons have an anomalous magnetic moment, a dynamical Higgs field appears. In the Lagrangian (5), the function $e\overline{F}_{ik}$, which describes the interaction of the anomalous moment with the external field, plays the part of the square of the mass for the components $W_i^{\mp}$ $(i=1, 2)$, among which only one is independent on account of the constraint $W_1^{-2}+W_2^{-2}=0$ found in the derivation of the expression (102). An analogous condition is used in the model of the Copenhagen vacuum.[41,42] It is found that the field mass has a tachyon nature, and therefore $W_i^{\mp}$ realize a Higgs field. The coupling constant $e^2/2$ plays the part of the constant λ. From this it follows that the external magnetic field can lead only to ordering of the system and formation of the Higgs phase in the vacuum by virtue of nonlinearity. Its effect in Yang-Mills theory is opposite to the effect of H in Abelian theories, in which the field occurs only in the kinetic energy and leads to disordering.

The dynamical origin of the tachyon mass has the consequence that the potential energy in Yang-Mills theory is dominant at all values of H.[14)] This makes it possible when determining the vacuum to ignore the kinetic energy in the external field for the $W_i^{\pm}$ components and to obtain a solution analogous to (102) for $\Phi_c=0$. The structure of the solution is similar to the case of a superconductor of the first kind in a weak field. Since the dynamical mass of the components $W_i^{\pm}$ and the screening mass acquired after condensation by the neutral gluon field have the order $(eH)^{1/2}$,[20,21] the Ginzburg-Landau parameter is $\varkappa\approx 1$. The possibility is therefore not ruled out that there exists a solution corresponding to a superconductor of the second kind. Such a vacuum was constructed in Refs. 40–42 by analogy with superconductivity theory near the critical field H_{c_2} on the basis of the solutions of the linearized equations of motion for the $W_i^{\pm}$ fields. However, in contrast to the case of superconductivity, there are no grounds for linearizing the Yang-Mills equations. As the results of the present section and also the results of Refs. 40–42 show, the amplitude of the condensate $W_{ic}^{\pm}$ is not small; it is comparable with the amplitude of the external field. Therefore, the nonlinearity is important and the vacuum structure obtained in Ref. 42, of the type of an Abrikosov lattice but of tubes with unquantized magnetic flux, evidently does not reflect the real situation (for comments on the impossibility of the occurrence of a lattice in Yang-Mills theory, see Ref. 62). It is more probable that the macroscopic structure of the vacuum recalls a superconductor of the second kind in a field far from H_{c_2}, when the amplitude of the Higgs field is not small. Such a picture arises at the classical level. It is difficult to choose between the structures which we have described, since $\varkappa \approx 1$. To obtain a complete picture of the properties of the vacuum that arises through the elimination of the tachyon instability, it is necessary to take into account the quantum corrections. A large numerical value of e^2 can in general significantly change the nature of the static picture which we have described in both cases, in contrast to the Higgs model, in which λ is small.[17]

We now turn to theories of the electroweak interactions. Here there is an additional order parameter Φ_c and an interaction of order $\Phi^2 W^+ W^-$, which plays an important part. In the study of the phase transition, the use of the U gauge is important, as is the treatment of the equations over the nonsymmetric vacuum with $\Phi_c = \delta(0) \neq 0$. Formally, there arises only one tachyon mass of field origin and mathematically (but not with respect to the physical properties) the theory recalls Ginzburg-Landau theory. The nature of the phase transition and the vacuum properties depend on the value of K. The role of the thermodynamic critical field H_c is played by $H_{cr} = H_0\sqrt{K}$, which describes the threshold of the replacement of the condensate $\delta(0)$ by the condensate $W_c^{\pm}$. The role of the field H_{c_2} is played by H_0.[15)] If $K<1$, the field H_0 is not attained, and there is a phase transition of the first kind. In the case of large K, the field H_0 is attained and a tachyon is formed above the vacuum with a scalar condensate. For values of H somewhat exceeding H_0, the classical field $W_c^{\pm}$ resulting from the tachyon condensation will be small. Therefore, to describe its structure we can linearize Eq. (98) in complete agreement with the case of a superconductor of the second kind in fields that differ little from H_{c_2}. It is therefore clear that it is to this case that one must apply the results obtained in Refs. 40–42 in QCD, although for an understanding of the resulting macroscopic structure it is necessary to take into account the coupling of the order parameters, which has not yet been done. The influence of the quantum fluctuations has also not been studied, as in Yang-Mills theory.

CONCLUSIONS

The results of the investigations into the vacuum of the electroweak interaction in a strong electromagnetic field obtained by various methods lead to the conclusion that in many respects it is similar to the vacuum of Yang-Mills theory in a covariantly constant field $\overline{F}^a_{\mu\nu} = \overline{F}_{\mu\nu} n^a$ The properties of the effective potential as a function of H for fixed Φ_c^2 indicate the possible occurrence of spontaneous magnetization of the vacuum at small Φ^2, the magnetization disappearing with increasing Φ^2. The appearance of the Savvidi level in the theories of the electroweak interactions is due to the existence of a continuous limit $\Phi^2 \to 0$. It must evidently be taken into account in the study of the restoration of symmetry by temperature in the presence of a magnetic field. Spontaneous symmetry breaking shifts the region of H values in which properties analogous to the infrared properties of massless non-Abelian gauge theories appear to intermediate fields $\sim H_0 = M^2/e$, which cannot be studied perturbatively. The mass M plays the part of a normalization point in the field for the massless case.

The investigation of the effective potential $V^{(1)}(\Phi_c, H, K)$ as a function of Φ^2 for fixed H showed that its behavior is different for $K(0) < 9$–10 and $K(0) > 9$–10. In the first case the effective potential has two essential minima, and in the second case it has one, this reflecting the differences in the vacuum physics for theories with light and heavy Higgs bosons. The change in the nature of the behavior of the effective potential as K varies is not unexpected. Indeed, let us consider the expression (57) for the effective potential. For $K \ll 1$, the H-independent terms of order K have the order of smallness α/π, as due to the field. In this case, the theory does not actually have a small parameter: $\alpha/\pi K \sim 1$. If $K \gtrsim 1$, there is such a parameter, and this is reflected in the properties of the effective potential.

In the study of the radiative spectrum of the W boson in a magnetic field it was established that in the Georgi-Glashow model for $K(H) < 1.66$ the threshold for the appearance of the tachyon is shifted to the region of fields weaker than H_0. For $K(H) > 1.66$, and in the Weinberg-Salam theory foı $K(H) > 0.08$, the square of the effective mass in a field is positive. The radiative corrections stabilize the spectrum. However, a final solution of this problem requires a more drastic abandonment of perturbation theory.

Study of the phase transition in the Georgi-Glashow model for $K(0) > 10$, $1 < K(H) < 1.66$, described by means of the effective Lagrangian for the unstable mode, showed that in this case the behavior of the system near the transition threshold H_{cr} can be treated by perturbation theory and that one can consider the condensation of the W bosons in the framework of a self-consistent problem that takes into account renormalization-group summation. The results obtained here agree in large measure with the model of the Copenhagen vacuum for QCD until after the weak-coupling region has been left. Thus, if we do not take into account the restriction $e_V^2(\Phi^2, H)$, $e_M(\delta^2(H), H) < 1$, we can obtain for this model a "spaghetti"-type structure (see Refs. 41 and 42).

The solution of the classical equations of motion made it possible to establish a number of characteristic features of the phase transition in the non-Abelian gauge theories with spontaneous symmetry breaking for $K(0) < 1$. The most important are the existence of two rather than one order parameters, Φ_c and $(W_i^+ W_i^-)_c$, and the interaction between them. Because of this, the phase transition occurs at the tree level. The transition threshold $H_{cr} = H_0\sqrt{K}$ is shifted to the region of fields weaker than H_0. The critical field is here equal to the estimate that arises for the Higgs model in the case $m_\varphi/M_A < 2^{-1/2}$.[1] However, the results of the phase transition in the Abelian and non-Abelian theories are opposite. In the Higgs model when $H > H_{cr}$ the symmetry is restored, all the fields have zero vacuum expectation values, and the magnetic field in the system is equal to the applied external field. In the non-Abelian case, the scalar condensate $\Phi_c = \delta(0)$ is replaced by the condensate of the W bosons, and this lowers the energy of the system. Therefore, one cannot speak of restoration of the symmetry. The formation of the W condensate leads to partial or, possibly, complete screening of the external field.

In discussing in this review the behavior of the symmetry, we have concentrated all attention on the results relating to the boson sectors of the electroweak theory, since it is they that determine the main features of the vacuum dynamics in a magnetic field. To complete the picture, we briefly describe the role of the fermions. In the first place, we note that to a large degree the influence of the fermions is model-dependent. In the investigation in Ref. 2 of the effective potential in the case of weak fields ($\Phi^2/h \gg 1$), corresponding to the expression (61), it was noted that the effect of the charged fermions and scalars leads to an enhancement of the vacuum asymmetry. However, if the theory is asymptotically free, the influence of the gauge sector is predominant, and as a result of this the tendency for the vacuum expectation value $\Phi_c = \delta(H)$ to be reduced persists. The presence of fermions somewhat increases the numerical value of the critical field determined from Eqs. (62) and (63). In Ref. 65, the influence of the fermions on the properties of the effective potential was investigated in the approximation of strong fields ($\Phi^2/h \ll 1$), this being analogous to the asymptotic behavior (64). It was shown that the behavior of the part of the effective potential due to the fermions is determined by their bare mass. If in the tree approximation the fermions are massive (as in the Georgi-Glashow model), then there is a tendency to restoration of the symmetry. In the case of massless bare fermions (as in the Weinberg-Salam theory), their contribution leads to the existence of a nonvanishing vacuum expectation value $\delta(H) \neq 0$ irrespective of the effect of the scalars and the vector particles. However, whatever the part played by the fermions in the effective potential, the presence of Im $V^{(1)}$ is the most important feature of the phase transitions (for a similar conclusion, see Ref. 66).

We shall describe briefly the situation that arises in a strong electric field. Here there is also a tachyon instability of the vacuum, as can be shown by solving Eq. (69) for $E = \text{const}$. Similarly, there is a tachyon in a constant color electric field in Yang-Mills theory, as noted in Ref. 64.

The investigation of the phase transition in the Weinberg-Salam theory for $K(0) < 1$ shows that in an electric field there is replacement of the scalar condensate $\Phi_c = \delta(0)$ by a W-boson condensate $(W_i^+ W_i^+ W_k^- \times W_k^-)_c = (eE)^2/g^4$. Its energy density is $V^{(0)}(W_c^\pm) = -(E^2/2)\sin^2\theta$. The physical manifestation of the W condensate is evidently the same as in the case of a magnetic field.

[1)] *Translator's Note.* The Russian notation for the trigonometric, inverse trigonometric, hyperbolic functions, etc., is retained here and throughout the article in the displayed equations.

[2)] It should be noted that the Savvidi level arises in the region of h values in which the condition of applicability of the single-loop approximation is not satisfied. Our exposition of this question follows the point of view, common in the literature, according to which the phenomenon of spontaneous magnetization is possible and is not merely a reflection of the features of the employed approximation. For more detail on this matter, see the end of the second subsection of Sec. 3.

[3)] In Refs. 4 and 23, no allowance was made for the circumstance that the behavior of Re $v(\Phi^2, h, K)$ is qualitatively different for $K < 9$–10 and for $K \gtrsim 9$–10; this difference led to discrepancies in the description of the properties of the effective potential in the field h.

[4)] In Ref. 4, the calculation corresponds to $K \approx 1.2$ but with less accuracy than in the present paper; the function $\ln\Gamma_1(x)$ is calculated.

[5)] In the study of the effective potential, it is necessary to consider the system of coupled renormalization-group equations for the invariant charges $\bar{e}^2$ and $\bar{\lambda}$. Solving the renormalization-group equation for $\mathcal{L}'(M^2, H)$ as $H \to H_0$, we study only $\bar{e}^2$ in this limit.

[6)] Bearing in mind what we have said (see the second subsection of this section), we write M^2 instead of $M^2(H)$ for the tree-approximation mass.

[7)] In QED, divergences $\sim \bar{F}_{\mu\nu}$ are absent, and therefore a nontrivial renormalization group does not arise.[51,52]

[8)] In Ref. 56 there is a discussion of the tachyon problem by means of a renormalization-invariant mass, but no allowance is made for the restriction $K(0) > 9$–10 or the dependence of K on H.

[9)] In Ref. 56 the author used α instead of $\bar{\alpha}(H) = \bar{e}^2/4\pi$.

[10)] To simplify the arguments, we assume that H is strong and ignore the mass M^2.

[11)] The cubic equation (101) has three roots. The root (102) has the lowest energy.[20]

[12)] See the comments on the papers of the Copenhagen group in Ref. 62.

[13)] In Ref. 63, there is an interesting development of the effective-Lagrangian method for the unstable mode in chromodynamics.

[14)] It is shown in Ref. 27 that when one retains the interactions with the field H of only the anomalous magnetic moment the basic properties of non-Abelian gauge theories above the perturbation-theory vacuum hold—the tachyon instability and asymptotic freedom in the field.

[15)] The fields H_{c_2} and H_0 determine the thresholds for the appearance of the charged condensate in the vacuum. But the first is the upper critical field, and the second is the lower one.

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